On sellers’ cooperation in hybrid marketplaces

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Abstract
Hybrid marketplaces, such as Amazon’s and Zalando’s stores or Apple’s and Google’s app stores, which distribute their own products and services in competition with those of third-party sellers, play a significant and growing role in the Internet economy. This paper shows that, other things equal, such platforms would maximize their profits if they lowered the fees charged to sellers and the prices charged to consumers in response to cooperation agreements between third-party sellers: horizontal mergers or collusive agreements. It also shows that such cooperation can be pro-competitive when the platform is a vertically integrated gatekeeper, adopts the agency business model, is a close competitor to the third-party sellers it hosts, and observes (or correctly anticipates) the third-party sellers’ agreement. The discussion here is of significant policy relevance, since third-party sellers in online marketplaces may find it easier to collude and may respond to the bargaining power of certain gatekeeper platforms by merging their activities.

1 | INTRODUCTION

Online marketplaces represent an important distribution channel for products and services around the world, one whose importance is growing over time. Sellers find it profitable to distribute their products and services through marketplaces patronized by many consumers. Consumers in turn are attracted to marketplaces offering a wide selection of products and services, distributed efficiently, and sold at low prices. Thus, marketplaces have the incentive to offer a wide range of product varieties, including some which may be regarded as close substitutes, to increase product choice and encourage price competition and, therefore, attract a greater number of consumers. To do so, they enter into distribution agreements with many third-party sellers. Yet, those sellers may not have the incentive to offer as many varieties through marketplaces as they do through other channels, especially their own direct distribution channels, given that marketplaces typically charge per-transaction fees. Also, they may not have incentive to price competitively the varieties sold through the marketplace, especially when they possess market power. And, in any event, their and the marketplaces’ incentives are not fully aligned, as each third-party seller fails to internalize the external effect of its pricing and non-pricing decisions on the sales of other sellers in the marketplace and on the marketplace’s overall reputation and profits.

Because of this, some marketplaces, such as Amazon, Zalando, Walmart, Apple’s Appstore and Google’s Playstore, have adopted a “hybrid” business model, supplying directly their own products and services in addition to those of third-party sellers. By adding missing varieties, hybrid marketplaces seek to achieve selection parity with other distribution channels. By entering with close substitutes to those sold by third-party sellers, they may incentivize the
latter to price more competitively. Hybrid marketplaces profit from selling their own products and services and from the per-transaction fees they charge to third-party sellers. Because these two profit streams are interrelated when the marketplace and the sellers offer substitutes, the prices and fees applied by the hybrid marketplace are expected to respond to changes in the degree of competition or cooperation between the third-party sellers in the platform.

This paper investigates the nature and extent of such a response. Although related to the growing literature on hybrid marketplaces,1 this is, to the best of our knowledge, the first paper to consider (i) the optimal response of a hybrid marketplace to a horizontal merger or a collusive agreement between third-party sellers offering substitutes products or services through the marketplace; and (ii) the consumer and total welfare effects of horizontal mergers and collusive agreements amongst sellers distributing their products and services through gatekeeper hybrid marketplaces.

This is a discussion of significant policy relevance. Assistant Attorney General Jonathan Kanter recently noted “As American consumers increasingly turn to e-commerce, it is critically important to deter, detect and prosecute crimes that prevent fair and open competition in online marketplaces.”2 On the one hand, competition authorities on both sides of the Atlantic, as well as legal and economics scholars, have recently expressed concerns that competition in online marketplaces may increase the scope for anti-competitive practices by sellers.5 The greater price transparency which characterizes online marketplaces allows retailers to track more effectively the prices charged by their rivals, which may facilitate collusion, whether explicit or tacit (OECD, 2019). Moreover, the use of algorithmic prices among e-retailers may also foster collusion.4 On the other hand, the significant bargaining power of certain gatekeeper platforms may trigger changes in the ownership structure of third-party sellers distributing through such platforms. In particular, those sellers may decide to merge or their shares may end up in the hands of common shareholders.5 Competition agencies investigating such deals will have to take into account the specific context where such firms operate and, specifically, the role played by gatekeeper platforms.

We analyze a model where two competing firms sell exclusively through a hybrid marketplace which adopts the agency business model (Johnson, 2017)—that is, it charges sellers per-unit or ad-valorem fees—and commercializes a private label in competition with third-party sellers. We first characterize the hybrid marketplace’s private label price and fees and third-party sellers’ prices when sellers set prices non-cooperatively. We then characterize equilibrium prices and fees when third-party sellers maximize their joint profit, either as a result of a horizontal merger or because of collusion. The marketplace is a gatekeeper for the third-party sellers and, therefore, has all the bargaining power vis-à-vis third-party sellers in both scenarios.

We find that the hybrid marketplace responds to third-party sellers’ cooperation by reducing the prices of its private label as well as the fees applied to the sellers. Other things equal, cooperation among third-party sellers would lead to excessively high retail prices from the platform’s viewpoint. To offset the resulting reduction in sales, the platform has the incentive to cut its (linear or ad-valorem) fees to mitigate double marginalization and hence moderate the increase in third-party sellers’ prices. And because distributing third-party sellers’ products becomes less profitable when they cooperate, the platform also lowers the price of its private label, so as to steer consumers toward its relatively more profitable business. Being confronted with lower fees and expecting more aggressive competition from the platform’s private label, cooperating third-party sellers may end up setting lower retail prices. This will be the case when price competition between third-party sellers and the vertically integrated platform is fierce—that is, when products are relatively homogeneous. In such a case, consumers will be unambiguously better off under cooperation (as all products are available at a lower price), and allowing cooperation is also optimal from a total welfare standpoint.

A key feature of our model is that fees are secretly negotiated only after third-party sellers have decided to collude or merge. Admittedly, platform fees seem to be posted uniformly or, at best, not conditioned on the mode of competition and the ownership structure of the third-party sellers. Yet, we believe the timing in our game is most realistic, because platforms commonly negotiate firm-specific charges (e.g., logistical costs) and incentives (e.g., promotions and other forms of marketing support) secretly. Those charges and incentives are adjusted frequently and could naturally be changed in response to changes in sellers’ conduct and structure.

These results are robust to a number of changes in the underlying model—for example, the use of nonlinear contracts, different demand and cost specifications, a larger number of sellers in the marketplace, alternative timing, informational and belief assumptions. Yet, our results hinge on the specific nature of the platform’s business model. They do not hold for vertically disintegrated marketplaces or marketplaces operating a wholesale (or resale) business model.

A policy-relevant implication of our analysis is that, to the extent that policymakers are concerned that selling through e-commerce marketplaces might facilitate collusion among retailers (through data sharing or the use of pricing algorithms), the ability of the platforms to sell competing products mitigates this concern. This raises questions about
the potential effects of the Ending Platform Monopolies Act, recently proposed in US Congress, which would make it unlawful for certain platforms (such as Amazon) to operate as hybrid marketplaces.

