THE JOURNAL OF INDUSTRIAL ECONOMICS 0022-1821
Volume LXXI March 2023 No. 1

VERTICAL CONTRACT DISCLOSURE IN THREE-TIER INDUSTRIES*

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Consider a three-tier industry with a monopolist supplying a manufacturer which sells its product to final consumers through two retailers. Contracts are linear and secret. Hence, upon receiving an out-of-equilibrium offer, each retailer must form a belief about the identity of the deviating upstream firm. This beliefs' specification problem wipes out if an Open Book Accounting (OBA) policy is implemented, whereby the input price is disclosed to retailers. Under Cournot (Bertrand) competition, OBA increases industry profits and consumer surplus if retailers believe that any out-of-equilibrium offer is more likely to reflect a deviation by the upstream supplier (by the manufacturer).

I. INTRODUCTION

WHEN THE VERTICAL RELATIONS BETWEEN AN UPSTREAM FIRM AND MULTIPLE COMPETING DOWNSTREAM FIRMS are ruled by bilateral secret contracts, the behavior of each downstream firm depends not only on the contractual terms it has negotiated with the upstream firm, but also on its expectations on its competitors' behavior, which are in turn affected by its conjectures (*beliefs*) about the contractual conditions that they are facing. As a consequence, disclosing these vertical contracts affects the equilibrium prices. Therefore, disclosure may be beneficial or harmful to firms and consumers, depending on the nature of downstream competition (Cournot vs. Bertrand competition) and on the contracts' structure (linear contracts vs two-part tariffs), *inter alia*, as shown by Arya and Mittendorf [2011] and Gaudin [2019].

These contributions, as the much wider literature dealing with vertical relations ruled by secret contracts, which stems from the seminal contributions by Hart and Tirole [1990], O'Brien and Shaffer [1992] and

*For many useful comments, I would like to thank Alessandro Bonatti (the editor), two anonymous referees, Germain Gaudin, Jorge Padilla, Salvatore Piccolo, Patrick Rey, Enrico Salonia and Jean Tirole. Open Access Funding provided by Universita degli Studi di Bergamo within the CRUI-CARE Agreement.

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McAfee and Schwartz [1994], consider two-tier supply chains.[1] However, in the real world, supply chains often consist of several tiers, with different numbers of firms competing at each tier (e.g., Mena *et al.* [2013]).[2] Modeling the firms' behavior in such more complex vertical structures allows to analyze a larger set of disclosure standards of vertical contracts, which are often observed in reality, but also brings up novel technical issues.

In this paper, I build a simple model to emphasize both these aspects. Specifically, I consider a three-tier industry, with an upstream supplier, which sells an essential input to a manufacturer whose products reach final consumers through two horizontally differentiated retailers. The retailers compete (choosing quantities or prices) in the same market. Put differently, unlike in the previous literature, the manufacturer's production cost is endogenous, the input price being strategically set by an upstream monopolist. In line with the industry practice in many markets (see, e.g., Gaudin [2017] and references therein), all distribution contracts throughout the supply chain are linear.

In spite of the model's simplicity, when defining a Perfect Bayesian Equilibrium (PBE) of the game under secret contracts, a novel specification problem of retailers' off-equilibrium path beliefs arises. The reason is rather intuitive. Exactly as in the previous literature, upon receiving an out-of-equilibrium offer from the manufacturer, a retailer has to revise its beliefs about the manufacturer's behavior *vis-á-vis* the competing retailer.[3] However, unlike in a two-tier model in which its cost is common knowledge, here the manufacturer's expected behavior depends on whether or not it has in turn faced an out-of-equilibrium offer from the input supplier.

Put differently, in a three-tier model, upon being offered an out-of-equilibrium contract, the contract that a retailer believes its competitor is facing depends on its own beliefs concerning the identity of the deviating upstream player. If the unexpected offer were to reflect a deviation by the manufacturer (only), then the retailer's beliefs on its rival's contract can be specified as in the previous literature. In this case, I consider *wary beliefs* (McAfee and Schwartz [1994], Rey and Vergé [2004]). If, by contrast, the

[1] A notable exception is a recent paper by Bisceglia *et al.* [2021], who study a three-tier industry with vertical relations ruled by secret (linear) distribution contracts, but consider an agency (rather than a wholesale) business model and adopt Contract Equilibrium rather than PBE as solution concept.

[2] Notable examples include the automotive, consumer appliances, electronic equipment, and apparel industries (see, e.g., Corbett and Karmarkar [2001]).

[3] Three different beliefs' specifications have been usually considered in the literature from McAfee and Schwartz [1994] onwards. Upon receiving an out-of-equilibrium contract, a retailer: (i) under *passive beliefs*, believes that its rival is offered the equilibrium contract; (ii) under *wary beliefs*, believes that the manufacturer acts optimally with its rival, given the offer it just received; (iii) under *symmetric beliefs*, believes that the manufacturer offers the same contract to both retailers.

manufacturer itself received an out-of-equilibrium contract from its supplier, then the out-of-equilibrium contract it offered to the retailers may indeed be its *best response* to the unexpected input price. In this case, by symmetry between the retailers, the manufacturer will optimally offer the same (out-of-equilibrium) contract to both of them, so that they should have *symmetric beliefs*. Clearly, when the upstream contract between the input supplier and the manufacturer is kept secret, the retailers cannot distinguish between the two cases. Thus, it is natural to assume that they believe that an out-of-equilibrium offer reflects a deviation by the input supplier with a certain probability and by the manufacturer with complementary probability.[4]

As a consequence, the equilibrium prices and quantities under secret contracts depend on these (subjective) probabilities, which thus play an important role in the welfare comparison of different contract disclosure standards. In this respect, rather than focusing (as Arya and Mittendorf [2011] and Gaudin [2019]) on information sharing between the retailers, I consider *vertical* information sharing. Indeed, vertically related firms often share information about their costs and/or consumers' demand, as recognized by a wide operational research and management literature.[5] In particular, they often share sensitive cost information, which is usually kept secret by any firm. Such sensitive data are provided within an Open Book Accounting (hereafter, OBA) agreement (see, e.g., Agndal and Nilsson [2008], Hoffjan *et al.* [2011], and references therein).[6] Notably, a case study conducted by Alenius *et al.* [2015] shows that firms use OBA in managing the interdependencies with the first-tier supplier and down in the supply chain, thereby influencing second- and third-tier suppliers. In other words, vertical information sharing of cost-related information often takes place in multi-tier industries, whereby

[4] In principle, both upstream players may have deviated from their equilibrium behavior. However, there are good reasons to ignore this possibility. Of course, this belief specification problem wipes out when the manufacturer offers public contracts to the retailers. In this case, disclosing the input price is always welfare neutral. Moreover, most of the paper's insights are robust with respect to the chosen beliefs' specification.

[5] Some contributions specifically consider supply chains with competing retailers: see, e.g., Li [2002] and Li and Zhang [2008], who deal with vertical information sharing concerning firms' demand, and Yao *et al.* [2008], for a model of vertical sharing of cost-related information.

[6] As observed by Kajüter and Kulmala [2005], disclosing cost data to supply chain partners is a practice that appeared with the spread of lean production and supply in the 1990s, and is observed in Japanese as well as in Western companies in many (multi-tier) industries (e.g., electronic components, automotive, food grocery). Of course, in the reality the implementation of OBA is driven by (and serves to) multiple purposes, such as supporting Interorganizational Cost Management measures (so as to lead to cost reductions), facilitating price negotiations, informing various decision-making processes and creating mutual trust and commitment (Agndal and Nilsson [2008], Hoffjan *et al.* [2011]). This model abstracts from all these aspects, focusing only on the consequences of disclosing (endogenous) cost information on firms' pricing behavior, in the spirit of the available IO literature analyzing information sharing in vertical markets.

the relevant information is determined (at least in part) by the strategic choices of firms along the supply chain: for instance, a manufacturer sharing information with its retailers about its inputs' costs is *de facto* disclosing the terms of contracts signed with its suppliers.

Starting from these considerations, this paper sheds light on the welfare effects of OBA policies in rather common situations in which the cost-related information to be disclosed is endogenously chosen in equilibrium. The contribution of the paper is thus twofold. First, it extends the literature on vertical market relations ruled by secret contracts, by considering, more realistically, a multi-tier industry, which brings up novel issues as for the specification of downstream firms' off-equilibrium path beliefs. Second, it provides a stylized model of OBA policies, in which the disclosed cost-related information consists in a contract signed with a third party. More generally, to the best of my knowledge, no previous theoretic model has analyzed vertical information sharing between an upstream and a downstream firm concerning the upstream firm's costs.

The main finding of the paper is that, under Cournot competition, the implementation of OBA increases total industry profit and consumer surplus, as compared to the case in which all contracts are kept secret, if and only if retailers believe that any out-of-equilibrium offer is sufficiently likely to reflect a deviation by the upstream supplier rather than by the manufacturer, whereas the exact opposite result holds under Bertrand competition—that is, the implementation of OBA is total welfare increasing under Cournot competition if and only if it is total welfare decreasing when the retailers compete *á la* Bertrand.[7]

These results are driven by the role played by off-equilibrium path beliefs on the extent of the multiple marginalization problem arising in the industry.[8] Without OBA, each retailer's conjecture about its competitor's contract is more responsive to its own wholesale price under symmetric beliefs than under wary beliefs. This has opposite implications on the severity of the multiple marginalization problem depending on whether the retailers' instruments are strategic substitutes or complements. Suppose a retailer receives an unexpectedly high wholesale price. If it believes that its rival is facing the same (high) wholesale price, under Cournot competition, it expects its rival to order a lower quantity. Then, by strategic substitutability, it is willing to order a higher quantity as compared to the case in which it believes its rival is facing the (lower) equilibrium wholesale price. By contrast, under Bertrand competition, since higher wholesale prices translate into higher

[7] For the sake of tractability, these results are obtained under a linear demand system *á la* Singh and Vives [1984], though the main insights of the analysis are valid in more general settings.

[8] This follows from the assumption of linear contracts. Under two-part tariffs, the implementation of OBA solves the well-known supplier's opportunism problem (Hart and Tirole [1990]), thereby increasing total industry profits while harming consumers.

retail prices and prices are strategic complements, the price set by a retailer is increasing in the wholesale price it believes it has been offered to its rival. Thus, unlike in the Cournot model, under Bertrand competition, symmetric beliefs tend to exacerbate the multiple marginalization problem, as compared to wary beliefs.

By backward induction, when the retailers' instruments are strategic substitutes (complements), under symmetric beliefs (more generally, when the retailers believe that out-of-equilibrium offers are more likely to reflect a deviation by the upstream supplier), the upstream firms are less (more) concerned about multiple marginalization, thereby they find it optimal to charge higher (lower) mark-ups. The higher (lower) input and wholesale prices unambiguously translate into higher (lower) retail (or market clearing) prices. Through the described mechanisms, the social desirability of the implementation of an OBA policy depends on the retailers' beliefs on the identity of the deviating upstream firm.

Notably, the disclosure standard which maximizes industry-wide profits always coincides with the consumers' preferred one. This is not surprising as, due to the multiple marginalization problem, retail prices are always upward distorted compared to the industry-wide monopoly level. However, supposing that, at the outset of the game, the manufacturer efficiently bargains with both retailers over the disclosure standard, there will be under-adoption (over-adoption) of OBA policies compared to the social optimum under Cournot (Bertrand) competition, as the bargaining parties do not internalize the upstream supplier's profit.

Finally, considering three-tier industries yields further novel, rather counterintuitive, results. First, the gain in commitment power *vis-á-vis* the retailers when its offer is disclosed is likely to harm the upstream supplier, through the effect of OBA on the manufacturer's incentives to mark up the input price. Second, under secret contracts, the equilibrium retail price in the examined industry may be lower than the one arising in an industry with two dedicated manufacturers—that is, an upstream horizontal merger, despite eliminating competition between manufacturers, may be pro-competitive (absent efficiencies). Both these results hold true, when the retailers compete *á la* Bertrand, under certain conditions, which, again, involve the retailers' beliefs.

The article is organized as follows. After reviewing the related literature, I set up the model in Section II. The equilibrium analysis and the welfare results under Cournot and Bertrand downstream competition are shown in Section III and IV, respectively. Several implications and extensions of the baseline analysis are discussed in Section V. In Section VI, I consider upstream competition and an horizontal merger between manufacturers. Section VII concludes. The proofs of the main results and a microfoundation for the off-equilibrium path beliefs' specification are in the Appendix. Additional material is contained in the Online Appendix, available at the *Journal*'s editorial web site.

I(i). *Related literature*

In focusing on vertical relations ruled by bilateral secret contracts, this paper is related to the literature pioneered by Hart and Tirole [1990], O'Brien and Shaffer [1992] and McAfee and Schwartz [1994], who have shown the supplier's opportunism problem: a manufacturer who distributes its products through more competing retailers is not able to achieve the industry-wide monopoly outcome, even using nonlinear contracts, when retailers cannot observe their rivals' contracts and have passive or wary beliefs, as the manufacturer's incentives to choose each contract to maximize bilateral profits yields inefficiently low retail prices. As a consequence, under two-part tariffs, contract disclosure before downstream competition takes place solves this opportunism problem, thereby leading to higher prices.

This need not always be the case under linear contracts: as shown by Gaudin [2019], wholesale and retail prices are lower under public contracts when retailers compete *á la* Bertrand.[9] The key driver of this result rests in the double marginalization problem being more severe under public contracts, when retailers' instruments are strategic complements.[10] Similarly, in this paper, retailers' beliefs which, other things being equal, make the multiple marginalization problem more severe, induce a disciplining effect on the upstream firms, thereby leading to a more efficient equilibrium outcome (i.e., to lower wholesale and retail prices). However, unlike in the previous literature, in the three-tier industry considered in this model these beliefs crucially depend on the relative probability that the retailers assign to an (eventually observed) out-of-equilibrium offer to come as a result of the manufacturer's rather than the upstream supplier's deviation.[11]

Notably, the adopted specification of off-equilibrium path beliefs absent OBA can be interpreted as introducing *mixed beliefs*,[12] which have already

[9] The same result holds true, but under Cournot competition, in industries with competing vertical chains: see Arya and Mittendorf [2011]. While the mentioned papers, as the present one, examine the effects of vertical contract disclosure, Montez and Schutz [2021] focus on the role of disclosing the inventory levels chosen by retailers before engaging in price competition.

[10] The reason is that, when each retailer can observe that a large wholesale price is being charged also to its rivals, it has strong incentives to pass it on to consumers, by the strategic complementarity between retail prices. By contrast, under secret contracts, unless it holds symmetric beliefs, a retailer facing an unexpectedly large wholesale price believes that its competitors are facing better deals, which weakens its incentives to charge a large retail price. Anticipating this, the manufacturer optimally lowers its wholesale prices under public contracts as compared to the game in which contracts are kept secret (so to avoid inefficiently reducing sales), which in turn translates into lower retail prices as well.

[11] Of course, another significant difference between this paper and Gaudin [2019] is that the latter focuses on the welfare effects of disclosing to each retailer contracts that the manufacturer has signed with its competitors, rather than with an upstream supplier.

[12] Mixed beliefs are consistent with the experimental evidence in Martin *et al.* [2001], who conclude that "the existing models in which it is common knowledge that players' out-of-equilibrium

been considered by Pagnozzi and Piccolo [2011], in a model with two competing supply chains, each composed by an upstream firm (manufacturer) and a downstream firm (retailer), and secret contracts. In their setting, mixed beliefs are compatible with the definition of PBE, in particular, with the *no signaling what you don't know* condition (Fudenberg and Tirole [1991]), only if the upstream firms are privately informed about their correlated marginal costs. By contrast, in a multi-tier industry, mixed beliefs are compatible with this condition in the presence of *imperfect information* due to secret contracting only—that is, even without introducing *incomplete information* concerning exogenous parameters.

