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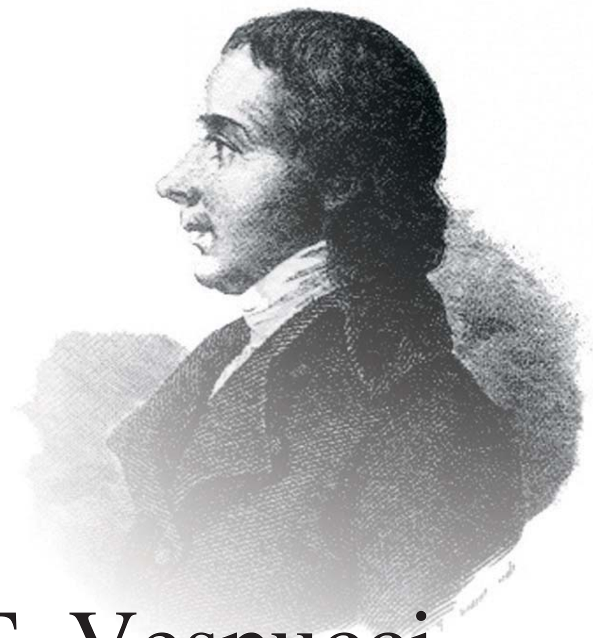

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A stochastic model for hedging electricity portfolio for an hydro-energy producer

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UNIVERSITÀ DEGLI STUDI DI BERGAMO

A stochastic model for hedging electricity portfolio for an hydro-energy producer [1]

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Abstract

A stochastic multi-stage portfolio model for a hydropower producer operating in a competitive electricity market is proposed. The portfolio includes its own production, a set of power contracts for delivery or purchase including contracts of financial nature as forwards to be able to hedge against risks. The goal of using such a model is to maximise the profit of the producer and reduce the economic risks connected to the fact that energy spot and forward prices are highly volatile. Our results show that, forward contracts can be used for hedging purposes if we assume that their price can be derived by the spot dynamics. In incomplete markets the relationship between spot and forward price is non uniquely determinate. If we explicitly model the spot and the forward dynamicss we can obtain consistent scenarios which allow for speculative behaviour. Beyond financial gains, the convenience of using financial contracts is a more efficient use of the hydroplant, taking advantage of the possibility of pumping water and ending up with a higher final value of the reservoir.

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Keywords: reservoir level in hydro plants, stochastic programming, forward contracts, portfolio model
2000 MSC: 90C15, 90C90

1. Introduction

As a result of the liberalization of energy markets, generation companies are exposed to higher uncertainties. Risk management becomes a more pressing issue for electricity consumers and producers and contracts for future delivery of electricity (i.e.forwards contracts) become a tool for hedging risk. The hydropower producer's portfolio includes his own production, a set of power contracts for delivery or purchase including contracts of financial nature as forwards to be able to hedge against various types of risks. In this paper we develop a stochastic portfolio model for a hydropower producer operating in a competitive electricity market.

The goal of using such a model is to reduce the economic risks connected to the fact that energy spot price may be highly volatile due to various different, unpredictable reasons (i.e. very cold winter) and to the possibility of a period of scarcity of raining or snowmelting. See [18] and [13] for discussion on the opportunity of using stochastic programming for such problem. The basis risk factors include the wholesale spot and forward price of electrical energy, which are supposed to be unaffected by the decision of the utility manager, and the uncertain inflow of hydro reservoirs, see also [8]. The model we propose differs from the one discussed in [8] since we want to concentrate on the advantage of using financial contracts and therefore we use as source of uncertainties both the electricity spot prices and the forward prices considering inflows as deterministic. We leave for future research the inclusion of stochastic inflows.

2. The producer daily scheduling model

Electricity generation is modeled at the level of detail common in medium-long term hydroplanning model with a granularity of one day at its finest, with start-up and shut-down costs considered not significant.

We start introducing the model of the hydroelectric system with hourly periods. The hydroelectric system consists of a number of cascades, i.e. sets of hydraulically interconnected hydro plants, pumped-storage hydro plants and reservoirs. It is mathematically represented by a directed multi-graph, where nodes represent water storages (reservoirs) and arcs represent water flows (either power generation, or pumping, or spillage). Let J denote the set of nodes and I denote the set of arcs. The arc-node incidence matrix, whose (i, j)-entry is denoted by $A_{i,j}$, represents the interconnections among water storages and water flows in the hydroelectric system ($A_{i,j} = -1$, if arc i leaves node j; $A_{i,j} = 1$, if arc i enters node j; $A_{i,j} = 0$, otherwise). For every arc $i \in I$ and for every node $j \in J$ the following data are relevant:

- k_i $[MWh/10^3m^3]$: energy coefficient ($k_i > 0$, if arc i represents generation; $k_i < 0$, if arc i represents pumping; $k_i = 0$, if arc i represents spillage)
- $\overline{q}_i$ $[10^3m^3/h]$: maximum water flow in arc i
- $\overline{v}_j$ $[10^3m^3]$: maximum storage volume in reservoir j
- $v_{j,0}$ $[10^3m^3]$: initial storage volume in reservoir j
- $\underline{v}_{j,T}$ $[10^3m^3]$: minimum storage volume required in reservoir j at the end of hour T
- $f_{j,t}$ $[10^3m^3/h]$: natural inflow in reservoir j in hour t

The power producer must schedule the production of each hydro plant, which is expressed as the product of the hydro plant energy coefficient times the turbined volume in hour t, as well as the hourly pumped and spilled volumes. The decision variables of the hydro scheduling problem are

- $q_{i,t}$ $[10^3m^3/h]$: water flow on arc i in hour t (turbined volume, if arc i represents generation; pumped volume, if arc i represents pumping; spilled volume, if arc i represents spillage);
- $v_{j,t}$ $[10^3m^3]$: storage volume in reservoir j at the end of hour t.