The remainder of the paper is organized as follows. The baseline model is set up and analyzed in Sections 2 and 3, respectively. Section 4 discusses the robustness of the results with respect to the model assumptions. Section 5 concludes. Supplementary calculations and proofs of the main results are in Appendices A and B, respectively. Further material is contained in an Supporting Information: Appendix.

2 | MODEL

Two retailers \( R_i, i = A, B \), sell their products through an e-commerce platform \( P \). In addition to hosting these third-part-sellers, \( P \) also owns (and is vertically integrated with) a retail unit that distributes a private label in competition with them. The platform adopts the so called agency model of business (e.g., Johnson, 2017). Specifically, it charges a fee \( f_i \) to \( R_i \) for each unit sold through its web site. Following a recent literature (e.g., Etro, 2021; Padilla et al., 2022), we assume throughout that consumers can purchase retailers’ products only through \( P \)’s web site—that is, \( P \) is a gatekeeper platform (Alexiadis & de Streel, 2020)—or, more generally, they compete with each other only on the e-commerce marketplace owned by \( P \)—for example, as in Baye and Morgan (2001), they are monopolists in different (offline) local markets.

The products sold by the three firms are imperfect substitutes and (symmetrically) horizontally differentiated. Specifically, given retail prices \( p_A, p_B \) and \( p_P \), consumers’ demand for product \( h = A, B, P \) is given by

\[
D^h(p_h, \sum_{k=A,B,P, k \neq h} p_k) \triangleq \frac{1}{(1 - \gamma)(1 + 2\gamma)} \left( 1 - \gamma - (1 + \gamma)p_h + \gamma \sum_{k=A,B,P, k \neq h} p_k \right),
\]

where \( \gamma \in (0, \infty) \), with \( \gamma \approx 0.95 \), is an inverse measure of product differentiation. In what follows, let

\[
D^h_i(\cdot) \triangleq \frac{\partial D^h(\cdot)}{\partial p_h} = -\frac{1 + \gamma}{(1 - \gamma)(1 + 2\gamma)} < 0, \quad D^h_{ij}(\cdot) \triangleq \frac{\partial D^h(\cdot)}{\partial p_k} = \frac{\gamma}{(1 - \gamma)(1 + 2\gamma)} \in (0, |D^h_i(\cdot)|),
\]

All firms’ production costs are linear and normalized to zero. Hence, for \( i, j = A, B, j \neq i \), each \( R_i \)’s profit is given by

\[
\pi_{R_i} \triangleq (p_i - f_i)D^i(p_i, p_j + p_P),
\]

whereas \( P \)’s profit, normalizing to zero the cost of operating the marketplace, is

\[
\pi_P \triangleq \sum_{i=A, B} f_i D^i(p_i, p_j + p_P) + p_P D^P(p_P, p_A + p_B).
\]

The outside option value of each \( R_i \), when it does not distribute its products through \( P \)’s web site, is also normalized to zero.

Third-party sellers may either act non-cooperatively (in which case, each \( R_i \) maximizes \( \pi_{R_i} \)) or they can merge (or be acquired by the same entity) or collude—that is, set prices so as to maximize their joint profit. This decision is taken at the outset of the game. To simplify the exposition, in the baseline model we assume that this choice is observed by \( P \). Then, the game unravels as follows:

\[
t = 1 \quad P \text{ makes secret take-it-or-leave-it offers } f_i \text{ to third-party sellers;}
\]
\[
t = 2 \quad \text{Firms active on the platform (i.e., third-party sellers who accepted } P \text{'s offer and } P \text{ itself) set (retail) prices.}
\]

This timing is meant to capture in a simple game more complicated dynamic interactions whereby \( P \) is able to infer from third-party sellers’ pricing choices whether or not they are setting prices cooperatively, and to adjust its fees and price accordingly. Solution concept is Perfect Bayesian Equilibrium (PBE) with passive beliefs—that is, for any received offer \( f_i \), when choosing \( p_i \) in the non-cooperative regime \( R_i \) believes that \( R_j \) is facing the equilibrium fee. Secret
contracts and passive beliefs are standard assumptions in the literature on e-commerce marketplaces: see, for example, Johansen and Vergé (2017), Gomes and Mantovani (2020), Bisceglia et al. (2021), among many others. The results in Rey and Vergé (2004, 2020) show that any PBE with passive beliefs is a Sequential Equilibrium of a game in which firms who are involved in multiple simultaneous bargaining processes and have all the bargaining power (in this model, \( P \)) delegate the negotiations to partner-specific agents. In practice, even though platform fees are often posted uniformly, platforms commonly negotiate firm-specific charges (e.g., logistical costs) and incentives (e.g., promotions and other forms of marketing support) secretly. Moreover, those charges and incentives, which affect sellers’ distribution costs via the platform, are adjusted frequently and could naturally be changed in response to changes in their conduct and structure.

### 3 | ANALYSIS

In this section we derive the equilibrium fees and prices in the scenarios in which third-party sellers act non-cooperatively (hereafter, non-cooperative equilibrium), denoted by \((f^*, p^*_P, p^*)\), and cooperatively (cooperative equilibrium), denoted by \((f^c, p^c_P, p^c)\).\(^{11}\) Comparing the two equilibria then allows to establish whether sellers have incentives to set prices cooperatively (e.g., by merging their operations or forming a price-setting joint venture) and to assess the competitive and welfare effects of this conduct.

#### 3.1 | Non-cooperative equilibrium

When third-party sellers compete with each other, at stage \( t = 2 \), upon observing the unit fee \( f_i \), each \( R_i \) solves

\[
\max_{p_i} (p_i - f_i)D^i(p_i, p^*_j + p^*_P),
\]

where \( p^*_j \) and \( p^*_P \) are the prices that \( R_i \) expects to be set by \( R_j \) and \( P \), respectively. As contracts are secret, by the assumption of passive beliefs, \( R_i \) believes that \( f^*_j = f^* \), so that \( R_j \) is expected to set the equilibrium price (i.e., \( p^*_j = p^* \)), and \( P \) to optimally set \( p_P \) given the fees \((f_i, f^*)\).

