Strategic Inventories under Limited Commitment

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Strategic inventories under limited commitment

Fabio Antoniou∗ and Raffaele Fiocco†

Abstract

In a dynamic storable good market where demand changes over time, we investigate the producer’s strategic incentives to hold inventories in response to the possibility of buyer stockpiling. The literature on storable goods has demonstrated that buyer stockpiling in anticipation of higher future prices harms the producer’s profitability, particularly when the producer cannot commit to future prices. We show that the producer’s inventories act as a strategic device to mitigate the loss from the lack of commitment. Our results provide a rationale for the producer’s inventory behavior that sheds new light on the well-documented empirical evidence about inventories.

Keywords: buyer stockpiling, commitment, storable goods, strategic inventories.
JEL Classification: D21, D42, L12.

∗University of Ioannina, Department of Economics, and Humboldt University of Berlin, Institute for Economic Theory I. Email address: fabio.antoniou@wiwi.hu-berlin.de
†Universitat Rovira i Virgili, Department of Economics and CREIP, Avinguda de la Universitat 1, 43204 Reus, Spain. Email address: raffaele.fiocco@urv.cat
1 Introduction

Inventory management is essential for a firm’s viability in many industries. Although the share of inventory investment in the gross domestic product (GDP) is relatively small, changes in inventories constitute a significant component of economic fluctuations. During the recent financial crisis, the reduction in inventories accounted for 29% of the decline in GDP (Wang et al. 2014).

Traditional reasons for inventories are driven by technological features, such as production smoothing over time in the presence of convex production costs and stockout avoidance when production takes time and cannot be immediately adjusted to demand shocks (e.g., Aguirregabiria 1999; Anupindi et al. 2012; Arrow et al. 1951; Arvan and Moses 1982; Holt et al. 1960; Kahn 1987; Krishnan and Winter 2007, 2010; Nahmias 2008; Zipkin 2000). The empirical evidence indicates that inventories are procyclical and production is more variable than sales (e.g., Blanchard 1983; Blinder 1986; Ramey and West 1999; Wen 2005). Information about the production and inventory activities is generally available in industry reports, financial statements as well as balance sheets. Such information is also collected in accurate databases. For instance, Standard & Poor’s Compustat has provided since 1962 financial, statistical and market data about companies throughout the world.\(^1\)

Information about the 100 largest companies traded on the US stock exchanges can be regularly found at https://www.stock-analysis-on.net.

The presence of intermediaries and arbitrageurs in the commodity markets suggests that buyers also exhibit incentives to store. A recent strand of the empirical literature has systematically documented buyer stockpiling in anticipation of higher future prices in markets for various intermediate and final goods (e.g., Erdem et al. 2003; Hall and Rust 2000; Hendel and Nevo 2004, 2006a, 2006b; Pesendorfer 2002).

The storage activities of firms and their customers have been examined separately in the literature so far. In this paper, we provide a unified framework in order to investigate the inventory behavior of a producer vis-à-vis forward-looking buyers that are willing to store in anticipation of higher future prices. Abstracting from the aforementioned classical technological reasons for inventories, we show that a producer unable to commit to future prices has strategic incentives to hold inventories when facing the prospect of buyer stockpiling. Our results provide theoretical support for the main stylized facts about the firms’ inventory activities.

We focus our attention on storable goods, which are perishable in consumption but can be stored for future consumption. Typical examples include various intermediate goods (e.g., oil, coffee and wheat) and groceries that can be purchased in advance and stored. In order to characterize the strategic role of the producer’s inventories, we build on the seminal paper of Dudine et al. (2006), which considers a dynamic storable good market with deterministic time-varying demand where a monopolistic producer cannot commit to future prices and faces a continuum of competitive buyers available to stockpile the good in anticipation of higher future prices. Dudine et al. (2006) show that the excessively high future prices driven by the producer’s lack of commitment trigger buyer stockpiling, which is ex ante profit detrimental in that it reduces future sales occurring at higher prices. As a result of wasteful buyer stockpiling, the producer’s profits and consumer surplus are lower than under full commitment, which

\(^1\)Information about the 100 largest companies traded on the US stock exchanges can be regularly found at https://www.stock-analysis-on.net.
unambiguously reduces aggregate welfare. In this setting, we introduce the possibility for the firm to accumulate production in the form of inventories available for future sales. As a practical example, consider the oil market in the US, where a large oil producer (or refiner) generally supplies competitive distributors (or directly the petrol stations) and gathers some quantity in its depositories to cover future demand. The distributors (and petrol stations) are also endowed with storage capacities. In many other markets for storable goods — such as bananas, bauxite, coffee, copper, diamonds, iron ore, mercury, phosphates, and tin — supply is relatively concentrated, with some producers possessing large market power. Competitive speculators trade these goods and engage in stockpiling activities.\(^2\)

In our baseline framework, the firm’s production costs are linear and do not vary over time, while demand evolves deterministically. Therefore, there is no scope for the well-documented technological motives for inventories, and the producer shall engage in inventory activities only for strategic purposes. Under full commitment, the producer does not benefit from holding inventories, since it can credibly announce a price sequence that removes buyer stockpiling and inventories would only result in a mere loss due to their holding costs. However, this conclusion is no longer valid when the producer is unable to commit to future prices, and inventories can emerge in equilibrium. To understand the rationale for this result, it is important to realize that the costs of inventories are sunk once they have been incurred. A producer that cannot commit to future prices accumulates inventories as a strategic device to reduce future costs, which alleviates the producer’s temptation to charge high future prices and weakens the buyer stockpiling incentives.

We find that under certain circumstances, despite the lack of commitment, the producer holds the amount of inventories that maximizes the ex ante profits, and the constraint of sequential optimality is slack in equilibrium. As under full commitment, buyer stockpiling is fully removed, which occurs at the additional cost of holding inventories. Remarkably, this solution is implementable only if the unit cost of production is large enough. \(Prima\ facie,\) this might seem counterintuitive, since one could expect that a more efficient producer finds inventories more attractive. The rationale for this result arises from the strategic nature of inventories. To illustrate ideas, consider a two-period setting. The inventory level accumulated in the first period to serve the demand in the second period is associated with a second period price at which the quantity sold coincides with inventories. When the unit cost of production is large enough, the ex ante optimal level of inventories is relatively small and in the second period the producer does not succumb to the temptation to discard some inventories and to set a price above the level at which inventories are fully sold. Moreover, as long as the unit inventory holding cost is relatively small, the firm does not have any incentive to produce some additional quantity in the second period, either. Given that the producer shall charge the second period price at which the quantity sold coincides with inventories, the ex ante optimal inventory level is also sequentially optimal. Anticipating that the sunkness of inventory costs alleviates the producer’s temptation to charge high future prices, buyers abstain from any stockpiling activity. Therefore, inventories constitute a strategic device to mitigate the producer’s loss from the lack of commitment.

\(^2\)Governments can also implement stockpiling policies, especially in order to protect against future supply disruptions (e.g., Nichols and Zeckhauser 1977).
Notably, the producer accumulates in the first period the amount of inventories that covers the entire second period demand in order to remove buyer stockpiling, which constitutes only a portion of the second period demand. This suggests that inventories and buyer stockpiling are imperfect substitutes in equilibrium. As discussed in the sequel, inventories can also generate allocative benefits for the producer, since they generally lead to lower prices that approach the full commitment outcome. This implies that the threshold for the unit inventory holding cost below which the producer prefers to hold inventories is such that the aggregate inventory holding costs exceed the aggregate buyer stockpiling costs. When the inventory holding costs vanish, inventories become a fully effective instrument to prevent buyer stockpiling and the full commitment outcome is restored.

If the unit cost of production is small enough, the ex ante optimal level of inventories is relatively large and in the second period the producer cannot refrain from discarding a portion of the inventories accumulated in the first period and from setting a second period price above the level at which inventories are fully sold. In other words, the ex ante optimal level of inventories is not sequentially optimal and cannot be sustained in equilibrium. We show that, even in this case, inventories can play a strategic role in mitigating the buyer stockpiling incentives. It follows from our previous discussion that inventories must be distorted away from the ex ante optimal level, and the constraint of sequential optimality is binding in equilibrium. Inventories are not a fully effective instrument to alleviate the buyer stockpiling incentives, even when their holding costs are nil. As a result, the producer’s inventories and buyer stockpiling can now coexist in equilibrium.

When the unit cost of production is relatively large and therefore the ex ante optimal inventory level is also sequentially optimal, inventories generally lead to lower prices although they involve holding costs. The rationale for this apparently surprising result stems from the strategic nature of inventories, which mitigate the producer’s temptation to set high future prices and soften the buyer stockpiling incentives. Hence, despite being used in the producer’s private interest, inventories increase consumer surplus and aggregate welfare. This conclusion deserves some qualifications when the unit cost of production is relatively small and therefore the equilibrium inventory level departs from the ex ante optimal level. As shown in Section 6, the comparisons between equilibrium prices are driven by the demand curvature and the inventory holding costs in a non-trivial manner. Equilibrium prices exhibit peculiar features that merit some attention. For instance, we find that a lower unit inventory holding cost may lead to higher prices.

Our paper provides a novel strategic rationale for the firms’ inventory activities, which does not lie in the specific production technologies assumed by the traditional inventory theories. As discussed in Section 9, our results shed new light on the empirical evidence about inventories described at the beginning. The predictions of our model lend themselves to an empirical or experimental validation and can stimulate further investigation into the firms’ inventory management. A relevant market where our results can be applied is the oil market that exhibits relatively large costs of production and small inventory holding costs.\footnote{In 2015, with a crude oil price of around 48 dollars per barrel, in the US the oil production costs were 36 dollars per barrel, while the inventory holding costs only amounted to about 0.5 dollars per barrel.}
In practice, inventories are generally split into three different categories, namely, raw material inventories, work-in-progress inventories and final product inventories. For the sake of simplicity, in our framework inventories coincide with final products. Yet, the same rationale naturally carries over to any category of inventories as long as the associated costs are sunk. More generally, our analysis suggests that a producer can benefit from engaging in any (costly) prior action that credibly reduces future costs and mitigates the buyer stockpiling incentives. Common examples include different forms of investment activities — such as the purchase of equipment, plants, machines and any kind of future-cost-reduction technique — provided that they cannot be (easily) reversed. The commitment value arising from these actions is reminiscent of the strategic effect generated by a firm’s investment in capacities or inventories prior to the competition stage, which has been well studied in the economic literature, as discussed below. The novelty of our mechanism is that the purpose of the firm’s strategic behavior is to mitigate the stockpiling incentives of forward-looking buyers rather than to acquire a competitive advantage against the rivals.

Our analysis is conducted in a fairly general setting without imposing any unduly restrictive assumptions on the functional forms. As established in Section 7, our model is robust and the driving force of our results emerges in alternative scenarios. The extensions of our model deliver additional results of some interest. Specifically, we show that the producer still resorts to inventories for strategic purposes when they are not perfectly observable provided that the presence of inventories can be signalled to buyers in a sufficiently accurate manner. We also find that the strategic motive for inventories persists in the presence of convex production costs that induce the producer to use inventories in order to smooth production over time. Interestingly, the producer can employ buyer stockpiling as an alternative instrument for production smoothing. Once we generalize our results to a setting with more than two periods, we show that under certain circumstances the inventory accumulation to cover only the demand in the following period mitigates to a larger extent the buyer stockpiling incentives. Therefore, our analysis suggests a strategic rationale for “high inventory turnover”, which complements the standard technological motive associated with the reduction in inventory holding costs. Although our model is deterministic in the spirit of Dudine et al. (2006), our results can be extended to a setting where demand evolves stochastically over time. As argued in Section 9, our results provide potentially significant managerial, empirical and policy implications.

Related literature There exists a recent fast-growing literature on strategic inventories in markets for storable goods. Anand et al. (2008) show that in a dynamic buyer-seller relationship the buyer uses strategic inventories to induce the seller to decrease its future price. Extending the model of Anand et al. (2008), Arya and Mittendorf (2013) investigate the role of consumer rebates in the presence of strategic inventories, and Arya et al. (2015) find that strategic inventories influence the choice between centralization and decentralization. Hartwig et al. (2015) provide experimental support for strategic inventories. Differently from these contributions, we investigate the producer’s strategic incentives to hold inventories in response to the possibility that the consumer’s stockpiling behavior can predict the buyer’s demand.

4For instance, a shock affects the expectation about future demand. The stochastic process may follow a mean reversion pattern, similarly to Antoniou et al. (2017). In section 9 we discuss the implications of allowing for uncertain demand.
of buyer stockpiling.

It has been recognized long ago in the economic literature that a firm can benefit from the investment in capacities or inventories as an irreversible commitment against the rivals (e.g., Arvan 1985; Dixit 1980; Driver 2000; Mollgaard et al. 2000; Saloner 1986; Ware 1985). Rotemberg and Saloner (1989) identify inventories as a means to sustain collusion. Deneckere et al. (1996) show that a manufacturer facing uncertain demand and selling through a competitive retail market may wish to support an adequate level of retail inventories. More recently, Mitraille and Moreaux (2013) consider a dynamic setting where Cournot competitors, after storing in the first period, may produce and sell in the second period.

As previously discussed, one of the most relevant papers in the literature on buyer stockpiling is Dudine et al. (2006). Hendel et al. (2014) characterize nonlinear pricing policies for storable goods. We discuss the contribution of Hendel et al. (2014) when allowing for nonlinear prices (see Section 7.4). Anton and Das Varma (2005) show in a Cournot setting that, if buyers are sufficiently patient, competition among firms to attract buyer stockpiling generates an increasing price path.\textsuperscript{5} Guo and Villas-Boas (2007) find that in a differentiated good market the preference heterogeneity translates into differential buyer stockpiling propensity, which exacerbates future price competition and may remove buyer stockpiling in equilibrium. Su (2010) incorporates buyer stockpiling into Su (2007)’s analysis of the optimal dynamic strategy of a seller that faces strategic buyers. Differently from our approach, the seller’s inventories are carried for standard technological reasons such as economies of scale and do not have any strategic role. Hendel and Nevo (2013) investigate intertemporal price discrimination when buyers differ in their storage abilities.

Our paper also pertains to the vast literature on durable goods that share some similarities with storable goods, as Dudine et al. (2006) point out. A relevant contribution for our purposes is Wolinsky (1991), which examines the firm’s strategic incentives to hold inventories in a durable good market with convex production costs. In Section 8, we contrast storable goods with durable goods and discuss the different role of the producer’s inventories. More recent papers include Board (2008), which solves the profit maximization problem of a durable good monopolist with time-varying demand, and Garrett (2016), which addresses the same problem in a setting where buyers arrive over time and have values for the good that evolve stochastically.

\textbf{Structure of the paper} The rest of the paper is structured as follows. Section 2 provides a stylized example that describes the rationale for the main results. Section 3 sets out the formal model. Section 4 identifies three relevant benchmarks: the producer’s unconstrained problem (when buyer stockpiling is not feasible), the producer’s problem under full commitment and the producer’s problem under limited commitment in the absence of inventories. Section 5 shows the main results about the producer’s strategic incentives to hold inventories under limited commitment. Section 6 is devoted to price comparisons. Section 7 investigates the robustness of the model and explores different extensions. Section 8 discusses the relevant differences between storable and durable goods. Section 9 concludes and illustrates some managerial, empirical and policy implications. The main formal proofs are collected in Appendices I and II.

\textsuperscript{5}When exploring the effects of competition among producers (see Section 7.5.1), we describe the relation between our paper and Anton and Das Varma (2005) and Mitraille and Moreaux (2013).
formal results and associated proofs are available in the Supplementary Appendix.

2 Linear example

To illustrate the rationale for our results in a transparent and intuitive manner, we provide a stylized example of a two-period setting with linear demand. In the spirit of the analysis conducted in Sections 4 and 5, we first present the producer’s unconstrained problem and the producer’s problem in the two commitment scenarios. Afterwards, we incorporate the possibility for the producer to hold inventories.

The first period demand is \( D_1 (p_1) = 80 - p_1 \) and the second period demand increases to \( D_2 (p_2) = 140 - p_2 \). There is no discounting on the second period. Suppose that the firm faces a large unit cost of production equal to 24. When buyer stockpiling is not feasible, the producer faces an unconstrained maximization problem, which reduces to a replica of the static problem. This gives the static monopoly prices \( p_1^m = 52 \) and \( p_2^m = 82 \). The aggregate static monopoly profits are \( \Pi^m = 4148 \).

When buyer stockpiling is feasible, buyers can store the good in the first period and consume it in the second period. Assume that the buyer stockpiling cost per unit of the good is 8. In this case, the static monopoly solution is no longer implementable, since the first period price inflated by the buyer stockpiling cost is lower than the second period price \( (52 + 8 < 82) \) and buyers are inclined to stockpile the good in the first period. Under full commitment, the producer can credibly announce a two-period pricing policy that removes buyer stockpiling. Using the no-arbitrage constraint \( p_2 = p_1 + 8 \), the producer’s maximization problem becomes

\[
\max_{p_1} (p_1 - 24) D_1 (p_1) + (p_1 + 8 - 24) D_2 (p_1 + 8),
\]

which yields the full commitment equilibrium prices \( p_1^c = 63 \) and \( p_2^c = p_1^c + 8 = 71 \). The producer’s aggregate profits are \( \Pi^c = 3906 \). We find that \( \Pi^c = 3906 < \Pi^m = 4148 \), since the opportunity of buyer stockpiling prevents the producer from maximizing its profits in each period. Buyer stockpiling is deterred in equilibrium. This increases the demand served at the second period price \( p_2^c = 71 \) rather than at the first period price \( p_1^c = 63 \). A producer with full commitment powers does not benefit from holding inventories.

