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Essays in Environmental Economics with an OLG Model

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A mia nonna

Because we don't think about future generations,
they will never forget us.

Henrik Tikkanen

Preface

A problem of growing concern is the threat to the environment resulting from polluting economic activity. Indeed, from economic theory we know that there exists an externality associated with the conservation of the environment. We also know that, in the presence of externalities, social benefits(costs) and private benefits (costs) differ. The divergence between private benefits (costs) and social benefits (costs) results in inefficiency in resource allocation.[1] In particular, in the presence of a pollution externality, the marginal social cost will be higher than the marginal private cost (price) and hence the private optimal level of output will be higher than the social optimal output. Government intervention is then needed to internalize agents' externalities, so that the social optimal level of output and the private optimal level will be the same. Moreover, environmental externalities exhibit two dimensions. First, there is an intragenerational dimension: the environment can be thought as a public good and as such its conservation suffers from the well-known free-rider problem. Second, there is an intergenerational dimension: since pollution typically accumulates, not yet born agents bear the costs of the actions of current agents.

Due to the intergenerational feature characterizing environmental problems, an overlapping generations (OLG) model seems to provide a more suitable demographic framework than an infinitely-lived agents model. Indeed, it allows to study both the impact of the free-rider behaviour on current agents and its consequences on those who are not yet born. Moreover, it enables to analyze optimal policies needed to achieve the first-best solution.

This dissertation develops insights concerning the relationship between economic activity and the environment through different perspectives. Each part of this work can be read as an autonomous paper, though they are interconnected by the general purpose of shedding some light on the role played by environmental externalities in shaping the market-driven output

[1] A resource allocation is inefficient according to Pareto if it is possible, by a reallocation, to improve the welfare of at least one individual without decreasing the welfare of the others.

and its comparison with the allocation chosen by a benevolent planner who cares about the welfare of current and future generations.
In particular, Chapter 3 can be considered, to some extent, as an extension of Chapter 2. Chapter 3 enriches the fundamental features of the OLG setup used in Chapter 2 by

- introducing into the model health care expenditure (in addition to environmental maintenance) and studying relationship between health status and environmental conditions.
- assuming that agents vote over health care expenditure and environmental maintenance, thus introducing a probabilistic voting mechanism.
- allowing for agents' endogenous lifespan: agents face an uncertain future and the length of the second period lived depends on their health status.
- introducing and studying the role played by annuity markets.

This dissertation is organized as follows:

Chapter 1 of the dissertation is devoted to review the main literature using an OLG model in the field of environmental economics, and more precisely dealing with environmental externalities stemming from pollution. This chapter seeks to:

- describe the main features of the standard OLG model as developed by Diamond (1965), and show why an OLG model can provide a suitable framework to study cumulative environmental problems.
- provide an overview of the main literature dealing with environmental externalities springing from pollution. The review is structured according to the degree of concern of agents towards the environment. In particular,
  - it first reviews papers where agents do not invest at all in environmental improvements,
  - then, papers assuming a positive level of environmental maintenance are analyzed: this level can be chosen by either the individuals or a short-lived government that levies a tax on agents in order to finance investment in environmental quality. A particular attention is paid to the seminal article by John and Pecchenino (1994), on which Chapter 2 is based.

- finally, the case of altruistic agents who care about the welfare of their offspring and may decide to leave either monetary or environmental bequests is depicted.

In Chapter 2 a theoretical model is presented, where some of the channels detected in Chapter 1 are explicitly taken into account: the environment is a public good passed on to future generations, degraded by current consumption and improved by environmental maintenance. Agents are selfish and take the others' decision as given, according to a subscription equilibrium. An OLG model is set up in order to compare the *laissez-faire* outcome with the solution of a social planner who cares about the welfare of current and future generations. The contribution of this Chapter consists in:

- taking into account both intra- and intergenerational externalities arising from consumption

- studying both the competitive equilibrium and the the steady state of the *laissez-faire* economy by means of reasonable assumptions on the functional forms of the utility and production functions.

- highlighting the role played by the parameter of the model on the steady-state levels of capital and the environment by means of comparative statics.

- comparing the with the allocation chosen by a social planner who maximizes the welfare of current and future generations.

- presenting three different tax schemes which succeed in decentralizing the social optimum.

In Chapter 3, a joint work with D. Dottori, the environmental externality arising from the production side of the economy is analyzed by drawing an OLG model in which agents' longevity depends endogenously on health, which is determined both by environmental conditions (positively affected by environmental maintenance) and health care expenditure. Health care expenditure and environmental maintenance are provided by the government and decided through probabilistic voting. However, the young and the elderly have different preferences towards the composition of public spending: while the young are more interested in the long lasting effects of environmental maintenance, the old - who will not enjoy future improvements in environmental quality - tend to devote more resources to health care expenditure. As lifespan increases, the size of the elderly electorate enlarges, affecting the political decision-making outcome. The main contribution of this part is

- to highlight the relationship between health and the environment form a theoretical point of view, allowing agents' health status to depend on environmental conditions
- to provide empirical evidence supporting the hypothesis that age is a significant determinant of agents' attitudes towards the environment
- to solve analytically the model. By making reasonable assumptions on the functional forms used in the model, both the intertemporal equilibrium and the steady-state solutions are analyzed in two different frameworks: in the political economy scenario and in the social planner regime. For a sufficiently low discount rate and a targeted lifespan, the relative intensity in the use of environmental quality rather than health-care expenditure is larger in the planner regime. In the political economy scenario, a crucial role is indeed played by the political mechanism that - in an aging society - shifts endogenously support towards health-care expenditures.
- to assess numerically the impact of several factors on the steady state in the two scenarios, by calibrating the parameters of the model.

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Chapter 1

The OLG Model in Environmental Economics: a Literature Review

This chapter aims at considering some of the most important papers using an overlapping generations (OLG) framework in the field of environmental economics, and more precisely dealing with environmental problems stemming from pollution.

This chapter is organized as follows. Section 1.1 sketches some reasons that make the overlapping generations model interesting for macroeconomic and environmental analysis. The main features of the basic overlapping generations set-up are then analyzed in Section 1.2. We then review papers dealing with environmental externalities springing from pollution. In Section 1.3 we discuss papers in which there is no pollution abatement at all, while in Section 1.4 we examine works in which there is some depollution activity, and we distinguish between papers in which the pollution abatement activity are chosen by the government and studies in which contributions to environmental quality are voluntarily chosen by the agents. In Section 1.5 we explore the role played by altruism and how an altruistic behaviour can effect the inefficiency of the *laissez-faire* equilibrium. Section 1.6 concludes.

1.1 Introduction

A huge amount of studies has analyzed the relationship between environmental externalities and economic growth over years.

In the early 60s, the traditional neoclassical point of view explained the root of environmental problems as a result from market failures. In the case of collective, public goods these failures are due to difficulties in establishing markets, while in the presence of negative externalities, the failures are due to the lack of well-defined property rights. The solution proposed by Coase (1960) to environmental problems would be to determine property rights as a basis for negotiation between involved parties, but because of transaction costs this would be only rarely feasible.

Works were thus mainly focused on the design of environmental regulations able to realize a social optimal level of pollution through the use of either taxes or tradeable permits. The main feature of this analysis was the evaluation of the costs and benefits of existing and proposed regulations (see Van der Straaten (1998)). Following John et al. (1995) we can say that such approach, being implicitly static, ignored important aspects related to environmental problems.

By acknowledging the macroeconomic, dynamical nature of environmental externalities, in more recent years researchers have then started investigating the conflict between environmental conditions and economic growth in a dynamic setting.

There are two main blocks in modern macroeconomics, widely used also in environmental economics. The first one assumes that agents have infinite horizon. The second one analyzes the case of an economy in which agents have finite lives. This latter approach consists in the so-called overlapping generations (OLG) model. Many features are common to both approaches, like the analysis of the factors influencing economic growth. [1] Several issues are, however, specific to the overlapping generations approach. In particular, they are related to the analysis of intergenerational and distributional issues (e.g. social security, public debt issues).[2]

Early dynamic studies assumed infinitely-lived agents, focusing on the anal-

[1] The model with a representative infinitely-lived agent can be seen as a special case of the overlapping generations model where agents are perfectly altruistic and care about the welfare of their offspring. In this case, the properties of the two models are the same.

[2] Three important properties of overlapping generations models are the non-neutrality of debt, the possibility of asset bubbles and the possibility for competitive equilibria to be inefficient.

ysis of the *intragenerational* conflict given by the existence of the well-known free-rider problem within a generation. However, once dynamics are introduced in models of environmental policy and the environment is thought as of a public good whose provision in the future is affected by decisions taken by current generations, then *intergenerational* issues become a major part of the analysis, as pointed out by Solow (1986).

The overlapping generations approach provides a more suitable demographical framework to study environmental problems, since it takes into account both intra- and intergenerational externalities. In particular, conversely to other models, such as static or dynamic models with infinitely-lived agents, it allows to study both the impact of the free-rider behaviour on current agents and its consequences on those who are not yet born. Moreover, it enables to analyze public policies needed to correct negative externalities. Finally, even if the horizon of agents is finite, they may be altruistic, in the sense that they value the welfare of their offspring. In an OLG framework it is possible to investigate how agents' altruistic behaviour can correct, completely or to some extent, environmental externalities.[3]

As pointed out by Fodha (1998) (pg. 238), the methodology followed by articles using an OLG framework in the field of environmental economics *"always proceeds as follows: it starts by determining the competitive equilibrium, then its inefficiency is proved by comparing it with the social optimum. Only the presence of a social planner guarantees the internalization of the intergenerational, environmental externalities. Finally, the optimum is decentralized thanks to different strategies like pollution abatement activities, fiscal policies or tradeable permits"*.

This chapter is organized as follows. The main features of the basic overlapping generations setup are sketched in Section 1.2. We then review papers dealing with environmental externalities springing from pollution. In Section 1.3 we discuss papers in which there is no pollution abatement at all, while in Section 1.4 we examine papers in which there is some depollution activity, and we distinguish between papers in which the pollution abatement activity is chosen by the government and studies in which contributions to environmental quality are voluntarily chosen by the agents. In Section 1.5

[3] In the altruism à la Barro (1974) there exists a unique efficient steady state and households offset any policy aiming at redistributing income between generations through public debt. However, as argued by Mankiw (2000), there exists empirical evidence against Barro (1974)'s assumption that individuals are perfectly linked as in a dynasty. Hence, it might be of some interest to study less drastic approaches, like the *joy-of-giving* hypothesis analyzed by Andreoni (1989) or the *family altruism* hypothesis introduced by Becker (1991), which could lead to an inefficient allocation.

we explore the role played by altruism and how an altruistic behaviour can effect the inefficiency of the *laissez-faire* equilibrium. Section 1.6 concludes.

1.2 The basic OLG model

The basic overlapping generations model with capital accumulation is due to Allais (1947) and Diamond (1965).[4]
Time t is discrete and goes from 0 to ∞, every decision is taken at points in time. At time $t = 0$ there are initial conditions reflecting the history of the economy. At each period t there are three goods: capital, labour and a homogeneous physical good produced using capital and labour. This good can be either consumed or saved and invested to build future capital.

At time t, N_t individuals are born, while n is the growth rate of population. In the standard OLG framework agents are exactly alike[5] and live for two periods, so that at each period in time two generations are alive and overlap: the young and the old.[6] Agents have perfect foresight concerning future economic conditions. In the first period (t), when young, agents work and receive a competitive wage, w_t, they consume a part of this wage and save the rest. In the second period ($t+1$), when old, they retire and their income comes from the return on savings made at time t. As they are not altruistic and do not care about events occurring after their death, they consume their income entirely, both interest and principal. Preferences are embodied in an intertemporal utility function $U(c_t, d_{t+1})$ showing the following properties: $U'_i() > 0$, $U''_{ij}() < 0$, where c_t and d_{t+1} are consumptions at time t and

[4] A continous time version of the overlapping generations model has been developed by Blanchard (1985).

[5] Even this simple OLG model allows for heterogeneity, in the sense that young and old individuals coexist at each point in time. A further step in introducing heterogeneity is to allow for idiosyncratic characteristics. These characteristics can be related both to preferences that vary across individuals or to different abilities to work. However, as long as preferences are homothetic the basic model can be easily extended to allow for heterogeneous agents (see de la Croix and Michel (2002)).

[6] De la Croix and Michel (2002) also consider the case of an economy in which agents live for three periods. During the first period they do not take any decisions and their consumption is included in that of their parents, during the second period they work and they retire in the third one. This framework is usually used when education and human capital are concerned.
One might be tempted to think that the results obtained in the standard model with two-period-lived agents can be generalized to models where the households live for n-periods, however one should be cautious as not all the properties of the two-period-lived agents model can be extended to models with n-period-lived agents.

$t+1$, respectively. Then, the maximization problem of the representative household born at time t can be summarized as follows:

$$\max U(c_t, d_{t+1})$$

$$s.t \begin{cases} c_t & = & w_t - s_t \\ d_{t+1} & = & (1+r_{t+1})s_t \end{cases}$$

where s_t are savings invested in the firms at time t and r_{t+1} is the interest rate on savings from time t to time $t+1$.
The necessary condition $\frac{\frac{\partial U(\cdot)}{\partial c_t}}{\frac{\partial U()}{\partial d_{t+1}}} = 1 + r_{t+1}$ defines the intertemporal arbitrage of the agent and determines his optimal level of savings $s(w_t, r_{t+1})$.

The representative firm is perfectly competitive, it produces the homogeneous good using capital, K, and labour, L, according to a homogeneous of degree one production function $F(K, L)$, which can be expressed in intensive terms: $F(K, L) = LF(k, 1) = Lf(k)$.[7] $f(k)$ satisfies the Inada conditions $f(k) > 0$, $f'(k) > 0$, $f''(k) < 0$, $\lim_{k\to 0} f'(k) = +\infty$ and $\lim_{k\to +\infty} f'(k) = 0$. The firm, in order to maximize its profits, equalizes the marginal productivity of the two factors, capital and labour, with their prices

$$\begin{cases} r_t = & f'(k_t) & = r(k_t) \\ w_t = & f(k_t) - k_t f'(k_t) & = w(k_t) \end{cases}$$

The goods market equilibrium requires that the demand for goods in each period is equal to the supply, or equivalently that investment is equal to savings

$$K_{t+1} - K_t = N_t s(w_t, r_{t+1}) - K_t$$

The left-hand side is net investment, the change in the capital stock between t and $t+1$. The right-hand side is net savings: the first term is the savings of the young, the second is dissavings of the old.
Eliminating K_t from both sides tells us that capital at time $t+1$ is equal to

[7] The assumption of a representative firm is not restrictive as, with constant returns to scale, the number of firms does not matter and production turns out to be independent of the number of firms which use the same technology.

the savings of the young at time t. Dividing both sides by N_t gives[8]

$$(1+n)k_{t+1} = s(w_t, r_{t+1})$$

In principle, the relation between capital per worker in two consecutive periods may be very complicated: notice that $s(w_t, r_{t+1})$ is a non-linear function and its arguments, $r(\cdot)$ and $w(\cdot)$, are also non-linear. To analyze the equilibrium graphically, we could represent values of k_t on the horizontal axis and values of k_{t+1} on the vertical axis. Then we could draw a 45 degree line which, by definition, represents points where k is constant over time, and we could represent the equilibrium mapping between k_t and k_{t+1}. Where the equilibrium schedule intersects the 45 degree line, so that $k_{t+1} = k_t$, capital per worker is constant over time. In other words, the economy is at *a steady state*. See Fig.1.1.

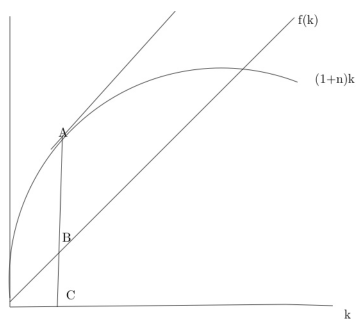

Figure 1.1: The steady state and the golden rule

Equality $(1+n)k_{t+1} = s(w_t, r_{t+1})$ gives an implicit relationship between k_t and k_{t+1}, which we will describe as the saving locus. The properties of the saving locus (and of the equilibrium) depend on the derivative

$$\frac{\partial k_{t+1}}{\partial k_t} = \frac{-s_w()k_t f''(k_t)}{1+n-s_r()f''(k_t)}$$

where s'_w and s'_r are the partial derivatives of $s(w_t, r_{t+1})$ with respect to wage and rate of return.

[8] This equality reflects the idea that the supply of funds by households equals investment by firms.

Under the assumption of separability and concavity of the utility function, the numerator of this expression is positive, reflecting the fact that an increase in the stock of capital in period t increases the wage, which increases savings. However, the denominator is of ambiguous sign because the effects of an increase in the interest rate on savings are ambiguous. An increase in the interest rate decreases the price of second-period consumption, leading individuals to shift consumption from the first to the second period (substitution effect). But it also increases the feasible consumption set, making it possible to increase consumption in both periods (income effect). The net effect is ambiguous. If the elasticity of substitution between consumption in both periods is greater than one, then in this two-period model the substitution effect dominates, and an increase in interest rates leads to an increase in savings.
Given a steady state, it will be stable if the slope of the mapping is less than 1 in absolute value. Conversely, when at the steady state the equilibrium schedule has a negative slope, the economy can feature oscillatory behavior. Moreover, for general forms of the utility function, and depending on the interplay between income and substitution effects, we have the possibility of multiplicity of steady-states and instability.

We define the *golden rule* as the level of capital per worker that maximizes steady-state consumption per worker. At the steady state $c = f(k) - (1+n)k$, thus the golden rule that maximizes c satisfies[9]

$$f'(k_{GR}) = 1 + n$$

When the steady-state level of capital per worker is above the golden rule, that is $k > k_{GR}$, there is overaccumulation of capital and the economy is dynamically inefficient. In such a situation it is possible to increase consumption per worker in some periods without reducing it in any other period.[10]

The social optimum is determined by an intertemporal social planner whose objective is to maximize the discounted sum of utilities of all, current and future, generations

$$\max V = U(d_0) + \sum_{t=1}^{\infty} \rho^t U(c_t, d_{t+1})$$

[9]The golden rule capital k_{GR} is represented in Fig.1.1 by the distance $A - B$.

[10]This possible inefficiency of the equilibrium contrasts sharply with the intertemporal efficiency of the competitive equilibrium in the Ramsey model.

$$\text{s.t.} \quad (1+n)k_{t+1} - k_t = f(k_t) - c_t - \frac{d_{t-1}}{1+n}$$

where ρ is the social discount factor, which defines the weight attached to the welfare of future generations relative to the present, and the initial conditions k_0, d_0 are given.[11]
The optimality conditions require a stock of capital such that the *modified golden rule* is satisfied

$$1 + f'(k^*) = (1+\rho)(1+n)$$

If ρ and n are not too large, this expression can be approximated to

$$f'(k^*) = \rho + n$$

The marginal product is equal to the sum of the social discount rate and the rate of growth of population. If $\rho = 1$, that is, if the central planner gives equal weight to the utility of each generation, the marginal product is simply equal to $1 + n$; the steady state is the *golden rule* steady state, at which the steady state level of per capita consumption is maximized.

Since ρ is an arbitrary parameter, unrelated to preferences as captured by the individual utility function, it is hardly surprising that k^* is generally not equal to the steady-state level chosen by the decentralized economy. This means that, as we have already stated earlier, the economy could experience overaccumulation of capital, which makes the equilibrium be inefficient and, hence, there would be room for a Pareto-improving policy. This potentially improving reallocation is feasible because there is an infinite number of generations. If there was a last generation, then consumption for this last generation would be actually smaller, because at the end of the world there would be no current young to compensate the current old and we could not find any Pareto improving allocation.[12]

In the following sections we will review the main papers using an OLG framework in the field of environmental economics, and in particular dealing with

[11]The choice of the social discount factor is an old debate. Michel (1990) argues that, in an utilitarian setup, one should choose the discount rate that allows the economy to converge to the golden rule. This discount rate is equal to the growth rate of population, and the corresponding social objective function is the undiscounted sum of Ramsey (1928).

[12]In other words, the equilibrium would not be dynamically inefficient: even if it was possible to increase total consumption, some (the old in the last generation) would lose.

the environmental problems related to pollution. The structure of the review will be based on the degree of responsibility of the agents: from the less to the more responsible attitude towards the environment. The review will be organized as follows: in Section 1.3, we will analyze the case of agents who do not care at all about pollution, externalities are internalized by a social planner by means of taxes and transfers. In Section 1.4, agents' utility is affected by the environmental quality and a short-lived government is in charge with environmental maintenance which is financed by a tax on consumption. Alternatively, agents can also decide to contribute voluntarily to improve the quality of the environment, they choose their contribution in order to realize an arbitrage between savings (investment in physical capital) and environmental maintenance (investment in depollution). Even if voluntary, this mechanism is selfish, as the agents decide to invest in environmental maintenance in order to improve the environmental quality which affects their own utility. In this case, it is shown that resources devoted to environmental maintenance are sub-optimal as decisions are taken in a non-cooperative way. Finally, in Section 1.5 we assume that agents care about the welfare of future generations. Their investment in environmental maintenance is then spurred also by altruistic reasons. Each household is a member of a dynasty, then the potential conflict between the horizon of the finitely-lived agent and that of the environment might disappear.

1.3 Pollution without environmental maintenance

A seminal paper in the field of OLG models applied to environmental issues is that by Howarth and Norgaard (1992).[13] They study how different social discount factors influence transfers and social welfare. To our best kwnoledge, this is the only model wherein the externality does not affect agents' utility but only the level of production.

The main hypotheses of the model are those of the standard OLG setting

[13]Howarth (1998) applies the approach used by Howarth and Norgaard (1992) to climate change. By working with a computational version of this model that is calibrated on the scientific and technological assumptions of DICE model by Nordhaus (1994), he establishes that rates of greenhouse gas emissions abatement should range from 16% to 25% in the *transfer baseline* case (where emissions taxes are chosen in order to achieve an efficient allocation of resources) and from 48% to 89% in the *utilitarian optimum* scenario (in which the government seeks to maximize the *undiscounted* sum of all current and future generations' life-cycle utility through the choice of both emission taxes and intergenerational transfers).

discussed above. Agents live two periods and population is constant. Preferences are defined over consumption in the first and second period. When young, agents work and receive a wage which is split into consumption and capital investment. When old, they retire and their income comes from the return on savings made in the previous period. In addition, in each period they receive a net (positive or negative) lump-sum income transfer financed by pigouvian taxes on pollution. Pollution is the by-product of firms activity: it comes from the energy consumed by firms during the production process and exhibits a cumulative character (greenhouse gases). Firms maximize their profits in a static way, taking the pollutant stock as given, that is without internalizing the negative externalities caused by their own emissions.
There is no voluntary investment in environmental maintenance, hence the only way to reduce pollution is to reduce production.

Not surprisingly, the *laissez-faire* equilibrium turns out to be inefficient. The authors then assume the existence of a social planner who maximizes the discounted sum of lifetime utility of all generations and sets a tax on energy consumption, in order to internalize the environmental externality. They show that the emissions tax at time t is set equal to the marginal present-value cost that current production imposes on the future economy. This optimal emissions tax ensures that the competitive equilibrium is Pareto efficient, so that it is impossible to improve the welfare of one generation without leaving another worse off.
Howarth and Norgaard (1992), however, show that a Pareto efficient allocation of resources does not ensure that consumption or utility is sustained over time. For an allocation to be sustainable, the weight attached by the social planner to the welfare of future generations, δ, must be sufficiently high.[14]

Table 1 summarizes the results of simulations carried out by the authors for two different values of δ. For $\delta = 0.7$, both total consumption and capital stock fall over time, while the pollutant stock grows at a rapid rate before achieving a steady state. For $\delta = 0.9$, consumption rises slowly over time, while the capital stock first rises to support higher output levels. Pollutant emissions are permanently less for $\delta = 0.9$ than for $\delta = 0.7$, hence environmental quality is higher. As δ is raised from 0.7 to 0.9, the firm's output rises over the long term, exacerbating the marginal impact of the pollution

[14] The equilibrium and the optimum are not solved analytically however, by means of numerical simulations, different trajectories are considered for different values of the social discount factor.

stock on current production. Increasing the assets transferred to future generations also lowers the interest rate. The net result is that the present value of the marginal environmental damage caused by current energy use, which is equal to the efficient emissions tax, increases substantially.

	$\delta = 0.9$	$\delta = 0.7$
Total consumption	$\nearrow$	$\searrow$
Capital stock	$\nearrow$	$\searrow$
Pollutant stock	$\nearrow$	$\nearrow\nearrow$
Energy consumption	$\searrow$	$\searrow$
Net revenue transfers	$\nearrow$	$\searrow$
Interest rate	$\searrow$	$\nearrow$
Emissions tax	$\nearrow$	$\searrow$
Marginal impact of externality	$\searrow$	$\searrow$

Table 1.1: Simulation results

While in the model by Howarth and Norgaard (1992) pollution does not affect directly the utility of the representative agent, Jouvet, Michel, and Rotillon (2005) assume that environmental quality enters the individual's utility. Moreover, instead of using proportional taxes, they analyze a market of permits for decentralizing the optimal growth path.
Agents live two periods and population is constant. Any agent born in period t derives utility from consumption and the quality of the environment in both periods, as well as leisure in the first period.
Firms produce a homogeneous good according to a constant returns to scale production function using capital and labour.
At each time $t + 1$ the environmental quality is degraded by pollution generated by firms at time t through the productive process.

The authors first characterize the problem of a central planner who chooses the level of consumptions, the level of labour supply and wants to maximize the discounted welfare of all current and future generations. They also explicit the different trade-offs faced by the planner: trade-off between generations, trade-off between consumption and leisure, trade-off between consumptions on life cycle and trade-off between environmental quality and consumptions.
They then study the case of an economy with a market of tradeable permits

of pollution, where the government policy consists of issuing a quantity of permits, $\overline{P}_t$, and allocating a part of them to firms, $\overline{P}_t^F$, while the difference, $\overline{P}_t - \overline{P}^F$, is auctioned. It also makes a transfer to the young and to the old. Agents take the environment as given. When young, they supply labour to firms and their total income is made up of the labour wage and a transfer (which can be positive or negative) from the government. Total income is divided into consumption and savings. When old, they retire and receive a transfer (either positive or negative) in addition to their savings. They spend all their income.
Firms are perfectly competitive, each firm is endowed with a stock of capital and a stock of permits, $\overline{P_t^F}$. The net revenue includes the net gain on the permit market, $q_t(\overline{P}_t - \overline{P}_t^F)$, where q_t is the price on the pollution permits market.
The competitive equilibrium is inefficient. The authors show that it is possible to decentralize the *laissez-faire* equilibrium with lump-sum transfers and a market of permits. However, the necessary condition to realize such a decentralization is to attribute zero permits to firms, that is $(P_t^F = 0)$, differently from what postulated by practices like grandfathering. The rational behind this property of optimal decentralization is that the equilibrium market of permits gives the optimal rate of interest. A consequence of this property is that at the decentralized equilibrium the interest rate is equal to the marginal productivity of capital.

1.4 Pollution with environmental maintenance

OLG models with pollution and environmental maintenance are quite recent. Seminal works are those Michel (1993), John and Pecchenino (1994) and John et al. (1995). Environmental quality is thought as of an amenity which positively affects consumers' utility. In this section, we first present the case in which the government is entitled of environmental maintenance and we then study the case in which investment in environmental quality is voluntarily chosen by the agents, as a result of the arbitrage between consuming (and hence polluting) and investing in environmental maintenance (and hence enjoying a better environmental quality).

1.4.1 Environmental maintenance chosen by the government

The externality arising from pollution affects the utility of the agents, the level of environmental maintenance is however chosen by a government or a social planner.

John et al. (1995) compare the steady-state equilibrium determined by a sequence of short-lived planners with that of a long lived planner who maximizes the utility of a representative generation.
Agents live two periods, population grows at rate n. Young agents have preferences defined over consumption in old age, d_{t+1}, and an index of environmental quality, E, when they consume[15]

$$U_t = U_t(d_{t+1}, E_{t+1}) \quad U_i'() > 0, U_{ij}''() \geq 0, U_{ii}''() \leq 0 \quad \forall i, j = 1, 2$$

At time t, young agents provide one unit of labour to firms inelastically and they receive a wage which is split between savings for old age and the payment of an environmental maintenance tax, m_t. When old they supply savings to firms and earn a gross rate of return.

The environmental quality at time $t+1$ (measured by the environmental index E) is degraded by consumption of the old at time t and improved by environmental maintenance programs, m_t.

$$E_{t+1} = (1-b)E_t - \beta d_t + \gamma m_t$$

where the parameter $b \in [0, 1]$ measures the autonomous evolution of environmental quality, β is the negative effect of consumption on the environment and γ represents the efficiency of the investment in environmental maintenance.[16]

[15]Since the authors want to study the choice between investment in the environment and investment in physical capital, they abstract from the free-rider problems within a generation and assume that individuals do not consume in the first period.

[16]In the environmental economics literature dealing with pollution, this law of motion is widely used. Some scholars, like John and Pecchenino (1994) or John et al. (1995), assume a positive natural decay, $b > 0$, arguing that, even without any human activity the index of environmental quality, due to some kind of entropy, converges spontaneously to infinite. Others, like Ono (2003) or Ono (2005), prefer to set the natural decay parameter equal to zero. In the literature there is not a plain definition of the environmental index E: it is usually said that it can encompass the inverse of the concentration of chlorofluorocarbons in the atmosphere, the inverse of other greenhouse gases, the quality of groundwater or

Firms are perfectly competitive, they produce using a constant returns to scale production function where the two factor inputs are labour and capital, k.

The authors first assume that at the beginning of each period agents elect a government for one-period term to enact policies for the agents alive during its term. The government levies lump-sum taxes on the young to achieve the desired level of environmental quality, subject to the condition that the old cannot be made worse off by these actions. It chooses the stock of capital k_{t+1} to equate the sum across agents of their marginal rates of substitution between consumption and environmental quality to the marginal rate of transformation. It then chooses the level of the environmental tax m_t such that the young are indifferent between consuming and investing in the environment. However, it internalizes only the current period externality (*intragenerational* externality) that the young impose upon themselves when they are old by not taking account of the effects of their actions on environmental quality; it does not internalize the *intergenerational* externality imposed by the current generation on future generations via the bequeathed environmental quality.

The authors then consider the case of a long-lived government treating all generations symmetrically. The planner sets capital at the golden rule and the optimal maintenance tax such that the marginal rate of transformation is equal to the social marginal rate of substitution between consumption and environmental quality. By doing so, it internalizes the intergenerational consumption and maintenance externalities. Indeed, the planner can translate an increase in output into increased consumption of the old and increased maintenance tax on the young, while keeping environmental quality constant. This mechanism is ignored by the short-lived government, the competitive steady-state for this economy is thus inefficient (overaccumulation of capital or overmaintenance of the environment).
The authors show that Pareto-improving reallocations are possible when the decisions of short-lived governments lead to dynamically inefficient allocations. The long-lived planner can set taxes on the net return on capital and on wages to induce the short-lived government to the steady-state social optimum. However, if the economy is dynamically efficient in both capital and

an index of biodiversity. Such an interpretation is so broad that that both specifications, with or without any positive natural decay rate, are accepted. The use of a positive decay rate might also have a mathematical explanation, in the sense that it avoids unit root in environmental dynamics. Notice that in Chapter 2, following the article by John and Pecchenino (1994), a positive natural decay rate is introduced; while in Chapter 3 it is set equal to zero.

environmental quality (neither overaccumulation of capital nor overmaintenance of environmental quality), a Pareto-improving tax-transfer scheme cannot exist, as it would imply transfers from not yet born generations (who benefit from maintenance) to currently alive generations (who bear the cost of maintenance).

