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THERMALIZATION OF A COUPLED OSCILLATOR CHAIN

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Abstract. *Coupled pendula show complex and unpredictable collective motions and provide a suitable physical model for complex dynamical systems. Starting from the well-known Fermi-Pasta-Ulam experiment coupled oscillators are expected to undergo spontaneous thermalization, typical of multi-body systems with non-linear interactions, and have been studied in order to investigate energy equipartition and second principle of thermodynamics. By means of an automated videotracking apparatus we have monitored both single and collective motions occurring in a chain of 24 non-linearly coupled pendula on varying the initial conditions (anharmonicity level, number and energy of excited pendula, etc.). Compared to the original FPU model our chain is highly and quickly dissipative and thermalizes very early. Moreover, we have observed other noticeable phenomena as, e.g., l oscillation asymmetry and intrinsic localized modes.*

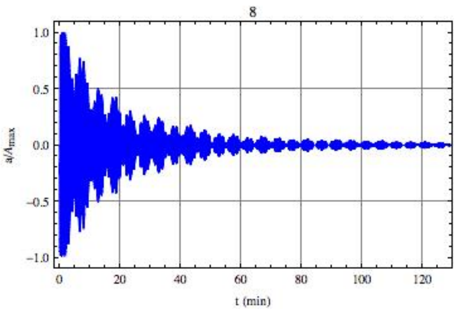
1 Introduction

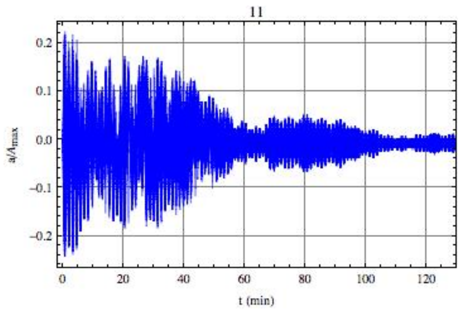
The well-known "Fermi-Pasta-Ulam (FPU) experiment" [1, 2, 3, 4, 5, 6, 7, 8] investigated the physical reasons of the open problem of the second law of thermodynamics, in particular the energy equipartition, by studying the complex dynamics of non-linearly coupled oscillators. We have built up and studied a FPU-like linear chain composed by 24 coupled pendula by monitoring in space and time the single and collective motions of the oscillators. Actually we have investigated the energy equipartition expected by the second principle of thermodynamics, the coexistence of oscillation modes at both small and large wavelengths, the asymptotic synchronisation, peculiar coherence and interference phenomena between oscillators, evidence of chaotic and non-linear behaviour typical of complex dynamical systems. Each pendulum consists of a red coloured iron sphere (mass 500 g and diameter 5 cm) connected to a 1.145 m length nylon rope. All oscillators are connected through that rope which is hung on a 50 mm steel cable: that cable in its turn is clamped to a metallic support structure. 24 electromagnets hold the corresponding pendula to a desired initial excitation angle. When the electromagnets are switched off, the pendula are released and we start monitoring in space and time all motions which depend on the initial conditions. For that purpose we use a *GoPro Hero4 Black* webcam in fullHD with a frame rate of 120 fps. The videotracking data have then been elaborated by means of *Wolfram Mathematica*.

2 First experimental results

2.1 Thermalization

We studied each oscillator motion for the whole run plotting the space coordinates as a function of time. Pulses on a 5 minutes scale can be recognized in the initial excited pendulum (8^{th}) plot, but larger scale pulses (20 to 30 minutes) can be noticed in not excited pendulum plots (e.g. 11^{th}).





