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# An NLP model for evaluating the impact of Italian liberalized electric energy market rules on independent power producers

Mario Innorta and MariaTeresa Vespucci  
Department of Management Engineering, Bergamo University,  
Via Marconi 5, Dalmine 24044, Italy

**Abstract.** This paper presents a procedure for simulating the hourly bidding process of an Independent Power Producer (IPP), who aims at maximizing his own profit in the competitive context resulting from the electric energy market liberalisation. In the new context, market rules define how the IPP interacts with other competing power producers and with the Market Operator in the process of producing and transmitting electricity as well as of determining the "market clearing price". Aim of the work is to study how market rules affect these interactions, in order to detect conditions under which the interaction results happen to be in contrast with the liberalisation aim, i.e. the improvement of economic efficiency. Therefore the model developed in this paper is thought of as a tool for analysing how market rules affect the IPP profits and for detecting whether the IPP may exert market power. In the numerical experiments three cases are discussed, with reference to IPPs of different dimensions.

**Keywords:** deregulation, Italian electricity market, zonal price, inter-area constraints, hourly thermal scheduling.

## 1 Introduction.

In the new scenario introduced by the deregulation process, generating companies sell their own production by presenting bids on the day-ahead market for each hour of the following day. The Market Operator has to satisfy the hourly energy demand by choosing the cheapest suppliers, taking into account, at the same time, constraints on power flow exchanges among the master areas in which the network is divided (security network constraints). This process generates a list of accepted bids, the clearing zonal prices and the power flow levels among master areas.

The Independent Power Producer (IPP) operation planning decision is hierarchically articulated in long, medium and short-term levels, with hourly supply curves being the final output of the planning process. In this paper we analyse how market rules (on the basis of which power producers' bids are either accepted or refused and both power flows among zones and zonal prices are determined) affect the IPP position with respect to competitors and therefore the IPP profits. A model of the IPP profit problem, which takes into account interactions among IPP, competitors and Market Operator, is developed, based on the following assumptions:

1. the network is divided in master areas, or zones, connected by transmission links; the transmission system topology is radial and, for security reasons, the power flows among zones are subject to lower and upper bounds;
2. the energy demand in each zone is price independent;
3. in each zone the Market Operator has to feed the hourly energy demand by choosing the cheapest bids while satisfying the power transmission limits among the network master areas: this process also determines the clearing zonal prices;
4. in each zone the IPP offers the production of his own thermal units at a price lower than the clearing zonal price: therefore all IPP sell bids are accepted by the Market Operator and the zonal clearing prices can be considered as functions of the IPP production scheduling in all areas;
5. in order to satisfy the zonal residual load demand, the Market Operator chooses the cheapest competitor bids; an estimate is available of the competitor bid structure: a linear function is known that, for each offered energy level between zero and a maximum value, gives the price at which the energy is offered.

Concerning assumption 1. we note that the radial topology is used by the Transmission System Operator to describe the Italian system.

Since the unit commitment for all thermal units and the amount of hydraulic resources have been previously defined by the short term (weekly or daily) operation planning [1], [4], [8], the proposed hourly procedure has to find the optimum power level for each thermal plant in activity. It takes into account not only the

operating costs analysis [2], [3], [10] but also the rules employed by the Market Operator and the competitors' behaviours. This is done by modeling the Market Operator problem as a convex quadratic programming problem, which also includes competitor bids estimates: the optimality conditions of the Market Operator problem are then introduced as constraints in the hourly IPP scheduling problem. Our procedure, rather than being conceived as a tool for the actual IPP planning operation (which would require a much more detailed model, with time-related constraints, as in [11]), is thought of as an instrument for analysing how market rules may affect the IPP profits and for detecting whether the IPP has a dominant position. An alternative approach for analysing interactions among different agents on the electricity market is based on game theory, see [6], [7].

