LP active benchmarking strategies based on performance measures and stochastic dominance constraints

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Abstract

The active benchmark tracking portfolio problem is a investment strategy which aims to exceed the performance of a selected target benchmark and it is sometimes referred to as to active portfolio management. It is well known that many professional investors achieve this benchmarking strategy: The aim of this work is to solve the benchmark tracking problem implementing active strategies to manage a portfolio with the aim to outperform the benchmark index. We develop linear formulation portfolio optimization problems which maximize some performance measures. Then, introducing first and second order stochastic dominance constraints, we evaluate their impact in the invested portfolio wealth path in a high dimensionality framework.

Keywords

Active Strategy, Portfolio Selection, Performance Measure, Linear Programming, Stochastic Dominance.

JEL Classification: C61, G11.

1. Introduction

The problem to identify the "best" portfolio composition to beat a given benchmark or market portfolio is still an open question in the financial literature. The benchmark tracking problem covers a wide range of portfolio strategy with the aim to replicate, enhance or beat a given benchmark. In particulat, since portfolio managers present different goals they introduce several portfolio strategies to achieve them.

In this paper, we propose some linear programming portfolio selection problems based on the maximization of four performance ratios. Since this type of formulation results to be non-linear we review the methodology to linearize the STARR and the Rachev Ratio (Stoyanov et al., 2007) and we develop the theoretical structure to linearize the Sharpe Ratio and the proposed Semideviation Ratio. Then, we introduce and empirically test two orders of stochastic dominance constraints (Kuosmanen, 2004; Kopa, 2010) in the portfolio selection models. These constraints should enhance the wealth path of the invrested portfolio as empirically tested in the last section.

This paper is organized as follow. In the next session we review the four performance measure introducing the proposed Semideviation Ratio. In Section 3, we define the linear

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programming portfolio selection problems which maximize the performance measure and we develop the problem formulation to take into account first and second order stochatic dominance constraints. Section 4 present an empirical analysis while in the last section we briefly summarize the main results of this paper.

2. Performance measures and different investors' profiles

In the active strategy the goal of portfolio managers is to maximize their future or final wealth considering the performance of an index or benchmark. In particular, maximizing future investors wealth, we generally use a reward/risk portfolio selection applied either to historical series or to simulated scenario models (see, among others, Rachev et al., 2008 and Biglova et al., 2004). Given N assets with returns $r = [r_1, \dots, r_N]'$, the return portfolio is $x_t\beta = \sum_{n=1}^{N} x_{n,t}\,\beta_n$, where $x = [x_1, \dots, x_N]'$ is the vector of the positions taken in the N assets. To maximize the performance of a portfolio in the reward/risk framework, we provide the maximum expected reward μ per unit of risk ρ. This optimal portfolio is commonly called the market portfolio and it can be obtained with several possible reward/risk performance ratios (Cogneau and Hubner, 2009a, Cogneau and Hubner, 2009b) defined as:

$$G(X) = \frac{\mu(X)}{\rho(X)}$$

Recall that a performance ratio must be isotonic with investors' preferences, i.e. if X is preferable to Y, then $G(X) \geq G(Y)$ (Rachev et al., 2008). Although the financial literature agrees that investors are non-satiable, there is no common vision about their risk aversion. Investors' choices should be isotonic with non-satiable investors' preferences, i.e. if $X \geq Y$ then $G(X) \geq G(Y)$, and several behavioral finance studies suggest that most investors are neither risk averse nor risk seeking (Rachev et al., 2008, Cogneau and Hubner, 2009a, Cogneau and Hubner, 2009b).