The standard first-order condition (hereafter, FOC) of this problem,

\[
D^i(\cdot) + (p_i - f_i)D^i_1(\cdot) = 0, \tag{2}
\]

which trades off the gain on infra-marginal consumers and the demand reduction following an increase in \( p_i \), yields \( R_i \’s \) strategy, which is increasing in the unit distribution cost (i.e., \( f_i \)) and, by strategic complementarity among prices, in \( p^*_j \) and \( p^*_P \) as well.\(^{12}\)

Simultaneously, \( P \), who of course knows both \( f_A \) and \( f_B \) it offered to third-party sellers, chooses the price of its private label solving

\[
\max_{p_P} \sum_{i=A,B} f_i D^i(p^*_i, p^*_j + p_P) + p_P D^P(p_P, p^*_A + p^*_B),
\]

where \( p^*_i, i = A, B \), is the price optimally set by \( R_i \) given the offer \( f_i \) made by \( P \) in the previous stage. The FOC of this problem can be written as follows:

\[
\sum_{i=A,B} f_i D^i_1(\cdot) + D^P(\cdot) + p_P D^P_1(\cdot) = 0. \tag{3}
\]

When setting \( p_P \), besides taking into account the standard trade-off of a single-product oligopolist, \( P \) also internalizes the positive effect that a larger price of its private label has on the demand of third-party sellers, hence on
its revenues from their fees. As a consequence, the price of its private label is larger than the one it would set absent revenues from third-party sellers, and even more so when fees are larger, so that distributing third-party sellers’ products is more profitable. In addition, larger fees translate into a larger $p_p$ also because they imply larger (expected) retail prices by third-party sellers, and prices are strategic complements. 13

Moving backward to stage $t = 1$, $P$’s problem can be written as follows:

$$\max_{f_i, f_j} \sum_{i=A, B} f_i D_i^i \left( p_i^*(f_i), p_j^*(f_j) + p_p^*(f_A, f_B) \right) + p_p^*(f_A, f_B) D_p^p \left( p_p^*(f_A, f_B), p_A^*(f_A) + p_B^*(f_B) \right),$$

where $p_i^*(f_i)$ and $p_p^*(f_A, f_B)$ are the retail prices optimally charged by $R_i$ and $P$ when actual fees are $f_A$ and $f_B$ but each $R_i$ holds passive beliefs on $f_j$ (see Appendix A). The FOC with respect to $f_i$ writes as

$$D_i^i(-) + f_i \left( \frac{\partial p_i^*(\cdot)}{\partial f_i} + \frac{\partial p_p^*(\cdot)}{\partial f_i} \right) + f_j D_j^i(-) \left( \frac{\partial p_i^*(\cdot)}{\partial f_j} + \frac{\partial p_p^*(\cdot)}{\partial f_i} \right) = 0.$$  (4)

In words, $P$ takes into account that, by charging a larger $f_i$, $R_i$ is induced to set a larger price $p_i$, and, by the mechanisms detailed above, $P$ itself will optimally increase its price $p_p$. As a result, holding $f_j$ fixed, an increase in $f_i$ induces a drop in $R_i$’s and $P$’s demand, and an increase in $R_i$’s sales. Hence, by offering a larger $f_i$: (i) $P$ collects higher unit fees from $R_i$, but on a lower amount of sales; (ii) it obtains larger revenues from $R_i$, whose contract and behavior is fixed, thereby its sales increase; and (iii) it sells lower quantities of its private label at a larger price.

Solving the system of FOCs (4) for $i = A, B$ and imposing $p^* = p_i^*(f^*)$ yields the equilibrium fees and prices set by third-party sellers, from which also $p_p^* = p_p^*(f^*, f^*)$ can be computed.

### 3.2 Cooperative equilibrium

When third-party sellers choose prices so as to maximize their joint profit, their optimization problem in $t = 2$ writes as

$$\max_{p_i, p_j} \sum_{i=A, B} (p_i - f_i) D_i^i \left( p_i, p_j + p_p^c \right),$$

where $p_p^c$ is the optimal price of $P$’s private label (that they expect to be set by $P$) given the fees $f_A$ and $f_B$. The FOC with respect to $p_i, i = A, B$, writes as

$$\frac{D_i^i(-) + (p_i - f_i) D_i^i(-)}{\text{(Non-Cooperative Oligopoly Rule)}} + \frac{(p_j - f_j) D_j^i(-)}{\text{Externality on $R_i$’s profit (+)}} = 0.$$  (5)

When setting $p_i$, a merged entity or a cartel take into account not only the standard trade-off between an increase in margins and a drop in the demand for $R_i$’s product, but also internalize the positive effect of a larger $p_i$ on the demand (hence, profit) of the other seller $R_j$. Of course, other things equal, this implies that retailers have stronger incentives to raise prices in the cooperative scenario compared to the non-cooperative game analyzed above: a standard competition-softerning effect 14.

This anti-competitive force is magnified by $P$’s behavior at the retail pricing stage. To see this, notice that $P$’s problem in $t = 2$ is as in Section 3.1. Hence, ceteris paribus, since prices are strategic complements, anticipating a larger retail price by third-party sellers, $P$ would optimally increase its price compared to the non-cooperative equilibrium so as to take profit from the softer product market competition. 15 In turn, again by strategic complementarity between retail prices, a larger expected price set by $P$ would give third-party sellers further incentives to raise their prices, 16 and so on. Therefore, if fees were set at the same level in the two considered scenarios,
consumers would be unambiguously harmed by third-party sellers’ cooperative behavior, as it would cause an increase of all products’ prices.

Moving backward to the fee-setting stage \((t = 1)\), \(P\) solves:

\[
\max \sum_{f_i \in F_i} \left[ D^i (f_i^* (f_j, f_i), f_i^* (f_j, f_i)) + p^c (f_{A_i}, f_{B_i}) D^p (f_{A_i}, f_{B_i}, p^c (f_j, f_i)) + p^c (f_{A_i}, f_{B_i}) D^p (f_{A_i}, f_{B_i}, p^c (f_j, f_i)) \right],
\]

where \(p^c (f_j, f_i)\) and \(p^c (f_{A_i}, f_{B_i})\) are the equilibrium prices of third-party sellers and \(P\), respectively, given the fees offered in \(t = 1\) (see Appendix A). The FOC of this problem with respect to \(f_i\) writes as

\[
D^i (\cdot) + f_i \left[ D^i_1 (\cdot) \frac{\partial p^c (\cdot)}{\partial f_i} + D^i_2 (\cdot) \left( \frac{\partial p^c (\cdot)}{\partial f_i} + \frac{\partial p^c (\cdot)}{\partial f_i} \right) \right] + f_i \left[ D^i_1 (\cdot) \frac{\partial p^c (\cdot)}{\partial f_i} + D^i_2 (\cdot) \left( \frac{\partial p^c (\cdot)}{\partial f_i} + \frac{\partial p^c (\cdot)}{\partial f_i} \right) \right] = 0.
\]

Notably, when setting each fee, \(P\) anticipates that now a larger value of \(f_i\) translates into larger retail prices of all products: \(P\) itself will optimally increase the price of the private label as distributing \(R_i’s\) products becomes more profitable, and both third-party sellers will do the same, as prices are strategic complements.

### 3.3 Competitive and welfare analysis

Comparing the equilibria derived above yields the following results.