A general model for the IPP hourly operation planning is described in section 2. The behaviour of the Market Operator when the IPP scheduled energy is offered at a price lower than the clearing zonal price is modeled as a convex quadratic programming problem which is detailed, together with its optimality conditions, in section 3. In section 4 the IPP Maximum Profit Problem (MPP) is reformulated as a non convex quadratic programming problem which from a computational point of view can be more easily solved. Finally the proposed procedure has been tested on a system that represents a likely scenario of the Italian situation as a result of the deregulation process currently under way. The results reported in section 5 show that

- the designed method is capable of dealing with large-scale systems;
- the inter-area constraints strongly affect the zonal prices and as a consequence the IPP profits;
- in particular peak load hours the price levels induced by the IPP hourly bidding could be so high that the energy cost becomes unbearable by the overall economic system.

## **2 The Independent Power Producer optimization problem.**

The Maximum Profit Problem (MPP) for an Independent Power Producer requires not only the knowledge of the production costs for each power plant but

also a forecast of the competitor bids as well as a model for the Market Operator behaviour. Both the IPP optimization model and the Market Operator behaviour model, developed in the next section, are based on the assumptions detailed in the previous section.

The following notation will be used:

$K$  is the number of network master areas, or zones, in which the network is divided;

$k$  is the zone index;

$H$  is the number of transmission links among zones;

$h$  is the transmission link index;

$TR_h$ ,  $\underline{TR}_h$  and  $\overline{TR}_h$  are the power flow on link  $h$ , its lower bound and its upper bound;

$A$  is the network incidence matrix, i.e.

$$A_{kh} = \begin{cases} 1 & \text{if network link } h \text{ is oriented toward area } k \\ -1 & \text{if network link } h \text{ is oriented from area } k \\ 0 & \text{otherwise;} \end{cases}$$

$CA_k$  is the load demand (MWh) in zone  $k$  for the hour in consideration;

$I$  is the number of the IPP thermal groups that, for the hour in consideration, have been determined to be in activity by the short term operation planning;

$i$  is the IPP (in activity) thermal group index;

$G(k)$  is the set of indices of IPP (in activity) thermal groups in area  $k$ ;

$P_i$ ,  $\underline{P}_i$  and  $\overline{P}_i$  are the hourly production (MWh) of thermal unit  $i$ , its lower bound and its upper bound;

$TP_k$  is the total IPP production in zone  $k$  for the hour in consideration, i.e.

$$TP_k = \sum_{i \in G(k)} P_i; \tag{1}$$

$Cost_i$  is the production cost function associated to IPP thermal unit  $i$ ;

$J$  is the number of competitor bids;

$j$  is the competitor bid index;

$\Pi_j$  and  $\bar{\Pi}_j$  are the production offered (MWh) in bid  $j$  and its upper bound;

$b_{0j}$  and  $b_{1j}$  are intercept and slope respectively of the linear function that estimates the price at which energy levels between 0 and  $\bar{\Pi}_j$  are offered in bid  $j$ ; the function is  $Price_j(\Pi_j) = b_{0j} + b_{1j}\Pi_j$  with  $0 \leq \Pi_j \leq \bar{\Pi}_j$ ;

$O(k)$  is the subset of indices of competitor bids presented in zone  $k$ ;

$\mu_k$  is the clearing zonal price determined by the Market Operator for area  $k$ .

The IPP Maximum Profit Problem is formulated as follows:

$$\max \sum_{k=1}^K \left[ \mu_k \left( \sum_{i \in G(k)} P_i \right) \right] - \sum_{i=1}^I Cost_i(P_i) \quad (2)$$

subject to

- lower and upper bounds on production of IPP thermal group  $i$

$$\underline{P}_i \leq P_i \leq \bar{P}_i \quad \forall i \quad (3)$$

- relations between zonal clearing prices and total IPP productions in all zones (see Assumption 4 above)

$$\mu_k = \mu_k \left( \sum_{i \in G(1)} P_i, \dots, \sum_{i \in G(K)} P_i \right) \quad \forall k \quad (4)$$

This constraint is quite complex, since the evaluation of the functions

$$\mu_k \left( \sum_{i \in G(1)} P_i, \dots, \sum_{i \in G(K)} P_i \right)$$

requires the computation of the optimal dual variables of the optimization problem which describes the behaviour of the Market Operator. For this reason the authors replace constraint (4) with the Kuhn-Tucker optimality conditions of the Market Operator problem. In section 3 the Market Operator optimization problem and the corresponding optimality conditions are presented, while in section 4 it is discussed how the IPP maximum profit problem can be transformed into a non convex quadratic programming problem. In section 5 numerical results are reported for three case studies related to IPPs of different dimensions.

