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Agent-Based Modeling
Exploring Sovereign CDS Spreads in Europe

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To my mama, in memorial

To my loving family

Declaration of Autorship

I, Irina Simakova, declare that this thesis titled, Agent-Based Modeling Exploring Sovereign CDS Spreads in Europe, and the work presented in it are my own. I confirm that:

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- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

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Abstract

Agent-Based Modeling

Exploring Sovereign CDS Spreads in Europe

Simakova Irina

In this thesis, we suggest the application of the agent-based modeling in finance to the detection of the linkage between two complex systems on the macro- and micro-levels – the European market of sovereign Credit Default Swaps (macro-level) and the Italian equity market (micro-level). To execute this, we develop a framework for the capacity to calibrate the simulated dynamics of the first market to the changes in the dynamic direction of the second market in the long term by the transformation of the $-game slaving algorithm in the part of the choice of the optimal strategy. Moreover, using the Tremor Price Dynamics, we explain the practicality of the global market dynamics for the predictions of the local market fluctuations. Then, we use the effect of decoupling to obtain the knowledge about the time of (negative) bubble occurrence in the European market of sovereign CDS, and on its base, we divide the time series of two markets analytically into separate bubble-periods. As a result, we obtain the information confirming the lead-lag linkage between the European market of sovereign Credit Default Swaps and the Italian equity market which could not be caught using the standard approach. Finally, we demonstrate a simple example of how to make the predictions of the changes in the dynamic direction of the lagged local market on the base of the bubble-periods appearing on the leading global market.

Symbols and Notations

Here we insert definitions of major symbols and notations used within the current thesis.

Symbol	Definition
P_t	price of asset
P_t^*	fundamental value of the asset
ε_t	random term; bias
r	interest rate considered as constant
$\frac{D(t+1)}{[1+r]^t}$	cash flow at next time step discounted on constant interest rate
I_{t-1}	information available at time (t-1)
P_0	proper price of stock
R_f	risk-free rate of return
P_T	payoff of asset or portfolio
R_M	return of market
R_i	return of asset
β_i	coefficient measuring risk of asset
B_t	mispricing caused by factors of behavioral origin
ε_1	random variable resulting from aggregate errors in the processing of information
ε_2	random variable resulting from aggregate representativeness errors
ε_3	random variable resulting from biases in investor preferences
C	measure of market's ability to self-correction, $A \in [0, 1]$
s_i^I	return of stock *i* belonging to index *I*
R_{-i}^I	return of the remaining *N – 1* stocks in index *I*
s^I	return of index *I*
AT	average over given time window size
$R(t)$	return of market at time *t*
$A(t)$	excess demand of market at time *t*, order imbalance

Symbol	Definition
$a_i(t)$	decision of agent *i* to buy or to sell at time *t*
λ	liquidity of market
N	number of agents interacting in market
S	total number of strategies
s	set of strategies available for agent during game, $s \in S$
m	set of last market moves
T	the time horizon,
$g_i^{Majority}$	payoff of agent *i* in Majority game
$g_i^{Minority}$	payoff of agent *i* in Minority game
σ^2	variance
α	control parameter, ratio between number of uncorrelated strategies in use and total number of strategies that *N* agents adopt
$g_i^{\$}$	payoff of agent *i* in \$-game
$A_{coupled}(t)$	excess demand of market at time *t* from decisions of agents within coupled strategies
$A_{decoupled}(t)$	excess demand of market at time *t* from decisions of agents within decoupled strategies
$A_{decoupled}^{+bub}$	positive excess demand from decisions of agents within decoupled strategies
$A_{decoupled}^{-bub}$	negative excess demand from decisions of agents within decoupled strategies
t_b^0	time of bubble occurrence
t_b	time of bubble formation
t_b^1	time of next to t_b^0 bubble occurrence
$p_{decoupled}^{+bub}$	percentage of positively decoupled strategies
$p_{decoupled}^{-bub}$	percentage of negatively decoupled strategies
M_{bubble}	order parameter for phase transition
Ʈ	temperature parameter of market
R_i	return of market *i*

Symbol	Definition
$R_i^{transfer}$	cumulative part of return of market *i* from interconnection of stock markets on base of global economic news
η_i	the part of return of the market *i* from local economic news
PD	probability of default
LGD	loss given default
EAD	exposure at default
$\mathrm{R}_{\mathrm{CDS}}^{\mathrm{input}}$	real return from the CDS time series
P_{CDS}	value of sovereign CDS spread
$\mathrm{R}_{\mathrm{CDS}}^{\mathrm{ABM}}$	return from the CDS simulations of an agent-based model.
$R_i^{FTSE}(t)$	return of FTSE price index in a particular market *i* between time *t* – *1* and *t*.
R^{CDS}	R^{CDS} – return of European sovereign CDS market
P_i^{FTSE}	price FTSE index
$R_i^{FTSE,transfer}$	effect of external global news with large price movements of FTSE index
η_i^{FTSE}	effect of internal local news relevant only for specific index *i* of FTSE
η_i^{CDS}	effect of internal local news relevant only for specific market *i* of CDS
t'	advance in time of CDS market towards the return of FTSE index
lag	the delay of a FTSE toward CDS

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Introduction

Nowadays, the information flow which becomes available every day is so vast that no computer-based system, not to speak of a human being, is capable of analyzing it in all its diversity. Financial news falls into the category of specific information which requires high-level analytical skills and a practical scientific approach to deal with it.

Throughout last decades with the progression of global financial shocks, a standard approach toward the modeling and prediction of financial fluctuations has shown its non-efficiency. As it turned out, the shock of local origin may cause the break of a global financial system by force of the contagion effect. A financial system demonstrating the attributes of a complex self-emergent system requires specific methods of analysis and control. To understand hidden properties of the system, to build the expectations toward its behavior in a variable environment, we should make an emphasis on the representation of the ensemble of the agents with their inherent heterogeneity and mobility. This evidence points to the necessity of the socialization of science.

The agent-based modeling (ABM) provides a necessary toolbox to deal with a complex system. In this thesis, we systematize the knowledge and recent studies in the field of financial ABM. We show that the application of the socio-finance approach within ABM represents a favorable technique to simulate the interaction between two complex systems on the macro- and micro-levels.

To work out an efficient prediction mechanism for financial crises, we need some indicators of state stability which will be subject to ABM. The market of sovereign Credit Default Swap (CDS) is a very informative source for the evaluation of the steadiness of a particular country or the global system in the whole. Data of CDS swaps provide time series with daily frequency, which are very sensitive to the global fluctuations. This makes it a favorable and almost instantaneous indicator of the information changes, which can be used for predictions in the connected markets.

This ability of CDS to serve for predictions of financial dynamics was noticed several years ago. Since that time, many studies appeared of the possibility to use the dynamics of CDS swaps as an indicator of global market changes. We will show that some studies confirm this hypothesis, but some disprove it. Within the following thesis, we demonstrate the empirical evidence of the sovereign CDS market information value and its ability to predict the fluctuations of the equity markets.

In this thesis, we introduce the application of the ABM to the simulations of two complex financial systems on the macro- and micro-levels and show the linkage between the European market of sovereign CDS (macro-level) and the Italian equity market (micro-level). We develop the $-game framework for the capacity to calibrate the simulated dynamics of the first market to the changes in the dynamic direction of the second market by transformation of the $-game slaving algorithm. The term of "slaving" is used in current work to name the calibration technique when the real price history is used to determine the optimized set of parameters. In this connection, we consider that the actions of the agents in the ABM conditioned by choice of optimal strategies are "slaved" to the price history.

In Chapter 1, we provide a systematized overview of the ABM studies. We specify the main categories of ABM with their attributes and peculiarities. Within the citation of the science areas covered with ABM, we single out recent central tendencies in interdisciplinary and transdisciplinary studies together with the surveys in finance which are of particular interest for the thesis.

In Chapter 2, we discuss the development of the theoretical approaches toward the concept of a financial market. After standard approaches are criticized, it is explained how a social representation of financial reality through different types of communication between heterogeneous agents within ABM may help in the understanding of the behavior of a complex financial system in a short and long-term perspective. The game-theoretical framework is introduced in its variety of the Majority game, Minority game, and $-games. For these multi-agent games, we give a description and make a comparison.

In Chapter 3, we describe in brief the European sovereign CDS market stressing the connection of the extreme events occurred in the market with the recent financial crisis. Then, we apply the $-game framework for the simulations of the sovereign CDS spread market and its slaving to the FTSE index dynamics. We transform the slaving algorithm in point of the choice of the optimal strategy which gives us the possibility to simulate the long-term sovereign CDS dynamics. After that, we explain using the Tremor Price Dynamics (TPD) why the fluctuations of a global sovereign CDS market explain the fluctuations of a local market more efficiently, than if we would use just the time series from two local markets of CDS and equity. Finally, we insert the result of the model calibration, whereby the effect of decoupling, (negative) bubbles appear, which divide the time series into separate bubble-periods. As an example, we give a simple exercise of the prediction of the Italian FTSE index dynamic direction in the correspondence with the changes of the bubble-periods in the European sovereign CDS market.

In Chapter 4, we conclude this thesis with the final discussion about the influence of previous studies on our agent-based computational model and its application to the relationships between the CDS market and the Italian Equity market. We notice the limitations of the suggested model and determine possible extensions of our approach in the future work.

Chapter 1

Main Pillars of Agent-based Modeling and Recent Studies Review

1.1 Introduction

Agent-based Modeling was introduced a long time ago as a useful tool for modeling the dynamics of complex adaptive systems. However, only during past couple of decades, it started to be considered as an everyday scientific alternative to the purely mathematical modeling [15, 31, 64]. Different from many traditional approaches toward system modeling, an agent-based concept implies a bottom-up approach to the construction of a self-organized system appearing from the behavior of its elements. It is necessary to single out that an emergent system is not merely the summation of autonomous agents or the representation of a multiple behaviors of resembling elements, but the joint result of interactions of autonomous heterogeneous entities in a particular environment.

To execute an efficient simulation of a complex system using the ABM, the specification of its main categories is required. The set of the ABM elements described in details below forms the simulated system as close to the real system as the target of the modeling requires it. We show that ABM is a favorable tool to represent different complex systems owing to its ability to simplify the inherent complexity of the system with maintaining the required level of accuracy.

ABM is defined as a bottom-up approach to model a complex spatially-defined system as the outcome of dynamic interactions of individual agents [4, 9, 21, 32, 34, 73, 77, 92]. The definition of ABM has become more general with times, as soon as it covers many applications in completely different areas of science, such as demography, energy, management, medicine, psychology, economy, etc. This became possible due to the development of ABM tools, the availability of micro-data subject to simulations, and advances in computational techniques. The generalization of science promotes the transference of ABM findings and resent tendencies between disciplines, increasing the level of research.

As soon as the ABM of financial systems is at the core of this thesis, we present a detailed literature review in this field, describing recent studies, main tendencies, and scientific discoveries. We also consider the interdisciplinary knowledge demonstrating how the tendencies in the ABM for finance affect other fields of science, and also which findings of the global science are appeared to be the most applicable for ABM of complex financial systems.

This chapter is organized as follows. In Section 1.2 we specify the main categories of ABM with their attributes and peculiarities and suggest the procedure of the category determination for a particular ABM. Section 1.3 is dedicated to the science areas covered with ABM, where economics and finance deserve detailed description in line with the current research issue. In Section 1.4 we discuss the resent tendencies in ABM which are of particular interest for almost all areas of science. Section 1.5 concludes the chapter.

1.2 Main Pillars of Agent-Based Modeling

The emergent order of the self-organized system is maintained by the core categories forming ABM. They are represented in their interactions in Figure 1.1 and include:

- agents (goals, attributes, behavior rules, information, resources);
- agent interactions (agent – agent, agent – system, system – agent, system – system; competition, cooperation, and coalition);
- agent environment;
- complex emergent system (decentralized, accumulative, scalable).

The determined set of the elements listed above represents the characteristics of a particular agent-based model (AB-m). An agent with its attributes and behavioral rules applicable during the direct and indirect interactions with other agents inside the environment represents the simulation of a real agent. The simulations of a complex emergent system as a joint behavior of simulated agents are the mapping of relevant characteristic features of a real system. A precise specification of each category is crucial for building up a useful AB-m.

The purpose of a particular model determines the choice of the level of detail to construct an in-depth model with many variables and parameters or a simplified model with a compact set of parameters. Detailed models are needed to study the micro-scale of the system with all its vast set of variables, constraints, and assumptions. In turn, simple models may help us to understand the mechanism of a global system formation, showing its dynamical transformation.

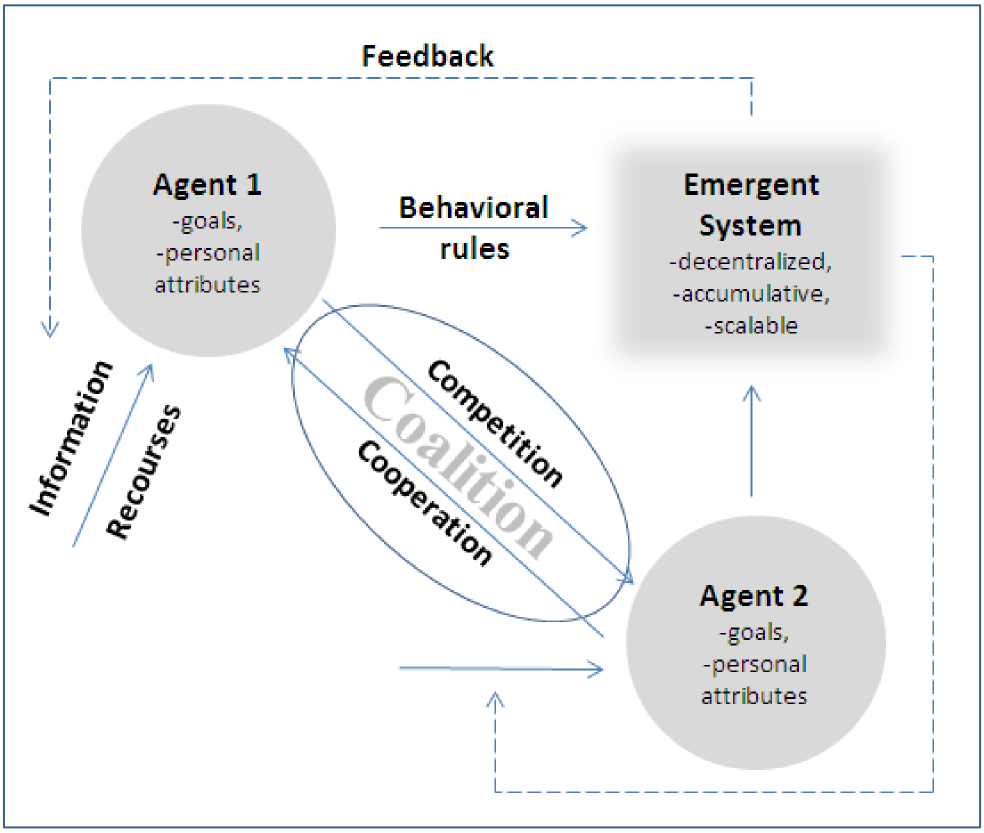

Figure 1.1: Main categories of ABM and their interactions.

Before building up an AB-m, the researcher should determine the target of modeling and its future applications within the following framework:

- the area of an interest showing the detailed representation of the empirical evidence to model;
- the target of agent-based simulations (predictions, system description, system interactions);
- specifications of the attributes of an agent relevant to the AB-m;
- decision for the required model accuracy concerning agent behavioral patterns, environmental restrictions, etc.;
- specifications of the scientific hypothesis regarding the underlying complex system behavior.

The preliminary determination of an AB-m by the described framework may clarify the simulation mechanism and help to define the relevant determinants of ABM categories for a particular scientific phenomenon. We present a detailed description of the ABM attributes represented schematically in Figure 1.1.

1.2.1 Heterogeneous autonomous agents

Agents constituting the base of ABM nevertheless represent a category that is not well defined, as it was mentioned in [74]. However, most researchers agree that an agent is an autonomous entity (software, model, individual, etc.) which has a set of attributes, initially fixed and acquired, and which interacts with its environment and other agents.

There are attributes of each agent which are formed in different ways and different levels of system appearance. The first group of attributes is input attributes such as information and recourses available; they are essential for an agent to start interacting with the environment. Then, the agent decides to set specific goals considering the input and the personal attributes. On the stage of interaction between agents, behavioral rules appear which include base-level rules to act in initial conditions and upper-level rules to change the rules when the environment has changed. As a result of the determination of all agent attributes, we get the simulation of a real agent inside the model.

The availability of the upper-level rules which may change the agent behavior introduces the adaptability of an agent. An agent can learn from the interactions and the feedback of the system and adapt its behavior to the changing conditions. Agent behavior rules may be represented by a simple set of “if-then-else” rules, sophisticated artificial intelligence techniques, and even by sub-models [1]. Thus, an agent can learn from the environments and change the behavior rules in response to agent experiences [65].

The agents are considered to be heterogeneous, i.e., each agent can be distinguished from every other agent by the attributes. Usually, inside one agent-based model, the set of attributes is fixed for all agents, but the value of them may vary, providing a diversity of the agents simultaneously with the system determination.

An agent is autonomous, which can act by itself during the interactions with other agents inside the environment. For every piece of information coming from the other agents and the system, the agent uses the pattern of behavior corresponding to the attributes determined for this particular agent. At any point, the agent may be distinguished from its environment through the instrumentality of its attributes. This concept of autonomy gives the possibility for heterogeneity of the agents inside one complex system.

1.2.2 Agent interactions

Although agents are autonomous, they are not independent. The concept of the agent entity implies its sociability in the sense that every agent has dynamic interactions with other agents and the environment, and these interactions affect the state of every agent and the emergent system in general [32]. The behavioral rules mentioned above represent the agent template tooling for interactions.

The emergent system comes from the interactions of agents representing the bottom up approach instead of top down. As soon as, the system is decentralized by nature, i.e., there is no central governance which would provide the information and manage the relationships inside the system; only local information is available for an agent. A set of agent relationships defines how the agent interacts with its environment and other agents, also it demonstrates the type of connections inside the system. It is not an obligatory case when all agents interact directly with all the other agents. Usually, agents gain the local information from their neighborhood and mediated interactions through the localized environment.

Taken from the modeling of different systems, the theory concludes that interactions may represent or competition or cooperation. During the competition, the agents have the target to act better than others and to get maximum individual benefits without a collaboration component. Cooperation, contrariwise, implies the collaboration of agents to achieve a common purpose maximizing their shared target. Unlike in theory, recent surveys showed that there is almost no pure competition or cooperation, as classical studies posted before [52]. For example, in [71] it was shown that the world of business is a mixture of cooperation and competition. The term of *coopetition* describes a network of key players who cooperate and compete with each other to create maximum profitability. Also, another word *coalition* is used for the combination of competition and cooperation in the same strategy of interacting agents (individuals or organizations) [69]. In the coalition, by analogy with temporary political alliances of parties, competing agents are willing to engage in collaboration, and such partnership can induce optimal individual gains for both parties without losing sight of the defense of their own interests. In general, the decision of agents results partly from communication with others (involving competition, cooperation, and coalition) and partly from independent decision-making [4].

1.2.3 Agent environment

As it was mentioned above, ABM is characterized by a multitude of autonomous agents which interact with each other and with the environment, and by a numerical aggregation [78]. Here the heterogeneity of agents plays a significant role defining methodology of ABM but not only the presence of micro-foundations or the computational nature which is inherent also to some other approaches.

The agent environment is represented by the set of global variables which will determine the surrounding conditions. The agent bases its decisions on those conditions, thereby evolving to a new stage as soon as the environment has changed. Moreover, the state of a whole ABM is the mapping of the collective states of all agents encased in the state of the environment. Unlike agent interactions which determine an active part of the system dynamics, the agent environment is passive by nature, as soon as it represents the general conditions of the system where the agents interact and evolve.

The environment of the system poses the boundaries for the interactions of the agents. It determines the possibilities of the agents to interact directly (for example, the infrastructure for transportation models) or indirectly (the availability of social networks for social models), it is responsible for providing the information to localized groups of agents.

1.2.4 Emergent system

ABM presents an emergent system as the result of agent interactions inside the environment. The system is called “emergent” since it is endogenously emerging from micro-interactions of agents instead of being assumed on the micro-level and then simply summed up to the macro-level [57]. Therefore it acquires the properties which arise from the joint behavior of the agents and which did not belong to the agents on the micro-level. The system does not represent a central or “top-down” coordination device [77], but is a dynamic representation of the agent interactions.

An emergent system may be characterized by scale components: spatial, temporal and behavioral [9]. These components emerge from the specificity of agent interactions. A spatial component goes from mobility: agents stay in constant interaction with each other and with the system in spatially defined conditions. A temporal constituent comes from dynamics: agents are self-organized and though self-improving. Thus, constituting the global structure of the system, an agent represents a self-structure with its regulatory motives and dynamics [4]. A behavioral scale component appears from individuality and diversity: agents are autonomous and considered to be heterogeneous.

The inherent characteristics of ABM make it appropriate to study complex systems, which have many associated entities subject to non-linear interactions. We should consider the particularities of a complex system to take them into account during the ABM procedure [1]:

- a complex system is not just a result of the summation of its components but the result of joint agent behavior inside the environment, the emergent system will have its properties which have not been inherent to its components individually;
- a complex system is self-organized which means that a stable stage appears from the interactions of the agents inside the environment;
- a complex system may have several stationary states with different equilibrium conditions;
- it may stand out of the equilibrium stage demonstrating nonstationarity;
- it is usually resistant to small perturbations, i.e., maintaining the inner stability, but a particular small exposure may cause an unexpected phase transition of the system when it shifts and obtains substantially differing characteristics;
- together with the system phase transition, the cascading effects may appear. During these effects, local factors, very slight at first sight, may acquire a systemic significance. Cascading effects are used as one of the possible explanations for the stylized fact of a fat-tailed distribution when the probability of "extreme events" to occur is substantially higher than it is expected according to a normal distribution.

It is an intricate task to model a complex system with all its variables, constraints and assumptions. But the ability of ABM to generalize the system descriptions without losing the accuracy and to represent complex systems with a compact set of relevant parameters makes it attractive for the elaboration of the applications in many different fields of science.

1.2.5 Derogations

After the introduction of mail pillars of ABM, we should clarify some basic assumptions concerning the categories of ABM such as heterogeneous agents and the emergent system:

- *heterogeneity of agents combines with possible homogeneity inside of their autonomous groups which may also interact in the system*, as in [52] where it has been noticed that according to crowd psychology, commonalities lead to group formation, thus the behaviors of the resulting units are homogeneous and biased inside the group even though different groups of agents are themselves heterogeneous;
- *top-down approach which is not main for ABM is also usable in the stage when emergent system affects the agents.* As discussed in [81], for complex adaptive systems the micro-interactions translate into macro dynamics through bottom-up mechanisms, followed by top-down feedback between macro- and micro-levels.

1.3 Science areas covered with Agent-Based Modeling

Ten years ago ABM has been mentioned as a new approach to complex system modeling [64, 65, 77], but nowadays it is a field of high interest among different sciences. The annual rate of publications using ABM has increased substantially, especially after 2007 if to compare with 2000 year [21]. The development of modeling tools and the availability of micro-data have made possible a growing number of agent-based applications across a variety of disciplines. Once generally accepted by a certain field, ABM brings valuable advances within the overall research paradigm. Among areas of science, covered with ABM there are (in alphabetic order):

- *archeology*: the Artificial Anasazi model represents one of the icon AB-m which described the population dynamics in the Long House Valley in Arizona between 800 and 1350 [30]. By simple household rules on choosing locations for farms and settlements, archaeological records on the occupation of the Anasazi in Long House Valley were reproduced. Nevertheless, just in 2014 it was stated that "simulation has only become unremarkable in certain fields of archaeological enquiry, in particular evolutionary archeology and the study of human evolution, and cannot therefore yet be considered a ‘mainstream’ tool in the way that, say, geographical information systems are used” [21]. But in 2016 the role of ABM in archeology has been rethought, it is successfully applicable to a wide range of archeological areas. Many of the surveys being conducted in the fields of social, biological, geological, and engineering sciences are of direct connection to archeological inquiry and are applied in this field.

- *demography* [20, 51, 76, 84]: ABM gives capabilities of analyzing micro-interactions among individuals and environments building the population behavior, it provides a dynamic solution for demographic research which is difficult in micro-simulations. As an example, the Population Structure Model represents a population in terms of demographic characteristics for evaluating the population impacts of changes in individual behavior using cigarette smoking [84]. A marriage model, called the “Wedding Ring”, represents a bottom-up approach to simulate the individuals whose desire to get married is affected by the social pressure that a social circle exerts on them. The hazard rates of marriage of this simulated society emerge from the individual behaviour and the interactions between the individuals [20]. The Marriage Age Model [76] in turn addresses the influence of social norms to the age at which people marry, and specifically explores the role of social networks (peer groups) in influencing marriage age. Demography science within ABM approach coexists with other social sciences as, for example in the Sugarscape model [31]. Each individual agent has simple local rules: governing movement, sexual reproduction, trading behavior, combat, interaction with the environment, and the transmission of cultural attributes and diseases. When an initial population of such agents is released into an artificial environment, the resulting artificial society links demography, economics, cultural adaptation, genetic evolution, combat, environmental effects, and epidemiology.

- *ecology* [76]: ABM has a long development in ecology with the attempts to model the adaptive behavior of individuals as a product of natural selection. For example, AB-m of stream fish has been developed to model natural selection and the consequent response of populations to flow alteration; the model tried to explain the important processes determining survival, growth, and reproduction of individual fish, and how these processes are affected by river flow.

- *economics*: financial market [35, 52, 53, 79, 81, 91], macroeconomics (fiscal and monetary policies, bank regulation, and central bank independence) [43, 57, 72, 78], systemic risk [5, 60, 81], computational economics [25], microfinance [55]. According to the overview made in [78], agent-based macro-models have covered the problems of analyzing systemic risk and macroprudential regulations, the role of innovation policies, endogenous business cycles and stabilization policies in economies with imperfect information and incomplete credit markets, the stability of general equilibrium. The ABM of complex systems which have common emergent aspects allows the application of the interdisciplinary studies in the field of economic science, as, for example, in the Econophysics (a mixture of economics and physics) or the Social Finance (a mixture of social science and finance) [4, 22].

