

Bayesian modeling for spatially misaligned health and air pollution data through the INLA-SPDE approach

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Abstract

In air pollution studies a key issue concerns the change of support: pollutant concentrations are continuous phenomena in space but their measurements are typically available at a finite number of point-referenced monitoring stations or result from numerical models. When linking exposure to health outcomes, the latter are usually available at administrative level, hence on an irregular lattice, providing challenges in terms of data misalignment.

In this paper we tackle the change of support problem for air pollution and health studies through a two-stage Bayesian approach; in the first stage our model estimates the air pollution concentration at the area level and then in the second stage it links the exposure to the health outcome, accounting for the uncertainty on the exposure estimates. We show through an extensive and realistic simulation that our model is able to predict the concentration accurately at the administrative level as well as estimate the association between exposure and health outcome. We use the Integrated Nested Laplace Approximation, coupled with the Stochastic Partial Differential Equation method for model implementation. Finally we apply the proposed model to evaluate the effect of NO₂ concentration on hospital admissions for respiratory diseases in the Piemonte region (Italy). We found that the upscaling method and the approach used to propagate uncertainty from the first to the

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second stage has an impact on the posterior distribution of the relative risk. Moreover, we found a significant increased risk of 1.6% and 1.8% associated to an increase of 10 $\mu\text{g}/\text{m}^3$ in NO_2 concentration.

Keywords: Spatial misalignment; Integrated Nested Laplace Approximation (INLA); Stochastic Partial Differential Equations (SPDE); Hierarchical modeling; Uncertainty propagation; Air pollution

1. Introduction

Air pollution is both an environmental and social criticality and it represents the single largest environmental health risk in Europe today (Lim et al., 2012). The recent report of the European Environmental Agency (EEA, 2017a) states that 19% of the urban population in the EU-28 was exposed in 2015 to PM_{10} (particulate matter with an aerodynamic diameter of less than 10 μm) concentrations above the EU daily limit value of 50 $\mu\text{g}/\text{m}^3$; the same percentage rises to 53% if the WHO stricter threshold (set to 20 $\mu\text{g}/\text{m}^3$) is considered. The same report states that 9% of the EU-28 urban population lived in areas with concentrations of NO_2 (nitrogen dioxide) exceeding the annual EU limit value of 40 $\mu\text{g}/\text{m}^3$ in 2015.

In terms of health impact, in 2014 long-term exposure to $\text{PM}_{2.5}$ was responsible for about 428,000 premature deaths in Europe (of which around 399,000 were in the EU-28), mainly due to heart and lung diseases (EEA, 2017a), while NO_2 concentration accounted for about 78,000 premature deaths per year (about 75,000 in the EU-28). A large number of epidemiological studies have shown short and long term effects of air pollution on mortality (see for instance Raaschou-Nielsen et al. 2012; Faustini et al. 2014; Atkinson et al. 2016; Halonen et al. 2016; Carugno et al. 2016) or hospital admissions (among the others see for instance Halonen et al. 2016; Carey et al. 2016; Sanyal et al. 2018). Recently some work has appeared suggesting even a link with drug prescriptions for chronic diseases like asthma and COPD, in a primary care perspective (Blangiardo et al., 2016; Lee, 2018). Air pollution has also a considerable economic impact in terms of increased medical costs, reduced productivity and decrease in crop yields: the OECD estimates that these costs will gradually increase to 1% of global worldwide GDP (around USD 2.6 trillion annually) by 2060 (OECD, 2016).

Substantive methodological work has been published to assess the presence of health effects associated with air pollution, including cohort, time

30 series and small area studies (e.g. see the recent review by Bruno et al.,
31 2016). In this paper we focus on the latter case and consider ecological spa-
32 tial regression models for aggregated health data consisting of mortality or
33 morbidity counts at the small area level (typically administrative, e.g. elec-
34 toral wards, district, etc.) together with pollutant measurements available
35 for a set of monitoring stations or grid cell centroids.

36 In this modeling framework the first statistical challenge regards the spa-
37 tial misalignment between health and exposure data, with the consequence
38 that pollutant concentration has to be upscaled at the area level, while being
39 measured at a finite number of point-referenced monitoring stations, a pro-
40 cedure known as *change of support* (Gelfand, 2010). The simplest solution
41 consists in averaging the concentration point measurements available for each
42 area, possibly using distance- or population-based weights (e.g. Elliott et al.,
43 2007; Madsen et al., 2008; Young et al., 2009). However this approach is not
44 feasible where the network is sparse, i.e. there are areas without monitoring
45 stations, or when the considered pollutants show a strong spatial heterogene-
46 ity. As a solution, it is possible to estimate the area-level concentration by
47 computing weighted averages over grid level concentrations available from
48 deterministic atmospheric dispersion models (see e.g. Bell, 2006; Rushworth
49 et al., 2014; Lee and Sarran, 2015). Alternatively, in a model-based per-
50 spective, a spatial statistical model can be built which combines pollutant
51 measurements from stations with the output of numerical dispersion models
52 in a data fusion approach (see for instance Fuentes et al., 2006; Peng and Bell,
53 2010; Sahu et al., 2010; Berrocal et al., 2010; Pannullo et al., 2015; Moraga
54 et al., 2017). Under this modeling framework, beside accounting for spatial
55 correlation and measurement error, it is easy to include additional covariates
56 like meteorological variables. In this paper we adopt a model-based approach
57 and we deal with the spatial misalignment by computing concentration at the
58 area level through a weighted mean with two different types of weights (lin-
59 ear combination with neighbourhood intersections or simple mean). This
60 approach requires to use the exposure model to obtain predictions for a set
61 of points belonging to a regular grid which covers the region of interest.

62 The second statistical challenge concerns how to link the exposure with
63 the health outcomes. Commonly a *two-stage* approach is used: the pollutant
64 concentrations estimated at the first stage are then averaged at the level of
65 the irregular lattice where the health data are available; then the posterior
66 mean or median for each area is included as covariate in the second stage (see
67 e.g. Lee and Shaddick, 2010; Huang et al., 2015; Lee et al., 2015; Pannullo

68 [et al., 2016](#); [Liu et al., 2016](#)). The advantage of this approach is mainly
69 computational since the exposure and health models are fitted separately.
70 The crucial issue with a two-stage approach is that it typically treats the
71 area predicted exposures as known and constant, without accounting for the
72 uncertainty in the prediction of the first stage. This may result in overprecise
73 estimates of the risk effect associated to air pollution concentration, or even
74 biased results if the exposure is also assumed to suffer from measurement
75 error.

76 Some solutions have been recently proposed in the literature and re-
77 gard the propagation of the uncertainty from the first to the second stage
78 through: *i*) multiple simulation of exposure values (from the pollutant con-
79 centration posterior predictive distributions) followed by iterative fits of the
80 health model ([Blangiardo et al., 2016](#); [Liu et al., 2016](#); [Lee et al., 2017](#)); *ii*)
81 considering the exposure as a random variable in the health model with an
82 informative prior obtained from the posterior distributions of the exposure
83 model ([Warren et al., 2012](#); [Powell and Lee, 2014](#); [Lee et al., 2017](#); [Huang](#)
84 [et al., 2017](#)). However no papers have explicitly compared the performance
85 of these different strategies to account for uncertainty from the exposure into
86 the health model.

87 In this paper we are framed in the same perspective as [Liu et al. \(2016\)](#)
88 and [Lee et al. \(2017\)](#), and build a two-stage model to predict air pollution
89 at a regular grid and to evaluate its health effects at small (administrative)
90 area level. In particular in the first stage we estimate the pollutant concentra-
91 tion via the integration of data from monitoring stations as well as numerical
92 model output and additional covariates, while in the second stage we link the
93 estimated exposure to the health outcome. The novel aspect of the paper
94 consists in the thorough evaluation of the impact of the averaging from grid to
95 small area on the exposure estimates as well as of the uncertainty propagation
96 from stage 1 to stage 2 of the modeling framework on the health outcomes. In
97 order to do so we develop an extensive and realistic simulation study, which
98 we believe will be useful for other researchers working on environmental and
99 health studies at the area level. We then use the proposed framework to study
100 the relationship between NO_2 concentration and hospitalisations for respira-
101 tory causes for each communality of the Piemonte region (Italy) for the year
102 2011. We implement the two-stage model by means of the Integrated Nested
103 Laplace Approximation (INLA; [Rue et al., 2009](#)) and Stochastic Partial Dif-
104 ferential equations (SPDE; [Lindgren et al., 2011](#)) approach, as a computa-
105 tionally effective alternative to the standard approach based on Markov chain

106 Monte Carlo methods (MCMC). The R code to reproduce the simulation anal-
107 ysis is available at https://github.com/michelacameletti/INLA_COSP.

108 The rest of the paper is structured as follows: in section 2 we introduce
109 the case study on air pollution and hospital admissions in the Piemonte re-
110 gion in Italy; in section 3 we present the modeling framework, while section
111 4 briefly describes the INLA-SPDE approach used for the implementation.
112 Section 5 introduces the simulation design and presents its results, then sec-
113 tion 6 presents the results of the real data application on NO₂ and hospital
114 admissions in Piemonte, while in section 7 we raise discussion points and
115 concluding remarks.

116 **2. Motivating problem: air pollution and hospitalizations in Pie-** 117 **monte regions, Italy**

118 Piemonte is located in the North-Western part of Italy (see Figure 1,
119 left). Together with Lombardia, Veneto and Emilia Romagna, it is part of
120 the Po Valley, a densely populated and heavily industrialised area located
121 at the footstep of the Alps and characterized by a wide variety of pollution
122 sources mainly related to traffic, domestic heating, farming activities, etc.
123 Its particular geographic position, with the Alps acting like a shelter, leads
124 to frequent occurrence of stagnant meteorological conditions with absence of
125 wind and reduction of pollutant dispersion. For these reasons, Po Valley has
126 been identified as one of the most polluted European regions where pollutant
127 standards, set for human health protections, are regularly exceeded (EEA,
128 2017a,b).

129 For this work we consider the annual NO₂ mean concentrations (in $\mu\text{g}/\text{m}^3$)
130 for 2011, obtained from the 55 monitoring stations depicted with red (for
131 training sites) and green (for validation sites) points in the left plot of Figure
132 1. NO₂ values range from 16 to 71.88 $\mu\text{g}/\text{m}^3$ with a median of 35.63 $\mu\text{g}/\text{m}^3$.
133 In order to make the distribution of NO₂ approximately normal, we use a
134 logarithmic transformation. In addition, in the exposure model the follow-
135 ing covariates are considered: precipitation (in mm), mixing height (in m),
136 temperature (in K), windspeed (in m/s) and NO₂ emissions (in g/s), that
137 are obtained from a nested system of deterministic computer-based models
138 implemented by the environmental agency *ARPA Piemonte*. These deter-
139 ministic models provide data at the monitoring station sites and for all the
140 points of a 4 km \times 4 km grid covering Piemonte region (Cameletti et al.,

141 [2011](#), [2013](#)). This grid, which has a resolution of $56 \times 72 = 4032$ points (see
142 the blue points in Figure 1, left) will be used for spatial prediction.

143 To assess the impact of air pollution exposure on human health, we con-
144 sider hospitalizations data provided by the Ministry of Health. For each
145 patient discharged from either a public or private healthcare facility, we have
146 data about socio-demographic variables (e.g. gender, age) and the hospi-
147 talization event (e.g., diagnosis, dates of admission and discharge). In this
148 paper we consider cardio-respiratory hospitalizations occurred in Piemonte
149 region during 2011 and aggregated at the municipality level. The Standard-
150 ized Morbidity Ratio (adjusted by age and gender) is reported in the right
151 plot of Figure 1 and shows substantial spatial variability, with higher risks
152 around large cities such as Turin and Alessandria, while the more rural cen-
153 tral part as well as most of the mountain region is characterized by risks
154 lower than averages.

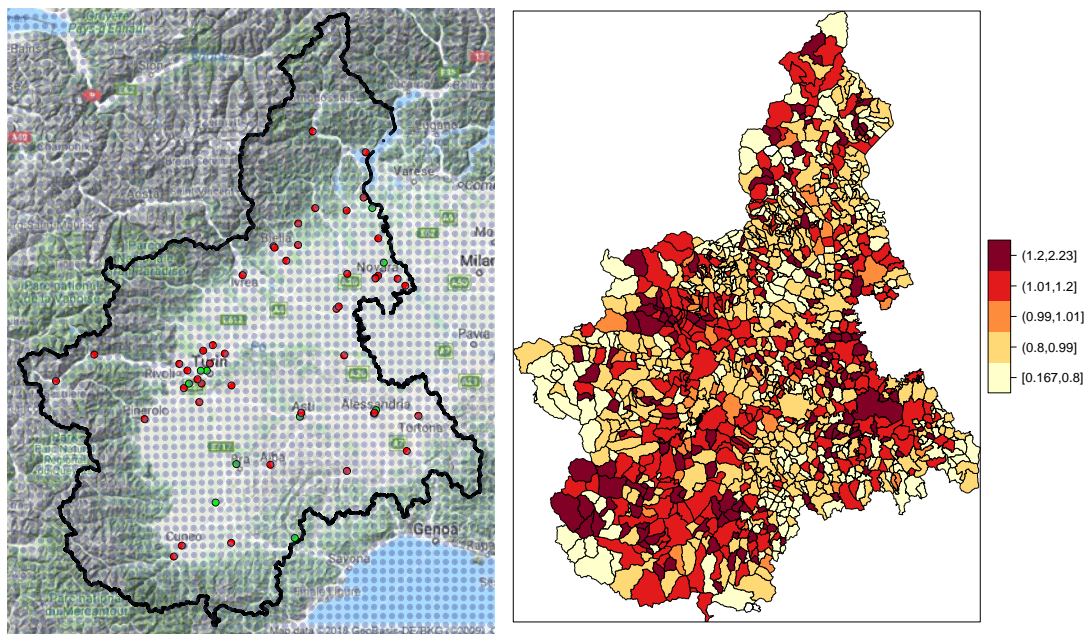


Figure 1: Left: Piemonte map: the red and green dots denote the NO_2 monitoring stations used for model estimation and validation, respectively. The blue dots represent the centroids of the regular grid. Right: Standardized Morbidity Ratio (adjusted by age and gender) for cardio-respiratory hospitalizations in the Piemonte municipalities in 2011.

