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Abstract

In this paper we develop two decision support procedures for the short-term hydro-thermal resource scheduling problem of a power producer operating in a liberalized market. These procedures, based on mixed integer LP models, determine the unit commitment of thermal units and the production levels of committed thermal units and available hydro plants in each hour so as to maximize profits, while satisfying constraints describing the hydro system, the thermal system and the market. The thermal system is modelled in great detail as it allows start-up and shut-down manouvres in every hour of the planning horizon, taking into account minimum up-time and down-time constraints as well as ramp-up and ramp-down constraints. In both models the annual scheduling decisions are given (optimal maintenance plans of hydro and thermal plants, optimal weekly discharge of seasonal basins) as well as forecasts on basin natural inflows. In the first model the power producer is assumed to be a price taker: energy prices do not depend on his own production decisions, i.e. are exogenous to the decision model, and the optimal schedule is determined on the basis of price forecasts. In the second model the power producer is assumed to be a price maker: assuming that competitors' supply curves can be estimated, the model takes explicitly into account the Market Operator activity of choosing the cheapest bids on the basis of the aggregated supply curve and the hourly energy prices are therefore endogenous variables.

1 Introduction

In the last decade the electric power industry has undergone a fundamental transformation from one dominated by a regulated vertically integrated monopoly to an industry where electricity is produced and traded as a commodity. In the liberalized electricity market each power producer, in competition with other producers, aims at maximizing his own profit; production is sold by power producers either directly to consumers, on the basis of bilateral contracts, or by presenting sell bids to the Market Operator for each hour of the following day. Analogously, electricity is purchased by consumers either on the basis of bilateral contracts or by presenting purchase bids to the Market Operator for each hour of the following day. The Market Operator determines the aggregated supply and demand curves and the equilibrium point of supply and demand, taking into account transmission system constraints defined by the Transmission System Operator. The electricity price is then a "market clearing price", resulting from the interactions among all market participants.

In the previous monopolistic context, production resource scheduling aimed at minimising production costs while satisfying given security standards. In liberalized markets each power producer aims at maximizing his own profit and resource scheduling must take into account both other producers' decisions and the Market Operator's rules for determining the electricity price.

Different time horizons are considered in the production resource scheduling problem. A time horizon of at least one year (medium-term scheduling) is considered when determining the optimal maintenance plans of hydro and thermal plants and the optimal weekly discharge of seasonal basins. A time horizon of a week (short-term scheduling) is considered for determining the unit commitment of thermal groups, i.e. the start-up and shut-down manouvres of the available (not in maintenance) thermal groups, as well as the production levels of the committed thermal units and of the available hydro plants in each hour. Authomatic decision support procedures for scheduling production resources have been developed with respect to the different time horizons: for the monopolistic case see for example Medina et al.(1998), Read (1999), Ouyang and Shahidehpour (1991), Burelli et al. (1990), Innorta et al. (1997a), Innorta et al. (1997b), Cazzol et al. (1998); for the case of liberalized market see Gross and Finlay (1996), Richter and Sheblé (1997), Li et al. (1999), Sheblé (1999), Zhang et al. (2000), Martini et al. (2001) and Garzillo et al. (2002).

In this paper we develop two decision support procedures for the short-term hydro-thermal resource scheduling problem of a power producer operating in a liberalized market. These procedures, based on mixed integer LP models, determine the unit commitment of thermal units and the production levels of committed thermal units and available hydro plants in each hour so as to maximize profits, while satisfying constraints describing the hydro system, the thermal system and the market. In both models the annual scheduling decisions are given (optimal maintenance plans of hydro and thermal plants, optimal weekly discharge of seasonal basins) as well as forecasts on basin natural inflows. In the first model the power producer is assumed to be a price taker: prices do not depend on his own production decisions, i.e. are exogenous to the decision model, and the optimal schedule is determined on the basis of price forecasts. In the second model the power producer is assumed to be a price maker: assuming that competitors' supply curves can be estimated, the model takes explicitly into account the Market Operator activity of choosing the cheapest bids and the hourly energy prices are therefore endogenous variables.

