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Niccolò Tartaglia
Re–Thinking the Role Played by Science of Weights in the Sixteenth–Century.
Selections from *Quesiti et inventioni diverse*:
Books VII–VIII

Raffaele PISANO

Supervisor
Prof. Enrico GIANNETTO, University of Bergamo, Italy

Co-Supervisor
Associate Prof. Rémi FRANCKOWIAK, University of Lille 1, France

University of Bergamo, Italy

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Preface

Monsieur Raffaele Pisano présente ici une thèse de Doctorat de l'Université de Bergame intitulée « Niccolò Tartaglia : Re–Thinking the Role Played by Science of Weights in the Sixteenth–Century. Selections from *Quesiti et inventioni diverse*: Books VII–VIII ». Ce travail de recherche, magistralement mené durant trois années académiques (2012-2014), a été dirigé par le Professeur Enrico Giannetto de l'Université de Bergame et co-encadré par moi-même, Rémi Franckowiak, Maître de Conférences HDR à l'Université de Lille 1, France. C'est un travail d'une grande rigueur et d'un intérêt scientifique incontestable qui s'inscrit dans un champ de recherche plus large travaillé par M. Pisano, à savoir l'histoire de la mécanique d'Archimède à Torricelli. . Une partie de la thèse fera l'objet d'un développement qui sera prochainement publié chez Springer, en collaboration avec Danilo Capecchi de l'Université de Rome, La Sapienza.

Il me faut, avant toute chose, préciser le cadre dans lequel a été mené ce travail. M. Raffaele Pisano a débuté sa thèse à Bergame avant d'être accueilli come doctorant–enseignant depuis deux ans au Centre d'Histoire des Sciences et d'Épistémologie de l'Université de Lille 1 – Sciences et Technologies où il mène des recherches en histoire des sciences et en didactique de la physique, et enseigne l'histoire de la physique à différents niveaux de la Licence de Sciences Exactes et de Sciences pour l'Ingénieur, et développe des réflexions sur les sciences en société dans le Master « Journaliste et Scientifique » (partenariat entre l'Université de Lille 1 et l'École Supérieure de Journalisme de Lille). C'est ainsi, en tant qu'Attaché Temporaire d'Enseignement et de Recherche à Lille mais toujours en relation et dans l'échange avec M. le Prof. Giannetto, que M. Pisano a poursuivi et achevé sa thèse de doctorat. J'ajouterai que M. Pisano, bien que très investi dans la rédaction de son travail doctoral, a fait preuve de beaucoup de disponibilité, de pédagogie, d'initiative dans tous ses enseignements, et d'une très forte implication dans les thèmes de recherche du laboratoire. C'est ainsi, par exemple, qu'il a encadré les mémoires de 'découverte de la recherche' d'étudiants inscrits en 3^{è} année de Licence de Physique Fondamentale, développé un enseignement spécifique en épistémologie à destination d'étudiants désirant devenir professeurs des écoles, mis sur pied un workshop avec le CERN de Genève dans la foulée de la découverte du boson de Higgs, développé des partenariats internationaux (Lituanie, Slovénie, Belgique) pour le compte du laboratoire, mis en place une école d'été internationale sur la didactique de la physique, co-organisé des sessions de colloque internationaux et co-rédigé des articles et posters avec moi-même et d'autres membres du laboratoire ; tout cela s'ajoutant aux activités de recherche et d'édition propres à M. Pisano. Aussi le mérite de M. Pisano quant au travail qu'il nous présente ici – et je souhaite bien insister sur ce point – est-il d'autant plus grand. Je crois pouvoir dire qu'il est devenu une force pour notre équipe.

La présente thèse de Doctorat concerne de manière générale l'épistémologie et l'histoire des sciences des XV^{e} et XVI^{e} siècles, en particulier les relations entre physique, géométrie et mathématiques. Inscrit dans la tradition de l'épistémologie historique des sciences, M. Pisano est préoccupé par la question des fondements de la science ; ce qui l'amène à proposer quelques clés d'interprétation du développement de la science à la Renaissance en prenant fermement appui sur des faits historiques replacés dans leur contexte et des documents tout à fait originaux. La thèse est articulée autour de quatre parties : la première est une présentation biographique de Niccolò Tartaglia et de l'imbrication de la science et de la technique dans la société dans laquelle celui-ci évoluait. La seconde représente le cœur même du travail de M. Pisano ; il s'agit de l'examen – inédit – à la fois historique et épistémologique des Livres VII et VIII des *Quesiti et inventioni diverse* de 1554 sur lesquels se concentrent précisément sa recherche doctorale. La troisième partie est constituée, à la suite de quelques considérations philologiques générales mais aussi plus précises quant à la langue employée par Tartaglia, du *fac-simile*, de la traduction en anglais avec reconstitution des figures, et de la transcription en italien, accompagnées de leur appareil critique, des deux livres cités. On y trouve également la version originale et les traduction et transcription critiques, pour le premier cas en anglais et pour le second en latin, du *Iordani opusculum de ponderositate* de 1565, texte publié de manière posthume par l'éditeur de Tartaglia, Curtio Troiano, dont la source serait un manuscrit de Jordanus de Nemore, augmenté de corrections et de figures de la main de Tartaglia et qui aurait fortement inspiré le Livre VIII de son traité *Quesiti et inventioni diverse*. La dernière partie présente une recherche très originale sur les multiples éditions des *Quesiti* jusqu'en 2010. Enfin, une très complète bibliographie clôt ce volumineux travail de thèse.

M. Raffaele Pisano présente la mécanique des XV^{e} et XVI^{e} siècles comme relevant de nos jours de la statique. Elle était alors nommée la *Scientia de ponderibus* et était appréhendée suivant deux points de vue ; Tartaglia – de son vrai nom Niccolò Fontana (1500 ?-1557) – tenant pour sa part, dans son traité *Quesiti et inventioni diverse* (1546 pour la première édition à Venise), une position médiane entre une approche aristotélicienne, c'est-à-dire celle considérant l'équilibre d'un corps comme le résultat de deux tendances opposées se contrecarrant l'une l'autre, et une approche archimédienne, c'est-à-dire celle assimilant le problème de l'équilibre à l'évaluation d'un centre de gravité pour le corps considéré. Voilà la thèse présentée et développée avec une grande rigueur par M. Pisano dans son travail qui se concentre plus particulièrement sur les Livres VII et VIII, jusqu'à présent jamais sérieusement étudiés, de l'ouvrage en italien (qui contient 9 livres) de Tartaglia qui mêle en réalité, sous forme d'un dialogue entre personnes issues de différentes catégories sociales et avec la volonté de rapprocher science et pratique, physique, architecture, géométrie et mathématiques ; ces deux livres concernent pour leur part davantage la statique.

Tartaglia est loin d'être un inconnu. Et M. Pisano le rappelle très bien en consacrant une partie de son travail à une minutieuse biographie de ce personnage, couvrant aussi bien les étapes de sa vie que ses productions intellectuelles. Celui-ci est entre autres l'auteur de deux autres ouvrages (*Nova scientia*, 1537-1550, et *General trattato di numeri et misure*, 1556-1560) sur des questions de physique et de mathématiques, y compris dans leurs applications à la balistique, à l'architecture et la localisation de fortifications ainsi qu'à la mesure des calibres. On a aussi retenu son nom pour la résolution d'équations du 3e degré, ses échanges avec Cardan et l'édition en italien des *Éléments* d'Euclide. Son parcours scientifique est parfaitement analysé par l'auteur de la présente thèse qui n'oublie pas de le resituer dans le courant conceptuel de la Renaissance. Je soulignerai ici le passionnant passage concernant les études balistiques de la *Nova Scienta* de Tartaglia qui, au lieu de s'inscrire dans la physique aristotélicienne, fait plutôt le choix – innovant – de suivre la tradition archimédienne faisant pour le coup de la *Nova Scienta* un ouvrage sur une science réellement nouvelle. Il est, par ailleurs, à noter que M. Pisano a la délicatesse de rendre ici hommage à Arnaldo Masotti, l'éditeur peu connu des écrits de Tartaglia au XXe siècle, grâce auquel Tartaglia nous est pour une bonne part accessible ; nous rappelant ainsi, par la même occasion, tout l'intérêt que revêt encore aujourd'hui Tartaglia pour l'histoire des sciences.

Le chapitre 1.5 de la partie 1 amène l'analyse des Livres VII et VIII des *Quesiti et inventioni diverse* de la partie suivante de la thèse. Les *Quesiti*, qui traitent de sujets allant des applications aux arts militaires (tirs d'artillerie, dimension des boulets, poudre à canon, armes à feu, tactiques d'infanterie, données topographiques) de la mécanique à la question de l'équilibre des corps, la théorie des centres de gravité et des équations du 3e degré, en passant par des considérations sur les fortifications, sont édités en italien à deux reprises du vivant de Tartaglia (1546 et 1554) puis une fois de manière posthume en 1562. Ils ont la particularité d'avoir été dédicacés au roi Henri VII d'Angleterre, excommunié en 1533 (les contacts entre Tartaglia et son élève, Richard Wentworth, expliqueraient pour une part ce choix). Un grand nombre d'élèves, de savants et autres correspondants sont cités dans le traité. M. Pisano les répertorie tous, avant de s'arrêter plus longuement sur des points philologiques de son travail et sur une étude comparée des différentes éditions des *Quesiti* et en relation avec les autres écrits de Tartaglia, l'amenant à poser quelques hypothèses sur le développement historique de la mécanique à la Renaissance ; tout cela ouvrant tout naturellement sur le chapitre suivant qui expose dans un temps long le passage à la Renaissance des conceptions anciennes de la statique aux modernes, par une contextualisation partant des savoirs du Moyen Age et de l'apport des Arabes au renouveau de l'Humanisme qui voit émerger un nouveau type de savants : l'ingénieur (c'est alors l'occasion pour l'auteur de la thèse d'exposer la science des poids de Léonard de Vinci). Le chapitre est important puisque M. Raffaele Pisano montre parfaitement la part capitale que prend Tartaglia dans la transition entre la science des poids et la statique moderne.

Avec la partie 2 de la thèse, nous entrons dans le vif du sujet. Sur une centaine de pages, les Livres VII et VIII des *Quesiti et inventioni diverse* de Niccolò Tartaglia sont scrupuleusement étudiés. La discussion est serrée et est menée sur deux plans : historique et épistémologique. M. Pisano se concentre sur le raisonnement proposé par Tartaglia à l'encontre des arguments aristotéliciens développés dans les *Problemata mechanica* au sujet de la précision et de la stabilité d'un équilibre (Livre VII) et sur les principes de la science des poids (Livre VIII). Ce dernier point est mis en regard avec le corpus de Jordanus de Nemore sur la science des poids afin d'évaluer l'étendue et la nature de la connaissance de la statique moderne à la Renaissance ; M. Pisano pouvant alors discuter des relations entre physique et mathématiques (observations physiques et modèle mécanique théorique). L'édition commentée des textes étudiés nous est alors offerte, dans la partie suivante du travail, dans leur forme originale, leur traduction anglaise et leur transcription italienne.

Voilà une très belle thèse de Doctorat menée, à partir de sources documentaires originales, avec toute la rigueur exigée à ce niveau.

M. Raffaele Pisano fait par ailleurs la preuve de sa parfaite maîtrise de l'histoire et de l'épistémologie des sciences – physique, géométrie et mathématiques – de la Renaissance ainsi que de sa connaissance des arts mécaniques et du contexte culturel de l'époque. Je terminerai en précisant qu'il m'a été très agréable de participer à l'encadrement de ce travail.

Rémi Franckowiak
Université de Lille 1, Villeneuve d'Ascq
Octobre 2014

Acknowledgments

The genesis of such a lengthy book has deep roots, and the result has been a long time in the making. Therefore, to all the directors and staff members of libraries and archives cited within the book, I express my profound appreciation for their collaboration.

I express my warm and pleasant gratitude to Claudia Masotti for her friendly homages of Uncle Arnaldo Masotti's images. I thank Paolo Bussotti (Berlin Alexander von Humboldt Foundation, Germany), Gérard Hamon (IREM Rennes, France) and Lucette Degryse (University of Boulogne–sur–Mer, France) for their friendly readings and collaboration. Further, Caroline Duroselle–Melish (Harvard Printing and Graphic Arts Department, USA), Tricia Buckingham (Bodleian Oxford Libraries, UK), Marie–Lise Faget (Service Patrimoine Bibliothèque de Bordeaux, France), Hermann Hunger (Österreichischen Akademie der Wissenschaften, Austria) and Luigi Pizzamiglio (Biblioteca Carlo Viganò e Fondo Tartaglia, Italy) for their collaboration with manuscripts and editions.

I want to thank Prof. Enrico Giannetto (University of Bergamo, Italy), Prof. Rémi Franckowiak (University of Lille 1, France) and Dr. Davide Cremaschi (University of Bergamo, Italy) for their professional and friendly collaboration, respectively as tutor, co-tutor and administrative doctoral coordinator.

Finally, of great importance, I address my acknowledgments to Marco Ceccarelli (University of Cassino, Italy), Nathalie Jacobs and Anneke Pot (Springer book Series Editor, The Netherland) for their good positive reception of my editorial project (with Danilo Capecchi, University of Roma La Sapeinza, Italy) on Tartaglia's science of weights and mechanics as new forthcoming book (2014. *Tartaglia's Science of Weights and Mechanics in Sixteenth–Century. Selections from Quesiti et inventioni diverse: Books VII–VIII*) which rest on my left side when writing this dissertation.

Lille (France)–Bergamo (Italy)
2014, October

Remarks for the Reader

This Ph.D. Thesis is devoted to the history and historical epistemology of science, in particular to the fields of geometry, mathematics, physics and Western civilization of the 15^{th}–16^{th} centuries). The latter is mainly viewed as a branch of the combined history of science, related to the history of foundations of sciences. I have convinced it as an integrated history and epistemology of scientific methods, combining epistemological and historical approaches to clearly identify significant historical hypotheses. I contend that such hypotheses should always be subject to epistemological interpretation by means of declared keys of investigations based on historical facts of scientific activities and original documents that trace their historical development. For, bibliographical references, the relationships between physics–mathematics and physics–geometry, and the role played by science in context are strongly stressed.

In order to recall Masotti's edition both "Tartaglia 1554" and "Tartaglia [1554] 1959" are cited. In the References section both "de Nemore 1565" and "Tartaglia 1565", as editor, are listed for the readers convenience. Both the names "Galileo" and "Galilei" are used to recognise their international adoption.

The Volume is many pages long so I have relied on numerous recalls of dates and names to help guide the reader to correct documents.

For the English translations of the Tartaglia's text I assumed as a model – with several technical variations – that of Stillman Drake (Drake and Drabkin 1969) and seldom Marshall Clagett (Moody and Clagett 1952; Brown 1967–1968; Clagett 1959). They were of most helpful.

In order to make the reader comfortable reading in composite Latin, *vulgare*, Italian and English languages presented in the book, never losing historical rigour, I made some choices for multiple forms of names (e.g., Nicolo–Nicolò–Niccolò) and subjects (e.g., quaestio–questions–propositions). I conserved the original style of numeration to identify chapters (e.g., XIIII, XIX, etc.)

I have dedicated one chapter to original texts. In order to present fac–simile texts, transcriptions and translations, to best advantage, my critical comments are reported in endnotes, as well.

In the perspective of the Springer forthcoming book on Tartaglia (2014, as above cited in the *Acknowledgments* section) some parts of this dissertation were performed in collaboration with Danilo Capecchi (University La Sapienza, Italy) my friend and co-author of numerous papers and books in the latest ten years.

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Introduction

The practice of science, as well as its history, has for centuries been a leading component of the scholarly work of both the Eastern and Western world. The results of these efforts have mainly depended on individual scientific and disciplinary ambitions that lead to their technological innovations. Scientific traditions over the years and contributions by these scientists created a scientific framework in which to interpret celestial and terrestrial phenomena.

The development of astronomy, geometry, physics, mathematics, and science, generally speaking, is also a social phenomenon because it is influenced both by the needs of the labour market and by the basic knowledge of laws of nature. Therefore, the way in which science is framed changes according to modifications of the social environment and the attribute referred to as “know–how”.

In the period considered in the book in Europe, a series of wars required new financial supports and new knowledge. Moving of soldiers from one country to another permitted the spread of know–how and competence in practices that were necessary for these people to be recruited: i.e., *Tercio* in Spain, *Légion* in France, and *Regiment* in England. For this reason, and among many social factors, the military literature of the 16th and 17th century was particularly rich (fortifications, strategy, weapons, etc.). The organization and production of gunpowder evidently created a bridge towards structured recruitments, army training and attack–defences strategies. Therefore, a certain body of knowledge started to spread within early military handbooks (constructions and maintenance of war machines, mathematical and geometrical rules for weapons, battle projects, Pythagorean tables, fortifications projects, measurements and devices, etc.) in which a minimum of mathematical (calculus) basic education was required. For that reason, the scientific education of the solders & gunners played an important role within the art of war. In the beginning, this social dynamic was randomly undefined and only later became more structured. A prime example was one of the first organized English military education schools, *Honourable Artillery Company* (1087; 1537). The company built its first *Armoury House in London* at the site of the *Old Artillery Gardens* (1622). Consequently, mathematical education and early physical arguments were provided for *Fire Master* and *Master Gunner* abilities. The latter were busy with deployment of cannon, as well as both practical and technological considerations: i.e., brass rather than iron cannonballs, geometrical dimension of a cannon's mouth, angle of fire, use of instruments (i.e., Tartaglia' *quadrante*). Traditions of families of Italian metalworkers such as Alberghetti, Gioardi, Morando, Borgognoni et al., were representative of this expertise. Thus, standards were evidently sought due to previous unsatisfactory productions of, for example, replicating a series of cannonballs. As a result, a basic but complex scientific and applied knowledge (mathematical, geometrical, physical) was required because, as is still the case today, education in the field of weapons requires more than simply expertise in artillery schools. In my opinion, new advanced *geometrization* and *mathematization* of nature were, and still are, needed.

During the long period between the second half of the 12th century and the first half of the 16th century, Italian cities–states were among the most advanced countries with respect to economic structure and development of science. Fundamental to the opening of new perspectives in the development of science was however the development and spread of mathematical knowledge. Starting in the 13th century in some Italian regions, an organized mathematical education was developed connected to the prevailing economic and social structure. The way in which mathematics education was structured in Italy between the 13th and the end of the 15th century is significant and paradigmatic to highlight the influence society can have on education. Mathematical education was organized around the so–called *Scuole d'abaco* (*Abacus schools*). Their heritage was influential for mathematical education and important mathematicians who lived in the late Middle Ages and in the Renaissance. An emblematic case is that of Luca Pacioli (1445–1517) who, in turn, had a fundamental role in Leonardo da Vinci's (1452–1519) mathematical education. Furthermore the *Abacus schools* had connections with mathematicians such as Scipione dal Ferro (1465–1526), Niccolò Tartaglia (1499–1557), Gerolamo Cardano (1501–1576), Lodovico Ferrari (1522–1565), Rafael Bombelli da Bologna (1526–1572), who developed algebra and in particular studied the solutions of third and fourth degree equations. The relations among these mathematicians are significant from a scientific, social and anthropological point of view. The present book is concentrated on one of those mathematicians, Niccolò Tartaglia.

The writing of *dialogues* was not exclusive to Tartaglia. I have dedicated a section below to that topic (Chapter 4). Of further interest are his distinguished interlocutors, his *honorando* disciples, and anonymous personages such as a "pescatore" (fisherman), an "architettore" (architect), an “inzegnero” (engineer), and a “capo dei bombardieri” (artillerymen head), etc. Tartaglia’s language was not only a way to write differently from the official scientific language at that time (Latin), but it was a tentative effort to establish a closer relationship between the traditions of scientists and the traditions citizens, as well; quite correctly, Gosselin entitled his *L'Arithmétique de Nicolas Tartaglia Brescian, Grand Mathematicien, et Prince des Praticiens* (Gosselin [1578] 1613). In this sense, by including both amateurs and experts from other not necessarily scientific disciplines, he established clear evidence that the proposed “science–in–practice” would be subjected to sufficiently enquiring criticism from a wide–ranging set of perspectives. Thus, without using the current language of scientists, Tartaglia chose a simpler form of communication that is the *dialogue* (as both Plato and Lucian did in the Renaissance) between a specialist and a practitioner. There is ample evidence; e.g., at the beginning of the *Quesiti et invention diverse*, within the dedicatory letter to Henry VIII, King of England:

> Which thought made me wish (although I lack that eloquence and polish of speech which is requisite to the hearing of your Majesty) that these questions or inventions of mine, with their replies and solutions, might be offered and dedicated – not as something necessary to your Majesty (for indeed even things of profound learning, set forth and explained in elegant and lucid style, could not add to your Majesty’s high perfection; let alone these of mine, that are mechanical things, plebeian, and written, as spoken, in rough and low style) but only as new things – I offer them and dedicate them to you […][1]

and in the *General Trattato*:

> I am sure that many will be astonished why I wrote the above proportions, both in Latin, within the tradition of our ancient mathematicians, and vulgar, and vulgar and Latin together.[2]

The whole *Quesiti et inventioni diverse*, which is the main purpose of this book, is presented in the form of a dialogue; further, in *Book IX* (Tartaglia 1554, Pr. XXVII–XLII) an added method of communication appears, the epistolary. The questions among mathematicians evidently revolved

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[1] Tartaglia 1554, 4v; see also *Alli Lettori*, 3v. *Idem* in: Tartaglia 1546, 1v.

[2] Tartaglia 1556–1560, II, 103r. The translations is mine. See also many passages within Tartaglia’s answers in *I sei scritti di matematica disfida di Lodovico Ferrari coi sei contro–cartelli in risposta di Niccolò Tartaglia* (Tartaglia 1876, 2nd Tartaglia’s answer).

around the problem of solution of the third degree equation; often, the tune echoed mediaeval disputes.

The Ph.D. volume comprises four main parts within six main chapters.

At the beginning (Part I, chapter 1) biographical sketches and philological–historical–epistemological reflections are reported.

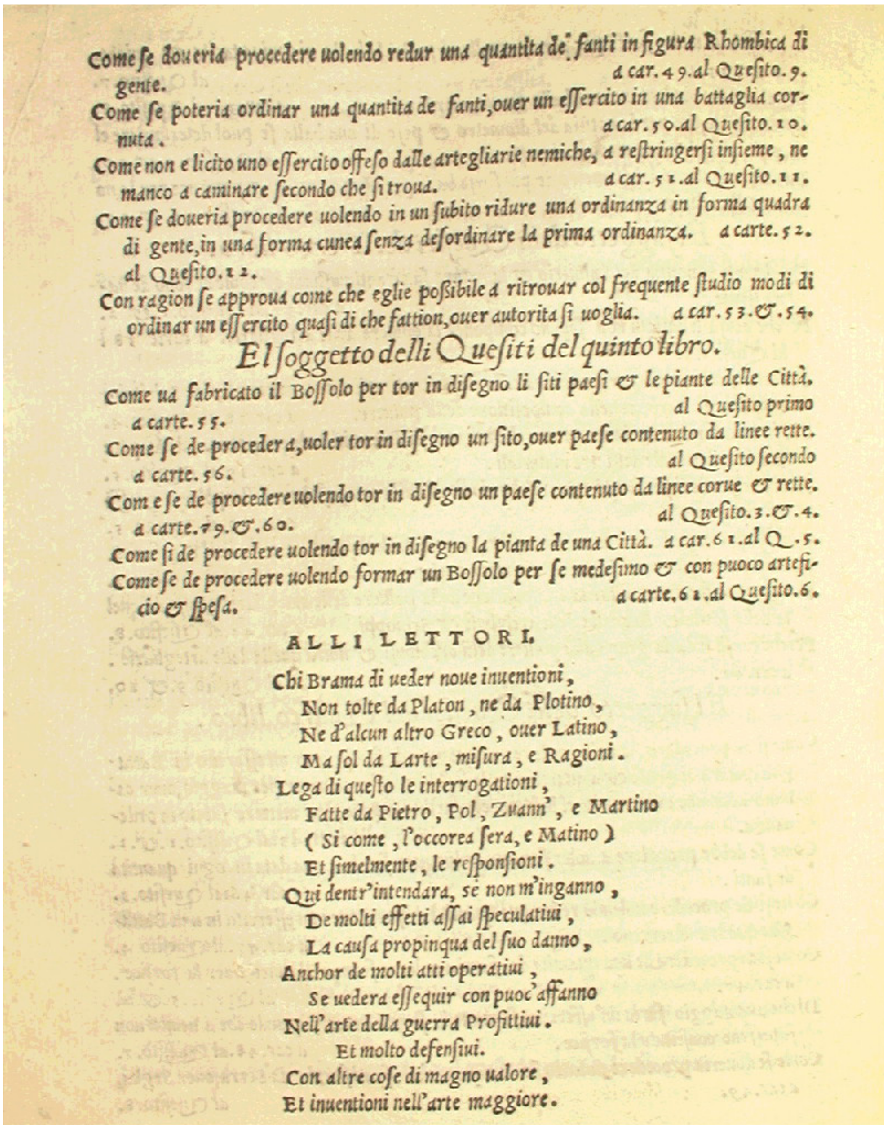
In the Chapter 2 (Part I) an historical account of S*cientia de ponderibus* (*Science of Weights*) and statics during ancient time and the Renaissance is presented. Consequently I extensively analyse Niccolò Tartaglia's Books VII and VIII of the *Quesiti et inventioni diverse* (Part II, chapter 3) from historical and epistemological standpoints. Particularly, this Chapter is also devoted to *historical epistemology of science* presenting an integrated history and epistemology of scientific methods, which combine epistemological and historical approaches to identify significant historical hypotheses within the relationship between physics and mathematics (physical observations and theoretical mechanical modeling).

In Chapter 4 (Part III) I report on translations into English and transcriptions of the main works studied for my research.

Part IV is composed of two chapters. In Chapter 5, I list foreign editions of *Quesiti et invention diverse* as a component of the history. Bibliographical notes and alleged editions are commented.

Finally, in Chapter 6, final remarks end the book. After the reference section, a list of main *Quesiti* accounts is presented.

My Tutor, Co-tutor and I think that the composition of the Ph.D. thesis makes absorbing reading for historians and philosophers of science, as well as for scientists themselves.



Come ſe doueria procedere uolendo redur una quantita de' fanti in figura Rhombica di gente. a car. 49. al Queſito. 9.

Come ſe poteria ordinar una quantita de fanti, ouer un eſſercito in una battaglia cornuta. a car. 50. al Queſito. 10.

Come non e licito uno eſſercito offeſo dalle arteglliarie nemiche, a reſtringerſi inſieme, ne manco a caminare ſecondo che ſi troua. a car. 51. al Queſito. 11.

Come ſe doueria procedere uolendo in un ſubito ridure una ordinanza in forma quadra di gente, in una forma cunea ſenza deſordinare la prima ordinanza. a carte. 52. al Queſito. 12.

Con ragion ſe approua come che eglie poßibile a ritrouar col frequente ſtudio modi di ordinar un eſſercito quaſi di che fattion, ouer autorita ſi uoglia. a car. 53. & . 54.

El ſoggetto delli Queſiti del quinto libro.

Come ua fabricato il Boſſolo per tor in diſegno li ſiti paeſi & le piante delle Città. a carte. 55. al Queſito primo

Come ſe de procedera, uoler tor in diſegno un ſito, ouer paeſe contenuto da linee rette. a carte. 56. al Queſito ſecondo

Com e ſe de procedere uolendo tor in diſegno un paeſe contenuto da linee corue & rette. a carte. 79. & . 60. al Queſito. 3. & . 4.

Come ſi de procedere uolendo tor in diſegno la pianta de una Città. a car. 61. al Q. 5.

Come ſe de procedere uolendo formar un Boſſolo per ſe medeſimo & con puoco artefi-cio & ſpeſa. a carte. 61. al Queſito. 6.

ALLI LETTORI.

Chi Brama di ueder noue inuentioni,
Non tolte da Platon, ne da Plotino,
Ne d'alcun altro Greco, ouer Latino,
Ma ſol da Larte, miſura, e Ragioni.
Lega di queſto le interrogationi,
Fatte da Pietro, Pol, Zuann', e Martino
(Si come, l'occorea ſera, e Matino)
Et ſimelmente, le reſponſioni.
Qui dentr'intendara, se non m'inganno,
De molti effetti aſſai ſpeculatiui,
La cauſa propinqua del ſuo danno,
Anchor de molti atti operatiui,
Se uedera eſſequir con puoc'affanno
Nell'arte della guerra Profittiui.
Et molto defenſiui.
Con altre coſe di magno ualore,
Et inuentioni nell'arte maggiore.

Tartaglia 1554, *Quesiti et inventioni diverse,* 3v

PART I

Biographical Sketches & Science in Context

Chapter 1

Niccolò Tartaglia and the Reinassance Society between Science and Technique

Ma poi fra me pensando un giorno, mi parve cosa biasimevole, vituperosa e crudele & degna di non puoca punitione appresso Iddio & alli uomini a voler studiare di assottigliare tal essercitio dannoso al prossimo, anzi destruttore della specie umana & massime de Cristiani in lor continue guerre.
(Tartaglia 1537, 4rv, line 37).

In this section, biographical sketches and philological–historical–epistemological reflections are reported. In particular, I present Tartaglia's study of mathematics, geometry, arithmetic, ballistics and fortifications.

1 Niccolò Fontana called Tartaglia

Tartaglia produced crucial contributions to mathematics, physics, and to the application of architecture scientific foundations of ballistics, criticism to Aristotle's lever, statics, the measurement of calibres and land surveying and fortifications. He discussed them principally in *General trattato di numeri et misure* (Venice, 1556–1560), *Nova scientia* (Venice, 1537, Books III; 1550 with a *Gionta* to 3rd Book) and in *Quesiti et inventioni diverse* (1546, 1554). He is also well–known for the resolution of third–degree equations and his *discussions* with Cardano and also as editor of the Italian translation of Euclid's *Elements* titled *Euclide Megarense* (Venice, 1543a). His contribution on *science of weights*–mechanics mainly concerns *Scientia de ponderibus*: Book VII recalls a question of *Mechanical Problems*, Book VIII is inspired by Book I of the *Liber Jordani de Nemore de ratione ponderis,* and is both an epitome and a paraphrase of it.

According to the title page of *Quesiti et inventioni diverse* Tartaglia was 45 years old.

1.1 Biographical and Scientific Sketches

Niccolò Tartaglia[1] was born in Brescia, and presumably (not historically proved) between the end of 1499 and the beginning of 1500, and died in Venice "[...] poor and alone [...]" (Masotti 1970–1980, 13, 259), during the night between the 13th and 14th of December 1557, "[...] in the *Calle del Sturion* near *Ponte di Rialto*.

In the Venezia notary's archive, a document (Filza 168.VII; N.119) exists. It includes his last will and testament (Boncompagni 1881) written on Friday 10th December 1557, by "Nicolai Tartalea Doctoris Mathematicarum" (*Ivi*). It notes the extact date of Tartaglia's death: "Obijt die Lune hora septima noctis. xiij xbris", that it is *the hour* (italic) *seventh of night* (midnight) *on Monday 13 towards Tuesday 14 December.*

In previous studies, Antonio Favaro (1847–1922) found a civil status certification (*Archivio di Stato di Verona*) attesting that the mathematician was 30 years old in 1529; thus, Tartaglia's date of birth was consequently inferred[2]. Concerning the date of death, it is indicated in his testament (10th

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[1] For a recent biographical *excursus* see Pizzamiglio (Pizzamiglio 2012).

[2] Cfr.: Favaro 1913, 335–372. See also: "Introduzione" by Masotti (Tartaglia 1554, XIX–XXII). A selected list of works on Tartaglia is reported in the Reference section.

December 1557), as subsequently added on by the Venetian notary Rocco de Benedetti (fl. 1556–1582) who also edited the certificate:

> MDLVII. Die Veneris Decimo m(ensi)s. Xbris […] objt. Die Lunae hora septima Noctis. Xiij. Xbris supti.[3]

The original testament states:

> I Nicolo Tartaglia Doctor of Mathematics [...] being now in bed diseased by a serious illness, list my personal belongings.[4]

He left his belongings to his heirs, including his publisher Curtio Troiano Navò[5], also called "Troian Navò librer all'insegna del Lion" (*Ivi*) and named "commissioner and executor of this my last testament [commissario et executor di questo mio ultimo testamento]" concerning his notes, manuscripts and latest books which had not yet been sold. According to his testament, at the end of 1557 (*Ivi*) Tartaglia had Parts I and II of his *General trattato di numeri et misure* published by Curtio Troiano Navò (1537–1566); and in 1556 he already had Parts III and IV as well, which were posthumously published in 1660. At this stage, Favaro (Favaro 1882, 32–32) contested a publishing problem[6] that concerned the title page and contents of the work: the replacement of the Parts III and IV belonging to the original title page and to the *colophon*, and other random pages with new pages (reporting dates, supposedly, 1556 or also 1557, as effectively is written in the *colophon* of Part IV, would attribute the publishing to Comin da Trino, in 1557), having posthumously dated the manuscript as being published in 1560, as most surviving specimanuscripts show.

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[3] Tartaglia 1554, XXII, footnote 5. The translation is above in the running text.

[4] "Io Nicolo Tartaia Dottor di Mathematice [...] ritrovandomi hora in letto aggravato da molto male, ho deliberato ordinar i fatti miei." (The translation is mine; see also Filza 168.VII; N.119; Boncompagni 1881).

[5] Curzio Troiano Navò (or de Navò) was one of the most important editors and book sellers during the 16th century in Venice. His French origins are not clear. Some historians report about a family–publishing composed of him and his brothers. They and their heirs edited and published ca. 30 books between 1537 and 1599.

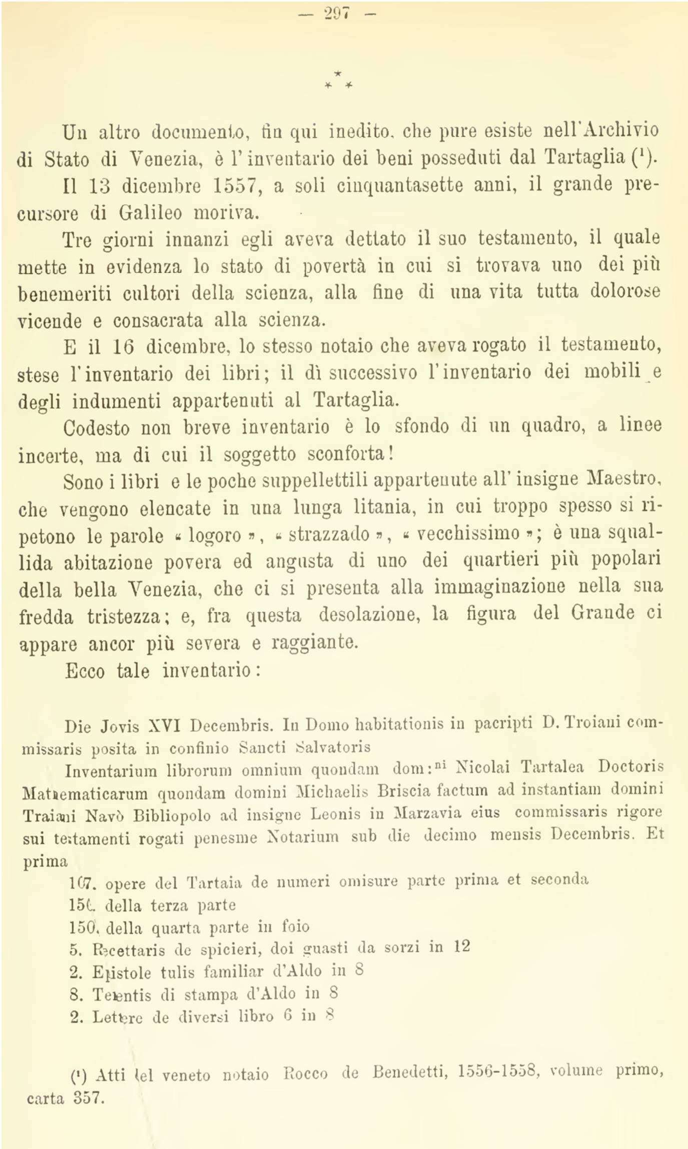
[6] Cuzio Troiano Navò posthoumously published two other works by Tartaglia: *Iordani Opusculum de ponderositate* (de Nemore [Tartaglia's editor] 1565) and *Esperienze fatte da Nicolo Tartalea* from 1541, 14 April to 1551, 7 April (Tartaglia 1541–1551). Philological notes regarding this point are provided in the following paragraphs concerning *Book VI* and *Gionta* in the *Quesiti et inventioni diverse* (Tartaglia [1554] 1959)

With regard to his legacy, Tartaglia wrote:

> I have books [manuscripts] of my general trattato de numeri et misure (first) [part] 2.[nd] (second [part]) 3.[rd] (third [part]) and 4.[th] (fourth) part, and my Quesiti et invention diverse around four hundred copies [...] Idem I have around .60. books of the travagliata inventione et ragionamenti [...] Idem several books used for my research, [cost] estimated around one hundred [Italian] ducati [...] Idem I have around forty books of the nuova scientia [...] I have a collection of several books from Paris, which I am going to sell.[7]

The notary (1557, 16 December) upon request by the executor, Curtio Troiano Navò, first wrote up the inventory (Tonni–Bazza 1904b, 7–8, 297–298) regarding the books belonging toTartaglia and the following day (17 December) wrote up the inventory concerning furnishings and belongings (*Ibidem*). In the following section, we present the early notary's quotation as regards books possessed by Tartaglia:

[7] "Io mi attrovo libri del mio general trattato de numeri et misure p.[a] (prima) 2.[da] (seconda) 3.[a] (terza) et 4.[a] (quarta) parte, et di miei Quesiti et invention diverse circa quatro cento [...] Item mi attrovo circa .60. opere della travagliata inventione et ragionamenti [...] Item libri de diverse sorte per lo mio studiare, per la valuta di cento ducati in circa [...] Item mi attrovo circa quaranta libri di nuova scientia [...] Io mi attrovo una balla de libri de Paris di diverse sorte, quali io sto per vendere". (The translation is mine. In the *Notary Archive of Venezia*, a document (Filza 168.VII; N.119) which includes the testament exists (see also Boncompagni 1881; Pizzamiglio 2007, 40).



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Un altro documento, fin qui inedito, che pure esiste nell'Archivio di Stato di Venezia, è l'inventario dei beni posseduti dal Tartaglia (1).

Il 13 dicembre 1557, a soli cinquantasette anni, il grande precursore di Galileo moriva.

Tre giorni innanzi egli aveva dettato il suo testamento, il quale mette in evidenza lo stato di povertà in cui si trovava uno dei più benemeriti cultori della scienza, alla fine di una vita tutta dolorose vicende e consacrata alla scienza.

E il 16 dicembre, lo stesso notaio che aveva rogato il testamento, stese l'inventario dei libri; il dì successivo l'inventario dei mobili e degli indumenti appartenuti al Tartaglia.

Codesto non breve inventario è lo sfondo di un quadro, a linee incerte, ma di cui il soggetto sconforta!

Sono i libri e le poche suppellettili appartenute all'insigne Maestro, che vengono elencate in una lunga litania, in cui troppo spesso si ripetono le parole « logoro », « strazzado », « vecchissimo »; è una squallida abitazione povera ed angusta di uno dei quartieri più popolari della bella Venezia, che ci si presenta alla immaginazione nella sua fredda tristezza; e, fra questa desolazione, la figura del Grande ci appare ancor più severa e raggiante.

Ecco tale inventario:

Die Jovis XVI Decembris. In Domo habitationis in pacripti D. Troiani commissaris posita in confinio Sancti Salvatoris

Inventarium librorum omnium quondam dom:ni Nicolai Tartalea Doctoris Mathematicarum quondam domini Michaelis Briscia factum ad instantiam domini Traiani Navò Bibliopolo ad insigne Leonis in Marzavia eius commissaris rigore sui testamenti rogati penesme Notarium sub die decimo mensis Decembris. Et prima

167. opere del Tartaia de numeri omisure parte prima et seconda
156. della terza parte
150. della quarta parte in foio
5. Recettaris de spicieri, doi guasti da sorzi in 12
2. Epistole tulis familiar d'Aldo in 8
8. Terentis di stampa d'Aldo in 8
2. Letere de diversi libro 6 in 8

(1) Atti del veneto notaio Rocco de Benedetti, 1556-1558, volume primo, carta 357.

Fig. 1.1 The number of the works cited by Notary[8]

[8] Tonni–Bazza 1904b, pp 297–300; see also the document in Venezia as above cited (Filza, 168.VII; N.119).

