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A NEW SEISMIC DAMAGE PARAMETER BASED ON THE HILBERT-HUANG TRANSFORM

S. Castelli[1], P. Riva[1], A. Belleri[1], L. Danesi[1], L. Rota[1]

[1] University of Bergamo, Department of Engineering and Applied Science, Dalmine, Italy

Abstract: *The objective of this study is to assess the performance of a new damage index based on the Hilbert-Huang Transform (HHT) to evaluate the possible expected structural damage in the case of seismic events. Earthquakes generally occur with a non-predictable time history, non-stationary and nonlinear in nature, therefore, a transform capable of analysing such signals such as HHT is used herein for damage detection. The goal is to obtain the most accurate estimate of potential seismic damage on existing structures. The study proposes a novel formulation for seismic damage identification and a validation through a finite element model representing a column in a 3-storey building. The results show the potential benefit of the use of HHT as earthquake damage index in existing buildings.*

1. Introduction

The objective of this study is to assess the performance of a new damage index based on the Hilbert-Huang Transform (HHT) to evaluate the possible expected structural damage in the case of seismic events. Earthquakes generally occur with a non-predictable time history, non-stationary and nonlinear in nature, therefore, a transform capable of analyzing such signals such as HHT is used herein for damage detection. The goal is to obtain the most accurate estimate of potential seismic damage on existing structures. The study proposes a novel formulation for seismic damage identification and a validation through a finite element model representing a column in a 3-storey building. The results show the potential benefit of the use of HHT as earthquake damage index in existing buildings.

2. Hilbert-Huang Transform

In this section, we briefly review the main mathematical aspects of the Hilbert-Huang trans-form (HHT) and then develop and test the associated index. The HHT is useful for performing time-frequency analysis of nonstationary and nonlinear data. The procedure consists of the following steps:

- Decompose the seismic signal into a set of intrinsic mode functions (IMF);
- For each IMF x_i perform:
- the Hilbert transform (HT) for calculating the analytical signal $z_i = x_i(t) + j\mathcal{H}\{x_i(t)\}$;
- express z_i as $z_i(t) = a_i(t)e^{j\theta_i(t)}$, where $a_i(t)$ represents the instantaneous amplitude and $\theta_i(t)$ its instantaneous phase;
- calculate the instantaneous energy $\|a_i(t)\|^2$ and the instantaneous frequency, defined as $\omega_i(t) \equiv \frac{d\theta_i(t)}{dt}$.

1.1. Empirical Mode Decomposition

Empirical mode decomposition (EMD) allows any complex signal to be decomposed into non-sinusoidal oscillatory signals [1], each of which is referred to as an IMF with these properties:

- throughout the signal, the number of extremes and zero steps must be the same or differ by at most one from the original signal;
- at any point in the signal, the average value of the envelope defined by the local maxima and the envelope of local minima must be zero.

At this point, what is stated in [1] and [2] is carried out. All local maxima are identified and connected by a cubic spline to create the upper envelope, called $u_{max}(t)$,, of the signal $X(t)$. A very similar procedure is performed to create the envelope of the local minimum of the signal called $u_{min}(t)$. The two envelopes found include the entire signal between them. Their average is defined as $m_1(t)$ and is calculated as follows:

$$m_1(t) = \frac{\left(u_{max}(t) - u_{min}(t)\right)}{2} \tag{1}$$

In addition, the difference between the original seismic signal and m_1 (t) is expressed as:

$$h_1(t) = X(t) - m_1(t) \tag{2}$$

then it becomes:

$$h_{11}(t) = h_1(t) - m_{11}(t) \tag{3}$$

Where $m_{11}(t)$ is the new average between the envelopes of $h_1(t)$. This process will be iterated k times until we obtain

$$h_{1k}(t) = h_{1(k-1)}(t) - m_{1k}(t) \tag{4}$$

Where $h_{1k}(t) = c_1(t)$ should contain the shortest period. After that, since there is a residual called $r(t)$, the first IMF is derived from the original seismic signal as

$$r_1(t) = X(t) - c_1(t) \tag{5}$$

The residual $r(t)$ contains components with higher periods and is considered as a new original signal. We repeat the original signal going through the same process as just described until we obtain all the functions $r(t)$

$$r_j(t) = r_{(j-1)}(t) - c_j(t), \qquad j = 2,3,\dots,n \tag{6}$$

The recursive procedure thus composed is interrupted through the following criteria:

- the value of the component $c_n(t)$ or the value of the residual $r(t)$ is less than a given value;
- the residual $r(t)$ is presented as a monotone function with only one extreme or a constant function, therefore, it is no longer possible to extract IMFs.