**Proposition 1.** When third-party sellers act cooperatively, \(P\) optimally lowers both the unit fees \((f^c < f^*)\) and the price of its private label \((p^c < p^*)\). Third-party sellers charge a lower price in the cooperative equilibrium if and only if products are sufficiently homogeneous—that is, \(p^c < p^*\) for \(\gamma > \hat{\gamma}\), with \(\hat{\gamma} \in (0, 1)\).

The intuition is as follows. The double marginalization problem, arising in the industry under unit platform fees, bites more when third-party sellers set prices cooperatively: third-party sellers’ pass-through rate is always larger under cooperation. This is because each \(R_i\) is not afraid of losing market shares at the benefit of \(R_j\), which gives it stronger incentives to charge a larger mark-up or, equivalently, to pass on fees to consumers. Thus, in the cooperative regime \(P\) anticipates that, by charging the unit fee \(f^*\) it sets in the non-cooperative equilibrium, third-party sellers would set excessively large prices, which would inefficiently reduce their sales, and accordingly result in lower revenues for \(P\) as a marketplace owner. As products are differentiated (i.e., consumers exhibit preferences for variety), these lower revenues cannot be fully compensated through the increase in the demand for \(P’s\) private label. Hence, its optimal strategy consists in lowering unit fees in response to the third-party sellers’ cooperative behavior—that is, it optimally sets \(f^c < f^*\). Since lower fees imply that the profitability of hosting third-party sellers drops, \(P\) has stronger incentives to compete aggressively in the marketplace when third-party sellers act cooperatively, so to divert market shares toward the relatively more profitable business. As a result, \(p^c < p^*\).

The comparison between the prices set by third-party sellers in the two equilibria is in general ambiguous. This is because, when they set price cooperatively, on the one hand they can profitably increase their mark-ups (which, other things equal, would imply \(p^c > p^*\), but on the other hand they face lower distribution costs (which, all else equal, would imply \(p^c < p^*\)). Importantly, their choices are also driven by the price of \(P’s\) private label that they expect to be set in equilibrium: anticipating \(P’s\) incentives to cut \(p_p\) when they set prices cooperatively, by the strategic complementarity between retail prices, third-party sellers have a further reason to charge a lower price in the cooperative scenario. This effect is more relevant when products are sufficiently homogeneous (\(\gamma\) large), so that strategic complementarity forces are strong enough. To put it differently, when products are sufficiently homogeneous, the presence of \(P’s\) private label constitutes a strong competitive constraint on the merged entity’s or cartel’s choices.
This implies that third-party sellers cannot increase too much their mark-ups in the cooperative regime, as otherwise they would lose too many market shares at the benefit of \( P \)'s private label. As a result, since they also face lower distribution costs, they optimally reduce their prices compared to the non-cooperative equilibrium.

The equilibrium of the whole game, in which third-party sellers endogenously choose whether or not to cooperate, and the welfare effects of allowing their cooperation are summarized in the following proposition.

**Proposition 2.** Third-party sellers’ profits are always larger in the cooperative equilibrium. Hence, they cooperate in equilibrium, whenever possible. Moreover, there are thresholds \( 0 < \gamma^{CS} < \gamma^{TW} < \hat{\gamma} \) such that the cooperative equilibrium maximizes consumer surplus if and only if \( \gamma > \gamma^{CS} \) and increases total welfare if and only if \( \gamma > \gamma^{TW} \).

Third-party sellers always benefit from setting prices cooperatively, whether as a result of a merger or due to collusion, the reason being that (i) they benefit from the reduction in fees, and (ii) the competitive response of the hybrid retailer \( P \) is insufficient to overcome the incremental profits resulting from cooperation. As a consequence, unless competition authorities make such cooperation infeasible—either by blocking their merger or joint venture or aggressively enforcing the prohibition to collude—third-party sellers will prefer to cooperate.

Interestingly, allowing these conducts may be optimal from a consumer surplus or total welfare viewpoint, provided that \( P \) is able to observe (or correctly anticipate: see Section 4.5) the sellers’ cooperative behavior, and to adjust its fees (and price) accordingly. If this is not the case—that is, \( P \) sets its fees before third-party sellers make pricing decisions, including the decision to set prices independently or cooperatively, and is not able to modify them afterwards—then, as shown in Section 3.2, the sellers’ cooperative behavior always results in larger prices of all products, thereby harming consumers and reducing total welfare. Thus, the platform’s ability to react to changes in the competitive environment of the marketplace it manages is a relevant force which may outweigh the standard anti-competitive effects of horizontal integration or collusion.

Specifically, in order for consumers to be better off in the cooperative equilibrium, competition in the marketplace must be sufficiently intense. In particular, \( P \)'s private label must be a relatively close substitute of third-party sellers’ products. This is because, when cross-price elasticities are small, \( P \) has weaker incentives to reduce the price of its private label, and also third-party sellers are less concerned about \( P \)'s pricing behavior and thus less afraid of setting large prices. In these cases, the standard competition-softening force is bound to prevail. In particular, for \( \gamma < \gamma^{CS} \) the harm caused to consumers by the larger price of third-party sellers’ products is not compensated by the availability of \( P \)'s private label at a lower price, as products are rather differentiated, and two out of the three available varieties are sold at a higher price. By contrast, for \( \gamma \in (\gamma^{CS}, \gamma^{TW}) \), cooperation is beneficial to consumers, but still decreases total welfare. This is due to the fact that the lower margins earned by \( P \), both as a marketplace owner and as a seller, are not outweighed by the beneficial effect of sellers’ cooperation on their profits and consumer surplus. However, due to the double marginalization problem, the cooperative equilibrium always maximizes total welfare when it lowers prices of all products in the marketplace \( (\gamma^{TW} < \hat{\gamma}) \).

Notably, these results are obtained abstracting from the potential effect that concentration may have on the bargaining power of third-party sellers vis-à-vis powerful gatekeeper platforms. Rather than reflecting a different allocation of bargaining power, the lower unit fees charged by the platform in our model are a by-product of its concerns about the excessive double marginalization problem induced by third-party sellers’ cooperative pricing behavior.\(^{18}\) Of course, allowing for buyer power forces (Galbraith, 1952), cost-efficiencies generated by the merger (Williamson, 1968) or efficiencies in sellers’ pricing brought up by the adoption of collusive algorithmic pricing tools (Miklós-Thal & Tucker, 2019) would increase the likelihood that consumers are better off in the cooperative equilibrium.

### 4 | ROBUSTNESS AND DISCUSSION

In this section we discuss the robustness of the results shown above along several dimensions. All proofs are in the Supporting Information: Appendix.