### 3 Optimality conditions of the Market Operator optimization problem.

#### 3.1 The Market Operator behaviour model.

In order to satisfy the residual load demand in each zone  $k$ , while taking into account the network security constraints, the Market Operator has to determine the quantity  $\Pi_j$  of energy to be accepted for each competitor bid, as well as the power flows  $TR_h$  among zones, so as to minimize the total energy cost. Therefore the Market Operator behaviour model is as follows:

$$\min \sum_{j=1}^J \int_0^{\Pi_j} Price_j(\pi) d\pi = \sum_{j=1}^J \left( b_{0j}\Pi_j + \frac{1}{2}b_{1j}\Pi_j^2 \right) \quad (5)$$

subject to

- lower and upper bounds on competitor productions

$$0 \leq \Pi_j \leq \bar{\Pi}_j \quad \forall j \quad (6)$$

- lower and upper bounds on power flows among zones

$$\underline{TR}_h \leq TR_h \leq \overline{TR}_h \quad \forall h \quad (7)$$

- zonal balance constraints

$$\sum_{j \in O(k)} \Pi_j + \sum_{h=1}^H A_{kh} TR_h = CA_k - TP_k \quad \forall k \quad (8)$$

The above problem is called Minimum Financial Effort Problem (MFEP), see [5]. The zonal clearing prices determined by the Market Operator, to be used in the IPP Maximum Profit Problem, are the Lagrange multipliers of the zonal balance constraints. The IPP can include in his own MPP the Market Operator behaviour, as well as the competitor bid structure estimate, by substituting constraint (4) with the Kuhn-Tucker optimality conditions of MFEP, which we describe in the following section.

### 3.2 The Market Operator problem optimality conditions.

The Lagrangian function  $L$  of the Market Operator optimization problem (MFEP) can be written as

$$\begin{aligned}
L = & \sum_{j=1}^J \left[ b_{0j} \Pi_j + \frac{1}{2} b_{1j} \Pi_j^2 - \underline{\lambda}_j \Pi_j - \bar{\lambda}_j (\bar{\Pi}_j - \Pi_j) \right] + \\
& - \sum_{h=1}^H \left[ \underline{\eta}_h (TR_h - \underline{TR}_h) + \bar{\eta}_h (\bar{TR}_h - TR_h) \right] + \\
& - \sum_{k=1}^K \left[ \mu_k \left( \sum_{j \in O(k)} \Pi_j + \sum_{h=1}^H A_{kh} TR_h - CA_k + TP_k \right) \right]
\end{aligned}$$

where

$\underline{\lambda}_j$  and  $\bar{\lambda}_j$  are the dual variables associated to the technical lower and upper limitations on bid  $j$  respectively;

$\underline{\eta}_h, \bar{\eta}_h$  are the dual variables associated to the technical lower and upper bounds on the transport capability of link  $h$  respectively;

$\mu_k$  is the Lagrange multiplier of the  $k$ -th zonal balance constraint (8).

The MFEP optimality conditions can be written as follows:

•

$$\frac{\partial L}{\partial \Pi_j} = b_{0j} + b_{1j} \Pi_j - \underline{\lambda}_j + \bar{\lambda}_j - \mu_k = 0 \quad \forall j \quad (9)$$

where  $k$  is the index of the zone where the thermal group  $j$  is located;



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$$\frac{\partial L}{\partial TR_h} = -\underline{\eta}_h + \bar{\eta}_h - \sum_{k=1}^K \mu_k A_{kh} = 0 \quad \forall h \quad (10)$$

- the MFEP constraints (6), (7), (8);

- the nonnegativity conditions for the dual variables

$$\underline{\lambda}_j \geq 0 \quad \bar{\lambda}_j \geq 0 \quad \forall j \quad (11)$$

$$\underline{\eta}_h \geq 0 \quad \bar{\eta}_h \geq 0 \quad \forall h \quad (12)$$