- *energy*: wholesale power markets [39, 73], renewable energy studies [38], oil market [90], rooftop solar adoption [92]. Standard approaches were limited to model appearing attributes: technological and economic constraints, several agents, and multiple interactions. ABM helps to model complex systems which are composed of many autonomous and heterogeneous components with myriad interrelationships, especially for new sources of energy, which are of great interest nowadays;

- *insurance*: at 2008 on the UK Actuarial Profession Risk and Investment Conference the concepts of ABM to actuaries has been introduces, after that ABM has been widely applied to model insurance cycles [94]. It is useful to model the heterogeneity of individual players who are not necessary full rational and it helped to understand the insurance cycle endogenously;

- *management*: construction safety management [61]. A bottom-up approach of agent-based modeling helps to model a complex system defined by interactions among a worksite, individual construction workers, and different safety investments. ABM provides a practical framework to investigate how different safety investments interacting with different parameters such as human and environmental factors could affect safety performance;

- *medicine*: epidemiology [36], AIDS research, vaccination [41]. In 2011 it was mentioned that ABM has been rarely applied as a public health tool [56]. But the concept of ABM became crucial for several medicine areas where individual information and its interpretation are of a great importance for surveys. Also some kind of the information required for a certain research may be not available from empirical sources, as an example [42] where it is mentioned that the available empirical data for smallpox is limited, so scientists have to build computer models that simulate smallpox outbreaks in relevant contexts;

- *political science*: [16] A model analyzing the formation of coalitions in different domains has been successfully used to reproduce the alignment of European countries in the Second World War;

- *psychology*: initially having social focus ABM is considered by many psychologists as a substitute of social experiments. In [87], authors distinguished the personality- and situation-oriented modeling approaches. In this case, ABM has been applied to determine their significant differences and to measure of the importance of either personal traits, or the situation, or both having impact on the social behavior of agents;

- *transportation*: traffic discipline [28], transit system infrastructure [56]. The main concept of ABM – considering of an individual agent's activity – lets analyze transportation problems from the point of view of individual citizens and model the dynamic system. ABM has been used to explain a discrepancy between road user behavior and traffic rules, to assess the effect of increased Bus Rapid Transit system infrastructure on walking for transportation, to model the line discipline on the roads with heterogeneous traffic represented by two vehicle types.

As can be seen from the above list which is far from being comprehensive, ABM applies to a wide variety of scientific fields. An interdisciplinary advantage is that once an issue has been rigorously tested in an AB-m, it becomes a part of the toolbox of "generative" ABM. An AB-m can be used as a building block for other AB-ms addressing different systems or questions [32]. As soon as the ABM focuses on what is general, not on what is specific, it is helpful in exploring interdisciplinary issues. Among interdisciplinary issues there is the path dependency, the effects of adaptive versus rational behavior, the consequences of heterogeneity among agents, the design of institutional mechanisms to achieve specific goals in a population of autonomous agents, the effects of network structure, cooperation among egoists, etc. [15]. A particular model developed within one field of science may be applied to the general problem of understanding and modeling how a system's dynamics emerge from the characteristics of its individuals.

The development of the bottom-up approach after the prevalence of the classical top-down approach to model a complex system helped to elaborate a complex theory that links the individual and higher levels. A system simulated with ABM of a particular science can be viewed as a subset of the Complex Adaptive System (CAS), dynamic systems consisted of many simple, typically nonlinearly interacting parts possessing capabilities of adaptation to their constantly changing environment [76].

In many fields, be it demography, political science or economics, the ABM includes a social component for simulating a complex system. The simulation of agents and their interactions is known by several names, including agent-based modeling, individual-based modeling, bottom-up modeling, and artificial social systems. Whatever name is used, the purpose of agent-based modeling is to understand properties of complex social systems through the analysis of simulations of different types of communication between heterogeneous agents [15].

Core aspects of ABM as an interdisciplinary approach were formulated in [15] more than ten years ago. Moreover, till now the scientific community is welcome to remember and develop these useful features of ABM:

- an AB-m designed for a specific problem in one discipline can sometimes be applied directly to an apparently quite different problem in another discipline facilitating interdisciplinary collaboration;
- ABM can reveal unity across disciplines addressing many problems seen in many fields of science;
- it provides a useful multidisciplinary tool when the mathematical equation models are intractable;
- convergence in the ABM methodology within the scientific community can enhance its interdisciplinary value.

As it was demonstrated, the experience in different fields of science shows how ABM can address research questions common to many disciplines. Facilitating of interdisciplinary collaboration may provide a useful multidisciplinary tool when the mathematical equation models are intractable, and reveal unity across disciplines. Being of the main interest within this report, the field of financial research also experiences the tendencies of “generative” ABM. Among them there is the complexity and social influence of the simulated systems. In the next subsection the literature review is presented dedicated to the ABM of complex financial systems.

1.3.1 Agent-Based modeling in finance

An active role in the development of ABM applied in economics and finance has been played by such large-scale projects as EURACE3, aimed at developing an agent-based software platform for European economic policy design, POLHIA4, analyzing monetary, fiscal and structural policies, and CRISIS5, aimed at understanding systemic instabilities [90]. Recently, the SYRTO project has been implemented aimed at systemic risk studying and prevention where the concept of ABM is among leading ones [7, 8, 60, 81, 86].

Many different surveys and simulations of financial time series go against a standard theoretical approach in economics. Thus, empirical evidence shows that the Efficient Market Hypothesis (EMH) is not applicable to the understanding of the real behavior of individual market agents and their attitude toward risk and information available [19]. Also the development of ABM approach in finance has been followed by the criticism of the standard approach of Dynamic Stochastic General Equilibrium Models (DSGE) with its concept of the Representative agent (Representative Agent Hypothesis, RAH) [81]. The followers of ABM unveil disadvantages of this standard model, such as its over-simplistic models, the hyper-rational behavior of individuals, and compulsory assumption of equilibrium. Standard approaches do not presume any market disequilibria [72]. They are therefore severely equipped to study how the economy behaves during crises when non-normal distributions with fat tails of many macro-variables are observed [78].

The approach of ABM is opposite to the standard one to a large extent. ABM lets avoid a theoretical requirement of general equilibrium and completely rational behavior by taking into account the bounded rationality of agents and market disequilibria and by showing how the emergent system appears from the interactions of heterogeneous individuals [57]. Moreover, it does not require any assumptions about the global system: it can show how collective actions emerge from individual behavior and constitute the emergent system in the long term.

Analyzing the tendencies in ABM for economics and finance it is worth to derive several particularities, which will be discussed in details below, such as:

- an effective AB-m usually requires a compact set of parameters;
- the modeling of different markets implies the combination of micro- and

macro-levels;

- the concept of agent heterogeneity and different inequalities in society (in wealth, income, resource ownership, gender, information, etc.) is one of the central in the ABM for finance;
- ABM is used to work with pathologies, i.e., the phases of the market out of equilibrium which may be caused by limits to arbitrage or psychological biases leading to the deviations from full rationality.

Compact set of parameters. In general, ABM corresponds to the principle in physics called Renormalization Group Theory telling that detailed description for a system is irrelevant for an understanding of its nature. So to model a complex system, it is necessary to identify essential factors in the form of relevant variables at the expense of trying to describe all aspects in detail [4, 81].

In [81], the transitions in the market price dynamics have efficiently been represented by only three relevant variables: the number of traders, the number of strategies, and the amount of information used by agents. These variables together give the core measure of the complex market dynamics named the temperature parameter which forces the transition from a speculative to a fundamental behavior of traders and back.

Combination of micro- and macro-levels. Agent-based models show the way for introducing behavioral patterns at the micro-level of each agent, and then deriving traits and rules of interaction among the agents at a system macro-level. Moreover, here the interactions between agents play a crucial role in the forming the emergent system but not the existence of micro-level of single agents itself [4]. Aggregate decision making between individuals in a system may be connected with different phenomena which are not detected in micro-level and needed to be studied for an understanding of financial systemic risk. Thus, synchronization in the behavior of individual agents may lead to the creation of collective risk when a price formation is far from the fundamental value of the asset [60].

Analyzing the effectiveness of monetary policy the authors of [52] have proved that in a system that focuses just on a macro-monetary policy, steep fluctuations occur. However, in a system that focuses on a micro-monetary policy or both a micro- and a macro-monetary policy, a stable trend is derived. This fact can be explained by the reaction of agents which has been not taken into account in a macro-monetary policy. Such reaction of individuals may have an unheeded effect which will change the parameters of a macro-model before its results will be applied for a real market, so the results of the policy intervention will be belated and invalidated.

Here it is worth mention that the macro-lever of the system is not equal just to the sum of micro-behaviors as it was considered in the traditional economic theory, where the rationalized microeconomic behavior of a representative agent has been used in a synthesis of macro-behaviors. In ABM the decisions of individual heterogeneous agents going through the stage of interactions form the choice of the system in the form of aggregate decision making [81].

Heterogeneity of agents. ABM requires typification of laws of interactions and decision rules [78]. The interaction among formation of agents with different behavioral rules affects the aggregate behavior of the market. It helps to avoid the concept of a Representative Agent (RAH) in the formation of micro-foundations. Agents usually are divided into several groups which represent heterogeneity. Group formation and the characteristic behavior of each group are relevant variables for analyzing and solving financial problems. The division of groups can be executed according to the inequalities of the market conditions and irrationality level of a financial problem prevailing in the financial market [52].

The diversification of agents in the economy which requires ABM is well described in [72] with informational asymmetries, and unequal distribution (of incomes, resources). For example, the modeling of credit contracts interactions needs to take into account the information asymmetries between two distinct agents: the borrower and the lender. By assuming a representative agent, the analysis of informational asymmetries is not possible.

Agent-based models can represent facts that DSGE models fail to reproduce concerning inequality in the distribution. For example, in [78] ABM took into account the fact that some individuals are fully employed while others remain unemployed. Also in [85], the degree of heterogeneity of agents has been supported by the assumption that they may be affected by different shocks. Authors of [91] divided the investors in the market into three types: informed traders, uninformed traders, and noise traders, with an additional type of spot-future arbitragers who trade between the markets. Each group is characterized by the level of wealth and risk and trading mechanisms.

We should also mention the characterization of the agents separately in financial markets by two macro-configurations shaped by the spatiotemporal interactions of single agents: fundamental and speculative behavior of agents [41, 81]. In a speculative state, single traders take the same directions what can lead to the appearance of bubbles. Oppositely, being in a fundamental state, traders put different orders to buy and sell to lead the price path to move around its fundamental value and leading markets to the stability.

Behavioral finance for pathologies. It has been documented that investors interacting in financial markets do not behave rationally and effectively as EMH assumptions have required. Instead, they systematically violate the principles of expected utility, Bayesian learning, and rational expectations [19]. In fact, the symmetry of the markets can be easily broken with different kinds of pathology leading to bubble or anti-bubble scenarios.

Studies of pathology in finance may be generally divided into 2 categories [19]. The first one considering the limits of arbitrage is attempting to show that the arbitrage mechanism in financial markets is not perfect, i.e., it is not always effective in allowing asset prices to remain connected to their fundamental values. Prices deviate from fundamental value, and these deviations are brought about by the presence of traders who are not fully rational. As soon as the irrational traders cause deviations from fundamental value, rational traders will often be powerless to stimulate the arbitrage. The strategies designed to correct it can be both risky and costly, thereby allowing the mispricing to survive. The second direction of studies explores the psychological reasons and patterns of decision-makers which are often not logical and optimal. This may lead to systematic errors of the agents because of uncertainty, i.e., they deviate from neoclassical assumptions in terms of maximization of utility, stable preferences, and optimal information processing. Despite of a formal division in the studies of financial pathologies, usually in practice these two categories coexist and combine an effective approach to model different financial markets.

Thus, price movements of a given stock index may be generated externally by the aggregation of the world’s stock exchanges reacting together like a complex system or internally, due to specific economic news of the country. Moreover, the look on the combining model may provide new insight into the origins and thereby also prevent systemic risks in the global financial network [5]. For studying a leverage effect when past negative returns increase future volatilities both the investors' asymmetric trading and herding effect are essential generation mechanisms which should be included in the model [24].

In recent conditions, when the volume of the information available is dramatically significant, agents hardly can act all the time rationally. In [52], the "irrationality level" has been defined as an index of the degree to which economic units attempt to behave irrationally. They are affected by "psychological traps": panic, herding effect [24, 41], overconfidence [19], change-blindness [5], and others. Results of [41] confirm the idea of behavioral finance that herding interactions may be dominant over agent rationality and contribute towards bubble occurrence. In [79], authors show how spontaneous collective "moods" or "biases" emerge dynamically among human participants playing a trading game in a simple model of the stock market through the information acceptance by the agents.

Overconfidence is considered as one of the possible biases related to the cognitive psychology of the decision-maker and has received a great deal of attention in financial studies [19]. Under high complexity and uncertainty of market conditions, agents rely on fixed decision-making patterns and intuition when making financial decisions. Studies of the psychology of agents combined with financial studies have unveiled the tendency of traders to feel high confidence in their ability to predict the future and overestimate the accuracy of their data.

Often a fundamental principle in finance losses its actuality stating that as soon as new information is revealed, it immediately becomes reflected in the price of an asset through the reaction of agents. People tend to make systematic cognitive errors when analyzing the information and making decisions. Sometimes the reaction of marker agents tends all humans to reply in a nonlinear fashion to changes, placing emphasis on events with significant changes and disregarding events with modest information content [5]. This phenomenon is called change blindness.

Under some specific market history, it is possible that no matter what the next step of the market response, the strategy of the investor will recommend an indifferent decision [79]. In such situation, an investor is no longer influenced by the incoming information and takes the decision independently from the current market change. This cognitive mechanism of "decoupling," when experienced by the majority of investors, stops the natural mechanism of consolidation and synchronization, forces rapid changes of market dynamics and may lead to the bubble creation.

1.4 Tendencies in Agent-Based Modeling of particular interest

One of the main challenges which the current science is faced with is the volume of the information available. It is so vast and diverse, that the question which information is usable and in which extent is crucial for the beginning of every survey. There are several tendencies in global science which show the attitude toward the global information in the field of ABM:

- spatially defined surveys;
- data-driven ABM;
- interdisciplinary and transdisciplinary approaches;
- hybridization.

The tendencies may arise inside a particular area of science, but with time they cover all the disciplines to a varying degree. Below we describe the appearing tendencies in the field of ABM mode detailed.

1.4.1 Spatially defined surveys

Recently ABM has begun to model the behavior of agents which are "grounded," i.e., they represent entities which can sense and act in a representation of the real world [9]. In [73], it is described that the EU has embarked on a project to create an Integrated Electricity Market by facilitating a cross-border trade to increase competition within each Member State. This initiative needs the technics of ABM to gather diverse spatial data.

The system of patrolling in [93] considers the possibility of intersectional interaction. When the in-district demand exceeds in-district supply, police patrols begin crossing boundaries to meet demand in other police sectors at a very high frequency. Here ABM has shown higher effectiveness in comparison with two other technics. The agent-based model for interdisciplinary science in [34] has used spatial network formation to identify the potential impact of research success on the structure of scientific communities.

In the field of economic geography, ABM helped to discover spatial market formation [58]. Authors considered only locations without geographic advantages or disadvantages that can influence economic development. Here the spatial clustering has been replicated using the agent-based market model that considers a spectrum of different information-sharing mechanisms: no information sharing, information sharing among friends and pheromone-like information sharing.

Spatial economics brings benefits associated with a covering of more diversified risk. One central set of results in [85] concerns risk diversification under the phenomenon of "contagion," when separate markets of different countries adopt patterns of behavior, such that a downturn in one country could lead to that in another. Contagion forces the volatility of the outcome and thus reduces the ability of the financial networks to provide the benefits expected from risk sharing.

1.4.2 Data-driven agent-based modeling

The emergence of "Big Data" offers new opportunities to develop ABM in a way that is entirely data-driven, both in terms of model calibration and validation [92]. The availability of big data leads to the appearance of complex models, which can be tackled with ABM technics. There are several methods of data calibration which differs from the level of calibration: agent level and model level.

To the data-driven ABM, we may also classify the combination of simulated and read data, which leads to the better simulation of the real system. First attempt to do it was in medicine in [36] where the agent-based system has been proposed to use social interactions and individual mobility patterns extracted from call detail records to accurately model virus spreading.

1.4.3 Interdisciplinary and transdisciplinary approaches

ABM has historical roots in complex adaptive systems (CAS) and strong roots in the fields of multi-agent systems and robotics in the part of designing of artificial systems. In the same time, it is directly attached to the modeling of human social and organizational behavior and individual decision-making [65]. Also, by multiple studies in different fields it is possible to notice that ABM theoretical foundations have connections to many other fields including complexity science, computer science, management science, statistical physics, and traditional modeling and simulation [41].

The phenomenon laying in the basis of ABM system formation is what characterizes a phase transition in physics when a system changes its states from a symmetric but random state into an ordered asymmetric state and back. For example, in [81] markets are assumed as thermodynamically systems where energy (order) and entropy (disorder) are contradicting and thus moving the markets from one state to another.

According to [41], statistical physics has got the preference over social sciences in the understanding of complex systems. This happened because physicists were able to build simple models for the understanding of simple phenomena and later add the complexity together with the increasing complexity of the considered phenomena. On the other hand, socio-economic sciences have considered any system as complex from the very beginning, thereby weighting down and complicating the process of system modeling.

However, also initially scientists have been discussing the "socialization" of ABM [4, 8, 45]. Most applications of ABM in such areas as economics, demography, medicine, etc. include social impact, as soon as the consideration of individual agent interests is the underlying assumption of this approach. In [56], authors have studied the effect of transportation on walking activity. They have started from the field of transportation and concluded with the effect of decreasing the death level (demography). In [25], Agent-based Computational Economics (ACE) as a part of computational social science, a newly integrated and overarching framework of social sciences, has been developed. Authors hope that ABM has the potential to become a useful approach for social sciences and interdisciplinary science.

Introduction of a socio-financial model in finance helps to understand pricing in financial markets not only through individual investment behavior but also considering different kinds of interactions (communication, competition, cooperation, etc.) between agents supporting the general idea that social dynamics drives financial markets [4, 8]. A socio-finance AB-m allows studying cognitive processes with multiple motives, moods and psychological traps discussed above lead to biases.

Among different disciplines the main concepts of policy formulation are applicable [36, 51, 52, 84, 92]. ABM is a useful tool to model several policy directions to analyze them before approbation in real life. The modeling of policy and implementation will not be effective if not to combine two different directions: macro and micro [43, 52, 57, 81]. One good example of it can be taken from economics and finance where ABM of macroeconomics allows for absolutely consistent micro-foundations.

1.4.4 Hybridization

Authors of [84] proved that a hybrid discrete-event AB-m has many advantages in projecting population and health outcomes compared with any other single type of modeling approach for smoking behavior. In demography, ABM is very usable, but sometimes there can be a problem of computational complexity [51]. As the population increases or the complexity of interactions is getting higher, the computation issue becomes more and more critical. In this regard, the High-Performance Computing (HPC) and distributed computing application to agent-based modeling should be carefully considered as a future study.

The Agent-based Framework for Renewable Energy Studies (ARES) was suggested in [21]. Authors suggested an integrated approach that adds an AB-m of industry actors to PLEXOS software and combines the strengths of the two to overcome their shortcomings. In [44], the authors integrate concepts from social networks (friendship, scale-free and single-scale networks), social science (opinion exchange, media impact, and irrationality), complexity theory (emergent behavior and adaptation), diffusion theory (adaptability and innovativeness), and decision theory (utility) to build their comprehensive ABM. This approach helped them to integrate optimization rules and build a comprehensive social layer to analyze and simulate details of social behaviors in the US power system. The approach of [75] has presented a new technique called Test-Driven Simulation Modeling (TDSM) as a combination of ABS and Discrete-Event Simulation (DES) gathering advantages of both approaches.

In finance, for the introduction of agent-based herding model of the financial markets in [41], authors felt the necessity to combine microscopic, agent-based, and macroscopic, phenomenological modeling. As a result, the integrated agent-based and stochastic model of the financial markets has appeared. The idea to combine two approaches came from the properties of nonlinear stochastic differential equations generating power law statistics and possibility to derive these equations as the macroscopic outcome of microscopic herding model.

To show the dependence of monetary policy and economic units in the financial market, authors of [52] combined an ABM, the Hurst exponent, and the Shannon entropy. First, ABM is used to analyze the behavior of the agent groups with different levels of irrationality. Second, the Hurst exponent is applied to analyze the peculiarities of the trend-following irrationality group. Third, the Shannon entropy is used to analyze and measure the degree of randomness and unpredictability of group behavior.

1.5 Conclusion

In this chapter, we discussed the main pillars of ABM and reviewed some recent studies relevant to this field. After that, we may notice that the ABM as a bottom-up approach to model complex systems represents a very perspective direction of research in different areas of science. The understanding of the ABM mechanism helps in the efficient investigations conducting.

It is difficult to make a correct decision about the agent attributes of an AB-m, their interactions and the particularities of the environment without in-depth knowledge of ABM pillars. We described the core categories of ABM, such as the autonomous heterogeneous agents with their attributes, direct and indirect interactions of the agents, the environment conditioning the boundaries of interactions, and a complex emergent system as the result of all the elements functioning together. We showed that ABM is appropriate to study complex systems, which have many peculiarities caused by non-linear interactions: several stationary states, standing of the system out of equilibrium, sudden phase transition shifts with cascading effects, etc. The degree of accuracy of the agent-based simulations of a complex system is determined by the level of detail during the construction of the categories for a particular model and depends on the target of the model construction.

In this chapter, we introduced several applications of ABM in a diverse range of science areas. We showed that the outstanding feature of ABM to create a complex system in a simplified way without losing the accuracy makes it a favorable tool for modeling complex dynamic systems in archeology, demography, energy, insurance, management, medicine, psychology, transportation, and finance. The emphasis on modeling the heterogeneity of interacting agents and the emergence of a self-organized system are two of the distinguishing features of agent-based simulation as compared to other simulation techniques in global science.

We made our emphasis on the demonstration the recent scientific discoveries in the ABM for finance. ABM applied to financial markets overcomes the faults of standard approaches to model financial systems in their complexity, with the real behavior of individual market agents, not always rational, during crises with the stages of disequilibria. When a standard approach fails to represent a financial system as a dynamic emergent mechanism characterized by non-normal distributions with fat tails, ABM provides all the necessary tools to simulate it through the interactions of heterogeneous individuals without deriving the assumptions about the global system. In the agent-based simulations, it is possible to consider micro- and macro-levels of the system in one model describing it with a compact set of parameters. In the next Chapter, we enter into details in introducing an agent-based representation of a financial market.

We believe that findings of recent studies in the field of finance may be attractive to the researchers working in different areas of science. Moreover, the practical experience in one field may be of great interest to a wide spread of applications. Modern attitude allows breaking the borders between strictly distinguished disciplines and building up an adequate interdisciplinary model catching all the relevant variables of the system.

Chapter 2

Agent-Based Modeling Representation of Financial Market

2.1 Introduction and Description of Financial Market System

A financial market is a complex mechanism which can be depicted as an aggregate of the agents with their attributes, interactions inside the environment and the origination of a dynamic system. Approaches toward the financial market modeling have evolved through the time introducing different core concepts and theories. Some of those theories have been falsified with empirical findings or subsequent studies, and some concepts have proved their efficiency for the financial market simulations. Before the review is provided of the central theoretical concepts in the representation of a financial market, we would like to define the primary attributes of it.

Simply defined, a financial market is a way of interacting between participants connected with an asset exchange (buying or selling). The groups of participants represent the main categories describing a particular financial market, types of interactions and features of assets being exchanged. Participants may be divided into sellers, buyers, intermediaries, financial advisors, etc., each with their motivations (profitability, risk aversion, etc.) and behavior rules. Interactions include trading, information exchange and consultations with different inter-individual and inter-group types of interactions such as cooperation, coalition or competition. The assets being traded may be characterized with liquidity, duration, prices with their fluctuations, price policies and so on. The combination of all these elements moves a market dynamics in the form of trading operations between a certain number of agents with specific prices and volumes of assets diversified by quantity and quality and followed by regulations and risks.

It is evident that even such a schematic variant of a financial market description includes a wide range of parameters and elements. It is important to single out key aspects and assumptions required to model a financial market system. Different theories determine various issues being studied within the financial market modeling: predictability, fluctuations, information to use, etc. Some theories may coexist supplementing each other, and some of them contradict in the critical aspects of the financial market representation.

Chapter 2 is organized as follows. In Section 2.2, we introduce the development of the theoretical approaches toward the concept of a financial market which precede the ABM. In Section 2.3, we discuss the agent-based framework for financial market modeling. It will be explained how a social representation of financial reality through different types of communication between heterogeneous agents may help in the understanding of the behavior of a complex financial system in a short and long-term perspective. In subsections 2.3.1 – 2.3.3, we describe the game-theoretical framework within ABM and introduce multi-agent games with unique payoff functions: the Majority game, the Minority game, and the $-game.

In Section 2.4, we give the information concerning some recent applications of the multi-agent game framework. The concept of systemic risk is taken into account, when the agent-based simulations can detect the risk of the failure of the entire system on the base of the non-rational behavior of the agents followed by the effects of change blindness, synchronization, decoupling, etc. Section 2.5 concludes the chapter.

2.2 Theoretical Representation of Financial market

In an attempt to trace the development of theoretical background for the ABM of a financial market we cover the following theoretical approaches toward the financial market modeling:

1 – rational representation of financial market (with permanent equilibrium state, the representative agent, and information availability);

2 – behavioral representation of financial market (appending individual human self-structure with its motives, emotions, and biases);

3 – social representation of financial market (appending interactions between homogeneous and heterogeneous individuals).

In Appendix 1, the results of comparative analysis are represented. To give the detailed information concerning the approaches mentioned above we follow several steps. After starting with the description of the classical approach toward pricing in the financial market in Section 2.2.1, we proceed with the behavioral representation of a financial market in Section 2.2.2. The discussion of a social representation within complex socio-finance modeling is separated out in Section 2.3.

2.2.1 Rational Representation of Financial Market

We start the overview with the Rational Expectation Theory which has been served as a foundation for many scientific tendencies for decades starting from the 1970s. Leading the standard approaches, Rational Expectation Theory [62, 70] postulates that the price of an asset depends only on two variables: interest rates and dividends, and changes only corresponding to the information shocks:

$$P_t = P_t^* + \varepsilon_t = \sum_{t+1}^{\infty} E\left[\frac{D(t+1)}{[1+r]^t} | I_{t-1}\right] + \varepsilon_t. \tag{2.1}$$

General financial market model of classical rational theory may be described as the behavior of a homogenized rational agent leading to the formation of a fundamental price of the asset with some biases which are not possible to predict. The Rational Expectation Hypothesis (REH) is closely connected with the financial concept of the market efficiency. The Efficient Market Hypothesis (EMH) was formulated in 1960's by Samuelson and Fama. Assumptions of REH together with EMH imply rational behavior of individuals who react only to the new information relevant for interest rates or earnings leading to the change of the fundamental price. Agents are homogeneous in the sense that all they are considered to have similar behavioral rules and rational reactions to the information incoming. Information is readily and immediately available with three levels of availability:

− weak: all the public information on past prices and volumes is reflected in the current price at every time;

− intermediate: all public information of any kind (earnings, political news, etc.) is included in the price at every time;

− strong: all kinds of information, including secret information, are fully reflected in the price at every time.