155 **3. Poisson health model**

156 The standard spatial model for the observed number of health events y_i
 157 in the area \mathcal{A}_i ($i = 1, \dots, n_{\mathcal{A}}$), when there is a relatively low count of disease
 158 and/or the area is small, is given by

$$y_i \sim \text{Poisson}(E_i \rho_i)$$

159 where E_i represents the expected number of events obtained applying stan-
 160 dard rates from the whole study region (by age and gender) to the population
 161 of each area (Shaddick and Zidek, 2016). The term ρ_i represents the area
 162 specific relative risk and a linear predictor is defined on its logarithmic trans-
 163 formation as follows:

$$\log(\rho_i) = \gamma_0 + x_i \gamma_1 + \mathbf{z}'_i \boldsymbol{\gamma}_2 + \phi_i. \quad (1)$$

164 This linear predictor includes an intercept γ_0 (representing the average rate
 165 in the entire study region), the area air pollution concentration x_i , random
 166 effects ϕ_i and a vector of measured confounders \mathbf{z}'_i . In the application pre-
 167 sented in Section 6 we include the social vulnerability index provided by the
 168 Italian National Institute of Statistics (ISTAT) for 2011, which measures the
 169 deprivation level of individuals within municipalities. To evaluate the health
 170 risk associated to air pollution, the parameter of interest is γ_1 or the cor-
 171 responding relative risk given by $\exp(\gamma_1 \delta)$, which represents the change in
 172 the risk of experiencing the considered health outcome when air pollution
 173 concentrations increases by $\delta \mu\text{g}/\text{m}^3$.

174 The random effects ϕ_i capture any overdispersion and potential residual
 175 spatial correlation in the health data after the covariate effects have been
 176 accounted for. An additive specification can be adopted such that $\phi_i = u_i + v_i$,
 177 and several structures can be assumed on these two terms (see Lee, 2011 for a
 178 review). In the application of Section 6 we use the specification by Besag et al.
 179 (1991), which places an exchangeable random effect on v_i and a conditional
 180 autoregressive structure on u_i so that

$$\begin{aligned} v_i &\sim \text{Normal}(0, \sigma_v^2) \\ u_i | u_{-i} &\sim \text{Normal}\left(\frac{\sum_{j \in D_i} u_j}{|D_i|}, \frac{\sigma_u^2}{|D_i|}\right), \end{aligned}$$

181 where D_i represents the set of areas sharing borders with the i -th area and
 182 $|D_i|$ its cardinality. This assumes that only the areas close to each other can

183 influence one another and provides some local smoothing to the estimates
 184 of the relative risks. In addition some global smoothing is provided by the
 185 spatially unstructured random effects v_i .

186 3.1. Exposure estimation through upscaling

187 The term x_i in Eq. (1) represents the exposure level for area \mathcal{A}_i ; it can not
 188 be measured directly as air pollutant concentrations are available only for a
 189 finite number of spatial points with coordinates $\mathbf{s}_1, \dots, \mathbf{s}_n$. The set of point-
 190 referenced concentration measurements is denoted by $(x(\mathbf{s}_1), \dots, x(\mathbf{s}_n))$ and
 191 is a realization of the latent stochastic process $x(\mathbf{s})$ representing the true air
 192 pollution field, which is continuous in space. The average exposure level for
 193 area \mathcal{A}_i would be given by

$$x_i = \int_{\mathbf{s} \in \mathcal{A}_i} x(\mathbf{s}) p(\mathbf{s}) d\mathbf{s} \quad (2)$$

194 where $p(\mathbf{s})$ is a weight for a generic spatial point $\mathbf{s} \in \mathcal{A}_i$ such that $\int_{\mathbf{s} \in \mathcal{A}_i} p(\mathbf{s}) d\mathbf{s} =$
 195 1 (Gelfand, 2010).

196 This stochastic integral can be estimated using the set of measurements
 197 from the n monitoring stations by simply averaging the concentration data
 198 from the stations falling within each area \mathcal{A}_i . However, given that monitoring
 199 networks are typically sparse, some areas could end up with no monitoring
 200 stations; a possible solution would consist in estimating the concentration
 201 only within a specific distance from monitoring stations and then evaluating
 202 the health effects only on the population within the same areas (as in Zhu
 203 et al. 2003). Alternatively, in order to cover all the spatial domain, the area
 204 exposure level x_i can be computed through Monte Carlo integration using a
 205 set of additional points, denoted by \mathbf{s}^* , which are the centroids of a regular
 206 grid covering the region of interest (see e.g. Lee and Shaddick, 2010; Lee and
 207 Sahu, 2016). With this approach the exposure value for area \mathcal{A}_i is estimated
 208 through the following weighted mean:

$$x_i = \sum_{j=1}^{N_i} x(\mathbf{s}_{ij}^*) p(\mathbf{s}_{ij}^*), \quad (3)$$

209 where $x(\mathbf{s}_{ij}^*)$ is the pollutant concentration value for the generic location \mathbf{s}_{ij}^* ,
 210 which is one of N_i regular grid centroids inside area \mathcal{A}_i . The corresponding
 211 weights are normalized so that $\sum_{j=1}^{N_i} p(\mathbf{s}_{ij}^*) = 1$. The exposure values $x(\mathbf{s}_{ij}^*)$

212 can: *i*) be provided by an air pollution (deterministic) dispersion model, such
 213 as the Community Multiscale Air Quality Modeling System (CMAQ), used
 214 by the US Environmental Protection Agency, or the Atmospheric Dispersion
 215 Modeling System (ADMS), particularly useful for urban areas; *ii*) be the
 216 output of simple spatial interpolation (e.g. inverse distance weighting, krig-
 217 ing) using the monitoring network data; *iii*) be derived by spatial prediction
 218 from an *exposure model* which can fuse different sets of data, account for the
 219 measurement error and include explicitly a continuous spatial process (see
 220 e.g. Sahu, 2011; Lee et al., 2017) as described in Section 3.2; in the Bayesian
 221 framework, this means that an exposure posterior predictive distribution is
 222 available for each grid point \mathbf{s}_{ij}^* and it can be used to derive the area level ex-
 223 posure posterior distribution. In particular, the prediction of the exposure at
 224 the area level is performed in two steps: firstly the exposure posterior distri-
 225 bution is obtained for a set of points \mathbf{s}_{ij}^* belonging to a regular grid. Secondly,
 226 for each area \mathcal{A}_i , the exposure average is computed using two methods:

- 227 1. Method 1 (*linear combination with neighbourhood intersections*): the
 228 area exposure is computed using Eq. (3), with $x(\mathbf{s}_{ij}^*)$ being the exposure
 229 estimates available at the centroid \mathbf{s}_{ij}^* of the N_i cells which have an
 230 intersection with the considered area \mathcal{A}_i ($j = 1, \dots, N_i$). Note that
 231 we assume that the generic weight $p(\mathbf{s}_{ij}^*)$ is given by the proportion
 232 of the j -th grid cell overlapping with area \mathcal{A}_i . For example, Figure 2
 233 (left) represents a generic area of the considered region which intersects
 234 11 cells of the regular grid ($N_i = 11$) and with corresponding weights
 235 ranging from 0.001 to 0.236.
- 236 2. Method 2 (*simple mean*): the area exposure is computed using Eq. (3),
 237 but considering $x(\mathbf{s}_{ij}^*)$ as the exposure estimates available at the grid
 238 cell centroids \mathbf{s}_{ij}^* that lie in the considered area. If no grid centroids
 239 are located inside the area, then the closest grid point is used. In this
 240 case we consider a system of equal weights for all the considered grid
 241 points used in the linear combination. For example, for the generic
 242 area shown in Figure 2 (right) 3 grid points lie inside the area and their
 243 weights are all equal to $1/3$.

244 It would also be possible to estimate the area exposure x_i by averaging
 245 grid predictions with weights $p(\mathbf{s}_{ij}^*)$ proportional to the population at-risk
 246 (e.g. Wakefield and Shaddick, 2006). This approach requires to have high-
 247 resolution information about the population size (for example the LandScan™
 248 project provides global population estimates at 1km spatial resolution) and

249 to perform some geoprocessing to align the spatial datasets (e.g. Shaddick
 250 et al., 2018).

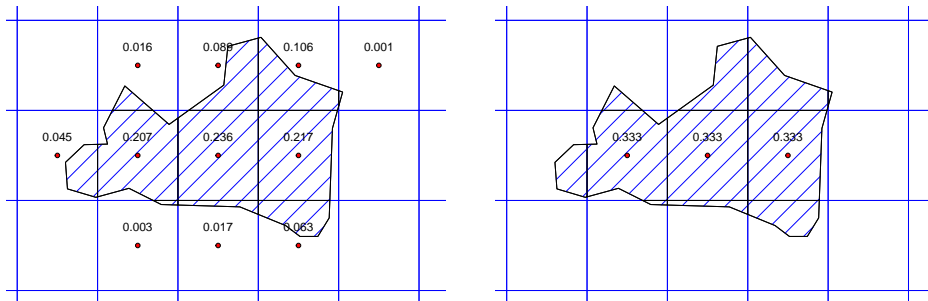


Figure 2: Method1: intersections between a generic area and the regular grid and corresponding weights (left). Method2: grid points inside a generic area and corresponding weights (right).

251 *3.2. Gaussian exposure model*

252 Let $z(\mathbf{s}_j)$ ($j = 1, \dots, n$) be the set of air pollution data measured by n
 253 monitoring stations. The pollutant concentration is assumed to be centred
 254 on the true concentration $x(\mathbf{s}_j)$ and depends on the measurement error $e(\mathbf{s}_j)$:

$$z(\mathbf{s}_j) = x(\mathbf{s}_j) + e(\mathbf{s}_j), \quad (4)$$

255 where $e(\mathbf{s}_j) \sim \text{Normal}(0, \sigma_e^2)$ independently for each location. The true con-
 256 centration is defined by the following linear predictor

$$x(\mathbf{s}_j) = b_0 + \mathbf{v}(\mathbf{s}_j)' \mathbf{b} + \omega(\mathbf{s}_j) \quad (5)$$

258 which includes an intercept b_0 (i.e. the average level of pollution for the
 259 considered area), a set of site specific covariates $\mathbf{v}(\mathbf{s}_j)$ (e.g. meteorological
 260 and geographical variables) with \mathbf{b} vector of coefficients (assumed to be site-
 261 invariant) and a latent process $\omega(\mathbf{s}_j)$ representing the residual spatial field.
 262 The n -dimensional process $\boldsymbol{\omega} = (\omega(\mathbf{s}_1), \dots, \omega(\mathbf{s}_n))$ is assumed to be Normally
 263 distributed with zero mean vector and spatially structured covariance matrix
 264 which is defined by the following Matérn covariance function

$$\text{Cov}(\omega(\mathbf{s}_j), \omega(\mathbf{s}_{j'})) = \frac{\sigma_\omega^2}{\Gamma(\lambda) 2^{\lambda-1}} (\kappa \|s_j - s_{j'}\|)^\lambda K_\lambda(\kappa \|s_j - s_{j'}\|), \quad (6)$$

265 where $\|\mathbf{s}_j - \mathbf{s}_{j'}\| \in \mathbb{R}$ is the Euclidean spatial distance, σ_ω^2 is the spatial
266 variance and κ is the scaling parameter. The term $K_\lambda(\cdot)$ denotes the modified
267 Bessel function of second kind and order $\lambda > 0$. The parameter λ , which is
268 usually kept fixed, measures the degree of smoothness of the process and
269 its integer value determines the mean square differentiability of the process.
270 Instead, $\kappa > 0$ is a scaling parameter related to the range r , i.e. distance
271 at which the spatial correlation is close to 0.1 for each $\lambda \geq 1/2$; [Lindgren](#)
272 [et al. \(2011\)](#) proposed an empirically derived definition for the spatial range
273 r , given by $r = \sqrt{8\lambda}/\kappa$. Many other exposure models are available in the
274 literature including also extensions to the spatio-temporal case (see e.g. [Sahu,](#)
275 [2011](#); [Cameletti et al., 2011, 2013](#); [Fassò and Finazzi, 2011](#); [Pirani et al., 2014](#);
276 [Pannullo et al., 2015](#)).