#### 4.1 | Nonlinear contracts

In the baseline model, following the literature (e.g., Boik & Corts, 2016; Johansen & Vergé, 2017; Mariotto & Verdier, 2020), we considered per-unit platform fees, which allowed to derive simple closed-form solutions for the
equilibrium values. However, in practice, many e-commerce platforms charge ad-valorem fees\textsuperscript{19} (e.g., Johnson, 2017; Wang & Wright, 2017)—that is, \( P \) collects \( \phi_i p_i \), with \( \phi_i \in [0,1] \), for each unit sold by \( R_i \) (at price \( p_i \)).\textsuperscript{20}

In Supporting Information: Appendix A we show (using numerical simulations) that ad-valorem fees magnify the pro-competitive effects of third-party sellers’ cooperative behavior: consumer surplus and total welfare are always larger in the cooperative equilibrium. As in the baseline model, cooperation among third-party seller induces \( P \) to set lower fees and reduce the price of its private label. However, under ad-valorem fees also third-party sellers unambiguously set lower prices in the cooperative equilibrium. The reason is that firms do not fully appropriate an increase in the cooperative price, since a fraction \( \phi^c \) of such an increase is appropriated by \( P \). In spite of this, also with ad-valorem fees cooperation softens competition and leads to lower platform fees, thereby third-party sellers prefer to act cooperatively.

The qualitative results of the baseline model also hold when considering two-part tariff contracts—that is, \( P \) charges a unit fee \( f_i \) and a fixed (listing) fee \( F_i \) to each \( R_i \) (see Supporting Information: Appendix B). This is perhaps more surprising since, as it is well known since Hart and Tirole (1990), the use of such contracts exposes the platform to an opportunism problem that wipes out when third-party sellers cooperate, to the detriment of consumers. However, the presence of \( P \)'s private label weakens the opportunism problem in the non-cooperative regime (see Pan, 2018). Moreover, sellers' cooperative behavior is efficient from \( P \)'s viewpoint with two-part tariffs, as it can appropriate the increase in their joint profit through the fixed fees. As distributing third-party sellers’ product becomes more efficient, \( P \) optimally lowers its unit fees in the cooperative regime, thereby all firms' trade-offs at the final pricing stage are as in the baseline model.

### 4.2 Alternative demand and cost functions

The foregoing analysis was carried out under a linear demand system à la Singh and Vives (1984). This linear specification of the model has the advantage of simple closed-form solutions, which cannot be obtained under more general demand systems. Moreover, the linear specification does not limit the generality of our main ‘possibility result’ that the welfare effects of sellers’ cooperation may be positive or negative depending on the strength of competition between the third-party sellers’ brands and \( P \)'s private label. In any event, this result appears to be fairly robust with respect to the demand specification. First, it holds under a linear demand specification à la Shubik and Levitan (1980): see Supporting Information: Appendix F. Second, it also holds when product differentiation is asymmetric, provided again that \( P \)'s product is a relatively close substitute to the third-party sellers’ products: see Supporting Information: Appendix C.

Finally, in Supporting Information: Appendix D we show that our main result also holds when \( P \) suffers from a cost disadvantage—that is, it incurs a unit production cost \( c_P > 0 \). In this case, the threshold \( \gamma^\text{CS} \) above which consumer surplus is maximized in the cooperative equilibrium is an increasing function of \( c_P \), since, as \( c_P \) grows larger, the market share of \( P \)'s private label drops, so that \( P \) can discipline the pricing of third-party sellers only if its product is a sufficiently close substitute to theirs.

### 4.3 Increased competition in the marketplace

Supporting Information: Appendix E shows that our main result holds also when we extend the baseline model to \( N \geq 2 \) symmetric third-party sellers. In this case, we find that \( \gamma^\text{CS} \) is increasing in \( N \). As \( N \) grows larger, the market share of \( P \)'s private label drops, and thus the share of consumers harmed by third-party sellers’ higher prices under cooperation (for \( \gamma < \gamma^c \)) is larger in less concentrated marketplaces.\textsuperscript{21}

These results, together with those discussed in the previous section, show that, in order for third-party sellers’ cooperative behavior to be pro-competitive, they must face fierce competition from \( P \)'s private label.

### 4.4 Alternative timing and beliefs

In the baseline model, \( P \) first makes offers to third-party sellers and then sets the price \( p_p \) in competition with the sellers themselves, who hold passive beliefs. Our main result is qualitatively unchanged under a slightly different
timing of the game, in which $P$ simultaneously chooses $f_A, f_B$ and $p_P$ before third-party sellers set prices, when they hold passive beliefs.

This is also true under (affine) wary beliefs (e.g., Bisceglia, 2022; Gaudin, 2019; Rey & Vergé, 2004)—that is, if each $R_i$ believes that $P$ has optimally set the other prices ($p_p$, and also $f_j$ in the non-cooperative regime), given the observed offers ($f_i$, and also $f_j$ in the cooperative regime). In this case, while $P$’s fees and price are still lower in the cooperative equilibrium, third-party sellers’ prices are always lower when they compete with each other—that is, $p^c > p^h$ for all $\gamma$. The reason is that $P$ has less incentives to cut its fees under cooperation.22 Yet, consumer surplus is still larger in the cooperative equilibrium when products are sufficiently homogeneous, even though the threshold $\gamma^{CS}$ is larger than in the base model. All the details are in Supporting Information: Appendix G.

4.5 Alternative informational assumptions

In the baseline model we made the simplifying assumption that $P$ is able to observe whether third-party sellers cooperate before the fee-setting stage. This is a reasonable assumption when considering that cooperation is implemented via merger or through a joint venture, but it is more questionable when the cooperative price emerges through collusion. For this reason, we investigate whether our conclusions hold under weaker informational requirements.

More precisely, consider a version of the model where $P$ cannot observe whether the third-party sellers cooperate, but is able to observe whether their cooperation is viable and tolerated by the competent antitrust authority. In this environment, in any PBE of the game, $P$ correctly anticipates whether third-party sellers will act cooperatively at the final pricing stage. In Supporting Information: Appendix H we show that, in the unique PBE of this game: (i) the third-party sellers choose to cooperate whenever this is feasible, (ii) $P$, anticipating this choice, optimally sets $f^c$ and $p_P^c$, and (iii) sellers set $p^c$—that is, the cooperative equilibrium characterized in Section 3.2 emerges. This is because, regardless of whether $P$ expects third-party sellers to cooperate, they find it optimal to do so and, hence, it is optimal for $P$ to set its fees and price as if the sellers were committed to cooperate.

4.6 Platform’s business model

So far, we have shown that the key result of the paper is robust to a number of modeling assumptions. As we explain in what follows, there are some assumptions that are, however, critical.

First, $P$ must adopt an agency business model. Suppose instead that $P$ operates a wholesale (or resale) business model—that is, it first purchases the products from third-party sellers (at wholesale prices set by the sellers themselves), and then sets retail prices. In this case, a merger or cooperative agreement among the third-party sellers will result in higher wholesale prices, which $P$ then passes on to consumers. Since (under linear demand) the price of $P$’s private label is set at the same level regardless of the wholesale prices, it follows that cooperation is always anti-competitive under the wholesale business model (a formal analysis is contained in Supporting Information: Appendix I).