The equilibrium state is only one state of a financial system which is of interest for the model of the standard approach. It postulates that the expected outcomes do not differ from the market equilibrium value, i.e., the system stays in a permanent equilibrium state, and all the deviations do not matter having a random and temporary nature. Showing good performance characteristics during stable periods of economics, the expectation of rational behavior of the representative agent and equilibrium state of the system is entirely unjustified in the periods of global prolonged financial shocks.

As it was mentioned above, the Rational Expectation Theory assumes that there is only one type of agents operating in the market – a Representative Agent (RA) who is always rational with his behavior and expectations. Technically speaking, the concept of RA merely implies the summation of identical agents into one entity which represents a subject of a standard modeling approach. Even if the difference between agents is allowed, for the concept of RA, such agents are considered to be homogeneous if not strictly identical, and they may also be summed up into one entity. This assumption makes a theoretically represented system not dynamic and deprives it of the closeness to a real complex financial system.

The fact that Rational Expectation Theory has no relevance to the real conditions of the market and that it is not robust to big financial shocks makes it not applicable in the modern economics. Expanding the idea of the rational behavior of agents with the idea that higher risk is a price for a higher return, the Modern Portfolio Theory (MPT) has appeared with Markowitz's portfolio theory and the Capital Asset Pricing Model (CAPM) [59, 66, 82].

Markowitz developed a mean-variance analysis in the context of selecting a portfolio of common stocks. The theory formalizes the concept of diversification in mathematical terms and suggested that rational investors seek to make investment decisions that maximize portfolio return for a given level of risk.

Based on the Markowitz's portfolio theory, CAPM was developed one decade later which has provided a tool to find the proper price of a stock. The model calculates the required rate of return for an asset using the expected return on both the market and a risk-free asset, and the asset's correlation or sensitivity to the market.

Under the assumptions of Markowitz's Portfolio theory and CAPM, a financial market model is represented by the homogenized optimal portfolio as the combination of riskiness and profitability where the price of the asset is determined by the behavior of the market over one period with all the information available at the same time to all investors.

The combination of Markowitz's Portfolio theory with the assumption of CAPM about borrowing/lending at a risk-free rate gives the idea of the proper price of a stock P_0 [82]:

$$P_0 = \frac{1}{1+R_f}\left[E(P_T) - \frac{COV(P_T, R_M)(E(R_M) - R_f)}{VAR(R_M)}\right]. \tag{2.2}$$

This formula goes from the assumption that all investors in the market have rational behavior and they will form their portfolio based on the same optimal solution. As a result, it will be equal to the market portfolio $t = M$. The conclusion that the return of the asset is the same as the return of the market is clarified through the formula (2.4):

$$E(R_i) = R_f + \beta_i\left[E(R_M) - R_f\right], i = 1, \dots, N, \tag{2.3}$$

$$\beta_i = \frac{COV(R_i, R_M)}{VAR(R_M)}. \tag{2.4}$$

Thus, the return of the asset is represented as a combination of the return from the safe assets (government bond) plus the additional return taken from the risky assets. The coefficient β, according to CAPM, is the only relevant measure of an asset's risk. It measures a stock's relative volatility – that is, it shows how much the price of a particular stock jumps up and down compared with how much the stock market as a whole jumps up and down. And for the above-mentioned assumption, $COV(R_i, R_M) = 1$, so the return of the asset is the same as the return of the market with their time series following each other. Correlations between the different shocks are considered as constant. Shocks here are predictable only in small or intermediate scales without taking into account long-term big risks affecting the whole system.

Nowadays, the usefulness of CAPM is under debates, as soon as the coefficient β has not proved its total efficiency in capturing the market dynamics [33]. Nevertheless, the CAPM has suggested the idea of a self-consistent and self-organized market which is organizing itself in the convergence to an equilibrium point, representing the simplified idea of an emergent system.

Standard approaches laid down the foundation of a financial market theoretical model. Nevertheless, they do not take into account crucial empirical evidence of global financial shocks and the concept of systemic risk. Systemic risk as a risk of the collapse of the entire financial system appears in the global scale, and its modeling requires specific features of a simulated complex system. The standard theoretical approach considers a system in an equilibrium state where a representative agent responds rationally toward the incoming information. Such approach has some drawback from the point of understanding the sources of systemic risk. The risk may lay into the diversification of the agents and their natural tendency to interact in a heterogeneous way, not always rational.

The level of the market representation also matters for the consideration of systemic risk. The modern portfolio theory based on the traditional thinking implies only the risks of individual assets or portfolios. There are no descriptions of the emergent behavior at the system level which would demonstrate a joint dynamic mechanism subject to systemic risk.

2.2.2 Behavioral Representation of Financial Market

The complex of open questions which is not solved by standard approaches is actively working out by different modern approaches. In this connection, behavioral finance which underlines the need for cognitive process analysis for the financial market modeling is of big interest. The idea of behavioral representation has come from the evidence of anomalies in agent behavior and excess volatility of returns which the standard approach is not able to explain.

The concept of the agent becomes better specified and diversified. There is not only one generalized agent, but every agent has the right to be different from others in the attributes and behavioral rules toward the incoming information. From psychology, the fact is well-known that different individuals may affect in completely different ways to the same influence or shock. By considering this fact not as an isolated case but as a concept of agent representation, we may distinguish several cognitive processes on the individual level which diversify the agent behavior correspondingly to the motives, emotions, and biases [4].

Behavioral finance adds the relaxation to the assumption of the standard approach about one rational target of an agent to maximize the outcome. Psychological aspect assumes that the agents may have several motives simultaneously. Emotions also represent a new component of the agent attributes which the rational agent is not supposed to have. Human emotions often run counter to the logical conclusions which an agent could get from the environment. A well-known susceptibility of the participants in the financial market may be one example of the emotional agent attribute.

The next cognitive process characterizing the agents in the behavioral model is self-evaluation, the phenomenon when an individual evaluates himself and the environment with his inherent behavior rules corresponding to his self-structure. It means that agents not always react to the matters of facts but their perception and thinking of it. The example of cognitive closure should be introduced here, when after making a decision individuals stop to react to the incoming information which runs counter to it. Finally, the processes mentioned above may lead to the behavioral biases of the agents which cause them to act in a non-rational way, for example, framing, or overconfidence.

Those deviations from the standard approach increased significantly the ability of the behavioral models to simulate the activity of the agents in the financial market. The Behavioral Asset Pricing Model [17, 63, 83] focuses on the deviations from the fundamental values arising from rational behavior and links them to psychological biases. Unlike classical assumptions where there is only one category of rational agents which is represented in the market, within behavioral approach two categories of investors are assumed to be present in the market: rational traders and irrational ones who are subject to psychologically driven heuristics and biases. For behavioral impact clarification three crucial categories of errors made by irrational investors are introduced [17]:

$$P_t = P_t^* + B_t + \varepsilon_t = P_t^* + [(\varepsilon_1(x_t) + (\varepsilon_2(x_t) + (\varepsilon_3(x_t)) * (1 - C)] + \varepsilon_t \quad (2.5)$$

The Behavioral Asset Pricing Model assumes that the level of asset prices is affected by fundamental value and three behavioral variables resulting from errors in the processing of informational signals, representativeness errors, and unstable preferences. The non-rational behavior of agents causing errors of investors may result in considerable deviations from the fundamental value, thus leading to temporary mispricing (overpricing or underpricing) of assets. The final scale of mispricing depends on the market's ability to self-correct which is measured by the coefficient A introduced to the model.

Prospect theory [50] as a branch of behavioral finance has added important assumptions of psychological origin to the standard approach, such as the effect of the relative wealth to the decision making, risk aversion of agents, and subjective evaluation of the probability of events. The theory of average behavior, prospect theory models how an individual or a group of homogenized individuals behave in a world of uncertainty. Not covering all the aspects of behavioral finance, it focuses on the risky choice behavior and considers individuals' behavior rational in the conditions of imperfect and asymmetric information and the rules of the game in financial markets.

Though the theories of behavioral finance can explain a vast range of market anomalies, it is characterized by a high level of generality. The primary purpose is to quantify the relationships between psychological biases from rational behavior and asset pricing. In this connection, models can be used more successful to model real behavioral processes and to explain financial events *ex-post*, than for pricing and precise future predictions. Classical and behavioral finance complement each other with the classical model of rational and efficient market performance and the behavioral model explaining why empirical findings differ from theoretical rational expectations.

The idea of pricing an asset correspondingly to the performance of the market used in the CAPM is also acceptable for the behavioral finance, where pricing of a given stock can be expressed in terms of the general sentiments of the market. The return s of stock i in a given stock index I may be expressed through the formulation (2.3):

$$E(s_i^I) = \frac{E(R_{-i}^I)}{\sqrt{\langle VAR(R_{-i}^I)\rangle_{AT}}}\sqrt{\langle VAR(s^I\rangle_{AT}}. \tag{2.6}$$

The model of pricing stocks by yardsticks and sentiments described with (2.6) is similar to the model of CAPM (2.2). However, the main difference between them is the necessity of standard deviations rather than the covariance between the stock return and index return when the link between the stocks and market sentiments is modeled.

Behavioral finance has added a critical component into the financial market model – the impact of an individual non-rational behavior, – but it has at least one significant disadvantage: it does not include interactions between individuals and their diversification. The next step toward the efficient market modeling is including the impact of communication on the pricing which the socio-finance model lets us do in the network of agent-based modeling.

2.3 Agent-Based Representation of Financial Market

In compliance with a social nature of ABM, a financial market includes four associated components as it was shown in Chapter 1.2: agents, interactions, environment, and the emergent system. Given the knowledge of a collective behavior of individuals, social finance results in the emergence of complex aggregate-level behavior. The study of the system emergence in the macro-level from micro-level behavioral models supplemented with social interactions allows bridging the micro- and macro-level perspective from "the bottom-up." Therefore, ABM approach to the emergence of a social financial system provides a means of explaining the links between individual traits and financial market system behavior. Unlike in behavioral finance, specific individual traits and biases themselves are not at the core of the AB-ms. Instead, the individual traits are interesting that give rise to the specific system behavior.

Further, ABM gives the possibility to model a two-sided link in the market. Social interactions of the agents lead to the market formation and at the same time agents react to changes in the system by adapting to them.

To compare a social representation of financial market inside ABM with two previously described theories, rational theory, and behavioral finance, we may make the following notices. The rational representation determined the four categories but without a strong correspondence to the real financial system. All the agents are introduced through one representative agent with a bounded set of attributes; behavioral rules are represented only with a rule to behave rationally maximizing the profit. The relationships are executed through the environment by the information and prices under equilibrium; there is no concept of a complex system. Behavioral finance has added the diversity of agent attributes including individual cognitive processes with the assumption of heterogeneity. The environment affects the decision process of an individual heterogeneous agent. However, the self-organized system has not appeared yet because the agents exist independently without interactions through the system, they are not able to exchange the information within the environment.

In an AB-m, many agents are operating according to simple behavioral rules, but the resulting interactions between agents lead to the emergence of a complex aggregate-level behavior. The emergence of qualitatively different behavior at the aggregate-level than at the level of its constituent parts is a particularity of complex systems. The emphasis on the system appearance means that in ABM we are not interested in theory that explains individual behavior in detail (unlike in behavioral finance). Contrariwise, we try to model individual behaviors that are readily used in AB-ms to explain financial dynamics of social origin. Often, an AB-m is a highly simplified representation of the real behavior of individuals. Further, the formation of a dynamic collective behavior will be detailed with the idea of cognitive closure, synchronization effect and decoupling, and change blindness.

In other words, ABM allows us to study micro-to-macro mapping. For the modeling of a social financial system, it is important to begin with solid foundations regarding the individual behavior of the agents. Herewith, the behavioral finance may be useful. However, a technique still is needed for transforming the individual traits to the macro-level through the social interactions, where ABM provides a useful tool [32].

In an AB-m, the structure of a financial market model contains all four elements:

− a set of agents with their behavior rules who are heterogeneous with respect to their various properties like financial strategies, wealth, timescale, etc.;

− a set of agent interactions and methods of communication for the purpose to buy or to sell an asset, to insure against risks, etc.;

− an agent environment represented by the conditions of the financial market, infrastructure, etc.;

− a complex financial system as the result of agent interactions. This complex origin of a collective behavior gives rise to systemic risk studies within ABM.

For ABM, a financial market may be viewed as a complex social system appeared from the interactions of multiple heterogeneous agents with bounded rationality and behavioral traits, and trading strategies. Agents interact directly and indirectly with each other and with the environment in the form of competition, cooperation, and coalition, and can adapt their behavior correspondingly to the changes in the environment. Thus, the bottom-up mechanism of a complex financial system creation within ABM implies the aggregation of agent actions at the micro-level and the appearance of a macro-structure called the emergent system. The system obtains its peculiarities which are not inherent to separate agents. Finally, the dynamic macro-structure represents the fluctuations of asset prices in the market.

Problems in social sciences are often characterized by many variables needed to describe a given system. Reducing the set of all required parameters into the subset of key variables, the financial market system may be represented as follows. There are two main groups of agents (with bullish or bearish sentiments) who interact in the form of communication of subgroups of different sizes and thus lead to the evolution of sentiments in the population. Effects of interactions include the appearance of the emergent system which represents the market performance and affects sentiments in a nonlinear way. The price P of an asset formed by the emergent system may be represented as in [5]:

$$P(t) = P(t-1)\exp(R(t)), \tag{2.7}$$

with $R(t)$ being the return of the market defined as:

$$R(t) = \frac{A(t)}{\lambda} = \frac{\frac{1}{N}\sum_{i=1}^{N} a_i(t)}{\lambda}. \tag{2.8}$$

The formulation explains to us that the order imbalance $A(t)$ representing the difference between the amount of agents decided to buy ($a_i(t) = 1$) and the agents decided to sell ($a_i(t) = -1$) an asset, leads the price in a certain direction. The liquidity λ is used as a scale factor determining the size of the order flow necessary to move the price by a certain amount.

In the agent-based model of a financial market, agents use the available information every time step when deciding to buy or to sell an asset. The agents possess the information in the form of technical or fundamental economic analysis. Behavioral rules show the patterns of the price policy each agent is willing to apply: is he waiting for low or high prices (short and long position), at what state of the market he wants to buy, to sell, or to abstain from action. The interaction of the agents leading to the appearance of the collective behavior occurs through the price and usually has the form of a game.

Pure Game Theory [37] assumes that a few agents operating in the market are entirely rational; they try to use the best strategy considering the possible reaction of other participants to the agent’s action. However, on behalf of the financial market modeling, a game may be introduced as a minimal model to describe and predict financial market dynamics made by multiple heterogeneous agents.

The idea of such a minimal game-theoretical model for a financial market comes from the El Farol Bar Game when some agents must decide each week independently whether to go to their favorite bar or not [11]. The problem is that people will feel them comfortable in the bar only if it will not be crowded, so they should be in the minority to appear in the bar. Otherwise, it will be better for them to stay at home. To make a decision, the agents demonstrate adaptive behavior using past data to predict future attendance.

One reason for discussing this model is that it gives an example when the RET is not able to solve the problem. If agents have similar expectations about the bar attendance and behave accordingly, these expectations are going to prove wrong. The El Farol model dedicated to the inductive-reasoning path to equilibrium inspired multi-agent games dealing with the fluctuations around the equilibrium.

There are several multi-agent games with fundamentally different views on how to behave in the market which was introduced in ABM to study market price dynamics [4, 23, 68]. The behavioral difference of agents following these games relates to the deriving the payoff from each strategy. The variation of games includes a majority, minority, and dollar-games. Below, the general mechanism of gaming in the sense of agent-based modeling is described.

Following the standard specification of a multi-agent game, the assumption is valid that markets are purely speculative: agents use technical analysis for making the decisions and profit only from changes in the prices. The information available for agents in decision making is represented via the information vector of ones and zeros representing ups and downs of the market correspondingly. N agents use the same memory m of the last market moves (the number m of last digits of the information vector) in the set s of strategies. A strategy specifies for the agent what to do whatever the market behavior was in the past. For example, one strategy may tell the agent to buy (1) or to sell (–1) knowing the last 2 information days: 00 → 1; 01 → 1; 10 → –1; 11 → –1. Based on the information set, each agent chooses the dominant strategy (which differs subject to the type of the game) from a set s and uses it to make a decision. At each t < T period each agent i follows the strategy chosen and decides to buy or to sell $a_i(t) = \pm 1$, where +1 means buying and –1 – selling, thus determining in the aggregate the direction of the market represented by the excess demand, $A(t) = \sum_{i=1}^{N} a_i(t)$.

At first sight, the described model seems simplistic, but its complexity is hidden in the calculation of the total number of possible strategies and the model nonlinearity, as it was initially observed in the El Farol Bar Game [11]. The number of information combinations depends on the number of information days and equal to 2^m. The total number of strategies is given by S = 2^{2^m}, what could make the computational algorithm of an agent-based model too much complex. But in [23], it was shown that the appropriate implementation of the model can be obtained from a much smaller set of 2^m independent vectors instead of the total set *S* of strategies. The nonlinearity of the models within a game-theoretical approach is appeared due to the fact that the agents try to react to the market changes by choosing the optimal strategy out of the available subset *s* of strategies at each time step. These actions together provide a nonlinear feedback of the market which is changing due to the changes of the optimal strategies by the agents. Such nonlinearity of a complex financial system provides a close connectedness of the model with a real financial market.

2.3.1 Majority Game

The definition of the model within the Majority game corresponds to the description represented above with *N* agents, *s* strategies available for each agent, and *m* last digits of information vector. In a Majority game, agents believe that making the same decision as a majority of agents will be the most beneficial behavior. This pattern represents the standard strategy of trend followers and the concept of the increasing return in the financial market. At each time *t*, every agent uses his most successful strategy in the term of payoff to decide whether to buy or to sell an asset. The payoff of an agent *i* in the Majority game is given in [68] by:

$$g_i^{Majority}(t) = a_i(t)A(t). \tag{2.9}$$

The tendency of agents to follow the mainstream of the market leads, in fact, to the escalating of the market swings, because the trend-follower technique completely disregards the underlying value of the asset. Prices must go up as other agents employ the same line of thought, forcing the effect of herding.

The mechanism of Majority game does not require the assumption for heterogeneity of agent behavior, repeating the drawback of the theories based on an effective representative agent. Also, it misses the competition which is considered to be one of a key type of relationships between the agents in the financial market. To overcome these shortcomings the Minority game has been introduced.

2.3.2 Minority Game

In a Minority game which has been proposed as the formalization of the El-Farol bar problem [12], agents try to be anti-imitative and to make a decision as the minority of all agents. The payoff of an agent i in the Minority game is given in [23] by:

$$g_i^{Minority}(t) = -a_i(t)A(t). \tag{2.10}$$

The adaptability of the Minority game for the modeling of a financial market is explained by the fact that fluctuations and heterogeneities are the key ingredients of such simulations which are intrinsically frustrated. The theory of an effective representative agent applicable to the Majority game is failed for the Minority game which can catch heterogeneity of agent strategies and behavioral rules in the conditions of limited resources.

Despite the characteristics of the agents in the model being anti-imitative the property of Minority game agent-based framework is the fact that agents cooperate. A relative measure of cooperation is evaluated with the order imbalance $A(t)$ [23]:

$$\sigma^2 = \langle A^2 \rangle. \tag{2.11}$$

The volatility σ^2/N representing the fluctuations of the order imbalance of the market (Figure 2.1) under certain conditions can be smaller than the volatility of the system where the agents trade randomly. The control parameter $\alpha = \sigma^2/N$ giving the ratio between the number of uncorrelated strategies in use and the total number of strategies that N agents adopt distinguishes the state of the financial market in three groups:

− small α indicates the crowded state when the agents display a herding behavior;

− intermediate α: the state of the highest coordination between agent decisions;

− large α: the variance verges toward the volatility of a system where the agents trade randomly.

The mechanism of the Minority game is simple for understanding, but for that, it can simulate the nature of competitive interactions between the agents in the financial market. As well, it gives very significant results in the simulations of the inherent particularity of a complex system concerning phase transitions during shifts. It provides the tool to detect the system transition from the crowded symmetric phase to the uncrowded phase with the random behavior of the agents.

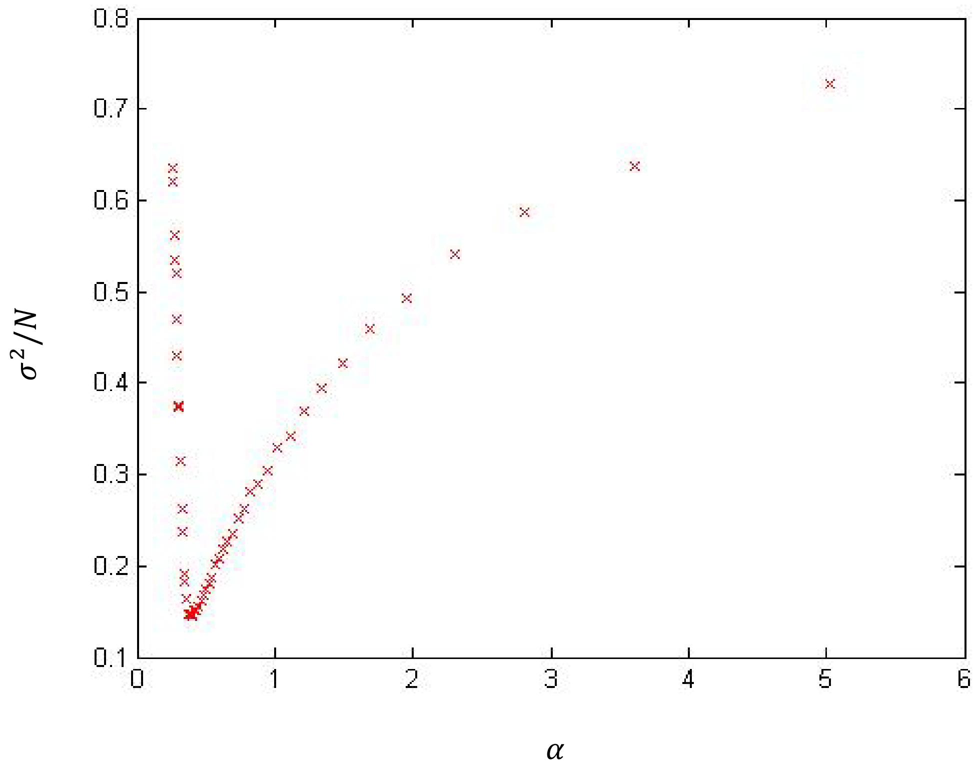

Figure 2.1: Normalized fluctuations versus the control parameter $\alpha = \sigma^2/N$ for s = 2, m = 8, average over 50 samples (the example of the code for the Minority game which has been used to run this exercise is available in Appendix 2). Minimum value separates the state of the system into two different phases: symmetric phase and asymmetric phase.

2.3.3 $-Game

The $-game concept appeared when the main goal of the agents has been prioritized. It emphasizes that the main goal of the agents not to belong to the group of trend-followers or contrarians but to maximize their profit. Instead of being either minority or majority players, the agents change adaptively from trend-followers to contrarians and vice versa.

This adaptive behavior demonstrates emergent properties of the price dynamics and the wealth of agents which are different from those of the Minority game. The payoff is determined with the same notation as for Majority and Minority games in [6] by:

$$g_i^{\$}(t+1) = a_i(t)A(t+1). \tag{2.12}$$

The defining difference of the \$-game payoff function with respect to the previous games is that the payoff function now depends on two different times. The agent tries to predict the trend before it happens, over next two time steps ahead (t and $t + 1$). In fact, the target of the agent is to gain profit on the difference of the price $|p(t+1) - p(t)|$.

The intent of agents to cooperate and take the same decision during the \$-game rises the mechanism of the intrinsic spontaneous bubble generation which does not appear in the Majority and Minority games. The effect of a bubble is defined as the sequence of m time step increases in the market, while a negative bubble means m consecutive time step decreases in the market. The optimal state of the \$-game corresponds the conditions when the price deviates exponentially in time from the fundamental value of the asset. All agents profit from the speculative state of the market when permanent price increases or decreases during the (negative) bubble state [4].

2.3.3.1 Decoupling and Bubble Occurrence

The joint behavior of the agents with their intention to cooperate leads to the appearance of speculative moments in the market when humans treat socially created reality as objective reality. In this case, a collective opinion in dynamics will determine the price of a given asset, without clear fundamental knowledge of the agents about the exact equilibrium value.

The effect of decoupling may appear which means that under certain conditions, people stop considering previous changes in the market dynamics and start following some socially created biased reality. Thus, the convergence of humans to mainstream may force agents to make decisions independently from real market changes. As it was shown in [4], the presence of decoupling indicates the hearding behavior of agents who locked in their positions and may generate the pockets of predictability, i.e., the possibility to predict the future global market behavior in several time steps ahead without knowing the future dynamics.

Following the formalism of [4], a strategy of an agent is decoupled if at time t it determines the action of the agent at time $t + 2$, independently from the behavior of the market at time t + *1*. In other words, the knowledge of the price movement at the next time step t + *1* is not required to determine what the strategy will recommend at time *t* + *2*. Knowing this, at any time *t*, the excess demand of the market $A(t)$ is consistent of the agent decisions within coupled and decoupled strategies:

$$A(t) = A_{coupled}(t) + A_{decoupled}(t). \tag{2.13}$$

To use the effect of decoupling, it is necessary to simulate the situation when the majority of the agents will be decoupled in the same direction. Thus, the condition for certain predictability one-time step ahead is as follows:

$$\left|\mathrm{A_{decoupled}}(\mathrm{t}+2)\right| > \mathrm{N}/2. \tag{2.14}$$

In turn, the excess demand $A_{decoupled}$ formed by the decouples strategies can be divided into positive $A^{+bub}_{decoupled}$, which is made by the agents whose decisions go in line with the direction of the market and negative $A^{-bub}_{decoupled}$, which is provided by the agents whose decisions go against the direction of the market.

The bias of the agents' expectations gives the rise of speculative moments in the market: bubbles and negative bubbles. As it was mentioned before, the effect of a bubble is defined as the sequence of *m* time step increases in the market, while a negative bubble means *m* consecutive time step decreases in the market.