2.1 The Sharpe Ratio

The Sharpe ratio is a commonly used measure of portfolio performance. However, because it is based on the mean-variance theory, it is valid only for either normally distributed returns or quadratic preferences. In other words, the Sharpe ratio is a meaningful measure of portfolio performance when the risk can be adequately measured by standard deviation. When return distributions are non-normal, the Sharpe ratio can lead to misleading conclusions and unsatisfactory paradoxes, see, for example, (Bernardo and Ledoit, 2000; Ortobelli et al., 2005). According to the Markowitz mean-variance analysis, Sharpe (1994) suggests that investors should maximize what is now referred to as the Sharpe Ratio (SR) (Farinelli et al., 2008) given by:

$$SR = \frac{\mathbb{E}\left[r_t^p - r_t^b\right]}{\sigma\left(r_t^p - r_t^b\right)}$$

where $r_t^p = x_t\beta$ is the active portfolio, r_t^b is a benchmark return and $\sigma\left(r_t^p - r_t^b\right)$ is the standard deviation of excess returns. By maximizing the Sharpe Ratio, we obtain a market portfolio that should be optimal for non-satiable risk-averse investors, but that is not dominated in the sense of second-order stochastic dominance. This performance measure is fully compatible with elliptically distributed returns, but it leads to incorrect investment decisions when the returns distribution presents heavy tails or skewness.

2.2 The Rachev Ratio

The Rachev Ratio (Biglova et al., 2004) is based on tail measures and it is isotonic with the preferences of non-satiable investors that are neither risk averse nor risk seekers. The Rachev Ratio (RR) is the ratio between the average of earnings and the mean of losses; that

$$RR = \frac{CVaR_{\beta}(X)}{CVaR_{\alpha}(X)}$$

where the Conditional Value-at-Risk (CVaR), is a coherent risk measures (Artzner et al., 1999; Rockafellar and Uryasev, 2002) defined as:

$$CVaR_{\alpha} = \frac{1}{\alpha}\int_{0}^{\alpha} VaR_q(X)dq$$

and

$$VaR_q(X) = -F_X^{-1}(q) = -inf\{x|\mathbb{P}(X \leq x) > q\}$$

is the Value-at-Risk (VaR) of the random return X. If we assume a continuous distribution for the probability law of X, then $CVaR_{\alpha}(X) = -\mathbb{E}[X|X \leq VaR_{\alpha}(X)]$ and, therefore CVaR can be interpreted as the average loss beyond VaR. Typically, we use historical observations to estimate the portfolio return and a risk measures. A consistent estimator of $CVaR_{\alpha}(X)$ is given by:

$$CVaR_{\alpha}(X) = \frac{1}{\lceil \alpha T \rceil}\sum_{t=1}^{\lceil \alpha T \rceil} X_{\{t:T\}}$$

where T is the number of historical observations of X, $\lceil \alpha T \rceil$ is the integer part of αT and $X_{\{t:T\}}$ is the t-th observation of X ordered in increasing values. Similarly an approximation of $VaR_q(X)$ is simply given by $-X_{(\lceil qT \rceil:T)}$.

2.3 The STARR

In 2005, Martin et al. introduce a different reward risk measure: the Stable Tail Adjusted Return Ratio ($STARR_{\alpha}$). This measure is a generalization of the Sharpe Ratio but it allows to overcome the drawbacks of the standard deviation as a risk measure (Artzner et al., 1999). In particular, STARR focus on the downside risk and it is not a symmetric and unstable measure of risk when returns present heavy-tailed distribution. Thus, let a random variable \$X\$ be the difference between two random variables representing the portfolio and benchmark returns, the STARR at the confidence level \$\alpha\$ is expressed as:

$$STARR_{\alpha} = \frac{\mathbb{E}[X]}{CVaR_{\alpha}(X)}$$

The STARR differently from the Sharpe Ratio consider a coherent risk measure and not a deviation one as risk sources.

2.4 The Semideviation Ratio

Finally, we develop a measure of performance based on the two sides of the mean absolute semideviation measure. Thus, the Semideviation Ratio (SDR) is defined as:

$$SDR = \frac{\mathbb{E}\left[\left(r_t^p - r_t^b\right)\mathbb{I}_{\{r_t^p \geq r_t^b\}}\right]}{\mathbb{E}\left[\left(r_t^p - r_t^b\right)\mathbb{I}_{\{r_t^p < r_t^b\}}\right]} = \frac{\mathbb{E}\left[\max\left(r_t^p - r_t^b, 0\right)\right]}{\mathbb{E}\left[\max\left(r_t^b - r_t^p, 0\right)\right]} = \frac{\mathbb{E}\left[\max\left(r_t^p - r_t^b, 0\right)\right]}{\mathbb{E}\left[-\min\left(r_t^p - r_t^b, 0\right)\right]}$$

where $\mathbb{I}$ is the indicator function. This measure consider the ratio between positive and negative difference between invested and benchmark portfolio. Differently from the previous ratios which are defined in the interval $[-\infty, +\infty]$, the Semideviation Ratio has only positive outcomes (i.e. $SDR: \mathbb{R}^N \to [0, +\infty]$).