277 In a fully Bayesian approach, we denote by $\boldsymbol{\theta} = \{b_0, \mathbf{b}, \boldsymbol{\omega}\}$ the latent
278 Gaussian field and by $\boldsymbol{\psi} = \{\sigma_e^2, \sigma_\omega^2, \kappa\}$ the vector of hyperparameters. This
279 identifies a three-level hierarchical model with the first stage given by ob-
280 served data distribution $\pi(\mathbf{z} \mid \boldsymbol{\theta}, \boldsymbol{\psi})$ with $\mathbf{z} = (z(\mathbf{s}_1), \dots, z(\mathbf{s}_n))$, the second
281 stage specified by the latent field distribution $\pi(\boldsymbol{\theta} \mid \boldsymbol{\psi})$ and the last level de-
282 voted to the hyperparameter prior distribution $\pi(\boldsymbol{\psi})$. Within this modeling
283 framework, the exposure distribution for a new spatial point \mathbf{s}_{ij}^* not included
284 in the set of monitoring stations, is simply given by substituting \mathbf{s}_j with \mathbf{s}_{ij}^*
285 in Eq. (4) and (5). The corresponding posterior predictive distribution is
286 then denoted by $\pi(x(\mathbf{s}_{ij}^*) \mid \mathbf{z})$ and is given by

$$\begin{aligned} \pi(x(\mathbf{s}_{ij}^*) \mid \mathbf{z}) &= \int \int \pi(x(\mathbf{s}_{ij}^*), \boldsymbol{\theta}, \boldsymbol{\psi} \mid \mathbf{z}) d\boldsymbol{\theta} d\boldsymbol{\psi} \\ &= \int \int \pi(x(\mathbf{s}_{ij}^*) \mid \boldsymbol{\theta}, \boldsymbol{\psi}, \mathbf{z}) \pi(\boldsymbol{\theta} \mid \boldsymbol{\psi}, \mathbf{z}) \pi(\boldsymbol{\psi} \mid \mathbf{z}) d\boldsymbol{\theta} d\boldsymbol{\psi}. \end{aligned} \quad (7)$$

287 When performing Bayesian inference through Markov chain Monte Carlo
288 methods, samples from the posterior predictive distribution (7) are drawn by
289 composition ([Sahu, 2011](#)). In this paper instead we adopt the INLA approach
290 for jointly estimating the parameters and performing spatial prediction both
291 at the grid point and area level.

292 3.3. Linking the exposure and health model

293 The easiest and most commonly used method for estimating the adverse
294 effect of air pollution on human health is through a *plug-in* (PI) approach:
295 first the exposure model is estimated (see Section 3.2) and the pollutant

296 concentration is upscaled at the area level as described in Section 3.1; from
 297 this first stage a summary statistic is computed for each area (e.g. exposure
 298 posterior mean or median). The second stage consists in including such value
 299 as the term x_i in the linear predictor of the health model (see Eq. (1)), which
 300 is then fitted separately from the first stage. While being computationally
 301 advantageous, this approach does not consider the uncertainty intrinsic in the
 302 prediction of the pollutant concentration as only the summary statistics is
 303 plugged in. Thus, the resulting risk estimate tends to be unnaturally precise
 304 and might even be biased if the concentration suffers from measurement error.

305 Here we implement two ways of propagating the uncertainty from the
 306 exposure into the health model. The first, which we call *feed-forward* (FF)
 307 approach, consists in sampling J samples (e.g. $J = 100$) from the joint
 308 posterior predictive distribution of the pollutant concentration at the area
 309 level and to fit the health model for each one of these simulated exposure
 310 values. The posterior distribution of the risk estimate γ_1 (see Eq.(1)) will then
 311 be obtained by combining all the results across the J runs. This approach
 312 has been adopted also by Blangiardo et al. (2016), Liu et al. (2016) and Lee
 313 et al. (2017) and it represents a relatively computationally cheap solution for
 314 taking into account the variability of the exposure estimates.

315 Alternatively, we consider a *prior-exposure* (PE) approach, which specifies
 316 an informative prior distribution for the exposure area level x_i in Eq.(1) as
 317 follows

$$x_i^A \sim \text{Normal}(\mu_i, \sigma_i^2) \quad (8)$$

318 where μ_i and σ_i^2 are given by the posterior means and variances from the
 319 area level exposure posterior predictive distributions (see Section 3.1). This
 320 corresponds to a multivariate Normal distribution for $x^A = (x_1^A, \dots, x_{n_A}^A)$
 321 with spatially structured mean vector and diagonal covariance matrix with
 322 values given by σ_i^2 .

323 Note that the product of two Gaussian distributions for the term $\gamma_1 x_i^A$
 324 follows by assuming a vague Normal prior distribution also for γ_1 .

325 4. Implementation: the INLA-SPDE approach

326 We performs Bayesian inference using the integrated nested Laplace ap-
 327 proximations (INLA) (Rue et al., 2009; Blangiardo and Cameletti, 2015; Rue
 328 et al., 2017), a computationally efficient alternative to MCMC methods for
 329 latent gaussian models which can be implemented through the R-INLA library
 330 (see <http://www.r-inla.org>).

331 For the first stage of our modeling framework, we couple INLA with
 332 SPDE approach proposed by [Lindgren et al. \(2011\)](#) required when Bayesian
 333 inference is needed on a spatial process defined over a continuous domain.
 334 The SPDE method represents a Gaussian field with Matérn spatial covari-
 335 ance function (see Eq.(6)) as a discrete indexed Gaussian Markov random
 336 field (GMRF), which is characterized by a sparse precision matrix and en-
 337 joys computational benefits in terms of fast inference. This representation
 338 is based on a finite combination of piecewise linear functions defined over
 339 a triangulation (or mesh) of the domain of interest and with basis weights
 340 defined by a GMRF with sparse precision matrix explicitly depending on the
 341 Matérn parameters ([Lindgren and Rue, 2015](#)). Spatial prediction in a given
 342 location belonging to the considered spatial domain is straightforward since
 343 SPDE provides the approximation of the entire spatial process; it is just a
 344 matter of including in the INLA model the locations where predictions are
 345 required as missing values observations ([Lindgren and Rue, 2015](#)).

346 Note that the PE modeling described in Section 3.3 cannot be run in
 347 INLA as it involves the product of two Gaussian distributed parameters (γ
 348 and x_i^A) which breaks normality of the latent field. However, by conditioning
 349 on γ_1 , it is possible to rewrite the product $x_i\gamma_1$ as follows:

$$x_i\gamma_1 = (\mu_i + z_i\sigma_i)\gamma_1 = \mu_i\gamma_1 + \gamma_1\sigma_i z_i$$

350 where $z_i \sim N(0, 1)$. This model conditioned on γ_1 can be estimated using
 351 INLA: in particular, in the R-INLA setting the term $\mu_i\gamma_1$ must be consid-
 352 ered as an offset, while the term $\gamma_1\sigma_i$ is the weight of the i.i.d. random
 353 effect given by z_i (see [Gómez-Rubio and Rue, 2018](#)). Thus, it is possible
 354 to obtain the posterior conditional marginals of all the remaining paramet-
 355 ers in $\boldsymbol{\theta}$ (including the parameters from the exposure and health model),
 356 i.e. $\pi(\boldsymbol{\theta} \mid \gamma_1, \mathbf{y})$, and the conditional likelihood $\pi(\mathbf{y} \mid \gamma_1)$. To draw values
 357 for γ_1 the Metropolis-Hastings (MH) algorithm could be used: after a suit-
 358 able number of iterations (say L), the MH algorithm will produce samples
 359 from $\pi(\gamma_1 \mid \mathbf{y})$ denoted by $\{\gamma_1^{(j)}\}_{j=1}^L$. Finally, it is possible to get the pos-
 360 terior marginals of all the remaining parameters in $\boldsymbol{\theta}$ by combining all the
 361 conditional marginals as follows (see [Gómez-Rubio and Rue, 2018](#)):

$$\pi(\theta_i \mid \mathbf{y}) = \int \pi(\theta_i \mid \mathbf{y}, \gamma_1)\pi(\gamma_1 \mid \mathbf{y}) d\gamma_1 \simeq \frac{1}{L} \sum_{j=1}^L \pi(\theta_i \mid \mathbf{y}, \gamma_1^{(j)}).$$

362 4.1. Priors

363 In R-INLA the smoothness parameter λ of the Matérn covariance function
364 in Eq.(6), which is usually kept fixed to ensure model identifiability, is by
365 default equal to 1. The SPDE parameters are represented as $\log(\tau) = \theta_1$
366 (τ is related to the variance through the relationship $\sigma_\omega^2 = 1/(4\pi\kappa^2\tau^2)$) and
367 $\log(\kappa) = \theta_2$, with θ_1 and θ_2 being given independent Normal(0,1) prior dis-
368 tributions (for more details see [Blangiardo and Cameletti, 2015](#)). Moreover,
369 weakly informative Normal priors centered on 0 and with a small precision
370 equal to 0.01 are specified for the fixed effects parameters b_0 , \mathbf{b} and γ_0 and
371 γ_1 . Finally all the log precisions are assigned inverse Gamma distributions
372 with parameters equal to 1 and 0.00005.

373 5. Simulation study

374 In this section we describe the simulation study which has a twofold aim:
375 firstly, it evaluates the goodness of exposure predictions at the area level ob-
376 tained through the change of support by using Method 1 and Method 2 (see
377 Section 3.1); secondly it assesses the effect of different ways for incorporating
378 exposure in the health model on the relative risk estimate (i.e. the PI, the
379 FF and the PE approach described in Section 3.3). For the simulation study
380 we use the Belo Horizonte region shapefile, available through the `spdep R`
381 package ([Bivand and Piras, 2015](#)); this has a smaller number of areas com-
382 pared to our Piemonte case study ($n_A = 98$ vs $n_A = 1206$) hence it is more
383 computationally manageable.

384 5.1. Simulation of the exposure field

385 In order to create a continuous spatial field, we simulate exposure at a
386 large number m of locations ($m = 4009$) which are aligned in space and
387 cover completely the considered region (see for example left plot in Figure
388 3). The model used for simulating exposure is based on Eq. (4) and (5) for
389 $j = 1, \dots, m$. In particular, we assume to have just one covariate v , simulated
390 from a Normal(0,1) distribution, with coefficient $b = 2$ and an intercept equal
391 to $b_0 = 10$. Regarding the spatial parameters (see Eq. (6) and recall that
392 in R-INLA $\lambda = 1$ and $\kappa = \sqrt{8}/r$), we set the spatial variance σ_ω^2 equal to 0.5
393 and consider a range r given by 1.63 degrees, corresponding to the 40% of
394 the maximum distance.

395 Finally, the *true* exposure at the area level is denoted by \tilde{x}_i ($i = 1, \dots, n_A$)
 396 and computed for each area \mathcal{A}_i by averaging the exposure values of the sites
 397 located inside the area (see right plot in Figure 3):

$$\tilde{x}_i = \frac{\sum_{\mathbf{s}_j \in \mathcal{A}_i} x(\mathbf{s}_j)}{m_i}, \quad (9)$$

398 where m_i denotes the number of sites inside area \mathcal{A}_i ($\sum_i^{n_A} m_i = m$). Note
 399 that the cardinality ranges from 7 to 174 points with a median of 30 points
 400 per area.

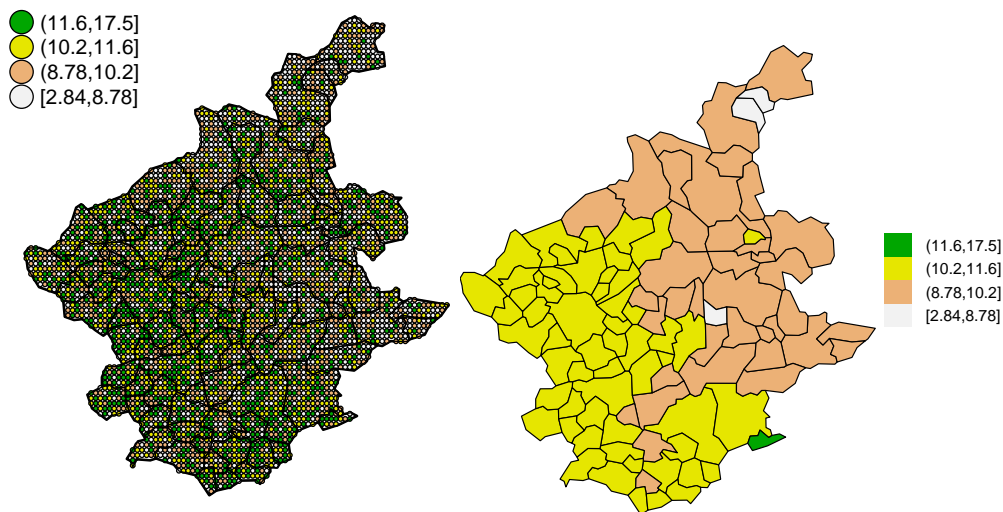


Figure 3: Example of true exposure values simulated at 4009 points inside the Belo Horizonte region (left) and corresponding area averages computed for the 98 areas (right).