Finally, the initial seismic signal $X(t)$ is rewritten as the sum of all IMFs plus the residual $r(t)$, as in Equation 7.

$$X(t) = \sum_{j=1}^{n} c_j(t) + r_n(t) \tag{7}$$

1.2. Hilbert Spectral Analysis

To generate the Hilbert spectrum or Hilbert spectral analysis (HSA), the HT [1] [2] is applied to each IMF $c(t)$

$$y_j(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{c_j(\tau)}{t - \tau} d\tau \tag{8}$$

where P represents the Cauchy principal value of the integral. The IMFs $c_j(t)$ and the HT $y_j(t)$ form a new analytic signal $z_j(t)$:

$$z_j(t) = c_j(t) + iy_j(t) = a_j(t)e^{i\theta_j(t)} \tag{9}$$

In which:

- $a_j(t)$ is the radius of rotation of the analytical signal defined as

$$a_j(t) = \sqrt{c_j^2(t) + y_j^2(t)} \tag{10}$$

- $\theta_j(t)$ is the instantaneous phase function defined as

$$\theta_j(t) = arctan\left(\frac{y_j(t)}{c_j(t)}\right) \tag{11}$$

The instantaneous angular velocity $\omega_j(t)$ is calculated as the derivative of the instantaneous phase function, from which in addition the instantaneous frequency can be calculated as:

$$\omega_j(t) = \frac{d\theta_j(t)}{dt} = 2\pi \cdot f_j(t) \rightarrow f_j(t) = \frac{\omega_j(t)}{2\pi} = \frac{d\theta_j(t)}{dt} \tag{12}$$

Thus, the IMF components can be described as:

$$c_j(t) = Re\left(a_j(t)e^{i\theta_j(t)}\right) = a_j(t)\cos\theta_j(t) \tag{13}$$

Therefore, the initial signal can be rewritten as:

$$X(t) = Re\left[\sum_{j=1}^{n} a_j(t)\cos\left(\int \omega_j(t)\,dt\right)\right] \tag{14}$$

It should be made clear that the residual term $r_n(t)$ of the original signal, Equation 7, is omitted as a constant or at most monotonic function. Equation 14 shows how amplitude and frequency are functions of time and can be represented in a three-dimensional time-frequency-amplitude diagram; this graphical representation is referred to as the Hilbert amplitude spectrum or simply the Hilbert HSA spectrum. The quantities instantaneous amplitude and instantaneous frequency thus refer to the three-dimensional Hilbert spectrum and not to the individual IMF.

3. Index formulation

Extracting a damage index is the process of converting the raw response recorded on the structure into an index, which is usually scalar and can reflect the state of the structure. There are already many indices in the literature that try to give an interpretation of post-seismic damage [3], [4]. The occurrence of structural damage is usually associated with a change in structural properties such as stiffness and energy. Damage-induced energy dissipation is usually the result of material nonlinearity. For example, cracks in concrete or yielding of steel change the stiffness and energy response of the structure under consideration. Therefore, the goal is to develop indices that can detect such changes and correlate them with a damage metric. The correlation between response change and damage extent can follow very complex laws that depend on the type of material (or several materials such as reinforced concrete) and the properties of the seismic input.

Thanks to HSA, we have the possibility to calculate the instantaneous energy, which we will call I_i in each DOF. If we schematize an SDOF system and impose a seismic input on it, we can obtain I_i^g i.e., the instantaneous energy of the ground and I_i^g i.e., the instantaneous energy in the DOF. I_i^g is not sufficient because, although it is fundamental, it does not contain information about the dynamical response (or energy in this case) of the system. In fact, two different excitations may contain similar energies but affect the structure differently and vice versa.

The basic hypothesis of this damage index is as follows: the nonlinearity, if present, alters the relationship between the instantaneous energies recorded in the different DOFs and is localized between them.