Under limited commitment, the producer succumbs to the temptation to revise the pricing policy in order to maximize its continuation profits. For the time being, suppose that the producer does not accumulate any inventories. The amount of buyer stockpiling \( D_s (p_1) \) inflates the demand in the first period and decreases it in the second period. Proceeding backwards, given the buyer stockpiling \( D_s (p_1) \) inherited from the first period, the producer’s maximization problem in the second period is given by

\[
\max_{p_2} (p_2 - 24) [D_2 (p_2) - D_s (p_1)].
\]

The second period price is \( p_2 (D_s) = 82 - \frac{D_s}{2} \). It follows from the no-arbitrage constraint \( p_2 = p_1 + 8 \) that buyer stockpiling is \( D_s (p_1) = \max \{148 - 2p_1, 0\} \). The producer’s maximization
problem can be written as

\[
\max_{p_1} \left( (p_1 - 24) [D_1 (p_1) + D_s (p_1)] + (p_1 + 8 - 24) [D_2 (p_2) - D_s (p_1)] \right). \tag{2}
\]

In the absence of producer’s inventories, the limited commitment equilibrium prices are \( p_{ns1} = 67 \) and \( p_{ns2} = p_{ns1} + 8 = 75 \). Buyer stockpiling amounts to \( D_{ns} = 14 \). The producer obtains profits \( \Pi_{ns} = 3762 \). A comparison between the solutions to the maximization problems in (1) and (2) indicates that the producer’s lack of commitment entails higher prices, i.e., \( p_{ns1} = 67 > p_c1 = 63 \) and \( p_{ns2} = 75 > p_c2 = 71 \), and lower profits, i.e., \( \Pi_{ns} = 3762 < \Pi_c = 3906 \). This unambiguously reduces aggregate welfare.

We now show that a producer with limited commitment powers has strategic incentives to hold inventories. In the first period the producer accumulates the amount of inventories that covers the demand in the second period in order to maximize its ex ante profits, neglecting the constraint of sequential optimality. The costs of producing (and holding) inventories incurred in the first period are sunk once the second period has commenced. Given the no-arbitrage constraint \( p_2 = p_1 + 8 \), when holding inventories is costless, the producer’s maximization problem coincides with the full commitment problem in (1). Despite the producer’s lack of commitment, buyer stockpiling is removed and the full commitment outcome is restored. The inventory level corresponds to the second period full commitment demand, i.e., \( I^* = D_{c2}^* = 140 - p_{c2} = 69 \). This solution is implementable only if the producer does not want to sell in the second period a quantity that departs from the inventory level accumulated in the first period, which ensures that the constraint of sequential optimality is slack in equilibrium. Note that inventories generate a second period marginal revenue equal to \( 140 - 2I^* = 140 - 2 \cdot 69 = 2 \). Since the marginal revenue from inventories is positive (\( 2 > 0 \)), the producer loses from discarding some inventories in the second period, whose costs are zero once the second period has commenced. Moreover, given that the marginal revenue from inventories is lower than the unit cost of production (\( 2 < 24 \)), additional production in the second period is not profitable, either. This implies that the ex ante optimal level of inventories is also sequentially optimal. Inventories remove buyer stockpiling in equilibrium and allow a producer with limited commitment powers to approach the full commitment outcome. This can be restored when inventory holding costs vanish.

Suppose now that the firm’s unit cost of production decreases from 24 to 12. Now, the full commitment equilibrium prices are \( p_c1 = 57 \) and \( p_c2 = p_c1 + 8 = 65 \). Under limited commitment, the equilibrium prices in the absence of inventories are \( p_{ns1} = 61 \) and \( p_{ns2} = p_{ns1} + 8 = 69 \), which leads to buyer stockpiling \( D_{ns} = 14 \) and profits \( \Pi_{ns} = 4866 \). In this case, if the firm accumulates the inventory level that corresponds to the second period full commitment demand, the second period marginal revenue from inventories is negative \( (140 - 2D_{c2}^* = 140 - 2 \cdot (140 - 65) = -10 < 0) \). In the second period the producer cannot refrain from charging a price at which a portion of inventories remains unsold. This implies that the full commitment outcome is no longer achievable. The producer can still benefit from holding inventories to serve the second period demand net of buyer stockpiling, but the constraint of sequential optimality is now binding in equilibrium. It follows from the second period profit maximization problem with zero costs that the second period price is \( p_2 (D_s) = 70 - \frac{D_s}{2} \). Using the no-arbitrage constraint \( p_2 = p_1 + 8 \), buyer stockpiling is \( D_s (p_1) = \max \{ 124 - 2p_1, 0 \} \). Since the costs of producing inventories for
the second period net demand are incurred in the first period, the producer’s maximization problem can be expressed as

$$\max_{p_1} \left( p_1 - 12 \right) \left[ D_1 (p_1) + D_s (p_1) \right] - 12 \left[ D_2 (p_2) - D_s (p_1) \right] + (p_1 + 8) \left[ D_2 (p_2) - D_s (p_1) \right].$$

When the producer holds inventories, the limited commitment equilibrium prices are $p_1^{es} = 61$ and $p_2^{es} = p_1^{es} + 8 = 69$. Buyer stockpiling is $D_s^{es} = 2$. The producer achieves profits $\Pi^{es} = 4962$. Prices are the same as in the absence of inventories, i.e., $p_1^{ns} = p_1^{es} = 61$ and $p_2^{ns} = p_2^{es} = 69$ (which is, however, an artefact of the assumptions of linear demand and zero inventory holding costs). More relevantly, buyer stockpiling declines, i.e., $D_s^{es} = 2 < D_s^{ns} = 14$. This leads to higher profits, since $\Pi^{es} = 4962 > \Pi^{ns} = 4866$. Inventories mitigate the buyer stockpiling incentives and make the producer better off.

3 The model

3.1 Setting

Buyers We consider a two-period monopoly market for a storable good where in each period $\tau \in \{1, 2\}$ the producer faces a (continuously differentiable) demand $D_\tau (p_\tau)$, which decreases with the price $p_\tau$, i.e., $D_\tau' (p_\tau) < 0$. Following Dudine et al. (2006), the demand changes deterministically over time, although this is not necessary for our results. As it will become clear in the sequel, we are mainly interested in the case where the demand rises in the second period. For the sake of simplicity, there is no discounting on the second period. In Section 7.5.3 we concisely discuss the role of the discount factor.

The producer serves a continuum of competitive buyers, which operate as arbitrageurs or intermediaries in the market and purchase the good from the producer in order to resell it to the final consumers at zero profits. Therefore, the buyer demand corresponds to the final consumer demand. Buyers can purchase in advance and stockpile the good in the first period at a unit cost $s_b > 0$.

Denoting by $p_2^{e}$ the expected price in the second period, the buyer stockpiling demand writes as

$$D_s (p_1) = \begin{cases} D_2 (p_1 + s_b) & \text{if } p_1 + s_b < p_2^{e} \\ \left[0, D_2 (p_1 + s_b)\right) & \text{if } p_1 + s_b = p_2^{e} \\ 0 & \text{if } p_1 + s_b > p_2^{e} \end{cases}. \quad (3)$$

If $p_1 + s_b < p_2^{e}$, the first period price inflated by the buyer stockpiling cost is smaller than the second period expected price. Buyers prefer to purchase in advance and stockpile the good. Conversely, if $p_1 + s_b > p_2^{e}$, buyer stockpiling is strictly dominated. If $p_1 + s_b = p_2^{e}$, buyers are indifferent between stockpiling or not. Under buyer rational expectations and perfect foresight
(no uncertainty), the second period expected price coincides with the second period equilibrium price.

**Producer** In each period the firm decides on the amount of production and on the level of sales, or equivalently the price for the good. Let $c \geq 0$ be the (constant) unit cost of production. This cost formulation isolates the strategic inventory incentives under investigation and neutralizes the standard technological inventory reasons. In Section 7.2 we analyze the case of nonlinear production costs.\(^8\) The quantity produced net of the current sales represents the producer’s inventories, which are available for sale in the following period. Since the game consists of two periods, it is straightforward to see that both the producer and the buyers do not store in the second period. Therefore, we restrict our attention to the producer’s inventories and buyer stockpiling in the first period. We denote by $I \geq 0$ the producer’s first period inventory level available for sale in the second period. This involves a unit carrying cost of inventory, henceforth inventory holding cost, $s_p \geq 0$. We assume that $s_p \leq s_b$. Generally, in supply chain management the costs associated with the perishability of the good (obsolescence and depreciation) constitute only a fraction of the total storage costs. In particular, storage costs include warehousing, logistic and insurance costs, as well as the opportunity cost of capital, which can differ substantially between the producer and the buyers. It is plausible that a large producer can store at a lower unit cost than the buyers, which act as arbitrageurs or intermediaries in the market.\(^9\) For the sake of convenience, we treat the unit inventory holding cost $s_p$ and the unit buyer stockpiling cost $s_b$ as independent parameters. In practice, however, they are likely to move in the same direction. Our results are unaffected if we allow for interdependence between $s_p$ and $s_b$.

The producer’s aggregate profits are $\Pi = \Pi_1 + \Pi_2$, where

$$\Pi_1 = (p_1 - c) \left[ D_1(p_1) + D_s(p_1) \right] - (c + s_p) I$$

and

$$\Pi_2 = p_2 \left[ D_2(p_2) - D_s(p_1) \right] - c \left[ D_2(p_2) - D_2(p_1) - I \right] \cdot 1_{Q_2}$$

\(^4\) denote the profits in the first and second period, respectively. The indicator function $1_{Q_2} \in \{0,1\}$ in (5) assumes value 1 if production takes place in the second period, i.e., $D_2(p_2) - D_s(p_1) - I > 0$, and value zero otherwise. The buyer stockpiling demand $D_s(p_1)$ inflates the demand faced by the producer in the first period but depresses it in the second period. The first period costs $(c + s_p) I$ of producing and holding inventories are sunk once the second period has commenced. The producer’s profits $\Pi_\tau$, $\tau \in \{1,2\}$, satisfy the following standard assumption.

**Assumption 1** $\Pi_\tau |_{\tau \tau} < 0$, $\tau \in \{1,2\}$.

\(^8\)A change in production costs over time would affect the producer’s inventory behavior in a predictable manner.

\(^9\)Large producers can benefit from holding inventories for additional reasons (e.g., the use of inventories as an input in the production process). In any case, it will become apparent throughout the analysis that, in equilibrium, the producer may face larger aggregate storage costs than the buyers. This suggests that the removal (or the reduction) of buyer stockpiling through inventories turns out to be significantly costly for the producer.
This ensures that the producer’s profits \( \Pi_\tau \) are concave in the price \( p_\tau \) and the second-order conditions for profit maximization are satisfied.

### 3.2 Timing and equilibrium concept

Each period of the game includes the following two stages.

(I) The producer chooses the amount of production and the price for the good.

(II) Buyers purchase a quantity of the good and decide on the amount to be stockpiled.

The solution concept we adopt is the subgame perfect Nash equilibrium. The difference between production and sales in each period determines the producer’s inventories available for sale in the following period. In line with some relevant literature (e.g., Anand et al. 2008; Arvan 1985; Arya et al. 2015; Arya and Mittendorf 2013; Mitraille and Moreaux 2013; Mollgaard et al. 2000; Krishnan and Winter 2010; Ware 1985), production and inventory decisions are observable and can therefore affect the buyer behavior. As discussed in the introduction, reliable data about the production and inventory activities are publicly available, at least for large firms.\(^{10}\) In Section 7.1 we show that our qualitative results still hold when the producer’s inventories are not perfectly observable.

### 4 Relevant benchmarks

#### 4.1 Unconstrained problem

When buyer stockpiling is not feasible, the producer does not have any incentive to hold inventories. In each period the amount of production meets the current buyer demand. Formally, the producer’s unconstrained problem is given by

\[
\max_{p_1, p_2} \left( (p_1 - c) D_1(p_1) + (p_2 - c) D_2(p_2) \right). \tag{6}
\]

This yields the static monopoly price \( p_m^\tau = c - \frac{D_m^\tau}{D_{1\tau}} \) in period \( \tau \in \{1, 2\} \). Setting \( \tau = 2 \) in the first-order condition \( D_\tau(p_\tau) + (p_\tau - c) D_{\tau\vert\tau}(p_\tau) = 0 \), we obtain the following auxiliary function

\[
\phi_2(p_2) \equiv D_2(p_2) + (p_2 - c) D_{2\vert2}(p_2), \tag{7}
\]

which is helpful for the subsequent analysis. Note from the buyer stockpiling demand \( D_s(\cdot) \) in (3) that for \( p_1^m + s_b \geq p_2^m \) buyers do not exhibit any (strict) incentive to stockpile. The producer’s problem is trivial and reduces to a replica of the static monopoly problem. In line with Dudine et al. (2006), hereafter we impose the following assumption, which ensures that the opportunity of buyer stockpiling affects the producer’s intertemporal pricing problem.

**Assumption 2** \( p_1^m + s_b < p_2^m \iff s_b < \frac{D_m^1}{D_{11}} - \frac{D_m^2}{D_{22}} \).

The buyer stockpiling cost \( s_b \) must be sufficiently small that buyers prefer to stockpile when the producer charges the static monopoly price in each period. As \( s_b > 0 \), Assumption 2 requires

\(^{10}\)For instance Shell, one of the largest oil companies in the world, systematically publishes information about the production and inventory levels and their variations. Details are available at http://www.annualreports.com/Company/royal-dutch-shell-plc.
that the demand (for a given slope) grows over time.

4.2 Full commitment

To better appreciate the forces at play under limited commitment, we start with the case where the producer is able to commit to a two-period pricing policy. The following lemma formalizes the equilibrium under full commitment.

Lemma 1 The full commitment equilibrium exhibits the following features:

(i) no producer’s inventories, i.e., $I^c = 0$;
(ii) no buyer stockpiling, i.e., $D_s^c = 0$;
(iii) prices $p_1^c = c - \frac{D_1^c + \phi_2}{D_1^c}$ and $p_2^c = p_1^c + s_b$, where $p_1^c > p_1^m$ and $p_2^c < p_2^m$.

Lemma 1 indicates that the producer prefers to commit to a price sequence that fully removes buyer stockpiling. The reason is that buyer stockpiling is profit detrimental since it reduces the future sales that occur at higher prices. To prevent buyer stockpiling, the producer increases the first period price and reduces the second period price compared to the static monopoly level, i.e., $p_1^c > p_1^m$ and $p_2^c < p_2^m$. The no-arbitrage constraint is binding, i.e., $p_2^c = p_1^c + s_b$, and buyers pay in the second period the cost they would incur for stockpiling. This allows the producer to extract the increased surplus. Indeed, buyers are indifferent between stockpiling or not, but no buyer stockpiling takes place in equilibrium. This is because the producer could slightly increase the first period price (or alternatively reduce the second period price), which removes buyer stockpiling and yields a discontinuous increase in profits. The possibility of buyer stockpiling harms the producer, since it cannot equalize marginal revenues and marginal costs in the two periods. Remarkably, under full commitment, the producer does not benefit from holding inventories, since they involve a mere loss associated with the inventory holding costs.

4.3 Limited commitment without inventories

Turning to the case where the producer cannot commit to future prices, the following lemma establishes the results in the absence of the producer’s inventories.

Lemma 2 Under limited commitment, in the absence of the producer’s inventories, one of the following outcomes arises:

(a) buyer stockpiling, i.e., $D_s^{ns} = \phi_2^{ns}$, and prices $p_1^{ns} = c - \frac{D_1^{ns} + \phi_2^{ns} - s_b}{D_1^{ns}}$, $p_2^{ns} = p_1^{ns} + s_b$;
(b) no buyer stockpiling, i.e., $D_s^{nn} = 0$, and prices $p_1^{nn} = p_2^{nn} = s_b$.

Lemma 2 replicates the results of Dudine et al. (2006), where the producer does not engage in inventory activities.\textsuperscript{11} Notably, the full commitment outcome is no longer achievable under limited commitment. To see this, recall from Lemma 1 that the producer finds it optimal to commit to a second period price below the static monopoly level in order to prevent buyer stockpiling. When the producer cannot commit to future prices, it succumbs to the temptation to charge the second period static monopoly price if buyers did not store in the first period.

\textsuperscript{11}Since we show the results of Dudine et al. (2006) in a manner that better fits our purposes, the proof of Lemma 2 is included in Appendix I.
Anticipating this, buyers are eager to store, and the full commitment outcome breaks down. As Lemma 2(a) reveals, buyer stockpiling emerges in equilibrium. This occurs when the demand in the second period is relatively high or buyer stockpiling is not too costly. To mitigate buyer stockpiling, the producer sets the price in the first period above the full commitment level. Since the no-arbitrage constraint is binding, the price exceeds the full commitment level in the second period as well. Buyers are indifferent to the quantity stored in equilibrium, and buyer stockpiling is endogenously determined by taking into account the producer’s response.

Lemma 2(b) indicates that, when the demand growth is less pronounced or the buyer stockpiling cost is large enough, buyer stockpiling does not take place in equilibrium, although the no-arbitrage constraint is still binding. The price in the second period is set at the static monopoly level but the price in the first period is distorted upwards (by Assumption 2). As in Lemma 2(a), prices are higher than under full commitment in the two periods.

5 Strategic inventories

We are now in a position to address the main issue of this paper and investigate the producer’s strategic incentives to hold inventories under limited commitment. Since the producer’s inventories correspond to the difference between the quantity produced and the current sales, the producer’s problem reduces to the choice of the price in each period and of the inventory level. Using (4) and (5), the producer’s maximization problem can be formulated as

\[
\max_{p_1, I} (p_1 - c) [D_1 (p_1) + D_s (p_1, I)] - (c + s_p) I
\]

\[
+ p_2 [D_2 (p_2) - D_s (p_1, I)] - c [D_2 (p_2) - D_s (p_1, I) - I] \cdot 1_{Q_2}
\]

subject to the following constraint of sequential optimality

\[
p_2 (D_s (p_1, I), I) \equiv \arg \max_{\tilde{p}_2} \tilde{p}_2 [D_2 (\tilde{p}_2) - D_s (p_1, I)] - c [D_2 (\tilde{p}_2) - D_s (p_1, I) - I] \cdot 1_{Q_2}.
\]

The salient feature of the producer’s problem is the interdependence of the choice variables across periods. Similarly to the setting of Dudine et al. (2006) described in Lemma 2, the first period price \(p_1\) can influence the second period profits through the buyer stockpiling demand \(D_s (\cdot)\) in (3). The innovative aspect of our framework is the possibility that the producer engages in inventory activities. The producer accumulates inventories in the first period, which are available for sale in the second period. Inventories affect the producer’s aggregate profits through two channels. A first effect of inventories arises from the fact that the costs of producing and holding inventories are sunk once they have been incurred. A second, more subtle, effect of inventories — that we investigate in the sequel — is their impact on the buyer behavior and specifically on the buyer stockpiling demand \(D_s (\cdot)\) in equilibrium.

With constant marginal costs, the producer’s choice of the inventory level reduces to a binary decision. Either the producer abstains from inventory activities at all, i.e., \(I = 0\), or it accumulates the amount of inventories that covers the second period demand net of buyer stockpiling, i.e., \(I = D_2 (\cdot) - D_s (\cdot)\), which drives the producer’s second period costs to zero. Any production in the second period in addition to inventories would entail a marginal cost equal
to \( c \) and a price at the second period static monopoly level. Anticipating this, the firm would prefer to produce the whole quantity in the second period and to avoid the inventory holding costs. Clearly, any inventory level above the second period (net) demand is also suboptimal.