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2. Oribasi di stampa d'Aldo in 8 un rotto
2. Epistole de Tulio d'Aldo vulgar in 8
2. Hieronysmi Ragazzoni in epistolis Ciceronis in 8
2. De Auctoritate Pontificis
2. Ettiche del Figliuzzi in 8
4. Virgili d'Aldo in 8°.
4. Ricchezze della Lingua vulgar in foglio
1. 2ª parte dell'historie del Jonio in 4 strapazza
2. Consilia Boeris in 8
3. Hieronimi Vida in 16
4. Amoni in 16
2. Montan in Aphorismos in 8
3. libri del battizar in 8
10. Gioan Gierson in 16
10. Dialettiche Cesaris in 8
1. Gioan Forneli in medecina in 8
1. Quisdem medendi ratio in 8
2. Ovedo die in i officis in 8
3. Floratis con com:ᵗᵒ a un li manca in fine in foio
3. Pratiche Farneli una imbrattà assai in 8
3. Pratiche del Valeriola in medicina in 8
1. Gian Batta Montan. in Artemp rimam Galleni in 8
1. Opera del Montan. in 8
3. Sacerdotalie in 4.ᵗᵒ
2. Lexicon in greco in foglio
5. Almanach uno ruinato in 4
5. Testamenti novi in 16
1. Dialogo della Sanità in 8
1. Suetonio vulgar in 8
1. Marco Marulo di fatti d'hercule in 8
1. Historia di Marco Ruffo p.° in 8
1. Dialogo della musica in 4.°
4. Motteti di Francesco Lupino in 4.°
1. Logica del Piccolomini in 8
1. prima parte della filosofia eiusdem in 8
2. Costantin Cesari vulgar in 8
2. Summa Conciliorum in 8
2. Epistole Ovidis con comento in foglio
2. Lasoni in artem peticam horatis in 8
2. Palmerin d'Inghiltera in 8
1. Marco Aurelio in 4.°
1. Opera del Mechiaveli in 4.°
4. Natalis comit um de horis in 8
4. eiusdem de venatione in 8
1. Ragionamenti del Caggio in 8
1. 15 libri di Euclide latino in 8
1. Dialogo dell'amor divino in 8

Una balla de libri da Paris nominata nel testamento.

Fig. 1.2 The number of the works cited by Notary – Continued[9]

[9] *Ibidem*

The document alo reports other belongings:

In calce. Testes. Michael specularius ad insigne pomi aurei in marzaria quondam ser Symonis–Ser Octavianus de Ripa a coloribus insigne Rose in calle ab aquis testibus vocatis et rogatis.

Die Veneris XVII dicti. In domo habitationis defuncti posita in confinio Sancii Silvestri. Aliud inventarium rerum mobiliura suprascripti quondam domini Nicolai repertarum in eius domo. Et prima

In la sua Camereta 2 casse depente. In una cassa. Dieso camise tra vecchie e nuove da homo, quatro lenzuoli usadi. Doi strazze grande, sei fazzuoli da viso di tela grossa, 4 calaori piceli, 4 mantilli vecchi, sei brazza di tela in circa da entimelle, 4 entimelle usade 3 depente di negro, 5 tovaiuoli usadi. Un luti mela vecchia con do scuffie di bombaso, do fasse, cinque scarpete, una masseta del fil, una porcetera, una chiare, nove lire de fil de la grossa. La sua vesta ingraspata vecchia.

In altra cassa: Una vestizuola di mocaiaro vecchio fodrà di volpe vecchie. Un saggio di veludo vecchio. Un tabareto di panno negro vecchio. Una vestizzola fodrà di dossi pelai vecchissima. Una vestizzola strazzada di mocaiaro vecchissima fodrà d'Albertoni vecchi. 4 barette alla forestiera vecchie. Non so che privilegij di sue opere.

In un altra casseta:

4 pera di calzoni di panno vecchio scavezzi. Doi Ziponi di mocaiaro vecchi. Un Zipon d'ormesin di certo colore vecchio. Un Zipon vecchissimo di fostagno. Un Zipon con il casso mezo di vaso. Una vesta, et una vestina di ciambelotto usade. Una vesta usada, et una vesteta di moariaro strazza vecchia. Una vesteta di panno vecchio. Una strazza di sarza da donna. Una vesteta di mocaiaro vecchio, 3 calcete. Un mazzetto di strazze.

In una cassetta:

Do pera di scarpette di rassa. 4 colari di tela. 5 scuffie. Un rechin di bombaso. 3 para di scarpete, un mazzeto de cordette de tela.

In un banco da letto.

3 lenzuoli sporchi vecchi, 4 camise sporche vecchie, et na bona. Un saccho. Un mantil vecchissimo. Un pezzo de canevazza et duci fazzuoli da man vecchi. 12 tovaioli sporchi vecchissimi, 2 camisuole di bombaso, 4 scuffie sporche, et 2 pera di scarpete. Un intimella usada. 20 fazzoleti sporchi fra boni e cattivi.

In un forciereto. In un coffaneto coverto di cuore, drente cinque bossoli tra grandi, e piceli, in un di queli vi son 4 anneli per quel si vede d'oro uno scavezzo et una vera, et in un altre alcune piere et una capota dorada, un fiaschette picelo, et un pezeto de lapis. Un scritto di Giordan Zileti librer de D. 100 de di 12 Decembrio 1556. Un scritto de D. 74 de ser Santo Guerin librare sette di 29 Novembrio del 55 ed una sententia sette di 10 Marzo 1557 fatta sopra esse scritto. Un pesete de lin circa 8 enze. Una lettiera di negherà vecchia. 4 lenzueleti strazzadi vecchissimi. Doi cussineti di piuma con la sua intimela. Do coltre bianche usade bone. Un altra vecchia. Do cussini vecchi di piuma. Tre cavazzali de piuma beni. Un paiarizzo. Un letto di piuma vecchissimo. Un letto di piuma buono. Una carioleta de negherà col sue lette de piuma.

Una credenza de noghera con vasi, et altre bagaie con un pezze di banchal vecchissime. Panni vecchissimi vergadi della camera con 2 pezzi a torno il letto. Un tapedo vecchissimo strazzado. Un bancheto in foggia di scagno con sqnarzasoi drente. Un mortareto di bronzo. Un specchie. Una pezza di tela intorno al camin.Una balla de libri da Paris nominata nel testamento. Una foghereta di rame. Un trapie. Un banchetto con diverse cassellete con squarza foi. Una Zangola.

In cosina:

Una staiera, una fersora con una fersoreta, et un altra fersora col manego, do caene da fuogo, una gradela, un coverchieto da farsora di rame. Una saliera dì legno, 4 cazze de ferro. Do lavezzi, una calderuola, una cazza, 4 secchi mezzani, una caldiera di rame de do secchi, do tamisi, do pitari da oio, 3 tondini de laton, 6 sculieri de laton rotti. Una rassaora, uno scolaor da pozzo, 18 tra scuole e piadene 2 quarte in una circa, 3 secchi de vin bianco. Una mezaruola. Do secchieti da vin. Una paleta. Do candelieri de laton. Un banco, 2 pignate, una tecchia, un intian, meza corba de carbon. Un banco e do scagni.

In Portegheto: Un Forcier con squarzafoi, un altro forcier con alcuni gotti. Un scagneto da magnar al fuogo. Un bancho con do banchi. Un scaldaleto picolo.

In magazen:

Cinque carra in circa di legna.

ser Aloysius Georgij sutor Eivalti in dorai Sancti Marci domini Joanis Lipomano.

ser Marius Brixiensis fo ser Ioanis lacobi Cozzerij in Briscia.

Domina Helena Zambelli quondam domini Hieronimi uxor ser Joanis aurisicis.

Domina Marieta uxor domini Benedicti Alexandri staierarij in presentis dominis penes domuni defunti.

Fig. 1.2bis The belongings held by Tartaglia and cited by Notary[10]

Among Tartaglia's unsold books and the collection of Parisian books, there is a quotation concerning 51other books, for a final collection of 134 volumes, which – according to the testament – was worth approximately one hundred ducati.

> Therefore it seems noteworthy to us that Tartaglia, at the time of his death, was not in possession of either of the two Latin editions of *Euclide* that he he used, which were in–f°, neither the edition edited by B. Zamberti [see 1505], nor G. Campano–L. Pacioli's edition [see edition of the 1509].[11]

[10] Tonni–Bazza 1904b, 299–300.

[11] "Degna di nota ci sembra di conseguenza la circostanza per cui il Tartaglia, al momento della sua morte, non fosse in possesso di nessuna delle due edizioni latine dell'*Euclide* da lui utilizate, che erano in–f°, cioè nè quella di B. Zamberti

An early and very short biography on Tartaglia was written by Bernardino Baldi's (1553–1617). Neverthless he referred to an oversight concerning the date of 1567 (since Tartaglia died in 1557):

> 1567. Nicolò Tartaglia Bresciano of humble birth studied mathematics and particularly Geometry & Arithmetic with so much genius that he excelled with respect to other scholars of his time. He wrote Euclid's Elements in vulgare [Italian] language and also gave lectures in Venice on this subject. He wrote many works concerning the motion of heavy bodies, artillery shots [ballistics], fortifications, measurements by sight, & other [scientific] similar things, and finally he wrote two huge volumes regarding all necessary aspects of Arithmetics and Geometry as both theory and practice. He was an adversary of Girolamo Cardano and disagreed with some of Cardano's works. He paid so little attention to language that it brings a smile to the face of those who read of his works.[12]

It is possible to find Tartaglis'a biographical sketches and quotations on his science throughout history. For the sake of brevity, I concisely report some of them below[13].

Table 1.1. Tartaglia's main biographies and mentions on his science in history

Date	Author	Source/Title	Refs.(*folia*/p)
1707	Baldi	*Cronica de'matematici ovvero Epitome dell'istoria delle vite loro*	

[v. 1505] nè quella di G. Campano–L. Pacioli [v. 1509]". Pizzamiglio in Tartaglia 2007, XXXIII (Author's brackets and Italics).

[12] "1567. Nicolò Tartaglia Bresciano d'humile nascimento attese alle cose Matematiche e particolarmente alla Geometria & all'Aritmetica con tanto genio, che si lasciò molti adietro. Trasferì costui in lingua volgare gl'Elementi d'Euclide, ch'egli leggeva publicamente in Venetia. Scrisse molte opere appartenenti al moto de corpi gravi, a' tiri dell'Artigliarie, a fortificationi de luoghi, a misurar con la vista, & altre cose tali, e finalmente scrisse due gran volumi, ne quali raccolse tutto quello che s'appartiene ad una compita specolatione e pratica delle cose dell'Aritmetica e della Geometria. Fu egli grand'avversario di Girolamo Cardano e scrisseli contro alcune opere. Attese nondimeno così poco alla bontà della lingua, che muove a riso talhora chi legge le cose sue." (Baldi, 1707, 133).

[13] With regard to the second half of the past century, we should include works by Bortolotti and, of course, the crucial works by Masotti and recently by Pizzamiglio. For the sake of brevity we refer the reader to Pizzamiglio's works, particularly his last publication on *Tartaglia in history*.

1564	Castriotto–Maggi	*Della fortificazione delle città*	7r; 11v.
1581	Del Monte	*Le Meccaniche dell'Illustrissimo Sig. Guido Ubaldo dè Marchesi del Monte*	5v; 6v; 8v; 9r.
1585	Benedetti	*Diversarum speculationum Mathematicarum et Physicarum Liber*	92–96; 105; 111–112; 114–115; 148–151. In particular, he mentions the wrong Aristotelian assumption on free fall shared by Tartaglia, as well (168); and "bombardae diversas elevations (258–259).
1644	Torricelli	*Opera Geometrica* (*Book II*)	227.
1797–99	Cossali	*Origine, trasporto in Italia, primi progressi in essa dell'Algebra*; *Scritti di Pietro Cossali*.	96–158. In particular, he cites passages on the *Book IX* of *Quesiti et invetioni diverse*.
1810	Marini	*Biblioteca istorico–critica di fortificazione permanente*	XII. In particular, he cites Tartaglia as the first to publish innovations on fortifications with bastions.
1841	Di Giorgio Martini	*Trattato di architettura civile e militare* (by Carlo Promis)	Vol. I, Parte I, 248, footnote 1; Vol. II, Parte II, 5, 77–78, 88, 104; 151; 165; 207; 293. In particular he writes a short biography ("Memoria I", chap. XXVI, 69–71).
1854	D'Ayala	*Bibliografia militare italiana e moderna*	123; 155–156, 180.
1941–43	Uccelli	*Enciclpedia storica delle scienze e delle loro applicazioni*	Vol. I, 31–34
1891–00	Caverni	*Storia del metodo sperimentale in Italia*	Vol. I, 52–54
1880–19	Favaro	*Lo Studio di Padova al tempo di Niccolò Coppernico; Le Matematiche nello Studio di*	Many quotations.

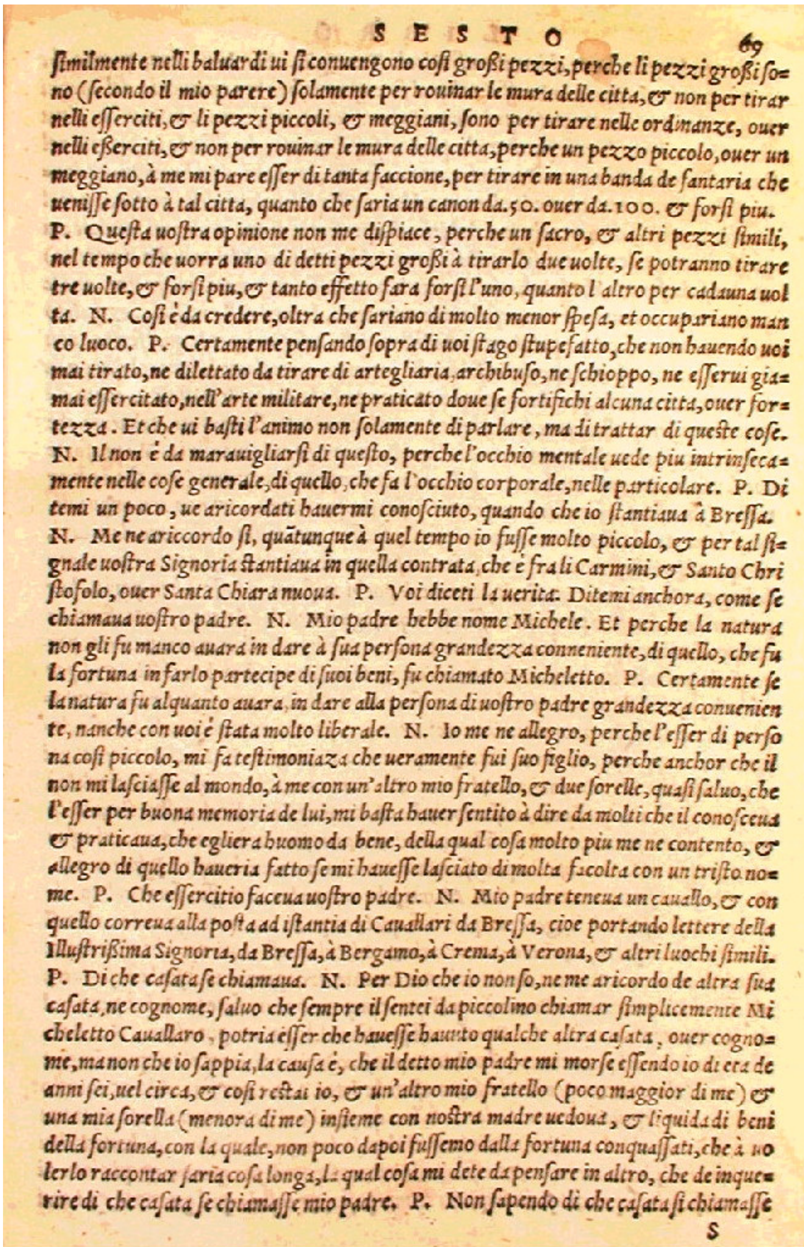
		Padova dal principio del secolo XIV alla fine del XVI; Intorno al testamento inedito di Niccolò Tartaglia pubblicato da D. B. Boncompagni; Per la biografia di Niccolò Tartaglia; Di Niccolò Tartaglia e della stampa di delle sue opere con particolare riguardo alla Travagliata Inventione; Niccolò Tartaglia e la determinazione dei specifici; Leonardo Da Vinci e Niccolò Tartaglia, in Scoprendosi il monumento a N. Tartaglia; A proposito della famiglia di Niccolò Tartaglia; Notizie storico–critiche sulla divisione delle aree	
1897	Vailati	*Dal concetto di Centro di Gravità nella Statica di Archimede; Il principio dei lavori Virtuali da Aristotele a Erone d'Alessandria; Per la preistoria del principio dei momenti virtuali.*	101–112; 113–128. In particular, he cites the lack of quotations (concerning *De ponderibus*) by Tartaglia (*Quesiti*, 1554) versus de Nemore (122, ft. 2); 225–232
1919	Marcolongo	*Lo sviluppo della meccanica sino ai discepoli di Galileo.*	95; 98; 108; 112–113; 114, ft. 1; In particular he discusses the lack of quotations (concerning *Elementa Iordani*) by Tartaglia (*Quesiti*, 1554) versus de Nemore (95).
1914–33	Loria	*Le scienze esatte nel'antica Grecia;* *Pagine di storia della scienza;* *Storia delle matematiche;*	193–194; 291–292; 592; 84–87; 287; 299; 302–306; 309–314;

1.1.1 The Roots

Due to some uncertainty of the information on Tartaglia's birth, the origin of his lineage is also unknown. He experienced a tragedy in 1512 when the French invaded Brescia during the War of the League of Cambrai. The militia of Brescia defended their city for seven days. When the French finally broke through, they took their revenge by massacring the inhabitants of Brescia. By the end of battle, over 45,000 residents had been killed. During the massacre, some French soldier by Gaston de Foix–Nemours (1489–1512) sliced Niccolò's jaw and palate with a saber. Concerning this event, a suggestive autobiographical tale, with Signor Priore di Barletta as interlocutor, can be found in *Book* VI dei *Quesiti et invetioni diverse*[14] (Tartaglia 1554, Q VIII). In the tale, Tartaglia's father is mentioned, and the author reports that he can remember hearing his name, "Micheletto Cavallaro"[15], an employee riding horses for the postal service, he also reports the frightful battle (sack) of Brescia (19th February 1512) which made him an orphan, and which also caused him five serious wounds on face and head. Such injuries generated a temporary speech impediment, which seems be the origin of the surname Tartaglia (stammer). He was alone with his mother, two siblings and impoverished.

[14] Hereafter *Quesiti*.

[15] "Micheletto" due his low stature. "Cavallo" in English is "horse". "Cavallaro" is an ancient Italian word derived from "Cavallo" and means, more or less, a man busy with horses or using horses.



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ſimilmente nelli baluardi ui ſi conuengono coſi großi pezzi, perche li pezzi großi ſo=
no (ſecondo il mio parere) ſolamente per rouinar le mura delle citta, & non per tirar
nelli eſſerciti, & li pezzi piccoli, & meggiani, ſono per tirare nelle ordinanze, ouer
nelli eßerciti, & non per rouinar le mura delle citta, perche un pezzo piccolo, ouer un
meggiano, à me mi pare eſſer di tanta faccione, per tirare in una banda de fantaria che
ueniſſe ſotto à tal citta, quanto che ſaria un canon da.50. ouer da.100. & forſi piu.
P. Queſta uoſtra opinione non me diſpiace, perche un ſacro, & altri pezzi ſimili,
nel tempo che uorra uno di detti pezzi großi à tirarlo due uolte, ſe potranno tirare
tre uolte, & forſi piu, & tanto effetto fara forſi l'uno, quanto l'altro per cadauna uol
ta. N. Coſi è da credere, oltra che ſariano di molto menor ſpeſa, et occupariano man
co luoco. P. Certamente penſando ſopra di uoi ſtago ſtupefatto, che non hauendo uoi
mai tirato, ne dilettato da tirare di artegliaria, archibuſo, ne ſchioppo, ne eſſerui gia=
mai eſſercitato, nell'arte militare, ne praticato doue ſe fortifichi alcuna citta, ouer for=
tezza. Et che ui baſti l'animo non ſolamente di parlare, ma di trattar di queſte coſe.
N. Il non è da marauigliarſi di queſto, perche l'occhio mentale uede piu intrinſeca=
mente nelle coſe generale, di quello, che fa l'occhio corporale, nelle particolare. P. Di
temi un poco, ue aricordati hauermi conoſciuto, quando che io ſtantiaua à Breſſa.
N. Me ne ariccordo ſi, quãtunque à quel tempo io fuſſe molto piccolo, & per tal ſi=
gnale uoſtra Signoria ſtantiaua in quella contrata, che è fra li Carmini, & Santo Chri
ſtofolo, ouer Santa Chiara nuoua. P. Voi diceti la uerita. Ditemi anchora, come ſe
chiamaua uoſtro padre. N. Mio padre hebbe nome Michele. Et perche la natura
non gli fu manco auara in dare à ſua perſona grandezza conneniente, di quello, che fu
la fortuna in farlo partecipe di ſuoi beni, fu chiamato Micheletto. P. Certamente ſe
la natura fu alquanto auara, in dare alla perſona di uoſtro padre grandezza conuenien
te, nanche con uoi è ſtata molto liberale. N. Io me ne allegro, perche l'eſſer di perſo
na coſi piccolo, mi fa teſtimoniaza che ueramente fui ſuo figlio, perche anchor che il
non mi laſciaſſe al mondo, à me con un'altro mio fratello, & due ſorelle, quaſi ſaluo, che
l'eſſer per buona memoria de lui, mi baſta hauer ſentito à dire da molti che il conoſceua
& praticaua, che eglie era huomo da bene, della qual coſa molto piu me ne contento, &
allegro di quello haueria fatto ſe mi haueſſe laſciato di molta facolta con un triſto no=
me. P. Che eſſercitio faceua uoſtro padre. N. Mio padre teneua un cauallo, & con
quello correua alla poſta ad iſtantia di Caualari da Breſſa, cioe portando lettere della
Illuſtrißima Signoria, da Breſſa, à Bergamo, à Crema, à Verona, & altri luochi ſimili.
P. Di che caſata ſe chiamaua. N. Per Dio che io non ſo, ne me aricordo de altra ſua
caſata, ne cognome, ſaluo che ſempre il ſentei da piccolino chiamar ſimplicemente Mi
cheletto Cauallaro, potria eſſer che haueſſe hauuto qualche altra caſata, ouer cogno=
me, ma non che io ſappia, la cauſa è, che il detto mio padre mi morſe eſſendo io di eta de
anni ſei, uel circa, & coſi reſtai io, & un'altro mio fratello (poco maggior di me) &
una mia ſorella (menora di me) inſieme con noſtra madre uedoua, & liquida di beni
della fortuna, con la quale, non poco dapoi fuſſemo dalla fortuna conquaſſati, che à uo
lerlo raccontar ſaria coſa longa, la qual coſa mi dete da penſare in altro, che de inque=
rire di che caſata ſe chiamaſſe mio padre. P. Non ſapendo di che caſata ſi chiamaſſe

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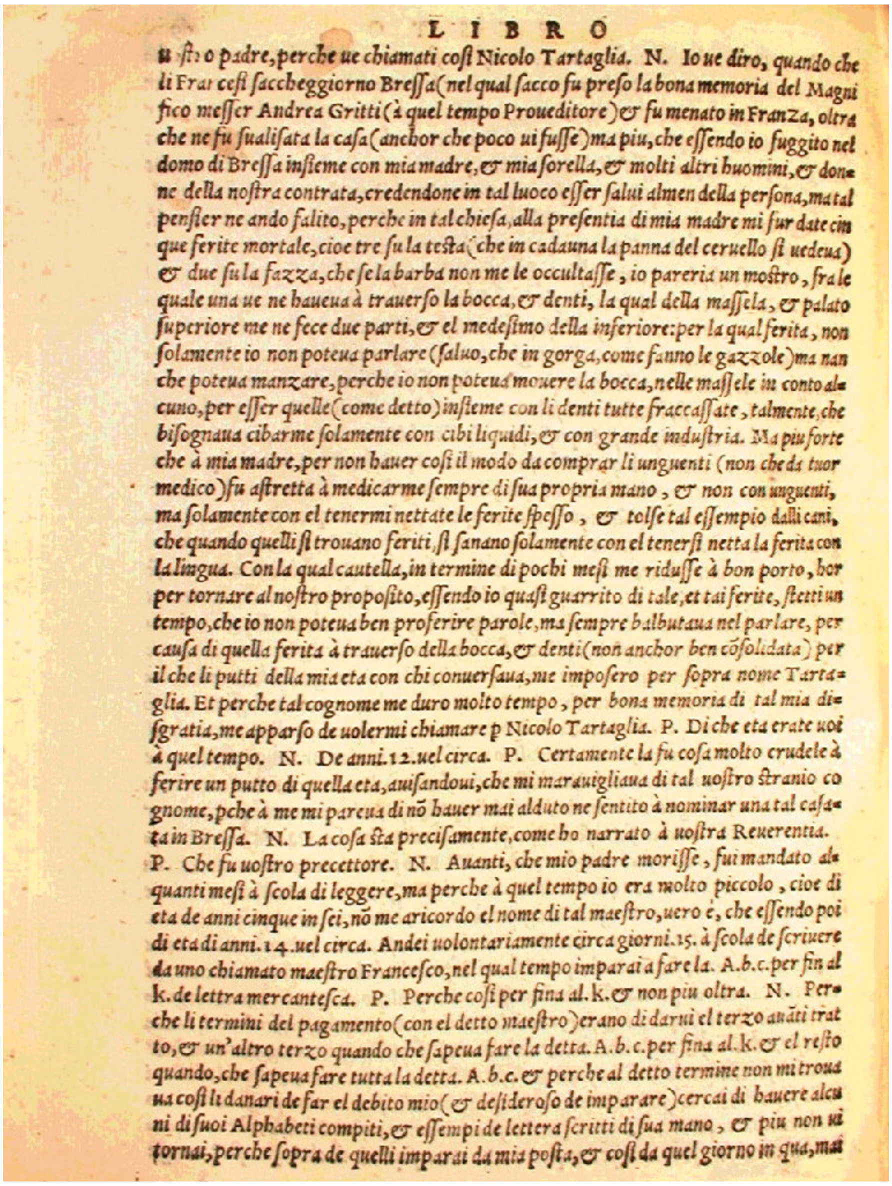
Fig. 1.3 Plates on speeches by Tartaglia aroud his childhood[16]

[16] Tartaglia 1554, VI, Q VIII, pp 68rv–69r, from line 18.

1.1.2 Tartaglia's Education

Concerning his childhood education cited above, it is important to note that in *the National Archive of Verona* (*Archivio di Stato di Verona*), where his testament (Bittanti 1871; Tartaglia 1554, Q XX) is preserved, Tartaglia mentions his brother with the surname "Fontana".[17] As also emerges from the following passage from *Quesiti*, after the loss of his father, Tartaglia was left alone with his mother, conserving the memory of a difficult period in which he was also forced to abandon his studies due to a lack of money to pay the teacher (Tartaglia 1554, *Book VI*, Q VIII). Therefore, he learned the rest on his own which makes him twice as worthy of attention.

[17] The surname Fontana appears in his testament: Zuampiero Fontana.



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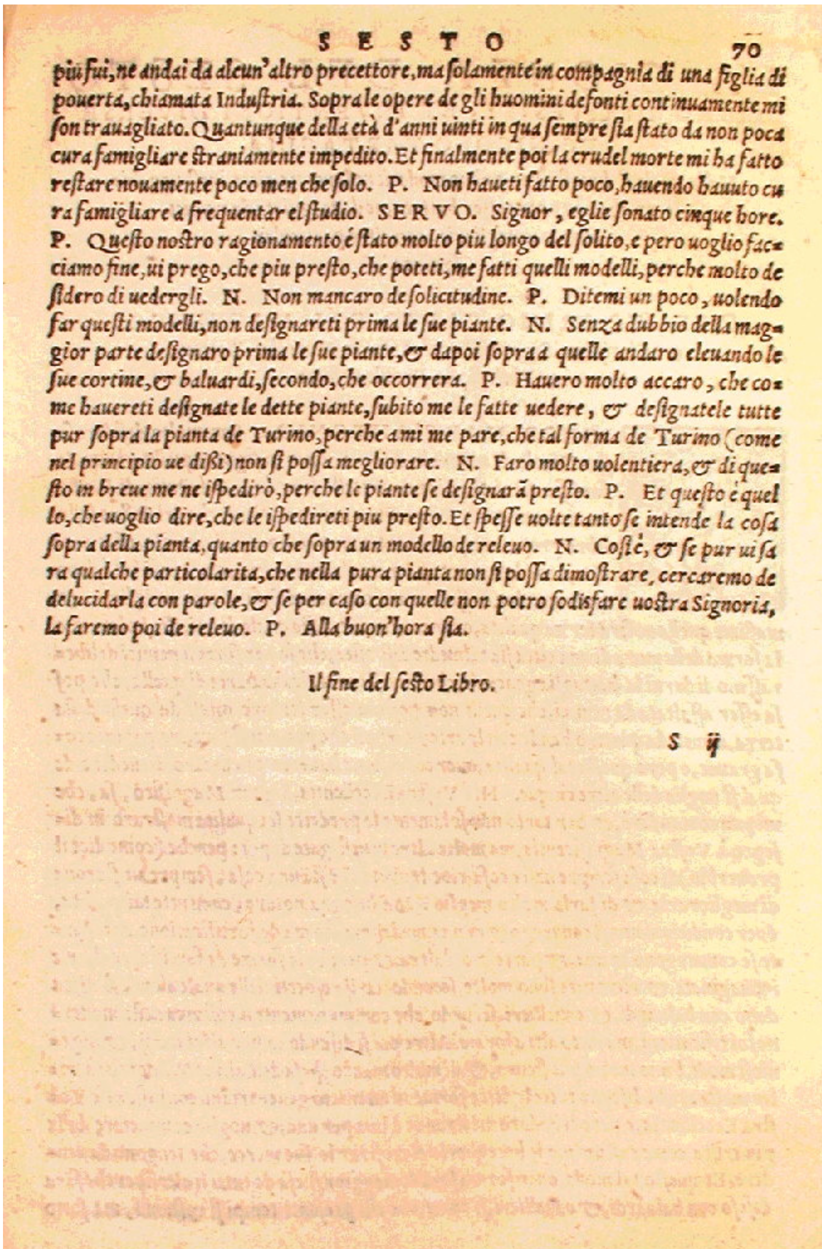
u stro padre, perche ue chiamati cosi Nicolo Tartaglia. N. Io ue diro, quando che
li Frai cesi saccheggiorno Bressa (nel qual sacco fu preso la bona memoria del Magni
fico messer Andrea Gritti (à quel tempo Proueditore) & fu menato in Franza, oltra
che ne fu sualisata la casa (anchor che poco ui fusse) ma piu, che essendo io fuggito nel
domo di Bressa insieme con mia madre, & mia sorella, & molti altri huomini, & don-
ne della nostra contrata, credendone in tal luoco esser salui almen della persona, ma tal
pensier ne ando falito, perche in tal chiesa, alla presentia di mia madre mi fur date cin
que ferite mortale, cioe tre su la testa (che in cadauna la panna del ceruello si uedeua)
& due su la fazza, che se la barba non me le occultasse, io pareria un mostro, fra le
quale una ue ne haueua à trauerso la bocca, & denti, la qual della massela, & palato
superiore me ne fece due parti, & el medesimo della inferiore: per la qual ferita, non
solamente io non poteua parlare (saluo, che in gorga, come fanno le gazzole) ma nan
che poteua manzare, perche io non poteua mouere la bocca, nelle massele in conto al-
cuno, per esser quelle (come detto) insieme con li denti tutte fraccassate, talmente, che
bisognaua cibarme solamente con cibi liquidi, & con grande industria. Ma piu forte
che à mia madre, per non hauer cosi il modo da comprar li unguenti (non che da tuor
medico) fu astretta à medicarme sempre di sua propria mano, & non con unguenti,
ma solamente con el tenermi nettate le ferite spesso, & tolse tal essempio dalli cani,
che quando quelli si trouano feriti, si sanano solamente con el tenersi netta la ferita con
la lingua. Con la qual cautella, in termine di pochi mesi me ridusse à bon porto, hor
per tornare al nostro proposito, essendo io quasi guarrito di tale, et tai ferite, stetti un
tempo, che io non poteua ben proferire parole, ma sempre balbutaua nel parlare, per
causa di quella ferita à trauerso della bocca, & denti (non anchor ben cõsolidata) per
il che li putti della mia eta con chi conuersaua, me imposero per sopra nome Tarta-
glia. Et perche tal cognome me duro molto tempo, per bona memoria di tal mia di-
sgratia, me apparso de uolermi chiamare p Nicolo Tartaglia. P. Di che eta erate uoi
à quel tempo. N. De anni. 12. uel circa. P. Certamente la fu cosa molto crudele à
ferire un putto di quella eta, auisandoui, che mi marauigliaua di tal uostro stranio co
gnome, pche à me mi pareua di nõ hauer mai alduto ne sentito à nominar una tal casa-
ta in Bressa. N. La cosa sta precisamente, come ho narrato à uostra Reuerentia.
P. Che fu uostro precettore. N. Auanti, che mio padre morisse, fui mandato al-
quanti mesi à scola di leggere, ma perche à quel tempo io era molto piccolo, cioe di
eta de anni cinque in sei, nõ me aricordo el nome di tal maestro, uero è, che essendo poi
di eta di anni. 14. uel circa. Andei uolontariamente circa giorni. 15. à scola de scriuere
da uno chiamato maestro Francesco, nel qual tempo imparai a fare la. A.b.c. per fin al
k. de lettra mercantesca. P. Perche cosi per fina al. k. & non piu oltra. N. Per-
che li termini del pagamento (con el detto maestro) erano di darui el terzo auãti trat
to, & un'altro terzo quando che sapeua fare la detta. A.b.c. per fina al. k. & el resto
quando, che sapeua fare tutta la detta. A.b.c. & perche al detto termine non mi troua
ua cosi li danari de far el debito mio (& desideroso de imparare) cercai di hauere alcu
ni di suoi Alphabeti compiti, & essempi de lettera scritti di sua mano, & piu non ui
tornai, perche sopra de quelli imparai da mia posta, & cosi da quel giorno in qua, mai

Fig. 1.4 Plates on speeches by Tartaglia aroud his education[18]

[18] Tartaglia 1554, 69v–70r, from line 15.



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più fui, ne andai da alcun'altro precettore, ma solamente in compagnia di una figlia di pouerta, chiamata Industria. Sopra le opere de gli huomini defonti continuamente mi son trauagliato. Quantunque della età d'anni uinti in qua sempre sia stato da non poca cura famigliare straniamente impedito. Et finalmente poi la crudel morte mi ha fatto restare nouamente poco men che solo. P. Non haueti fatto poco, hauendo hauuto cura famigliare a frequentar el studio. SERVO. Signor, eglie sonato cinque hore. P. Questo nostro ragionamento é stato molto piu longo del solito, e pero uoglio facciamo fine, ui prego, che piu presto, che poteti, me fatti quelli modelli, perche molto desidero di uedergli. N. Non mancaro de solicitudine. P. Ditemi un poco, uolendo far questi modelli, non designareti prima le sue piante. N. Senza dubbio della maggior parte designaro prima le sue piante, & dapoi sopra a quelle andaro eleuando le sue cortine, & baluardi, secondo, che occorrera. P. Hauero molto accaro, che come haureti designate le dette piante, subito me le fatte uedere, & designatele tutte pur sopra la pianta de Turino, perche a mi me pare, che tal forma de Turino (come nel principio ue dissi) non si possa megliorare. N. Faro molto uolentiera, & di questo in breue me ne ispedirò, perche le piante se designarã presto. P. Et questo é quello, che uoglio dire, che le ispedireti piu presto. Et spesse uolte tanto se intende la cosa sopra della pianta, quanto che sopra un modello de releuo. N. Cosi è, & se pur ui sara qualche particolarita, che nella pura pianta non si possa dimostrare, cercaremo de delucidarla con parole, & se per caso con quelle non potro sodisfare uostra Signoria, la faremo poi de releuo. P. Alla buon'hora sia.

Il fine del sesto Libro.

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Fig. 1.4 Continued[19]

[19] *Ibidem*.

Thus, it seems that Tartaglia only learned half the alphabet from a private tutor, called Maestro Francesco Feliciano[20] (da Lazise: fl 1500's), before funds ran out "[…] but by the time he reached "k", he was no longer able to pay the teacher."[21] Thus, he had to learn the rest for himself. Be that as it may, he was essentially self–taught and

> […] never returned to a tutor, but continued to labor by myself over the works of dead men, accompanied only by the daughter of poverty that is called industry.[22]

He and his contemporaries, working outside the academies, were responsible for the spread of classic works in modern languages among the educated middle class.

Finally, Tartaglia was a mathematician, an architect (designing fortifications), a surveyor (nowadays we can speak of topography, seeking the best means of defense or offense) and a bookkeeper from the Republic of Venice. He published many books, including the first Italian translations of Archimedes and Euclid, and an acclaimed compilation of mathematics. MaybeTartaglia was one of the first to apply mathematics to the investigation of the paths of cannonballs (Capecchi and Pisano 2010a, 2010b; Pisano 2007; Pisano and Capecchi 2010a). His work was later validated by Galilei's studies on falling bodies. He also published a treatise on retrieving sunken ships. His edition of Euclid in 1543, the first translation of the *Elements*[23] into any modern European language, was especially significant. It is know that sme current Latin translations (mostly taken from an Arabic source) contained errors in *Book V*, the Eudoxian theory of proportion, which rendered it unusable. Tartaglia based on Zamberti's Latin translation of an uncorrupted Greek text, and rendered *Book V* correct. He also wrote the first modern and useful commentary on the theory. Later, the theory was an essential tool for Galileo, just as it had been for Archimedes (Pisano and Bussotti 2012).

An important collection of Tartalean works was studied and archived by Arlando Masotti, distinguished scholar. His works and archives constitute a great contribution to the history of science, among which the biography in the *Dictionary of Scientific Biography* (Masotti 1970–1980, 13, pp 158–262), the *Archivio Niccolò Tartaglia*, made up of card catalogues and historiographical binders divided by theme, photocopies,

[20] Tartaglia 1554, Book IX, Q. I.

[21] Masotti 1970–1980, 13, 258. (Author's quotation marks).

[22] Tartaglia 1554, Book VI, Q 8.

[23] OnTartaglia's Euclid, see Tartaglia (2007).

and of the *Fondo Arnaldo Masotti*, which today is preserved at the *Biblioteca Centrale del Politecnico di Milano*.

1.1.3 Arnaldo Masotti,Tartaglia's Editor

Arnaldo Masotti was born in Milano (Italy) on November 18th 1902 and died on July 11th 1989. He attended "C. Cattaneo" a technical Institute (secondary school) within the physics–mathematics section. Then he studied *Industrial engineering* (1924, R. Polytechnic of Milan) and *Applied mathematics* (1926, R. University of Milano) delivering a dissertation in hydrodynamics. Mentored by Umberto Cisotti (1882–1946), he became a professor of rational mechanics at the Faculty of Architecture of the Polytechnic (1933). Despite his early works on hydromechanics based on his studies with Cisotti, subsequent works dealt with potential theory of electrostatics, electrodynamics, and thermo–electronics. Masotti intensely worked on the history of mathematics, rediscovering some Italian mathematicians such as Matteo Ricci (1552–1610), Bonaventura Cavalieri (1598–1647), Maria Gaetana Agnesi (1718–1799) and Paolo Frisi (1728–1784). His works on Nicolaus Copernicus (1473–1543) and the monograph on "Mathematics and mathematicians in the history of Milan" (for the Foundation of Treccani Alfieri enciclopedia) are very early distinguished productions. Starting in the 1930*s,* he published works on astronomy and on Giovanni Schiaparelli (1835–1910). His wife, Giuseppina Biggiogero Masotti[24] (1894–1977) was a professor of geometry at Politecnico di Milano. Masotti wrote several papers in Italian and International magazines. Just to mention the ardent interest in his and his wife's research, *Archive for History of Exact Sciences*, whose editor–in–chief at the time was Clifford Ambrose Truesdell (1919–2000), dedicated the entire volume n. 14 (ed. 1974–1975) to their works. Most of Masotti's life was devoted to Niccolò Tartaglia (1499?–1557) and Lodovico Ferrari (1522–1565) producing vast national and international literatures (e.g. see his contribution to Gillipise's *Dictionary*). The first "Commemoration of Niccolò Tartaglia" by Masotti was at *Ateneo di Brescia* in the afternoon of Saturday, 14 December 1957, at *Palazzo Tosio*. On that occasion, Masotti proposed the project of a commented new edition of the *Tartalea corpus.* After the first new edition of *Quesiti* (1959), in 1974, "Cartelli di sfida matematica" also appeared. In 1979,

[24] She was Oscar Chisini's (1889–1967) pupil and collaborated closely on historical studies with Masotti. She wrote two important memoirs on Luca Pacioli (1445–1517; Pisano 2013).

Ateneo di Brescia decided to prepare a new edition of *Euclide Megarense*. Masotti could not conclude his work (even though the work was in an advanced stage).

> It is precisely that initiative, which now comes to fruition, during the celebration of the 450th anniversary of the death of the great mathematician from Brescia, and is therefore right and proper that this volume of "Opere di Niccolò Tartaglia" is properly dedicated to Professor Arnaldo Masotti[25].

It is thanks to the great competence and passion of Pierluigi Pizzamiglio that the edition of *Euclide Megarense* lives on.