The paper is organized as follows. In Section 2 we describe the hydro system constraints and in section 3 the thermal system constraints (the start-up and shut-down manouvres possible in every hour t, the minimum on-time and off-time constraints and a ramp-up and ramp-down constraints). Moreover we describe the market constraints and the objective function (in this case the PM model contains a representation of competitors'aggregate offer function)

The planning horizon is short term (typically a week or 10 days) with a discretization period of 1 hour. Let T denote the number of periods considered and let t, $0 \leq t \leq T$, denote the period index, where $t = 0$ denotes the last hour of the planning horizon immediately preceeding the one in consideration.

2 Model of the hydroelectric subsystem

The hydroelectric subsystem consists of a number of sets, called *valleys*, of hydraulically interconnected hydro plants, pumped-storage hydro plants and basins. Each valley is mathematically represented by a directed graph: each node represents a basin, with a given storage capacity, and each arc may represent either a hydro plant (power generation) or a hydro pump (power storage) or a basin spillage (water flow to a downstream basin for keeping water storage within the storage capacity limit). Let J denote the set of nodes and I denote the set of arcs. An example of valley and of the corresponding arc-node incidence matrix A is reported in Figure 1.

The following data describe the hydroelectric system: for $i \in I$ and $j \in J$

A_{ij} : (i, j)-entry of network arc-node incidence matrix

$$A_{ij} = \begin{cases} -1 & \text{if arc } i \text{ leaves node } j \\ 1 & \text{if arc } i \text{ enters node } j \\ 0 & \text{if arc } i \text{ and node } j \text{ are not incident} \end{cases}$$

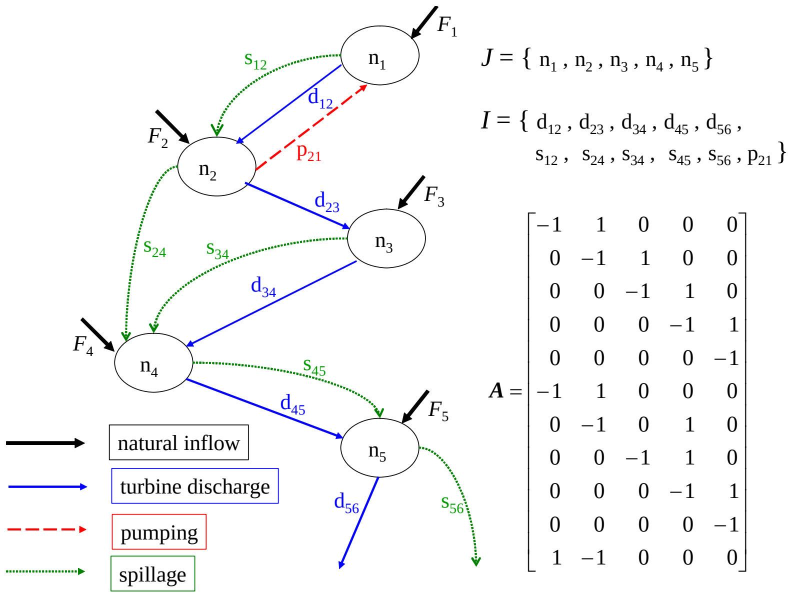

Figure 1:

k_i	$\left[MWh/10^3 m^3\right]$	energy coefficient $\cdot\, k_i > 0$, if arc i represents generation $\cdot\, k_i < 0$, if arc i represents pumping $\cdot\, k_i = 0$, if arc i represents spillage
$\overline{q}_i$	$\left[10^3 m^3/h\right]$	maximum water flow in arc i
ρ_i	$[h]$	delay on arc i
$F_{j,t}$	$\left[10^3 m^3/h\right]$	natural inflow in basin j in hour t
$\overline{v}_j$	$\left[10^3 m^3\right]$	maximum storage volume in basin j
$v_{j,0}$	$\left[10^3 m^3\right]$	initial storage volume in basin j
$\underline{v}_{j,T}$	$\left[10^3 m^3\right]$	minimum storage volume in basin j at the end of hour T

Note that $v_{j,0}$ and $\underline{v}_{j,T}$ are determined by the annual resource scheduling.