Lemma 1 indicates that inventories are not beneficial to a producer with full commitment powers. However, when the producer cannot commit to future prices, it succumbs to the temptation to adjust the price in the second period in response to the buyer behavior. In the following proposition, we characterize the producer’s incentives to hold inventories as a strategic device to mitigate the loss from the lack of commitment.

**Proposition 1** Suppose \( c \geq \tilde{c} \), where \( \tilde{c} \) is defined by (15) in Appendix I. Then, there exists a threshold \( \tilde{s}_p > 0 \) for the unit inventory holding cost such that for \( s_p \leq \tilde{s}_p \) the limited commitment equilibrium exhibits the following features:

(i) producer’s inventories, i.e., \( I^* = D_2^* \);

(ii) no buyer stockpiling, i.e., \( D_1^* = 0 \);

(iii) prices \( p_1^* = c - D_1^* |_{c=0} \) and \( p_2^* = p_1^* + s_b \).

For \( s_p = 0 \), the full commitment outcome is restored.

Proposition 1 shows that under limited commitment the producer can benefit from holding inventories for strategic purposes. The amount of inventories covers the second period demand and maximizes the producer’s ex ante profits in (8), ignoring the constraint of sequential optimality in (9). As under full commitment, buyer stockpiling is fully removed. The producer resorts to inventories as a complementary instrument to prices. This outcome can be sustained in equilibrium only if selling exactly the ex ante optimal amount of inventories is also sequentially optimal. Put differently, the producer must not have any incentive to revise its decision and to set a second period price at which the quantity sold differs from the inventory level.

The solution in Proposition 1 is implementable when the unit cost of production \( c \) is above the threshold \( \tilde{c} \) defined by (15) in Appendix I. *Prima facie*, this could seem counterintuitive, since one might expect that a high cost of production undermines the desirability of inventories. The rationale for this result stems from the strategic nature of inventories under limited commitment. We know from Lemma 2 that, if buyers did not store in the first period, a producer with limited commitment powers cannot refrain from setting an excessively high price in the second period. In the extreme case where \( c = 0 \), the producer’s sequentially optimal strategy is to charge the second period price such that marginal revenues fall to zero, irrespective of the amount of inventories. For \( c > 0 \), inventories are potentially beneficial, since they reduce the second period costs, which alleviates the producer’s temptation to charge a high price in the second period and therefore softens the buyer stockpiling incentives.

As Figure 1 illustrates, the ex ante optimal inventory level that covers the second period demand maximizes the second period profits if and only if \( D_2 (p_2^{\text{opt}} |_{c=0}) \leq I^* = D_2^* \leq D_2 (p_2^{\text{opt}} |_{c=0}) \). In this case, the constraint of sequential optimality is slack in equilibrium. When the unit cost of production is relatively large, i.e., \( c \geq \tilde{c} \), the ex ante optimal inventory level is \( I^* \leq D_2 (p_2^{\text{opt}} |_{c=0}) \). Put differently, the second period price at which the quantity sold corresponds to the inventory level is higher than the static monopoly price at zero costs, i.e., \( p_2^* \geq p_2^{\text{opt}} |_{c=0} \). The producer does not have any incentive to increase the price above \( p_2^* \) and to discard some inventories, since the
marginal revenue at $I^* \leq D_2(p_2^m|c=0)$ is positive. The lower sales due to a higher price reduce the producer’s revenues, while costs remain at zero. This is because the inventory costs were incurred in the first period and they are sunk once the second period has commenced. Hence, a sufficiently large cost of production curbs the amount of inventories so that the producer does not succumb to the temptation to set a price higher than $p_2^*$ and to leave some inventories unsold. Moreover, for $p_2^* \leq p_2^m$ the ex ante optimal inventory level is $I^* \geq D_2(p_2^m)$. The marginal cost $c$ of the additional production associated with a price reduction from $p_2^*$ outweighs the corresponding marginal revenue. In the second period the producer does not have any incentive to set a price below $p_2^*$ and to sell a quantity that exceeds the inventory level. We know from Lemma 1 that the second period full commitment price is lower than the static monopoly level, i.e., $p_c^2 < p_m^2$. When holding inventories is costless, i.e., $s_p = 0$, we find that $p_2^* = p_2^m$. The producer is able to allocate the second period full commitment quantity at no additional cost. Consequently, the full commitment outcome characterized in Lemma 1 is restored under limited commitment. If $s_p$ is sufficiently small, the producer can still benefit from holding inventories, although this entails a price distortion above the full commitment level. As mentioned in the introduction, the oil market is a particularly suitable example for our purposes, since it exhibits relatively large costs of production and negligible inventory holding costs.

Our analysis indicates that the inventory cost sunkness reduces the producer’s future costs and alleviates the producer’s temptation to charge high future prices. Anticipating this, buyers abstain from stockpiling. Inventories act as a strategic device to mitigate the producer’s loss from the lack of commitment.\footnote{It follows from our previous discussion that the presence of a cost at which the producer can discard or destroy its inventories reinforces this result.} Note that the amount of inventories that the producer accumulates to remove buyer stockpiling corresponds to the entire second period demand, while buyer stockpiling is only a portion of it. This suggests that there exists imperfect substitutability between inventories and buyer stockpiling in equilibrium. Inventories are not a fully effective instrument to prevent buyer stockpiling, unless holding inventories is costless. As we will see in Proposition 3, when the ex ante optimal inventory level is also sequentially optimal, inventories generally lead to lower prices that approach the full commitment outcome. Hence, inventories can also generate allocative benefits to the producer. As formally shown in the proof of Proposition 1 in Appendix I, these allocative benefits imply that the threshold value $\tilde{s}_p$ for...
the unit inventory holding cost below which the producer prefers to hold inventories is such that the aggregate inventory holding costs exceed the aggregate buyer stockpiling costs, i.e., \( \bar{s}_p D_2(p_1^* + s_b) > s_b D^m_s(p_2^*) \).

Given that the no-arbitrage constraint is binding, buyers are indeed indifferent to storing. As under full commitment, no buyer stockpiling takes place in equilibrium, since a slight rise in the first period price fully removes buyer stockpiling and yields a discontinuous increase in profits while preserving sequential optimality, whose constraint is slack in equilibrium. A similar result would obtain with a slight rise in the producer’s inventories, which translates into a lower second period price.

As shown in Figure 1, any ex ante optimal inventory level such that either \( I^* < D_2(p_2^m) \) or \( I^* > D_2(p_2^m | c = 0) \) is not sequentially optimal and cannot be sustained in equilibrium. When the unit cost of production is relatively small, i.e., \( c < \tilde{c} \), the inventory level is \( I^* > D_2(p_2^m | c = 0) \). The marginal revenue at \( I^* > D_2(p_2^m | c = 0) \) is negative, which implies that inventories are excessive in the second period. The second period price at which the quantity sold coincides with inventories is lower than the static monopoly price at zero costs, i.e., \( p_2^* < p_2^m | c = 0 \). The producer succumbs to the temptation to set the second period price at \( p_2^m | c = 0 \) and to discard some inventories. When the unit inventory holding cost \( s_p \) is sufficiently large, the inventory level \( I^* < D_2(p_2^m) \) does not suffice to maximize the second period profits, since the marginal revenue at \( I^* < D_2(p_2^m) \) is higher than the marginal cost of production \( c \). The second period price at which the quantity sold coincides with inventories is higher than the static monopoly level, i.e., \( p_2^* > p_2^m \). The firm prefers to set the price at \( p_2^m \) and to produce some additional quantity in the second period.

Since the result in Proposition 1 holds for a relatively large unit cost of production, a natural issue is whether the strategic role of inventories persists when the unit cost of production is sufficiently small that the ex ante optimal inventory level violates the constraint of sequential optimality. To this end, in the following lemma we characterize the possible outcomes in the presence of inventories.

**Lemma 3** Suppose \( c < \tilde{c} \). Then, under limited commitment, if the producer holds inventories, one of the following outcomes arises:

(a) producer’s inventories and buyer stockpiling, i.e., \( I^{is} = -p_2^{is} D^{is}_{2|1}, D^{is} = \phi_2^{is} + c D^{is}_{2|1} \), and prices \( p_1^{is} = c - \frac{D^{is}_{2|1} + \phi_2^{is} - s_p D^{is}_{2|1} - (s_b - s_p)(\phi_2^{is} + c D^{is}_{2|11})}{D^{is}_{2|11}}, p_2^{is} = p_1^{is} + s_b \).

(b) producer’s inventories but no buyer stockpiling, i.e., \( I^m = D_2^m \), \( D_2^m = 0 \), and prices \( p_1^{im} = p_2^m - s_b, p_2^{im} = p_2^m | c = 0 \).

We know from the discussion following Proposition 1 that, if the cost of production is relatively small, i.e., \( c < \tilde{c} \), the amount of inventories that maximizes the producer’s ex ante profits is excessive in the second period and cannot be sustained in equilibrium. The inventory level must be distorted away from the ex ante optimal level, and the constraint of sequential optimality is binding in equilibrium. Inventories still mitigate the buyer stockpiling incentives since the reduction in the producer’s future costs associated with the inventory cost sunkness makes the producer less prone to charge high future prices. However, buyer stockpiling is not necessarily removed. Similarly to Lemma 2(a), buyer stockpiling emerges in Lemma 3(a) when...
the second period demand is relatively high or the buyer stockpiling cost is small enough. Since
the no-arbitrage constraint is binding, buyers are indeed indifferent to storing in equilibrium,
and buyer stockpiling is endogenously derived by taking into account the producer’s response.

Interestingly, we find that a lower unit inventory holding cost \( s_p \) leads to a higher price in
the first period (and in the second period as well, given the binding no-arbitrage constraint) if
the second period demand is not too convex.\(^ {13} \) To understand the rationale for this surprising
result, it is important to note from Lemma 3(a) that a higher first period price reduces the buyer
stockpiling \( D_{s}^i \cdot \), which translates into a higher second period net demand \( D_2 (\cdot) - D_{s}^i (\cdot) \) when
the second period demand \( D_2 (\cdot) \) is not too convex.\(^ {14} \) Since the producer’s inventories coincide
with the second period net demand, a lower unit inventory holding cost makes it more attractive
for the producer to stimulate the second period net demand, which requires a higher first period
price.

Lemma 3(b) reveals that, in line with Lemma 2(b), when the second period demand is
relatively low or the buyer stockpiling cost is high enough, buyer stockpiling does not take place
in equilibrium. Since inventories cover the entire second period demand and their costs are
sunk, sequential optimality implies that the producer sets the second period price at the static
monopoly level with zero costs.

We can now present the following result.

**Proposition 2** Suppose \( 0 < c < \bar{c} \). Then, under limited commitment, as long as in the absence
of the producer’s inventories buyer stockpiling arises, holding inventories is profitable for the
producer if the unit inventory holding cost \( s_p \) and the unit production cost \( c \) are small enough.

The producer can benefit from holding inventories even when the solution in Proposition 1
is not implementable. One of the outcomes characterized in Lemma 3 emerges in equilibrium.
The strategic benefit of inventories still consists in the reduction in the producer’s second period
costs, which alleviates the producer’s temptation to charge a high second period price and softens
the buyer stockpiling incentives. It follows from Lemma 3 that, differently from Proposition 1,
in addition to the mere inventory holding costs, the producer incurs the costs associated with the
distortion from the ex ante optimal inventory level to ensure sequential optimality. When the
unit cost of production is small enough, inventories are a less effective instrument to mitigate the
buyer stockpiling incentives and cannot work perfectly even when their holding costs vanish.
Consequently, buyer stockpiling can persist in equilibrium. We refer to the Supplementary
Appendix (Section 4) for a full characterization of the results with linear demand.

As proved in Appendix I, Proposition 2 provides sufficient, albeit not necessary, conditions
for the profit superiority of inventories, which ensure that the producer can replicate the equi-
librium pricing policy without inventories described in Lemma 2(a) and be better off due to

\[^{13}\text{Differentiating the left-hand side of the first-order condition for } p_1^* \text{ in (16) in Appendix I with respect to}
\] s_p \text{ yields after some manipulation } \left( p_1 + s_b \right) D_{2|11} (p_1 + s_b) + D_{2|1} (p_1 + s_b). \text{ It follows from the implicit function}
\text{ theorem that } \frac{\partial p_1^*}{\partial s_p} < 0 \text{ if and only if this expression is negative, i.e., } D_{2|11} (p_1 + s_b) < - \frac{D_{2|1} (p_1 + s_b)}{p_1 + s_b}.
\[^{14}\text{Using (7), we find that } D_{s}^i (p_1) = 2D_{2|1} (p_1 + s_b) + (p_1 + s_b) D_{2|11} (p_1 + s_b) < 0, \text{ where the inequality}
\text{ follows from Assumption 1 (second-order condition for the maximization of second period static profits with zero costs). The sign of the derivative of } D_2 (p_1 + s_b) - D_{s}^i (p_1) = -(p_1 + s_b) D_{2|11} (p_1 + s_b) \text{ with respect to } p_1 \text{ is}
\text{ positive if and only if } D_{2|11} (p_1 + s_b) < \frac{D_{2|1} (p_1 + s_b)}{p_1 + s_b}. \]
a reduction in buyer stockpiling. Note from Lemmas 2(a) and 3(a) that the buyer stockpiling difference evaluated at \( p^1_\tau \) is \( D^u_\sigma (p^1_\tau) - D^d_\sigma (p^1_\tau) = -cD_{1|1}^u (p^1_\tau) > 0 \) (for \( c > 0 \)). For given prices, the producer’s inventories induce lower buyer stockpiling. A sufficiently small (but positive) unit production cost \( c \) guarantees that, for any buyer stockpiling level \( D^u_\sigma (p^1_\tau) > 0 \) without inventories, there exists a feasible buyer stockpiling level \( D^u_\sigma (p^1_\tau) \geq 0 \) with inventories. \( A \text{ fortiori} \), the producer’s benefits of holding inventories are larger when equilibrium prices are considered.

6 Price comparisons

The following proposition formalizes the price comparisons across different scenarios. Some threshold values are derived in the proof available in Appendix I.

**Proposition 3** For \( \tau \in \{1, 2\} \), the following price orderings hold:

(i) \( p^s_\tau \leq p^s_\tau < p^m_\tau < p^m_\tau \), where \( p^s_\tau = p^s_\tau \) if and only if \( s_p = 0 \) and \( p^s_\tau < p^m_\tau \) if and only if \( s_p < \bar{s}_p \);  
(ii) \( p^l_\tau < p^m_\tau < p^m_\tau \), where \( p^m_\tau = p^m_\tau \) if and only if \( c = 0 \);  
(iii) \( p^s_\tau < p^m_\tau \) if and only if one of the following conditions holds: (a) \( D^u_\sigma \leq 0 \) and \( s_p > \hat{s}_p \); (b) \( 0 < D^u_\sigma \leq \hat{D}_{2|1} \); (c) \( D^u_\sigma > D^u_\sigma \) and \( s_p < \hat{s}_p \).

Proposition 3 delivers results of some interest. We begin with the price comparisons collected in point (i). As mentioned in Section 5, the solution in Proposition 1, where the ex ante optional inventory level is also sequentially optimal, leads to prices distorted above the full commitment level as long as holding inventories is costly, i.e., \( p^s_\tau < p^s_\tau \) for \( s_p > 0 \). This is because the producer passes a part of these costs on to the buyers. The full commitment solution can be replicated if and only if holding inventories is costless, i.e., \( p^s_\tau = p^s_\tau \) for \( s_p = 0 \).

Inventories allow a producer with limited commitment powers to generally charge lower prices, i.e., \( p^s_\tau < p^m_\tau \) and \( p^s_\tau < p^m_\tau \). The result that, despite being costly, inventories lead to lower prices stems from their strategic role in mitigating the buyer stockpiling incentives. Specifically, we find that \( p^s_\tau < p^m_\tau \) for \( s_p < \bar{s}_p \). Using \( D^u_\sigma \) in Lemma 2(a), it follows from (17) in Appendix I that this condition corresponds to \( s_p D^u_\sigma > s_b D^u_\sigma \). The term on the left-hand side captures the inventory holding cost effect, whose sign is negative. This is because a higher price reduces the second period demand, which coincides with the producer’s inventories, and therefore the aggregate inventory holding costs decline. This effect is more pronounced when the unit inventory holding cost \( s_p \) is higher. The term on the right-hand side captures the buyer stockpiling effect, whose sign is also negative (\( D^u_\sigma = \phi^u_\sigma < 0 \), where the inequality follows from Assumption 1). The idea is that a higher price reduces the buyer stockpiling \( D^u_\sigma \) and the associated loss for the producer. The comparison between these two effects implies that for \( s_p < \bar{s}_p \) the buyer stockpiling effect dominates (in absolute terms) and prices are lower in the presence of inventories. It turns out that \( s_p < \bar{s}_p \) under fairly general conditions. However, we cannot dismiss the opposite case, which seems at first sight less intuitive. To see this, note that when the second period demand is sufficiently convex the price impact on the buyer stockpiling \( D^u_\sigma \) in Lemma 2(a) is negligible (\( D^u_\sigma = \phi^u_\sigma \rightarrow 0 \)). Hence, the inventory
holding cost effect dominates (in absolute terms) even for relatively low values of $s_p$ and prices are higher when the producer holds inventories. The reason is that, with sufficiently convex demand, higher prices are more beneficial to the reduction in the aggregate inventory holding costs than to the reduction in buyer stockpiling. It is important to realize that, as long as the unit inventory holding cost is not too large, inventories are profitable for the producer since they remove buyer stockpiling, although equilibrium prices might rise. The fact that buyer stockpiling disappears despite higher prices is only apparently a contradiction. In equilibrium, the no-arbitrage constraint is binding irrespective of the producer’s inventories, and therefore the buyers are indeed indifferent between stockpiling or not.

