SELF-PREFERENCING IN MARKETS WITH VERTICALLY INTEGRATED GATEKEEPER PLATFORMS*

JORGE PADILLA†
JOE PERKINS‡
SALVATORE PICCOLO§

The competitive strategies of ‘gatekeeper’ platforms are subject to enhanced scrutiny. For instance, Apple and Google are being accused of charging excessive access fees to app providers and privileging their own apps. Some have argued that such allegations make no economic sense when the platform’s business model is to sell devices. In this paper, we build a model in which a gatekeeper device-seller facing potentially saturated demand for its device has the incentive and the ability to exclude from the market third-party suppliers of a service that consumers buy via its devices. Foreclosure is more likely if demand growth for the platform’s devices is slow or negative, and can harm consumers if the device-seller’s services are inferior to those offered by the third parties.

I. INTRODUCTION

The economic significance of online marketplaces, such as Apple’s App Store and Google Play, has increased over time. Apple’s App Store and Google Play earned gross revenues of around €70 billion in 2019, of which almost €10 billion was in Europe. Access to consumers via such platforms has stimulated rapid innovation; over 2.5 million apps are available on Google Play, and more than 1.8 million on the App Store. This is the bright side of their ‘gatekeeper’ role. Because of their broad and loyal customer bases, Apple’s App Store and Google Play constitute critical distribution channels. App developers distributing through them can reach a large number of users at once. But

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†Authors’ affiliations: Compass Lexecon, Madrid, Spain.
e-mail: jpadilla@compasslexecon.com
‡Compass Lexecon, London, UK.
e-mail: jperkins@compasslexecon.com
§University of Bergamo (Dept. of Economics), Compass Lexecon and CSEF, Bergamo, Italy.
e-mail: salvapiccolo@gmail.com

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this also allows platform providers to charge app providers significant listing fees and \((ad\ \text{valorem})\) commissions. On the App Store and Google Play, these amount to 30% of revenues in the first year, and 15% in subsequent years.

Some app developers have complained against these charges. Others have argued that gatekeeper stores restrict their commercial ability so that they cannot avoid these costs. These complaints have attracted considerable policy and regulatory interest and media attention. For example, the European Commission is investigating whether Apple’s rules for the App Store violate antitrust laws.\(^1\) In the U.S., Epic Games filed lawsuits against Apple and Google in August, 2020, alleging that restrictions on possible payment methods for apps violate the Sherman Act and harm consumers.\(^2\) The Dutch competition authority carried out a market study into mobile app stores in 2019, and recommended further investigation into either \textit{ex ante} regulation or greater use of competition law in the sector.\(^3\) More broadly, reviews of digital competition have supported a more active approach to regulation and antitrust enforcement, including Crémer \textit{et al.} [2019], U.S. House Committee on the Judiciary [2020] and Digital Competition Expert Panel [2019].

Complainants argue that some platforms exploit their gatekeeper status to extract excessive rents from app developers and/or to favor their own apps to the detriment of their rivals. Some commentators have dismissed these allegations as illogical, arguing that such conduct would be counterproductive for the platforms themselves, since they benefit from the availability of highly valuable third-party apps in their stores. Our paper investigates the plausibility of this argument, seeking to identify when a gatekeeper platform might have an incentive to abuse its market position. To this end, we build a stylized model showing that the incentive of platforms selling devices to abuse their gatekeeper role relates to the evolution of demand for these devices. We set up a two-period game where a monopolist selling a device (e.g., a smartphone) has the option (in the second period) of restricting access to its users by the competitive suppliers of complementary products (e.g., apps) or, in other words, privileging its own product relative to third-party alternatives.

We find that when the growth of demand for the electronic device is healthy, foreclosure in the complementary market is less likely. It becomes more profitable as demand for devices becomes saturated, and the service offered by the device seller is not too inferior compared to the third-party competitors. In our model, foreclosure occurs at equilibrium as an optimal response of the device seller to a slowing down or a decline of its primary business. Under

\(^1\) See European Commission [2020\textit{a}] and European Commission [2020\textit{b}].


\(^3\) Netherlands Authority for Consumers and Markets [2019].
these conditions, foreclosing rivals from the complementary service market enables the device seller to monetize the user base acquired in the first period. Consumers will lose out from foreclosure if the monopolist’s service is inferior to those provided by third-party developers. Such harm is relatively more pronounced as demand for devices becomes saturated.

We also show that, in addition to being detrimental to consumers, the ability to foreclose harms the device seller’s profits \textit{ex ante}. The device-seller would like to be able to commit not to foreclose in order to increase prices in the first period. Greater profits from a higher period 1 price outweigh the profits gained from foreclosing the service market in period 2. However, there is a time-inconsistency problem; when it makes its decision in period 2, the device-seller may prefer to foreclose to exploit its captive market. Notably, this time-inconsistency problem is more severe when the device seller has a greater incentive to foreclose in the second period — i.e., when demand for devices becomes saturated — which is also the case in which foreclosure harms consumers more. Hence, policies enabling (or forcing) device sellers to overcome the time-inconsistency problem are relatively more important in industries that feature declining demand dynamics.

Several tech giants, such as Apple and Google, seem to have understood the time-inconsistency problem highlighted in our paper, which they have been unable to avoid. For example, when launching the App Store in 2008, Steve Jobs announced that Apple did not intend to profit from it, and that all the money would be given to the developers (Cohen [2008]). Google’s founders initially promoted their search engine’s impartiality on the basis that it was not subject to the potentially harmful influence of advertising (see, e.g., White [2013]).

As of today, however, Apple makes considerable revenues from the App Store, and Google monetises its search engine through paid advertising. Such examples are not universal; for instance, Padilla \textit{et al.} [2017] discuss how Adobe’s commitment to an open standard for pdf’s enabled it to maintain the attractiveness of the format for third-party developers and end users. But they do provide some evidence of time-inconsistent incentives on platforms, which cannot easily be overcome by unilateral commitments. Explicit regulatory constraints may therefore be required to protect consumers against the risk of hold-up. For instance, Gilbert [2021] argues that a mix of antitrust enforcement and regulation may be required to address concerns about the abuse of market power by dominant platforms. Entry by these platforms into aftermarket businesses could be made less harmful by requiring mandatory quality standards to protect consumers and to compensate them for losses in the event of rivals’ foreclosure.