Second, $P$ must be vertically integrated. Indeed, if only third-party sellers distribute their products in the online marketplace owned by $P$, then cooperation, even if limited to a subset of sellers, always damages consumers. This is because a vertically disintegrated $P$ has weaker incentives to lower its fees, since such lower fees do not lead to a lower price for the private label with which to discipline the sellers. Then, since retail prices of all sellers are strategic complements, the cooperative regime unambiguously features higher prices (detailed proofs are in Supporting Information: Appendix J). This finding is perfectly in line with the results summarized in Sections 4.2 and 4.3, which clearly pointed out that, in order for consumers to be better off in the cooperative regime, third-party sellers must face tough competition from $P$’s private label.

Of course, the platform’s incentives to sell a private label are affected by the third-party sellers’ behavior. In Supporting Information: Appendix K we prove that $P$ has a greater incentive to sink a fixed cost to introduce a private label when it expects sellers to set prices cooperatively—that is, there is a range of entry costs for which $P$ enters the marketplace if and only if it expects cooperation among third-party sellers.23 Cooperation results in softer competition and, therefore, increases the profitability of entry or, in other words, makes a hybrid-marketplace business model more
attractive. Such a move will benefit consumers by mitigating the potential anticompetitive effects of cooperation among third-party sellers.

5 CONCLUSION

Motivated by the growing importance of online hybrid marketplaces in the distribution of products and services, and by the concerns that competition in online marketplaces may increase the scope for anti-competitive practices by sellers, we have built a simple model to assess the competitive and welfare effects of horizontal mergers and other joint selling agreements or collusive agreements among third-party sellers distributing through such platforms. We have shown that such agreements need not be anticompetitive when (a) third-party sellers compete with each other only on the gatekeeper’s platform; (b) the platform adopts the agency business model; (c) the platform is vertically integrated and is a close competitor to the third-party sellers it hosts; and (d) the platform observes (or correctly anticipates) the third-party sellers’ agreement, and adjusts its fees and price accordingly.

These findings should be taken into account by competition agencies when assessing, for example, mergers between app developers or e-retailers. When conditions (a) to (d) hold, such mergers (and likewise other joint selling and cooperation agreements) need not harm consumer welfare. The paper leaves some interesting issues for further research. One issue of practical relevance is the potential spillover effect of the gatekeeper’s response to cooperation on competing distribution channels. Another extension worth pursuing is to investigate whether the platform’s choice of a business model—agency versus wholesale and hybrid versus vertically disintegrated—may be affected by the degree of cooperation among third party sellers.

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ENDNOTES

1 See, for example, Etro (2021), Anderson and Bedre-Defolie (2021), and Hagiu et al. (2022), who study the welfare implications of allowing e-commerce platforms to sell private labels on their marketplaces, and Padilla et al. (2022), who examine the circumstances under which a platform can use self-preferencing to foreclose third-party sellers. Unlike the present paper, these articles focus on platforms’ strategies and their welfare implications under the assumption of sellers’ competition in the marketplace.


3 In a recent antitrust case (CMA case 2015 n. 50223), the CMA successfully challenged a pricing software that was allegedly designed to coordinate the price of posters by multiple online sellers. In brief, Trod Limited has admitted agreeing with one of its competing online sellers (GB eye Limited) that they would not undercut each other’s prices for posters and frames sold on Amazon’s UK web site. Such agreement was implemented by using algorithmic pricing software which each of the parties configured to give effect to the illegal cartel. The same conduct was successfully challenged also in the United States. See: https://www.justice.gov/atr/case-document/file/628891/download.

4 For evidence that a significant fraction of sellers in a large online marketplace (namely, Amazon US) adopt algorithmic pricing, see, for example, L. Chen et al. (2016). For discussions on algorithmic pricing and collusion, see Harrington (2018), Calvano et al. (2019), and references therein. For related experimental evidence that relatively simple pricing algorithms systematically learn to play collusive strategies, see Calvano et al. (2020) and Klein (2021).

5 Private equity groups are massively acquiring successful Amazon’s third-party sellers: see, for example, https://www.ecommercetimes.com/story/Amazon-Seller-Acquisition-Market-Seeing-Unprecedented-Activity-87199.html.

6 The analysis is unchanged if, rather than being vertically integrated, $P$ purchases the products from a competitive fringe of suppliers—that is, if it acts as a first-party retailer.

7 This demand system is derived from a linear-quadratic specification of a representative consumer’s utility à la Singh and Vives (1984): see Appendix B.
The restriction \( \gamma \leq \mathcal{F} \) ensures that all second-order conditions (SOCs) of the maximization problems considered in the analysis hold. Details are in Appendix A.

Thus, as standard in the literature (e.g., Salant et al., 1983), both merged and colluding sellers are assumed to behave as a multi-product firm. This assumption is not entirely consistent with reality when collusion is implemented through the adoption of algorithmic pricing tools, because in that case (i) stricto sensu, sellers still act as independent firms when setting prices; and (ii) even though competing pricing algorithms may reach supra-competitive prices, they typically fail to converge to the fully cooperative outcome (see Calvano et al., 2020). Although, as a consequence, our theoretical findings do not necessarily translate to a setting in which pricing algorithms are colluding, we can argue that by considering explicit collusion we are amplifying the anti-competitive effects of collusion or, in other words, analyzing a worst-case benchmark.

Notice that, while in principle all firms would benefit if also \( P \) were to merge or collude with third-party sellers, we do not account for this possibility since, as far as we know, the policy circles to date are not concerned about platforms acquiring or colluding with sellers they host.

The restriction to symmetric equilibria, in which \( P \) offers the same fee to both third-party sellers, who then set the same retail price, is without loss of generality under the demand system (1).

To see this, notice that the left-hand side of (2) is increasing in \( f_i, p^*_i \) and \( p^*_e \) (as \( D^i_1(\cdot) < 0 < D^i_2(\cdot) \)), whereas it must be decreasing in \( p_e \) at the optimum (by the SOC). Using the implicit function theorem immediately yields these comparative statics results.

To see that \( p_e \) is increasing in the fees and third-party sellers’ expected prices notice that the left-hand side of (3) is increasing in \( f_i + f_j \) and \( p^*_A + p^*_B \) and decreasing in \( p_e \) (by the SOC) at the optimum. The results then immediately follow from the implicit function theorem.

Formally, if \( f_A = f_B = f^* \) and \( p^*_e = p^*_p \),

\[
D^i\left(p^*, p^*_e + p^*_p\right) + (p^* - f^*)D^i_1(\cdot) + (p^* - f^*)D^i_2(\cdot) > 0,
\]

which, by the SOC, immediately implies that the solutions to the FOCs (5) for \( i = A, B \) must be larger than \( p^* \).