The values assigned to the decision variables must satisfy the following constraints that describe the hydroelectric system:

- flow on arc i in hour t is nonnegative and bounded above by the maximum volume that can be either turbined, or pumped, or spilled

$$0 \leq q_{i,t} \leq \overline{q}_i \qquad i \in I, \quad 1 \leq t \leq T \tag{1}$$

- the storage volume in reservoir j at the end of hour t is nonnegative and bounded above by the maximum storage volume

$$0 \leq v_{j,t} \leq \overline{v}_j \qquad j \in J, \quad 1 \leq t \leq T \tag{2}$$

- at the end of hour T, the last hour of the planning period, the storage volume in reservoir j is bounded below by the minimum storage volume required at the end of the current planning period, so as to provide the required initial storage volume at the beginning of the following planning period

$$\underline{v}_{j,T} \leq v_{j,T} \qquad j \in J \tag{3}$$

- the storage volume in reservoir j at the end of hour t must be equal to the reservoir storage volume at the end of hour $t-1$ plus the sum of inflows in hour t minus the sum of outflows in hour t

$$v_{j,t} = v_{j,t-1} + f_{j,t} + \sum_{i \in I} A_{i,j} \cdot q_{i,t} \qquad j \in J, \quad 1 \leq t \leq T \tag{4}$$

  where $v_{j,0}$ is a data representing the initial storage volume in reservoir j. Reservoir inflows are natural inflows, turbine discharge from upstream hydro plants, pumped volumes from downstream hydro plants, spilled volumes from upstream reservoirs. Reservoir outflows are turbine discharge to downstream hydro plants, pumped volumes to upstream hydro plants and spilled volumes to downstream reservoirs. In this paper, the values of natural inflows $f_{j,t}, j \in J, 1 \leq t \leq T$, are assumed to be known with certainty.

- the value of the reservoir at the end of the horizon $V(v_{j,T})$, is a function of reservoir level that has to be specified to avoid end effects.

This is the general model used by the producer for daily scheduling where the time unit is the hour. When we want to include financial contracts we will assume to work on a daily basis. Thus, all the previous equations will be transformed to fit the new time period and t will represent the day. From now on, T represents the time horizon expressed in multiple of days.

3. The use of forward contracts

An electricity forward contract is the obligation to buy or sell a specified amount of power -1 MegaWatt (MW) during every hour (i.e. base-load) - at a predetermined delivery price, the forward price, during a delivery period fixed at the issue time of the contract. Additional to these so-called base-load contracts, there are peak-load contracts, which deliver 1MW every hour during working days from 8 am to 8 pm, in the delivery period only. In this paper we will not consider them, at this stage.

The forwards contracts are standardized by the following characteristics: volume, delivery period and settlement. The volume is the number of MWh underlying the contract. For contracts with a fixed rate (energy amount per hour) of 1 MW, it is equivalent to the number of hours in the delivery period. As an example, for a April contract with monthly delivery period (in the following we will specify the days in the delivery period as DP, hence $DP = 30$ days in our example), this means a total of $1MW \times 30days \times 24h/day =$ 720MWh. The quoted forward price is the price at which the owner of the contract will buy/sell energy during the delivery period per 1MWh. The value of the contract is the product of the quotation and the volume. For each buying or selling of the contract, we consider a transaction fee of 0.01 per MW and an estimated bid-ask spread of 3% of the forward price. In our example the transaction fee is $0.01EUR \times 30days \times 24h/day =$ 7.2EUR

The delivery periods are fixed to each of the 12 calendar months (M1, M2, .., M12), to the four quarters (Q1, Q2, Q3, Q4) of the calendar year or to the whole calendar year. There are also shorter delivery period of one day, one week, and the week-end. For each contract we can distinguish between a trading period and a delivery period. The trading in a given contract stops when it enters the delivery period. An other relevant characteristic is the settlement. We can distinguish between financial contracts and physical contracts. The former requires a cash settlement of forward price against the realized spot prices during the delivery period. The latter requires energy's delivery at the delivery price during the delivery period.

Let us consider, for modeling reasons, the accounting of a forward contract. This specification is relevant to understand the optimisation model we propose in the following section, where we want to select which contract hold and which contract close before the delivery period. Assume that we enter in a long forward position at time T_b, and that we maintain the contract till the delivery period which starts in T_e and holds for DP days. See Figure (1) for an illustration of all these quantities. We can decomposed the loss/gain on the contracts into two components: the mark to market, during the trading period, and the settlement, during the delivery period. The first mechanism implies that from the purchase day till the last trading day, we close the position every day and immediately we reopen it at the new forward price. The daily gain/loss is given by the price variations. At the end of the trading period the holder of the long position has a gain/loss of $24 \cdot DP \cdot (F_{T_e} - F_{T_b})$. The last position of the trading period is a forward with the quote F_{T_e}, called the reference price. During the delivery period we distinguish between cash settlement and physical delivery. The former is computed considering the daily variations between the reference price and the spot price without a physical exchange of electricity. The latter consists in the payment of the reference price against the physical delivery of electricity. In equation (5) we show that this mechanism is equivalent to the classical settlement where the exchange is between the forward price fixed at the purchase of the contract and the spot price during the delivery period.