\footnote{4 Apple's 2020 annual report shows net services sales of $54 billion, and states that the 16% increase from 2019 was 'due primarily to higher net sales from the App Store, advertising and cloud services.'}
I(i). **Related Literature**

The economic literature on vertically-integrated gatekeeper platforms competing with third party sellers is very recent and still developing. Lynskey [2017] and Alexiadis and de Street [2020] discuss the role of platform gatekeepers and their potential harms to competition. Newman [2019] argues that digital platforms are in a unique position to close or mimic startups’ features while discouraging entry. Hagiu and Spulber [2013] suggest that platforms facing unfavorable demand conditions have more incentive to enter into the seller product space. The competitive and consumer welfare impact of a platform’s entry in competition with third-party sellers distributing through that platform is discussed by Hagiu *et al*. [2020], Dryden, Khodjamirian and Padilla [2020] and Etro [2020]. These authors find that such entry typically enhances consumer surplus. In contrast, our dynamic model clarifies that such an impact depends on the growth and, hence, the expected future profitability of the platform’s primary product (e.g., the device). Gatekeeper platforms may have an incentive to exploit the installed user base of consumers and foreclose competitors when demand growth is slow. This is the case even if the platform offers a worse alternative to the sellers’ product, thereby hurting consumers.

The empirical literature on the economic effects of the strategies adopted by vertically-integrated platforms competing with third party sellers is also sparse and, thus far, provides ambiguous results. Zhu and Liu [2018] and Wen and Zhu [2019] find evidence that the entry of a platform into the seller’s product space reduces innovation incentives of third party sellers. The opposite result is found in Dryden, Khodjamirian, Rovegno and Small [2020]. Foerderer *et al*. [2018] and Li and Agarwal [2017] also find that platform entry increases innovation and benefits consumers. Foerderer *et al*. [2018] find that Google’s entry into the market for photography apps increased consumer attention in that market, leading to greater innovation. Li and Agarwal [2017] show that tighter integration of a platform’s application increases consumer demand, which benefits big third party applications while smaller third party applications are hurt.

The model in this paper is also related to the durable goods literature; the device provided by the gatekeeper can be interpreted as a durable good, with services being linked to non-durable goods. The dilemma faced by the gatekeeper in this setting can then be seen as analogous to a durable goods monopolist subject to the Coase conjecture. The gatekeeper would like to be able to commit not to foreclose in order to sell more devices in the first period, just as a durable goods monopolist would like to be able to commit not to lower prices in future. A classic reference is Bulow [1986], who shows that durable goods producers in concentrated markets may have an incentive to manipulate the durability of their products. In particular, a monopolist will typically wish to reduce durability below the efficient level, as doing so enables it to
capture a greater share of the surplus. In our model, the monopolist’s foreclosure of the linked market reduces total surplus, but enables the monopolist to capture a greater share of surplus in the second period.\(^5\)

Kühn and Padilla [1996] show that the Coase conjecture does not hold when a durable-goods monopolist also sells nondurable goods that are demand-related to the durable — i.e., the presence of nondurable complements or substitutes reduces the rate at which the monopolist introduces the durable into the market. Laussel et al. [2015] reach similar conclusions in a continuous-time dynamic game where a monopolist producing a durable good is also involved in the corresponding aftermarket and consumers benefit from subsequent expansions of the network.\(^6\) Fethke and Jagannathan [2000] instead examine the endogenous degree of durability that a monopolist would choose when it produces both durable and complementary non-durable services. All these models assume that the durable goods monopolist faces no competition on the linked non-durable good market, and therefore, unlike this paper, do not consider strategic foreclosure (see also Goering [2007]). The literature examining the relationship between demand uncertainty and the business strategies of durable good monopolists is also rather developed (see Cason and Sharma [2001], Bhatt [1989], Desai et al. [2007], among others). Yet none of these models examines the link between demand dynamics and anti-competitive conduct, as we do in what follows.

II. THE MODEL

Consider a two-period model (with \(t = 1, 2\) indicating time) in which a monopolist (\(M\) hereafter) sells a durable good (e.g., an electronic device). In the first period, there is a unit mass of consumers with a heterogeneous willingness to pay \(v\) (type) for the device. We assume, for simplicity, that \(v\) is uniformly distributed over the unit support \([0, 1]\).\(^7\) In the second period, the willingness to pay of a consumer whose type was \(v\) in the first period is \(\theta v\). The parameter \(\theta \geq 0\), which is common to all consumers, captures taste dynamics: \(\theta < 1\) means that consumers’ willingness to pay for the durable good declines over time, \(\theta > 1\) means that it increases over time, and \(\theta = 1\) means it remains stable over time. For example, \(\theta\) can be seen as a parameter that reflects (anticipated)

\(^5\) Similar dynamics have been studied in the search engine literature. Search engines may have an incentive to integrate with publishers to compete more aggressively with nonintegrated publishers to attract advertisers. See, e.g., Burguet et al. [2015], de Cornière and Taylor [2014] and White [2013].

\(^6\) See also Dana Jr. and Spier [2015], Dana Jr. and Spier [2018] and de Cornière and Taylor [2019], for related models where quality, insurance provisions and bundling can facilitate foreclosure.

\(^7\) Results generalize to a generic, continuously differentiable cdf \(F (v)\) with increasing hazard rate.
changes in the ecosystem attached to a device, or the rate at which it becomes
laggard and obsolete over time.

If they wish, consumers can bundle the device with a complementary service
(e.g., an app for their device with a regular subscription payment) offered by
$N$ suppliers (each denoted by $i = 1, \ldots, N$). To isolate the device seller’s incen-
tive to foreclose in the clearest possible way, we assume that these suppliers
compete à la Bertrand (in Section V we discuss the dynamics of an imper-
fectly competitive service market where $M$ can also charge third-party seller
access fees). We denote by $\Delta \geq 0$ the additive extra utility obtained by a con-
sumer that bundles the device with the service sold by any of these suppliers,
and assume that $\Delta \leq 1$; this implies that, in period 1, no consumer values the
complementary service more highly than the device itself. In period $t = 1, 2,$
the price of the device is $p_t$, while the price of the service is $q_t^i$, with $t = 1, 2$
and $i = 1, \ldots, N$. For simplicity, and without loss of insight, we assume that
all firms have a constant marginal cost of production, which is normalized
to 0.

At the beginning of period 2, $M$ can prevent the suppliers of the service
from accessing the users of its device and selling them their products.$^8$ In this
case, $M$ develops its own service, which requires an investment $I \geq 0$ and is
valued at $\beta \Delta$ by consumers, with $\beta \in [0, 1]$ capturing the idea that $M$ might
not be able to produce the same quality as its specialized rivals.$^9$

The timing of the game is as follows. Each period features two stages (see
the timeline in Figure 1). In period $t = 1$:

1. $M$ posts a price $p_1$ for the device. Consumers decide whether or not to buy
   it now.
2. The suppliers of the service post prices ($q_t^i$ with $i = 1, \ldots, N$) to attract the
   users of $M$’s product, who then decide whether or not to buy it.

In period $t = 2$:

1. $M$ decides whether to develop its own service and foreclose the third-party
   suppliers selling a competing service through its platform. If so, the $N$ rivals
   are foreclosed. $M$ posts a price $p_2$ for the device. Consumers who have not
   purchased in period 1 decide whether to buy it.

$^8$ We do not explicitly incorporate the possibility that foreclosure occurs in period 1. But we
show in Section IV that the monopolist would never choose to foreclose then, provided $\beta < 1$.