In the game simulations, a bubble or a negative bubble appears at the time $t_b^0 = t_b + m$, when the last *m* price movements were all positive or negative, respectively, what makes a game "trapped", i.e. fixed in its current state. The moment t_b at wich the price begins to constantly increase or decrease has a core role in the forecasting of market dynamics. The split in the percentage $p^{+bub}_{decoupled}$ of positively decoupled strategies and the percentage $p^{-bub}_{decoupled}$ of negatively decoupled strategies indicates the bubble or negative bubble several time steps its appearance in the price history and let us define the order parameter M_{bubble} for the phase transition:

$$M_{bubble} = \frac{p^{+bub}_{decoupled} - p^{-bub}_{decoupled}}{p^{+bub}_{decoupled} + p^{-bub}_{decoupled}}, \tag{2.15}$$

with $M_{bubble} = 0$ corresponding to the high temperature phase, when the bubble or negative bubble is not in the aggregation, and $M_{bubble} \neq 0$ goes with the low temperature phase, when the agents have entered the bubble or negative bubble state, i.e. when $t > t_b$.

Table 2.1: Differences and commonalities between three multi-agent games

Particularities	**Majority Game**	**Minority Game**	**$-game**
Parameters used	*N* agents, *s* strategies, *m* information days		
Interactions	Cooperation	Competition	Diverse
Goal of an agent	Be in majority	Be in minority	Gain profit
Type of strategy	Trend-follower	Contrarian	Mixed adaptive strategy
Best strategy	Which produced the highest success rate for taking the majority action at every time step	Which produced the highest success rate for taking the minority action at every time step	Which produced the highest profit based on the history of the market
Payoff assignment	To all strategies *s*, even to those who are not active		
Payoff function dependence	Upon time *t*	Upon time *t*	Upon two times: *t* and *t* + 1
Intrinsic spontaneous bubble generating property	No	No	Yes

The described features of the $-game model single it out from the community of multi-agent games. It emphasizes the main goal of financial agents to gain profit (unlike in Majority or Minority games) but implies bounded rationality in decision making (unlike the Rational Expectation Theory). The emergent properties of the price dynamics are entirely different from those determined for Majority or Minority games. The best strategy is determined by the agent not in compliance with the target to be in the majority or minority but in terms of profitability. Finally, there is a crucial difference in the behavior of the system simulated via $-game: there appears an inherent capacity of the system to generate bubbles which help to model system phase transition and to find the pockets of predictability.

2.4 Applications of Multi-Agent Games for Market State Detection

In the previous chapter, we have introduced the definitions and discrepancies between different multi-agent games. Schematically, we have summarized the information in Table 2.1

The analysis of different ideas of applying ABM to real financial market has demonstrated that there is a wide range of empirical evidence where a multi-agent game framework has given interesting results. One of the applications has been described above concerning the effect of decoupling and the ability of the simulated system to predict market dynamics on the base of (negative) bubble formation. Below we insert several other findings in the field of ABM in finance which confirm the fact that socio-finance is of great value for the studies of the complex financial system especially in the periods of global crises.

2.4.1 Temperature Parameter of Market

A financial market system can be represented as the dynamic transition of fundamental and speculative states. During a fundamental state a system is in equilibrium and shows symmetry, the speculative state appears when the symmetry is broken leading to (negative) bubble scenarios.

When analyzing the information for decision making an agent can use technical or fundamental analysis, trying to profit from past price trends within the first one or the expectation of future dividends within the second one. This ability of agents to use different analytical techniques let us divide the market dynamics into a speculative state (with technical analysis strategies) and a fundamental state (with fundamental analysis strategies).

It was proven in [81] that the critical parameter that moves the market from symmetric (fundamental) state with "high temperature" to unstable (speculative) state with "low temperature" and back is the "temperature parameter" $Ʈ = \frac{2^m+1}{N \times s}$, which depends on the core parameters of the financial market game model: the number N of agents, the number s of strategies available for each agent, and the memory m of past movements of the market.

The described method provides a framework to simulate the financial market dynamics in a symmetric state when agents do not create a trend over time, and unstable non-symmetric state when trend followers appear. It gives the ability to catch the moment of phase transition from "high temperature" to "low" one and vice versa.

2.4.2. Price-Quakes in Financial market

The stylized facts of agent-based models such as synchronization, hearding, and change blindness together with the tools of the game theory help to model a real financial market with a compact set of parameters and a required level of the accuracy providing clear parameterization of behavior rules. On the micro-level, the behavioral traits and rules of interactions are introduced which lead to a strategic decision. Agents adapt to the price movement of the market by choosing the most successful strategy at each time step and thus form the emerging system on the macro-level which represents complex behavior and drive the interactions between individuals through the price.

Large movements in the global markets have a particular role in the clustering of market movements. The effect of change blindness implies the fact that humans react disproportionally to significant changes. Thus, during the periods with slow price movements worldwide (symmetrical state), there is a weak correlation between the dynamics of separate financial markets, but a hearding effect is detected during the periods with intensive worldwide fluctuations. This situation has got the name of price-quakes [5]. In general, it can be described as follows:

$$R_i(t) = R_i^{transfer}(t) + \eta_i(t). \tag{2.16}$$

The term of $\mathrm{R_i^{transfer}}$ evaluates for a given local market i a part of the return which is caused by the large movements in other local markets. The excess demand $A(t)$ of a global market which represents a global shift may be determined as an average of the reactions of local markets to the global news shocks, $\frac{1}{N}\sum_{\mathrm{i}} \mathrm{R_i^{trans\mathit{f}er}} \equiv \mathrm{A(t)}$. In compliance with this, the sign of $\mathrm{R_i^{transfer}}$ is used to predict the sign of the return of a given stock exchange.

This representation of a financial system implies the division of the stage of the market into a symmetric state and a state with broken symmetry. The phenomenon of symmetry breaking is related to a phase transition when the emergent behavior of a complex system changes its regimes itself without necessary external influence. [4] In a long-term, this particularity of the emergent behavior with self-organized criticality at the system level let us study such a phenomenon as systemic risk, which refers to the risk of collapse of an entire system.

2.5 Conclusion

In this chapter, we have discussed theoretical approaches toward a financial market. A standard approach based on the Rational Expectation Theory and the concept of the Representative Agent failed to explain the complex and volatile nature of financial markets. With assumptions of a permanent equilibrium and completely rational agents obtaining all the information, it cannot deal with the simulations of a real complex non-stable system during the economic crises.

Considering agents to be heterogeneous, behavioral finance has introduced a fundamental agent attribute – cognitive behavior rules including motives, emotions, and biases. It relaxed the assumption of one rational agent with the concept of bounded rationality of multiple agents. One peculiarity of a real financial market which has not been caught with the behavioral finance approach is the ability of agents to interact and create a complex system in a decentralized manner.

Agent-based representation of a financial market considers the interactions of the agents as a driving force for creating an emergent system. It includes all the postulates from earlier theories but with specific corrections bringing a simulated system closer to reality. In agent-based models, the market is filled with heterogeneous, boundedly rational agents with different expectations and behavior traits. The participants execute trading operations; interact inside the environment at the micro-level, and the aggregation of this activity with the bottom-up method gives rise to a complex financial system at the macro-level. ABM can explain main statistical observations of the financial time series which got the name of "stylized facts": excess volatility, temporary bubbles, fat-tailed data distributions, phase transitions with mean reversion, etc.

The described modeling approach toward financial markets usually has the representation of games, which underlines the emergent properties of the price dynamics and the behavior of the agents. The compact set of parameters used in the games (the number N of agents, the memory m that each agent uses for decision-making, and the number s of strategies used by each agent) gives the tool to understand the mechanism of speculative collective behavior in the market forcing the market fluctuations and formation changes. This representation provides non-linearity and stochastic component, what makes the models more realistic.

The comparative analysis of three multi-agent games was executed which helped to derive commonalities and distinguish differences between the Majority game, the Minority game, and the $-game. The $-game framework combines the advantages of two other games and brings important novelties in the representation of a financial market, such as aiming of the agents in the maximization of their profit but not just in being in the majority or minority, the intrinsic ability of the simulated system to form (negative) bubbles.

Several applications of ABM for the simulations of real financial data have been discussed, as the determination of the “temperature parameter” of the market, which can detect the phase transition between fundamental and speculative states of the market. Another application gives the idea of “price-quakes” in the market when significant movements in the global markets have a particular value for the system shifts.

Clearly, many interesting applications remain beyond the scope of this chapter, but the tendency has been determined that the studies in the field of ABM for finance are dedicated to the global system problems. They unveil hidden phenomena of complex financial systems which are hardly possible to detect using a standard theoretical framework. This ability of ABM is considered as the most valuable in the period of sequential global financial shocks.

Chapter 3

Simulations of Credit Default Swap Market and its Connection With Equity Market

3.1 Introduction

A Credit Default Swap (CDS) is a relatively new financial instrument appeared in 1994 [89] which has been under precise attention and debates between the financial market participants and regulators from its origination. A CDS represents an agreement to compensate a possible default (a credit event on an underlying reference entity) of a protection buyer (a creditor) by a protection seller of the CDS in exchange for the premium paid (CDS spread) to the protection seller, and by nature creates credit risk transfer mechanisms.

The years of the CDS regulatory foundations have been marked by the financial crises, which uncovered the shortcomings of the CDS contracts and promoted substantial changes in the CDS market [13]. Nevertheless, this financial instrument is not completely discovered heretofore, and as soon as the operations with CDS are becoming better spread in the financial markets, a non-effective treatment with it may cause further global financial problems in addition to the financial difficulties of separate market participants.

The observations of the tendencies in the CDS market may help us to deal with a risk of the collapse of an entire financial system, i.e., systemic risk. The connection between CDS dynamics and a systemic risk may be traced as follows. The spread of CDS evaluated by the market contains, among other things, the Expected Loss component [14] which is determined through Probability of Default (PD), Loss Given Default (LGD), and Exposure at Default (EAT) as follows:

$$\text{Expected Loss} = \text{PD x LGD x EAD}. \tag{3.1}$$

CDS contracts insuring against defaults pay off only as long as the seller of protection itself is solvent. Therefore, CDS spreads provide information about the probability of joint default of both the issuer and the protection seller. By assuming that the prices of CDS are written by different financial institutions against each other, together with the underlying entity prices, the information about all pairwise default probabilities across the financial network may be gained and analyzed. Thus, the CDS initially assigned to cover the risk of a certain creditor also affects the stability of all financial system at a global level reflecting the susceptibility to systemic risk.

Concurrent to the external regulation, market origin of CDS leads to the necessity of self-regulation performed by the market makers. For the financial stability, the main goal is to detect the appearance of critical moments before they will happen, considering the interaction of the participants as a complex dynamic system.

Hereinbefore we gave some information about the nature of CDS. It can help to understand the further description of sovereign CDS which are of particular interest for this study. In Section 3.1.1 we describe in brief the European sovereign CDS market stressing the connection of the extreme events occurred in the market with the recent financial crisis. In Section 3.1.2 we provide the definition and main matter concerning the calibration algorithms used for our simulated time series. These algorithms have proved their efficiency in solving the problem of an AB-m optimization.

Section 3.2 is dedicated to the simulations of sovereign CDS spread time series within the $-game framework defined in the Section 3.2.1. We describe in details the transformation of a classical $-game algorithm in Section 3.2.2 in point of the choice of the optimal strategy which gives us the possibility to simulate the sovereign CDS dynamics of many years.

In Section 3.3, we explain the decision to choose global European market data of CDS and Italian FTSE for the calibration technique of slaving using the Tremor Price Dynamics (TPD) and insert the result of the model calibration. Section 3.4 contains a preliminary statistical analysis of the time series. Then an exercise is described demonstrating the application of our simulation results to the predictions of the real estate market dynamics. Section 3.5 concludes the chapter.

3.1.1 Sovereign Credit Default Swap Market

Sovereign CDS offering the protection against the default of an entire sovereign government are of particular interest for systemic risk studies. The consequences of the European sovereign debt crisis have shown a possible scale of the disorder in the sovereign CDS market and its affection to a global financial system. The monitoring of sovereign CDS spreads can be considered both as an adequate measure of credit and systemic risks. It gives a pure measure of risks regardless liquidity, reserves and so on, and in the scale of global financial system predictions of CDS market fluctuations it can provide a valid indicator of global systemic shocks [13, 14]. The activity in sovereign

CDS markets allows us to obtain datasets with high frequency which are less dependent on the country-specific fundamentals than publicly traded sovereign bonds, and which may help to study the mechanism of the systemic risk origination [10].

The yearly information of 5-year European sovereign CDS spreads in the period 2009 – 2016 is represented in Appendix 4. As soon as a country becomes less creditworthy, the dynamics of its CDS spreads becomes more chaotic, and the volatility increases substantially. From 2008 to 2010 of the sovereign debt crisis such counties as Greece, Ireland, and Portugal, which have appeared to be in the middle of it, have demonstrated the highest spread volatility. While 5-year European sovereign CDS spreads have increased their value on 248 percent since the end of 2009 (113.14 euro) till the end of 2011 (393.95 euro), the same indicator for Greece have reached 3730 percent (262.91 euro at the end of 2009 and 10070.53 euro at the end of 20011), for Portugal – 1246 percent (80.07 – 2009, 1078.51 – 2011), for Ireland – 371 percent (141.55 – 2009, 667.39 – 2011). Also, the tendency for comovement is traced in the dynamics of sovereign CDS spreads: global financial shocks affect the creditworthiness of a global financial system, and sovereign CDS spreads indicate these changes almost instantly. Those global financial shocks may have their origin from global or local financial news, reflecting the effect of contagion or the consequence of a local financial system failure. Thus, sovereign CDS spreads demonstrate the force of the global and country-specific factors together. In [13], the conclusion has been made that for a particular country during stable economic situation the variation in spreads is mostly driven by global risk factors period with upward sloping term structure, while during the shaky economic period country fundamentals lead the sovereign CDS spread variation.

Credit derivatives represent only a small fraction of the derivatives market [13, 14], but the attention and discussion of this financial instrument are continuously increasing. This fact may be explained by the substance for the financial market predictions of the hypothesis toward a lead-lag linkage between the CDS and equity markets [13, 14, 67, 74]. The dynamics of the equity prices and CDS spreads are affected by the same fundamental shocks, but CDS is naturally more sensitive to negative information representing insurance on the potential default. So, the informed participants of the CDS market may react to the information with a few days' in advance to the participants of the equity markets, what allows to predict stock returns over the following few days [74]. It was detected that the pattern of the lead-lag linkage is asymmetric with respect to the negative and positive information flows, and the predictions of the equity returns by the CDS dynamics are effective for the negative information shocks. However, the hypothesis of predictability mentioned above runs counter to several research findings documenting that the equity market leads the CDS market at daily and weekly frequencies [47].

Further, we describe our approach to the sovereign CDS market simulations for the purpose to detect a possible connection between the market of CDS and the equity market. Our survey will prove the existence of the lead-lag linkage between these two markets and will demonstrate a simple “rule of thumbs” to model this connection.

3.1.2 Calibration Method

To execute the calibration of our agent-based model we use the following methods: Genetic algorithm and Reverse-Engineering technique. These two methods work together representing an efficient mechanism of financial data calibration.

Genetic Algorithm (GA) was proposed with the purpose to mimic the processes observed in a natural evolution – the Darwinian principle of natural selection [48]. The GA represents an adaptive heuristic search algorithm based on the evolutionary ideas of natural selection and genetics. It is the variation of a random search used to calibrate the optimal parameters of the model. Introducing a random component to the optimization, in fact, the GA is not random itself, but it explores historical information in a randomized way to detect the optimal value. The final aim of the algorithm is to find the optimal answer, but it is not necessarily the best. GA typically consists of three genetic operators of reproduction, crossover, and mutation.

In a GA, a set of candidate optimal solutions is evolving toward better solutions. The process of calibration usually starts from the first generation – an initial population of randomly generated individuals. Then each *chromosome* (a candidate parameter value) in the population is evaluated by the fitness function to test how well it solves the problem. The initialization of a population of candidate solutions instead of a single value is an advantage of GA which is useful in searching for a global optimum (not just local one).

As the result of iteration, the generations of variations of the initial population appear: the group of individuals with better objective functions is formed out of the initial set. Gradually, each new generation inherits the better traits and is, therefore, more fit than the previous one. The next iteration is executed under the evolved set of candidates. The fitter a chromosome is, the more likely it is to be selected.

GAs are iterated until the fitness value of the "best-so-far" chromosome stabilizes and does not change for many generations. Typically, the final generation is composed entirely of new offspring and the first generation is completely replaced. Also, commonly, the algorithm terminates when either a satisfactory level of optimization is achieved or a maximum number of iterations has been produced.

In the applications to a game-theoretical AB-m, the GA helps to calibrate simulations to real-time series by exploring the different parameters of games as well as different initial strategies attributed to the agents [4, 6, 92]. Applicable within a reverse engineering technique it helps to find a set of model parameters which can reproduce real-time series in the best way.

Reverse engineering is the process of the system modeling by analyzing a final result. The knowledge is extracted from a final representation of the system; often the process involves disassembling the resulting product and analyzing its components and working mechanisms in detail. This technique is applicable to the complex systems when it is difficult to model the behavior of it without detailed analysis of a final representation.

The origin of reverse engineering lays in the analysis of hardware for commercial or military advantage [26]. In fact, the reverse engineering process is not dedicated to creating a copy of a final product. It is beneficial to conduct anlyses to deduce design features from a final system with little or no additional knowledge about the procedures of its creation. In this connection, reverse engineering is a favourite tool for the calibrations of complex financial systems, when an emergent system appears spontaneously and has a hidden function mechanism.

The calibration using reverse engineering with the application of the GA involves evolving the set of parameters within defined ranges until a termination condition is satisfied. The real financial market gives the base for calibration permitting to evaluate the effectiveness of calibrations and determination of the optimal parameters of the model. The optimized set of parameters means the best option evaluated from the point of minimization of the difference between the simulated and real-time series measured with different standard norms.

3.2 Simulations of Sovereign Credit Default Swaps

The goal of the following empirical studies is to simulate the joint behavior of agents in the CDS market and to analyze a possible connection of the sovereign CDS spread dynamics and the dynamics of the price indices learned by the example of Italy. Our hypotesis assumes that there is an inverse dependence between CDS spreads and equity prices (Figure 3.1). The initial idea was to simulate the behavior of the Italian sovereign CDS market using $-game theoretical modeling with the real input and to analyze the possibility for the Financial Times-Stock Exchange price index (FTSE) predictions on its base.

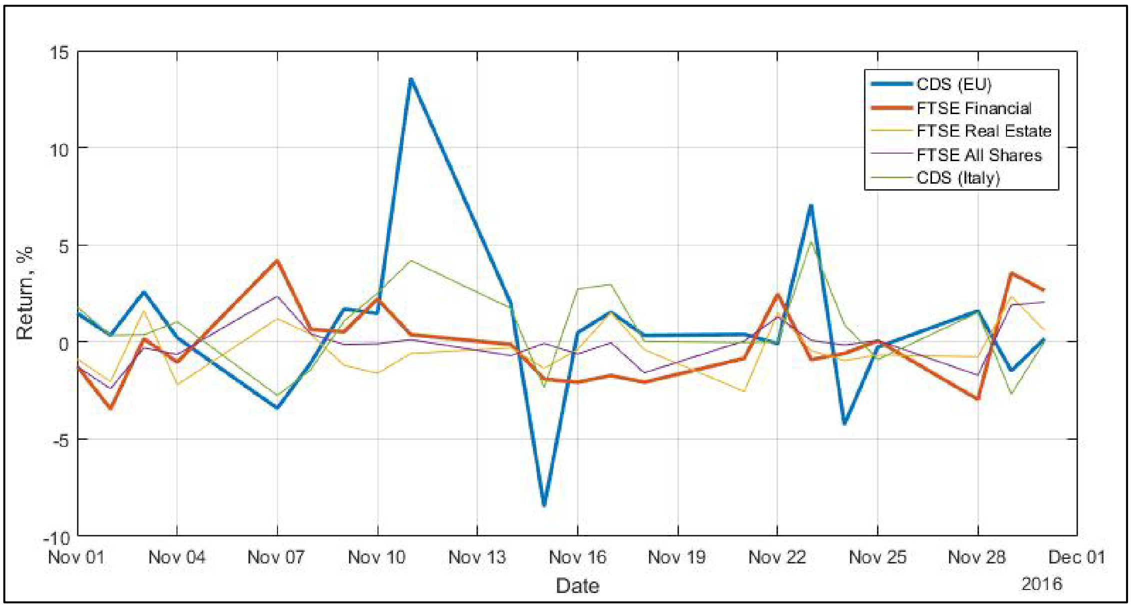

Figure 3.1: The representation of an inverse dependence between CDS spreads (time series of European and Italian sovereign CDS spreads) and equity prices (time series of FTSE financial, real estate and all shares indices).

The main steps leading to the application of this idea to a real financial market are:

1) to construct the Monte Carlo $-game simulations with the real input of the

CDS market data;

2) to use a reverse engineering technique evaluation of the optimal parameters of the \$-game model for the purpose to simulate the dynamics of CDS for the determination of the FTSE index dynamics;

3) to observe the results of (negative) bubble occurrence caused by synchronization in decision making and change blindness for the sake of FTSE index predictions.

3.2.1 Definition of Model

A \$-game model of the sovereign CDS spreads includes 3 parameters within a general multi-agent game framework described in the previous chapter: the number *N* of agents operating in the market, the number *s* of strategies available for each player during the game, and the number *m* of the last information days available for the agents to make the decision. The information vector with the length *m* is represented by zeros and ones, where ones indicate the rise of the CDS spread, and zeros indicate its stagnation or reduction. Agents are considered to be speculators, i.e., they use technical analysis of past realization of prices to make a decision. Each agent possesses the history of past *m* price movements and a fixed number *s* of strategies, which are initially assigned randomly from all the set of possible strategies $S = 2^{2^m}$. In the example from Table 3.1, a set of specific strategies for an agent tells whether to buy or to sell an asset depending on *m* = 3 information vector.

At each time $t < T$ every agent makes the decision to buy or to sell the CDS. Agent design assumes that all agents have unbounded wealth, i.e., they are unbounded in the amount of assets to hold, to buy or to sell within the time horizon *T*. An agent *i* follows the strategy chosen and makes the decision $a_i(t)$ to buy or to sell: $a_i(t) = \pm 1$, where +1 means buying and –1 – selling, thus determining in the aggregate the direction of the market represented by the excess demand, $A(t) = \sum_{i=1}^{N} a_i(t)$. The choice of the strategy is made by an agent for the reason of using the best strategy available, i.e. the strategy with the highest payoff accumulated during the game. The payoff $g_i^{\$}$ is given in [6] by:

$$g_i^{\$}(t+1) = a_i(t)A(t+1). \tag{3.2}$$

Table 3.1: Example of agent strategy set for s = 3 and m = 3. A strategy suggests a specific action for an agent to buy (1) or to sell (-1) an asset for all possible combinations of the past market movements.

Information vector	Action		
	Strategy 1	Strategy 2	Strategy 3
000	1	-1	1
001	-1	-1	1
010	1	1	-1
011	1	-1	-1
100	-1	1	1
101	1	1	-1
110	-1	1	1
111	-1	1	-1

The representation of major building blocks of the model gives a better understanding of the model complexity. The financial system simulated inside the AB-m is a *social* representation of a complex system created in *indirect interactions* among individual agents with their *behavioral* traits. The model characteristics are as follows according to major building blocks:

1 – agent design. The model is a 2-type design model represented by joint behavioral expectations of agents with no risk preferences and unconstrained wealth. The 2-type design is based on the chartist feedback, but with two agent categories, i.e. trend-followers and contrarians. Behavioral rules of the agents are represented using randomly initialized set of strategies for every agent. This building block is based on the assumption of behavioral finance about the heterogeneity of the agents;

2 – agent evolution. Randomly initialized strategies are updated using an evolutionary algorithm in conformity with the payoff function. The switching mechanism between two agent categories is evolutionary, being dependent on the performance of the respective strategies with the goal to maximize the profit. Acting in the market, agents have no direct information of the actions of other market participants. The indirect communication is performed through the information about the aggregate actions in the past incorporated in the past price behavior;

3 – price finding mechanism. During the indirect communication, agents make decisions corresponding to their current optimal strategies and last market changes. The price change is a function of the aggregate excess demand. The optimal state of the $-game is observed during speculative periods (bubbles), which requires coordination between the agents to enter and stay in the speculative state. The coordination is organized indirectly as a sum of independent decisions of agents using optimal strategies. This particularity adds an indirect socialization component to the model building blocks.

The AB-m represents the mechanism for incorporating behavioral traits in the micro-level and the interactions of the individuals into the joint complex behavior at the system level. Behavioral component is concerned with how individual decision-making takes place, in a static setting. Whereas price-setting in financial markets is a collective and dynamic process related to the appearance of the complex social phenomenon.

3.2.2 Simulation Algorithm

The period of the database has covered 8 years, from 2009 till 2016, the time series with daily frequencies come from DataStream. The CDS data are represented by the time series of European and Italian 5-year sovereign CDS spreads. The FTSE data are represented by Italian FTSE financial, real estate and all share price indices.

To use the time series of the Italian sovereign CDS spreads as a real input for agent memory m, the sovereign CDS spreads' time series have been reduced down to just binary indicators for up and down moves into the sequence R_{CDS}^{input}, where:

$$\begin{cases} R_{CDS}^{input}(t) = 1, if P_{CDS}(t) > \ P_{CDS}(t-1), \\ R_{CDS}^{input}(t) = 0, if P_{CDS}(t) \leq \ P_{CDS}(t-1). \end{cases} \tag{3.3}$$

The parameter values used for the calibration are as follows:

- the number of agents in the game, $N \epsilon \{3, \dots, 150\}$,
- the number of information days, $m \epsilon \{2, \dots, 8\}$,
- the number of agents' strategies, $s \epsilon \{2, \dots, 18\}$.

The calibration itself implies evolving the determined set of parameters within defined intervals until a target condition has been satisfied. Real CDS time series reduced to a binary representation are used to reverse engineer to achieve the target of the calibration is to minimize the objective function:

$$\min \sum_{t=1}^{T} \left(R_{CDS}^{input} - R_{CDS}^{ABM}\right)^2. \tag{3.4}$$

The results of the simulations have the representation demonstrated in Table 3.2. During the simulations (the example of code is available in Appendix 3), every time step a strictly one best strategy, i.e., the strategy with maximum payoff, is chosen for every agent corresponding to the payoff function (3.2). This means, that if there is only one best strategy (not several strategies with the same maximum payoff from which it is possible to choose randomly), there is no way for the simulated game, but to choose the action recommended by this strategy. If the actions of the majority of the agents for whom only one best strategy is available do not correspond with the real direction of the market, there is a certain discrepancy between simulated and real return of the market. If there is a sequence of constant increases (decreases) in the market during several time steps, which run counter to the real market behavior, we may tell about the (negative) bubble appearance. It means that the game is "trapped", i.e. fixed in its current state and cannot leave it.