3. LP problem for active strategies with stochastic dominance constraints

In the Modern Portfolio Theory, the problem of choice to maximize the performance of an investor with different preferences is still an open question in the financial literature. In recent time, one of the main issue is to deal with portfolio problem characterized by high dimensionality and numerous assets. In particular, the first concept relates with the situation when the number of assets are greater than the historical observation. Considering these type of problems is essential to have an easily solvable problem and the class of LP problem results to be suitable. In fact, linear programming implies the possibility to find an optimal solution reducing the computational time. For this reason, the research of optimization techniques in order to obtain linear formulations leads to solvable and efficient portfolio optimization model and it is essential addressing with the high dimensionality.

Moreover, several problems need a preliminary reduction when the number of assets is still numerous. The main technique for a preliminary reduction is achieved introducing the pre-selection step before the portfolio optimization. Pre-selection consist to active only the assets that satisfy a given condition to reduce the number of portfolio's inputs. In this essay, we pre-select the “best” assets ordering with respect to their performance measure computed on the historical observation of the rolling window. Then, we select the first d assets such that $d \leq N$ with higher value. In the next subsection, we describe the general formulation of LP portfolio problem where we maximize a given performance measure and we introduce the first and second order stochastic dominance constraints (Kuosmanen, 2004; Kopa, 2010).

Let $r_t^p = x_t\beta$ be the return of the invested portfolio at time t where x_t is the raw of the asset returns and let r_t^b be the returns of the benchmark, we could define the common portfolio selection problem as follow:

$$\begin{aligned} \max_{\beta} \quad & \frac{\mu\left[r_t^p - r_t^b\right]}{\rho\left(r_t^p - r_t^b\right)} \\ s.t. \quad & \sum_{n=1}^{N} \beta_n = 1 \\ & lb \leq \beta_n \leq ub \quad \forall n = 1, \dots, N \end{aligned}$$

where β_n for $n = 1, \dots, N$ is the portfolio weight vector and optimal solution of the minimization problem, lb the lower bound and ub the upper bound as maximum amount invested in a given asset. In particular, fixing the value of the upper bound it is possible to define the number of minimum active assets in the portfolio selection problem.

We could notice that the objective function is non-linear since in the ratio the variable β appears both to the numerator and to the denominator. For this reason analyzing the nature of different reward and risk measure, we linearize these objective functions following (Stoyanov et al., 2007) for the STARR and Rachev Ratio and developing the linear formulation for the Sharpe Ratio and the Semideviation Ratio.

3.1 Maximization of the Sharpe Ratio

The maximization of the Sharpe Ratio in the common portfolio selection problem should be obtained following the fractional integral property in the minimization of the standard deviation (Ortobelli et al., 2013)

Proposition 3.1

The maximization of the Sharpe Ratio should be solve in a linear programming way considering that the reward measure satisfy the positive homogeneity property and it is a concave measure while the risk measure is positive homogenous and sub-addictive:

$$\min_{w,v,u,g} \quad \sum_{i=1}^{M-1} \frac{1}{T} \sum_{k=1}^{T} v_{k,i} + u_{k,i}$$