Therefore, the damage index for an SDOF can be defined by the following equation

$$ID_{Hilbert-Huang} = \frac{I_i^1}{I_i^g} \tag{15}$$

4. Case study

To test this new damage index, it was decided to apply it to a simple system such as a reinforced concrete column. The column shown in Figure 1 is 12 m high and was discretized every 3 m. A nodal mass of 150 kN/g was used for each node in the two horizontal directions. The nonlinearity was inserted by phenomenological formulation with concentrated plastic hinges of Takeda type at the ends of the beam elements, whose properties are calculated on the cross section (constant for the whole height) consisting of a square geometry with a side of 30 cm with a longitudinal reinforcement of 8 φ 16 and φ 8 2-arm brackets with 30 cm spacing. To make the damage detection more difficult, the nonlinearities at the first level are not considered. We want to investigate the ability of the new index to correctly localize the damage (which occurs on the second level) even though the first level has a larger amount of energy applied to it.

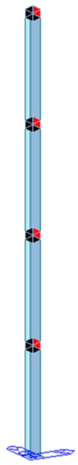
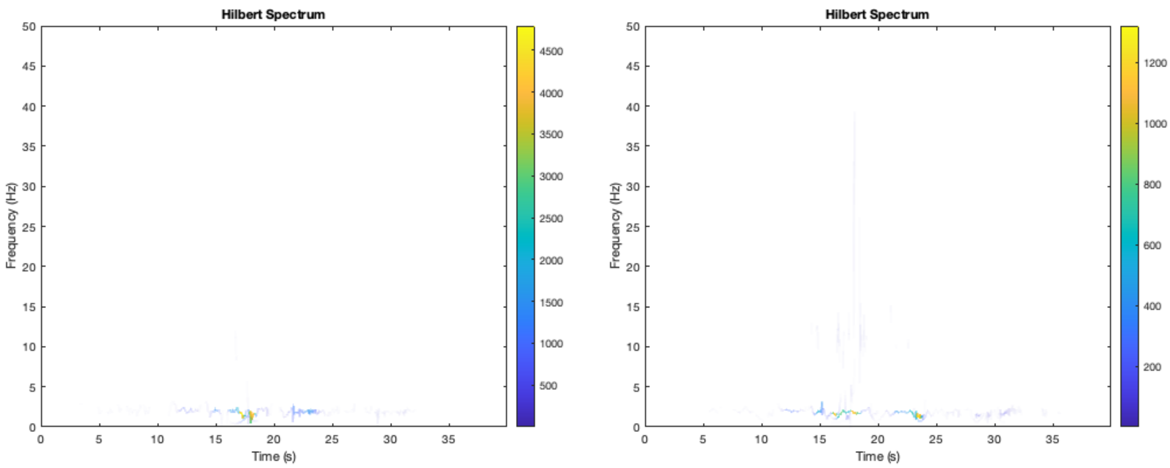
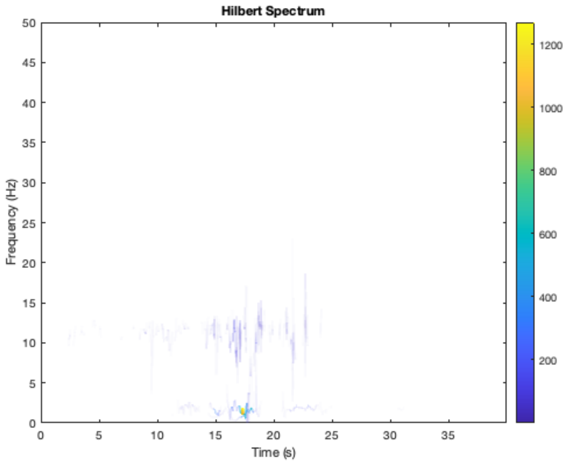


Figure 1. Case study of a reinforced concrete column with the position of the masses drawn in.

