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Decision support models for short term hydro-thermal resource scheduling of a price-taking power producer

by

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Decision support models for short term hydro-thermal resource scheduling of a price-taking power producer

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Abstract

In this paper we develop a decision support procedure for the short-term hydro-thermal resource scheduling problem of a power producer who operates in the liberalized electric energy market and aims at maximizing his own profit. The resources owned by the producer are hydroelectric plants and/or thermoelectric plants. It is assumed that weekly discharge of seasonal basins and maintenance plans of hydro and thermal plants have been determined so as to maximize medium-term (one year) profit. In the short-term horizon (one week or ten days), the power producer has to solve the unit commitment problem for the thermal units and the dispatchment problem for the available hydro plants and the committed thermal units, aiming at maximizing short-term profit. His decisions must be compatible with both technical constraints, i.e. inherent the production technologies, and market constraints. The procedure proposed in this paper is based on a mixed integer linear programming model, where the objective function represents the total profit and the constraints describe the hydro system, the thermal system and the market. The thermal system is modelled in great detail as it allows start-up and shut-down manouvres in every hour of the planning horizon, taking into account minimum up-time and down-time constraints as well as ramp-up and ramp-down constraints. The power producer is assumed to be unable to influence the market price, therefore energy prices are parameters exogenous to the decision model and the optimal schedule is determined on the basis of price forecasts.

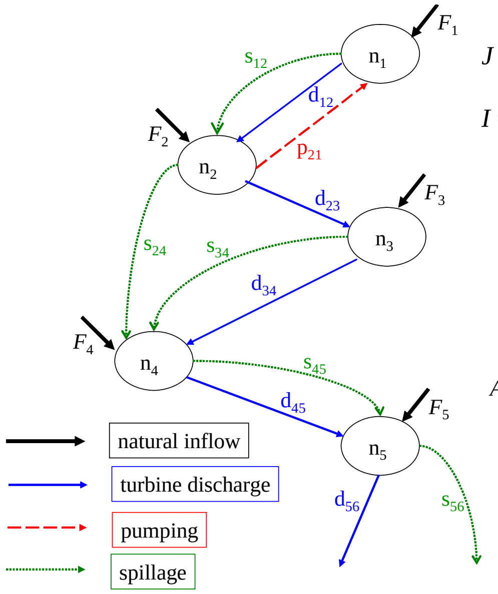
1 Introduction

In the last decade the electric power industry has undergone a fundamental transformation from one dominated by a regulated vertically integrated monopoly to an industry where electricity is produced and traded as a commodity. In the liberalized electricity market each power producer, in competition with other producers, aims at maximizing his own profit; production is sold by power producers either directly to consumers, on the basis of bilateral contracts, or by presenting sell bids to the Market Operator for each hour of the following day. Analogously, electricity is purchased by consumers either on the basis of bilateral contracts or by presenting purchase bids to the Market Operator for each hour of the following day. On the basis of the aggregated supply and demand curves, the Market Operator determines the equilibrium point of supply and demand, taking into account transmission system constraints defined by the Transmission System Operator. The electricity price is then a "market clearing price", resulting from the interactions among all market participants.

In the previous monopolistic context, production resource scheduling aimed at minimising production costs while satisfying given security standards. In liberalized markets each power producer aims at maximizing his own profit: if the producer is a price-taker, resource scheduling must take into account the price at which electricity may be sold; if the producer is a price maker, resource scheduling decisions must take into account both other producers' decisions and the Market Operator's rules for determining the electricity price.