We can also see from point (i) that $p^*_s < p^{\text{nn}}_t$, where $p^{\text{nn}}_t = p^{\text{nn}}_2$ by Lemma 2(b). As discussed after Proposition 1, the second period price with inventories cannot be higher than the static monopoly level in order to be implementable. Clearly, if the two prices were the same, the producer could not benefit from holding inventories. Given the binding no-arbitrage constraint, the first period price with inventories is lower as well. Moreover, we have $p^{\text{ns}}_t < p^{\text{nn}}_t$, since buyer stockpiling allows the producer to mitigate prices through a reduction in the second period demand. An immediate implication of the results in point (i) is that, when the unit cost of production is sufficiently large that the ex ante optimal level of inventories is also sequentially optimal, the producer’s inventories generally lead to lower prices. Therefore, inventories increase the producer’s profits and consumer surplus, which unambiguously enhances aggregate welfare.

As point (ii) of Proposition 3 reveals, when the producer’s inventories must be distorted from the ex ante optimal level to ensure sequential optimality, the coexistence of the producer’s inventories and buyer stockpiling leads to lower prices than in the absence of buyer stockpiling, i.e., $p^{\text{is}}_t < p^{\text{in}}_t$. Moreover, the producer’s inventories without buyer stockpiling ensure that prices are lower with respect to the situation where no storing occurs in the economy, i.e., $p^{\text{in}}_t \leq p^{\text{nn}}_t$, where the equality holds if and only if $c = 0$.

Point (iii) of Proposition 3 indicates that the conclusions drawn from point (i) deserve some qualifications when inventories are distorted from the ex ante optimal level and coexist with buyer stockpiling. Using $D^{\text{ns}}_s$ and $D^{\text{is}}_s$ in Lemmas 2(a) and 3(a), it follows from (18) in Appendix I that $p^{\text{is}}_t < p^{\text{ns}}_t$ if and only if $s_p \left[ D^{\text{ns}}_{21} (p^{\text{ns}}_t) - D^{\text{is}}_{11} (p^{\text{is}}_t) \right] > s_b \left[ D^{\text{is}}_{21} (p^{\text{is}}_t) - D^{\text{is}}_{11} (p^{\text{is}}_t) \right]$. Differently from point (i), the inventory holding cost effect on the left-hand side is now positive as long as the second period demand is not too convex, i.e., $D^{\text{ns}}_{21} < \hat{D}_{211}$, where $\hat{D}_{211} > 0$. To understand why, recall from the discussion following Lemma 3 that higher prices decrease buyer stockpiling and, provided that the second period demand is not too convex, this translates into a higher second period demand net of buyer stockpiling, which coincides with the producer’s inventories. Hence, lower prices mitigate the aggregate inventory holding costs. A higher $s_p$ strengthens the inventory holding cost effect. Furthermore, differently from point (i), since buyer stockpiling persists in the presence of inventories, the buyer stockpiling effect on the right-hand side is now given by the price impact on the buyer stockpiling difference between the two inventory scenarios. It follows from Lemmas 2(a) and 3(a) that the buyer stockpiling difference evaluated at $p^{\text{is}}_t$ is $D^{\text{is}}_s (p^{\text{is}}_t) - D^{\text{is}}_s (p^{\text{is}}_t) = -cD^{\text{is}}_{21} (p^{\text{is}}_t)$. The positive (negative) sign of the buyer stockpiling effect means that a higher price is more (less) helpful for the reduction in buyer stockpiling when the producer holds inventories. If the second period
demand is concave, i.e., \( D_{ns}^{2|11} \leq 0 \), the buyer stockpiling effect is also positive. The comparison between the two effects implies that the inventory holding cost effect dominates and prices are lower when the producer holds inventories if the second period demand is concave and the unit inventory holding cost is high enough, i.e., \( p_{i}^{ns} < p_{n}^{ns} \) if \( D_{ns}^{2|11} \leq 0 \) and \( s_{p} > \hat{s}_{p} \), as point (iii-a) of Proposition 3 indicates.

If the second period demand is convex, i.e., \( D_{ns}^{2|11} > 0 \), the buyer stockpiling effect is negative. This implies that, if the second period demand is only moderately convex, i.e., \( 0 < D_{ns}^{2|11} < \hat{D}_{2|11} \), the two effects push in the same direction and prices are lower when the producer holds inventories, i.e., \( p_{i}^{ns} < p_{n}^{ns} \), as point (iii-b) reveals.

If the second period demand is sufficiently convex, i.e., \( D_{ns}^{2|11} \geq \hat{D}_{2|11} \), the producer’s inventories decrease rather than increase with prices, and the inventory holding cost effect is now negative. Since the buyer stockpiling effect is also negative with convex demand, we find that the latter effect dominates (in absolute terms) and the producer sets lower prices when holding inventories if the second period demand is sufficiently convex and the unit inventory holding cost is small enough, i.e., \( p_{i}^{ns} < p_{n}^{ns} \) if \( D_{ns}^{2|11} \geq \hat{D}_{2|11} \) and \( s_{p} < \hat{s}_{p} \), as point (iii-c) establishes. Differently from point (i), the inventory distortion from the ex ante optimal level implies that buyer stockpiling is not fully removed, which makes price comparisons more convoluted.

### 7 Robustness and extensions

#### 7.1 Inventory observability

Throughout our analysis we assume that the producer’s inventories are perfectly observable. Buyers can be thought of as competitive arbitrageurs that should possess adequate information about the producer’s economic situation. In reality, however, this information may not be perfect. Now, we show that there still exists scope for strategic inventories in a setting where inventories are not perfectly observable, provided that the producer can signal their presence to buyers. As discussed in the introduction, signals about the firms’ production and inventory activities can be inferred from different sources, such as industry reports, midyear financial statements as well as balance sheets. To preserve the simplicity of our framework and focus on the essence of our results, we introduce a reduced form game with discrete choices that incorporates the main elements of the baseline model. The game involves two players, a producer and a representative buyer.

The timing of the game unfolds as follows.

(I) The producer decides whether to hold inventories (I) or not (O).
(II) Nature determines that, in the presence of the producer’s inventories, the buyer observes inventories with probability \( \sigma \in (0,1) \) and does not observe any inventories with the complementary probability \( 1 - \sigma \). In the absence of the producer’s inventories, the buyer does not observe any inventories.
(III) The buyer decides whether to stockpile the good (S) or not (N).
(IV) The producer’s and the buyer’s payoffs materialize.

This framework reflects in a simple and natural manner the imperfect observability of inventories. When the producer holds inventories, the buyer can discover this decision only with
probability \( \sigma \). Therefore, \( \sigma \) captures the accuracy of the signal \( s \in \{I, O\} \) that the buyer receives about the producer’s choice between holding inventories (\( I \)) or not (\( O \)). The idea is that the producer attempts to inform the buyer about its inventories but some noise may prevent communication from being successful. When the producer does not hold any inventories, the buyer receives the signal that correctly identifies the producer’s choice, i.e., \( s = O \). This assumption is not necessary and is only imposed for the sake of convenience. What matters for our purposes is that the buyer cannot perfectly observe the producer’s inventories. Given the signal \( s = O \), the buyer does not know the producer’s inventory choice when deciding whether to stockpile the good (\( S \)) or not (\( N \)).

Figure 2 illustrates the extensive form of this game. Let \( \Pi \) be the producer’s profits and \( V \) the buyer surplus. The subscripts of the players’ payoffs indicate the players’ actions that lead to these payoffs. The producer’s profits are such that (i) \( \Pi_{IN} > \Pi_{IS} \), (ii) \( \Pi_{IN} > \Pi_{OS} \), (iii) \( \Pi_{OS} > \Pi_{IS} \) and (iv) \( \Pi_{ON} > \Pi_{IN} \). The rationale for the inequalities in points (i)-(iv) follows from the baseline model. Points (i) and (ii) state that the profits of a producer that accumulates inventories and removes buyer stockpiling are higher than the profits in the presence of buyer stockpiling, regardless of whether the producer holds inventories or not. Points (iii) and (iv) indicate that, for a given strategy of the buyer (stockpiling or not), the producer prefers to abstain from (costly) inventory activities. As regards buyer surplus, we have (v) \( V_{IN} > V_{IS} \) and (vi) \( V_{OS} > V_{ON} \). Points (v) and (vi) show that the producer’s inventories mitigate the buyer stockpiling incentives.

There exists a threshold \( \tilde{\sigma} \equiv \frac{\Pi_{OS} - \Pi_{IN}}{\Pi_{IN} - \Pi_{IS}} \in (0, 1) \) such that for \( \sigma \leq \tilde{\sigma} \) the producer never accumulates inventories. Intuitively, when the signal about inventories is sufficiently noisy, the producer cannot benefit from holding inventories. Hereafter, we focus on the more relevant case \( \sigma > \tilde{\sigma} \). The game does not exhibit any perfect Bayesian equilibrium in pure strategies. The rationale is the following. Suppose that the producer chooses strategy \( I \) with probability 1. If the non-singleton information set in Figure 2 is reached, the buyer attaches a belief
1 to strategy \( I \) and therefore to being at the right decision node. This gives the producer \( \sigma \Pi_{IN} + (1 - \sigma) \Pi_{IN} = \Pi_{IN} \). However, the producer has an incentive to deviate by selecting strategy \( O \), which yields \( \Pi_{ON} > \Pi_{IN} \) (see the inequalities in points (i)-(vi) above). Similarly, the producer cannot choose strategy \( O \) with probability 1 in equilibrium, either. Suppose that, by contradiction, this is an equilibrium. The buyer assigns a belief 1 to strategy \( O \) and therefore to being at the left decision node of the non-singleton information set in Figure 2. This gives producer \( \Pi_{OS} \). However, the producer prefers to switch to strategy \( I \), which yields \( \sigma \Pi_{IN} + (1 - \sigma) \Pi_{IS} > \Pi_{OS} \), where the inequality follows from \( \sigma > \tilde{\sigma} \). The game admits an equilibrium in mixed strategies. We define \((\nu, 1 - \nu)\) as the probability distribution according to which the producer randomizes over the pure strategies \( I \) and \( O \). The non-singleton information set in Figure 2 is reached with probability \( 1 - \nu + \nu (1 - \sigma) = 1 - \nu \sigma \). Applying Bayesian updating, the probability that the buyer is at the right decision node of the non-singleton information set given that it has been reached is \( \lambda = \frac{\nu (1 - \sigma)}{1 - \nu \sigma} \). Put differently, \( \lambda \) denotes the belief that the buyer assigns to the producer’s choice of the strategy \( I \) upon receiving the signal \( s = O \). The buyer is indifferent between the pure strategies \( S \) and \( N \) in the non-singleton information set if and only if \( V_{IS} + (1 - \lambda) V_{OS} = \lambda V_{IN} + (1 - \lambda) V_{ON} \). This yields \( \lambda = \frac{V_{OS} - V_{ON}}{V_{OS} - V_{ON} + V_{IN} - V_{IS}} \in (0, 1) \) (see the inequalities in points (v)-(vi) above). As \( \lambda = \frac{\nu (1 - \sigma)}{1 - \nu \sigma} \), we find after some manipulation that the equilibrium probability that the producer holds inventories is \( \nu^* = \frac{V_{OS} - V_{ON}}{V_{OS} - V_{ON} + (1 - \sigma) (V_{IN} - V_{IS})} \in (0, 1) \). Intuitively, an increase in \( \sigma \) leads to a higher \( \nu^* \), since more accurate communication about inventories makes it more likely that the producer selects inventories in equilibrium. In the limit where \( \sigma \to 1 \) and therefore communication tends to be perfect, the producer prefers to accumulate inventories, i.e., \( \nu^* \to 1 \), consistently with the baseline model.

Turning to the buyer, we define \((\mu, 1 - \mu)\) as the probability distribution according to which the buyer randomizes over the pure strategies \( S \) and \( N \) when being at the non-singleton information set in Figure 2. Since in equilibrium the producer must be indifferent between the pure strategies \( I \) and \( O \), we have \( \sigma \Pi_{IN} + (1 - \sigma) [\mu \Pi_{IS} + (1 - \mu) \Pi_{IN}] = \mu \Pi_{OS} + (1 - \mu) \Pi_{ON} \). The equilibrium probability of buyer stockpiling is \( \mu^* = \frac{\Pi_{ON} - \Pi_{IN}}{\Pi_{ON} - \Pi_{OS} + (1 - \sigma) (\Pi_{IN} - \Pi_{IS})} \in (0, 1) \) (recall the inequalities in points (i)-(iv) above and \( \sigma > \tilde{\sigma} \)). Therefore, the perfect Bayesian equilibrium of the game prescribes that the producer holds inventories with probability \( \nu^* \in (0, 1) \) and the buyer stockpiles the good with probability \( \mu^* \in (0, 1) \).

Our stylized game indicates that the producer still resorts to inventories when they are only partially observable. As Mitraille and Moreaux (2013) suggest, it is sufficient that the producer is endowed with a communication technology that signals its actions in a relatively accurate manner (Schelling 1960). The cost of this technology can be captured in our model by the inventory holding cost. These results possibly contribute to explaining the practical evidence that firms reveal significant information about their production and inventory decisions either directly through communication activities or indirectly through specialized intermediaries. Krishnan and Winter (2010) describe these channels in their expanded working paper (Krishnan and Winter 2009). Firms often advertise their product availability and allow customers to view inventories online.\(^{15}\) Moreover, intermediaries operate in several markets to collect information.

\(^{15}\)We also refer to Yin et al. (2009) for the firms’ inventory disclosure to their customers.
about the firms’ inventory activities.

Following the approach recently elaborated by In and Wright (2018), inventories can be used for strategic purposes even when they are fully unobservable provided that the producer is able to signal the presence of inventories to buyers through some observable actions, such as prices. This model pertains to the class of games defined by In and Wright (2018) as “endogenous signalling games”, which have been underinvestigated in the economic literature so far. The prominent feature of these games is that, differently from the standard signalling games, a sender (e.g., a producer) endogenously determines an action (e.g., inventory accumulation) that is not observed by receivers (e.g., buyers) but can be signalled through observable actions (e.g., prices). Specifically, our framework can be included in the category of “monotone endogenous signalling games”, where the producer is able to affect the buyers’ choices via its signal. The monotone structure of the game must be such that (i) the producer benefits from a reduction in buyer stockpiling, (ii) the buyer stockpiling incentives are mitigated by the producer’s inventories, and (iii) the producer chooses an observable action (e.g., the pricing policy) that is monotonically related to inventory accumulation. As In and Wright (2018) demonstrate, under these conditions, despite being unobservable, inventories can still be used as if they are observable, at least to some extent. This is because the producer’s private decision about inventories can have some commitment value in terms of affecting the buyers’ choices. Moreover, “signal exaggeration” occurs in equilibrium, which may involve in our framework further price distortions relative to the case where inventories are observable.

7.2 Nonlinear production costs

It is well established that production technology is a significant determinant of inventories. The presence of increasing returns to scale, typically associated with fixed costs of production, induces the firm to accumulate production in one period since a larger quantity produced translates into lower unit costs. This could be easily accommodated in our setting by adding fixed costs of production in each period. Intuitively, the producer’s incentives to hold inventories are exacerbated by the magnitude of fixed costs.

Definitely more interesting is the situation where production technology exhibits decreasing returns to scale, namely, the firm’s production costs are (strictly) convex. Let \( C(Q_\tau) \) be the firm’s cost function, where \( Q_\tau \) denotes the quantity produced in period \( \tau \in \{1, 2\} \). The associated marginal costs \( C|_Q(Q_\tau) \geq 0 \) (with \( C|_Q(Q_\tau) > 0 \) for \( Q_\tau > 0 \)) are (strictly) increasing, i.e., \( C|_QQ(Q_\tau) > 0 \). In the first period the firm’s production corresponds to the demand for consumption inflated by buyer stockpiling and inventories, i.e., \( Q_1 = D_1(\cdot) + D_s(\cdot) + I \), while in the second period the firm’s production covers the demand for consumption net of buyer stockpiling and inventories, i.e., \( Q_2 = D_2(\cdot) - D_s(\cdot) - I \). Familiar cost efficiency considerations lead to production smoothing over time. Since in the baseline model with linear production costs a firm unable to commit to future prices has an incentive to overproduce in the first period, one might be tempted to think that the strategic role of inventories could disappear with decreasing returns to scale that make the concentration of production in one period more costly. Indeed, we show that the strategic motive for holding inventories still holds — and is even reinforced — in the presence of convex production costs. Formal details are provided in Appendix II, which
includes the characterization of the solutions under full commitment and limited commitment. The following proposition emphasizes that the producer’s lack of commitment creates incentives to hold inventories for mere strategic reasons, abstracting from any cost efficiency rationale.

**Proposition 4** **In the presence of convex production costs, the producer’s strategic incentives to hold inventories persist.**

To understand the result in Proposition 4, it is helpful to start with the full commitment case, where buyer stockpiling is removed, as in the baseline model. With convex costs, the producer prefers to accumulate inventories despite their holding costs (if \( s_p \) is not too large), since this allows production smoothing over time in anticipation of higher future demand. The producer holds inventories only for efficiency reasons and chooses the amount of inventories at the cost minimization level by equalizing marginal costs across periods, i.e., \( C_{|Q}(Q_1) + s_p = C_{|Q}(Q_2) \). Under limited commitment, we find that the producer has an additional strategic benefit of holding inventories, since a higher amount of inventories leads to lower buyer stockpiling, i.e., \( D_{s_{stockpiling}}(p_1, I) < 0 \), where \( D_{s_{stockpiling}}(\cdot) \) denotes buyer stockpiling with convex costs. This implies that the amount of inventories accumulated in the first period is above the level associated with cost minimization, i.e., \( C_{|Q}(Q_1) + s_p > C_{|Q}(Q_2) \), differently from the full commitment case. In line with the baseline model, the producer’s inventories act as a strategic device to reduce future costs, which alleviates the producer’s temptation to charge high future prices and softens the buyer stockpiling incentives. Since production generally occurs in each period as a consequence of convex costs, the constraint of sequential optimality is binding in equilibrium, and therefore the producer’s inventories and buyer stockpiling can coexist.

Our analysis indicates that an increase in inventories leads to a reduction in buyer stockpiling to a smaller extent, i.e., \( \left| D_{s_{stockpiling}}^{convex}(p_1, I) \right| < 1 \), irrespective of the existence of the inventory holding costs. In other terms, inventories and buyer stockpiling are imperfect substitutes in equilibrium. Inventories do not constitute a fully effective instrument to mitigate buyer stockpiling, even when inventory holding costs vanish, i.e., \( s_p = 0 \). This result resembles the case of a sufficiently small (constant) unit cost of production (characterized in Proposition 2), where the constraint of sequential optimality is also binding. The main difference is that with convex costs the producer does not store the entire future demand in order to reduce buyer stockpiling.