The power producer must schedule the hourly production of each hydro plant, which is expressed as the product of the hydro plant energy coefficient times the turbined volume in hour t, as well as the hourly pumped and spilled volumes. The decision variable $q_{i,t}$ $\left[10^3 m^3/h\right]$ represents the water flow on arc i in hour t (turbined volume, if arc i represents generation, pumped volume, if arc i represents pumping, and spilled volume, if arc i represents spillage). Moreover, the decision variable $v_{j,t}$ $\left[10^3 m^3\right]$ represents the storage volume in basin j at the end of hour t. The decision variables must satisfy the following constraints that describe the hydroelectric subsystem:

- flow on arc i in hour t is nonnegative and bounded above by the maximum volume that can be either turbined or pumped or spilled

$$0 \leq q_{i,t} \leq \overline{q}_i \qquad i \in I \ , \quad 1 \leq t \leq T \tag{1}$$

- the storage volume in basin j at the end of hour t is nonnegative and bounded above by the maximum storage volume

$$0 \leq v_{j,t} \leq \overline{v}_j \qquad j \in J \ , \quad 1 \leq t \leq T \tag{2}$$

- at the end of hour T the storage volume in basin j is bounded below by the minimum storage volume required at the end of the current planning horizon, so as to provide the required initial storage volume at the beginning of the following planning horizon

$$\underline{v}_{j,T} \leq v_{j,T} \qquad j \in J \tag{3}$$

- the storage volume in basin j at the end of hour t must be equal to the basin storage volume at the end of hour $t-1$ plus the sum of basin inflows in hour $t - \rho_i$ minus the sum of basin outflows in hour $t - \rho_i$

$$v_{j,t} = v_{j,t-1} + F_{j,t} + \sum_{i \in I} A_{i,j} \cdot q_{i,t-\rho_i} \qquad j \in J \ , \quad 1 \leq t \leq T \tag{4}$$

  where $v_{j,0}$ is a data representing the initial storage volume in basin j. Basin inflows are natural inflows, turbine discharge from upstream hydro plants, pumped volumes from downstream hydro plants, spilled volumes from upstream basins. Basin outflows are turbine discharge to downstream hydro plants, pumped volumes to upstream hydro plants and spilled volumes to downstream basins.

3 Model of the thermal subsystem

Let K denote the set of thermal plants owned by the power producer. For every thermal unit $k \in K$ the power producer must solve

- a unit commitment problem, i.e. decide in which hours of the planning horizon start-up and shut-down manouvres have to take place, taking into account technical constraints, such as minimum up-time and down-time constraints;
- a scheduling problem, i.e. decide the production level in every hour of the planning horizon of each committed unit, taking into account technical constraints, such as ramp-up and ramp-down constraints.

In the unit commitment problem the power producer decisions are represented by three sets of binary variables:

$$\alpha_{k,t} = \begin{cases} 1 & \text{start-up of thermal plant } k \text{ in hour } t \\ 0 & \text{no start-up of thermal plant } k \text{ in hour } t \end{cases}$$

$$\beta_{k,t} = \begin{cases} 1 & \text{shut-down of thermal plant } k \text{ in hour } t \\ 0 & \text{no shut-down of thermal plant } k \text{ in hour } t \end{cases}$$

$$\gamma_{k,t} = \begin{cases} 1 & \text{thermal plant } k \text{ is ON in hour } t \\ 0 & \text{thermal plant } k \text{ is OFF in hour } t \end{cases}$$

The following four cases are relevant

1. thermal unit k is OFF in hours $t-1$ and t (no manouvre in hour t): the corresponding values of the binary variables are
$$\gamma_{k,t-1} = 0, \quad \alpha_{k,t} = 0, \quad \gamma_{k,t} = 0, \quad \beta_{k,t} = 0 \tag{5}$$
2. thermal unit k is ON in hours $t-1$ and t (no manouvre in hour t): the corresponding values of the binary variables are
$$\gamma_{k,t-1} = 1, \quad \alpha_{k,t} = 0, \quad \gamma_{k,t} = 1, \quad \beta_{k,t} = 0 \tag{6}$$
3. thermal unit k is OFF in hour $t-1$ and is started up in hour t: the corresponding values of the binary variables are
$$\gamma_{k,t-1} = 0, \quad \alpha_{k,t} = 1, \quad \gamma_{k,t} = 1, \quad \beta_{k,t} = 0 \tag{7}$$
4. thermal unit k is ON in hour $t-1$ and is shut down in hour t: the corresponding values of the binary variables are
$$\gamma_{k,t-1} = 1, \quad \alpha_{k,t} = 0, \quad \gamma_{k,t} = 0, \quad \beta_{k,t} = 1 \tag{8}$$