Different time horizons are considered in the production resource scheduling problem. A time horizon of at least one year (medium-term scheduling) is considered when determining the optimal maintenance plans of hydro and thermal plants and the optimal weekly discharge of seasonal basins. A time horizon of a week (short-term scheduling) is considered for determining the unit commitment of thermal groups, i.e. the start-up and shut-down manouvres of the available (not in maintenance) thermal groups, as well as the production levels of the committed thermal units and of the available hydro plants in each hour. Authomatic decision support procedures for scheduling production resources have been developed with respect to the different time horizons: for the monopolistic case see for example Medina et al.(1998), Read (1999), Ouyang and Shahidehpour (1991), Burelli et al. (1990), Innorta et al. (1997a), Innorta et al. (1997b), Cazzol et al. (1998); for the case of liberalized market see Gross and Finlay (1996), Richter and Sheblé (1997), Li et al. (1999), Sheblé (1999), Zhang et al. (2000), Martini et al. (2001) and Garzillo et al. (2002). In this paper we develop a decision support procedure for the short-term hydro-thermal resource scheduling problem of a power producer who is assumed to be a price taker: prices do not depend on his own production decisions, i.e. are exogenous to the decision model, and the optimal schedule is determined on the basis of price forecasts. The decision support procedure, based on mixed integer LP models, determines the unit commitment of thermal units and the production levels of committed thermal units and available hydro plants in each hour so as to maximize profits, while satisfying constraints describing the hydro system, the thermal system and the market. The annual scheduling decisions (optimal maintenance plans of hydro and thermal plants, optimal weekly discharge of seasonal basins) are given, as well as forecasts on basin natural inflows.

The paper is organized as follows. In Sections 2 and 3 the model constraints describing the hydroelectric system and the thermoelectric system, respectively,



$$J = \{ n_1, n_2, n_3, n_4, n_5 \}$$

$$I = \{ d_{12}, d_{23}, d_{34}, d_{45}, d_{56}, s_{12}, s_{24}, s_{34}, s_{45}, s_{56}, p_{21} \}$$

$$A = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 \\ 1 & -1 & 0 & 0 & 0 \end{bmatrix}$$

Figure 1: an example of cascade

are discussed. In Section 4 a linearized model for thermal production costs is discussed and in section 5 market constraints and model objective function are presented. In section 6 some considerations are presented related to the model validation and in section 7 some lines for further research are discussed.

The planning horizon is short term (typically a week or 10 days) with a discretization period of 1 hour. Let T denote the number of periods considered and let t, $0 \leq t \leq T$, denote the period index, where $t = 0$ denotes the last hour of the planning horizon immediately preceeding the one in consideration.

2 Model of the hydroelectric subsystem

The hydroelectric subsystem consists of a number of cascades. A cascade is a set of hydraulically interconnected hydro plants, pumped-storage hydro plants and basins. Each cascade is mathematically represented by a directed graph: each node represents a basin, with a given storage capacity, and each arc may represent either a hydro plant (power generation) or a hydro pump (power storage) or a basin spillage (water flow to a downstream basin for keeping water storage within the storage capacity limit). Let J denote the set of nodes and I denote the set of arcs. An example of valley and of the corresponding arc-node incidence matrix A is reported in Figure 1.

The following data describe the hydroelectric system: for $i \in I$ and $j \in J$

A_{ij} : (i,j)-entry of network arc-node incidence matrix

$$A_{ij} = \begin{cases} -1 & \text{if arc } i \text{ leaves node } j \\ 1 & \text{if arc } i \text{ enters node } j \\ 0 & \text{if arc } i \text{ and node } j \text{ are not incident} \end{cases}$$

k_i $\left[MWh/10^3 m^3\right]$ energy coefficient
- $k_i > 0$, if arc i represents generation
- $k_i < 0$, if arc i represents pumping
- $k_i = 0$, if arc i represents spillage

$\overline{q}_i$ $\left[10^3 m^3/h\right]$ maximum water flow in arc i

ρ_i $[h]$ delay on arc i

$F_{j,t}$ $\left[10^3 m^3/h\right]$ natural inflow in basin j in hour t

$\overline{v}_j$ $\left[10^3 m^3\right]$ maximum storage volume in basin j

$v_{j,0}$ $\left[10^3 m^3\right]$ initial storage volume in basin j

$\underline{v}_{j,T}$ $\left[10^3 m^3\right]$ minimum storage volume in basin j at the end of hour T

Note that $v_{j,0}$ and $\underline{v}_{j,T}$ are determined by the annual resource scheduling.