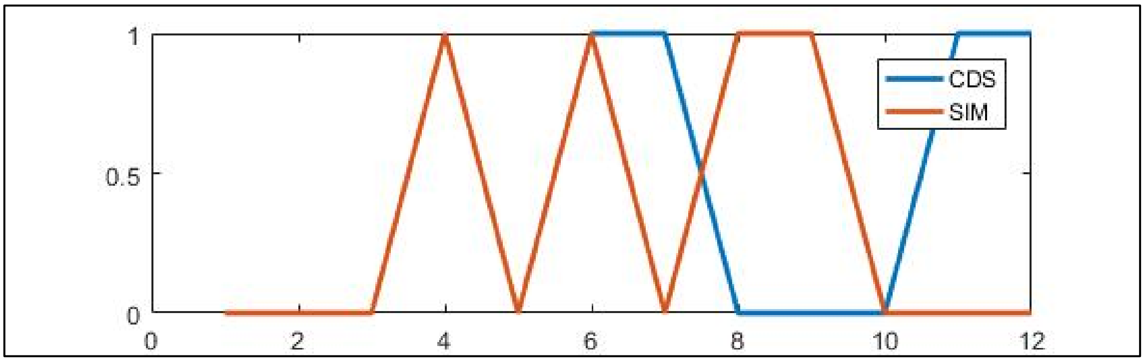

Figure 3.2: The difference between real binary indicators (blue line) of sovereign CDS dynamics and simulated ones (red line).

Figure 3.2, 3.3 and Table 3.2 illustrate this effect: after a system has entered a (negative) bubble state, it took several time steps to bring in conformity a simulated output with a real input substantially deflecting the simulated time series for the time of a (negative) bubble. It is more crucial for small values of the parameter *s* when the probability is high enough that all possible choices for $a_i(t)$ of each agent will have the same sign and will not let us slave a real and ABM output by simulation manipulations.

Several time steps spent to bring the system back to the conformity with the real input has substantially decreased the simulation accuracy.

Table 3.2: The demonstration of the difference between real binary indicators (CDS) of sovereign CDS spread dynamics and simulated ones. The results of simulations are expressed in terms of the excess demand A (SIM(A)), and the binary indicators of the simulated excess demand SIM(bin). SIM(bin) = 1 if SIM(A) > 0, and SIM(bin) = 0 otherwise. The times of mismatches are marked in pink.

t	CDS	Accumulated CDS	SIM (A)	SIM (bin)	Accumulated SIM (bin)
1	0	0	-5	0	0
2	0	0	-1	0	0
3	0	0	-1	0	0
4	1	1	7	1	1
5	0	1	-3	0	1
6	1	2	1	1	2
7	1	3	-7	0	2
8	0	3	3	1	3
9	0	3	1	1	4
10	0	3	-3	0	4
11	1	4	-5	0	4
12	1	5	-1	0	4

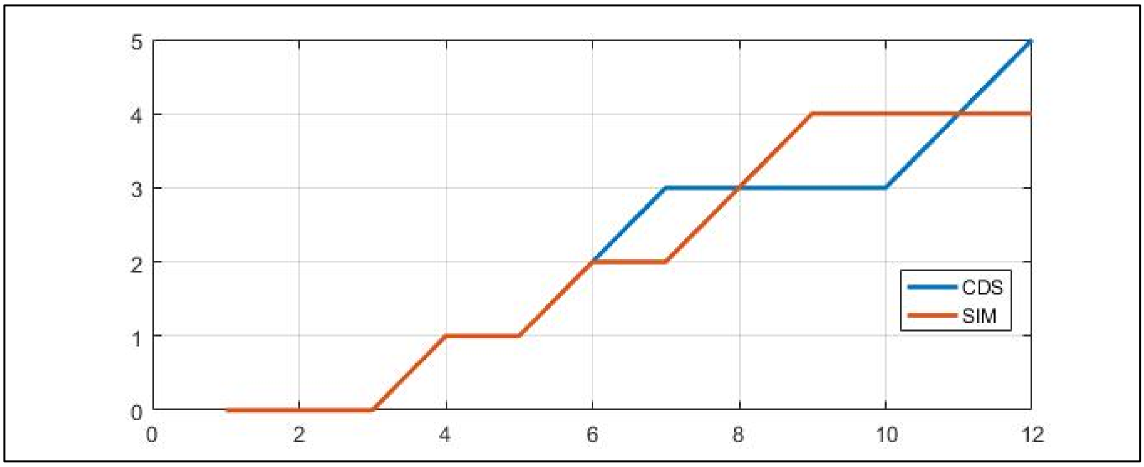

Figure 3.3: The accumulated difference between real binary indicators (blue line) of sovereign CDS dynamics and simulated ones (red line).

To simulate the behavior of the market in the long term for our research, we felt the need to avoid the described drawback connected with the periods of discrepancies between simulations and the real market. The idea has been tested to modify the choice of the strategy by an agent during the period of slaving. The algorithm implies the assumption to choose between first two strategies with a higher payoff every time step and to repeat this choice until the correspondence between simulated and real market return is reached. Multiple iterations provide a high-level correspondence between real and simulated returns. The case of the discrepancy in this algorithm indicates the exact time of a (negative) bubble appearance, but the algorithm let overcome a several time step discrepancy and reduce it to one-time step. The algorithm is represented below, and an example of the code is available in Appendix 5.

<u>Algorithm of slaving for real and simulated data of CDS:</u>

- for each agent determine the strategy s_i used:
    - if a set of strategies with maximum accumulated payoff contains only one strategy, use it;
    - if a set of strategies with equal maximum accumulated payoff consists of several elements, choose one strategy randomly;
- for each agent determine the decision $a_i(t)$ corresponding the strategy s_i and the information vector *m(t)* ;
- calculate $A(t) = \sum_{i=1}^{N} a_i(t)$, determining the sign of the simulated outcome R_{CDS}^{ABM} of the market:
    - if $\mathrm{A(t)} > 0, \mathrm{R}_{\mathrm{CDS}}^{\mathrm{ABM}}(\mathrm{t}) = 1$,
    - otherwise $\mathrm{R}_{\mathrm{CDS}}^{\mathrm{ABM}}(\mathrm{t}) = 0$;
- if $R_{CDS}^{input}(t) = R_{CDS}^{ABM}(t)$, the simulation for the time step *t* was successful, go to the next time step;
    - if $R_{CDS}^{input}(t) \neq R_{CDS}^{ABM}(t)$, keep the choices $a_i(t)$ satisfying conditions $sign[a_i(t)] = signR_{CDS}^{input}(t)$ and choose another s_i (among the set of strategies with equal maximum accumulated payoff) for agents whose $sign[a_i(t)] \neq signR_{CDS}^{input}(t)$:
        - form a set of strategies with maximum accumulated payoff adding the strategy with the second maximum payoff, choose one element randomly;

- for each agent determine the decision $a_i(t)$ corresponding to the strategy s_i and the information vector m(t) ;
- calculate $A(t) = \sum_{i=1}^{N} a_i(t)$, determining the sign of the simulated outcome R_{CDS}^{ABM} of the market: if $A(t) > 0, R_{CDS}^{ABM}(t) = 1$, otherwise $R_{CDS}^{ABM}(t) = 0$;
- if $R_{CDS}^{input}(t) = R_{CDS}^{ABM}(t)$, the simulation for the time step *t* was successful, go to the next time step;
  - if $R_{CDS}^{input}(t) \neq R_{CDS}^{ABM}(t)$, consider it as the time t_b^0 of a (negative) bubble appearance in the market, when agents don't follow the real dynamics of the market, but experience change blindness.

Table 3.3: Success rate for the identity between binomial representations of real and simulated Italian and European sovereign CDS spread time series (for some choices of parameter sets).

Parameter set			Success rate of identity between real and simulated binomial representation of time series, %	
s	*m*	*N*	Italian sovereign CDS spreads	European sovereign CDS spreads
3	2	25	84.32	88.45
7	4	25	90.40	93.97
9	7	25	97.35	97.43

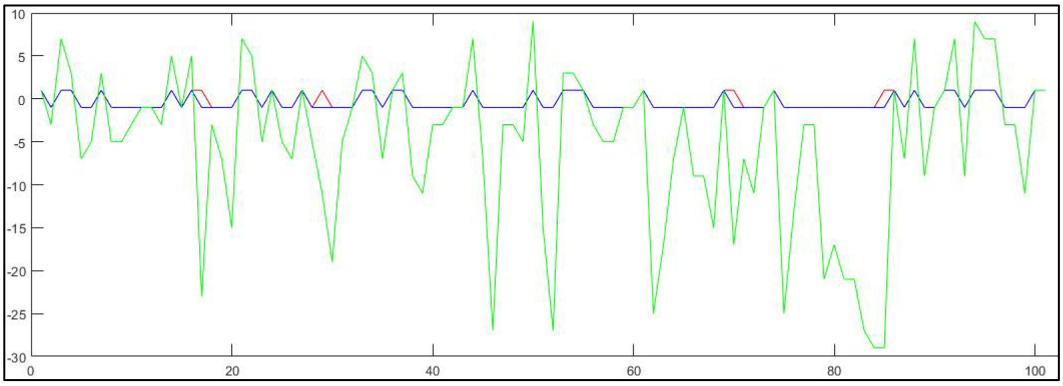

Figure 3.4: Simulations of sovereign CDS spread market (*N* = 25, *s* = 7, *m* = 4). Real binary indicators of Italian sovereign CDS spread time series are marked in blue. The simulated excess demand A is marked in green, determining SIM(bin), simulated binary indicators (red line) of sovereign CDS time series (SIM(bin) = 1 if SIM(A) > 0, and SIM(bin) = 0 otherwise). The time moments of mismatches in simulated and real

market data due to the speculative moments indicating the time of the bubble or negative bubble appearance are marked in red.

The described algorithm applied for the Italian sovereign CDS spread time series gives 84.32 – 97.30 % success rate (depending on the choice of the parameter set) for the identity between corresponding binomial representations of real and simulated sovereign CDS spreads dynamics (Table 3.3). The example is represented in Figure 3.4. An empirical analysis of the simulated CDS data and corresponding to time FTSE index data demonstrated a very interesting ability of CDS simulations to predict the changes in FTSE market dynamics, i.e., the time t_b^0 of a (negative) bubble appearance in the CDS market switches the FTSE index dynamics from negative to positive and vice versa. The possibility to make predictions on this base will be discussed in the next section and now the choice of the optimal database and ABM parameters will be explained.

3.3 Slaving of CDS Market Simulations to FTSE Real Time Series

The described simulation algorithm has been applied first to the Italian sovereign CDS spread time series. After that, the idea appeared to trace the correlation between sovereign CDS spreads of Europe and Italian FTSE index to detect a global interconnectedness of the markets. The supposition that fluctuations of FTSE index of Italy are explained in a better way by the dynamics of global European CDS prices is corroborated (Table 3.3).

The supposition can be explained by the nature of complex systems and the mechanisms of interactions between their components in the application to financial markets. If to consider a global financial market as a complex system where country markets are its elements, there are indirect interactions between the country markets through the changes in the global prices. A global price is affected by significant changes in country markets, implying the concept of systemic risk in the world’s financial markets, which may be useful in describing a sociological phenomenon behind the way price formation propagates across markets [4]. In turn, after incorporating significant local changes, a global price may cause the following changes in the local markets.

Intuitively, it would be expected that the fluctuations in Italian FTSE indices are driven mostly by country-specific fundamentals which first may be reflected in the fluctuations of Italian sovereign CDS spreads. However, empirical evidence shows that the variation in the Italian FTSE index is explained better by global financial dynamics of sovereign CDS spreads. Graphically the difference between Italian and European sovereign CDS spreads dynamics is demonstrated in Figure 3.5, where it may be noticed that the trends of the European market are steadier than of the Italian one.

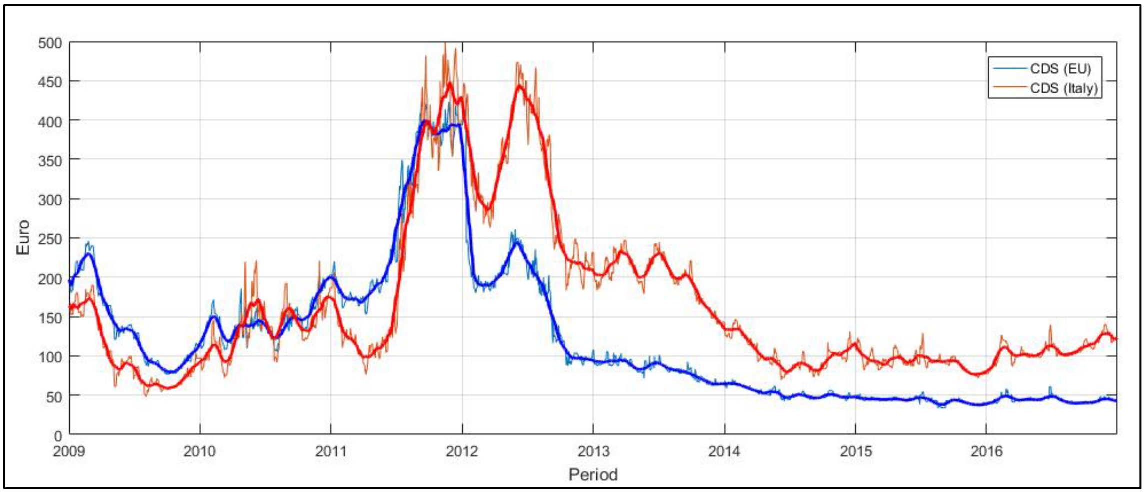

Figure 3.5: The dynamics of Italian (red line) and European (blue line) sovereign CDS spreads followed by the corresponding trend lines.

To explain theoretically the obtained empirical evidence of the FTSE major dependence on the global financial movements we use the Tremor Price Dynamics (TPD) model which implies the ideas from finance, physics, and psychology [5]. At time *t* a trader of FTSE at a given European market *i* (Italian, in our example) estimates the price index P_i^{FTSE}:

$$P_i^{FTSE}(t) = P_i^{FTSE}(t-1)\exp(R_i^{FTSE}(t)). \tag{3.5}$$

The return is determined in turn as a joint effect of global shocks and local news:

$$R_i^{FTSE}(t) = R_i^{FTSE,transfer}(t) + \eta_i^{FTSE}(t), \tag{3.6}$$

with the first term $R_i^{FTSE,transfer}(t)$ showing the *effect of external global news* with large price movements of the FTSE index. The second term $\eta_i^{FTSE}(t)$ indicates the *effect of internal local news* relevant only for the specific market *i*.

With the assumption that CDS dynamics leads FTSE market with inverse relation and that it has a mediated effect through the global European market with the appearance of systemic risk, we may write:

$$R_i^{FTSE,transfer}(t) \equiv -R^{CDS}(t-t') =$$

$$= -\sum_{i=1}^{N} R_i^{CDS}(t-t') = -(\sum_{i=1}^{N}[R_i^{CDS,transfer}(t-t') + \eta_i^{CDS}(t-t')]). \quad (3.7)$$

Finally, we derive the determinant of the FTSE return dynamics as follows:

$$R_i^{FTSE}(t) = -R^{CDS}(t-t') + \eta_i^{FTSE}(t). \quad (3.8)$$

As it was demonstrated in the representation of major building blocks in the definition of the model in Section 3.2.1, the AB-m of the CDS market includes behavioral and social components representing the model of a complex financial system. Additionally, the application of the AB-m to the relationships between CDS market and Italian Equity market represents the interaction between two complex systems of different levels: the local equity market which represents the result of indirect interactions between agents, and the global market of CDS which represents the result of social interactions between national markets.

The reasoning behind (3.5) – (3.8) may give us the understanding of the fact that the joint changes in European sovereign CDS spreads have a more determined effect on the Italian FTSE dynamics than just changes in the Italian sovereign CDS spreads. Thus, a certain dependence between CDS and FTSE dynamics may be applied for the joint analysis of two markets. The target of the calibration advances from just minimizing the difference between real and simulated outcomes to the anticipation in several time steps the changes of the FTSE market regimes.

The reverse engineering a real sovereign CDS spread time series with the parameter of ABM selected by using a genetic algorithm is described in Section 3.1.2 of this chapter. During the in-sample period, the input to the information vector *m* is generated from the real European sovereign CDS time series. This technique of slaving demonstrates the ability of agents to learn what goes on in the real market. The final target of the calibration is to try out different parameter sets within a determined interval ($\mathrm{N} \in \{3, \dots, 150\}$, $\mathrm{m} \in \{2, \dots, 8\}$, $\mathrm{s} \in \{2, \dots, 18\}$) during the simulations and optimize the choice of agent-based model parameters via the conditions described above.

The results of the calibration using a genetic algorithm have given the optimal parameter values equal to: $N = 25$, $m = 4$ and $s = 7$. For the sake of only slaving the simulated data to the real one the result with $N = 25$, $m = 7$ and $s = 9$ would seem to be preferable because it gives almost 100% success rate of identity (see Table 3.3). However, in this case, the simulated time series would be devoid of its inherent ability to form (negative) bubbles – the ability which lets us make predictions of the market future dynamics. The example of the calibrated simulations is represented in Table 3.4 providing an insight into a future possibility to make predictions of the FTSE dynamics on the base of sovereign CDS simulations.

Table 3.4: The correlation of the European sovereign CDS market and the market of Italy represented by the returns of FTSE indices (financial, real estate and all shares) with the ABM parameter values equal to: $N = 25$, $m = 4$ and $s = 7$. The synchronization effect in the global CDS market switches the direction of the FTSE dynamics. The moments of simulated and real market data mismatches (when the simulated excess demand A is positive while CDS dynamics is negative, or vice versa, i.e., when CDS and SIM are not equal) due to the speculative moments indicating the time of the bubble or negative bubble appearance are marked in green. Substantial positive and negative dynamics of FTSE indices are marked in blue/light-blue and red/pink respectively (marked values are higher than 1%). The returns of FTSE financial index show the biggest sensitivity to the changes in the CDS market. Switching the regimes in the CDS market leads to the delayed switching (with the opposite sign) of the dynamic direction of the FTSE returns.

Date	CDS	CDS (bin)	SIM (A)	SIM (bin)	FTSE fin., %	FTSE r/est., %	FTSE all/sh, %
01.08.2016	42.25	0	-13	0	-3.49	0.49	-1.59
02.08.2016	42.27	1	-9	0	-4.91	-3.69	-2.64
03.08.2016	42.67	1	7	1	-0.29	-0.55	0.13
04.08.2016	41.77	0	-5	0	0.69	0.46	0.62
05.08.2016	41.34	0	-3	0	4.19	2.35	2.30
08.08.2016	40.82	0	-7	0	1.60	-0.51	0.64
09.08.2016	40.35	0	-13	0	0.81	0.03	0.34
10.08.2016	40.28	0	-21	0	0.38	0.24	-0.06
11.08.2016	40.03	0	-21	0	0.94	0.12	1.01
12.08.2016	40.03	1	-9	0	0.27	-0.50	0.15

Date	CDS	CDS (bin)	SIM (A)	SIM (bin)	FTSE fin., %	FTSE r/est., %	FTSE all/sh, %
15.08.2016	39.40	0	-5	0	0.00	0.00	0.00
16.08.2016	39.50	1	3	1	-1.88	-1.40	-1.18
17.08.2016	39.59	1	5	1	-2.48	-1.51	-1.50
18.08.2016	40.34	1	11	1	0.55	1.14	0.82
19.08.2016	41.69	1	9	1	-3.77	-2.57	-2.00
22.08.2016	39.79	0	13	1	0.92	-2.04	0.32
23.08.2016	40.20	1	3	1	4.88	0.53	2.23
24.08.2016	40.46	1	3	1	2.41	1.22	0.69
25.08.2016	43.70	1	11	1	-1.90	-0.69	-1.00
26.08.2016	40.28	0	-1	0	0.59	0.48	0.74
29.08.2016	40.28	0	-1	0	-1.15	-0.91	-1.04
30.08.2016	40.30	1	5	1	2.25	2.14	1.29
31.08.2016	40.03	0	-3	0	2.62	-0.54	0.31
01.09.2016	39.36	0	-5	0	0.54	-0.10	-0.07
02.09.2016	38.89	0	-3	0	1.72	0.37	1.42
05.09.2016	38.62	0	-13	0	-0.30	0.13	0.07
06.09.2016	39.99	1	-9	0	-1.47	0.12	-0.68
07.09.2016	37.69	0	-1	0	1.18	0.60	1.33
08.09.2016	38.31	1	3	1	1.52	0.26	0.43
09.09.2016	38.65	1	5	1	-0.75	-3.05	-1.17
12.09.2016	38.97	1	11	1	-2.54	-2.35	-1.80
13.09.2016	38.79	0	-1	0	-2.22	-1.22	-1.58
14.09.2016	38.96	1	3	1	-0.50	0.00	-0.03
15.09.2016	39.07	1	3	1	-0.13	1.73	0.30
16.09.2016	39.19	1	11	1	-3.70	-2.43	-2.20

Multiple iterations of European sovereign CDS spread simulations with fixed optimal parameter values have shown that the division of time series into bubble-periods is stable from simulation to simulation. An evident lagged dependence is demonstrated of the positive/negative period changes of the FTSE indices from the bubbles in the CDS market during the agent-based simulations (Table 3.4).

The result of the simulations includes 166 bubbles within 8 years of the time series: 70 positive and 96 negative ones. We will concentrate our further studies on the last 1.5 years of the CDS simulations and the FTSE observations (in the period 01/07/2015 – 30/12/2016), which include 10 bubbles (4 positive and 6 negative). Table 5 demonstrates two tendencies in the correlation between CDS and FTSE which will help us to build a prediction model:

1) a (negative) bubble in the CDS market simulations changes the direction of the FTSE dynamics, switching the regimes of the financial market from positive to negative and vice versa;

2) the lag effect is observed in the information connection between two markets: the dynamics of CDS is advanced toward the dynamics of FTSE for several time steps.

In the next paragraph, the linkage between European sovereign CDS spreads, and Italian FTSE indices will be investigated which may help to detect FTSE market dynamics using CDS time series.

3.4 Detection of FTSE market dynamics using CDS simulations

After the linkage between European sovereign CDS spreads and Italian FTSE indices has been caught empirically, we need to find its explanation in and around a traditional approach or agent-based modeling. Here the studies will be described which help to determine the nature of the correlation between the dynamics of the sovereign CDS spreads and FTSE indices.

We try to find a methodological explanation for the tendencies in the linkage between CDS and FTSE demonstrated in Table 3.4. First, we review the descriptive statistics of European sovereign CDS spreads and its correlation with Italian FTSE price indices. The entire time series of the European sovereign CDS spreads including 8 years of observations have a coefficient of variation, or relative standard deviation, equal to 74%, which means that the time series are not homogeneous. For a standard correlation analysis, we divide the entire time series into 7 homogeneous periods, the statistical results are represented in Table 3.5.

The coefficients of correlation between the time series of European sovereign CDS spreads and Italian FTSE indices indicate mostly moderate and high negative correlation, not stable from period to period. The type of connection for the periods with high correlation represents a nonlinear inverse dependence demonstrated in the scatter plots in Figure 3.6. It is important to single out that the correlation evaluation made for the 7th sub-period of the observations, which is under our scientific interest, disproves the hypothesis about the possibility to predict one time series with another showing the absence of any significant correlation.

As it is formulated in (3.5) the price of financial assets is stochastic by nature. Therefore, the analysis of price returns has a more significant practical value for the sake of predictions. The correlation between the returns of European sovereign CDS spreads and Italian FTSE indices is represented in Table 3.6. The dependence of returns is negative, mostly moderate and more uniform through the periods and different indices than the correlation of spreads and prices.

Table 3.5: Descriptive statistics of European sovereign CDS spreads and its correlation with Italian FTSE price indices in the period of 01/01/2009 – 30/12/2016.

Period	Date range	Max	Min	Med	Var, %	Correlation with CDS		
						FTSE fin	FTSE real estate	FTSE all shares
all	01/01/2009 – 30/12/2016	431.70	33.69	90.46	74	-0.25	0.14	-0.50
1	01/01/2009 – 20/07/2009	246.00	115.65	164.29	23	-0.76	-0.43	-0.73
2	21/07/2009 – 12/01/2010	116.57	76.24	91.75	13	-0.43	-0.71	-0.24
3	13/01/2010 – 07/07/2011	266.38	105.49	155.50	20	-0.75	0.18	-0.18
4	11/07/2011 – 12/01/2012	431.70	269.08	386.14	9	-0.87	-0.74	-0.81
5	13/01/2012 – 04/09/2012	260.94	169.10	202.77	11	-0.62	-0.49	-0.69
6	12/09/2012 – 03/06/2012	138.94	49.24	82.78	22	-0.90	-0.72	-0.88
7	04/06/2014 – 30/12/2016	60.97	33.69	44.33	10	-0.04	-0.24	-0.18

The results of a superficial correlation analysis of the European sovereign CDS spreads and Italian FTSE scarcely give us an explanation of the dependence caught by the agent-based simulations. The standard analysis considers the market dynamics as time series, dependent or independent, and gives the set of results in the form of coefficients, mathematical functions, etc. While the AB-m considers the data as ordered sequence of the dynamic results of the complex system behavior. It can show the mechanism of interactions generating the market dynamics with pure collectively created speculative behavior. Here the time is a critical element of AB-ms, as soon as ABM models time in discrete units, and may reproduce patterns of behavior that unfold over time.