A beliefs' specification problem which is closer to the one arising in this model (absent OBA) is present in a few contributions that, stemming from the seminal work by Janssen and Shelegia [2015], combine the literature streams on consumer search and vertical contracting in two-tier industries. In particular, Janssen and Shelegia [2020] consider a consumer search problem in which each consumer, upon visiting a retailer and observing its price, has to decide whether to buy its product or instead sink a search cost to visit a competing retailer before taking her purchase decision. Since the retailers are supplied by a common manufacturer, assuming that consumers do not observe the wholesale contracts, upon observing an out-of-equilibrium price, their decision as to whether or not to incur the search cost depends on their belief about the price set by the other retailer, which, as in this model, in turn depends on their belief regarding the identity of the deviating player along the supply chain. Accordingly, they consider a specification for the expected price charged by the other retailer equivalent to the one considered in this paper. However, they consider symmetric rather than wary beliefs when the deviation is attributed to the manufacturer, as they assume that a common two-part tariff must be offered to both retailers.[13]

Indeed, wary beliefs coincide with passive beliefs under Cournot competition (see, e.g., Nocke and Rey [2018]) but, as argued by McAfee and Schwartz [1994] and Rey and Vergé [2004], they are more compelling under Bertrand competition.[14] A strong game-theoretic justification for wary beliefs is provided by In and Wright [2018]. They study a class of *endogenous signaling games*, in which a sender (in this model, the manufacturer) takes more simultaneous actions and different receivers (retailers) have different information on the sender's actions. They construct *reordered games* in which

beliefs are homogeneous may not be sufficiently rich to capture the complicated strategic environment in practical settings".

[13] Within a similar framework, linear contracts are considered by Janssen [2020], who considers both passive and symmetric consumers' beliefs, but, as all the other contributions within this stream of the literature, does not allow for any kind of *mixed beliefs*.

[14] Wary beliefs have been considered also by Avenel [2012] and Miklós-Thal and Shaffer [2016], though in different environments.

observed actions are modeled as if they are chosen before unobserved actions, and find that the PBE with wary beliefs in the vertical contracting game is a sequential equilibrium satisfying *reordering invariance*—that is, each retailer's strategy and beliefs at all of its non-singleton information sets are also a part of a sequential equilibrium in every reordered game of the original game.

II. THE MODEL

In this section, I show the set-up of the baseline model (Section II(i)), and carefully discuss the specification of retailers' off-equilibrium path beliefs (Section II(ii)).

II(i). *Set-up*

Markets and players. Consider an upstream supplier (S), selling an essential input (e.g., a raw material or an intermediate good) to a monopolist manufacturer (M). The manufacturer produces a final good which is distributed through two retailers (R_i, $i = 1, 2$). The industry vertical structure is represented in Figure 1. The goods sold by the retailers are perceived as horizontally differentiated by final consumers.[15] Following the literature and consistently with most real-world *brick and mortar industries* (e.g., Corbett and Karmarkar [2001]), I assume that the wholesale (or resale) business model is adopted in the considered (three-tier) supply chain.

Contracts and payoffs. Throughout the baseline analysis, I consider linear distribution contracts.[16] Outside options' values of each firm, if distributions contracts are not signed, are normalized to zero. Firms' payoffs are specified as follows.

Input supplier. The input supplier S produces at a constant marginal cost, normalized to zero, and supplies the manufacturer according to a linear contract specifying a price $t \geq 0$ for every unit purchased by M. Thus, S's profit is simply

$$\pi^S(t) \triangleq tq,$$

with q denoting the quantity purchased by M.

Manufacturer. The manufacturer needs one unit of input purchased from S (and other inputs, whose cost is common knowledge and normalized to zero) to produce one unit of the final good. Denoting by q_i the quantity of M's

[15] This is the case either because retailers are *per se* differentiated (e.g., they offer different ancillary products or services) or because M produces at the same cost (using the same input, purchased by S) two differentiated products, and each of them is distributed by one retailer.

[16] The analysis under two-part tariffs contracts is summarized in Section V(iv).

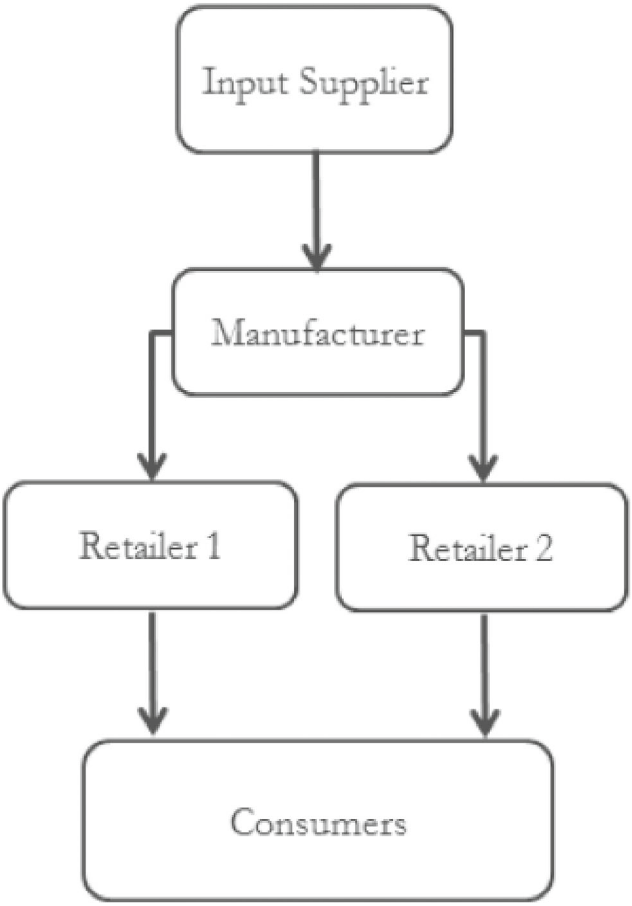

FIGURE 1
Industry Vertical Structure

product purchased by each retailer R_i, $i = 1, 2$, at a unit wholesale price $w_i \geq 0$, M's profit is

$$\pi^M(w_1, w_2, t) \triangleq \sum_{i=1,2} (w_i - t)q_i.$$

Hence, the total quantity sold by M is $q = q_1 + q_2$.

Retailers. Two horizontally differentiated retailers compete in the product market to attract consumers. I consider both quantity and price competition. Under quantity competition, each R_i, $i = 1, 2$, orders a quantity q_i from M, which is then sold to final consumers at the market clearing price p_i. By contrast, under price competition, each retailer sets a retail price p_i at which its consumers can buy M's product, and then orders from M the demanded quantity q_i.[17] Normalizing to zero the retailing costs, R_i's profit is thus given by

$$\pi_i^R(p_i, w_i) \triangleq (p_i - w_i)q_i.$$

Retailers are risk neutral: R_i maximizes its expected profit, the expectation being taken with respect to its conjecture on its rival's marginal cost w_{-i} upon observing w_i and eventually t (more below).

Consumers' demand. For the sake of tractability, in what follows, I consider the following linear-quadratic specification of the consumers' utility function *á la* Singh and Vives [1984]:[18]

$$U(q_1, q_2) \triangleq \sum_{j=1,2}\left(q_j - \frac{1}{2}q_j^2\right) - \gamma q_1 q_2 - \sum_{j=1,2} p_j q_j + M,$$

where $M > 0$ is the utility from income and $\gamma \in (0, 1)$ is an inverse measure of product differentiation.[19] Optimization yields the following linear inverse demand functions

$$P(q_i, q_{-i}) \triangleq 1 - q_i - \gamma q_{-i},$$

from which it is easy to obtain the direct demand functions

$$Q(p_i, p_{-i}) \triangleq \frac{1}{1+\gamma} - \frac{1}{1-\gamma^2}p_i + \frac{\gamma}{1-\gamma^2}p_{-i}.$$

Contract disclosure and timing. At the outset of the game, an OBA policy can be introduced, whereby the manufacturer's cost, namely the unit input price t it pays for purchases from S, will be disclosed to both retailers. The disclosure standard is common knowledge.[20] Then, the game unravels as follows:

1. The input supplier sets the input price t.
2. The manufacturer sets the wholesale prices w_i, $i = 1, 2$.
3. Upon observing its own wholesale price w_i,[21] and also t if OBA has been implemented, each retailer takes its quantity or price setting decision.
4. Consumers allocate their demand and firms' profits realize.

[17] The equilibria under Cournot and Bertrand competition are compared in Section V(i).

[18] In Section V(v), I argue that the qualitative results are robust with respect to the demand specification.

[19] It can be easily checked that all the second-order conditions of firms' optimization problems are satisfied under this linear demand specification, though, to simplify the exposition, they are omitted throughout.

[20] See Section V(ii) for a discussion of these assumptions and the analysis of endogenous OBA adoption.

[21] The implications of information sharing between retailers are discussed in Section V(iii).

Equilibrium concept. The game is dynamic and there is imperfect information, due to the presence of secret contracts along the supply chain. Accordingly, following the literature, solution concept will be PBE.

As for retailers' off-equilibrium path beliefs, I consider *wary beliefs* (McAfee and Schwartz [1994], Rey and Vergé [2004])—that is, upon receiving an unexpected offer, each retailer believes that M acts optimally with its rival, given the offer it just received.[22] As detailed below, in the game without vertical information sharing, a full specification of the off-equilibrium path beliefs of a downstream firm requires to specify its conjectures concerning the identity of the deviating upstream firms along the supply chain. In this respect, throughout the analysis, I maintain the following assumption.[23]

Assumption (A). In the game without OBA, it is common knowledge that each retailer believes that an out-of-equilibrium offer $w_i \neq w^*$ reflects a deviation by S with probability $\varphi \in [0, 1]$ and a deviation by M with complementary probability $1 - \varphi$.

In Sections III and IV, I carry out the equilibrium analysis in the two considered disclosure standards under Cournot and Bertrand competition, respectively. Throughout the analysis I restrict attention to symmetric PBE in which: (i) S sets an input price $t^*_{k,j}$; (ii) M offers the same wholesale price $w^*_{k,j}$ to both retailers; (iii) retailers in turn charge the same final price $p^*_{k,j}$. The subscript $k = 1$ ($k = 0$) refers to the equilibrium in the presence (in the absence) of OBA, whereas the subscript $j = C, B$ refers to Cournot and Bertrand competition. It can be easily checked that the restriction to symmetric equilibria is without loss of generality in the considered linear demand setting.[24] For convenience, the comparison between equilibrium prices is summarized in Table I.

Before moving forward to the equilibrium analysis, a thorough discussion on the beliefs' specification problem and Assumption (A) is presented in Section II(ii) below.

II(ii). *Off-equilibrium path beliefs*

In the game in which the input price t is disclosed to the retailers, any contract offered by S defines a proper subgame, in which wary beliefs can be

[22] As discussed in Section I(i), a strong game-theoretic justification for considering wary beliefs in this game is provided by In and Wright [2018].

[23] Robustness of the qualitative results with respect to the beliefs' specification is discussed in Section V(vi).

[24] More precisely, given the specification of off-equilibrium path beliefs (more below), in each disclosure scenario, under Cournot competition, the game admits a unique PBE, which is a symmetric equilibrium. Similarly, under Bertrand competition, the unique PBE within the class of polynomial strategies is symmetric.

TABLE I
COMPARISON OF EQUILIBRIUM PRICES.

	$\varphi \in \left(0, \frac{1}{3}\right)$	$\varphi \in \left(\frac{1}{3}, 1\right)$
Cournot competition	$t^*_{0,C} < t^*_{1,C}$ $w^*_{0,C} < w^*_{1,C}$ $p^*_{0,C} < p^*_{1,C}$	$t^*_{0,C} < t^*_{1,C}$ $w^*_{0,C} > w^*_{1,C}$ $p^*_{0,C} > p^*_{1,C}$
Bertrand competition	$t^*_{0,B} > t^*_{1,B}$ $w^*_{0,B} > w^*_{1,B}$ $p^*_{0,B} < p^*_{1,B}$	$t^*_{0,B} > t^*_{1,B}$ $w^*_{0,B} > w^*_{1,B}$ $p^*_{0,B} < p^*_{1,B}$

specified as in the previous contributions modeling two-tier industries. Formally, for every offer w_i received by M, and for every (observed) value of t, R_i believes that its rival has been offered $w^e_{-i} = \mathcal{W}_{-i}(w_i, t)$, where $\mathcal{W}_{-i}(w_i, t) \triangleq \arg\max_{w_{-i} \geq 0} \pi^M(w_i, w_{-i}, t)$.

By contrast, in the game without vertical information sharing, the three-tier vertical structure complicates the specification of off-equilibrium path beliefs, as compared to the previous literature.[25] The reason is as follows. When a retailer R_i receives an out-of-equilibrium offer $w_i \neq w^*_i$, it faces the problem of conjecturing which player has deviated along the supply chain: does the unexpected offer reflect a deviation by M or by S? The answer has fundamental implications on the beliefs of R_i concerning the offer received by its competitor, since w^e_{-i} depends on the input price t, which is not observed by the retailers. Formally, $w^e_{-i} = \mathcal{W}_{-i}(w_i, t^e)$, where t^e is the value of the input price conjectured by R_i upon observing w_i.

If R_i believes that, despite having received an out-of-equilibrium offer, the input supplier has offered the equilibrium contract t^* to M (i.e., the deviating player along the supply chain is M), then its belief on its competitor's contract is given by $w^e_{-i} = \mathcal{W}_{-i}(w_i, t^*)$. If, by contrast, R_i believes that the out-of-equilibrium offer reflects a deviation by S (i.e., $t \neq t^*$), then its belief on w_{-i} is given by $w^e_{-i} = \mathcal{W}_{-i}(w_i, t^e)$, where now the value for $t^e \neq t^*$ is characterized as follows. Let $w_i = w^*_i(t)$ denote M's optimal offer w_i as a function of the input price t—that is, $w^*_i(t) \triangleq \text{argmax}_{w_i} \pi^M(w_i, w_{-i}, t)$. Thus, if R_i believes that w_i is M's best reply to its input price (i.e., $w_i = w^*_i(t)$), it follows that R_i's belief on the input price faced by M is $t^e = w^{*-1}_i(w_i)$.[26] In words, R_i believes that its rival has been offered by M the optimal contract (given w_i) when the input price is t^e—that is, $w^e_{-i} = \mathcal{W}_{-i}(w_i, t^e)$, with $t^e = w^{*-1}_i(w_i)$.

Notice that, since retailers are symmetric, one can restrict attention to equilibria in which, for any t, M optimally offers the same wholesale price

[25] This specification problem is not specific to the case in which the solution concept is PBE. In particular, it also arises when considering Contract Equilibrium (see the Online Appendix).

[26] Notice that this inverse function is always well-defined since, under the linear demand specification, all equilibrium strategies turn out to be affine functions.

$w_1^*(t) = w_2^*(t) = w^*$ to both retailers.[27] The reason is simple: a deviation by S implies a cost shock in the production function of M, which M finds it optimal to pass through to the same extent to both (symmetric) retailers. Accordingly, if the out-of-equilibrium wholesale price is attributed to a deviation by S, *symmetric beliefs* arise as a particular case of wary beliefs: formally, $w_{-i}^e = \mathcal{W}_{-i}(w_i, t^e) = w_i$ for every t^e.

Moreover, when downstream firms compete in a Cournot fashion, it is well known (e.g., Rey and Vergé [2004], Nocke and Rey [2018]) that wary beliefs coincide with *passive beliefs*. To be more precise, in this three-tier industry, if R_i conjectures that the out-of-equilibrium offer reflects a deviation by M, it then believes that its rival remains under the equilibrium contract w^*—that is, under Cournot competition, $w_{-i}^e = \mathcal{W}_{-i}(w_i, t^*) = w^*$ for every w_i.[28] This is not the case when the retailers compete *á la* Bertrand.