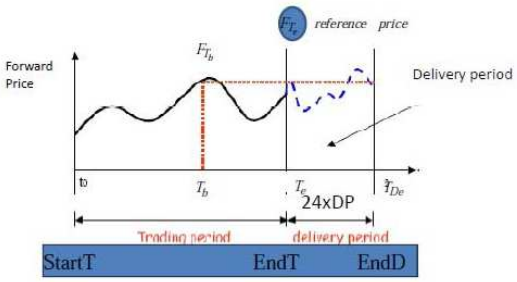

Figure 1: The Evolution of Forward Price

$$24 \cdot DP \cdot \sum_{T_b \le t \le T_e} (F_t - F_{t-1}) + \sum_{T_e < t \le T_e + 24 \cdot DP} (S_t - F_{T_e}) = \sum_{T_e \le t \le T_e + 24 \cdot DP} (S_t - F_{T_b}) \tag{5}$$

If we consider physical settlement, the exchange is between the forward price fixed at the purchase of the contract and the physical delivery of energy during the delivery period.

$$24 \cdot DP \cdot \sum_{T_b \le t \le T_e} (F_t - F_{t-1}) + \sum_{T_e < t \le T_e + 24 \cdot DP} (S_t - F_{T_e}) = \sum_{T_e \le t \le T_e + 24 \cdot DP} -F_{T_b} \tag{6}$$

In this work we consider base-load contracts with physical settlement, since we decide to model the daily movements of the spot process. The introduction of peak-load contracts would require to model hourly prices.

4. The deterministic financial model

The power producer is assumed to be a price taker, i.e. not able to influence the electricity market price, which is therefore exogenous to the model. We suppose to be able to hedge the producer's portfolio by buying and selling forward base-load contracts; the variable $buy_{l,t}$ ($sell_{l,t}$) is used to denote the number of positions at time t and l represents the tipology of the contract to be bought (sold) on the forward market. The market price of these contracts are indicated by $F_{l,t}$. Transaction costs (tc) and bid-ask spread (ba) are as indicated in section 3.

Generally, the producer can buy or sell forward contracts with different tipology of delivery period, i.e. forward contracts with weekly, monthly, quarterly and yearly delivery. If the producer has a planning horizon T of one year (expressed in days), he can decide to hedge his risk by buying or selling forward contracts with different delivery periods in the different days of his planning horizon. For contract l, $T_e(l)$ indicates the maturity of the contract, $T_d(l)$ the end of the delivery period, $T_s(l)$ indicates the time when the contract is firstly traded on the market.

We model the objective function representing the producer's profit: it consists of various parts taking into accounts net sales in the spot market, selling and buying forward contracts, and the value of end reservoir:

$$\begin{aligned} \max \;=\; & \sum_t (1+r)^{T-t}((S_t \sum_i k_i q_{i,t} + \sum_l x_{l,t} \cdot 1_{t\in[T_e(l),T_d(l)]}) + \\ - & \sum_l F_{l,t} \cdot x_{l,t} \cdot 1_{t\in[T_e(l),T_d(l)]} + \sum_l DP(l)(F_{l,t} - F_{l,t-1}) \cdot x_{l,t} \cdot 1_{t\in[T_s(l),T_e(l))} + \\ - & \sum_l (buy_{l,t} + sell_{l,t})(tc + ba \cdot F_{l,t}) \cdot 1_{t\in[T_s(l),T_e(l))}) + \sum_j V(v_{j,T}). \end{aligned} \tag{7}$$

subject to the following constraints:

$$DP(l) = T_d(l) - T_e(l) \tag{8}$$

$$\sum_i k_i q_{i,t} + \sum_l \cdot x_{l,t} \cdot 1_{t\in[T_e(l),T_d(l)]} = D_t \tag{9}$$

where D_t is the production scheduling at time t; the number of open positions in forward contracts at time t for contract l

$$x_{l,t} = x_{l,t-1} + (buy_{l,t} - sell_{l,t}) 1_{t\in[T_s(l),T_e(l))}; \tag{10}$$

where, at every time t, $buy_{l,t}$ and $sell_{l,t}$ must be nonnegative.

The decision variables $q_{i,t}$ and $v_{j,t}$ represent respectively the flow on arc i in day t and the storage volume in reservoir j at the end of day t. Thus, we have to add constraints similar to (1)-(4) where the time unit t is the day instead of the hour:

$$0 \leq q_{i,t} \leq \overline{q}_i \qquad i \in I, \quad 1 \leq t \leq T \tag{11}$$

$$0 \leq v_{j,t} \leq \overline{v}_j \qquad j \in J, \quad 1 \leq t \leq T \tag{12}$$

$$\underline{v}_{j,T} \leq v_{j,T} \qquad j \in J; \tag{13}$$

$$v_{j,t} = v_{j,t-1} + f_{j,t} + \sum_{i \in I} A_{i,j} \cdot q_{i,t} \qquad j \in J, \quad 1 \leq t \leq T. \tag{14}$$

5. The stochastic model

In this paper we assume that the natural inflows are known with certainty and we concentrate our attention on the financial aspects, i.e. the uncertainty in the electricity spot prices and in the forward prices and we introduce a stochastic version of the model discussed in the previous section.

A scenario tree (see for instance [6]) represents the information on the daily energy spot price and contract forward price, where each path from the root to a leaf of the tree corresponds to one scenario. The stochastic model is written in terms of the nodes $\{1, \ldots, n, \ldots, N\}$ of the scenario tree and the tree structure is described by giving each node n the probability P_n, $1 \leq n \leq N$, and a pointer to its parent $pred(n)$, $2 \leq n \leq N$ (i.e. except the root of the tree). The planning horizon is divided in K stages, where each stage k, $1 \leq k \leq K$, is associated to the number of days T_k and to the set of nodes N_k, where $k(n)$ is the stage associated to node n. The model can be extended to any time length.