The above four cases satisfy constraints

$$\gamma_{k,t-1} + \alpha_{k,t} = \gamma_{k,t} + \beta_{k,t} \qquad k \in K \ , \quad 1 \le t \le T \tag{9}$$

where $\gamma_{k,0}$ is a binary data representing the status of thermal unit k (0: OFF, 1: ON) in the last hour of the planning period immediately preceeding the one in consideration. Note that constraints (2) are also satisfied by the 4-uples of binary variables

$$\gamma_{k,t-1} = 0, \quad \alpha_{k,t} = 1, \quad \gamma_{k,t} = 0, \quad \beta_{k,t} = 1 \tag{10}$$

and

$$\gamma_{k,t-1} = 1, \quad \alpha_{k,t} = 1, \quad \gamma_{k,t} = 1, \quad \beta_{k,t} = 1 \tag{11}$$

where both start-up and shut-down manouvres in hour t are associated to a thermal unit k which is ON (or OFF) both in $t-1$ and in t. We note, however, that the 4-uples of binary variables (10) or (11) will never appear in the optimal solution. Indeed, suppose a feasible solution contains the 4-uple (10) and consider the solution that differs from it only for the values of $\alpha_{k,t}$ and $\beta_{k,t}$, i.e. $\alpha_{k,t} = \beta_{k,t} = 0$: sthe producer problem is a profit maximization problem, therefore the solution with $\alpha_{k,t} = \beta_{k,t} = 1$ is more costly than the one with $\alpha_{k,t} = \beta_{k,t} = 0$, because of the presence of start-up and shut-down costs in hour t.

In order to take into account minimum up-time and minimum down-time constraints, the following data are needed for every thermal unit $k \in K$:

ta_k $[h]$: minimum number of hours unit k must be ON after start-up

ts_k $[h]$: minimum number of hours unit k must be OFF after shut-down

$\gamma_{k,0}$ $[0/1]$: unit k was OFF/ON in the last hour of the scheduling period immediately preceeding the current one

nh_k $[h]$: in the scheduling period immediately preceeding the current one the last manouvre for unit k took place in hour $T - nh_k$

The minimum up-time constraints are as follows

- if unit k was ON in the last hour of the previous scheduling period, it must be ON at least for the first $ta_k - nh_k$ hours of the current scheduling period: this is obtained by the assignment

$$\text{if } \gamma_{k,0} = 1, \quad \gamma_{k,t} = 1 \quad \text{for } 1 \le t \le ta_k - nh_k \tag{12}$$

- if a start-up manouvre takes place in hour t, then unit k must be ON either for $ta_k - 1$ subsequent hours, if $ta_k - 1 \le T - t$, or for the $T - t$

subsequent hours, otherwise: this is obtained by constraints

$$\sum_{\tau=t+1}^{\min(t+ta_k-1,T)} \gamma_{k,\tau} \geq \alpha_{k,t} \cdot \min(ta_k - 1, T - t) \qquad k \in K \ , \quad 1 \leq t \leq T \tag{13}$$

The minimum down-time constraints for unit k are

- if unit k was OFF in the last hour of the previous scheduling period, it must be OFF at least for the first $ts_k - nh_k$ hours of the current scheduling period: this is obtained by the assignment

$$\text{if } \gamma_{k,0} = 0, \quad \gamma_{k,t} = 0 \quad \text{for } 1 \leq t \leq ts_k - nh_k \tag{14}$$

- $\gamma_{k,t} = 0 \quad \text{for } 1 \leq t \leq ts_k - nh_k \quad \text{if } \gamma_{k,0} = 0$;