401 5.1.1. Monitoring station sampling

402 From the set of m spatial locations used for simulating the true expo-
 403 sure field (see the previous Section 5.1), we randomly select n sites which
 404 correspond to the monitoring stations where exposure concentration is mea-
 405 sured. We consider three cases with the number of stations in each area
 406 n_i ($\sum_{i=1}^{n_A} n_i = n$) being 2%, 10% or 30% of the total number of available

407 points m_i in each area. This leads to a total number of monitoring sites
 408 equal to 80, 403 and 1200, respectively. The distribution of points across
 409 areas is reported in the left plot of Figure 4: note that the median number
 410 of monitoring stations for each area is equal to 1, 3 and 9 for the three cases,
 411 respectively.

412 We assume that the exposure is measured with an error, as specified in
 413 Eq. (4). For this reason we add a term to the monitoring station exposure
 414 $x(\mathbf{s}_j)$, simulated independently from a Normal distribution with mean zero
 415 and variance equal to $\sigma_e^2 = 0.1$. This set of data will be used to estimate the
 416 exposure field and to predict exposure first at the grid level and then at the
 417 area level, as described in the next section.

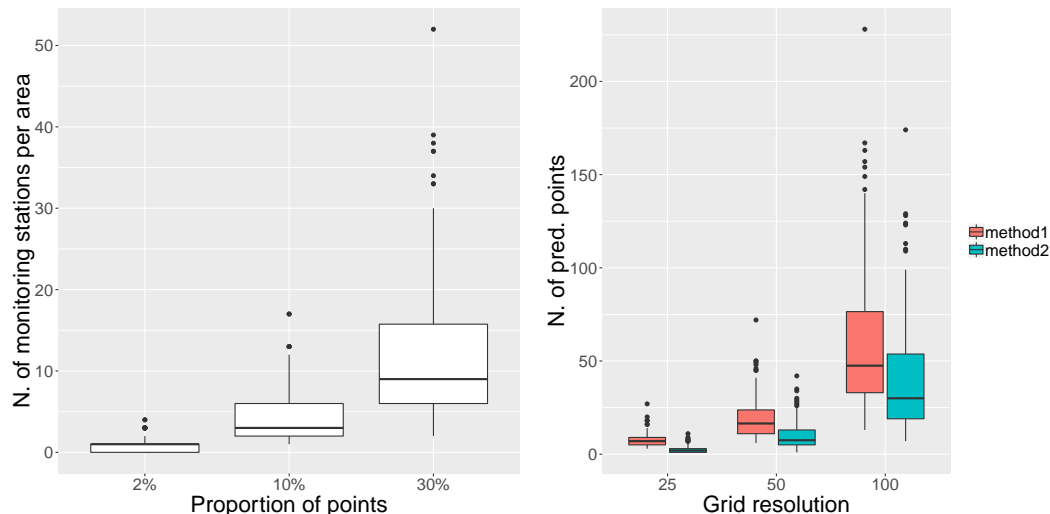


Figure 4: Left: distribution of the number of monitoring stations across areas for the three considered cases (number of monitoring stations equal to 2%, 10% or 30% of the total number of available grid points inside each area, respectively). Right: distribution of the number of prediction points across areas according to the regular grid resolution (25^2 , 50^2 and 100^2 points) and the upscaling method (Method 1 and Method 2).

418 5.1.2. Regular grid for prediction

419 Spatial prediction of exposure is performed considering a regular square
 420 grid with n_g cells covering the entire region and extending also slightly outside
 421 (see for example Figure 5 for the case with $n_g = 25^2 = 625$ points). This
 422 regular grid is employed for predicting exposure at the grid cell centroids \mathbf{s}_{ij}^*

423 using the exposure model described in Section 3.2. As in Section 5.1, the
 424 values of the covariate v for the grid cell centroids are drawn independently
 425 from a Normal(0, 1) distribution. For the simulation study we consider three
 426 grid resolutions with n_g equal to 625, $50^2 = 2500$ and $100^2 = 10000$ points,
 427 respectively. Note that the latter corresponds exactly to the grid used for
 428 simulating the true exposure field with $m = 4009$ points inside the region
 429 (see Section 5.1).

430 The grid resolution is strictly related to the number of prediction points
 431 used for computing the exposure level at the area level using Method 1 and
 432 Method 2 described in Section 3.1. The right plot of Figure 4 displays the
 433 distribution of the number of prediction points across area according to the
 434 regular grid resolution and the upscaling method. It can be observed that,
 435 as expected, Method 1 - that considers the intersections between prediction
 436 grid and area - employs a higher number of prediction points than Method
 437 2: the median number of points, according to the three grid resolutions, is
 438 equal to 7, 16.5 and 47.5 for Method 1 and 2, 7.5 and 30 for Method 2. The
 439 total number of prediction points in the whole region is equal to 774, 1957
 440 and 5831 for Method 1 and 257, 1001 and 4009 for Method 2. We expect
 441 to be able to predict exposure more accurately at the area level by using a
 442 higher number of prediction points, especially for small areas which do not
 443 contain any grid square centroids when the grid is coarse.

444 5.2. Simulation of the health data

445 Given the true exposure \tilde{x}_i ($i = 1, \dots, n_A$) at the area level, it is possible
 446 to simulate the health count data y_i using the Poisson model introduced in
 447 Section 3. In particular, for the linear predictor of Eq. (3.1) we set $x_i = \tilde{x}_i$
 448 and $\log(\gamma_1)$ equal to $\log(1.05)$, which would be realistic for the impact of air
 449 pollution on hospital admissions or mortality in Europe (see for instance Lee
 450 and Sarran, 2015; Moore et al., 2016). The corresponding value for the inter-
 451 cept γ_0 is fixed equal to -0.4, a value which guarantees a reasonable spread of
 452 the Poisson simulated data. We include also a spatially unstructured random
 453 effect $v_i \sim \text{Normal}(0, \sigma_\phi^2)$ with $\sigma_\phi^2 = 0.05$ but for the sake of simplicity omit
 454 the spatially structured random effect u_i (see Section 3). The choice of the
 455 value for σ_ϕ^2 is done in order to avoid too much variability in the random
 456 effect given that the relative risk is small. Finally, we assume that the ex-
 457 pected number of cases E_i is fixed and equal to 100 for all the areas (Lee and
 458 Sarran, 2015; Wang et al., 2019).

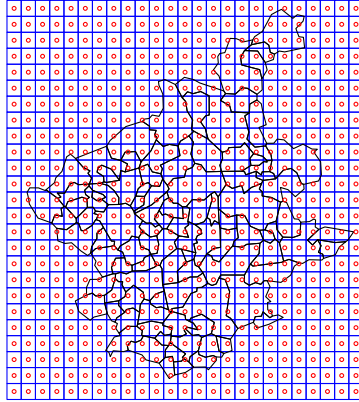


Figure 5: Regular prediction grid with $n_g = 625$.

459 *5.2.1. Simulation scenarios and performance assessment*

460 Combining together the proportions of monitoring stations (n_i equal to
 461 2, 10% and 30% of the sites m_i in each area) with the grid resolution n_g (25^2 ,
 462 50^2 and 100^2 grid centroids) we obtain 9 scenarios which are summarized in
 463 Table 1.

n_g	% of monitoring stations		
	2%	10%	30%
25^2	①	②	③
50^2	④	⑤	⑥
100^2	⑦	⑧	⑨

Table 1: Simulation scenarios considering different values for the resolution of the regular grid (n_g) and the percentage of sampled points as monitoring stations.

464 For the h -th scenario ($h = 1, \dots, 9$) we run 500 simulations which differ
 465 for the true simulated exposure (at the point and area level) and the Poisson
 466 data. The monitoring station coordinates are instead fixed across simula-
 467 tions to avoid that their locations influence the estimation and prediction
 468 results. Within a specific scenario, the k -th simulation run ($k = 1, \dots, 500$)
 469 is structured as follows:

- 470 1. **Simulation:** (i) the exposure field is simulated using $m = 4009$ points
471 and then the true exposure at the area level is computed (see Sec-
472 tion 5.1); (ii) the n_i monitoring stations are assigned exposure values
473 equal to the true exposure plus the measurement error realisation, as
474 described in Section 5.1.1; (iii) the health data y_i ($i = 1, \dots, n_A$) are
475 simulated from the Poisson model using the true area exposure (see
476 Section 5.2).
- 477 2. **Estimation and prediction:** (i) Using the INLA-SPDE approach the
478 spatial parameters of the exposure model ($b_0, b, \sigma_e^2, \sigma_\omega^2, r$) are estimated
479 and the exposure field is predicted for the grid prediction points. At the
480 same time, the exposure is estimated at the area level using Method 1
481 and Method 2 for upscaling; (ii) using the three methods for linking the
482 exposure with the health model (PI, FF and PE), the Poisson model
483 with parameters $\gamma_0, \gamma_1, \sigma_\phi^2$ is estimated.

484 Let $\tilde{\theta}$ denote the true value for the generic parameter of interest θ . Given
485 a scenario h and a simulation run k , for each parameter we simulate 100
486 values, denoted by $\{\hat{\theta}_{hkl}\}$ ($l = 1, \dots, 100$) from the corresponding posterior
487 distribution. Then for each scenario, simulation and parameter we compute
488 the bias and root mean square error (RMSE) as follows:

$$\text{bias}(\theta) = \frac{1}{100} \sum_{l=1}^{100} (\hat{\theta}_{hkl} - \tilde{\theta}) \quad (10)$$

$$\text{RMSE}(\theta) = \sqrt{\frac{1}{100} \sum_{l=1}^{100} (\hat{\theta}_{hkl} - \tilde{\theta})^2} . \quad (11)$$

489 The same performance indexes are used in order to evaluate the goodness
490 of fit of the exposure predictions for each area \mathcal{A}_i , by comparing the true
491 exposure area value \tilde{x}_{hki} and the corresponding estimates \hat{x}_{hkli} which is the
492 l -th value drawn from the exposure posterior predictive distribution of area
493 \mathcal{A}_i .

494 5.2.2. Simulation results

495 The goodness of fit of the area exposure predictions depends strongly on
496 the number of prediction grid points: as shown in the left and middle plots
497 of Figure 6 the performance indexes (bias and RMSE) are worse for scenarios

498 1-3 (with 25^2 prediction points), improve for scenarios 4-6 (with 50^2 predic-
 499 tion points) and reach the best values for scenarios 7-9 (with 100^2 prediction
 500 points). The latter was expected as the number of prediction points for sce-
 501 narios 7-9 coincides with the number of points used for simulating the true
 502 area exposure ($m = 4009$). Regarding the two upscaling methods (Method 1
 503 and Method 2) it is important to note that Method 2 for scenarios 7-9 rep-
 504 represents the benchmark as it is exactly the same method used for simulating
 505 the true area exposure (computed as average of the exposure values observed
 506 in the prediction points inside each area, with a grid resolution of 100^2 pre-
 507 diction points). The corresponding bias, RMSE and correlation reported in
 508 the plots of Figure 6 are not exactly equal to 0 and 1 as expected only due
 509 to sampling variability. At the same time, it seems that the performance
 510 of Method 1 for scenarios 7-9 is quite similar to the benchmark (Method 2).
 511 This holds especially for the bias and RMSE, even if for Method 1 we observe
 512 a higher variability of the results. Moreover, for scenarios 1-3 (low resolution
 513 prediction grid) Method 1 seems to have larger median biases but lower med-
 514 ian RMSEs and higher correlation values. For the remaining scenarios 3-5,
 515 the two methods behave very similarly and the indexes' medians basically
 516 coincide. Finally, it is worth to note that from the computational point of
 517 view Method 1 and Method 2 require the same time to run.

518 The effect of the number of monitoring stations can be assessed by evalu-
 519 ating differences in the indexes' distribution within groups of scenario 1-3, 4-6
 520 and 7-9: while it seems uninfluential for the prediction bias, increasing the
 521 number of monitoring stations helps improve the area predictions in terms
 522 of RMSE and correlation, especially within the scenarios 7-9.

523 The number of monitoring stations has an effect also on the bias and
 524 RMSE of the spatial model parameters (b_0 , b_1 , σ_e^2 , σ_ω^2 and r) since the ac-
 525 curacy of the posterior distributions increases when more locations are used
 526 for estimation. As expected, there is no effect of the prediction size grid and
 527 the parameter estimates for scenario 1-4-7, 2-5-8, 3-6-9 coincide. The results
 528 reported in Table A.1 show that the RMSE is lower for the scenarios 3, 6
 529 and 9, which are the ones with the highest number of stations. This is not
 530 true only for the spatial variance σ_ω^2 which shows the lowest RMSE for the
 531 intermediate scenarios (2, 5 and 8). Regarding the bias, we observe that, on
 532 average, we overestimate the spatial variance σ_ω^2 and the range r , with the
 533 lowest values obtained in the intermediate scenarios (2, 5 and 8) with 403
 534 monitoring stations. It is worth noting that in general the spatial variance
 535 and the range are the most difficult parameters to be estimated and more

536 informative prior could be adopted to help the inferential procedure (Bakar
 537 and Sahu, 2015). For the remaining parameters the values of the bias are
 538 quite small, especially for b_1 and σ_e^2 . Overall the mean bias (RMSE), aver-
 539 aged across all the scenarios, is -0.022 (0.9) for b_0 , -0.001 (0.033) for b_1 , -0.003
 540 (0.019) for σ_e^2 , 0.179 (0.676) for σ_ξ^2 and 0.19 (1.115) for r .