The variables $q_{i,t}$, $v_{j,t}$, $f_{j,t}$, $buy_{l,t}$, $sell_{l,t}$ in the deterministic model correspond to variables $q_{i,t,n}$, $v_{j,t,n}$, $f_{j,t,n}$, $buy_{l,t,n}$, $sell_{l,t,n}$, with $n \in N_k$ if $t \in T_k$, $1 \leq k \leq K$. in this version of the model we consider only one contract typology, i.e. quarterly contracts. We indicate with r the risk-free interest rate, $W(t, n)$ the cumulative wealth at time t and node n, $U(W)$ an increasing concave utility function of wealth describing the producer risk aversion.

The stochastic model finds values of the decision variables $q_{i,t,n}$, $v_{j,t,n}$, $x_{l,t,n}$,$f_{j,t,n}$, $buy_{l,t,n}$, $sell_{l,t,n}$, for $1 \leq k \leq K$, $n \in N_k$, $t \in T_k$, $i \in I$ and $j \in J$, so as to

$$\max \sum_{n \in N_K} P_n U(W_{T,n} + \sum_j V(v_{j,T,n}))$$

subject to

for every $2 \leq k \leq K$, $t \in T_k$, $n \in N_k$

$$\begin{aligned} W_{t,n} &= W_{t-1,\nu}(1+r) + (S_{t,n}(\sum_i k_i q_{i,t,n} + \sum_l x_{l,t,n} \cdot 1_{t \in [T_e(l), T_d(l)]}) + \\ &- \sum_l F_{l,t,n} \cdot x_{l,t,n} \cdot 1_{t \in [T_e(l), T_d(l)]} + \\ &+ \sum_l DP(l)(F_{l,t,n} - F_{l,t-1,n}) \cdot x_{l,t,n} \cdot 1_{t \in [T_s(l), T_e(l))}) + \\ &- \sum_l (buy_{l,t,n} + sell_{l,t,n})(tc + ba \cdot F_{l,t,n}) \cdot 1_{t \in [T_s(l), T_e(l))}) \end{aligned} \tag{15}$$

where

$$W_{0,0} = 0; \quad F_{l,0,1} = 0 \quad \forall l \tag{16}$$

$$DP(l) = D_e(l) - T_e(l) \tag{17}$$

$$\sum_i k_i q_{i,t} + \sum_l x_{l,t} \cdot 1_{t \in [T_e(l), D_e(l)]} = D_t \tag{18}$$

where D_t is the production scheduling at time t;

$$0 \leq q_{i,t,n} \leq \overline{q}_i \qquad i \in I,\ t \in T_k,\ n \in N_k,\ 1 \leq k \leq K \tag{19}$$

$$0 \leq v_{j,t,n} \leq \overline{v}_j \qquad j \in J,\ t \in T_k,\ n \in N_k,\ 1 \leq k \leq K \tag{20}$$

$$v_{j,t,n} = v_{j,t-1,\nu} + f_{j,t,n} + \sum_{i \in I} A_{i,j} \cdot q_{i,t,n} \qquad j \in J,\ t \in T_k,\ n \in N_k,\ 1 \leq k \leq K \tag{21}$$

$$\underline{v}_{j,T} \leq v_{j,T,n} \qquad j \in J, \quad n \in N_K \tag{22}$$

$$x_{l,t,n} = x_{l,t-1,\nu} + (buy_{l,t,n} - sell_{l,t,n})1_{t\in[T_s(l),T_e(l))} \qquad t \in T_k, \qquad n \in N_k \quad (23)$$

$$buy_{l,t,n} \geq 0 \ \ , \qquad sell_{l,t,n} \geq 0 \qquad t > t_{k(n)}, \qquad n \in N_k \qquad (24)$$

In the wealth equation (16), in the mass balance equations (21) of the hydro system model and in the financuial balance equation (23) $\nu = n$, if $t-1, t \in T_k$, and $\nu = pred(n)$, if $t-1 \in T_{k-1}$ and $t \in T_k$. The objective function in (5) represents the expected utility of the final wealth on the scenarios and the constraints are the equivalent of the deterministic constraints related to the structure of the hydroplant and to the financial contracts.

6. Electricity Spot and Forward prices Scenarios Generation

The Italian electricity spot market was opened in 2003, its activity has been increasing during the last years and can be considered as a liquid market with many daily transactions. In our analysis we consider the daily base-load spot prices time series from 1/1/2008 to 9/9/2009. After removing the daily and weekly seasonal components, we analyze the log prices data and we find stationarity but no strong presence of spikes: only four observations are larger then 3 times the standard deviation on the whole period. The log spot price exhibits autocorrelation, heteroschedasticity but not a dramatic kurtosis. In line with recent researches, we fit a regime switching model able to capture different market conditions, in terms of changing mean and volatilities. We assume that y_t, the log price process, follows an AR(1) model depending on the state variables s_t :

$$y_t = \mu_{s_t} + \Phi_{s_t} y_t + \epsilon_t \quad \text{where} \quad \epsilon_t \sim i.i.d\ N(0, \sigma^2_{s_t}) \qquad (25)$$

where s_t changes through time and takes values $j = 1, \ldots, J$. The changes in s_t are described by a Markov chain $P(s_{t+1} = j | s_t = i) = p_{ji}$. We do not observe s_t directly, but we only infer it through the observed behaviour of y_t.

Using the complete data set we find the evidence of the presence of two regimes. The parameters necessary to fully describe the probability law governing y_t are then the volatility of the Gaussian innovation, the autoregressive coefficients, the two intercepts and the two state transition probabilities, p_{11} and p_{22}. In Table 1 and 2 we present the estimated parameters with the t-statistics in parenthesis.