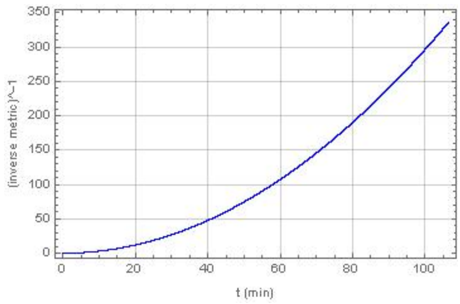
We calculated the so-called "energy inverse metric" Ω for the only potential energy U, which measures the chain termalization degree as a function of time:

$$\langle U_j(t)\rangle = \frac{1}{t}\sum_{i=0}^{i=N} U_j(t_i)\,[t_{i+1}-t_i]$$

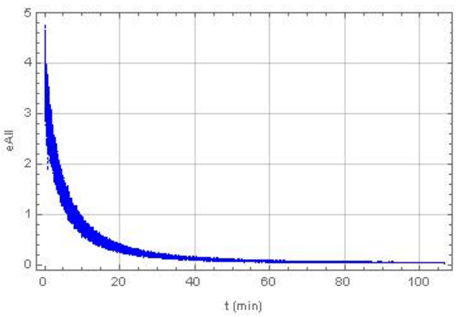
$$\overline{U(t)} = \frac{1}{24}\sum_{j=1}^{j=24}\langle U_j(t)\rangle$$

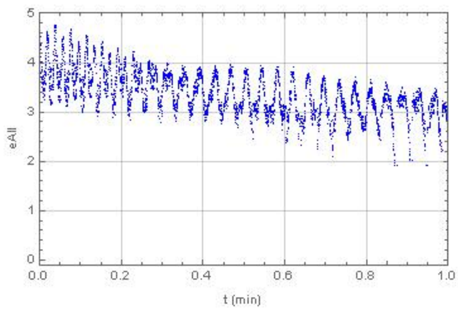
$$\Omega(t) = \frac{1}{24}\sum_{j=1}^{j=24}\left[\overline{U(t)} - \langle U_j(t)\rangle\right]^2$$

For ergodic systems this quantity is expected to decrease at least as t^{-1}: here we plot the inverse of it as a function of time and we see that it grows faster that linearly, indicating higher thermalization rate.



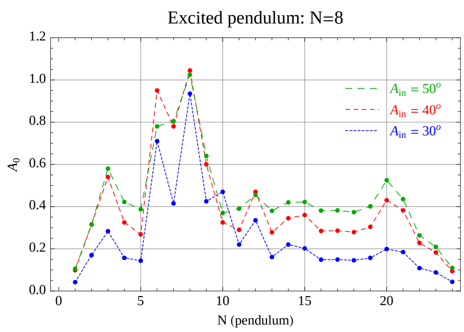
Dissipation plays an important rôle in our environment, because the initial total energy is quickly lost by the excited pendula. Most of it is dissipated at the fixed end points, because the rope bends at the clamping point, as is shown in short time and long time plots below quoted. The first plot shows that, when high initial angles are considered, 90% of the initial energy is lost in the first 20 minutes. The second plot shows the beginning of the run when the system experiences a periodical energy exchange between pendula and support cable.

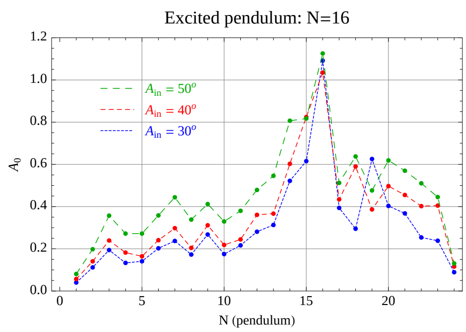




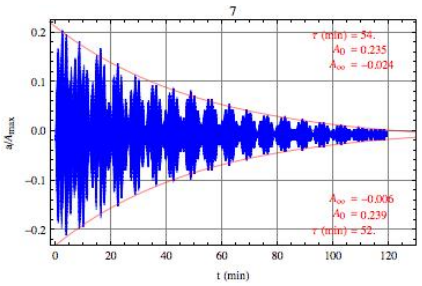
2.2 Energy fluxes and coherence effects

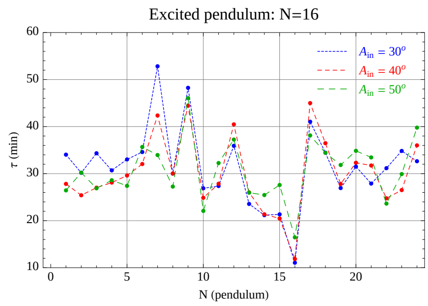
We also calculated the maximum amplitude for each pendulum in three different runs with the same excited pendulum (8^{th} or 16^{th}), but with different initial angles (30°, 40° and 50°), as plotted in the following figure using different colours. Notice that the excited pendulum does transfer most its energy to the near oscillators, but often not to the next.