1.4.2 Environmental maintenance chosen by the agents

John and Pecchenino (1994) study the potential conflict between economic growth and environmental quality. The basic set-up is the following: agents live two periods, at each time t a new generation is born, there is no population growth. Preferences are defined over consumption in old age, d_{t+1}, and an index of the environmental quality, E when they consume[17]

$$U_t = U_t(d_{t+1}, E_{t+1}) \quad U_{ij} \geq 0, U_i > 0, U_{ii} \leq 0 \quad \forall i, j = 1, 2$$

Young agents supply labour to firms inelastically. They divide their wage between savings and investment in environmental maintenance. When old they supply inelastically their savings to firms from which they receive a gross return and they consume entirely their income.
The environment, E, is a public good. Exactly as in John et al. (1995), at time $t+1$ it is degraded by consumption of the old at time t, but it can be improved through environmental maintenance, m, of the young at t.

$$E_{t+1} = (1-b)E_t - \beta d_t + \gamma m_t$$

Firms are perfectly competitive profit maximizers, they produce output using labour and capital. The production function exhibits increasing returns to scale: it is multiplied by $\Psi(k)$, a productivity parameter depending on last period's social capital. Hence, the investment in capital by one agent improves the productive efficiency of all.
The authors assume the existence of a one-period government who represents those alive at each period in time and whose responsibility is the provision of environmental quality for the benefit of agents alive during its period of office. In particular, it levies lump-sum taxes on the young to achieve

[17]As in John *et al.* (1995) (John et al. 1995) , since the authors want to study the choice between investment in the environment and investment in physical capital while the consumption-savings decision is secondary to their main concern, they assume that individuals do not consume in the first period.

their desired level of environmental quality, leaving the welfare of the old unchanged.

John and Pecchenino (1994) first analyze the equilibrium in absence of external increasing returns.[18] By solving the model, they obtain two nonlinear difference equations:

- one (named SSE) representing the dynamics of environmental quality as a function of the stock of capital, $E_{t+1} = \varphi(k_{t+1})$
- one (named FOC) representing the arbitrage condition between savings ant maintenance in equilibrium. It turns out to be an implicit relationship between E_{t+1} and k_{t+1}, that is $E_{t+1} = \phi(k_{t+1})$

In the steady state they obtain:

$$\begin{cases} \overline{E} &= \varphi(\overline{k}) \\ \overline{E} &= \phi(\overline{k}) \end{cases}$$

The economy is in steady state when $\varphi(\overline{k}) = \phi(\overline{k})$, that is when the two functions intersect. The authors illustrate the economy in the $E - k$ space by using a Cobb-Douglas function.

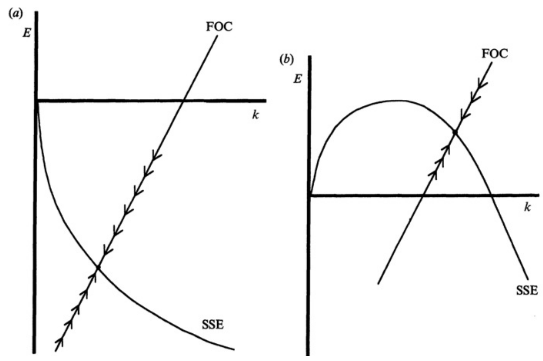

Figure 1.2: Interior equilibrium without external increasing returns (a) ($\rho < 0$), (b) ($\rho > 0$)

There is no steady state if the FOC line lies everywhere above the SSE line, and there are two steady-state equilibria if the two curves intersect twice. In

[18]Put it differently, they set the productivity parameter multiplying the production function, $\Psi(k)$, equal to one for all levels of capital.

this case, the equilibrium with higher capital stock and better environmental quality is stable.[19]
If, at time $t = 0$, the economy lies under the SSE line, then both the stock of capital and the environmental quality will increase along the equilibrium path. Conversely, if at time $t = 0$ the economy lies above the SSE line, then both capital stock and environmental maintenance will decrease along the path towards the steady state.

In economies with little capital or high environmental quality the agents may choose not to invest in environmental maintenance, that is $m_t = 0$. In this case, the equilibrium path is defined by the same conditions as in the previous case, but with $m_t = 0, \forall t$.
The dynamic adjustment of the economy entails increases in k and decreases in E, implying a negative correlation between environmental quality and economic growth under zero maintenance, in contrast to the positive correlation at interior equilibrium. The rational behind this result is the following: as agents accumulate capital, the consumption externality causes degradation of the environment; successive generations, being at the zero maintenance corner, do not find maintenance worthwhile.

The authors then consider the more general case of equilibrium with increasing returns. As before, the evolution of the economy is described by two nonlinear difference equations:

- the arbitrage condition between savings and environmental maintenance, $E_{t+1} = \varphi(k_{t+1}, k_t)$
- the law of motion of the environment, $E_{t+1} = \phi(k_{t+1}, k_t)$

If the external increasing returns are low (that is, if $\Phi(k)f(k)$ is strictly concave in k), then the qualitative behaviour of the model is substantively identical to Fig.1.2. If the external increasing returns are sufficiently high (that is , $\Phi(k)f(k)$ is convex in k), then sustained growth is possible.

The model can exhibit a low-level equilibrium trap, as illustrated in Fig.1.3a: economies with sufficient capital and environmental quality can take advantage of increasing returns and experience sustained growth, while economies with worse initial conditions will move either towards shutdown or towards a low-level stable equilibrium. If multiple equilibria arise, the one with lower environmental quality and capital is stable.

[19]The condition for the stability of the steady state is $\varphi'(\bar{k}) = \phi'(\bar{k})$.

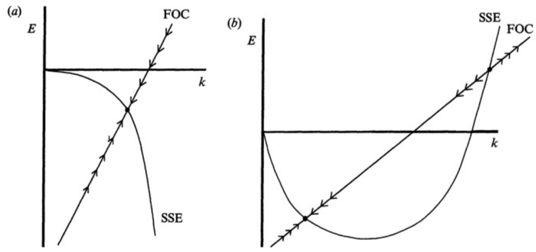

Figure 1.3: Interior equilibrium with external increasing returns (a) ($\rho < 0$), (b) ($\rho > 0$)

The authors then study the case of a social planner who treats all generations symmetrically. The social planner maximizes output and divides it optimally: he sets capital at the level at which net output is maximized, satisfying then the golden-rule condition; then he equates the marginal rate of substitution of consumption for maintenance to the marginal rate of transformation. He translates any increase in output into increased consumption and maintenance such that the environmental quality is constant. By doing so, he unambiguously increases utility. The planner internalizes the consumption externality as well as the maintenance externality, establishing the optimal E for given k. This value attains a maximum at the golden rule capital stock, as shown in Fig.4.

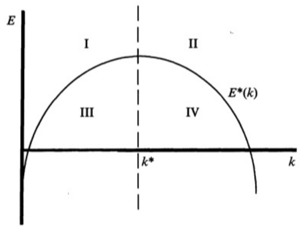

Figure 1.4: The golden rule allocation

The optimum is in (E^{*}, k^{*}), but the competitive equilibrium can be inefficient: there could be overaccumulation of capital (regions II and IV) or overmaintenance of the environment (regions I and III). Pareto improvements can be implemented through the reallocation of resources between generations. In particular, if the economy is experiencing:

- Overmaintenance of the environment and underaccumulation of capital: agents invest in maintenance more than needed, generations can increase their welfare by investing less in environmental maintenance and consuming more; moreover savings must increase.

- Overmaintenance of the environment and overaccumulation of capital: Pareto-improving policies must decrease both maintenance and savings. The planner shall tax the wages of the young and distribute the tax revenues to the old.

- Undermaintenance of the environment and underaccumulation of capital: in this case Pareto-improving policies are not easily found. However, the authors show that they may exist, the social planner must implement the optimal fiscal policy, consisting of intergenerational transfers from not yet born generations to currently alive generations (i.e. *backward transfers*).

- Undermaintenance of the environment and overaccumulation of capital: agents must invest more in the environment and save less. The social planner must implement a scheme of transfers from the young to the old.

Jouvet (1998) presents an OLG model with pollution technology, where each agent can voluntarily contribute in order to reduce pollution by financing depollution activities. In this setting he introduces uncertainty, meaning that it is not always clear or certain what effects will result from a reduction in pollution. Therefore agents are not certain about the real impact of their contributions to the quality of the environment.

Agents live two periods, at each time t there are N identical individuals. In the first period, agents work and receive a competitive wage which is entirely devoted to savings, s_t. When old, they receive savings from the previous period and divide this amount between consumption, d_{t+1}, and a voluntary contribution to the environmental quality, m_{t+1}, while taking as given the contribution of all the other agents. Preferences are represented by the following utility function

$$U(d_{t+1}, P_{t+1})$$

where P_{t+1} is an index of pollution. The authors assume that pollution has a negative effect on consumption, since an increase in pollution induces a decrease in marginal utility of consumption, that is $U_{dP} < 0$.

Perfectly competitive maximizers firms use capital, K, and labour, L, to produce a homogeneous good according to a constant returns to scale production function.

Pollution is a by-product of the productive activity and evolves according to the following law of motion

$$P_t = bY_t + (1-\delta)P_{t-1} - g\sum_{i=1}^{N} m_t^i$$

where $\delta \in (0,1]$ is the absorption rate, $b > 0$ is the contribution of current production Y_t to pollution, g is the marginal efficiency of linear depollution technology and m_t^i is, as we have already said, the voluntary contribution of agent i to environmental quality.

Marginal efficiency of depollution technology, g, is uncertain, that is it depends on a random variable $\tilde{\epsilon}$

$$\tilde{g} = a\tilde{\epsilon} + \bar{g}$$

where $a > 0$.

Each agent maximizes his *expected* utility function, $E(U(d_{t+1}, P_{t+1}))$ subject to his intertemporal budget constraint and the evolution of pollution. The author considers both a change in the average marginal efficiency of the linear depollution technology, g, and a change in the uncertainty level of this technology, a. He shows that agents' contribution choice depends on both the substitution effect between consumption and contribution and the pollution effect on agents' utilities. In particular, contribution could increase or decrease with the average marginal efficiency of depollution technology. Moreover, the comparison between the substitution effect between consumption and contribution and the pollution effect on agents' utilities is affected by agents' risk aversion degree. The relative risk aversion A_r is defined as follows

$$A_r = g\frac{U''_{PP}}{U'_P}\sum_{i=1}^{N} m_t^i$$

Jouvet (1998) proves that if the agent is sufficiently risk averse against environmental degradation, that is $A_r < 1$, then the substitution effect is stronger than the disutility pollution effect and contribution is a decreasing function of average efficiency, $\hat{g}$.

Then the author analyzes the change in contribution due to an increase or decrease in risk.[20] In doing so, he defines a prudent agent as an agent characterized by a positive absolute prudence degree.[21] In particular, a sufficiently risk aversion agent is prudent if and only if

$$\sum_{i=1}^{N} m_{t+1}^{i}(U'''_{dPP} + U'''_{PPP}) - 2U''_{PP} > 0$$

A prudent attitude has two possible consequences, which depend on agent's risk aversion. If the agent is sufficiently risk averse, then an increase in the uncertainty of pollution implies an increase in contribution. Conversely, if the agent is not sufficiently risk averse, then an increase in risk implies a decrease in his contribution.

Hence, risk aversion and precautionary attitude are interlinked: on one hand to understand the implications of precaution one must consider agents' risk aversion characterization, on the other hand to study the effect of an increase in risk aversion one must have a knowledge of agents' prudent attitude.

Finally, he studies the welfare consequences of uncertainty, where the welfare function is defined as the sum of individual *expected* lifetime utilities, that is

$$W = \sum_{i=1}^{N} E(U^{i}(d_t, P_t))$$

Social welfare increases with consumption and depollution, then an increase in average efficiency of depollution technology implies a higher expected depollution and then a lower expected level of pollution. Considering a risk increase, welfare decreases since uncertainty rises to a high potential pollution level.

1.5 Altruism, pollution and intergenerational externalities

In the previous sections we have seen that in economies with intergenerational externalities the competitive equilibrium is inefficient since current

[20] A shift in risk is represented by a shift in the parameter a.

[21] That means that the agent is willing to pay a positive premium in order to decrease risk.

generations do not take into account the welfare of future generations and do not internalize their own externalities. May intergenerational altruism ensure that the *laissez-faire* equilibrium is efficient?

The macroeconomic implications of altruistic links between generations have been widely studied. Indeed, since the seminal works by Becker (1965) (Becker 1991) and Barro (1974) (Barro 1974), many papers studying fiscal and distribution policies have been devoted to the OLG models with altruism (see for example Burbidge (1983), Weil (1987) and Abel (1987)). Also in environmental issues the role of bequests is crucial[22], because the environment can be thought as of an asset which is passed on to future generations.[23] Monetary bequests may not be, in fact, the only way to improve offspring's welfare when agents are altruistic and intergenerational externalities are at work. If production causes pollution, there is a trade-off between bequests and resources devoted to pollution abatement, between the transmission of wealth embodied in productive capital an that of environmental assets.

1.5.1 Altruism with monetary bequests

Howarth and Norgaard (1995) use the framework of their previous work, Howarth and Norgaard (1992), without pollution and show that altruism - when reproduction is parthenogenetic - makes the equilibrium be optimal. However, this result is strongly related to the demographic structure of the model and does not hold if each household is composed by two agents who give birth to two children. In this case, the competitive equilibrium is not optimal since it underestimates offspring's welfare. The amount of intergenerational transfers is chosen by the government, elected by a social planner who cares about intergenerational equity. Individuals are selfish, and this makes the framework paradoxical, since the social preferences on which the planner bases his choices should spring from agents' preferences. If agents are allowed to be altruistic, then the existence of an exogenous agency is no longer needed, since agents make transfers voluntarily. The authors then assume the existence of a representative altruistic agent who cares about the welfare of his/her child, V_{t+1} which is in turn an argument of his/her own utility function,V_t

[22]Nevertheless, despite the importance of bequests in the environment - growth framework (see Lofgren (1991)), there are few studies that incorporate them into the analysis (see, for example, Hultkrantz (1992), Howarth and Norgaard (1995) and Amacher et al. (1999)).

[23]For instance, altruistic motives are important in recycling activities, since there is often a lack of traditional (non-altruistic) economic incentives.

$$V_t = U_t(c_t, d_{t+1}) + \gamma V_{t+1}$$

where γ is the degree of intergenerational altruism. By iteration, they show that the representative household behaves as an infinitely-lived agent who tries to maximize the discounted sum of all current and future lifetime utilities

$$\max \sum_{t=s}^{T} \gamma^t U_t(c_t, d_{t+1})$$

if the first-order necessary conditions are satisfied and the tax set by the planner to correct intergenerational externalities is efficient, then the representative household turns out to be a planner and the competitive equilibrium is the same as that that would be chosen by a social planner. The result is then optimal, since it maximizes the welfare of a member of the current generation taking into account the welfare of the future ones.

However, this result strongly depends on the demographic structure of the model. If it was assumed that a household is composed by two individuals who give birth, at the end of the first period, to two children who - once grown up - settle down and start a new household with children from other families, then a dynastic sequence would not exist anymore. The utility function of the household then would become

$$V_t = U_t(c_t, d_{t+1}) + \widetilde{\gamma} \sum_{i=1}^{2} V_{t+1}^i$$

where $\widetilde{\gamma}$ is the relative weight attached to the welfare of child i ($i = 1, 2$). By iteration, the household's welfare can be rewritten as

$$V_t = U_t(c_t, d_{t+1}) + \sum_{g=1}^{T-t} \sum_{i=1}^{2g} \widetilde{\gamma}^g U_{t+g}^i(c_{t+g}^i, d_{t+g+1}^i)$$

where 2^g is the number of children at the $g - th$ descent and $T - t$ is the number of children born between t and T.

The authors find the arbitrage condition that determines the transfer that each household leaves to its child i (they consider only the case $g = 1$). This condition is called *intergenerational condition*, and in the case of a

decentralized economy with altruism is the same as in the centralized setting with selfish agents and parthenogenesis.
The authors then consider the case of a planner who allows the households of the same generation to have the same level of consumption (hence, there is no reason anymore to distinguish between welfare of the households and welfare of the children). The maximization problem faced by a household at time t is then

$$\max V_t = \sum_{i=0}^{T-t} (2\widetilde{\gamma})^i U_{t+i}(c_{t+i}, d_{t+i+1})$$

By comparing the intergenerational arbitrage in the *laissez-faire* scenario with that of the planner, Howarth and Norgaard (1995) show that without any interventions by an exogenous planner, households tend to underestimate the welfare of the children by a factor 2^t for the $t - th$ generation.
This is because in the *laissez-faire* case there exists an externality that is not taken into account and weakens the motivation of parents to make monetary transfers to their children: bequests benefit also the child partner's parents.

1.5.2 Altruism with environmental bequests

If generations are connected through altruism and intergenerational environmental externalities are at work, then bequests are not the only way to increase children's welfare, since the environment is a transferable good as well.
This is the scenario analyzed by Jouvet, Michel, and Vidal (2000). In their model pollution affects agents' utility, while agents can decide to invest in environmental maintenance. Without population growth, at each time t $2N$ individuals are alive: N young agents who work, earn the competitive wage and save; and the altruistic old agents who are retired and allocate the proceeds of their savings between their own consumption, d_{t+1}, the bequests to their child, x_{t+1}, and the voluntary contribution to the abatement of pollution, m_{t+1}. Individuals care about their offspring's welfare; they do so by weighting their offspring's utility in their own utility function which is therefore recursively defined by[24]

[24] As in Jouvet (1998), the utility function $U(\cdot)$ verifies the assumption of disutility from pollution: an increase in pollution determines a decrease in the marginal utility of consumption, that is $U_{dP} < 0$

$$V_t = U(d_{t+1}, P_{t+1} + \gamma V_{t+1}) = \sum_{s=t}^{\infty} \gamma^{s-t} U(d_{s+1}, P_{s+1})$$

where $\gamma \in [0, 1)$ denotes the intergenerational degree of altruism.

Jouvet, Michel, and Vidal (2000) assume that pollution results from production. Firms have access to the same constant returns to scale production function using labour and capital, k, as inputs. The emission of pollutants at time t is a linear function of the output level, aY_t, where $a > 0$. Pollution abatement occurs according to a linear technology which is invariant through time, $b > 0$. bM_t is then the amount of resources devoted to pollution abatement at time t. At each time a part $h \in (0, 1]$ of pollution is absorbed, so that the dynamics of pollution are thus given by

$$P_t = aY_t + (1 - h)P_{t-1} - bM_t$$

Investment in environmental maintenance is decided according to a Cournot-Nash *subscription equilibrium*: each individual takes the other's supply of the public good as given, and there will be a voluntary contribution to environmental maintenance only if the level of pollution is above a critical threshold.

The authors focus on steady-state solution and they prove that both voluntary contributions to pollution abatement and bequests are operative if and only if:

- the level of pollution with no contribution $\left(S^\gamma = \frac{aNf(k)^\gamma}{k}\right)$ exceeds a pollution threshold $\overline{P}^\gamma$.[25] $\overline{S}^\gamma$ is the solution to the arbitrage condition between consumption and pollution abatement, that is

  $$\frac{U'_d(d, S)}{U'_s(d, S)} = \frac{b}{\gamma(1 - h) - 1}$$

  Below this threshold $\overline{S}^\gamma$, agents leave bequests but are not willing to contribute to pollution abatement. Since the marginal utility of consumption is higher than the marginal disutility of pollution, the individual's optimal choice is not to contribute to pollution abatement.

[25] The pollution level positively depends on the stock of capital whose value is related to the intergenerational degree of altruism.

- bequest motive is sufficiently high, that is $\gamma \geq \frac{1}{1+R^d} = \gamma^\omega$, where R^d is the interest rate in an economy without pollution ($a = 0$) nor altruism ($\gamma = 0$) (as in Diamond's (1965) (Diamond 1965) model). γ^ω is the lowest bequest motive in order to have operative bequests. Main results are summarized in Table1.2.

	Positive pollution abatement	No pollution abatement
Positive bequests ($\gamma > \gamma^\omega$)	$S^\gamma > \overline{S}^\gamma$	$S < \overline{S}^\gamma$
No bequests ($\gamma \leq \gamma^\omega$)	$S^d > \overline{S}^d$	$S^d < \overline{S}^d$

Table 1.2: Main Results

An increase in the degree of altruism has two effects working in opposite directions. On one hand, stronger altruistic feelings result in a higher steady-state stock of capital. Hence, there are more resources available for consuming and clearing of pollution. On the other hand, an increase in capital stock results, *ceteris paribus*, in a higher level of pollution. The overall effect is ambiguous.
However, if bequests are not operative ($\gamma < \gamma^\omega$), then consumption and pollution decrease with the degree of intergenerational altruism whereas voluntary contributions increase.[26]

The *laissez-faire* equilibrium is characterized by two externalities which are not taken into account by the agents. The first one springs from the *subscription equilibrium*: each individual takes the others' decision as given and there is no cooperation amongst them. The second externality comes from pollution caused by production. This proves that the optimal growth model does not replicate the behaviour of altruistic individuals: the two approaches clearly differ in the modelling of behaviours. As a consequence, despite individuals' altruistic tendencies, there is room for public policies.

Hence the authors consider the case of a central planner who wants to maximize a utilitarian social welfare function consisting of the discounted sum of individuals' lifetime utilities

[26]The decrease in consumption is at variance with the result obtained in the Barro model, wherein consumption positively depends on the degree of altruism.

$$\max_{\{d_t, m_t, P_t, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \gamma^t U(d_t, P_t)$$

subject to the resource constraint of the economy and the pollution law of motion. They assume that the social discount rate equates the private degree of intergenerational altruism, γ.

Interior solutions are defined by the arbitrage condition between operative bequests and pollution abatement, that is

$$\frac{U'_d(d,S)}{U'_S(d,S)} = \frac{Nb}{\gamma(1-h)-1}$$

and by the optimal stock of capital that takes into account the environmental externality of production

$$f'\left(\hat{k}^{\gamma}\right) = \frac{1}{(1-\frac{a}{b})\gamma}$$

The technology of pollution abatement is efficient if $a < b$. However, if production results in a low level of pollution, the central planner's optimal choice is not to devote resources to pollution abatement ($m = 0$). In this case, the stationary capital stock at the social optimum *without* pollution abatement lies between the modified golden rule and the stationary capital stock of the optimum *with* pollution abatement, that is

$$\frac{1}{(1-\frac{a}{b})\gamma} \geq f'(k) > \frac{1}{\gamma}$$

Even if the central planner is not willing to finance pollution abatement, he takes into account the pollution externality caused by production, the term $(1-\frac{a}{b})$.

Increasing the social rate of discount results in two opposing effects. The central planner cares more about future generations' level of consumption and is therefore willing to devote more resources to capital accumulation. At the same time it is more concerned with the implications of capital accumulation for the environmental quality and he can monitor the level of pollution only by decreasing the level of capital accumulation (since we are considering the case of no pollution abatement). Capital accumulation and consumption can either increase or decrease with the social rate of discount, the total effect depends on the value of the parameters.

The authors then show that, when there is pollution abatement, the social optimum can be decentralized. Since two externalities are at work, the decentralization requires two policy instruments. The social planner subsidizes the private contributions to pollution abatement, σ, and sets a tax on savings, τ^s.[27]

At the steady state, the central planner sets the tax rate on savings such that the economy accumulates the optimal capital stock. This optimal tax rate depends on whether or not bequests are operative. If bequests are operative, then the optimal tax rate is

$$\tau^s = \frac{a}{b}$$

if bequests are not operative, then the optimal tax rate on savings becomes

$$\tau^s = 1 - \frac{\hat{k^\gamma}}{w(\hat{k^\gamma})}$$

where $\hat{k^\gamma}$ is the optimal capital stock. Finally, the optimal rate of subsidization is

$$\sigma = 1 - \frac{1}{N}$$

Without pollution abatement, the decentralization of the optimum only requires the tax on savings, since the individual are never willing to contribute to pollution abatement if the central planner is not.

1.6 Conclusions

All these works tell us that is important to take into account intergenerational externalities in the field of environmental economics and policy. More precisely, costs caused by these decisions have to be compared with the benefits that all generations, and not only the current one, will enjoy. However, it seems that the internalization of environmental or intergenerational externalities does not guarantee the international equity.

Indeed, time preference and how the well-being amongst generations is shared seem to be two distinct notions. The distribution of resources

[27] As usual in OLG models, the government's budget is balanced by a lump-sum transfer to the old.

amongst generations determines whether or not efficient allocations of resources allow for a not-decreasing welfare across generations. The ethical reason is that future generations have right to the same level of welfare as current generations: it is an implicit intergenerational contract based on the redistribution of either goods or natural capital. A social planner might look for an intergenerational equity through transfers given to future generations as a compensation for environmental damages, though it might be difficult to implement such transfers.

John et al. (1995) compare the consequences of a policy chosen by a short lived-government with the one that would be chosen by a social planner who has infinite lifetime. The short-lived government fails to internalize the intergenerational externalities and the resulting equilibrium is inefficient as current generations do not care about the welfare of future ones. Without altruism but with preferences towards the environment, the competitive equilibrium path can be characterize by intergenerational and non voluntary transfers represented by investment in pollution abatement, as agents are worried (only) about their future. Hence, these transfers do not allow for an optimal distribution of welfare amongst generations, which hinders the capabilities of development of future generations. This a standard result in welfare economy which stems from the hypothesis made on the life cycle: by looking only for his well-being, every agent carries the economy towards a worse situation for everybody. When agents are altruistic, intergenerational and voluntary transfers are at work, but they are not enough to bring the economy to the optimum.

Chapter 2

Environmental Externalities and Fiscal Policy in an Overlapping Generations Model

We study on overlapping generations (OLG) model of growth and environment à la Diamond in order to compare the centralized and decentralized equilibria and to analyze the effects of different fiscal policies. Environmental quality is a public good negatively affected by consumption activity and positively affected by maintenance investment. Households derive utility from both consumption of physical goods and environmental quality. Due to the negative externalities that characterize the model, it is shown that the competitive equilibrium is inefficient. Three different fiscal policies designed by the government in order to correct the market failure and achieve the social optimum are analyzed.

2.1 Introduction

In recent years, researchers have investigated the conflict between environmental preservation and economic growth in a dynamic setting. Environmental policy should therefore pay attention to both the efficiency question of *how much pollution to allow* and the intergenerational equity question of *which generation pays for the environment and by how much*. Unfortunately, most of the early formal literature analyzing the environment-economy interaction (see Solow (1974) and more recently Tahvonen and Kuuluvainen (1991)) employs an infinitely-lived representative agent model, which makes it hard to study intergenerational issues.
Following this approach all future impacts are treated as if they happened to current agents, ignoring that society is composed of mortal individuals of different generations whose actions have consequences that outlive them.
Authors such as Solow (1986) suggest that it is useful to capture these intergenerational aspects in the economic analysis of environment and natural resource. Moreover, there exists strong empirical evidence rejecting that members of extended families are altruistically linked in the way postulated by standard infinitely-lived agents models.

Recent articles have then addressed environmental issues using overlapping generations (OLG) models. For instance, John and Pecchenino (1994), in their seminal paper, analyze the potential conflict between economic growth and the maintenance of environmental quality in a context where consumption degrades the environment. By using the same model, John et al. (1995) and Ono (1996) examine the optimal tax policies that must be implemented by a long-lived government which lasts longer than the representative agent of the economy, in order to internalize the intergenerational externalities produced by competitive behaviour. Guruswamy, Kumar, and Murthy (1977) analyze the relationship between resource exhaustion and pollution by using an OLG framework and show that Pareto inefficient outcomes arise (due to the lack of intragenerational coordination) and is propagated across generations. Bovemberg and Heijdra (1998) study the effects of environmental taxation within an OLG economy in which the quality of the environment is considered as a durable consumption good. They show that if attention is used on Pareto improving policies, it is harder to introduce pollution taxes than to increase those taxes when they already exist. Ono and Maeda (2002) analyze the effects of population aging on economic growth and the environment and the conditions under which aging is beneficial to the environment.

We also focus on environmental issues and use an OLG model both to examine the effects that pollution can produce on the economic growth and to explore concrete environmental policies designed by the social planner to assure long-run compatibility between economic activity and the environment. In general, environmental externalities can arise from either production or consumption, and can enter either preferences and technologies.[1] We model the externality as a product of consumption.[2] When the environment enters agents' utilities, changes in environmental quality alter the marginal rate of substitution between consumption and environment, causing agents to alter their allocation of resources between consumption and environmental improvement. Conversely, when external effects operate through the technology, changes in environmental quality affect the total resources available.[3]

Our modelling framework is related to the work by John and Pecchenino (1994), who study optimal environmental policy employing the Samuelson and Diamond (1965) approach.[4] We consider an infinite-horizon economy composed of perfectly competitive firms and finitely-lived agents. A new generation is born in each period and lives two periods, youth and old age. Households derive utility from both consumption of physical goods and environmental quality, which is modeled as a renewable resource (for instance, air or soil quality). Since the environmental amenities are treated as public good they do not enter the budget constraint. As we have already said, environmental quality is negatively affected by pollution, which is generated as a by-product of consumption. Without government intervention, the decentralized market outcome is likely to result in too much pollution, because individuals fail to internalize its societal cost. Consequently, the stock of natural resources bequeathed to future generations is degraded. To correct the environmental externality, the government can employ different fiscal policies.

Even if our model is related to the model by John and Pecchenino (1994),

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[1]First, production might directly affect welfare. Second, production might affect current or future production possibilities, for instance pollution can reduce the amount of resources available. Third, consumption might be a source of disutility and have external effects on welfare. Finally, consumption might have external effects on production, i.e. CO_2 emissions cause climate change and so influence agricultural output.

[2]By changing the technology for environmental improvement we could equivalently view the externality as coming from production.

[3]For instance, in Ono (2003) the environment affects technology. Bovemberg and Smulders (1995) set up a model in which environment affects both utility and technology.

[4]Another approach has been recently used in studying environmental issues. In the Yaari-Blanchard framework agents face an exogenous probability of death, equal to the birth rate, so as to yield a constant population size. This framework has been used, for instance, by Marini and Scaramozzino (1995).

it differs from theirs in some respects. Firstly, the two authors assume that agents consume only when old. We instead assume that households consume in both periods, while working only in the first period. By making this assumption, we are able to analyze the consumption-savings decisions and study the trade-off between present and future consumption.
Secondly, John and Pecchenino (1994) are interested only in intergenerational externalities caused by consumption, leaving out the free-rider problem. We instead take into account both the intra- and the intergenerational negative effects of consumption.[5]
The effects within a generation have been largely ignored by the OLG literature. Many papers normalize the size of each generation to unity (see, for instance, Ono (2003), Ono and Maeda (2002) and Toshida (2002)). Many others assume that individuals are represented by a short-lived government whose sole responsibility is the provision of the public good for the benefit of agents alive during its period of office (see, for instance, John and Pecchenino (1994)). In this way, the intragenerational effects are not taken into account in the analysis of the model.
Conversely, we assume that the i-th representative consumer takes others' actions as given, like in a non-cooperative framework. To our knowledge there exists only a paper that analyzes intragenerational externalities in a non-cooperative framework. Jouvet, Michel, and Vidal (2000) set up an OLG model of pollution externality in which individuals are altruistically linked to their offspring as in Barro (1974). They show that, despite the individuals' altruism, the competitive equilibrium is suboptimal because of the intragenerational externality characterizing the model.
Finally, we extend the work by John and Pecchenino (1994) by analyzing the economy from the point of view of a social planner who wants to maximize the utility of current and future generations, and we introduce three tax-transfer schemes that succeed in carrying the economy to the social optimum.