In principle, retailers could believe that an out-of-equilibrium offer reflects deviations by both upstream firms. Yet, the beliefs' specification given in Assumption (A) implies that each R_i considers multiple deviations along the supply chain as zero-probability events. A formal microfoundation, contained in Appendix B, shows that the *belief parameter* φ can be seen as a measure of the relative probability of each upstream firm deviating from the equilibrium behavior, thereby it reflects the retailers' assessment of the (relative) reliability of the upstream firms' pricing decision processes. Alternatively, φ can be interpreted as a measure of the relative probability of S and M facing an exogenous (zero-probability) cost shock in the production process: see the Online Appendix for a formal microfoundation. From a methodological viewpoint, the most important reason why it is worth positing $\Pr[t \neq t^*, w_i \neq w_i^*(t) | w_i \neq w^*] = 0$ is that, if R_i were to believe that an out-of-equilibrium offer $w_i \neq w^*$ reflects deviations of both upstream firms, then it would not be able to form a (point) belief on the offer received by its competitor, which would greatly complicate the analysis.

Notice that Assumption (A) also requires that (i) retailers have identical beliefs; and (ii) these beliefs are common knowledge. These are very standard assumptions in the literature. However, in this three-tier model, (i) amounts to impose that also the value of the *belief parameter* φ is the same for both the retailers, which is indeed reasonable, since in this model they are fully symmetric. Analogously, (ii) entails that the value of φ is common knowledge. Although perhaps more disputable, this assumption can be justified in view of the above-mentioned interpretations behind the parameter φ, which suggest that its value depends on markets' and/or firms' organizational features

[27] The restriction to symmetric equilibria is maintained in all the previous literature. Moreover, for any observed input price, it is with no loss of generality under the considered linear demand specification (see, e.g., Gaudin [2019]).

[28] Similarly, when OBA is implemented, for all w_i and t: $w_{-i}^e = \mathcal{W}_{-i}(w_i, t) = w^*(t)$, where $w^*(t)$ denotes the (symmetric) SPNE wholesale price (given t).

that may well be common knowledge. In addition, it is not easy to come up with a compelling way of relaxing this assumption—that is, of introducing asymmetric information about the *belief parameter* φ. The reason is that, by assuming that the retailers are privately informed about the value of φ, one should then consider the upstream firms' beliefs on its value, which requires to specify a prior distribution from which (the upstream firms believe) φ is drawn. However, to maintain tractability, one would need to assume that such a distribution is common knowledge, which is a disputable assumption as well.

III. QUANTITY COMPETITION

In this section, I characterize the equilibrium of the model under Cournot competition, first with OBA (Section III(i)), and then when all contracts are kept secret (Section III(ii)). The welfare comparison between the two disclosure standards is carried out in Section III(iii).

III(i). *Equilibrium with OBA*

When OBA is in place, at the last stage of the game, after observing t and w_i, R_i's problem is as follows:

$$\max_{q_i \geq 0} q_i(P(q_i, q^e_{-i}(t)) - w_i),$$

where $q^e_i(t)$ is the quantity R_i believes its rival will order (more below). The (standard) first-order condition (henceforth, FOC) for the above problem,

$$P(q_i, q^e_{-i}(t)) + q_i P_1(q_i, q^e_{-i}(t)) = w_i,$$

which equates R_i's (expected) marginal revenue with its marginal cost, yields R_i's best-response function, which is clearly decreasing in the wholesale price w_i and, by strategic substitutability between the retailers' instruments, decreasing in q^e_{-i} as well:

$$q^*_i(w_i, t) \triangleq \frac{1}{2}(1 - \gamma q^e_{-i}(t) - w_i).$$

Moving backward to the previous stage, the manufacturer sets the wholesale prices solving the following maximization problem:

$$\max_{w_1, w_2 \geq 0} \sum_{i=1,2} (w_i - t) q^*_i(w_i, t),$$

whose FOC with respect to w_i,

$$q^*_i(w_i, t) + (w_i - t)\frac{\partial q^*_i(w_i, t)}{\partial w_i} = 0,$$

does not contain w_{-i}. As a consequence, for any w_i offered to R_i, M finds it optimal to offer the SPNE contract, denoted by $w^*(t)$, to the other retailer R_{-i}: this shows why, when downstream firms compete *á la* Cournot, wary beliefs coincide with passive beliefs (e.g., Rey and Vergé [2004], Nocke and Rey [2018]). Put it another way, M faces, *vis-á-vis* each retailer, the problem of a monopolist (serving final consumers and) facing the demand function $q_i^*(\cdot)$. Thus, $w^e_{-i} = w^*(t)$, thereby $q^e_{-i}(t) = q^*(t)$, with $q^*(t)$ denoting the quantity ordered by each retailer in the SPNE, where the SPNE strategies $w^*(\cdot)$ and $q^*(\cdot)$ have to be determined.

The above FOC defines M's best reply to the input price charged by S:

$$w_i^*(t) \triangleq \frac{1}{2}(1 - \gamma q^e_{-i}(t) + t).$$

It can be easily seen that, given the linear specification of the demand, if its offers to the retailers were publicly observed, M would find it optimal to pass through the input price at a constant rate $\frac{1}{2}$. However, under secret contracts between M and each R_i, the pass-through rate turns out to be larger. This is because a higher value of t, observed by R_i, shifts upwards its belief on the wholesale price faced by its competitor, whereby $q^e_{-i}(t)$ decreases. All else equal, by strategic substitutability, this induces R_i to order a higher quantity, which makes the multiple marginalization problem less severe, thereby allowing M to increase its pass-through rate. Then, by a backward induction logic, knowing that its offer t will be disclosed to the retailers, when fixing the input price, S takes into account that the value of t affects the retailers' behavior not only through its direct effect on the wholesale prices charged by M but also through its impact on their beliefs on their competitors' contracts and choices: a *belief effect*.

Formally, using the best-response functions derived above and imposing symmetry,[29] it is easy to obtain the SPNE strategies:

$$w^*(t) \triangleq \frac{2 + (2+\gamma)t}{4+\gamma}, \qquad q^*(t) \triangleq \frac{1-t}{4+\gamma}.$$

Finally, using $q^*(t)$, one can easily solve the input supplier's problem—that is, find $t^*_{1,C} = \text{argmax}_{t \geq 0} 2t\, q^*(t)$. The equilibrium outcome can then be summarized as follows.

Lemma 1. In the presence of OBA, the input supplier sets $t^*_{1,C} = \frac{1}{2}$. Each retailer purchases M's product at a wholesale price $w^*_{1,C} = \frac{6+\gamma}{2(4+\gamma)}$ and sells to final consumers a quantity $q^*_{1,C} = \frac{1}{2(4+\gamma)}$. The resulting market clearing price is $p^*_{1,C} = \frac{7+\gamma}{2(4+\gamma)}$.

[29] That is, $q_i^*(w_i^*(t), t)|_{q^e_{-i}(t)=q^*(t)} = q^*(t)$ and $w_i^*(t)|_{q^e_{-i}(t)=q^*(t)} = w^*(t)$.

The disclosure of its offer gives the input supplier a commitment power *vis-á-vis* the retailers. Specifically, in the linear demand setting, contract disclosure enables S to sell the input at the price that would be set by a (vertically integrated) multi-product monopolist. Nonetheless, S still makes less profits as compared to a vertically integrated monopolist because of the multiple marginalization problem, whereby the aggregate quantity sold in the final market is inefficiently low.

III(ii). *Equilibrium without OBA*

I now turn to the case in which all contracts are kept secret. Assumption (A) implies that, in this scenario, for every offer w_i received by M, a retailer believes that its rival has been offered the same contract (i.e., w_i) with probability φ, and the equilibrium contract (i.e., $w^*_{0,C}$) with complementary probability. In the former case, R_{-i} is thus expected to best-reply to w_i, selling a quantity $q^S \triangleq q^*(w_i)$, whereas in the latter case it is expected to order the equilibrium quantity $q^*_{0,C}$. Accordingly, upon observing w_i, R_i's expected profit is

$$\mathbb{E}[q_i(P(q_i, q_{-i}) - w_i)|w_i] = q_i(\varphi P(q_i, q^S) + (1-\varphi)P(q_i, q^*_{0,C}) - w_i).$$

Profit maximization yields the following FOC:

$$(1)\quad \varphi[P(q_i, q^S) + q_i P_1(q_i, q^S)] + (1-\varphi)[P(q_i, q^*_{0,C}) + q_i P_1(q_i, q^*_{0,C})] = w_i,$$

which, equating R_i's (expected) marginal benefit and the marginal cost of increasing q_i, defines R_i's best reply as a function of the wholesale price w_i and the quantities that its rival is expected to sell. Notice that, for any $\varphi > 0$, a change in w_i not only has a direct effect on R_i's marginal cost, but also entails a *belief effect*, since it affects R_i's expected market price. Specifically, since R_{-i}'s optimal quantity is a decreasing function of w_{-i}, by the logic of symmetric beliefs, a higher wholesale price w_i received by M induces R_i to believe that, for every quantity q_i it decides to sell, the market price will be higher.[30] In other words, this *belief effect* induced by an increase in w_i (which is stronger for higher values of φ) points in the direction of increasing R_i's marginal revenue, which in part offsets the increase in the marginal cost, thereby leading it to expand its quantity as compared to the case with $\varphi = 0$ (i.e., with passive beliefs). Formally, imposing symmetry (in q^S), equation (1) yields

$$q^*(w_i) \triangleq \frac{1 - (1-\varphi)\gamma q^*_{0,C} - w_i}{2 + \varphi\gamma}.$$

[30] This is because, with probability φ, R_{-i} is also facing a higher wholesale price, thereby its expected ordered quantity is lower.

Hence, due to the mentioned *belief effect*, as φ increases, R_i's best-reply is less responsive to the wholesale price w_i. Not surprisingly, this *belief effect* is more pronounced if market competition is fiercer (i.e., if γ is larger), given that the strategic substitutability between the retailers' instruments is stronger if their products are closer substitutes.

Moving backward to the previous stage, the manufacturer's problem is as in the scenario with OBA, once the difference in the retailer's behavior is accounted for. Maximizing M's profit with respect to w_i yields

$$w^*(t) \triangleq \frac{1}{2}(1-\gamma(1-\varphi)q^*_{0,C}+t). \tag{2}$$

Notice that, unlike in the game with OBA, here (given that $q^*_{0,C}$ does not depend on t) it holds $\frac{\partial w^*(\cdot)}{\partial t}=\frac{1}{2}$. This is because, when its offer is not disclosed to the retailers, S's choice of the input price cannot directly impact the retailers' beliefs,[31] thereby t is passed through to the extent of a simple cost parameter.

Next, substituting (2) into R_i's best-reply gives

$$q^*(w^*(t)) = \frac{1-\gamma(1-\varphi)q^*_{0,C}-t}{2(2+\varphi\gamma)}.$$

The trade-off shaping the input supplier's choice of t is the standard one: a higher input price has a direct positive revenue effect, but also reduces equilibrium quantities. However, this latter effect, which refrains S from setting a high input price, is weaker when φ is relatively large, since in this case R_i's optimal quantity is less responsive to the wholesale price, whereas M's pass-through rate does not depend on t. Solving S's problem and imposing the equilibrium condition $q^*(w^*(t^*_{0,C}))=q^*_{0,C}$, it is easy to obtain what follows.

Lemma 2. In the absence of OBA, the input supplier sets a price $t^*_{0,C}=\frac{2(2+\varphi\gamma)}{8+(1+3\varphi)\gamma}$. Each retailer purchases the product at a wholesale price $w^*_{0,C}=\frac{3(2+\varphi\gamma)}{8+(1+3\varphi)\gamma}$ and sells a quantity $q^*_{0,C}=\frac{1}{8+(1+3\varphi)\gamma}$. The resulting market clearing price is $p^*_{0,C}=\frac{7+3\varphi\gamma}{8+(1+3\varphi)\gamma}$.

[31] Indeed, t affects the retailers' beliefs only indirectly, that is through its impact on the wholesale prices consequently offered by M, as expressed by (2), and provided that $\varphi<1$. Roughly speaking, without OBA the *belief effect* is under the control of M (rather than of S, as it happens with OBA). However, the result that the pass-through rate $\frac{\partial w^*(\cdot)}{\partial t}$ does not depend on φ and coincides with the pass-through rate under public contracts is specific to the linear demand setting (see Section V(v)).

Due to the *belief effect* discussed above, input and wholesale prices are increasing in φ. This is because, as φ increases, both upstream firms know that a larger wholesale price w_i makes R_i relatively more aggressive, because it believes that it is more likely that its competitor also faces a large marginal cost. Hence, ordered quantities are less elastic to wholesale prices when φ is larger. As a consequence, M can charge a larger mark-up,[32] and also S, knowing that t affects retailers' behavior only through its impact on the wholesale price, can increase the input price. These higher input and wholesale prices then unambiguously translate into lower ordered quantities in equilibrium. Finally, notice that $t^*_{0,C}|_{\varphi=1} = t^*_{1,C}$. Taken together, these observations imply that, for all $\varphi < 1$, $t^*_{0,C} < t^*_{1,C}$, which will be key to the welfare results shown in the next section.

III(iii). *Welfare*

The foregoing analysis has established that (i) absent OBA, as φ grows larger, prices along the supply chain increase, which ultimately leads retailers to sell a lower quantity, thereby harming consumers; and (ii) in the presence of OBA, the input supplier is able to set a higher price, which, *ceteris paribus*, is detrimental to consumers. However, this anticompetitive effect of contract disclosure is less relevant when φ is large (recall that $t^*_{0,C} \to t^*_{1,C}$ as $\varphi \to 1$). Moreover, the multiple marginalization problem arising under linear contracts harms the industry as a whole as well as final consumers. Taken together, these facts imply that a higher value of the belief parameter φ unambiguously makes the implementation of OBA more attractive from consumers' and industry-wide profit maximization viewpoint. Yet, firms at different level of the supply chain have rather conflicting preferences as far as the choice of the disclosure standard is concerned. In particular, the following welfare results hold.

Proposition 1. When the retailers compete á la Cournot, the implementation of OBA increases total industry profits and consumer surplus if and only if $\varphi > \frac{1}{3}$.[33] Moreover:

- The input supplier always benefits from OBA;
- The manufacturer is always worse off when OBA is in place;

[32] Indeed, $w^*_{0,C} - t^*_{0,C} = \frac{2+\gamma\varphi}{8+\gamma(1+3\varphi)}$ is increasing in φ.

[33] Notably, under the considered linear demand system, the social desirability of the implementation of OBA does not depend on the degree of product differentiation, the threshold $\varphi = \frac{1}{3}$ being independent on γ. Indeed, this result carries over to a more general oligopoly model, with N symmetric retailers competing *á la* Cournot. Specifically, considering a linear demand system $p_i = 1 - q_i - \gamma \sum_{j \neq i} q_j$, for every $N \geq 2$ and $\gamma \in (0, 1)$, $\varphi > \frac{1}{3}$ is a necessary and sufficient condition in order for the implementation of OBA to be welfare increasing. Proofs are available upon request.

- The retailers are better off if OBA is implemented if and only if $\varphi > \frac{1}{3}$.

The intuition behind these results can be more easily understood considering the two extreme cases—that is, $\varphi \in \{0, 1\}$. If $\varphi = 0$, since the retailers hold passive beliefs regardless of t being observable or not, OBA has no effect on the retailers' and M's behavior, but it does alter S's behavior. In fact, with OBA the downstream quantity is less responsive to an increase in t than without OBA. This is because, in the latter case, each R_i cannot update the equilibrium quantity it expects its rival to set in response to an increase in t (and, consequently, in w_{-i}), thereby it reduces quantity to a larger extent than it would do if the increase in t were observed. Thus, absent OBA, the upstream supplier's incentives to increase t are weakened, hence $t^*_{0,C}|_{\varphi=0} < t^*_{1,C}$. Accordingly, wholesale and retail prices are also lower without OBA. As a consequence, S benefits from OBA, whereas M, retailers and consumers are clearly harmed.

By contrast, if $\varphi = 1$, since the retailers' beliefs are symmetric absent OBA, their behavior is the same in both disclosure scenarios, given that the input price does not enter R_i's belief on w_{-i} (i.e., $w^e_{-i}(\cdot)|_{\varphi=1} = w_i$). However, unlike in the previous case, M's behavior is different depending on whether OBA is in place (alongside passive beliefs) or not (alongside symmetric beliefs). This is because, if M were to charge a higher w_i, with symmetric beliefs downstream quantity would shrink less, which gives M stronger incentives to charge a higher mark-up. Under the considered linear demand specification, this higher mark-up does not induce a change in the input price: $t^*_{0,C}|_{\varphi=1} = t^*_{1,C}$. Therefore, retail prices are lower with OBA, due to the lower mark-up chosen by M. As a consequence, OBA makes consumers, S and retailers better off, while harming M.