$^9$ Note that, if there is efficient bargaining between the device seller and one or more app or ser-
vice providers, the device seller might be able to extract all surplus in the service market without
foreclosing. However, such bargaining is unlikely to occur in reality, as is suggested by the costs
of the current disputes over Apple’s interactions with app providers. $\beta < 1$ could alternatively
be interpreted as the result of bargaining frictions following the incumbent’s attempts to extract
surplus from the services market, rather than the device-seller providing a lower-quality prod-
uct itself. For instance, the device seller might reduce the functionality of a third-party service
provider’s app, or make it more difficult for consumers to access it.

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2. Without foreclosure, the sellers of the service post prices to attract the users of M’s product as in the first period. Consumers decide whether to buy the complementary product. By contrast, with foreclosure, M charges the users of its durable product — i.e., all consumers that at the end of period 2 have purchased M’s product — a price $q_2$ for its non-durable product, and consumers decide whether to buy.

The equilibrium concept is Subgame Perfect Nash Equilibrium (SPNE). An equilibrium must specify M’s decision of foreclosing or not, which will be denoted by $k = F$ in case of foreclosure and $k = NF$ in case of no foreclosure, a set of prices $(p^k_t, q^k_t)$ chosen by M in each period $t = 1, 2$, the prices $(q^1_i, q^2_i)_{i=1}^N$ chosen by the suppliers of the non-durable good in each period, and the mass of consumers that buy the device in each period.

Throughout we assume that

$$\theta \geq \theta \triangleq \Delta + \frac{\Delta \beta + \sqrt{4(1 - \Delta)^2 + \beta \Delta (\beta \Delta + 4 (3 + \Delta))} - 1}{2},$$

in order to guarantee that the equilibrium price of the device in the second period is always positive. Notice that $\theta$ is always positive for every $(\beta, \Delta) \in (0, 1)^2$. In words, this means that consumers value the durable good relatively more than the non-durable good (i.e., they never buy the device simply because they want to use the app).

We assume that the monopolist and the consumers have a common discount factor, normalized to 1 without loss of generality, and that they have perfect foresight. As standard in the durable good literature, we conjecture (and verify later) that consumers’ behavior follows a cut-off strategy according to which consumers with high willingness to pay purchase the device in the first period, while consumers with a low willingness to pay purchase the device in the second period. Formally, this means that with (resp. without) foreclosure every consumer with type $v \geq v^F \in (0, 1)$ (resp. $v^{NF}$) purchases the device.
in the first period, while consumers with type \( v < v^F \) (resp. \( v^{NF} \)) purchase the device in the second period, if at all. Of course, the thresholds \( v^F \) and \( v^{NF} \) will be determined endogenously at equilibrium. Finally, we assume that, due to regulatory or practical constraints, the device seller cannot price discriminate among its consumers on the basis of their heterogeneous willingness to pay for the device.

III. ANALYSIS

In this section we characterize the equilibrium of the game. We shall consider two types of pure strategy equilibria; one in which \( M \) forecloses, the other in which it does not. In the equilibrium with foreclosure it is easy to verify that \( q_1^F = 0 \) because of Bertrand competition for the complementary service in period 1, and \( q_2^F = \beta \Delta \) since \( M \) is the only provider of the service in period 2. By contrast, in the equilibrium without foreclosure \( q_2^{NF} = q_1^{NF} = 0 \) because of Bertrand competition between the complementary service providers in both periods. The analysis is developed in three steps. We first characterize \( M \)'s second-period foreclosure decision for a given mass of consumers buying the device in the first period. Second, assuming that a given equilibrium (with or without foreclosure) exists, we characterize the associated price pattern and optimal strategy for consumers. Then we close the circle by checking that the candidate equilibrium under consideration is indeed immune to deviations — e.g., \( M \) cannot profit by not foreclosing if consumers expect it to foreclose, and vice versa. In doing so, we follow a backward induction logic.

III(i). The Foreclosure Decision

We first describe \( M \)'s optimal second-period foreclosure decision for a given mass of consumers purchasing the device in that period. Since we assumed that consumers behave according to a cut-off strategy, assume that there exists a threshold \( v^k \in (0, 1) \) such that consumers with type \( v \geq v^k \) buy the device in the first period, and those with type \( v < v^k \) buy in the second period, if at all (a conjecture that will be verified \textit{ex post}). In order to characterize \( M \)'s foreclosure decision we begin by determining the optimal prices and expected profit in the second period with and without foreclosure.

III(i)(a). Optimal Pricing with Foreclosure

With foreclosure, a consumer with valuation \( v < v^k \) — i.e., who has not bought the device in period 1 — purchases the device in period 2 if and only if

\[
\theta v - p_2 \geq 0 \quad \iff \quad v \geq v^F (p_2) \triangleq \frac{p_2}{\theta}.
\]

Hence, assuming that the mass of late device users is \( v^k \) as before, \( M \) solves the following maximization problem

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(1) \[
\max_{p_2 \geq 0} \left( p_2 + \beta \Delta \right) \Pr \{ v \geq v^F (p_2) \mid v < v^k \} = \max_{p_2 \geq 0} \int_{v^k}^{v^F (p_2)} \left( p_2 + \beta \Delta \right) \frac{dv}{v^k},
\]
which accounts for the profit $\beta \Delta$ that $M$ expects to receive from each sale of the complementary service in the second period. The first-order condition (in an interior solution) is

\[
\theta v^k - p_2 - (p_2 + \beta \Delta) = 0
\]

which reflects the trade-off between the positive effect of a higher price on $M$’s second-period margin and the negative effect on second-period demand, which also accounts for the loss of profit $\beta \Delta$ that $M$ suffers from missed sales in the service market when the price of the device increases (and its demand drops). The above first-order condition yields

\[
p^F_2 (v^k) = \frac{\theta v^k - \beta \Delta}{2},
\]

which is always positive as long as $v^k \geq \frac{\beta \Delta}{\theta}$ (which will be verified ex post once we characterize the equilibrium).

Hence, recalling that we have normalized the marginal cost of production to 0, $M$’s expected profit with foreclosure is

\[
\pi^F_2 (v^k) \triangleq \int_{v^F (p^F_2 (v^k))}^{v^k} \left( p^F_2 + \beta \Delta \right) \frac{dv}{v^k} + (1 - v^k) \beta \Delta - I
\]

\[
= \frac{(\theta v^k + \beta \Delta)^2}{4\theta v^k} + (1 - v^k) \beta \Delta - I.
\]

The first term in the above equation corresponds to $M$’s total (durable and non-durable) revenues collected from late device users. The second term corresponds to $M$’s second period revenue from the provision of the non-durable good to early device users. Finally, the third term is the investment cost required for $M$ to develop and supply its non-durable service.