Let us also point out that pendula with higher energy get damped more quickly than the others and that the pendula at the end of the chain get less energy than the others (this phenomenon could be related to the nearby constraints) as is shown in the plots below where the relaxation time τ (i.e. the time constant of the exponential decrease of the oscillation amplitude) of each pendulum in the chain is compared in the different runs. For most of the pendula τ is around 30 minutes, but it also assumes much smaller or larger values for specific pendula. Moreover we notice that a small variation on the initial angle produces as a result that a local minimum becomes a local maximum or viceversa.





From the analysis of the laboratory data we have also deduced the Fourier spectrum for each oscillator motion. The dominant period T_1 is approximately of 2.2 s, corresponding to $2\pi\sqrt{L/g}$, as expected for a simple pendulum. Smaller Fourier components are obtained for other time scales, corresponding to superior harmonics or to combinatory frequencies occurring in anharmonic (with cubic or quartic potential) oscillations, or related to beats, observed in any run, characterized by large time periods.

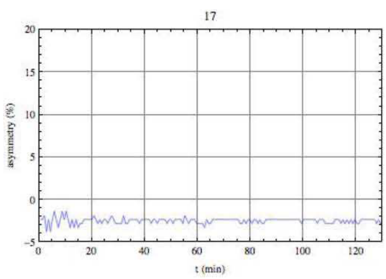
2.3 Spatial oscillation asymmetry

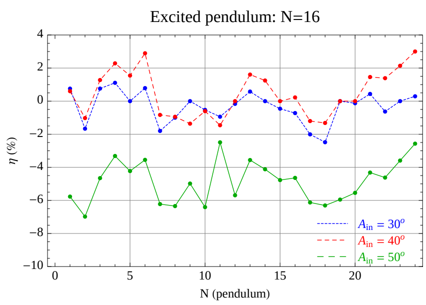
The mean position of each pendulum over the whole run is approximately equal to the resting point, that is the lowest height with minimum gravitational potential energy. Quite surprisingly we have found that any pendulum oscillates around a position rather far from the chain halfway (placed at $x = 0$) as much in the positive direction as in the negative one. Furthermore, the above vertical asymmetry varies in amplitude and sign for each oscillator from a run to another or by varying the initial conditions. This is an interesting unexpected phenomenon, not yet reported in the FPU literature. Some oscillation asymmetry could in principle be observed in collective resonance phenomena and in particular in bridges, buildings or mechanical machines endowed with proper modes due to distributed coupled oscillators.

To evaluate such an effect we have estimated the average oscillation asymmetry for the j^{th} pendulum integrating from the beginning of the run up to a time $t > 0$

$$\langle x_j \rangle(t) = \frac{1}{t} \sum_{t_i=0}^{t_i=t} x_j(t_i)\,[t_{i+1} - t_i]$$

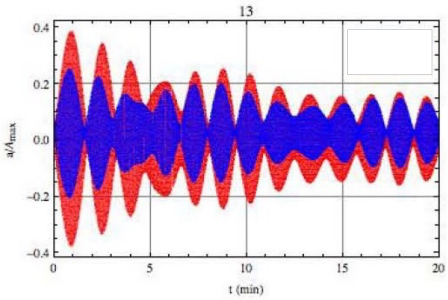
where t_i indicates the generic i^{th} time instant at which our optical setup detects the pendulum space position according to the adopted frame rate. In the following figures the oscillation asymmetry percentage is quoted as a function of time and compared among the pendula along the chain.