Thus, when the retailers' instruments are strategic substitutes, S always benefits from the commitment power it gains *vis-á-vis* the retailers when its offer to M is publicly observed, since the higher input price it is able to charge when OBA is in place forces M to lower its mark-up,[34] which also explains why M is always harmed by the implementation of OBA. Also notice that the retailers' interests over the disclosure standard are perfectly aligned with those of consumers. This is because wholesale prices are lower when OBA is in place if and only if $\varphi > \frac{1}{3}$ (see Table I): in this case, the welfare-decreasing effect of OBA on the input price is outweighed by its welfare-increasing effect on M's mark-up. Lower wholesale prices unambiguously benefit retailers and translate into higher sold quantities, thereby making also consumers better off.

Interestingly, in the accounting and management literature it is often stated that buyers (retailers in this case) want to obtain cost-related information

[34] Indeed, it holds: $w^*_{1,C} - t^*_{1,C} = \frac{1}{4+\gamma} < w^*_{0,C} - t^*_{0,C} = \frac{2+\gamma\varphi}{8+\gamma(1+3\varphi)}$, for all $\varphi \in [0, 1]$.

from their supplier (in this model, M), that the latter is reluctant to provide (see, e.g., Agndal and Nilsson [2008] and references therein), and there is considerable evidence that buyers use OBA to the disadvantage of their supplier (see Hoffjan *et al.* [2011] and references therein). Thus, this simple model, even without giving retailers any bargaining power over wholesale prices, thereby excluding the use of OBA in order to increase their negotiation pressure on the manufacturer, provides an intriguing rationale for the negative effect of OBA on M's profit, provided that downstream firms directly set quantities or, more realistically, order quantities first and then compete in prices.

IV. PRICE COMPETITION

In this section, I characterize the equilibrium of the model under Bertrand competition, first with OBA (Section IV(i)), and then when all contracts are kept secret (Section IV(ii)). The two equilibrium outcomes are then compared in Section IV(iii).

IV(i). *Equilibrium with OBA*

When OBA is implemented, at the last stage of the game, upon observing t and w_i, each R_i's best-reply function is characterized as follows:

$$\mathcal{P}_i(w_i, t) \triangleq \arg\max_{p_i \geq 0} (p_i - w_i) Q(p_i, \mathcal{P}_{-i}(\mathcal{W}_{-i}(w_i, t), t)).$$

Hence, it must satisfy the FOC:

$$\begin{aligned} (3) \qquad & Q(\mathcal{P}_i(w_i, t), \mathcal{P}_{-i}(\mathcal{W}_{-i}(w_{-i}, t))) \\ & + (\mathcal{P}_i(w_i, t) - w_i) Q_1(\mathcal{P}_i(w_i, t), \mathcal{P}_{-i}(\mathcal{W}_{-i}(w_{-i}, t))) = 0. \end{aligned}$$

Notice that in the formulation of R_i's problem it is assumed that it believes that, given w_i and t, M optimally sets the wholesale price to charge its rival (wary beliefs)—that is, $w^e_{-i} = \mathcal{W}_{-i}(w_i, t)$, hence $p^e_{-i} = \mathcal{P}_{-i}(\mathcal{W}_{-i}(w_i, t), t)$. Formally, taking as given t and w_i, and anticipating each R_i's best reply $\mathcal{P}_i(w_i, t)$, M's optimal offer to R_{-i} is characterized as follows:

$$\begin{aligned} \mathcal{W}_{-i}(w_i, t) \triangleq \arg\max_{w_{-i} \geq 0} & (w_i - t) Q(\mathcal{P}_i(w_i, t), \mathcal{P}_{-i}(w_{-i}, t)) \\ & + (w_{-i} - t) Q(\mathcal{P}_{-i}(w_{-i}, t), \mathcal{P}_i(w_i, t)). \end{aligned}$$

Hence, $\mathcal{W}_{-i}(w_i, t)$ solves the FOC:

$$(4) \qquad \frac{\partial \mathcal{P}_{-i}(\cdot)}{\partial w_{-i}}(w_i - t)Q_2(\mathcal{P}_i(w_i, t), \mathcal{P}_{-i}(\mathcal{W}_{-i}(w_i, t), t)) + Q(\mathcal{P}_{-i}(\mathcal{W}_{-i}(w_i, t), t), \mathcal{P}_i(w_i, t)) + \frac{\partial \mathcal{P}_{-i}(\cdot)}{\partial w_{-i}}(\mathcal{W}_{-i}(w_i, t) - t)Q_1(\mathcal{P}_{-i}(\mathcal{W}_{-i}(w_i, t), t), \mathcal{P}_i(w_i, t)) = 0.$$

Notice that this FOC (taken with respect to w_{-i}) contains w_i, which implies that, as well known since McAfee and Schwartz [1994], wary beliefs do not coincide with passive beliefs when downstream firms compete *á la* Bertrand. However, differently from previous works (e.g., Rey and Vergé [2004], Gaudin [2019]), in this multi-tier setting (and with OBA in place) the equilibrium strategies are functions of two variables (w and t)—that is, they solve a system of PDEs rather than ODEs. More precisely, for any given input price t, a symmetric SPNE is given by a pair of functions $(w^*(t), p^*(t))$ such that $w^*(t) = \mathcal{W}(w^*(t), t)$ and $p^*(t) = \mathcal{P}(w^*(t), t)$, where $\mathcal{W}(w, t)$ and $\mathcal{P}(w, t)$ solve the PDE system (3)-(4).[35] Finally, $p^*(t)$ can be substituted into the (standard) input supplier's problem. It is then rather easy to obtain the following equilibrium values.

Lemma 3. If OBA is implemented, in equilibrium, the input supplier sets $t^*_{1,B} = \frac{1}{2}$. The manufacturer supplies the final good at price $w^*_{1,B} = 1 - \frac{2-\gamma}{2(4-\gamma-\gamma^2)}$. Finally, the retail price is $p^*_{1,B} = 1 - \frac{1}{2(4-\gamma-\gamma^2)}$ and each retailer sells $q^*_{1,B} = \frac{1}{8+6\gamma-4\gamma^2-2\gamma^3}$.

Perhaps surprisingly, the wholesale price is an inverted U-shaped function of γ. As $\gamma \to 0$, M is an upstream monopolist in two independent markets operating at a unit cost $t^*_{1,B} = \frac{1}{2}$, whereas for $\gamma \to 1$, since Bertrand competition drives retailers' mark-ups to zero, M is *de facto* a vertically-integrated monopolist, facing the same cost and selling two homogeneous products. Under linear demand, it optimally charges the same wholesale price in these two extreme cases—that is, $w^*_{1,B}|_{\gamma\to 0} = w^*_{1,B}|_{\gamma\to 1} = \frac{3}{4}$. More generally, the inverted U-shaped relation reflects the following trade-off. As product market competition intensifies (i.e., γ grows larger), each R_i on the one hand is bound to reduce its mark-up and on the other hand becomes more responsive to the belief on its competitor's price.[36] The first effect enables M to offer a larger wholesale price without reducing too much demand. By

[35] Following the literature (e.g., Rey and Vergé [2004], Gaudin [2019]), I restrict attention to polynomial solutions of this system. All the details are in Appendix A.

[36] To see this, notice that solving the FOC of R_i's problem (3) yields $\mathcal{P}_i(\cdot) = \frac{1}{2}\left(1 - \gamma + w_i + \gamma p^e_{-i}(\cdot)\right)$.

contrast, the second effect leads to an increase in retailers' pass-through rates, which exacerbates the marginalization problem and weakens M's incentives to charge large wholesale prices.

IV(ii). *Equilibrium without OBA*

If OBA is not implemented, upon observing w_i only, R_i's best-reply function $\mathcal{P}_i(w_i)$ is characterized as follows:

$$\mathcal{P}_i(w_i) \triangleq \arg\max_{p_i \geq 0}(p_i - w_i)\left(\varphi Q(p_i, \mathcal{P}_{-i}(w_i)) + (1-\varphi) Q(p_i, \mathcal{P}_{-i}(\mathcal{W}_{-i}(w_i, t^*_{0,B})))\right).$$

In words, by the logic of wary beliefs, R_i believes that M always sets w_{-i} optimally, given w_i and t. This means that, if also w_i is set optimally (given t), then, by symmetry, M is expected to offer R_{-i} the same wholesale price w_i. If, by contrast, w_i is not set optimally (given t), then M is expected to offer R_{-i} a price $\mathcal{W}_{-i}(w_i, t)$. Next, by Assumption (A), upon observing an out-of-equilibrium offer $w_i \neq w^*_{0,B}$, R_i believes that, with probability φ, $w_i \neq w^*_{0,B}$ is M's best response to $t \neq t^*_{0,B}$, whereas, with complementary probability $1-\varphi$, S has set the equilibrium price $t = t^*_{0,B}$ and M has deviated from its equilibrium behavior in its offer to R_i (and then has optimally set $w_{-i} = \mathcal{W}_{-i}(w_i, t^*_{0,B})$).

Thus, $\mathcal{P}_i(\cdot)$ solves the following FOC:

$$(5)\quad (\mathcal{P}_i(w_i) - w_i)\left(\varphi Q_1(\mathcal{P}_i(w_i), \mathcal{P}_{-i}(w_i)) + (1-\varphi) Q_1(\mathcal{P}_i(w_i), \mathcal{P}_{-i}(\mathcal{W}_{-i}(w_i, t^*_{0,B})))\right)$$
$$+ \varphi Q(\mathcal{P}_i(w_i), \mathcal{P}_{-i}(w_i)) + (1-\varphi) Q(\mathcal{P}_i(w_i), \mathcal{P}_{-i}(\mathcal{W}_{-i}(w_i, t^*_{0,B}))) = 0.$$

Hence, unlike in the game with OBA, here R_i's equilibrium strategy does not contain t, but, for every $\varphi < 1$, it contains the equilibrium value $t^*_{0,B}$.

Similarly to the scenario in which OBA is in place, by the logic of wary beliefs, for any t and w_i, M's optimal offer to R_{-i} is characterized as follows:

$$\mathcal{W}_{-i}(w_i, t) \triangleq \arg\max_{w_{-i} \geq 0}(w_i - t) Q(\mathcal{P}_i(w_i), \mathcal{P}_{-i}(w_{-i})) + (w_{-i} - t) Q(\mathcal{P}_{-i}(w_{-i}), \mathcal{P}_i(w_i)).$$

Thus, it must satisfy the following FOC:

$$(6)\quad \frac{\partial \mathcal{P}_{-i}(\cdot)}{\partial w_{-i}}(w_i - t) Q_2(\mathcal{P}_i(w_i), \mathcal{P}_{-i}(\mathcal{W}_{-i}(w_i, t))) + Q(\mathcal{P}_{-i}(\mathcal{W}_{-i}(w_i, t)), \mathcal{P}_i(w_i))$$
$$+ \frac{\partial \mathcal{P}_{-i}(\cdot)}{\partial w_{-i}}(\mathcal{W}_{-i}(w_i, t) - t) Q_1(\mathcal{P}_{-i}(\mathcal{W}_{-i}(w_i, t)), \mathcal{P}_i(w_i)) = 0.$$

In a symmetric equilibrium, $\mathcal{W}(\cdot)$ and $\mathcal{P}(\cdot)$ solve the PDE system (5)-(6), and the optimal wholesale price $w^*(t)$ is pinned down imposing $w^*(t) = \mathcal{W}(w^*(t), t)$. Clearly, for all $\varphi < 1$, these solutions also contain the equilibrium value $t^*_{0,B}$, which is then obtained solving the input supplier's problem, taking as given the downstream players' equilibrium strategies (which result in the retail prices $\mathcal{P}(w^*(t))$), and imposing that the downstream players correctly conjecture the equilibrium input price (i.e., that the solution to S's problem, given that the other firms expect it to charge $t^*_{0,B}$ in equilibrium, is indeed $t^*_{0,B}$). The equilibrium can be summarized as follows.[37]

Lemma 4. Without OBA, in equilibrium, the input supplier sets $t^*_{0,B} = \frac{2(2-\gamma\varphi-\gamma^2(1-\varphi))}{8-\gamma(1+3\varphi)-3\gamma^2(1-\varphi)}$. The manufacturer supplies the final good at price $w^*_{0,B} = 1 - \frac{2-\gamma}{8-\gamma(1+3\varphi)-3\gamma^2(1-\varphi)}$. Finally, the retail price is $p^*_{0,B} = 1 - \frac{1}{8-\gamma(1+3\varphi)-3\gamma^2(1-\varphi)}$ and each retailer sells $q^*_{0,B} = \frac{1}{8-\gamma(1+3\varphi)-3\gamma^2(1-\varphi)}$.

Notably, input and wholesale prices are decreasing in φ. The reason is as follows. As φ increases, both upstream firms know that a larger wholesale price w_i makes R_i relatively less aggressive (i.e., softens price competition), because it believes that its competitor is more likely facing a larger marginal cost. Indeed, if $\varphi = 0$, given w_i, M is expected to offer $w^e_{-i} = \mathcal{W}_{-i}(w_i, t^*_{0,B})$, where its equilibrium strategy is such that $\frac{\partial \mathcal{W}_{-i}(\cdot)}{\partial w_i} = \gamma \in (0,1)$ (see Appendix A), whereas, when $\varphi = 1$, R_i expects the same wholesale price w_i to be faced by its rival—that is, $\frac{\partial w^e_{-i}(\cdot)}{\partial w_i} = 1$. As a consequence, for any w_i, the expected wholesale price $w^e_{-i}(\cdot)$ is an increasing function of φ. Due to this *belief effect*, retail prices are more responsive to wholesale prices when φ is larger—that is, given that final prices are strategic complements and are increasing in the wholesale prices, retailers have stronger incentives to charge high final prices as φ grows larger. Such retailers' behavior, exacerbating the multiple marginalization problem, refrains the upstream firms from charging high prices. This explains why $t^*_{0,B}$ and M's mark-up are decreasing in φ.[38] In equilibrium, the lower input and wholesale prices, as φ grows larger, unambiguously translate into lower retail prices.[39] Finally, notice that $t^*_{0,B}|_{\varphi=1} = t^*_{1,B}$. Taken together, these observations imply that, for all $\varphi < 1$, $t^*_{0,B} > t^*_{1,B}$ (see Table I), which will be key to the welfare results shown in the next section.

IV(iii). *Welfare*

From the foregoing analysis, it follows that a higher value of the belief parameter φ unambiguously makes the implementation of OBA less attractive

[37] Once again, I restrict attention to polynomial equilibrium strategies (see Appendix A).

[38] Indeed, $\frac{\partial\left(w^*_{0,B}-t^*_{0,B}\right)}{\partial\varphi} = -\frac{\gamma(1-\gamma)(2-\gamma)}{8-\gamma(1+3\varphi)-3\gamma^2(1-\varphi)} < 0$.

[39] Moreover, for every $\varphi < 1$, $t^*_{0,B}$, and $w^*_{0,B}$ are inverted U-shaped functions of γ. The intuition is as in the game with OBA.

from a consumer surplus viewpoint, since, for the reasons pointed out above: (i) without OBA, prices along the supply chain are decreasing in φ; and (ii) the pro-competitive effect of OBA in lowering the input price weakens as φ grows larger, since, as in the Cournot model, $t^*_{0,B} \to t^*_{1,B}$ for $\varphi \to 1$. Once again, due to the multiple marginalization problem, the industry as a whole has preferences aligned with consumers' interests, but individual firms have very conflicting preferences. Specifically:

Proposition 2. When the retailers compete á la Bertrand, the implementation of OBA increases total industry profits and consumer surplus if and only if $\varphi < \frac{1}{3}$. Moreover:

- There is a threshold $\underline{\varphi} < \frac{1}{3}$ such that the input supplier is better off without OBA for every $\varphi > \underline{\varphi}$;
- The manufacturer is always better off when OBA is in place;
- The retailers are better off with OBA if and only if $\varphi < \frac{1}{3}$.