- the complementary slackness conditions

$$\underline{\lambda}_j \Pi_j = 0 \quad \bar{\lambda}_j (\bar{\Pi}_j - \Pi_j) = 0 \quad \forall j \quad (13)$$

$$\underline{\eta}_h (TR_h - \underline{TR}_h) = 0 \quad \bar{\eta}_h (\bar{TR}_h - TR_h) = 0 \quad \forall h \quad (14)$$

By using appropriate algebraic manipulations (see Appendix), the nonlinear "complementary slackness" conditions (13) and (14) can be converted into the nonlinear constraint

$$\begin{aligned} \sum_{j=1}^J (b_{0j} \Pi_j + b_{1j} \Pi_j^2 + \bar{\lambda}_j \bar{\Pi}_j) - \sum_{k=1}^K \mu_k (CA_k - TP_k) + \\ + \sum_{h=1}^H (\bar{\eta}_h \bar{TR}_h - \underline{\eta}_h \underline{TR}_h) = 0 \end{aligned} \quad (15)$$

## 4 The non convex quadratic model for the IPP.

The results of the previous section allow a new formulation of the IPP Maximum Profit Problem to be obtained, where constraint (4) is replaced by the linear constraints (6), (7), (8), (9), (10), (11), (12), (1) and the nonlinear constraint (15). Note that constraint (1) has been included as the quantity  $TP_k$  is a variable in MPP (while it is a constant in MFEP). From a computational point of view it is convenient to eliminate constraint (15) by introducing a suitable penalty in the objective function: since the left hand side of constraint (15) is always greater or equal to zero, provided that the remaining linear constraints are satisfied, constraint (15) can be replaced with a linear penalty in objective function (2).

Therefore the MPP final form can be written as:

$$\begin{aligned} \max \sum_{k=1}^K \left[ \mu_k \left( \sum_{i \in G(k)} P_i \right) \right] - \sum_{i=1}^I [Cost_i(P_i)] + \\ -M \left\{ \sum_{j=1}^J (b_{0j}\Pi_j + b_{1j}\Pi_j^2 + \bar{\lambda}_j\bar{\Pi}_j) - \sum_{k=1}^K \mu_k (CA_k - TP_k) + \right. \\ \left. + \sum_{h=1}^H (\bar{\eta}_h \overline{TR}_h - \underline{\eta}_h \underline{TR}_h) \right\} \end{aligned}$$

subject to

- technical bounds on the IPP power productions (3)
- the constraints of MFEP (6), (7) and (8)
- the MFPE optimality conditions (9) and (10)
- the IPP total production definition (1)
- the nonnegativity conditions (11) and (12).

Note that  $M$  is a very large penalty coefficient. The above obtained IPP Maximum Profit Problem is a non convex quadratic problem, with nonconvexity of the objective function due to the terms  $\mu_k TP_k$ . Its solution yields the optimal hourly bidding for the IPP.

## 5 Numerical results.

The methodology described in the previous sections has been implemented and tested on a large scale model corresponding to the Italian system, requiring small computational resources (few seconds on a personal computer with 128MB RAM and 500MHz).

The electrical production and transmission test system is not too far from the Italian situation that is emerging in the deregulation process under way: the production system consists of 165 thermal groups belonging to four IPPs and the transmission system consists of six network master areas. Both topology and limitations on power flows among zones are shown in Figure 1.

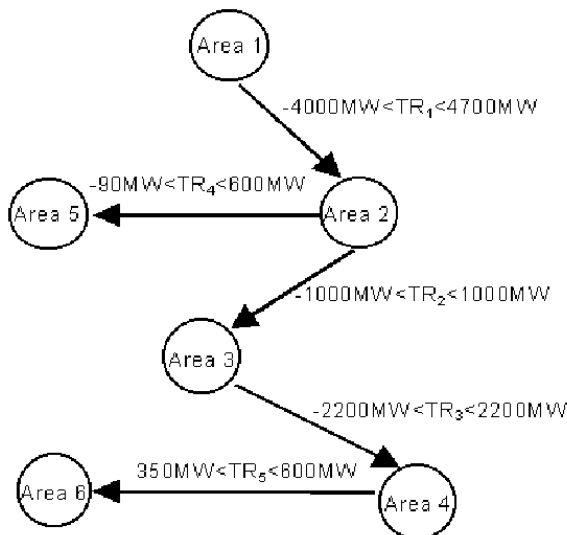


Figure 1: Master area topology and transmission limits in the test network

In the following we describe three of the performed simulations. In the first one, we consider an Independent Power Producer with excessive market power, who jeopardizes a real competition among power producers and gives rise to an intolerably high energy price. In the second one the impact of the network congestion on the zonal price levels and on the profits of a small power producer is assessed. In the third case it is shown, for an IPP of even smaller dimension, that the possibility of exerting excessive market power is detected by solving his Maximum Profit Problem by a global optimisation routine.