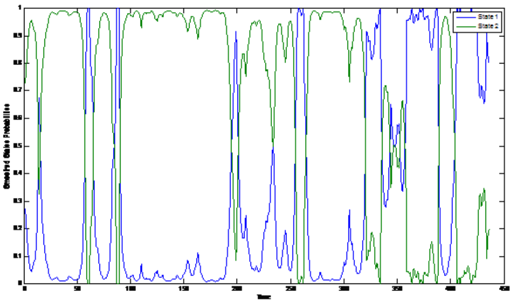

Figure 2: The Spot Switching Model

Table 1: Table related to 2.

	μ	Φ_{s_t}	σ
State 1	0.0140	0.1510	0.0760
	(2.4140)	(1.9610)	(18.5366)
State 2	-0.0210	-0.0507	0.1409
	(-1.8584)	(0.6225)	(19.8451)

Table 2: Transition probability between the two regimes in 2.

Transition	State 1	State 2
State 1	0.96 (0.05)	0.04 (0.02)
State 2	0.05 (0.03)	0.95 (0.04)

For a multistage stochastic programming model a scenario tree is needed where information is revealed. Hence, we generate 100 independent scenarios describing the evolution of spot prices on a time horizon of one year and we aggregate them in a recombining tree using the backward scenario reduction technique proposed by Pflug [15] and Pflug and Hochreither [10]. By reducing the 100 scenarios we derive a three stage tree and the real probability measure, P, describing the probability for each scenario. In order to maintain consistency with the market, the spot prices scenarios are adjusted so that the expected average spot price in a period is equal to the current market price of a forward with delivery in that period.

$$F(t_0, T_1, T_2) = \sum_s P_s \cdot \frac{\sum_{t=T_1,T_2} Spot_s(t)}{T_2 - T_1} \quad (26)$$

where $Spot_s(t)$ is the spot price in scenario s at time t and $F(t_0, T_1, T_2)$ is the today observed forward quotation for a contract with delivery period in $(T_1;\, T_2)$.

The theoretical dynamic of the forward prices can be derived from the spot dynamic. The price at time t of the forward contract with delivery period $(T_1;\, T_2)$ can be constructed on the spot scenarios as the expected value of the average spot prices in the delivery period under an equivalent Q-martingale measure, conditional on the information set available up to time t. In a complete market this measure is unique and it assures a unique arbitrage free price of the forward. In incomplete markets, as the electricity market, this measure is not unique. An approach common in litterature is to assume that the real probability measure coincides with the risk neutral probability measure and perform the pricing directly. This is the approach similar to the approach of Fleten and al.[18]. [2]

Following this approach, the consistency constraints on the spot scenarios combined with the direct pricing of the forward contract on the spot scenarios implies that the model supports only hedging aspects of trading in contracts. The source of stochasticity is only one in this case: the spot dynamics. See figure 3 for an example of scenario tree using this approach.

However, if the decision maker expects the average spot price to be different, in any time t along the scenarios, from the forward price then we may introduce the possibility of speculation. In reality, the electricity spot and futures prices are not closely related, as it is typical for other commodities, such as crude oil. Electricity spot and forward prices can be very far from each other. Often, the spot and futures markets are so dissimilar that the relationship between spot and futures prices breaks down. If we compare, ex post, the forward quotations with the realised spot prices we observe that the difference does not necessarily tend to zero for contracts approaching their maturity.

"‘For example, for the Nordpool data, the historical correlation (com-

[2] An alternative approach is to remain under the real probability measure and estimate and incorporate a market price of risk in the drift. However, this approach requires a liquid market for forward contracts.

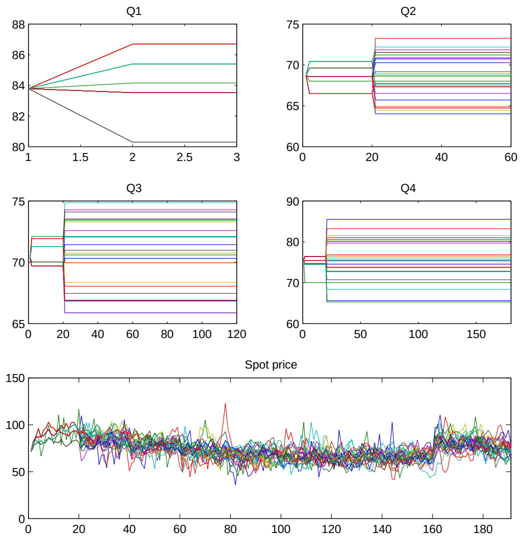

Figure 3: Scenarios for spot and related forward prices

puted using a moving window of the past 60 days) between the electricity spot and the nearby futures price ranges from 0.65 to −0.15, indicating that the futures price is a poor proxy for the electricity spot price."' see Borovkova and Geman [3],[4].

For this reason we decide to model explicitly the dynamic of the forward curve imposing that the scenarios generated for the spot and forward contracts are correlated and consistent with the today observed contracts quotations, i.e we impose that

1. the spot scenarios are consistent with the observed quotations of the forward contracts (26) and
2. expected forward price at each time equals the today observed quotation

$$F(t_0, T_1, T_2) = \sum_s P_s \cdot F_s(t, T_1, T_2) \quad \forall t \tag{27}$$

where $F_s(t, T_1, T_2)$ is the forward price along the scenario s at time t.

The sources of stochasticity are 2 in this case: the spot dynamics and the forward curve dynamics.