Once again, to gain intuition it is useful to consider the two extreme cases $\varphi \in \{0, 1\}$. If $\varphi = 0$, since the retailers hold wary beliefs regardless of t being observable or not, OBA has no effect on the retailers' and M's behavior, but it does alter S's behavior, since retailers' prices are more responsive to an increase in t with than without OBA. In fact, absent OBA, retailers always believe t to be set at the equilibrium value. Hence, when facing an unexpectedly large wholesale price, even when this is due to an increase in the input price, R_i believes that its rival is being confronted with a lower wholesale price ($w^e_{-i}(\cdot) < w_i$), since, as noted above, $\frac{\partial w^e_{-i}(\cdot)}{\partial w_i}\Big|_{\varphi=0} = \frac{\partial W_{-i}(\cdot)}{\partial w_i} < 1$. By contrast, with OBA R_i can observe any increase in the input price, and correctly infer that it is passed through by M to the same extent to its competitor as well, which induces it to charge a larger retail price. Thus, absent OBA, the marginalization problem is less pronounced, thereby the upstream supplier has stronger incentives to charge a larger t (i.e., $t^*_{0,B}|_{\varphi=0} > t^*_{1,B}$), which is passed on throughout the supply chain.

By contrast, if $\varphi = 1$, since the retailers' beliefs are symmetric absent OBA, their behavior is the same under both disclosure standards, given that $w^e_{-i}(\cdot) = w_i$ regardless of t being disclosed or not. However, unlike in the previous case, M's behavior is different depending on whether OBA is in place (alongside wary beliefs) or not (alongside symmetric beliefs). This is because, by the mechanism described above, if M were to charge higher wholesale prices, under symmetric beliefs downstream prices would increase more than under wary beliefs, which weakens M's incentives to charge a higher mark-up absent OBA. Under the linear demand specification, M's mark-up does not affect the input price optimally set by S: $t^*_{0,B}|_{\varphi=1} = t^*_{1,B}$.

The larger mark-up charged by M when OBA is in place then results in larger retail prices.

Interestingly, the upstream firm is likely to be harmed by its gain in commitment power *vis-á-vis* the retailers following the implementation of OBA. This is a novel implication of this three-tier model. Indeed, by a simple revealed preference argument, it can be immediately seen that vertical contract disclosure always benefits the upstream firm in a two-tier industry: even though, as shown by Gaudin [2019], under Bertrand competition the upstream firm charges higher prices when its offers are not disclosed, it is then harmed by the excessive double marginalization so induced. By contrast, in a three-tier supply chain, there is an additional effect brought up by M's mark-up, which, for all $\varphi \in [0,1]$, is higher when OBA is implemented.[40] This is because, by being able to charge a higher input price when OBA is not in place (for the reasons pointed out above), S *de facto* forces M to add a lower mark-up, so as to avoid exacerbating the multiple marginalization problem. For this reason, M always benefit from OBA, whereas S is harmed by the disclosure provision unless φ is sufficiently small, since in that case M's mark-up is relatively high in both disclosure regimes and OBA mitigates the multiple marginalization problem. Lastly, as in the Cournot model, the retailers are better off under the disclosure standard which minimizes their costs, and their preferences are perfectly aligned with consumers surplus maximization.

V. EXTENSIONS AND ROBUSTNESS

In this section, I discuss some implications and extensions of the foregoing analysis. To begin with, I compare the equilibrium outcomes under Cournot and Bertrand competition (Section V(i)) and endogenize the implementation of OBA (Section V(ii)). I then consider information sharing between the retailers (Section V(iii)) and nonlinear contracts (Section V(iv)). Finally, I argue that the *belief effects* driving the main results are not specific to the linear demand setting (Section V(v)), and discuss the robustness of the main findings with respect to the beliefs' specification (Section V(vi)). Proofs of all the results presented in this section are in the Online Appendix.

V(i). *Price vs quantity competition*

The welfare analysis carried out in the previous sections has shown that, as in Gaudin [2019], the disclosure of vertical contracts has completely

[40] Indeed, $w^*_{1,B} - t^*_{1,B} = \frac{2-\gamma^2}{2(4-\gamma-\gamma^2)} > w^*_{0,B} - t^*_{0,B} = \frac{2-\gamma^2(1-\varphi)-\gamma\varphi}{8-\gamma(1+3\varphi)-3\gamma^2(1-\varphi)}$, for all $\varphi \in [0,1]$.

opposite welfare effects depending on the kind of downstream competition. This is because, by the *belief effects* discussed above, the impact of φ on the extent of the multiple marginalization problem (and hence on the upstream firms' incentives to charge large prices) crucially depends on whether the retailers' instruments are strategic substitutes or complements.

While the foregoing analysis focused on the welfare comparison of contract disclosure standards taking as given the competition model, in what follows I compare, for any given disclosure standard, the equilibrium outcomes under Cournot and Bertrand competition.

When OBA is implemented, the two competition models compare as follows:

$$t^*_{1,C} = t^*_{1,B}; \qquad w^*_{1,C} \leq w^*_{1,B}; \qquad p^*_{1,C} < p^*_{1,B} \iff \gamma < \sqrt{3} - 1.$$

When the input price is disclosed to the retailers, with linear demand, S finds it optimal to set the (integrated) multi-product monopolist's price. Hence, since M faces the same marginal cost regardless of the kind of downstream competition, and makes secret offers to the retailers, the comparison of the two models is as in a two-tier model with secret contracts. Since the retailers' mark-ups are, *ceteris paribus*, higher under Cournot than under Bertrand competition, in the former case M, being more concerned about the multiple marginalization problem, finds it optimal to charge a lower wholesale price. Hence, under Cournot competition, the retailers face a lower marginal cost but have stronger incentives to charge higher mark-ups as compared to the Bertrand model, and the latter effect prevails when products are sufficiently homogeneous.

Similarly, in the scenario without OBA implementation,

$$t^*_{0,C} \leq t^*_{0,B}; \quad w^*_{0,C} \leq w^*_{0,B}; \quad p^*_{0,C} < p^*_{0,B} \Leftrightarrow \gamma < \hat{\gamma} \triangleq \frac{\sqrt{22 - 18(2-\varphi)\varphi} - 2}{3(1-\varphi)}.$$

As in Gaudin [2019], the upstream monopolist charges a lower price when its contract is secret than when it is publicly observed if and only if downstream firms' instruments are strategic substitutes, hence $t^*_{0,C} \leq t^*_{1,C} = t^*_{1,B} \leq t^*_{0,B}$. Then, also M charges a lower (wholesale) price in the Cournot model, since it faces a lower cost and, as noticed above, it is more concerned about the retailers' mark-ups. The comparison between the retail prices is then as in the case with OBA, but here also the belief parameter φ plays a role. Specifically, the threshold $\hat{\gamma}$ above which equilibrium retail prices are larger under Cournot competition is decreasing in φ since, for the reasons pointed out above, prices are increasing (decreasing) in φ under Cournot (Bertrand) competition.

V(ii). *Endogenous disclosure standard*

Following the literature on contract disclosure in vertical markets (e.g., Arya and Mittendorf [2011], Gaudin [2019]), I assumed that the disclosure standard is chosen before the contracting stages and is common knowledge. These assumptions capture in a stylized way the idea that the implementation of OBA relies on and fosters long-term supplier-buyers relationships (see, e.g., Ellström and Larsson [2017]), whereas the terms of distribution contracts are negotiated on a short-term basis (as they can be linked to transient demand or costs conditions). Moreover, in the real world, the supplier (in this case, M) typically negotiates with its buyers (in this model, retailers) the access to its cost data: they define how to structure OBA agreements (including what types of data to disclose, what intended usages of data disclosed, etc.), and find an agreement on how to share benefits and risks associated with OBA implementation (see, e.g., Romano and Formentini [2012] and references therein). Based on these considerations, in what follows, I endogenize the implementation of OBA under the assumption that, at the outset of the game, M efficiently bargains with both retailers over the disclosure standard. Moreover, for simplicity and without loss of insights, I also assume that OBA can be implemented at no cost. The results can be summarized as follows.

Under Cournot competition, there is a threshold $\widehat{\varphi} > \frac{1}{3}$ such that the implementation of OBA increases the joint profit of M and both retailers if and only if $\varphi > \widehat{\varphi}$. Hence, when φ is sufficiently high, M and the retailers efficiently agree to introduce OBA, and the retailers make a transfer to M to compensate it for its loss. Notice that, since $\widehat{\varphi} > \frac{1}{3}$, even in the presence of efficient bargaining, there will be under-adoption of OBA from a social welfare point of view (as, for all $\varphi \in (\frac{1}{3}, \widetilde{\varphi})$, a welfare-increasing OBA policy is not implemented). Clearly, this is because the positive effect of OBA on the upstream supplier's profit is not internalized in the bargaining process.

By contrast, in the game with Bertrand competition, there is a threshold $\widetilde{\varphi} > \frac{1}{3}$ such that, under efficient bargaining, OBA will be implemented if and only if $\varphi < \widetilde{\varphi}$. Hence, there will be over-adoption of OBA from a social welfare point of view (as, for all $\varphi \in (\frac{1}{3}, \widetilde{\varphi})$, a welfare-decreasing OBA policy is implemented), since M always benefits from OBA and the negative effect of this disclosure standard on the upstream supplier's profit is not internalized in the bargaining process.

V(iii). *Horizontal information sharing and MFN clauses*

In this section, I consider the game under public contracts. First, it is immediate to see that, once the wholesale prices are publicly observed, the equilibrium of the game is the same regardless of the input price being disclosed or not to the retailers—that is, the presence of OBA agreements does not play a role in the analysis. Second, by symmetry between the retailers, the equilibrium under public contracts can be implemented if, at the outset of the game: (i) retailers agree to share (verifiable) information about the wholesale prices before taking their price or quantity setting decisions; or (ii) each of them negotiates with the manufacturer a MFN (most favoured nation) clause (e.g., McAfee and Schwartz [1994]), whereby M (credibly) commits to supply R_i at no worse conditions than those offered to its competitor (i.e., $w_i \leq w_{-i}$).

It can be easily shown that, regardless of the kind of downstream competition, the equilibrium outcome under public contracts is the same arising under secret contracts and symmetric beliefs (i.e., in the game without OBA and $\varphi = 1$). This is because, under public contracts, retailers' choices only depend on the wholesale prices. Accordingly, the input supplier's choice does not entail any *belief effect*, exactly as it happens when the retailers hold symmetric beliefs.

From the foregoing analysis, it follows that, as in the two-tier model of Gaudin [2019], when downstream firms compete *á la* Cournot, the outcome under public contracts yields a lower industry profit and damages consumers as compared to both the disclosure standards examined in this model (which entail secret downstream contracts), whereas the converse is true under Bertrand competition.

V(iv). *Two-part tariffs*

Up until now, I considered linear distribution contracts. The reasons are as follows. First, such contracts are employed in reality in many industries (see, e.g., Li and Zhang [2008], Gaudin [2017, 2019], and references therein), and OBA is often used in practice to negotiate *cost-plus deal* (see, e.g., Alenius *et al.* [2015])—that is, the information on the upstream firm's cost is used in the negotiation over its unit mark-up. Second, as argued by Inderst [2010], the use of linear contracts in simple models of vertical market relations under secret contracts often provides *less extreme outcomes* as compared to more general nonlinear contracts, which are likely to lead either to perfect monopolization or to an extreme opportunism problem. Relatedly, under linear contracts all firms' profits are endogenously determined (i.e., they do not depend on exogenously specified bargaining weights), which allowed to analyze firms' conflicting preferences over the presence of OBA and to endogenize its adoption.

As seen above, regardless of the disclosure standard, the equilibrium of the game under linear contracts features a multiple marginalization problem. As well known, this inefficiency can be solved by allowing firms to use two-part tariffs—that is, the distribution contract between S and M takes the form $T + tq$, and similarly the contract between M and each R_i takes the form $W_i + w_i q_i$. For the sake of simplicity, in each bilateral negotiation, the upstream firm is assumed to have all the bargaining power,[41] and off-equilibrium path beliefs, in the game without OBA, are specified as in the baseline model: for every W_i,[42] upon receiving an out-of-equilibrium wholesale price $w_i \neq w_0^*$, R_i believes that its rival has been offered the same contract with probability φ, and M's optimal contract, given w_i and $t_{-i}^e = t_0^*$, otherwise.

It is easy to prove that, under both quantity and price competition, the implementation of OBA leads to the industry-wide monopoly outcome. Indeed, even with OBA, since its offers to the retailers are secret, the well-known opportunism problem arises (e.g., Hart and Tirole [1990], O'Brien and Shaffer [1992], McAfee and Schwartz [1994]). However, unlike in a two-tier model, here S can exploit its commitment power to set the input price at a level which induces the retailers to optimally choose to implement the industry-wide monopoly outcome. Hence, the implementation of OBA increases the industry profit but lowers consumer surplus. This is because it plays a similar role of horizontal information sharing or MFN clauses in solving the opportunism problem arising under private contracts, thereby making it achievable the integrated monopolist's outcome.

Indeed, even when all contracts throughout the supply chain are kept secret, the opportunism problem wipes out if the retailers hold symmetric beliefs ($\varphi = 1$). This is because, as already shown by Rey and Tirole [2007] in a two-tier model, with symmetric beliefs, M can behave as if its offers were publicly observed. Therefore, there is no scope for opportunistic behavior from M, thereby it can set a wholesale price so as to induce the retailers to implement the industry-wide monopoly outcome.

V(v). *General demand function*

The *belief effects* absent OBA, discussed in the foregoing analysis, which are the key driver of the main results of the paper, are not specific to a linear

[41] That is, S makes a take-it-or-leave-it (TIOLI) offer (T, t) to M, which in turn (upon accepting this offer) simultaneously makes a TIOLI offer (W_i, w_i) to each R_i. This assumption can be easily relaxed in the upstream contracting problem: one can equivalently assume that S and M agree on the input price that maximizes their joint profit, which is then split according to some bargaining weights. However, things are more complicated in the downstream contracting problem, since not only (as in the previous literature) M simultaneously bilaterally bargains with two retailers, but on top of this, in the absence of OBA, M and R_i have asymmetric information (since only the former observes t), implying that they might disagree on the wholesale price which maximizes their joint profit. Hence, giving all the bargaining power to the upstream firm (M) allows to avoid modeling a more complex bargaining process under asymmetric information.