## 5.1 Simulation 1.

This simulation considers an Independent Power Producer (IPP1), whose production is necessary in order to satisfy the load demand. For each network master area the load levels corresponding to a peak hour of the Italian system, the upper bounds to competitor productions and the lower and upper bounds to the IPP1 productions are reported in Table 1.

$k$	$CA_k$ [MWh]	$\sum_{j \in O(k)} \bar{\Pi}_j$ [MWh]	$\sum_{i \in G(k)} \underline{P}_i$ [MWh]	$\sum_{i \in G(k)} \bar{P}_i$ [MWh]
1	11437	8387	2983	6922
2	5410	0	883	2156
3	8400	1071	2655	6029
4	4544	1611	1883	4689
5	948	640	260	437
6	1741	1202	558	1500
total	32480	12911	9222	21733

Table 1: load level, IPP1 minimum and maximum active power and competitor maximum active power in the network master areas

As regard to the competitor bid structure, it is assumed that each competitor thermal group offers its own production at marginal cost. Moreover for each master area a further bid is considered, whose price corresponds to the maximum price,  $\bar{\mu}$ , allowed by the market rules. The algorithm used to compute the optimal values of the decision variables is based on the sparse non convex implementation of the active set strategy due to Gould [9]. The same algorithm is also used for computing the optimal values of the decision variables in Simulation 2. We report in Table 2 the optimal IPP1 total productions and the optimal competitor total productions for each zone and in Table 3 the optimal power flows among zones. In both tables an upper (lower) asterisk near a variable value indicates that the optimal value equals the variable upper (lower) bound.

$k$	1	2	3	4	5	6
$\sum_{i \in G(k)} \bar{P}_i$ [MWh]	6922*	2156*	5598	4301	403	558*
$\sum_{j \in O(k)} \bar{\Pi}_j$ [MWh]	8387*	0*	1071*	1611*	640*	833

Table 2. Optimal IPP1 total productions and optimal competitors' total productions for each zone.

$h$	1	2	3	4	5
$TR_h$	3871	713	-1018	-95	350*

Table 3. Optimal power flows among zones.

In order to maximize his own profit, IPP1 has to set his own production in each area at the level which forces the Market Operator to accept all competitor bids, which in turn forces the zonal clearing price to be set at the maximum allowed value  $\bar{\mu}$ . The simulation results show that this strategy may be pursued in Master Areas 1 to 5 but not in Area 6, where, because of the lower bounds on the IPP1 production and on the power flow among Master Areas 4 and 6, the residual load demand is 833  $MWh$ , which in turn implies that the competitor bids cannot be saturated. In order to impose the maximum price  $\bar{\mu}$  also in Area 6, IPP1 should shut down some operative units, but these manoeuvres could conflict with his own daily or weekly unit commitment. It is evident that the presence of a power producer with an excessive market power, capable of imposing the maximum price in most of the areas, is dangerous for the economic system. Therefore such a situation is not allowed in a competitive energy market. It is worth noticing that if IPP1 becomes aware of his dominant position, he can activate this mechanism himself by offering in each area the production levels determined by the optimization procedure at the maximum price  $\bar{\mu}$ .

## 5.2 Simulation 2.

This simulation shows that the network security constraints, via the different zonal prices which they give rise to, may greatly affect the profit of an Independent Power Producer. We consider a small power producer (IPP2), whose contribution is not indispensable, since the other competitors have enough resources to feed the demand in each area.

A first computation has been performed taking into account the network security constraints: the zonal clearing prices, the IPP2 production levels and the IPP2 profits for each area are reported in Table 4. We note that the lower zonal prices in areas 5 and 6 are due to the congestion of their links with the remaining part of the system.