- if a shut-down manouvre takes place in hour t, then unit k must be OFF either for the $ts_k - 1$ subsequent hours, if $ts_k - 1 \leq T - t$, or for the $T - t$ subsequent hours, otherwise: this is obtained by constraints

$$\sum_{\tau=t+1}^{\min(t+ts_k-1,T)} \gamma_{k,\tau} \leq (1-\beta_{k,t}) \cdot \min(ts_k - 1, T - t) \qquad k \in K \ , \quad 1 \leq t \leq T \tag{15}$$

As regard to the production level $p_{k,t}$, to be determined for every hour t, $1 \leq t \leq T$ and every thermal unit $k \in K$, the following constraints are considered:

1. if unit k is OFF in hour t, its production $p_{k,t}$ must be zero;

2. if unit k is ON in hour t, its production $p_{k,t}$ must be bounded below by a minimum value $\underline{p}_k$ $[MWh]$;

3. if unit k is ON in hour t, its production $p_{k,t}$ must be bounded above by a maximum value $\overline{p}_k$ $[MWh]$;

4. if thermal unit k is OFF in hour $t-1$ and is started up in hour t, i.e. values of $\alpha_{k,t}$, $\beta_{k,t}$ and $\gamma_{k,t}$ given in (**??**) apply, power production in hour t cannot be greater than vsu_k $[MWh]$, the maximum production of unit k at start-up;

5. if thermal unit k is ON in hour $t-1$ and is shut down in hour t, i.e. values of $\alpha_{k,t}$, $\beta_{k,t}$ and $\gamma_{k,t}$ given in (7) apply, then production in hour $t-1$ cannot be greater than vsd_k $[MWh]$, the maximum production of unit k at shut-down;

6. if thermal unit k is ON in hours $t-1$ and t, then production in hour t must satisfy constraints

$$p_{k,t-1} - \delta v_k \leq p_{k,t} \leq p_{k,t-1} + \delta u_k,$$

   where δu_k and δd_k $[MWh]$ denote the maximum power production increase per hour and the maximum power production decrease per hour, respectively, in unit k.

Constraints 1 and 3 are imposed by the inequalities

$$0 \leq p_{k,t} \leq \gamma_{k,t} \cdot \overline{p}_k \qquad k \in K \ , \quad 1 \leq t \leq T \tag{16}$$

impose zero production, if $\gamma_{k,t} = 0$; if $\gamma_{k,t} = 1$ power production is nonnegative and bounded above by $\overline{p}_k$ production, if $\gamma_{k,t} = 1$.

Constraints 4, 5 and 6 are imposed by the inequalities

$$p_{k,t} - p_{k,t-1} \geqslant -\delta d_k + \beta_{k,t}(-vsd_k - \delta d_k) \qquad k \in K \ , \quad 1 \leq t \leq T \tag{17}$$

and

$$p_{k,t} - p_{k,t-1} \leq \delta u_k + \alpha_{k,t}(vsu_k - \delta u_k) \qquad k \in K \ , \quad 1 \leq t \leq T \tag{18}$$

where $p_{k,0}$ $[MWh]$ is a data representing the power production of unit k in the last hour of the previous scheduling period. Indeed, the left-hand sides of the inequalities take the following forms, depending on the values of $\gamma_{k,t-1}$ and $\gamma_{k,t}$

$$p_{k,t} - p_{k,t-1} = \begin{cases} 0 & \text{if } \gamma_{k,t-1} = \gamma_{k,t} = 0 \\ p_{k,t} - p_{k,t-1} & \text{if } \gamma_{k,t-1} = \gamma_{k,t} = 1 \\ p_{k,t} & \text{if } \gamma_{k,t-1} = 0 \text{ and } \gamma_{k,t} = 1 \\ -p_{k,t-1} & \text{if } \gamma_{k,t-1} = 1 \text{ and } \gamma_{k,t} = 0. \end{cases}$$