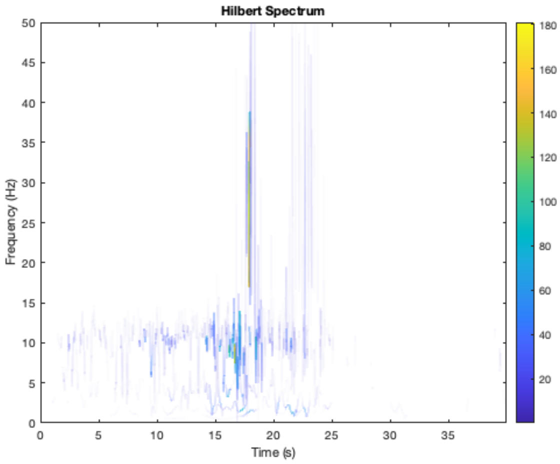
To simulate the seismic event and then generate the synthetic signals (extracted at each node) to test the index, nonlinear dynamic analyzes were performed. The results for the 1980 Irpinia earthquake are shown here. The seismic input was input in only one horizontal direction for better control of the results when benchmarking the index. At this point we proceed with the HSA analysis for all recorded signals, in order: EMD and HHT. Figure 2 shows the Hilbert spectra for the signals at the different levels.



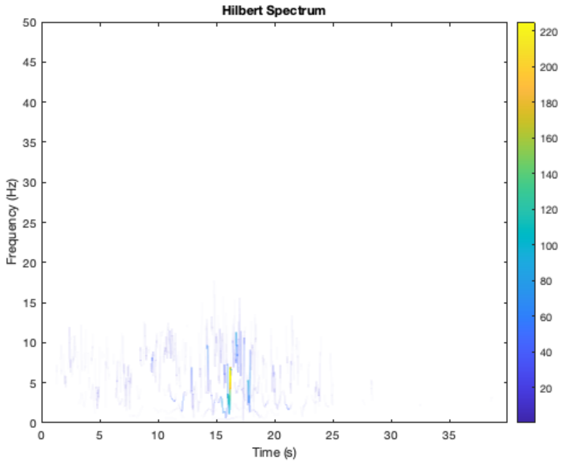
a) Hilbert spectrum for the node at 12m height. *b) Hilbert spectrum for the node at 9m height.*



b) *Hilbert spectrum for the node at 9m height.*



d) Hilbert spectrum for the node at 3m height.



e) Hilbert spectrum for the node at the base.

Figure 2. Hilbert spectra for various recorded signals.

Once all instantaneous energies have been extracted, we proceed to calculate the actual index, i.e., we relate these energies between the different levels by constructing a transfer function of the instantaneous energies. Figure 3 shows the value of the damage index evolved as a function of time. As can be clearly seen, the index never falls below a value of 1. Since the index is a ratio, it tends to ∞ when the denominator tends to 0. To avoid this, it was decided to set the value of the index to 1 as soon as the denominator falls below a certain threshold, knowing that it is highly unlikely that damage will occur at a time associated with infinite energy. The value of the index is shown on the ordinate. In this case, we can see a considerable increase with respect to the second level, a level that actually reports damage in the finite element model, as can be seen in Figure 4, which shows the extracted energies and the hysteretic cycles of the plastic hinges: the activation of plasticity for the second level hinge is easily seen, unlike the others, which remain perfectly in the elastic range (the first level plastic hinge is not shown, since it was modeled elastically). In particular, the kinetic energy associated with the motion and the energy derived by the nonlinearities present. The comparison of Figure 3 with Figure 4 shows the synchronization between the developed damage index and the time when the energy dissipation in the plastic hinges starts, while the comparison of Figure 3 with Figure 5 shows the goodness of the index in locating the damage, in this case the second level of the column.