Area	Zonal prices [ <i>Euro/MWh</i> ]	Production [ <i>MWh</i> ]	Profit [ <i>Euro</i> ]
1	55.705	3585	101 533
2	55.705	0	0
3	55.705	0	0
4	55.705	1200	37 108
5	25.240	0	0
6	23.374	702	394

Table 4: zonal prices, IPP2 productions and profits for each master area

A second computation has been performed neglecting the inter area power flow limits and the market clearing price of 37.185 *Euro/MWh* has been obtained. In Table 5 a comparison is reported of optimal IPP2 production, production cost and profit and optimal system production cost with and without security transmission constraints.

	IPP2 production [ <i>MWh</i> ]	IPP2 production cost [ <i>Euro</i> ]	System production cost [ <i>Euro</i> ]	IPP2 profit [ <i>Euro</i> ]
with constraints	5 487	143 923	815 533	139 035
without constraints	5 716	142 039	791 505	71 472

Table 5: comparison between scheduling with and without transmission constraints.

It can be seen that the effects of the transmission constraints on IPP2 production costs and on system production costs are moderate, while they are quite remarkable on IPP2 profits: this is due to the fact that almost 90% of the IPP2 optimal production is located in areas where the zonal prices become sensibly higher as the transmission constraints become active.

### 5.3 Simulation 3.

In this simulation a power producer IPP3 of quite small dimension is considered, in a situation where the load demand is higher than in both previous simulations

and transmission constraints are neglected. In Table 6 load level, competitors' maximum active power and IPP3 minimum and maximum active power for the hour in consideration are reported. When the IPP3 maximum profit problem is solved by a local optimiser, the results, reported in Table 7, indicate that IPP3 maximizes his own profit by using all his own production capacity: in this case the market clearing price is 53.96 *Euro/MWh*. However, the global optimization results (also reported in Table 7) show that there exists a much more advantageous solution for IPP3 which corresponds both to a lower production level and to the maximum price  $\bar{\mu}$  allowed by the system: in this solution the IPP3 profits are considerably higher. In other words, the global solution detects the possibility for IPP3 of becoming indispensable in satisfying the hourly load demand and therefore of exerting excessive market power. In order to do that, IPP3 must reduce his own production to the level that forces the Market Operator to accept all competitors' bids and set the market clearing price to the maximum value  $\bar{\mu}$ . We notice that analogous results have been obtained when transmission constraints were considered. The local optimization results have been obtained by using solver MINOS in GAMS framework (version 21.5), while the global optimization results have been obtained by using solver BARON in GAMS framework.

$CA_k$ [MWh]	$\sum_{j \in O(k)} \bar{\Pi}_j$ [MWh]	$\sum_{i \in G(k)} \underline{P}_i$ [MWh]	$\sum_{i \in G(k)} \bar{P}_i$ [MWh]
35200	33159	1525	2701

Table 6: load level, competitors' maximum active power and IPP3 minimum and maximum active power

local minimizer		global minimizer	
$\sum_{j \in O(k)} \bar{\Pi}_j$ [MWh]	$\sum_{i \in G(k)} \bar{P}_i$ [MWh]	$\sum_{j \in O(k)} \bar{\Pi}_j$ [MWh]	$\sum_{i \in G(k)} \bar{P}_i$ [MWh]
32499	2701	33159	2041

Table 7: local and global optimal IPP3 total productions and competitors' total productions.

## 6 Conclusions.

The proposed approach can be considered as the last step of an operation planning decision for an Independent Power Producer who aims at maximizing

his own profit in an open market environment.

Since the chosen time scale is the hour, the hydro scheduling and the unit commitment for the thermal system are assumed to be completely defined.

Great care has been devoted in modeling the transmission system, the market rules and the competitor bids, with their precise location within the network topology.

The procedure is a useful tool for the IPP hourly bidding for analysing how market rules affect the IPP profits. It can also be used, in a simulation context, by the Transmission System Operator for the assessment of the economical effects due to the required security levels. Finally, the procedure is able to detect excessive market power, which can be an effective support to the decisions of the Italian Regulatory Authority for Electricity and Gas.