It is worth discussing the impact that production technology exhibits on the scope for strategic inventories captured by \( D_{s_{stockpiling}}^{convex}(p_1, I) \). A higher degree of cost convexity magnifies the producer’s strategic incentives to hold inventories. The idea is that a given rise in the producer’s inventories entails a higher reduction in the future marginal costs, which mitigates the buyer stockpiling incentives to a larger extent. This can be easily verified in a framework with linear demand and quadratic production costs. Notably, this rationale goes along the same lines as our analysis with linear production costs, which shows that inventories are a more effective instrument to alleviate buyer stockpiling in the presence of higher unit costs of production.

Analogously to inventories, buyer stockpiling increases production in the first period and decreases it in the second period. The economic literature has already recognized that buyer stockpiling allows a producer that cannot hold inventories to smooth production over time (e.g., Mitraille and Thille 2014). Interestingly, we find that buyer stockpiling can be preferred
to inventories as an instrument for production smoothing even when the producer’s inventories are available at a lower unit storage cost. Naturally, under full commitment, the producer chooses inventories instead of buyer stockpiling for production smoothing purposes as long as the unit inventory holding cost is lower than the unit buyer stockpiling cost, i.e., $s_p < s_b$. For $s_p = s_b$, the producer is indifferent between the two options. Under limited commitment, things change substantially. For $s_p = s_b$, the unit storage costs associated with the two alternatives are the same. However, we know from the previous discussion that inventories and buyer stockpiling are imperfect substitutes, i.e., $|D_{sl}^{isc}(p_1, I)| < 1$. This implies that the producer strictly prefers buyer stockpiling to inventories. By continuity, this holds when $s_p$ is sufficiently close to $s_b$. Consequently, the producer forgoes inventories although they involve a lower unit storage cost, and buyer stockpiling is employed as an instrument to smooth production over time.

Having established the result in Proposition 4, we compare in the following corollaries the price patterns in the two commitment scenarios.

**Corollary 1** In the presence of convex production costs, under limited commitment the producer has incentives to set higher prices than under full commitment.

Corollary 1 is a natural extension of the result in the baseline model. For a given inventory level, a producer with limited commitment powers is inclined to charge higher prices than under full commitment in order to reduce buyer stockpiling.

**Corollary 2** In any linear-quadratic framework, equilibrium prices are higher under limited commitment than under full commitment.

Corollary 2 reinforces Corollary 1 and indicates that the producer’s incentives to set higher prices under limited commitment for a given inventory level translate into higher equilibrium prices at least in any framework with linear demand and quadratic production costs. The reason is that a higher level of inventories makes it profitable for the producer to increase prices. An amount of inventories above the cost minimization level driven by strategic incentives implies that the first period costs tend to increase due to the imperfect substitutability between inventories and buyer stockpiling, while the second period costs tend to decrease. In response to the associated cost inefficiency, the producer increases prices, since the consequential reduction in buyer stockpiling helps to smooth production over time.

### 7.3 Longer time horizon

A natural extension of our framework is to allow for a longer time horizon. There are different reasons why a parsimonious two-period model is suitable for our purposes aside from its analytical tractability. Demand predictions are likely to be accurate only in the near future. Moreover, storable goods tend to depreciate and can be stockpiled only for a limited amount of time. Nonetheless, it is worth incorporating a longer time horizon into our analysis, which delivers results of some interest. Technical details are provided in the Supplementary Appendix (Section 2). To fix ideas, consider a setting with $T \geq 2$ periods, where in each period...

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16We refer to the Supplementary Appendix (Section 5) for additional formal details on this result.
and therefore the producer does not benefit from increasing the price in period $\tau$. In case of strict inequality, the producer has an incentive to set a price $p^*_\tau$ at

$$D_\tau(p^*_\tau) + p^*_\tau D_{\tau|\tau}(p^*_\tau) + \sum_{t=\tau+1}^T \{D_t(p^*_t) + [p^*_t - (t-\tau)s_p] D_{t|t}(p^*_t)\} \leq 0 \quad (10)$$

and

$$D_\tau(p^*_\tau) + (p^*_\tau - c) D_{\tau|\tau}(p^*_\tau) + \sum_{t=\tau+1}^T \{D_t(p^*_t) + [p^*_t - c - (t-\tau)s_p] D_{t|t}(p^*_t)\} \geq 0. \quad (11)$$

Condition (10) states that the derivative of the producer’s continuation profits in period $\tau \in \{2, \ldots, T\}$ (where the production costs of inventories are sunk) with respect to $p^*_\tau$ evaluated at $p^*_\tau$ must be nonpositive. If the equality follows in condition (10), the price $p^*_\tau$ is clearly sequentially optimal. In case of strict inequality, the producer has an incentive to set a price $p^*_\tau$ in period $\tau$ below $p^*_\tau$. In line with the baseline model, we find that condition (10) holds and therefore the producer does not benefit from increasing the price in period $\tau$ above $p^*_\tau$, and

$$\tau \in \{1, 2, \ldots, T\} \text{ the demand } D_\tau(p^*_\tau) \text{ increases over time so that buyers are willing to store across periods when the producer charges the static monopoly price, i.e., } p^m_\tau + s_b < p^m_{\tau+1} \text{ for any period } \tau \in \{1, 2, \ldots, T - 1\}. \text{ This constitutes a natural extension of Assumption 2. We focus on the case where the no-arbitrage constraint is binding between periods, i.e., } p_\tau + s_b = p_{\tau+1}. \text{ As shown in the Supplementary Appendix, our qualitative results carry over to the case where the no-arbitrage constraint is not binding in some periods. Before proceeding, it is important to emphasize that, in the spirit of Dudine et al. (2006), we focus on a finite time horizon, which preserves the nonstationarity of the model. An infinite time horizon setting would still exhibit some degree of nonstationarity, since the demand changes over time. The presence of periods where the no-arbitrage constraint is not binding exacerbates the nonstationary nature of the model.}$$

17 If the no-arbitrage constraint were always binding, the demand should increase forever without any interruption and the equilibrium price would become infinitely large, which would violate the transversality condition.

We begin with the case where the producer accumulates in the first period the amount of inventories that covers the aggregate demand for the following $T - 1$ periods and maximizes the producer’s ex ante profits, ignoring the constraint of sequential optimality. As under full commitment, no buyer stockpiling emerges in equilibrium. Since the producer must incur inventory holding costs in each period where it holds inventories, the producer’s aggregate profits are $\Pi^*(p_1) = \sum_{\tau=1}^T (p_\tau - c) D_\tau(p_\tau) - s_p \sum_{\tau=2}^T (\tau - 1) D_\tau(p_\tau)$. Let $\{p^*_\tau\}_{\tau=1}^T$ be the pricing policy that maximizes the producer’s aggregate profits. A producer with limited commitment powers may have an incentive in period $\tau$ to set a price different from $p^*_\tau$ and to sell a quantity that departs from the ex ante optimal level. In particular, the producer can reallocate its inventories across periods given that the associated production costs are sunk. The producer’s continuation profits in period $\tau \in \{2, \ldots, T\}$ are given by

$$p_\tau D_\tau(p_\tau) + \sum_{t=\tau+1}^T [p_t - (t-\tau)s_p] D_t(p_t).$$

The pricing policy $\{p^*_\tau\}_{\tau=1}^T$ is sequentially optimal when for any period $\tau \in \{2, \ldots, T\}$ the following two conditions hold

$$D_\tau(p^*_\tau) + p^*_\tau D_{\tau|\tau}(p^*_\tau) + \sum_{t=\tau+1}^T \{D_t(p^*_t) + [p^*_t - (t-\tau)s_p] D_{t|t}(p^*_t)\} \leq 0 \quad (10)$$

and

$$D_\tau(p^*_\tau) + (p^*_\tau - c) D_{\tau|\tau}(p^*_\tau) + \sum_{t=\tau+1}^T \{D_t(p^*_t) + [p^*_t - c - (t-\tau)s_p] D_{t|t}(p^*_t)\} \geq 0. \quad (11)$$

Condition (10) states that the derivative of the producer’s continuation profits in period $\tau \in \{2, \ldots, T\}$ (where the production costs of inventories are sunk) with respect to $p^*_\tau$ evaluated at $p^*_\tau$ must be nonpositive. If the equality follows in condition (10), the price $p^*_\tau$ is clearly sequentially optimal. In case of strict inequality, the producer has an incentive to set a price $p^*_\tau$ in period $\tau$ below $p^*_\tau$. In line with the baseline model, we find that condition (10) holds and therefore the producer does not benefit from increasing the price in period $\tau$ above $p^*_\tau$, and
provided that the unit cost of production $c$ is above a certain threshold, i.e., $c \geq \tilde{c}_\tau$. This implies that for $c \geq \tilde{c}_\tau \equiv \max_{\tau \in \{1, \ldots, T\}} \tilde{c}_\tau$ the producer does not succumb to the temptation to revise the price upwards in any period $\tau \in \{2, \ldots, T\}$.\(^{18}\) If the cost of production is relatively large, i.e., $c \geq \tilde{c}_\tau$, which satisfies condition (10), the pricing policy $\{p^*_\tau\}_{\tau=1}^T$ is sequentially optimal when condition (11) holds as well. To understand why, note that a deviation price $p'_\tau$ in period $\tau$ below $p^*_\tau$ leads to a price sequence such that the deviation prices in the following periods are also lower than the ex ante optimal prices, i.e., $p'_t < p^*_t$ for any $t \geq \tau$.\(^{19}\) Since the amount of inventories in period $\tau$ is not sufficient to cover the aggregate demand after deviation, the producer anticipates that a price reduction involves additional production costs, which are given by $c\left(\sum_{t=\tau}^TD_t(p'_t) - \sum_{t=\tau}^TD_t(p^*_t)\right)$. Condition (11) ensures that the marginal benefits of a price reduction in period $\tau$ captured by the left-hand side of condition (10) do not outweigh the associated marginal costs $c\sum_{t=\tau}^TD_t(p'_t)$. In order to fulfill condition (11), the price in the final period $T$ must be below the static monopoly level, i.e., $p^*_T \leq p^m_T$, as in the baseline model. In a long time horizon framework, this condition might not be sufficient anymore. In particular, with more than three periods ($T > 3$), an additional sufficient (albeit not necessary) condition is that $p^*_\tau > p^m_{\tau+1}$ in the early periods $\tau \in \{2, \ldots, T-2\}$.\(^{20}\) Intuitively, this occurs when the demand increases sufficiently over time, which makes a price reduction less attractive for the producer. Therefore, conditions (10) and (11) guarantee that the pricing policy $\{p^*_\tau\}_{\tau=1}^T$ can be sustained in equilibrium. In the following remark, we show that our results naturally extend to a setting with more than two periods.

**Remark 1** Let $T \geq 2$. Suppose that conditions (10) and (11) hold, which ensures that the pricing policy $\{p^*_\tau\}_{\tau=1}^T$ is sequentially optimal. Then, there exists a threshold $\tilde{s}^T_p > 0$ for the unit inventory holding cost such that for $s_p \leq \tilde{s}^T_p$ the limited commitment equilibrium exhibits the following features:

(i) producer’s inventories, i.e., $I^* = \sum_{\tau=2}^TD^*_\tau$;

(ii) no buyer stockpiling, i.e., $D^*_\tau = 0$, $\tau \in \{1, \ldots, T\}$;

(iii) prices $p^*_1 = c - \frac{D^*_1}{D^*_1 + \sum_{\tau=2}^T\phi^*_\tau - s_p\sum_{\tau=1}^T\bar{\phi}^*_\tau}$ and $p^*_\tau = p^*_1 + (\tau - 1)s_b$, $\tau \in \{2, \ldots, T\}$.

For $s_p = 0$, the full commitment outcome is restored.

Remark 1 provides a generalization of Proposition 1. The producer has strategic incentives to hold inventories to cover the aggregate future demand when the unit inventory holding cost is below a certain threshold, i.e., $s_p \leq \tilde{s}^T_p$. In line with the baseline model, strategic inventories can also create allocative benefits and lead to lower prices that approach the full commitment outcome. As shown in the Supplementary Appendix, these allocative benefits imply that the producer is willing to incur aggregate inventory holding costs that exceed the aggregate buyer

\(^{18}\)The profit concavity implies that the unique value for $p^*_\tau$ that satisfies condition (10) with equality identifies the threshold $\tilde{c}_\tau$. As $p^*_\tau$ increases with $c$, condition (10) holds for any $\tau \in \{2, \ldots, T\}$ when $c \geq \tilde{c}_\tau \equiv \max_{\tau} \tilde{c}_\tau$.

\(^{19}\)This follows from $p'_t < p^*_t$, $p^*_t = p^*_1 + (t - \tau)s_b$ for $t \geq T$ and $p'_t + s_b \geq p^*_t + s_b$. To see why $p'_t + s_b \geq p^*_t + s_b$, suppose by contradiction that $p^*_t + s_b < p^*_t + s_b$. Anticipating this, buyers store $D_{t+1}(p^*_t + s_b)$ in period $t$. However, the producer can do better by setting $p'_t$ such that $p'_t + s_b = p^*_t + s_b$ because it can serve (at least partially) $D_{t+1}(p'_t + s_b)$ at $p'_t + s_b$ in period $t + 1$ rather than at $p^*_t$ in period $t$.

\(^{20}\)In line with the baseline model, it follows from $p^*_T < p^m_T$ (as $p^m_1 + s_b < p^m_{T+1}$) that $p^*_T \leq p^m_T$ when $s_p$ is not excessively large. The first-order condition for profit maximization (provided in the Supplementary Appendix) and $p^*_1 > p^m_1$ (as $p^*_1 \geq p^*_1 > p^m_1$) imply that for $T > 3$ a further sufficient condition for (11) to hold is that $p^*_\tau > p^m_{\tau+1}$ for $\tau \in \{2, \ldots, T-2\}$.
stockpiling costs. Notably, the producer’s incentives to hold inventories persist for values of $s_p$ above the threshold $\tilde{s}_T^p$. Although it is too expensive to accumulate inventories in order to serve the aggregate future demand, the producer can still use inventories to mitigate buyer stockpiling in some periods.

If the unit cost of production is small enough, i.e., $c < \tilde{c}_T$, condition (10) is violated. The pricing policy $\{p^\tau\}_{\tau=1}^T$ is such that there exists at least one period $\tau$ where the producer cannot refrain from increasing the price above $p^\tau$ and from carrying the additional inventories to the following period $\tau + 1$. Anticipating this, buyers are eager to store in the previous period $\tau - 1$. This implies that the pricing policy $\{p^\tau\}_{\tau=1}^T$ and the associated inventories cannot be sustained in equilibrium. Following the same rationale as in the baseline model, inventories can still be profitable for the producer since they mitigate the buyer stockpiling incentives, but the amount of inventories must be distorted from the ex ante optimal level to satisfy the constraint of sequential optimality. In this case, buyer stockpiling can persist in equilibrium.

Interestingly, we find that under certain circumstances the firm has strategic incentives to produce inventories in each period to serve only the demand in the following period. This can be interpreted as “high inventory turnover”, since the inventories accumulated in one period are fully exhausted in the following period. The rationale for this result is rather subtle. In the light of our discussion so far, one might believe that the production of inventories in the first period to serve the aggregate demand in the following periods should always be the producer’s most effective strategy to mitigate the buyer stockpiling incentives because the production costs of inventories are sunk. To understand the rationale for this result, it is helpful to realize that, when the constraint of sequential optimality is binding, the production of inventories in each period interacts with the producer’s pricing policy over time. Specifically, as long as the price in period $\tau$ affects positively the amount of inventories produced in period $\tau$ to cover the demand in the following period $\tau + 1$, the producer is inclined to mitigate the price in period $\tau$ to reduce the production costs of inventories. Anticipating this, buyers are less keen to store in the previous period $\tau - 1$ compared to the situation where the entire amount of inventories is produced in the first period. A positive relation between the price and inventories in period $\tau$ can arise from the fact that a lower price in period $\tau$ inflates buyer stockpiling and therefore reduces the demand in the following period $\tau + 1$, which coincides with the inventories in period $\tau$. To illustrate ideas, in the following remark we formalize the main results in a three-period setting.

**Remark 2** Let $T = 3$. Then, it holds $D^s_{x1} (\cdot) = D^{ish}_{x1} (\cdot) + c D^{ish}_{22} (\cdot)$ and $D^s_{x\tau} (\cdot) = D^{ish}_{x\tau} (\cdot)$ for $\tau \in \{2, 3\}$.

With high inventory turnover, the firm produces inventories in the second period to cover the third period demand net of buyer stockpiling, i.e., $I^{ish}_{x2} (\cdot) = D_3 (\cdot) - D^{ish}_{x2} (\cdot)$. If the second period inventory level $I^{ish}_{x2} (\cdot)$ increases with the second period price, i.e., $I^{ish}_{22} (\cdot) > 0$, in the first period buyers anticipate that the producer is inclined to mitigate the second period price in order to reduce the production costs of inventories. Therefore, high inventory turnover alleviates the buyer stockpiling incentives in the first period, i.e., $D^s_{x1} (\cdot) > D^{ish}_{x1} (\cdot)$ at given prices (for $c > 0$). We find that $I^{ish}_{22} (\cdot) > 0$ if and only if the increase in the second period buyer stockpiling
Due to a lower price translates into a lower third period net demand \( D_3(\cdot) - D_{s2}^{ish}(\cdot) \), which coincides with the second period inventories. This is the case when the third period demand \( D_3(\cdot) \) is not too convex. Since buyer stockpiling in period \( \tau - 1 \) depends on the impact of the price in period \( \tau \) on the inventories for period \( \tau + 1 \), high inventory turnover does not alter the buyer stockpiling incentives in the last two periods. Our results suggest a strategic rationale for high inventory turnover, which complements the traditional technological motive that lies in the reduction in inventory holding costs.

### 7.4 Nonlinear pricing

It is also interesting to extend our analysis to circumstances where the producer can resort to nonlinear pricing policies. Two main alternatives are investigated. We start with the case where the producer offers the buyers a quantity-payment schedule for each period, which cannot be contingent on the past history of purchases. This typically occurs in markets with anonymous transactions. Then, we move to markets where transactions are not anonymous and buyers pay a membership (or subscription) fee that allows them to purchase products at a unit price.