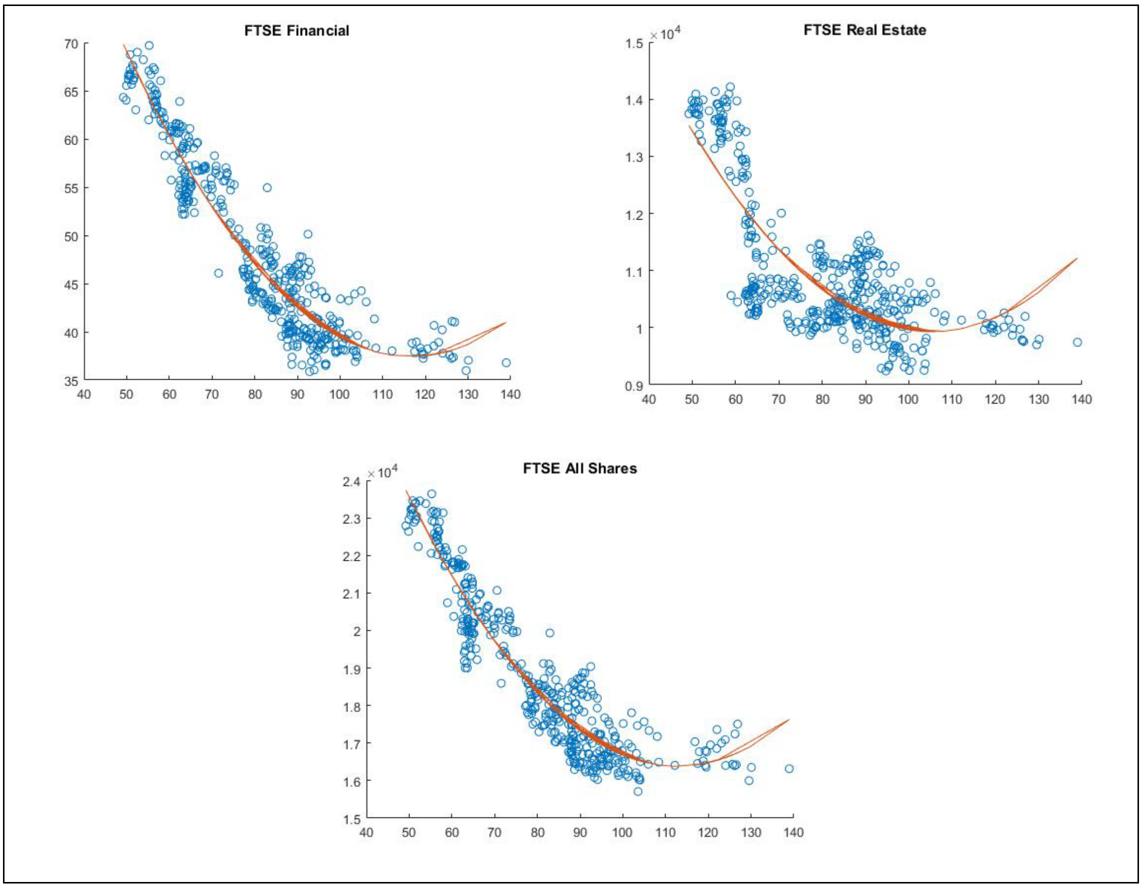

Figure 3.6: Scatter plots demonstrating the type of correlation between the European sovereign CDS spreads and Italian FTSE indices: financial, real estate and all shares during the Period 6; trends are represented with red lines.

A stable pattern emerging from the heterogeneous interactions between the agents may be used for the representation of complex system behavior in the long term. Two-time series which have failed to have a stable significant correlation evaluated by a standard approach demonstrate a stable connection as two complex systems within an ABM. The time of a (negative) bubble occurrence divides the time series into separate windows where the type of the linkage between CDS and FTSE time series is determined by the direction of the bubble and the amplitude of the fluctuations during the previous bubble period.

With an attempt to explain the second tendency of European sovereign CDS spread dynamics to be anticipatory toward Italian FTSE index dynamics, the coefficient of delay between two-time series has been estimated for each of the sub-periods. A possible delay between two-time series has been estimated via the cross-correlation. The hypothesis that changes in European sovereign CDS spreads are advanced with respect to changes in Italian FTSE indices has been rejected, as soon as for the majority of sub-periods the delay between two time series has not been detected. For some sub-periods, the changes in the European sovereign CDS spreads were delayed with respect to changes in FTSE indices. Thus, a traditional analysis of lag-dependence between two-time series through cross-correlation is incapable of explaining the linkage ascertained within agent-based simulations.

Table 3.6: The correlation coefficients between the returns of European sovereign CDS spreads and the returns of Italian FTSE indices in the period of 01/01/2009 – 30/12/2016.

Period	Date range	FTSE-financial	FTSE-real estate	FTSE-all shares
all	01/01/2009 – 30/12/2016	-0.35	-0.25	-0.34
1	01/01/2009 – 20/07/2009	-0.43	-0.08	-0.37
2	21/07/2009 – 12/01/2010	-0.25	-0.25	-0.24
3	13/01/2010 – 07/07/2011	-0.51	-0.44	-0.49
4	11/07/2011 – 12/01/2012	-0.69	-0.49	-0.64
5	13/01/2012 – 04/09/2012	-0.34	-0.32	-0.35
6	12/09/2012 – 03/06/2012	-0.31	-0.28	-0.31
7	04/06/2014 – 30/12/2016	-0.30	-0.26	-0.32

Ability of an emergent system simulated via ABM to generate (negative) bubbles may be declared to be crucial in the prediction mechanism for FTSE dynamics for a certain period in advance. Below we present an example of the CDS simulation application to the analysis of a future FTSE market dynamic tendency by giving an example of the return prediction of the FTSE financial price index.

Initially, an intuitive assumption has been made that a (negative) bubble in the market of European sovereign CDS spreads switches the dynamics of Italian FTSE indices in the opposite side from the bubble, i.e., after a positive bubble has appeared in the simulations (*SIM*), FTSE market will go down, and vice versa. To measure the amplitude of the future FTSE dynamics, the moving average has been used to achieve the invention to predict the sign and the approximate average value of the FTSE fluctuations during the period till the next bubble.

We are at the time t_b^0 and attempt to model the average fluctuations of financial FTSE during the future period $t_b^1 - t_b^0$ using a span p for the moving average and the *lag* as the delay of a FTSE switch after a CDS switch:

$$\Delta\left(R_i^{FTSE}(t_b^1) - R_i^{FTSE}(t_b^0)\right) =$$

$$= -sign(SIM(t_b^0)) \times MA(FTSE(t_b^0 - p - lag):(t_b^0 - lag)). \qquad (3.9)$$

Table 3.7: Results of CDS simulation application for the prediction of the return of Italian FTSE financial price index.

№	Period	Real average	Predicted average	Commentaries for predicted values
1	01/07/2016 – 25/07/2016	0.68	5,09	Correct sign, upper bound
2	26/07/2016 – 02/08/2016	-1.07	-0,20	Correct sign
3	03/08/2016 – 12/08/2016	1.07	0,81	Correct sign
4	15/08/2016 – 22/08/2016	-1.11	-1,82	Correct sign, lower bound
5	23/08/2016 – 06/09/2016	0.93	1,02	Correct sign, upper bound
6	07/09/2016 – 12/10/2016	-0.18	-1,06	Correct sign, lower bound
7	13/10/2016 – 07/11/2016	0.22	-0,97	Wrong sign
8	08/11/2016 – 15/11/2016	0.29	2,03	Correct sign, upper bound
9	16/11/2016 – 22/11/2016	-0.85	-1,90	Correct sign, lower bound
10	23/11/2016 – 30/12/2016	0.68	1,46	Correct sign, upper bound

For a period of 01.07.2016 – 30.12.2016 the optimal values for *n* and *lag* have been evaluated equal to: *n* = 4, *lag* = 3. The model lets us forecast a correct direction of the FTSE financial index for a period of 01.07.2016 – 30.12.2016; the results are demonstrated in Table 3.7. After a system has entered a steady state in the simulations, an almost absolute correspondence has appeared of the sign between real and predicted dynamics of the financial FTSE index (9 out of 10 bubble sub-periods). In 7 out of 9 sub-periods the predicted value acts as an upper bound for a positive dynamics, and a lower bound for a negative dynamics.

This simple example demonstrates a possible variant of the rule of thumb method for the application of the ABM simulation to the price return predictions with its inherent ability to form the bubbles. ABM can catch the hidden interactions and relationships within the dynamics of collective decision making. Because of the complex nature of the emergent patterns, and the nonlinear interactions between these parameters, the outputs of AB-ms can rarely be characterized by mathematical functions and statistical coefficients. Therefore, formal analytic methods often prove insufficient. For two time series where a preliminary traditional statistical analysis did not help to determine the linkage and to derive the model of the CDS and FTSE dependence, an ABM approach helped to catch the hidden linkage. Agent-based simulations have splitted non-homogeneous time series into separate bubble-periods with switching dynamics from positive to negative and vice versa.

3.5 Conclusion

In this chapter, we showed that a game-theoretical framework within the ABM provides the means to build up a minimal model of a financial market which requires only a set of three parameter values (the number N of agents operating in the market, the number s of strategies available for each player during the game, and the number m of the last information days available for the agents to make the decision). It was demonstrated that such a simplified approach is capable of detecting not just the inherent behavior of a complex system in the long term in the way of modeling the joint behavior of its elements. Besides, in addition to the complex simulated financial system of European sovereign CDS, we were capable of slaving this simulation to the Italian FTSE indices and tracing their connection, enlarging the agent-based simulations with the ability to represent a joint behavior of two complex systems in micro-macro levels.

At the beginning of this chapter, we described the European sovereign CDS market stressing the fact that it is a very informative source for the evaluation of financial stability of a country, in particular, or the global system, in the whole. We discussed yearly information concerning European sovereign CDS spreads during recent sequential financial shocks when high volatility has been observed.

The agent-based computational model was introduced which could capture the underlying structure of the CDS price dynamics. Model building blocks were represented with the behavioral component of an agent design and the complex social component of agent evolution during the joint emergent process of price finding.

During the application of the AB-m, we postulated the hypothesis that the fluctuations in the sovereign CDS market lead the dynamics of the equity market. Some previous studies have shown that, conversely, the equity market leads CDS market in the scale of a particular country. Also, previous studies have been dedicated to the modeling of a single market or the dynamics of a global market represented as an aggregation of county markets in the emergent system.

We found that a (negative) bubble in the European CDS market simulations changes the direction of the Italian FTSE dynamics, switching the regimes of the financial market from positive to negative and vice versa. Also, the lag effect of the CDS dynamics has been observed when the dynamics of CDS is anticipatory toward the dynamics of FTSE for several time steps.

In the final part of this chapter, we have noticed that the results of a superficial correlation analysis of the European sovereign CDS spreads and Italian FTSE did not provide an explanation of the dependence caught by the agent-based simulations. While the standard analysis gives the set of results in the form of coefficients, mathematical functions, ABM may reproduce patterns of behavior that unfold over time. Using this capacity of ABM, we demonstrated a simple example of the rule of thumb method for the application of the ABM simulation to the price return predictions with its inherent ability to form the bubbles.

Chapter 4

Conclusions and Future Directions

Agent-based computational models include behavioral and social components which provide an efficient representation of a complex financial system. Based on microscopic behaviors of financial agents with heterogeneous strategies, these models can explain the origination of the temporal and spatial correlations of the financial markets.

In this thesis, we made an attempt for representation of the global interconnectedness of two separate markets at two levels – the global European market of CDS (macro-level) and the local Italian equity market (micro-level). We confirmed the hypothesis of the temporal and spatial connection between markets using the ABM when a standard approach could not recognize it.

At the beginning of the thesis, we discussed main pillars of ABM and previous studies in this field which provided a better understanding of the AB-m applications in finance. We concluded that ABM is an interdisciplinary science. In many fields, the ABM includes a social component for simulating a complex system. The purpose of agent-based modeling is to understand properties of complex social systems through the simulation of different types of communication between heterogeneous agents. Based on this, we derived several particularities of ABM which are useful for the simulations of a financial market. Among them, there is the combination of micro- and macro-levels, agent heterogeneity, and the concept of agent bounded rationality with behavioral traits.

In Chapter 2, we itemized the ABM representation of a financial market. Firstly, we compared rational, behavioral, and social concepts of financial markets in their evolution. The conclusion was made that social finance adds essential complexity aspects to complex system representation. Secondly, we discussed three multi-agent games with fundamentally different views on how to behave in the market: a Majority game, a Minority game, and a $-game. We noticed that the $-game framework combines the advantages of two other games and brings important novelties in the representation of a financial market. Finally, we described several applications of multi-agent games for the market state detection. Though we did not cover all recent studies, we could understand that ABM applications unveil hidden phenomena of complex financial systems with inherent global shocks which are hardly possible to detect using a standard theoretical framework.

Out of three multi-agent games described in Chapter 2 we have chosen the $-game framework and applied it to the simulations of the sovereign CDS spread market. Based on the $-game payoff function, we transformed the calibration algorithm of slaving in the part of the choice of the optimal strategy. The reverse engineering technique with the genetic algorithm has been used to get the optimal parameters of the model. As a result, we obtained the simulations of long-term sovereign CDS dynamics. We were capable of simulating the dynamics of the market for eight years without the preparatory statistical procedures which a standard approach requires. Using the Tremor Price Dynamics, we explained why the fluctuations of a global European sovereign CDS market explain the fluctuations of a local Italian equity market more efficiently than if we would use the time series from both local markets of CDS and equity.

We showed that the market response mismatches between simulated and real-time series, in fact, represent the effect of decoupling and inherent ability of $-game simulations to create bubbles. Analytically we have found the set of model optimal parameters (s = 7, m = 4, N = 25). Simulations were able to detect the time of the changes of the market direction: the time of (negative) bubbles creation, which divided the time series into separate bubble-periods. The most interesting empirical evidence is that the simulations of one market could detect the changes in the direction of another market.

As a result, we confirmed the lead-lag linkage between the European market of sovereign CDS and the Italian equity market which could not be caught using the standard approach. We realized that ABM could catch the hidden interactions and relationships within the dynamics of collective decision making. Because of the complex nature of the emergent patterns, and the nonlinear interactions between these parameters, the outputs of AB-ms can rarely be characterized by mathematical functions and statistical coefficients. Finally, we demonstrated a simple example of the rule of thumb method based on the division of time series into bubble-periods. We applied ABM simulations of European CDS market to predict a price return of Italian equity market.

In conclusion, the following should be mentioned that we have introduced the new inter-level of ABM, the dependence of one complex local system from another complex but global system, in the application to the simplest AB-m involving only three parameters. Therefore, many opportunities are left outside the scope of this thesis for the extensions in the future work. The following directions seem to be more relevant in conformity with the main direction of this thesis:

− the increment of a prediction period. To predict the dynamic direction of the lagged market, we used the time of the (negative) bubble occurrence t_b^0 in the leading market – the time when bubble-period changes and so does the dynamics of the lagged market. We suggest using the time of the (negative) bubble formation $t_b = t_b^0 - m$, the time for which *m* subsequent price movements have the same sign. This is a more complicated task, but it provides more significant prediction power to the model; in [4] the formula to determine t_b has been suggested;

− the modeling of interconnection between two global systems. The concept of ABM allows us to deal with a complex system origination unveiling its hidden inherent particularities. In the application to finance this means that we may extend the model to trace the connection of two global markets and to understand the origination of systemic risk;

− including of the liquidity and dividend components to evaluate the temperature parameter of the market. The $-game algorithm applies to measure the "temperature parameter" of the market as an indicator of a fundamental and a speculative phase transitions [81]. To extend our model, it is necessary to consider the levels of market liquidity and possible scenarios for the expected dividends. As an application of the previous extension we suggest to determine the "temperature parameter" which can detect the phase transition of one global market and to translate it to another connected global market;

− delimiting positive and negative bubbles. Many studies have been dedicated to the fact that negative shocks usually have a stronger effect on the financial dynamics. We suggest to trace this tendency in the application to the bubble-periods within our modeling framework and to segregate the effect of negative and positive market switches.

Table 3.8: Theoretical Approaches to Financial Market Representation

Approach	Time	Assumptions and main idea	Main terms and novelties	Problems	Attitude toward risk
Rational Expectations Theory	1960s – 1970s	The behavior of market participants is rational; price of the market is at any instant time in equilibrium; deviations from fundamental price are random; prices don't change without new information about fundamental price; all the information in the market is available; fundamental price depends only on 2 variables: interest rates and dividends	Fundamental price, Efficient market hypothesis	Doesn't determine price changes; Doesn't represent the behavior of real agents	Doesn't consider high risks, robust only for stable periods
Markowitz's Portfolio Theory and Capital Asset Pricing Model	1960s – 1970s	Risk is compensated with higher return; Standard mean-variance approach; considers investments over one time period; correlations between the different stocks remain constant; Borrowing and lending at a risk-free rate; Provide a tool to find the proper price of a stock; Variance is a measure of risk;	Efficient frontier, Two-fund separation theorem; Diversification	Works only under normal distribution;	Introduces different types of risk (specific and systematic risks) but not in the scale of the whole system; Treating individual risks (small and intermediate) but doesn't prevent large risks; Systemic risk is lying beyond the control
Behavioral finance, Prospect Theory	1980s	Implies the effect of cognitive processes and human biases onto the market; Individuals may need to maximize more than one outcome; Self-interest is combined with communication (cooperation, competition); Individuals hardly can take into account all the relevant information; Rationality is affected by human biases (framing, overconfidence, anchoring, etc.); The relative wealth matters in decision making; People are risk averse; Deals with human decision-making in isolation	Cognitive processes, Cognitive closure, Human biases, Sentiment indices, Disposition effect	Treats only individual behavior without taking into account collective effects; Doesn't take into account that the price formation in financial markets is a collective phenomenon with dynamic nature; Doesn't take into account past price dynamics	Only the risk of individual assets or portfolios of assets is considered, not the risk concerning the financial system as a whole
Complexity Theory	2000s	Socio-finance complex approach	Self-organized system, Temperature parameter of the market, Synchronization, Decoupling	Is not well spread among scientists, doesn't have clear methodology	Systemic risk is studied and modeled as the outcome of the self-organized system dynamics

```
clear
% Determine initial parameters
SSIZE = 10000;
S = input ...
    ('Enter the number of strategies available for each
player during the game: ');
M = input ('Enter the number of days of information
available: ');
G = input ('Enter the number of games: ');
myv = 51:20:1001;
sigma = nan(1,numel(myv));
X     = nan(1,numel(myv));
Y     = nan(1,numel(myv));
binf = zeros(1,M);
iinf = zeros(1);
index= zeros(max(myv));
decision= zeros(max(myv));
A = zeros(200*2.^M+SSIZE+1,1);
E_A = nan(numel(myv),G);
V_A = nan(numel(myv),G);
AVol = zeros(G,1);
binmap = (2.^[M-1 : -1 : 0]);
for kk = 1:numel(myv)
    N = myv(kk);
    P = zeros(S,N);
    eqv = zeros(S,N);
    for g = 1:G
        A(:) = 0;
        P(:) = 0;
        decision(:) = 0;
        binf(:) = 0;
        iinf(:) = 0;
        index(:) = 0;
% the array of random strategies chosen for all players
        B = randi(2, 2^M, S, N) - 1;
        B(B==0)=-1;
% determine initial vector of information randomly
        binf(:) = randi(2, 1, M) - 1;
        for t = 2:(200*2.^M+SSIZE+1) % production case
% do mapping from binary data to integer
            iinf(1) = sum(binf.*binmap);
            eqv(:) = 0;
% game started: choose the strategy for each agent, with
max rating or
% randomly
            for i = 1:N
```

```
                mmax = max(P(:,i));
                eqv(P(:,i)==mmax,i) = 1;
            end
% choose the decision (inf, i, j): the cross of the best
strategy of each agent and the information set
             for i = 1:N
                 pos = find(eqv(:,i)==1);
                 index(i) = pos(randi(numel(pos),1));
                 decision(i) = B(iinf(1)+1,index(i),i);
             end
             A(t) = sum(decision(:));
% update the inf vector (queue LIFO)
             if A(t) > 0
                 new_inf = 1;
             else
                 new_inf = 0;
             end;
             binf(:) = [binf(2:M) new_inf];
% update the rating  of winning strategies in P vector
(updating of the payoff % function)
            P = P - reshape(B(iinf(1)+1,:,:),S,N)*A(t);
        end; % for t
        V_A(kk,g) = var(A(end-SSIZE:end));
     end; % for g
     Gen_sigma = mean(V_A(kk,:));
     Y(kk) = Gen_sigma ./ N;
     X(kk) = 2.^M/N;
     save(['myWS' num2str(kk)], 'V_A', 'Gen_sigma', 'Y',
'X');
 end; % for kk
 plot(X,Y,'xr')
```

$-Game (part of code)

```
B = randi(2, 2^M, S, N) - 1;
B(B==0)=-1;
binf(:) = randi(2, 1, M) - 1;
iinf(1) = sum(binf.*binmap);
eqv(:) = 0;
for i = 1:N
    index(i) = randi(S);
    decision(i) = B(iinf(1)+1,index(i),i);
end;
A(1) = sum(decision(:));
if A(1) > 0
    new_inf = 1;
else
        new_inf = 0;
end;
binf(:) = [binf(2:M) new_inf];
for t = 2:(T)
    old_iinf(1) = iinf(1);
    iinf(1) = sum(binf.*binmap);
    eqv(:) = 0;
    for i = 1:N
        mmax = max(P(:,i));
        eqv(P(:,i)==mmax,i) = 1;
    end;
    for i = 1:N
        pos = find(eqv(:,i)==1);
        index(i) = pos(randi(numel(pos),1));
        decision(i) = B(iinf(1)+1,index(i),i);
    end
    A(t) = sum(decision(:));
    if A(t) > 0
        new_inf = 1;
    else
        new_inf = 0;
    end;
    binf(:) = [binf(2:M) new_inf];
    P = P + reshape(B(old_iinf(1)+1,:,:),S,N)*A(t);
```

Appendix 4

Table 3.9: Yearly Information of 5-year European Sovereign CDS Spreads

Country	5 year sovereign CDS, euro									Annualized volatility, %								
	2009		2010	2011	2012	2013	2014	2015	2016	2009	2010	2011	2012	2013	2014	2015	2016	**2009 - 2016**
	initial	final	final															
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Europe	196.94	113.14	203.03	393.95	108.03	64.72	46.9	37.48	42.88	40.30	68.34	50.30	93.40	73.51	66.88	64.11	69.41	**67.29**
France	53.00	24.97	87.23	143.19	48.95	31.53	34.09	17.84	24.91	84.51	91.93	89.07	91.62	34.56	72.80	48.63	82.27	**76.95**
Italy	164.00	96.92	186.62	399.63	231.61	142.53	117.32	77.08	122.82	72.66	109.71	92.50	60.19	42.10	59.02	58.88	44.57	**70.72**
Germany	45.18	26.99	44.78	62.44	19.75	14.09	9.14	6.66	13.39	69.59	104.46	94.50	89.42	67.48	53.06	86.89	180.90	**99.93**
UK	107.00	76.13	68.18	93.01	35.18	28.26	17.71	18.20	30.43	66.70	65.05	70.91	54.19	56.85	83.27	132.00	104.52	**82.80**
Greece	236.00	262.91	977.27	10070.53	14904.36	14904.36	14904.36	14904.36	14904.36	73.02	96.89	97.31	160.31	0.00	0.00	0.00	0.00	**94.45**
Portugal	92.00	80.07	419.41	1078.51	392.46	315.77	165.20	133.65	220.87	81.96	124.21	66.01	50.53	43.81	53.16	65.62	50.59	**71.24**
Ireland	175.00	141.55	544.36	667.39	170.33	95.50	37.78	29.30	47.77	66.52	93.39	52.12	67.22	26.02	36.92	23.38	39.72	**55.42**
Poland	245.00	134.00	128.48	247.31	71.83	70.41	63.28	66.84	71.62	73.92	80.53	53.56	43.45	41.52	26.69	20.35	44.93	**51.85**
Denmark	118.50	23.98	35.90	120.31	22.18	13.56	14.12	11.99	14.73	73.34	82.17	80.50	56.88	55.26	87.12	140.91	121.76	**91.66**
Sweden	125.00	49.24	27.27	65.13	10.79	11.25	10.10	8.65	14.32	71.79	67.16	107.65	117.66	169.63	129.71	166.21	127.42	**124.61**
Austria	124.00	65.71	78.99	120.91	24.70	22.70	13.36	15.52	17.79	71.01	86.04	143.35	71.32	74.53	72.00	64.37	70.02	**79.18**
Estonia	545.00	190.00	84.63	144.61	68.31	57.42	63.81	58.02	59.14	70.73	61.61	48.55	30.39	25.11	13.98	11.65	47.40	**43.67**
Latvia	817.30	552.00	240.24	332.29	107.14	103.17	101.73	72.29	54.51	64.86	43.24	37.98	37.40	19.73	24.37	25.01	45.79	**39.68**
Hungary	435.30	244.00	352.17	551.65	246.51	233.85	162.54	149.89	114.85	58.21	68.47	42.08	41.55	45.50	24.97	23.22	33.71	**44.56**
Cyprus	144.00	116.85	236.15	1218.44	1027.95	816.31	366.68	241.08	227.05	64.99	48.49	89.75	41.85	47.33	64.80	29.99	29.80	**55.43**
Slovenia	110.00	75.00	66.00	357.58	217.82	208.47	124.91	103.35	93.93	73.80	77.42	61.56	44.49	41.27	32.23	13.17	18.15	**50.56**
Romania	658.50	290.00	296.00	401.93	194.07	170.90	120.51	118.06	100.29	58.80	58.07	45.03	36.91	23.81	17.13	17.98	19.79	**38.34**
Slovakia	150.00	79.00	73.04	278.02	90.47	76.50	51.86	45.46	41.05	84.84	75.72	87.05	44.50	21.00	28.15	14.16	21.59	**55.15**
Lithuania	600.00	310.00	232.96	329.61	93.74	106.79	101.37	72.54	53.81	60.56	55.09	37.37	33.43	21.03	26.11	20.95	31.13	**38.30**
Bulgaria	515.15	233.00	226.59	366.10	85.94	108.87	167.21	150.54	132.78	57.99	69.22	47.22	45.29	39.58	22.77	22.44	20.70	**43.96**
Iceland	1025.00	418.94	310.28	301.34	178.45	167.19	126.91	114.45	90.73	37.60	36.04	36.43	20.82	14.69	29.96	50.61	31.54	**33.77**
Czech R.	170.00	94.00	82.81	155.96	57.42	54.83	49.82	45.52	38.49	86.08	78.69	59.52	32.73	25.89	27.14	11.29	20.63	**50.12**
Croatia	445.00	234.00	239.33	494.19	250.92	349.55	265.05	299.51	202.06	65.68	63.06	38.88	36.29	22.84	22.46	17.65	18.34	**39.97**