The power producer must schedule the hourly production of each hydro plant, which is expressed as the product of the hydro plant energy coefficient times the turbined volume in hour t, as well as the hourly pumped and spilled volumes. The decision variable $q_{i,t}$ $\left[10^3 m^3/h\right]$ represents the water flow on arc i in hour t (turbined volume, if arc i represents generation, pumped volume, if arc i represents pumping, and spilled volume, if arc i represents spillage). Moreover, the decision variable $v_{j,t}$ $\left[10^3 m^3\right]$ represents the storage volume in basin j at the end of hour t. The decision variables must satisfy the following constraints that describe the hydroelectric subsystem:

- flow on arc i in hour t is nonnegative and bounded above by the maximum volume that can be either turbined or pumped or spilled

$$0 \leq q_{i,t} \leq \overline{q}_i \qquad i \in I \ , \quad 1 \leq t \leq T \tag{1}$$

- the storage volume in basin j at the end of hour t is nonnegative and bounded above by the maximum storage volume

$$0 \leq v_{j,t} \leq \overline{v}_j \qquad j \in J \ , \quad 1 \leq t \leq T \tag{2}$$

- at the end of hour T the storage volume in basin j is bounded below by the minimum storage volume required at the end of the current planning horizon, so as to provide the required initial storage volume at the beginning of the following planning horizon

$$\underline{v}_{j,T} \leq v_{j,T} \qquad j \in J \tag{3}$$

- the storage volume in basin j at the end of hour t must be equal to the basin storage volume at the end of hour $t-1$ plus the sum of basin inflows in hour $t - \rho_i$ minus the sum of basin outflows in hour $t - \rho_i$

$$v_{j,t} = v_{j,t-1} + F_{j,t} + \sum_{i \in I} A_{i,j} \cdot q_{i,t-\rho_i} \qquad j \in J \ , \quad 1 \leq t \leq T \tag{4}$$

  where $v_{j,0}$ is a data representing the initial storage volume in basin j. Basin inflows are natural inflows, turbine discharge from upstream hydro plants, pumped volumes from downstream hydro plants, spilled volumes from upstream basins. Basin outflows are turbine discharge to downstream hydro plants, pumped volumes to upstream hydro plants and spilled volumes to downstream basins.

3 Model of the thermal subsystem

Let K denote the set of thermal plants owned by the power producer. For every thermal unit $k \in K$ and for every hour $t \in T$, the power producer must decide

- the state (ON-OFF) of unit k in hour t and whether or not a manouvre (start-up or shut-down) has to take place, while taking into account minimum up-time and minimum down-time constraints;
- the production level of each committed unit in every hour of the planning horizon, while taking into account upper and lower bounds on production levels as well as ramp constraints.

The power producer decisions are represented by the set $p_{k,t}$ of nonnegative real variables and by the three sets of binary variables $\alpha_{k,t}$, $\beta_{k,t}$ and $\gamma_{k,t}$, with the following meaning respectively:

$$\alpha_{k,t} = \begin{cases} 1 & \text{start-up of thermal plant } k \text{ in hour } t \\ 0 & \text{no start-up of thermal plant } k \text{ in hour } t \end{cases}$$

$$\beta_{k,t} = \begin{cases} 1 & \text{shut-down of thermal plant } k \text{ in hour } t \\ 0 & \text{no shut-down of thermal plant } k \text{ in hour } t \end{cases}$$

$$\gamma_{k,t} = \begin{cases} 1 & \text{thermal plant } k \text{ ON in hour } t \\ 0 & \text{thermal plant } k \text{ OFF in hour } t \end{cases}$$

In order to take into account minimum up-time and minimum down-time constraints, the following data are needed for every thermal unit $k \in K$:

ta_k $[h]$: minimum number of hours unit k must be ON after start-up

ts_k $[h]$: minimum number of hours unit k must be OFF after shut-down

$\gamma_{k,0}$ $[0/1]$: state of unit k at the beginning of the scheduling period

nh_k $[h]$: in the scheduling period immediately preceeding the current one the last manouvre for unit k took place at hour $T - nh_k$

The following data are needed for every thermal unit $k \in K$, in order to take into account constraints on production levels $p_{k,t}$:

$\underline{p}_k$ $[MWh]$: minimum production level when unit k is ON

$\overline{p}_k$ $[MWh]$: maximum production level when unit k is ON

vsu_k $[MWh]$: maximum production of unit k at start-up

vsd_k $[MWh]$: maximum production of unit k at shut-down

δu_k $[MWh]$: maximum production increase per hour of unit k

δd_k $[MWh]$: maximum production decrease per hour of unit k

$p_{k,0}$ $[MWh]$: production of unit k at the beginning of scheduling period

The following constraints describe the thermoelectric unit k:

- relation among manouvres in t, state in t and state in $t-1$:

$$\gamma_{k,t-1} + \alpha_{k,t} = \gamma_{k,t} + \beta_{k,t} \qquad k \in K \ , \quad 1 \le t \le T \ ; \tag{5}$$

- minimum up-time constraints:

$$\text{if } \gamma_{k,0} = 1, \quad \text{then} \quad \gamma_{k,t} = 1 \qquad\qquad \text{for } 1 \leq t \leq ta_k - nh_k \tag{6}$$

i.e. if unit k was ON in the last hour of the previous scheduling period, it must be ON at least for the first $ta_k - nh_k$ hours of the current scheduling period;

$$\sum_{\tau=t+1}^{\min(t+ta_k-1,T)} \gamma_{k,\tau} \geq \alpha_{k,t} \cdot \min\left(ta_k - 1, T - t\right) \qquad k \in K \ , \quad 1 \leq t \leq T \tag{7}$$

i.e. if a start-up manouvre takes place in hour t, then unit k must be ON either for $ta_k - 1$ subsequent hours, if $ta_k - 1 \leq T - t$, or for the $T - t$ subsequent hours, otherwise;

- minimum down-time constraints:

$$\text{if } \gamma_{k,0} = 0, \quad \text{then} \quad \gamma_{k,t} = 0 \qquad\qquad \text{for } 1 \leq t \leq ts_k - nh_k \tag{8}$$

i.e. if unit k was OFF in the last hour of the previous scheduling period, it must be OFF at least for the first $ts_k - nh_k$ hours of the current scheduling period;

$$\sum_{\tau=t+1}^{\min(t+ts_k-1,T)} \gamma_{k,\tau} \leq (1-\beta_{k,t}) \cdot \min(ts_k-1, T-t) \qquad k \in K \ , \quad 1 \leq t \leq T \tag{9}$$

i.e. if a shut-down manouvre takes place in hour t, then unit k must be OFF either for the $ts_k - 1$ subsequent hours, if $ts_k - 1 \leq T - t$, or for the $T - t$ subsequent hours;

- upper bounds on production level:

$$0 \leq p_{k,t} \leq \gamma_{k,t} \cdot \overline{p}_k \qquad\qquad k \in K \ , \quad 1 \leq t \leq T \tag{10}$$

i.e. production must be zero, if $\gamma_{k,t} = 0$; if $\gamma_{k,t} = 1$ production is nonnegative and bounded above by $\overline{p}_k$ production, if $\gamma_{k,t} = 1$;

- ramp constraints and upper bounds to production at start-up and shut-down:

$$p_{k,t} - p_{k,t-1} \geqslant -\delta d_k + \beta_{k,t}(-vsd_k - \delta d_k) \qquad k \in K \ , \quad 1 \leq t \leq T \tag{11}$$

and

$$p_{k,t} - p_{k,t-1} \leq \delta u_k + \alpha_{k,t}(vsu_k - \delta u_k) \qquad k \in K \ , \quad 1 \leq t \leq T \tag{12}$$

4 Thermal production costs.

Two types of costs are associated to thermal production: costs of manouvres and generation costs. For every unit k the cost csu_k is associated to each start-up manouvre and the cost csd_k is associated to each shut-down manouvre. The thermal generation cost $C_{k,t}$ of unit k in hour t is assumed to be a convex quadratic function of the production level $p_{k,t}$

$$C_{k,t}\left(p_{k,t}\right) = c_{2k} \cdot p_{k,t}^2 + c_{1k} \cdot p_{k,t} + c_{0k}$$

where c_{2k}, c_{1k} and c_{0k} are the cost coefficients for unit k. Since the model we develop is of Mixed Integer type, we choose to linearize the thermal generation costs, so as to obtain a Mixed Integer Linear Programming model.