The lower bounds on production variations, i.e. the right-hand sides of inequalities (17), depend on the value of $\beta_{k,t}$

$$-\delta d_k + \beta_{k,t} \cdot (-vsd_k - \delta d_k) = \begin{cases} -\delta d_k & \text{if } \beta_{k,t} = 0 \\ -vsd_k & \text{if } \beta_{k,t} = 1. \end{cases}$$

The upper bounds on production variations, i.e. the right-hand sides of inequalities (18), depend of the value of $\alpha_{k,t}$

$$\delta u_k + \alpha_{k,t} \cdot (vsu_k - \delta u_k) = \begin{cases} \delta u_k & \text{if } \alpha_{k,t} = 0 \\ vsu_k & \text{if } \alpha_{k,t} = 1 \end{cases}$$

In the following section we show that constraint 2 can be imposed in the context of the linearization of thermal generation costs.

3.1 Computing thermal production costs.

Two types of costs are associated to thermal production: costs of manouvres and generation costs. For every unit k the cost csu_k is associated to each start-up manouvre and the cost csd_k is associated to each shut-down manouvre. The thermal generation cost $C_{k,t}$ of unit k in hour t is assumed to be a convex quadratic function of the production level $p_{k,t}$

$$C_{k,t}\left(p_{k,t}\right) = c_{0k} + c_{1k} \cdot p_{k,t} + c_{2k} \cdot p_{k,t}^2$$

where c_{0k}, c_{1k} and c_{2k} are the cost coefficients for unit k. Since the model we develop is of Mixed Integer type, we choose to linearize the thermal generation costs, so as to obtain a Mixed Integer Linear Programming model.

In order to linearize the generation cost function of thermal unit k, the interval $\left[\underline{p}_k, \overline{p}_k\right]$ is divided in H subintervals of width $\overline{pl}_{k,h}$, $1 \leq h \leq H$. Let $p_{k,t,h-1}$ and $p_{k,t,h}$ denote the extreme points of subinterval h: the straight line segment passing through the points $(p_{k,t,h-1}, C_{k,t}(p_{k,t,h-1}))$ and $(p_{k,t,h}, C_{k,t}(p_{k,t,h}))$ is associated to each subinterval h. Let $cl_{k,h}$ denote the slope of the line segment associated to subinterval h: since the quadratic function is convex, it holds that

$$cl_{k,h-1} < cl_{k,h} \qquad \text{for } 2 \leq h \leq H.$$

Finally, let $pl_{k,t,h}$ denote the real variable associated to subinterval h. For each production level $p_{k,t}$, $\underline{p}_k \leq p_{k,t} \leq \overline{p}_k$, there exist a unique $\widehat{h}$, $1 \leq \widehat{h} \leq H$, and a unique $pl_{k,t,\widehat{h}}$, $0 < pl_{k,t,\widehat{h}} \leq \overline{pl}_{k,\widehat{h}}$, such that

$$p_{k,t} = \underline{p}_k + \sum_{h=1}^{\widehat{h}-1} \overline{pl}_{k,h} + pl_{k,t,\widehat{h}}.$$

Given a production level $p_{k,t}$, the corresponding (linearized) generation costs are correctly computed if variables $pl_{k,t,h}$ take the values

$$pl_{k,t,h} = \overline{pl}_{k,h} \qquad \text{for } 1 \leq h \leq \widehat{h} - 1. \tag{19}$$

This is obtained by introducing the constraints

$$p_{k,t} = \underline{p}_k \cdot \gamma_{k,t} + \sum_{h=1}^{H} pl_{k,t,h} \qquad k \in K \ , \quad 1 \leq t \leq T \tag{20}$$

and

$$0 \leq pl_{k,t,h} \leq \overline{pl}_{k,h} \qquad k \in K \ , \quad 1 \leq t \leq T \ , \quad 1 \leq h \leq H. \tag{21}$$

This guarantees that, in the optimal solution, variables $pl_{k,t,h}$ take the values (19).