$$s.t. \quad \begin{aligned} &\mathbb{E}[x_t w] - g\mathbb{E}[y_t] = 1 \\ &\sum_{n=1}^{N} w_n = g \\ &g\, lb \leq w_n \leq g\, ub \\ &g \geq 0 \\ &v_{k,i} \geq c + \frac{i}{M}\left(\frac{1}{T}\sum_{t=1}^{T} x_t w - g y_t - c\right) - x_k w + g y_k \\ &u_{k,i} \geq c + \frac{i}{M}\left(-c - \frac{1}{T}\sum_{t=1}^{T} x_t w - g y_t\right) + x_k w - g y_k \\ &u_{k,i}, v_{k,i} \geq 0 \\ &\forall k = 1, \ldots, T; \forall i = 1, \ldots, M;\ \forall n = 1, \ldots, N; \forall t = 1, \ldots, T \end{aligned}$$

where M is a large integer and $c = -max\left(\left|min_\beta\, min_k(x_k\beta - y_k)\right|, \left|max_\beta\, max_k(x_k\beta - y_k)\right|\right)$. Thus, the optimal portfolio weight $\beta_n = \frac{w_n}{t}$ for $n = 1, \ldots, N$.

The previous portfolio optimization problem is linear and could be efficiently solved since it is not time consuming (Cassader et al., 2015).

3.2 Mixed-Integer linear programming to maximize the Rachev Ratio

Differently, Stoyanov et al., (2007) develop the following mixed-integer lienar programming to maximize the Rachev Ratio:

$$
\begin{array}{ll}
\underset{w,g,d,\gamma,\lambda,f}{min} & -\frac{1}{\lceil \beta T \rceil}\sum_{t=1}^{T} f_t \\
 & f_t \le B\lambda_t \\
 & f_t \ge x_t w - B(1-\lambda_t) \\
 & f_t \le x_t w + B(1-\lambda_t) \\
 & \sum_{t=1}^{T} \lambda_t = \lceil \beta T \rceil \\
s.t. & \gamma + \frac{1}{\lceil \alpha T \rceil}\sum_{t=1}^{T} d_t \le 1 \\
 & -x_t w - \gamma \le d_t \\
 & \sum_{n=1}^{N} w_n = g \\
 & g\ lb \le w_n \le g\ ub \\
 & g \ge 0; d_t \ge 0; f_t \ge 0 \\
 & \lambda_t \in \{0,1\} \\
 & \forall t = 1, \dots, T; \forall n = 1, \dots, N
\end{array}
$$

where α and β are the risk and the return level of confidence. Solving the previous problem, we obtain a global maximum of the Rachev Ratio.

3.3 Portfolio with maximum STARR

It is also possible to solve the problem to maximize the STARR with a linear formulation (Stoyanov, 2007):

$$
\begin{array}{ll}
\underset{w,g,d,\gamma}{min} & \gamma + \frac{1}{\alpha T}\sum_{t=1}^{T} d_t \\
 & \mathbb{E}[x_t w] - g\mathbb{E}[y_t] = 1 \\
 & \sum_{n=1}^{N} w_n = g \\
s.t. & g\ lb \le w_n \le g\ ub \\
 & g \ge 0; d_t \ge 0 \\
 & -x_t w + g y_t - \gamma \le d_t \\
 & \forall t = 1, \dots, T; \forall n = 1, \dots, N
\end{array}
$$

where α is the confidential level.

3.4 LP problem to maximize the Semideviation Ratio

Finally, we propose a portfolio optimization problem maximizing the proposed Semideviation Ratio

Proposition 3.2

The maximization of the Sharpe Ratio should be solve in a linear programming way considering that the reward measure satisfy the positive homogeneity property and it is a concave measure while the risk measure is positive homogenous and sub-addictive:

$$\min_{w,d,g} \quad \sum_{t=1}^{T} d_t$$

$$\text{s.t.} \quad \begin{gathered} \mathbb{E}[x_t w] - g\mathbb{E}[y_t] = 1 \\ \sum_{n=1}^{N} w_n = g \\ g\, lb \le w_n \le g\, ub \\ g \ge 0; d_t \ge 0 \\ d_t \ge g y_t - x_t w \\ \forall t = 1, \dots, T; \forall n = 1, \dots, N \end{gathered}$$

In this case, the introduction of a slack variable d_t allow to linearly solve the problem to maximize the Semideviation Ratio (Cassader et al., 2015).