Our main results are as follows: for an infinite horizon economy where there are many finitely-lived agents per generation, the Pareto inefficiency arises due to the lack of intragenerational coordination and it is propagated across generations because of intergenerational externalities. In such a context, effective tax schemes exist and may be successfully implemented by a long-lived government agency whose planning horizon is the environment lifetime.

The remainder of this paper is organized as follows. Section 2.2 outlines the overlapping generations model of the environmental externality wherein each

[5] For an analysis of the intragenerational conflict, see Pigou (1920) and Varian (1995).

generation is comprised of N identical individuals. Section 2.3 studies the optimization problem faced by households and firms. Section 2.4 derives the competitive equilibrium, whereas in Section 2.5 we characterize the steady state and its comparative static behaviour. In Section 2.6 we consider the solution to the long-lived social planner's problem. Section 2.7 develops three optimal tax programmes. Section 2.8 concludes.

2.2 The model

An overlapping generations model provides the framework for the analysis. The rationale for using this kind of model for analyzing an environmental externality follows from the fact that - as we have already argued - economic activities of a generation born at time t cause environmental degradation not only in period t, but also in all future periods $t+1, t+2, ..., t+n$. However, if a finitely-lived generation born at t is alive only for $k < n$ periods, it would not consider environmental degradation beyond period $t+k$, that is it does not take into account the external costs caused for generations still alive after period $t+k$. Hence, the use of a infinitely-lived agents model would obscure the relationship between generations that cause and generations that bear the costs of environmental degradation.

2.2.1 Households

We consider a perfectly competitive overlapping generations economy with discrete time $t = 0, 1, ..., \infty$. Each generation is alive for two periods and has perfect foresight, this implies that $r^e_{t+1} = r_{t+1}$, where r^e_{t+1} is the expected value at time t of the interest rate at time $t+1$. We assume that there are N identical individuals of each generation at time t and no population growth, that is $N_t = N \quad \forall t$.[6] Since agents are identical (except for their ages), we consider a *representative* individual in each generation.[7]

When young, the agent supplies inelastically to firms one unit of labour.[8] He receives a wage, w_t, which is allocated between consumption, c_t, savings

[6] John et al. (1995) develop a model with environmental externalities and population growth. They show that a higher population growth rate lowers environmental quality per capita but could raise the aggregate quality of the environment.

[7] By doing so, we abstract from the typical problem of aggregation of interacting (even though identical) agents. We thank an anonymous referee for pointing out this assumption.

[8] Our long term view allows us to assume full employment. The assumption that

s_t - which are supplied inelastically to firms - and maintenance investment m_t - whose role is discussed later on.

$$w_t = c_t + s_t + m_t \tag{2.1}$$

In period $t+1$, when old, the representative individual retires and earns the gross return $R_{t+1} = (1 + r_{t+1})$ on the savings he supplied in period t.

$$d_{t+1} = (1 + r_{t+1})\, s_t \tag{2.2}$$

Variable d_{t+1} denotes consumption of physical goods of the representative agent in the second period of life. From equation (2.1) and (2.2), the intertemporal budget constraint of the representative agent born at time t can be written as follows

$$w_t = c_t + \frac{d_{t+1}}{1 + r_{t+1}} + m_t \tag{2.3}$$

Preferences of each individual are defined by the following life-cycle utility function[9]

$$U_t = u\,(c_t) + \delta v\,(d_{t+1}; E_{t+1}) \tag{2.4}$$

where $\delta > 0$ represents the psychological discount factor, and it is the same for all agents.[10]

The representative household derives utility from consumption in both periods: c_t, d_{t+1} and enjoys a high environmental quality, E, in the second period in life.[11] A plausible justification for the fact that agents care about environment when they are old might be found in the well-known relationship between pollution and health costs, as in Williams (2002). In his model, an

households supply their labour inelastically also means that they receive no utility from leisure.

[9] In our paper we implicitly make use of the assumption that a given level of consumption provides the same level of utility, independent of one's own consumption history and independent of the consumption level of other households. Frames where individual consumption relative to the average (status) and the household's own consumption history (habits) affect an individual's well-being are developed by Ng and Wang (1993), and Howarth (1996). It is shown that habit formation leads to a rising consumption profile over time. Moreover, if status seeking has a stronger impact on the consumption of that generation that has a higher status utility than on consumption of the other generation, then it raises aggregate consumption and lowers environmental quality.

[10] Higher values of δ represent a larger preference for future compared to current consumption.

[11] In the context of the model, households enjoy leisure time when old only. Environmental quality is seen as an important factor of leisure quality in this context.

increase in pollution deteriorates consumers' health. Thus, consumers care indirectly about the environmental quality through the health costs that enter in the consumers' budget constraints. However, although households derive utility from the environment, the old generation has no incentive to care for environmental quality in future period.[12] This formulation is also motivated by the contribution of John and Pecchenino (1994).

The representative household chooses c_t; s_t; m_t and d_{t+1} to maximize utility (2.4) subject to the intertemporal budget constraint (2.3). Utility (2.4) satisfies the following assumption

Assumption 1 *The utility function* $U : \Re^3_+ \Rightarrow \Re$ *is defined over non-negative consumption during the first and second period of life as well as over environmental quality. It is twice continuously differentiable, increasing in all arguments and strictly concave. In particular, it satisfies:* $u'(c_t) > 0$; $v'_d(d_{t+1}, E_{t+1}) > 0$; $v'_E(d_{t+1}, E_{t+1}) > 0$ *and* $u''(c_t) < 0$; $v''_{dd}(d_{t+1}, E_{t+1}) < 0$; $v''_{EE}(d_{t+1}, E_{t+1}) < 0$. *Finally, in order to have interior solutions, we assume that* $\lim_{c\to 0} u'() = +\infty$, $\lim_{d\to 0} v'_d() = +\infty$ *and* $\lim_{E\to 0} v'_E() = +\infty$.

2.2.2 Environment

We model the index of environmental quality as evolving according to the following state equation[13]

$$E_{t+1} = (1-b)\,E_t - \beta \sum_{i=0}^{N} (c_{it} + d_{it}) + \gamma \sum_{i=0}^{N} m_{it} \tag{2.5}$$

where parameter $b \in [0; 1]$ stands for the natural decay rate, $\beta > 0$ indicates the strength of the impact of agent i's consumption on the environment and $\gamma > 0$ is the efficiency of agent i's environmental expenditures.
This index may include the inverse of the concentration of chlorofluorocarbons in the atmosphere, the inverse of other greenhouse gases, the quality of groundwater or an index of biodiversity.

This specification for the evolution of the environmental quality is based on the work by John and Pecchenino (1994), but differs from that in two

[12] See Jouvet, Michel, and Vidal (2000) for a model in which parents are altruistic.

[13] A similar linear specification for the evolution of the environmental quality, introduced by John and Pecchenino (1994), has been widely used in the recent literature. See for example Jouvet, Michel, and Vidal (2000) among others.

respects. As we have already said, the authors are not interested in the intragenerational externality of consumption, they focus only on intergenerational issues. For this reason they abstract from the free-rider problem.Conversely, as we are interested in studying both intra- and intergenerational externalities, we introduce in the dynamics of environmental quality the parameter N, which stands for the size of each generation. In this way we will be able both to characterize the effect of N on the equilibrium and to take into account the intragenerational externality.
Moreover, conversely to John and Pecchenino (1994), we assume that agents consume in both periods of life. In this way, we are able to characterize the agent's consumption-savings decision.

Under the hypothesis that individuals are identical, we can rewrite (2.5) as

$$E_{t+1} = (1-b)\,E_t - \beta N\,(c_t + d_t) + \gamma N m_t \tag{2.6}$$

Without economic activity, the *index* of environmental quality tends to zero: $E_{t+1} = (1-b)\,E_t$. Thtat is, each period, environmental quality changes by b towards the autonomous level of zero: $\lim_{t\to\infty} E_t = \lim_{t\to\infty} E_0\,(1-b)^t = 0$. When $b = 1$ environmental quality lasts only one period and consumption has not effect at all on the environment, otherwise actions of those alive at time t have environmental externalities on the next generations.
Environmental quality is affected by two economic activities: consumption and maintenance investment. As (2.6) shows, the environmental quality at time $t+1$ is affected by the consumption of the old households born at time $t-1$, that is the variable Nd_t, and by the consumption of the young households born at t, Nc_t.[14] While young households account for environmental degradation in their second period of life, they do not consider the deterioration of the environment and its impact on future generations' utility in periods thereafter. Put it differently, although consumption of those now alive produces an environmental externality that impinges on future generations' utility, this effect is ignored by current generations.
On the other hand the environment is also positively affected by the payment of environmental maintenance and improvement, Nm_t. However, while agents' efforts reduce the existing stock of waste for themselves and for future generations, the present generation is concerned only with its own welfare and ignores any benefits bestowed on its progeny. Finally, the assumption $\gamma > 0$ assures that investment in the environment has a positive effect on environmental quality. The coefficients β and γ are assumed to be time

[14]We assume that the pollution coefficient of c_t is the same as that of d_t because these consumption goods are homogeneous.

invariant.

2.2.3 Firms

There are a large number of competitive firms producing a homogeneous good that, at each period t, have access to the same production technology given by the following production function: $F(K_t, L_t) = Y_t$, where K_t is the stock of capital, L_t is the labour supply and Y_t is the level of output. We assume that $F(.)$ displays constant returns to scale, so that it can be expressed in intensive terms to give $F(K_t, L_t) = f(k_t)$, where $k_t(= K_t/L_t)$ is the per capita level of capital. We also assume that the per capita production function, $f(k_t)$, satisfies the standard neoclassical properties

Assumption 2 *The production function $f : \Re_+ \Rightarrow \Re_+$ has the following properties. It is twice continuously differentiable, increasing and strictly concave. In particular, $f'(k) > 0$; $f''(k) < 0$ $\forall k > 0$. Moreover, we impose that $\lim_{k\to\infty} f'(k) = 0$, $\lim_{k\to 0} f'(k) = \infty$ and $f(0) = 0$*

At each time t, the output per unit of labour $y_t = f(k_t)$ is allocated to maintenance investment, consumption and capital investment

$$f(k_t) = c_t + d_t + m_t + s_t \tag{2.7}$$

Notice that the rate of depreciation of capital is set equal to one in (2.7).[15]

Equilibrium conditions between investment and savings imply

$$K_{t+1} = Ns_t \tag{2.8}$$

This condition states that savings from the young households are transformed into productive capital for the next period, as in Diamond (1965). In intensive terms the accumulation rule for capital can be written as

$$k_{t+1} = s_t \tag{2.9}$$

[15]Since the length of a period in a two-generations-context is about thirty years, this may be considered a realistic assumption.

2.3 The optimization problem

2.3.1 The representative agent's optimization problem

The representative agent takes as given the wage, w_t, the return on savings, r_{t+1} and the stock of environment at the beginning of period t, E_t. He also takes as given the decisions of the other $N-1$ consumers of each generation, $\overline{c}_t, \overline{d}_t, \overline{m}_t$ and the environmental parameters β, γ and b. Thus, the competitive life-cycle choice problem of the representative agent is to choose c_t, d_{t+1}, m_t and s_t according to the following programme

$$\max u(c_t) + \delta v(d_{t+1}; E_{t+1}) \tag{2.10}$$

subject to

$$E_{t+1} = (1-b)\,E_t - \beta[(N-1)\left(\overline{c}_t + \overline{d}_t\right) + (c_t + d_t)] + \gamma[(N-1)\overline{m}_t + m_t] \tag{2.11}$$

$$w_t = s_t + m_t + c_t \tag{2.12}$$

$$d_{t+1} = (1 + r_{t+1})\,s_t \tag{2.13}$$

$$c_t, d_{t+1}, s_t, m_t \geq 0 \tag{2.14}$$

Hence, the agent maximizes the intertemporal utility function (2.10) subject to the evolution of environmental quality (2.11) and the constraints (2.12) – (2.14).

We focus on the case of $m > 0$.[16] As John and Pecchenino (1994) show in their paper, if the successive generations choose not to invest in the environment, capital accumulation occurs, whereas the quality of the environment decreases constantly in the future because of both the accumulation of pollution and the lack of maintenance investment. At some period t, agents born in that period will find it worthwhile to invest in the environment.

By supposing that $s_t, m_t > 0$ and given Assumption1, the problem admits a solution.
We can use the constraints (2.11) – (2.14) to write down the objective function of the representative household as

[16] Notice that there is nothing in the model that precludes the possibility that $m_t = 0$. For the case $m = 0$ see John and Pecchenino (1994).

$$u(c_t)+\delta v\left[(1+r_{t+1})(w_t-c_t-m_t);(1-b)E_t-\beta[(N-1)(\overline{c}+\overline{d})+(c_t+d_t)]+\gamma[(N-1)\overline{m}+m_t]\right] \tag{2.15}$$

By deriving $U(\cdot)$ with respect to c_t and m_t (notice that E_t is not a control variable), we get

$$u'(c_t)+\delta\left[v'_d(d_{t+1},E_{t+1})(-(1+r_{t+1}))-\beta v'_E(d_{t+1},E_{t+1})\right]=0 \tag{2.16}$$

and

$$v'_d(d_{t+1},E_{t+1})(-(1+r_{t+1}))-\gamma v'_E(d_{t+1},E_{t+1})=0 \tag{2.17}$$

from (2.17) we obtain

$$v'_d(d_{t+1},E_{t+1})=\frac{\gamma}{1+r_{t+1}}v'_E(d_{t+1},E_{t+1}) \tag{2.18}$$

By plugging (2.18) into (2.16) we get

$$u'(c_t)=\delta(\gamma+\beta)v'_E(d_{t+1},E_{t+1}) \tag{2.19}$$

Equation (2.19) implies that, in the case of $m>0$, the representative household chooses consumption when young in order to equate the marginal rate of substitution between consumption in youth and environmental quality in old age, that is $u'(c_t)/\delta v'_E(d_{t+1},E_{t+1})$, and the marginal rate of transformation $(\gamma+\beta)$.
Equivalently, (2.18) gives us a simple arbitrage between the rate of return on the private savings, $(1+r_{t+1})$, and the rate of return on the investment in environmental quality, γ. This condition states that, in the case of $m>0$, the representative household chooses savings to equate the marginal rate of substitution between consumption in old age and environmental quality in old age, that is $v'_d(d_{t+1},E_{t+1})/v'_E(d_{t+1},E_{t+1})$, and the rate of transformation $\gamma/(1+r_{t+1})$.

2.3.2 The representative firm's optimization problem

Profits of the representative firm which produces at time t are

$$\pi_t = F(K_t, L_t) - w_t L_t - R_t k_t \tag{2.20}$$

where L_t is the labour input paid at a wage w_t and $R_t \equiv 1 + r_t$ is the return factor on savings from time $t-1$ to time t. Equation (2.20) is maximized with respect to L_t, hence the labour demand L_t which maximizes this expression is obtained by equalizing the marginal productivity of labour with the wage rate.[17]

$$F'_L(\cdot) = f(k_t) - k_t f'(k_t) = w_t \tag{2.21}$$

or alternatively

$$\omega\left(\frac{K_t}{L_t}\right) = w_t \tag{2.22}$$

The equilibrium wage that equalizes the labour demand L_t solution to (2.22) with the inelastic labour supply N_t is

$$w_t = \omega\left(\frac{K_t}{N_t}\right) = \omega(k_t) \tag{2.23}$$

where $k_t \equiv K_t/N_t$ is the stock of capital per young person.

By assumption, profits are distributed to the owners of the capital stock. Given the equilibrium of labour market (2.23), the realized profits are

$$\pi_t = f'(k_t)K_t \tag{2.24}$$

while the distributed profits can be written as

$$\pi_t = NR_t s_{t-1} = R_t K_t \tag{2.25}$$

where $K_t \equiv Ns_{t-1}$.
The equality between (2.24) and (2.25) implies that the rate of return R_t is equal to the marginal productivity of capital, that is

$$R_t = f^{'}(k_t) \tag{2.26}$$

[17]Assumption2 implies that the productivity of labour is increasing and positively valued.

Hence, the first-order conditions (FOCs) solution to the representative firm's maximization problem are (2.23) and (2.26).

2.4 Competitive equilibrium

Most of models dealing with environmental issues restrict the analysis to steady states, ignoring the transitional dynamics towards the long-run positions.[18]

In this Section we will try to analyze the competitive equilibrium of the economy under analysis. We can define a competitive equilibrium as follows

Definition 1 A competitive equilibrium for the economy under analysis is a sequence $\left\{c_t^*, d_{t+1}^*, m_t^*, w_t^*, r_t^*, s_t^*, k_t^*, E_t^*\right\}_{t=0}^{\infty}$ such that, given the initial conditions of the state variables k_0 and E_0: i) firms maximize profits; ii) consumers maximize their utility function; iii) markets clear.

The competitive equilibrium of the model can be summarized by the following equations

$$c_t^* = w_t^* - S_t\left(w_t^*, r_{t+1}^*, E_t^*\right) \tag{2.27}$$

$$d_{t+1}^* = \left(1 + r_{t+1}^*\right) s_t\left(w_t^*, r_{t+1}^*, E_t^*\right) \tag{2.28}$$

$$k_{t+1}^* = s\left(w_t^*, r_{t+1}^*, E_t^*\right) \tag{2.29}$$

$$w_t^* = f\left(k_t^*\right) - f^{'}\left(k_t^*\right) k_t^* \tag{2.30}$$

$$\left(1 + r_{t+1}^*\right) = f^{'}\left(k_{t+1}^*\right) \tag{2.31}$$

$$E_{t+1}^* = (1-b) E_t^* - \beta N\left(c_t^* + d_t^*\right) + \gamma N m_t^* \tag{2.32}$$

where $S_t\left(w_t^*, r_{t+1}^*, E_t^*\right) = s_t^* + m_t^*$ is the optimal aggregate savings function, with $S_t \in (0, w_t)$, defined as a function of wage, interest rate and environmental quality; and implicitly derived from the optimal conditions $(2.18) - (2.19)$ and the restrictions $(2.11) - (2.14)$.

[18]The exception is Bovenberg and Smulders (1995), who find sharp differences between short-run and long-run effects of environmental policy. However, since transitional paths in the Bovenberg and Smulders' model eventually converge to a balanced growth path, the dynamics can still be regarded as simple dynamics.

If we know the equilibrium paths for the capital level k and the environmental quality E, then we can obtain all the equilibrium sequences of our model. Thus, we can focus on the dynamics of the capital accumulation and of the environmental quality.

By substituting conditions (2.23) and (2.26) into equation (2.9) we obtain that the capital stock evolves according to

$$k_{t+1} = s_t\left(f\left(k_t\right) - f^{'}\left(k_t\right)k_t, f^{'}\left(k_{t+1}\right), E_t\right) \tag{2.33}$$

which is a nonlinear first-order difference equation that defines implicitly k_{t+1} as a function of k_t and E_t.

The evolution of the environmental quality can be found in a similar way

$$\begin{aligned} E_{t+1} = (1-b)\,E_t - \beta N\left\{\left(f\left(k_t\right) - f^{'}\left(k_t\right)k_t - S\left(.\right)\right) + f^{'}\left(k_t\right)k_t\right\} + \ldots \\ +\gamma Nm\left(f\left(k_t\right) - f^{'}\left(k_t\right)k_t, f^{'}\left(k_{t+1}\right), E_t\right) \end{aligned} \tag{2.34}$$

Equations (2.33) and (2.34) form a system of nonlinear first-order difference equations that describes the dynamics of the capital accumulation and the evolution of the environmental quality along the competitive equilibrium path of the model. The solution of this system characterizes the competitive equilibrium path for capital and environmental quality $\{k_t^*, E_t^*\}_{t=0}^{\infty}$. Once these paths are known, $\left\{c_t^*, d_{t+1}^*, s_t^*, w_t^*, r_t^*\right\}$ can be obtained by using (2.12) - (2.26).

The law of capital accumulation (2.33) can be written as

$$k_{t+1} - s_t\left(f\left(k_t\right) - f^{'}\left(k_t\right)k_t, f^{'}\left(k_{t+1}\right), E_t\right) \equiv \Psi\left(E_t, k_t, k_{t+1}\right) = 0 \tag{2.35}$$

Similarly, the evolution of the environmental quality can be defined as

$$E_{t+1} \equiv \Omega\left(E_t, k_t, k_{t+1}\right) \tag{2.36}$$

Studying the dynamics of the capital accumulation and the evolution of the environmental quality along the competitive equilibrium path of the model might be very complicated. Indeed, as Zhang (1999) suggests, there are at least two reasons to believe that transitions in environmental growth models may be rather complex. Firstly, although these models resemble the otherwise standard growth models, where the concept of an environmentally sustainable state corresponds to that of a steady state, in the models dealing with environmental issues there exist two opposing forces that balance each other towards the sustainable level: on one hand, economic agents' consumption activities hurt the process; whereas, on the other hand, their commitments to environmental preservation can aid the process. The interplay of these two effects, which is generally absent in traditional growth models, may give rise to nontrivial dynamics. Secondly, the steady-state analysis is based on a hypothesis that the sustainable level is achievable. Thus, in these models, convergence eventually prevails. However, studies in nonlinear dynamics suggest that, given the above two effects, the convergence prediction appears oversimplified. In particular, bounded equilibrium paths that never converge to the steady state may arise.

Under plausible assumptions on the functional forms of the utility and production functions, Zhang (1999) finds that the model developed by John and Pecchenino (1994) can produce perfect foresight equilibria, in which the level of environmental quality and other endogenous variables fluctuate forever.[19]

In the remainder of this Section we will study the possibility of a sustainable long-run growth. In order to simplify our analysis, we make a technical hypothesis on the functional form of the utility. Following Zhang (1999)'s analysis, we define the following elasticity parameters as

$$\eta_c \equiv \frac{E}{c}\frac{v'_E(d,E)}{u'(c)} > 0 \tag{2.37}$$

$$\eta_d \equiv \frac{E}{d}\frac{v'_E(d,E)}{v'_d(d,E)} > 0 \tag{2.38}$$

where we assume that they are *constant* and identical across agents.[20] These parameters represent the agents' attitude towards preserving the environments when young and old, respectively.

An intuitive example of an utility function that satisfies this hypothesis is

$$Z = (1/\eta_c)\ln c + \delta[(1/\eta_d)\ln d + \ln E]$$

[19]Zhang (1999) finds that, when agents' concerns towards greener preferences and the maintenance efficiency relative to degradation are not sufficiently high, cyclically or chaotically fluctuating equilibria are more likely to exist. These findings suggest that the dynamics towards the sustainable state may be complex and government policy can be implemented to induce a smooth convergence.

[20]Notice that, under this assumption, the savings function is independent of the return on savings, that is there is no substitution effect.

where $1/\eta_c$ and $1/\eta_d$ can be thought of as the weights attached by the household to consumption c and d relative to E.

Under assumptions (2.37) - (2.38) and the hypothesis that $m > 0$, the first-order conditions of the household's maximization problem (2.18) and (2.19) follows

$$E_{t+1} = \delta(\gamma + \beta)\eta_c c_t \tag{2.39}$$

$$(1 + r_{t+1})E_{t+1} = \gamma\eta_d d_{t+1} \tag{2.40}$$

These two equations state that there exists a fixed proportion between the consumption level of the representative household and the index of environmental quality.

Equation (2.40) can be solved for d_{t+1}

$$d_{t+1} = \frac{E_{t+1}(1 + r_{t+1})}{\gamma\eta_d} \tag{2.41}$$

By bearing in mind the savings market clearing condition (2.8), we can define a relation between k_{t+1} and E_{t+1}

$$k_{t+1} = \frac{E_{t+1}}{\gamma\eta_d} \tag{2.42}$$

Now, by getting rid of m_t from the budget constraint (2.12) and substituting it along with (2.39) and (2.42) (lagged one period) into the environmental law of motion (2.6), we obtain

$$m_t = \left[f(k_t) - k_t f^{'}(k_t)\right] - \frac{E_{t+1}}{\gamma\eta_d} - \frac{E_{t+1}}{\delta(\gamma + \beta)\eta_c} \tag{2.43}$$

and

$$\begin{aligned} E_{t+1} = \quad & (1-b)E_t - \beta N \left[\tfrac{E_{t+1}}{\delta(\gamma+\beta)\eta_c} + \tfrac{(1+r_t)E_t}{\gamma\eta_d}\right] + \\ & + \gamma N \left[f(k_t) - f^{'}(k_t)k_t - \tfrac{E_{t+1}}{\gamma\eta_d} - \tfrac{E_{t+1}}{\delta(\gamma+\beta)\eta_c}\right] \end{aligned} \tag{2.44}$$

By rearranging (2.44) we get

$$\begin{gathered} E_{t+1}\left[1+\tfrac{\beta N}{\delta(\gamma+\beta)\eta_c}+\tfrac{\gamma N}{\delta(\gamma+\beta)\eta_c}+\tfrac{N}{\eta_d}\right]= \\ (1-b)E_t-\beta N(f^{'}(k_t)k_t)+\gamma N\left[f(k_t)-f^{'}(k_t)k_t\right] \end{gathered} \tag{2.45}$$

$$E_{t+1}\left[1+\frac{N}{\delta\eta_c}+\frac{N}{\eta_d}\right]=(1-b)E_t-\beta N(f^{'}(k_t)k_t)+\gamma N\left[f(k_t)-f^{'}(k_t)k_t\right] \tag{2.46}$$

We define the capital's share of output as $\alpha(k_t)=k_tf^{'}(k_t)/f(k_t)$. To further simplify the analysis, we assume that the parameter α is *constant*. Under this assumption, (2.45) reduces to

$$E_{t+1}\left[\frac{\delta\eta_c\eta_d+N\eta_d+\delta N\eta_c}{\delta\eta_c\eta_d}\right]=(1-b)E_t+Nf(k_t)\left[\gamma(1-\alpha)-\beta\alpha\right] \tag{2.47}$$

As a consequence, the dynamic equilibrium is described by the following nonlinear first-order difference equation

$$E_{t+1}=\frac{\delta\eta_c\eta_d}{\delta\eta_c\eta_d+N\eta_d+\delta N\eta_c}\left\{(1-b)E_t+Nf(k_t)\left[\gamma(1-\alpha)-\beta\alpha\right]\right\} \tag{2.48}$$

Obviously, some general forms of production function satisfy this assumption; for our study we use a Cobb-Douglas function. By substituting $f(k)=Ak^{\alpha}$ into (2.48) and by using the relationship found in (2.42), we obtain

$$E_{t+1}=\frac{\delta\eta_c\eta_d}{\delta\eta_c\eta_d+N\eta_d+\delta N\eta_c}\left\{(1-b)E_t+AN\left[\gamma(1-\alpha)-\beta\alpha\right]\left(\frac{E_t}{\gamma\eta_d}\right)^{\alpha}\right\} \tag{2.49}$$

$$E_{t+1}=a_0E_t+a_1(E_t)^{\alpha}\equiv G(E_t,\eta_d,\eta_c,N,\beta,\gamma,b,\alpha) \tag{2.50}$$

where the two constant coefficients are defined as

$$a_0=\frac{\delta\eta_c\eta_d(1-b)}{\delta\eta_c\eta_d+N\eta_d+\delta N\eta_c}$$

and

$$a_1 = \frac{AN\delta\eta_c\eta_d^{1-\alpha}[\gamma(1-\alpha)-\beta\alpha]}{\gamma^\alpha[\delta\eta_c\eta_d + N\eta_d + \delta N\eta_c]}$$

Now we look at the behaviour of a_0 and a_1 as N goes to infinity

$$\lim_{N\to\infty} a_0 = 0 \tag{2.51}$$

and

$$\lim_{N\to\infty} a_1 = \frac{A\delta\eta_c\eta_d^{1-\alpha}[\gamma(1-\alpha)-\beta\alpha]}{\gamma(\eta_d + \delta\eta_c)} \tag{2.52}$$

It means that, as N goes to infinity, the equation $E_{t+1}(E_t)$ can be graphically represented in the space E_t-E_{t+1} as a concave curve passing through the origin.

By bearing in mind the relation between environment and capital as shown in (2.42), we can derive from (2.50) a dynamic equation for the evolution of k.

$$\gamma\eta_d k_{t+1} = a_0(\gamma\eta_d k_t) + a_1(\gamma\eta_d k_t)^\alpha \tag{2.53}$$

Since the environment and capital are related throughout a linear equation that holds for all t in (2.42), the behaviour of $k(t)$ can be deduced from that of $E(t)$. Hence, as $N \to \infty$, $k_{t+1}(k_t)$ can be represented as a concave curve passing through the origin.

a_0 is always larger than or equal to zero, whereas a_1 is of ambiguous sign; moreover, it can be readily verified that $a_0 < 1$. These properties open up the possibility of a rich dynamic behaviour. In particular, the existence of a non-trivial steady state crucially depends on the sign of a_1. A necessary condition for a_1 to be positive is: $\gamma > \beta[\alpha/(1-\alpha)]$. For this to hold, either better maintenance technologies (higher γ) or lower environmental degradation by consumption (lower β) are required. Put it differently, in order to have a growth path that converges to a non-trivial steady state, maintenance efficiency relative to degradation must be sufficiently high. Conversely, neither agents' attitude towards the environment (parameters η_c and η_d) nor the size of the economy seem to affect the *existence* of the steady state.

It is also of interest to study the *stability properties* of the long-run equilibrium. By exploiting the linear relationship between E and k highlighted in eq. (2.42) the stability of the long-run equilibrium can be studied by solving (2.50).
It is easy to show that there are two steady states. The first one is trivial, that is

$$(\overline{E}_0, \overline{k}_0) = (0, 0)$$

The second one is readily found after some computations.

$$\overline{E}_1 = \left(\frac{a_1}{(1-a_0)}\right)^{\frac{1}{1-\alpha}}$$

Hence, the non trivial fixed point of the induced map is

$$(\overline{E}_1, \overline{k}_1) = \left(\left(\frac{a_1}{(1-a_0)}\right)^{\frac{1}{1-\alpha}}, \left(\frac{a_1/(\gamma\eta_d)^{1-\alpha}}{(1-a_0)}\right)^{\frac{1}{1-\alpha}}\right)$$

The stability properties of the long-run equilibrium can be studied easily by proving that the partial derivative of E_{t+1} evaluated at the steady state has modulus less than one. That is

$$|a_0(1-\alpha) + \alpha| < 1 \tag{2.54}$$

and it is easily shown that this is true. Actually we have already shown that a_0 is always positive, hence we can write (2.54) as

$$a_0(1-\alpha) + \alpha < 1 \tag{2.55}$$

after some computation (2.55) reduces to

$$a_0 < 1$$

but this is always true, as we have proved previously.
Hence we can conclude that, under the assumptions we have made on the

functional form of the utility and production function and the size of the relevant parameters γ and β, the system converges to a long-run equilibrium which is locally stable.