Summarizing, the linearized generation costs of thermal plant k in hour t are given by

$$LC_{k,t}\left(p_{k,t}\right) = \left(c_{0k} + c_{1k} \cdot \underline{p}_k + c_{2k} \cdot \underline{p}_k^2\right) \cdot \gamma_{k,t} + \sum_{k,h} cl_{k,h} \cdot pl_{k,t,h} \tag{22}$$

and the total thermal generation costs of unit k in hour t are

$$TC_{k,t}\left(p_{k,t}\right) = LC_{k,t}\left(p_{k,t}\right) + csu_k \cdot \alpha_{k,t} + csd_k \cdot \beta_{k,t} \tag{23}$$

with $LC_{k,t}\left(p_{k,t}\right)$ given by (22).

4 Market constraints and objective function of the Price Taker model.

In this section we introduce the market constraints and the objective function for a producer who cannot influence the market price. It is assumed that in every hour t of the planning period the Price Taker must satisfy the load car_t deriving from his bilateral contracts. If his total production exceeds the load from bilateral contracts, the Price Taker sells the excess quantity, $sell_t$, on the spot market; if his total production is less than the load from bilateral contracts, the Price Taker must buy on the market the amount of energy, buy_t, necessary to meet the load car_t. Therefore the market constraints are

$$\sum_{i \in I} k_i \cdot q_{i,t} + \sum_{k \in K} p_{k,t} + buy_t - sell_t = car_t \qquad 1 \leq t \leq T$$

The objective function represents the Price Taker profits

$$\sum_{t=1}^{T} \left[\lambda_t \cdot sell_t - \mu_t \cdot buy_t - \sum_{k \in K} TC_{k,t}\left(p_{k,t}\right) \right]$$

where λ_t is the market sell price in hour t, μ_t is the market purchase price in hour t and $TC_{k,t}\left(p_{k,t}\right)$ are the total (linearized) thermal generation costs given by (6).

5 Market constraints and objective function of the Price Maker model.

In this section we introduce the market constraints and the objective function for a producer who can influence the market price. It is assumed that in every hour t the Price Maker knows the load CAR_t required by the system and therefore, depending on his total production, he can determine the residual demand Π_t, to be satisfied by his competitors. The market constraints are

$$\sum_{i \in I} (k_i \cdot q_{i,t}) + \sum_{k \in K} p_{k,t} + \Pi_t = CAR_t \qquad 1 \leq t \leq T$$

It is also assumed that the Price Maker knows (or can estimate) the sell bids presented in hour t by his competitors. On the basis of the sell bids, the competitors' aggregated offer function is then constructed and used to determine the hourly energy price. The Price Maker revenues are therefore computed in the model by means of electricity prices endogenously determined.

The electricity price in hour t is determined by the following procedure. Let S_t denote the number of sell bids presented by the competitors in hour t and let $QV_{s,t}$ and $PV_{s,t}$ denote quantity and price, respectively, declared in bid s, $1 \leq s \leq S_t$. Suppose that sell bids have been reordered in merit order, i.e.

$$PV_{s,t} \leq PV_{s+1,t} \qquad 1 \leq s \leq S_t - 1.$$

In order to construct the competitors' aggregated offer function, we first determine its $P_t = 2 \cdot S_t$ points of discontinuity: for $1 \leq p \leq P_t$

$$(aqv_{p,t}, apv_{p,t}) = \begin{cases} (QV_{p-1,t}, PV_{p,t}) & \text{if } p = 2 \cdot s - 1 \\ (QV_{p,t}, PV_{p,t}) & \text{if } p = 2 \cdot s \end{cases}$$

with $QV_{0,t} = 0$. For values of Π_t such that $aqv_{p,t} < \Pi_t < aqv_{p+1,t}$, with p odd (in which the competitors' aggregated offer function is continuous), the corresponding price value is

$$Price_t = apv_{p,t} = apv_{p+1,t} \ ;$$

for values of Π_t such that $\Pi_t = aqv_{p,t} = aqv_{p+1,t}$, with p even (in which the competitors' aggregated offer function is discontinuous), any value between $apv_{p,t}$ and $apv_{p+1,t}$ may be taken as the price at hour t.