Appendix 5

Transformed $-Game Slaving Algorithm (part of code)

```
clear
data = xlsread('CDS.xlsx');
CDS = zeros(1,numel(data) - 1);
s = 7; %number of strategies available for each player
during the game
m = 4; %number of days of information available
N = 25;
for t = 1:numel(data) - 1
    if data(t+1) > data(t),
        CDS(t)= 1;
    else
        CDS(t)= 0;
    end;
end;
iinf = zeros(1);
old_iinf = zeros(1);
Err = zeros(1,4);
Err_SIM = zeros(1,4);
index = zeros(1,N);
decision = zeros(1,N);
binmap = (2.^[m-1 : -1 : 0]);
A = zeros(numel(CDS),1);
SIM = zeros(numel(CDS),1);
binf = zeros(1,m);
P = zeros(s,N);
eqv = zeros(s,N);
A(:) = 0;
P(:) = 0;
decision(:) = 0;
binf(:) = 0;
iinf(:) = 0;
index(:) = 0;
B = randi(2, 2^m, s, N) - 1;
B(B==0)=-1;
binf = CDS(1:m);
iinf(1) = sum(binf.*binmap);
eqv(:) = 0;
for i = 1:N
    index(i) = randi(s);
    decision(i) = B(iinf(1)+1,index(i),i);
end;
A(m+1) = sum(decision(:));
if A(m+1) > 0
    SIM(m+1) = 1;
else
    SIM(m+1) = 0;
```

```
end;
while SIM(m+1) ~= CDS(m+1)
    for i = 1:N
        index(i) = randi(s);
        decision(i) = B(iinf(1)+1,index(i),i);
    end;
    A(m+1) = sum(decision(:));
    if A(m+1) > 0
        SIM(m+1) = 1;
    else
        SIM(m+1) = 0;
    end;
end; % for while
binf(:) = CDS(2:m+1);
for t = (m+2):T
    old_iinf(1) = iinf(1);
    iinf(1) = sum(binf.*binmap);
    eqv(:) = 0;
    for i = 1:N
        mmax = max(P(:,i));
        eqv(P(:,i)==mmax,i) = 1;
    end;
    for i = 1:N
        pos = find(eqv(:,i)==1);
        index(i) = pos(randi(numel(pos),1));
        decision(i) = B(iinf(1)+1,index(i),i);
    end;
    A(t) = sum(decision(:));
    if A(t) > 0
        SIM(t) = 1;
    else
        SIM(t) = 0;
    end;
    loopCounter = 1;
    allagents = 1:N;
    candidates = allagents;
    for i = candidates
        mmax = max(P(:,i));
        eqv(P(:,i)==mmax,i) = 1;
    end;
    while SIM(t) ~= CDS(t) && loopCounter < 10000
        for i = candidates
            pos = find(eqv(:,i)==1);
            if numel(pos) > 1
                index(i) = pos(randi(numel(pos),1));
            else
                [sortedValues,sortIndex] =
sort(P(:,i),'descend');
                maxIndex = sortIndex(1:2);
```

```
                index(i) =
maxIndex(randi(numel(maxIndex),1));
            end;
            decision(i) = B(iinf(1)+1,index(i),i);
        end;
        A(t) = sum(decision(:));
        if A(t) > 0
            SIM(t) = 1;
        else
            SIM(t) = 0;
        end;
        loopCounter = loopCounter + 1;
        if 0 == CDS(t)
            candidates = allagents(decision == 1);
        else
            candidates = allagents(decision == -1);
        end
    end; % for while
    binf(:) = CDS((t-m+1):t);
    P = P + reshape(B(old_iinf(1)+1,:,:),s,N)*A(t);
end; % for T
```

Appendix 6

Table 3.10: Slaved Simulations of Sovereign CDS Market to FTSE Indices in Period 01.01.2015 – 30.12.2016

Date	CDS (EU)	CDS (EU)	A (EU) m=6	A (EU) m=4	FTSE fin., %	FTSE r/es., %	FTSE all/sh, %	A (It) m=4	CDS (It)	CDS (It)
01.01.2015	47.51	1	3	5	0.00	0.00	0.00	-7	0	117.32
02.01.2015	44.65	0	-7	-5	1.31	1.14	0.69	-5	0	107.01
05.01.2015	49.05	1	7	7	-5.14	-2.71	-4.47	7	1	112.09
06.01.2015	50.92	1	3	3	-1.21	-0.11	-0.35	1	1	117.99
07.01.2015	48.09	0	-3	-5	-0.67	1.05	-0.16	3	1	123.21
08.01.2015	47.34	0	-1	-3	4.61	1.08	3.39	-7	0	116.52
09.01.2015	48.07	1	1	5	-4.21	-1.06	-2.97	3	1	120.63
12.01.2015	51.36	1	3	1	1.21	0.81	0.79	-5	0	117.27
13.01.2015	51.36	0	-1	-7	1.98	0.03	1.88	-1	0	115.96
14.01.2015	50.93	0	-1	-1	-0.81	-1.68	-1.41	-3	0	114.17
15.01.2015	46.25	0	-5	-7	1.81	1.37	2.15	-21	0	109.20
16.01.2015	45.61	0	-3	-13	1.96	0.84	2.00	-23	0	104.69
19.01.2015	44.94	0	-9	-21	2.16	1.67	1.25	-23	0	99.63
20.01.2015	44.90	0	-17	-21	1.52	1.29	0.91	-23	0	98.30
21.01.2015	45.11	1	-15	-9	1.50	1.19	1.55	-23	0	95.85
22.01.2015	44.18	0	-3	-5	2.40	2.20	2.35	-23	0	89.08
23.01.2015	43.02	0	-7	-1	-0.92	5.13	0.35	-23	0	83.63
26.01.2015	42.67	0	-3	-1	0.55	1.41	1.23	-23	0	82.08
27.01.2015	42.74	1	5	1	-0.26	-0.31	-0.47	-23	1	83.98
28.01.2015	43.80	1	5	7	-1.75	-0.61	-0.73	3	1	87.59
29.01.2015	44.08	1	3	1	0.87	-0.49	0.49	1	1	89.00
30.01.2015	43.64	0	-3	-1	-1.18	0.26	-0.31	-7	0	86.90
02.02.2015	44.81	1	3	9	-0.18	2.37	-0.02	7	1	93.11
03.02.2015	44.40	0	-1	-1	3.15	3.13	2.43	-1	0	91.30
04.02.2015	43.99	0	-3	-7	-0.48	1.58	-0.24	-1	0	88.49
05.02.2015	44.05	1	3	5	-1.09	1.98	-0.43	3	1	88.91
06.02.2015	44.44	1	3	1	0.06	2.67	-0.25	3	1	91.71
09.02.2015	46.00	1	1	1	-2.44	-0.08	-1.75	5	1	98.98
10.02.2015	45.96	0	-1	-1	2.40	-0.29	1.72	5	1	98.99
11.02.2015	46.40	1	1	5	-1.20	-2.05	-0.77	5	1	101.36
12.02.2015	46.71	1	1	3	2.90	2.42	2.08	5	1	104.51
13.02.2015	46.80	1	3	11	2.06	0.81	0.84	5	1	105.50
16.02.2015	48.04	1	5	9	-0.26	0.51	-0.13	7	1	110.79
17.02.2015	47.61	0	-1	15	0.76	-1.20	0.45	-5	0	107.99
18.02.2015	47.54	0	-3	-1	1.90	0.86	1.73	-7	0	105.26
19.02.2015	46.60	0	-5	-7	0.81	1.25	0.61	-5	0	100.14

20.02.2015	46.76	1	1	1	0.15	-0.25	0.24	-21	0	99.17
23.02.2015	45.43	0	-3	-3	0.56	1.58	0.52	-23	0	93.20
24.02.2015	44.96	0	-1	-3	0.73	1.23	0.80	-23	0	90.32
25.02.2015	44.96	0	-3	-5	-1.46	-1.22	-0.87	-23	1	91.79
26.02.2015	44.22	0	-5	-13	1.56	2.51	1.13	-1	0	89.39
27.02.2015	44.47	1	1	-9	0.67	0.44	0.74	5	1	89.42
02.03.2015	45.26	1	1	7	-0.72	0.18	-0.10	-3	0	87.64
03.03.2015	45.44	1	3	1	-1.40	-0.72	-1.35	1	1	90.03
04.03.2015	45.78	1	1	9	0.69	-1.18	0.65	5	1	90.32
05.03.2015	44.26	0	-7	15	1.52	1.41	1.27	-1	0	84.58
06.03.2015	43.56	0	-3	-9	0.86	1.44	0.30	-3	0	81.85
09.03.2015	42.53	0	-3	-7	0.56	0.50	0.56	3	1	82.58
10.03.2015	42.45	0	-7	-13	-0.89	0.09	-0.89	3	1	83.76
11.03.2015	43.68	1	1	-9	1.47	1.84	2.05	-7	0	82.77
12.03.2015	44.00	1	1	7	-0.17	-0.03	-0.05	5	1	85.15
13.03.2015	44.20	1	3	1	0.00	-0.16	-0.30	1	1	86.06
16.03.2015	43.73	0	-7	-1	1.76	3.28	0.93	-3	0	85.09
17.03.2015	43.01	0	-11	-1	-1.30	-1.60	-0.96	5	1	88.38
18.03.2015	44.92	1	7	3	-1.50	-1.04	-0.69	1	1	96.75
19.03.2015	44.56	0	-3	-1	1.08	0.79	1.02	-3	0	95.77
20.03.2015	46.24	1	1	3	1.67	-0.56	1.51	-1	0	94.52
23.03.2015	45.30	0	-1	-1	-0.36	-1.63	-0.37	-7	0	91.72
24.03.2015	46.31	1	3	9	2.13	3.10	1.18	-1	1	95.76
25.03.2015	46.65	1	3	1	-0.50	0.44	-0.68	-1	0	95.76
26.03.2015	45.32	0	-1	-1	-1.27	-0.24	-0.98	5	1	95.78
27.03.2015	45.09	0	-5	-3	0.07	-1.55	0.25	-3	0	93.89
30.03.2015	44.27	0	-9	-7	1.71	-0.33	1.23	-3	0	91.22
31.03.2015	43.95	0	-7	-13	-0.67	-0.63	-0.41	-3	0	88.49
01.04.2015	43.62	0	-9	-21	0.84	1.34	0.83	-1	1	88.55
02.04.2015	44.96	1	-9	-9	0.00	-0.30	-0.13	-1	0	88.32
03.04.2015	44.96	1	5	7	0.00	0.00	0.00	-1	0	88.32
06.04.2015	44.96	0	-5	-5	0.00	0.00	0.00	-1	0	88.32
07.04.2015	44.39	0	-1	-1	1.24	1.14	1.60	-21	0	85.58
08.04.2015	42.90	0	-5	-7	-1.14	1.08	-0.44	-23	1	86.09
09.04.2015	44.45	1	3	1	0.52	0.89	0.94	1	1	86.91
10.04.2015	43.01	0	-1	-3	0.01	-0.27	0.40	-7	0	86.91
13.04.2015	42.79	0	-1	-3	0.07	-0.24	0.59	-1	0	84.30
14.04.2015	43.46	1	1	5	-1.83	-0.65	-1.09	3	1	88.77
15.04.2015	43.63	1	1	1	1.20	0.14	1.10	3	1	89.69
16.04.2015	45.46	1	3	1	-2.22	-1.77	-1.74	5	1	98.03
17.04.2015	47.74	1	1	9	-3.24	-2.61	-2.42	3	1	111.15
20.04.2015	47.67	0	-5	15	2.13	0.69	1.19	-1	0	109.24
21.04.2015	48.11	1	1	11	-1.25	-0.81	-0.38	1	1	114.32

22.04.2015	47.58	0	-7	-1	1.13	0.14	0.26	-1	0	110.90
23.04.2015	47.32	0	-3	-5	-0.68	-3.49	-0.54	-1	0	109.57
24.04.2015	47.49	1	3	5	1.17	2.21	0.95	1	1	111.79
27.04.2015	46.88	0	-3	-5	2.25	2.12	1.59	-1	0	107.80
28.04.2015	46.58	0	-5	-1	-1.58	0.50	-1.04	-1	0	106.04
29.04.2015	46.58	0	-5	-7	-2.43	-2.11	-2.22	1	1	106.23
30.04.2015	44.53	0	-5	-13	1.30	0.57	0.23	-1	0	96.72
01.05.2015	44.33	0	-19	-21	0.00	0.00	0.00	-7	0	95.84
04.05.2015	44.33	1	-9	-9	0.62	-2.58	0.66	-5	0	95.84
05.05.2015	45.90	1	5	7	-3.23	-2.98	-2.63	-1	1	102.91
06.05.2015	45.11	0	-3	-1	1.52	2.60	0.28	-3	0	101.15
07.05.2015	44.74	0	-9	-5	2.06	-3.88	0.80	-3	0	97.61
08.05.2015	44.03	0	-3	-7	1.50	3.16	2.01	-3	0	94.96
11.05.2015	43.85	0	-1	-13	0.44	1.62	0.12	-1	1	95.40
12.05.2015	43.94	1	1	-9	-1.46	-1.39	-0.88	-3	0	95.40
13.05.2015	43.47	0	-1	-5	1.09	-2.00	0.41	-3	0	91.35
14.05.2015	43.03	0	-5	-1	1.42	0.61	1.32	-3	0	89.59
15.05.2015	42.65	0	-1	-5	0.08	-0.57	-0.26	-21	0	88.70
18.05.2015	43.11	1	3	1	-1.37	-0.06	-1.07	-23	1	93.02
19.05.2015	42.34	0	-1	-5	2.74	1.74	2.09	-3	0	91.05
20.05.2015	42.68	1	1	3	0.17	1.11	0.27	-5	0	90.83
21.05.2015	42.00	0	-1	-1	-0.59	0.45	-0.12	-3	0	88.21
22.05.2015	42.22	1	3	9	0.26	-1.52	0.11	-21	0	88.21
25.05.2015	42.21	0	-3	-1	-1.83	-1.67	-1.84	-23	0	88.21
26.05.2015	43.55	1	1	9	-0.85	-0.72	-0.09	-23	1	93.46
27.05.2015	42.39	0	-5	-1	3.21	2.10	2.17	-3	0	87.78
28.05.2015	42.78	1	3	9	-0.85	-1.41	-0.40	3	1	90.40
29.05.2015	42.59	0	-5	-1	-1.37	-0.98	-0.98	-1	0	89.10
01.06.2015	47.64	1	5	9	-0.51	-1.19	-0.20	1	1	96.11
02.06.2015	43.70	0	-7	-1	1.11	-0.63	0.54	5	1	96.54
03.06.2015	44.36	1	7	9	0.46	0.66	0.18	-1	0	89.96
04.06.2015	43.82	0	-3	-1	-0.70	-0.31	-1.05	-3	0	86.89
05.06.2015	43.67	0	-3	-5	-2.63	-1.19	-2.08	1	1	87.76
08.06.2015	42.88	0	-13	-5	-1.41	-2.02	-0.90	3	1	90.39
09.06.2015	44.48	1	1	1	-0.18	-0.89	-0.52	-7	0	89.96
10.06.2015	43.92	0	-1	-5	2.63	0.05	2.34	-5	0	86.46
11.06.2015	44.03	1	1	3	0.43	1.44	0.35	-5	0	86.45
12.06.2015	45.50	1	7	1	-1.52	-2.29	-1.27	-1	1	93.89
15.06.2015	47.61	1	3	11	-3.25	-2.09	-2.45	5	1	102.65
16.06.2015	46.53	0	-1	-1	0.49	-2.00	0.28	-7	0	102.21
17.06.2015	48.72	1	5	3	-0.78	2.19	-0.65	5	1	105.26
18.06.2015	49.36	1	3	3	1.03	0.65	0.89	3	1	107.45
19.06.2015	49.41	1	3	11	1.23	-0.87	1.04	-1	0	107.44

22.06.2015	45.19	0	-3	-3	4.85	5.71	3.41	-1	0	95.22
23.06.2015	45.66	1	1	1	-0.01	1.05	0.44	-5	0	89.96
24.06.2015	46.14	1	3	3	-0.81	-0.83	-0.52	-1	1	92.15
25.06.2015	43.79	0	-5	-5	1.46	-0.87	0.76	-3	0	89.53
26.06.2015	45.87	1	1	7	0.63	1.72	0.65	-3	0	89.52
29.06.2015	38.34	0	-3	-1	-6.36	-2.10	-4.99	-5	0	89.52
30.06.2015	51.07	1	7	9	-0.20	0.08	-0.37	1	1	113.11
01.07.2015	50.30	0	-5	-1	3.06	2.12	2.08	-3	0	109.09
02.07.2015	49.98	0	-3	-5	-2.16	-0.75	-1.39	-7	0	107.75
03.07.2015	50.22	1	-1	5	-0.62	0.10	-0.49	1	1	109.09
06.07.2015	52.66	1	1	1	-5.56	-2.91	-3.85	3	1	120.11
07.07.2015	54.80	1	1	1	-3.29	-0.88	-2.86	5	1	129.05
08.07.2015	53.63	0	-1	-1	3.25	0.46	2.37	-7	0	122.06
09.07.2015	52.39	0	-9	-1	4.33	2.00	3.43	-7	0	115.07
10.07.2015	48.70	0	-3	-7	3.87	1.67	2.88	-5	0	100.59
13.07.2015	47.02	0	-7	-13	1.41	2.12	1.05	-21	0	92.67
14.07.2015	45.55	0	-9	-21	-0.34	-0.43	-0.39	-23	1	93.54
15.07.2015	43.52	0	-17	-21	1.08	0.52	1.24	-3	0	92.23
16.07.2015	42.43	0	-25	-21	2.08	2.14	1.72	-1	0	86.96
17.07.2015	41.72	0	-25	-21	0.32	0.18	0.00	-5	0	84.75
20.07.2015	41.50	0	-25	-21	1.19	2.51	1.15	1	1	84.76
21.07.2015	42.75	1	-17	-9	-1.70	-0.16	-1.20	5	1	91.35
22.07.2015	43.12	1	5	7	0.15	0.03	-0.04	5	1	92.67
23.07.2015	42.82	0	-7	-7	-0.03	3.24	-0.11	-7	0	91.79
24.07.2015	42.96	1	3	7	-0.51	-1.84	-0.48	1	1	92.67
27.07.2015	43.78	1	1	3	-3.29	-0.86	-2.87	3	1	95.75
28.07.2015	43.35	0	-1	-3	2.70	-0.26	2.07	-5	0	94.43
29.07.2015	43.57	1	3	7	-0.90	-0.69	0.06	-5	0	94.43
30.07.2015	43.56	0	-1	-5	0.57	0.31	0.57	-5	0	94.43
31.07.2015	47.12	1	3	9	0.87	-0.86	0.61	-21	0	93.98
03.08.2015	43.62	0	-5	-1	0.40	0.94	0.82	-23	1	94.20
04.08.2015	43.80	1	5	9	-1.71	-0.57	-0.98	7	1	95.31
05.08.2015	43.25	0	-5	-1	2.43	1.66	1.72	-7	0	91.35
06.08.2015	43.29	1	1	9	0.04	-0.12	-0.45	-1	0	91.35
07.08.2015	43.36	1	1	3	-0.93	-0.48	-0.55	1	1	91.79
10.08.2015	43.15	0	-1	-1	1.47	0.26	1.03	-3	0	91.35
11.08.2015	43.21	1	3	7	-0.72	0.01	-1.03	3	1	92.22
12.08.2015	43.56	1	1	3	-2.93	-1.72	-2.88	5	1	93.54
13.08.2015	43.06	0	-3	-1	1.67	-0.23	1.55	-3	0	92.67
14.08.2015	42.79	0	-1	-1	-0.47	1.66	-0.38	-3	0	92.23
17.08.2015	42.84	1	1	5	0.95	0.76	0.65	-5	0	91.35
18.08.2015	33.87	0	-1	-3	0.50	1.98	-0.03	1	1	92.66
19.08.2015	33.89	1	3	3	-1.30	-0.39	-1.66	-3	0	92.23

20.08.2015	34.66	1	7	5	-2.92	-1.80	-2.56	3	1	94.43
21.08.2015	35.10	1	1	11	-3.53	-2.84	-2.78	5	1	96.62
24.08.2015	36.42	1	3	9	-5.91	-5.31	-5.70	13	1	102.77
25.08.2015	34.97	0	-3	13	6.62	3.28	5.60	-7	0	95.75
26.08.2015	34.99	1	1	3	-0.36	-1.78	-0.76	-7	0	95.75
27.08.2015	34.27	0	-9	-3	2.64	1.70	3.21	-5	0	92.23
28.08.2015	34.28	1	5	9	-1.50	-1.94	-0.90	1	1	92.67
31.08.2015	34.29	1	3	3	-0.24	1.87	-0.20	-3	0	92.67
01.09.2015	34.25	0	-7	-3	-2.73	-2.72	-2.22	3	1	93.98
02.09.2015	34.22	0	-3	-7	1.35	0.51	0.61	-3	0	93.55
03.09.2015	33.83	0	-9	-7	2.51	2.80	2.48	-7	0	91.90
04.09.2015	33.92	1	3	1	-4.10	-2.03	-2.97	1	1	92.33
07.09.2015	33.93	1	1	7	0.45	-0.09	0.63	-1	0	92.33
08.09.2015	33.69	0	-7	-1	1.80	0.57	1.49	-1	0	91.04
09.09.2015	42.34	1	3	7	0.80	1.09	0.75	-5	0	89.29
10.09.2015	42.85	1	1	3	-0.73	-0.10	-0.89	-21	0	89.29
11.09.2015	43.11	1	1	11	-0.99	-2.32	-0.60	-23	1	90.58
14.09.2015	43.12	1	5	9	-0.98	-0.99	-0.94	-3	0	90.58
15.09.2015	43.06	0	-1	13	1.76	1.17	1.45	-3	0	89.72
16.09.2015	42.89	0	-3	-1	-0.31	-0.24	0.64	-1	0	89.29
17.09.2015	42.92	1	1	3	0.51	-0.44	0.20	-21	0	87.10
18.09.2015	42.42	0	-1	-1	-2.87	-0.67	-2.39	-23	1	87.96
21.09.2015	43.76	1	3	3	1.49	1.00	1.03	1	1	91.39
22.09.2015	44.56	1	1	1	-3.06	-1.43	-3.16	5	1	94.06
23.09.2015	44.69	1	1	11	-1.00	0.84	0.21	3	1	95.83
24.09.2015	45.15	1	3	9	-1.29	0.20	-2.17	5	1	99.29
25.09.2015	45.11	0	-1	13	3.93	2.25	3.45	5	1	99.30
28.09.2015	44.90	0	-5	-1	-3.11	-2.63	-2.63	-1	0	97.15
29.09.2015	45.28	1	7	3	-0.09	-0.34	-0.30	9	1	99.29
30.09.2015	44.67	0	-1	-1	2.91	1.37	2.52	3	1	99.90
01.10.2015	45.28	1	9	3	-0.57	1.21	-0.62	-5	0	99.29
02.10.2015	45.28	0	-1	-1	1.09	1.22	1.09	-1	0	99.29
05.10.2015	45.12	0	-5	-5	2.75	2.20	2.63	3	1	99.30
06.10.2015	43.74	0	-13	-7	0.67	0.51	0.91	-5	0	94.94
07.10.2015	43.72	0	-3	-13	-1.37	0.08	-0.76	-1	0	94.07
08.10.2015	43.78	1	3	-9	0.48	-1.30	0.64	1	1	94.50
09.10.2015	43.51	0	-1	-5	0.34	-0.21	0.32	-1	0	93.64
12.10.2015	43.51	1	9	3	-0.62	-0.43	-0.70	3	1	94.07
13.10.2015	43.39	0	-3	-1	-0.35	-0.29	-0.15	-3	0	93.20
14.10.2015	43.68	1	1	9	-1.14	-0.15	-0.80	1	1	94.51
15.10.2015	43.47	0	-1	-1	1.83	1.85	1.64	-3	0	93.64
16.10.2015	47.28	1	3	9	0.98	0.26	0.47	-3	0	92.33
19.10.2015	42.82	0	-3	-1	0.53	0.64	0.45	-3	0	91.46

20.10.2015	42.40	0	-3	-1	-0.01	-0.80	-0.60	-21	0	90.16
21.10.2015	42.42	1	-1	5	-0.17	-2.48	-0.42	-23	0	90.16
22.10.2015	41.95	0	-3	-1	1.75	1.78	1.83	-23	0	87.56
23.10.2015	44.86	1	1	3	-0.21	1.99	0.59	-23	0	83.62
26.10.2015	40.89	0	-3	-1	-0.55	0.21	-0.36	-23	0	83.20
27.10.2015	40.72	0	-5	-3	-0.40	-0.86	-1.10	-23	0	83.20
28.10.2015	40.38	0	-13	-5	1.36	2.03	1.31	-23	0	81.45
29.10.2015	40.23	0	-3	-13	-1.78	0.65	-0.89	-23	0	81.01
30.10.2015	40.35	1	1	-9	-0.32	0.76	0.05	-23	1	81.45
02.11.2015	40.41	1	1	7	0.13	-0.80	0.22	5	1	82.58
03.11.2015	40.36	0	-7	-7	-0.65	0.36	-0.12	-7	0	81.46
04.11.2015	39.83	0	-1	-1	-0.90	-0.34	-0.55	-3	0	80.61
05.11.2015	39.55	0	-5	-7	-0.20	-2.04	-0.38	-5	0	78.85
06.11.2015	39.63	1	3	1	2.39	0.64	1.33	-1	1	79.28
09.11.2015	39.80	1	1	7	-1.92	-0.41	-1.69	-3	0	78.83
10.11.2015	39.63	0	-7	-7	1.28	0.17	1.38	3	1	79.28
11.11.2015	39.63	0	-5	-5	0.17	0.48	-0.18	-3	0	77.56
12.11.2015	39.75	1	1	7	-2.86	-1.59	-2.19	1	1	78.41
13.11.2015	39.92	1	5	1	-0.24	-0.61	-0.09	-5	0	78.41
16.11.2015	39.88	0	-1	-5	-0.34	-0.57	-0.20	1	1	78.84
17.11.2015	39.15	0	-3	-5	1.89	1.10	2.15	-5	0	77.54
18.11.2015	38.62	0	-5	-7	-1.07	-0.49	-0.86	-1	0	76.23
19.11.2015	38.88	1	3	1	0.47	1.35	0.46	1	1	76.24
20.11.2015	38.76	0	-3	-7	-0.94	-0.66	-0.20	3	1	76.67
23.11.2015	38.38	0	-7	-1	0.96	-0.48	0.65	-7	0	76.67
24.11.2015	38.35	0	-3	-1	-1.45	-0.44	-1.54	5	1	77.54
25.11.2015	38.23	0	-5	-13	1.88	1.04	1.76	-3	0	77.54
26.11.2015	38.14	0	-19	-21	0.61	-0.10	0.94	-5	0	77.54
27.11.2015	38.05	0	-17	-21	-0.07	0.55	0.00	-1	0	76.68
30.11.2015	36.89	0	-25	-21	0.76	1.28	0.64	-21	0	75.80
01.12.2015	36.97	1	-13	-9	-0.43	-0.51	-0.46	-23	1	75.81
02.12.2015	37.47	1	5	7	-0.59	0.59	-0.08	-3	0	74.06
03.12.2015	37.34	0	-3	-7	-2.66	-3.25	-2.32	-3	0	74.01
04.12.2015	37.37	1	5	7	0.54	0.22	0.13	1	1	75.78
07.12.2015	37.20	0	-3	-1	0.15	-0.50	0.11	-5	0	74.47
08.12.2015	36.78	0	-5	-1	-2.44	-0.85	-2.10	-1	0	71.85
09.12.2015	36.39	0	-7	-3	-0.34	0.33	-0.23	1	1	72.73
10.12.2015	36.43	1	3	1	-0.75	-2.26	-0.46	3	1	73.16
11.12.2015	37.25	1	5	7	-1.56	-1.72	-1.75	1	1	75.79
14.12.2015	37.72	1	3	1	-2.37	-1.14	-2.30	3	1	78.82
15.12.2015	37.40	0	-7	-1	4.19	2.55	3.51	-5	0	78.39
16.12.2015	36.97	0	-11	-1	-0.28	-0.68	-0.20	-7	0	77.10
17.12.2015	36.67	0	-3	-7	1.44	4.61	1.44	-5	0	76.66