[25] "È proprio quell'iniziativa che giunge ora a compimento, in occasione della celebrazione del 450° anniversario della morte del grande matematico bresciano, ed è quindi giusto e doveroso che questo volume delle "Opere di Niccolò Tartaglia" venga dedicato proprio al prof. Arnaldo Masotti. [Transl.: mine]. See also: Tartaglia 2007. "1990. *Rendiconti dell'Istituto Lombardo*, col. 124, pp 157–166 (L. Amerio) Nastasi, *Lettera matematica*, 23.

(1954)

Fig. 1.5 Inedited Arnaldo Masotti's image. Plate from the original portraits (*Masotti archive*) conserved by Madame Claudia Masotti, with her kind authorization, member of Masotti's family

Masotti edited an edition of *Quesiti et inventioni diverse* (1554), published by the Ateneo di Brescia (*Supplemento ai Commentari dell'Ateneo*) in 1959 (Tartaglia [1554] 1959), Lodovico Ferrari and Niccolò Tartaglia, *Cartelli di sfida matematica*, facsimile reproduction (1547–1548) published by the same editor in 1974 (Masotti 1960b, 1962).

1.2 Tartaglia's Conceptual Stream in the Renaissance

Tartaglia produced crucial and important contributions to mathematics, physics, and fortifications: equations, scientific foundations of ballistics, criticism to Aristotle's lever, statics, the measurement of calibers and land surveying and fortifications. He discussed them principally in *General trattato di numeri et misure* (Venice, 1556–1560), *Nova scientia* (Venice, 1537) and in *Quesiti et invention diverse* (herafter *Quesiti*).

Thanks to his mathematical studies at an early age, Tartaglia went to Verona[26] (fl. 1516–1518) where he had a job as a *teacher of the abacus* at a school in Palazzo Mazzanti. In 1534 he moved once again to Venice[27] to give public lectures in mathematics, e.g., at the Church of San Zanipolo. Venice would be the most important setting for his main scientific works. In fact, all of his studies were published in this city where he essentially spent all of his life[28].

1.2.1 Mathematics: the Third Degree Equations

Generally speaking, the affair *third–degree–equation* dates back to Archimedes' *Proposition IV* in *On the Sphere and Cylinder*:

> To cut a given sphere by a plane so that the volumes of the segments are to one another in a given ratio.[29]

Many succeeding authors worked on both geometrical and mathematical (after Algebra's invention) standopoints without a definitive solution.

The resolution of third degree equations (Tartaglia 1554, book IX) and his subsequent controversy with Girolamo Cardano (1501–1576) (and Lodovico Ferrari (1522–1565)), surely represent one of the most significant subjects in history related to Tartaglia's name. Cardano knew of the innovations directly through Tartaglia himself (1539); then he published them in his *Ars magna* (1545). Generally speaking, the

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[26] We specify that Masotti reported the existence of some documents (*Archivio di Stato di Verona*) that declared his stay in Verona to be around 1529–1533 (Masotti 1970–1980, 13, 259). In this period 17 *Quesiti* concerning Book IX were proposed to him to solve.

[27] Until 1557 and except a short stay in Brescia (March 1548–October 1549).

[28] With the exception of his return to Brescia from 1548 to 1549 (ca. 18 months) he taught at Sant'Afra, San Barnaba, San Lorenzo and at the Academy near Rezzato, a small village.

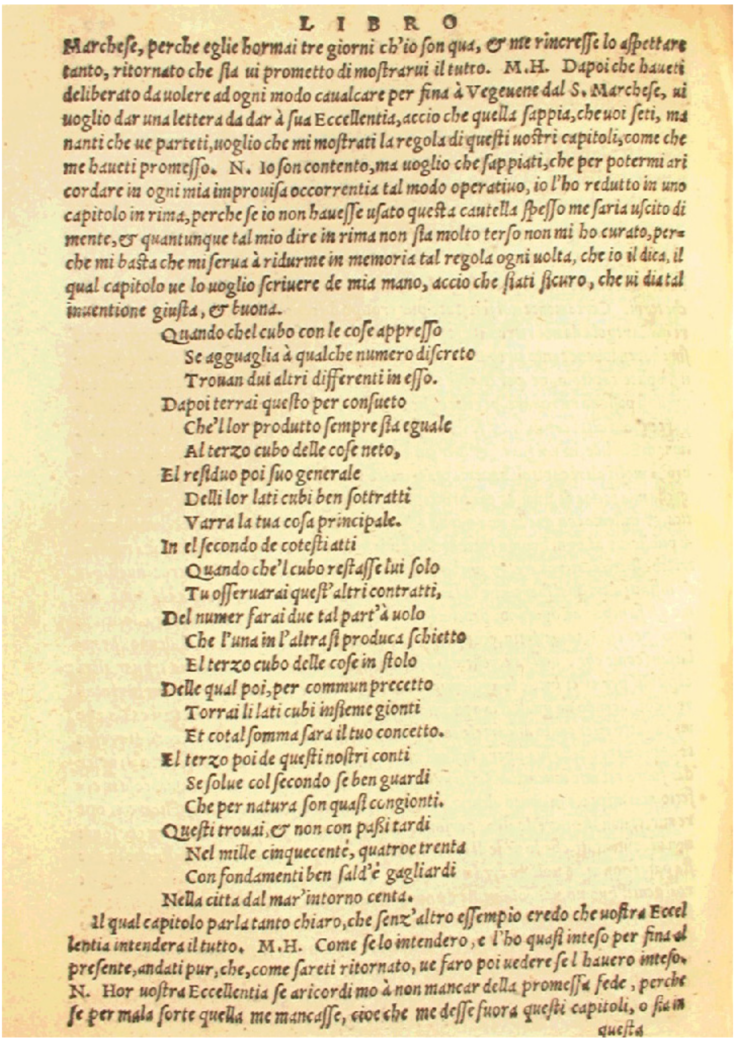
[29] Heath 2002, *On the Sphere and Cylinder*, Book II, 62.

resolution (which at the end of the 15th century Luca Pacioli (Pisano 2013a) considered impossibile with only the use of known calculations of the time) was studied and separately proved by both Scipione del Ferro and Tartaglia. Cardano and Ferrari improved the method. Book IX of *Quesiti et invention diverse* (Tartaglia Book IX, 1554) explains this procedure. It is known that the solution of third–degree equations was acknowledged in one of Tartaglia's poems:

Quando chel cubo con le cose appresso	$x^3 + px$
Se agguaglia à qualche numero discreto	$= q$
Trovan dui altri differenti in esso.	$u - v = q$
Dapoi terrai questo per consueto	
Che'l lor produtto sempre sia eguale	$uv = (p/3)^3$
Al terzo cubo delle cose neto,	
El residuo poi suo generale	
Delli lor lati cubi ben sottratti	$\sqrt[3]{u} - \sqrt[3]{v} = x$
Varra la tua cosa principale.	
In el secondo de cotesti atti	
Quando che'l cubo restasse lui solo	$x^3 = px + q$
Tu osservarai quest'altri contratti,	$u + v = q$
Del numer farai due tal part'à volo	
Che l'una in l'altra si produca schietto	$uv = (p/3)^3$
El terzo cubo delle cose in stolo	
Delle qual poi, per commun precetto	
Torrai li lati cubi insieme gionti	$\sqrt[3]{u} + \sqrt[3]{v} = x$
Et cotal somma sara il tuo concetto.	
El terzo poi de questi nostri conti	
Se solue col secondo se ben guardi	$x^3 + q = px$
Che per natura son quasi congionti.	
Questi trovai, & non con passi tardi	
Nel mille cinquecente, quatroe trenta	
Con fondamenti ben sald'è gagliardi	
Nella citta dal mar'intorno centa.	

Fig. 1.6 Adapted from Tartaglia's poem solution of the third–degree equation[30]

[30] Cosa/cose refers to unknown variables. In brief: When the cube and its things ("cose") near. Add a new, discrete number. Determine two new, different



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Marcheſe, perche eglie hormai tre giorni ch'io ſon qua, & me rincreſſe lo aſpettare tanto, ritornato che ſia ui prometto di moſtrarui il tutto. M.H. Dapoi che haueti deliberato da uolere ad ogni modo caualcare per fina à Vegeuene dal S. Marcheſe, ui uoglio dar una lettera da dar à ſua Eccellentia, accio che quella ſappia, che uoi ſeti, ma nanti che ue parteti, uoglio che mi moſtrati la regola di queſti uoſtri capitoli, come che me haueti promeſſo. N. Io ſon contento, ma uoglio che ſappiati, che per potermi aricordare in ogni mia improuiſa occorrentia tal modo operatiuo, io l'ho redutto in uno capitolo in rima, perche ſe io non haueſſe uſato queſta cautella ſpeſſo me ſaria uſcito di mente, & quantunque tal mio dire in rima non ſia molto terſo non mi ho curato, perche mi baſta che mi ſerua à ridurme in memoria tal regola ogni uolta, che io il dica, il qual capitolo ue lo uoglio ſcriuere de mia mano, accio che ſiati ſicuro, che ui dia tal inuentione giuſta, & buona.

Quando chel cubo con le coſe appreſſo
Se agguaglia à qualche numero diſcreto
Trouan dui altri differenti in eſſo.
Dapoi terrai queſto per conſueto
Che'l lor produtto ſempre ſia eguale
Al terzo cubo delle coſe neto,
El reſiduo poi ſuo generale
Delli lor lati cubi ben ſottratti
Varra la tua coſa principale.
In el ſecondo de coteſti atti
Quando che'l cubo reſtaſſe lui ſolo
Tu oſſeruarai queſt'altri contratti,
Del numer farai due tal part'à uolo
Che l'una in l'altra ſi produca ſchietto
El terzo cubo delle coſe in ſtolo
Delle qual poi, per commun precetto
Torrai li lati cubi inſieme gionti
Et cotal ſomma ſara il tuo concetto.
El terzo poi de queſti noſtri conti
Se ſolue col ſecondo ſe ben guardi
Che per natura ſon quaſi congionti.
Queſti trouai, & non con paſſi tardi
Nel mille cinquecente, quatro e trenta
Con fondamenti ben ſald'è gagliardi
Nella citta dal mar'intorno centa.

Il qual capitolo parla tanto chiaro, che ſenz'altro eſſempio credo che uoſtra Eccellentia intendera il tutto. M.H. Come ſe lo intendero, e l'ho quaſi inteſo per fina al preſente, andati pur, che, come ſareti ritornato, ue faro poi uedere ſe l haurero inteſo. N. Hor uoſtra Eccellentia ſe aricordi mo à non mancar della promeſſa fede, perche ſe per mala ſorte quella me mancaſſe, cioe che me deſſe fuora queſti capitoli, o ſia in

queſta

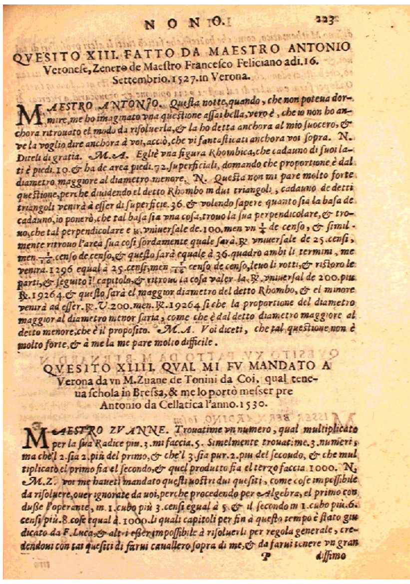
Fig. 1.6*bis* Plates from original Tartaglia's poem[31]

numbers. By that one; this feat will be kept as a rule. Their product always equals, the same, to the cube of a third. Of the number of things ("cose") named. Then, the remaining amount. Of the cube roots subtracted will be our desired count. When a cube and its things near. Add to a new, descrete number. Determine two new, different numbers. By that one, then, the remaining amount of the cube roots subtracted will be our desired count". This is the solution in the poem, not the demonstration Tartaglia sent to Cardano. The last verse could allude to the fact that Tartaglia found the formula while he was in Venice.

[31] Tartaglia [1554] 1959, Book IX, Q XXXIIII, 120v.

Tartaglia studied the following equations (modern notation with all the the terms positive):

$$x^3 + px^2 = q \qquad\qquad x^3 + q = px^2$$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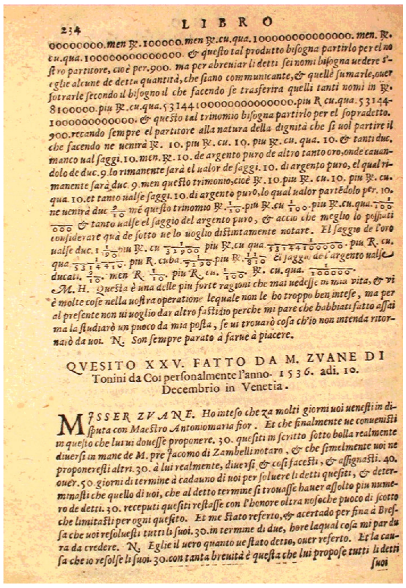
QVESITO XIII. FATTO DA MAESTRO ANTONIO Veronese, Zenero de Maestro Francesco Feliciano adi.16. Settembrio.1527.in Verona.

QVESITO XIIII. QVAL MI FV MANDATO A Verona da vn M.Zuane de Tonini da Coi, qual tenea la schola in Bressa, & me lo portò messer pre Antonio da Cellatica l'anno.1530.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QVESITO XXV. FATTO DA M. ZVANE DI Tonini da Coi personalmente l'anno 1536. adi. 10. Decembrio in Venetia.

Fig. 1.7 Plates from Tartaglia's reasoning on the third–degree equation[32]

His studies is aimed at building new equations – as previously mentioned – having (in modern style notation) roots in the following form:

$$x = \sqrt{a} - b \qquad\qquad x = \sqrt{a} + b$$

Based on this and in modern terms, Tartaglia could also study the following type of equations:

$$x^3 + px = q\,, \qquad x^3 = px + q\,, \qquad x^3 + q = px\,.$$

The events and reasons surrounding the origin of the *matematica disfida* (Masotti 1974a, 1974b), which arose between 1547 and 1548 between

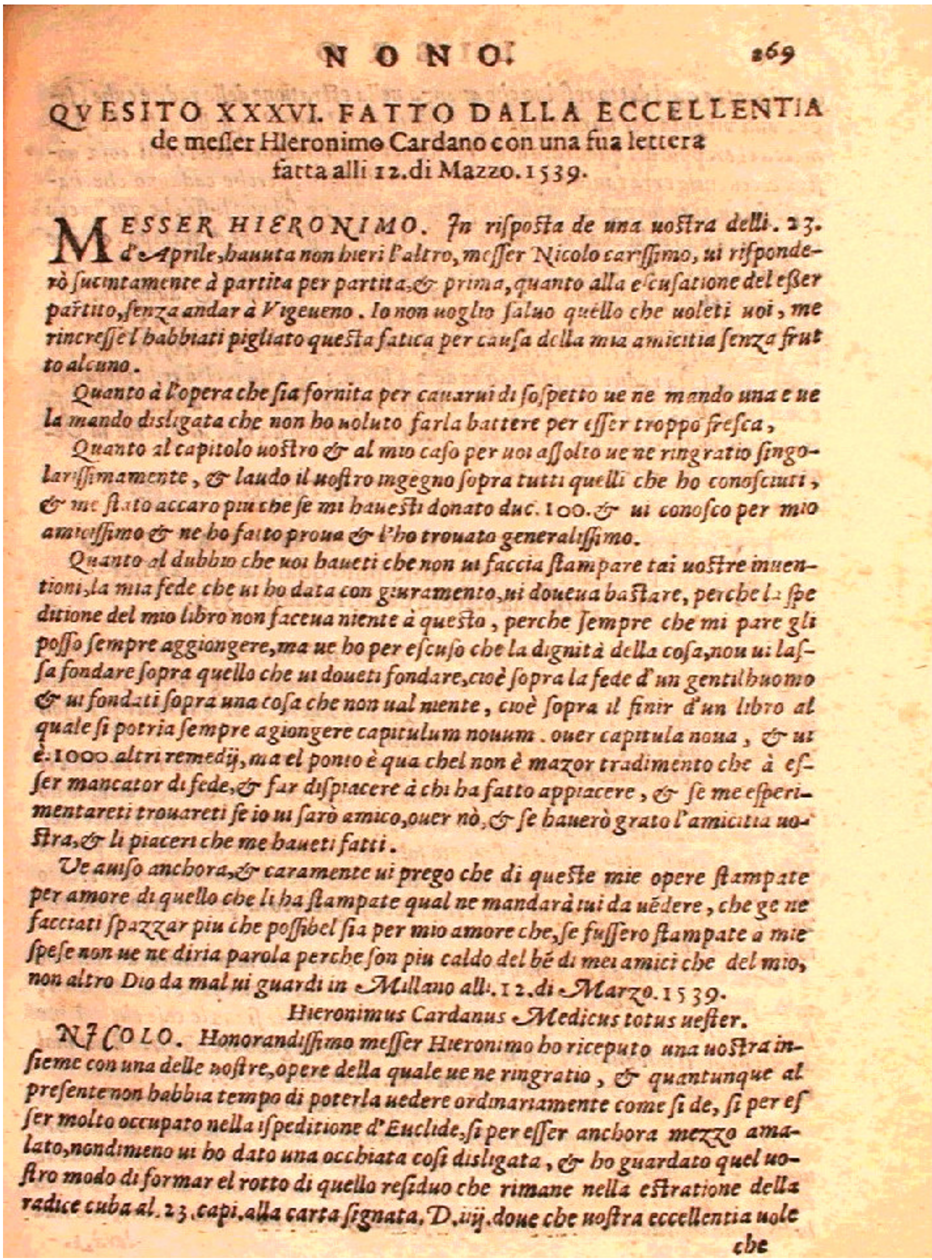
[32] Tartaglia 1554, Book IX, Q 14, 101rv and Q 25, 106rv.

Ferrari, who sought to defend his mentor Cardano, and Tartaglia are well known. The scientific dispute began with *cartelli* and six *controcartelli* in which 62 mathematical problems referring to Euclidean geometry were put forth and partially solved. Neverthless, a concise timeline with presumably historical discoveries related to the evidence of the rule for solving third–degree equations is here below:

Scipione del Ferro (1465–1526) in the 1510s (fl. 1520s) but never published.
Tartaglia's solution[33] (Tartaglia 1535) since his methatical debate with Anton Maria del Fiore, Ferrari's scholar. Tartaglia did not publish his solution.
Lodovico Ferrari (1522–1565) and his six "Cartelli" (Pamphlets) (1547–48) against Tartaglia
Tartaglia's *Risposte* (Replies) to *Lodovico Ferrari*, Venezia 1547 (1–4) and Brescia 1548 (5–6).

Let us see the main details (Tartaglia 1554, Qs. 20, 25, 26, 28, 29, 31–41). Tartaglia, after great insistance, relayed the solution to Girolamo Cardano (25 March 1539) who, in addition to being a very famous doctor, was also an excellent mathematician (Bolletti 1958, pp 93–111). Fortunately, for Cardano, despite the fact that Tartaglia's solution was expressed in coded verses, his skills helped him to decipher the solution and publish it before Tartaglia. There is an interesting exchange between Cardano and Tartaglia (4 August 1539) (Tartaglia 1554, Book IX, Q 38), in *Quesiti et invention diverse* (Tartaglia 1554) not only regarding the solution of the superior degree equation but also geometrical topics. In this exchange, Cardano put forth a specific request concerning a geometrical problem:

[33] The news spread and a mathematical contest made up of thirty problems was organized (12 febbraio 1535). Only Tartaglia succeeded in solving these problems in the allotted time.



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QVESITO XXXVI. FATTO DALLA ECCELLENTIA de messer Hieronimo Cardano con una sua lettera fatta alli 12. di Mazzo. 1539.

MESSER HIERONIMO. In risposta de una vostra delli. 23. d'Aprile, hauuta non hieri l'altro, messer Nicolo carissimo, ui risponderò sucintamente à partita per partita, & prima, quanto alla escusatione del esser partito, senza andar à Vigeueno. Io non uoglio saluo quello che uoleti uoi, me rincresse l habbiati pigliato questa fatica per causa della mia amicitia senza frutto alcuno.

Quanto à l'opera che sia fornita per cauarui di sospetto ue ne mando una e ue la mando disligata che non ho uoluto farla battere per esser troppo fresca,

Quanto al capitolo uostro & al mio caso per uoi assolto ue ne ringratio singolarissimamente, & laudo il uostro ingegno sopra tutti quelli che ho conosciuti, & me stato accaro piu che se mi hauesti donato duc. 100. & ui conosco per mio amicissimo & ne ho fatto proua & l'ho trouato generalissimo.

Quanto al dubbio che uoi haueti che non ui faccia stampare tai uostre inuentioni, la mia fede che ui ho data con giuramento, ui doueua bastare, perche la spedition del mio libro non faceua niente à questo, perche sempre che mi pare gli posso sempre aggiongere, ma ue ho per escuso che la dignità della cosa, non ui lassa fondare sopra quello che ui doueti fondare, cioè sopra la fede d'un gentilhuomo & ui fondati sopra una cosa che non ual niente, cioè sopra il finir d'un libro al quale si potria sempre agiongere capitulum nouum. ouer capitula noua, & ui è. 1000. altri remedij, ma el ponto è qua chel non è mazor tradimento che à esser mancator di fede, & far dispiacere à chi ha fatto appiacere, & se me esperimentareti trouareti se io ui sarò amico, ouer nò, & se hauerò grato l'amicitia uostra, & li piaceri che me haueti fatti.

Ve auiso anchora, & caramente ui prego che di queste mie opere stampate per amore di quello che li ha stampate qual ne mandarà lui da uedere, che ge ne facciati spazzar piu che possibel sia per mio amore che, se fussero stampate a mie spese non ue ne diria parola perche son piu caldo del bé di mei amici che del mio, non altro Dio da mal ui guardi in Millano alli. 12. di Marzo. 1539.

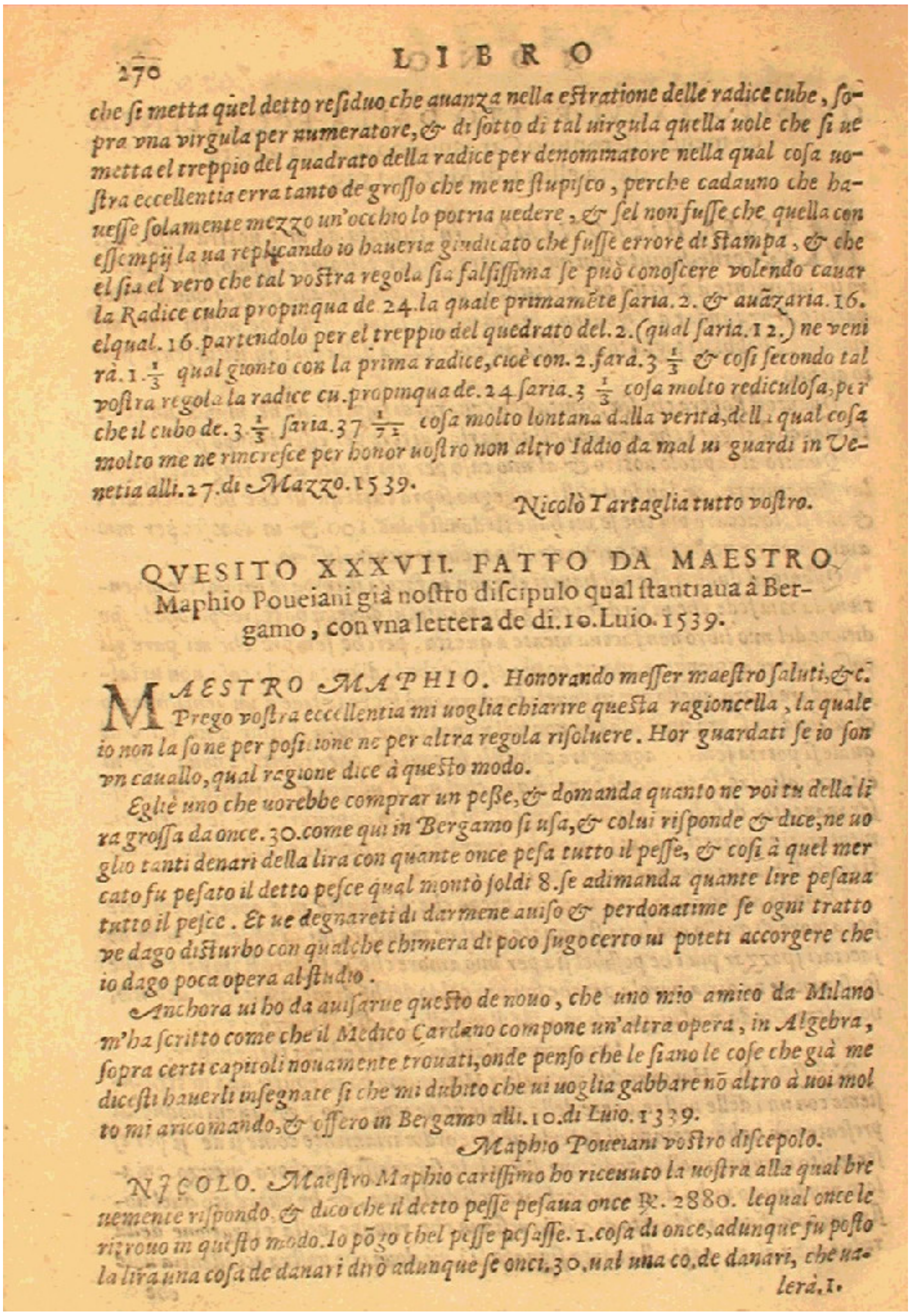
Hieronimus Cardanus Medicus totus uester.

NICOLO. Honorandissimo messer Hieronimo ho riceputo una uostra insieme con una delle uostre, opere della quale ue ne ringratio, & quantunque al presente non habbia tempo di poterla uedere ordinariamente come si de, si per esser molto occupato nella ispeditione d'Euclide, si per esser anchora mezzo amalato, nondimeno ui ho dato una occhiata cosi disligata, & ho guardato quel uostro modo di formar el rotto di quello residuo che rimane nella estratione della radice cuba al. 23. capi. alla carta signata, D. iiij. doue che uostra eccellentia uole

che

Fig. 1.8 Plates from Tartaglia's reasoning on Cardano[34]

[34] Tartaglia 1554, Book IX, Qs. 36–38.



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che si metta quel detto residuo che auanza nella estratione delle radice cube, sopra vna virgula per numeratore, & di sotto di tal uirgula quella uole che si ue metta el treppio del quadrato della radice per denommatore nella qual cosa uostra eccellentia erra tanto de grosso che me ne stupisco, perche cadauno che hauesse solamente mezzo un'occhio lo potria uedere, & sel non fusse che quella con essempij la ua replicando io haueria giudicato che fusse errore di stampa, & che el sia el vero che tal vostra regola sia falsissima se può conoscere volendo cauar la Radice cuba propinqua de 24. la quale primamēte saria. 2. & auāzaria. 16. elqual. 16. partendolo per el treppio del quedrato del. 2. (qual saria. 12.) ne veni rà. 1. $\frac{1}{3}$ qual gionto con la prima radice, cioè con. 2. farà. 3 $\frac{1}{3}$ & cosi secondo tal vostra regola la radice cu. propinqua de. 24 saria. 3 $\frac{1}{3}$ cosa molto rediculosa, per che il cubo de. 3 $\frac{1}{3}$ saria. 37 $\frac{1}{72}$ cosa molto lontana dalla verità, della qual cosa molto me ne rincresce per honor uostro non altro Iddio da mal ui guardi in Venetia alli. 27. di Mazzo. 1539.

Nicolò Tartaglia tutto vostro.

QVESITO XXXVII. FATTO DA MAESTRO Maphio Poueiani già nostro discipulo qual stantiaua à Bergamo, con vna lettera de di. 10. Luio. 1539.

MAESTRO MAPHIO. Honorando messer maestro saluti, &c. Prego vostra eccellentia mi uoglia chiarire questa ragioncella, la quale io non la so ne per positione ne per altra regola risoluere. Hor guardati se io son vn cauallo, qual ragione dice à questo modo.

Eglie uno che uorebbe comprar un peße, & domanda quanto ne voi tu della lira grossa da once. 30. come qui in Bergamo si usa, & colui risponde & dice, ne uoglio tanti denari della lira con quante once pesa tutto il pesse, & cosi à quel mercato fu pesato il detto pesce qual montò soldi 8. se adimanda quante lire pesaua tutto il pesce. Et ue degnareti di darmene auiso & perdonatime se ogni tratto ve dago disturbo con qualche chimera di poco sugo certo ui poteti accorgere che io dago poca opera al studio.

Anchora ui ho da auisarue questo de nouo, che uno mio amico da Milano m'ha scritto come che il Medico Cardano compone un'altra opera, in Algebra, sopra certi capitoli nouamente trouati, onde penso che le siano le cose che già me dicesti hauerli insegnate si che mi dubito che ui uoglia gabbare nō altro à uoi molto mi aricomando, & offero in Bergamo alli. 10. di Luio. 1539.

Maphio Poueiani vostro discepolo.

NICOLO. Maestro Maphio carissimo ho riceuuto la uostra alla qual breuemente rispondo. & dico che il detto pesse pesaua once R. 2880. lequal once le ritrouo in questo modo. Io pōgo chel pesse pesasse. 1. cosa di once, adunque fu posto la lira una cosa de danari dirò adunque se onci. 30. ual una co. de danari, che ualerà. 1.

Fig. 1.8 Continued[35]

35 *Ibidem*.

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lerà.1.cosa de once multiplico.1.cosa.de once fia.1.cosa de danari farà.1.censo de danari da partir per.30.qual partendolo me ne uien.1.censo, estimo de 30.& qsto tal rotto sarà equale à danari.96.cioè a soldi.8.fatto in danari, leuo li rotti & seguito el capitolo trouo la cosa ualer ℞.2880.& tante once pesaua el detto pesse,come di sopra dissi,& anchora tāti danari fu posto la lira à once.30.p lira, onde facēdo el conto mōtaria pcisamēte danari.96.cioè soldi.8.ch'è il proposito.

Circa alla noua che me scriueti hauer inteso del Medico Cardano da Milano,certamente ne hò riceuuto fastidio assai,perche s'eglie il uero che lui dice di uoler dar fora capitoli nouamente ritrouati,el non può esser altramente di quello che haueti detto, e però il prouerbio non mentisse, qual dice. Quello che tu non uoi che si sappia nol dir ad alcuno, stati attento se intendereti altro sopra di questo datimene auiso non altro Iddio da mal ui guardi in Venetia alli.19.di Luio.1539.

QVESITO XXXVIII. FATTO CON VNA LETTERA dalla eccellentia de messer Hieronimo Cardano riceputa alli.4.di Agosto.1539.

MESSER HIERONIMO. Per auiso del nostro ben stare, & de molte altre lettere quale ue hò scritte anchor non ue siati degnato di riscriuermi, & tanto più io ue ho mandato adomandare la resolutione de diuersi quesiti alli quali non mi haueti risposto,& tra li altri quello di cubo equale à cose, e numero,eglie ben uero che hò inteso tal regola, ma quando che il cubo della terza parte delle cose eccede il quadrato della mità del numero, all'hora non posso farli seguir la equatione,come appare,però haueria appiacere me soluesti questa.1. cubo egual à.9.cose più.10.& di questo mi fareti sommo appiacere.

Vi prego anchora che mi uogliati mandarme quel uostro modo da descriuere Geometrice uno quadrato in un triangolo de lati diuersi, però che circa à tal cosa me gli sono affaticato assai,& mai hò potuto ritrouar modo da saperlo fare,offerendomi anchora mi per uoi,se posso,e uaglio.

Ve auiso anchora qualmente io indrizzai da uoi il Signor Don Diego de Mendocia Ambasciatore della maestà dell'Imperatore,qual se diletta di queste scientie,qual penso non ui saria inutile,& gli dissi dell'altezza delle uirtù uostre, come meritati.

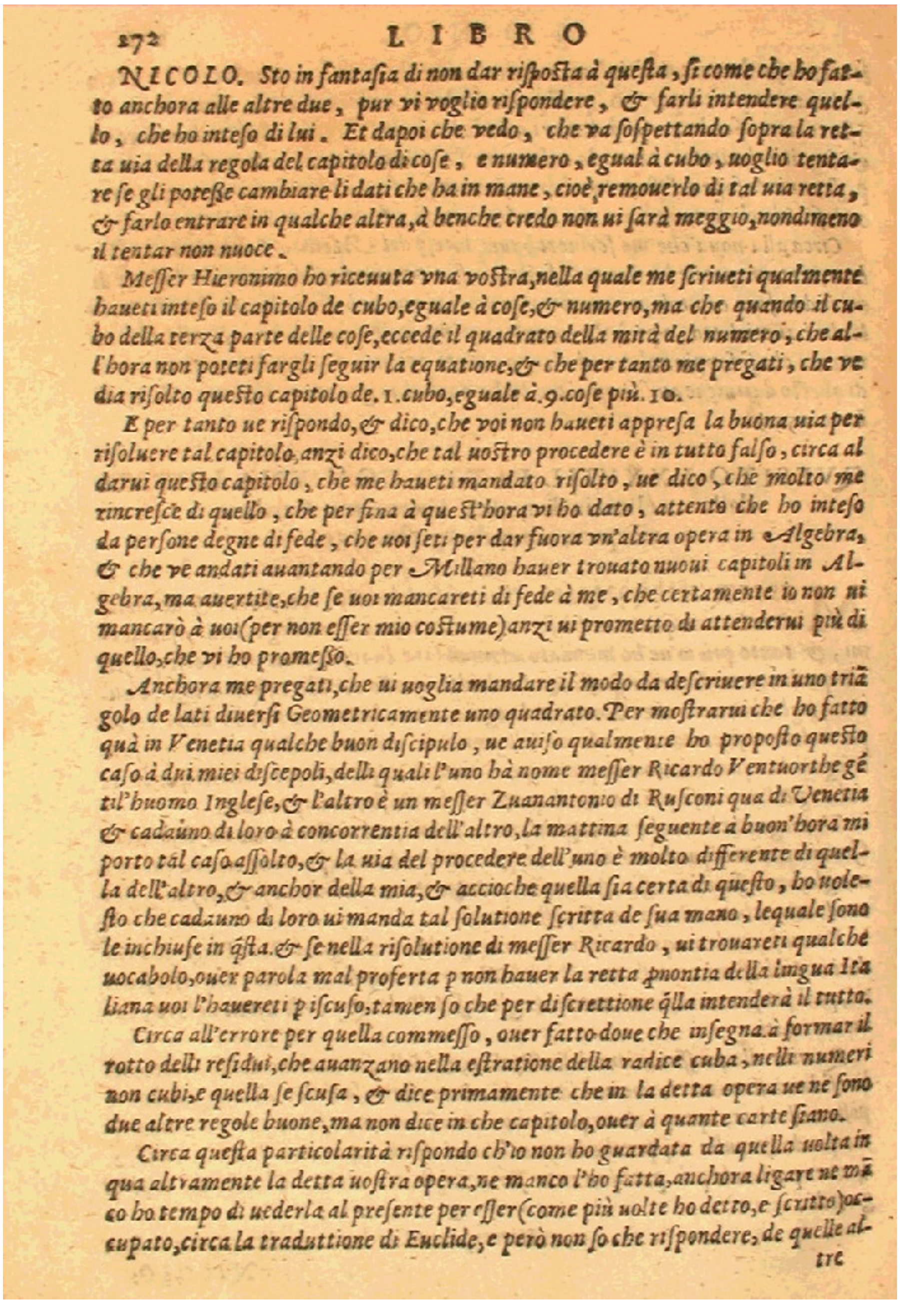
Quanto alla prossimatione della Radice, & della formatione del suo rotto,nel li residui delli numeri, che non sono cubi. Dico che ne sono due altre regole buone poste nella detta opera, & in quella non ui casca errore, saluo che nel detto essempio de ℞.cuba 24 perche la ℞.cuba del detto.24.reuera sarebbe circa.2. $\frac{1}{4}$ ouer parlando più precisamente saria.2 $\frac{20}{81}$.non altro. Christo da mal ui guardi.

Hieronymus Cardanus medicus totus uester.

S NICOLO.

Fig. 1.8 Continued[36]

[36] *Ibidem*.



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NICOLO. Sto in fantasia di non dar risposta à questa, si come che ho fatto anchora alle altre due, pur vi voglio rispondere, & farli intendere quello, che ho inteso di lui. Et dapoi che vedo, che va sospettando sopra la retta uia della regola del capitolo di cose, e numero, egual à cubo, uoglio tentare se gli potesse cambiare li dati che ha in mane, cioè, remouerlo di tal uia retta, & farlo entrare in qualche altra, à benche credo non ui sarà meggio, nondimeno il tentar non nuoce.

Messer Hieronimo ho riceuuta vna vostra, nella quale me scriueti qualmente haueti inteso il capitolo de cubo, eguale à cose, & numero, ma che quando il cubo della terza parte delle cose, eccede il quadrato della mità del numero, che all'hora non poteti fargli seguir la equatione, & che per tanto me pregati, che ve dia risolto questo capitolo de. 1. cubo, eguale à. 9. cose più. 10.

E per tanto ue rispondo, & dico, che voi non haueti appresa la buona uia per risoluere tal capitolo, anzi dico, che tal uostro procedere è in tutto falso, circa al darui questo capitolo, che me haueti mandato risolto, ue dico, che molto me rincresce di quello, che per fina à quest'hora vi ho dato, attento che ho inteso da persone degne di fede, che uoi seti per dar fuora vn'altra opera in Algebra, & che ve andati auantando per Millano hauer trouato nuoui capitoli in Algebra, ma auertite, che se uoi mancareti di fede à me, che certamente io non ui mancarò à uoi (per non esser mio costume) anzi ui prometto di attenderui più di quello, che vi ho promesso.

Anchora me pregati, che ui uoglia mandare il modo da descriuere in uno triãgolo de lati diuersi Geometricamente uno quadrato. Per mostrarui che ho fatto quà in Venetia qualche buon discipulo, ue auiso qualmente ho proposto questo caso à dui miei discepoli, delli quali l'uno hà nome messer Ricardo Ventuorthe gétil'huomo Inglese, & l'altro è un messer Zuanantonio di Rusconi qua di Venetia & cadauno di loro à concorrentia dell'altro, la mattina seguente à buon'hora mi porto tal caso assolto, & la uia del procedere dell'uno è molto differente di quella dell'altro, & anchor della mia, & accioche quella sia certa di questo, ho uolesto che cadauno di loro ui manda tal solutione scritta de sua mano, lequale sono le inchiuse in qsta. & se nella risolutione di messer Ricardo, ui trouareti qualche uocabolo, ouer parola mal proferta p non hauer la retta pnontia della lingua Italiana uoi l'haureti p iscuso, tamen so che per discrettione qlla intenderà il tutto.

Circa all'errore per quella commesso, ouer fatto doue che insegna à formar il rotto delli residui, che auanzano nella estratione della radice cuba, nelli numeri non cubi, e quella se scusa, & dice primamente che in la detta opera ue ne sono due altre regole buone, ma non dice in che capitolo, ouer à quante carte siano.

Circa questa particolarità rispondo ch'io non ho guardata da quella uolta in qua altramente la detta uostra opera, ne manco l'ho fatta, anchora ligare ne mãco ho tempo di uederla al presente per esser (come più uolte ho detto, e scritto) occupato, circa la traduttione di Euclide, e però non so che rispondere, de quelle altre

Fig. 1.8 Continued[37]

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[37] *Ibidem*.

Tartaglia's response arrived on 7 August 1539 with obvious merit for his solution.

Tartaglia begins by proposing 17 problems for Ferrari[38] which involve using a compass with fixed opening (Tartaglia, *Seconda risposta*, Venice, 21 April 1547), [15–18], 53–56). Ferrari responds by solving these problems, adding that not only Tartaglia's problems but all of the Euclidean propositions can be solved by using a compass with fixed opening[39]. The subject was translated into Latin and published by Cardano in *De subtilitate* (Cardano 1550, Book XV, 296–302) [40]. Ferrari and Cardano's solution methods are too complex for Tartaglia, who introduces one of his future publications (*Sesta Risposta*, Brescia, 24 July 1548). In fact, Tartaglia goes back to the 17 problems and risolves them in *General Trattato* (Tartaglia 1556–1560, Part V, 63v–83v). Today both Ferrari and Tartaglia's merits in their conclusive and demonstrative procedures are recognized (Bortolotti 1935, 75–76). Most importantly, Tartaglia and Ferrari are recognized, thanks to the *cartelli di sfida matematica,* for creating a conclusive approach using a straightedge and compass with fixed opening (assigned at will) which became a public use.

After a long written diatribe, the two rivals faced each other in Milan on 10 August 1548. The outcome of this encounter was subject to opposing judgements (Masotti 1974, pl XXXIV–XL). The fact that some problems discussed in Tartaglia and Ferrari's dispute concerned Euclidean geometry is noteworthy. These problems concerning plane geometry were quite significant (Masotti 1974, XXI–XXIII and footnotes 104–107) since they were always solved by using a straightedge and compass, the latter using the fixed opening technique (*Ivi*).