$$apv_{p,t} \leq Price_t \leq apv_{p+1,t} \ .$$

In order to model the competitors' aggregated offer function, a real variable $\eta_{p,t}$, $0 \leq \eta_{p,t} \leq 1$, is associated to every discontinuity point p for $1 \leq t \leq T$. The set of variables $\eta_{p,t}$, $1 \leq p \leq P_t$, is a special ordered set of type 2 (SOS2), see Beale and Tomlin (1969), i.e. an ordered set within which at most two adjacent variables can be nonzero. We also introduce the convexity constraints, for $1 \leq t \leq T$,

$$\sum_{p=1}^{P_t} \eta_{p,t} = 1$$

The constraints that define Π_t, for $1 \leq t \leq T$, is

$$\Pi_t = \sum_{p=1}^{P} aqv_{p,t} \cdot \eta_{p,t} \tag{24}$$

Consider a value of Π_t such that

$$aqv_{\widehat{p},t} < \Pi_t < aqv_{\widehat{p}+1,t}$$

with $\widehat{p}$ odd. Since the variables $\eta_{p,t}$ are of SOS2 type and because of the convexity constraints, it follows that the following values are assigned to variables $\eta_{p,t}$:

$$\begin{aligned}
\eta_{\widehat{p}+1,t} &= \frac{\Pi_t - aqv_{\widehat{p},t}}{aqv_{\widehat{p}+1,t} - aqv_{\widehat{p},t}} \\
\eta_{\widehat{p},t} &= 1 - \eta_{\widehat{p}+1,t} \\
\eta_{p,t} &= 0 \qquad \text{for } p \neq \widehat{p}, \widehat{p}+1 \ .
\end{aligned}$$

Note that expression (24) is also valid if Π_t is a point of discontinuity of the aggregated offer function, i.e. $\Pi_t = aqv_{\widehat{p},t} = aqv_{\widehat{p}+1,t}$, with $\widehat{p}$ even.

The revenues of the Price Maker in hour t may then be expressed as

$$Revenues_t = \sum_{p=1}^{P} apv_{p,t} \cdot (CAR_t - aqv_{p,t}) \cdot \eta_{p,t} \ :$$

and the objective function, representing the profits of the Price Maker, is

$$\sum_{t \in T} \left\{ \sum_{p \in P} \left[apv_{t,p} \cdot (CAR_t - aqv_{t,p}) \cdot \eta_{t,p} \right] - \sum_{k \in K} \left[TTG_{k,t} \left(p_{k,t} \right) \right] \right\},$$

where $TTG_{k,t}(p_{k,t})$ represents the total thermal generation costs and is given by (6).

6 Conclusions

We have developed two decision support procedures for the short-term hydro-thermal resource scheduling problem of a power producer operating in a liberalized market. These procedures, based on mixed integer LP models, determine the unit commitment of thermal units and the production levels of committed thermal units and available hydro plants in each hour so as to maximize profits, while satisfying constraints describing the hydro system, the thermal system and the market. In both models the annual scheduling decisions are given (optimal maintenance plans of hydro and thermal plants, optimal weekly discharge of seasonal basins) as well as forecasts on basin natural inflows.

The time horizon of the final phase of the planning procedure is the single hour, for which the Power Producer must decide his own generation units' sell bids. In this short horizon it is possible to include in the model a detailed representation of bids and of production units' localisation for each competitor. The hourly resource planning procedure will require to explicitly introduce in the Power Producer model the optimality conditions (Kuhn-Tucker conditions) of the Market Operator problem, which will be modelled under different hypotheses, such as inelastic or elastic energy demand, no transmission constraints or transmission network with either radial or grid topology. A Mathematical Programming problem with Equilibrium Constraints (MPEC) will then have to be solved. Each producer must define for each hour of the following day the so called "production bids", i.e. pairs "energy quantity - price".

The power producer decision support procedures will also allow both to analyse the behaviour of the electricity market, i.e. the process by which the hourly energy price is determined, and to detect whether a producer has an excessive market power, incompatible with the aim of the liberalisation. The above described planning procedures for the weekly, daily and hourly time horizon are also able to detect the existence of an independent producer with excessive market power. Indeed, if one producer turned out to be indispensable to meet the demand, profit maximisation models would determine energy sell bids with arbitrarily high prices. The planning tools that will be developed in this research project will make it possible to estimate the maximum dimensions of a single producer above which genuine competition in the market is hindered.

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