In order to linearize the generation cost function of thermal unit k, the interval $\left[\underline{p}_k, \overline{p}_k\right]$ is divided in H subintervals of width $\overline{pl}_{k,h}$, $1 \leq h \leq H$. Let $p_{k,t,h-1}$ and $p_{k,t,h}$ denote the extreme points of subinterval h: the straight line segment passing through the points $(p_{k,t,h-1}, C_{k,t}(p_{k,t,h-1}))$ and $(p_{k,t,h}, C_{k,t}(p_{k,t,h}))$ is associated to each subinterval h. Let $cl_{k,h}$ denote the slope of the line segment associated to subinterval h: since the quadratic function is convex, it holds that

$$cl_{k,h-1} < cl_{k,h} \qquad \text{for } 2 \leq h \leq H.$$

Finally, let $pl_{k,t,h}$ denote the real variable associated to subinterval h. For each production level $p_{k,t}$, $\underline{p}_k \leq p_{k,t} \leq \overline{p}_k$, there exist a unique $\widehat{h}$, $1 \leq \widehat{h} \leq H$, and a unique $pl_{k,t,\widehat{h}}$, $0 < pl_{k,t,\widehat{h}} \leq \overline{pl}_{k,\widehat{h}}$, such that

$$p_{k,t} = \underline{p}_k + \sum_{h=1}^{\widehat{h}-1} \overline{pl}_{k,h} + pl_{k,t,\widehat{h}}.$$

Given a production level $p_{k,t}$, the corresponding (linearized) generation costs are correctly computed if variables $pl_{k,t,h}$ take the values

$$pl_{k,t,h} = \overline{pl}_{k,h} \qquad \text{for } 1 \leq h \leq \widehat{h} - 1. \tag{13}$$

This is obtained by introducing the constraints

$$p_{k,t} = \underline{p}_k \cdot \gamma_{k,t} + \sum_{h=1}^{H} pl_{k,t,h} \qquad k \in K\ , \quad 1 \leq t \leq T \tag{14}$$

and

$$0 \leq pl_{k,t,h} \leq \overline{pl}_{k,h} \qquad k \in K\ , \quad 1 \leq t \leq T\ , \quad 1 \leq h \leq H. \tag{15}$$

This guarantees that, in the optimal solution, variables $pl_{k,t,h}$ take the values (13).

Summarizing, the linearized generation costs of thermal plant k in hour t are given by

$$LC_{k,t}(p_{k,t}) = \left(c_{2k} \cdot \underline{p}_k^2 + c_{1k} \cdot \underline{p}_k + c_{0k}\right) \cdot \gamma_{k,t} + \sum_{k,h} cl_{k,h} \cdot pl_{k,t,h} \tag{16}$$

and the total thermal generation costs of unit k in hour t are

$$TC_{k,t}(p_{k,t}) = LC_{k,t}(p_{k,t}) + csu_k \cdot \alpha_{k,t} + csd_k \cdot \beta_{k,t} \tag{17}$$

with $LC_{k,t}(p_{k,t})$ given by (16).

5 Market constraints and objective function of the Price Taker model.

In this section we introduce the market constraints and the objective function for a producer who cannot influence the market price. It is assumed that in every hour t of the planning period the Price Taker must satisfy the load car_t that derives from his bilateral contracts. If his total production exceeds the load from bilateral contracts, the Price Taker sells the excess quantity, $sell_t$, on the spot market; if his total production is less than the load from bilateral contracts, the Price Taker must buy on the market the amount of energy, buy_t, necessary to meet the load car_t. Therefore the market constraints are

$$\sum_{i \in I} k_i \cdot q_{i,t} + \sum_{k \in K} p_{k,t} + buy_t - sell_t = car_t \qquad 1 \le t \le T$$

The objective function represents the Price Taker profits

$$\sum_{t=1}^{T} \left[\lambda_t \cdot sell_t - \mu_t \cdot buy_t - \sum_{k \in K} TC_{k,t}(p_{k,t})\right]$$

where λ_t is the market sell price in hour t, μ_t is the market purchase price in hour t and $TC_{k,t}(p_{k,t})$ are the linearized total thermal generation costs given by (17). It is supposed that $\mu_t > \lambda_t$, as it also includes transaction costs.