#### 7.4.1 Sequence of bundles

Following Hendel et al. (2014), in a setting where transactions are anonymous and take place on the spot market, a nonlinear pricing policy specifies a (history-independent) sequence of quantity-payment bundles. In the absence of the producer’s inventories, our results share significant similarities with Hendel et al. (2014). In particular, product storability substantially alters the buyer participation constraints since buyers have an incentive to store in anticipation of skipping a purchase event. Hereafter, we provide a descriptive analysis that conveys the intuition behind the results. Technical details are available in the Supplementary Appendix (Section 3). We first consider the full commitment case where the producer is able to adhere to a two-period pricing policy. When the unit buyer stockpiling cost \( s_b \) is sufficiently large, the producer prefers to deter buyer stockpiling. This is because buyers must be compensated for costly stockpiling, which reduces the amount of the buyer surplus that the producer can extract. The optimal nonlinear pricing policy dictates a first period quantity below the static efficient level as long as buyers must be prevented from purchasing only in the first period and from smoothing consumption over time through storage. This allows buyers to enjoy a positive surplus. The second period quantity is set at the efficient level. Under limited commitment, the producer cannot fully remove buyer stockpiling when buyers benefit from smoothing consumption across periods. Similarly to the full commitment case, we find that the first period consumption is below the efficient level, so that buyer stockpiling is mitigated. Sequential optimality requires that the second period consumption is at the efficient level. Anticipating buyer stockpiling, in the first period the producer finds it optimal to charge a payment for the good that includes the associated buyer surplus. In the second period the sequentially optimal payment extracts the additional buyer surplus. Hence, the producer fully appropriates buyer surplus in equilibrium. Since the producer’s ex ante profits are maximized conditionally upon the presence of buyer stockpiling, a producer with full commitment powers replicates the limited commitment equilibrium pricing policy when the buyer stockpiling cost is sufficiently small and therefore buyer
stockpiling is desirable for the producer.

Our results suggest that nonlinear pricing affects the nature and the magnitude of the producer’s limited commitment problem. If the buyer stockpiling cost is sufficiently small, the full commitment outcome coincides with the one under limited commitment, so that the producer’s commitment powers are inconsequential. Otherwise, the producer’s lack of commitment implies that buyer stockpiling cannot be removed despite being ex ante profit detrimental, but the second period consumption is at the efficient level, as under full commitment. In this setting, the producer’s inventories cannot help to approach the full commitment outcome. We expect that there is still some scope for inventories under nonlinear pricing policies when production costs are convex. As previously discussed, if the buyer stockpiling cost is large enough, nonlinear pricing implies that under full commitment the amount of production is distorted downwards in the first period but is set at the efficient level in the second period. Given the dispersion of production between periods, inventories can be used to smooth production over time in the presence of convex costs. Under limited commitment, buyer stockpiling inflates the firm’s production in the first period. In line with the results in Section 7.2, buyer stockpiling can be an alternative instrument to achieve production smoothing.

7.4.2 Membership fee

In an increasing number of markets, transactions do not take place anonymously. Technological advances facilitate the identification of buyers according to their past history of purchases. In these markets, nonlinear pricing policies generally consist of a membership (or subscription) fee that enable buyers to purchase products at a unit price. Membership fees typically last for some time. For illustrative purposes, suppose that the membership fee is only paid in the first period. Under full commitment, the producer’s problem is rather simple. The producer can commit to set the unit price in each period at its marginal cost, which delivers the efficient level of consumption. Buyer surplus is fully extracted via the membership fee. Buyers do not stockpile in equilibrium, since they cannot gain from smoothing consumption over time. Contrary to the case where the anonymity of transactions implies the use of a sequence of bundles, the adoption of a membership fee in markets with nonanonymous transactions allows a producer with full commitment powers to achieve the efficient outcome. Under limited commitment, the producer’s problem is along the same lines as in our baseline model, and inventories can be used in equilibrium. In the second period the producer succumbs to the temptation to raise the price to the static monopoly level. Anticipating this, buyers refuse to pay the (full commitment) membership fee because they would incur losses ex post. The producer must reduce the membership fee in order to induce buyers to participate in the market. Clearly, this makes the producer worse off. To highlight the strategic role of inventories, it is helpful to consider the case where the unit cost of production is relatively large. Following the same rationale as in Proposition 1, in the first period the producer chooses the level of inventories that maximizes the ex ante profits. In the absence of inventory holding costs, it is sequentially optimal for the producer to set the second period price at the (unit) marginal cost, and the

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21 On-demand economy services and online marketplaces have recently experienced a substantial increase in the adoption of membership fees. A popular example is Amazon’s loyalty program, Amazon Prime. Alibaba charges membership fees to intermediaries that resell products to the final consumers.
full commitment outcome is restored. As long as the inventory holding costs are relatively small, the producer has strategic incentives to hold inventories. In line with the intuition for our main results, inventories reduce the producer’s future costs, which alleviates the producer’s temptation to charge excessively high future prices and mitigates the loss from the lack of commitment.

7.5 Further extensions
We now discuss further extensions that deserve some attention.

7.5.1 Competition
As mentioned in the introduction, competitive markets for storable goods have been investigated in the literature. For our purposes, it is helpful to consider two significant contributions that focus on the storing activities conducted by firms or alternatively by their customers. In a two-period Cournot duopoly model, Anton and Das Varma (2005) show that the possibility of buyer stockpiling exacerbates competition and leads to lower prices. The firms’ incentives to attract buyer stockpiling in the first period are stronger when the demand growth is smaller, since this reduces the relevance of the second period market. Conversely, in our setting, as in Dudine et al. (2006), buyer stockpiling is profit detrimental when the demand growth is high enough. Therefore, it is reasonable that, if the demand does not increase significantly over time, the firms’ incentives to compete for buyer stockpiling à la Anton and Das Varma (2005) will prevail. However, when the demand growth is high enough, the firms’ incentives to mitigate buyer stockpiling should be dominant. Inventories allow firms to reduce their future costs and to soften the buyer stockpiling incentives. This effect is expected to be stronger in the presence of a higher degree of market concentration or product differentiation, since firms are able to internalize to a larger extent the strategic benefits of inventories.

Abstracting from buyer stockpiling, Mitraille and Moreaux (2013) consider a two-period model where identical firms accumulate inventories in the first period and engage in Cournot competition in the second period. Inventories help some firms to exert endogenously a Stackelberg leadership over the rivals. The main results are derived under the condition of relatively low demand, differently from our framework. Therefore, we provide a complementary explanation for inventories that generally applies to different market conditions. Notably, when the two incentives to hold inventories coexist, they are mutually reinforcing.

7.5.2 Forward contracts
In practice, a potentially valuable instrument to which a firm can resort in some markets in order to restore its commitment powers is a forward or futures contract that specifies the trade of a good at a given price, with delivery and payment occurring at a future point in time. We argue that in our framework forward contracting cannot remove the limited commitment problem. The presence of buyers that act in the market as arbitrageurs or speculators and are not interested in final consumption generates ex post incentives for contract renegotiation

\[22\] This may also apply to the setting of Dudine et al. (2006).
between the producer and the buyers. To understand the rationale for this result, consider a contract that commits the producer to deliver to the buyers in the second period a quantity \( D^c_2 \) at a unit price \( p^c_2 \), which potentially replicates the full commitment outcome and removes buyer stockpiling. If the contract is enforced, buyers resell the good to the final consumers at the same unit price \( p^c_2 \) and make zero profits. Contract renegotiation can take place at the beginning of the second period. In response to the absence of buyer stockpiling, the producer has an incentive to propose in the second period a new contract according to which it provides the second period static monopoly quantity \( D^m_2 \) at the unit price \( p^m_2 \). Since buyers still make zero profits by reselling the good to the final consumers at the same unit price \( p^m_2 \), there are gains from renegotiation. Anticipating that the second period price will rise to \( p^m_2 > p^c_1 + s^b = p^c_2 \) (see Lemma 1), buyers are eager to store in the first period and the full commitment outcome breaks down.

7.5.3 Discount factor

In our model the producer attaches the same weight to the current and future profits, namely, the discount factor is equal to 1. In line with Dudine et al. (2006), this captures in a simple manner the relevance of the producer’s limited commitment problem. Our qualitative results naturally extend to the case of a discount factor lower than 1, provided that future profits are sufficiently important for the producer. Since an increase in the discount factor makes the producer’s limited commitment problem more relevant, we expect that a higher discount factor (or a lower interest rate) will amplify the producer’s incentives to hold inventories, analogously to the impact of a lower unit inventory cost.

8 Storable and durable goods

Our analysis focuses on markets for storable goods, where buyers exhibit demand anticipation incentives and are willing to stockpile in expectation of higher future prices. The nature of the intertemporal demand incentives changes significantly in markets for durable goods, where a monopolist with limited commitment powers faces the classical Coase problem and succumbs to the temptation to charge lower future prices in order to capture the buyers with lower valuations. This leads buyers to postpone their purchases in anticipation of future better deals and erodes the firm’s monopoly power. According to the Coase conjecture, when a durable good monopolist can adjust its prices very quickly over time, buyers expect prices to be close to marginal costs in the future and, if they are sufficiently patient, refuse to purchase at any higher price. In equilibrium, consistently with the buyer expectations, prices converge to marginal costs. The demand postponement incentives imply that the firm would prefer to exhaust all sales in the first period by credibly promising not to lower its future prices. Various remedies have been proposed in the economic literature to escape the Coase problem and to restore the firm’s commitment powers. The firm can offer its customers a money-back guarantee (sometimes called the “most-favored-nation” clause) in case of a future price fall. This price protection policy reimburses customers for any loss associated with the firm’s opportunistic future behavior and enables the firm to commit not to lower its future prices, which neutralizes the demand postponement
incentives. Another solution consists in leasing (or renting) rather than selling the good, which allows the firm to maintain the property of the good and eliminates the firm’s temptation to cut future prices in order to attract buyers with lower valuations. Alternatively, the firm can resort to planned obsolescence, which reduces the durability of the good and mitigates the firm’s time inconsistency problem. An extreme, essentially theoretical, possibility — which highlights to a larger extent the difference with respect to the firm’s limited commitment problem in a storable good market — is the destruction of the firm’s factory after the current production, provided that another factory cannot be (easily) constructed. This drastically increases the firm’s future costs and prevents the firm from “flooding the market” in the future. Clearly, the remedies to the Coase problem are worthless for a producer of storable goods. On the other hand, inventory accumulation cannot help a durable good monopolist vis-à-vis the Coase problem. In addition, differently from a storable good market, the presence of increasing marginal production costs or of new customers that inflate the future demand reduces the firm’s propensity to cut future prices and therefore mitigates the firm’s limited commitment problem.

Inventories can be used in a durable good market with a finite time horizon à la Wolinsky (1991) where, in contrast to the Coase problem, a monopolist would like to postpone all sales to the last period (in the absence of time costs). With a production technology characterized by constant (or increasing) returns to scale, this outcome can be costlessly achieved. Buyers simply wait to purchase until the last period and the producer attains the static monopoly profits. It is apparent that in this case there is no scope for inventories. However, in the presence of convex production costs, if the monopolist postpones the entire production and sales to the last period, it incurs excessively high production costs. The monopolist clearly prefers to smooth production over time for cost efficiency considerations. The accumulation of inventories for production smoothing purposes facilitates at the same time the postponement of sales. Contrary to our setting, the role of inventories as a commitment to postpone sales is directly driven by technological features and only emerges with convex production costs. The rationale for inventories in a durable good monopoly setting à la Wolinsky (1991) is substantially different from the one in our framework. In a market for storable goods, the producer holds inventories to credibly reduce future costs, which weakens the stockpiling incentives of forward-looking buyers available to store in anticipation of higher future prices. This holds true irrespective of the production technology.

Naturally, inventories alter the firm’s future cost structure in both settings. However, in Wolinsky (1991) this additional effect is detrimental to the firm, since a larger amount of inventories increases future sales due to lower future costs and depresses the current price. This is what the producer tries to avoid. In our framework, the firm resorts to inventories exactly as a strategic device to reduce future costs, which mitigates the buyer stockpiling incentives and enhances the firm’s profits. To appreciate even further the difference between the two settings, it is helpful to recall from Section 7.2 that, in the presence of convex production costs, inventories are an imperfect instrument to alleviate buyer stockpiling and full production smoothing never occurs, even when holding inventories is costless. Conversely, in Wolinsky (1991) inventories perfectly work in the absence of holding costs, since they allow the monopolist to evenly smooth production over time and to postpone all sales to the last period.
9 Conclusions: managerial, empirical and policy implications

A full understanding of the firms’ inventory behavior is a challenging task that goes well beyond our study. In this paper, we unveil a novel strategic channel for inventories that complements the technological reasons provided by the traditional inventory theories. Specifically, we characterize the producer’s strategic incentives to hold inventories in a dynamic storable good monopoly framework à la Dudine et al. (2006) where the producer is unable to commit to future prices and forward-looking buyers are willing to stockpile in expectation of higher future prices. Anticipating that the producer cannot refrain from charging excessively high future prices, buyers engage in stockpiling activities, which harm the producer’s profits. We show that the producer’s inventories act as a strategic device to mitigate the loss from the lack of commitment. Since the costs of inventories are sunk once they have been incurred, the producer hold inventories to reduce future costs, which alleviates the producer’s temptation to charge high future prices and softens the buyer stockpiling incentives. With linear production technology, if the unit cost of production is relatively large, the producer chooses the level of inventories that maximizes the ex ante profits, and the constraint of sequential optimality is slack in equilibrium. Despite the producer’s lack of commitment, buyer stockpiling is fully removed. If the unit cost of production is small enough, inventories can be still beneficial to the producer although their amount is distorted from the ex ante optimal level to ensure sequential optimality. This implies that buyer stockpiling can persist in equilibrium. We also find that equilibrium prices exhibit nontrivial features. Notably, our results can be extended in different directions. Our model suggests that the normalization of production costs to zero usually adopted in the literature is not innocuous in storable good markets, since it undermines the firms’ strategic incentives to hold inventories. Although our attention is devoted to storable goods, similar storage incentives may hold for durable goods, as Dudine et al. (2006) emphasize. Therefore, the predictions of our model can apply to durable good markets.

The plainness of our analysis allows us to identify in a transparent and intuitive manner a strategic rationale for inventories that leads to potentially significant managerial, empirical and policy implications. Our results recommend that managers recognize the strategic benefits of holding inventories in anticipation of higher future demand. We also provide theoretical corroboration for the anecdotal evidence about the firms’ advertisement and disclosure of product availability (e.g., Krishnan and Winter 2009, 2010; Yin et al. 2009).

Our analysis sheds new light on the well-established empirical findings about inventories discussed in the introduction. In particular, since in our framework inventories are accumulated in periods of demand expansion, we establish microfoundations for the empirical observation of inventory procyclicality. Furthermore, our investigation of the incentives to accumulate inventories by a firm with (monopoly) market power can contribute to explaining the empirical evidence documented by Amihud and Mendelson (1989) that firms with greater market power hold a higher level of inventories. The empirical implications of our results extend to the impact of inventories on the firm’s performance. Using a panel data set of 54 US manufacturing industries in the period 2003–2008, Eroglu and Hofer (2011) show that the inventory-performance relationship varies significantly across industries. They suggest that the use of inventories can depend on industry-specific features, such as product and demand conditions. Our study
proposes the empirically testable explanation based on buyer product storability and demand expansion. A related financial implication potentially helpful for investors in the stock and commodity markets is the role of inventories as a predictor of higher profitability in storable good markets in periods of demand growth. A stochastic version of our model can capture other relevant pieces of the empirical evidence about inventories. In a setting à la Kahn (1987) where demand is uncertain and exhibits positive serial correlation, a positive demand shock increases production more than sales since inventories cover the higher expected future demand. Conversely, a negative demand shock implies that inventories are not accumulated and production coincides with sales in expectation of lower future demand. This makes the volatility of production larger than the volatility of sales, consistently with the empirical evidence.

We bring to the public debate a number of additional findings that are suitable for empirical or experimental corroboration and can promote further research on the relevant issue of inventory management. Our analysis suggests that, when production technology is linear, inventories are more likely to be observed in industries with a relatively large unit cost of production. In the presence of convex costs, inventories and buyer stockpiling contribute to production smoothing across periods. We also find that inventories can lead to higher aggregate production and lower prices over time. This generates policy implications of some interest, especially in terms of inventory taxation.

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Appendix I

This Appendix collects the proofs of Lemmas 1, 2 and 3 as well as the proofs of Propositions 1, 2 and 3.

Proof of Lemma 1. Using (3), we identify the following three cases: (a) \( p_1 + s_b = p_2 \), which implies \( D_s(\cdot) \in [0, D_2(p_1 + s_b)] \); (b) \( p_1 + s_b > p_2 \), which implies \( D_s(\cdot) = 0 \); (c) \( p_1 + s_b < p_2 \), which implies \( D_s(\cdot) = D_2(p_1 + s_b) \). We start with case (a). At the end of the proof, we show that the remaining cases (b) and (c) cannot be an equilibrium. The producer’s maximization problem is given by

\[
\max_{p_1, I, D_s} (p_1 - c) [D_1(p_1) + D_s] - (c + s_p) I \\
+ (p_1 + s_b) [D_2(p_1 + s_b) - D_s] - c [D_2(p_1 + s_b) - D_s - I] \cdot 1_{Q_s}.
\]
Since the objective function decreases with $I$ and $D_s$, we have $I^c = D^c_s = 0$ in equilibrium. The producer’s maximization problem becomes

$$\max_{p_1} (p_1 - c) D_1 (p_1) + (p_1 + s_b - c) D_2 (p_1 + s_b).$$

Taking the first-order condition for $p_1$ yields

$$D_1 (p_1) + (p_1 - c) D_{1|1} (p_1) + \phi_2 (p_1 + s_b) = 0,$$  \hspace{1cm} (12)

where $\phi_2 (\cdot)$ is defined by (7). This implies $p_1^c = c - \frac{D_{1|1}'}{D_{1|1}''}$ and $p_2^c = p_1^c + s_b$. Now, we show that $p_1^c > p_1^m$ and $p_2^c < p_2^m$. To show $p_1^c > p_1^m$, we substitute the first-order condition for $p_1^m$, i.e., $D_1 (p_1) + (p_1 - c) D_{1|1} (p_1) = 0$, into the left-hand side of the first-order condition for $p_1^c$ in (12). This yields $\phi_2 (p_1^c + s_b) > 0$, where the inequality follows from Assumption 2. Then, Assumption 1 implies $p_1^c > p_1^m$.