It is planned to extend the investigation in the following directions:

- introduce a Direct Current network model in MFPE, in order to handle nodal prices and limits on physical network links;
- extend the time horizon to one day, in order to include the analysis of the hydroelectric system.

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## 7 Appendix

In MFEP the lower and upper bound constraints (6) and (7) on the primal variables  $\Pi_j$  and  $TR_h$  and the nonnegativity conditions (11) and (12) on the dual variables  $\underline{\lambda}_j$ ,  $\bar{\lambda}_j$ ,  $\underline{\eta}_h$  and  $\bar{\eta}_h$  imply that

$$\underline{\lambda}_j \Pi_j \geq 0 \quad \bar{\lambda}_j (\bar{\Pi}_j - \Pi_j) \geq 0 \quad \forall j \quad (16)$$

$$\underline{\eta}_h(TR_h - \underline{TR}_h) \geq 0 \quad \bar{\eta}_h(\overline{TR}_h - TR_h) \geq 0 \quad \forall h. \quad (17)$$

Because of (16) and (17), the complementary conditions (13) and (14) are satisfied if and only if the relation

$$\sum_{j=1}^J [\underline{\lambda}_j \Pi_j + \bar{\lambda}_j (\bar{\Pi}_j - \Pi_j)] + \sum_{h=1}^H [\underline{\eta}_h (TR_h - \underline{TR}_h) + \bar{\eta}_h (\overline{TR}_h - TR_h)] = 0 \quad (18)$$

holds. The left-hand side of (18) can be rewritten as

$$\sum_{j=1}^J [(\underline{\lambda}_j - \bar{\lambda}_j) \Pi_j + \bar{\lambda}_j \bar{\Pi}_j] + \sum_{h=1}^H [(\underline{\eta}_h - \bar{\eta}_h) TR_h] + \sum_{h=1}^H (\bar{\eta}_h \overline{TR}_h - \underline{\eta}_h \underline{TR}_h). \quad (19)$$

By rearranging (9) the relation

$$\underline{\lambda}_j - \bar{\lambda}_j = b_{0j} + b_{1j} \Pi_j - \mu_k, \quad (20)$$

is obtained, where  $k$  takes the value of the zone index in which the competitor thermal group is located. Therefore, by using (20), the first term of (19) becomes

$$\sum_{j=1}^J [(\underline{\lambda}_j - \bar{\lambda}_j) \Pi_j + \bar{\lambda}_j \bar{\Pi}_j] = \sum_{j=1}^J [b_{0j} \Pi_j + b_{1j} \Pi_j^2 + \bar{\lambda}_j \bar{\Pi}_j] - \sum_{k=1}^K \mu_k \sum_{j \in O(k)} \Pi_j. \quad (21)$$

Analogously, by rearranging (10) the relation

$$\underline{\eta}_h - \bar{\eta}_h = - \sum_{k=1}^K \mu_k A_{kh}$$

is obtained, which enables us to express the second term of (19) as

$$\sum_{h=1}^H [(\underline{\eta}_h - \bar{\eta}_h) TR_h] = - \sum_{k=1}^K \mu_k \sum_{h=1}^H A_{kh} TR_h. \quad (22)$$

By using (21) and (22), condition (18) can be written as

$$\begin{aligned} \sum_{j=1}^J [b_{0j} \Pi_j + b_{1j} \Pi_j^2 + \bar{\lambda}_j \bar{\Pi}_j] - \sum_{k=1}^K \mu_k \left( \sum_{j \in O(k)} \Pi_j + \sum_{h=1}^H A_{kh} TR_h \right) + \\ + \sum_{h=1}^H (\bar{\eta}_h \overline{TR}_h - \underline{\eta}_h \underline{TR}_h) = 0 \end{aligned}$$

and, by using the balance equation (8), the nonlinear constraint (15) is obtained, which is equivalent to the nonlinear complementary slackness conditions (13) and (14).

**Redazione**

Dipartimento di Matematica, Statistica, Informatica ed Applicazioni  
Università degli Studi di Bergamo  
Via dei Caniana, 2  
24127 Bergamo  
Tel. 0039-035-2052536  
Fax 0039-035-2052549

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