III(i)(b). **Optimal Pricing without Foreclosure**

Without foreclosure a consumer with valuation $v < v^k$ buys the device in the second period if and only if

\[
\theta v + \Delta - p_2 \geq 0 \iff v \geq v^{NF} (p_2) \triangleq \frac{p_2 - \Delta}{\theta}.
\]

As intuition suggests, the mass of consumers that buys the device in the second period is increasing in $\theta$ and $\Delta$. As a result, in the second period, $M$ solves the following maximization problem
\[
\max_{p_2} \Pr \left[ v \geq v^N_F \left( p_2 \right) \mid v < v^k \right] = \max_{p_2 \geq 0} \int_{v^N_F \left( p_2 \right)}^{v^k} \frac{d v}{p_2}.
\]

That is, \( M \) maximizes the second-period (conditional) expected profit. The first-order condition (in an interior solution) is

\[
\theta v^k - (p_2 - \Delta) - p_2 = 0,
\]

which reflects the standard trade-off between the positive effect of a higher price on \( M \)'s second-period margin and the negative effect on the second-period demand (volume) for the device. The above first-order condition yields

\[
p_2^N \left( v^k \right) = \frac{\theta v^k + \Delta}{2},
\]

which, as intuition suggests is, increasing in the maximal willingness to pay of the consumers that purchase in the second period — i.e., \( \theta v^k \) according to the strategy conjectured above. The price \( p_2^N \left( v^k \right) \) is also increasing in the consumer’s willingness to pay for the non-durable good; without foreclosure, Bertrand competition in the market for the subscription service allows consumers to appropriate fully the utility \( \Delta \). This, in turn, stimulates demand for the device and raises its price.

Hence, without foreclosure, for given \( v^k \), \( M \)'s expected (conditional) profit in the second period is

\[
\pi^F_2 \left( v^k \right) = \int_{v^N_F \left( p_2^N \left( v^k \right) \right)}^{v^k} p_2^N \left( v^k \right) \frac{d v}{v^k} = \frac{\left( v^k \theta + \Delta \right)^2}{4 v^k \theta},
\]

which corresponds to \( M \)'s revenue from the sales of the durable good in the second period (recall that we have normalized the marginal cost of production to 0).

III(i)(c). Foreclosure Decision

The characterization obtained above is key to examine \( M \)'s foreclosure decision in the second period. Comparing \( \pi^F_2 \left( v^k \right) \) with \( \pi^N_2 \left( v^k \right) \) we have

\[
\pi^F_2 \left( v^k \right) - \pi^N_2 \left( v^k \right) = \left( 1 - v^k \right) \beta \Delta - \left( 1 - \beta \right) \frac{\left[ 2 \theta v^k + (1 + \beta) \Delta \right] \Delta}{4 v^k \theta} + I.
\]

Gain from foreclosure

Loss from foreclosure

Notice that the gain from foreclosure is decreasing in \( v^k \), while the loss from foreclosure decreases with \( v^k \). Hence, we can conclude the following.

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Lemma 1. For a given mass $v^k \in (0, 1)$ of late device users, $M$ forecloses if and only if

$$I \leq I(v^k) \triangleq \Delta \max \left\{ 0, (1 - v^k) \beta - (1 - \beta) \left( \frac{2 \theta v^k + (1 + \beta) \Delta}{4 v^k \theta} \right) \right\}.$$  

For given $v^k$, $M$’s incentive to foreclose is shaped by three effects. First, since foreclosure is costly, $M$ will find it optimal to foreclose if and only the required investment cost $I$ is not too large. Second, foreclosure secures $M$ a unit profit $\beta \Delta$ on each early device user — i.e., on each consumer that has purchased the device in the first period. Third, since $M$’s service is of inferior quality compared to that produced by its specialized competitors, the device sales profit that $M$ obtains with foreclosure is lower than that it can extract without foreclosure — i.e., there is a disincentive effect given by the difference

$$\left( \frac{\theta v^k + \beta \Delta}{4 \theta v^k} \right)^2 - \left( \frac{v^k \theta + \Delta}{4 \theta} \right)^2 = -\Delta (1 - \beta) \left( \frac{2 \theta v^k + (1 + \beta) \Delta}{4 v^k \theta} \right) \leq 0.$$  

This effect is obviously equal to zero when the quality of $M$’s service matches the quality of the product supplied by its rivals (i.e., when $\beta = 1$).

Notice that the impact of $v^k$ on $I(v^k)$ is ambiguous. First, when $v^k$ increases, the mass of early device users drops, reducing $M$’s incentive to foreclose because there are fewer customers who only buy the service in the second period. Second, a larger $v^k$ also means that the mass of late device users increases, and the willingness to pay for the device increases so that the relative impact of selling an inferior complement on $M$’s profits is reduced. Clearly, the first effect dominates for $\beta$ large (e.g., $\beta \to 1$) while the second dominates for $\beta$ small (e.g., $\beta \to 0$).

Of course, since $v^k$ will be determined endogenously, for each equilibrium under consideration the relationship between $\theta$ and $I(v^k)$ will depend on the impact of taste dynamics on the effects described above, as well as on its impact on the pattern of consumers’ purchasing decisions.

III(ii). First-Period Behavior

We now turn to characterize $M$’s first-period optimal behavior and the consumers’ optimal purchasing behavior for a given foreclosure decision in the second period.

III(ii)(a). Price Pattern and Consumers’ Strategy with Foreclosure

Consider an equilibrium in which $M$ forecloses and charges $p^F_2$ in the second period. For a given price $p_1$ charged by $M$ in the first period, a consumer with valuation $v$ does not delay consumption of the device to the second period if and only if

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\[ v + \Delta - p_1 + \theta v \geq v - p_F^2 \iff v \geq p_1 - \Delta - p_F^2. \]

For given \( p_1 \) and \( p_F^2 \), a consumer who is indifferent between purchasing the device in period 1 and period 2 must have valuation

\[ v = v^F(p_1, p_F^2) \triangleq p_1 - \Delta - p_F^2. \]

Using the optimality condition (3) we have

\[ p_F^2 = p_F^2(v^F(p_1, p_F^2)) = \frac{\theta v^F(p_1, p_F^2) - \beta \Delta}{2}. \]

Solving for \( p_F^2 \) as a function of \( p_1 \) we then have

\[ p_F^2(p_1) \triangleq \frac{\theta (p_1 - \Delta) - \beta \Delta}{2 + \theta}. \]

Hence, the optimal first-period price charged by \( M \) in an equilibrium with foreclosure (assuming that it exists) must solve the following maximization problem

\[ \max_{p_1 \geq 0} \int_0^1 v^F(p_1, p_F^2(p_1)) (p_1 + \beta \Delta) \, dv + \int_{v^F(p_1, p_F^2(p_1))}^{v^F(p_1, p_F^2(p_1))} p_F^2(p_1) (p_1 + \beta \Delta) \, dv, \]

whose first-order condition, by the Envelope Theorem, is

\[ \int_0^1 v^F(p_1, p_F^2(p_1)) \left( \frac{\partial v_F(p_1, p_F^2)}{\partial p_1} \right) \, dv - p_1 \frac{\partial v_F(p_1, p_F^2)}{\partial p_1} + p_F^2(p_1) \left( \frac{\partial p_F^2}{\partial p_1} (p_1 - p_F^2(p_2)) \right) + \frac{\partial p_F^2}{\partial p_1}(p_1 - p_F^2(p_2)) = 0. \]

The first term in this condition reflects the standard static trade-off between volume reduction and profit margin expansion. The second term captures the intertemporal link between the first-period price and second-period demand (recall that the discount factor has been normalized to 1). A higher price today induces relatively more consumers to purchase in the second period, which calls for a higher price in the second period. This higher profit in the second period, other things being equal, induces \( M \) to be more willing to increase the price of the device in the first period relative to a static context.