18.12.2015	37.18	1	3	1	-1.11	-2.02	-1.19	1	1	77.94
21.12.2015	37.83	1	13	7	-0.05	-0.81	-0.66	3	1	80.12
22.12.2015	41.09	1	7	1	-0.02	0.77	-0.22	1	1	80.56
23.12.2015	37.89	0	-1	-1	0.71	1.28	1.74	-7	0	80.12
24.12.2015	37.89	0	-11	-1	0.00	0.00	0.00	-7	0	80.12
25.12.2015	41.00	1	7	5	0.00	0.00	0.00	-5	0	80.12
28.12.2015	38.52	0	-1	-3	-0.47	-1.18	-0.27	-21	0	80.12
29.12.2015	37.90	0	-3	-3	1.49	1.59	1.32	-23	1	80.13
30.12.2015	37.90	1	1	5	-1.20	-1.33	-0.93	-3	0	80.13
31.12.2015	37.48	0	-5	-5	0.00	0.00	0.00	-9	0	77.08
01.01.2016	37.43	0	-7	-1	0.00	0.00	0.00	-1	0	77.08
04.01.2016	38.02	1	1	5	-3.65	0.04	-3.01	1	1	77.51
05.01.2016	37.96	0	-5	-5	0.97	1.41	1.04	-3	0	77.07
06.01.2016	40.92	1	1	3	-3.09	-0.24	-2.46	3	1	78.37
07.01.2016	41.81	1	1	7	-1.64	-3.75	-1.22	5	1	80.56
08.01.2016	39.19	0	-5	-1	-1.01	2.18	-1.49	-5	0	80.14
11.01.2016	39.06	0	-5	-1	-1.35	-2.30	-0.67	5	1	80.57
12.01.2016	39.27	1	1	5	1.07	0.20	1.11	3	1	82.33
13.01.2016	41.99	1	3	1	1.51	0.12	0.83	-3	0	82.33
14.01.2016	42.34	1	3	1	-2.30	-2.35	-1.70	5	1	82.76
15.01.2016	39.79	0	-1	-3	-3.09	-2.17	-2.90	3	1	84.05
18.01.2016	40.44	1	1	5	-4.66	-3.84	-2.68	13	1	84.90
19.01.2016	40.59	1	3	3	-0.73	-2.21	1.01	1	1	86.68
20.01.2016	42.50	1	1	11	-6.10	-4.83	-4.79	5	1	94.90
21.01.2016	42.01	0	-3	-1	5.53	1.89	3.94	5	1	94.93
22.01.2016	44.94	1	3	5	0.60	1.98	1.74	-1	0	91.45
25.01.2016	41.28	0	-1	-1	-3.49	0.75	-1.78	3	1	91.90
26.01.2016	40.33	0	-3	-1	1.34	-0.62	1.34	-1	0	88.41
27.01.2016	39.66	0	-9	-3	-1.81	-1.98	-0.33	-5	0	85.80
28.01.2016	40.07	1	3	1	-5.24	-1.56	-3.22	1	1	88.85
29.01.2016	40.22	1	7	7	2.93	2.80	2.46	3	1	91.02
01.02.2016	40.42	1	7	1	-1.36	0.65	-0.74	7	1	92.33
02.02.2016	41.60	1	1	9	-3.60	-2.39	-2.91	3	1	97.98
03.02.2016	45.51	1	5	23	-4.94	0.29	-2.82	5	1	98.41
04.02.2016	42.67	0	-1	13	2.51	-1.89	0.88	5	1	101.46
05.02.2016	42.99	1	3	5	-2.51	-2.04	-2.11	5	1	103.65
08.02.2016	48.33	1	1	3	-5.35	-7.09	-4.72	5	1	114.92
09.02.2016	47.42	0	-5	-1	-6.07	-3.12	-2.96	5	1	118.80
10.02.2016	50.77	1	1	7	10.69	4.74	4.72	-3	0	116.24
11.02.2016	54.50	1	5	3	-7.15	-1.13	-5.17	9	1	126.58
12.02.2016	51.53	0	-5	-1	6.29	2.10	4.20	-3	0	126.22
15.02.2016	49.75	0	-1	-3	3.69	3.00	3.19	-7	0	120.12
16.02.2016	49.29	0	-7	-7	0.04	0.47	-0.41	-1	0	120.12

17.02.2016	49.25	0	-3	-13	2.39	2.55	2.52	-1	1	120.58
18.02.2016	48.81	0	-7	-21	-3.96	-0.07	-1.31	-3	0	118.84
19.02.2016	49.05	1	-9	-9	-2.03	-1.65	-1.15	3	1	119.70
22.02.2016	49.02	0	-5	-5	4.48	2.16	3.36	-7	0	118.86
23.02.2016	48.76	0	-7	-1	-2.03	-2.04	-1.84	-7	0	117.11
24.02.2016	58.32	1	1	5	-2.38	-1.62	-2.39	1	1	119.71
25.02.2016	57.72	0	-5	-9	1.82	4.17	2.19	-5	0	117.16
26.02.2016	52.79	0	-3	-5	2.19	-1.29	2.01	-1	0	116.32
29.02.2016	56.50	1	1	5	0.63	-0.46	0.80	-3	0	114.18
01.03.2016	51.01	0	-5	-1	1.94	1.23	2.13	-21	0	110.35
02.03.2016	46.53	0	-1	-3	3.48	3.81	0.97	-23	0	106.45
03.03.2016	46.05	0	-3	-1	1.88	0.87	0.68	-23	0	106.45
04.03.2016	48.94	1	7	1	-1.78	-0.22	-0.31	-23	0	105.20
07.03.2016	49.01	1	1	7	-2.41	-0.99	-1.17	-23	0	104.78
08.03.2016	49.09	1	3	1	0.79	-1.21	-0.30	-23	0	104.76
09.03.2016	49.03	0	-1	-1	0.68	-0.24	0.89	-23	0	104.37
10.03.2016	51.43	1	1	1	0.91	1.31	-0.47	-23	0	96.10
11.03.2016	41.59	0	-1	-1	7.83	1.98	4.41	-23	0	87.67
14.03.2016	41.61	1	3	9	-0.87	2.40	0.15	-23	0	87.21
15.03.2016	42.32	1	3	5	-1.37	-1.45	-1.04	-23	1	91.03
16.03.2016	42.07	0	-3	-1	-0.83	0.77	-0.13	5	1	92.28
17.03.2016	41.26	0	-5	-1	-2.87	1.77	-0.64	-7	0	90.62
18.03.2016	40.88	0	-7	-7	-0.86	0.74	0.13	-5	0	90.62
21.03.2016	43.14	1	3	1	0.89	-0.55	0.40	3	1	97.44
22.03.2016	43.84	1	1	7	-0.64	0.19	0.00	3	1	99.38
23.03.2016	43.94	1	3	1	-1.96	-1.98	-1.15	-7	0	99.38
24.03.2016	43.84	0	-1	-3	-2.33	-0.89	-1.49	5	1	101.14
25.03.2016	43.84	1	5	5	0.00	0.00	0.00	-7	0	101.14
28.03.2016	43.84	0	-1	-3	0.00	0.00	0.00	-1	0	101.14
29.03.2016	43.75	0	-3	-5	-0.11	0.54	0.00	1	1	102.41
30.03.2016	43.17	0	-9	-5	-0.78	0.17	1.11	-3	0	100.26
31.03.2016	43.19	1	1	3	-1.02	-0.44	-1.26	3	1	100.70
01.04.2016	43.12	0	-3	-7	-2.00	-0.50	-1.63	-5	0	99.90
04.04.2016	43.19	1	3	3	-1.98	-0.99	-0.69	1	1	100.70
05.04.2016	44.83	1	3	7	-3.83	-3.03	-2.86	5	1	107.43
06.04.2016	44.69	0	-5	-1	0.84	0.41	0.64	13	1	107.45
07.04.2016	45.84	1	5	7	-3.76	-0.02	-2.15	3	1	112.45
08.04.2016	44.84	0	-3	-3	6.00	0.61	3.66	-1	0	107.01
11.04.2016	44.76	0	-5	-1	2.17	1.67	1.24	5	1	107.04
12.04.2016	44.46	0	-9	-1	-3.04	-1.99	-1.39	-5	0	106.24
13.04.2016	43.53	0	-3	-13	7.29	1.40	3.73	-9	0	102.39
14.04.2016	43.71	1	-1	-9	1.51	1.73	0.82	1	1	103.20
15.04.2016	43.84	1	3	7	0.17	-0.73	-0.35	-1	0	103.19

18.04.2016	43.40	0	-1	-1	-0.02	2.71	0.55	-3	0	101.95
19.04.2016	43.65	1	5	7	-0.84	-1.51	0.39	-3	0	100.23
20.04.2016	44.33	1	1	3	2.24	0.49	1.01	-1	1	101.04
21.04.2016	43.21	0	-5	-3	1.53	0.07	0.33	-3	0	99.02
22.04.2016	43.53	1	1	7	-0.10	-0.39	-0.24	-1	0	99.02
25.04.2016	43.35	0	-3	-1	-2.73	-0.78	-1.33	-3	0	99.02
26.04.2016	47.24	1	1	9	2.89	-0.42	1.26	-21	0	99.02
27.04.2016	43.64	0	-5	-1	-0.53	0.25	0.45	-23	0	99.01
28.04.2016	47.11	1	5	9	1.43	0.25	1.09	-23	0	97.73
29.04.2016	44.31	0	-7	-1	-3.13	0.25	-1.78	-23	1	98.17
02.05.2016	47.28	1	3	9	-1.98	-1.59	-0.86	-3	0	98.17
03.05.2016	47.90	1	3	5	-3.17	-0.92	-2.33	3	1	100.69
04.05.2016	44.51	0	-3	-1	-0.68	1.13	-0.11	5	1	101.51
05.05.2016	44.72	1	5	7	-1.48	-0.34	-0.06	13	1	102.76
06.05.2016	44.72	0	-3	-3	0.11	0.76	-0.44	-7	0	102.35
09.05.2016	44.40	0	-5	-3	-0.85	-0.81	-0.76	-7	0	101.54
10.05.2016	44.11	0	-3	-3	1.22	0.58	1.22	-5	0	99.42
11.05.2016	44.00	0	-3	-13	-2.01	-3.77	-1.16	-21	0	99.30
12.05.2016	44.26	1	-1	-9	-0.80	0.10	-0.23	-23	1	100.15
13.05.2016	44.26	0	-1	-3	-0.11	0.10	0.46	-3	0	100.15
16.05.2016	44.43	1	7	3	-0.44	0.65	0.09	3	1	101.00
17.05.2016	44.19	0	-1	-1	-1.29	-0.11	-1.20	-3	0	101.00
18.05.2016	43.94	0	-7	-3	2.08	1.32	1.09	-7	0	101.00
19.05.2016	44.12	1	-3	5	0.32	-0.65	-0.85	1	1	101.84
20.05.2016	44.01	0	-1	-3	2.15	1.04	1.38	-7	0	101.41
23.05.2016	43.96	0	-3	-3	-4.45	0.24	-2.38	-5	0	101.21
24.05.2016	43.92	0	-9	-7	5.45	1.38	2.99	-5	0	101.21
25.05.2016	43.58	0	-7	-13	2.46	1.47	1.55	-21	0	98.87
26.05.2016	43.43	0	-19	-21	-0.74	1.02	0.10	-23	0	98.05
27.05.2016	43.42	0	-19	-21	-0.15	-0.05	-0.08	-23	1	98.06
30.05.2016	43.43	1	-15	-9	0.50	0.36	0.59	-3	0	98.06
31.05.2016	43.54	1	5	7	-2.57	0.28	-1.30	3	1	99.31
01.06.2016	43.59	1	5	1	-1.54	-0.42	-1.05	5	1	99.72
02.06.2016	43.66	1	1	9	-0.84	-0.26	-0.22	13	1	100.16
03.06.2016	43.76	1	5	23	-2.90	0.12	-1.42	3	1	100.54
06.06.2016	44.15	1	5	23	-0.31	-0.76	0.54	5	1	103.55
07.06.2016	44.16	1	5	23	2.27	0.26	1.89	-3	0	103.55
08.06.2016	44.18	1	3	23	-1.20	-0.31	-0.37	-7	0	103.54
09.06.2016	44.32	1	5	23	-0.89	-0.22	-0.71	3	1	104.82
10.06.2016	44.90	1	1	23	-4.81	-1.08	-3.38	3	1	107.36
13.06.2016	46.18	1	3	23	-5.08	-3.64	-2.95	1	1	116.27
14.06.2016	48.08	1	5	23	-3.08	-3.84	-2.04	3	1	118.75
15.06.2016	46.98	0	-1	15	1.91	0.65	1.24	-1	0	113.36

16.06.2016	47.04	1	7	5	-1.16	-3.66	-1.10	-7	0	110.74
17.06.2016	47.34	1	1	3	5.53	2.48	3.42	-5	0	110.02
20.06.2016	45.51	0	-5	-5	2.89	3.32	2.43	-21	0	107.77
21.06.2016	45.15	0	-3	-5	1.50	-0.33	0.41	-23	0	107.76
22.06.2016	44.98	0	-7	-7	0.22	0.72	-0.52	-23	0	106.13
23.06.2016	45.38	1	3	3	5.25	0.18	3.39	-23	0	105.65
24.06.2016	60.97	1	5	7	-19.73	-7.95	-11.75	-23	1	130.57
27.06.2016	60.75	0	-7	-1	-8.91	-4.46	-4.07	7	1	139.80
28.06.2016	58.80	0	-1	-3	3.02	3.42	3.26	-7	0	131.80
29.06.2016	50.85	0	-5	-7	1.04	-0.14	2.04	-3	0	125.88
30.06.2016	48.39	0	-1	-13	1.99	1.34	1.51	-5	0	117.24
01.07.2016	53.06	1	1	-9	-0.71	-0.19	0.67	-21	0	111.93
04.07.2016	47.01	0	-3	-5	-3.37	-1.70	-1.67	-23	1	115.32
05.07.2016	50.80	1	3	3	-2.01	-5.98	-1.53	5	1	115.57
06.07.2016	47.70	0	-1	-1	-1.92	-3.88	-2.24	-7	0	115.56
07.07.2016	47.74	1	1	9	0.29	4.19	0.17	5	1	115.97
08.07.2016	47.15	0	-5	-1	8.68	3.66	3.78	-1	0	111.79
11.07.2016	47.19	1	5	9	1.00	-0.58	1.25	-5	0	110.99
12.07.2016	46.71	0	-7	-1	6.86	1.25	2.64	-5	0	108.02
13.07.2016	50.11	1	9	9	-1.80	1.41	-1.01	-21	0	103.55
14.07.2016	44.85	0	-5	-1	3.20	3.20	1.59	-23	0	99.00
15.07.2016	44.95	1	1	9	-0.35	-1.70	-0.37	-23	1	99.84
18.07.2016	44.83	0	-9	-1	0.73	2.23	0.17	5	1	104.82
19.07.2016	44.55	0	-5	-3	-0.75	-0.55	-0.49	-7	0	103.54
20.07.2016	44.53	0	-13	-3	1.19	2.06	0.62	5	1	103.96
21.07.2016	43.56	0	-3	-13	0.45	1.24	0.20	-5	0	102.31
22.07.2016	42.97	0	-19	-21	-0.45	-1.08	-0.13	-3	0	101.50
25.07.2016	46.33	1	-9	-9	-0.83	-0.46	-0.42	-7	0	101.50
26.07.2016	46.55	1	5	7	-0.35	0.10	0.00	1	1	102.71
27.07.2016	46.18	0	-1	-3	0.73	2.75	1.12	-3	0	102.30
28.07.2016	45.86	0	-1	-3	-2.79	-0.31	-1.85	-1	0	102.30
29.07.2016	42.30	0	-5	-7	4.42	0.15	1.76	-1	0	101.89
01.08.2016	42.25	0	-3	-13	-3.49	0.49	-1.59	-21	0	101.48
02.08.2016	42.27	1	1	-9	-4.91	-3.69	-2.64	-23	1	102.35
03.08.2016	42.67	1	3	7	-0.29	-0.55	0.13	-3	0	101.49
04.08.2016	41.77	0	-11	-5	0.69	0.46	0.62	-1	0	99.82
05.08.2016	41.34	0	-3	-3	4.19	2.35	2.30	-1	0	99.40
08.08.2016	40.82	0	-5	-7	1.60	-0.51	0.64	-21	0	98.99
09.08.2016	40.35	0	-1	-13	0.81	0.03	0.34	-23	0	98.99
10.08.2016	40.28	0	-7	-21	0.38	0.24	-0.06	-23	1	100.65
11.08.2016	40.03	0	-19	-21	0.94	0.12	1.01	-3	0	100.23
12.08.2016	40.03	1	-13	-9	0.27	-0.50	0.15	-3	0	100.22

15.08.2016	39.40	0	-3	-5	0.00	0.00	0.00	-3	0	99.23
16.08.2016	39.50	1	5	3	-1.88	-1.40	-1.18	1	1	99.84
17.08.2016	39.59	1	1	5	-2.48	-1.51	-1.50	3	1	100.67
18.08.2016	40.34	1	1	11	0.55	1.14	0.82	-7	0	100.66
19.08.2016	41.69	1	3	9	-3.77	-2.57	-2.00	5	1	101.10
22.08.2016	39.79	0	-1	13	0.92	-2.04	0.32	3	1	102.71
23.08.2016	40.20	1	1	3	4.88	0.53	2.23	13	1	103.97
24.08.2016	40.46	1	1	3	2.41	1.22	0.69	1	1	104.80
25.08.2016	43.70	1	3	11	-1.90	-0.69	-1.00	5	1	104.81
26.08.2016	40.28	0	-3	-1	0.59	0.48	0.74	-3	0	104.80
29.08.2016	40.28	0	-7	-1	-1.15	-0.91	-1.04	-7	0	104.80
30.08.2016	40.30	1	7	5	2.25	2.14	1.29	3	1	106.36
31.08.2016	40.03	0	-1	-3	2.62	-0.54	0.31	-1	0	105.95
01.09.2016	39.36	0	-7	-5	0.54	-0.10	-0.07	-7	0	105.94
02.09.2016	38.89	0	-1	-3	1.72	0.37	1.42	-3	0	104.26
05.09.2016	38.62	0	-7	-13	-0.30	0.13	0.07	1	1	104.68
06.09.2016	39.99	1	3	-9	-1.47	0.12	-0.68	-3	0	103.83
07.09.2016	37.69	0	-3	-1	1.18	0.60	1.33	-9	0	103.41
08.09.2016	38.31	1	9	3	1.52	0.26	0.43	1	1	103.43
09.09.2016	38.65	1	3	5	-0.75	-3.05	-1.17	3	1	104.27
12.09.2016	38.97	1	5	11	-2.54	-2.35	-1.80	-7	0	104.26
13.09.2016	38.79	0	-3	-1	-2.22	-1.22	-1.58	5	1	105.09
14.09.2016	38.96	1	3	3	-0.50	0.00	-0.03	-11	0	104.46
15.09.2016	39.07	1	1	3	-0.13	1.73	0.30	1	1	104.47
16.09.2016	39.19	1	1	11	-3.70	-2.43	-2.20	5	1	105.54
19.09.2016	38.78	0	-3	-1	2.29	1.06	1.17	-1	0	104.71
20.09.2016	41.42	1	1	9	-1.57	0.13	-1.09	5	1	108.09
21.09.2016	41.15	0	-3	-1	1.84	-0.88	0.78	-3	0	106.34
22.09.2016	40.32	0	-1	-7	0.73	1.66	1.69	1	1	107.18
23.09.2016	39.52	0	-5	-3	-1.47	-1.01	-1.04	-9	0	107.18
26.09.2016	39.69	1	1	1	-2.14	-0.70	-1.47	1	1	108.46
27.09.2016	41.51	1	7	7	-1.05	0.37	-0.41	5	1	115.60
28.09.2016	41.87	1	3	1	0.45	0.32	0.56	13	1	116.01
29.09.2016	41.72	0	-1	-5	-0.31	0.43	0.61	3	1	118.13
30.09.2016	41.88	1	5	3	0.87	0.08	0.31	5	1	119.80
03.10.2016	42.10	1	5	3	-1.41	2.69	-0.60	5	1	121.07
04.10.2016	42.05	0	-5	-1	0.00	-0.52	0.21	-3	0	121.07
05.10.2016	42.13	1	1	7	3.35	-1.44	0.92	7	1	121.91
06.10.2016	41.51	0	-1	-1	0.98	-1.45	0.03	-1	0	118.57
07.10.2016	41.18	0	-7	-1	-0.46	-1.02	-0.56	-7	0	118.13
10.10.2016	40.23	0	-9	-1	0.81	-0.45	1.25	-3	0	113.95
11.10.2016	40.10	0	-3	-13	-1.40	-1.18	-0.87	-21	0	113.09
12.10.2016	40.64	1	3	-9	1.01	-0.51	-0.01	-23	0	113.09

13.10.2016	40.81	1	3	7	-2.51	0.63	-1.08	-23	1	113.92
14.10.2016	39.96	0	-1	-5	2.19	1.43	1.83	-3	0	111.82
17.10.2016	40.23	1	5	7	1.38	-0.74	0.27	3	1	112.23
18.10.2016	40.05	0	-5	-3	2.11	1.29	1.83	-1	0	110.13
19.10.2016	39.50	0	-7	-3	0.78	1.27	0.51	-5	0	110.13
20.10.2016	39.23	0	-5	-5	0.91	0.98	0.57	-9	0	109.71
21.10.2016	39.44	1	1	1	0.59	1.56	0.10	-21	0	109.29
24.10.2016	39.69	1	3	7	2.25	0.97	0.73	-23	0	108.03
25.10.2016	40.70	1	1	1	-0.85	-0.17	-0.41	-23	1	110.56
26.10.2016	39.87	0	-1	-3	0.66	0.13	0.29	-3	0	110.55
27.10.2016	39.78	0	-11	-1	1.25	-0.41	0.76	-3	0	109.31
28.10.2016	40.56	1	5	9	-1.31	-0.95	-0.54	1	1	114.36
31.10.2016	40.61	1	5	1	-2.11	-0.73	-1.01	3	1	116.88
01.11.2016	41.21	1	3	1	-1.26	-0.88	-1.26	3	1	118.98
02.11.2016	41.35	1	1	9	-3.45	-2.04	-2.41	7	1	119.39
03.11.2016	42.42	1	3	23	0.16	1.62	-0.30	5	1	119.83
04.11.2016	42.51	1	7	23	-1.05	-2.20	-0.64	5	1	121.07
07.11.2016	41.06	0	-13	13	4.20	1.19	2.35	-3	0	117.74
08.11.2016	40.61	0	-1	-1	0.65	0.39	0.42	-7	0	116.06
09.11.2016	41.30	1	5	3	0.52	-1.20	-0.13	3	1	117.31
10.11.2016	41.91	1	5	1	2.21	-1.61	-0.09	3	1	120.25
11.11.2016	47.60	1	3	1	0.38	-0.59	0.11	7	1	125.29
14.11.2016	48.54	1	1	9	-0.13	-0.30	-0.70	9	1	127.46
15.11.2016	44.42	0	-3	15	-1.91	-1.35	-0.08	-1	0	124.46
16.11.2016	44.63	1	1	9	-2.07	-0.39	-0.62	3	1	127.84
17.11.2016	45.32	1	7	3	-1.72	1.51	-0.04	3	1	131.63
18.11.2016	45.47	1	1	11	-2.07	-0.40	-1.59	13	1	131.67
21.11.2016	45.65	1	1	9	-0.84	-2.55	0.05	-7	0	131.63
22.11.2016	45.61	0	-3	15	2.46	1.49	1.28	-7	0	131.63
23.11.2016	48.84	1	1	5	-0.91	-0.44	0.08	3	1	138.44
24.11.2016	46.76	0	-7	-1	-0.59	-0.96	-0.16	3	1	139.67
25.11.2016	46.64	0	-3	-3	0.05	-0.67	0.08	-7	0	138.41
28.11.2016	47.38	1	1	5	-2.95	-0.76	-1.71	5	1	140.50
29.11.2016	46.68	0	-1	-9	3.55	2.33	1.89	-5	0	136.73
30.11.2016	46.75	1	1	3	2.64	0.60	2.05	1	1	136.76
01.12.2016	46.02	0	-3	-1	2.26	0.11	0.86	-5	0	133.83
02.12.2016	45.71	0	-7	-1	-0.51	-0.86	-0.16	-7	0	131.23
05.12.2016	45.72	1	-3	5	-1.29	-0.30	-0.13	1	1	131.28
06.12.2016	45.25	0	-1	-5	7.43	3.93	3.98	-3	0	128.32
07.12.2016	48.45	1	5	3	4.38	2.25	2.06	-1	0	127.06
08.12.2016	44.49	0	-3	-1	3.34	0.73	1.59	-9	0	125.38
09.12.2016	45.28	1	3	9	-2.03	1.74	-0.66	1	1	130.45
12.12.2016	44.35	0	-1	-1	-0.65	-0.63	0.35	-3	0	124.55

13.12.2016	43.10	0	-7	-3	4.32	1.36	2.27	-3	0	117.36
14.12.2016	43.01	0	-15	-3	-2.36	-2.07	-1.04	-1	0	116.52
15.12.2016	43.16	1	5	3	3.49	0.66	1.94	1	1	116.54
16.12.2016	43.23	1	1	7	0.31	1.39	0.11	3	1	119.02
19.12.2016	43.19	0	-5	-9	-1.71	-0.29	-0.17	-7	0	118.59
20.12.2016	43.44	1	5	7	1.79	-0.83	1.34	5	1	121.15
21.12.2016	43.02	0	-5	-1	0.15	1.21	-0.07	3	1	121.56
22.12.2016	43.16	1	1	9	-0.24	0.42	-0.39	13	1	122.82
23.12.2016	43.04	0	-1	-1	0.81	0.08	1.00	-7	0	121.99
26.12.2016	43.04	0	-7	-5	0.00	0.00	0.00	-7	0	121.99
27.12.2016	43.04	0	-15	-9	-0.20	0.57	0.31	-5	0	121.99
28.12.2016	42.95	0	-3	-13	-1.18	0.04	-0.69	-21	0	121.14
29.12.2016	42.84	0	-17	-21	-0.95	-0.55	-0.17	-23	1	122.38
30.12.2016	42.88	1	-11	-9	0.18	1.89	0.23	1	1	122.82

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