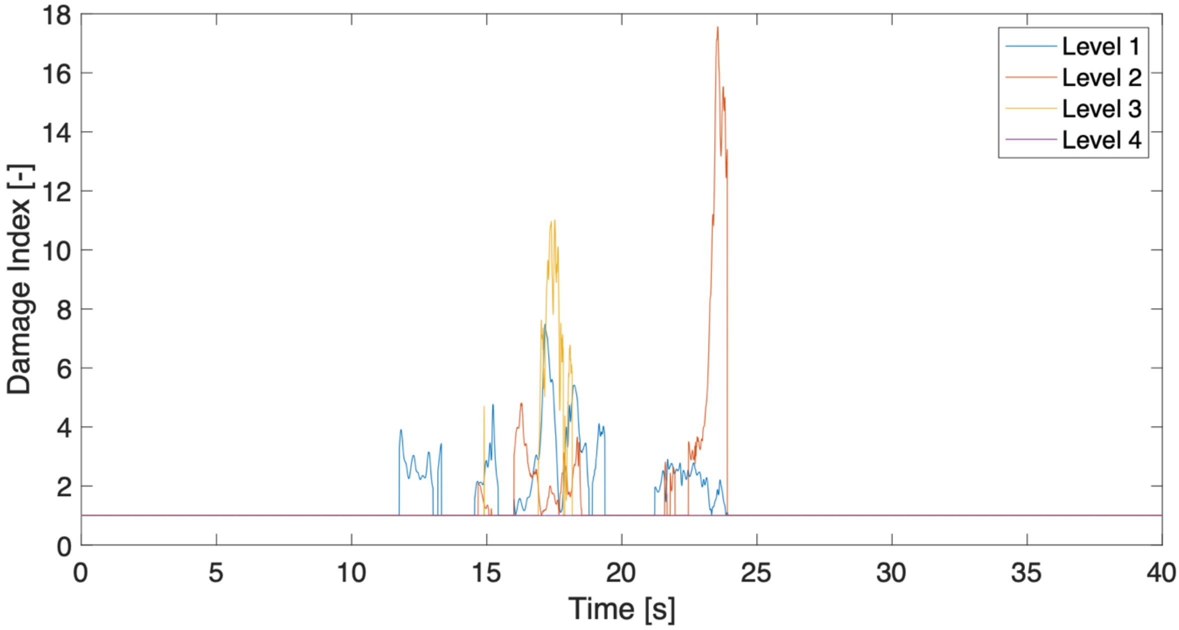


Figure 3. Value of the damage index for different levels as a function of time.

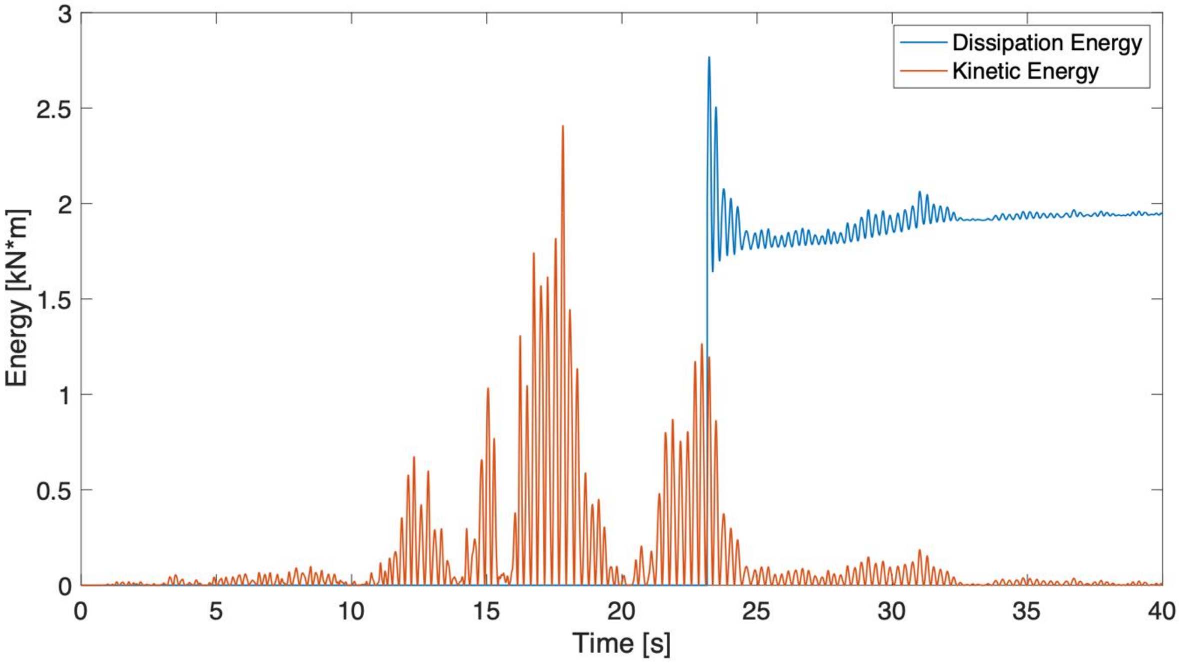
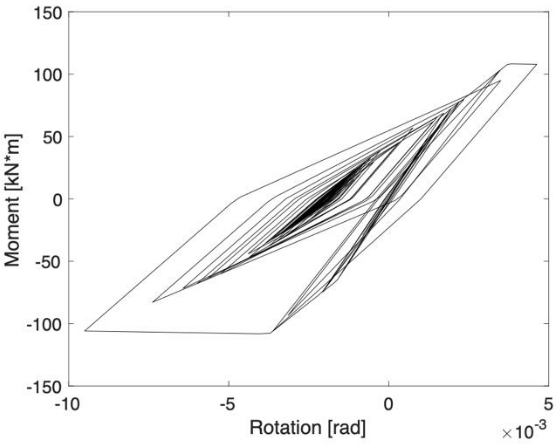
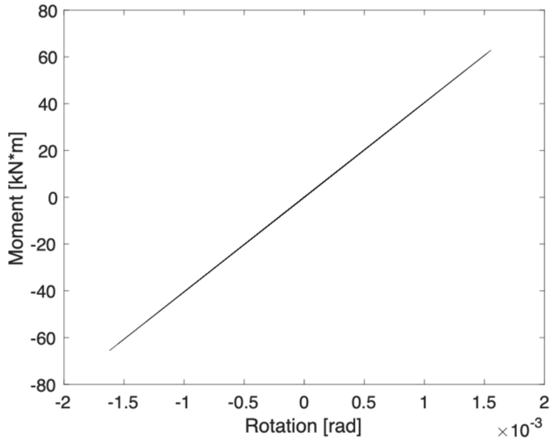