[42] Thus, following the literature, I rule out that R_i's beliefs also depend on the fixed fee W_i. Admittedly, in this model such an assumption is more problematic. The reason is as follows. While in the two-tier models "franchise-dependent beliefs could self-sustain themselves, but such dependence is not triggered by fundamental variables" (Rey and Vergé [2004]), in a three-tier industry, an out-of-equilibrium fixed fee can (exactly as the wholesale price) be the consequence of an out-of-equilibrium input price faced by M—that is, W_i, as well as w_i, can *signal* a deviation of the upstream supplier.

demand environment. To see this, consider (for the sake of exposition) the Cournot model and the two extreme cases—that is, $\varphi \in \{0,1\}$.[43] From (1), it is easy to obtain that, under standard assumptions on the demand function, the slope of each retailer's demand $q^*(\cdot)$ is, in absolute value, higher under passive beliefs than under symmetric beliefs—that is,

$$\left| \left. \frac{\partial q^*(\cdot)}{\partial w_i} \right|_{\varphi=0} \right| > \left| \left. \frac{\partial q^*(\cdot)}{\partial w_i} \right|_{\varphi=1} \right|.$$

Next recall that M faces, *vis-á-vis* each retailer, the problem of a monopolist facing the demand $q^*(\cdot)$, thereby the standard monopoly pricing rule applies—that is, $\frac{w^*(\cdot)-t}{w^*(\cdot)} = \frac{1}{\eta}$, where $\eta \triangleq \left|\frac{\partial q^*(\cdot)}{\partial w_i}\right| \frac{w^*(\cdot)}{q^*(\cdot)}$ is the price elasticity of the retailers' demand $q^*(\cdot)$. Since η positively depends on $\left|\frac{\partial q^*(\cdot)}{\partial w_i}\right|$, other things being equal, M charges a lower wholesale price under passive beliefs. By the same reasoning, the input price set by the upstream monopolist is *ceteris paribus* lower when the slope of the retailers' demand as function of t (i.e., from the viewpoint of S),

$$\frac{\partial q^*(\cdot)}{\partial t} = \left|\frac{\partial q^*(\cdot)}{\partial w_i}\right| \cdot \frac{\partial w^*(\cdot)}{\partial t},$$

is relatively large. Differentiating the FOC of M's problem, it is easy to see that M's pass-through rate can be written as

$$\frac{\partial w^*(\cdot)}{\partial t} = \frac{1}{2-E},$$

where $E \triangleq q^*(\cdot)\frac{\partial^2 q^*(\cdot)}{\partial w_i^2}\left(\frac{\partial q^*(\cdot)}{\partial w_i}\right)^{-2}$ denotes the curvature of the retailers' demand (e.g., Gaudin [2016]). With linear demand,[44] $E=0$, which implies that, as pointed out above, $\frac{\partial w^*(\cdot)}{\partial t} = \frac{1}{2}$ for all φ. However, under a different specification of the demand function, E (hence, M's pass-through rate) may depend on φ. Therefore, our qualitative results are even reinforced (namely, S is expected to charge a lower input price under passive beliefs) whenever M's pass-through rate is higher under passive beliefs—that is, when E is larger at $\varphi = 0$. In turn, under relatively mild assumptions, this is the case when the demand function $P(\cdot)$ is convex.

[43] A similar analysis for the Bertrand model is developed in the Online Appendix. It can be argued that the qualitative results for any $\varphi \in (0,1)$ are in between of the results in the extreme cases $\varphi \in \{0,1\}$.

[44] Indeed, a linear specification of the consumers' demand function $P(\cdot)$ yields a linear best response $q^*(\cdot)$ of each retailer.

V(vi). *Alternative beliefs and "best" PBE*

As noted by Segal and Whinston [2003], *inter alia*, "once retailers are allowed to hold arbitrary beliefs after observing out-of-equilibrium offers, a large set of outcomes can be sustained in a weak perfect Bayesian equilibrium. This is also true for stronger solution concepts such as sequential equilibrium."[45] Thus, while (as discussed in Section II(ii)) there are good reasons to specify beliefs as in the foregoing analysis, one may wonder about the robustness of the main results with respect to the beliefs' specification.

To begin with, it is easy to prove that, regardless of retailers' off-equilibrium path beliefs, in any symmetric equilibrium, under Cournot or Bertrand competition: (i) market-clearing or retail prices are increasing in wholesale prices; and (ii) retailers' profit is decreasing in wholesale prices. Since consumer surplus is decreasing in the equilibrium prices, it then follows that the equilibrium which maximizes retailers' profit coincides with the consumer-preferred equilibrium. Moreover, as long as the equilibrium of the game features larger equilibrium or market clearing prices as compared to the industry-wide monopoly outcome, the equilibrium with the lowest wholesale and retail prices also maximizes aggregate industry profits. In what follows, I refer to this equilibrium as the "best" PBE.

This equilibrium can be characterized as follows. Under Bertrand competition, M's profit (for any input price) following out-of-equilibrium contract offers is decreasing in the retailers' mark-ups, and each R_i's price and mark-up is increasing in its belief $p^e_{-i}(\cdot)$ on the price charged by its rival. Thus, the higher $p^e_{-i}(\cdot)$, the lower M's profit following any out-of-equilibrium offer w_i. Therefore, M's *deviation profit* is minimized when $p^e_{-i}(\cdot)$ takes its maximum value—that is, for $p^e_{-i}(\cdot) \geq 1$ or, equivalently, when R_i behaves as a monopolist.[46] In turn, given that R_{-i} is supposed to play rationally, $p^e_{-i}(\cdot) \geq 1$ is an admissible off-equilibrium path belief whenever $w^e_{-i}(\cdot) \geq 1$. It then immediately follows that $w^e_{-i}(\cdot) \geq 1$ for all $i = 1, 2$ and out-of-equilibrium offers w_i is the off-equilibrium path belief which sustains the lowest possible equilibrium wholesale price (hence, the best PBE), as it minimizes M's incentives to deviate from the candidate equilibrium and offer larger wholesale prices.

By a similar logic, under Cournot competition, M's out-of-equilibrium profit is increasing in the retailers' quantities and each R_i's quantity is decreasing in $q^e_{-i}(\cdot)$, thereby the belief system which attains the best PBE is that in which each R_i expects its rival to optimally sell the largest possible

[45] Indeed, consider a symmetric equilibrium wholesale price w^*. Then, upon receiving an offer w_i, any function $f(w_i)$ such that $f(w^*) = w^*$ specifies admissible beliefs of R_i on the offer faced by its rival, with $f(\cdot)$ which, in this model, can also depend on t in the game with OBA, and contain the equilibrium value t^*_0 in the game without OBA.

[46] Recall that the market dimension under the considered demand specification is equal to one, thereby, when R_{-i} charges any $p_{-i} \geq 1$, R_i is a monopolist.

quantity.[47] Thus, exactly as in the baseline analysis, retailers' beliefs which exacerbate the marginalization problem following out-of-equilibrium offers weaken M's incentive to offer large wholesale prices, thereby sustaining more efficient equilibrium outcomes. Then, under both competition models, the wholesale contract in the best PBE is pinned down as the (lowest) value which makes M not willing to deviate to a larger wholesale price and induce more marginalization (via the retailers' beliefs).

Moving backward to the first stage of the game, as seen in the foregoing analysis, when OBA is in place S's offer t defines a proper subgame—that is, the wholesale price that retailers expect to receive depends on the observed input price. Hence, S chooses t so as to maximize its profit taking into account how the input price translates into equilibrium wholesale and retail prices/quantities. Hence, even in the best PBE S it is able to commit to a large input price (namely, $t^*_{1,B} = \frac{1}{2} \leq t^*_{1,C}$), thereby all PBE feature inefficiently high prices.

By contrast, absent OBA, S's equilibrium offer in the best PBE is pinned down as the smallest input price such that it obtains a weakly larger profit in equilibrium than following any deviation (which, by construction, induces more marginalization). Thus, unlike with OBA, S is unable to commit to a large input price *vis-á-vis* the retailers, thereby the best PBE features lower prices throughout the supply chain when S's contract is kept secret. Actually, when γ is large enough, the best PBE features inefficiently low retail prices from an industry-wide profit maximization viewpoint. Yet, the considered system of beliefs can also sustain the industry-wide monopoly outcome as an equilibrium.

To conclude, similarly to the baseline analysis, not observing the input price leaves retailers more freedom as to the way they can form their beliefs, which results in a larger set of possible equilibrium outcomes and in the possibility of achieving a more efficient outcome.

[47] Actually, to derive the maximum value of $q^e_{-i}(\cdot)$ under the constraint that R_{-i} plays rationally, and the belief system which attains this value, is rather tricky. The reason is that $q^e_{-i}(\cdot)$ takes its maximum value when (R_i believes that) R_{-i} faces the minimum feasible wholesale price and (by strategic substitutability) it expects R_i to face the largest wholesale price. Since the PBE concept imposes no restrictions of off-equilibrium path beliefs in this game, it is not straightforward how to specify a lower bound on w^e_{-i}: it is natural to impose a non-negativity constraint $w^e_{-i} \geq 0$, but one can also require M's *individual rationality constraint* $w^e_{-i} \geq t$. For simplicity and with no loss of insights, I just impose $w^e_{-i} \geq 0$. It is then easy to see that $q^e_{-i}(\cdot) \leq \frac{1}{2}$—that is, R_i can never expect its rival to sell more than a monopolist facing zero marginal cost. Moreover, $q^e_{-i}(\cdot) = \frac{1}{2}$ for all (positive) out-of-equilibrium w_i under the following beliefs' specification:

$$w^e_{-i}(w_i) = \begin{cases} 1 & \text{if } w_i = 0 \\ w^*_{d,C} & \text{if } w_i = w^*_{d,C} \\ 0 & \text{otherwise} \end{cases}$$

for all $i = 1, 2$ and disclosure regimes $d = 0, 1$. This belief system thus attains the best PBE under non-negativity constraints on contract offers.

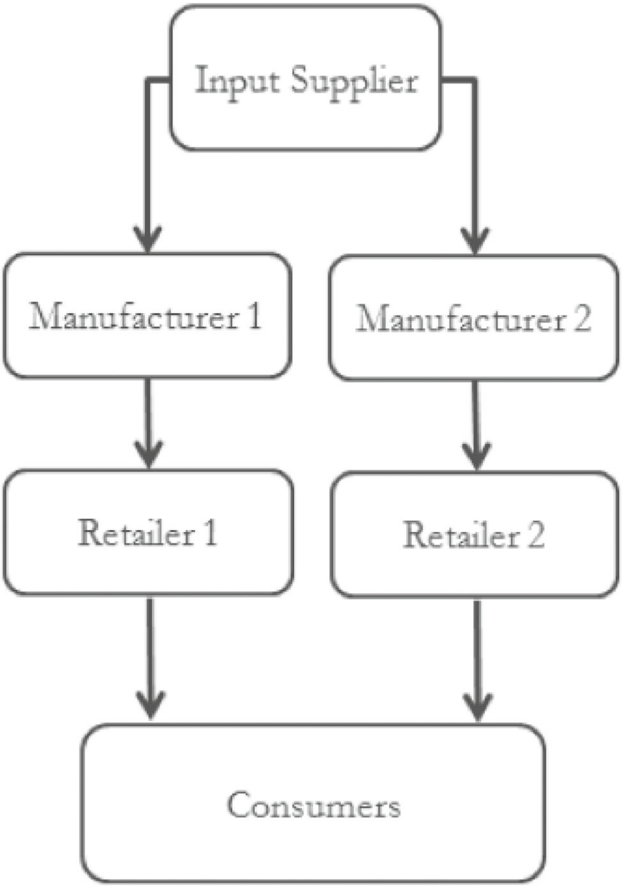

FIGURE 2
Industry Structure with Downstream and Upstream Competition

VI. UPSTREAM COMPETITION AND MERGER

In this section, I consider a slightly more complex industry structure. Specifically, suppose that the upstream firm (S) supplies two manufacturers (M_i, $i = 1, 2$), each producing a final good which is distributed through an exclusive retailer (R_i, $i = 1, 2$): see Figure 2.[48,49]

[48] Considering interlocking relations would bring up more complex beliefs' specification problems, as discussed in the Online Appendix.

[49] The goods sold by retailers are perceived as horizontally differentiated by final consumers either because of manufacturer-specific product's characteristics or because of retailer-specific services. Demand functions faced by retailers are as in the baseline model.

Thus, normalizing again to zero production and retailing costs, S's profit is now given by $\pi^S\left(t_1, t_2\right) \triangleq \sum_{i=1,2} t_i q_i$, with q_i denoting the quantity purchased by M_i at price t_i. Hence, each M_i's profit is $\pi_i^M(w_i, t_i) \triangleq (w_i - t_i)q_i$. Finally, each R_i's profit is $\pi_i^R(p_i, w_i)$, defined as in the baseline model.

Once again, I compare two alternative contract disclosure standards, depending on the implementation of OBA. In the presence of OBA, a manufacturer M_i discloses to its exclusive retailer R_i its input cost t_i. For the sake of tractability (i.e., to maintain symmetry), in what follows I restrict attention to the case in which OBA is implemented either by both manufacturers or by none of them.

VI(i). *Off-equilibrium path beliefs*

Unlike in the baseline model, since here the input supplier makes multiple offers, off-equilibrium path beliefs need to be specified not only at the retailers' level but also for the manufacturers. Once again, I consider wary beliefs: for every offer t_i received by S, M_i believes that its competitor M_{-i} has been offered $t^e_{-i} = \mathcal{T}_{-i}(t_i)$, where $\mathcal{T}_{-i}(t_i) \triangleq \arg\max_{t_{-i}\geq 0} \pi^S(t_i, t_{-i})$.

Clearly, in the scenario with OBA being implemented (by both manufacturers), upon observing t_i, M_i and R_i have the same beliefs—that is, they conjecture the same value for t^e_{-i}. Next consider the case in which OBA policies have not been implemented. Also in this case, upon observing t_i, M_i has wary beliefs on the input price t_{-i} faced by its competitor. However, as for the downstream retailers, by the same logic behind Assumption (A), I assume that there exists $\varphi \in [0, 1]$ such that, upon receiving an out-of-equilibrium offer $w_i \neq w_i^*$, R_i believes that:

- With probability φ, the out-of equilibrium offer reflects a deviation by S. In this case, R_i believes that $w^e_{-i} = \mathcal{W}_{-i}(t^e_{-i})$, with $\mathcal{W}_{-i}(\cdot)$ denoting M_{-i}'s equilibrium strategy—that is, $\mathcal{W}_{-i}(t_{-i}) \triangleq \text{argmax}_{w_{-i}} \pi^M_{-i}(w_{-i}, t_{-i})$—and t^e_{-i} is obtained as follows. First, R_i can infer M_i's cost from its equilibrium strategy: indeed, as M_i follows its equilibrium strategy, $w_i = \mathcal{W}_i(t_i)$, thereby $t^e_i = (\mathcal{W}_i)^{-1}(w_i)$.[50] Next, by the logic of wary beliefs, $t^e_{-i} = \mathcal{T}_{-i}(t^e_i)$.
- With complementary probability $1 - \varphi$, the out-of equilibrium offer reflects a deviation by M_i. In this case, from the assumption of exclusive relation between M_i and R_i it follows that, according to the *no signaling what you don't know* condition (Fudenberg and Tirole [1991]), R_i must have passive beliefs concerning the offer w_{-i} received by its rival—that is, $w^e_{-i} = w^*_{-i}$ (since $t^e_{-i} = t^*_{-i}$).

[50] As in the baseline model, this inverse function is always well-defined under the restriction to affine strategies (which, once again, is without loss of generality in the class of polynomial strategies).

Notice that, when the retailers compete *á la* Cournot, the equilibrium of the game is the same regardless of OBA being implemented or not. This is because, also in this model, wary beliefs coincide with passive beliefs under quantity competition: hence, in both the disclosure standards, both M_i and R_i always believe that S has offered M_{-i} the equilibrium contract (i.e., $t^e_{-i} = t^*_{-i}$). By contrast, the implementation of OBA affects the equilibrium outcome of the game when retailers compete in a Bertrand fashion.

VI(ii). *Equilibrium and welfare*

Given the specification of off-equilibrium path beliefs, proceeding as in the baseline analysis, one can characterize and compare the equilibrium outcomes under the two disclosure standards.[51] The main welfare result is shown in the following Proposition.

Proposition 3. Under Bertrand competition, the implementation of OBA (by both manufacturers) increases total industry profits and consumer surplus if and only if $\varphi < \varphi^* \triangleq \frac{1}{3-\gamma(1+\gamma)}$.

Thus, the welfare comparison between the two contract disclosure standards yields the same qualitative results shown in Section IV.[52] Namely, the implementation of OBA is total welfare increasing if and only if φ is relatively low. Once again, this result is driven by the role played by the *belief effects* on the extent of the multiple marginalization problem.

In particular, absent OBA, prices along the supply chain are decreasing in φ (see the Appendix). The intuition is rather similar to the baseline model. By the logic of wary beliefs, upon receiving an unexpectedly high input price t_i, each M_i believes that also its rival is facing a higher input price as compared to the equilibrium level. However, the incentives of M_i to pass through this higher t_i to R_i are affected by φ. To gain intuition, it is again useful to consider the extreme cases. If $\varphi = 0$, then, regardless of the offer w_i, R_i always believes that its competitor is facing the equilibrium wholesale price (passive beliefs). By contrast, if $\varphi = 1$, R_i believes that the unexpectedly higher w_i unambiguously reveals an unexpectedly high input price t_i, which in turn (by the logic of wary beliefs) is expected to induce a higher t_{-i} and, as a consequence, a higher w_{-i}. By the strategic complementarity between the retailers' instruments, it then follows that, for given wholesale prices, retailers have stronger incentives to set higher final prices when φ is larger. Hence, as φ increases, the multiple

[51] Once again, I restrict attention to a symmetric equilibrium, which is with no loss of generality within the class of polynomial strategies.