2.5 The steady state

Before studying the steady state in a more general framework, we characterize the comparative static behaviour of the steady state we have found under the particular hypotheses made on the utility and production functions. For the sake of brevity, and exploiting the relation found in eq (2.42), only the comparative static behaviour of $\overline{E}$ will be studied.

Let us write (2.50) as

$$\overline{E} = \left(\frac{\frac{AN\delta\eta_c\eta_d^{1-\alpha}[\gamma(1-\alpha)-\beta\alpha]}{\gamma^\alpha[\delta\eta_c\eta_d+N\eta_d+\delta N\eta_c]}}{\frac{\delta\eta_c\eta_d+N\eta_d+\delta N\eta_c-\delta\eta_c\eta_d+b\delta\eta_c\eta_d}{\delta\eta_c\eta_d+N\eta_d+\delta N\eta_c}} \right)^{\frac{1}{1-\alpha}} \tag{2.56}$$

After some computations this reduces to

$$\overline{E} = \left(\frac{AN\delta\eta_c\eta_d^{1-\alpha}[\gamma(1-\alpha)-\beta\alpha]}{\gamma^\alpha[N\eta_d+\delta N\eta_c+b\delta\eta_c\eta_d]} \right)^{\frac{1}{1-\alpha}} = \Omega^{\frac{1}{1-\alpha}} \tag{2.57}$$

$$\text{where } \Omega = \left(\frac{AN\delta\eta_c\eta_d^{1-\alpha}[\gamma(1-\alpha)-\beta\alpha]}{\gamma^\alpha[N\eta_d+\delta N\eta_c+b\delta\eta_c\eta_d]} \right)$$

Now we characterize the comparative static behaviour of (2.57).

Proposition 1 *Economies with better destruction technologies (higher γ) have a better environmental quality in steady state and more capital than economies with worse technologies.*[21]

Proof: An economy's destruction technology is better if γ is higher. From eq. (2.57) characterizing the steady state, we have

[21] Changes in the productivity of the destruction technology are naturally irrelevant in any zero-destruction equilibrium.

$$\begin{aligned}\frac{\partial \overline{E}}{\partial \gamma} = \frac{1}{1-\alpha}\Omega^{\frac{\alpha}{1-\alpha}} \Bigg(& \frac{AN\delta\eta_c\eta_d^{1-\alpha}(1-\alpha)[\gamma^\alpha(N\eta_d+\delta N\eta_c+b\delta\eta_c\eta_d)]}{[\gamma^\alpha(N\eta_d+\delta N\eta_c+b\delta\eta_c\eta_d)]^2}+ \\ & -\frac{\alpha\gamma^{\alpha-1}(N\eta_d+\delta N\eta_c+b\delta\eta_c\eta_d)[AN\delta\eta_c\eta_d^{1-\alpha}(\gamma(1-\alpha)-\beta\alpha)]}{[\gamma^\alpha(\eta_d+\delta\eta_c+b\delta\eta_c\eta_d)]^2}\Bigg)\end{aligned} \tag{2.58}$$

For (2.58) to be positive we have to set some conditions on the numerator, in particular, we have to impose

$$(1-\alpha)\gamma^\alpha - \alpha\gamma^{\alpha-1}[\gamma(1-\alpha)-\beta\alpha] > 0$$

By simplifying and rearranging this expression we obtain

$$\alpha^2\beta\gamma^{\alpha-1} > 0$$

which is always satisfied under our hypotheses.

Ceteris paribus, an economy with a more productive maintenance technology can devote less to maintenance to achieve a given environmental quality. These resources can be allocated to capital accumulation, thus sustaining at higher levels both the environment and capital. This result mimics that found by John et al. (1995).

■

Proposition 2 *More wasteful economies (higher β) accumulate less capital and have a worse environmental quality.*

Proof: Let us consider eq. (2.57).

$$\frac{\partial \overline{E}}{\partial \beta} = \frac{1}{1-\alpha}\Omega^{\frac{\alpha}{1-\alpha}} \left(\frac{-AN\alpha\delta\eta_c\eta_d^{1-\alpha}[\gamma^\alpha(N\eta_d+\delta N\eta_c+b\delta\eta_c\eta_d)]}{[\gamma^\alpha(N\eta_d+\delta N\eta_c+b\delta\eta_c\eta_d)]^2} \right) \tag{2.59}$$

Since, under the assumptions we have made on the parameters of the model, the denominator of (2.59) is positive and the numerator is negative, we can state that the marginal effect of β on the steady state value $\overline{E}$ is negative.

If each unit of consumption causes greater environmental degradation, then the young must devote more resources to environmental maintenance, leaving fewer resources for saving and consumption. This result goes in the same direction of that obtained by John et al. (1995) but differs, for instance, from that obtained by Gutierrez (2004). In her model, pollution makes agents incur health costs when they are elderly. Hence, in that case, a greater environmental degradation implies larger health costs that make agents save more for old age and accumulate more capital. ■

Proposition 3 : *Economies with higher values of speed of adjustment (higher b) accumulate less capital and have worse environmental quality.*

Proof: Let us consider eq. (2.57).

$$\frac{\partial \overline{E}}{\partial b} = \frac{1}{1-\alpha}\Omega^{\frac{\alpha}{1-\alpha}}\left(\frac{-\gamma^{\alpha}\delta\eta_c\eta_d[AN\delta\eta_c\eta_d^{1-\alpha}[\gamma(1-\alpha)-\beta\alpha]]}{[\gamma^{\alpha}(N\eta_d+\delta N\eta_c+b\delta\eta_c\eta_d)]^2}\right) \tag{2.60}$$

It is readily seen that the denominator of (2.60) is always positive while the numerator is negative: hence, the marginal effect of b on $\overline{E}$ is negative.

Ceteris paribus, a lower rate of natural purification, $(1-b)$, implies a lower future environmental quality for any given stock of capital, and therefore the young must face a higher maintenance investment to keep the environmental quality constant. This larger cost makes agents save less and this in turn implies that the society accumulates less capital and degrades the environment more. John et al. (1995) find a similar result.

■

Now let us consider the effects of the elasticity parameters on the steady state value.[22]

Proposition 4 : *Economies with higher values of the elasticity parameter η_c accumulate more capital and have a better environmental quality.*

Proof: Consider eq. (2.57).

[22] To our knowledge, there is no paper which addresses this issue.

$$\frac{\partial\overline{E}}{\partial\eta_c} = \frac{1}{1-\alpha}\Omega^{\frac{\alpha}{1-\alpha}}\left(\frac{AN\delta\eta_d^{1-\alpha}[\gamma(1-\alpha)-\beta\alpha][\gamma^{\alpha}(N\eta_d+\delta N\eta_c+b\delta\eta_c\eta_d)]}{[\gamma^{\alpha}(N\eta_d+\delta N\eta_c+b\delta\eta_c\eta_d)]^2}+ -\frac{\gamma^{\alpha}\delta(N+b\eta_d)(AN\delta\eta_c\eta_d^{1-\alpha}[\gamma(1-\alpha)-\beta\alpha])}{[\gamma^{\alpha}(N\eta_d+\delta N\eta_c+b\delta\eta_c\eta_d)]^2}\right) \tag{2.61}$$

(2.61) is positive if

$$(N\eta_d + \delta N\eta_c + b\delta\eta_c\eta_d) - \delta N\eta_c - b\delta\eta_c\eta_d > 0$$

and this holds if $N\eta_d > 0$, which is always satisfied under our assumption on η_d.

A larger value of η_c implies that agents prefer to substitute consumption in the first period of life with environmental quality during the second period. On one hand, if consumption is lower, then the negative effect of consumption on the environment is lower and the environmental quality is higher, *ceteris paribus*. On the other hand, if the level of consumption during the first period is lower, then savings are higher and the economy accumulates more capital. ■

Proposition 5 *Economies with higher values of the elasticity parameter η_d accumulate less capital, while the effect on the environmental quality is positive if and only if $\eta_d < \frac{\delta N\eta_c(1-\alpha)}{\alpha(N+b\eta_c)}$.*

Proof: Consider eq. (2.57).

$$\frac{\partial\overline{E}}{\partial\eta_d} = \frac{1}{1-\alpha}\Omega^{\frac{\alpha}{1-\alpha}}\left(\frac{AN\delta\eta_c(1-\alpha)\eta_d^{-\alpha}[\gamma(1-\alpha)-\beta\alpha][\gamma^{\alpha}(N\eta_d+\delta N\eta_c+b\delta\eta_c\eta_d)]}{[\gamma^{\alpha}(N\eta_d+\delta N\eta_c+b\delta\eta_c\eta_d)]^2} -\frac{\gamma^{\alpha}(N+b\delta\eta_c)(AN\delta\eta_c\eta_d^{1-\alpha}[\gamma(1-\alpha)-\beta\alpha])}{[\gamma^{\alpha}(N\eta_d+\delta N\eta_c+b\delta\eta_c\eta_d)]^2}\right) \tag{2.62}$$

for (2.62) to be positive we must check

$$(1-\alpha)\eta_d^{-\alpha}(N\eta_d + \delta N\eta_c + b\delta\eta_c\eta_d) - \eta_d^{1-\alpha}(N - b\delta\eta_c) > 0 \tag{2.63}$$

After some computations, we get

$$-\alpha\eta_d(N + b\delta\eta_c) + \delta N\eta_c(1-\alpha) > 0$$

Which holds if $\eta_d < \frac{\delta N \eta_c (1-\alpha)}{\alpha(N+b\eta_c)}$.

If the value of η_d is sufficiently low, then the households prefer to consume less in the second period, which means that savings are lower and the economy accumulates less physical capital. At the same time, the effect on the environment is ambiguous because, *a priori*, we cannot say if the households prefer more consumption in the first period or more environmental quality in the second period. Put it differently, if the second-period consumption decreases (due to a higher η_d), then the households might decide to consume more during the first period or to invest in environmental quality, or both. In particular, this decision is related to the value of η_c: if η_c is high, then they prefer to invest in environmental quality. Hence, we can say that the effect on the environment is positive only if η_d is sufficiently smaller than η_c. ■

Proposition 6 *Economies with higher values of N accumulate more capital and have a better environmental quality.*

Proof: Consider eq. (2.57).

$$\frac{\partial \overline{E}}{\partial N} = \frac{1}{1-\alpha}\Omega^{\frac{\alpha}{1-\alpha}} \left(\frac{A\delta\eta_c\eta_d^{1-\alpha}[\gamma(1-\alpha)-\beta\alpha][\gamma^\alpha(N\eta_d+\delta N\eta_c+b\delta\eta_c\eta_d)]}{[\gamma^\alpha(N\eta_d+\delta N\eta_c+b\delta\eta_c\eta_d)]^2} - \frac{\gamma^\alpha(\eta_d+\delta\eta_c)(AN\delta\eta_c\eta_d^{1-\alpha}[\gamma(1-\alpha)-\beta\alpha])}{[\gamma^\alpha(N\eta_d+\delta N\eta_c+b\delta\eta_c\eta_d)]^2} \right) \tag{2.64}$$

In order to determine the marginal effect of N on $\overline{E}$ we have to check whether the following inequality holds

$$N(\eta_d + \eta_c) + b\delta\eta_c\eta_d > N(\eta_c + \eta_d)$$

which turns out to be true $\forall b > 0$.

An increase in the number of agents raises total consumption and total environmental expenditures. The former effect is harmful, whereas the latter is beneficial. Equation (2.64) states that the net effect is positive. This is because we are considering the case where $\gamma > [\beta\alpha/(1-\alpha)]$, that is γ is sufficiently higher than β.

This result can be shown also by computing the limit value of the steady state for $N \to 1$ and $N \to \infty$.

Let us compute the limit of $\overline{E}$, $\overline{k}$ as $N \to 1$.

$$\lim_{N\to 1} \overline{E} = \left(\frac{A\delta\eta_c\eta_d^{1-\alpha}[\gamma(1-\alpha)-\beta\alpha]}{\gamma^{\alpha}[\eta_d+\delta\eta_c+b\delta\eta_c\eta_d]} \right)^{\frac{1}{1-\alpha}} \tag{2.65}$$

Now we compute the limits as $N \to \infty$.

$$\lim_{N\to \infty} \overline{E} = \left(\frac{A\delta\eta_c\eta_d^{1-\alpha}[\gamma(1-\alpha)-\beta\alpha]}{\gamma^{\alpha}[\eta_d+\delta\eta_c]} \right)^{\frac{1}{1-\alpha}} \tag{2.66}$$

As we can see, compared to the case of $N \to 1$, as $N \to \infty$ the economy experiences a steady state with more capital accumulation and a better environmental quality. ■

Now we study the steady state equilibrium in the more general framework suggested by John and Pecchenino (1994). Let us start by providing the definition of steady-state equilibrium.

Definition 2 A *steady-state* equilibrium in this OLG economy is established by an intertemporal equilibrium that implies the two following sequences: $\{k\}_{t=0}^{\infty} = \{k\}$ and $\{E\}_{t=0}^{\infty} = \{E\}$.

A competitive equilibrium allocation in a steady state is a quintuple **CE**$=$ $\{c; d; m; k; E\}$ characterized by (2.67) to (2.71)

$$u'(c) = \delta\left(\beta+\gamma\right) v'_E(d, E) \tag{2.67}$$

$$v'_d(d, E) = \frac{\gamma}{f'(k)} v'_E(d, E) \tag{2.68}$$

$$f(k) - kf'(k) = c + k + m \tag{2.69}$$

$$d = kf'(k) \tag{2.70}$$

$$E = N\left[\frac{\gamma}{b}m - \frac{\beta}{b}(c+d)\right] \tag{2.71}$$

Equation (2.67) states that, in the case of $m > 0$, the representative agent chooses consumption when young in order to equate the marginal rate of substitution between consumption in youth and environmental quality in old age, that is $u'(c)/\delta v'_E(d, E)$, with the marginal rate of transformation, that is $(\beta + \gamma)$. At the utility maximum, a decrease in utility due to falling consumption during youth is equal to an increase in utility due to the sum of an increase in maintenance effort, γ, and a decrease in consumption externality, β. Equation (2.68) gives us a simple arbitrage between the rate of return on the private savings, $f'(k)$, and the rate of return on the investment in environmental quality, γ. This condition states that, in the case of $m > 0$, the representative household chooses savings in order to equate the marginal rate of substitution between consumption in old age and environmental quality in old age, $v'_d(d, E)/v'_E(d, E)$, with the marginal rate of transformation, $\gamma/f'(k)$. At the utility maximum, a decrease in utility due to falling consumption during old age, $f'(k)$, is equal to an increase in utility due to an increase in maintenance effort, γ.

2.6 The social optimum

The decentralized steady-state economy summarized by (2.67) to (2.71) shows two kinds of market failures, also called intragenerational and intergenerational externalities. It is well known from economic theory (see Baumol and Oates (1988)) that the presence of externalities hinders the efficiency of the market mediated outcome.[23] In this framework, taxes and transfers would be effective in internalizing both kinds of externalities (see Atkinson and Sandmo (1980)). In order to evaluate the normative role of fiscal policies, it is useful to investigate the first-best solution of this economy where agents are identical.

Hence, in this Section a socially optimal steady-state allocation chosen by a central planner will be analyzed.
For all $t \geq 0$, the social planner chooses c_t, d_t, k_t, m_t (and implicitly E_t), where k_0 and E_{-1} are given. He takes into account the resource constraint (2.7) and the environmental constraint (2.6) in each period t.

Following Jouvet et al. (2000) we assume that the central planner adopts a utilitarian social welfare function, W, consisting of the discounted sum of

[23] As we will show later on, in the economy under analysis the consumption level is too high, while the environmental quality (or the voluntary contribution to improve the environment) is too low.

individuals' lifetime utilities.

$$W = \sum_{t=0}^{\infty} \rho^t \left\{ u\left(c_t\right) + \delta v\left(d_{t+1}, E_{t+1}\right) \right\} \tag{2.72}$$

where $0 < \rho < 1$ denotes the social discount factor.

Then the planner's optimization problem is

$$\max_{c_t, d_t, k_t, m_t, E_t} L = \sum_{t=-1}^{\infty} \rho^t \left\{ \left[U(c_t) + \delta V(d_{t+1}, E_{t+1}) \right] + \dots \right. \\ \left. + \lambda_t \left(f\left(k_t\right) - c_t - d_t - k_{t+1} - m_t \right) + \mu_t \left(E_{t+1} - (1-b)E_t + \beta N \left(c_t + d_t\right) - \gamma N m_t \right) \right\} \tag{2.73}$$

where k_0 and E_{-1} are exogenously given. In (2.73) λ and μ are the current shadow prices of k and E respectively.

We first consider the first-order condition with respect to k_{t+1}

$$-\rho^t \lambda_t + \rho^{t+1} \lambda_{t+1} f'(k_{t+1}) = 0 \tag{2.74}$$

From (2.74) we obtain λ_{t+1} as a function of λ_t

$$\lambda_{t+1} = \frac{\lambda_t}{\rho f'(k_{t+1})} \tag{2.75}$$

In a similar way, we compute the first-order condition with respect to m_t

$$-\lambda_t = \gamma N \mu_t \tag{2.76}$$

by plugging (2.76) into (2.74) for t and $t+1$, we obtain

$$\rho^t \gamma \mu_t = \rho^{t+1} \gamma \mu_{t+1} f'(k_{t+1}) \tag{2.77}$$

which gives us m_{t+1} as a function of m_t. By simplifying equation (2.77) we get

$$\mu_{t+1} = \frac{\mu_t}{\rho f'(k_{t+1})} \tag{2.78}$$

We now compute the first-order condition with respect to the environmental variable E

$$\begin{aligned} \delta v'_{t+2,E}(d_{t+2}, E_{t+2}) &= \mu_{t+2}(1-b) - \mu_{t+1} \quad (2.79) \\ \delta v'_{t+1,E}(d_{t+1}, E_{t+1}) &= \mu_{t+1}(1-b) - \mu_{t} \quad (2.80) \end{aligned}$$

and we derive the following relationship between $v'_{t+1,E}(d_{t+1}, E_{t+1})$ and $v'_{t+2,E}\,(d_{t+2}, E_{t+2})$, as shown in equation (2.81)

$$\frac{v'_{t+1,E}(d_{t+1}, E_{t+1})}{v'_{t+2,E}(d_{t+2}, E_{t+2})} = \frac{1-b-f'\,(k_{t+2})}{\rho f'\,(k_{t+2})\,[1-b-f'(k_{t+1})]} \quad (2.81)$$

By computing the first-order condition for c_t we obtain

$$\rho^t\,(u'(c) - \lambda_t + \beta N \mu_t) = 0 \quad (2.82)$$

Moreover, by exploiting the relation we have found in equation (2.76), we can rewrite (2.82) as follows

$$u'(c) = N\,(\beta+\gamma)\,\mu_t = N\delta\,(\beta+\gamma)\,\frac{f'\,(k_{t+1}) v'_{t+1,E}(d_{t+1}, E_{t+1})}{1-b-f'(k_{t+1})} \quad (2.83)$$

The optimal condition with respect to d_{t+1} amounts to

$$\left(\frac{\delta}{\rho} v'_{t+1,d}(d_{t+1}, E_{t+1}) - \lambda_t + \beta N \mu_t\right) = 0 \quad (2.84)$$

By rearranging and substituting for λ_t, we get

$$v'_{t+1,d}(d_{t+1}, E_{t+1}) = \frac{\rho N\,(\beta+\gamma)}{\delta}\mu_t = \frac{\rho N\,(\beta+\gamma)}{\delta}\frac{\delta f'\,(k_{t+1}) v'_{t+1,E}(d_{t+1}, E_{t+1})}{1-b-f'(k_{t+1})} \quad (2.85)$$

By comparing (2.83) with (2.19) and (2.85) with (2.18), we can readily notice that they differ from the ones we have found in the decentralized context. Indeed, the social planner, in deciding the optimal allocation of capital, takes into account the negative effect of consumption on the environmental quality and, as a consequence, on the utility function of future generations. It means that, in the decentralized economy, each generation tends to consume more than the social desirable level implied by (2.83) and (2.85).

We now characterize the social optimum in the steady state. Eq. (2.81) becomes

$$\frac{v'_E(d,E)}{v'_E(d,E)} = \frac{1-b-f^{'}(k)}{\rho f'(k)\left[1-b-f'(k)\right]} \tag{2.86}$$

From this, it is easy to derive:

$$f^{'}(k) = \frac{1}{\rho} \tag{2.87}$$

By evaluating these conditions and the restrictions on the problem in the steady state, we can define

Definition 3 The *social planner solution* for a given discount rate ρ is a quintuple $\mathbf{SO} = \{c, d, k, m, E\}$ characterized by

$$u'(c) = \frac{\delta N(\beta+\gamma)}{\left[1-\rho(1-b)\right]} v'_E(d,E) \tag{2.88}$$

$$v'_d(d,E) = \frac{N(\beta+\gamma)}{\frac{1}{\rho}\left[1-\rho(1-b)\right]} v'_E(d,E) \tag{2.89}$$

$$f^{'}(k) = \frac{1}{\rho} \tag{2.90}$$

$$f(k) = k + c + d + m \tag{2.91}$$

$$E = N\left[\frac{\gamma}{b}m - \frac{\beta}{b}(c+d)\right] \tag{2.92}$$

Equations (2.88) and (2.89) are the first-order conditions and determine, together with the resource constraint, consumption c and d. Equation (2.91) represents the resource constraint of the economy. Equation (2.90) governs the optimum level of k and (2.92) determines the environmental quality steady-state.

A comparison of the two first-order conditions between the competitive equilibrium (**CE**) and the social optimum (**SO**) allows us to identify the externalities springing from consumption and environmental investment. Let us then compare (2.67) with (2.88).

From (2.88) we have the optimal marginal rate of substitution between E and c in the **SO**

$$\frac{v'_E(d,E)}{u'(c)} = \frac{[1-\rho\,(1-b)]}{\delta N\,(\beta+\gamma)} \tag{2.93}$$

Notice that, in the decentralized market economy, the marginal rate of substitution between E and c is given by

$$\frac{v'_E(d,E)}{u'(c)} = \frac{1}{\delta\,(\beta+\gamma)} \tag{2.94}$$

which is greater than (2.93), since $\rho > 0$ and $N > 1$. In the *laissez-faire* economy c is too high and E is too small, due to intragenerational externalities ($N > 1$) and intergenerational externalities ($\rho(1-b) > 0$).

Put it differently, when young, the representative household takes into account the impact of his own consumption on the environment when elderly, that is the effect of β on E. He also bears in mind the positive effect γ of his own maintenance investment. However he does not consider the effect that his current decisions have on the welfare of the other $N-1$ young agents alive at time t. Moreover, due to his finite lifetime and his selfish behaviour, he also does not consider his consumption's impact on the environment for periods after his death. In eq. (2.88) these two different externalities are corrected by multiplying the term $v'_E(\beta+\gamma)$ of eq. (2.67) by the factor $N/[1-\rho(1-b)]$.

From (2.88) we can say that the consumption level chosen by the social planner is lower than in the competitive equilibrium (that is the marginal utility of consumption is higher), because the intra- and intergenerational effects of c on E are taken into account. This condition does not appear in John and Pecchenino (1994), because they assume that households do not consume at all during the first period.
Moreover, eq. (2.88) tells us that the smaller is the autonomous regeneration rate b, the higher is the intergenerational externality. In particular, if we set $b = 1$, then the long-run effects of the environment on future generations do not exist and the social planner does not need to correct the intergenerational externality, but as $N > 1$ the intragenerational externality still exists and must be corrected. Finally, if $N = 1$ and $b = 1$, then (2.88) reduces to (2.67).

Let us compare eq. (2.68) with eq. (2.89). In the steady state, we can derive from (2.89) the optimal marginal rate of substitution between E and d in the **SO**

$$\frac{v'_E(d,E)}{v'_d(d,E)} = \frac{\frac{1}{\rho}\left[1-\rho\left(1-b\right)\right]}{N\left(\beta+\gamma\right)} \tag{2.95}$$

Whereas in the **CE**, the marginal rate of substitution between E and d is given by

$$\frac{v'_E(d,E)}{v'_d(d,E)} = \frac{\frac{1}{\rho}}{\gamma} \tag{2.96}$$

which is greater than (2.95) since the old generation does not take into account the intra- and intergenerational externalities of the model and tends to consume too much from a social optimum point of view. Indeed, a comparison between (2.68) and (2.89) allows us to notice that the elderly do not think at all about the impact of their consumption on the environment, because they are not alive in the next period and hence they do not suffer from a low environmental quality. Put it differently, like the young, the old households do not consider the intragenerational effect of their own consumption on the other $N-1$ agents of their cohort. Moreover, not only they do not care about the effect of their consumption on the environmental quality at time $t+2$, $t+3$, ..., $t+n$, but they also fail to internalize the environmental externality at time $t+1$: indeed, in (2.68) the term β does not appear. Because of this *short-run* intergenerational externalitiy, even when $b=1$ and $N=1$ equations (2.68) with (2.89) do not coincide. Once again, the smaller is the value of b and the higher is N, the higher is the externality to be corrected.

Unlike the households, the social planner internalizes both the negative intragenerational effect of consumption on the environment, that is $N\beta$, and the long-run effect, $1/[1-\rho(1-b)]$. This condition is similar to that found by John and Pecchenino (1994), wherein the environment is deteriorated by consumption of the old and the central planner treats each generation symmetrically. Nevertheless our condition differs from theirs because we consider not only the intergenerational externality, as they do, but also the intragenerational one arising from the non-cooperative behaviour of each agent.

We have clearly shown that the **CE** solution differs from the **SO**, because each household, in solving his optimization problem, does not take into account the effects that his own decisions have on the welfare of current and future generations. Hence, in this framework Pareto improving policies are possible.

Recent literature has focused on optimal tax schemes that internalize environmental externalities in order to attain an efficient allocation of resources in the decentralized economy. For instance, John et al. (1995) assume that the long-lived planner taxes the net return on capital and wages in order to internalize the intergenerational externality. Ono (2003), by using a model in which the environment is negatively affected by production, shows that there are different schemes that can be used to reach the social optimum. Gutierrez (2004), in a model in which the environment is degraded by pollution stemming from the production activity and agents bear health costs when elderly, designs two different fiscal policies in order to put the economy into the golden rule allocation: one based on production taxes and the other one based on capital and wages taxes. By considering consumers as the only citizens from the electoral point of view, she shows that young and elderly agents pay more taxes in the capital-wage system than in the production tax scheme.

Rangel (2003) shows that it is possible to improve the environmental quality for future generations by creating a link between the investment in environmental maintenance by present generation and a pay-as-you-go security system. The main reason why this tool can work is that the contribution to the social security system can be used by future generations as a threat to force present generations to invest in environmental quality.

In the next Section we shall characterize optimal fiscal policies that succeed in carrying the economy to the social optimum. From the welfare point of view, young and elderly agents are indifferent as to which tax scheme they prefer.

2.7 Tax schemes

In this Section, we study how to implement optimal tax schemes in order to achieve efficient allocations for economies whose competitive equilibrium is dynamically inefficient. We first present the general framework, that is the policy instruments that the social planner can choose in order to correct environmental externalities.

In an insightful article, Ono (1996) investigates two optimal tax programmes and proves that the externality caused by consumption can be internalized either by a consumption tax program with a consumption tax-rate higher on the old than on the young, or by a tax programme with a uniform consumption tax plus a capital income tax. This Section is an extension of Ono's work (1996). In particular, if we set $\rho = 1$ and $N = 1$, then Ono's optimal tax schemes would follow.

We assume that the social planner can implement: *i*) a tax on consumption conditioned on age, *ii*) a tax on savings and consumption or *iii*) a subsidy on maintenance investment combined with a tax on savings. Before deriving and discussing these three different cases in the coming subsections, we set up the lifetime budget constraint of the consumer in the more general framework.

The budget constraints of the young and the old become respectively

$$c_t(1+\tau_c) + s_t + (1-\tau_m)m_t = w_t - T_t \tag{2.97}$$

$$d_{t+1}(1+\tau_d) = (1+r_{t+1})s_t(1-\tau_k) + \sigma_{t+1} \tag{2.98}$$

where τ_c and τ_d stand for the tax-rate on consumption of young and old households respectively, τ_m is the subsidy on the investment in environmental quality, τ_k is the tax-rate on savings, while T_t is a lump-sum tax levied on the young and σ_{t+1} denotes the net tax revenue assumed to be given to the old as a transfer.[24]

$$\sigma_{t+1} = \tau_c c_{t+1} + \tau_d d_{t+1} + \tau_k(1+r_{t+1})s_t + T_{t+1} - \tau_m m_{t+1} \tag{2.99}$$

From (2.97) and (2.98) we can derive the intertemporal budget constraint of the representative household born at time t

$$c_t(1+\tau_c) + \frac{d_{t+1}(1+\tau_d) - \sigma_{t+1}}{(1+r_{t+1})(1-\tau_k)} + (1-\tau_m)m_t = w_t - T_t \tag{2.100}$$

Within this general framework, we now analyze three alternative tax schemes. The first one is characterized by consumption taxes and transfers, where the consumption tax-rate can be distinguished between generations. In the second one the long-lived government cannot use differentially consumption taxes, hence a tax on savings is required as an additional tax

[24] Our starting point is to consider optimal tax programmes wherein the government neither runs a surplus nor a deficit. When considering this class of fiscal policies, any net tax surplus needs to be given to someone. We assume that the old generation receives the net tax revenue. Certainly this need not be the case. There are at least two alternatives. First, it could be the young who receives the lump-sum transfer. Second, any net surplus could be shared among members of the young and the old generations.

instrument. The third one uses a subsidy on maintenance investment and a tax on savings.

2.7.1 Consumption taxes

We consider the case where differentiated consumption taxes are available for the long-lived government. Moreover, young households pay a lump-sum tax. To prevent the public sector from running a surplus or a deficit, the net tax revenue is transferred to the old generation.