In *Ars magna* (1545) Cardano also published the solution to the fourth degree equation. It must be noted, however, that Cardano cites Tartaglia as author of the solution of the cubic equation and Ludovico Ferrari (1522–1565) as the person who discovered the solution to the fourth degree equation. Therefore, Cardano's error in regard to Tartaglia (which he avoided mentioning) was not keeping his promise not to divulge the secret of the solution. On can image that Tartaglia – with such a discovery –

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[38] We remark that among the 31 inquiries which Ferrari sent to Tartaglia in *Terzo Cartello di matematica disfida* (1547–1548), there are two inherent to the inscription and reciprocal circonscription of regular polygons, which can also be found in *Commentaria in Euclidis Elementa geometrica* by Cardano (Cardano 1574; see also Masotti 1974c, pp 66–68; 1974b).

[39] Ferrari, *Quinto cartello* (Milan, October 1547), [25–39], 141–155.

[40] Which was also translated into Latin (Masotti 1974, plates XXX–XXXVI; Cardano 1663, *Opera omnia*, III, 589–592).

could have acquired a certain visibility in the academic and professional panorama. This occurrence engendered a series of disputes between the two mathematicians that lasted two years and a ferocious dispute between Tartaglia and Cardano's student, Ludovico Ferrari. Obviously, whatever the historical truth about such misdeeds is not what interests us. We note only that on the first page *of Ars Magna* (*The Great Art*) Cardano attributes (Baldi and Canziani) the solution of the cubic equation to Scipione del Ferro[41] (ca. 1465–1526), instructor of mathematics at the Medieval University of Bologna –a solution it seems he had already studied in 1515:

Fig. 1.9 Plates on Cardano's speeches concerning the solutions of 3rd equation[42]

[41] Although he didn't publish his discovery, before his death, Scipione dal Ferro revealed it to one of his students, the Venetian Anton Maria Florido (*Floridus*).

[42] "CAPVT PRIVM. De duabus equationibus in singulis capitalis. Haec ars olim a Mahomete, Molis Arabus filio initium fumpsit. [...]. Domum etiam ex primis, alia tria deriuatiua, a quodam ignoto viro inunenta legi, haec, tamen

In fact, as regards the formula that gave the solution to the cubic equation, both Tartaglia's version and Scipione dal (or del) Ferro's previous version were not immediately reducible since both contained a quadratic term that neither mathematician initially knew how to eliminate. It seems that Tartaglia was not able to overcome this obstacle before Cardano's publication of *Ars Magna*. Some maintain that this publication was justified both because six years[43] had passed since Cardano's promise to Tartaglia and because Cardano was not expected to respect a promise based on a discovery belonging to del Ferro and not to Tartaglia. Tartaglia responded to such claims by *Quesiti*, where – in in addition to the disputes with Cardano – he lists some others. Ferrari did the same in a pamphlet entitled "matematica disfida". In *Cartelli* an extreme value is proposed which seems to refer to Ferrari but Tartaglia solved it without sufficient proof (Masotti 1970–1980, p 259).

In the end the historical legend concerning an eventual plagiarism and other accusations directed to Tartaglia made his ascent into the academic world difficult even though his works, today, are impartially seen as a milestone in the history of mathematics and an important contribution to statics. Tartaglia stayed in Brescia for a period of time (1548–1549), teaching at S. Afra, S. Barnaba, S. Lorenzo and at the Accademy of Rezzato. In the last years of his life he had thriving scientific activities in Venice.

1.2.2 On the Geometry: Euclid's *Elements*

Concerning this subject, Tartaglia's calculation of the volume of a tetrahedron from the length of its sides and inscribing within a triangle three of circles tangent to each others is very important. Not less important

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minime in lucem prodierant, cum essent alijslong. Utiliora nam cubi & numeri & cubi quadrati aestimationem docebant. Verum temporibus nostris, Scipio Ferreus Bnonoiensis, capitulum cubi & rerum numero aequalium inuenit, rem sanè pulchram & admirabiliem. Cum omnem humanam sublititatem, omnis ingenij mortalis claritatem ars haec superet, donum profecto coeleste, experimentum autem virtutis animorum, atque adeo illistre ut qui haec attigerit, nihil non intelligere posse se credat. Huius aemulatione Nicolaus Tartalea Brixellensis, amicus noster, cum in certamen cum illius discipulo Antonio Maria Florido venniset, capitulum idem, ne vinceretur, inuenit, qui mihi ipsum multis precibus exoratus tradit" (Cardano 1663, chap 1, cl–left, line 1).

[43] It must be noted that a different historiographyc opinion exists according with Cardano who waited for six years so that Tartaglia could have the chance to publish it.

were the studies on the division of areas (see *Cartelli* against Ferrari) and on geometry of the compass (before Galilei'studies) which he presented in his *General trattato di numeri e misure*. Tartaglia's work also possesses extraordinary cultural and scientific significance since he is also known for being an editor of classical geometry: he translated Euclid's *Elements* even if with the unhappy title *Euclide Megarense* (Tartaglia 1543a; see also: 1565–*Euclid*; 1569; 1585).

According to Tartaglia's biography (1567) by Bernardino Baldi (1553–1617), Tartaglia lectured on Euclid's *Elements* in *SS. Giovanni e Paolo* church (Venice, starting in 1536). In fact, he was mainly a teacher–researcher first in Verona as an *Abacus' Master* (starting in 1518) and then in Venice[44] as a *Pubblico lettore di Matematica* (Lecturer of mathematics, 1536–1548).

Tartaglia's Euclidean translation is at the center of a renewed scientific debate within an extensive sixteenth–century movement of geometric revival and geometric practice (Masotti 1980a; Pizzamiglio 2007). At the time Euclide from Megara (fl. V–IV B.C.) was considered to be the author of *Elements* (Euclid from Alexandria (fl. 325–265 B.C.); see also Cuomo 2004).

[44] Differently from other opinions (Gabrieli 1986, p 30) – based on no historical proof – Tartaglia not *substituted* for Giovanni Battista Memo (1550–1575) in mathematics teaching in Venice, but he was only a successor (1536) as one can read in *Book IX* (Tartaglia 1554, *Book IX*, Quesito XXII, 104v).

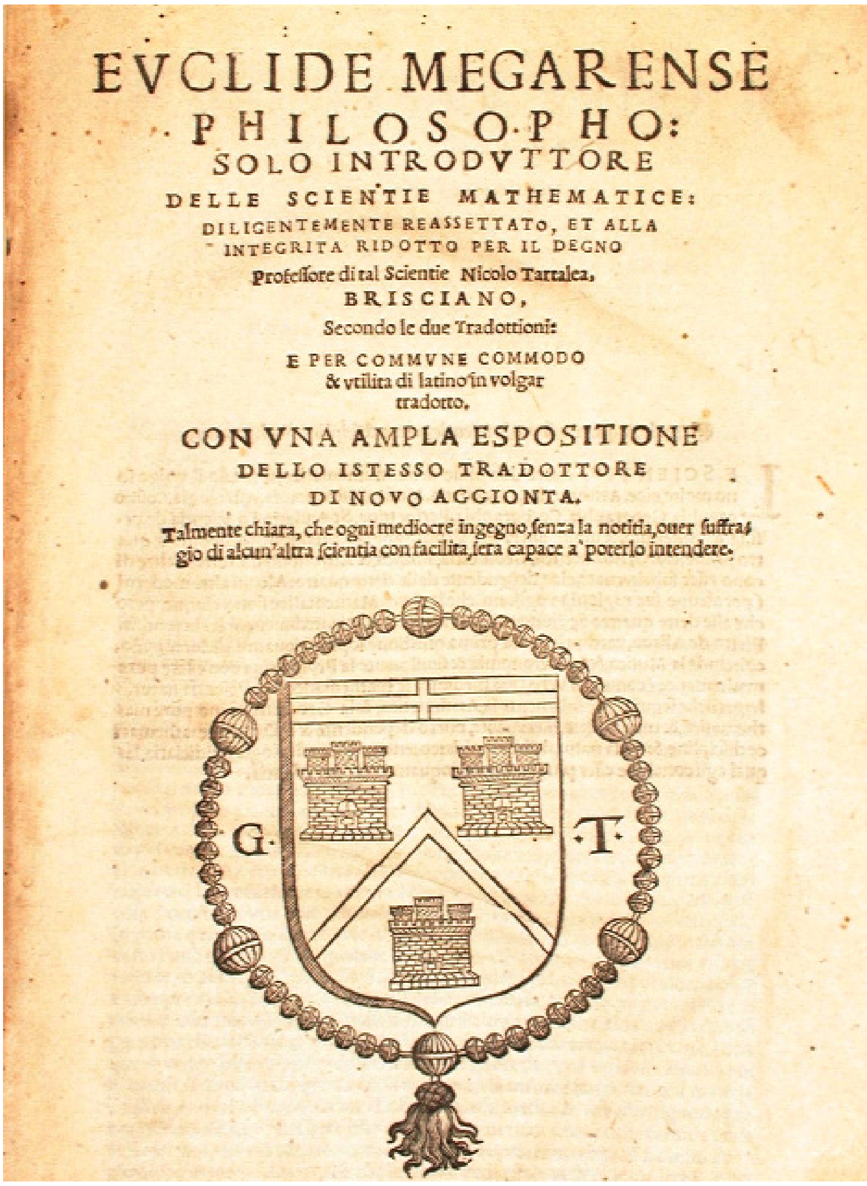

Fig. 1.10 Plate from the cover of Euclide Megarense by Tartaglia[45]

[45] Tartaglia 1543a. Pierluigi Pizzamiglio recently edited an excellent historical–critical work on Tartaglia's Euclide Megarense (Tartaglia 2007).

After the Willem van Moerbeke (1215–1286) edition, Archimedes was republished both in *Opera Archimedis* (Tartaglia 1543), and in the final parts of some of Tartaglia's other works (Tartaglia 1551, 1565).

In a recent work (Pizzamiglio 2007) the editorial and didactic character of Tartaglia's Euclidean operation was reconstructed as an operation essentially within the field of teaching (Pisano 2013d). Based on the historiography on Euclid by Tartaglia (Pizzamiglio 2007), in the end, four main approaches can be found:

1. The precarious nature of the various integral or partial editions of Euclid's text (Thomas–Stanford, 21–31, ft 1–25) and the relative more or less ample comments.
2. Partial texts were present among the various contributions, which contained statements of the Euclidean propositions (Thomas–Stanford, 35–37, ft 26–33). Thus, only Euclid's problems and theorems were considered interesting. The demonstrations of the latter would have been elaborated by Theon and other Euclidean commentators (Pizzamiglio 2007). This could have depended also on the scholastic use of Euclidean manuals which left the intstructor the choice of which geometric statements to demonstrate and which to consider simply as declarations of properties which were more or less evident. The fact that less lengthy texts cost less for students with limited means was also of considerable importance (*Ivi*).
3. The revival (*Ivi*) of the Tartalean text to meet the demands of new emerging classes in *vulgare* Italian instead of classical language. Tartaglia began (*Ibidem*) analagous editorial intitiatives in vulgar national languages which, in the course of the 16th century, interested all of Europe (Thomas–Stanford, 41–45, ft 34–45).
4. The revival in non–classical language also favoured (*Ivi*) a noteworthy secondary literature in mathematics and geometry by way of amplification and elaboration (Thomas–Stanford, 49–62, fts. I–XXXVIII).

In brief, we provide a timeline of Euclidean subjects–editions in history concering Tartaglia's lifetime[46]

Date	Event
1505	After Giovanni Campano's edition, Bartolomeo Zamberti (fl. 15th–16th) 25 October (*VIII Kalendas Novembris*) 1505 published in Venice, with editor Ioannes Tacuinus (240 *foli*): *Euclidis Megarensis philosophi platonicj, Mathematicarum disciplinarum Ianitoris*. It included: Zamberti's translation from Greek to Latin

[46] Cfr.: Pizzamiglio 2007.

of various works [47] of an "Euclide Megarense, platonic philosopher", known, however, in the title as "Introducer to the mathematical disciplines"—a heading Tartaglia subsequently used. Zambetti's monumental Euclidean edition was plagiarized and reprinted in various editions which are not always easy to discern one from each other[48].

1509 Luca Pacioli (1445c.–1517) published a re–release, revisited and corrected, of the medieval version of Campano by the title: *Euclidis megarensis philosophi acutissimi mathematicorumque omnium sine controversia principis opera a Campano interprete fidissimo traslata*. The text takes up a little more than half the width of the page, while the rest is reserved for the 129 geometric figures. Campano's Euclidean comments are re–used until Tartaglia's Italian translation which also used, as did many in this period, Zamberti's translation.

1528–1550 In 1528 in Vienna, in 1529 in Strasbourg, in 1534 in Paris and in Frankfurt, in 1536 in Writtenberg, in 1539 in Venice, in 1548 in Frankfurt, in 1550 in Paris l'*Elementale geometricum ex Euclidis Geometria* by Johann Voegelin (fl. 15^{th}–16^{th}) is repeatedly reprinted.

1529 Giovanni Battista Politi (XV–XVI centuries) publishes (Siena Simone Nicolò de' Nardi editor) a booklet: *Expositio super definitiones et propositiones quae supponuntur ab Euclide in Quinto Elementorum eius*.

1532 Tartaglia asks for and obtains from the Venetian Senate 11 December 1532, a printing license and the concession of exclusive privileges for the translation and revision of *Elements,* as well as for the writings of Archimedes, Heron and Luca Pacioli (*Archivio di Stato di Venezia*: Senato, Terra, reg. 32, cc. 94r–v). However, in the end he will only able to produce editorial interventions on Euclid and Archimedes.

1534–1547 Tartaglia teaches in Venice[49]at the Church of San Zanipolo, presenting Euclid [50] and various books.

1543 In February 1543, Niccolò Tartaglia's translation of Euclid is published in Venice: *Euclide Megarense philosopho, solo introduttore delle scientie mathematice*. The Tartalean edition has three more editions in Venice: 1565–66, 1569 and 1585.

1546–1548 Between 1546 and 1548 Giovanni Battista Benedetti (1530–1590) studied Tartaglia's edition of the first four books from Euclid's *Elements*.

[47] Directly on Greek codes, as yet unidentified, however of rather low quality.
[48] We refer the reader to Pizzamiglio's literature for these details (Pizzamiglio 2007).
[49] Gabrieli 1986, 29–67.
[50] Tartaglia 1554, Book IX, Q 22.

1554 Study in the form of a dialogue of scientific problems from ballistics is re–edited and widened to the fortifications of statics in the mathematics of *Quesiti et inventioni diverse* (1554), already edited by Tartaglia in a shorter form in 1546. A version from 1562 will be published posthumously.

Moreover, the geometry is included (Tartaglia 1554, Book IX) as well. The arguments concern triangules and squaring the circle (Tartaglia 1554, Book IX, Qs. 15, 32, 38) as one of the main mathematical and historical problems proposed by ancient geometers. It is the challenge of constructing a square with the same area as a given circle by using only a finite number of steps with a compass and straightedge. More abstractly and more precisely, it may be asked whether specified axioms of Euclidean geometry concerning the existence of lines and circles entail the existence of such a square.

1.2.3 On the Arithmetics: Tartaglia's Triangle

Other mathematical subjects Tartaglia studied are linked to his contributions to arithmetics: numerical calculations, extraction of roots, denominator's rationalization, combinatorial analysis and other methods to solve arithmetical and measurement problems. "Tartaglia's triangle"[51] presented in *General trattatto* di *numeri e misure* (Tartaglia 1556–1560) aimed at finding a general formula for solving cubic polynomials.[52] It is quite interesting that his handbook for arithmetics and physical measurements was entitled "Trattato" instead of the more common word "Summa"[53], typical of the late Middle Ages so making clearer the novelties and purposes of the research. The same consideration could concern the word "Generale" which explains Tartaglia's didactic nature.

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[51] The triangular method by means of a different configuration is possibile to see in other early scholarly works, e.g., in Pascal's *Traité du triangle arithmétique* (1653). Nevertheless, the earliest explicit depictions of a triangle of binomial coefficients occur in the 10th century in commentaries on the Chandas Shastra, an Ancient Indian book on Sanskrit prosody written (fl. 2nd century BC) by Pingala. (Edwards 2002, 30–31).

[52] Two years before his death (1556), Tartaglia worked on his larger compendium, which unfortunately, he was unable to finish and publish.

[53] Generally speaking, the *Trattato* was intended (at that time) as research work not necesserely large, and well structured mostly based on known principles. The *Summa*, tpically within Meddle Ages, had the prerogative to be a largely and oraganically exhaustive for monastic schools and universities (Pisano 2013; Pisano 2014-submitted).


LA SECONDA PARTE
DEL GENERAL TRATTATO DI
NVMERI, ET MISVRE DI NICOLO TARTAGLIA,
NELLAQVALE IN VNDICI LIBRI SI NOTIFICA LA
PIV ELLEVATA, ET SPECVLATIVA PARTE DELLA PRATICA
Arithmetica, laqual è tutte le regole, & operationi praticali
delle progressioni, radici, proportioni,
& quantita irrationali.

MALIGNITA'
NOIAR NON PVO
A' FORTEZZA

Con priuilegio della santita di Papa Paolo IIII. Della Illustrissima Signoria di Venetia, & dell'eccellentissimo signor Duca d'Vrbino.

In Vinegia per Curtio Troiano dei Nauò.
M D LVI.
Appresso dell'Auttore.

Fig. 1.11 Plate from *General Trattato* on Tartaglia's triangular method[54]

[54] Tartaglia 1556–1560, pt II, Frontespice. Pascal's *Traité du triangle arithmétique* (*Treatise on Arithmetical Triangle*) was published posthumously in 1665. Pascal collected several results then known about the triangle, and employed them to solve problems in probability theory. Recently for the 450th Anniversary ofTartaglia's death, Pierluigi Pizzamiglio organized a Colloquium

LIBRO

8 ET il principio di numeri triangolari li nostri antichi filosofi vogliono, che sia la vnita, & dapoi quella il 3. dapoi il 6. dapoi il 10. dapoi il 15. & cosi tutti quelli, che assettati secondo l'ordine de gli essempi figurati in margine formano vna figura triangolare equilatera.

9 SImilmente il principio di tutti li numeri quadrati vogliono che sia pur la vnita, & dapoi quella il 4. dapoi il 9. dapoi il 16. dapoi il 25. & cosi tutti quelli, che assettati secondo l'ordine, che in margine appar formino vna figura quadrata.

10 SImilmente il principio di tutti li numeri penthagonali vogliono che sia pur la vnita, & dapoi quella il 5. poi il 12. poi il 22. poi il 35. & cosi tutti quelli, che assettati secondo l'ordine posto in margine venghino in forma, ouer figura penthagonale.

11 SImilmente il principio di tutti li numeri essagonali vogliono che sia pur la vnita, & dapoi quella il 6. dapoi il 15. dapoi il 28. & cosi tutti gli altri, che assettati sotto a vn certo suo ordine formino vna figura essagonale, & cosi vanno procedendo nelli numeri settagonali, ottogonali, & altri liquali per non esser materia molto al nostro proposito, perche questi numeri triangolari, penthagonali, essagonali, settagonali. &c. non rispondeno a tai figure geometrice, & tengo che per questa causa Euclide non fece mention saluo che di quelli, che corrispondeno a tai figure geometrice, cioe li numeri quadrati.

Della penultima specie, atto, ouer passione del algoritbmo, cioe della pratica di numeri, detta progressioni. Cap. VI.

1 SEguita la penultima specie, atto, ouer passione della pratica di numeri, chiamata progressione, laquale (per non esser materia molto necessaria a mercanti) fu pretermessa nella prima parte detta regole negotiarie, abenche tal specie, ouer atto non sia molto accadente, ouer necessario nelle pratiche mercantile, nondimeno molte, & molte questioni nella general pratica di numeri, & anchora in quella di misure occorrer possono, che senza l'aiuto, ouer suffragio di tal specie, & delle sue regole saria impossibile di poter risoluere, & pero furno astretti li nostri antichi a ritrouar tal specie con le sue conuenienti regole, ma nanti che procediamo piu oltra voglio dichiarir, che cosa sia progressione.

Che cosa sia progressione.

2 PRogressione nõ é altro in questo luogo, che vn certo ordine di piu numeri, che l'uno va eccedendo il suo antecedente egualmente di mano in mano talmente, che l'ultimo vien a esser maggiore di qual si voglia delli intermedi, & il primo vien a esser il minimo di tal ordine.

Delle specie delle progressioni arithmetici principianti dalle vnita dette continoue.

3 LE specie delle progressioni sono molte, ma quelle che in questo libro trattar intendo sono due, cioe progressioni Arithmetici, & progressioni Geometrici, ma prima diremo delle Arithmetici, lequali principiano dalla vnita, & si vanno augumentando, & dilatando continuamente in egual differentie, cioe se il secondo termine eccede il primo in vna vnita medesimamente il terzo eccede il secondo pur per vna vnita, & cosi il quarto eccede il terzo, & il quinto il quarto, & il sesto il quinto, & cosi procedendo di mano in mano, & similmente se il secondo eccede il primo per due vnita medesimamente il terzo eccede il secondo per due vnita, & il quarto eccede il terzo, & il quinto eccede il quarto, & cosi vanno procedendo, & se il secondo eccede

Fig. 1.11 Continued[55]

(2007, December 13) at the Ateneo di Brescia (Italy). The proceedings mainly deal with Tartaglia's teaching and "General Trattato" (Pizzamiglio 2007; Gavagna 2007).

[55] *Ibidem*.

The *General trattatto di numeri e misure* (1556–1560) is composed of 740 folia (1480 pages total). It is perhaps the largest known comprehensive mathematical contribution produced in the 16th century, including arithmetic, geometry, mensuration, and algebra as far as quadratic equations. The work is divided into six main parts, four of them were printed before Tartaglia's death. A general panorama is:

I part	17 Books	On the arithmetics and practical arguments
II part	11 Books	Mainly onTartaglia's triangle
III part	5 Books	On the geometric figures and unit mesurements
IV part	3 Books[56]	On the theoretical geometry and Archimdean books
V part	3 Books[57]	On the compass–and–straightedge rules and on Euclidean problems by different methods of solution
VI part	96 pages	On the Algebra

The *General trattatto di numeri e misure* presents Tartaglia's artithmetic triangle (Part II) having coefficients of the first twelve line powers, that is until *cu.ce.ce*. (the cube of the quadrate of the quadrate), the calculation of expressions with radicals, the rules for extracting cube roots, quarters, fifths, etc. (*Ivi*). However, there are also Fibonacci and Luca Pacioli's congruent numbers, perfect Euclidean numbers, irrational numbers, the theory of proportions, descriptions, tables and many practical problems executed, and corrections of "errors in *Summa* by Pacioli and Cardano's errors" (Tartaglia 1556–1560, pp 41–42).

Table 1.3. Tartaglia's first six triangle lines

Line	Tringle	$(a+b)^n$
0	1	$(a+b)^0=1$
1	1 1	$(a+b)^1=1a+1b=a+b$
2	1 2 1	$(a+b)^2=1a^2+2ab+1b^2$
3	1 3 3 1	$(a+b)^3=1a^3+3a^2b+3ab^2+1b^3$
4	1 4 6 4 1	$(a+b)^4=1a^4+4a^3b+6a^2b^2+4ab^{3+}1b^4$
5	1 5 10 10 5 1	$(a+b)^5=1a^5+5a^4b+10a^3b^2+10a^2b^{3+}5ab^4+1b^5$
[…]	[…]	[…]

[56] At the end of the book, this part includes the following quotation "in Vinagia per Comin da Tridino MDLVI" even though the title page reads "1560". It circulated after Tartaglia's death. An even more interesting fact is that in the inventory this book is cited "in folio", that is, printed but not in hardcover.

[57] The correlation between Euclid's propsitions (IV: 1–16) and respectively Tartaglia's propositions (Tartaglia 1556–1560, Part V, IX: 1–17, 13r–16r) is an interesting historical matter.

Other reasonings on *Quesiti* in arithmatics and algebraeic calculations are present, in particular in Book IX (Tartaglia 1554, *Book IX*, Qs 32, 36–38) where the rationalization of the denominator of a fraction (*Ivi*, Q 32) and the extraction of a cube root of a binomial are found (*Ivi*, Q 40):

1.2.4 On Physics: Ballistics

Tartaglia also presented contributions to the art of warfare in *Nova scientia* (Tartaglia 1537, *Books I–II*) and *Quesiti et invention diverse* (Tartaglia 1554, *Books I–III*). We should say that this subject is centred on the art of defence by means of fortifications that he regularly published in *Quesiti* (Tartaglia 1554, *Book VI* and *Gionta*). The arguments presented by Tartaglia (Tartaglia 1537, 1554) are algebraic and geometrical and specifically concering military artillery, cannonballs, gunpowder and other related subjects. The famous problematic argument on the trajectory of a cannoball and its maximum range, for any given degree–measure is dealt with in *Quesiti* (Tartaglia 1554, *Books I–III;* Tartaglia 1537, *Books I–II*). The Venetian period was still disciplinarily bitter. Consequently, Tartaglia was not able to formulate a modern theory of projectiles through, e.g., a correct mathematical interpretation (nowadays) of a parabolic trajectory. In effect, (see figures below), the path was curved but not parabolic. We know that Galileo could only be able to do this in 1609 (Naylor 1976, 153–172). This involves a case–study on the trajectory of projectiles which Tartaglia had not yet sufficiently theoretically developed (see, for example, the following images); perhaps it was also not yet sufficiently theoretically developed by others at this time, who more or less based their reasoning on the medieval theory of *impetus*[58]. Particularly, Jean Buridan's[59] medieval theory (Buridan 1509):

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[58] Buridan also into Latin Johannes Buridanus (ca. 1300 – ca. 1360). The historical genesis of the theory of *impetus*—later applied to the motion of projectiles— is quite complex and varied. Aside from Aristotle's initial theory (384–322 B.C.), among the scholars who dealt with the topic, we note: Johannes Philoponus (active in VI century), Pūr Sina' (Persian) son of Sina called Avicenna (980–1037), Roger Bacon (1214–1292), Thomas Aquina (1225–1274), Pierre Jean Olivi (1248–1298), Francesco of Marchia or of Esculo, of Ascoli (fl. XIV century), William of Ockham (ca. 1280 – ca. 1349), and for some considerations, Jordanus de Nemore, too. Here, for the sake of brevity, and since there is already a vast literature on the topic, we refer only to that which historians consider a true cultural background of projectile theory until the Renaissance.

[59] Buridan 1509. Nicolas d'Oresme's (ca. 1320/1325–1382) version should also be considered. An Eglish study is in *The Science of Mechanics in the Middle Ages*

- ✓ *The impetus varies with the speed of the projectile and with its mass.* Paraphrasing Buridan, we can say, more speed impressed by the motor on the mobile, stronger the transmitted impetus.
- ✓ *The impetus is a permanent quality different and distinct from the motion and the mass of the projectile.* A characteristic of permanence dell'impetus might be weak by the movement, i.e., by the air resistance and the degree of inclination of the launch.

Buridano also attempts a more accurate reasoning of the *impetus* without, however, producing any formal and or mathematical language. It is quite probable that before 1607, Galilei had not yet clarified the theory on the composition of vertical and horizontal motion. This non–clarification was mostly likely due to a lack of sufficient experimental proof and the known caution with which Galilei avoided affirmations devoid of *sensate esperienze*; this was the case until he wrote his notes in the famous code *Ms. 72*, precisely on foglio *116v*[60] (1609) in which he outlined the solution. It should be noted that in Galilei's time, typically Aristotelian motion was supported by the dichotomy of violent and natural motion.

In *Book VII* of *Quesiti* Tartaglia (1554) was able to produce noteworthy criticism[61]of Aristotle. This distinct scientific significance also emerges from Tartaglia's ability to theorize on the curved trajectory of projectiles. This topic must have been of great interest to him since he developed reasoning and drawings (see images below) in *Nova scientia* (1537) and other similar graphic developments and details by way of dialogue–problems (see passage below) in the subsequent *Quesiti et inventioni diverse* (1554). The law of elevation at a 45 degree angle materializes between these two works where both, *Nova scientia* and *Quesiti et inventioni diverse*, refer to one (although partial) final curved projection (today we could say semi–parabolic curve) of the projectile. Therefore, in this case, he was confined to the division of motion into two parts: one part due to a *virtù impressa*, and one *naturale,* which had the property to

(Clagett 1959) and in turn reproduced by Maier (1968) which, in turn, includes – with some modifications— the Parisian edition from 1509. For the comments of *Subtilissimae Quaestiones*, at first glance, one can see Clagett (Clagett 1959 and secondary literatures cited). Clagett dated Buridan's manuscript around 1357. It is archived at Vatican Library in Roma (Vat. Lat. 2136, 1r.).

[60] Galilei *Ms. 72*, 116v; see also: Hill 1986, 283–291.

[61] Tartaglia criticized Aristotle's theory of the lever in regard to the sensitivity of a scale according to which (wrongly) the Stagirite supported that the greater the length of the arms, the greater the sensitivity of the instrument (Tartaglia 1554, *Book VII*, Qs IV–V–VI, 80–82).

overcome the initial force that, in time, became weaker and allowed the projectile to fall. However, this did not prevent Tartaglia from informing the reader of his correct idea of a curved trajectory for this type of motion; although with certain approximation according to which, in time and for certain cases, he himself tended to identify with a straight line:

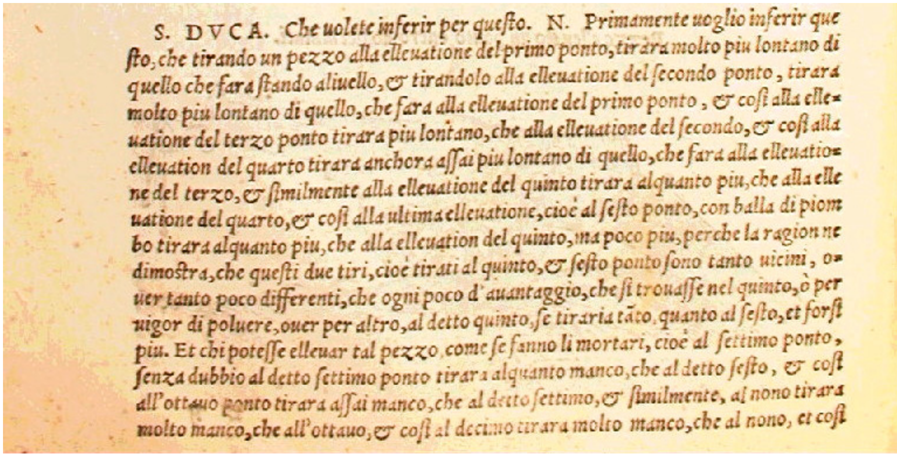

S. DVCA. Che uolete inferir per questo. N. Primamente uoglio inferir questo, che tirando un pezzo alla elleuatione del primo ponto, tirara molto piu lontano di quello che fara stando aliuello, & tirandolo alla elleuatione del secondo ponto, tirara molto piu lontano di quello, che fara alla elleuatione del primo ponto, & cosi alla elleuatione del terzo ponto tirara piu lontano, che alla elleuatione del secondo, & cosi alla elleuation del quarto tirara anchora assai piu lontano di quello, che fara alla elleuatione del terzo, & similmente alla elleuatione del quinto tirara alquanto piu, che alla elleuatione del quarto, & cosi alla ultima elleuatione, cioè al sesto ponto, con balla di piombo tirara alquanto piu, che alla elleuation del quinto, ma poco piu, perche la ragion ne dimostra, che questi due tiri, cioè tirati al quinto, & sesto ponto sono tanto uicini, ouer tanto poco differenti, che ogni poco d'auantaggio, che si trouasse nel quinto, ò per uigor di poluere, ouer per altro, al detto quinto, se tiraria tãto, quanto al sesto, et forsi piu. Et chi potesse elleuar tal pezzo, come se fanno li mortari, cioè al settimo ponto, senza dubbio al detto settimo ponto tirara alquanto manco, che al detto sesto, & cosi all'ottauo ponto tirara assai manco, che al detto settimo, & similmente, al nono tirara molto manco, che all'ottauo, & cosi al decimo tirara molto manco, che al nono, et cosi

Fig. 1.12 Plate from Tartaglia*'s Quesiti* around the straight line trajectory and general law at 45–degrees[62]

In effect, we should remark that the problem of the physical and mathematical knowledge of the projectile trajectory was complicated in this period. In fact, it was known that a cannon ball in the air proceeded in a *non–rectilinear*[63] way. However, the idea that an equation (of motion) of the second degree could indeed mathematically interpret the physical motion of a cannon shot along the trajectory was hardy mature 16th century. It was also mature for Tartaglia who, as we showed in the previous paragraphs, was versed in the mathematical study of higher–order equations. In addition to the theoretical problem there was also the practical problem of the military art of fortified defense and later that of

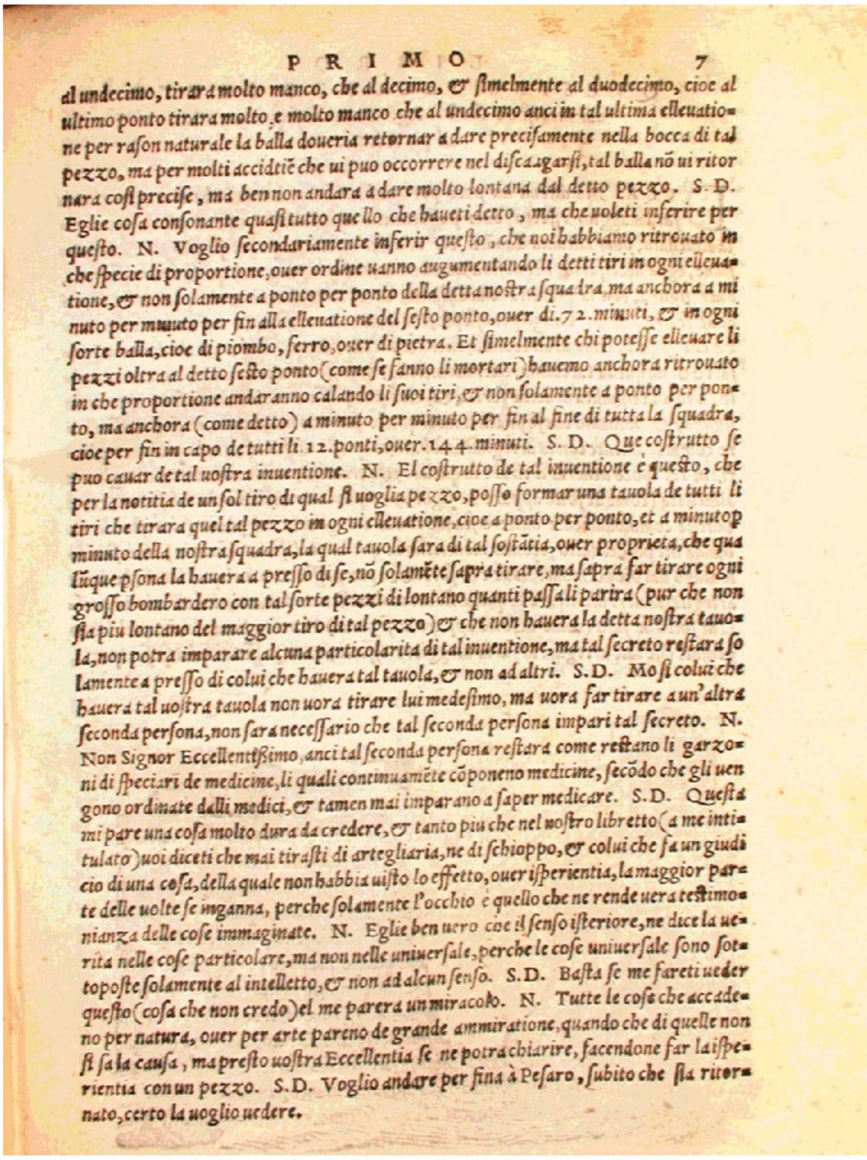
[62] Tartaglia 1554, *Book I*, Q I, 6v–7r. In total, see *Ivi*, Qs. I–II–III–VI, 5rv–14r. (Author's rounded parentheses). It must be noted that in almost all of the parts of *Books I–II–III* of *Quesiti* (*Ivi*, 5rv–40rv) there are considerations and figures on the semi–parabolic trajectory of projectiles to which the applications to war machinery and to artillery "squads" are added (*Ivi*, Q I, 5r). He had also extensively discussed these considerations in *La Nouva scientia* (Tartaglia 1537).

[63] 1504. Mortar' model (Codex Atlanticus, 33r.). See also: Gille 1964, 219; Pisano and Capecchi 2010a, 2009).

the architectural design of fortification walls[64]. Essentially, it was crucial to know that, the curvilinear trajectory followed, for example, by a cannon ball in the air was one thing, the rectilinear distance that interjected itself between the cannon–artillery and the walls to hit was another. Such knowledge favored the artilleryman versed in the subject that, thanks to Tartaglia's discovery of the 45–degree elevation, prepared the shot with precision.

In order to improve the study around the trajectory and correlated piece ("pezzo") Tartaglia was interested in both theory and experience. His idea regarding the relationship between an inclined "pezzo" angle and the trajectory, nowadays, is considered a general law independent from technical and technological manufacturing. Therefore, Tartgalia stated a general law for any kind of "pezzo" paying attention to the practical and shared knowledge of his time. In his words:

[64] Here, Tartaglia also gave his contribution, which we will later discuss.


PRIMO 7

al undecimo, tirara molto manco, che al decimo, & similmente al duodecimo, cioe al ultimo ponto tirara molto, e molto manco che al undecimo anci in tal ultima elleuatione per rason naturale la balla doueria retornar a dare precisamente nella bocca di tal pezzo, ma per molti accidētiē che ui puo occorrere nel discaargarsi, tal balla nō ui ritornara cosi precise, ma ben non andara a dare molto lontana dal detto pezzo. S. D. Eglie cosa consonante quasi tutto quello che haueti detto, ma che uoleti inferire per questo. N. Voglio secondariamente inferir questo, che noi habbiamo ritrouato in che specie di proportione, ouer ordine uanno augumentando li detti tiri in ogni elleuatione, & non solamente a ponto per ponto della detta nostra squadra, ma anchora a minuto per minuto per fin alla elleuatione del sesto ponto, ouer di. 72. minuti, & in ogni sorte balla, cioe di piombo, ferro, ouer di pietra. Et similmente chi potesse elleuare li pezzi oltra al detto sesto ponto (come se fanno li mortari) hauemo anchora ritrouato in che proportione andaranno calando li suoi tiri, & non solamente a ponto per ponto, ma anchora (come detto) a minuto per minuto per fin al fine di tutta la squadra, cioe per fin in capo de tutti li 12. ponti, ouer. 144. minuti. S. D. Que costrutto se puo cauar de tal uostra inuentione. N. El costrutto de tal inuentione è questo, che per la notitia de un sol tiro di qual si uoglia pezzo, posso formar una tauola de tutti li tiri che tirara quel tal pezzo in ogni elleuatione, cioe a ponto per ponto, et a minuto p minuto della nostra squadra, la qual tauola sara di tal sostātia, ouer proprieta, che qualūque psona la hauera a presso di se, nō solamēte sapra tirare, ma sapra far tirare ogni grosso bombardero con tal sorte pezzi di lontano quanti passa li parira (pur che non sia piu lontano del maggior tiro di tal pezzo) & che non hauera la detta nostra tauola, non potra imparare alcuna particolarita di tal inuentione, ma tal secreto restara solamente a presso di colui che hauera tal tauola, & non ad altri. S.D. Mo si colui che hauera tal uostra tauola non uora tirare lui medesimo, ma uora far tirare a un'altra seconda persona, non sara necessario che tal seconda persona impari tal secreto. N. Non Signor Eccellentissimo, anci tal seconda persona restara come restano li garzoni di speciari de medicine, li quali continuamēte cōponeno medicine, secōdo che gli uengono ordinate dalli medici, & tamen mai imparano a saper medicare. S.D. Questa mi pare una cosa molto dura da credere, & tanto piu che nel uostro libretto (a me intitulato) uoi diceti che mai tirasti di artegliaria, ne di schioppo, & colui che fa un giudicio di una cosa, della quale non habbia uisto lo effetto, ouer isperientia, la maggior parte delle uolte se inganna, perche solamente l'occhio è quello che ne rende uera testimonianza delle cose immaginate. N. Eglie ben uero che il senso isteriore, ne dice la uerita nelle cose particolare, ma non nelle uniuersale, perche le cose uniuersale sono sottoposte solamente al intelletto, & non ad alcun senso. S.D. Basta se me fareti ueder questo (cosa che non credo) el me parera un miracolo. N. Tutte le cose che accadeno per natura, ouer per arte pareno de grande ammiratione, quando che di quelle non si sa la causa, ma presto uostra Eccellentia se ne potra chiarire, facendone far la isperientia con un pezzo. S.D. Voglio andare per fina à Pesaro, subito che sia ritornato, certo la uoglio uedere.