The electricity forward curve is a non-trivial object and requires special attention, mainly for two reasons. Firstly, forward contracts are subject to a seasonal effect related to the delivery period and secondly, the term structure cannot be constructed simply by interpolating between points in the price maturity space because electricity forward contracts concern delivery of electricity during a given time interval - week, month, year - in the future, not a single day. Consequently, the methods developed for fixed income markets cannot be applied directly to electricity price data. We restrict our analysis to one segment of the term structure, the quarterly contracts, and leave a more comprehensive analysis to future research. Here we apply the model to the forward prices of quartely contracts in the period 1/01/09-9/09/09 on Italian data of OTC contracts quoted at the TFS, a private platform which at present is more liquid than the standardised market, the IDEX (Italian Derivatives Energy Exchange) just started at the beginning of 2009.

The idea is to compute from the daily quotations of the forward contracts, the forward term structure for fixed key rates and analyse the dynamics of the term structure.

In order to derive the forward curve we have to remove the seasonal effect associated with forward contracts. We follow Borovkova and Geman [3],[4]. They observe that seasonal effects in the spot price and in the futures contracts are significantly different and that the main feature of electricity forward curves is the seasonality attached to the delivery period, not to the trading day. Let $F(t, T, T+Q)$ be the day-t price of the forward contract expiring in T with $T = (T1, T2, T3, T4)$. We fix the beginning of the delivery period at 4 dates (January 1st , April 1st, July 1st and September 1st) and the length of delivery period to 3 months in order to represent the quarterly contracts $Q1, Q2, Q3, Q4$.

We estimate the deterministic seasonal forward premium, $\pi(T)$, for each delivery date assuming that the forward price is the product of two components, a seasonal component (the premium) and a component which depends on the time to maturity

$$F(t, T, T+Q) = \bar{F}(t)e^{\pi(T)-\gamma(t,T-t)(T-t)} \quad (28)$$

where the deseasonalized forward price $F^{DS}(t, T, T+Q)$ is

$$F^{DS}(t, T, T+Q) = F(t, T, T+Q)e^{-\pi(T)} \quad (29)$$

The seasonal premium is defined as the average deviation fom the mean value of the log forward quotations

$$\hat{\pi}(T) = \frac{1}{n}\sum_t (\ln(F(t,T,T+Q)) - \ln(\bar{F}(t))) \quad T = T1, \ldots, T4 \tag{30}$$

where

$$\ln(\bar{F}(t)) = \frac{1}{4}\sum_{T=T1,\ldots,T4} \ln(F(t,T,T+Q)) \tag{31}$$

The estimated seasonal premia for quartely electricity forwards are shown in Figure 4.

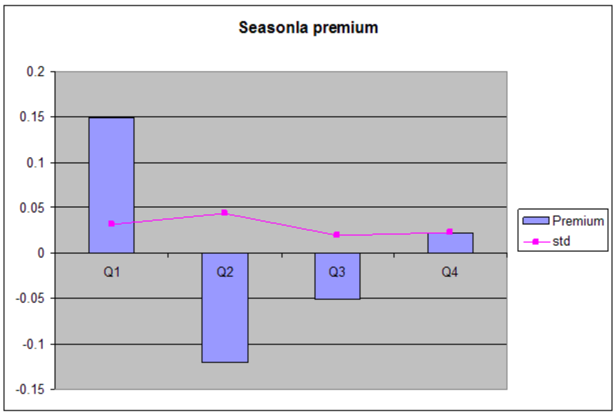

Figure 4: The Forward Seasonal Premia

As expected, forwards expiring in winter are at a premium with respect to the average price level, and summer forwards at a discount. For electricity the January premium is the highest, at 15%, while the April is −12%.

Once we have removed the seasonal premium, we can derive the terms structure for quarterly contracts. More precisely we concentrate our attention on four key rates relative to the four quarterly maturities. The procedure used in the forecasting approach for scenario generation basically involves two distinct steps. By principal component analysis (PCA) on the daily deseasonalized historical returns of forwards key rates, we find the orthogonal factors. The first three factors explain 92% for the forward curve and correspond to a parallel shift (first factor which explains 61%) a tilting (second factor which explains 17%) and a curvature effect (third factor explains 14%), see Figure 5. We consider as relevant the first two factors that have an explanatory power of about 80% of the variability.

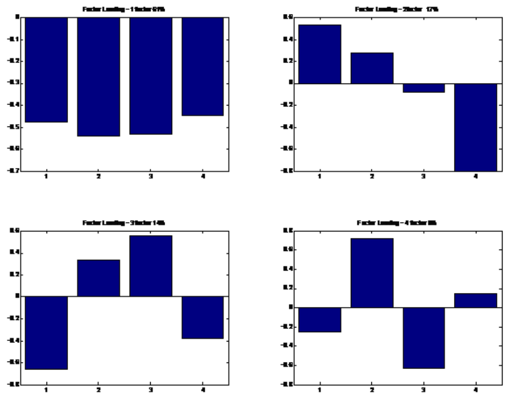

Figure 5: Factor loadings

The residuals contains all the information related to the residual 20% of variability and the correlation among the returns of the different maturities. Thus, we compute the residuals not explained by the first two factors obtained from the PCA and we then model the variance of the residuals with a GARCH model in order to capture the dependence of returns. This model incorporates:

- a dependence effect given by the relevance of the observations of the immediate past (conditional term)

- a feedback mechanism through which past observations are taken into consideration to explain the present volatility value (autoregressive part).

Roughly speaking, if a time series exhibits GARCH effects, it means it is heteroskedastic; that is, its variance may be well described by a time-varying process.