[52] Indeed, also the qualitative results on firms' individual profits shown in the baseline model (with Bertrand competition) carry over to this more complex market structure: see the Online Appendix.

marginalization problem magnifies, which in turn induces M_i to pass through the input price to a lesser extent and, by a backward induction logic, also the upstream supplier to set a lower input price. Finally, due to the multiple marginalization problem, the lower prices along the supply chain induced by a higher value of φ increase industry profits and benefit consumers.

Finally, notice that, when final products are more homogeneous, the implementation of OBA is more likely to be welfare enhancing (φ^* being increasing in γ). This is because, as γ grows larger, the strategic complementarity forces weaken, thereby reducing the above mentioned concerns which refrain the upstream firms from charging relatively low (input and wholesale) prices when all contracts are kept secret.

VI(iii). *Upstream horizontal merger*

Comparing the equilibrium outcomes of the baseline analysis and of the extension examined in this section, one can easily analyze the welfare effects of an upstream horizontal merger.[53] Thus, suppose that the status quo industry structure is as in Figure 2 and that the two manufacturers agree to merge, so that (if the merger is cleared by the competent Antitrust authority) the post-merger industry structure would be the one represented in Figure 1. Then, the welfare effects of such a merger are likely to depend also on the downstream players' beliefs.[54] Specifically, assuming for simplicity that the same contract disclosure standard prevails pre- and post-merger, it is easy to obtain what follows.

Proposition 4. A merger between manufacturers is always anticompetitive when OBA policies are in place. By contrast, under secret contracts, it is pro-competitive whenever $\varphi > \overline{\varphi} \triangleq \frac{2-\gamma}{3-6\gamma+\gamma^2+\gamma^3} (> 0)$, with $\overline{\varphi} < 1$ for $\gamma < \underline{\gamma} \approx 0.21$.

The intuition is as follows. On the one hand, the considered merger eliminates upstream competition, which, *ceteris paribus*, induces higher wholesale prices.[55] As a consequence, absent any *belief effect*, the merger

[53] Upstream horizontal mergers in two-tier industries have been extensively analyzed in the literature: see, e.g., Horn and Wolinsky [1988] and Ziss [1995].

[54] Indeed, it is not straightforward to assume that the belief parameter φ takes the same value pre- and post-merger. This is because in the pre-merger industry it is the input supplier to offer multiple contracts, whereas post-merger is the manufacturer. Put it another way, from the retailers' viewpoint, the relative probabilities of each upstream player deviating from its equilibrium behavior may well depend on the industry structure itself. However, let me abstract for simplicity from these further complications, and consider the same φ under both industry structures. Moreover, I also rule out *efficiencies* brought up by the merger or an increase in the *buyer power* of the merged entity *vis-á-vis* the input supplier, so as to isolate the role played by the retailers' beliefs in the welfare effects of the merger.

[55] For this reason, under both the disclosure standards, manufacturers always benefit from merging (see the Online Appendix).

unambiguously leads to higher retail prices. On the other hand, under secret contracts there is a countervailing force driven by the retailers' beliefs, whereby (by strategic complementarity) the multiple marginalization problem bites more in the post-merger industry structure, which, all else equal, disciplines the merged entity. On the balance, the procompetitive effect driven by the retailers' beliefs outweighs the anticompetitive effect stemming from the elimination of upstream competition when the former effect is relatively strong whereas the latter is relatively weak. This is the case when, respectively: (i) φ is relatively high, so that, by the logic of symmetric beliefs, the merged entity is more seriously concerned by the excessive retail prices induced by increasing the wholesale prices;[56] and (ii) γ is relatively low, since in this case manufacturers have a strong market power even in the pre-merger scenario.

VII. CONCLUSIONS

Modeling multi-tier supply chains in which distribution contracts are secret and downstream firms compete in the same market brings up novel nontrivial issues as for the specification of players' off-equilibrium path beliefs, which do not arise in the simpler two-tier industries analyzed by a number of previous contributions. Specifically, upon receiving an out-of-equilibrium offer, the beliefs of a downstream firm about the offers received by its rivals may well depend on its conjecture about the identity of the deviating upstream firm along the supply chain.

To shed light on these issues, I have considered the simplest possible model, namely a three-tier supply chain with two symmetric (competing) retailers supplied by a common manufacturer, which in turn purchases an essential input from an upstream monopolist. In this setting, retailers' beliefs can be specified as in the previous literature if the unexpected offer reflects the out-of-equilibrium behavior of the manufacturer. However, an out-of-equilibrium offer made by the input supplier is passed through by the manufacturer to the same extent to both retailers, thereby they hold symmetric beliefs when the deviation is attributed to the upstream supplier.

Rather than being a purely technical modeling choice, the way these beliefs are specified plays a crucial role in the evaluation of the welfare effects of contract disclosure provisions often employed in the real world, such as OBA

[56] This is because $\varphi = 1$ corresponds to symmetric retailers' beliefs (on w^e_{-i}) in the post-merger industry, and to wary retailers' beliefs (on t^e_{-i}) in the post-merger industry. Slightly abusing notation, while with symmetric beliefs $\frac{\partial w^e_{-i}(\cdot)}{\partial w_i} = 1$, with wary beliefs $\frac{\partial w^e_{-i}(\cdot)}{\partial w_i} = \frac{\partial t^e_i(\cdot)}{\partial t_i} \frac{\partial \mathcal{W}_{-i}(\cdot)}{\partial t_{-i}} \in (0, 1)$, which entails that the multiple marginalization problem is more severe in the post-merger industry.

policies. Indeed, in this simple model, whether an OBA policy implemented by the manufacturer mitigates or exacerbates the multiple marginalization problem arising under linear contracts depends on the relative probability that the retailers assign to an (eventually observed) out-of-equilibrium offer to come as a result of the manufacturer's rather than the upstream supplier's deviation. Specifically, under Cournot (Bertrand) competition, the implementation of OBA increases total industry profit and consumer surplus, as compared to the case in which all contracts are kept secret, if and only if retailers believe that any out-of-equilibrium offer is sufficiently likely to reflect a deviation by the upstream supplier (the manufacturer). Moreover, if the manufacturer and the retailers efficiently bargain over the choice of the disclosure standard, there will be under-adoption (over-adoption) of OBA compared to the social optimum under Cournot (Bertrand) competition.

More generally, as suggested by an extension analyzed in this paper, these subjective beliefs of downstream firms may also play a crucial role in the assessment of vertical restraints or horizontal mergers, *inter alia*. The complex set of market variables and institutional aspects shaping these beliefs thus constitute important elements that competition authorities must take into consideration in their assessments.

APPENDIX A

PROOFS

Proof of Lemma 1. The result is obtained solving the FOCs derived in Section III(i). ■

Proof of Lemma 2. The result is obtained solving the FOCs derived in Section III(ii). ■

Proof of Proposition 1. Comparing the equilibrium outputs yields $q^*_{1,C} > q^*_{0,C} \Leftrightarrow \varphi > \frac{1}{3}$, for all $\gamma \in [0,1]$, from which the result concerning consumer surplus trivially follows. Equilibrium profits in the presence of OBA are as follows:

$$\pi^S_{1,C} = \frac{1}{2(4+\gamma)}, \quad \pi^M_{1,C} = \frac{1}{(4+\gamma)^2}, \quad \pi^R_{1,C} = \frac{1}{4(4+\gamma)^2}.$$

Analogously, firms' profits when OBA is not implemented are

$$\pi^S_{0,C} = \frac{4(2+\varphi\gamma)}{(8+(1+3\varphi)\gamma)^2}, \quad \pi^M_{0,C} = \frac{2(2+\varphi\gamma)}{(8+(1+3\varphi)\gamma)^2}, \quad \pi^R_{0,C} = \frac{1}{(8+(1+3\varphi)\gamma)^2}.$$

The results concerning firms' and industry profits easily follow by direct comparisons, noting that all firms' profits without OBA are decreasing in φ. ■

Proof of Lemma 3. From equations (3) and (4), imposing symmetry, it follows that the equilibrium strategies $\mathcal{P}(w,t)$ and $\mathcal{W}(w,t)$ must solve:

$$\begin{cases} Q(\mathcal{P}(\cdot),\mathcal{P}(\mathcal{W}(\cdot))) + (\mathcal{P}(\cdot)-w)Q_1(\mathcal{P}(\cdot),\mathcal{P}(\mathcal{W}(\cdot))) = 0, \\ \frac{\partial \mathcal{P}(\cdot)}{\partial w}\left((w-t)Q_2(\mathcal{P}(\cdot),\mathcal{P}(\mathcal{W}(\cdot))) + (\mathcal{W}(\cdot)-t)Q_1(\mathcal{P}(\mathcal{W}(\cdot)),\mathcal{P}(\cdot))\right) \\ \qquad + Q(\mathcal{P}(\mathcal{W}(\cdot)),\mathcal{P}(\cdot)) = 0. \end{cases}$$

Following the literature (e.g., Rey and Vergé, 2004; Gaudin, 2019), I restrict attention to polynomial solutions of this system. Within this class of functions, by replicating the analysis carried out in Gaudin (2019), it can be shown that any solution must be affine:[57]

$$\mathcal{P}(w,t) = \rho_0(t) + \rho_1 w, \qquad \mathcal{W}(w,t) = \omega_0(t) + \omega_1 w,$$

with $\rho_1, \omega_1 \in \mathbb{R}$ and $\rho_0, \omega_0 : t \in \mathbb{R}^+ \to \mathbb{R}$ to be determined in order for the considered functions to solve the above PDE system.

After substituting these functions, the system rewrites

$$\begin{cases} 1-\gamma-2(\rho_0+\rho_1 w)+\gamma(\rho_0+\rho_1(\omega_0(t)+\omega_1 w))+w=0, \\ \rho_1\left((w-t)\gamma-(\omega_0(t)+\omega_1 w-t)\right)+1-\gamma-(\rho_0+\rho_1(\omega_0(t)+\omega_1 w)) \\ \qquad +\gamma(\rho_0+\rho_1 w)=0, \end{cases}$$

which must hold for any w. Collecting terms containing w yields

$$\begin{cases} -2\rho_1+\gamma\rho_1\omega_1+1=0 \\ 2\rho_1(\gamma-\omega_1)=0 \end{cases}$$

which, since the second-order condition of M's problem is satisfied if and only if $\rho_1 > 0$, gives

$$\rho_1 = \frac{1}{2-\gamma^2}, \qquad \omega_1 = \gamma.$$

Next, collecting the other terms gives

$$\rho_0(t) = \frac{(1-\gamma)(4+\gamma(2-\gamma(2+\gamma)+t))}{(2-\gamma^2)(4-\gamma-\gamma^2)}, \qquad \omega_0(t) = \frac{(1-\gamma)(2-\gamma^2+(2-\gamma)t)}{4-\gamma-\gamma^2}.$$

For every t, the SPNE value for the wholesale price solves $w^*(t) = \omega_0(t) + \omega_1 w^*(t)$, and the retail price is $p^*(t) = \rho_0(t) + \rho_1 w^*(t)$. The result is given by

$$w^*(t) \triangleq \frac{2-\gamma^2+(2-\gamma)t}{4-\gamma-\gamma^2}, \qquad p^*(t) \triangleq \frac{3-\gamma-\gamma^2+t}{4-\gamma-\gamma^2}.$$

Moving backward to the first stage of the game, the input supplier's problem is

$$\max_{t\geq 0} 2tQ(p^*(t),p^*(t)),$$

[57] Details are available upon request.

whose FOC,

$$Q(p^*(t), p^*(t)) + t\frac{\partial p^*(t)}{\partial t}\left(Q_1(p^*(t), p^*(t)) + Q_2(p^*(t), p^*(t))\right) = 0,$$

yields $t^*_{1,B} = \frac{1}{2}$, from which the equilibrium values $w^*_{1,B} = w^*(t^*_{1,B})$ and $p^*_{1,B} = p^*(t^*_{1,B})$ can be easily obtained. ■

Proof of Lemma 4. From equations (5) and (6), imposing symmetry, it follows that the equilibrium strategies $\mathcal{P}(w)$ and $\mathcal{W}(w,t)$ must solve:

$$\begin{cases} \varphi Q(\mathcal{P}(\cdot), \mathcal{P}(\cdot)) + (1-\varphi)Q(\mathcal{P}(\cdot), \mathcal{P}(\mathcal{W}(\cdot))) + (\mathcal{P}(\cdot) - w)\big(\varphi Q_1(\mathcal{P}(\cdot), \mathcal{P}(\cdot)) \\ \qquad + (1-\varphi)Q_2(\mathcal{P}(\cdot), \mathcal{P}(\mathcal{W}(\cdot)))\big) = 0, \\ \frac{\partial \mathcal{P}(\cdot)}{\partial w}\left((w-t)Q_2(\mathcal{P}(\cdot), \mathcal{P}(\mathcal{W}(\cdot))) + (\mathcal{W}(\cdot) - t)Q_1(\mathcal{P}(\mathcal{W}(\cdot)), \mathcal{P}(\cdot))\right) \\ \qquad + Q(\mathcal{P}(\mathcal{W}(\cdot)), \mathcal{P}(\cdot)) = 0. \end{cases}$$

Once again, I restrict attention to symmetric and affine solutions of this PDE system, which can be done without loss of generality within the class of polynomial solutions. Specifically, similarly to the previous case, consider

$$\mathcal{P}(w) = \zeta_0 + \zeta_1 w, \qquad \mathcal{W}(w,t) = \psi_0(t) + \psi_1 w,$$

with $\zeta_0, \zeta_1, \psi_1 \in \mathbb{R}$, and $\psi_0 : t \in \mathbb{R}^+ \to \mathbb{R}$ to be determined. As before, $\zeta_1 > 0$ is required in order for the second-order condition of M's problem to be satisfied.