Fig. 1.13 Plate from Tartaglia*'s Quesiti* around the "pezzo" [65]

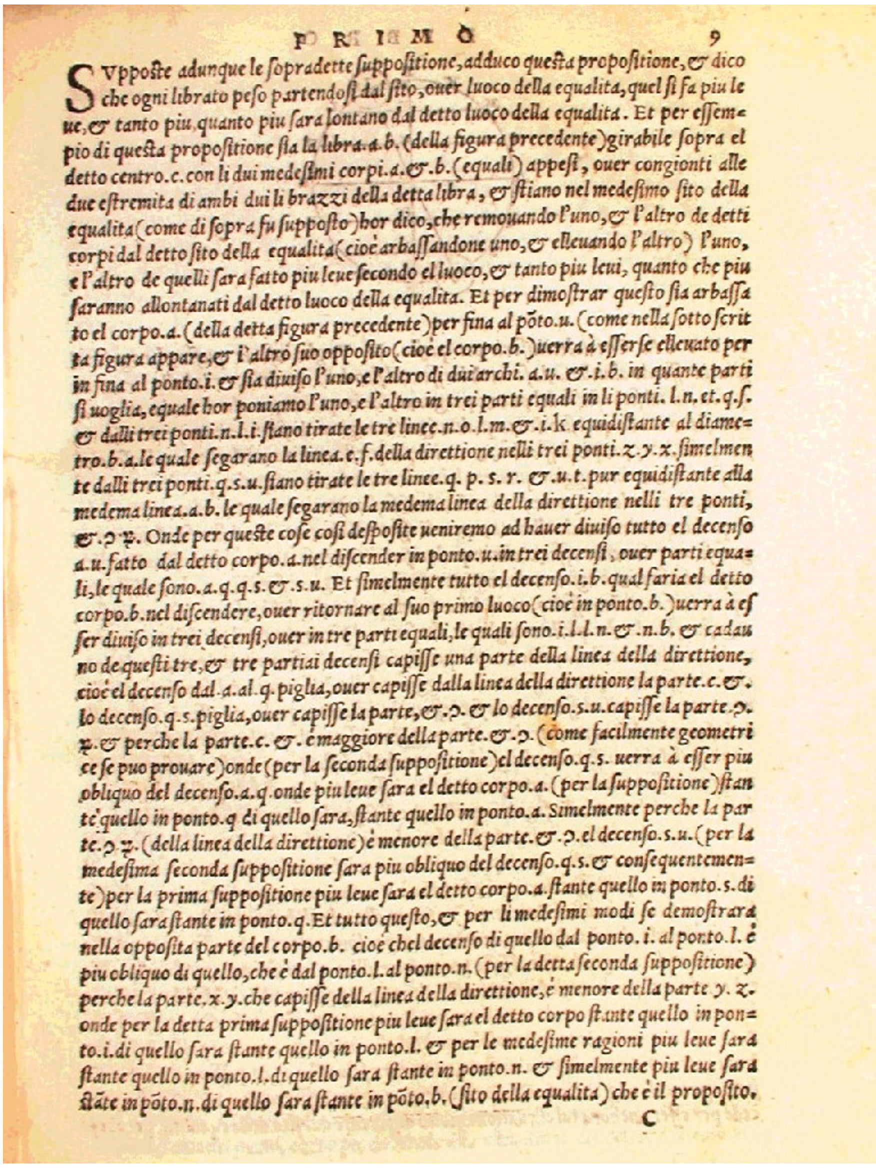
In my opinion, Tartaglia was both one of the first to use physical elements and mathematical interpretation (clearly in contrast with the Aristotelian school and partially with the *impetus theory*) to investigatate the physical law of the maximum range of the projectile and related path of the

[65] Tartaglia 1554, *Book* I, Q I, 7r.

cannonsballs. With few arguments (both in *Nova scientia* and *Quesiti*), he claimed that *the maximum range of a projectile*[66] *is attained when the firing elevation is 45 degrees*.[67] On the theoretical side, he argued his genereal law; and only later, he reasoned on Jordanus de Nemore's classical *gravitas secundum situm* demonstration. By following his discourse:

[66] Later, other scholars took up the questions of the range of projectile motion. Mainly (17th centuries): Galilei (*The Dialogues Concerning the Two New Sciences*), Torricelli (*De Motu*) on the geometrical way of calculating the range of a projectile, and Newton (*Principia*) on the proportion between air resistance and the square of the speed of the projectile (Shawnee and Rickey 2011). Recently on Newton and Jesuit edition see: Pisano 2013b, 2014; Bussotti and Pisano 2014a,b).

[67] See also Riccardi's quotation in his *Biblioteca Matematica Italiana dalla origine della stampa ai primi anni del secolo* (Riccardi 1870–1880, II, 497).



PRIMO 9

Supposte adunque le sopradette suppositione, adduco questa propositione, & dico che ogni librato peso partendosi dal sito, over luoco della equalita, quel si fa piu le ue, & tanto piu, quanto piu sara lontano dal detto luoco della equalita. Et per essem= pio di questa propositione sia la libra.a.b. (della figura precedente) girabile sopra el detto centro.c. con li dui medesimi corpi.a.&.b. (equali) appesi, over congionti alle due estremita di ambi dui li brazzi della detta libra, & stiano nel medesimo sito della equalita (come di sopra fu supposto) hor dico, che removando l'uno, & l'altro de detti corpi dal detto sito della equalita (cioe arbassandone uno, & ellevando l'altro) l'uno, e l'altro de quelli sara fatto piu leve secondo el luoco, & tanto piu levi, quanto che piu saranno allontanati dal detto luoco della equalita. Et per dimostrar questo sia arbassa to el corpo.a. (della detta figura precedente) per fina al pŏto.u. (come nella sotto scrit ta figura appare, & l'altro suo opposito (cioe el corpo.b.) verra à esserse ellevato per in fina al ponto.i. & sia diviso l'uno, e l'altro di dui archi. a.u. &.i.b. in quante parti si voglia, equale hor poniamo l'uno, e l'altro in trei parti equali in li ponti. l.n. et.q.s. & dalli trei ponti.n.l.i. siano tirate le tre linee.n.o.l.m.&.i.k. equidistante al diame= tro.b.a. le quale segarano la linea.e.f. della direttione nelli trei ponti.z.y.x. simelmen te dalli trei ponti.q.s.u. siano tirate le tre linee.q. p. s. r. &.u.t. pur equidistante alla medema linea.a.b. le quale segarano la medema linea della direttione nelli tre ponti, &.ɔ. ꝝ. Onde per queste cose cosi desposite veniremo ad haver diviso tutto el decenso a.u. fatto dal detto corpo.a. nel discender in ponto.u. in trei decensi, over parti equa= li, le quale sono.a.q.q.s.&.s.u. Et simelmente tutto el decenso.i.b. qual faria el detto corpo.b. nel discendere, over ritornare al suo primo luoco (cioe in ponto.b.) verra à es ser diviso in trei decensi, over in tre parti equali, le quali sono.i.l.l.n.&.n.b. & cadau no de questi tre, & tre partiai decensi capisse una parte della linea della direttione, cioe el decenso dal.a.al.q. piglia, over capisse dalla linea della direttione la parte.c.&. lo decenso.q.s. piglia, over capisse la parte, &.ɔ. & lo decenso.s.u. capisse la parte.ɔ. ꝝ. & perche la parte.c.&. è maggiore della parte.&.ɔ. (come facilmente geometri ce se puo provare) onde (per la seconda suppositione) el decenso.q.s. verra à esser piu obliquo del decenso.a.q. onde piu leve sara el detto corpo.a. (per la suppositione) stan te quello in ponto.q di quello sara, stante quello in ponto.a. Simelmente perche la par te.ɔ.ꝝ. (della linea della direttione) è menore della parte.&.ɔ. el decenso.s.u. (per la medesima seconda suppositione sara piu obliquo del decenso.q.s. & consequentemen= te) per la prima suppositione piu leve sara el detto corpo.a. stante quello in ponto.s. di quello sara stante in ponto.q. Et tutto questo, & per li medesimi modi se demostrara nella opposita parte del corpo.b. cioe chel decenso di quello dal ponto.i. al ponto.l. è piu obliquo di quello, che è dal ponto.l. al ponto.n. (per la detta seconda suppositione) perche la parte.x.y. che capisse della linea della direttione, è menore della parte y. z. onde per la detta prima suppositione piu leve sara el detto corpo stante quello in pon= to.i. di quello sara stante quello in ponto.l. & per le medesime ragioni piu leve sara stante quello in ponto.l. di quello sara stante in ponto.n. & simelmente piu leve sara stăte in pŏto.n. di quello sara stante in pŏto.b. (sito della equalita) che è il proposito.

C

Fig. 1.14 Plate from Tartaglia*'s Quesiti* around the general law at 45 degrees"[68]

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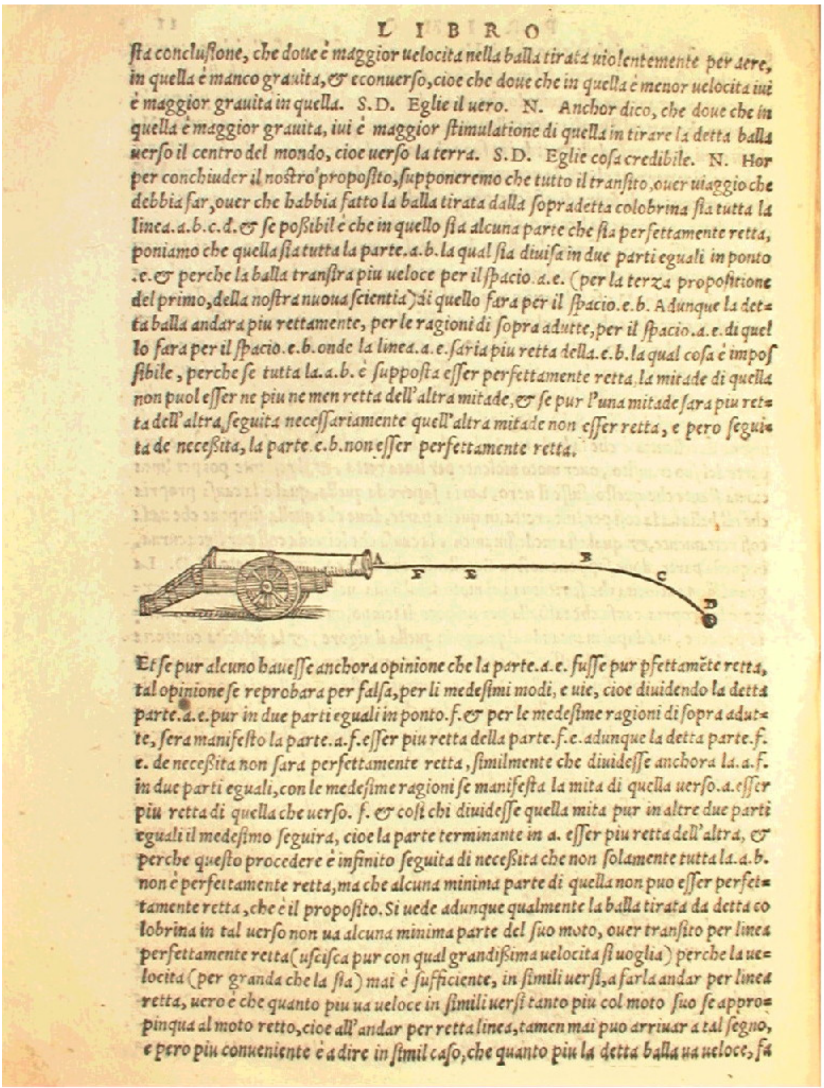
[68] "Supposte adunque le sopradette suppositione, adduco questa propositione, & dico che ogni librato peso partendosi dal sito, over luoco della equalita, quel si fa piu leve, & tanto piu quanto piu sara lontano dal detto luoco della equalita. Et per essemepio di questa propositione sia la libra.a.b. (della figura precedente) girabile sopra el detto centro .c. con li dui medesimi corpi .a. & .b. (equali) appesi, over

Fig. 1.15 Plate from *Nova scientia* around 45–degree elevation and bombardier's quadrant[69]

congionti alle due estremita di ambi dui li brazzi della detta libra, & stiano nel medesimo sito della equalita (come di sopra fu supposto) hor dico, che removando l'uno, & l'altro de detti corpi dal detto sito della equalita (cioè arbassandone uno, & ellevando l'altro) l'uno, e l'altro de quelli sara fatto piu leve secondo el luoco, & tanto piu levi, quanto che piu saranno allontanati dal detto luoco della equalita". (Tartaglia 1554, Q II, 9r).

[69] Tartaglia 1537, 4r.



LIBRO

ſta concluſione, che doue è maggior uelocita nella balla tirata uiolentemente per aere, in quella è manco grauita, & econuerſo, cioe che doue che in quella è menor uelocita iui è maggior grauita in quella. S.D. Eglie il uero. N. Anchor dico, che doue che in quella è maggior grauita, iui è maggior ſtimulatione di quella in tirare la detta balla uerſo il centro del mondo, cioe uerſo la terra. S.D. Eglie coſa credibile. N. Hor per conchiuder il noſtro propoſito, ſupponeremo che tutto il tranſito, ouer uiaggio che debbia far, ouer che habbia fatto la balla tirata dalla ſopradetta colobrina ſia tutta la linea.a.b.c.d.& ſe poßibil è che in quello ſia alcuna parte che ſia perfettamente retta, poniamo che quella ſia tutta la parte.a.b.la qual ſia diuiſa in due parti eguali in ponto .e.& perche la balla tranſira piu ueloce per il ſpacio.a.e. (per la terza propoſitione del primo, della noſtra nuoua ſcientia) di quello fara per il ſpacio.e.b. Adunque la detta balla andara piu rettamente, per le ragioni di ſopra adutte, per il ſpacio.a.e.di quello fara per il ſpacio.e.b.onde la linea.a.e.ſaria piu retta della.e.b.la qual coſa è impoſſibile, perche ſe tutta la.a.b. è ſuppoſta eſſer perfettamente retta, la mitade di quella non puol eſſer ne piu ne men retta dell'altra mitade, & ſe pur l'una mitade ſara piu retta dell'altra, ſeguita neceſſariamente quell'altra mitade non eſſer retta, e pero ſeguita de neceßita, la parte.e.b.non eſſer perfettamente retta.

Et ſe pur alcuno haueſſe anchora opinione che la parte.a.e. fuſſe pur pfettamēte retta, tal opinione ſe reprobara per falſa, per li medeſimi modi, e uie, cioe diuidendo la detta parte.a.e.pur in due parti eguali in ponto.f.& per le medeſime ragioni di ſopra adutte, ſera manifeſto la parte.a.f.eſſer piu retta della parte.f.e.adunque la detta parte.f.e. de neceßita non ſara perfettamente retta, ſimilmente che diuideſſe anchora la.a.f. in due parti eguali, con le medeſime ragioni ſe manifeſta la mita di quella uerſo.a.eſſer piu retta di quella che uerſo. f. & coſi chi diuideſſe quella mita pur in altre due parti eguali il medeſimo ſeguira, cioe la parte terminante in a. eſſer piu retta dell'altra, & perche queſto procedere è infinito ſeguita di neceßita che non ſolamente tutta la.a.b. non è perfettamente retta, ma che alcuna minima parte di quella non puo eſſer perfettamente retta, che è il propoſito. Si uede adunque qualmente la balla tirata da detta colobrina in tal uerſo non ua alcuna minima parte del ſuo moto, ouer tranſito per linea perfettamente retta (uſciſca pur con qual grandißima uelocita ſi uoglia) perche la uelocita (per granda che la ſia) mai è ſufficiente, in ſimili uerſi, a farla andar per linea retta, uero è che quanto piu ua ueloce in ſimili uerſi tanto piu col moto ſuo ſe appropinqua al moto retto, cioe all'andar per retta linea, tamen mai puo arriuar a tal ſegno, e pero piu conueniente è a dire in ſimil caſo, che quanto piu la detta balla ua ueloce, fa

Fig. 1.16 Plate from *Quesiti* on trajectories[70]

70 Tartaglia 1554, *Book I*, Q III, 11rv; Qs I–II–III–VI, 5rv–13rv.

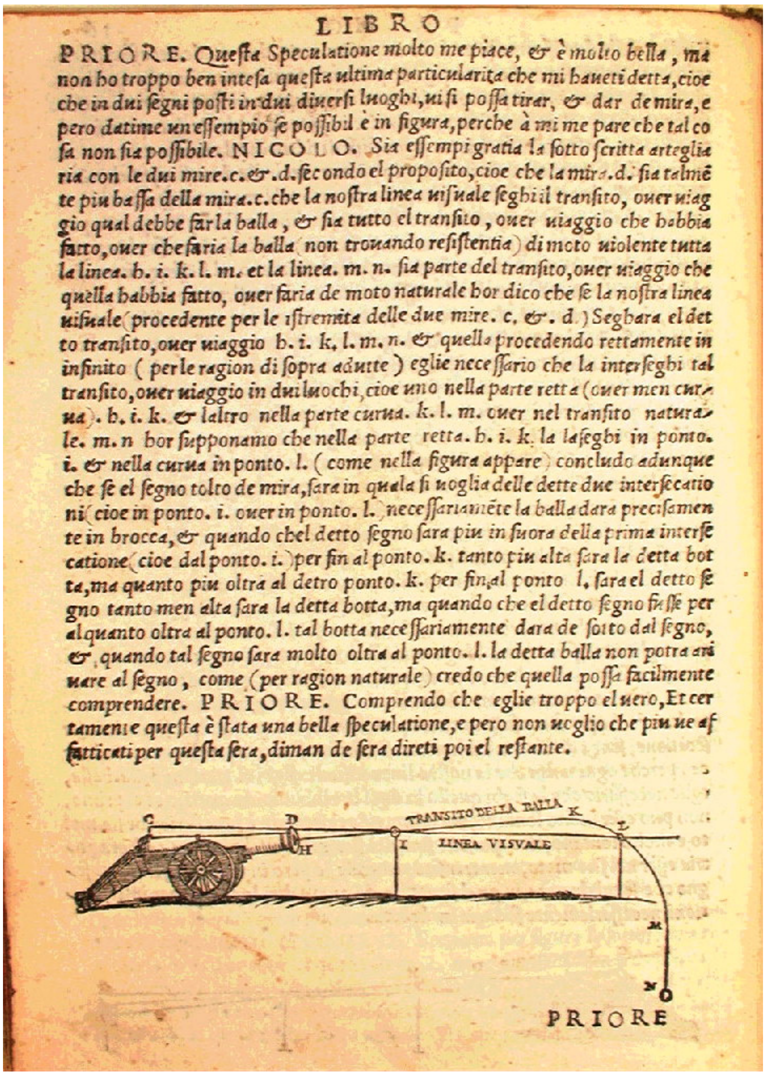
LIBRO

Pezzo elleuato al ſeſto ponto, ouero alli ſettantadue minuti.

S. DVCA. Che uolete inferir per queſto. N. Primamente uoglio inferir que
ſto, che tirando un pezzo alla elleuatione del primo ponto, tirara molto piu lontano di
quello che fara ſtando aliuello, & tirandolo alla elleuatione del ſecondo ponto, tirara
molto piu lontano di quello, che fara alla elleuatione del primo ponto, & coſi alla elle-
uatione del terzo ponto tirara piu lontano, che alla elleuatione del ſecondo, & coſi alla
elleuation del quarto tirara anchora aſſai piu lontano di quello, che fara alla elleuatio-
ne del terzo, & ſimilmente alla elleuatione del quinto tirara alquanto piu, che alla elle
uatione del quarto, & coſi alla ultima elleuatione, cioè al ſeſto ponto, con balla di piom
bo tirara alquanto piu, che alla elleuation del quinto, ma poco piu, perche la ragion ne
dimoſtra, che queſti due tiri, cioè tirati al quinto, & ſeſto ponto ſono tanto uicini, o-
uer tanto poco differenti, che ogni poco d'auantaggio, che ſi trouaſſe nel quinto, ò per
uigor di poluere, ouer per altro, al detto quinto, ſe tiraria tāto, quanto al ſeſto, et forſi
piu. Et chi poteſſe elleuar tal pezzo, come ſe fanno li mortari, cioè al ſettimo ponto,
ſenza dubbio al detto ſettimo ponto tirara alquanto manco, che al detto ſeſto, & coſi
all'ottauo ponto tirara aſſai manco, che al detto ſettimo, & ſimilmente, al nono tirara
molto manco, che all'ottauo, & coſi al decimo tirara molto manco, che al nono, et coſi

Fig. 1.17 Plate from *Quesiti* on inclined cannon for the maximum path and bombardier's quadrant[71]

[71] Tartaglia 1554, *Book I*, Q I, 6; *Ivi*, Q I, 5–7.



LIBRO

PRIORE. Queſta Speculatione molto me piace, & è molto bella, ma non ho troppo ben inteſa queſta ultima particularita che mi haueti detta, cioe che in dui ſegni poſti in dui diuerſi luoghi, ui ſi poſſa tirar, & dar de mira, e pero datime un eſſempio ſe poſſibil è in figura, perche à mi me pare che tal coſa non ſia poſſibile. NICOLO. Sia eſſempi gratia la ſotto ſcritta arteglia ria con le dui mire. c. & . d. ſecondo el propoſito, cioe che la mira. d. ſia talmẽte piu baſſa della mira. c. che la noſtra linea uiſuale ſeghi il tranſito, ouer uiaggio qual debbe far la balla, & ſia tutto el tranſito, ouer uiaggio che habbia fatto, ouer che faria la balla (non trouando reſiſtentia) di moto uiolente tutta la linea. h. i. k. l. m. et la linea. m. n. ſia parte del tranſito, ouer uiaggio che quella habbia fatto, ouer faria de moto naturale hor dico che ſe la noſtra linea uiſuale (procedente per le iſtremita delle due mire. c. & . d.) Segbara el detto tranſito, ouer uiaggio h. i. k. l. m. n. & quella procedendo rettamente in infinito (per le ragion di ſopra adutte) eglie neceſſario che la interſeghi tal tranſito, ouer uiaggio in dui luochi, cioe uno nella parte retta (ouer men curua). h. i. k. & laltro nella parte curua. k. l. m. ouer nel tranſito naturale. m. n hor ſupponamo che nella parte retta. h. i. k. la laſeghi in ponto. i. & nella curua in ponto. l. (come nella figura appare) concludo adunque che ſe el ſegno tolto de mira, ſara in quala ſi uoglia delle dette due interſecationi (cioe in ponto. i. ouer in ponto. l.) neceſſariamẽte la balla dara preciſamente in brocca, & quando chel detto ſegno ſara piu in fuora della prima interſecatione (cioe dal ponto. i.) per fin al ponto. k. tanto piu alta ſara la detta botta, ma quanto piu oltra al detto ponto. k. per fin al ponto l. ſara el detto ſegno tanto men alta ſara la detta botta, ma quando che el detto ſegno fuſſe per alquanto oltra al ponto. l. tal botta neceſſariamente dara de ſotto dal ſegno, & quando tal ſegno ſara molto oltra al ponto. l. la detta balla non potra ariuare al ſegno, come (per ragion naturale) credo che quella poſſa facilmente comprendere. PRIORE. Comprendo che eglie troppo el uero, Et certamente queſta è ſtata una bella ſpeculatione, e pero non uoglio che piu ue affatticati per queſta ſera, diman de ſera direti poi el reſtante.

PRIORE

Fig. 1.18 Plate from *Quesiti* on cannonball[72]

Finally, history tells us that from the principles of conservation (*inertia*) and the composition of motion and from having understood that the speeds were proportional to the squares of times, through various hypotheses of heights (Drake 1973, 291–305) Galileo wrote important notes in folio *116v*

[72] Tartaglia 1554, *Book I*, Q VII, 16rv.

(shown below).[73] On statics, the science of weights and mechanics, in general, we refer the reader to the following paragraphs in which my discussion is centered specifically on the aim of my Thesis.

1.3 Physics and Architecture: On Ballistics & Fortifications

The *Book sesto* together with *Gionta*, a sort of technical appendix, are presented in *Quesiti* before the topics of statics (Tartaglia 1554, *Book VII* and *Book VIII*). The topic addressed will not give explicit technical reasoning on the science of weights. In fact, it develops essentially according to geometric reasoning on the choice of materials and military strategies. In particular, it should be noted that in the study of fortifications, Tartaglia considers geometry to be of primary importance for the choice of buildings–materials (Tartaglia 1554, Book VI). Consequently, from the beginning (*Ivi*, *Book VI,* Qs I–III).Tartaglia dedicates a significant amount of space to the study of maps of some important cities such as Turin to emphasize how the geometric shape of the fortifications bears on their efficacy and therefore on the security of the besieged. His studies on military–guard and initial approaches to bastioned fortifications (Pisano 2013c) that traversed the history of science are of great importance.[74]

1.3.1 On Ballistics & Technical Instruments in the *Nova Scienta*

According to previous historians, Tartaglia's first printed work was entitled *Nuova scientia*, *inventa da Nicolo Tartalea B.*[risciano] (Tartaglia 1537). The book is devoted to a discussion of ballistic arguments and correlated tecnhiniques–instruments of measurements (Cuomo 1997, 1998; Guidera 1994) in order to search a general law useful both (at that time to early) mechanical–ballistic theory and practical–weapon science. The organization of the argumentation is (like in the *Quesiti*) very far from axiomatic[75] structure, or by principles; only partially did he adopt

[73] Galileo's notes were made more legible by transcribing the content of the *folio* (Drake 1985, 3–14, 1992, 113–116).

[74] *Biblioteca istorico–critica di fortificazione permanente* (Marini 1810, p XII).

[75] Nowadays we find an undue use of the term *axiomatization* concerning non-modern theories in the history of science. For, some lines are necessary. Usually, in mathematics and mathematical physics, the term *axiomatization* of a *scientific* theory represents a formulation of a *scientific* system of statements (e.g.,

Aristotelian forms; Euclidean forms appears more frequently in the *Nova scientia*; rather he seems to follow Archimedean tradition (Pisano 2008; Pisano 2009b; Pisano and Capecchi 2009b). Thus no surprise for the novelty of science as "*Nova"*.

The manuscript is composed of *incipt*, a usual dedicatory letter and four main books and deals with the theory and practice of gunnery. Neverthless, his early mathematical studies applied to ballistics, particularly to the trajectories of cannonballs, and were explained in *epistola dedicatoria* (20 December 1537, Venice) as a preface to *Nova scientia* and addressed to Francesco Maria Feltrese della Rovere, Duke of Urbino and Captain of the Venetian Senate:

axioms/primitive terms) in order to build a consistent-coherent *corpus* of statements (e.g., propositions) which may be logically and deductively derived from these statements; and the proof of any statement (i.e., theorems) should be taken into account and traceable back to these axioms. Of course, the latter is a difficult condition to be universally claimed: i.e., see the case–study of Archimedean's *On the equilibrium of planes* (Capecchi and Pisano 2007, 2010b, Pisano 2009b, Pisano and Capecchi 2008, 2010b), and non–Euclidean geometry. Therefore, the use of axioms (in the history of science) as self–evident statements in a theory does not mean that this theory-system is axiomatically built (Pisano 2008). In fact, three fundamental properties should be fomally respected: 1) an axiomatic system is said to be consistent if it lacks contradiction, i.e. the ability to derive both a statement and its denial from the system's axioms; 2) in an axiomatic system, an axiom is called independent if it is not a theorem that can be derived from other axioms in the system; a system will be called independent if each of its underlying axioms is independent. Although independence is not a necessary requirement for a system, consistency is; 3) An axiomatic system will be called complete if for every statement, either itself or its negation is derivable. For example, Euclid of Alexandria authored the earliest extant axiomatic geometry and number theory presentation that can be formally considered: an axiomatic system, a model theory, and mathematical proofs within a formal system. All of that evidently is lacking in Tartaglia. Therefore a random use of axioms (i.e., in Tartaglia) only means a tentative step toward ordering a new theory – or simply to order a scientific reasoning extrapolated from a known theory – by means of primitive statements and eventually derived propositions.

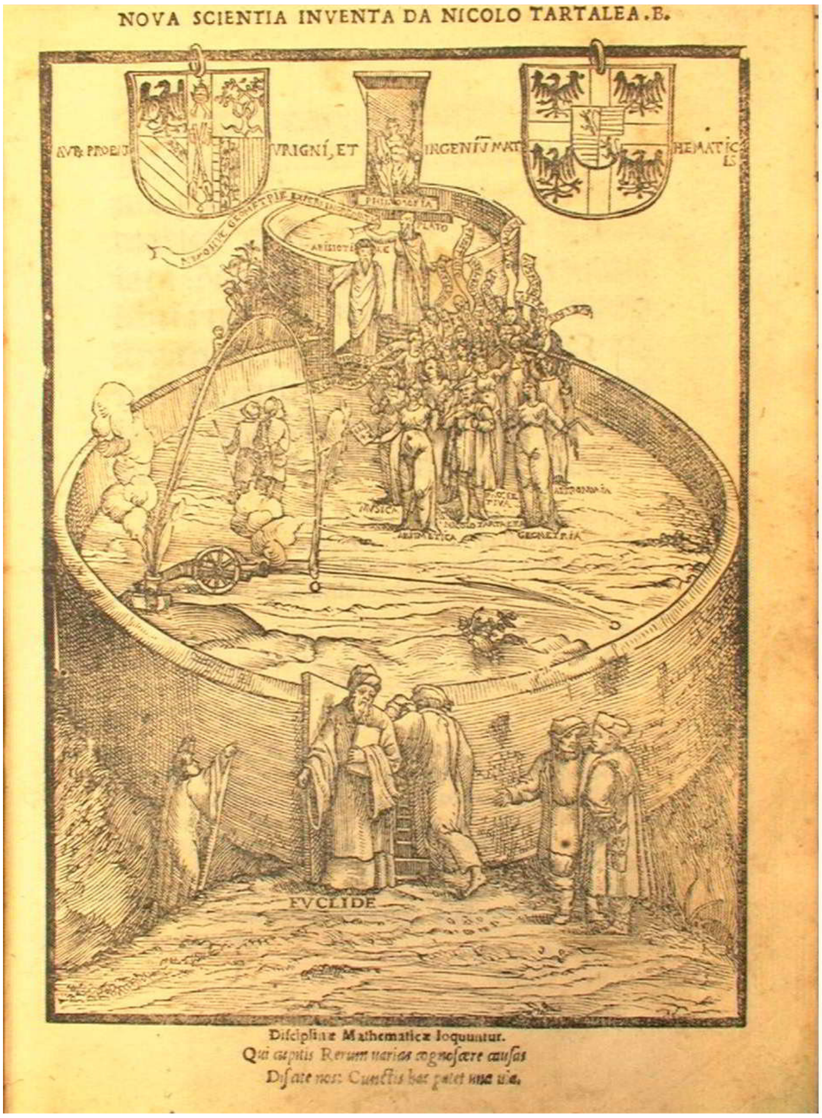
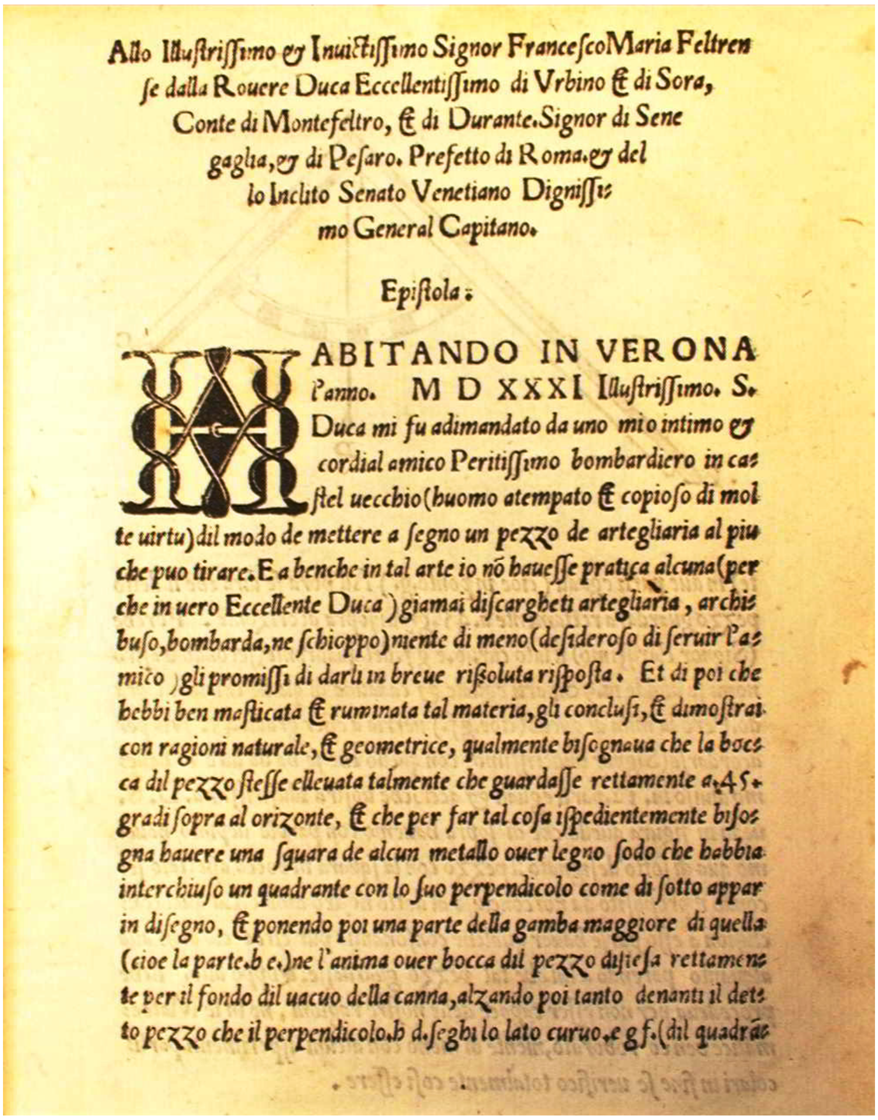


Fig. 1.19 Plate from *Nova scientia* – Frontespice[76]

[76] Tartaglia 1537. By frontespice and the curved path, the role played by his studies on trajectories in his aims is evident. A summary of the main topics of the *Nova scientia* is important for my aim because some crucial arguments discussed are then reworked/represented by Tartaglia in his *Quesiti*. A recent edition is available published (Tartaglia 2013; on that see also Arend 1988).



Allo Illustrissimo & Inuictissimo Signor Francesco Maria Feltren se dalla Rouere Duca Eccellentissimo di Vrbino & di Sora, Conte di Montefeltro, & di Durante. Signor di Sene gaglia, & di Pesaro. Prefetto di Roma. & del lo Inclito Senato Venetiano Dignissi mo General Capitano.

Epistola.

HABITANDO IN VERONA l'anno. M D XXXI Illustrissimo. S. Duca mi fu adimandato da uno mio intimo & cordial amico Peritissimo bombardiero in castel uecchio (huomo atempato & copioso di molte uirtu) dil modo de mettere a segno un pezzo de artegliaria al piu che puo tirare. E a benche in tal arte io nõ hauesse pratica alcuna (per che in uero Eccellente Duca) giamai discarghetti artegliaria, archibuso, bombarda, ne schioppo) niente di meno (desideroso di seruir l'amico) gli promissi di darli in breue risoluta risposta. Et di poi che hebbi ben masticata & ruminata tal materia, gli conclusi, & dimostrai con ragioni naturale, & geometrice, qualmente bisognaua che la bocca dil pezzo stesse elleuata talmente che guardasse rettamente a.45. gradi sopra al orizonte, & che per far tal cosa ispedientemente bisogna hauere una squara de alcun metallo ouer legno sodo che habbia interchiuso un quadrante con lo suo perpendicolo come di sotto appar in disegno, & ponendo poi una parte della gamba maggiore di quella (cioe la parte.b c.) ne l'anima ouer bocca dil pezzo distesa rettamente per il fondo dil uacuo della canna, alzando poi tanto denanti il detto pezzo che il perpendicolo.b d.seghi lo lato curuo.e g f.(dil quadrãte

Fig. 1.20 Plate from *Nova scientia*[77]

Below, we clearly provide some passages from *Nova scientia* regarding the aforementioned *epistola dedicatoria* where the spirit with which Tartaglia wrote his "operina", as Tartaglia himself cites his work in *Nova scientia*, can be inferred, as well as some original innovations on trajectories of shots that will later be revisited in *Quesiti*.

[77] Tartaglia 1537, *Book I*, 3r.

Very important are the definitions of *equally bodies* and *time* as measure of the motion concerning *natural* and *violent motions* and related studies where Tartaglia argued about the influence of air – as opposition to the motion (nowadays thinking of friction) – during the path made by projectile:

> First Definition. An equally [uniformly] heavy[78] body is said to be a body which, according to the heaviness and shape of the matter, is not perceptibly influenced by air opposition during its motion.[79]

---

[78] Stillman Drake translated it as "A body is called uniformly heavy […]" (Drake and Drabkin 1960, p 70). A remark is necessary. Now, following Tartaglia's text (just after *First definition*) we note his recalls Avicenna's work (see "Fen", that is a section of the *Liber canonis*). ParticularlyAverroes' fourth book of the *De caelo et mundo, text 29* is cited by Tartaglia (*Ibidem*). In addition, the tentative correlation with geometric forms of bodies, the kind of the matter of bodies, the concept of shared gravity where "[...] each body, compounded of four elements, one of which is air, shares gravity […]" with bodies' qualities (*Ibidem*), make evident his difficulties to distinguish *equally bodies* from – as Drake proposed – *uniformly bodies*. Of course athe knowledge of a physical magnitude lacks: let us think to uniformly term which can be addressed (ambiguously) both constant velocity and no–acceleration. Moreover, one should also add equally bodies *between them* like i.e., Tartaglia correctly wrote "Equally heavy bodies are said to be similar and equal when they do not show [among each other] any substantial or accidental differences" (Tartaglia 1537, *Book I*, def. II, 9v). On that we would add that we prefer both *equally* and *uniformly* or more simply *constant* bodies since at that time the concept of *constant gravity* was already proposed in many works during the 1300's –1400's, i.e., one can see *Subtilissimae Quaestiones super octo Physicorum libros Aristotelis* (Buridanus 1509; 1513, 1942) by Johannes Buridanus and *Tractatus de configurationibus qualitatum et motuum* by Nicole Oresme also edited by Clagett as *A treatise on the configuration of qualities and motions* (Oresme 1968; of course see Clagett 1959; Brown 1967–1968; Moody and Clagett ([1952] 1960). Now, by avoiding Latinism–and–*vulgare* philological analysis since within a dictionary the term "egualmente" can be translated by "uguale a se", "uniforme", "costante" (equally itself, uniform, constant) we remark that an *a posteriori* reflection related to physical proprieties of a body during the motion, i.e. an ideal rigid geometric body and its tendency to fall down, may suggest, at that time, the idea of *constant*, that is a sort of invariant of the motion. In Tartaglia's words: "[...] is not perceptibly influenced by air opposition during its motion" (*Ibidem*). On the contrary let us think about a paper or a leaf falling down. Finally in our opinion, since he refers to ancient conceptions of the fifth elements, Aristotelian and Medieval streams (i.e., *gravitas ex figura*), early attempts to formalize the friction as resistance by *corpo offeso* (offended bodies) concerning weapons etc., we prefer to literally translate it with *equally heavy* adding the term *uniformly* to both to give the idea that some

> Definition IIII. Time is a measure of motion and of the state of rest; its ends are two instants.[80]
> Definition VI. The natural movement of equally [uniformly] heavy bodies is the movement they accomplish from a higher place to a lower one perpendicularly and without any violence.[81]
> Definition VII. The violent movement of equally [uniformly] heavy bodies is the movement they accomplish with effort either upwards or downwards, to the right or the left, and is caused by a moving power.[82]

After the definitions follows five hypotheses called by Tartaglia *Suppositione* (Tartaglia 1537, *Book I*, 11v). After the *Suppositions* and just before the *Propositions* and *Corollaries* (*Ivi*, 12r et s.) follow four sentences called *Comune Sententie*[83] (common assumptions or axioms) by

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physical substance (not clear at that time) does not change and for the modern–specialist–reader, avoiding attribution to Tartaglia – at this stage –of advanced mathematical abstract concepts within physics –mathematics relationships of subjects that are still hard to make historically and epistemologically clear and since the mathematization of the nature was still far from complete. (Pisano 2011; Pisano, Capecchi and Lukešová 2013; on the relationship between physics and mathematics in the 19th century see: Pisano and Bussotti 2014; Pisano 2014a,b,c,d,e; 2012; Pisano and Capecchi 2013; Barbin and Pisano 2013).

79 "Diffinitione Prima. Corpo egualmente graue è detto quello, che ſecondo la grauita della materia, et la figura di quella è atto à non patire ſenſibilmente la oppoſition di l'aere in alcun ſuo moto." (Tartaglia 1537, *Book I*, 9r).

80 "Diffinitione. IIII. Il Tempo e una miſura del mouimento, et della quiete, li termini del quale ſon dui iſtanti." (Tartaglia 1537, *Book I*, 9v).

81 "Diffinitione. VI. Mouimento naturale di corpi egualmente graui e quello che naturalmente fanno da un luogo ſuperiore a un'altro inferiore perpendicularmente ſenza uiolenza alcuna." (Tartaglia 1537, *Book I*, 10v).