The variables to be modelled are the residuals series obtained by the PCA. We applied the following univariate GARCH(1,1) to each of them:

$$r_t = \epsilon_t \tag{32}$$

$$h_t^2 = \alpha_1 \epsilon_{t-1}^2 + \alpha_2 h_{t-1}^2 \tag{33}$$

where h_t^2 is the conditional variance process of the residuals and ϵ_t is the innovation of the time series process, with $\epsilon_t = z_t h_t$ and z_t is a Gaussian *i.i.d.*

process with zero mean and unit variance. In Table 3 we report the estimate of the GARCH(1,1) models and asymptotic t-statistics.

Table 3: Garch(1,1) parameters and t-statistics.

	α_1	α_2
Residual Q1	0.0376 (3.2478)	0.9499(54.0736)
Residual Q2	0.0233 (3.3218)	0.9654(97.9361)
Residual Q3	0.1274 (2.7203)	0.8492(21.4536)
Residual Q4	0.1125 (1.6657)	0.8589(13.5745)

In order the generate correlated scenarios, we combine together the standardized residuals of the GARCH(1,1) model and the residuals from the regime switching model for the same days. We do not impose any parametric assumption on the marginal distributions and use the the empirical cumulative distribution to fit a Gaussian copula of the historical residuals vectors. We simulate a vector of correlated innovations from the Gaussian copula and reconstruct the forecasted scenarios using the estimated principal factors for the forward price scenario and the regime switching for the log spot price. Following this procedure we generate correlated scenarios for spot and forward prices

Hence, we generate 100 correlated scenarios and we aggregated them in a recombining tree using the backward scenario reduction technique proposed by Pflug and Hochreither [10] (See figure 3. Finally, we adjust the multivariate tree, as described above, in order to guarantee market consistency.

7. Numerical results

In this section we discuss the numerical results obtained by solving the stochastic model on two cases studies. The simulation framework is based on MATLAB release 12 and on GAMS release 21.5, for modeling and solving the optimization problem by non linear optimization package (MINOS). The hydro system is composed by one cascade with three basins and three hydro plants, one of these is a pumped storage hydro plant as shown in Figure 7 (see Tables 4 and 5 for input data of the hydro system). In order to represent the scenarios we introduce the following notation, see also Vespucci et al. [17], $T_1 = 1$, $T_2 = \{t : 2 \leq t \leq 20\}$, $T_3 = \{t : 21 \leq t \leq 191\}$. We have considered

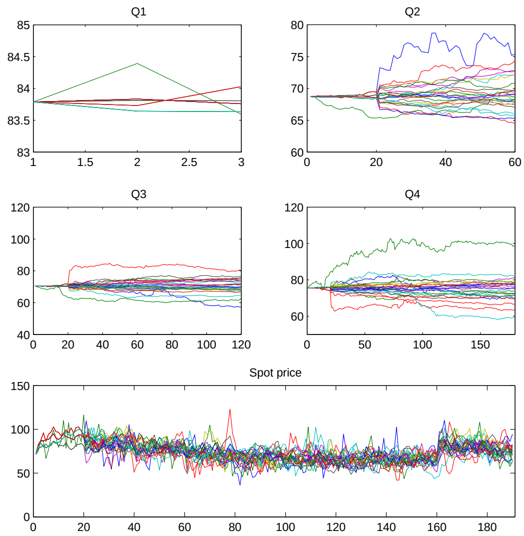

Figure 6: Multivariate scenarios for spot and forward prices

twenty five scenarios represented by means of a scenario tree where the nodes are as follows: $N_1 = 1$, $N_2 = \{2, \ldots, 6\}$, $N_3 = \{7, \ldots, 31\}$.

Table 4: Hydro basin data: capacity, initial and minimum final storage volumes.

basin	$\overline{v}_j$	$v_{j,0}$	$\underline{v}_{j,T}$
v_1	1000	100	0
v_2	2000	1000	500
v_3	2000	1000	500

Table 5: Hydro arc data: energy coefficient and capacity.

arc	k_i	$\overline{q}_i$
c_1	1.0	100
c_2	-1.7	50
c_3	1.1	150
c_4	0.9	120

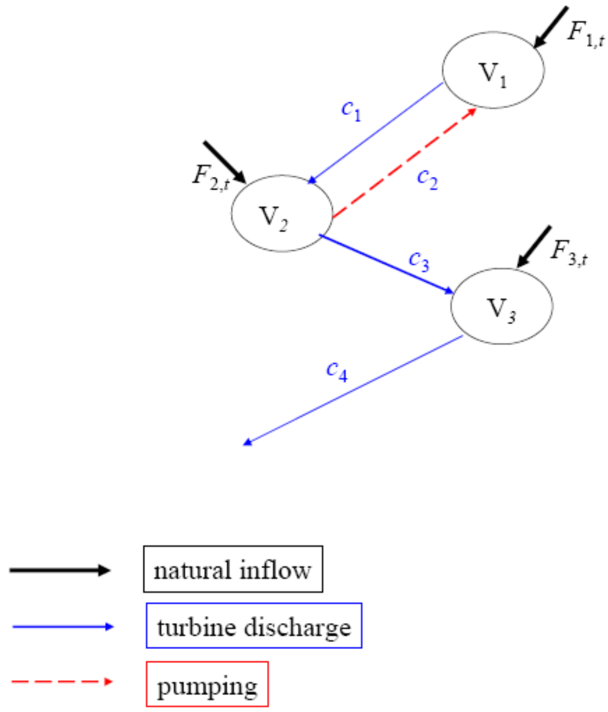

Figure 7: The hydro system

In order to assess the value of modeling uncertainty for three stage problems, we follow the procedure introduced in the literature by Vespucci et al. [17] for evaluating the value of the stochastic solution for three stage problem.