Solving (7) with respect to \( p_1 \), we can characterize the optimal price pattern \((p_F^1, p_F^2)\) and the consumers’ optimal strategy in a (candidate) equilibrium with foreclosure (i.e., the cut off \( v_F \)).
**Proposition 1.** If an equilibrium with foreclosure exists, it features the following price pattern:

\[
p_F^1 = \frac{4(1 + \Delta) + (4 + \theta)(\theta - \beta\Delta)}{2(4 + \theta)} > p_F^2 = \frac{2\theta(1 - \Delta) + \theta(\theta - \beta\Delta) - 4\beta\Delta}{2(4 + \theta)} \geq 0.
\]

The mass of late device users is

\[
v_F = \frac{2(1 - \Delta) + \theta}{4 + \theta} \in (0, 1),
\]

with \(v_F\) increasing in \(\theta\) and decreasing in \(\Delta\).

This proposition shows that, in equilibrium with foreclosure, \(M\)'s optimal price falls over time — i.e., \(M\) has an incentive to increase the price in the first period to benefit from higher demand in the second period, as in standard durable goods models. The comparative statics shows that consumers are relatively more likely to delay consumption when demand grows relatively more over time — i.e., as \(\theta\) grows. On the contrary, consumers are less likely to delay consumption as the extra utility that they obtain from the service, \(\Delta\), grows large. By consuming the device only in the second period, they forego the utility associated with the service in the first period.

**III(ii)(b). Price Pattern and Consumers’ Strategy without Foreclosure**

Consider now a (candidate) equilibrium without foreclosure. Assume that \(M\) charges \(p_{NF}^2\) in the second period. Then, for given price \(p_1^1\) charged in the first period, a consumer with valuation \(v\) buys the device in the first period if and only if

\[
v + \theta v + \Delta \geq \theta v + \Delta - p_{NF}^2 \iff v \geq p_1^1 - \Delta - p_{NF}^2.
\]

For given \(p_1^1\) and \(p_{NF}^2\), the indifferent consumer must have valuation

\[
v_{NF} (p_1^1, p_{NF}^2) \triangleq p_1^1 - \Delta - p_{NF}^2.
\]

Using the optimality condition (5) we have

\[
p_{NF}^2 = p_{NF}^2 (v_{NF} (p_1^1, p_{NF}^2)) = \frac{\theta v_{NF} (p_1^1, p_{NF}^2) + \Delta}{2}.
\]

Solving for \(p_{NF}^2\) as a function of \(p_1^1\) we have

\[
p_{NF}^2 (p_1^1) = \frac{\theta (p_1^1 - \Delta) + \Delta}{2 + \theta}.
\]

Hence, the optimal first-period price charged by \(M\) in an equilibrium without foreclosure (if it exists) must solve the following maximization problem
\[
\max_{p_1 \geq 0} \int_{vNF(p_1, pNF_2(p_1))}^{1} p_1 \, dv + \int_{pNF_2(p_1) - \Delta}^{vNF(p_1, pNF_2(p_1))} p_2^{NF}(p_1) \, dv,
\]
whose first-order condition, by the Envelope Theorem, is

\[
\int_{vNF(p_1, pNF_2(p_1))}^{1} dv - p_1 \frac{\partial vNF(p_1, p_2^{NF}(p_1))}{dp_1}
\]
\[
+ p_2^{NF}(p_1) \frac{\partial vNF(p_1, p_2^{NF}(p_1))}{dp_1} + (p_1 - p_2^{NF}(p_1)) \frac{\partial p_2^{NF}(p_1)}{dp_1} = 0,
\]

which again reflects the static monopoly trade-off and the intertemporal link between the first-period price and the mass of late users.

Solving (8) with respect to \(p_1\), we can characterize the optimal price pattern \((p_1^{NF}, p_2^{NF})\) and the consumers’ optimal strategy in an equilibrium without foreclosure (i.e., the cut off \(v^{NF}\)).

Proposition 2. If an equilibrium without foreclosure exists, it features the following price pattern:

\[
p_1^{NF} = 4 \left(1 + \Delta\right) + (4 + \theta)(\theta + \Delta) \geq p_2^{NF} \geq \frac{2\theta(1 - \Delta) + \theta(\theta + \Delta) + 4\Delta}{2(4 + \theta)} > 0.
\]

The mass of late device users is \(v^{NF} = v^{F}\).

As intuition suggests, even without foreclosure, the equilibrium features a declining price pattern. The reason why the intertemporal allocation of consumers is the same across equilibria (i.e., \(v^{F} = v^{NF}\)) hinges on the additive structure of the consumers’ utility function. Two effects, which perfectly balance out, are at play. With foreclosure, consumers pay a (relatively) low second-period price for the device because they anticipate that \(M\) will fully extract the utility \(\beta\Delta\) created by the consumption of the service. Without foreclosure, instead, consumers pay nothing for the service because of Bertrand competition, but this in turn enables \(M\) to charge a (relatively) high price for the device in order to extract the surplus \(\Delta\).

III(iii). Existence and Uniqueness of Equilibria

Finally, applying Lemma 1, we can state the conditions under which equilibria with and without foreclosure exist and are unique within the class of consumers’ cut-off strategies considered above.
Proposition 3. There exists a threshold $\beta^* \in [0, 1]$ such that the game features a unique, cut-off strategy equilibrium with foreclosure if $\beta \geq \beta^*$ and only if $I \leq I^* \triangleq I(v^F) = I(v^{NF})$. Otherwise, the unique cut-off strategy equilibrium features no foreclosure.

Clearly, $M$’s incentive to foreclose tends to be higher when the utility that it can extract from selling the service to its users increases — i.e., when $\beta$ is large enough — and when the investment cost required to develop the non-durable service is not too large.