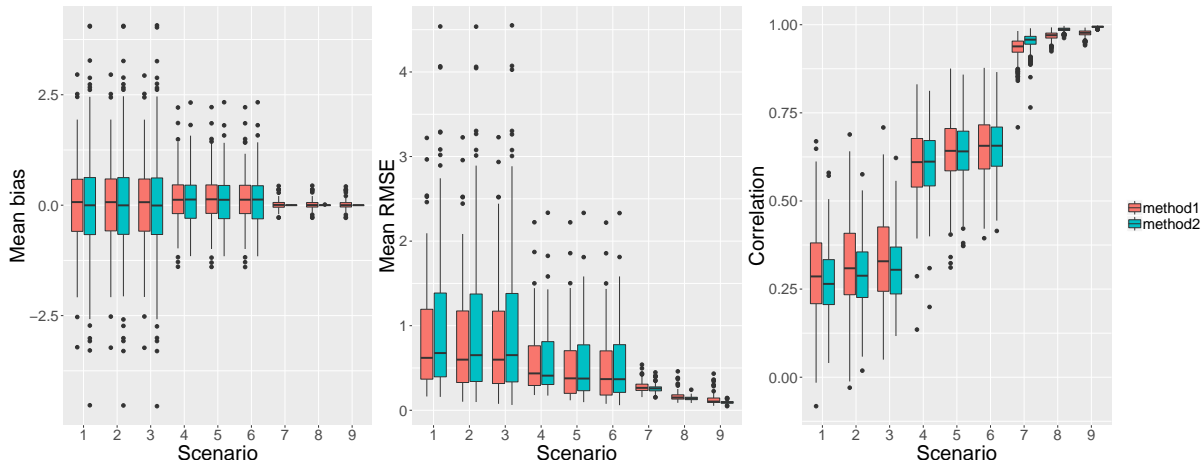


Figure 6: Distribution over areas of the exposure prediction mean bias (left) and mean RMSE (centre), averaged across simulations (see Eq.(10) and (11)), by scenario and upscaling method (Method 1 and Method 2). Right: distribution over simulations of the correlation coefficient between area true exposure values and corresponding predictions, by scenario and upscaling method.

541 The analysis for the log relative risk parameter γ_1 is based on the results
 542 reported in Figure 7. It can be observed that the higher the resolution of
 543 the prediction grid (moving from scenarios 1-3 to scenarios 7-9) the lower the
 544 parameter bias, independently from the upscaling method. The RMSE index
 545 has a slightly different behaviour because the lowest values of the index are
 546 observed for scenarios 4-6; in any case all the RMSE medians are small and
 547 lower than 0.051. No strong differences are observed across the three Pois-
 548 son methods (PI, FF and PE) or across upscaling methods (Method 1 and
 549 Method 2), with the exception of the bias values for scenarios 1-3 which are
 550 always slightly lower for Method 1 even if showing higher variability across
 551 simulations. These results are confirmed by the plots of the γ_1 posterior
 552 distributions reported in Figure A.1 which show very similar patterns across
 553 the 6 cases (3 propagation combined with 2 upscaling methods), in terms of

554 variability and location, especially for the scenarios from 4 to 9. We expected
555 that the three different methods (PI, FF and PE) employed to acknowledge
556 the exposure uncertainty would have led to differences in the posterior distri-
557 bution of the Poisson parameter γ_1 . This effect is not evident from the plot
558 of Figure A.1 or from the posterior standard deviation (SD) values reported
559 in Table A.2, as the three methods return similar posterior distribution with
560 the same level of precision. Nevertheless, it is worth noting that the upscaling
561 method seems to have an effect on the uncertainty: Method 1 (linear com-
562 bination with neighbourhood effect) is always associated with a less precise
563 estimation of γ_1 (higher SD), especially for scenarios 1-6.

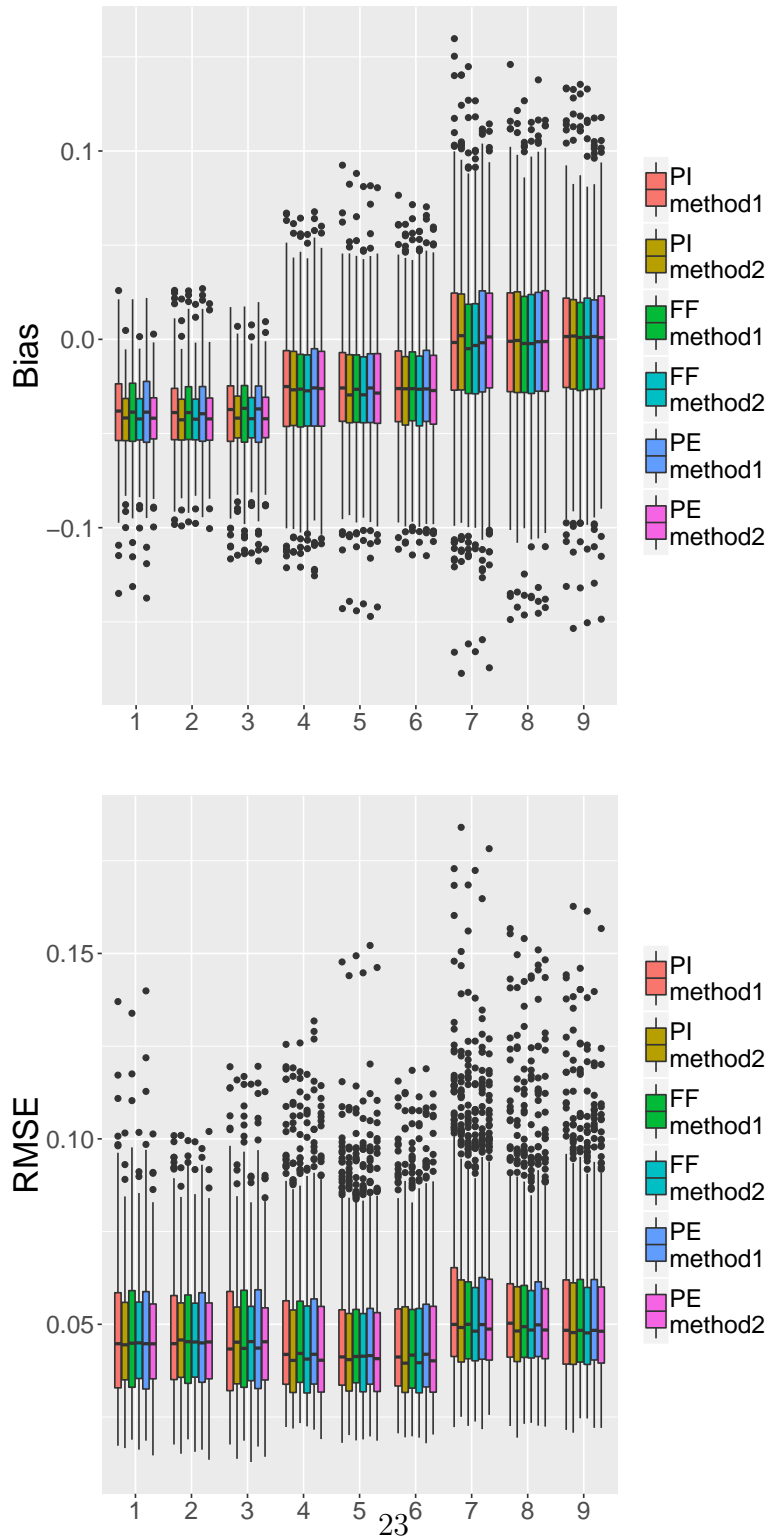


Figure 7: Distribution over simulations of the bias (left) and RMSE (right) for the γ_1 Poisson parameter, by scenario, uncertainty propagation method (PI, FF or PE) and exposure upscaling approach (Method 1, Method 2) for the 9 considered scenarios.

564 6. Motivating example: results

565 Going back to the NO₂ and hospitalisations data for the municipalities
566 in the Piemonte region (presented in Section 2), we first estimate the NO₂
567 exposure model described in Section 3.2 by considering the data from 45 mon-
568 itoring stations among the 55 available sites (10 sites are randomly chosen
569 and set apart for validation purposes) and a SPDE mesh with 533 vertexes.
570 The vector \mathbf{b} of Eq.(5) contains 6 coefficients for the standardized covari-
571 ates (NO₂ emissions, wind speed, temperature, precipitation, mixing height
572 and altitude). The parameter posterior estimates are reported in Table 2 for
573 Method 1 as upscaling method (the results for Method 2 are not reported
574 because they coincide). It can be observed that NO₂ emissions is the only
575 regressor showing a small but positive posterior mean, with the 95% credible
576 interval completely above zero - while for all the meteorological and geo-
577 graphical variables there is not strong evidence of an effect. The intercept
578 estimate, equal to 3.52 on the log scale, corresponds to a posterior median
579 pollution level of 34.54 $\mu\text{g}/\text{m}^3$, after adjustment for covariates. The esti-
580 mate of the measurement error variance σ_ϵ^2 is small (posterior median equal
581 to 0.02) while the variability related to the spatial process is higher with a
582 posterior median for σ_ω^2 equal to 0.10. The posterior mean for σ_ω^2 is 0.12 and
583 this denotes a right skewed posterior distribution; the same happens for the
584 range r which shows a posterior median and mean of about 81 km and 104
585 km, respectively (consider that the maximum distance in the region is 274
586 km).

587 The data of the 10 validation stations are used for computing the pre-
588 diction performance indexes: the correlation between NO₂ observed and pre-
589 dicted measurements is equal to 0.835, the bias is -0.00003 and the RMSE is
590 equal to 0.231. As a basis for comparison, consider that the universal kriging
591 model with the same covariates and Matèrn variogram performs as follows:
592 correlation 0.732, bias 0.026 and RMSE 0.285. Thus, the exposure model
593 outperforms the universal kriging for NO₂ point prediction.

594 Figure 8 reports the maps of the posterior medians of NO₂ concentration
595 (transformed back to the original scale in $\mu\text{g}/\text{m}^3$) for the 56×72 regular
596 grid (left plot) and for the 1206 areas in Piemonte (right plot). As expected,
597 higher concentration are predicted in the areas close to the biggest cities
598 and the main highways that connect Piemonte with Lombardia region on
599 the east. All the municipalities located near the mountain areas surrounding
600 the region on the northern, western and southern side are characterized by

601 concentration lower than $15 \mu\text{g}/\text{m}^3$. These area predictions are obtained
 602 using Method 1 for upscaling; Method 2 returns similar patterns, as it can
 603 be seen in the difference map reported in Figure A.2. However, we observe
 604 that some areas are characterized by different concentration estimates under
 605 Method 1 and Method 2 and this may have an effect on the results of the
 606 Poisson health model. These differences could be related to the geographical
 607 structure of Piemonte region and the number of points used for predicting
 608 exposure with Method 1 and Method 2. In this regard, consider that the
 609 median (mean) number of prediction points is 4 (5.4) for Method 1 and 1
 610 (1.6) for Method 2 (see also Figure A.3 for the distribution of the number of
 611 prediction points across areas).

Parameter	Mean	Sd	0.025quant	Median	0.975quant
Intercept b_0	3.52	0.21	3.06	3.52	3.98
$b_{\text{NO}_2\text{emissions}}$	0.09	0.04	0.01	0.09	0.17
$b_{\text{windspeed}}$	-0.03	0.05	-0.13	-0.03	0.07
$b_{\text{temperature}}$	-0.06	0.06	-0.18	-0.06	0.06
$b_{\text{precipitation}}$	0.10	0.05	0.00	0.10	0.18
$b_{\text{mixingheight}}$	0.12	0.07	-0.01	0.12	0.25
b_{altitude}	-0.19	0.10	-0.39	-0.19	0.02
σ_e^2	0.02	0.01	0.01	0.02	0.05
σ_ω^2	0.12	0.08	0.04	0.10	0.34
r (in metre)	104249.25	76087.92	31034.37	81308.65	311351.38

Table 2: Posterior summaries (mean, standard deviation (sd), 2.5%, 50% and 97.5% quantiles) of the parameters of the NO_2 exposure model estimated using data from 45 monitoring stations.