3.5 First and second order stochastic dominance constraints

To obtain portfolios that dominate the benchmark, we consider first and second order stochastic dominance constraints in the portfolio selection problem. This formulation is proposed in Kuosmanen (2004) and Kopa (2010) with the introduction of a permutation matrix P with realization $p_{r,c}$ for $r = 1, \dots, T$ and $c = 1, \dots, T$ to have the first order stochastic dominance constraints set and with a double stochastic matrix Z with realization $z_{r,c}$ for $r = 1, \dots, T$ and $c = 1, \dots, T$ for the second order stochastic domiance. In the portfolio optimization problem, we have the following set of constraints to add to the previous optimization problems:

$$\begin{aligned} & x_t\beta \ge P y_t \quad && \forall t = 1, \dots, T \\ & \sum_{c=1}^{T} p_{r,c} = 1 && \forall r = 1, \dots, T \\ & \sum_{r=1}^{T} p_{r,c} = 1 && \forall c = 1, \dots, T \\ & p_{r,c} \in \{0,1\} \end{aligned}$$

for the first order stochastic dominance. In this case, the problem becomes a mixed-integer linear programming since the permutation matrix allows only integer values. The set of constraints for the second order stochastic dominance is the follow:

$$\begin{aligned} & x_t\beta \ge Z y_t \quad && \forall t = 1, \dots, T \\ & \sum_{c=1}^{T} z_{r,c} = 1 && \forall r = 1, \dots, T \\ & \sum_{r=1}^{T} z_{r,c} = 1 && \forall c = 1, \dots, T \\ & z_{r,c} \in [0,1] \end{aligned}$$

In this case the linearity of the common portfolio problem is still hold.

4. Empirical Application

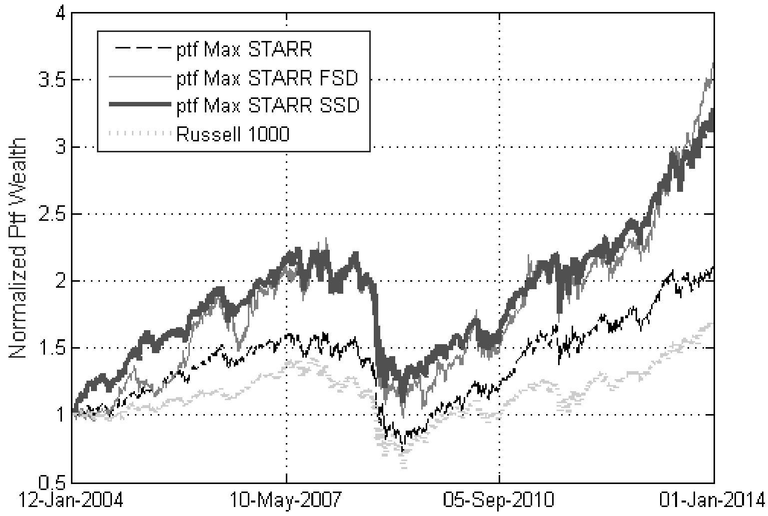
The proposed methodologies are applied to the active management strategy solving the problem to find a portfolio composition which beat the benchmark in the last ten years. For this reason, we compute an empirical application where the benchmark stock index is the Russell 1000 from 31st December 2002 to 31st December 2013 and we investigate how different portfolio selection problems could outperforms its performances. Solving the previous linear programming optimization problems with or withour first or sencond order stochastic dominance constraints we develop three optimization strategies for each of the presented ratios. Thus, we consider a historical moving window of 260 observations while for the mixed-integer linear programming we take into account 120 time series data. Every strategy starts on 12th January 2004. Then, at each optimization step, we compute the $STARR_{5\%}$ and the $RACHEV_{5\%,2\%}$ for a total number of 125 optimization since we change portfolio composition every month (20 days). Finally, transaction costs of 30 bps are included.

4.1 Active strategies in the benchmark tracking problem with stochastic dominance constraints

In this section, we report the results of an empirical analysis solving six portfolio selection problem involving the maximization of the STARR and the Rachev Ratio with or without first or second order stochastic domiance constraints. Then, we compare and empirically test the different optimization methods and the impact to introduce stochastic dominance constraints in the problem formulation.