The effect of $\theta$ on the incentive to foreclose can be non-monotonic and thus deserves a more detailed discussion. To see why, notice that the derivative of $I^*$ with respect to $\theta$ can be decomposed into two terms with opposing sign — i.e.,

$$\frac{dI^*}{d\theta} = -\frac{\partial v^F}{\partial \theta} \beta + \frac{(1 - \beta^2) \Delta}{4(v^F \theta)^2} \left( \frac{\partial v^F}{\partial \theta} \theta + v^F \right),$$

where

$$\frac{\partial v^F}{\partial \theta} = \frac{2(1 + \Delta)}{(4 + \theta)^2} > 0.$$

The first negative effect captures the effect of higher $\theta$ on $M$’s gains from foreclosure — i.e., the profit that it makes by selling the service to each early user. Other things being equal, the larger the willingness to pay for the device in the second period — i.e., the higher $\theta$ — the more late-adopters there will be, meaning that foreclosure becomes less important for $M$. Hence, $I^*$ drops. The second term represents the impact of $\theta$ on the loss that $M$ incurs by selling an inferior product to late users. This effect consists of two forces: a direct one and an indirect one, both pointing in the direction of making foreclosure more profitable. First, the higher consumers’ willingness to pay for the device in the second period is, the smaller is the impact of foreclosure on reducing second-period sales; the service is less important to the consumer relative to the device. Second, since a higher $\theta$ implies that there are relatively more late adopters ($v^F$ increases), there are more device-owners to exploit by foreclosing. The balance between these two effects is, a priori, unclear. However, we can show the following.

Corollary 1. For $\beta$ sufficiently large, $\frac{dI^*}{d\theta} < 0$.

Intuitively, when consumers perceive $M$’s and its rivals’ non-durable services as not too differentiated, foreclosure has little effect in reducing $M$’s sales to late adopters. As a result, the second term in (9) matters relatively less, making foreclosure more appealing for low values of $\theta$. 

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For low and intermediate values of $\beta$ both effects matter and $I^*$ exhibits an inverted-U shaped pattern with respect to $\theta$. In Figure 1 we simulate $I^*$ as a function of $\theta$ for different values of $\beta$ and $\Delta$. Specifically, $\beta = 0.9$ in Panel a, $\beta = 0.8$ in Panel b and $\beta = 0.7$ in Panel c; while in each panel the black curve corresponds to $\Delta = \frac{1}{4}$, the green to $\Delta = \frac{1}{6}$ and the red one to $\Delta = \frac{1}{8}$.10

Figure 2 shows that the incentive to foreclose is more pronounced — i.e., $I^*$ is higher — when demand for the device in the second period grows at a relatively low rate (i.e., for $\theta$ not too large). In this case, $M$ is willing to expand the range of its business projects to cope with the decline of its primary activity. Of course, for $\theta$ very small, consumers’ willingness to pay is so low in the second period that $M$ has no incentive to pay the cost of foreclosure $I$.

Notably, in all panels, the region of parameters in which $I^*$ decreases with $\theta$ (that is, the region of parameters where a drop in the consumers’ second period willingness to pay for the device spurs $M$’s incentive to foreclose rivals in the service market) is wider for large values of $\beta$. This is because (other things being equal) the service it can provide becomes a close substitute for that offered by its specialized competitors and thus more valuable. Other things being equal, for values of $\theta$ not too small, foreclosure is less likely for low values of $\Delta$ because consumers value the service less highly, and thus there is less benefit to the firm from exploiting captive consumers in period 2.

IV. CONSUMER WELFARE AND $M$’S TIME INCONSISTENCY PROBLEM

In this section we examine the effect of foreclosure on consumer welfare and the device seller’s intertemporal profit. We show that, in addition to being detrimental to consumers, $M$’s ability to foreclose in the second period also reduces its intertemporal profits, creating a novel time inconsistency problem.

10 Results are robust to alternative parametric specifications.
IV(i). Consumer Surplus

We begin by comparing consumer surplus with and without foreclosure. The objective is to understand whether consumer protection may require limitations of the monopolist’s gatekeeper power.

The expression for consumer surplus with foreclosure is

\[
CS_F \equiv \int_{v_F}^{1} \left( v + \Delta - p_1^F + \theta v \right) dv + \int_{p_1^F}^{v_F} \left( \theta v - p_2^F \right) dv .
\]

Without foreclosure, instead, we have

\[
CS_{NF} \equiv \int_{v_{NF}}^{1} \left( v + \Delta - p_1^{NF} + (\theta v + \Delta) \right) dv + \int_{p_2^{NF}}^{v_{NF}} \left( \theta v + \Delta - p_2^{NF} \right) dv.
\]

Comparing these expressions (see the Appendix) we can show the following.

**Proposition 4.** For every \( \beta \leq 1 \), ex ante consumer surplus is lower with foreclosure than without foreclosure (at \( \beta = 1 \) consumers are indifferent). The consumer harm is larger for lower values of \( \theta \) and \( \beta \), and for higher values of \( \Delta \).

Foreclosure has two effects on consumer welfare. First, it reduces the number of consumers who buy the device in period 2.\(^{11}\) In particular, the gap between the mass of consumers who buy without and with foreclosure is given by

\[
\frac{p_2^{NF} - \Delta}{\theta} - \frac{p_2^F}{\theta} = \frac{\Delta(\beta - 1)}{2\theta} < 0,
\]

provided \( \beta < 1 \). So the lowest-valuation consumer who buys in period 2 has a lower valuation without foreclosure than with foreclosure. That is, more consumers buy in period 2 without foreclosure.

Second, foreclosure reduces the surplus received by a given consumer who buys the device. For a consumer with valuation \( \bar{v} \) who buys the device in period 1, the gap between his surplus without and with foreclosure is given by

\[
\bar{v} + \Delta - p_1^{NF} + (\theta \bar{v} + \Delta) - (\bar{v} + \Delta - p_1^F + \theta \bar{v}) = \frac{\Delta}{2}(1 - \beta) > 0.
\]

\(^{11}\) We have shown that in equilibrium \( v^{NF} = v^F \), so that the same number of consumers buy in period 1 in both cases.

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That is, the consumer’s welfare is higher without foreclosure, and the difference is increasing in the importance of the app (Δ) and decreasing in the quality of the monopolist’s app (β).¹²

The gatekeeper platform’s ability to foreclose suppliers of services from its platform can therefore harm consumer welfare (and cannot improve it). Because foreclosure is more likely with relatively low levels of θ,¹³ such consumer harm is more likely to occur in markets where demand is stagnant. A policy aimed at protecting consumer surplus might forbid a monopoly device-seller from exerting its gatekeeper power by denying access to its users to third party suppliers. It might also restrict the device seller’s ability to provide services that compete with third parties on its platform, or else impose binding quality standards for entry in the aftermarket.

IV(ii). Commitment and Time Inconsistency

In addition to being detrimental to consumers, the ability to foreclose also harms the device-seller’s ex ante profits. The device-seller would like to be able to commit in period 1 not to foreclose, in order to increase prices in that period. Greater profits from higher period 1 prices outweigh the profit gained from foreclosing the service market in period 2. However, there is a time-inconsistency problem; when it makes its decision in period 2, the device-seller may prefer to foreclose to exploit its captive market.