6 Model validation

On the basis of the above discussion, the Price Taker resource scheduling model is as follows

$$\max \sum_{t=1}^{T} \Bigg\{ \lambda_t \cdot sell_t - \mu_t \cdot buy_t - \sum_{k \in K} \Big[csu_k \cdot \alpha_{k,t} + csd_k \cdot \beta_{k,t} +$$

$$+ \left(c_{2k} \cdot \underline{p}_k^2 + c_{1k} \cdot \underline{p}_k + c_{0k}\right) \cdot \gamma_{k,t} + \sum_{h=1}^{H} cl_{k,h} \cdot pl_{k,t,h} \Big] \Bigg\}$$

subject to

$$0 \le q_{i,t} \le \overline{q}_i \qquad i \in I \qquad 1 \le t \le T$$

$$0 \le v_{j,t} \le \overline{v}_j \qquad j \in J \qquad 1 \le t \le T$$

$$\underline{v}_{j,T} \le v_{j,T} \qquad j \in J$$

$$v_{j,t} = v_{j,t-1} + F_{j,t} + \sum_{i \in I} A_{i,j} \cdot q_{i,t-\rho_i} \qquad j \in J \qquad 1 \le t \le T$$

$$\alpha_{k,t}, \quad \beta_{k,t}, \quad \gamma_{k,t} \quad \in \quad \{0,1\} \qquad k \in K \qquad 1 \le t \le T$$

$$\gamma_{k,t-1} + \alpha_{k,t} = \gamma_{k,t} + \beta_{k,t} \qquad k \in K \qquad 1 \le t \le T$$

$$\gamma_{k,t} = 1 \qquad \text{if} \quad \gamma_{k,0} = 1 \qquad k \in K \qquad 1 \le t \le ta_k - nh_k$$

$$\gamma_{k,t} = 0 \qquad \text{if} \quad \gamma_{k,0} = 0 \qquad k \in K \qquad 1 \le t \le ts_k - nh_k$$

$$\sum_{\tau=t+1}^{\min(t+ta_k-1,T)} \gamma_{k,\tau} \ge \alpha_{k,t} \cdot \min(ta_k - 1, T - t) \qquad k \in K \qquad 1 \le t \le T$$

$$\sum_{\tau=t+1}^{\min(t+ts_k-1,T)} \gamma_{k,\tau} \le \left(1 - \beta_{k,t}\right) \cdot \min(ts_k - 1, T - t) \qquad k \in K \qquad 1 \le t \le T$$

$$0 \le p_{k,t} \le \gamma_{k,t} \cdot \overline{p}_k \qquad k \in K \qquad 1 \le t \le T$$

$$p_{k,t} - p_{k,t-1} \geqslant -\delta d_k + \beta_{k,t}(-vsd_k - \delta d_k) \qquad k \in K \qquad 1 \le t \le T$$

$$p_{k,t} - p_{k,t-1} \le \delta u_k + \alpha_{k,t}(vsu_k - \delta u_k) \qquad k \in K \qquad 1 \le t \le T$$

$$0 \le pl_{k,t,h} \le \overline{pl}_{k,h} \qquad 1 \le h \le H \qquad k \in K \qquad 1 \le t \le T$$

$$p_{k,t} = \underline{p}_k \cdot \gamma_{k,t} + \sum_{h=1}^{H} pl_{k,t,h} \qquad k \in K \qquad 1 \le t \le T$$

$$\sum_{i \in I} k_i \cdot q_{i,t} + \sum_{k \in K} p_{k,t} + buy_t - sell_t = car_t \qquad 1 \le t \le T$$