In this framework, the intertemporal budget constraint (2.100) becomes

$$c_t(1+\tau_c) + \frac{d_{t+1}(1+\tau_d) - \sigma_{t+1}}{(1+r_{t+1})} + m_t = w_t - T_t \qquad (2.101)$$

whereas the government budget constraint (2.99) simplifies to

$$\sigma_{t+1} = \tau_c c_{t+1} + \tau_d d_{t+1} + T_{t+1} \qquad (2.102)$$

The representative household chooses the optimal level of s_t and m_t in order to maximize the life-cycle utility function (2.4) subject to (2.101) and to the environmental law of motion (2.6). By using these two constraints, we can write the objective function as

$$\begin{gathered} u(c_t) + \delta v \left\{ \tfrac{\sigma_{t+1} + (1+r_{t+1})(w_t - c_t(1+\tau_c) - m_t - T_t)}{1+\tau_d}; \right. \\ \left. (1-b)E_t - \beta[(N-1)(\overline{c}+\overline{d}) + (c_t + d_t)] + \gamma[(N-1)\overline{m} + m_t] \right\} \end{gathered} \qquad (2.103)$$

By deriving with respect to c_t we get

$$u'(c_t) + \delta \left[\frac{-(1+r_{t+1})(1+\tau_c)}{1+\tau_d} v'_d(d_{t+1}, E_{t+1}) - \beta v'_E(d_{t+1}, E_{t+1}) \right] = 0 \qquad (2.104)$$

By deriving with respect to m_t we get

$$v'_d(d_{t+1}, E_{t+1}) = \frac{\gamma(1+\tau_d)}{1+r_{t+1}} v'_E(d_{t+1}, E_{t+1}) \qquad (2.105)$$

By substituting for $v'_d(d_{t+1}, E_{t+1})$ from (2.105), equation (2.104) amounts to

$$u'(c_t) = \delta\left[\gamma(1+\tau_c)+\beta\right] v'_E(d_{t+1}, E_{t+1}) \tag{2.106}$$

We can now define a competitive equilibrium in the steady state of the OLG economy with taxes and transfers as an tuple $\mathbf{CET}_A$=$\{c;d;m;k;E;\tau_c;\tau_d;T\}$ characterized by (2.107) to (2.111).

$$u'(c) = \delta\left[\gamma(1+\tau_c)+\beta\right] v'_E(d.E) \tag{2.107}$$

$$v'_d(d, E) = \frac{\gamma(1+\tau_d)}{f'(k)} v'_E(d, E) \tag{2.108}$$

$$c(1+\tau_c) + m + k = f(k) - kf'(k) - T \tag{2.109}$$

$$d(1+\tau_d) = kf'(k) + \sigma \tag{2.110}$$

$$E = N\left[\frac{\gamma}{b}m - \frac{\beta}{b}(c+d)\right] \tag{2.111}$$

By equating the FOC (2.67) with (2.88) and after some computations, we obtain the optimal level of τ_c such that, in the steady state, the competitive equilibrium with taxes $\mathbf{CET}_A$ mimics the social optimum.

$$\tau_c = \frac{[N-1+\rho(1-b)](\beta+\gamma)}{\gamma[1-\rho(1-b)]} = \frac{N(\beta+\gamma)}{\gamma[1-\rho(1-b)]} - \frac{\beta}{\gamma} - 1 \tag{2.112}$$

Similarly, by equating the FOC (2.68) with (2.89) and by considering the fact that $f'(k) = 1/\rho$, we obtain the optimal level of τ_d.

$$\tau_d = \frac{N(\beta+\gamma) - \gamma + \gamma\rho(1-b)}{\gamma[1-\rho(1-b)]} = \frac{N(\beta+\gamma)}{\gamma[1-\rho(1-b)]} - 1 \tag{2.113}$$

Finally, the social planner chooses the optimal level of the transfer T using (2.109) in order to obtain $f'(k) = 1/\rho$.

$$T = f(k) - k\left(\frac{1+\rho}{\rho}\right) - m - c(1+\tau_c) \tag{2.114}$$

Some remarks are worth noting. Given the assumptions we have made on the parameters, both τ_c and τ_d are larger or equal to zero.[25]

By comparing (2.112) with (2.113) one can easily notice that $\tau_d > \tau_c$. This inequality can be explained by considering the optimization problem of the representative household: while the young agent considers the externality that his current consumption c_t produces on the environmental quality of the period that follows, the old agent do not care about the impact of his current consumption d_t on the environmental quality in period $t+1$. Hence, the old must be taxed more than the young.

Not surprisingly, by analyzing (2.112) and (2.113), we can assert that the size of each generation, N, and the pollution caused by consumption, β, are concerned with the optimal consumption tax-rates on both generations: that is, the optimal values of τ_c and τ_d depend, *inter alia*, on the parameters β and N. The more relevant the negative impact of consumption on the environment or the greater the size of each generation, the larger the value of the consumption taxes needed to correct such externalities. Conversely, τ_c and τ_d are decreasing in the autonomous evolution of environmental quality, b, and with the productivity of environmental expenditures, γ.

Even if the long-run effects of the environment on future generations do not exist, that is if $b = 1$, consumption taxes on the young and the old are required (because $N > 1$). If $b = 1$ and $N = 1$, then the tax on the young is no longer needed, but the tax on the old is still required.

Finally, suppose that there exists a myopic short-lived government only concerned with the well-being of the current young generation. By substituting $\rho \to 0$ in (2.112) and (2.113) we obtain

$$\tau_{c_0} = \frac{(N-1)(\beta+\gamma)}{\gamma} \tag{2.115}$$

and

$$\tau_{d_0} = \frac{N(\beta+\gamma)-\gamma}{\gamma} \tag{2.116}$$

It is straightforward to notice that $\tau_{c_0} < \tau_c$ and $\tau_{d_0} < \tau_d$. When the government cares also about the welfare of future generations, the optimal consump-

[25]The rationale behind it is that consumption negatively affects the environmental quality. Moreover, an increase in consumption of the young indirectly implies a decrease in environmental maintenance, hence it should not be subsidized.

tion tax-rates must be raised to internalize the intergenerational externalities at work.

As Ono (2003) argues, it is often difficult to implement consumption tax-rates that differ between two generations. Hence, in the following subsection we will discuss a fiscal policy which allows the social planner to correct the environmental externality even in the presence of a unique consumption tax-rate.

2.7.2 The constrained Pigouvian tax system

We model a tax system characterized by a uniform consumption tax rate on consumption of young and old, τ_c^*, and a capital income tax, τ_k.

Under this assumption, the intertemporal budget constraint of the representative consumer becomes

$$c_t(1+\tau_c^*) + \frac{d_{t+1}(1+\tau_c^*) - \sigma_{t+1}}{(1+r_{t+1})(1-\tau_k)} + m_t = w_t - T_t \tag{2.117}$$

where

$$\sigma_{t+1} = \tau_c^*(c_{t+1} + d_{t+1}) + \tau_k(1+r_{t+1})s_t + T_{t+1} \tag{2.118}$$

By taking into account (2.117) and the environmental constraint (2.6), the objective function of the household's maximization problem can be written as follows

$$\begin{aligned} u(c_t) + \delta v \Big\{ &\tfrac{(1+r_{t+1})(1-\tau_k)}{1+\tau_c^*}[w_t - (1+\tau_c^*)c_t - m_t - \tau_t] + \sigma_{t+1}; \\ (1-b)E_t - \beta[(N-1)(\overline{c}+\overline{d}) &+ (c_t + d_t)] + \gamma[(N-1)\overline{m} + m_t] \Big\} \end{aligned} \tag{2.119}$$

The derivatives with respect to c_t and m_t amount respectively to

$$u'(c_t) + \delta\left[-(1+r_{t+1})(1-\tau_k)v'_d(d_{t+1}, E_{t+1}) - \beta v'_E(d_{t+1}, E_{t+1})\right] = 0 \tag{2.120}$$

and

$$v'_d(d_{t+1}, E_{t+1}) = \frac{\gamma(1+\tau_c^*)}{(1+r_{t+1})(1-\tau_k)} v'_E(d_{t+1}, E_{t+1}) \tag{2.121}$$

By substituting for $v'_d(d_{t+1}, E_{t+1})$ from (2.121), equation (2.120) can be simplified as follows

$$u'(c_t) = \delta\left[\gamma(1+\tau_c^*) + \beta\right] v'_E(d_{t+1}, E_{t+1}) \tag{2.122}$$

We can define a competitive equilibrium in the steady state of the OLG economy with taxes and transfers as an tuple $\mathbf{CET}_B$=$\{c;d;m;k;E;\tau_c^*;\tau_k;T\}$ characterized by (2.123) to (2.127).

$$u'(c) = \delta\left[\gamma(1+\tau_c^*) + \beta\right] v'_E(d, E) \tag{2.123}$$

$$v'_d(d, E) = \frac{\gamma(1+\tau_c^*)}{f'(k)(1-\tau_k)} v'_E(d, E) \tag{2.124}$$

$$c(1+\tau_c^*) + m + k = f(k) - kf'(k) - T \tag{2.125}$$

$$d(1+\tau_c^*) = kf'(k)(1-\tau_k) + \sigma \tag{2.126}$$

$$E = N\left[\frac{\gamma}{b}m - \frac{\beta}{b}(c+d)\right] \tag{2.127}$$

In order to equate the FOCs (2.67) and (2.88), the social planner sets the optimal level of τ_c^*

$$\tau_c^* = \frac{[N-1+\rho(1-b)](\beta+\gamma)}{\gamma[1-\rho(1-b)]} \tag{2.128}$$

Similarly, the optimal level of τ_k is set to obtain the equivalence between FOCs (2.68) and (2.89). By considering that $f'(k) = 1/\rho$, after some computations we obtain

$$\tau_k = 1 - \frac{\gamma(1+\tau_c^*)[1-\rho(1-b)]}{N(\beta+\gamma)} = \frac{\beta[1-\rho(1-b)]}{N(\beta+\gamma)} \tag{2.129}$$

where $0 < \tau_k < 1$. Given the values for τ_c^* and τ_k the social planner chooses the optimal level of the transfer T using (2.125) in order to obtain $f'(k) = 1/\rho$.

$$T = f(k) - k\left(\frac{1+\rho}{\rho}\right) - m - c(1+\tau_c^*) \tag{2.130}$$

From (2.128) and (2.129) one can observe that τ_c^* and τ_k move in opposite direction as the parameter b varies. The higher b, the smaller τ_c^* and the larger τ_k. In particular, when $b = 1$, τ_c^* attains its minimum at $[(N-1)(\beta+\gamma)/\gamma]$, whereas the capital income tax attains its maximum at $\beta/N(\beta+\gamma)$. Indeed, even when $b = 1$ and the long-run externality due to consumption of current generation disappears, the intragenerational externality is still at work. As in the previous tax scheme, τ_c^* and τ_k are increasing in the degree of environmental degradation, β, and decreasing in the return on environmental investment, γ. Moreover, the larger N, the higher τ_c^* and the lower τ_k.

We briefly compare these two tax schemes. As one can easily see, $\tau_c^* = \tau_c < \tau_d$. When the social planner cannot implement two different tax-rates on consumption of the young and the old generation, he chooses an optimal tax-rate τ_c^* which is equal to the tax-rate he would implement on consumption of the young, τ_c, if differentiated consumption taxes were available. Nevertheless, without an appropriate capital income tax, consumption of the old would be too high. Hence, under this constrained tax system, a capital income tax τ_k is needed to decrease the disposable income of the old and obtain the optimal level of consumption in old age.

2.7.3 An alternative tax system

In this subsection we study an alternative fiscal policy suggested by Ono (2003) (Ono 2003). In his paper Ono (p.289) claims (without proving it) that "*an optimal allocation can be achieved by a lump-sum tax T, a capital income tax τ_k and a subsidy for the environmental maintenance τ_m*". In the remainder, we will prove this proposition.

Under this assumption, the budget constraint of the young is

$$c_t + s_t + (1-\tau_m)m_t = w_t - T_t \tag{2.131}$$

where τ_m stands for the subsidy for the environmental maintenance and T_t denotes the lump-sum tax. The budget constraint of old households becomes

$$d_{t+1} = (1+r_{t+1})s_t(1-\tau_k) + \sigma_{t+1} \tag{2.132}$$

As before, to prevent a public deficit, the government sets the net tax revenue, σ_{t+1}, to be transferred to the old generation

$$\sigma_{t+1} = \tau_k(1 + r_{t+1})s_t + T_{t+1} - \tau_m m_{t+1} \tag{2.133}$$

From (2.131) and (2.132) we derive the intertemporal budget constraint of a generation born at time t

$$c_t + \frac{d_{t+1} - \sigma_{t+1}}{(1 + r_{t+1})(1 - \tau_k)} + m_t(1 - \tau_m) = w_t - T_t \tag{2.134}$$

By substituting (2.134) and (2.6) into the lifetime utility function (2.4), the representative household's maximization problem becomes

$$\begin{aligned} u(c_t) + \delta v\,\{&(1 + r_{t+1})(1 - \tau_k)(w_t - c_t - (1 - \tau_m)m_t - \tau_t) + \sigma_{t+1}; \\ &(1 - b)E_t - \beta[(N - 1)(\overline{c} + \overline{d}) + (c_t + d_t)] + \gamma[(N - 1)\overline{m} + m_t]\} \end{aligned} \tag{2.135}$$

The FOCs are

$$u'(c_t) = \delta \left[\frac{\gamma + \beta(1 - \tau_m)}{1 - \tau_m} \right] v'_E(d_{t+1}, E_{t+1}) \tag{2.136}$$

$$v'_d(d_{t+1}, E_{t+1}) = \frac{\gamma}{(1 - \tau_m)(1 + r_{t+1})(1 - \tau_k)} v'_E(d_{t+1}, E_{t+1}) \tag{2.137}$$

where (2.136) and (2.137) are the derivatives of (2.135) with respect to c_t and m_t.

In order to equate the FOCs (2.67) with (2.88) and the FOCs (2.68) with (2.89), the social planner sets the optimal levels of τ_m and τ_k respectively as follows

$$\tau_m = \frac{(\beta + \gamma)[N - 1 + \rho(1 - b)]}{N(\beta + \gamma) - \beta[1 - \rho(1 - b)]} = 1 - \frac{\gamma[1 - \rho(1 - b)]}{N(\beta + \gamma) - \beta[1 - \rho(1 - b)]} \tag{2.138}$$

$$\tau_k = 1 - \frac{\gamma[1 - \rho(1 - b)]}{N(\beta + \gamma)(1 - \tau_m)} = \frac{\beta[1 - \rho(1 - b)]}{N(\beta + \gamma)} \tag{2.139}$$

The social planner then chooses the optimal level of the lamp sum tax T such that $f'(k) = 1/\rho$.

$$T = f(k) - k\left(\frac{1+\rho}{\rho}\right) - c - m(1-\tau_m) \qquad (2.140)$$

One can notice that (2.139) is the same tax-rate on savings as we have derived in the previous tax programme, namely (2.129). Moreover, by comparing (2.139) with (2.140), one can readily see that they move in opposite direction as N grows. The higher N, the lower the optimum τ_k, while τ_m is increasing in N. As expected, τ_m is increasing in the strength of environmental degradation, β, and decreasing in the return on environmental maintenance, γ. The subsidy on the environmental expenditure leads the young to invest more in environmental quality and to consume less. Hence, τ_m yields the optimal consumption level of the young. When $b = 1$ and $N > 1$, τ_m and τ_k are still different from zero as, even if the long-run effect of current consumption on environmental quality is zero, the intragenerational externality is still at work. When $b = 1$ and $N = 1$, consumption does not affect the environmental quality in the long-run and the intragenerational externality disappears, therefore there is no reason to subsidize the level m_t chosen by the young. However, even in this case, the optimal level of the capital income tax is positive because the old still consume too much from a social point of view.

2.8 Conclusions

In our paper we have studied an OLG model with environmental externality wherein the environment is negatively affected by consumption activities and positively affected by maintenance investment. By assuming $N > 1$ and a fully decentralized economy, we have taken into account both the intra- and the intergenerational negative effects of consumption.

After setting up the OLG model, we have studied the competitive equilibrium. In order to simplify our analysis we have imposed some plausible assumptions on the functional form of the utility and production function. In this framework we have found that the existence of a long-run asymptotically stable equilibrium depends on the strength of both the externality consumption, β, and the investment in environmental quality, γ. For a sink to exist, better maintenance technologies or lower environmental degradation from consumption are needed.

Comparative statics have shown that physical capital and environmental conditions positively depend on N, γ and the elasticity parameter η_c. Parameters b and β have a negative impact on both the stock variables. Finally, the effect of the elasticity parameter η_d on capital is negative, while its effect on E is ambiguous.

We have then derived the competitive equilibrium in a steady state and compared it with the centralized economy of a social planner who cares about the welfare of current and future generations. We have showed that in the decentralized economy the consumption level is too high (and the environmental quality too low) because the representative household fails to internalize the external environmental costs for other, current and future, households.

The social planner can successfully implement a tax scheme to correct both kinds of externalities. Three different tax policies have been analyzed and discussed.

Our model is stylized and could be extended in several ways. Firstly, we could consider that the environmental externality is caused by the production activity, instead of consumption. Secondly, we could analyze the effect of population growth. Thirdly, we could assume the existence of an altruistic behaviour between generations and study the effect of operative bequests on both the steady state and the social optimum. Finally, an interesting research question might be to introduce households' heterogeneity (either in terms of differing preferences or in terms of differing initial endowments of labour or skills), and to analyze environmental externalities in such a richer context.

Chapter 3

Aging Society, Health and the Environment

Joint work with D. Dottori[a]

Both environmental quality and health care expenditure are determinants of health and life expectancy, but the support for them can be different according to the electors' age, with a relatively larger support for health expenditure among the elderly as it is generally effective on a shorter horizon than environmental maintenance. With population aging, the political support for health care expenditure is then self-reinforcing. We cast this issue in an overlapping generations model with endogenous longevity, where lifespan depends on health care expenditure and environmental quality. We compare the long-run outcomes for health care expenditure, environmental quality, lifespan, consumption and capital accumulation of an economy where agents vote over health spending and environmental maintenance, with those of a social planner who takes into account also the welfare of future generations. The role played by other parameters, such as the propensity for smoothing consumption or the degree of annuity markets is also highlighted. Empirical evidence of age-biased environmental care is provided.

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3.1 Introduction

Many Western and industrialized countries are nowadays experiencing population aging. One of the effects of an aging society is on the composition of the electorate by class of age and hence, as a consequence, also on the outcome of voting, which in a democracy is a determinant factor of the policy choice.[1] Elderly voters, who do not benefit from long-term public expenditures, tend to vote for spending which are more effective on a short-term horizon. This is the case, for example, of expenditures which affect health status, such as health care spending and environmental maintenance. The positive role of environmental quality on health is well-documented (see EEA, European Environmental Agency (2007)), but environmentally friendly policies, though yielding long-lasting impacts, tend to take more time to be fully effective. As the old do not enjoy future environmental improvements, they are more willing to vote for expenditures which affect their health in a more immediate way - namely health care expenditures. We are not claiming that the elderly are not interested at all in environmental maintenance, but that they are interested at a smaller extent than the young are. In a similar way, the young and the elderly voters are both interested in public spending that increase their life expectancy, but they have different preferences towards the composition of such expenditures.

In our model, we extend the Diamond (1965) overlapping generations (OLG) framework by allowing lifespan to be endogenous and to depend on health status. Health depends in turn on the current level of health care expenditure and on environmental quality, which is directly and positively affected by environmental maintenance. Two different solutions are compared. Firstly, we depict the case where health care and environmental expenditures are chosen through probabilistic voting, where agents must vote for both the level of taxes they are willing to pay and how tax-revenues have to be allocated between health care expenditure and environmental maintenance. Secondly, we consider the case of a social planner who cares about the welfare of current and future generations and has to maximize an utilitarian social welfare function consisting of the discounted sum of individuals' lifetime utilities. The political economic solution is likely to differ from the social optimum in several respects: firstly, the social outcome is determined by the currently living agents only. Secondly, the weight of the elderly component of the electorate is endogenously increasing as life expectancy improves. Finally, the availability and the completeness of financial markets on annuities can

[1] Other remarkable economic implications of aging are widely studied by the literature (e.g. Cremer and Pestieau (2000) and Fogel and Costa (1997)).

play a further role in shaping the outcome.

Our paper entwines different streams of the existing literature. One major feature of the model is that lifespan is endogenous and depends both on current health care expenditure and on environmental quality.
Hence, we relate to literature on endogenous longevity (e.g. Blackburn and Cipriani (1998), de la Croix and Licandro (1999), Hazan and Zoabi (2006) and Cervellati and Sunde (2005)), and in particular to papers which focus on health as the main determinant of lifespan. In this respect, Chakraborty and Das (2005), for instance, postulate a positive relationship between the probability of survival and private health investment and show that, in the absence of annuities markets, the resulting interplay between income and mortality can be instrumental in generating poverty traps. However, the crucial role of environmental quality on health is usually neglected. A longevity function depending on two arguments is used in Finlay (2006) where the second one is an exogenous baseline survival probability, whilst in our paper both the arguments are endogenously determined. Other works (see Chakraborty (2004), Osang and Sarkar (2005) and Bhattacharya and Qiao (2005)) model health as provided by the government through public measures. We share this feature but, whereas they assume an exogenously given tax rate to fund health expenditure, in our model agents vote over taxes and their use. Moreover, differently from the models mentioned above, following Pecchenino and Pollard (1997) and Ono and Maeda (2002), we allow for the existence of imperfect annuity markets.[2]

One of the effects of aging society is the intergenerational political divergence in determining the level and the composition of taxes. Previous studies on environmental policy in OLG models (see e.g. John and Pecchenino (1994) and John et al. (1995)), however, assume that political decision-making as regards environmental policy is undertaken by a short-lived government representing the currently living young, ignoring the more and more important political role played by the elderly, due to population aging. The relation between longevity and the environment is, partly, studied by Ono (2005) who analyzes the effect of increasing agents' lifespan on political decision-making as regards environmental policy and finds that, in the presence of imperfect annuity markets, greater longevity has no effect on the environmental tax levied by the government and a non-positive effect on the environmental quality. Ono (2005), however, posits that lifespan is exogenous.

[2]Generally perfect annuity markets are assumed. A remarkable exception is Chakraborty and Das (2005) where annuity markets do no exist at all.

Although environmental quality has a significant impact on health conditions, as confirmed by many state-of-the-art empirical studies (with respect to air pollution: see e.g. Bell and Davis (2001), Pope and al. (2002), Evans and Smith (2005) and Katsouyanni et al. (1997); with respect to poor water quality: see Sartor and Rondia (1983); with respect to climate change: see Kunst, Looman, and Mackenbach (1993)), much less has been said from a theoretical point of view. Two exceptions are articles by Jouvet, Pestieau, and Ponthière (2007) and Pautrel (2006). Jouvet, Pestieau, and Ponthière (2007) set up an OLG model in which the environmental quality is assumed to affect both longevity and (indirectly) the quantity of space available for each person and study the optimal public intervention where these two impacts are at work. They show that the first-best public intervention involves a positive taxation on capital income, as well as a positive taxation on health expenditures. The optimal second-best levels of the tax on capital income and on health spending depend instead on various factors, such as the intensity of preferences for environmental quality or the pollution process. However, they do not allow for environmental maintenance and they assume that health spending is undertaken by the young only.[3] Differently from their approach, in our model the effects of pollution are internalized by the government. Pautrel (2006), by using an OLG model à la Blanchard (1985) with human capital accumulation, explores the channels through which the detrimental effect of pollution on life expectancy can influence the optimal growth in the long-run. In his model health is determined by pollution and public health, but it is the share of health spending over GDP that matters and not its absolute amount.

This paper proceeds as follows. Section 3.2 provides some empirical evidence of age-biased environmental care. Section 3.3 sketches the model. Section 3.4 describes the equilibrium in the political economy. Section 3.5 deals with the case of the social planner and compares the two solutions. Section 3.6 provides numerical analysis. Section 3.7 concludes.

[3]The absence of environmental maintenance makes health spending be the only way to contrast pollution, thus leading to an under-provision of health care expenditure.

3.2 Empirical Analysis

3.2.1 Dataset and preliminary analysis

In this section an empirical analysis is carried out in order to test whether there exists some evidence supporting the idea that the young are more interested than the old in the quality of the environment.
Data we use are from *Special Eurobarometer 217/EB62.1/2004 'Attitudes of European citizens towards the environment'* (2005). The survey was carried out between 27th and 29th November 2004 by an independent consortium, on request of the European Commission, Directorate-General Press and Communication, opinion Polls. It covers the population of the 25 nationalities of the European Union Member States, resident in each of the Member States and aged 15 years and over.[4] In each EU country, a number of sampling points was drawn with probability proportional to population size and to population density.[5] The total sample is made from 24,787 respondents.[6]

Amongst the survey's questions, we are interested in the following one: "In your opinion, to what extent do the following factors influence your 'quality of life"'? (See Table 3.1).

Respondents could give only one answer for each factor. Hereafter we con-

[4]At that time Romania and Bulgaria were not Member States yet.

[5]In order to do so, the sampling points were drawn systematically from each of the "administrative regional units", after stratification by individual unit and type of area. They thus represent the whole territory of the countries surveyed according to the distribution of the resident population of the respective EU-nationalities in terms of metropolitan, urban and rural areas. In each of the selected sampling points, a starting address was drawn, at random. Further addresses were selected by standard random route procedures, from the initial address. In each household, the respondent was drawn, at random (following the closest *birthday rule*). All interviews have been conducted face-to-face in people's home and in the appropriate national language.

[6]The survey mainly focuses on three area: European citizens' perception of the environment, questions related to information on the environment and the importance of the environment in the political decision making process. As a whole, from the survey emerges a large agreement about the role that the environment should play in political decisions: almost nine out of ten Europeans believe that policy makers should take into account environmental concerns when developing policy in other areas such as economy and employment. (pg. 33 of the report). Moreover, Europeans are highly conscious of the determining role the environment plays as far as individual's quality of life is concerned: seven out of ten citizens believe that the environment influences their lives. Social factor have a similar impact, while economic factors are perceived as slightly more important (pg. 30). However, European citizens clearly state they lack information on environmental issues and would like to have more information on solutions (pgs. 14 and 16).

Table 3.1: Factors influencing the quality of life

		Very much	Quite a lot	Not much	Not a tall	Don't Know
1	State of the environment	1	2	3	4	5
2	Economic Factors	1	2	3	4	5
3	Social Factors	1	2	3	4	5

centrate our analysis only on the importance of the state of the environment for the quality of life, calling this variable *env_imp2*. Table 3.2 describes how answers are distributed in the population.

Table 3.2: Importance of the environment for the quality of life. Variable *env_imp2*

env_imp2	Freq.	Percent	Cum.
don't know	425	1.71	1.71
not at all	879	3.55	5.26
not much	5,162	20.83	26.09
quite a lot	11,236	45.33	71.42
very much	7,085	28.58	100.00
Total	24,787	100	

As it can be seen, the majority of Europeans think that the state of the environment is very (or at least quite) important in determining the quality of their life. For the purpose of our analysis it will be useful to create also another variable which summarizes the answers shown in Table 3.2. This new variable, that we call *env_imp*, groups the positive answers on one side and the negative ones on the other side; it hence takes three values: a lot or more, not much or less and don't know, as shown in Table 3.3.

Table 3.3: Variable *env_imp*

env_imp	Freq.	Percent	Cum.
don't know	425	1.71	1.71
not much or less	6,041	24.37	26.09
a lot or more	18,321	73.91	100.00
Total	24,787	100.00	

As long as age is concerned, we can see from Table 3.4 that the sample is well balanced, being each age group roughly equally represented. As before, it is

Table 3.4: Age distribution of the sample. Variable *age*.

Age	Freq.	Percent	Cum.
15-24	3,112	12.55	12.55
25-34	3,996	16.12	28.68
35-44	4,393	17.72	46.40
45-54	4,254	17.16	63.56
55-64	3,975	16.04	79.60
65+	4,983	20.10	99.70
refusal	74	0.30	100.00
Total	24,787	100.00	

useful to aggregate variable *age* in order to take into account the demographic structure of our theoretical framework. Therefore we group respondent aged 25 to 54 years old and we label them as "the young"; while respondents aged 55 years or more are grouped together as "the old". The new variable *agegroup* can thus take only 4 values: refusal, less than 25, 25-54, 55 or more, as shown in Table 3.5.[7]

Table 3.5: Aggregate age distribution of the sample. Variable *agegroup*.

Agegroup	Freq.	Percent	Cum.
refusal	74	0.30	0.30
less than 25	3,112	12.55	12.85
25-54	12,643	51.01	63.86
55+	8,958	36.14	100.00
Total	24,787	100.00	

In Table 3.6, the distribution of *env_imp* across age groups is shown. It is interesting to notice that the percentage of the young for whom the environment is an important factor of the quality of life (75.7%) is actually larger than the old's (71.52%).

In order to check whether the difference in the proportion of respondents who show a high care for the environment is statistically significant between

[7]From Table 3.5 it can be observed that a half of the population is represented by those we called the young (51.01%), while only a third of respondents fall in the group we called the old (36.14%). Although splitting the sample in more precise age groups would have been preferable, the aggregation shown in Table 3.5 is the most accurate one, given the structure of the data.

Table 3.6: Importance of the environment in relation to age group.
(Terms on the second row of each block are column percentages)

env_imp	Agegroup				
	refusal	less than 25	25-54	55+	Total
don't know	1	38	164	222	425
	1.35	1.22	1.3	2.48	1.71
not much or less	15	789	2,908	2,329	6,041
	20.27	25.35	23	26	24.37
a lot or more	58	2,285	9,571	6,407	18,321
	78.38	73.43	**75.7**	**71.52**	73.91
Total	74	3,112	12,643	8,958	24,787
	100	100	100	100	100

the young and the old, we carry out the two-sample test of proportion. It performs tests on the equality of proportion in two groups of interest; in our case, it concerns the two percentages in bold in Table 3.6. The result, shown in Table 3.7, supports the idea that the difference is significant, so that the young seem to care more about the environment than the old.

Table 3.7: Two-sample test of proportion for *env_imp*= "A lot or more" between the young and the old.

Two-sample test of proportion x:Number of obs=12643
y:Number of obs=8958

Variable	Mean	Std. Err.	z	$P > \|z\|$	[95% Conf.	Interval]
x	0.757	0.003814			0.749524	0.764476
y	0.7152	0.004769			0.705854	0.724546
diff	0.0418	0.006106			0.029832	0.053768
	under Ho:	0.006060	6.8974	0.000		

diff =prop(x) - prop(y) z=6.8974
Ho: diff =0

Ha: diff <0	Ha: diff !=0	Ha: diff>0
Pr(Z < z) =1.0000	$Pr(\|Z\| < \|z\|)$ =0.0000	Pr(Z > z)=0.0000

However, the result could be due to data aggregation which, though useful at first sight, can hide information. Therefore, we look for evidence of a negative correlation between the most disaggregate, available variables for age (*age*) and for environmental care (*env_imp2*). As reported in Table 3.8, the correlation turns out to be negative (-0.0269) and statistically significant at 99%.[8] Moreover, if we exclude the youngest part of the sample (i.e. those who are younger than 25, since they are not considered in our model), the negative correlation coefficient jumps to -0.0417.

Table 3.8: Correlation between *env_imp2* and *age*

	env_imp2	env_imp2 25+
age	-0.0269^{***}	-0.0417^{***}
	(0.0000)	(0.0000)

$*** = 99\%$

As a further robustness check, we perform a t-test to compare the mean of the disaggregate variable *env_imp2* among the two groups of interest (variable *agegroup*). This test is more suitable than the two-sample test of proportion when the variable of interest can take several values with an ordinal meaning.[9] The results highlighted in Table 3.9 confirm that the means of the two groups of interest (the young (25-54) and the old (55+)) are statistically different from each other, with the young showing a greater concern for the environmental conditions.