Figure 1 and Figure 2 illustrate the normalized wealth path during the investment period from 12th January 2004 to 31st December 2013. In Figure 1 we notice how every active strategy outperforms the Russell 1000. In particular, the classical maximization of the STARR produce an extra-performance of the 40% at the end of the investment period. Then, it is evident the impact of the introduction of the stochastic dominance constraints in the portfolio decisional problem. In fact, during the first part of the investment period these two strategies reach about 2.2 gaining more than 120%.

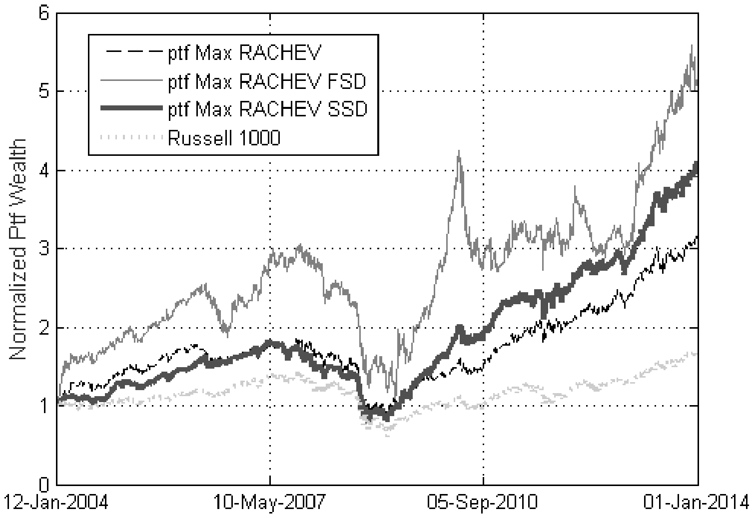
Figure 1: Portfolio Wealth, Active Strategies STARR



These active portfolio strategies with stochastic dominance constraints suffer the sub-prime crisis but the values of their portfolio does not fall down the initial wealth and they could have a solid base to make use of the following financial upturn. Therefore, starting from the 2009 the market increase forcefully and the strategies based on the maximization of the STARR significantly dominate the benchmark stock index. In particular, the portfolio strategies with first and second order stochastic dominance constraints have a final wealth more than 3.5 and 3.2, respectively.

Neither risk aversion nor risk seeking investors that maximize their utility solving a portfolio problem which involves the maximization of the Rachev Ratio obtain higher final value than the previous strategies. Figure 2 shows the normalized wealth path of the Russell 1000 (purple line), the maximization of the Rachev ratio, the strategy with the introduction of first order stochastic dominance and the maximization of the Rachev Ratio with second order stochastic dominance constraints. Also in this case every active managed portfolio outperforms the benchmark stock index for the overall period.

Figure 2: Portfolio Wealth, Active Strategies Rachev Ratio



Weather the common maximization of the Rachev Ratio produce a portfolio with more than 3 times the initial wealth allocation, the other two strategies have better results. In particular, the maximization of the performance measure with first order stochastic dominance constraints amplifies the market jumps during the entire period. In fact, this strategy reach 3 time the initial wealth in 2008 before the sub-prime crisis while investors holding this portfolio composition have also a peak of more than 400% of earnings in 2010 and after a period of stability during the European sovereign debt crisis the portfolio wealth path starts a increasing rally with a final value more than 5 times the original invested capital.

5. Conclusion

In this paper, we treat active management portfolio strategy proposing a portfolio selection problem consistent with the maximization of a performance measure and stochastic dominance constraints. Following the linear formulation proposed by Stoyanov et al., 2007, we implement LP portfolio selection problems, which maximize Rachev Ratio and STARR adding stochastic dominance constraints. Then, we develop LP portfolio formulation maximizing Sharpe Ratio and Semideviation Ratio. Finally, we empirically test this approach and we could notice how there is an increment in the portfolio wealth not only with respect to the index tracking and enhanced indexation strategies but also adding different orders of stochastic dominance in the active strategy management problem.

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