To show $p_2^c < p_2^m$, we substitute the first-order condition for $p_2^m$, i.e., $\phi_2 (p_2^c) = 0$, into the left-hand side of the first-order condition for $p_2^c$, which corresponds to the first-order condition for $p_1^c$ in (12). This yields $D_1 (p_2^m - s_b) + (p_2^m - s_b - c) D_{1|2} (p_2^m - s_b) < 0$, where the inequality follows from Assumption 2. Then, Assumption 1 implies $p_2^c < p_2^m$.

Finally, we consider cases (b) and (c) and show that they cannot be an equilibrium. In case (b) we have $p_1 + s_b > p_2$, which implies $D_s (\cdot) = 0$. The producer’s maximization problem reduces to the unconstrained problem in (6), which leads to the static monopoly prices $p_1^m$ and $p_2^m$. However, the supposition $p_1^m + s_b > p_2^m$ violates Assumption 2. For any price $p_2^c < p_2^m$ such that $p_1^m + s_b > p_2^c$, there exists another price $p_2^c$ such that $p_2^c > p_2^c$ and $p_1^m + s_b > p_2^c$, which makes the producer better off (due to an increase in the second period profits). Then, case (b) cannot be an equilibrium. In case (c) we have $p_1 + s_b < p_2$, which implies $D_s (\cdot) = D_2 (p_1 + s_b)$. Buyers store in the first period and abstain from purchasing in the second period. However, the producer can do better by setting $p_2^c = p_1 + s_b$, which implies from case (a) that the producer can serve $D_2 (p_1 + s_b)$ at $p_1 + s_b$ rather than at $p_1$. Then, case (c) cannot be an equilibrium, either.

**Proof of Lemma 2.** Using (3), we identify the same three cases as in the proof of Lemma 1. We start with case (a) where $p_1 + s_b = p_2$, which implies $D_s (\cdot) \in [0, D_2 (p_1 + s_b)]$. At the end of the proof, we show that the remaining cases (b) and (c) cannot be an equilibrium. Using the sequential optimality constraint in (9) for $I = 0$, we obtain $p_2 = c - \frac{D_2(p_2) - D_2(p_1)}{D_2'(p_2)}$. This yields

$$D_s (p_1) = \max \{ \phi_2 (p_1 + s_b), 0 \},$$

where $\phi_2 (\cdot)$ is defined by (7). Suppose first that $D_s (p_1) > 0$. Substituting $D_s (p_1)$ into (8) for $I = 0$, the producer’s maximization problem becomes after some manipulation

$$\max_{p_1} (p_1 - c) D_1 (p_1) + (p_1 + s_b - c) D_2 (p_1 + s_b) - s_b \phi_2 (p_1 + s_b).$$

The first-order condition for $p_1$ is

$$D_1 (p_1) + (p_1 - c) D_{1|1} (p_1) + \phi_2 (p_1 + s_b) - s_b \phi_{2|1} (p_1 + s_b) = 0,$$  \hspace{1cm} (13)

which yields $p_1^{ns} = c - \frac{D_{1|1}'' + \phi_{2|1}'' - s_b \phi_{2|1}''}{D_{1|1}''} = 0$ and $p_2^{ns} = p_1^{ns} + s_b$. Buyer stockpiling is $D_s^{ns} = \phi_2^{ns}$. This

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is the result in point (a) of the lemma. If \( \phi_{2}^{ns} \leq 0 \), buyer stockpiling is \( D_{s}^{mn} = 0 \), with prices \( p_{1}^{*n} = p_{2}^{*n} - s_{b} \) and \( p_{2}^{*n} = c - \frac{D_{s}^{mn}}{D_{1}^{1/2}} = p_{2}^{m} \). This is the result in point (b) of the lemma.

Finally, we consider cases (b) and (c) and show that they cannot be an equilibrium. In case (b) we have \( p_{1} + s_{b} > p_{2} \), which implies \( D_{s} (\cdot) = 0 \). It follows from the sequential optimality constraint in (9) that the second period price is \( p_{2}^{m} \). For any price \( p_{1}' > p_{2}^{m} \) such that \( p_{1}' + s_{b} > p_{2}^{m} \), there exists another price \( p_{1}'' \) such that \( p_{1}'' < p_{1}' \) and \( p_{1}'' + s_{b} > p_{2}^{m} \), which makes the producer better off (due to an increase in the first period profits). Then, case (b) cannot be an equilibrium. In case (c) we have \( p_{1} + s_{b} < p_{2} \), which implies \( D_{s} (\cdot) = D_{2} (p_{1} + s_{b}) \). Buyers store in the first period and abstain from purchasing in the second period, which gives zero second period profits. However, the producer can do better by reducing \( p_{2} \) below \( p_{1} + s_{b} \) and serving the residual demand in the second period. Then, case (c) cannot be an equilibrium, either. ■

**Proof of Proposition 1.** Suppose that the producer chooses to hold inventories. We characterize the conditions under which this solution is optimal. Using (3), we identify the same three cases as in the proof of Lemma 1. We start with case (a) where \( p_{1} + s_{b} = p_{2} \), which implies \( D_{s} (\cdot) \in [0, D_{2} (p_{1} + s_{b})] \). At the end of the proof, we show that the remaining cases (b) and (c) cannot be an equilibrium. Since \( I = D_{2} (p_{1} + s_{b}) - D_{s} (p_{1}) \), ignoring the sequential optimality constraint in (9), the producer’s maximization problem is given by

\[
\max_{p_{1}} \left( p_{1} - c \right) D_{1} (p_{1}) + (p_{1} + s_{b} - c - s_{p}) D_{2} (p_{1} + s_{b}) - (s_{b} - s_{p}) D_{s} (p_{1})
\]

As \( s_{p} \leq s_{b} \), we have \( D_{s}^{*} = 0 \) in equilibrium, otherwise the producer could slightly increase \( p_{1} \), which removes buyer stockpiling and entails a discontinuous increase in profits. The first-order condition for \( p_{1} \) is

\[
D_{1} (p_{1}) + (p_{1} - c) D_{1|1} (p_{1}) + \phi_{2} (p_{1} + s_{b}) - s_{p} D_{2|1} (p_{1} + s_{b}) = 0,
\]

which yields \( p_{1}^{*} = c - \frac{D_{1}^{*} + \phi_{2}^{*} - s_{p} D_{2|1}^{*}}{D_{1|1}^{*}} \) and \( p_{2}^{*} = p_{1}^{*} + s_{b} \). We now show that selling \( I^{*} = D_{2}^{*} \) in the second period satisfies the constraint of sequential optimality if and only if \( p_{2}^{m} \mid_{c=0} \leq p_{2} \leq p_{2}^{m} \).

The second period cost function \( C_{2} (p_{2}) \) is \( c [D_{2} (p_{2}) - D_{2}^{*}] \) for \( p_{2} < p_{2}^{m} \) and zero for \( p_{2} \geq p_{2}^{m} \). The marginal cost \( C_{2|2} (p_{2}) \) is equal to \( c D_{2|2} (p_{2}) \) for \( p_{2} < p_{2}^{m} \) and zero for \( p_{2} > p_{2}^{m} \). The second period revenue function is \( R_{2} (p_{2}) = p_{2} D_{2} (p_{2}) \), which entails a marginal revenue \( R_{2|2} (p_{2}) = D_{2} (p_{2}) + p_{2} D_{2|2} (p_{2}) \). Assume first that \( p_{2}^{m} < p_{2}^{m} \mid_{c=0} \), where \( R_{2|2} (p_{2}^{m} \mid_{c=0}) = 0 \). Since the concavity (by Assumption 1) of the static second period profit function with zero costs implies that \( R_{2|2} (p_{2}) \) decreases with \( p_{2} \), we have \( R_{2|2} (p_{2}^{m}) > \lim_{p_{2} \rightarrow p_{2}^{m}+} C_{2|2} (p_{2}) = 0 \). Then, a price above \( p_{2}^{m} \) increases the producer’s second period profits. This implies that \( p_{2}^{m} < p_{2}^{m} \mid_{c=0} \) is not sequentially optimal.

Now, assume that \( p_{2}^{m} > p_{2}^{m} \), where \( R_{2|2} (p_{2}^{m}) = c D_{2|2} (p_{2}^{m}) \). Since the concavity (by Assumption 1) of the static second period profit function with marginal costs \( c \) (maximized at \( p_{2}^{m} \)) implies that \( R_{2|2} (p_{2}^{m}) < c D_{2|2} (p_{2}^{m}) \), we have \( R_{2|2} (p_{2}^{m}) < \lim_{p_{2} \rightarrow p_{2}^{m}-} C_{2|2} (p_{2}) = c D_{2|2} (p_{2}^{m}) \). Then, a price below \( p_{2}^{m} \) increases the producer’s second period profits. This implies that \( p_{2}^{m} > p_{2}^{m} \) is not sequentially optimal, either. The last step is to show that, if \( p_{2}^{m} \mid_{c=0} \leq p_{2} \leq p_{2}^{m} \), then \( p_{2}^{m} \) is sequentially optimal. We have \( R_{2|2} (p_{2}^{m}) \in \left[c D_{2|2} (p_{2}^{m}), 0\right] \), where the lower bound of the interval holds if and only if \( p_{2}^{m} = p_{2}^{m} \) and the upper bound holds if and only if \( p_{2}^{m} = p_{2}^{m} \mid_{c=0} \).

A price above \( p_{2}^{m} \) does not increase the producer’s second period profits, since \( R_{2|2} (p_{2}^{m}) \leq \)}
\[ \lim_{p_2 \to p_2^*} C_{2|2} (p_2) = 0. \] A price below \( p_2^* \) does not increase the producer’s second period profits, since \( 0 \geq R_{2|2} (p_2^*) \geq \lim_{p_2 \to p_2^*} C_{2|2} (p_2) = cD_{2|2} (p_2^*) \). Hence, \( p_2^* \) is sequentially optimal if and only if \( p_2^* \) is the unique price that satisfies \( p_2^* \leq p_2^m \).

We now derive the conditions under which it holds \( p_2^m \mid_{c=0} \leq p_2^* \leq p_2^m \). Substituting the first-order condition for \( p_2^m \mid_{c=0} \), i.e., \( D_2 (p_2^m) + D_2 D_{2|2} (p_2^m) = 0 \), into the left-hand side of the first-order condition for \( p_2^* \), which corresponds to the first-order condition for \( p_2^* \) in (14), and using (7), we find that

\[
D_1 (p_2^m \mid_{c=0} - s_b) + (p_2^m \mid_{c=0} - s_b - c) D_{1|2} (p_2^m \mid_{c=0} - s_b) - (c + s_p) D_{2|2} (p_2^m \mid_{c=0}) \geq 0
\]

if and only if

\[
c \geq \tilde{c} \equiv \frac{D_1 (p_2^m \mid_{c=0} - s_b) + (p_2^m \mid_{c=0} - s_b) D_{1|2} (p_2^m \mid_{c=0} - s_b) - s_p D_{2|2} (p_2^m \mid_{c=0})}{D_{2|2} (p_2^m \mid_{c=0} - s_b) + D_{2|2} (p_2^m \mid_{c=0})}.
\]

It follows from Assumption 1 that \( p_2^* \geq p_2^m \mid_{c=0} \) if and only if \( c \geq \tilde{c} \). We find from Assumption 2 that \( \tilde{c} > 0 \) for \( s_p \) small enough. Substituting the first-order condition for \( p_2^m \), i.e., \( D_2 (p_2) + (p_2 - c) D_{2|2} (p_2) = 0 \), into the left-hand side of the first-order condition for \( p_2^* \), which corresponds to the first-order condition for \( p_2^* \) in (14), and using (7), we find that

\[
D_1 (p_2^m - s_b) + (p_2^m - s_b - c) D_{1|2} (p_2^m - s_b) - s_p D_{2|2} (p_2^m) \leq 0
\]

if and only if

\[
s_p \leq \frac{D_1 (p_2^m - s_b) + (p_2^m - s_b - c) D_{1|2} (p_2^m - s_b)}{D_{2|2} (p_2^m)},
\]

where the expression on the right-hand side is positive by Assumption 2. It follows from Assumption 1 that \( p_2^* \leq p_2^m \) if and only if this condition is satisfied. A comparison with the solution in Lemma 1 reveals that for \( c \leq \tilde{c} \) the full commitment outcome is restored as long as \( s_p = 0 \). The producer’s equilibrium profits are \( \Pi^* (p_2^*) = \Pi^c (p_1^*) - s_p D_2 (p_1^* + s_b) \), where \( \Pi^c (\cdot) \) denotes the full commitment profits. Since the limited commitment profits in the presence of inventories coincide with the full commitment profits for \( s_p = 0 \) and decrease with \( s_p \) while the limited commitment profits in the absence of inventories are lower than the full commitment profits and do not depend on \( s_p \), there exists a threshold \( \tilde{s}_p > 0 \) such that for \( s_p \leq \tilde{s}_p \) the solution characterized in the proposition is optimal. Consider the most relevant case in the absence of inventories, namely, the buyer stockpiling outcome formalized in Lemma 2(a). The associated profits are \( \Pi^{ns} (p_1^{ns}) = \Pi^c (p_1^{ns}) - s_b D^{ns}_s (p_1^{ns}) \). In this case, the threshold \( \tilde{s}_p \) is defined by

\[
\Pi^c (p_1^*) - \Pi^c (p_1^{ns}) + s_b D^{ns}_s (p_1^{ns}) - \tilde{s}_p D_2 (p_1^* + s_b) = 0.
\]

Given that \( p_1^* < p_1^{ns} \) under relatively mild conditions (see the proof of Proposition 3), we have \( \Pi^c (p_1^*) > \Pi^c (p_1^{ns}) \) (as \( p_1^* \geq p_1^c \) and \( p_1^{ns} > p_1^c \)). This implies that \( \tilde{s}_p D_2 (p_1^* + s_b) > s_b D^{ns}_s (p_1^{ns}) \).

Finally, we consider cases (b) and (c) and show that they cannot be an equilibrium. In case (b) we have \( p_1 + s_b > p_2 \), which implies \( D_1 (\cdot) = 0 \) and \( I = D_2 (\cdot) \). This yields \( p_1 = p_1^m \) and \( p_2 = p_2^m \mid_{c=0} < p_1^m + s_b \), where \( p_2 = p_2^m \mid_{c=0} \) follows from the sequential optimality constraint.
in (9). However, the producer can do better by holding a lower inventory level \( I = D_2(p_2') \), where \( p_2' = p_2'' + s_b \) and \( p_2'' \mid c=0 < p_2' < p_2'' \) (by Assumption 2). This implies from the previous analysis that \( p_2' \) is sequentially optimal and makes the producer better off (by increasing the second period profits and reducing the inventory holding costs). Then, case (b) cannot be an equilibrium. It follows directly from the proof of Lemma 2 that case (c) cannot be an equilibrium, either. ■

**Proof of Lemma 3.** Using (3), we identify the same three cases as in the proof of Lemma 1.

We start with case (a) where \( p_1 + s_b = p_2 \), which implies \( D_s(\cdot) \in [0, D_2(p_1 + s_b)] \). It follows from the proof of Proposition 1 that the remaining cases (b) and (c) cannot be an equilibrium.

Since \( I = D_2(p_1 + s_b) - D_s(p_1) \), we find from the sequential optimality constraint in (9) that

\[
D_s(p_1) = \max \left\{ \phi_2(p_1 + s_b) + cD_{2|1} (p_1 + s_b), 0 \right\}.
\]

Suppose first that \( D_s(p_1) > 0 \). Substituting \( D_s(p_1) \) into the producer’s maximization problem in (8) yields after some manipulation

\[
\max_{p_1} (p_1 - c) D_1(p_1) + (p_1 + s_b - c - s_p) D_2(p_1 + s_b) - (s_b - s_p) D_s(p_1 + s_b).
\]

The first-order condition for \( p_1 \) is

\[
D_1(p_1) + (p_1 - c) D_{1|1}(p_1) + \phi_2(p_1 + s_b) - s_p D_{2|1}(p_1 + s_b)
- (s_b - s_p) \left[ \phi_{2|1}(p_1 + s_b) + cD_{2|11}(p_1 + s_b) \right] = 0,
\]

which yields \( p_1^* = c - \frac{\phi_1^* + \phi_2^* - s_p D_{2|1}^* - (s_b - s_p) \left( \phi_{2|1}^* + cD_{2|11}^* \right)}{D_{1|1}^*} \) and \( p_2^* = p_1^* + s_b \). The producer’s inventories and buyer stockpiling are respectively \( I^* = D_2^* - D_{2|1}^* = -p_2^* D_{2|1}^* \) and \( D_s^* = \phi_2^* + cD_{2|1}^* \). This is the result in point (a) of the lemma. If \( \phi_2^* + cD_{2|1}^* \leq 0 \), the producer’s inventories and buyer stockpiling are \( I^* = D_2^* \) and \( D_s^* = 0 \), with prices \( p_1^* = p_2^* = s_b \) and \( p_2^* = -\frac{D_{1|1}^m}{D_{2|1}^m} = p_2^m \mid c=0 \). This is the result in point (b) of the lemma. ■

**Proof of Proposition 2.** It follows from the proof of Lemma 2 that, if \( D_s^*(p_1^*) = \phi_2^* > 0 \), the profits of a producer with limited commitment powers in the absence of inventories are

\[
\Pi_s^*(p_1^*) = (p_1^* - c) D_1^* + (p_1^* + s_b - c) D_2^* - s_b \phi_2^*.
\]

We find from the proof of Lemma 3 that the profits of a producer with limited commitment powers that holds inventories and replicates the prices without inventories are given by

\[
\Pi_s^*(p_1^*) = (p_1^* - c) D_1^* + (p_1^* + s_b - c - s_p) D_2^* - (s_b - s_p) D_s^*(p_1^*),
\]

which is feasible for \( D_s^*(p_1^*) = \phi_2^* + cD_{2|1}^* \geq 0 \). Using (7), we obtain after some manipulation that

\[
\Pi_s^*(p_1^*) - \Pi_s^*(p_1^*) = -s_b c D_{2|1}^* - s_p \left( D_2^* - \phi_2^* - cD_{2|1}^* \right)
= \left[ s_p \left( p_1^* + s_b \right) - s_b c \right] D_{2|1}^* > 0,
\]

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where the inequality holds for $s_p < \frac{sc}{p_1}$. Since $D_s^ns(p_1^ns) - D_s^is(p_1^ns) = -cD_s^ns_{21} > 0$ (for $c > 0$), we find that for $D_s^ns(p_1^ns) > 0$ it holds $D_s^is(p_1^ns) \geq 0$ if the unit cost of production $c$ is small enough. ■

**Proof of Proposition 3.** The results in point (i) of the proposition follow from steps 1-4.
1. Substituting the first-order condition for $p_1^s$ in (12) into the left-hand side of the first-order condition for $p_1^s$ in (14) yields $-s_pD_s^c_{21} \geq 0$. Assumption 1 implies that $p_1^s \leq p_1^s$. Since $p_1 + s_b = p_2$, we find $p_1^s \leq p_1^s$, $\tau \in \{1, 2\}$, where the equality follows if and only if $s_p = 0$.
2. Substituting the first-order condition for $p_1^ns$ in (13) into the left-hand side of the first-order condition for $p_1^ns$ in (14) yields

$$s_b\phi_{21}^ns - s_pD_s^ns_{21}. \tag{17}$$

The sign of (17) is negative if and only if $s_p < \frac{s_b\phi_{21}^ns}{D_s^c_{21}} \equiv \bar{s}_p$, where $\bar{s}_p > 0$. It follows from Assumption 1 that $p_1^s < p_1^s$ if and only if $s_p < \bar{s}_p$. Since $p_1 + s_b = p_2$, we find $p_1^s < p_1^s$, $\tau \in \{1, 2\}$, if and only if $s_p < \bar{s}_p$.
3. It follows from the proof in Proposition 1 that $p_2^m \leq p_2^m$. A (necessary) condition for the optimality of inventories is that $p_2^s < p_2^m$. Since $p_2^m = p_2^m$ from Lemma 2(b) and $p_1 + s_b = p_2$, we find $p_1^s < p_1^m$, $\tau \in \{1, 2\}$.
4. Substituting the first-order condition for $p_2^ns$, i.e., $\phi_2(p_2) - D_s^ns(p_1) = 0$, into the left-hand side of the first-order condition for $p_2^ns$, i.e., $\phi_2(p_2) + cD_2^s(p_2) = 0$, yields $D_s^ns > 0$, where the inequality follows from Lemma 2(a). Assumption 1 implies $p_2^s < p_2^m$. Since $p_1 + s_b = p_2$, we find $p_1^s < p_1^m$, $\tau \in \{1, 2\}$.