Table 3.9: t-test for means of *env_imp2* between young and old

Two-sample t test with unequal variances

Group	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf.	Interval]
25-54	12479	1.766969	0.003785	0.422779	1.75955	1.774387
55 +	8736	1.733402	0.004731	0.442206	1.724128	1.742676
combined	21215	1.753146	0.002960	0.431191	1.747344	1.758949
diff		0.033566	0.006059		0.021691	0.045442

diff=mean(25-54) - mean(55 +) t =5.5403
Ho: diff=0 Satterthwaite's degrees of freedom=18257.4

Ha: diff <0 Ha: diff !=0 Ha: diff>0
Pr(T < t) =1.0000 Pr($\|T\| < \|t\|$) =0.0000 Pr(T> t)=0.0000

[8]Clearly, the test is restricted only to observations when both the variables do not take values such as "refusal" or "don't know".

[9]This test fits best for continuous variables. However, even if *env_imp2* is not continuous, it can be tested better by this test than by the test of proportion, where only two values can be compared.

The effect of age is so far always significant. However this result might be spurred by the presence of correlates that we did not take into account. In order to address this issue, in the following subsection we take advantage of other variables available from the survey to perform an econometric analysis which allows to control for the possible effects of other determinants of environmental care.

3.2.2 Probit Analysis

In the dataset are available information on other variables which may play a role in shaping the attitude towards the environmental care: the level of education, the gender, the living area, whether the respondent has children, etc.[10] Besides education and political opinions, which could plausibly affect the attitude towards environmental issues, one could expect that living in more polluted areas might make the environmental conditions a more pressuring issue. We also include a dummy for living in one of the countries that built the core of the European Union, because many of the new Members States have been experiencing a remarkable growth in pollution levels (especially Eastern Europe countries), while in the EU-15 members green laws have a longer tradition. Moreover, it is interesting to control whether having children has any effects, since altruistic agents might feel environmental conditions important not only for themselves but also for their offspring's well-being.[11]

Moreover, the sample is restricted to respondents older than 24 years. This allows to be closer to our theoretical implications and to simplify the treatment for some variables (e.g. education, see footnote 13). A synthetic representation of the remaining dataset is provided in Table 3.10.

The variable *education* consists of answers to the question "What age did you leave school?". We use this piece of information first by grouping values together into three categories and then by constructing a set of dummy variables. The first group is represented by those who attended no full time education or left school when they were younger than 15,[12] the second group

[10] The data are recoded as explained below and all the observations for which the answer was "don't know" or a refusal are set as missing values for that variable.

[11] In our model agents are assumed to be non-altruistic and altruism can be see as a way for departing from the implications arising from the political economy setting we put forward in Section 3.4 and 3.6.

[12] Grouping the class of those with no full time education with the immediately upper class is a matter of convenience since the no full time education class is made up only of

Table 3.10: Control variables.

Control variables		Freq.	Percent	Cum.
education	no-full time education	89	0.42	0.42
	15 or less	5,402	25.42	25.84
	16-19	8,941	42.07	67.91
	more than 20	6,561	30.87	98.78
	still studying	259	1.22	100.00
gender	male	9,487	43.92	43.92
	female	12,114	56.08	100.00
area	rural area or village	8,178	38.00	38.00
	small or middle sized town	7,830	36.39	74.39
	large town	5,511	25.61	100.00
offspring	0	15,347	71.05	71.05
	1	2,961	13.71	84.76
	2 or more	3,293	15.24	100.00
children	0	15,347	71.05	71.05
	1	6,254	28.59	100.00
polview	left	1,599	9.31	9.31
	centre-left	3,691	21.49	30.80
	centre	7,151	41.64	72.44
	centre-right	3,355	19.53	91.97
	right	1,379	8.03	100.00
leftright	left	5,290	30.80	30.80
	centre	7,151	41.64	72.44
	right	4,734	27.56	100.00
eu15	0	7,963	36.86	36.86
	1	13,638	63.14	100.00
Dependent Variables				
env_imp	not much or less	5,237	24.69	24.69
	a lot or more	15,978	75.31	100.00
env_imp2	not at all	760	3.58	3.58
	not much	4,477	21.10	24.69
	quite a lot	9,798	46.18	70.87
	very much	6,180	29.13	100.00

is made by those who left school when they were younger than 20, while the third group joins those who left school when they were older than 20 and those who are still studying.[13] Then the dummies *mededu* and *hiedu* are created, where the former is 1 if the respondent stopped full-time education when he was less than 20 years, and the latter is 1 when the respondent stopped full-time education when he was aged 20 years or more or if he is still studying.[14]

The variable *gender* takes the value 1 if the respondent is a female, the dummy variable *eu15* is 1 if the respondent is a citizen of one of the EU-15 countries, the dummy variable *children* is 1 if the respondent has at least 1 child less than 15 years old living in the household. By means of dummy variables we include the qualitative variables *area* (the living area of the respondent, which is classified either as "rural area or village", or "small or middle sized town", or "large town") and *leftright* (the political views, classified as left, center, right). See Table3.10.

Results

We consider a probit model where the dependent variable is *env_imp* and an order probit model where the dependent variable is *env_imp2*. The results on significance of coefficients are robust to the replacement of the assumption of a normal distribution with a logistic one.[15]

In the first two columns of Table 3.11 the results from the probit regression with the dependent variable *env_imp* can be observed. In the first column the regressor for age is represented by the dichotomic variable *agegroup*, while in the second column age is represented by the more analyitical variable *age*. All the results are robust to the use of different proxies for the control variables, they are omitted for the sake of brevity but are available upon request.[16] The coefficient of *agegroup* is negative and 99% significant, supporting the idea that the oldest group feels a relative lower concern for

89 observations out of a total of 21,252. The regressions results are robust with respect to keeping distinct these two classes.

[13] Including the answer "still studying" in the highest level of education is appropriate as the sample is restricted to people aged 25 or more. Out of 259 still-studying respondent after the sample's restriction, 210 are less than 35 years old, 37 between 35 and 44 years old, while only 12 are distributed in the eldest age classes.

[14] Therefore *hiedu* can be seen as a subset of *mededu* and should capture the marginal effect of the highest class of education better.

[15] All the logit regressions are available from the authors upon request.

[16] In particular, the regressions have been performed substituting *leftright* with a more detailed set of dummy variables for the political views and *children* with several alternative measures (log of number of children, number of children, a separate dummy when children are at least two, etc).

Table 3.11: Probit analysis

	Probit regressions env_imp				Order probit regressions env_imp2			
agegroup	-0.0822	***	-		-0.0708	***	-	
	(0.0253)				(0.0200)			
age	-		-0.0256	***	-		-0.0257	***
			(0.0091)				(0.0072)	
center-parties	-0.0360		-0.0366		-0.0292		-0.0298	
	(0.0255)		(0.0255)		(0.0199)		(0.0199)	
right-wing parties	-0.0697	**	-0.0712	**	-0.0354		-0.0359	
	(0.0280)		(0.0280)		(0.0220)		(0.0220)	
mededu	-0.0338		-0.0326		-0.0018		-0.0042	
	(0.0286)		(0.0288)		(0.0226)		(0.0228)	
hiedu	0.1364	***	0.1354	***	0.1596	***	0.1584	***
	(0.0251)		(0.0251)		(0.0197)		(0.0197)	
female	0.0410	*	0.0416	*	0.0291	*	0.0295	*
	(0.0214)		(0.0214)		(0.0169)		(0.0169)	
small town	0.0755	***	0.0753	***	0.0432	**	0.0431	**
	(0.0247)		(0.0247)		(0.0195)		(0.0195)	
big city	0.1459	***	0.1450	***	0.1441	***	0.1429	***
	(0.0276)		(0.0277)		(0.0217)		(0.0217)	
children	0.0252		0.0265		0.0202		0.0160	
	(0.0268)		(0.0274)		(0.0210)		(0.0214)	
eu15	-0.1901	***	-0.1085	***	-0.0405	**	-0.0406	**
	(0.0231)		(0.0231)		(0.0181)		(0.0181)	
const	0.8433	***	0.7474	***				
	(0.0862)		(0.0684)					
cut1					-2.3631		-2.3008	
					(0.0730)		(0.060)	
cut2					-1.7848		-1.7223	
					(0.0696)		(0.0560)	
cut3					-0.7304		-0.6679	
					(0.0682)		(0.05423)	
cut4					0.4968		0.5593	
					(0.0681)		(0.0542)	
Number of obs =	16678		16678		16678		16678	
LR $\chi^2_{(10)}$ =	118.00		105.27		188.01		186.90	
Prob > χ^2 =	0.0000		0.0000		0.0000		0.0000	
Pseudo R^2 =	0.0064		0.0057		0.0046		0.0046	

*: 90%; **: 95%; ***:99%

environmental conditions than the youngest one.

Another significant coefficient is the one of the highest class of education (but not those of the other classes), suggesting that only the highest level of education has a clear positive impact on environmental care. Moreover women appear to care significantly more than men about environmental conditions and citizens of the new Member States are more likely to be concerned with environmental care than citizens of EU-15 countries, probably because of the high levels of pollution that these countries have been experiencing. Furthermore, left-wing people are found to be significantly more concerned than right-wing voters (but not than voters of center-parties). Living in towns, especially if big, has an impact on environmental concern, plausibly because of the greater exposure to pollution. Finally, we notice with interest that having children turns out to have no significant impact (even if the sign is positive) on the environmental care as measured by *env_ imp*.

It is hence interesting to check whether the results are robust to a more disaggregate classification of age classes. This is done in column 2 by using *age* instead of *agegroup*. Although in absolute value the coefficient for the age variable gets smaller, it is still negative and significant. All the other results mentioned for column 1 are substantially confirmed. This provides support for the assumption that the concern for environmental problems is decreasing with age.

However it might be that, even if the old caring for environmental conditions are less, they could be more "enthusiastic". Therefore we use the variable *env_ imp2* which provides a more disaggregate statement of the environmental care. Since in *env_ imp2* four ordinal levels of environmental concern are expressed, we perform order probit regressions. Similarly to column 1, column 3 considers age as represented by *agegroup*, while in column 4 (similarly to column 2) *age* is used.[17] The negative signs and the statistical significance for the coefficient of the variable capturing the age effect are confirmed also by the order probit approach. age is found to have a negative impact on environmental care also when the latter is divided into 4 classes of opinion. Being left-wing people is no more significant in determining the attitude towards the environment. The results for the other coefficients are substantially confirmed as well.[18]

Hence, as we can see from the probit analysis, even after controlling for the

[17]Here again results turn out to be robust to the use of different variables as controls. See footnote 16.

[18]The coefficient of *eu15* goes from 99% to 95% significance, as well as the the one of living in small towns.

effects of other variables, age seems to be a significantly determinant of the attitudes of the respondents towards the environment.

3.3 The Model

As intergenerational issues are central in our analysis an overlapping generations model provides a suitable framework for the analysis. We consider an infinite horizon economy where agents live two periods. We denote by generation t the cohort of agents born at t with $t = 1, 2, 3, \ldots$. Each new-born generation has the same size, that we normalize to 1, and agents belonging to the same generation are assumed to be identical.

As in Bhattacharya and Qiao (2005), the length of the first period of life is given and normalized to 1, while the length of the second period, $p() \in (0; 1)$, depends on the current health condition h of the agent.[19] Therefore, the life expectancy of an agent born at t can be written as

$$1 + p(h_{t+1}) \tag{3.1}$$

Alternatively, as the number of agents is large, one can look at $p()$ as the probability of living throughout the whole second period. Moreover, $p(h_t)$ can be seen as the old-young ratio at time t.[20] Usually, in the literature on endogenous longevity $p(h)$ is assumed to exhibit the following properties[21]

$$p'(h) \geq 0 \,\forall h, \quad p''(h) \leq 0 \,\forall h, \quad \lim_{h \to 0} p(h) = \underline{p} \geq 0, \quad \lim_{h \to \infty} p(h) = \bar{p} \leq 1$$

where the notation $p'(h)$ ($p''(h)$) denotes the first (second) derivative of p with respect to h.

Combining the evidence for sizable effects on lifespan of both environmental quality and health care expenditure, we assume that health status is endogenously determined by the current environmental quality (that we denote by an environmental index E) and by current health care expenditure

[19] The fact that $p() \in (0; 1)$ implies that the span of second period lived is shorter than the first period. Despite one can argue that improvements in health can enlarge substantially life expectancy, it is expectable that the working age increases as well; hence it does not seem restrictive to impose $p()$ less than one.

[20] Differently from others in literature (e.g. Finlay (2006) and Chakraborty and Das (2005)), we do not interpret $p()$ as a mortality shock at the end of the first period. The dependence of $p()$ on the second period health status can be found also in Osang and Sarkar (2005).

[21] See Chakraborty and Das (2005), Osang and Sarkar (2005), Bhattacharya and Qiao (2005) and Kalemli-Ozcan, Ryder, and Weil (2000).

g_t: $h_t = h(E_t; g_t)$. The function $h()$ is assumed to be homogeneous of degree 1 and increasing in its arguments at decreasing rates, i.e. for all E, g[22]

$$h() \geq 0,\ h'_E() \geq 0,\ h'_g \geq 0,\ h''_{EE} \leq 0,\ h''_{gg} \leq 0$$

It is therefore possible to express the length of the second period as a function $\pi(E_t; g_t)$. Combining the properties of $p(h)$ and $h(g; E)$, the characteristics of the function $\pi(E_t; g_t)$ are summarized by Properties 1

Properties 1 (Properties of the longevity function) *For the longevity function:*

$$\pi(E_t; g_t) \equiv p(h(E_t, g_t)) \equiv \pi_t \tag{3.2}$$

the following properties are given:

1. $\pi'_x \geq 0,\ x = \{E, g\},\ \forall E_t, g_t$
2. $\pi''_{xx} \leq 0,\ x = \{E, g\},\ \forall E_t, g_t$
3. $\pi_t \in (0, 1)\ \forall E_t, g_t$

Properties 1.1-1.3 follow from assumptions on $p(h)$ and $h()$: they say that π is comprehended between 0 and 1 and increasing in its arguments but at decreasing rates.

Individuals

As in Ono (2005), since we are not interested in intragenerational issues, we assume that individuals derive utility only from second-period consumption. In this way individuals' decision-making can be left apart and we can focus only on the political economic equilibrium. The lifetime utility of the representative agent born at time t is

$$U_t = \pi_{t+1} u(c_{t+1}) \tag{3.3}$$

that is the utility he gets from consumption c_{t+1} weighted by the length of his life. The following assumptions on $u()$ are made

$$\forall c > 0: \quad u(c) > 0, \quad u'(c) \geq 0, \quad u''(c) \leq 0$$

where the first assumption is sufficient to have a worth-living life.[23]

[22] Often, only one determinant of health is considered. See Finlay (2006) for a model with a two input health function, where the same conditions given in the text are assumed.

[23] See Chakraborty and Das (2005). As it is always possible to add a positive constant to a utility function without altering the preference system this assumption is not too restrictive.

In the first period of life individuals supply to firms an inelastic amount of labour fixed to 1 and are paid at the competitive wage w_t. As agents do not consume in the first period, they fully save and in the second period they receive an interest rate r_{t+1} on top of their total savings s_t. So the second-period total income before taxation amounts to: $(1 + r_{t+1})s_t$.
In a deterministic set-up, individuals know their lifespan and, absent any altruism, they consume everything in the portion of the second period they live

$$\pi_{t+1}(c_{t+1} + \tau^o_{t+1}) = (1 + r_{t+1})(w_t - \tau^y_t) \tag{3.4}$$

where τ^y and τ^o are the lump-sum taxes levied by the government to fund health care spending and environmental maintenance (see below), and $(w_t - \tau^y_t)$ represents net savings.

When individuals do not know their lifespan with certainty, part of their savings can be "unrealized". In particular, out of the total risen amount $(1+r_{t+1})s_t$, a part $1-\pi()$ can be unrealized as it refers to a span where the agent is dead.[24] The unrealized amount is accordingly

$$(1 - \pi_{t+1})(1 + r_{t+1})s_t$$

When complete annuity markets are available, a rational non altruistic individual can neutralize the death risk by purchasing annuities (see Pecchenino and Pollard (1997)).[25] With annuity markets, as long as an agent is alive, he receives an extra rate of return μ on his savings funded through the unrealized savings of dead purchasers, so that the effective interest on savings is: $1 + r_{t+1} + \mu_{t+1}$. Basically, the portion of savings that would be unrealized is used to increase wealth as long as agents are alive. However, the degree of completeness of annuity markets is generally not perfect and can vary among countries and time.[26] Following Ono (2005) and Pecchenino and Pollard (1997) we allow for the possibility that only a fraction $\gamma \in (0, 1)$ of unrealized savings can be annuitised, while the remaining part, $1 - \gamma$, is passed on to the next generation as unintended bequest, b. In this general set-up the consumer is subject to the following budget constraint

$$c_{t+1} + \tau^o_{t+1} = (1 + r_{t+1} + \mu_{t+1})(w_t + b_t - \tau^y_t)$$

[24]Looking at π as the probability of surviving throughout the whole second period, there is a probability $(1 - \pi)$ of un-enjoying the savings. Under both interpretations the unrealized amount is $(1 - \pi_{t+1})(1 + r_{t+1})s_t$.

[25]A life annuity is an insurance product by which the insured person receives periodically a sum as long as he lives, in exchange for a premium charge. It allows agents to insure against the risk of outliving their savings, given the uncertainty of their remaining lifetime by pooling the mortality risk across annuity purchasers. See Mitchell et al. (1999).

[26]This may occur for a number of reasons which are surveyed by Brown and Wharshawsky (2001). See also Chakraborty and Das (2005).

By assuming perfect competition, equilibrium in the annuity markets requires

$$\mu_{t+1}\pi_{t+1}s_t = \gamma(1-\pi_{t+1})(1+r_{t+1})s_t \qquad (3.5)$$

which means that the total payment in extra return (the l.h.s.) is equal to the total amount of available funds (r.h.s.). Eq. (3.5) yields the equilibrium excess interest rate

$$\mu_t = \gamma\frac{1-\pi_t}{\pi_t}$$

The portion $(1-\gamma)$ of unrealized savings goes to the next generation as (unintended) bequests

$$b_{t+1} = (1-\gamma)(1-\pi_{t+1})(1+r_{t+1})s_t \qquad (3.6)$$

By using equation (3.5) and equation (3.6) the agent's budget constraint can be rewritten as

$$\begin{aligned} \tau^o_{t+1} + c_{t+1} &= (1+r_{t+1}+\mu_{t+1}) \quad \times \quad (w_t + b_t - \tau^{y_t}) \\ &= (1+r_{t+1})\left[\frac{(1-\gamma)\pi_{t+1}+\gamma}{\pi_{t+1}}\right]\times[\underbrace{w_t+(1-\gamma)(1-\pi_t)(1+r_t)s_{t-1}-\tau^y_t}_{s_t}] \end{aligned} \qquad (3.7)$$

In the two extreme cases of complete ($\gamma = 1$) and absent ($\gamma = 0$) annuity markets, the budget constraint simplifies to

$$\gamma = 1: \qquad \tau^o_{t+1} + c_{t+1} = \frac{1+r_{t+1}}{\pi_{t+1}}(w_t - \tau^y_t) \qquad (3.8)$$

$$\gamma = 0: \qquad \tau^o_{t+1} + c_{t+1} = (1+r_{t+1})[w_t + (1-\pi_t)(1+r_t)s_{t-1} - \tau^y_t] \qquad (3.9)$$

When annuity markets are complete the return on savings is increased in a way inversely proportional to the agents' survival probability. On the other hand, when annuity markets are absent, all the unrealized savings (which are higher the lower is π) are passed on to the next generation. Eq. (3.8) is the same as eq. (3.4): with complete annuity markets the death risk can be perfectly neutralized and each individual faces the same budget constraint as in a deterministic set-up. This happens because annuity markets basically shift income from the portion of life that is not lived to the portion of life that is lived. On average, this amounts to say that all the income risen from first-period savings can be used in the portion π of the second period, exactly as it happens under certainty. At the other extreme, when annuity markets are not present at all, the portion of savings that would have financed consumption in the portion $1-\pi$ of the second period is transferred to the next generation.[27] Hence, we can look at the case $\gamma = 1$ either as the case of certainty or as the case of perfect annuitisation in an uncertain framework.

Firms

We assume that the total output in the economy, Y , is produced by perfectly-competitive profit-maximizer firms adopting a constant returns to scale technology with labour L and capital K: $Y_t = F(K_t, L_t)$. Since labour supply is normalized to 1 and the production function is homogeneous of degree one, it is possible to rewrite it in intensive terms. Moreover, assuming that capital fully depreciates after one generation, the production function can be rewritten in intensive form as

$$y_t = f(k_t) \tag{3.10}$$

where y_t and k_t are output and capital per worker respectively, and $f(k) \equiv F(K, 1)$. Assuming perfect competition in the factor markets, the profit-maximization problem yields the following factor prices

$$w_t = f(k_t) - k_t f'(k_t) \tag{3.11}$$

$$r_t = f'(k_t) - 1 \tag{3.12}$$

Environment and Capital Dynamics

The state variables of the economy are the environmental quality E and capital k. The law of motion of capital is given by the equilibrium relation between savings and investment.[28] As we have assumed full depreciation of capital, savings determine the capital stock in the next period

$$k_{t+1} = s_t \tag{3.13}$$

As far as the environment is concerned, following Ono (2003) and Jouvet (1995) we assume that the environmental quality can be worsened by economic production and improved by environmental maintenance according to the following law of motion[29]

$$E_{t+1} = E_t - \eta y_t + \nu(m_t), \qquad \eta > 0 \tag{3.14}$$

[27] The rationale behind it is that, as agents do not know exactly when they die, they tend to keep something away which is then transferred to their children once they actually die. Although these bequests are not due to altruism (and are therefore often referred to as "unintended"), altruism can be easily incorporated as a negative shift in γ.

[28] Notice that we express everything per young and that the young cohort size is constant.

[29] Notice that eq. (3.14) resembles the environmental law of motion we use in Chapter 2, however it differs from that in some respects. First, here the environmental quality

where m_t is the investment in environmental maintenance. The parameter η is related to the impact of economic production on the environment (e.g. units of emission per unit of production). The function $\nu()$ is increasing in m_t and captures the efficiency of environmental investment. While Ono (2005) assumes a linear technology, we adopt a more general framework, imposing the less restrictive conditions

$$\forall m: \quad \nu'(m) \geq 0; \quad \nu''(m) \leq 0.$$

In case that $\lim_{m \to 0} \nu'(m) \to +\infty$, a minimum level of positive environmental maintenance is always necessary.

3.4 Political-Economic Equilibrium

In this section we consider an economy where health care expenditure and environmental maintenance are provided by the government and funded through lump-sum general taxation, as a result of electors' (probabilistic) voting. The government, hence, aims to maximize the overall utility of the electorate. We denote such a regime by PE, in contrast with PL, which will be used from Section 3.5 onwards to refer to a social planner economy where also the welfare of future generations is taken into account. In PE the expenditures on health care and environmental maintenance are usually chosen by the government under the constraints of limited time horizon and political support.

3.4.1 Government

We assume that the government is entitled to both health care and environmental expenditures. Such an assumption is fairly reasonable as one can think that each individual is small and see his contribution to maintenance

is worsened by the production of firms, while in Chapter 2 is agents' consumption that pollutes the environment. Second, in Chapter 2 the maintenance technology is linear, while here we adopt a broader framework and we allow the maintenance technology to exhibit decreasing returns to scale. Finally, here we impose that the natural decay rate of the environmental index is nil. Both specifications (with or without any natural decay rate) are widely assumed and accepted by the literature on natural resources. In particular, an environmental law of motion without any natural decay rate, as specified in (3.14), is shared, among others, by Ono (2003) and Ono (2005).

as negligible[30]. Moreover the environmental quality be thought of as a public good, being largely non rival and non excludable.[31] On the other hand, health care expenditure is mostly funded through public funds (with the remarkable exception of the USA, where the private share is larger).

In order to finance health care and environmental investment, the government levies taxes both on the young and the old to fulfill its budget constraint. For analytical tractability we make the assumption that taxes are lump-sum[32]

$$\tau_t^y + \pi_t \tau_t^o = m_t + \pi_t g_t \tag{3.15}$$

where τ^y, τ^o denote the tax levied on the young and the old, respectively. The level of τ_t^i, m_t, and g_t is chosen through probabilistic voting. Differently from majority voting where, as long as preferences are single-peaked, each agent votes with probability 1 for a policy and eventually it is the position of the median voter that matters, under probabilistic voting the electors have a probabilistic distribution for voting over different policy-variables (see de la Croix and Doepke (2007)). This assumption intuitively accounts for the role of "ideology" in determining the final vote. Probabilistic voting allows for a smoother aggregation of preferences and, differently from the median theorem concerning majority voting, can be applied either when preferences are single-peaked or not.[33] As in de la Croix and Doepke (2007), the probabilistic voting mechanism leads the winning party (the government) to implement a weighted social welfare function of the indirect lifetime utility of the voters. Accordingly to this result, we assume that the government implements the following social welfare function Ω

$$\Omega_t = \vartheta U_{t-1} + U_t = \vartheta \pi_t u(c_t) + \pi_{t+1} u(c_{t+1}) \tag{3.16}$$

[30] Here we follow the approach used by John and Pecchenino (1994). Notice that, on the contrary, in Chapter 2 we model the maintenance as privately chosen by the agents, sharing this view with Jouvet, Michel, and Vidal (2000).

[31] The sub-optimal provision of environmental maintenance in a decentralized setting is shown by Jouvet, Michel, and Vidal (2000) in the so-called "subscription equilibrium" where each agent takes as given the amount of public good provided by the others.

[32] Alternatively, one could assume, as in Ono (2005), that the government levies a proportional tax on production, which is then shared between workers and capitalists according to the inputs' share. However, this set-up makes dynamics more complicated as it involves expectations over future tax rates, thus making the problem more difficult to be treated analytically without providing any further substantial insights. Similarly, for the sake of analytical tractability, we abstract from the possibility that the government could run a deficit by issuing debt securities.

[33] Notice, moreover, that in our setting majority voting would induce the implausible outcome of a "dictatorship" of the young (workers), as they are undifferentiated and in a greater amount than the old (capitalists).

where ϑ stands for the weight that the government attaches to the old generation. Differently from Ono (2005), we do not postulate that the old have an exogenous political power, on the contrary we assume that each voter has the same weight, so that ϑ is equal to the share of old people in the economy, π_t. By using eq. (3.2) and (3.3), the maximization problem of the government can be rewritten as

$$\max_{\tau_t^y, \tau_t^o, m_t, g_t} \Omega_t = \pi^2(E_t, g_t)u(c_t) + \pi(E_{t+1}, g_{t+1})u(c_{t+1}) \tag{3.17}$$

While g_t has a beneficial effect on the lifespan of the old, m_t is beneficial for the life expectancy of the young since it positively affects E_{t+1} through eq. (3.14). In other words, health care expenditure has more short-term effects and it benefits more the agents with a shorter life-horizon (the old), whilst environmental investment has more effects in a longer term so that it is valued more by agents with a longer horizon (the young). In the economy under analysis, there are two contrasting interests in the use of tax-revenues: one group (the old) supports higher health care expenditure, whereas the other group (the young) is more environmentalist. The outcome of the voting process is a weighted sum of those two interests. Those weights are not constant but endogenously affected by the process of aging, which shifts political importance towards the elderly electorate.

3.4.2 Inter-temporal Equilibrium

The government aims to maximize its objective function stated in (3.17) in the context of a competitive economy in intertemporal equilibrium. By substituting into the right hand side (r.h.s.) of eq. (3.13) the expression for savings appearing as the last factor in eq. (3.7), replacing the factor prices by means of eq. (3.11) and (3.12), using eq. (3.13) lagged one period to replace for s_{t-1} and finally eq. (3.15) to take into account the government budget constraint, the following law of motion for capital holds

$$k_{t+1} = f(k_t) - k_t f'(k_t)[\gamma + (1-\gamma)\pi(E_t, g_t)] - [m_t + \pi(E_t, g_t)(g_t - \tau_t^o)] \tag{3.18}$$

The environmental law of motion is the same as in eq. (3.14).

From eq. (3.7), by replacing the market-clearing conditions (3.11) and (3.12) and using the government budget constraint (3.15) to get rid of τ_t^y, we obtain

$$c_{t+1} = f'(k_{t+1})k_{t+1}\left(1 - \gamma + \frac{\gamma}{\pi_{t+1}}\right) - \tau_{t+1}^o \tag{3.19}$$

As in Ono (2005) we define an economic intertemporal equilibrium for a given sequence of policy variables $\{\tau_t^y, \tau_t^o, g_t, m_t\}_{t=1}^{\infty}$ as a sequence of lifespan, allocations and prices $\{\pi_t, c_t, k_t, E_t, s_t, b_t, w_t, r_t, \mu_t\}_{t=1}^{\infty}$ with initial conditions $k_0 > 0, E_0 > 0$ such that individuals' utility is maximized, firms' profits are maximized, the government's budget constraint is met, markets clear and equations (3.18) and (3.14) hold.

3.4.3 Voting over health and environmental expenditure

We write the following Lagrangean for the government's maximization problem

$$\max_{c_t, c_{t+1}, k_{t+1}, E_{t+1}, m_t, g_t, \tau_t^o} \Psi \equiv \pi(E_t, g_t)^2 u(c_t) + \pi(E_{t+1}, g_{t+1}) u(c_{t+1}) +$$
$$+ q_{t+1}\Big\{f(k_t) - k_t f'(k_t)[\gamma + (1-\gamma)\pi(E_t, g_t)] - [m_t + \pi(E_t, g_t)(g_t - \tau_t^o)] - k_{t+1}\Big\} +$$
$$+ v_{t+1}\Big\{E_t - \eta A f(k_t) + \nu(m_t) - E_{t+1}\Big\} + \zeta_t^o \left\{f'(k_t)k_t\left(1 - \gamma + \frac{\gamma}{\pi_t}\right) - \tau_t^o - c_t\right\} +$$
$$\zeta_{t+1}^y \left\{f'(k_{t+1})k_{t+1}\left(1 - \gamma + \frac{\gamma}{\pi_{t+1}}\right) - \tau_{t+1}^o - c_{t+1}\right\} \quad (3.20)$$

The following first-order conditions (FOCs) are derived

$$\zeta_t^o = \pi_t^2 u'(c_t) \quad (3.21)$$
$$\zeta_{t+1}^y = \pi_{t+1} u'(c_{t+1}) \quad (3.22)$$
$$\zeta_t^o = q_{t+1}\pi_t \quad (3.23)$$
$$q_{t+1} = v_{t+1}\nu'(m_t) \quad (3.24)$$
$$q_{t+1} = \zeta_{t+1}^y\left[1 - \gamma + \frac{\gamma}{\pi_{t+1}}\right]\left[f'(k_{t+1}) + k_{t+1}f''(k_{t+1})\right] \quad (3.25)$$
$$v_{t+1} = \pi'_{t+1,E}\left[u(c_{t+1}) - \frac{k_{t+1}\gamma\zeta_{t+1}^y f'(k_{t+1})}{\pi_{t+1}^2}\right] \quad (3.26)$$
$$q_{t+1}\pi_t = \pi'_{t,g}\left[2\pi_t u(c_t) - \frac{\gamma k_t f'(k_t)\zeta_t^o}{\pi_t^2} - q_{t+1}[(1-\gamma)f'(k_t)k_t + g_t - \tau_t^o]\right] \quad (3.27)$$

From the first two equations, one can see that $\zeta_t^o = \pi(E_t, g_t)\zeta_t^y$. As ζ^o and ζ^y represent the shadow values of consumption of an old and a young agent respectively, it turns out that - as aging increases - the relative weight of consumption which the government assigns to the old increases as well. Eq. (3.24) states that the marginal improvement that a greater maintenance has on the environment must be equal to its relative "price", expressed by the ratio of the Lagrangean multipliers. Equation (3.25) shows that the marginal cost of accumulating one more unit of capital should equate the marginal benefit of enhancing next period consumption.