Formally, if \( f_A = f_B = f^* \), for \( p^*_i > p^* \) (for some \( i = A, B \)),

\[
\sum_{i = A, B} f^* D^i_1(\cdot) + D^p\left(p^*_A + p^*_B\right) + p^*_p D^p_1(\cdot) > 0,
\]

as the left-hand side is equal to zero at the non-cooperative equilibrium, which immediately implies that the \( P \)'s optimal price, which solves the FOC (3), must be larger than \( p^*_p \) whenever \( p^*_i > p^* \) (for some \( i = A, B \)).

This result immediately follows from the implicit function theorem, as the left-hand side of the FOC (5) is increasing in \( p^*_e \).

Formally, \( \frac{\partial y(\cdot)}{\partial e} > \frac{\partial y(\cdot)}{\partial i} \) (see Appendix A).

In this respect, these results (and the intuition behind them) are close to those of recent papers showing how the welfare effect of information sharing among retailers (Gaudin, 2019), vertical contract disclosure (Bisceglia, 2022) and wholesale price-parity clauses (Bisceglia et al., 2021) depend on the extent of the double marginalization problem arising in vertical industries under linear contracts.

See, for example: https://www.theverge.com/21445923/platform-fees-apps-games-business-marketplace-apple-google.

Of course, to make things interesting in this setting, firms’ (linear) production costs cannot be normalized to zero, as otherwise \( P \) would always optimally set \( \phi_i = 1 \) (for \( i = A, B \)) so as to extract all third-party sellers’ profits.

The negative effect of \( N \) on the competitive effects of cooperation is magnified under a demand system à la Shubik and Levitan (1980). In that case, while \( P \)'s fees and price are still always lower in the cooperative equilibrium for all \( N \), a larger degree of product substitutability is needed in order for \( p^*_i < p^* \), as \( N \) grows larger (see Supporting Information: Appendix F). The reason is that, since (in a symmetric equilibrium) aggregate demand does not depend on the number of firms (e.g., Motta, 2004), competition in the marketplace becomes much fiercer as \( N \) grows larger, which magnifies the anti-competitive effect of horizontal integration or collusion.

In the non-cooperative regime, the double marginalization problem is more pronounced than in the baseline model. This is because, confronted with a larger fee \( f_i \), unlike under passive beliefs, \( R_i \) believes that also \( R_j \) faces a larger fee, hence is expected to set a larger \( p_j \), which (by strategic complementarity) gives \( R_i \) more incentives to increase its price. Therefore, in this model sellers’ cooperation exacerbates double marginalization to a lesser extent compared to the model under passive beliefs, which in turn gives \( P \) weaker incentives to cut fees and compete more aggressively with cooperating third-party sellers.

Following the literature (Etro, 2021; Hagiu et al., 2022) and in line with industry practices, we assumed that \( P \)'s entry occurs through the introduction of a new product rather than through vertical integration with a third-party seller already present in the marketplace. For a model examining the welfare effects of platform’s vertical integration (taking as given that all sellers act non-cooperatively) in a setting à la Baye and Morgan (2001), see J. Chen (2008). Unlike in that paper, in which vertical integration harms consumers through its impact
on price transparency and prices in the (offline) local markets, in this model it can benefit consumers as it would reduce double marginalization when third-party sellers set prices cooperatively.

Indeed,

\[
\frac{\partial p^*(i)}{\partial f_i} - \frac{\partial p^*(\cdot)}{\partial f_i} = \frac{3\gamma(4 + 5\gamma)}{4(2 + \gamma)(2 + 3\gamma)(2 + 2\gamma - \gamma^2)} > 0 \quad \forall \gamma \in (0, 1).
\]

It is easy to check that the corresponding SOC is satisfied for all \( \gamma \in (0, 1) \).

REFERENCES
SUPPORTING INFORMATION
Additional supporting information can be found online in the Supporting Information section at the end of this article.


APPENDIX A: SUPPLEMENTARY CALCULATIONS

Non-cooperative equilibrium
Solving the FOCs (2) and (3) yields, respectively:

\[
p_i \left( f_i; p_j^e, p_p^e \right) = \frac{1 - \gamma + (1 + \gamma) f_i + \gamma \left( p_j^e + p_p^e \right)}{2(1 + \gamma)},
\]

and

\[
p_p \left( f_i, f_j; p_i^e, p_j^e \right) = \frac{1 - \gamma + \gamma \left( f_i + f_j + p_i^e + p_j^e \right)}{2(1 + \gamma)}.
\]

Hence, \( p_i^* (f_i) = p_i \left( f_i; p_i^*, p_p^e \right) \) and \( p_p^* = p_p \left( f_i, f_j^*; p_i^*(f_i), p_i^* \right) \), which gives

\[
p_i^* (f_i) = \frac{2 + \gamma - 3\gamma^2 + (2 + 4\gamma + 3\gamma^2)f_i + \gamma(2 + 3\gamma)p_i^* + \gamma^2f^*}{(2 + \gamma)(2 + 3\gamma)}.
\]

Similarly, \( p_p^* (f_A, f_B) = p_p \left( f_A, f_B; p_i^*, p_p^e \right) \) for \( p_i^* = p_i^* (f_i) \), which gives

\[
p_p^* (f_A, f_B) = \frac{(1 - \gamma)(2 + 3\gamma)^2 + 6\gamma(1 + \gamma)(f_A + f_B) + 2\gamma f^* + 2\gamma^2(2 + 3\gamma)p_i^*}{2(1 + \gamma)(2 + \gamma)(2 + 3\gamma)}.
\]

Using these strategies, one can solve \( P \)'s optimization problem in \( t = 1 \). It is easy to check that the corresponding SOC is satisfied for all \( \gamma \in (0, \bar{\gamma}) \), with \( \bar{\gamma} \approx 0.95 \) being the unique solution in \( (0, 1) \) of \( 16 + 32\gamma - 18\gamma^3(2 + \gamma) = 0 \). Following the steps detailed in Section 3.1, we find that, when third-party sellers act non-cooperatively, \( P \) charges them unit fees

\[
f^* = \frac{(2 + 3\gamma)(4 + 6\gamma + 3\gamma^2)}{2(2 + \gamma)(4 + 9\gamma + 6\gamma^2)}.
\]

and sells its private label at price

\[
p_p^* = \frac{1}{2} + \frac{2}{5(2 + \gamma)} - \frac{2(2 + \gamma)}{10(4 + 9\gamma + 6\gamma^2)}.
\]

Third-party sellers set price.
\[ p^* = \frac{1}{4} + \frac{1}{2 + \gamma} - \frac{\gamma}{4(4 + 9\gamma + 6\gamma^2)}, \]

with \( p^* > p_p^* \).