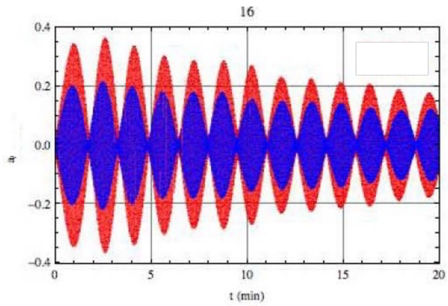




2.4 Chaos

In order to investigate the occurrence of chaos we compared two runs with identical initial conditions overlapping two plots of normalized amplitude as a function of time, as it is shown below for pendula 13^{th} and 16^{th}.





The strong difference between the two plots, observed already at very early instants, shows that the system is very sensitive to small variations in the initial conditions as expected for chaotic motions.

2.5 Localized modes

Finally, let us mention peculiar dynamical events, observed during some runs, which seems to show a temporary energy localization. Just to make an example, when 18^{th} oscillator was excited with an initial 50° angle the following interesting phenomena were observed for the first three oscillators:

- at minute 10 pendula 1 and 3 oscillate together for few seconds and then they give way to 2 and 3
- at minute 17 pendula 2 and 3 oscillate together and then give way energy to 1 and 2 (but with opposite phase)
- at minute 32 pendula 1 and 2 oscillate together and then give way energy to 2 and 3
- at minute 58 pendulum 1 is at rest and pendulum 2 oscillates in a complete asymmetrical way (only negative x)
- at minute 59 pendula 1, 2 and 3 oscillate together
- at minute 66, 71, 76, 81, 85 pendulum 1 remain at rest for at least 1 minute

In particular, the last phenomenon is related to the so-called "intrinsic localized modes" [9, 10, 11, 12, 13]: the chain just shows nodal-like stationary points where the motion energy vanishes, being temporarily migrated to other parts of the chain, as in the presence of a kind of an effective "potential barrier".

3 Conclusions

In the presence of non-linear forces (FPU experiment) coupled oscillators show complex and largely unpredictable collective motions, often chaotic or solitonic-like, and represent a physical model for quite different systems and phenomena (as. e.g., biological and artificial neural networks, financial markets, living organisms evolution, neutrino flavor oscillations, gravitational chaotic systems, non-linear interactions in atomic crystal lattices, etc.). We have built and studied a coupled pendula chain which does undergo a strong energy dissipation. The energy loss is due essentially to the cable torsion yielding the coupling force among all oscillators, which progressively moves mechanical energy from the pendula towards the support structure and then towards the surrounding environment, whilst the air friction becomes important only at large times with small oscillation amplitudes.

Monitoring in time and space the motion of each pendulum on varying the initial conditions we have found that the system approaches the thermalization predicted by second law of thermodynamics and equipartition theorem much faster compared to what expected in the absence of dissipation with constant total energy. Actually, the energy dissipation appears to play a rôle in the thermalization process accelerating the energy transfer from initially excited oscillator(s) to the whole chain. This seems to be a novel result, to our knowledge not yet present in the literature, which certainly deserves further experimental and theoretical investigations. As a matter of fact, some of the above-mentioned complex systems which show a spontaneous thermalization (or equivalently a strong tendency to uniformity among all subcomponents), not being closed, suffer also energy loss and motion damping. In order to better understand the connection between thermalization and dissipation, also generalizing the present results, we plan to perform further researches with suitable modifications and improvements of the actual experimental apparatus (for instance by changing the coupling strength or the rope length or the pendulum radius or mass).

Moreover we have analyzed the complex dynamics of single and collective motions by means of sensitive videotracking techniques with subsequent numerical elaboration of the experimental data. In so doing, in addition to the abovesaid initial chaotic behaviour, spatial oscillation asymmetry and intrinsic localized modes, we have observed other interesting phenomena as beats over (at least 3) different time scales, coherent effects on oscillators placed in positions corresponding to a rational ratio of the chain length and due to low frequency stationary waves propagating in a closed ends line; progressive phase and frequency sinchronization among the pendula motions.

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