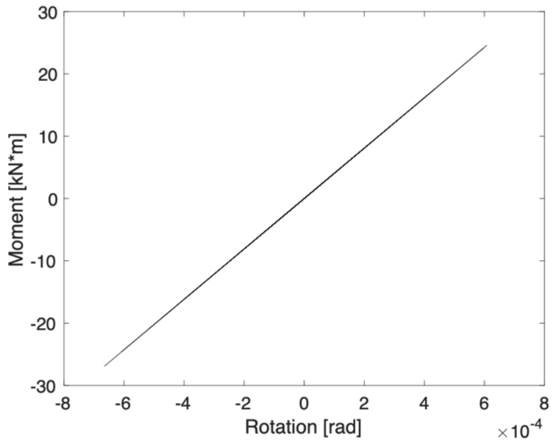
Figure 4. Energies extracted from the finite element model of the column.



a) Hysteretic cycle referred to the lower plastic hinge of the 2nd level.



b) Hysteretic cycle referred to the lower plastic hinge of the 3rd level.



c) Hysteretic cycle referred to the lower plastic hinge of the 4th level.

Figure 5. Hysteretic cycles at various levels of the case study column.

5. Conclusion

The damage index studied, based on an energy formulation of the HHT and tested on a simple system, allowed structural damage to be correctly localized both geometrically and temporally in a numerical benchmark of a reinforced concrete column. The HHT is used to calculate the energy required for the index formulation because it can analyze time histories of nonstationary and nonlinear seismic events. The quantification of damage is still the aspect that needs further research, since the establishment of thresholds is complicated and probably needs to be selected for each material and structure type; a good approach could be a statistical analysis of beam and column samples tested in the laboratory. On the other hand, geometric localization is particularly simple, since the index is calculated between two signals placed on different DOFs, so that in case of damage it is localized between the two DOFs considered.

One aspect not yet studied, but potentially relevant, is temporal localization. As we have seen in the example studied, the index is able to detect the time at which energy dissipation begins. This is not to be confused with the time at which the structure enters the plastic field, because energy dissipation begins when unloading and subsequent reloading occurs. As we know, the effects on a structure are quite different if it enters the plastic field at the beginning or at the end of the seismic event. Thanks to this index, we can know this information and possibly trace how many cycles it goes through in the plastic field, i.e., once energy dissipation starts.

However, further development and testing is needed before this new methodology can be reliably applied. The case history of structural types should also be extended numerically and then, if the results confirm its soundness, extended to experimental tests on shaking tables.

6. References

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