**Cooperative equilibrium**

Solving the system of FOCs (5) yields

\[ p_i(f_i; p_p^*) = \frac{1}{2}(1 - \gamma + f_i + \gamma p_p^*), \]

whereas \( P \)’s pricing strategy is \( p_P(f_i, f_j; p_i^c, p_j^c) \) as defined above. Using these strategies it is easy to find the equilibrium prices for any given fees \((f_A, f_B)\):

\[ p_i^c(f_i, f_j) = \frac{2(1 - \gamma)(2 + 3\gamma) + (2 + \gamma)^2f_i + 3\gamma^2f_j}{8 + 4(2 - \gamma)\gamma}, \]

\[ p_P^c(f_A, f_B) = \frac{2(1 - \gamma^2) + 3\gamma(f_A + f_B)}{4 + 2(2 - \gamma)\gamma}, \]

with \( \partial p_i^c(\cdot) \bigg/ \partial f_i > \partial p_P^c(\cdot) \bigg/ \partial f_i > 0 \quad \forall \gamma \in (0, \gamma). \)

Solving the system of \( P \)’s FOCs (6) at the fee-setting stage, using the strategies derived above, yields the equilibrium unit fee

\[ f^c = \frac{4 + 4\gamma + \gamma^3}{2(2 + \gamma)^2}, \]

which, substituted into the pricing strategies, gives the equilibrium prices

\[ p_P^c = \frac{1}{2} + \frac{\gamma(1 - \gamma)}{4 + 4\gamma + \gamma^2}, \]

and

\[ p^c = \frac{3 - \gamma}{4} + \frac{\gamma^2(1 - \gamma)}{4(4 + 4\gamma + \gamma^2)}, \]

with \( p^c > p_P^c \).

**APPENDIX B: PROOFS**

*Proof of Proposition 1.* Comparing equilibrium fees and prices immediately yields

\[ f^c - f^* = -\frac{\gamma(2 + 6\gamma + 7\gamma^2 - 3\gamma^4)}{(2 + \gamma)^2(4 + 9\gamma + 6\gamma^2)} < 0 \quad \forall \gamma \in (0, \gamma), \]

\[ p_P^c - p_P^* = -\frac{\gamma^2(1 + 5\gamma + 6\gamma^2)}{(2 + \gamma)^2(4 + 9\gamma + 6\gamma^2)} < 0 \quad \forall \gamma \in (0, \gamma), \]

and
\[ p^c - p^* = -\frac{\gamma^2(1 + 5\gamma + 6\gamma^2)}{(2 + \gamma)^2(4 + 9\gamma + 6\gamma^2)} < 0 \iff \gamma > \hat{\gamma}, \]

with \( \hat{\gamma} = \frac{10^{1/3} - 1}{3} \approx 0.38 \in (0, \gamma) \).

**Proof of Proposition 2.** Consumer surplus is given by the representative consumer’s utility from which the demand system (1) is obtained:

\[
U(\cdot) \triangleq \sum_{h=A,B,P} q_h - \frac{1}{2} \sum_{h=A,B,P} q_h^2 - \gamma \sum_{h,j=A,B,P,k \neq h} q_h q_j - \sum_{h=A,B,P} p_h q_h,
\]

where \( q_h, h = A, B, P \), are the consumed quantities. Substituting the equilibrium values yields

\[
U^* = \frac{96 + 576\gamma + 1492\gamma^2 + 2156\gamma^3 + 1833\gamma^4 + 2156\gamma^5 + 1833\gamma^6}{8(2 + \gamma)^2(1 + 2\gamma)(4 + 9\gamma + 6\gamma^2)^2}
\]

in the non-cooperative equilibrium, and

\[
U^c = \frac{24 + 56\gamma + 68\gamma^2 + 5\gamma^3(12 + 5\gamma + 2\gamma^2)}{8(2 + \gamma)^4(1 + 2\gamma)}
\]

in the cooperative equilibrium. Taking the difference gives

\[
U^c - U^* = \frac{\gamma(-16 - 38\gamma + 126\gamma^2 + 739\gamma^3 + 1523\gamma^4 + 1710\gamma^5 + 1140\gamma^6 + 450\gamma^7 + 90\gamma^8)}{2(2 + \gamma)^2(1 + 2\gamma)(4 + 9\gamma + 6\gamma^2)^2} > 0 \iff \gamma
\]

\[
> \gamma^{CS},
\]

with \( \gamma^{CS} \approx 0.24 < \hat{\gamma} \).

Each third-party seller’s profits in the two equilibria is given by

\[
\pi^*_R = \frac{(1 - \gamma^2)(2 + \gamma(4 + 3\gamma))^2}{(2 + \gamma)^2(1 + 2\gamma)(4 + 9\gamma + 6\gamma^2)^2}, \quad \pi^c_R = \frac{(1 - \gamma)(1 + \gamma + \gamma^2)^2}{(2 + \gamma)^4(1 + 2\gamma)}
\]

with

\[
\pi^c_R - \pi^*_R = \frac{\gamma(1 - \gamma)(8 + 61\gamma + 210\gamma^2 + 415\gamma^3 + 506\gamma^4 + 384\gamma^5 + 171\gamma^6 + 36\gamma^7)}{(2 + \gamma)^4(1 + 2\gamma)(4 + 9\gamma + 6\gamma^2)^2} > 0 \quad \forall \gamma \in (0, \overline{\gamma}),
\]

thereby sellers always find it optimal to commit, at the outset of the game, to set prices cooperatively.

Total welfare, defined as the unweighed sum of consumer surplus and industry profits \((\pi^c + 2\pi^*_R)\), in the two equilibria is

\[
TW^* = \frac{416 + 2432\gamma + 6076\gamma^2 + 8212\gamma^3 + 6267\gamma^4 + 2514\gamma^5 + 396\gamma^6}{8(2 + \gamma)^2(1 + 2\gamma)(4 + 9\gamma + 6\gamma^2)^2},
\]

and

\[
TW^c = \frac{104 + 232\gamma + 236\gamma^2 + 124\gamma^3 + 35\gamma^4 - 2\gamma^5}{8(2 + \gamma)^4(1 + 2\gamma)}.
\]
Taking the difference gives
\[ TW^c - TW^* = -\frac{3\gamma (16 + 46\gamma - 46\gamma^2 - 395\gamma^3 - 749\gamma^4 - 696\gamma^5 - 324\gamma^6 - 54\gamma^7 + 6\gamma^8)}{2(2 + \gamma)^4(1 + 2\gamma)(4 + 9\gamma + 6\gamma^2)^2} > 0 \iff \gamma > \gamma^{TW},\]

with \( \gamma^{TW} \approx 0.33 \in (\gamma^{CS}, \hat{\gamma}). \) \( \square \)