By eliminating Lagrangean multipliers, we get

$$\pi_t u'(c_t) = u'(c_{t+1})\Big[(1-\gamma)\pi_{t+1}+\gamma\Big]\Big[f'(k_{t+1})+k_{t+1}f''(k_{t+1})\Big] \quad (3.28)$$

$$\frac{\pi_t u'(c_t)}{\nu'(m_t)} = \pi'_{t+1,E}\left[u(c_{t+1}) - \frac{\gamma u'(c_{t+1})k_{t+1}f'(k_{t+1})}{\pi_{t+1}}\right] \quad (3.29)$$

$$\pi_t u'(c_t) = \pi'_{t,g}\left\{2u(c_t) - u'(c_t)\big[(1-\gamma)f'(k_t)k_t + g_t - \tau^o_t\big] - \frac{\gamma k_t f'(k_t)}{\pi_t}u'(c_t)\right\} \quad (3.30)$$

In what follows we denote by the symbol α the complement to 1 of the elasticity of marginal productivity of capital: $\alpha_t \equiv 1 - \left\|\frac{f''(k_t)k_t}{f'(k_t)}\right\|$.[34] Eq. (3.28) represents the Euler's equation in the PE case. It can be rewritten as follows

$$\pi_t \frac{u'(c_t)}{u'(c_{t+1})} = \pi_{t+1}\left[1-\gamma+\frac{\gamma}{\pi_{t+1}}\right]\alpha_t f'(k_{t+1}) \quad (3.31)$$

An increase in health care expenditure, g_t, has a negative cost in terms of foregone consumption represented by the left hand side (l.h.s.) of Eq. (3.30) (higher taxes are needed). On the other hand, it enlarges the lifespan of the currently old generation, thus increasing its total utility, its political weight (the first term in square brackets in eq. (3.27)) and enlarging the pool of taxpayers and health care receivers. With imperfect annuity markets two further effects are at work. First, the extra-return for the surviving old is lower because there are more people alive[35]. Second, there are fewer bequests accruing to the young, which lowers their savings. The former (latter) effect is greater the higher (lower) is the degree of completeness of annuity markets.

As far as environmental maintenance is concerned, Eq. (3.29) can be rewritten as follows

$$\nu'(m_t)\pi'_{t+1,E}\left[u(c_{t+1}) - \frac{\gamma u'(c_{t+1})k_{t+1}f'(k_{t+1})}{\pi_{t+1}}\right] = \pi_t u'(c_t) \quad (3.32)$$

[34] In the particular case of a Cobb-Douglas production function: $y_t = Ak_t^\alpha$, then $\forall t\ \alpha_t = \alpha$ and α represents the capital share of output. In Assumption 3.2 we refer to such a particular case for the sake of analytical tractability.

[35] Under the deterministic interpretation this can be seen as the reduction in consumption that agents face in exchange for living longer.

The l.h.s. of eq. (3.32) shows the net gain of increasing m_t, which is related to the increase in the lifespan of the young generation: this entails a benefit in terms of higher total utility and a negative effect on extra-return on savings in the presence of annuity markets.[36] The r.h.s. shows the cost, in terms of marginal utility of consumption, of devoting resources to maintenance. Notice that for the government such a cost is higher the higher is π_t, because it is related to political support from the old. Since the cost of maintenance is financed by taxation, the old face only a cost and no benefit from environmental maintenance.[37]

By replacing τ_t^o from the consumer's budget constraint, eq. (3.30) can be simplified as

$$[2u(c_t) - (c_t + g_t)u'(c_t)]\pi'_{t,g} = \pi_t u'(c_t) \tag{3.33}$$

By denoting by $\epsilon_{\pi_t,g}$ the elasticity of π_t with respect to g_t, eq. (3.33) can be rewritten as

$$\frac{g_t}{c_t} = \left(2\frac{u(c_t)}{u'(c_t)c_t} - 1\right)\left(\frac{\epsilon_{\pi_t,g}}{1+\epsilon_{\pi_t,g}}\right) \tag{3.34}$$

3.4.4 Analytical study of PE steady state

In the steady state, the environmental quality E must be constant. From eq. (3.14), this implies that the level of maintenance is determined only by the relation between cleaning and polluting technology

$$m = \nu^{-1}(\eta f(k))$$

From eq. (3.31), in the steady state we have

$$\pi = \pi\left(1 - \gamma + \frac{\gamma}{\pi}\right)\alpha f'(k)$$

In order to understand how longevity is related to the Euler's equation, we first consider the case of perfect annuity markets ($\gamma = 1$, i.e. the same as a deterministic set-up): the r.h.s. simplifies to $\alpha f'(k)$, which implies that any increase in utility obtained by living longer is offset by a corresponding reduction in consumption. Nevertheless, there is still another channel through which π plays a role: it enlarges the political weight of the part less interested in capital accumulation (the l.h.s.). On the contrary, when

[36]This can be read as a smaller consumption level in the deterministic framework where agents know the length of their life.

[37]As for the case of health care expenditure, this extreme assumption is done only for the sake of simplicity. The main qualitative effects still hold if the old benefit from m_t as well, as long as they do that at a smaller extent than the young.

annuity markets are completely absent ($\gamma = 0$, so that it is impossible to offset the risk of dying), the steady state level of capital is independent of lifespan: due to higher longevity, which lowers resources transferred to the next generation, the effect on the l.h.s. cancels out the one on the right hand side.

Since m can be expressed as a function of k only, the following system of 4 equations is sufficient to determine the steady state values of k, c, g and E

$$1 = \left(1 - \gamma + \frac{\gamma}{\pi}\right) \alpha f'(k) \tag{3.35}$$

$$f(k) - k - m(k) = \pi(E, g)(g + c) \tag{3.36}$$

$$\frac{g}{c} = \left(\frac{2u(c)}{u'(c)c} - 1\right) \frac{\epsilon_{\pi,g}}{1 + \epsilon_{\pi,g}} \tag{3.37}$$

$$\pi = \nu'(m)\pi'_E \left[\frac{u(c)}{u'(c)} - \frac{\gamma k f'(k)}{\pi}\right] \tag{3.38}$$

Eq. (3.36) can be obtained by the law of motion of capital (3.18) evaluated at the steady state and by taking into account the consumer's budget constraint (3.19). The l.h.s. of eq. (3.36) represents the PE feasibility set, i.e. what is available for consumption and health care expenditure, once capital and environmental quality are kept constant.

Now, in order to go further analytically, we make some assumptions to simplify and/or make explicit the functional forms used in the system (3.36)-(3.38). Then in Section 3.6 more general and realistic functional forms will be considered through numerical examples. In details, the assumptions are the following

Assumption 3 *The following explicit forms and values are assumed*

1. *Utility function is a constant intertemporal elasticity of substitution function (CIES):*

$$u(c_t) = \frac{c_t^{1-\sigma}}{1 - \sigma}, \quad \sigma \in (0, 1) \tag{3.39}$$

2. *The production function is:*

$$f(k) = Ak^{\alpha} \quad A > 0, \ \alpha \in (0, 1) \tag{3.40}$$

3. *The survival function π is:*

$$\pi(E_t; g_t) = \varpi g_t^{\epsilon} E_t^{1-\epsilon}, \qquad \epsilon \in (0, 1); \ \varpi \in \Re_+ \tag{3.41}$$

4. *The maintenance technology is expressed by a linear function:*

$$\nu(m) = \zeta m, \qquad \zeta > \eta > 0; \tag{3.42}$$

5. *Annuity markets are absent:*

$$\gamma = 0$$

Eq. (3.39) and (3.40) are quite familiar. The latter is the standard Cobb Douglas production function, while the former is a constant intertemporal elasticity of substitution (CIES) utility function, where the parameter σ represents the inverse of the intertemporal elasticity of substitution of consumption, thus being a measure of preference for consumption smoothing. In addition, in a non deterministic set-up, σ can also be interpreted as the coefficient of relative risk aversion and eq. (3.39) as a constant relative risk aversion (CRRA) utility. The restriction $\sigma \in (0, 1)$ is enforced in order to guarantee a positive value for human life, so that life is worth living.[38]

Assumption 3.3 is useful because it implies that the elasticity of π_t with respect to g_t is constant and equal to the coefficient ϵ. This specification still satisfies Properties 1.1 and 1.2, while Property 1.3 cannot be satisfied for all t as eventually it would imply a vanishing elasticity. However, since all variables (included g and E) take a finite value in the steady state, it is always possible to ensure that π is between 0 and 1 in a neighborhood of the steady state by considering a ϖ small enough.

Concerning Assumption 3.4, a linear maintenance technology is the most used functional form in the reference literature and it allows to have a constant marginal impact of maintenance on the environmental quality.[39] The restrictions on η and ζ are made to guarantee that investing in environmental maintenance is worthwhile and feasible.[40]

Finally, by Assumption 3.5, annuity markets are absent. Besides allowing some simplifications, such a case is not unrealistic as in many countries annuity markets are far from being complete.[41] Moreover, in the presence of an altruistic motive, it is reasonable to assume that a large amount of

[38]Such a restriction is made also in Chakraborty and Das (2005). This has also the further implication that the savings function is increasing in the gross interest rate. However, in our framework, with no consumption in the first period, this effect does not appear.

[39]With such a technology a corner solution with no maintenance cannot be excluded *a priori*.

[40]See eq. (3.44) afterward.

[41]See Brown (1999) and Milevskya and Young (2007). The very low choice of annuitised assets has been labeled as "annuity puzzle". Possible explanations are the presence of

unrealized savings is in fact transferred to the currently young generation. In Section 3.6 the role of annuity markets is assessed by allowing for a strictly positive γ.

Before tackling the system (3.36)-(3.38), which describes the PE steady state, we observe that under Assumption 3 a more general result, concerning also the intertemporal equilibrium, holds. We state it in the following proposition

Proposition 7 *Under Assumption 3, at any t, the ratio of health expenditure over consumption (g_t/c_t) is larger for a larger impact of health care expenditure on lifespan (ϵ) and/or for a larger preference for consumption smoothing (or an increase in risk aversion), σ.*

Proof: By rearranging eq. (3.34) and using Assumption 3:

$$\frac{g_t}{c_t} = \frac{1+\sigma}{1-\sigma}\frac{\epsilon}{1+\epsilon} \tag{3.43}$$

$\blacksquare$

Proposition 7 states that resources are diverted from consumption towards health care expenditure when health care spending is more effective to improve lifespan or when the agents have a greater preference for smoothing consumption or they are more risk averse (that is, they prefer to consume less in exchange for a lower risk).[42]

Now, by focusing on the steady state, we observe that under Assumption 3 maintenance can be solved explicitly to find

$$m = \frac{\eta}{\zeta} A k^{\alpha} \tag{3.44}$$

The system of equations which sufficiently determines the steady-state values

bequest motives, adverse selection issues, risk pooling within family and high load factors by companies, presence of Social Security, inflation risk, ignorance and regulatory impediments (see Brown (1999)). Alternatively we could have obtained some simplifications by assuming the opposite framework ($\gamma = 1$), which was however more unrealistic.

[42] This finding reminds of a similar conclusion regarding the dependence of the ratio on the curvature of the utility function found in Ono and Maeda (2001).

of k, c, g and E becomes

$$f'(k) = \frac{1}{\alpha} \tag{3.45}$$
$$\Theta^{PE} \equiv f(k) - k - m(k) = \pi(E,g)(g+c) \tag{3.46}$$
$$\frac{g_t}{c_t} = \frac{1+\sigma}{1-\sigma}\frac{\epsilon}{1+\epsilon} \tag{3.47}$$
$$\frac{\pi'_g}{\pi'_E} = \zeta\frac{1+\epsilon}{1+\sigma} \tag{3.48}$$

In eq. (3.46) we have denoted the feasibility set by Θ^{PE}. Since k and m can now be determined autonomously, the l.h.s. of eq. (3.46) is independent of c, g and E. As for capital and maintenance, the following Lemma holds

Lemma 1 *The constant share of maintenance over output is given by:*

$$m/y = \eta/\zeta$$

The feasibility set Θ^{PE} is given by:

$$\Theta^{PE} = (1 - \alpha^2 - \frac{\eta}{\zeta})\underbrace{(A^{\frac{1}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}})}_{f(k)} \tag{3.49}$$

Θ^{PE} is increasing in A and ζ and decreasing in η. Provided that the TFP A is large enough, Θ^{PE} is also increasing in α.

Proof: From (3.44) and (3.45) we have

$$k = A^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}} \tag{3.50}$$
$$m = \frac{\eta}{\zeta}A^{\frac{1}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}} \tag{3.51}$$

The rest of the proposition follows by replacing m and k in the definition of Θ^{PE} and by taking partial derivatives. The effect of α is positive for a sufficiently high A. The other effects are always unambiguous. ■

The sub-system (3.46)-(3.52) determines g, c and E as an allocation of Θ^{PE} between g and c, such that E is steady and consistent with the optimizing conditions. The following proposition establishes the relation between E and g

Proposition 8 *Under Assumption 3 the ratio between E and g in the PE steady state is:*

$$\frac{E}{g} = \frac{\zeta}{1+\sigma}\frac{(1-\epsilon)(1+\epsilon)}{\epsilon} \tag{3.52}$$

E/g is decreasing in ϵ and σ, while it is increasing in ζ.

Proof: The result comes from eq. (3.48) and partial derivatives. ■

If lifespan becomes more elastic with respect to health care expenditure (larger ϵ), then g will grow relatively to E. On the contrary, if maintenance becomes more efficient (larger ζ), then E will grow relatively to g. Moreover, we find that a greater σ reduces E relatively to g. The rational behind it is that when σ is large there is a preference for living longer (see Proposition 7); however in PE this goal implies a larger political weight of the old and hence a larger increase in health care spending.[43]

Under Assumption 3 all the steady-state values can be obtained explicitly. We present them in the following Lemma, leaving the intermediary steps in the proof

Lemma 2 *The steady-state values of health care expenditure (as share of output), consumption (as share of output), environmental quality, and lifespan variable π are given respectively by*

$$g = \sqrt{\frac{\epsilon^{2-\epsilon}(1+\sigma)^{2-\epsilon}}{\zeta^{1-\epsilon}(1-\epsilon^2)^{1-\epsilon}(2\epsilon+1-\sigma)}\Theta^{PE}} \tag{3.53}$$

$$c = \frac{(1+\epsilon)\ (1-\sigma)\ \sqrt{\Theta^{PE}}}{\sqrt{(\zeta-\epsilon^2\,\zeta)^{1-\epsilon}\,\epsilon\ (1+\sigma)^{\epsilon}\,(1+2\,\epsilon-\sigma)}} \tag{3.54}$$

$$E = \frac{\sqrt{(\zeta-\epsilon^2\zeta)^{1+\epsilon}\ \ \Theta^{PE}}}{\sqrt{\big[\epsilon(1+\sigma)\big]^{\epsilon}(1+2\epsilon-\sigma)}} \tag{3.55}$$

$$\pi = \varpi\sqrt{\frac{\epsilon\,[(1-\epsilon^2)\ \zeta]^{\frac{1-\epsilon}{2}}\ (1+\sigma)^{\frac{\epsilon}{2}}\,\Theta^{PE}}{1+2\,\epsilon-\sigma}} \tag{3.56}$$

Proof: By using (3.47) in (3.46) we get

$$g\pi\left(\frac{2\epsilon+1-\sigma}{(1+\sigma)\epsilon}\right) = \Theta^{PE}$$

[43] A similar reasoning could be made if σ was seen as related to risk aversion.

by replacing for π

$$gE\left(\frac{g}{E}\right)^{\epsilon}\left(\frac{2\epsilon+1-\sigma}{(1+\sigma)\epsilon}\right)=\Theta^{PE}$$

let us now divide and multiply by g the l.h.s., we get:

$$g^{2}\left(\frac{E}{g}\right)^{1-\epsilon}=\frac{(1+\sigma)\epsilon}{2\epsilon+1-\sigma}\Theta^{PE}$$

by using eq. (3.52), we can replace the ratio E/g and finally solve with respect to g

$$g=\left(\frac{\epsilon^{2-\epsilon}(1+\sigma)^{2-\epsilon}}{\zeta^{1-\epsilon}(1-\epsilon^{2})^{1-\epsilon}(2\epsilon+1-\sigma)}\Theta^{PE}\right)^{\frac{1}{2}}$$

As far as c and E are concerned, they are simply obtained by plugging eq. (3.53) into eq. (3.47) and eq. (3.52), respectively. Once E is known, π can be easily computed by replacing the values of g and E. ■

An increase in σ makes living longer more desirable. Therefore, if σ increases, c decreases (see eq. (3.54)) while π increases (see eq. (3.56)). Of the two main inputs of π, health care expenditure g unambiguously increases (see eq. (3.53)), while the effect on E is positive if and only if ϵ is not too high (see eq. (3.55)).[44] This occurs because, when ϵ is very large, the objective of living longer is pursued mainly through increasing health care expenditure. If maintenance technology improves (larger ζ), the effect on E is positive, the effect on g and c is instead ambiguous: on one hand, there is a negative substitution effect (better environmental quality becomes "cheaper"), on the other hand, there is a positive effect since, *ceteris paribus*, a lower level of m is needed to keep E steady, and hence resources are freed and the feasibility set is larger. The overall impact on π is however positive. This implies that an improvement in environmental maintenance technology is always beneficial (see eq. (3.56)) for life expectancy, though it does not necessarily increase health care expenditure.

The parameter ϵ yields more ambiguous effects. For instance, the impact on g of an increase in ϵ is not always positive: for small values of ζ, further increases of ϵ eventually lower g. Put it differently, when environmental maintenance is poorly effective, on one hand the feasibility set is low because maintenance absorbs many resources, on the other hand the environmental quality is low as well: in this context an increase in the elasticity with respect

[44] More precisely: $\partial E/\partial\sigma>0 \Leftrightarrow \epsilon<1/2(1+\sigma)$. All the partial derivatives are omitted for the sake of brevity, but are available from authors upon request.

to health care expenditure is not reflected into a net increase in g because the substitutability between g and E is limited and since E is too low also g must be reduced. As for E, it turns out that if ζ is large enough (good maintenance technology) an increase in ϵ initially spurs the environmental quality, while further increases eventually reduce E. The overall effect on π

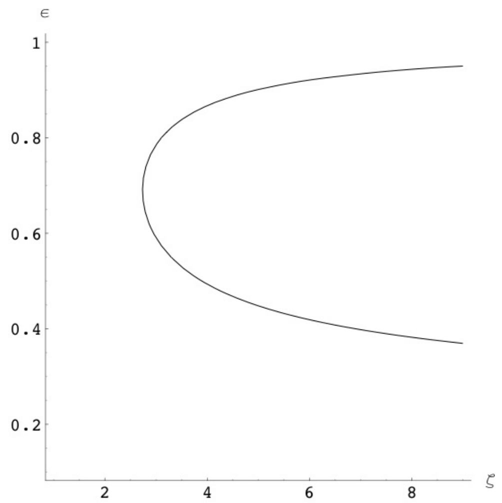

Figure 3.1: The partial effect of ϵ on π. Values used for other parameters: σ : 0.5, α : 0.3, A : 100, ϖ : 0.3, η : 0.1.

of a change in ϵ is therefore ambiguous, as shown in Fig. 3.1. For small values of ζ the effect of ϵ is always positive. However if ζ is high enough, there is a region of intermediate values of ϵ where the effect is negative. Recall that a larger ϵ (the elasticity of π to g) tends to move resources towards health care expenditure, but when investing in maintenance is effective enough, this reallocation can in fact hurt long run lifespan.[45]

In the next Section the case of a social planner economy (PL) is studied and compared with the PE case.

3.5 Social Planner Equilibrium

The PL case represents a first best approach that consists in solving the optimization problem of a social planner who takes care of current and future

[45] From Fig.3.1 we can see that this scenario is more likely to occur for intermediate values of ϵ.

generations. The planner treats each agent within a given generation equally and discounts future generations by an exogenous and constant factor $\rho < 1$.[46] The planner's optimization problem differs from the PE solution in three main respects: also future generations are taken into account; the weight attached to the welfare of each generation is not affected by the age structure of the population; and the planner is not constrained by market equilibria.

The objective function of the social planner is as follows

$$\max_{c_t,m_t,g_t,k_t,E_t} \sum_{t=0}^{+\infty} \rho^t \left(\frac{\pi_t u(c_t)}{\rho} \right) \tag{3.57}$$

Such a problem is subject to two resource constraints. The first one is represented by the resource feasibility constraint

$$y_t = \pi(E_t, g_t)(c_t + g_t) + m_t + k_{t+1} \tag{3.58}$$

which states that at each period the outcome has to be divided among consumption, health care expenditure, environmental maintenance and capital accumulation.

The other resource constraint is the environmental law of motion, given in eq. (3.14). Moreover E_0, k_0 are given and $m_t \geq 0,\ g_t \geq 0$. We write the Lagrangean as

$$\ell = \sum_{t=0}^{+\infty} \rho^t \Big\{ \pi(E_{t+1}, g_{t+1}) u(c_{t+1}) + \rho q_{t+1} \big(f(k_t) - \pi(E_t, g_t)(c_t + g_t) - m_t - k_{t+1} \big) + \\ + \rho v_{t+1}(E_t - \eta f(k_t) + \nu(m_t) - E_{t+1}) + z1_t m_t + z2_t g_t \Big\} \tag{3.59}$$

where q and v are the Lagrangean multipliers of capital and the environment respectively, thus representing the shadow values of relaxing the constraints on capital and environment; while $z1_t$ and $z2_t$ are the Khun-Tucker multipliers of environmental maintenance and health care expenditure respectively which, at interior solutions, are always zero. For the sake of simplicity we focus on interior solutions.[47] It is convenient to work with the Lagrangean

[46] The issue of social planner's discounting has been widely discussed since the seminal article by Ramsey (1928).

[47] Notice that when the assumption $\lim_{m \to 0} \nu'(m) \to +\infty$ is made, corner solutions are prevented from being optimal.

at time t

$$\ell_t = \frac{\pi(E_t, g_t)u(c_t)}{\rho} + \rho q_{t+1}\big(f(k_t) - \pi(E_t, g_t)(c_t + g_t) - m_t\big) - q_t k_t + \\ + \rho v_{t+1}(E_t - \eta f(k_t) + \nu(m_t)) - v_t E_t \quad (3.60)$$

Taking derivatives with respect to c_t, k_t, m_t and g_t, at interior solutions, we have

$$\frac{\partial \ell_t}{\partial c_t} \quad : \quad q_{t+1} = \frac{u'(c_t)}{\rho^2} \quad (3.61)$$

$$\frac{\partial \ell_t}{\partial E_t} \quad : \quad v_t - \rho v_{t+1} = \left(\frac{u(c_t)}{\rho} - q_{t+1}(c_t + g_t)\rho\right)\pi'_{t,E} \quad (3.62)$$

$$\frac{\partial \ell_t}{\partial k_t} \quad : \quad f'(k_t)(q_{t+1} - \eta v_{t+1})\rho = q_t \quad (3.63)$$

$$\frac{\partial \ell_t}{\partial m_t} \quad : \quad \frac{q_{t+1}}{v_{t+1}} = \nu'(m_t) \quad (3.64)$$

$$\frac{\partial \ell_t}{\partial g_t} \quad : \quad \pi_t q_{t+1}\rho = \left(\frac{u(c_t)}{\rho} - q_{t+1}\rho(c_t + g_t)\right)\pi'_{t,g} \quad (3.65)$$

Eq. (3.61) expresses the shadow-price of capital in terms of marginal utility of consumption. Combining eq. (3.23) and (3.21) yields $q_{t+1} = \pi(E_t, g_t)u'(c_t)$, which can be compared to (3.61). The shadow value of capital is always less in PE since $\rho^{-2} > \pi(E_t, g_t)$, given that $\rho, \pi \in (0, 1)$. In PL a larger value in terms of marginal utility is assigned to capital: relaxing the constraint on capital accumulation in order to consume more today is more costly, as more generations are taken into account. For the short-lived government such an implicit cost is less valued, nevertheless this difference is reduced by aging because, when a larger part of population expects to live longer, there is an increasing demand for a greater consumption in the next period, which means higher capital accumulation today.

Eq. (3.62) states that the current value of the depreciation of E must be equal to the marginal increase in lifespan due to a better environment, which has a positive effect in terms of utility and a negative one in terms of absorption of resources. Eq. (3.63) shows that the upfront cost of accumulating capital (the r.h.s.) must be equal to the discounted net gain which derives from a greater production, net of the negative effect of pollution. By comparing eq. (3.24) and (3.64), it can be seen that the PE relation between the shadow values of capital and maintenance is preserved also in the PL set-up.

Finally eq. (3.65) relates the price of devoting resources to health care expenditure to its effect in terms of higher lifespan. Again, this is twofold: on one side it is positive in terms of higher utility, on the other side it absorbs more resources.

Further insights can be obtained by eliminating the Lagrangean multipliers and properly shifting the time notation. The system of eq. (3.61)-(3.65) can be reduced to

$$\frac{f'(k_{t+1})\rho u'(c_{t+1})(\nu'(m_{t+1})-\eta)}{\nu'(m_{t+1})}=u'(c_t) \quad (3.66)$$

$$u'(c_t)[\pi_t+(c_t+g_t)\pi'_{t,g}]=u(c_t)\pi'_{t,g} \quad (3.67)$$

$$\nu'(m_t)\left\{\pi'_{t+1,E}\left[u(c_{t+1})-u'(c_{t+1})(c_{t+1}+g_{t+1})\right]+\frac{u'(c_{t+1})}{\nu'(m_{t+1})}\right\}=\frac{1}{\rho}u'(c_t) \quad (3.68)$$

By rearranging and using the short notation we can express the first two conditions as follows

$$\frac{u'(c_t)}{u'(c_{t+1})}=\rho\left(1-\frac{\eta}{\nu'(m_{t+1})}\right)f'(k_{t+1}) \quad (3.69)$$

$$\frac{g_t}{c_t}=\left(\frac{u(c_t)}{u'(c_t)c_t}-1\right)\left(\frac{\epsilon_{\pi_t,g}}{1+\epsilon_{\pi_t,g}}\right) \quad (3.70)$$

$$\nu'(m_t)\,\pi'_{t+1,E}\left[\frac{u(c_{t+1})}{u'(c_{t+1})}-c_{t+1}-g_{t+1}\right]=\frac{1}{\rho}\frac{u'(c_t)}{u'(c_{t+1})}-\frac{\nu'(m_t)}{\nu'(m_{t+1})} \quad (3.71)$$

Eq. (3.69) is the Euler's equation in an environmental economy with maintenance. Without environment the terms in parenthesis on the r.h.s. would simply amount to 1; with environment, instead, the polluting effect of production and the contrasting effect of maintenance are taken into account. Eq. (3.70) highlights the relation between health care expenditure and consumption, relating it to the elasticities of utility and lifespan. Eq. (3.71) shows the optimal condition concerning improving environment through m.

3.5.1 Comparing the PE and the PL solution

In this section the 3-equation systems for intertemporal capital accumulation, health care expenditure and environmental maintenance derived for PE (eqq. (3.31)-(3.33)) and for PL (eqq. (3.69)-(3.71)) are compared in the most general forms. Then, similarly to what has been done in Section 3.4.4, Assumption 3 is made and the PL steady state is studied.

As for capital, by comparing eq. (3.69) and (3.31) we can see how different are the forces behind capital accumulation in the two cases. Whereas in PL the polluting effect of production is taken into account, this is not the case in PE, as none of the electors is interested in E_{t+2}. This would enhance capital accumulation in PE, but there is also an effect going in the opposite direction and represented by α_t. In PE, factors' owners are remunerated according to the marginal productivity of the factor they own: as long as returns are decreasing this effect reduces the incentive in capital accumulation. Moreover, in PE the effects of annuity markets and of longevity should be taken into account as well. A higher degree of annuity markets tends to reduce the pool of intergenerational transfers through bequests, but increases the effective return on savings. Longevity of the young, which is endogenous, reduces the excess-return but enlarges the expected length of consumption. Longevity of the old, which is endogenous as well, increases the political importance of the old generation.

As for environmental maintenance, we compare PE eq. (3.32) with PL eq. (3.68). The last term on the l.h.s. of eq. (3.68) represents the current shadow-value of E_{t+2} (see eq. (3.61) and eq. (3.64)). Only the planner internalizes the long-lasting beneficial effects of investing in maintenance, whereas the short-lived government is interested only in the effects on E_{t+1}, since there is no current elector who is affected by the environment of two periods later. This effect tends to reduce the investment in maintenance relative to consumption made by the government. However, the planner takes into account also a negative effect of increasing lifespan of the current young generation, i.e. consumption and health care expenditure in the next period will increase as well, thus absorbing more resources which could be potentially accumulated for future generations (the last term in square brackets). In PE the government has no concern with such an issue because of its short-horizon but, on the other side, there is the constraint of competitive markets equilibria which, in the presence of annuity markets, implies that a higher lifespan has the partial effect of reducing extra-return on savings. Hence, also in PE a negative effect of increasing too much lifespan - despite quite different from the PL's one - is taken into account. Although these effects go in the same direction, it is not possible to say *a priori* which one is greater.

Finally, in order to compare the relations between health care expenditure and consumption, it is useful to refer to eq. (3.34) and eq. (3.70). They are pretty similar but on the r.h.s. of eq. (3.70) σ appears alone at the numerator of the first factor. It is convenient to introduce now Assumption 3 also for the PL case, as previously done in Section 3.4.4 for the PE framework. The following propostition states the relation between g and c in PL, in comparison to PE's

Proposition 9 *In PL, under Assumption 3, the health care expenditure to consumption ratio at any t is:*

$$\frac{g_t}{c_t} = \frac{\sigma}{1-\sigma}\left(\frac{\epsilon}{1+\epsilon}\right) \tag{3.72}$$

At any t the ratio of health care expenditure to consumption is larger in PE than in PL.

Proof: By using points 1 and 3 of Assumption 3 into eq. (3.70), equation (3.72) is obtained. By comparing eq. (3.72) with the corresponding expression in PE, that is eq. (3.43), it is straightforward to see that the r.h.s. of eq. (3.43) is always larger. ■

In PL as well as in PE, the g/c ratio is larger the higher is the propensity for consumption smoothing and lifespan elasticity to g. However, there exists a difference between the two cases. In the PE solution an increase in health care expenditure improves the lifespan of the old generation, thus also enlarging the amount of the electors who support health care expenditure the most. There is then a self-reinforcing effect between health care spending and political support, which pushes g_t high with respect to the PL solution, where the weight associated to the interest of each generation is exogenous (captured by the discount factor ρ and not dependent on π). When σ is low, the two solutions are more different because, whilst the planner substitutes easily health with consumption, in the PE framework this is not done since the support for spending in g is kept high by an aging population.