This model computes the values of the decision variables

$$q_{i,t}, \quad v_{j,t}, \quad p_{k,t}, \quad \alpha_{k,t}, \quad \beta_{k,t}, \quad \gamma_{k,t}, \quad pl_{k,t,h}, \quad sell_t, \quad buy_t$$

for $i \in I$, $j \in J$, $k \in K$, $1 \le t \le T$ and $1 \le h \le H$

given values of the parameters

$$k_i, \quad \overline{q}_i, \quad q_{i,-u} \qquad \text{for} \quad i \in I \quad \text{and} \quad 0 \le u \le \rho_i - 1$$

$$v_j, \quad v_{j,0}, \quad \underline{v}_{j,T}, \quad F_{j,t}, \quad \lambda_t, \quad \mu_t, \qquad \text{for} \quad j \in J \quad \text{and} \quad 1 \le t \le T$$

and

$$csu_k, \quad csd_k, \quad c_{0,k}, \quad c_{1,k}, \quad c_{2,k}, \quad cl_{k,h}, \quad \gamma_{k,0}, \quad ta_k, \quad ts_k,$$

$$\underline{p}_k, \quad \overline{p}_k, \quad \overline{pl}_{k,h}, \quad p_{k,0}, \quad \delta u_k, \quad \delta d_k, \quad vsu_k, \quad vsd_k$$

for $\quad k \in K, \quad 1 \le t \le T \quad$ and $\quad 1 \le h \le H.$

The model has been validated on a set of data referring to 17 Italian producers. In the following table we report some information on the production system of every producer.

Producer	$\lvert K \rvert$	$\sum_k \overline{p}_k \quad (MW)$	$\lvert I \rvert$	$\sum_i k_i \cdot \overline{q}_i \quad (MW)$	%
A	1	44	–	–	0.07
B	2	134	–	–	0.22
C	2	160	–	–	0.26
D	2	170	–	–	0.28
E	1	390	–	–	0.65
F	4	463	–	–	0.77
G	3	516	–	–	0.85
H	3	589	–	–	0.98
I	2	754	–	–	1.25
J	1	758	–	–	1.26
K	2	1000	–	–	1.66
L	4	1737	–	–	2.88
M	9	2728	–	–	4.52
N	10	3600	–	–	5.96
O	15	4434	10	620	8.37
P	17	6491	12(*)	928	12.28
Q	89	22945	173(**)	11937	57.75
Total	**167**	**46913**	**195**	**13485**	**100.00**

(*) out of the 12 hydro plants, 2 are pumped-storage hydro plants, with a pumping capacity of 120 MW

(**) out of the 173 hydro plants, 15 are pumped-storage hydro plants, with a pumping capacity of 5365 MW

Now we briefly discuss the results obtained by the model validation. Consider first the problem of determining, for thermal unit k, the production level

$p_{k,t}$ that maximizes profit in a single hour t. In this case intertemporal relations, imposed by minimum up-time/down-time constraints and by ramping constraints, are neglected, as well as start-up and shut-down costs. For thermal unit k such optimal level corresponds to the value of $p_{k,t}$ that maximizes the quadratic profit function $Q(p_{k,t}) = \lambda_t \cdot p_{k,t} - \left(c_{2k} \cdot p_{k,t}^2 + c_{1k} \cdot p_{k,t} + c_{0k}\right)$, where $c_{2k} \geq 0$, over the interval $\underline{p}_k \leq p_{k,t} \leq \overline{p}_k$. If we then consider a time horizon consisting of T hours, the hourly production level $p_{k,t}$ has to adjust to the variations of the hourly price λ_t, taking into account the restrictions in production level variations imposed by the ramping constraints. Moreover, if manouvre costs (for start-up and shut-down) are positive, a trade-off exists, in hours of low demand (i.e. in the night), among producing at a positive level, with production costs greater than revenues, and a zero production level, with costs due to the shut-down manouvre and the subsequent start-up manouvre, when the unit will be required to produce a positive level. Moreover, manouvres are subject to minimum up-time and minimum down-time restrictions. Concerning bilateral contracts, we note that a pure thermal problem without bilateral contracts is separable, i.e. the optimal solution for a set of thermal units may be obtained by optimizing each single thermal unit. If bilateral contracts are present, the thermal units must be optimized simultanously.

Consider now the pure hydro problem, in which case the objective function represents the total revenues in the planning horizon. In the optimal solution, the production levels are low when hourly prices λ_t are low, so that more water is available for production when hourly prices increases. If the hydro system contains also pumped-storage hydro plants, production in low price hours can be used for pumping.