612 The posterior distributions of the Poisson model parameters are reported
 613 in Figure 9. No evidence of substantial differences can be seen for the inter-
 614 cept γ_0 , the vulnerability parameter γ_2 and the variances σ_v^2 and σ_u^2 : all the
 615 6 compared cases (PI, FF and PE with Method 1 and Method 2) provide
 616 very similar posterior distributions (see also the posterior summary statistics
 617 reported in Table A.3). Some differences can be seen instead for the log-
 618 risk parameter γ_1 (for a $10 \mu\text{g}/\text{m}^3$ increase in NO_2 concentration), which is
 619 the main quantity of interest for assessing the impact of NO_2 exposure on
 620 the health outcome. First of all, we can observe that the upscaling method
 621 has an effect on the location of the posterior distribution: Method 2 (simple
 622 mean of exposure) shifts the distribution to the left (the posterior mean de-

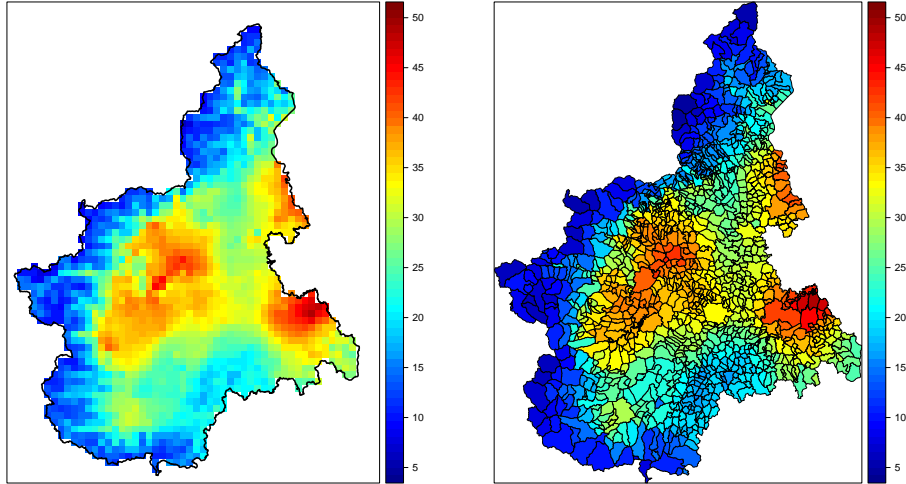


Figure 8: Map of the posterior median of NO_2 concentration (in $\mu\text{g}/\text{m}^3$) at the grid level (left) and at the area level using Method 1 for upscaling (right).

623 creases from 0.0016 to 0.0013 for the PI approach, from 0.0109 to 0.0087 for
 624 FF and from 0.0177 to 0.0145 for PE) while keeping the variability basically
 625 unchanged. The uncertainty propagation method (PI, FF or PE) has an ef-
 626 fect both on the location and on the dispersion of the posterior distributions:
 627 for the FF approach the γ_1 posterior distribution shows higher variability
 628 than the PI and PE strategies, in accordance with the results presented in
 629 [Blangiardo et al. \(2016\)](#), and the posterior medians are closer to zero. As
 630 a consequence, in the FF case there is not strong evidence that the log-risk
 631 parameter γ_1 is different from zero, both for Method 1 and Method 2. The
 632 PI approach is the one showing less variability and this is expected since we
 633 do not take into account the uncertainty of the exposure estimation. The
 634 PE case represents an intermediate situation between PI and FF in terms of
 635 variability (for example, considering Method 1, the posterior standard devi-
 636 ation is equal to 0.0072 for PI, 0.0076 for PE and 0.0103 for FF). Finally,
 637 note that the 95% credible interval for γ_1 does not include zero only in two
 638 cases: PI-Method 1 and PE-Method 1.

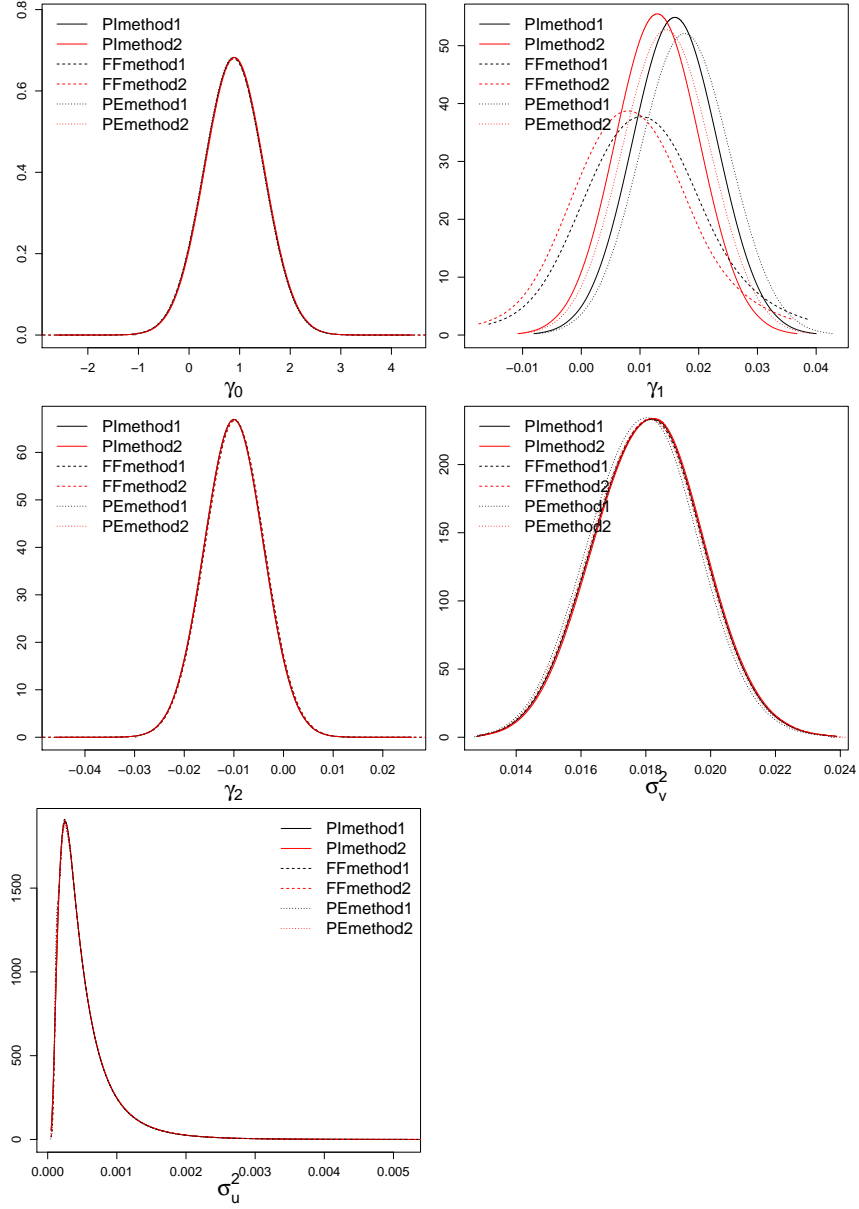


Figure 9: Posterior distributions of the Poisson model parameters: γ_0 , γ_1 for a $10 \mu\text{g}/\text{m}^3$ increase in NO_2 concentration, the vulnerability index parameter γ_2 , the variance σ_v^2 of the iid random effect and the variance σ_u^2 of the spatially structured random effect.

639 **7. Discussion and conclusions**

640 In this paper we presented a two-stage Bayesian model that predict air
641 pollution concentration at the small area level and evaluate its effect on
642 health outcomes. During the first stage the model integrates data from dif-
643 ferent sources characterized by spatial misalignment: monitoring stations are
644 available at point location, while meteorological variables and NO₂ emissions
645 are available at the monitoring station locations as well as for regular grid
646 points; we have then shown two different methods to aggregate the estimates
647 at the regular grid level to the irregular lattice, which is the spatial resolu-
648 tion available for the health outcome. The second stage links the predicted
649 concentration to the health outcome cases and we have presented different
650 ways of accounting for uncertainty from the first to the second stage. In
651 particular, with the feed-forward approach we draw some samples from the
652 joint NO₂ posterior predictive distributions at the area level and fit for each
653 sample the Poisson health model, thus obtaining a posterior distribution of
654 the risk parameter γ_1 . As an alternative, we propose the prior-exposure ap-
655 proach which assumes an informative Normal prior distribution for the area
656 pollutant concentration in the health model (see Eq.(8)). The mean param-
657 eters of these priors are spatially correlated as they are taken from the area
658 level exposure predictive distribution estimated by the SPDE approach. Con-
659 sequently, also the area exposure prior distribution will inherits the spatial
660 structure. Note that instead of using the posterior means and variances from
661 the first stage in the prior, we could sample from the posterior predictive
662 distribution, fit many models with INLA and then obtain a Bayesian model
663 average of the marginals from the fitted models (similarly to [Bivand et al., 2014](#)
664 and [Gómez-Rubio and Rue, 2018](#)). As computationally very intensive
665 we have not followed this approach. However, as noted by [Gómez-Rubio and
666 Palmí-Perales \(2019\)](#), fixing some of the parameters to the ML estimates or
667 posterior means may have little impact on the posterior marginals of the re-
668 mainder of the parameters when fitting (spatial) models with INLA. **Another
669 possible extension of the prior-exposure approach of Eq.(8) would consist in
670 using a proper multivariate Normal prior distribution where the mean vector
671 and covariance matrix are determined by posterior predictive distribution
672 samples from the exposure model (see e.g. [Warren et al., 2012](#); [Lee et al.,
673 2017](#)).**

674 The main challenge we faced in terms of model evaluation was how to
675 assess how accurate a model predicts area level exposure considering that it is

676 not possible to perform cross-validation as it is usually done with spatial point
677 prediction since there are no observed values for the pollutant concentration
678 at the area level. In addition, within a simulative approach, it is non trivial
679 to simulate exposure values for administrative regions given that pollutant
680 concentrations is a continuous spatial field which is usually measured in a
681 limited number of monitoring stations. To overcome this issue, in this paper
682 we simulate the *true* exposure for a very big number of spatial points covering
683 completely the considered region (as to recreate a continuous surface) and
684 then we averaged the values across areas.

685 From the simulation study we found that the exposure performance in-
686 dexes (bias and RMSE) for the predictions at the area level improves with
687 a higher resolution of the prediction grid. However, it is important to note
688 that commonly in real applications this resolution is fixed and given by the
689 regular grid of the numerical models that provide covariates included in the
690 model. Also the number (and position) of monitoring stations, which has a
691 positive effect on the RMSE and correlation indexes, cannot be chosen by
692 researchers but are set by environmental agencies. To deal with the issue
693 of covariate driven grid size, current work consists in a Bayesian space-time
694 model which integrates several numerical outputs characterised by different
695 spatial resolutions together with ground measurements, and is able to predict
696 pollutant concentration at the desired spatial resolution. With this approach
697 the resolution of the covariate grid does not represent a constraint anymore,
698 as the model reconstructs the spatial fields of the misaligned covariates jointly
699 with the latent field of pollutant concentration from ground measurements
700 in a data assimilation framework.

701 From the simulation results it can be observed that Method 1 and Method
702 2 do not seem to have a large effect on the risk parameter γ_1 , in terms of
703 bias, RMSE and posterior distribution pattern. However, the results may
704 depend on the spatial variation within and between the areas; as Method 1
705 averages across more points it should provide more stable estimates of the
706 concentration, particularly in case of local spatial variability. This is more
707 evident on the case study on Piemonte, where Method 2 shifts the posterior
708 distribution of the parameter towards zero. In addition, it is interesting to
709 note that while the FF method of uncertainty propagation results in estimates
710 of the health effect shifted towards 0 and more variable with respect to the
711 other approaches, the PE method leads to higher deviation from 0 with an
712 intermediate level of variability.

713 The choice of the methods to link stage 1 to stage 2 were motivated by

714 the fact that we wanted these to be easily adopted by the researchers' wide
715 community; hence we focused on methods which are easily implemented in
716 a readily available R package such as R-INLA without the need for algorithm
717 writing. An alternative to the two-stage specification will consist of a joint
718 model of the exposure and of its health effects; this would ensure that uncer-
719 tainty is directly propagated across it. However the level of computational
720 power required to run the model would increase substantially and at the
721 same time a two-stage approach proves more robust if the exposure model is
722 misspecified, hence we believe that a two-stage approach is to be preferred.
723 Future works will extend the current framework to fit spatio-temporal data
724 and to model multi-pollutant concentrations.

725 Acknowledgements

726 MC has been supported by the PRIN EphaStat Project (Project No.
727 20154X8K23, <https://sites.google.com/site/ephastat/>) provided by the
728 Italian Ministry for Education, University and Research. MB acknowledges
729 support from the MRC-PHE Centre for Environment and Health, funded
730 by the Medical Research Council (MR/L01341X/1) and Public Health Eng-
731 land (PHE). VGR has been supported by grant SBPLY/17/180501/000491,
732 awarded by Consejería de Educación, Cultura y Deportes (JCCM, Spain) and
733 FEDER, and grant MTM2016-77501-P, awarded by Ministerio de Economía
734 y Competitividad (Spain).

735 Atkinson, R. W., A. Analitis, E. Samoli, G. W. Fuller, D. C. Green, I. S.
736 Mudway, H. R. Anderson, and F. J. Kelly (2016). Short-term exposure to
737 traffic-related air pollution and daily mortality in London, UK. *Journal of*
738 *Exposure Science and Environmental Epidemiology* 26(2), 125.

739 Bakar, K. and S. Sahu (2015). sptimer: Spatio-temporal bayesian modeling
740 using r. *Journal of Statistical Software, Articles* 63(15), 1–32.

741 Bell, M. L. (2006). The use of ambient air quality modeling to estimate indi-
742 vidual and population exposure for human health research: A case study of
743 ozone in the Northern Georgia Region of the United States. *Environment*
744 *International* 32(5), 586–593.