The procedure is based on the idea of reproducing the decision process as the uncertainty reveals: this procedure is suitable for multistage problems and is not prone to infeasibility.

The optimal objective value obtained in stage 3 is called *modified EEV (MEEV)*. Technically, this is computed as follows.

1. Scenario tree $\mathscr{T}_{1,mean}$ (see Figure 8(a)), is defined by considering the expected value of the uncertantiy parameters (spot and forward prices); the stochastic model with scenario tree $\mathscr{T}_{1,mean}$ is solved and the optimal values of the first stage variables are stored. In this way the optimal solution of the EV problem is computed.

2. Scenario tree $\mathscr{T}_{2,mean}$ (see Figure 8(b)) is the expected value of the spot and forward prices on nodes belonging to N_3. The stochastic model with scenario tree $\mathscr{T}_{2,mean}$ is solved having assigned the value stored at step 1. to the first stage decision variables. The oprimal value of second stage variables are stored.

3. The stochastic model on benchmark tree $\mathscr{T}_1$ (see Figure 8(c)) is solved, assigning to the first stage decision variables the values stored at step 1 and to the second stage decision variables the values stored at step 2, (see Figure 8(c)).

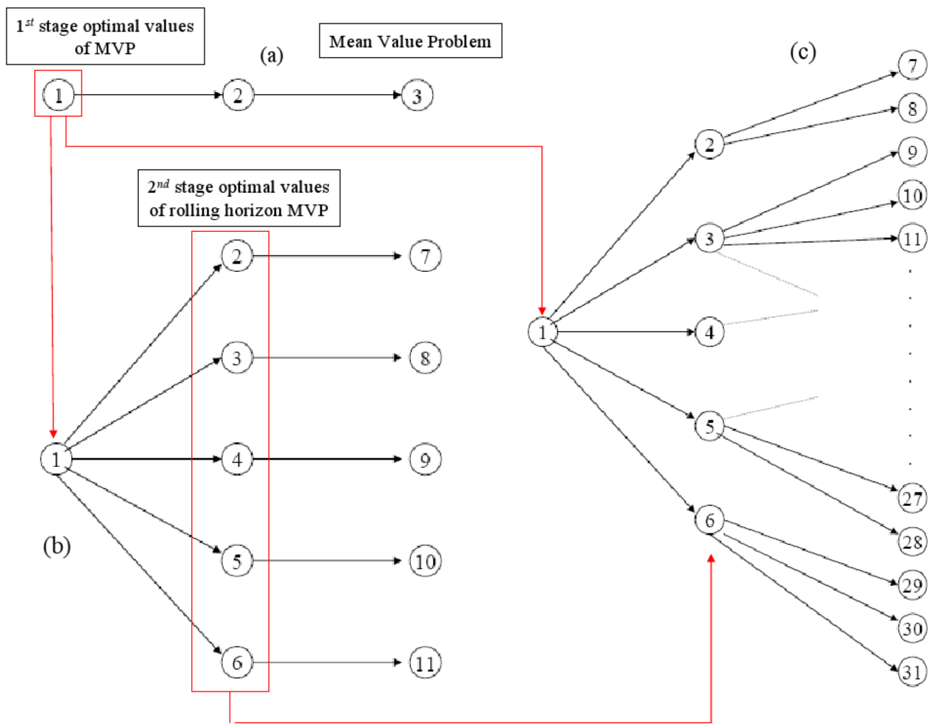

Figure 8: Scheme for the computation of MEEV

As first step, we solve the model considering only one source of variability, the spot price. If we consider only the financial components (buying and selling on the spot and forward markets) we notice a decrease both in the expected value and in the variance of the cumulated profit along the scenarios. It is natural since the contracts are used for hedging the risk related to movements of the spot prices. The main difference between the solutions with and without financial contracts is in the use of production scheduling. The introduction of financial contracts leads to a more efficient use of pumping, ending up with a higher expected value of water at the time horizon T. Overall, the effect of using forwards is an increase of the value of the objective function - from 267452.00 to 285248.42.

Finally, we consider all the sources of variability, the spot and the forward prices. If we consider only the financial part, we notice an increase both in the expected value of the profit and in the variance of the cumulated wealth along the scenarios. In this case we do not have pure hedging but speculative contracts. As before, the use of forward contracts allows a more efficient use of water pumping with a higher final level of water in the basins. Overall, the effects of the financial part and the production scheduling lead to an increase in the objective function, from 232,068.61 to 251,308.03. We report the certainty equivalent obtained by using a power utility function with risk aversion coefficient -0.5.

Finally, we compute the Modified EEV (MEEV=2445909.78). The value of the stochastic solution is 6798.25 and it allows us to obtain the goodness of the expected solution value when the expected values are replaced by the random values for the input variables.

Table 6: two sources of stochasticity.

Profit Value (Euro)	Certainty equivalent
Stochastic Model	251,308.03
Modified expected mean value model (rolling horizon)	244,509.78
Modified VSS	6,798.25

8. Conclusions

In this paper we have introduced a model for the daily hydro power system scheduling problem with scenarios on days power production. The model is a stochastic multi-stage non linear model where the profit comes from the direct production and financial operations in the forward energy market. At this stage we consider as random variables the spot energy price and the forward prices. When we consider only one source of stochasticity the forward contracts can be used for hedging purposes. When we consider two source of uncertainties, we allow for a speculative behaviour of the producer.

Our results show that, a part from the financial gains, the convenience of using financial contracts is a more efficient use of the hydroplant, taking advantage of the possibility of pumping water and ending up with a higher final value of the reservoir.

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