Considering now a hydro-thermal system, if there are neither bilateral contracts nor pumped-storage hydro plants, the problem is still separable, i.e. the optimal solution can be computed by optimizing each single thermal plant and each single cascade. However, in the presence of bilateral contracts, the resource scheduling model determines the optimal production plan for satisfying the fixed demand. In Figure 3 we report in graphical form the results obtained by the optimization model when pumped-storage hydro plants are present, given the price forecast showed in Figure 2. It can be observed that in high price hours the thermal production is constant, with hydro production fluctuating so as to follow price fluctuations; in low price hours a part of the thermal production is used for pumping, so that more water will be available in high price hours.

7 Conclusions

We have developed a decision support procedure for the short-term hydro-thermal resource scheduling problem of a price-taking power producer operating in a liberalized market. This procedure, based on a mixed integer linear programming model, determines the unit commitment of thermal units and the production levels of committed thermal units and available hydro plants in each hour so as to maximize profits, while satisfying constraints describing the hydro

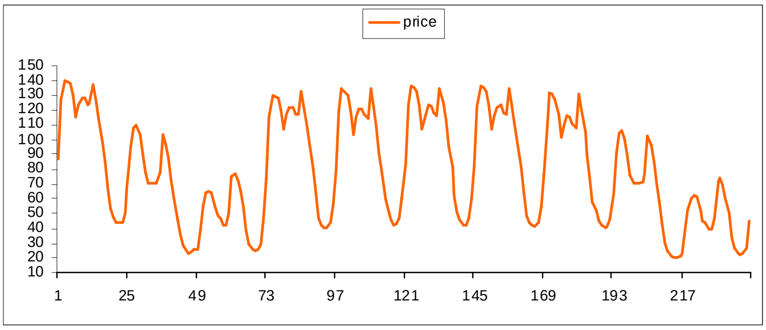


Figure 2: hourly price forecast

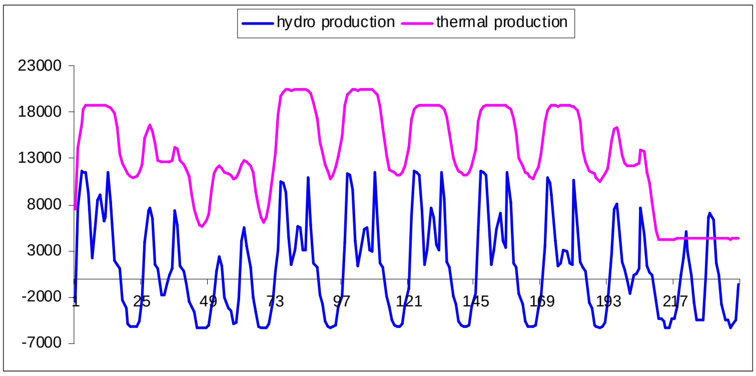


Figure 3: hydro and thermal optimal productions

system, the thermal system and the market. The annual scheduling decisions (optimal maintenance plans of hydro and thermal plants, optimal weekly discharge of seasonal basins) are given, as well as forecasts on basin natural inflows.

The time horizon of the final phase of the planning procedure is the single hour, for which the Power Producer must decide his own generation units' sell bids. In this short horizon it is possible to include in the model a detailed representation of bids and of production units' localisation for each competitor. The hourly resource planning procedure will require to explicitly introduce in the Power Producer model the optimality conditions (Kuhn-Tucker conditions) of the Market Operator problem, which will be modelled under different hypotheses, such as inelastic or elastic energy demand, no transmission constraints or transmission network with either radial or grid topology. A Mathematical Programming problem with Equilibrium Constraints (MPEC) will then have to be solved. Each producer must define for each hour of the following day the so called "production bids", i.e. pairs "energy quantity - price". The power producer decision support procedures will also allow both to analyse the behaviour of the electricity market, i.e. the process by which the hourly energy price is determined, and to detect whether a producer has an excessive market power, incompatible with the aim of the liberalisation. The above described planning procedures for the weekly, daily and hourly time horizon are also able to detect the existence of an independent producer with excessive market power. Indeed, if one producer turned out to be indispensable to meet the demand, profit maximisation models would determine energy sell bids with arbitrarily high prices. The planning tools that will be developed in this research project will make it possible to estimate the maximum dimensions of a single producer above which genuine competition in the market is hindered.

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