745 Berrocal, V. J., A. E. Gelfand, and D. M. Holland (2010). A spatio-temporal
746 downscaler for output from numerical models. *Journal of Agricultural,*
747 *Biological, and Environmental Statistics* 15(2), 176–197.

- 748 Besag, J., J. York, and A. Mollié (1991). Bayesian Image Restoration with
749 Two Applications in Spatial Statistics. *The Annals of the Institute of*
750 *Statistics and Mathematics* 43(1), 1–59.
- 751 Bivand, R. and G. Piras (2015). Comparing implementations of estimation
752 methods for spatial econometrics. *Journal of Statistical Software* 63(18),
753 1–36.
- 754 Bivand, R. S., V. Gómez-Rubio, and H. Rue (2014). Approximate Bayesian
755 inference for spatial econometrics models. *Spatial Statistics* 9, 146–165.
- 756 Blangiardo, M. and M. Cameletti (2015). *Spatial and Spatio-temporal*
757 *Bayesian Models with R-INLA*. Wiley.
- 758 Blangiardo, M., F. Finazzi, and M. Cameletti (2016). Two-stage bayesian
759 model to evaluate the effect of air pollution on chronic respiratory diseases
760 using drug prescriptions. *Spatial and Spatio-temporal Epidemiology* 18,
761 1–12.
- 762 Bruno, F., M. Cameletti, M. Franco-Villoria, F. Greco, R. Ignaccolo, L. Ip-
763 polito, P. Valentini, and M. Ventrucci (2016). A survey on ecological re-
764 gression for health hazard associated with air pollution. *Spatial Statis-
765 tics* 18(Part A), 276–299.
- 766 Cameletti, M., R. Ignaccolo, and S. Bande (2011). Comparing spatio-
767 temporal models for particulate matter in Piemonte. *Environ-
768 metrics* 22(8), 985–996.
- 769 Cameletti, M., F. Lindgren, D. Simpson, and H. Rue (2013). Spatio-temporal
770 modeling of particulate matter concentration through the SPDE approach.
771 *AStA Advances in Statistical Analysis* 97(2), 109–131.
- 772 Carey, I. M., H. R. Anderson, R. W. Atkinson, S. Beevers, D. G. Cook,
773 D. Dajnak, J. Gulliver, and F. J. Kelly (2016). Traffic pollution and the
774 incidence of cardiorespiratory outcomes in an adult cohort in London. *Oc-
775 cupational and Environmental Medicine* 73(12), 849–856.
- 776 Carugno, M., D. Consonni, G. Randi, D. Catelan, L. Grisotto, P. A. Bertazzi,
777 A. Biggeri, and M. Baccini (2016). Air pollution exposure, cause-specific
778 deaths and hospitalizations in a highly polluted Italian region. *Environ-
779 mental research* 147, 415–424.

- 780 EEA (2017a). Air quality in Europe. Technical report, European Envi-
781 ronmental Agency (EEA). [https://www.eea.europa.eu/publications/
782 air-quality-in-europe-2017/at_download/file](https://www.eea.europa.eu/publications/air-quality-in-europe-2017/at_download/file).
- 783 EEA (2017b). Urban PM2.5 Atlas: Air Quality in Eu-
784 ropean cities. Technical report, European Environmen-
785 tal Agency (EEA). [https://ec.europa.eu/jrc/en/
786 publication/eur-scientific-and-technical-research-reports/
787 urban-pm25-atlas-air-quality-european-cities](https://ec.europa.eu/jrc/en/publication/eur-scientific-and-technical-research-reports/urban-pm25-atlas-air-quality-european-cities).
- 788 Elliott, P., G. Shaddick, J. C. Wakefield, C. d. Hoogh, and D. J. Briggs
789 (2007). Long-term associations of outdoor air pollution with mortality in
790 Great Britain. *Thorax* 62(12), 1088–1094.
- 791 Fassò, A. and F. Finazzi (2011). Maximum likelihood estimation of
792 the dynamic coregionalization model with heterotopic data. *Environ-
793 metrics* 22(6), 735–748.
- 794 Faustini, A., R. Rapp, and F. Forastiere (2014). Nitrogen dioxide and mortal-
795 ity: review and meta-analysis of long-term studies. *European Respiratory
796 Journal* 44(3), 744–753.
- 797 Fuentes, M., H.-R. Song, S. K. Ghosh, D. M. Holland, and J. M. Davis
798 (2006). Spatial association between speciated fine particles and mortality.
799 *Biometrics* 62(3), 855–863.
- 800 Gelfand, A. (2010). *Handbook of Spatial Statistics*, Chapter Misaligned Spa-
801 tial Data: The Change of Support Problem. Chapman & Hall.
- 802 Gómez-Rubio, V. and F. Palmí-Perales (2019). Multivariate posterior in-
803 ference for spatial models with the integrated nested laplace approxima-
804 tion. *Journal of the Royal Statistical Society: Series C (Applied Statis-
805 tics)* 68(1), 199–215.
- 806 Gómez-Rubio, V. and H. Rue (2018). Markov chain Monte Carlo with the In-
807 tegrated Nested Laplace Approximation. *Statistics and Computing* 28(5),
808 1033–1051.
- 809 Halonen, J. I., M. Blangiardo, M. B. Toledano, D. Fehcht, J. Gulliver, H. R.
810 Anderson, S. D. Beevers, D. Dajnak, F. J. Kelly, and C. Tonne (2016).

- 811 Long-term exposure to traffic pollution and hospital admissions in London.
812 *Environmental Pollution* 208, 48–57.
- 813 Halonen, J. I., M. Blangiardo, M. B. Toledano, D. Fecht, J. Gulliver,
814 R. Ghosh, H. R. Anderson, S. D. Beevers, D. Dajnak, F. J. Kelly, et al.
815 (2016). Is long-term exposure to traffic pollution associated with mortal-
816 ity? A small-area study in London. *Environmental Pollution* 208, 25–32.
- 817 Huang, G., D. Lee, and M. Scott (2015). An integrated Bayesian model for
818 estimating the long-term health effects of air pollution by fusing modelled
819 and measured pollution data: a case study of nitrogen dioxide concentra-
820 tions in Scotland. *Spatial and Spatio-temporal Epidemiology* 14-15, 63–74.
- 821 Huang, G., M. Scott, and D. Lee (2017). Multivariate space-time modelling
822 of multiple air pollutants and their health effects accounting for exposure
823 uncertainty. *Statistics in Medicine*, 1–15.
- 824 Lee, A., A. Szpiro, S. Kim, and L. Sheppard (2015). Impact of preferential
825 sampling on exposure prediction and health effect inference in the context
826 of air pollution epidemiology. *Environmetrics* 26(4), 255–267.
- 827 Lee, D. (2011). A comparison of conditional autoregressive models used
828 in Bayesian disease mapping. *Spatial and Spatio-Temporal Epidemiol-
829 ogy* 2(2), 79–89.
- 830 Lee, D. (2018). A locally adaptive process-convolution model for estimating
831 the health impact of air pollution. *Annals of Applied Statistics*.
- 832 Lee, D., S. Mukhopadhyay, A. Rushworth, and S. K. Sahu (2017). A rig-
833 orous statistical framework for spatio-temporal pollution prediction and
834 estimation of its long-term impact on health. *Biostatistics* 18(2), 370–385.
- 835 Lee, D. and S. K. Sahu (2016). *Handbook of Spatial Epidemiology*, Chapter
836 Estimating the Health Impact of Air Pollution Fields. Chapman & Hall.
- 837 Lee, D. and C. Sarran (2015). Controlling for unmeasured confounding and
838 spatial misalignment in long-term air pollution and health studies. *Envi-
839 ronmetrics* 26, 477–487.
- 840 Lee, D. and G. Shaddick (2010). Spatial modeling of air pollution in studies
841 of its short-term health effects. *Biometrics* 66(4), 1238–1246.

- 842 Lim, S. S., T. Vos, et al. (2012). A comparative risk assessment of burden
843 of disease and injury attributable to 67 risk factors and risk factor clusters
844 in 21 regions, 1990-2010: a systematic analysis for the global burden of
845 disease study 2010. *The Lancet* 380(9859), 2224–2260.
- 846 Lindgren, F. and H. Rue (2015). Bayesian Spatial Modelling with R-INLA.
847 *Journal of Statistical Software* 63(19), 1–25.
- 848 Lindgren, F., H. Rue, and J. Lindström (2011). An explicit link between
849 Gaussian fields and Gaussian Markov random fields: the stochastic partial
850 differential equation approach. *Journal of the Royal Statistical Society:
851 Series B* 73(4), 423–498.
- 852 Liu, Y., G. Shaddick, and J. V. Zidek (2016). Incorporating high-dimensional
853 exposure modelling into studies of air pollution and health. *Statistics in
854 Biosciences*, 1–23.
- 855 Madsen, L., D. Ruppert, and N. S. Altman (2008). Regression with spatially
856 misaligned data. *Environmetrics* 19(5), 453–467.
- 857 Moore, E., L. Chatzidiakou, M.-O. Kuku, R. L. Jones, L. Smeeth, S. Beevers,
858 F. J. Kelly, B. Barratt, and J. K. Quint (2016). Global associations between
859 air pollutants and chronic obstructive pulmonary disease hospitalizations.
860 a systematic review. *Annals of the American Thoracic Society* 13(10),
861 1814–1827.
- 862 Moraga, P., S. M. Cramb, K. L. Mengersen, and M. Pagano (2017). A
863 geostatistical model for combined analysis of point-level and area-level data
864 using inla and spde. *Spatial Statistics* 21, 27 – 41.
- 865 OECD (2016). The economic consequences of outdoor air
866 pollution. Technical report, Organisation for Economic
867 Co-operation and Development (OECD). [http://www.
868 oecd.org/environment/indicators-modelling-outlooks/
869 Policy-Highlights-Economic-consequences-of-outdoor-air-pollution-web.
870 pdf](http://www.oecd.org/environment/indicators-modelling-outlooks/Policy-Highlights-Economic-consequences-of-outdoor-air-pollution-web.pdf).
- 871 Pannullo, F., D. Lee, E. Waclawski, and A. H. Leyland (2015). Improving
872 spatial nitrogen dioxide prediction using diffusion tubes: A case study in
873 West Central Scotland. *Atmospheric Environment* 118, 227–235.

- 874 Pannullo, F., D. Lee, E. Waclawski, and A. H. Leyland (2016). How robust
875 are the estimated effects of air pollution on health? accounting for model
876 uncertainty using bayesian model averaging. *Spatial and Spatio-temporal*
877 *Epidemiology* 18, 53–62.
- 878 Peng, R. D. and M. L. Bell (2010). Spatial misalignment in time series studies
879 of air pollution and health data. *Biostatistics (Oxford, England)* 11(4),
880 720–740.
- 881 Pirani, M., J. Gulliver, G. W. Fuller, and M. Blangiardo (2014). Bayesian spa-
882 tiotemporal modelling for the assessment of short-term exposure to particle
883 pollution in urban areas. *Journal of Exposure Science and Environmental*
884 *Epidemiology* 24(3), 319–327.
- 885 Powell, H. and D. Lee (2014). Modelling spatial variability in concentrations
886 of single pollutants and composite air quality indicators in health effects
887 studies. *Journal of the Royal Statistical Society: Series A (Statistics in*
888 *Society)* 177(3), 607–623.
- 889 Raaschou-Nielsen, O., Z. J. Andersen, S. S. Jensen, M. Ketzel, M. Sørensen,
890 J. Hansen, S. Loft, A. Tjønneland, and K. Overvad (2012). Traffic air
891 pollution and mortality from cardiovascular disease and all causes: a danish
892 cohort study. *Environmental Health* 11(1), 60.
- 893 Rue, H., S. Martino, and N. Chopin (2009). Approximate Bayesian inference
894 for latent Gaussian models by using integrated nested Laplace approxima-
895 tions. *Journal of the Royal Statistical Society: Series B* 2(71), 1–35.
- 896 Rue, H., A. Riebler, S. H. S., J. B. Illian, D. P. Simpson, and F. K. Lind-
897 gren (2017). Bayesian computing with inla: A review. *Annual Review of*
898 *Statistics and Its Application* 4(1), 395–421.
- 899 Rushworth, A., D. Lee, and R. Mitchell (2014). A spatio-temporal model for
900 estimating the long-term effects of air pollution on respiratory hospital ad-
901 missions in Greater London. *Spatial and spatio-temporal epidemiology* 10,
902 29–38.
- 903 Sahu, S. (2011). Hierarchical Bayesian models for space-time air pollution
904 data. In C. Rao (Ed.), *Handbook of Statistics-Time Series Analysis, Meth-*
905 *ods and Applications*, Volume 30 of *Handbook of Statistics*. Elsevier Pub-
906 lishers, Holland.