Since $\mathcal{W}(w, t^*_{0,B}) = \psi_0(t^*_{0,B}) + \psi_1 w$, the above system rewrites as

$$\begin{cases} \varphi(1-\gamma-(1-\gamma)(\zeta_0+\zeta_1 w)) + (1-\varphi)(1-\gamma-(\zeta_0+\zeta_1 w) \\ \qquad + \gamma(\zeta_0+\zeta_1(\psi_0(t^*_{0,B})+\psi_1 w))) - (\zeta_0+\zeta_1 w - w) = 0, \\ \zeta_1\left(\gamma(w-t) - (\psi_0(t)+\psi_1 w - t)\right) + 1 - \gamma - (\zeta_0 + \zeta_1(\psi_0(t)+\psi_1 w)) \\ \qquad + \gamma(\zeta_0+\zeta_1 w) = 0. \end{cases}$$

Imposing that these two equations must hold for any w yields

$$\zeta_1 = \frac{1}{2-\gamma\varphi-\gamma^2(1-\varphi)}, \qquad \psi_1 = \gamma,$$

and, for any given candidate equilibrium value $t^*_{0,B}$,

$$\zeta_0 = \frac{1}{2-\gamma}\frac{1-\gamma(1-(1-\varphi)\psi_0(t^*_{0,B}))}{2-\gamma\varphi-\gamma^2(1-\varphi)},$$

$$\psi_0(t) = \frac{1-\gamma}{2(2-\gamma)}\left(\gamma(\varphi(\gamma-1+\psi_0(t^*_{0,B})) - \psi_0(t^*_{0,B}) - t - \gamma) + 2(1+t)\right).$$

Next, imposing $w^*(t) = \psi_0(t) + \psi_1 w^*(t)$ gives

$$w^*(t) \triangleq \frac{t}{2} + \frac{2+\gamma(\varphi(\gamma-1+\psi_0(t^*_{0,B})) - \psi_0(t^*_{0,B}) - \gamma)}{2(2-\gamma)}.$$

The FOC of the input supplier's problem,

$$\max_{t\geq 0} 2tQ(\mathcal{P}(w^*(t))), \mathcal{P}(w^*(t)))),$$

then writes as

$$Q(\mathcal{P}(w^*(t)), \mathcal{P}(w^*(t))) + t\zeta_1 \frac{\partial w^*(t)}{\partial t} \left(Q_1(\mathcal{P}(w^*(t)), \mathcal{P}(w^*(t)) + Q_2(\mathcal{P}(w^*(t)), \mathcal{P}(w^*(t)))\right) = 0,$$

where $\mathcal{P}(w^*(t)) = \zeta_0 + \zeta_1 w^*(t)$. Imposing $t = t^*_{0,B}$ and solving this equation yields the equilibrium value for the input price, from which, by substitutions, the other equilibrium values can be easily found. ■

Proof of Proposition 2. Comparing the equilibrium prices yields $p^*_{1,B} > p^*_{0,B} \Leftrightarrow \varphi > \frac{1}{3}$, for all $\gamma \in [0, 1]$, from which the result concerning consumer surplus trivially follows. Equilibrium profits in the presence of OBA are as follows:

$$\pi^S_{1,B} = \frac{1}{8 + 6\gamma - 4\gamma^2 - 2\gamma^3}, \ \pi^M_{1,B} = \frac{2 - \gamma^2}{2(1 + \gamma)(4 - \gamma - \gamma^2)^2}, \ \pi^R_{1,B} = \frac{1 - \gamma}{4(1 + \gamma)(4 - \gamma - \gamma^2)^2}.$$

Analogously, firms' profits when OBA is not implemented are

$$\pi^S_{0,B} = \frac{4(2 - \gamma^2(1 - \varphi) - \gamma\varphi)}{(1 + \gamma)(8 - \gamma - 3\gamma^2(1 - \varphi) - 3\gamma\varphi)^2}, \ \pi^M_{0,B} = \frac{2(2 - \gamma^2(1 - \varphi) - \gamma\varphi)}{(1 + \gamma)(8 - \gamma - 3\gamma^2(1 - \varphi) - 3\gamma\varphi)^2},$$
$$\pi^R_{0,B} = \frac{1 - \gamma}{(1 + \gamma)(8 - \gamma - 3\gamma^2(1 - \varphi) - 3\gamma\varphi)^2}.$$

Hence, $\pi^S_{0,B} > \pi^S_{1,B}$ if and only if $\varphi > \underline{\varphi}$, where

$$\underline{\varphi} \triangleq \frac{8 + \gamma - 5\gamma^2 - \sqrt{8(8 + 2\gamma - 11\gamma^2 + \gamma^3 + 2\gamma^4)}}{9\gamma(1 - \gamma)}.$$

The other results concerning firms' profits easily follow by direct comparisons, noting that all profits without OBA are increasing in φ. ■

Proof of Proposition 3. The equilibrium of the game with upstream competition, in the two disclosure standards, is obtained as follows.

Equilibrium with OBA. First consider the case in which both manufacturers implement an OBA policy with their exclusive retailers. As pointed out above, in this game, upon observing t_i, M_i and R_i have the same beliefs about t_{-i}. As a consequence, they both believe $p^e_{-i}(t_i) = \mathcal{P}_{-i}(\mathcal{W}_{-i}(t^e_{-i}), t^e_{-i})$, with $t^e_{-i} = \mathcal{T}_{-i}(t_i)$.

Thus, upon being offered w_i, and knowing the input price t_i, R_i's equilibrium strategy is given by

$$\mathcal{P}_i(w_i, t_i) \triangleq \frac{1}{2}(1 + w_i - \gamma(1 - p^e_{-i}(t_i))).$$

Notice that, since p^e_{-i} does not depend on w_i, when OBA are implemented, the retailers' pass-through rate is simply $\frac{\partial \mathcal{P}_i(\cdot)}{\partial w_i} = \frac{1}{2}$, given that the value of w_i does not entail any *belief effect*.

Moving backward to the previous stage, M_i's best reply to the offer t_i received by S is then

$$\mathcal{W}_i(t_i) \triangleq \frac{1}{2}(1 + t_i - \gamma(1 - p^e_{-i}(t_i))),$$

which, substituted into $\mathcal{P}_i(\cdot)$, gives

$$\mathcal{P}_i(\mathcal{W}_i(t_i), t_i) \triangleq \mathcal{P}_i(t_i) = \frac{1}{4}t_i + \frac{3}{4}(1 - \gamma(1 - \mathcal{P}_{-i}(\mathcal{T}_{-i}(t_i)))),$$

where $\mathcal{P}_{-i}(\mathcal{T}_{-i}(t_i)) \triangleq \mathcal{P}_{-i}(\mathcal{W}_{-i}(\mathcal{T}_{-i}(t_i)), \mathcal{T}_{-i}(t_i))$. Notably, unlike in the baseline model, here in the presence of OBA the offers made by S do not define a proper subgame, since there is still imperfect information, from M_i's and R_i's viewpoint, concerning the input price t_{-i}. In particular, by the logic of wary beliefs, for any t_i offered to M_i, S optimally sets t_{-i}—that is,

$$\mathcal{T}_{-i}(t_i) \triangleq \arg\max_{t_{-i}\geq 0}\ t_i Q\left(\mathcal{P}_i(t_i), \mathcal{P}_{-i}(t_{-i})\right) + t_{-i} Q\left(\mathcal{P}_{-i}(t_{-i}), \mathcal{P}_i(t_i)\right).$$

Hence, the strategy $\mathcal{T}_{-i}(\cdot)$ satisfies the FOC

$$\begin{aligned} t_i \frac{\partial \mathcal{P}_{-i}(t_{-i})}{\partial t_{-i}} Q_2\left(\mathcal{P}_i(t_i), \mathcal{P}_{-i}(\mathcal{T}_{-i}(t_i))\right) + Q\left(\mathcal{P}_{-i}(\mathcal{T}_{-i}(t_i)), \mathcal{P}_i(t_i)\right) \\ + \mathcal{T}_{-i}(t_i)\frac{\partial \mathcal{P}_{-i}(t_{-i})}{\partial t_{-i}} Q_1\left(\mathcal{P}_{-i}(\mathcal{T}_{-i}(t_i)), \mathcal{P}_i(t_i)\right) = 0. \end{aligned}$$

A symmetric equilibrium is given by a pair of functions $\mathcal{P}(t)$ and $\mathcal{T}(t)$ such that $t_1^* = \mathcal{T}(t_1^*)$ and $p_1^* = \mathcal{P}(\mathcal{T}(t_1^*))$.[58]

The unique equilibrium in polynomial strategies is such that the input supplier charges both manufacturers $t_1^* = \frac{4-3\gamma^2}{8-3\gamma(1+\gamma)}$, each manufacturer supplies the final good at a wholesale price $w_1^* = 1 - \frac{2-\gamma}{8-3\gamma(1+\gamma)}$, and the retailers set a final price $p_1^* = 1 - \frac{1}{8-3\gamma(1+\gamma)}$. Details are in the Online Appendix.

Equilibrium without OBA. I now turn to the case in which all contracts are kept secret. At the final price setting stage, for any offer w_i received by M_i, R_i's best reply characterized as follows:

$$\mathcal{P}_i(w_i) \triangleq \arg\max_{p_i\geq 0}\left(p_i - w_i\right)\left(\varphi Q\left(p_i, \mathcal{P}_{-i}(\mathcal{W}_{-i}(\mathcal{T}_{-i}(t_i^e)))\right) + (1-\varphi)Q\left(p_i, p_0^*\right)\right),$$

where, $t_i^e = \mathcal{W}_i^{-1}(w_i)$ and $p_0^* = \mathcal{P}_{-i}(\mathcal{W}_{-i}(t_0^*))$. Therefore, in equilibrium, the following FOC must be satisfied:

$$\begin{aligned} (\mathcal{P}_i(\mathcal{W}_i(t_i)) - \mathcal{W}_i(t_i))\left[\varphi Q_1\left(\mathcal{P}_i(\mathcal{W}_i(t_i)), \mathcal{P}_{-i}(\mathcal{W}_{-i}(\mathcal{T}_{-i}(t_i)))\right)\right. \\ \left. + (1-\varphi)Q_1\left(\mathcal{P}_i(\mathcal{W}_i(t_i)), p_0^*\right)\right] \\ + \varphi Q\left(\mathcal{P}_i(\mathcal{W}_i(t_i)), \mathcal{P}_{-i}(\mathcal{W}_{-i}(\mathcal{T}_{-i}(t_i)))\right) + (1-\varphi)Q\left(\mathcal{P}_i(\mathcal{W}_i(t_i)), p_0^*\right) = 0. \end{aligned}$$

58 Then, clearly, the equilibrium wholesale price is $w_1^* = \mathcal{W}_i(t_1^*)$.

Unlike in the game with OBA, here (for all $\varphi > 0$) $\frac{\partial \mathcal{P}_i(w_i)}{\partial w_i} \neq \frac{1}{2}$, since w_i entails a *belief effect*, being used by R_i to infer t_i (hence, t_{-i}).

Moving backward to the previous stage, upon receiving an offer t_i, M_i's equilibrium strategy is obtained as follows

$$\mathcal{W}_i(t_i) \triangleq \arg\max_{w_i \geq 0} \left(w_i - t_i\right) Q\left(\mathcal{P}_i(w_i), \mathcal{P}_{-i}(\mathcal{W}_{-i}(\mathcal{T}_{-i}(t_i)))\right).$$

Therefore, in equilibrium, the following FOC must be satisfied:

$$\begin{aligned} &Q\left(\mathcal{P}_i(\mathcal{W}_i(t_i)), \mathcal{P}_{-i}(\mathcal{W}_{-i}(\mathcal{T}_{-i}(t_i)))\right) \\ &\quad + \left(\mathcal{W}_i(t_i) - t_i\right) \frac{\partial \mathcal{P}_i(\cdot)}{\partial w_i} Q_1\left(\mathcal{P}_i(\mathcal{W}_i(t_i)), \mathcal{P}_{-i}(\mathcal{W}_{-i}(\mathcal{T}_{-i}(t_i)))\right) = 0. \end{aligned}$$

Finally, by the logic of wary beliefs, for any t_i offered to M_i, S optimally sets t_{-i}—that is,

$$\mathcal{T}_{-i}(t_i) \triangleq \arg\max_{t_{-i} \geq 0} t_i Q\left(\mathcal{P}_i(\mathcal{W}_i(t_i)), \mathcal{P}_{-i}(\mathcal{W}_{-i}(t_{-i}))\right) + t_{-i} Q\left(\mathcal{P}_{-i}(\mathcal{W}_{-i}(t_{-i})), \mathcal{P}_i(\mathcal{W}_i(t_i))\right),$$

whose FOC is as follows:

$$\begin{aligned} &t_i \frac{\partial \mathcal{P}_{-i}(\cdot)}{\partial w_{-i}} \frac{\partial \mathcal{W}_{-i}(\cdot)}{\partial t_{-i}} Q_2\left(\mathcal{P}_i(\mathcal{W}_i(t_i)), \mathcal{P}_{-i}(\mathcal{W}_{-i}(t_{-i}))\right) \\ &\quad + Q\left(\mathcal{P}_{-i}(\mathcal{W}_{-i}(\mathcal{T}_{-i}(t_i))), \mathcal{P}_i(\mathcal{W}_i(t_i))\right) \\ &\quad + \mathcal{T}_{-i}(t_i) \frac{\partial \mathcal{P}_{-i}(\cdot)}{\partial w_{-i}} \frac{\partial \mathcal{W}_{-i}(\cdot)}{\partial t_{-i}} Q_1\left(\mathcal{P}_{-i}(\mathcal{W}_{-i}(\mathcal{T}_{-i}(t_i))), \mathcal{P}_i(\mathcal{W}_i(t_i))\right) = 0. \end{aligned}$$

Summing up, the symmetric equilibrium strategies $\mathcal{P}(\cdot), \mathcal{W}(\cdot), \mathcal{T}(\cdot)$ solve the system formed by the ODEs displayed above, with $t_0^* = \mathcal{T}(t_0^*)$ and $p_0^* = \mathcal{P}(\mathcal{W}(t_0^*))$.

The unique equilibrium within the class of polynomial strategies is such that the input supplier charges both manufacturers $t_0^* = \frac{(2-\gamma^2)(2-\gamma^2\varphi)}{8-\gamma(3+2\gamma+\gamma\varphi(3-\gamma-\gamma^2))}$, each manufacturer supplies the final good at a wholesale price $w_0^* = 1 - \frac{2-\gamma}{8-\gamma(3+2\gamma+\gamma\varphi(3-\gamma-\gamma^2))}$, and the retailers set a final price $p_0^* = 1 - \frac{1}{8-\gamma(3+2\gamma+\gamma\varphi(3-\gamma-\gamma^2))}$. Details are in the Online Appendix.

The result in Proposition 3 then easily follows from comparing p_0^* and p_1^*. ■

Proof of Proposition 4. The results easily follow from comparing, for any contract disclosure standard, the pre-merger and the post-merger retail prices. ■

APPENDIX B

ASSUMPTION (A): A MICROFOUNDATION

This appendix provides a microfoundation for the off-equilibrium path beliefs' specification given in Assumption (A) based on (uncorrelated) *mistakes* committed by players.

Fix $n \in \mathbb{N}$ and two constants $\alpha, \beta \in (0, n)$, and suppose that (retailers believe that) S plays according to the following completely mixed strategy over the interval $[0, 1]$: it sets $t = t^*$ with probability $1 - \frac{\alpha}{n}$, and draws t from a uniform distribution over $[0, 1]$ with complementary probability $\frac{\alpha}{n}$. Analogously, for every t, M charges R_i a wholesale price $w_i = w^*(t)$ with probability $1 - \frac{\beta}{n}$, and draws w_i from a uniform distribution over $[0, 1]$ with complementary probability $\frac{\beta}{n}$.[59] In both cases, (R_i believes that) M sets w_{-i} optimally given w_i (wary beliefs).

In words, the upstream firms are supposed to play *rationally* with a certain probability, which depends on firm-specific coefficients α and β. With complementary probability, they make random offers. Specifically, each R_i believes that, conditional on playing randomly, a firm selects any price with the same probability: roughly speaking, *small deviations* are as much likely to occur as *big* ones. As a consequence, R_i's off-equilibrium path belief on the identity of the deviating upstream firm is exogenous—that is, it does not depend on w_i nor on w^*.

As $n \to \infty$, for all α and β, the described pair of completely mixed strategies converges to the equilibrium strategies—that is, the probability of any deviation tends to zero. Moreover, upon observing any out-of-equilibrium offer $w_i \neq w^*$, R_i's beliefs, obtained by Bayes rule, are such that

$$\Pr[t \neq t^*, w_i = w^*(t) | w_i \neq w^*] \xrightarrow{n\to\infty} \frac{\alpha}{\alpha + \beta},$$

$$\Pr[t = t^*, w_i \neq w^*(t) | w_i \neq w^*] \xrightarrow{n\to\infty} \frac{\beta}{\alpha + \beta},$$

and $\Pr[t \neq t^*, w_i \neq w^*(t) | w_i \neq w^*] \xrightarrow{n\to\infty} 0$. Hence, setting $\varphi = \frac{\alpha}{\alpha+\beta}$ yields the beliefs specified by Assumption (A)—that is, the *belief parameter* φ measures the relative likelihood of each upstream firm making a (zero-probability) *mistake*.

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[59] Notice that the restriction of S's and M's action spaces to the interval $[0, 1]$ is without loss of generality in the considered linear demand setting: prices cannot be negative, and if an upstream firm were to offer any price higher than 1, the downstream firm would find it optimal not to accept the contract (recall that the market dimension is equal to 1), hence the game would end (with a zero profit for each firm) and there would be no need to specify off-equilibrium path beliefs. This formalization can be immediately generalized to any downward-sloping demand curve admitting a finite market dimension.

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