82 "Diffinitione. VII. Mouimento uiolente di corpi egualmente graui e quello che fanno sforzatamente di giuſo in ſuſo, di ſuſo in giuſo, di qua et di la, per cauſa di alcuna poſſanza mouente." (Tartaglia 1537, *Book I*, 10v).

83 Based on previous comments on axiomatization, we note that, in order to argue on statics in his "Scientia di Pesi" (Science of Weights) only in the *Book VIII* of the *Quesiti et invention diverse* (Tartaglia 1554, *Book VIII*, 83rv–97rv; see below Chapter 3) Tartaglia proposes a sort of prologue to the statics writing his definitive conceptual ideas concerning the role played by *Proper principles* (also called *Proper Principles* by Aristotle as sentences strictly related to the subject of theory: Aristotle 1853, *On the Definition and Division of Principles, Book I, Chap. X,* p 266)*, Propositions* (or also called by him conclusions which can confirm the science of weights), *Suppositions* (also called by him *true principles*) and *Petitions* (as sentences which can go against science of weights). We will return to that idea (see below Chapter 3). For further readings see Pisano and Capecchi 2010a, 2010b; Pisano 2009b.

him (Tartaglia 1537, *Book I*, 11v–12r). The *Comune Sententie* do not refer to a particular magnitude of one kind such as, e.g., lines, angles, figures etc. (Pisano 2005–2008). In fact, although this part of the *Tartalean* context seems typically (and generally speaking) organized like a traditional Aristotelian/Euclidean structure (*Definitions*, *Common notions* and *Propositions*) the *Comune Sententie* did not play precisely the role of *necessary elements* of the theory typically, i.e., within axiomatic Euclidean[84] organization of the theory (*Ibidem*).

The Definition IIII is addressed to a concept of measure that makes clear Tartaglia's empirical approach to the study of *natural* problems. Moreover, we want to remark – particularly important – his concept concerning *heavy equally bodies*. Certainly, it was not an original concept[85] at that time. Recent studies have shown how already Archimedes had argued on the *heavy equally bodies* and *bodies in equilibrium* concerning studies of the lever (Pisano and Bussotti 2012; Capecchi and Pisano 2010; Pisano 2007).

The following passage addresses his lack of experience in artillery, introduces the reader to the maximal range for projectiles of 45–degrees for all weapons and presents the genius intuition of using algebra and geometry together, to attempt (we would say today) the composition of horizontal and vertical motion in the visibly non–rectilinear trajectory of projectiles; In his words:

> Epistle. When I dwelt at Verona in MDXXXI [1531], Illustrious Mr. Duke, I had a very close and cordial friend, an expert bombardier at castel vecchio (and aged man blessed with many virtues) who asked me about the manner of aiming a given artillery piece for its farthest shot. Now I had no actual practice in that art (for truly, Excellent Duke, I never fired artillery, arquebus, mortar, or musket), nevertheless (desiring to serve my friend) I promised to give him shortly a definitive answer. And after I had chewed over and ruminated on this matter, concluded et proved to him by natural[86] and geometrical reasoning, how the mouth of the piece must be elevated in such a way as to point straight at an angle of 45 degrees avobe horizon, and to do this most expeditiously, you must have a square made of metal or hard wood that includes a quadrant with its vertical pendant, as appears below in th figure […].
>
> Nevertheless more during MDXXXII [1532], when the Prefect at Verona was the Magnifico [noble] Misser [Mr] Leonardo Iiustiniano [Giustiniano],

[84] i.e., one can see Book I–*The foundations: theories of triangles, parallels, and area* of the Euclid's *Elements* where after an initial 23 *Definitions* follow 5 *Postulates*, 5 *Common Notions* and 48 *Propositions*.

[85] A difference with regard to bodies in motion with respect to Archimedean statics studies.

[86] By physical reasonings.

> A chief of bombardiers, who was very close to that friend of ours, [...] one day it happened that the two of them took up the same problem which our friend proposed to us, that is how a cannon should be pointed in order to shoot as far as possible over plain[[87]]. [...].
> And, you should know, Vostra Magnimita [Your Magnanimity] having once gone this matter, I thought seriously of a further trial, and I began (not without reason) to investigate the kinds of motions that take place in a heavy body [cannon ball]. I thus found that there are two such motions, the natural and the violent, and I found these to be totally contrary in events ["accidenti"] through their contrary actions ["effetti" and] similarly I also found by reasons evident to intellect, that it is impossible for a heavy body to move with natural motion and violent motion mixed together. I then (Mr Serenissimo[[88]]) with demonstrative geometrical reasons the quality [character] of the trajectories ["transiti"], or violent motions of heavy bodies according to the various ways in which they may be ejected or thrown violently [artificially by artillery] through air. [...].
> There I found a new method of investigating quickly the heights, the hypotenusal (or diametral) distances, and also the horizontal distances of visible things. This is not completely a new thing, for indeed Euclid in his perspective shows it briefly, theoretically and in part.[89]

Continuing ahead in the letter, we can glimpse a sort of conscientious self–reflection on that fact that he is beginning to be aware of the danger of the general law that he was about to describe in the book: affirming that a law exists which is valid for all the pieces ("pezzi") produced –therefore, for everyone's use. Nevertheless, how would they have used such a law? It would have been used both by whoever was trying to defend himself and by he who was attacking, and therefore would have contributed to the elimination of human beings in either case. Here below, we provide interesting passages of this dedication in which a desire not to divulge this information emerges:

> One day, however, I was thinking to myself, Very Magnanimous Duke, and it seemed to me that working toward the perfection of such an art, harmful to the neighbor or even destructive for the human species & especially for the Christians because of their continuous wars, was a reproachful, vituperative and cruel thing, worthy of heavy punishment by God and by men. For this reason, Oh Very Excellent Duke, not only did I completely postpone the investigation of such matters and begin to work on

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[87] "[...] cioè a che segno si dovesse assettare un pezzo de arteglieria che facesse il maggior tiro che far possa sopra un piano". (Tartaglia 1537, *Book I,* 3rv).

[88] "Serenissimo" is, e.g., a title for some Principe and Doge of the Republic of Venice. "Altezza" is also commonly used.

[89] Tartaglia 1537, *Book I,* 3rv, line 1.

another subject, I also shredded and burned all the calculations and writings that I had annotated concerning such matters. I was very upset & ashamed about the time I had spent [working on] this subject, [also] I did not want to tell anyone of those particular things that remained on my mind (against my will), neither because of friendship nor reward (though I was asked by many people to do so) and this was because, had I taught them, it seemed to me that I would be making a big mistake. But now, seeing that the wolf [Turkish emperor Suleiman[90]] is anxious to ravage our flock while all our shepherds [Pope Paolo III, Imperator Charles V king of Spain, Francesco I king of France and la Venetian Republic] hasten to the defense, it no longer appears permissible to me at present to keep these things hidden. I have hence resolved to publish them partly in writing and partly by word of mounth, to every faithful Chrstian, so that each may be better fitted in offense as well as defense. And I am very sorry, Very Magnanimous Lord, that I ever abandoned this study, since I am certain that if I had kept on without pause I should bave discovered things of more value, as l hope soon to do. But since the present is certain, Most Illustrious Lord, time is short, and the future is always doubtful, I want to speed first that which I now have; and to carry this out in part, I bave hastily composed the present Iittle work. And like every river tha flows to approach and unite with the sea, this will seek to approach and unite with your greatness, your Excellency being the greatest of mortals in warlike virtue. For just as the abundant sea, which has no need of water, does not disdain to receive a little stream, so I hope tbat your Excellency will not distai to accept this, in order that tbe expert bombardiers of tbis our most illustrious duca) dominion, subjected to your Excellency, in addition to tbeir fine and practical skill, may be better instructed by reason and able to carry out your mandates. And if in tbese tbree books I bave not fully satisfied your Excellency together with tbe said expert bombardiers, I bope in a short time to do so with the practice of the fourtb and if books, not indeed in print (for many reasons) but in writin or by word of mouth; to satisfy, in part, them and your Excellency, to wbom I devotedly recommend myself.

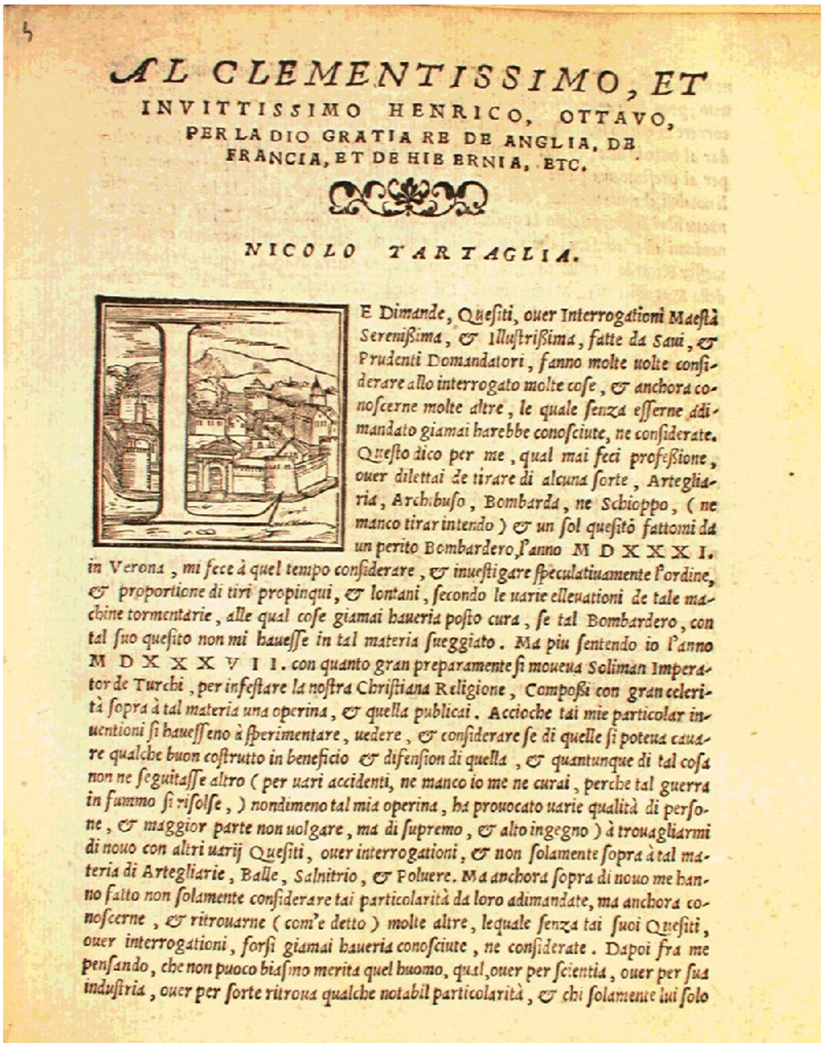
Date in Venice, at the new houses in San Salvatore XX.
[20th] December, MDXXXVII [1537]
Your Excellency's bumblest D.S.
Nicolo Tartaglia Brisciano [Nicolò Tartalea from Brescia].[91]

[90] On invasion of Italy, particularly North–East (especially Venezia). Since Francesco Maria della Rovere, Duke of Urbino (interlocutor of Tartaglia's letter) was employed by the Venetian Republic to organize a defense, Tartaglia's words are particularly important at this stage of the *Quesiti*. We note that in *Book I* of the *Quesiti*, Tartaglia also describes technical results on 20–pound culverin as being 10 feet in length (ca. 3 mt.), and weighing 4300 pounds (ca. 1950 Kg).

[91] Tartgalia 1537, 4rv, line 37.

His words in the last lines of this passage like a re–thinking, has to do with the last ballistic results obtained and only then proposed in the first three books of *Quesiti* (Tartgalia 1546, 1554).

Below we provide the first part of another *epistola dedicatoria*[92] to King Henri VIII of England, which serves as a preface to *Quesiti* (Tartaglia 1554).



AL CLEMENTISSIMO, ET INVITTISSIMO HENRICO, OTTAVO, PER LA DIO GRATIA RE DE ANGLIA, DE FRANCIA, ET DE HIBERNIA, ETC.

NICOLO TARTAGLIA.

LE Dimande, Quesiti, ouer Interrogationi Maestà Serenissima, & Illustrissima, fatte da Saui, & Prudenti Domandatori, fanno molte uolte considerare allo interrogato molte cose, & anchora conoscerne molte altre, le quale senza esserne addimandato giamai harebbe conosciute, ne considerate. Questo dico per me, qual mai feci professione, ouer dilettai de tirare di alcuna sorte, Arteglia-ria, Archibuso, Bombarda, ne Schioppo, (ne manco tirar intendo) & un sol quesitó fattomi da un perito Bombardero, l'anno M D X X X I. in Verona, mi fece à quel tempo considerare, & inuestigare speculatiuamente l'ordine, & proportione di tiri propinqui, & lontani, secondo le uarie ellevationi de tale machine tormentarie, alle qual cose giamai haueria posto cura, se tal Bombardero, con tal suo quesito non mi hauesse in tal materia sueggiato. Ma piu sentendo io l'anno M D X X X V I I. con quanto gran preparamente si moueua Soliman Imperator de Turchi, per infestare la nostra Christiana Religione, Composi con gran celerità sopra à tal materia una operina, & quella publicai. Acciochè tai mie particolar inuentioni si hauessено à sperimentare, uedere, & considerare se di quelle si poteua cauare qualche buon costrutto in beneficio & difension di quella, & quantunque di tal cosa non ne seguitasse altro (per uari accidenti, ne manco io me ne curai, perche tal guerra in fummo si risolse,) nondimeno tal mia operina, ha prouocato uarie qualità di persone, & maggior parte non uolgare, ma di supremo, & alto ingegno) à trouagliarmi di nouo con altri uarij Quesiti, ouer interrogationi, & non solamente sopra à tal materia di Artegliarie, Balle, Salnitrio, & Poluere. Ma anchora sopra di nouo me hanno fatto non solamente considerare tai particolarità da loro adimandate, ma anchora conoscerne, & ritrouarne (com'e detto) molte altre, lequale senza tai suoi Quesiti, ouer interrogationi, forsi giamai haueria conosciute, ne considerate. Dapoi fra me pensando, che non puoco biasmo merita quel huomo, qual, ouer per scientia, ouer per sua industria, ouer per sorte ritroua qualche notabil particolarità, & chi solamente lui solo

Fig. 1.21 Plate of the *Quesiti editions* and dedicatory to Henry VIII King of England[93]

[92] The figure and other observations are below.

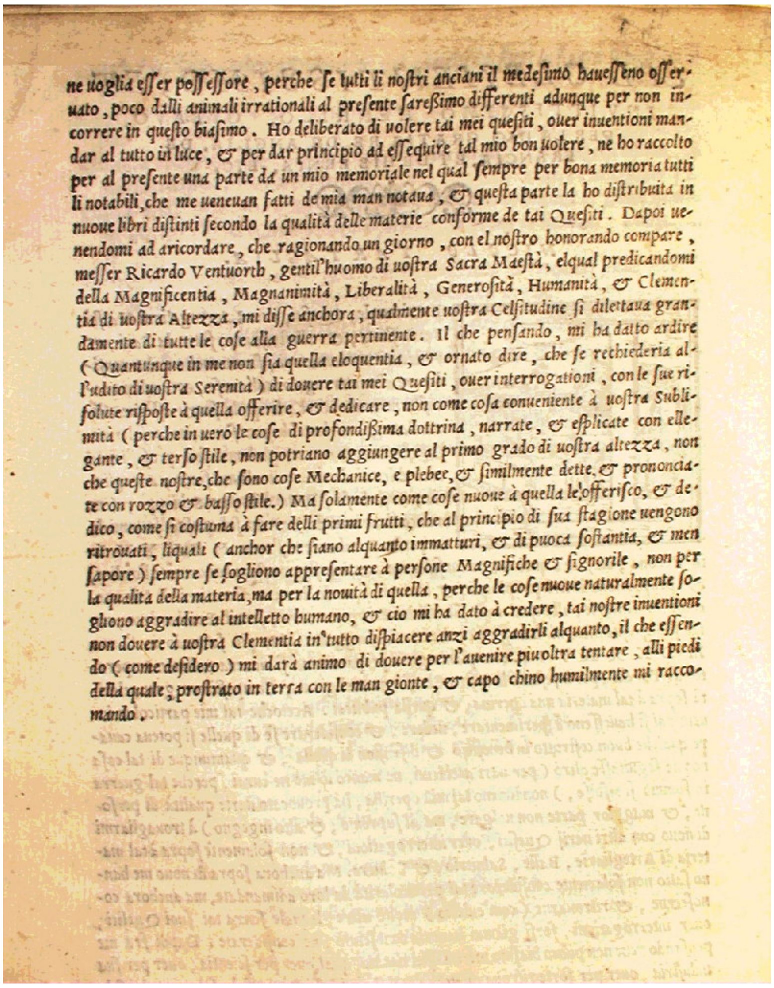

ne uoglia esser possessore, perche se tutti li nostri anciani il medesimo hauesseno osser-
uato, poco dalli animali irrationali al presente saresimo differenti adunque per non in-
correre in questo biasimo. Ho deliberato di uolere tai mei quesiti, ouer inuentioni man-
dar al tutto in luce, & per dar principio ad essequire tal mio bon uolere, ne ho raccolto
per al presente una parte da un mio memoriale nel qual sempre per bona memoria tutti
li notabili, che me ueneuan fatti de mia man notaua, & questa parte la ho distribuita in
nuoue libri distinti secondo la qualità delle materie conforme de tai Quesiti. Dapoi ue-
nendomi ad aricordare, che ragionando un giorno, con el nostro honorando compare,
messer Ricardo Ventuorth, gentil'huomo di uostra Sacra Maestà, elqual predicandomi
della Magnificentia, Magnanimità, Liberalità, Generosità, Humanità, & Clemen-
tia di uostra Altezza, mi disse anchora, qualmente uostra Celsitudine si dilettaua gran-
damente di tutte le cose alla guerra pertinente. Il che pensando, mi ha datto ardire
(Quantunque in me non sia quella eloquentia, & ornato dire, che se rechiederia al-
l'udito di uostra Serenità) di douere tai mei Quesiti, ouer interrogationi, con le sue ri-
solute risposte à quella offerire, & dedicare, non come cosa conueniente à uostra Subli-
mità (perche in uero le cose di profondissima dottrina, narrate, & esplicate con elle-
gante, & terso stile, non potriano aggiungere al primo grado di uostra altezza, non
che queste nostre, che sono cose Mechanice, e plebee, & similmente dette, & prononcia-
te con rozzo & basso stile.) Ma solamente come cose nuoue à quella le offerisco, & de-
dico, come si costuma à fare delli primi frutti, che al principio di sua stagione uengono
ritrouati, liquali (anchor che siano alquanto immatturi, & di puoca sostantia, & men
sapore) sempre se sogliono appresentare à persone Magnifiche & signorile, non per
la qualita della materia, ma per la nouità di quella, perche le cose nuoue naturalmente so-
gliono aggradire al intelletto humano, & cio mi ha dato à credere, tai nostre inuentioni
non douere à uostra Clementia in tutto dispiacere anzi aggradirli alquanto, il che essen-
do (come desidero) mi darà animo di douere per l'auenire piu oltra tentare, alli piedi
della quale, prostrato in terra con le man gionte, & capo chino humilmente mi racco-
mando.

Fig. 1.21 Continued[94]

[93] Tartaglia 1546, 1r; see also 1554, 1r, 4rv. I note that it would not be prudent to homage the book to King of England, previously excommunicated (1533), so it is very probable that the essential reason of the dedicatory was an homage to the English *gentleman* and Tartaglia's pupil Richard Wentworth cited in the dedicatory letter to *Quesiti* (Tartaglia [1554] 1959, 4rv) and in the Books V and IX (Tartaglia 2010, 9). Some sources report that Wentworth is (eventually) the author of an Italian manuscript archived in England (*Oxford Bodleian Library, Ms 584*, UK), as well. It seems that Tartaglia is often cited. Then, if so and maybe, it might be the true reason of a dedicatory to the King of England as well.

[94] *Ibidem*.

Finally, Tartaglia was overcomed by a guilty conscience, typical of scientists involved in activities with important social repercussions.[95]
Here, Tartaglia also describes his instruments of measurement and calculation for the 45 degree elevation. We note that in *Nova scientia*, the studies on the elevation of a piece ("pezzo") at a 45–degree angle and related images are included within the aforementioned *epistola dedicatoria* (Tartaglia 1537, 1r–4v). In *Quesiti* the *questione balistica* (see above) appears more organized since it is inserted starting in *Book I* (Tartaglia 1554, *Book I*, from folio 5r). Below, we provide the images present in the *epistola dedicatoria* of the *Nova scientia*[96]

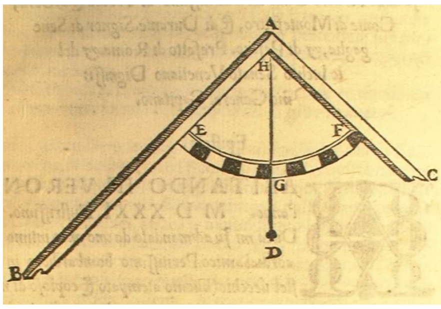


Fig. 1.22 Gunner's Square or *Tartaglia's Quadrant*[97]

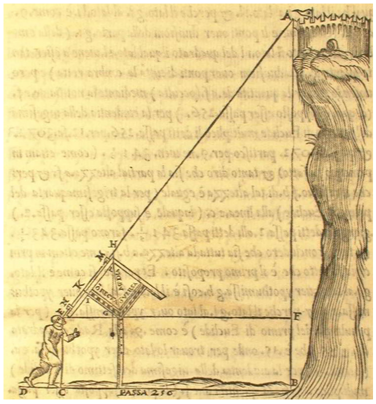


Fig. 1.23 Measuring-Calculating heights by Tartaglia's Quadrant[98]

> Beyond this, I made certain, by means of demonstrative geometrical reasons, that all shots with every kind of artillery, large and small, [whatever form they have] equally elevated above the plane of horizon, or equally oblique, or along the plane of the horizon, are similar to one another and consequentely proportional, as are [their] distances also.[99]

[95] From the beginning of the last century until today, we have not been exempt from seeing similar situations faced by Nobel Prize winners and involved scholars.

[96] We note that in the subsequent pages we can also see the explicit observation against the Aristotelean conception of violent and natural motion in effect at the time. (Tartaglia 1554, *Book I*, Q I, 5rv–7rv; Q. III, 11; Qs I–II–III–VI, 5rv–13rv, Q I, 6rv).

[97] Tartaglia 1537, *Book I*, 3v. Below we also provide those present in *Quesiti* (Tartaglia 1554, *Book I*).

[98] Tartaglia 1537, *Book III*, 40rv. A smilar argumentation and images will also be proposed by Galilei (Galilei 1606, Appendice II; *Id*., 1640, pp 61–80)

[99] Tartaglia 1537, *Book I*, 5rv, line 23.

Tartaglia found the elevation giving the greatest range to be 45°. Even if his proof was not satisfactory, he surely proposed a general law within the history of physics that was valid for every kind of gun. In fact, he inaugurated the scientific treatment of the subject. The argument was again studied, occupying two out (*Book I* and *Book II*) of nine books of *Quesiti.*

The *Nova Scientia* had a certain approval among those who practiced the art of the *bombardieri* (artillerymen) as can also be deduced from the dedication of the work. It should be noted that *Nova Scientia* is a treatment that was published in a very peculiar period for the history of Renaissance mechanics and that of fortifications inferred by several cultural events of the beginning of the 16th century:[100] It is the

> [...] first witing on ballistics [...] based firmly on the live, concrete experience of the facts and carried out with the aid of geometry and numerical calculation […].[101]

After the first 1537 editon, still three main volumes were published in 1550 where some reworked lines can be read (e.g., Book III), then posthumously in 1558 and in 1562 (reprinted[102] 1583 and 1606). Particularly, after the second edition, a new book *Gionta al Terzo Book* was included in the following edition. According to the above–cited development his ballistic research (from*Nova scientia* to *Quesiti et inventioni diverse*) we note that Tartaglia found the right relationship between the range and 45° angle, even though his reasoning was too weak to demonstrate the accuracy of his intuition. His ability and interest in the study of trajectories is noteworthy since he seems to have understood that the path is not entirely rectilinear. In this regard, we feel free to consider Tartaglia as one of the first to apply a scientific treatment to the subject.

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[100] The following works during 1531–1532 which, in general, from a historical point of view, had a certain influence on society should also be noted. Gerolamo Fracastoro observes the tails of comets and concludes that they are always facing opposite the Sun; 1535–38. Fracastoro publishes *Homocentricorum sive de stellis*, in which the system of the world starting with the geometric motion of the planets defined by the uniform rotations of homocentric spheres is discussed; 1536. Calvino publishes *Istituzioni della religione cristiana*.

[101] "[…] primo scritto di balistica […] basato saldamente sull'esperienza viva e concreta dei fatti e svolto con l'ausilio della geometria e del calcolo numerico […]." (Bolletti 1958, 14, line 8).

[102] The 1562 edition lacks of Book IX (Cfr.: Cuomo 1997, 1998).

He informed the Duke of Urbino of the remarkable general result of his research:

> *All pieces of artillery, of any size, firing bullets which describe trajectories curved and of the same geometric shape*.[103]

Therefore, this involves a theorem as a general proposition to demonstrate. Nevertheless, in addition to the law of the elevation of the cannon, it was also necessary to know – as Tartaglia correctly notes – how far away the target was. To this aim, he suggests a practical method to calculate *con la vista* and with two different types of "square ruler" with quadrants[104] the distances that are impossible to measure directly between the artilleryman and the target.

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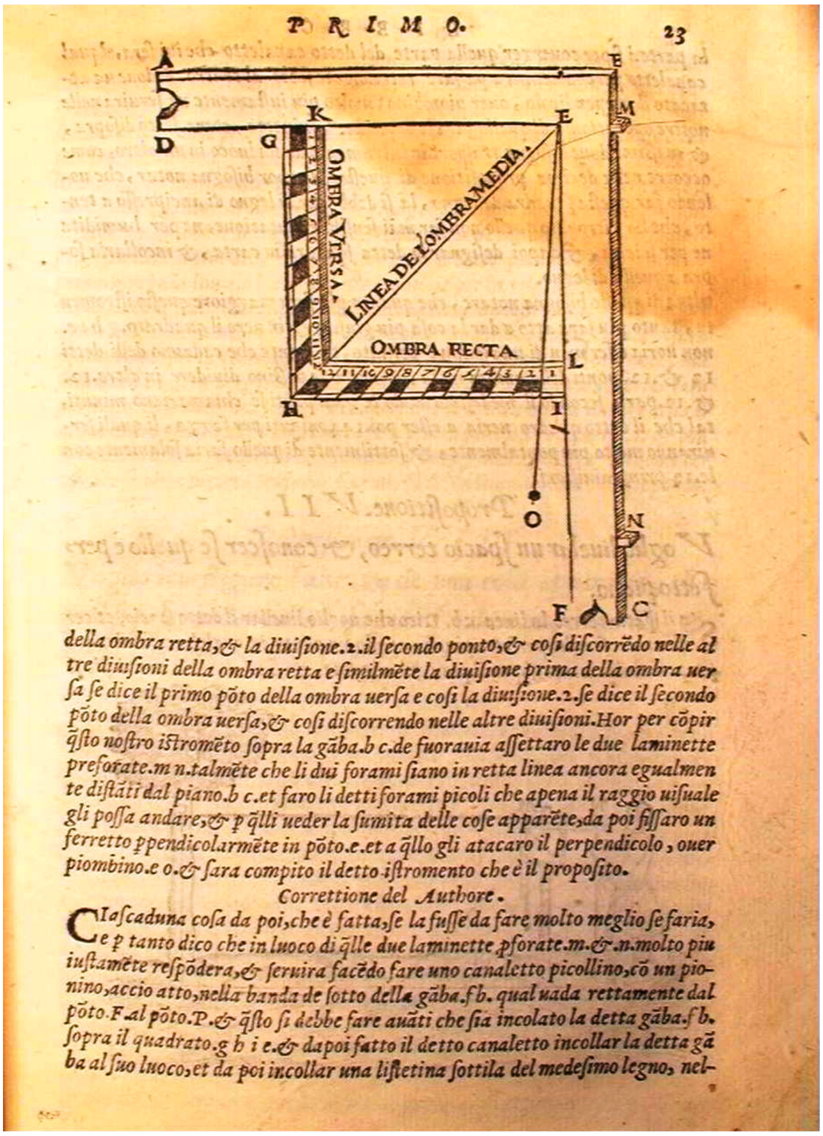
[103] A clarification. Within 7rv folia (in–between *Book I* and *Book II*) of the *Nova scientia*, Tartaglia proposed his main arguments concerning the 3 parts–composition of the trajectory of a projectile: rectilinear segment, arc of circumference and a final rectilinear segment towards the centre of the Earth (Tartaglia 1537, *Book I*, 13rv–20rv; see also *Book I,* IV–V Props., 14r–15r; for the representation of various distances with respect to various inclinations see *Ivi*, 20v). These parts are described by some figures (Tartaglia 1537, *Book I*, 15r, 16r) which are divided into letters corresponding to natural motion, violent motion and mixed natural motion. Of course without a modern vectorial and mathematical interpretation of a composed motion (particularly along a curved path where the change of vectorial orientation produces an acceleration), then it is obvious that in Tartaglia's context a body cannot assume (in a point long the path) negative and positive values at the same time.

[104] At that time many practical instruments were in use, so it is reasonable to think that the instruments often cited by Tartaglia were not originally invented by himself. For example, Tartaglia cites a frequent use of the *quadrant* at that time and without mentioning which version of *quadrant* he preferred. For sure we do not have *historical* proof if he really did or did not invent the *quadrant* that he often cited in his own manuscripts. Thus, even if similar instruments are reported in secondary literature (e.g., see: Alberti fl. 15th, 10rv–11rv (retrieved via web); Essenwein and Germanisches 1873), we cannot claim an historical hypothesis within history and historical epistemology of science studies concerning his eventual (or not) invention.



della ombra retta,& la diuiſione.2.il ſecondo ponto,& coſi diſcorrẽdo nelle al tre diuiſioni della ombra retta e ſimilmẽte la diuiſione prima della ombra uer ſa ſe dice il primo põto della ombra uerſa e coſi la diuiſione.2.ſe dice il ſecondo põto della ombra uerſa,& coſi diſcorrendo nelle altre diuiſioni.Hor per cõpir qſto noſtro iſtromẽto ſopra la gãba.b c.de fuorauia aſſettaro le due laminette preforate.m n.talmẽte che li dui forami ſiano in retta linea ancora egualmen te diſtãti dal piano.b c.et faro li detti forami picoli che apena il raggio uiſuale gli poſſa andare,& p qlli ueder la ſumita delle coſe apparẽte,da poi fiſſaro un ferretto ppendicolarmẽte in põto.e.et a qllo gli atacaro il perpendicolo, ouer piombino.e o.& ſara compito il detto iſtromento che è il propoſito.

Correttione del Authore.

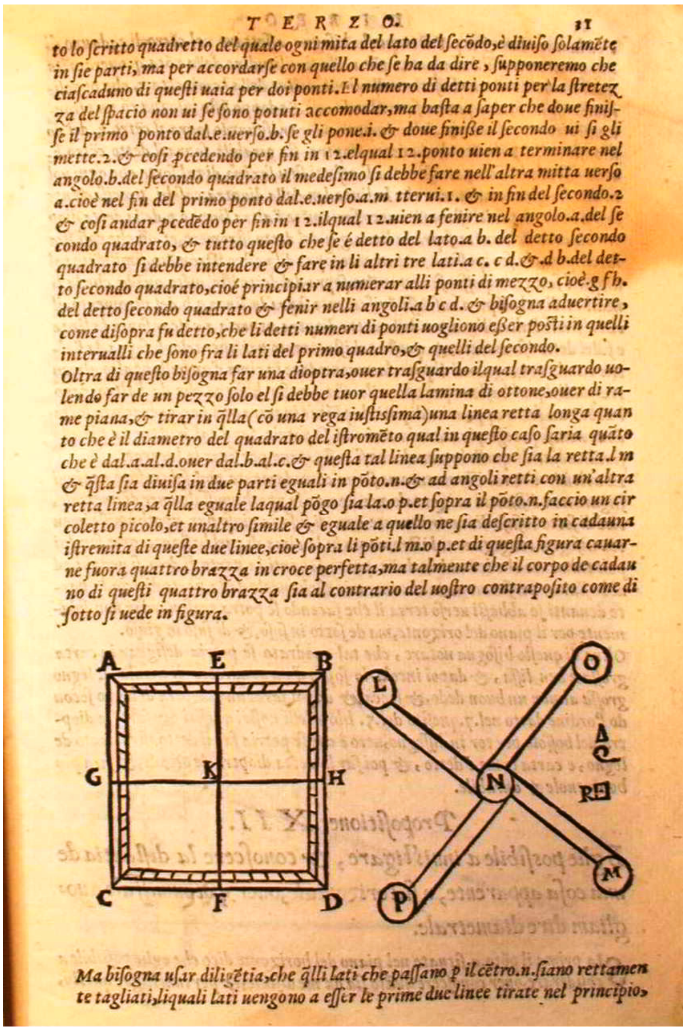
Ciaſcaduna coſa da poi,che è fatta,ſe la fuſſe da fare molto meglio ſe faria, e p tanto dico che in luoco di qlle due laminette pforate.m.& n.molto piu iuſtamẽte reſpõdera,& ſeruira facẽdo fare uno canaletto picollino,cõ un pionino,accio atto,nella banda de ſotto della gãba.f b. qual uada rettamente dal põto.F.al põto.P.& qſto ſi debbe fare auãti che ſia incolato la detta gãba.f b. ſopra il quadrato.g h i e.& dapoi fatto il detto canaletto incollar la detta gã ba al ſuo luoco,et da poi incollar una liſtetina ſottila del medeſimo legno, nel-

Fig. 1.24 Plate from *Nova scientia*, first instrument: *squadra a gnomone*[105]

The first square in which from the apex of the right angle a plumb line descends is useful for keeping the instrument perpendicular and therefore for evaluating possible slants. Inside the right angle, there is also a small

[105] Tartaglia 1537, *Book III*, 22v–23r.

graduated ruler on the smaller side of the cross. This is for determining the vertical distances, i.e., elevations, far from the observer. The second instrument is used to determine horizontal distances far from the observer.


TERZO. 31

to lo scritto quadretto del quale ogni mita del lato del secõdo, è diuiso solamẽte in sie parti, ma per accordarse con quello che se ha da dire, supponeremo che ciascaduno di questi uaia per doi ponti. El numero di detti ponti per la strettezza del spacio non ui se sono potuti accomodar, ma basta a saper che doue finisse il primo ponto dal.e.uerso.b.se gli pone.1.& doue finisse il secondo ui si gli mette.2.& cosi pcedendo per fin in 12.elqual 12.ponto uien a terminare nel angolo.b.del secondo quadrato il medesimo si debbe fare nell'altra mitta uerso a.cioè nel fin del primo ponto dal.e.uerso.a.m tterui.1.& in fin del secondo.2 & cosi andar pcedẽdo per fin in 12.ilqual 12.uien a fenire nel angolo.a.del secondo quadrato, & tutto questo che se é detto del lato.a b. del detto secondo quadrato si debbe intendere & fare in li altri tre lati.a c. c d.& d b.del detto secondo quadrato, cioé principiar a numerar alli ponti di mezzo, cioè.g f h. del detto secondo quadrato & fenir nelli angoli.a b c d.& bisogna aduertire, come disopra fu detto, che li detti numeri di ponti uogliono esser posti in quelli interualli che sono fra li lati del primo quadro, & quelli del secondo.

Oltra di questo bisogna far una dioptra, ouer trasguardo ilqual trasguardo uolendo far de un pezzo solo el si debbe tuor quella lamina di ottone, ouer di rame piana, & tirar in q̃lla (cõ una rega iustissima) una linea retta longa quanto che è il diametro del quadrato del istromẽto qual in questo caso saria quãto che è dal.a.al.d.ouer dal.b.al.c.& questa tal linea suppono che sia la retta.l m & q̃sta sia diuisa in due parti eguali in põto.n.& ad angoli retti con un'altra retta linea, a q̃lla eguale laqual põgo sia la.o p.et sopra il põto.n.faccio un circoletto picolo, et unaltro simile & eguale a quello ne sia descritto in cadauna istremita di queste due linee, cioè sopra li põti.l m.o p.et di questa figura cauarne fuora quattro brazza in croce perfetta, ma talmente che il corpo de cadauno di questi quattro brazza sia al contrario del nostro contraposito come di sotto si uede in figura.

Ma bisogna usar diligẽtia, che q̃lli lati che passano p il cẽtro.n.siano rettamente tagliati, liquali lati uengono a esser le prime due linee tirate nel principio.

Fig. 1.25 Plate from *Nova scientia*, second instrument: *squadra a traguardi mobili*[106]

It consists of a square positioned horizontally on a post and strips divided into twelve equal parts. In the center it is possible to move the alidade inserted at a right angle with appropriate paddles with slits.

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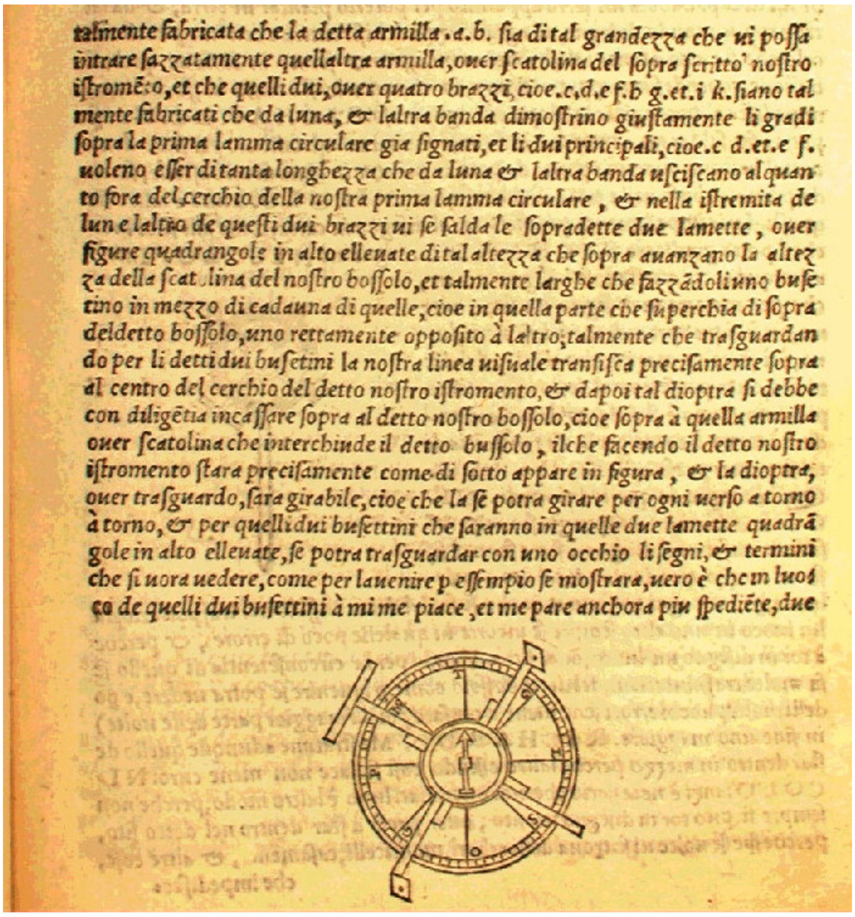
[106] Tartaglia 1537, *Book III*, 30rv.

Tartaglia describes these two instruments specifically designed for artillerymen. In *Quesiti* he describes a similar instrument for surveyors.



talmente fabricata che la detta armilla .a.b. sia di tal grandezza che ui possa
intrare sazzatamente quellaltra armilla, ouer scatolina del sopra scritto nostro
istromẽto, et che quelli dui, ouer quatro brazzi, cioe .c,d,e f.b g.et.i k. siano tal
mente fabricati che da luna, & laltra banda dimostrino giustamente li gradi
sopra la prima lamma circulare gia signati, et li dui principali, cioe .c d.et.e f.
uoleno esser di tanta longhezza che da luna & laltra banda uscis cano alquan
to fora del cerchio della nostra prima lamma circulare, & nella istremita de
lun e laltro de questi dui brazzi ui se salda le sopradette due lamette, ouer
figure quadrangole in alto elleuate di tal altezza che sopra auanzano la altez
za della scatolina del nostro bossolo, et talmente larghe che fazzãdoli uno buse
tino in mezzo di cadauna di quelle, cioe in quella parte che superchia di sopra
del detto bossolo, uno rettamente opposito à laltro; talmente che trasguardan
do per li detti dui busetini la nostra linea uisuale transisca precisamente sopra
al centro del cerchio del detto nostro istromento, & dapoi tal dioptra si debbe
con diligẽtia incassare sopra al detto nostro bossolo, cioe sopra à quella armilla
ouer scatolina che interchiude il detto bussolo, ilche facendo il detto nostro
istromento stara precisamente come di sotto appare in figura, & la dioptra,
ouer trasguardo, sara girabile, cioe che la se potra girare per ogni uerso a torno
à torno, & per quelli dui busettini che saranno in quelle due lamette quadrã
gole in alto elleuate, se potra trasguardar con uno occhio li segni, & termini
che si uora uedere, come per lauenire p essempio se mostrara, uero è che in luoi
co de quelli dui busettini à mi me piace, et me pare anchora piu spediẽte, due

Fig. 1.26 Plate from *Nova scientia*, third instrument: *Bossolo*[107]

Tartaglia uses the name, *Bossolo*, most likely derived from the fact that it is used somewhat like a compass (bussola). In fact, it can be maintained that the *Bossolo* is the predecessor of the *grafometro a bussola* (compass graphometer) with an internal circle. It has a large graduated metallic circle on the large circumference with a small compass in the middle and two alidades mobile amongst themselves at a right angle and can move around the same center for the final determination. Therefore, it is an

[107] Tartaglia 1537, *Book V,* 54rv.

application of the compass to topography. Consequently, it can be asserted that it was studied in order to provide an orientation rather than to measure angles.