The results in point (ii) of the proposition follow from steps 5-6.
5. Substituting the first-order condition for $p_2^s$, i.e., $\phi_2(p_2) + cD_2^s(p_2) = 0$, into the left-hand side of the first-order condition for $p_2^s$, i.e., $\phi_2(p_2) + cD_2^s(p_2) = 0$, yields $D_s^is > 0$, where the inequality follows from Lemma 3(a). Assumption 1 implies $p_2^s < p_2^m$. Since $p_1 + s_b = p_2$, we find $p_1^s < p_1^m$, $\tau \in \{1, 2\}$.
6. Substituting the first-order condition for $p_2^m$, i.e., $\phi_2(p_2) + cD_2^s(p_2) = 0$, into the left-hand side of the first-order condition for $p_2^m$, i.e., $\phi_2(p_2) + cD_2^ms(p_2) = 0$, yields $-cD_2^ns_{21} \geq 0$. Assumption 1 implies $p_2^m \leq p_2^m$. Since $p_1 + s_b = p_2$, we find $p_1^m \leq p_1^m$, $\tau \in \{1, 2\}$, where the equality holds if and only if $c = 0$.

To show the results in point (iii) of the proposition, we substitute the first-order condition for $p_1^ns$ in (13) into the left-hand side of the first-order condition for $p_1^ns$ in (16). This yields after some manipulation

$$s_p \left[ D_s^ns_{21} + (p_1^ns + s_b)D_s^ns_{211} \right] - s_bD_s^ns_{211}. \tag{18}$$

The sign of (18) is negative if and only if one of the following conditions holds: (a) $D_s^ns_{21} \leq 0$ and $s_p > \frac{s_bD_s^ns_{211}}{D_s^ns_{21} + (p_1^ns + s_b)D_s^ns_{211}} \equiv \tilde{s}_p$, where $\tilde{s}_p > 0$; (b) $0 < D_s^ns_{211} < -\frac{D_s^ns_{211}}{p_1^ns + s_b} \equiv \hat{D}_2|11$; (c) $D_s^ns_{211} \geq \hat{D}_2|11$ and $s_p < \tilde{s}_p$. Assumption 1 implies that, if one of these conditions holds, we have $p_1^ns < p_1^ns$. This yields $p_1^ns < p_1^ns$, $\tau \in \{1, 2\}$, since $p_1 + s_b = p_2$. ■
Appendix II

This Appendix provides the formal details for the case of convex production costs (see Section 7.2). When buyer stockpiling is not feasible, the producer’s maximization problem is given by

$$\max_{p_1, p_2, I} p_1 D_1 (p_1) - C (D_1 (p_1) + I) + p_2 D_2 (p_2) - C (D_2 (p_2) - I) - s p I.$$  \hfill (19)

Taking the first-order conditions for $p_r, \tau \in \{1, 2\}$, and $I$, we obtain respectively $D_\tau (p_r) = (p_r - C (Q / (Q / (Q / Q))) D_\tau (p_r) = 0$ and $C (Q / (Q / (Q / Q))) - C (Q / (Q / Q) - s p \leq 0$, where the equality holds if $I > 0$. This defines the static monopoly values $p^m \tau$, $\tau \in \{1, 2\}$, and $I^m$ with convex costs. We have $I^m > 0$ for $s p < s^m_p \equiv C (Q / (Q / (Q / Q))) - C (Q / (Q / (Q / Q)))$. As in the baseline model, we impose the following assumption, which ensures that the opportunity of buyer stockpiling affects the producer’s intertemporal pricing problem.

**Assumption 2’** $p^mc_1 + s_b < p^mc_2 \iff s_b < s_p + D^mc_1 / s_1 - D^mc_2 / s_2$ for $s_p < s^mc_p$.

The following lemma formalizes the full commitment equilibrium with convex production costs.

**Lemma 4** In the presence of convex production costs, the full commitment equilibrium exhibits the following features:

(i) producer’s inventories, i.e., $I^cc > 0$, for $s_p < s^cc_p$;

(ii) no buyer stockpiling, i.e., $D^cc_s = 0$;

(iii) prices $p^cc_1 = C (Q / (Q / (Q / Q))) - D^cc_s / s_1$ and $p^cc_2 = p^cc_1 + s_b$.

**Proof of Lemma 4.** Following the same rationale as in the proof of Lemma 1, we can restrict our attention to the case where $p_1 + s_b = p_2$, which implies $D_s (\cdot) \in [0, D_2 (p_1 + s_b)]$. The producer’s maximization problem becomes

$$\max_{p_1, I, D_s} p_1 [D_1 (p_1) + D_s] - C (D_1 (p_1) + D_s + I) + (p_1 + s_b) [D_2 (p_1 + s_b) - D_s]$$

$$- C (D_2 (p_1 + s_b) - D_s - I) - s p I.$$  \hfill (19)

The first-order conditions for $p_1$, $I$ and $D_s$ are respectively

$$D_1 (p_1) + [p_1 - C (Q / (Q / (Q / Q)))] D_{1|1} (p_1) + D_2 (p_1 + s_b) + [p_1 + s_b - C (Q / (Q / (Q / Q)))] D_{2|1} (p_1 + s_b) = 0 \hfill (20)

$$C (Q / (Q / (Q / Q)) - C (Q / (Q / (Q / Q)) - s_p \leq 0 \hfill (21)

$$C (Q / (Q / (Q / Q)) - C (Q / (Q / (Q / Q)) - s_b \leq 0. \hfill (22)

Using (21) and (22), we find that $D^cc_s = 0$ in equilibrium (as $s_p \leq s_b$). Moreover, it follows from (21) that for $s_p < s^cc_p \equiv C (Q / (Q / (Q / Q))) - C (Q / (Q / (Q / Q)))$ evaluated at $p^cc_1 |_{l=0}$ we have $I^cc > 0$ in equilibrium, which implies that (21) holds with equality. The Hessian matrix $H^cc$ must be negative definite to ensure that the second-order conditions for profit maximization are satisfied. Using the following auxiliary function

$$\Phi_2 (p_2) \equiv D_2 (p_2) + [p_2 - C (Q / (Q / (Q / Q)))] D_{2|2} (p_2), \hfill (23)$$
the first-order conditions for \( p_1 \) in (20) and \( I \) in (21) can be respectively rewritten as

\[
D_1(p_1) + \left[ p_1 - C_{|Q} (Q_1) \right] D_{1|1} (p_1) + \Phi_2(p_1 + s_b) = 0 \tag{24}
\]

\[
C_{|Q} (Q_2) - C_{|Q} (Q_1) - s_p = 0. \tag{25}
\]

It follows from (24) that \( p_1^{cc} = C_{|Q} (Q_1^{cc}) - \frac{D_{1|1} C_{|Q} (Q_1^{cc}) + \Phi_2^{cc}}{D_{1|1}} \) and \( p_2^{cc} = p_1^{cc} + s_b \), where \( Q_1^{cc} = D_1^{cc} + I^{cc} \) and \( Q_2^{cc} = D_2^{cc} - I^{cc} \).

The following lemma formalizes the limited commitment equilibrium with convex costs when buyer stockpiling arises.

**Lemma 5** In the presence of convex production costs, the limited commitment equilibrium with buyer stockpiling exhibits the following features:

(i) producer’s inventories, i.e., \( I^{isc} > 0 \), for \( s_p < \tilde{s}^{isc} \);

(ii) buyer stockpiling \( D_s^{isc} = \Phi_2^{isc} > 0 \), where \( D_s^{isc} (p_1, I) < 0 \) and \( -1 < D_s^{isc} (p_1, I) < 0 \);

(iii) prices \( p_1^{isc} = C_{|Q} (Q_1^{isc}) - \frac{D_{1|1} C_{|Q} (Q_1^{isc}) + \Phi_2^{isc}}{D_{1|1}} \) and \( p_2^{isc} = p_1^{isc} + s_b \).

**Proof of Lemma 5.** Following the same rationale as in the proof of Lemma 2, we can restrict our attention to case where \( p_1 + s_b = p_2 \), which implies \( D_s (\cdot) \in [0, D_2 (p_1 + s_b)] \). It follows from the first-order condition for the second period profit maximization (i.e., the constraint of sequential optimality) that

\[
D_s (p_1, I) = \max \left\{ \Phi_2 (p_1 + s_b, I), 0 \right\},
\]

where \( \Phi_2 (\cdot) \) is defined by (23). Suppose that \( D_s (p_1, I) = \Phi_2 (p_1 + s_b, I) > 0 \), which holds in equilibrium as long as \( s_b \) is not too large. Applying the implicit function theorem to the constraint of sequential optimality \( D_s (p_1, I) - \Phi_2 (p_1 + s_b, I) = 0 \) and using (23), we obtain that

\[
D_{s|1} (p_1, I) = -\frac{\Phi_{2|1} (p_1 + s_b, I)}{D_{2|1} (p_1 + s_b) C_{|QQ} (Q_2) - 1} < 0
\]

and

\[
-1 < D_{s|I} (p_1, I) = -\frac{D_{2|1} (p_1 + s_b) C_{|QQ} (Q_2)}{D_{2|1} (p_1 + s_b) C_{|QQ} (Q_2) - 1} < 0,
\]

where the inequalities follow from \( C_{|QQ} (\cdot) > 0 \) and \( \Phi_{2|1} (\cdot) < 0 \) (second-order condition for the second period profit maximization). The producer’s maximization problem can be written as

\[
\max_{p_1} D_1 (p_1) - C (D_1 (p_1) + D_s (p_1, I) + I) + (p_1 + s_b) D_2 (p_1 + s_b) - C (D_2 (p_1 + s_b) - D_s (p_1, I) - I) - s_b D_s (p_1, I) - s_p I,
\]

where \( D_s (\cdot) = \Phi_2 (\cdot) \) from the constraint of sequential optimality. The first-order conditions for \( p_1 \) and \( I \) are respectively

\[
D_1 (p_1) + p_1 D_{1|1} (p_1) - C_{|Q} (Q_1) \left[ D_{1|1} (p_1) + D_{s|1} (p_1, I) \right] + D_2 (p_1 + s_b) + (p_1 + s_b) D_{2|1} (p_1 + s_b) - C_{|Q} (Q_2) \left[ D_{2|1} (p_1 + s_b) - D_{s|1} (p_1, I) \right] - s_b D_{s|1} (p_1, I) = 0
\]

\[
(26)
\]
\[ [1 + D_{s|I} (p_1, I)] \left[ C_{|Q} (Q_2) - C_{|Q} (Q_1) \right] - s_b D_{s|I} (p_1, I) - s_p \leq 0. \]  
(27)

We find from (27) that for \( s_p < \tilde{s}^{isc}_p \equiv [1 + D_{s|I} (\cdot)] \left[ C_{|Q} (Q_2 (\cdot)) - C_{|Q} (Q_1 (\cdot)) \right] - s_b D_{s|I} (\cdot) \) evaluated at \( p^{isc}_1 = I^{isc} > 0 \) in equilibrium, which implies that (27) holds with equality. The Hessian matrix \( H^{isc} \) must be negative definite to ensure that the second-order conditions for profit maximization are satisfied. Using (23), the first-order conditions for \( p_1 \) in (26) and \( I \) in (27) become

\[
D_1 (p_1) + \left[ p_1 - C_{|Q} (Q_1) \right] D_{1|I} (p_1) + \Phi_2 (p_1 + s_b, I) - \left[ C_{|Q} (Q_1) - C_{|Q} (Q_2) + s_b \right] D_{s|I} (p_1, I) = 0
\]

(28)

\[
[1 + D_{s|I} (p_1, I)] \left[ C_{|Q} (Q_2) - C_{|Q} (Q_1) \right] - s_b D_{s|I} (p_1, I) - s_p = 0.
\]

(29)

It follows from (28) that \( p_1^{isc} = C_{|Q} (Q^{isc}_1) - \frac{D^{isc} + \Phi^{isc}_2 - \left[ C_{|Q} (Q^{isc}_1) - C_{|Q} (Q^{isc}_2) + s_b \right] D^{isc}_{s|I}}{D^{isc}_{1|I}} \) and \( p^{isc}_2 = p^{isc}_1 + s_b \), where \( Q^{isc}_1 \equiv D^{isc}_1 + D^{isc}_s + I^{isc} \) and \( Q^{isc}_2 \equiv D^{isc}_2 - D^{isc}_s - I^{isc} \). Buyer stockpiling is \( D^{isc}_s = \Phi^{isc}_2 \).

We are now in a position to provide the proofs of Proposition 4 and of the associated Corollaries 1 and 2.

**Proof of Proposition 4.** To identify the presence of the producer’s strategic incentives to hold inventories under limited commitment, we consider the first-order condition in (29) for inventories under limited commitment, which can be rewritten after some manipulation as

\[
[1 + D_{s|I} (p_1, I)] \left[ C_{|Q} (Q_2) - C_{|Q} (Q_1) - s_p \right] - (s_b - s_p) D_{s|I} (p_1, I) = 0.
\]

Imposing \( D_{s|I} (p_1, I) = 0 \) yields \( C_{|Q} (Q_1) + s_p = C_{|Q} (Q_2) \), which identifies the amount of inventories corresponding to the cost minimization level, as in the first-order condition in (25) under full commitment. Since under limited commitment it holds \(-1 < D_{s|I} (p_1, I) < 0 \) (see Lemma 5), we find that \( C_{|Q} (Q_1) + s_p > C_{|Q} (Q_2) \) (for \( s_p < s_b \)). Cost convexity implies that under limited commitment the producer has strategic incentives to increase the amount of inventories above the cost minimization level.

An alternative manner to identify the presence of the producer’s strategic incentives to hold inventories under limited commitment is as follows. Suppose that a producer with full commitment powers replicates the allocation under limited commitment in terms of prices and quantities. Substituting the first-order condition in (29) for inventories under limited commitment into the left-hand side of the first-order condition in (25) for inventories under full commitment yields \([C_{|Q} (Q^{isc}_1) - C_{|Q} (Q^{isc}_2) + s_b] D_{s|I} (p^{isc}_1, I^{isc}) < 0 \). The inequality holds since \( C_{|Q} (Q^{isc}_1) - C_{|Q} (Q^{isc}_2) + s_b > 0 \) and \( D_{s|I} (p^{isc}_1, I^{isc}) < 0 \), which follow from (29) (for \( s_p < s_b \) and Lemma 5). The concavity of the profit function with respect to inventories implies that a producer with full commitment powers has an incentive to reduce inventories relative to limited commitment. Put differently, under limited commitment, the producer has strategic incentives to hold inventories.

**Proof of Corollary 1.** Suppose that a producer with full commitment powers replicates the allocation under limited commitment in terms of prices and quantities. After substituting the first-order condition in (28) for prices under limited commitment into the left-
hand side of the first-order condition in (24) for prices under full commitment, we find that
\[
[C_{|Q}(Q^{isc}_1) - C_{|Q}(Q^{isc}_2) + s_b] D_{s|1}(p_1^{isc}, I^{isc}) < 0. 
\]
The inequality holds since \(C_{|Q}(Q^{isc}_1) - C_{|Q}(Q^{isc}_2) + s_b > 0\) and \(D_{s|1}(p_1^{isc}, I^{isc}) < 0\), which follow from (29) (for \(s_p < s_b\)) and Lemma 5. The concavity of the profit function with respect to prices implies that a producer with full
commitment powers is inclined to reduce prices relative to limited commitment. In other words,
under limited commitment, the producer has incentives to set higher prices. ■

Proof of Corollary 2. It follows from Proposition 4 that a producer with full commitment
powers that replicates the allocation under limited commitment has an incentive to reduce
inventories. To show that equilibrium prices are higher under limited commitment than under
full commitment, it suffices to investigate the sign of the second-order cross derivative \(\Pi_{|1|I}(\cdot)\)
of the profit function with respect to prices and inventories. If \(\Pi_{|1|I}(\cdot) > 0\), equilibrium prices
are lower under full commitment since, starting from the limited commitment equilibrium, a
producer with full commitment powers wants to reduce inventories, which makes it profitable to
reduce prices as well. Put differently, if \(\Pi_{|1|I}(\cdot) > 0\), equilibrium prices are higher under limited
commitment. In any framework with linear demand and quadratic production costs, we obtain
after some manipulation that
\[
\Pi_{|1|I} = -2D_{s|1} (p_1, I) \left[1 + D_{s|I} (p_1, I)\right] C_{|QQ} > 0, 
\]
where the inequality follows from Lemma 5 and \(C_{|QQ} > 0\). ■

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