3.5.2 Analytical study of PL Steady State

In the steady state, the subsystem

$$\nu(m) = \eta f(k) \tag{3.73}$$

$$1 = \rho\left(1 - \frac{\eta}{\nu'(m)}\right) f'(k) \tag{3.74}$$

(where eq. (3.74) is obtained from eq. (3.69)), identifies the levels of m and k autonomously from the other endogenous variables.[48] The following Lemma is then derived

[48] Note that, while in PE we had to make an assumption (that is Assumption 3.5) to isolate k and m from the other equations of the steady state system, this is not needed in PL.

Lemma 3 *In PL, the positive steady-state levels of k and m depend (differently than in PE) on the pollution/maintenance parameters.*
The steady-state level of k is positively affected by ρ and negatively by η; the steady-state level of m is positively affected by ρ, while the effect of η is ambiguous.

Under Assumption 3.4:

- *The steady-state values are:*

$$k = \left(A\alpha\left(1-\frac{\eta}{\zeta}\right)\rho\right)^{\frac{1}{1-\alpha}} \quad ; \quad m = \frac{A\,\eta}{\zeta}\left(A\alpha\left(1-\frac{\eta}{\zeta}\right)\rho\right)^{\frac{\alpha}{1-\alpha}}$$

  k is increasing in ζ, while m is decreasing in ζ (increasing in η) if $\eta/\zeta < 1-\alpha$.

- *The feasibility set in PL is:*

$$\Theta^{PL} \equiv f(k) - k - m = \left(1-\frac{\eta}{\zeta}\right)(1-\alpha\rho) \quad \underbrace{A\left(A\alpha\left(1-\frac{\eta}{\zeta}\right)\rho\right)^{\frac{\alpha}{1-\alpha}}}_{f(k)} \tag{3.75}$$

  For $\rho > \frac{\alpha\zeta}{\zeta-\eta}$ the output y and the feasibility set Θ are both larger in PL.

Proof: We consider total differentiation of eq. (3.73) and (3.74) with respect to k, m, ρ and η. The following matrix notation summarizes the effects

$$\begin{bmatrix} dk \\ dm \end{bmatrix} = \begin{bmatrix} \frac{f'(k)(\eta-\nu'(m))\nu'(m)^2}{-\rho(\eta-\nu'(m))\nu'(m)^2 f''(k)+\eta^2\rho f'(k)^2\nu''(m)} & \frac{f'(k)\left(\nu'(m)^2-\eta f(k)\nu''(m)\right)}{\nu'(m)^2 f''(k)(\nu'(m)-\eta)+\eta^2 f'(k)^2\nu''(m)} \\ \frac{\eta f'(k)^2(\eta-\nu'(m))\nu'(m)}{-\rho(\eta-\nu'(m))\nu'(m)^2 f''(k)+\eta^2\rho f'(k)^2\nu''(m)} & \frac{\nu'(m)\left(\eta f'(k)^2+f(k)(\nu'(m)-\eta)f''(k)\right)}{\nu'(m)^2 f''(k)(\nu'(m)-\eta)+\eta^2 f'(k)^2\nu''(m)} \end{bmatrix} \begin{bmatrix} d\rho \\ d\eta \end{bmatrix}$$

In order to have a positive steady state, $\nu'(m)$ must be larger than η.[49] Given the properties of $f()$ and $\nu()$, the following results hold

$$\frac{\partial k}{\partial \rho} > 0; \ \frac{\partial k}{\partial \eta} < 0; \ \frac{\partial m}{\partial \rho} > 0$$

Under Assumption 3.4, the first derivative of $\nu(m)$ is constant and the solution can be found explicitly.

[49]Notice that Assumption 3.4 satisfies this requirement.

If $\rho = \frac{\alpha\zeta}{\zeta-\eta}$, then both the output y and its portion to be used for consumption and health care expenditure (Θ/y) are the same in PE and PL, as can be checked by looking at eq. (3.75) and (3.49). For larger ρ, y and Θ, they are larger in PL (although Θ/y is larger in PE). ■

From Lemma 3 it follows that a planner who puts more weight on future generations brings about a steady state with a greater production (and hence a greater environmental maintenance as well to offset the greater level of pollution). Moreover, if production becomes more polluting, the planner reduces capital in the steady state and, under conditions identified in Lemma 3, increases the environmental maintenance in order to preserve the environment. Similarly, when maintenance technology becomes more effective, the level of capital can increase and (under certain conditions) environmental maintenance can decrease. The effect of ζ (η) on m is ambiguous because on one side it is necessary to spend more on maintenance in order to offset the pollution induced by the same level of production, but on the other side the output itself is reduced, so the amount of available resources is lower. Under Assumption 3.4 we can see that, unless the pollution coefficient is fairly high compared to the maintenance coefficient, an improvement in ζ allows to reduce m and to have a larger k.
Finally, Lemma 3 also implies that, unless ρ is fairly low, the PL steady state is characterized by a larger feasibility set ($\Theta^{PL} > \Theta^{PE}$) and hence by a larger amount of resources available for health care expenditure and consumption.

Under Assumption 3, the following system defines for PL the steady-state values of E, g and c as an allocation of Θ^{PL} between c and g, such that E is steady and optimization conditions are satisfied

$$\pi(E,g)(c+g) = \Theta^{PL} \tag{3.76}$$

$$\zeta\pi'_E\left(\frac{c}{1-\sigma} - c - g\right) = \frac{1-\rho}{\rho} \tag{3.77}$$

$$\frac{g}{c} = \frac{\sigma}{1-\sigma}\left(\frac{\epsilon}{1+\epsilon}\right) \tag{3.78}$$

The system can be solved explicitly. Solutions are given in the following Lemma

Lemma 4 *In PL's steady state, under Assumption 3, the variables E, g, c*

and π *are:*

$$E = \zeta \frac{\rho}{1-\rho} \frac{1-\epsilon}{1+\epsilon-\sigma} \Theta^{PL} \tag{3.79}$$

$$g = \left(\frac{1-\rho}{\rho(1-\epsilon)\zeta}\right)^{\frac{1-\epsilon}{1+\epsilon}} \left(\frac{\epsilon}{(1+\epsilon-\sigma)^{\epsilon}}\right)^{\frac{1}{1+\epsilon}} \left(\Theta^{PL}\right)^{\frac{\epsilon}{1+\epsilon}} \tag{3.80}$$

$$c = \frac{(1-\sigma)(1+\epsilon)}{\sigma} \left(\frac{1-\rho}{\rho(1-\epsilon)\zeta}\right)^{\frac{1-\epsilon}{1+\epsilon}} \left(\frac{1}{(1+\epsilon-\sigma)^{\epsilon}}\right)^{\frac{1}{1+\epsilon}} \left(\frac{\Theta^{PL}}{\epsilon}\right)^{\frac{\epsilon}{1+\epsilon}} \tag{3.81}$$

$$\pi = \left(\frac{\rho}{1-\rho}(1-\epsilon)\zeta\right)^{\frac{1-\epsilon}{1+\epsilon}} \left(\frac{\varpi^2 \epsilon^{\epsilon} \Theta^{PL}}{1+\epsilon-\sigma}\right)^{\frac{1}{1+\epsilon}} \tag{3.82}$$

Proof: By multiplying and dividing both sides of eq. (3.77) by g and π, and by using eq. (3.78), one obtains

$$\frac{E}{g} = \zeta \frac{\rho}{1-\rho} \frac{1-\epsilon}{\epsilon} \pi \tag{3.83}$$

Now, by dividing side by side eq. (3.83) by eq. (3.76), by multiplying both sides by g and finally by using c/g from eq. (3.78), it is possible to obtain the final equation for E, as in eq. (3.79). Once E is known, g can be obtained from eq. (3.83). Finally, with g known, c can be obtained from eq. (3.78).
Eq. (3.82) can be derived from eq. (3.83) after replacing the l.h.s. with the expression in eq. (3.84) (see the following Proposition 10) and some computations. ■

Lemma 4 shows that also in PL an increase in σ shifts resources from consumption to lifespan inputs (i.e.: E and g). A better environmental maintenance technology, for a given Θ^{PL}, shifts resources from c and g to E. A greater elasticity of π with respect to g (larger ϵ) lowers the environmental quality and increases health care expenditure. For a given Θ^{PL}, a larger ρ shifts resources from g and c towards a better environmental quality.
From eq. (3.82) it can be seen that the effects of parameters ρ, ζ and σ on π are all positive, while the effect of ϵ, as well as in PE, is ambiguous. Since π^{PE} does not depend on ρ, eq. (3.82) can also be read as suggesting that if ρ is sufficiently high, π^{PL} can be larger than π^{PE}.[50]

[50] This is confirmed by several numerical experiments under a wide range of reasonable values for the other variables. In particular the floor for ρ such that $\pi^{PL} > \pi^{PE}$ is decreasing in σ, ζ, and ϖ, increasing in η, while the effect of ϵ is non monotonic and hump-shaped.

It is now possible to compute the E/g ratio in PL and to contrast it with PE's in order to compare the relative intensity of environmental quality between the two regimes. The following Proposition states the results

Proposition 10 *Under Assumption 3, the ratio E/g in PL is:*

$$\frac{E}{g} = \left(\frac{\rho(1-\epsilon)\varpi\zeta}{1-\rho}\right)^{\frac{2}{1+\epsilon}} \left(\frac{\Theta^{PL}}{\epsilon(1+\epsilon-\sigma)}\right)^{\frac{1}{1+\epsilon}} \tag{3.84}$$

As in PE, the relative intensity of E with respect to g is increasing in ζ and decreasing in ϵ.
Differently from PE, the relative intensity of E with respect to g is increasing in σ.
In PL, the relative intensity of E with respect to g is increasing in ρ. With respect to PE, E is used more intensively than g for a sufficiently high ρ.

Proof: By replacing $\pi = \varpi g^{\epsilon} E^{(1-\epsilon)}$ in eq. (3.83) and rearranging we have

$$g^{(1+\epsilon)} = E^{\epsilon}\frac{\epsilon}{1-\epsilon}\frac{1-\rho}{\rho}\frac{1}{\zeta\varpi}$$

now let us divide both sides by g, multiply and divide the r.h.s. by E and rearrange

$$\left(\frac{E}{g}\right)^{\epsilon} = \frac{1-\epsilon}{\epsilon}\frac{\rho}{1-\rho}\zeta\varpi\frac{g}{E}E$$

which yields

$$\left(\frac{E}{g}\right)^{(1+\epsilon)} = \frac{1-\epsilon}{\epsilon}\frac{\rho}{1-\rho}\zeta\varpi E$$

finally, by replacing E on the r.h.s. with its steady-state value as in eq. (3.79), after some algebraical manipulations we get eq. (3.84).
The effect of marginal increases in σ, ρ, and ζ follows straightforwardly from respective partial derivatives applied to eq. (3.84).
In order to compare the E/g ratio between PE and PL we take the ratio between eq. (3.84) eq. (3.52). The E/g ratio is larger in PL if and only if:

$$(1-\epsilon)^{\frac{1-\epsilon}{1+\epsilon}}\epsilon^{\frac{\epsilon}{1+\epsilon}}(1+\epsilon)(1+\sigma)(1+\epsilon-\sigma)^{-\frac{1}{1+\epsilon}} > \left(\frac{1-\rho}{\varpi\rho}\right)^{\frac{2}{1+\epsilon}}\left(\frac{1}{\Theta^{PL}}\right)^{\frac{1}{1+\epsilon}}$$

The l.h.s. turns out to be always greater than 1 but for very low (and not realistic) values of σ, while the r.h.s. is always lower than 1 unless ρ and Θ^{PL} (which depends positively on ρ) are very low. Within a wide range of reasonable values for parameters the inequality turns out to be satisfied. ■

While the effects of ζ and ϵ are quite intuitive and confirms what found for the PE regime, σ has a positive effect on the E/g ratio while having a negative one in PE (see Proposition 8). A larger σ still implies a greater propensity for living longer despite giving up some consumption; this objective however is pursued by the planner through the health input with more long-lasting effects, i.e. E. In PE instead, because of political support, a greater propensity for living longer does come mainly through health care expenditure.

Moreover, the E/g ratio turns out to be larger in PL for a sufficiently high ρ. The intuition behind this result is related to the fact that improving environmental quality does take more time (with respect to g) to be effective as a determinant of health but then does produce long-lasting effects. Hence, the lower is discounted the future (the larger ρ) the greater is the relative advantage of improving environmental quality; while when the weight of current generations is high, a larger g is chosen. Unless the planner discounts heavily the future, in the long run the environmental quality will be higher in PL than in PE, where the optimization problem has a shorter horizon and it is constrained by the political claims of electors who care more for g, as a more immediately effective way to have good health.

3.6 Numerical Analysis

In this section some restrictions made in Assumption 3 are relaxed and the steady states in the two regimes are studied numerically. In this way, on one hand robustness of results obtained under Assumption 3 can be checked under more general functional forms for π and the law of motion of E, and on the other hand it can be assessed the effect of relevant parameters which were neglected for analytical tractability, such as the degree of annuity markets' completeness γ and the degree of substitutability between g and E.

Therefore, first, the strict linearity of the maintenance technology is relaxed by allowing for decreasing returns

$$\nu(m) = \zeta m^{\nu} \quad \zeta > \eta > 0,\ 0 < \nu \leq 1 \tag{3.85}$$

whenever $\nu < 1$, eq. (3.85) implies that an intertemporal equilibrium with zero maintenance cannot be optimal in either cases. In steady state, in both regimes, maintenance is given by

$$m = \left(\frac{\eta}{\zeta} A k^{\alpha}\right)^{1/\nu}$$

clearly, the particular case $\nu = 1$ yields eq. (3.44).

Then, the simplification of a Cobb Douglas survival function π is replaced in favor of a constant elasticity of substitution (CES) function for the health condition h[51]

$$\pi(E,g) = \frac{\lambda + h(E,g)}{1 + h(E,g)} \quad \lambda \in (0,1); \tag{3.86}$$

$$h(E,g) \equiv B\Big(\beta g^{-\theta} + (1-\beta)E^{-\theta}\Big)^{-1/\theta}; \; B > 0, \theta > -1 \tag{3.87}$$

The function $\pi()$ in eq. (3.86) is between 0 and 1, increasing and concave in the health condition h, which is in turn a CES function in g and E as eq. (3.87) shows. Differently from the specification in Assumption 3.3, eq. (3.86) satisfies all the requirements given in Properties 1. The parameter θ is related to the elasticity of substitution, which is equal to $\frac{1}{1+\theta}$.[52] Under this specification

$$\frac{\pi'_E}{\pi'_g} = \frac{h'_E}{h'_g} = \frac{1-\beta}{\beta}\left(\frac{g}{E}\right)^{1+\theta} \tag{3.88}$$

The elasticity of π with respect to each input is no more constant as under Assumption 3.3, in fact it is eventually decreasing in the respective argument, as implied by Properties 1.

As for the utility and the production functions, which do not play a central role in the model, the forms specified in Assumption 1.3 and 3.2 are kept. Given these explicit functional forms, the next step in order to perform a numerical analysis is assigning reasonable values to the parameters.

Numerical Experiments

As for the choice of numerical values for parameters, we proceed as follows. Since with no annuity markets ($\gamma = 0$), capital and maintenance can be determined autonomously, and since the evidence for annuity markets is of

[51] Assumption 3.3 was useful for analytical tractability, but it required to define a ϖ small enough to ensure π to be between 0 and 1. Under eq. (3.86), π is always between 0 and 1, increasing and concave in its inputs, which implies that elasticity to each input is eventually decreasing.

[52] The function expressed in eq. (3.86) is a particular case of that proposed by Blackburn and Cipriani (1998).

limited completeness,[53] we consider the extreme case $\gamma = 0$, which allows us to focus on the following technological parameters only: η, ν, A and α.

Table 3.12: Production and Environmental Parameters.

A	α	η	ν
100	0.3	0.01	0.8

The coefficient α is set as standard in literature, A is a chosen as a scale parameter. Return on maintenance ν and the polluting conversion factor η are such that the steady-state ratio m/y ratio is equal to a realistic value.[54] The other parameters are given the values as in Table 3.13.

Table 3.13: Health and Preference Parameters.

λ	θ	β	B	σ
.1	3	.6	.25	.5

The parameter λ represents the baseline survival probability throughout the second period, even with a very bad health. A low λ implies that a good health status is important to live longer; λ is set equal to 0.1. Assuming a period lasting 30 years, this implies a minimum life expectancy at the end of the first period of 3 years.[55] The parameter θ can range from -1 (perfect substitutability) to $+\infty$ (perfect complementarity), the Cobb-Douglas technology is equivalent to $\theta \to 1$. It seems reasonable to assume that health care expenditure and environmental quality are only imperfect substitutes and exhibit some complementarity to effectively improve health status. We take a value of 3 and see what happens by allowing for higher substitutability. The technical coefficient β for the health status is given a value of 0.6, which reflects that the health care expenditure has a greater weight in determining the health status; moreover it contributes to yield a realistic g/y ratio. Regarding σ, following Ehrlich and Lui (1991) we set $\sigma = 1/2$ in the benchmark case, which represents also the case of a quadratic utility function.[56] The parameter γ takes the benchmark value of 0.165,

[53] See Section 3.4.4 and footnote 41.

[54] In the literature linear maintenance functions have often been used for the sake of simplicity. It is hence preferable not to choose a too low value for ν.

[55] Assuming 20 years before the working age, this implies a floor of life expectancy of 53 years.

[56] Pestieau, Ponthiere, and Sato (2006) based on a survey by Browning, Hansen, and Heckman (1999) set a parameter analogous to our σ to 0.8. We assess what happens for larger σ when we perturb parameters from the baseline calibration.

the same used in Pecchenino and Pollard (1997): this reflects the value of private pension funds as a percentage of US household net wealth.[57] Finally, B is set as a scale parameter at 2.5 in order to fit realistic values for life expectancy and g/y ratio. Looking at the first row of Table 3.14, we can see that under this baseline parameterization the g/y ratio is at 10.13% which is reasonably close to the average of 11% in OECD high-income countries according to World Development Indicator (see World Bank (2007)). The life expectancy π is at 87.76%, which - by assuming each period made up of 30 years and about 20 years before entering the working age - implies a life expectancy of about 76-77, which is also fairly close to the average of 79 in OECD high-income countries according to World Development Indicator (see World Bank (2007)). The share of environmental maintenance over GDP is at about 1.27%, which is also realistic compared to available data for OECD countries.

The effect of perturbing parameters

Given the baseline specification, we perform a numerical comparison by changing the value for each parameter with respect to the benchmark. We consider a 20% shock on the benchmark value and address the impact on the variables of interest, which are primarily: health care's share of GDP, maintenance's share of GDP, lifespan, capital, the environment and welfare. Clearly the interest is mainly from a qualitative point of view rather than quantitive. Table 3.14 summarizes the results.

Table 3.14: The effect of perturbing parameters in the PE case

	g	E	k	m	π	$\pi g/y$ (%)	m/y (%)	U
Benchmark	29.88	21.81	23.8	3.28	0.88	10.13	1.27	26.86
$\gamma(+)$	-0.02%	-0.20%	0.65%	0.24%	-0.02%	-0.23%	0.05%	0.08%
$\theta(-)$	2.85%	-2.67%	-0.01%	0.00%	0.03%	2.88%	0.00%	-0.17%
$\eta(+)$	-0.87%	-2.00%	0.05%	25.62%	-0.19%	-1.07%	25.60%	-0.23%
$\nu = 1$	6.65%	19.80%	-0.39%	-21.26%	1.52%	8.40%	-21.17%	0.35%
$\sigma(+)$	13.63%	11.84%	-0.35%	-0.13%	1.36%	15.30%	-0.03%	
$\beta(+)$	5.50%	-7.74%	-0.06%	-0.02%	0.25%	5.78%	0.00%	-0.25%
$\lambda(+)$	-1.36%	-1.37%	-0.04%	-0.01%	0.15%	-1.12%	0.00%	0.15%
$B(+)$	-8.09%	-8.16%	-0.29%	-0.11%	1.13%	-6.96%	-0.02%	0.98%
$A(+)$	13.83%	11.86%	29.29%	38.30%	1.37%	-10.98%	6.70%	15.38%

[57] See also Auerbach, Kotlikoff, and Weil (1992). All changes are slightly magnified but not reversed for larger γ.

First of all, let us focus on the effect of increasing the degree of annuity markets (γ): on one side π becomes lower, as well as the health care's share of GDP and E; on the other side capital, the maintenance's share of GDP and welfare are larger. The rationale behind these results is that, with an annuity market system, agents can be better compensated for mortality risk; instead with a lower γ agents do not benefit from any extra-return which compensates them by increasing consumption. Therefore, *ceteris paribus*, agents prefer to live longer and to consume less.[58] Since with a greater γ a lower π is preferred, both g and E must decrease accordingly, thus freeing resources for capital accumulation.[59] The effect of greater consumption offsets the one of shorter lifespan as the welfare improves.

Then, let us look at the θ. When health inputs become more substitutable (a decrease in θ), the health care expenditure increases. This intuitively occurs because if the substitutability between g and E is higher, then spending in g is less constrained by the necessity of keeping an optimal ratio with E: there is more room for substituting E with g. Since the weight of the elderly in government's decision is related to g, when it becomes easier to compensate environmental degradation with health care expenditure, the government has the political support to enlarge health care expenditure accepting a lower environmental quality (in other terms, it is very unpopular in this scenario to reduce g). In this case, the elderly - by supporting spending on g - increase both their lifespan and their political weight more effectively. On the contrary, when health care spending and environmental quality get more complementary, the substitution of E with g is less efficient. In summary, the steady state is basically determined by a series of temporary equilibria where health care spending finds higher support the higher is the possibility of substituting it with environmental degradation. The overall result is however negative in terms of welfare.

Let us consider than the impact of parameters such as η and ν, which, differently from those considered before, affects also the m/y ratio.[60] A more polluting production (higher η) implies that is more costly to have a good quality of the environment. The economy in the steady state features

[58]Since, as we have already remarked, the case $\gamma = 1$ can be regarded also as a model with certainty, another interesting implication is that, in the case of certainty, a lower lifespan would be achieved.

[59]Notice that a greater k requires, however, a larger m in steady state in order to offset pollution and keep E steady: this is why m increases and also m/y does because of decreasing returns in production.

[60]As the law of motion of environment exhibits strong persistence (see eq. (3.14)), once the steady-state level of E is achieved, keeping it constant simply consists in offsetting the polluting effect of production.

consequently a worse environment and people live shorter. Also health care expenditure is lower, since the possibility of using it as a substitute is limited. The m/y ratio has to be grater to offset pollution. Despite a greater level of capital, the net effect on welfare turns out to be negative. If return to maintenance increases so to have a linear technology $\nu = 1$, the principal effect is that a better environment can be kept with a lower m. The economy can then afford to live longer (at the cost of a small reduction of the steady-state capital) and the overall effect on welfare is positive.

Finally, as for the other parameters, it can be observed that the results obtained under Assumption 3 are substantially confirmed by the numerical study of the general case. For instance, an increase in the risk-aversion parameter or in the preference for consumption smoothing (a larger σ) makes the agents desire to live longer: lifespan is larger and this requires both larger g and E. Since more resources are absorbed to keep life expectancy high, less capital can be accumulated.[61]
A greater TFP A implies a greater steady-state capital. Maintenance should increase as well to offset the polluting effect: the m/y ratio turns out to be higher. The agents afford both longer lifespan and greater consumption. Despite the growth on health care expenditure, the share of g/y is lower because output increases by more. When the coefficient β increases, health care expenditure becomes more important in determining the health status. Accordingly, the steady-state g and the g/y ratio are higher, whilst E is lower. The steady-state level of lifespan is slightly higher and it is achieved mainly by health care spending, but the welfare is lower. An improvement in the exogenous minimum lifespan parameter λ allows agents to afford higher life expectancy with less g and E. Since people live longer more resources are absorbed and there is less capital accumulation, but the overall impact on welfare turns out to be positive. The effect of an improvement in the health technology B is analogous: in the steady state agents live longer and are better off, by affording that level of life expectancy with a lower investment in health expenditure and environment.

The Planner Solution

In considering the PL case, our first objective is to compare the benchmark results under the baseline specification with the PE case. Hence, we keep the same values in Table 3.12 and 3.13; however in PL there is no need to specify any value for γ, but a new parameter must be fixed: we solve this issue by fixing ρ in order to match the same lifespan of the PL economy in

[61]Since σ directly affects the utility function, for this parameter it makes no sense comparing the welfare as the functions on which it is computed are inherently different.

the benchmark case. It turns out that this value is about 0.85. We also look at the effect of a larger ρ: from Section 3.5.2, a larger ρ carries out a higher level of capital since the planner discounts less future generations.[62] Table 3.15 summarizes the effect in PL.

Table 3.15: The effect of perturbing parameters in the PL case

	g	E	k	m	π	$\pi g/y$ (%)	m/y (%)	U
Benchmark	24.1	28	99.0	5.6	0.88	5.33	1.41	30.85
$\theta(-)$	-0.81%	1.85%	0.00%	0.00%	0.02%	-0.79%	0.00%	0.04%
$\eta(+)$	-0.61%	-1.75%	-0.65%	25.29%	-0.11%	-0.53%	25.53%	-0.32%
$\nu = 1$	3.84%	19.93%	1.11%	-28.91%	0.87%	4.40%	-29.15%	0.59%
$\sigma(+)$	22.57%	23.26%	0.00%	0.00%	2.29%	25.38%	0.00%	
$\beta(+)$	4.04%	-9.10%	0.00%	0.00%	-0.10%	3.93%	0.00%	-0.20%
$\lambda(+)$	-1.34%	-1.30%	0.00%	0.00%	0.15%	-1.19%	0.00%	0.12%
$B(+)$	-7.97%	-7.70%	0.00%	0.00%	1.16%	-6.89%	0.00%	0.85%
$A(+)$	14.60%	13.18%	29.53%	38.39%	1.52%	-10.29%	6.71%	15.15%
$\rho = .9$	3.30%	16.89%	9.22%	3.36%	0.76%	1.37%	0.66%	0.47%

Let us begin by focusing on the benchmark: it is confirmed that the PL solution provides the same steady-state lifespan by using more environmental quality and less health care expenditure. This allows to free resource for capital accumulation, therefore capital is larger in the PL case and, consequently, m must be larger in PL in order to offset pollution. Given that lifespan is the same but consumption is larger, welfare is higher in the PL case, which is not surprising given that it represents the first-best solution.

By perturbing parameters, it can be seen that changes are generally all in the same direction as in Table 3.14 but with some remarkable exceptions. In particular, increasing the substitutability between E and g (lower θ) in the PL case yields a higher value for E, a lower g and a lower share for health care expenditure. In other words, the planner substitutes g with a better environmental quality E. This occurs because the planner has a horizon that comprehends all the future generations, whereas in PE the objectives are relatively short-termed and there always exists a strong demand for g by the currently living old.[63] On the contrary, in PE the greater demand for g

[62] Note that this result was obtained with no need for Assumption 3 and *a fortiori* must still be true.

[63] Notice that, after a reduction in θ, the planner can increase welfare despite having a (small) decrease in life expectancy.

enlarges life expectancy but the overall effect on welfare is negative because of the greater reduction in consumption.

Another interesting discrepancy is represented by the effect of a greater η. As known from the analytical study, in PL an increase in η implies a reduction in the steady-state level of k, since the polluting effect of production is taken into account (see eq. (3.69)). In PE, instead, capital accumulation is not decreased; it actually increases. In addition, when ν is greater, in PL the increase in environmental quality is larger, while the increase in health care expenditure is lower than in the PE case. In both regimes a lower m/y ratio is possible but, whereas in the PE case the health care's share of GDP increases (thus implying a larger lifespan), this occurs by less in the PL case. The overall improvement in welfare is larger in the PL case. This happens because in the PE case a more effective environmental technology (combined with the larger demand for health care expenditure) pushes higher both lifespan and health care spending.

Among other parameters, let us consider σ: its impact is much larger in PL. This confirms a finding implied by Proposition 9, i.e. in PL it is indeed mainly σ to determine the allocation of resources with respect to the trade-off between consumption and lifespan, whereas in PE there is a further element that influences the allocation of resources toward health: this element is related to the political mechanism and it consists in increasing g as the old part of the population increases. Hence π is fairly less sensitive to σ in PE than it is in PL.

Technology-related parameters β, λ, B and A have a similar effect to the PE case, with the only exception of the impact of β on lifespan, which in the PL case is negative. This happens because the PL steady-state lifespan is obtained through a more intensive use of environmental quality: if health becomes more important, then the planner finds optimal to accept a slight reduction in life expectancy (notice that the reduction in welfare is in fact lower in the PL case). Finally, let us assess the effect of a greater ρ: when the weight assigned to future generations is larger, we find not surprisingly a greater k (and consequently a greater m), the level of π is improved as well through a larger g and above all E; the net effect on welfare is therefore clearly positive.[64]

[64]The steady-state share of health care expenditure and maintenance are both larger. Notice that in PL case we observe no effect on k and m of many parameters since, for fixed ρ, only productivity (A) and environmental parameters (η, ν) concur to determine them.

3.7 Conclusions

Health care expenditure and a good environmental quality contribute to improve agents' health conditions. However the relative importance of the two inputs seems to be age-based as the care for environmental quality appears to be higher among the young, while the support for health care expenditure is usually greater among the elderly part of population. This is likely to be related to the fact that the positive effect of environmental maintenance takes generally more time to be effective, but then it can last for a longer time. In the light of that, we have tackled the issue of whether and how an economy where public expenditure on health care and environmental maintenance (as well as taxation) are voted by agents departs from the solution that a social planner would implement.

In order to do that we have proposed a model where agents' longevity is endogenously dependent on health, which is determined by both environmental conditions and health care expenditure. In the political economy regime, health care expenditure and the environmental maintenance are provided by the government and decided through (probabilistic) voting, but the young and the elderly have different preferences towards the composition of public spending: health care expenditure finds larger support among the latter, while environmental maintenance among the former. As lifespan increases, the size of the elderly electorate enlarges as well, thus affecting the policies resulting from voting. In the social planner regime, the variables are chosen such as to maximize an utilitarian social welfare function consisting of the discounted sum of present and future individuals' lifetime utilities.

For a sufficiently low discount rate, the relative intensity in the use of environmental quality rather than health care expenditure in order to accomplish a targeted lifespan is larger in the planner regime, while the political economy sustains a more intensive use of health care expenditure. A crucial role is played by the political mechanism, which makes endogenously high the support toward health care expenditure in an aging society. This is also reflected in an allocation of resources between health care expenditure and consumption more biased towards the former in the political economy. By means of explicit functional forms and realistic calibration of parameters we have assessed numerically the impact of several factors: for instance, the degree of annuity market is found to reduce the health care's share of GDP, while the substitutability of health inputs tends to increase health care expenditure in the voting economy and to decrease it in the planner regime. Moreover we have found that for a sufficiently low discount rate the planner solution is welfare-improving.

Extensions of our model are possible in various directions, which can be suggested for further research. The assumption of a public health care system is congruous with most countries's health system and has allowed us to abstract from individual decision-making. Nevertheless, it seems promising to extend the model to encompass the case of a private health care system. Moreover, we have focused only on steady-state comparison, but the study of transitional dynamics seems interesting as well, in particular under the political economy set-up. In addition, the model has implications with respect to several factors, such as consumers' risk aversion, development of annuity markets, and technology of environmental cleaning and health production, which can be tested through careful empirical investigations.

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