In the *General trattato* he discusses another instrument for surveyors (Tartaglia 1560, *Parte III*, *Book III*). We describe the surveyor's cross in the bottom right corner of the following illustration.

Fig. 1.27 Plate from *General trattato,* Fourth instrument: *squadro*[108]

In addition to describing the instrument he writes: "[...] necessary land measurements called cross and how it is made and how to know if it is correct." (Tartaglia 1560, *Parte III*, *Book III*, line 4), he pauses at an interesting modification which he considers to be useful to apply to the

[108] Tartaglia 1560, *Parte III*, *Book III*, 24rv.

instrument in question. In practice, Tartaglia suggests the addition of the two vertical visual planes to the two visual lines, with the alidade of two vertical slits tracing each other between their perpendiculars. Therefore, in addition to his mathematical, geometrical, architectural and statics capabilities, he was also well–versed in the techniques of instruments.

Below, we provide a passage in which the author expresses all of his "Archimedean" capabilities, pointing out the theories that he will use in his calculations, such as geometry and algebra.

> Next (Signor humanissimo) I knew by Archimedean reasonings[109] that the distance of the aforementioned shot elevated at 45 degrees above the horizon was about ten times the straight carriage of a shot made in the plane of the horizon: which is called point blank ["ponto in bianco"] by bombardiers, which such evidence, Excellent Duke, I found by geometrical and algebraic reasons that a ball shot toward a point 45 degrees above the horizon goes about four times as far in a straight line as it goes when shot in the plane of the horizon, or (as I said) at point blank [that is, to shot horizontally].[110]

As already stated, since Tartaglia's studies on artillery in *Nova scientia* are also present in the *Quesiti*, for the sake of completeness, I also note that the crucial points of *Book I* of the *Quesiti* improved some of the theses presented in previous *Nova scientia*. In the following we list only the differences between *Nova scientia* and *Book I* of *Quesiti* around the matter above cited:

1. According to Tartaglia, the trajectory of the projectile is – in some points – curved so little that it can be thought of as straight. In fact, he draws it as a straight line, then traces a curved branch and in the end, draws a descending rectilinear branch. (Tartaglia 1537, *Book II*, Prop VI). This

[109] On that Drake (Drake and Drabkin 1969, 66) pointed out that in the next editions Tartaglia avoided the word *Archimedean* ("Archimedeane") and wrote "[…] con ragion natural […]" (by physical reasonings). In any case the relationship between Archmedean and natural reasoning is confirmed since the inductive method was adopted.

[110] "Da poi (Signor humanissimo) con ragion Archimedeane qualmente la distantia dil sopra ditto tiro elleuato alli 45 gradi sopra al orizonte, era circa decupla al tramito retto dun tiro fatto per il piano del orizonte: che da bombardiere è ditto tiro de ponto in bianco, con la qual evidentia, Magnanimo Duca, trovai con ragione geometrice e algebratice qualmente balla tirata vesro li detti 45 gradi sopra a l'orizzonte va circa a quattro volte tanto per l'aere di quello che va essendo tirato per il pian de l'orizzonte, che dà borbandieri è chiamato (come ho detto) tirar de punto in bianco [cioè tirare orizzontalmente]." (Tartaglia 1537, 5rv, line 28).

vision will be revisited in *Book I* of *Quesiti* in which the trajectory essentially appears curvilinear (Tartaglia 1554, *Book I, Qs. I–II–III–VI*).

2. The angle of maximum range of the projectile is 45 degrees (Tartaglia 1554, Q I, 6rv–7rv).
3. In the trajectory a point to which the minimum speed of the projectile corresponds is possible obtained (Tartaglia 1537, 5rv–9rv).
4. A target can be hit with two different angles of elevation of the "pezzo" provided that they are complementary (Tartaglia 1537, 5rv–10rv).
5. The angles of elevation of the "pezzi" of artillery on the horizon are measured with the "squadra" (Both in Tartaglia 1537, 5rv–6rv and in Tartaglia 1554, *Book I, Q I*).
6. The ranges in function of their angles are presented for practice for artillerymen (Both Tartaglia 1537, 5rv–8rv and in various parts of Tartaglia 1554, *Book I, Q I*, 5rv–7rv, *Book II,* 35rv–36rv, *Book III,* 39rv–40rv).

The word *point blank* ("punto bianco"[111]) was proposed by Tartaglia. He measured and calculated the elevation of a gun by means of a *gunners' quadrant*. In effect, if one thinks of an horizontal fire and consider the trajectory from *F* to *D* as proposed by Tartaglia in *Quesiti* (see Fig. 1.28), and if the distance EF is not too long, common sense suggests that the cannonball will not descend far from the cannon. From a strictly mathematical standpoint, this (horizontal) situation is called *point blank* (or blank point).

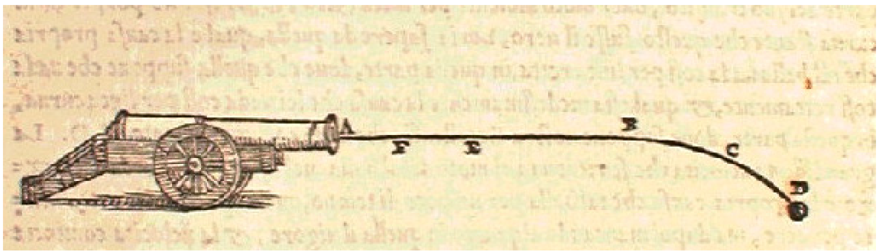
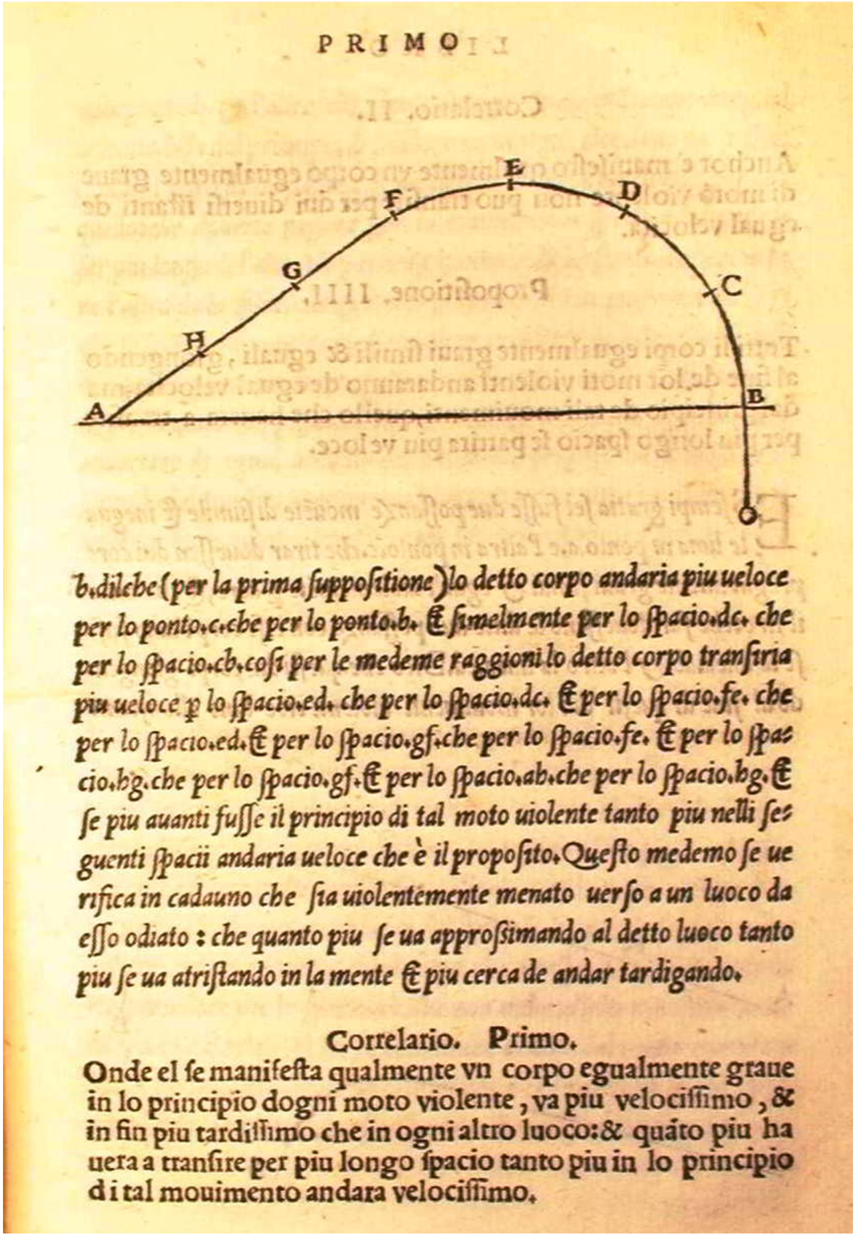


Fig. 1.28 Plate from *Quesiti* on the trajectory (out) of the cannon[112]

In the following I describe Tartaglia's first corollary in *Nova scientia* where he defines his main ideas on natural and violent motion realated to the trajectories of the projectiles.

[111] Tartaglia 1537, 5rv–9rv, line 7. Incidentally, literature on military arguments was current at that time, (i.e., see Alberti). Thus, Tartaglia's novelities might be merely part of these shared studies.

[112] Tartaglia 1554, *Book I*, Q III, 11v; see also Qs I–II–III–VI, 5rv–13rv.


PRIMO

b. dilche (per la prima ſuppoſitione) lo detto corpo andaria piu ueloce per lo ponto. c. che per lo ponto. b. & ſimelmente per lo ſpacio. dc. che per lo ſpacio. cb. coſi per le medeme raggioni lo detto corpo tranſiria piu ueloce p lo ſpacio. ed. che per lo ſpacio. dc. & per lo ſpacio. fe. che per lo ſpacio. ed. & per lo ſpacio. gf. che per lo ſpacio. fe. & per lo ſpacio. hg. che per lo ſpacio. gf. & per lo ſpacio. ab. che per lo ſpacio. hg. & ſe piu auanti fuſſe il principio di tal moto uiolente tanto piu nelli ſeguenti ſpacii andaria ueloce che è il propoſito. Queſto medemo ſe uerifica in cadauno che ſia uiolentemente menato uerſo a un luoco da eſſo odiato: che quanto piu ſe ua approſsimando al detto luoco tanto piu ſe ua atriſtando in la mente & piu cerca de andar tardigando.

Correlario. Primo.

Onde el ſe manifeſta qualmente vn corpo egualmente graue in lo principio dogni moto violente, va piu velociſſimo, & in fin piu tardiſſimo che in ogni altro luoco: & quáto piu ha uera a tranſire per piu longo ſpacio tanto piu in lo principio di tal mouimento andara velociſſimo.

Fig. 1.29 Plate from *Nova scientia* on the violent and natural motion[113]

[113] Tartaglia 1537, *Book I*, 15r.

The long arm may be laid in the cannon barrel. It was attributed to a shorter arm by a scale in the shape of a quarter circle, which was marked off with 12 points. For example, in order to fire at 6 points one should fire at 45°. In this sense, in order to fire horizontally, one obtains a *punto bianco*, that is, no useful points. Therefore, Tartaglia studied theoretical situations both inclined higher than 45° and inferior to 45°.

Later, the term *point blank* ("punto bianco") was also used in a Galilean didactic[114] work, postumely entitled (by Antonio Favaro in *Opere Nazionali di Galileo Galilei*) *Trattato di Fortificazione* and concerining Galileo's teaching speech on military architecture where *punto bianco* is taught within a paragraph *Delle diversità de' tiri* (*On several ways to shoot*):

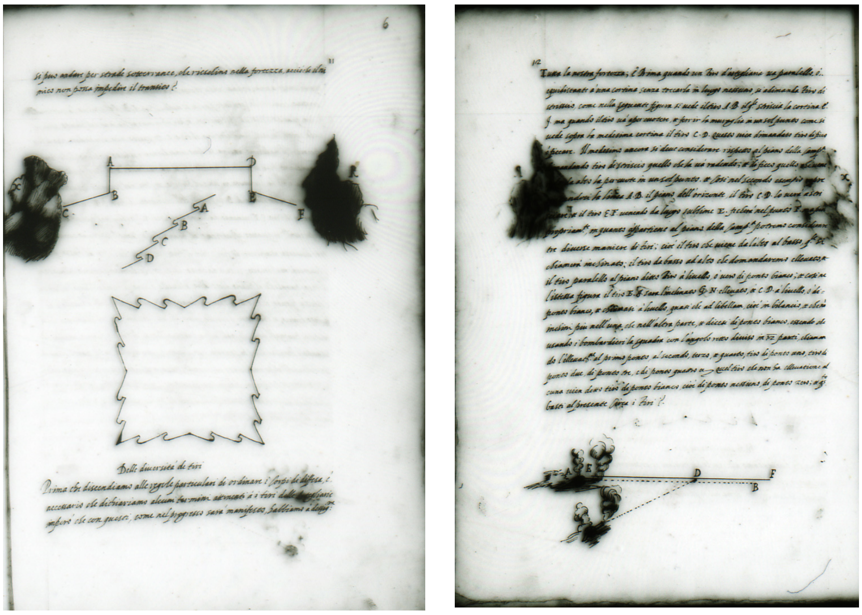


Fig. 1.30 Plates from Galilean manuscript on the military architecture teaching[115]

[114] Pisano 2009c, 2009d, Pisano and Capecchi 2010a; 2009; Pisano and Bussotti 2012.

[115] Galileo G, *Ms. B*. See also *Ms. m*. "DELLE DIVERSITÀ DE' TIRI. […] il tiro che viene da alto a basso, quale si chiamerà *inclinato*; il tiro da basso ad alto che domanderemo *elevato*; ed il tiro paralello al piano, detto *tiro a livello*, o vero *di punto bianco*. E così nell'istessa figura il tiro *EF* sarà *l'inclinato*, *GH elevato*, e *CD a livello* o *di punto bianco*. E chiamasi *a livello*, quasi che *ad libellam*; cioè in bilancio, e che non inchini più nell'una che nell'altra parte. E dicesi di punto bianco, essendo che, usando i bombardieri la squadra con l'angolo retto diviso in dodici punti, chiamando l'elevazione al primo punto, al secondo, terzo e quarto,

In the end, Tartaglia's re–examination (beginning in *Nova scientia*) of Aristotle's[116] *natural* and *violent* motion contributed to creating the cultural background that allowed him to write *Book I* of *Nova scientia* on the dynamics of projectiles,[117] and the subsequent *Libri VI*, *VII* e *VIII* of *Quesiti* on fortifications and the fundamental principles of the science of weights. In particular, his reasonings on the range of projectiles allowed him to write so many considerations on the geometry of fortifications (*Book VI* and its *Gionta*) showing himself to reader also to be a technician of architecture and military arts.

1.3.2 The *Sesto Libro* on Fortifications

In *Quesito III* of *Book VI* Tartaglia includes a sort of memorandum on the problems to solve within his arguments, or in his words, "quality over [or] condition" or "properties" most important to bear in mind for the design of a secure fortification:

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tiro di punto uno, di punto dua, di punto tre e di punto quattro etc., quel tiro, che non ha elevazione alcuna, vien detto tiro *di punto bianco*, cioè di punto nessuno, di punto zero." (Galilei 1890–1909, II, 92–93, line 17). In regard to analyzing the possible shots against *inclined, elevated* and point *blank strengths*, that is, at a zero degree elevation, he seems to follow an incorrect vision of the projectile trajectory since he draws rectilinear segments and not parabolic ones. (Pisano 2008, I, 225–231, 249; Pisano and Capecchi 2010; 2012).

[116] It should be noted that in the paradigm of Aristotelean science, it was necessary for the projectile trajectory to be composed of three parts: an inclined rectilinear branch ("violent motion"), a circular branch ("mixed motion") and a vertical branch ("natural motion"). That is to say, that as gravity prevails it decreases speed and the "balla" falls vertically. Subsequent developments of this vision hypothesized the decrease of the speed of the "balla" was due to the *impetus* action. In *Book I* of *Quesiti* (Tartaglia 1554, *Book I*, Q III), Tartaglia denies this thesis, affirming that gravity, which is always present, acts on the "balla" from the beginning (of the shot) of its path until it touches the ground. According to Bolletti, Tartaglia's explanation is essentially based on the fact that the "balla", shot with whatever initial speed, would favor the composition –so to speak— of gravity and of the impetus of the "balla" itself (Bolletti 1958, 61–62). It must be noted, however, that if this was Tartaglia's intention, in Q III of *Quesiti*, I don't believe he was as explicit and precise as it seems in Bolletti's analysis.

[117] We specify that in *Book I* Tartaglia suggests to the reader that, before proceeding in his ballistic theory, it is opportune to examine elements of the science of weights (Tartaglia 1554, *Book I*, Q II, 7rv–10rv).

Table 1.2. The "qualità" [qualities] of fortification designs according to Tartaglia[118]

Qualità	Tartaglia 1554, *Quesiti*, *Book VI*
1. Recoil [colpi di rimbalzo]	*Ivi*, Q III, 65rv
2. Bastions and curtains [Baluardi e cortine]	*Ivi*, Q IV, 66rv
3. Geometry of walls [Forma geometrica delle mura]	*Ivi*, Q V, 66rv
4. Defense with ruined walls [Difesa con le mura rovinate]	*Ivi*, Q VI, 66rv
5. Sentinels on walls [Sentinelle di guardia alle mura]	*Ivi*, Q VII, 67rv
6. Fortification of roads and expense estimate [Fortificazione delle strade e stima della spesa]	*Ivi*, Q VIII, 67rv

The first of the six qualities (in reality they are problems to address) appears to be particularly interesting, since it associates a type of physical to geometric skill also when studying the trajectories (even before Galileo) of recoil as, incidentally, Tartaglia had already argued on that in the *Nova scientia*.

Before proceeding with the analysis of *Book VI* of *Quesiti*, we think it useful to provide some reflections on the cultural and scientific context, from mechanical to astronomical new ideas, related to the period when Tartaglia wrote his work on fortifications.

1509. Luca Pacioli publishes *De divina proportione* (Pisano 2013) on the geometric principles and the study of the proportions of the human body. Da vinci's xylographies are included. .
1521. Cesare Cesariano translates *De architectura* by Vitruvio.
1527. Sack of Rome.
1527. Michele Sanmicheli (1484–1559) develops the *bastione angolare*
1533. *Liber Iordani Nemorarii viri clarissimi, de ponderibus [...]* edited by Petrus Apianus (1495–1552) who reproduced a manuscript of *Liber Jordani de ponderibus* (version P).
1534. Antonio da Sangallo il Giovane oversees the fortifications of *Fortezza da Basso* in Florence.
1535. Michele Sanmicheli in Venezia to construct the *lido* and the *forte di Sant'Andrea.* Perhaps the first example of an entirely bastioned system.

[118] The order follows the original order Tartaglia used.

1537. Antonio da Sangallo il Giovane oversees the fortifications of *Città del Vaticano* (*Vatican City*).
1537. Niccolò Tartaglia publishes *Nova scientia* on the geometric motion of projectiles
1540. In Venice *De la Pirotechnia* by Vannoccio Biringuccio is released posthumously. It is fundamental for the development of inorganic chemistry, mineralogy and metallurgy, but also for the improvement of firearms.
1543. *De revolutionibus orbium coelestium* by Copernicus is released.

Essentially, when Tartaglia wrote *Nova scientia* he could count on the ancient writings on mechanics that were available for consultation and on other important publications which, however, did not directly concern the study of statics or, more generally, mechanical tradition (Aristotle, Heron, Archimedes); he also counted on the first achievements of military architectural plans.

We will now see in detail *Book VI* of the *Quesiti* entitled *Sopra il modo di fortificar le Citta rispetto alla forma* (Tartaglia 1554, B*ook VI*). Before presenting his qualities, Tartaglia provided some examples of current problems at that time concering the state of art of fortifications in Italy; his arguments are related to the third quality (Tab. 1.2). He cites the fortifications of Torino. He speaks of the map of Torino, for which, Tartaglia raises, in no uncertain terms, his objections to the fragility of the fortifications of the city:

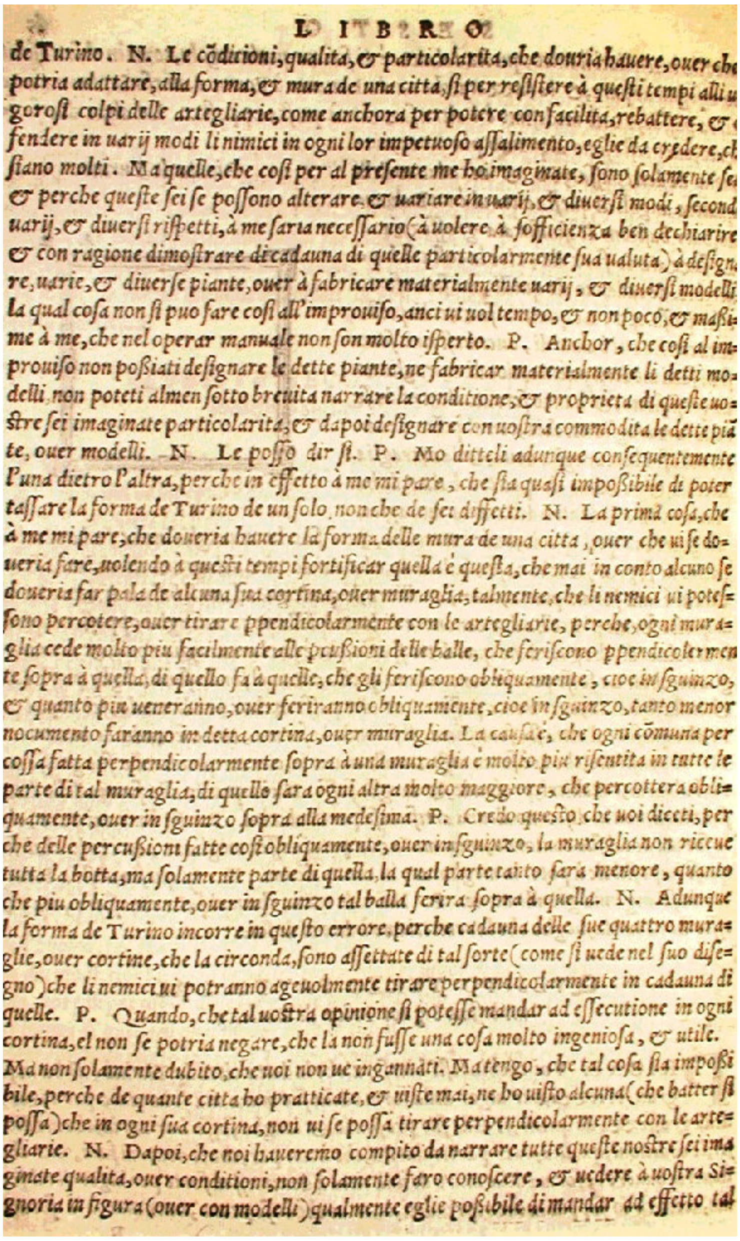
de Turino. N. Le côditioni, qualita, & particolarita, che douria hauere, ouer che potria adattare, alla forma, & mura de una citta, si per resistere à questi tempi alli rigorosi colpi delle artegliarie, come anchora per potere con facilita, rebattere, & offendere in uarij modi li nimici in ogni lor impetuoso assalimento, eglie da credere, che siano molti. Ma quelle, che cosi per al presente me ho imaginate, sono solamente sei, & perche queste sei se possono alterare, & uariare in uarij, & diuersi modi, secondo uarij, & diuersi rispetti, à me saria necessario (à uolere à sofficienza ben dechiarire & con ragione dimostrare di cadauna di quelle particolarmente sua ualuta) à designare, uarie, & diuerse piante, ouer à fabricare materialmente uarij, & diuersi modelli la qual cosa non si puo fare cosi all'improuiso, anci ui uol tempo, & non poco, & massime à me, che nel operar manuale non son molto isperto. P. Anchor, che cosi al improuiso non possiati designare le dette piante, ne fabricar materialmente li detti modelli non poteti almen sotto breuita narrare la conditione, & proprieta di queste uostre sei imaginate particolarita, & dapoi designare con uostra commodita le dette piãte, ouer modelli. N. Le posso dir si. P. Mo ditteli adunque consequentemente l'una dietro l'altra, perche in effetto à me mi pare, che sia quasi impossibile di poter tassare la forma de Turino de un solo, non che de sei diffetti. N. La prima cosa, che à me mi pare, che doueria hauere la forma delle mura de una citta, ouer che ui se doueria fare, uolendo à questi tempi fortificar quella è questa, che mai in conto alcuno se doueria far pala de alcuna sua cortina, ouer muraglia, talmente, che li nemici ui potessono percotere, ouer tirare ppendicolarmente con le artegliarie, perche ogni muraglia cede molto piu facilmente alle pcussioni delle balle, che feriscono ppendicolarmente sopra à quella, di quello fa à quelle, che gli feriscono obliquamente, cioe in sguinzo, & quanto piu ueneranno, ouer feriranno obliquamente, cioe in sguinzo, tanto menor nocumento faranno in detta cortina, ouer muraglia. La causa è, che ogni cõmuna percossa fatta perpendicolarmente sopra à una muraglia è molto piu risentita in tutte le parte di tal muraglia, di quello sara ogni altra molto maggiore, che percottera obliquamente, ouer in sguinzo sopra alla medesima. P. Credo questo che uoi diceti, perche delle percussioni fatte cosi obliquamente, ouer in sguinzo, la muraglia non riceue tutta la botta, ma solamente parte di quella, la qual parte tanto sara menore, quanto che piu obliquamente, ouer in sguinzo tal balla ferira sopra à quella. N. Adunque la forma de Turino incorre in questo errore, perche cadauna delle sue quattro muraglie, ouer cortine, che la circonda, sono assettate di tal sorte (come si uede nel suo disegno) che li nemici ui potranno ageuolmente tirare perpendicolarmente in cadauna di quelle. P. Quando, che tal uostra opinione si potesse mandar ad essecutione in ogni cortina, el non se potria negare, che la non fusse una cosa molto ingeniosa, & utile. Ma non solamente dubito, che uoi non ue ingannati. Ma tengo, che tal cosa sia impossibile, perche de quante citta ho pratticate, & uiste mai, ne ho uisto alcuna (che batter si possa) che in ogni sua cortina, non ui se possa tirare perpendicolarmente con le artegliarie. N. Dapoi, che noi haueremo compito da narrare tutte queste nostre sei imaginate qualita, ouer conditioni, non solamente faro conoscere, & uedere à uostra Signoria in figura (ouer con modelli) qualmente eglie possibile di mandar ad effetto tal

Fig. 1.31 Plate from *Quesiti* around qualities[119]

[119] "N. [Niccolò]. La prima cosa che à me mi pare, che doueria hauere la forma delle mura de una citta, ouer che ui se doueria fare, uolendo à questi tempi fortificar quella è questa, che mai in conto alcuno se doueria far pala de alcuna sua cortina, ouer muraglia, talmente, che li nemici ui potessono percotere, ouer tirare

In this passage, Tartaglia correlated his discourse to the walls of the fortifications and weapons, particularly with recoils caused by enemies' shots: the walls must not only resist new artillery shots but when they are hit, the shots must be diverted. This is possible with the construction of oblique and not vertical perimeter walls. In this way, the shot reaches the target not "perpendicularly with artillery, because every wall cedes much more easily to the shots […]"[120].

The second quality that he adds is also a "particularity" of fortifications, concerning the geometric shape of the curtains and bastions. The following passage considers how to find the best way and geometric shape (on the map) to then construct the perimeter walls of the city and those of the curtain (that is, the pieces of wall interposed between the bastions) to better prevent assailants from advancing too far and possibly being able to "find any place to be able to put their artillery".[121] According to Tartaglia, Torino, in this sense, was lacking in this protection (Tartaglia 1554, *Book VI*, Q IV, 66r).

In order to demonstrate his ability to be thoroughly familiar with certain military aspects of the defense of Italian cities, he returns to the walls of Turin, emphasizing the lack both of this *second quality* and also of the *third quality*. In particular, it is precisely this *third quality*, strictly correlated to the second, which addresses the study of the geometric shape of walls and the minimum artillery needed for defense. Moreover, he allows his influential interlocutor, the Prior of Barletta, to denounce the

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pendicolarmente con le artegliarie, perche, ogni muraglia cede molto piu facilmente alle cusioni delle balle, che feriscono pendicolermente sopra à quella, di quello fa à quelle, che gli feriscono obliquamente, cioe in sguinzo, & quanto piu ueneranno, ouer feriranno obliquamente, cioe in sguinzo, tanto menor nocumento faranno in detta cortina, ouer muraglia. La causa è, che ogni communa percossa fatta perpendicolarmente sopra à una muraglia è molto piu risentita in tutte le parte di tal muraglia, di quello sara ogni altra molto maggiore, che percottera obliquamente, ouer in sguinzo sopra alla medesima. P. Credo questo, che uoi diceti, perche delle percusioni fatte cosi obliquamente, ouer in sguinzo, la muraglia non riceue tutta la botta, ma solamente parte di quella, la qual parte tanto sara menore, quanto che piu obliquamente, ouer in sguinzo tal balla ferira sopra à quella. N. Adunque la forma de Turino incorre in questo errore, perche cadauna delle sue quattro muraglie, ouer cortine, che la circonda, sono assettate di tal sorte (come si uede nel suo disegno) che li nemici ui potranno ageuolmente tirare perpendicolarmente in cadauna di quelle". (Tartaglia 1554, *Book VI,* Q III, 65v, line 17).

[120] Tartaglia 1554, *Book VI*, Q III, 65r.

[121] Tartaglia 1554, *Book VI*, Q IV, 66r, line 10.

precariousness of the situation of the defensive system of certain Italian cities (Tartaglia 1554, *Book VI*, Q V, 66rv).

A discourse on the possible ruins of walls as further defense is introduced in the *fourth quality* (Tartaglia 1554, *Book VI*, Q VI, 66rv).

In these passages Tartaglia maintains that if enemies succeed in penetrating the walls, for example, by breaking through, the same ruined walls could produce yet another obstacle to their advancement, thanks to the particular way of constructing them. (Otherwise, they could also favor the passage of the assailants). This involves a rather well–known technique at the time, clearly also linked to the type of material with which the walls were constructed.[122] Moreover, to the incredulity of the Prior, Tartaglia then hypothesizes three different ways of dealing with the problems. He also creates a "modelletto"[123] to better explain the advantage of constructing walls with particularities innate in the previous qualities (Tartaglia 1554, Q. VI, *Book VI*, 66rv).

The fifth quality[124] is dedicated to a study typical of military strategy: the distribution of sentinels along the perimeter walls.[125] In regard to the Prior of Barletta's statement regarding the lack of adequate guards in Turin, but also in other Italian cities, Tartaglia undertakes a detailed discourse, indicating numbers of men useful for the armed defense of the walls when they are attacked from below or directly on the curtains (Tartaglia 1554, *Book VI,* Q. VII, 67rv; see also 74rv).

[122] Galileo, as we will see in the following paragraph, considers this "quality" without referring to Tartaglia (Galileo 1890–1909, II, pp 107–109, pp 118–120). In this sense, we will also see that the Galilean work feels the effects of the content from Tartaglia's *Book VI* and *Gionta*; even given the different historical period and different aim (also didactic) of Galilei's text compared to that of Tartaglia's, in *Trattato di Fortificazione*, important theoretical advances can be noted (Pisano and Capecchi 2012).

[123] The subject of the small model in Renaissance architecture will be dealt with later in the analysis of *Delle Fortificationi* by Lorini who considers the matter (Pisano and Capecchi 2009, II, 797–808; see also Pisano and Capecchi 2014de). On mechanics and architecture an indispensable work is *Entre Mécanique et Architecture* by Patricia Radelet–de Grave and Edoardo Benvenuto (Radelet–de Grave and Benvenuto 1995).

[124] The original text, which is not necessary to comment upon, is presented in the *Appendix* to this chapter. (see also Vol. II).

[125] Tartaglia 1554, *Book VI*, Q VII, 67rv.

In the *sixth* and last *quality*[126] of *Book VI*, Tartaglia discussed at length the fortification of roads, also incorporating the problem of those who came back to the city after working in the fields. Here (see also *Appendix*) he also includes estimates of the calculation of expenses, which in a city should be able to guarantee an effective organization of fortified defense, thereby also introducing a first approach to military economy (Tartaglia 1554, *Book VI*, Q VIII, 67rv).

1.3.3 The *Gionta del Sesto Libro*

The *Gionta del sesto libro* (hereafter referred to as the *Gionta*) is a very technical appendix, essentially founded on Euclidean geometry. It contains drawings and maps of the geometric shape of the fortifications. The *Gionta* is also, as the word itself suggests, an addition to *Book VI* of the *Quesiti* on fortifications. It is made up of six problems in the style of a dialogue[127] of *Quesiti*. In particular, from its content, we can also understand why Tartaglia seems to detect the need to add this topic to *Book VI*. In fact, from the beginning of the previous passage, as he makes his new interlocutor (the philosopher Marc'Antonio Morosini) say, he wants to better explore the qualities which were discussed in *Book VI*. Most likely, the then recent publications and constructions of new bastions would have suggested the necessity of elaborating on some techniques –as he himself writes "[…] which many were scandalized by […]"[128].

Tartaglia, focusing at length on the matter with elegant reasoning, succeeds in convincing philosopher Morosini, his interlocutor in *Gionta*, of the importance of constructing perimeter walls whose geometric shape is not, for example, square and therefore having right angles (like those of Torino), but have the shape of a polygon with obtuse angles.

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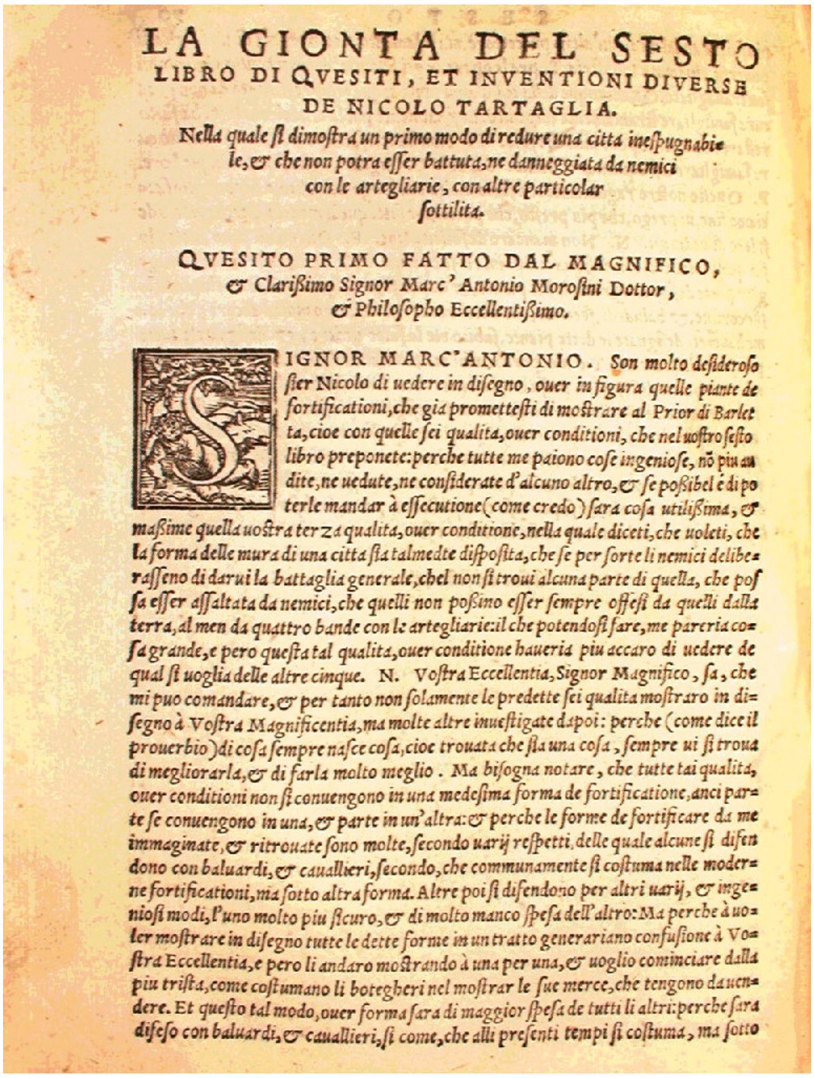
[126] The original text, which is not necessary to comment upon here, is presented in Chapter 4. It should be noted that the suggestive autobiographical information is found at the end of *Book VI*.

[127] The dialogue form (*Puer*'s questions and *Magister's* answers) was perfectly integrated in the typically Renaissance scientific context (Altieri Biagi 1984, 891–847) both as advanced research, and teaching science.

[128] Tartaglia 1554, *Gionta,* Q VI, 76rv, line 2.



LA GIONTA DEL SESTO LIBRO DI QVESITI, ET INVENTIONI DIVERSE DE NICOLO TARTAGLIA.

Nella quale si dimostra un primo modo di redure una citta inespugnabile, & che non potra esser battuta, ne danneggiata da nemici con le artegliarie, con altre particolar sottilita.

QVESITO PRIMO FATTO DAL MAGNIFICO, & Clarissimo Signor Marc' Antonio Morosini Dottor, & Philosopho Eccellentissimo.

SIGNOR MARC'ANTONIO. Son molto desideroso sier Nicolo di uedere in disegno, ouer in figura quelle piante de fortificationi, che gia promettesti di mostrare al Prior di Barletta, cioe con quelle sei qualita, ouer conditioni, che nel uostro sesto libro preponete: perche tutte me paiono cose ingeniose, nõ piu audite, ne uedute, ne considerate d'alcuno altro, & se possibel é di poterle mandar à essecutione (come credo) sara cosa utilissima, & massime quella uostra terza qualita, ouer conditione, nella quale diceti, che uoleti, che la forma delle mura di una citta sia talmedte disposita, che se per sorte li nemici deliberasseno di darui la battaglia generale, chel non si troui alcuna parte di quella, che possa esser assaltata da nemici, che quelli non possino esser sempre offesi da quelli dalla terra, al men da quattro bande con le artegliarie: il che potendosi fare, me pareria cosa grande, e pero questa tal qualita, ouer conditione haueria piu accaro di uedere de qual si uoglia delle altre cinque. N. Vostra Eccellentia, Signor Magnifico, sa, che mi puo comandare, & per tanto non solamente le predette sei qualita mostraro in disegno à Vostra Magnificentia, ma molte altre inuestigate dapoi: perche (come dice il prouerbio) di cosa sempre nasce cosa, cioe trouata che sia una cosa, sempre ui si troua di megliorarla, & di farla molto meglio. Ma bisogna notare, che tutte tai qualita, ouer conditioni non si conuengono in una medesima forma de fortificatione, anci parte se conuengono in una, & parte in un'altra: & perche le forme de fortificare da me immaginate, & ritrouate sono molte, secondo uarij respetti, delle quale alcune si difendono con baluardi, & cauallieri, secondo, che communamente si costuma nelle moderne fortificationi, ma sotto altra forma. Altre poi si difendono per altri uarij, & ingeniosi modi, l'uno molto piu sicuro, & di molto manco spesa dell'altro: Ma perche à uoler mostrare in disegno tutte le dette forme in un tratto generariano confusione à Vostra Eccellentia, e pero li andaro mostrando à una per una, & uoglio cominciare dalla piu trista, come costumano li boteghери nel mostrar le sue merce, che tengono da uendere. Et questo tal modo, ouer forma sara di maggior spesa de tutti li altri: perche sara difeso con baluardi, & cauallieri, si come, che alli presenti tempi si costuma, ma sotto

Fig. 1.32 Plate from *Gionta*[129]

In the *Gionta*, Tartaglia elaborates on the *third* and *fourth* "qualities" on geometry and the composition of perimeter walls. In addition, having assured the reader of the basic elements of ballistics, he can now focus more on the shape of walls (Euclidean geometry) and on the best way to build walls to obtain the deviation for recoil; almost wanting to construct a field of applicability for his previous dynamic theory. With this aim, he

[129] Tartaglia 1554, *Gionta,* Q I, 70rv, line 1.