- 907 Sahu, S., A. Gelfand, and D. Holland (2010). Fusing point and areal level
908 spacetime data with application to wet deposition. *Journal of the Royal*
909 *Statistical Society: Series C* 59(1), 77–103.
- 910 Sanyal, S., T. Rochereau, C. N. Maesano, L. Com-Ruelle, and I. Annesi-
911 Maesano (2018, 11). Long-term effect of outdoor air pollution on mor-
912 tality and morbidity: A 12-year follow-up study for metropolitan france.
913 *International journal of environmental research and public health* 15(11),
914 2487.
- 915 Shaddick, G., M. L. Thomas, A. Green, M. Brauer, A. van Donkelaar, R. Bur-
916 nett, H. H. Chang, A. Cohen, R. V. Dingenen, C. Dora, S. Gumy, Y. Liu,
917 R. Martin, L. A. Waller, J. West, J. V. Zidek, and A. Prüss-Ustün (2018).
918 Data integration model for air quality: a hierarchical approach to the
919 global estimation of exposures to ambient air pollution. *Journal of the*
920 *Royal Statistical Society: Series C (Applied Statistics)* 67(1), 231–253.
- 921 Shaddick, G. and J. V. Zidek (2016). *Spatio-Temporal Methods in Environ-*
922 *mental Epidemiology*. CRC Press.
- 923 Wakefield, J. and G. Shaddick (2006). Health-exposure modeling and the
924 ecological fallacy. *Biostatistics* 7(3), 438–455.
- 925 Wang, Y., M. Pirani, A. L. Hansell, S. Richardson, and M. Blangiardo (2019).
926 Using ecological propensity score to adjust for missing confounders in small
927 area studies. *Biostatistics (Oxford, England)* 20(1), 1–16.
- 928 Warren, J., M. Fuentes, A. Herring, and P. Langlois (2012, 12). Spatial-
929 temporal modeling of the association between air pollution exposure and
930 preterm birth: identifying critical windows of exposure. *Biometrics* 68(4),
931 1157–1167.
- 932 Young, L. J., C. A. Gotway, J. Yang, G. Kearney, and C. DuClos (2009).
933 Linking health and environmental data in geographical analysis: It’s so
934 much more than centroids. *Spatial and Spatio-temporal Epidemiology* 1(1),
935 73–84.
- 936 Zhu, L., B. P. Carlin, and A. Gelfand (2003). Hierarchical regression with
937 misaligned spatial data: relating ambient ozone and pediatric asthma ER
938 visits in Atlanta. *Environmetrics* 14(5), 537–557.

939 **Appendix A. Extra tables and figures**

Scenario	b_0		b_1		σ_e^2		σ_ξ^2		r	
	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
1	-0.044	1.033	-0.001	0.064	-0.012	0.039	0.181	0.708	0.188	1.450
2	0.001	0.845	-0.001	0.023	0.001	0.012	0.151	0.624	0.174	1.007
3	-0.018	0.830	-0.000	0.013	0.001	0.006	0.195	0.683	0.195	0.869
4	-0.044	1.032	-0.001	0.064	-0.012	0.039	0.180	0.707	0.188	1.450
5	-0.014	0.838	-0.001	0.023	0.001	0.011	0.155	0.638	0.186	0.997
6	-0.012	0.821	0.000	0.013	0.001	0.006	0.204	0.688	0.200	0.903
7	-0.044	1.039	-0.001	0.064	-0.012	0.040	0.181	0.708	0.196	1.461
8	-0.014	0.838	-0.001	0.023	0.001	0.011	0.154	0.636	0.185	0.996
9	-0.012	0.822	0.000	0.013	0.001	0.006	0.206	0.692	0.200	0.903

Table A.1: Bias and RMSE (averaged across simulations) for the spatial parameters.

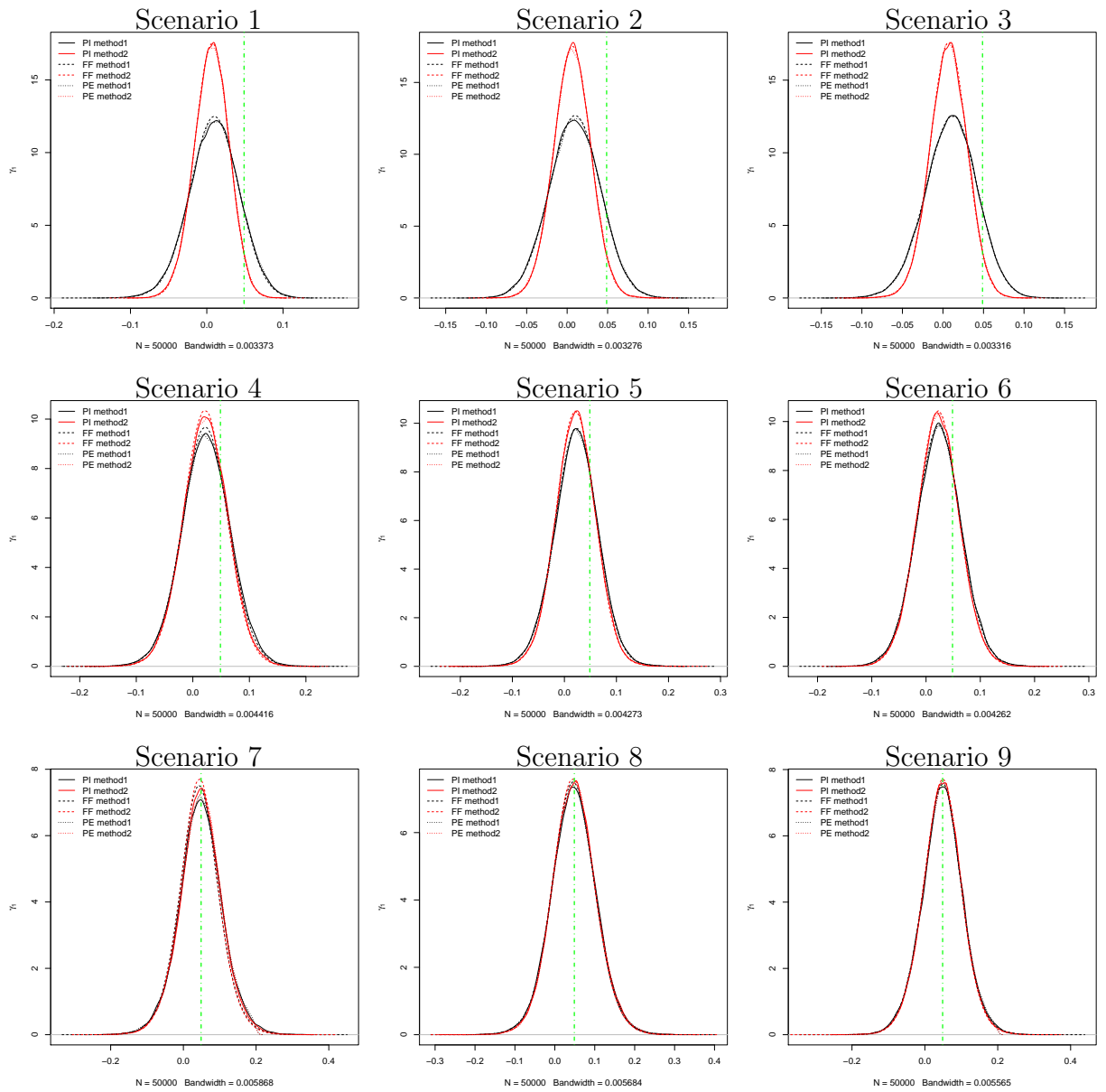


Figure A.1: Posterior distributions of the Poisson model parameter γ_1 for the 9 scenarios considered in the simulation study according to the upscaling method (Method 1, Method 2) and the uncertainty propagation approach (PI, FF, PE). The vertical green line represents the true parameter value.

Scenario	Propagation method	Upscaling method	
		Method 1	Method 2
1	PI	0.033	0.023
1	FF	0.032	0.023
1	PE	0.033	0.023
2	PI	0.032	0.023
2	FF	0.032	0.023
2	PE	0.032	0.023
3	PI	0.032	0.023
3	FF	0.032	0.023
3	PE	0.032	0.023
4	PI	0.043	0.040
4	FF	0.042	0.039
4	PE	0.043	0.040
5	PI	0.042	0.039
5	FF	0.042	0.039
5	PE	0.042	0.039
6	PI	0.042	0.039
6	FF	0.042	0.039
6	PE	0.042	0.040
7	PI	0.060	0.058
7	FF	0.058	0.056
7	PE	0.058	0.057
8	PI	0.057	0.055
8	FF	0.056	0.055
8	PE	0.057	0.055
9	PI	0.057	0.055
9	FF	0.056	0.055
9	PE	0.056	0.054

Table A.2: Posterior standard deviation for the Poisson model parameter γ_1 for the 9 scenarios considered in the simulation study according to the upscaling method (Method 1, Method 2) and the uncertainty propagation approach (PI, FF, PE).

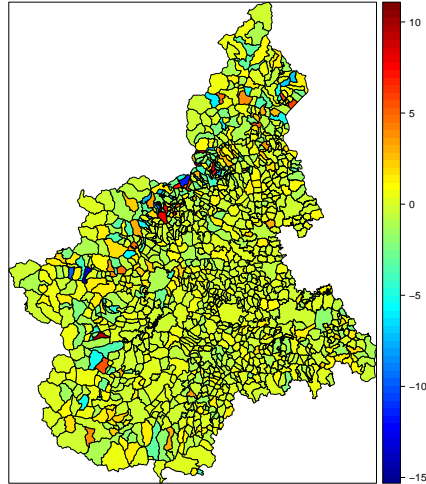


Figure A.2: Map of the differences between the posterior medians of NO₂ concentrations (in $\mu\text{g}/\text{m}^3$) obtained using Method 1 and Method 2.

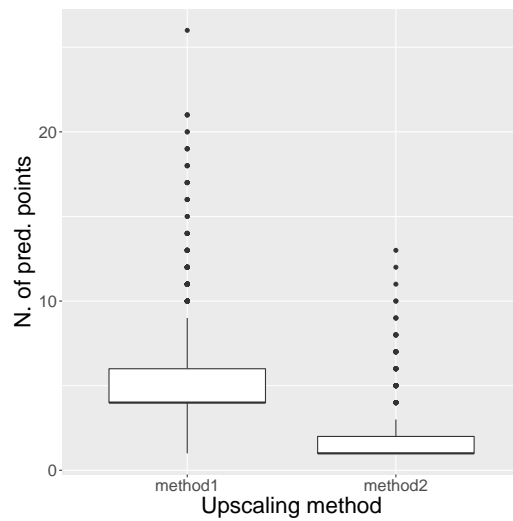


Figure A.3: Distribution of the number of prediction points across areas of the Piemonte region according to the upscaling method (Method 1 and Method 2).

Parameter	Method	Mean	Sd	Quant0.025	Median	Quant0.975
γ_0	PI method1	0.8866	0.5850	-0.2622	0.8851	2.0308
	PI method2	0.8989	0.5851	-0.2500	0.8974	2.0433
	FF method1	0.8824	0.5879	-0.2738	0.8798	2.0309
	FF method2	0.8916	0.5873	-0.2633	0.8890	2.0391
	PE method1	0.8741	0.5817	-0.2693	0.8733	2.0147
	PE method1	0.8887	0.5817	-0.2551	0.8880	2.0290
γ_1	PI method1	0.0159	0.0072	0.0018	0.0159	0.0301
	PI method2	0.0130	0.0072	-0.0011	0.0129	0.0270
	FF method1	0.0109	0.0103	-0.0088	0.0106	0.0326
	FF method2	0.0087	0.0100	-0.0105	0.0084	0.0298
	PE method1	0.0177	0.0076	0.0027	0.0176	0.0326
	PE method2	0.0145	0.0075	-0.0003	0.0145	0.0293
γ_2	PI method1	-0.0100	0.0060	-0.0217	-0.0100	0.0017
	PI method2	-0.0100	0.0060	-0.0217	-0.0101	0.0016
	FF method1	-0.0098	0.0060	-0.0216	-0.0098	0.0019
	FF method2	-0.0098	0.0060	-0.0216	-0.0099	0.0019
	PE method1	-0.0099	0.0059	-0.0216	-0.0099	0.0017
	PE method2	-0.0100	0.0059	-0.0216	-0.0100	0.0016
σ_v^2	PI method1	0.0181	0.0017	0.0148	0.0181	0.0214
	PI method2	0.0181	0.0017	0.0149	0.0181	0.0214
	FF method1	0.0181	0.0017	0.0148	0.0181	0.0214
	FF method2	0.0181	0.0017	0.0148	0.0181	0.0214
	PE method1	0.0179	0.0017	0.0147	0.0179	0.0212
	PE method2	0.0180	0.0017	0.0148	0.0180	0.0213
σ_u^2	PI method1	0.0005	0.0004	0.0001	0.0004	0.0017
	PI method2	0.0005	0.0004	0.0001	0.0004	0.0017
	FF method1	0.0005	0.0004	0.0001	0.0004	0.0017
	FF method2	0.0005	0.0004	0.0001	0.0004	0.0017
	PE method1	0.0005	0.0004	0.0001	0.0004	0.0017
	PE method2	0.0005	0.0004	0.0001	0.0004	0.0017

Table A.3: Posterior summary statistics (mean, standard deviation (sd), 2.5%, 50% and 97.5% quantiles) for the Poisson model parameters according to the upscaling method (Method 1, Method2) and the propagation approach (PI, FF, PE). Recall that γ_1 refers to a $10 \mu\text{g}/\text{m}^3$ increase in NO_2 concentration.