Proposition 5. M’s ex ante profits are lower with foreclosure than without foreclosure.

The difference between M’s intertemporal profit without and with foreclosure — i.e., \( \Delta \pi \equiv \pi^{NF} - \pi^{F} \) — measures the strength of the time-inconsistency problem faced by M — i.e., its incentive to commit not to exclude rivals from the service market.

The next result shows how this incentive varies with the primitives of the model — i.e., θ, β and Δ.

Corollary 2. M’s time-inconsistency problem is more severe — i.e., \( \Delta \pi \) is larger — when θ and β are low, and when Δ is large.

Other things being equal, the more consumers value the complementary service, the larger is the loss that M incurs when it introduces an inferior

¹² The gap in consumer surplus for a customer who buys the device in period 2 is identical \( \left( \frac{\Delta}{2} (1 - \beta) \right) \). This is not coincidental; in the non-foreclosure equilibrium, the monopolist must offer a period 1 consumer greater utility in the first period to encourage them to buy now rather than delaying.

¹³ Unless θ is so low as to make the development of the gatekeeper’s own services unprofitable.
service, since it can monetize this greater willingness to pay by increasing the price of the device in both periods. In addition, \( M \) has a weaker incentive to foreclose when the service it can provide is of sufficiently lower quality (and thus valued less by consumers) than the service provided by its specialized rivals. Finally, \( M \)'s time inconsistency problem is stronger for low levels of \( \theta \). The reason is that when growth in demand for the device is weak or stagnant (\( \theta \) is low) the first-period profit matters relatively more than the second-period profit for \( M \), and with foreclosure the first-period price is lower than the first-period price without foreclosure — i.e.,

\[
p'_{NF1} = \frac{4(1+\Delta) + (4+\theta)(\theta+\Delta)}{2(4+\theta)} > p'_F \frac{4(1+\Delta) + (4+\theta)(\theta-\beta\Delta)}{2(4+\theta)}.
\]

With foreclosure, consumers buying the device in the first period anticipate that \( M \) will fully extract the utility associated with the consumption of the complementary service in the second period, reducing their willingness to buy the device in the first period. Notably, for low values of \( \theta \), consumer harm is large.

V. FURTHER REMARKS

The model developed in the previous section hinges on some simplifying assumptions that have been made merely for expositional purposes.

First, we have assumed that players do not discount the future — i.e., we have normalized the discount factor to 1. None of the welfare effects discussed above depends on this assumption. Of course, when firms and consumers assign a lower weight to the future, both the consumer harm associated with foreclosure and the monopolist’s time inconsistency problem tend to be less critical because the model’s outcome gets closer to a static benchmark.

Second, to make our point in the clearest possible way, we assumed that the suppliers providing the non-durable service offer homogenous products and compete à la Bertrand. Of course, when this is not the case, e.g., when their services are perceived as differentiated by consumers, new incentives arise. On the one hand, other things being equal, the introduction of its own non-durable service is still profitable for the device monopolist because, by doing so, it can profit by charging early users. On the other hand, the monopolist has a lower incentive to foreclose as long as it can appropriate the service providers’ profits via access fees. The net effect on consumer surplus is ambiguous too. While the introduction of an inferior product unambiguously harms consumers, without foreclosure, a double marginalization effect arises when the monopolist charges a per-consumer access fee to third-party service suppliers, which also reduces consumer welfare. Overall, the qualitative impact of the changing taste parameter will still be present since the monopolist would still have a greater incentive to foreclose rivals in the complementary market if it expects...
demand in its primary business to grow at low rates and the degree of differentiation between the third-party sellers is not too high (analytical details are available upon request).

Third, none of our effects seems to depend on the assumption of consumer types being uniformly distributed. As long as profit functions are concave, dealing with non-linear demand would not affect our conclusions qualitatively.

Finally, while we assumed that the dynamics of taste shocks is deterministic and known by the players, one could extend our model by assuming that $\theta$ is a random variable that is realized only at the end of the first period. In such a model, our second-period analysis still applies, and foreclosure may or may not occur along the equilibrium path depending on the realized taste shock. Hence, the main message of our stylized model remains valid since the likelihood of foreclosure would be higher when the distribution of the taste shock places more mass on adverse events, or equivalently when $\theta$ is low.

VI. CONCLUSION

How to regulate digital markets effectively while encouraging beneficial innovation is one of the key economic questions that societies face in coming years. In this paper, we show that there can be significant consumer harm as a result of market abuse by gatekeeper platforms, and that such harm is more likely as markets for devices become saturated. This implies that, whatever the structure of future regulation, competition authorities will have to remain vigilant for potential abuses, including in apparently more mature markets.

While there should not be a blanket presumption against device-funded gatekeepers providing their own products on their platforms (any more than one should block supermarkets from offering their own labels), the potential for abuse is there. In our model, consumer welfare would be increased by preventing the device seller from selling its own apps and associated services in competition with third-party apps. Regulators may therefore wish to place a special responsibility on gatekeeper firms to ensure that consumers are not harmed through activities such as self-preferencing or raising rivals’ costs.

We have not attempted to compare how results differ depending on whether platforms are funded by device sales or by advertising. However, our model suggests there should be no a priori view that device-funded models are less likely to be anti-competitive. Device-funded gatekeepers may abuse their positions to extract value from their installed user bases, an incentive that is typically absent from advertising-funded gatekeepers.

APPENDIX

Proof of Lemma 1. The proof follows immediately from direct comparison of $\pi^{E}_2(v^k)$ and $\pi^{NF}_2(v^k)$. ■
Proof of Proposition 1. \( p_1^F \) is obtained as a solution of (7). Substituting \( p_1^F \) into \( p_2^F (p_1) \) we have \( p_2^F \). It is easy to verify that

\[ p_1^F - p_2^F = (1 + \Delta) \frac{2 + \theta}{4 + \theta} > 0. \]

Finally, substituting \( p_1^F \) and \( p_2^F \) into \( v^F (p_1, p_2^F (p_1)) \) we obtain

\[ v^F = \frac{2 (1 - \Delta) + \theta}{4 + \theta} , \]

which is clearly decreasing in \( \Delta \) and increasing in \( \theta \) — i.e.,

\[ \frac{\partial v^F}{\partial \theta} = \frac{2 (1 + \Delta)}{(4 + \theta)^2} > 0. \]

\[ \blacksquare \]

Proof of Proposition 2. \( p_1^{NF} \) is obtained as a solution of (8). Substituting \( p_1^{NF} \) into \( p_2^{NF} (p_1) \) we have \( p_2^{NF} \). It is easy to verify that

\[ p_1^{NF} - p_2^{NF} = (1 + \Delta) \frac{2 + \theta}{4 + \theta} > 0. \]

Finally, substituting \( p_1^{NF} \) and \( p_2^{NF} \) into \( v^{NF} (p_1, p_2^{NF} (p_1)) \) we obtain \( v^F = v^{NF} \). \[ \blacksquare \]

Proof of Proposition 3. The result follows from Lemma 1 and the fact that \( v^F = v^{NF} \). Given consumers' strategy and expectations, \( M \) has no incentive to deviate from an equilibrium with foreclosure (by not foreclosing in period 2) if and only if \( I \leq I^* \equiv I (v^F) \). Mutatis mutandis, \( M \) is not willing to deviate from an equilibrium without foreclosure (by foreclosing in period 2) if and only if \( I \geq I^* \).

Substituting \( v^F = v^{NF} = v^k \) into \( I (v^k) \) we have

\[ I^* = \Delta \max \left\{ 0, 2 \beta \frac{1 + \Delta}{4 + \theta} - (1 - \beta) \left( \frac{1}{2} + \frac{\Delta (\beta + 1) (\theta + 4)}{4 \theta (\theta + 2 (1 - \Delta))} \right) \right\} . \]

Solving \( I^* = 0 \) for \( \beta \), the unique root in the support \([0, 1]\) is

\[ \beta^* = \frac{\eta - \theta (\theta + 2 (1 - \Delta)) (\theta + 4 \Delta + 8)}{\Delta (4 + \theta)^2} \in (0, 1), \]

where

\[ \eta \equiv \sqrt[4]{\frac{64 \theta^2 \Delta^4 + 32 \theta^2 (4 - \theta) \Delta^3 + (256 - 224 \theta^3 - 288 \theta^2 - 15 \theta^4) \Delta^2}{2 \theta (2 + \theta) (64 - 16 \theta + 20 \theta^2 + 3 \theta^3) \Delta + \theta^2 (8 + \theta)^2 (2 + \theta)^2}} . \]

Hence, \( I^* \geq 0 \) if and only if \( \beta \geq \beta^* \).

Uniqueness of the equilibrium (within the class of cut-off strategies for consumers) follows directly from the fact that the price pattern for a given foreclosure decision is unique.

Showing that the conjectured cut-off strategy is indeed part of the equilibrium is straightforward since, holding \( M \)'s strategy constant, consumers buy in the first period.
if and only if $v \geq v^F$ regardless of whether they expect foreclosure in equilibrium. Finally, verifying the conjecture $v^F = vNF \geq \frac{\beta \Delta}{\theta}$ is immediate since $\theta \geq \theta$.

Proof of Corollary 1. The proof follows from Equation 9 and the fact that $v^F$ does not depend on $\beta$.

Proof of Proposition 4. With foreclosure consumer surplus is

$$CS^F = \int_{vF}^{1} (v + \Delta - p_1^F + \theta v) \, dv + \int_{p_2^F}^{vF} (\theta v - p_2^F) \, dv$$

$$= \frac{4\theta + 8\theta^2 + \theta^3 + 4\Delta^2 \beta^2 + 8\theta \Delta + 4\theta \Delta^2 + 4\theta^2 \Delta + 2\theta^2 \Delta \beta + \theta \Delta^2 \beta^2 + 12\theta \Delta \beta}{8\theta (4 + \theta)}.$$

Without foreclosure consumer surplus is

$$CS^{NF} = \int_{vN}^{1} (v + \Delta - p_1^{NF} + (\theta v + \Delta)) \, dv + \int_{p_2^{NF}}^{vNF} (\theta v + \Delta - p_2^{NF}) \, dv$$

$$= \frac{4\theta + 8\theta^2 + \theta^3 + 4\Delta^2 + 20\theta \Delta + 9\theta \Delta^2 + 6\theta^2 \Delta}{8\theta (4 + \theta)}.$$

Let $\Delta CS = CS^{NF} - CS^F$. We have

$$\Delta CS = \frac{1}{8} \Delta (1 - \beta) \frac{12\theta + 4\Delta + 2\theta^2 + 5\theta \Delta + 4\Delta \beta + \theta \Delta \beta}{\theta (\theta + 4)},$$

which is always positive, except at $\beta = 1$ where it is 0.

Moreover, it is immediate to show that

$$\frac{\partial \Delta CS}{\partial \theta} = -\frac{1}{8} \Delta (1 - \beta) \frac{16\Delta + 4\theta^2 + 8\theta \Delta + 16\Delta \beta + 5\theta^2 \Delta + \theta^2 \Delta \beta + 8\theta \Delta \beta}{\theta^2 (\theta + 4)^2} < 0,$$

$$\frac{\partial \Delta CS}{\partial \Delta} = \frac{1}{4} (1 - \beta) \frac{6\theta + 4\Delta + \theta^2 + 5\theta \Delta + 4\Delta \beta + \theta \Delta \beta}{\theta (\theta + 4)} > 0,$$

and

$$\frac{\partial \Delta CS}{\partial \beta} = -\frac{1}{4} \Delta \frac{6\theta + \theta^2 + 2\theta \Delta + 4\Delta \beta + \theta \Delta \beta}{\theta (\theta + 4)} < 0.$$

Hence, the consumer harm is decreasing in $\theta$ and $\beta$ and increasing in $\Delta$.

Proof of Proposition 5. M’s expected intertemporal profit under foreclosure is

$$\pi^F = \int_{vF}^{1} (p_1^F + \beta \Delta) \, dv + \int_{p_2^F}^{vF} (p_2^F + \beta \Delta) \, dv$$

$$= \frac{4\theta + 4\theta^2 + \theta^3 + 4\Delta^2 \beta^2 + 8\theta \Delta + 4\theta \Delta^2 + 2\theta^2 \Delta \beta + \theta \Delta^2 \beta^2 + 8\theta \Delta \beta}{4\theta (\theta + 4)}.$$
Without foreclosure, $M$’s expected intertemporal profit is

$$\pi^{NF} = \int_{v_{NF}}^{1} p_{1}^{NF} dv + \int_{v_{NF}}^{p_{2}^{NF} - \Delta} p_{2}^{NF} dv$$

$$= \frac{\theta^3 + 2 (2 + \Delta) \theta^2 + (5 \Delta^2 + 16 \Delta + 4) \theta + 4 \Delta^2}{4 \theta (4 + \theta)}.$$

Let $\Delta \pi \triangleq \pi^{NF} - \pi^{E}$. We have

$$\Delta \pi = \Delta (1 - \beta) \frac{2 \theta + \Delta (1 + \beta)}{4 \theta},$$

which is clearly positive.

Theorem 1.6

Proof of Corollary 2. Showing that $\Delta \pi$ is increasing in $\Delta$ and decreasing in $\beta$ and $\theta$, is immediate.

REFERENCES


Dryden, N.; Khodjamirian, S.; Rovegno, L. and Small, I., 2020, ‘Another Look at the Impact of Amazon Retail’s Entry on Third-Party Innovation at Amazon’s Marketplace,’ (available on request from the authors).

Etro, F., 2020, ‘Product Selection in Online Marketplaces’ (Available at SSRN 3641307).


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