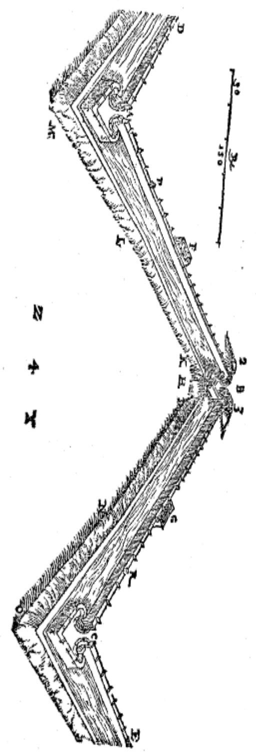
examines the third "quality", giving a concrete example that he presents in an entertaining analogy:

> N [Niccolò Tartaglia]. But since to show in the drawing all those forms into a sudden way should generate confusion at your Eccellency, so I will be showing them one by one, and I want to start from the more complicated, as the traders do, who want to sell their merchandise. […].[130]

At this point, a lengthy discourse ensues on the bastions and curtains in order "[...] to follow the modern use of strengthening [...]" (Tartagglia 1554, *Gionta*, Q I, 71rv, line 27) and on the importance of the "parianette"[131] to be built with a certain thickness ("grossezza"), that are often built to absorb the energy of the cannon balls ("balle"). These arguments are just prior to Tartaglia's presentation of walls with obtuse angles for the drawings of which the ratio of scale mentioned above is associated with the idea of building walls having oblique rather than medieval vertical parameters.

[130] Tartaglia 1554, *Gionta,* Q I, 70rv, line 34. The translation is ours.

[131] The *parianette*, also called *traverse*, are structural elements placed along the walls of the curtains. They are usually arranged vertically. The aim was to limit the effects of enfilade fire. As is clear from the text, Tartaglia shows personal innovation for the construction of the *traverse* by assuming an inclination of and a height greater than that of a man.



N [Niccolò Tartaglia]. Because I want too that, in the top of each curtain, many *parianette* are made, of joists planted and good planks, quite high over the height of a man, which *parianette* traverse the whole top of the curtain, but this crossing should not be orthogonal, but I want them to proceed with the outer part somewhat toward the city, and the inner part toward the country side as you see drawn in this figure. It is true that the *parianette* want to be somewhat more oblique than the figure for the same reasons that I say below. Being this made. I want from the side that looks towards the country of each of these *parianette*, a small earth embankment of such a size, which cannot be damaged by enemies with their artillery, under each such small embankment, I want there, a falconetto[132] with 6 or 3 lbs balls […].[133]

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[132] A small gun.

[133] Tartaglia 1554, *Gionta,* Q I, 71rv, line 10. The image in the quoted text is suitably enlarged and rotated. Fig.: *Ivi*, Q I, 71rv; this is the detail, enlarged and rotated, of the relationship scaled in figure A.

We note that in the previous quotation the figure is actually to scale. This is a historically important point for the analyses of the Gionta and Tartaglia's science.

1.3.4 The *Gionta del Sesto Libro* and Architecture

In Galilean *Trattato di fortificazione*, "Della scala" is in a short section on the relationship of scale (Galilei 1890–1909, II, 102; Galilei *Ms B*; Galileo *Ms m*; see Pisano and Capecchi 2012). These are arguments about units of measurement and their proportionality ratio, highlighting a nontrivial problem. Consider, e.g., the case of a designer who, far from his own country, was going to draw a certain design in lands in which could not adopt his own units of measurement. Below, we present some images of the Galilean paragraph on scale from the two Ambrosian manuscripts:

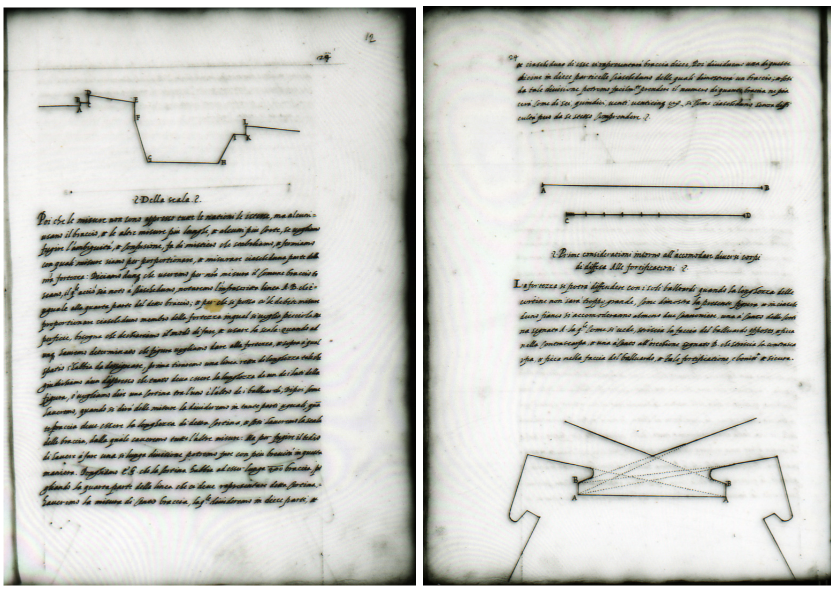


Fig. 1.33 Plate from Galilean wording on ratio[134]

The Galilean manuscript is assuming the possibility of inserting what normally today one does and that Tartaglia had already done in his *Quesiti* (Tartaglia 1554, *Book VI*, Q I, 71rv)[135]. The plan of a fortress was draw on paper, showing to the reader, near it, a unit of measurement: it is a graphical scale. The graphical scale is not autographically reported

[134] Galilei Ms B

[135] See also Tartaglia's figure in the text.

regularly in the *Galilean manuscripts* (Galilei *Ms B*, *Ms m*). For details on that, we refer to my recents work (i.e., Pisano and Capecchi 2014). However, even earlier than Galileo, and differently from comments from some scholars, Tartaglia was already specifying the scale relationship of the figures on fortifications in *Gionta del Sesto Libro.* From scaled figura (Tartaglia 1554, *Gionta*, Q I, 71rv) we can also see the presence of the obtuse angle (*Ivi*, 72rv, line 3).

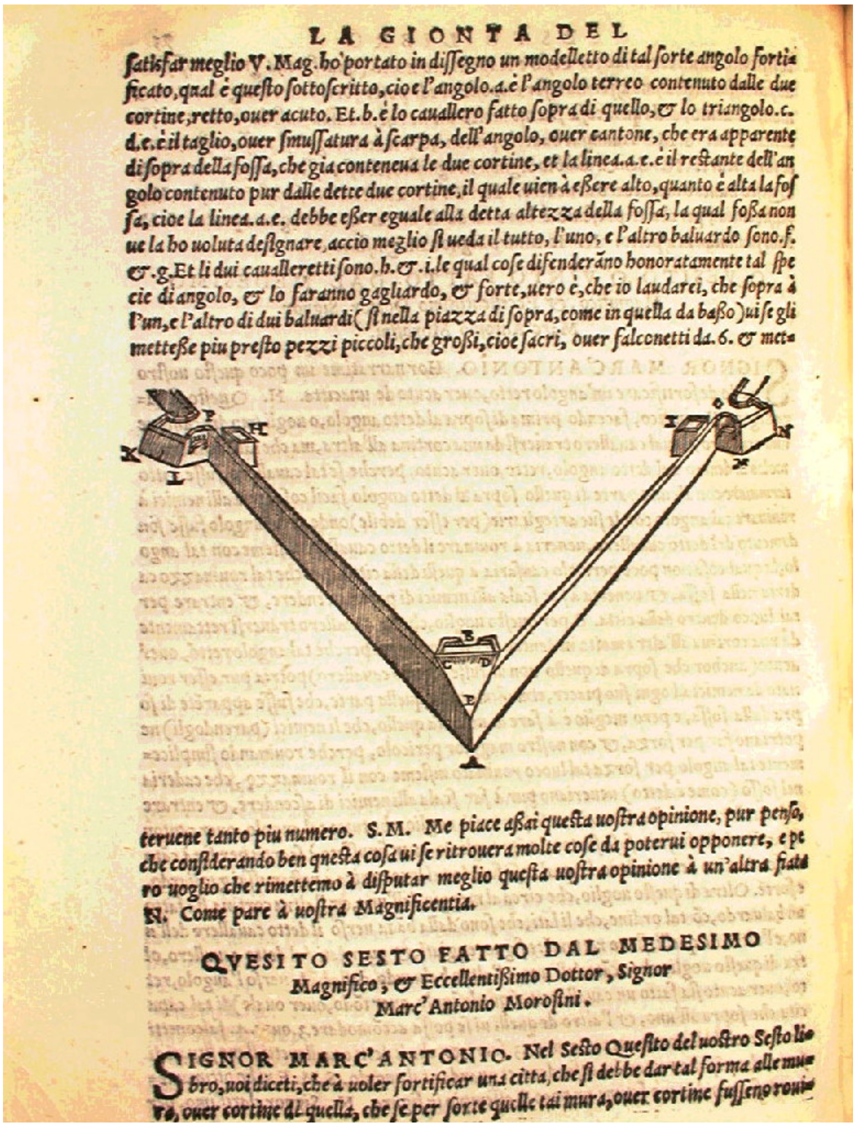
In addition to "fake doors" he also presents –as previously acknowledged in *Book VI* – protection and security of citizens returning from the country after work. The issue of the scale of measurement is also brought up in the following passage (Tartaglia 1554, *Gionta*, Q I) to which he adds considerations on the so–called "fake doors". This involves disguised entrances positioned along the external sides of the obtuse angle. Moreover, in reference to the scale of measurement problem, he informs the reader of the lack of "false doors" in the design since they are too small to include due to the chosen proportionality. (*Ibidem*, line 33).

During the dialogue, Tartaglia interlocutor defiantly argues that even regarding the elegance of his fortifications, Tartaglia's response emphasizes a cautious attitude. That is to say, perhaps, given the historical period in which he lived, he felt the necessity to take a position from a technical standpoint even in regard to the beauty of fortifications (Tartaglia 1554, *Gionta*, Q I, 72rv).

Further, ahead, in the second problem, Tartaglia provides details regarding what he refers to as the "first shape" of the walls, which, however, with "falconetti", "bastions", "curtains" with obtuse angles and "false doors", appears to his interlocutor as rather elaborate (*Ivi*, Q II, 73rv, line 38). However this also seems to be a way to emphasize the originality sought.

The *Quesito terzo* of *Gionta* concerns the *strade coperte* with attention also paid to citizens' paths when returning home from the country (*Ivi*, Q III, 73rv, line 1).

In the *Quesito quarto* and *Quesito quinto* Tartaglia focuses on the geometric motivation of the choice of the obtuse angle of the bastion and on the difficulty of fortifying with right and acute angles. The reason is of a strictly military nature. The protruding, angular shape (which will be perfected in the following years as an angular bastion), allows for protection without dead angles. To this aim artillery for *tiri di fianco*, *tiri di rovescio* e *tiri di infilata* is placed along the sides of what Tartaglia refers to as a "baluardo", thereby obtaining a defensive system of *fuoco incrociato* effective enough for short to medium distances from the curtain. In particular, the following passage which addresses these details is of a geometric nature; Tartaglia references Euclid's *Elements* several times.



LA GIONTA DEL

ſatisfar meglio V. Mag. ho portato in diſſegno un modelletto di tal ſorte angolo fortificato, qual è queſto ſottoſcritto, cioe l'angolo. a. è l'angolo terreo contenuto dalle due cortine, retto, ouer acuto. Et. b. è lo cauallero fatto ſopra di quello, & lo triangolo. c. d. e. è il taglio, ouer ſmuſſatura à ſcarpa, dell'angolo, ouer cantone, che era apparente di ſopra della foſſa, che gia conteneua le due cortine, et la linea. a. e. è il reſtante dell'angolo contenuto pur dalle dette due cortine, il quale uien à eſſere alto, quanto è alta la foſſa, cioe la linea. a. e. debbe eſſer eguale alla detta altezza della foſſa, la qual foſſa non ue la ho uoluta deſignare, accio meglio ſi ueda il tutto, l'uno, e l'altro baluardo ſono. f. & g. Et li dui caualleretti ſono. h. & i. le qual coſe difenderãno honoratamente tal ſpecie di angolo, & lo faranno gagliardo, & forte, uero è, che io laudarei, che ſopra à l'un, e l'altro di dui baluardi (ſi nella piazza di ſopra, come in quella da baſſo) ui ſe gli metteſſe piu preſto pezzi piccoli, che groſſi, cioe ſacri, ouer falconetti da. 6. & metteruene tanto piu numero. S. M. Me piace aſſai queſta uoſtra opinione, pur penſo, che conſiderando ben queſta coſa ui ſe ritrouera molte coſe da poterui opponere, e però uoglio che rimettemo à diſputar meglio queſta uoſtra opinione à un'altra fiata. N. Come pare à uoſtra Magnificentia.

QVESITO SESTO FATTO DAL MEDESIMO Magnifico, & Eccellentiſſimo Dottor, Signor Marc'Antonio Moroſini.

SIGNOR MARC'ANTONIO. Nel Seſto Queſito del uoſtro Seſto libro, uoi diceti, che à uoler fortificar una citta, che ſi debbe dar tal forma alle mura, ouer cortine di quella, che ſe per ſorte quelle tai mura, ouer cortine fuſſeno roui-

Fig. 1.34 Plate from *Gionta* on the geometrical reasoning[136]

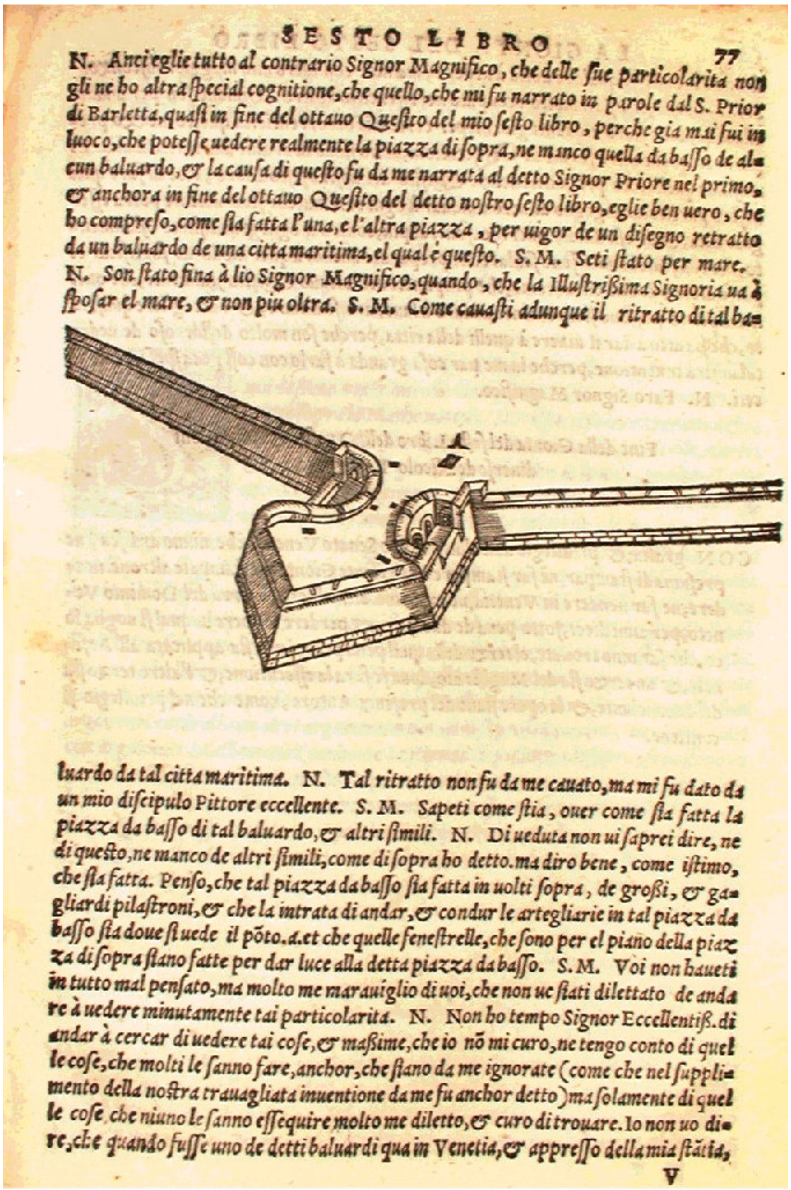
As a final consideration about *Quesito quarto*, we note that Tartaglia – as brefly above stated – cites Euclid ("commune scientia"), by referring to his *Libri*. In particular, for one of Euclid's axioms he writes, "for the converso

[136] Tartaglia 1554, *Gionta*, Q V, 75v, line 1.

modo of the fifth petition of our Euclid"; that is, he cites the axiom by associating the words "converso modo" and putting it in the form of a petition. Moreover, he also refers to a military architect, Cesare Napolitano Zotto, from whom he is supposed to have had the inspiration for his ideas on angular bastions (Tartaglia 1554, *Gionta*, Q IV, 75r).

The *Quesito sesto* is dedicated specifically to the methods of constructing walls. The considerations however always refer to information already given in *Book sesto* on the defensive system based on walls ruined by artillery shots. That is to say, the walls that fell due to such shots should not allow assailants to use the ruins as a passage to cross the walls and enter the city. Therefore, it is necessary to correctly choose the type of material and build the walls so that the ruins fall in such a way that they do not facilitate passage. In this regard, Tartaglia specifies that the foundations of the walls are never referred to, only the higher part which is more susceptible to fire from "cannonerie". Also citing the qualities shown in *Book sesto* (*Ivi*, Q VI, 76rv).

To this aim, in the following passage of *Quesito sesto*, Tartaglia suggests a structural remedy as well as one involving the type of material. Today we could say one based on the science of constructions, the other on that of materials. He suggests wall construction in an oblique manner and facing the internal part, that is, toward the city. In this way, the ruins that ruin the assailants' shots will fall into the city and therefore the attackers will not be able to use them as a sort of ladder. Let us see his reasoning on the structural remedy (*Ibidem*, line 18).



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N. Anci eglie tutto al contrario Signor Magnifico, che delle ſue particolarita non gli ne ho altra ſpecial cognitione, che quello, che mi fu narrato in parole dal S. Prior di Barletta, quaſi in fine del ottauo Queſito del mio ſeſto libro, perche gia mai fui in luoco, che poteſſe uedere realmente la piazza di ſopra, ne manco quella da baſſo de alcun baluardo, & la cauſa di queſto fu da me narrata al detto Signor Priore nel primo, & anchora in fine del ottauo Queſito del detto noſtro ſeſto libro, eglie ben uero, che ho compreſo, come ſia fatta l'una, e l'altra piazza, per uigor de un diſegno retratto da un baluardo de una citta maritima, el qual è queſto. S. M. Seti ſtato per mare. N. Son ſtato fina à lio Signor Magnifico, quando, che la Illuſtrißima Signoria ua à ſpoſar el mare, & non piu oltra. S. M. Come cauaſti adunque il ritratto di tal baluardo da tal citta maritima. N. Tal ritratto non fu da me cauato, ma mi fu dato da un mio diſcipulo Pittore eccellente. S. M. Sapeti come ſtia, ouer come ſia fatta la piazza da baſſo di tal baluardo, & altri ſimili. N. Di ueduta non ui ſaprei dire, ne di queſto, ne manco de altri ſimili, come di ſopra ho detto. ma diro bene, come iſtimo, che ſia fatta. Penſo, che tal piazza da baſſo ſia fatta in uolti ſopra, de großi, & gagliardi pilaſtroni, & che la intrata di andar, & condur le arteglíarie in tal piazza da baſſo ſia doue ſi uede il pōto. a. et che quelle feneſtrelle, che ſono per el piano della piazza di ſopra ſiano fatte per dar luce alla detta piazza da baſſo. S. M. Voi non haueti in tutto mal penſato, ma molto me marauiglio di uoi, che non ue ſtati dilettato de andare à uedere minutamente tai particolarita. N. Non ho tempo Signor Eccellentiß. di andar à cercar di uedere tai coſe, & maßime, che io nō mi curo, ne tengo conto di quelle coſe, che molti le ſanno fare, anchor, che ſiano da me ignorate (come che nel ſupplimento della noſtra trauagliata inuentione da me fu anchor detto) ma ſolamente di quelle coſe, che niuno le ſanno eſſequire molto me diletto, & curo di trouare. Io non uo dire, che quando fuſſe uno de detti baluardi qua in Venetia, & appreſſo della mia ſtātia,

V

Fig. 1.35 Plate from *Gionta* on the material used[137]

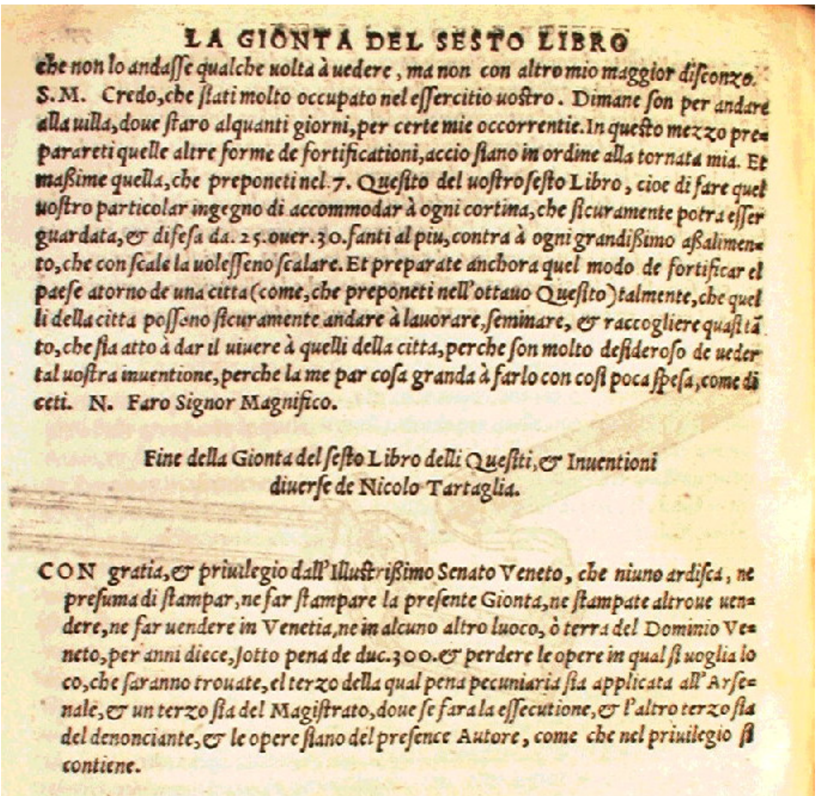
[137] Tartaglia 1554, *Gionta*, Q V, 75v, line 1.

Concerning his consideration on the choice of materials, Tartaglia suggests stone and mortar for the weak (high) part of the curtain and earth for the rest of the walls.[138]

In the *Quesito settimo*, the last in *Gionta*, to conclude his discourse, Tartaglia presents concrete examples, citing some defensive systems of the "maritime" city of Venice, to which he associates the figure of a bastion done by one of his students whose name is not given. In his words "image not mine, but given to me by one of my excellent Painter disciples"[139]. A sort of summary ends *Book sesto* and *Gionta*:

[138] This is an important fact for this work. It involves the ability to absorb kinetic energy from the "ball" in reference to the type of material used; in this case the dirt should absorb the shot better than the stone. (Tartaglia 1554, *Gionta*, Q VI, 76rv). Tartaglia also provides an entertaining geometric analogy with the moon (*Ivi*, Q VI, 76rv, line 19).

[139] In accordance with Masotti's unverified hypothesis (Tartaglia 1554, Qs L–LI) it could be about Rusconi (di) Zanantonio, architect and painter, student of Tartaglia who, in *Quesiti*, Tartaglia explicitly names when introducing a problem on artillery and on ballistics in Vitruvio's work (Tartaglia 1554, *Book II*, Q X, 34–35; here the entire "quesito" is dedicated to him: "done by [...]"); and from a solution to a geometric problem (*Ivi*, *Book IX,* Q XXXVIII, 123; see also: *Ivi*, Q VII, 76rv, line 1).



LA GIONTA DEL SESTO LIBRO

che non lo andasse qualche uolta à uedere, ma non con altro mio maggior disconzo.

S.M. Credo, che stati molto occupato nel essercitio uostro. Dimane son per andare alla uilla, doue staro alquanti giorni, per certe mie occorrentie. In questo mezzo prepareti quelle altre forme de fortificationi, accio siano in ordine alla tornata mia. Et masime quella, che preponeti nel. 7. Quesito del uostro sesto Libro, cioe di fare quel uostro particolar ingegno di accommodar à ogni cortina, che sicuramente potra esser guardata, & difesa da. 25. ouer. 30. fanti al piu, contra à ogni grandissimo assalimento, che con scale la uolessono scalare. Et preparate anchora quel modo de fortificar el paese atorno de una citta (come, che preponeti nell'ottauo Quesito) talmente, che quel li della citta possono sicuramente andare à lauorare, seminare, & raccogliere quasi tanto, che sia atto à dar il uiuere à quelli della citta, perche son molto desideroso de ueder tal uostra inuentione, perche la me par cosa granda à farlo con cosi poca spesa, come di ceti. N. Faro Signor Magnifico.

Fine della Gionta del sesto Libro delli Quesiti, & Inuentioni diuerse de Nicolo Tartaglia.

CON gratia, & priuilegio dall'Illustrissimo Senato Veneto, che niuno ardisca, ne presuma di stampar, ne far stampare la presente Gionta, ne stampate altroue uendere, ne far uendere in Venetia, ne in alcuno altro luoco, ò terra del Dominio Veneto, per anni diece, sotto pena de duc. 300. & perdere le opere in qual si uoglia loco, che saranno trouate, el terzo della qual pena pecuniaria sia applicata all'Arsenale, & un terzo sia del Magistrato, doue se fara la essecutione, & l'altro terzo sia del denonciante, & le opere siano del presence Autore, come che nel priuilegio si contiene.

Fig. 1.36 Plate from the end of the *Gionta*[140]

[140] "[… S.M.]. Dimane son per andare alla uilla, doue staro alquanti giorni, per certe mie occorrentie. In questo mezzo preparareti quelle altre forme de fortificationi, accio siano in ordine alla tornata mia. Et masime quella, che preponeti nel. 7. Quesito del uostro sesto Book, cioe di fare quel uostro particolar ingegno di accommodar à ogni cortina, che sicur amente potra esser guardata, & difesa da. 25. ouer. 30. fanti al piu, contra à ogni grandisimo asalimento, che con scale la uolesseno scalare. Et preparate anchora quel modo de fortificar el paese atorno de una citta (come, che preponeti nell'ottauo Quesito) talmente, che quel li della citta posseno sicuramente andare à lauorare, seminare, & raccogliere quasi tamto, che sia atto à dar il uiuere à quelli della citta, perche son molto desideroso de ueder tal uostra inuentione, perche la me par cosa granda à farlo con cosi poca spesa, come di ceti. N. Faro Signor Magnifico. Fine della Gionta del sesto Libro delli Quesiti, & Inuentioni diuerse de Nicolo Tartaglia" (Tartaglia 1554, *Gionta*, Q VII, 76rv, line 2).

1.4 On the *Opera Archimedis* and *Archimedis de insidentibus aquae*

1.4.1 On the *Opera Archimedis* (1543)

It is known that during the Middle Ages/early Renaissance Archimedean ideas were known within *Abacus schools* (Pisano and Bussottti 2013, 2014; Grendler 1995; Clagett 1971). In fact, they involved practical studies of geometric problems and the measurement of surfaces: e.g., let us think of practical measurements and calculations of pieces of breads–surfaces. We also know that only three authentic translations (by Moerbeke) were produced and we can presume that they were not widely read. Particularly, one of these copies concerned the priest Andreas Coner (fl XVI century). In Pietro Barozzi's (1441–1507) library, bishop in Padova, Coner read and copied many of Moerbeke's diagrams (*Codex O*) thereby creating his own personal but partial version (*Codex M*). It contained mechanical Archimedean works (fl. second half of the 15th century):

> *The quadrature of the parabola*
> The two books *on the equilibrium of planes* and with Eutocius of Ascalon's (fl. 480–fl.540) comments
> The first book of *on the floating bodies*
> *Measurement of a circle*

Later, Luca Gaurico (1475–1558) used this *Codex M* to publish a treatise on the quadrature of the circle entitled *Tetragonismus idest circuli quadratura per Campanum Archimedem Syracusanum atque Boetium mathematicae perspicacissimos adinuenta* (Archimedes 1503; Gaurico's preface in *Epistola,* 2rv and Camapano's *Conclusia* as low as 3r):

Fig. 1.37 Plate from the first page of the *Tetragonismus* (1503)[141]

[141] Archimedes 1503. In the *Opera archimedis syracusani* (Tartaglia 1543b) see also by Tartaglia: *Archimedis siracusani tetragonismus* (Tartaglia 1543c, 19v–29r), *Archimedis syracusani liber* (Tartaglia 1543d, 29v–31r) and *Archimedis de insidentibus aquae* (Tartaglia 1543e, *Book I*, 31v–[36r]).

Gaurico's text is the first known printed version of Archimedean works. It seems quite certain that in 1543 Tartaglia knew this codex/Moerbeke's version (Codex M), or had a copy of it.

Tartaglia's *Opera Archimedis Syracusani philosophi et mathematici ingeniosissimi per Nicolaum tartaleam Brixianum (mathematicarum scientiarum cultorem) multis erroribus emendata, expurgata, ac in luce posita, multisque necessariis additis, quae plurimis locis intellectu difficillima erant, commentariolis sane luculentis et eruditissimi aperta, explicata atque illustrata existunt. Appositisque manu propria figuris quae graeco exemplari deformatae ac depravatae erant, ad rectissimam Symetriam omnia instaurata reducta et reformata elucent* concerns the earliest version from Greek of some of the main works of Archimedes and was published by Tartaglia in Venice (Tartaglia 1543). It includes the following Archimedean books:

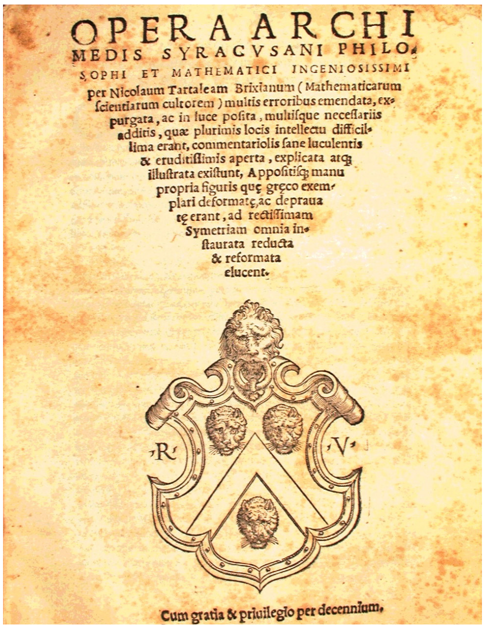


Fig. 1.38 Plate from Opera Archimedis on the contents (1543)[143]

The quadrature of the parabola.

The two books *on the equilibrium of planes* and without Eutocius of Ascalon's (fl. 480–fl.540) commentary.

The first book *on the floating bodies*

Measurement of a circle[142].

The *Opera Archimedis* by Tartaglia reflects of a way of working on ancient scholars, which was typical of Tartaglia's time. Today there are still differing opinions among historians[144]: for example, on the language

[142] For Archmedes works see Heath 2001.

[143] *Viganò Library Collection Archive.*

[144] We do not have space to comment significantly on the history of

and allusions to the "deformatae" figures, typical of the Greek language. Therefore, since Tartaglia was not a man of classical culture, his *Opera Archimedis* was in Latin and from the title page, it seems that he was really the translator. Moreover, in 1560, Tartaglia himself discussed important Archimedean documents during his stay in Verona[145].

He was interested in sharing his significant knowledge of the matter. Therefore, from the quote we can deduce that he found (and then possibly produced the Latin version) the text *On the Sphere and Cylinder* by Archimedes from Siracusa (fl. III B.C.). This attitude was tipical of Tartaglia in other works, as well. For example, in *Quesiti et inventioni diverse*, beginning in the the initial passages, he explained to the reader his skilled background on the Greek and Latin Mechanical problems (Tartaglia 1554, Q I, *Book VII*, 78r; Q XLII, Book IX, 126v). We would also like to point out that he (with respect to *Codex M*) did not include Eutocius' comments. Clagett's studies showed many important results, e.g., the glaring of errors of the *Codex M* reported in Tartaglia's edition without comments and corrections. In this way he formulated the hypothesis that Tartaglia had utilized *Codex M* (or a copy of it) for his Archimedes and maybe also Gaurico edition (Heath 2001, XXVIII; Clagett 1964–1984, 556–571). On the other hand, we should give scientific justification to Tartaglia for deleting the second book *On the floating bodies* from his editorial job. In fact, it is known that Moerbeck's version was full of nonsense and difficult passages. Thus Tartaglia, being a very good mathematician, avoided publishing it[146].

Archimedean works during Italian Renaissance. The secondary literature is extensive so for the sake of brevity we refer the reader to it. Mainly, see both Heath (2001, XXVII–XXX) and Clagett (1964–1984).

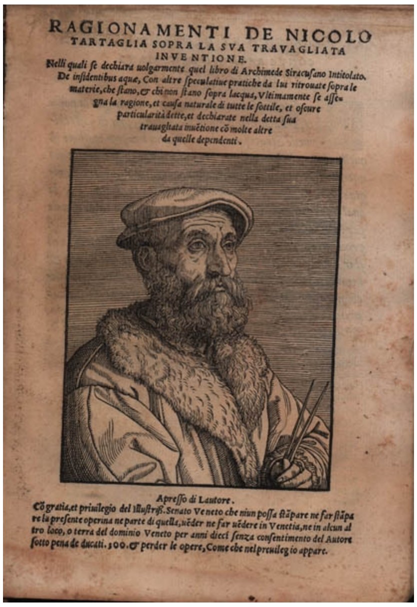
[145] "Il primo libro di Archimede Siracusano, da me trovato & tradotto da uno latinamente scritto, qual era andato quassi in strazzaria & in mano di un salzizaro in Verona l'anno 1531. Del qual libro molte parti erano totalmente rotte & annullate, onde accioche una così degna sua opra non restasse del tutto morta, mi sono sforzato di redrizzarla & d'interpretar le parti che mancavano, talmente che ogni commune impegno potrà gustar dimostrativamente la sua gran dottrina in tal materia". (Tartaglia 1560, *Parte IV*, *Book III*, 43v–44r).

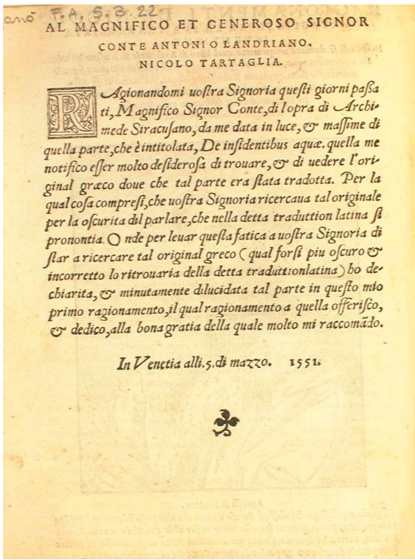
[146] We note that Tartaglia did not mention the existence of the second book. Later (Tartaglia 1565) his editor, Curtio Troiano, published both the Archimedean books *on the floating bodies* as credited manuscripts from Tartaglia for his editorial job. (Heath 2001, XXVII–XVIII). Some historians have conjectured that Tartaglia had all Archmedean works and did not publish some of them freely. Nevertheless, this only means that Curtio Troiano produced an editorial job after Tartaglia's death, and this it is not sufficient to claim (historically) that Tartaglia truly had the whole Archimedean *corpus*.

1.4.2 On the *Archimedis de insidentibus aquae* (1543; 1565)

In another occasion, within *Ragionamento Primo* of the *Ragionamenti sopra la sua Travagliata inventione* (Tartaglia 1551a) Tartaglia stated:

> Where in vulgare language is claimed that insidentibus aquae by Archimedes was an important subject & of an intellectual interest.[147]





AL MAGNIFICO ET GENEROSO SIGNOR
CONTE ANTONIO LANDRIANO.
NICOLO TARTAGLIA.

RAgionandomi uostra Signoria questi giorni passati, Magnifico Signor Conte, di l'opra di Archimede Siracusano, da me data in luce, & massime di quella parte, che è intitolata, De insidentibus aquæ. quella me notifico esser molto desiderosa di trouare, & di uedere l'original græco doue che tal parte era stata tradotta. Per la qual cosa compresi, che uostra Signoria ricercaua tal originale per la oscurita dil parlare, che nella detta traduttion latina si prononntia. Onde per leuar questa fatica a uostra Signoria di star a ricercare tal original greco (qual forsi piu oscuro & incorretto lo ritrouaria della detta traduttion latina) ho dechiarita, & minutamente dilucidata tal parte in questo mio primo ragionamento, il qual ragionamento a quella offerisco, & dedico, alla bona gratia della quale molto mi raccomãdo.

In Venetia alli.5.di mazzo. 1551.

Fig. 1.39 Plates from *Ragionamenti sopra la sua Travagliata inventione* (1551a)[148]

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[147] "Si dichiara volgarmente quell libro di Archimede Siracusano, ditto, de insidentibus aquae, materia di non poca speculation, & intellettual dilettatione" (Tartgalia 1551a, [part of the subtitle of] *Ragionamento Primo*). Translation is mine.

[148] Viganò Library Collection Archive. For the image on the left: "RAGIONAMENTI DE NICOLO Tartaglia sopra la sua Travagliata inventione. Nelli quali se dechiara uolgarmente quel libro di Archimede Siracusano intitolato. De insidentibus aquae, con altre speculatiue pratiche da lui ritrouate sopra le materie, che stano, & chi non stano sopra lacqua ultimamente se assegna la ragione et causa naturale di tutte le sottile et oscure particularità dette et dechiarate nella detta sua Trauagliata inue[n]tione co[n] molte altre da quelle dependenti". For the image on the right: "AL MAGNIFICO ET GENEROSO SIGNOR CONTE ANTONIO LANDRIANO. NICOLO TARTAGLIA Ragionandomi vostra Signoria questi giorni pasati, Magnifico Signor Conte, di sopra di Archimede Siracusano, da me data in luce, & massime di quella parte, che è

Nevertheless, as cited in the previous paragraph, the *Opera Archimedis* only included *Book I* of the *Archimedis de insidentibus aquae* (Tartaglia 1543b, 31v–[36r]). Therefore, firstly a Latin translation of *On the floating bodies* (*Book I*) along with three other Archimedean works was published (Tartaglia 1543b). Secondly, *Book I* was also published within *Travagliata Inventione* (Tartaglia 1551a). Thirdly, *Book II* – together with *Book I* and in the same essay – was published *postumo* after Tartaglia's death by Curtio Troiano as *Archimedis de insidentibus aquae* (Tartaglia 1565–*insidentibus*) in which his Latin replaced the lost Greek text. According to Rose, his translation was essentially a transcription of Moerbeke's translation (Rose 1975, 152–154; see also Biagioli 1989).

On this point, Heath took up the following philological study:

> It is next necessary to consider the probabilities as to the MSS. used by Nicolas Tartaglia for his Latin translation of certain of the works of Archimedes. [...] But it is established, by a letter written by Tartaglia himself eight years later (1551) that he then had no Greek text of the Books *de insidentibus aquae*, and it would be strange if it had disappeared in so short a time without leaving any trace. Further, Commandinus in the preface to his edition of the same treatise (Bologna, 1565) shows that he had never hoard of a Greek text of it. Hence it is most natural to suppose that it reached Tartaglia from some other source and in the Latin translation only*. The fact that Tartaglia speaks of the old MS. which he used as "fracti et qui vix legi poterant libri," at practically the same time as the writer of the preface to C was giving a similar description of Valla's MS., makes it probable that the two were identical; and this probability is confirmed by a considerable agreement between the mistakes in Tartaglia and in Valla's versions.[149]

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intitolata, De insidentibus aquae. quella me notifico esser molto desiderosa di trovare, & di vedere l'original Greco dove che tal parte era stata tradotta. Per la qual cosa compresi, che vostra Signoria ricercava tal originale per la oscurita del parlare, che nella detta traduttion latina si prononția. Onde per levar questa fatica a vostra Signoria di star a ricercare tal orognal Greco (qual forsi più oscuro & incoretto lo ritrovai della detta traduttion[ ]latina) ho dechiarata, & minutamente dilucidata tal parte in questo mio primo ragionamento, il qual ragionamento a quello ofeerisco, & dedico, alla bona gratia della quale molto mi raccoma[n]do. In Venetia alli.5.di mazz[ggi]o. 1551." (Tartaglia 1551a).

[149] Heath 2001, p XXVII, line 10. (Author's italics and quotations marks). The codices mentioned by Heath are: B=Codex Parisinus 2360, olim Mediceus; C=Codex Parisinus 2361, Fonteblandensis. Others codexes are mentioned, so we refer to Heath for a full reading. (Author's symbol and quotations).

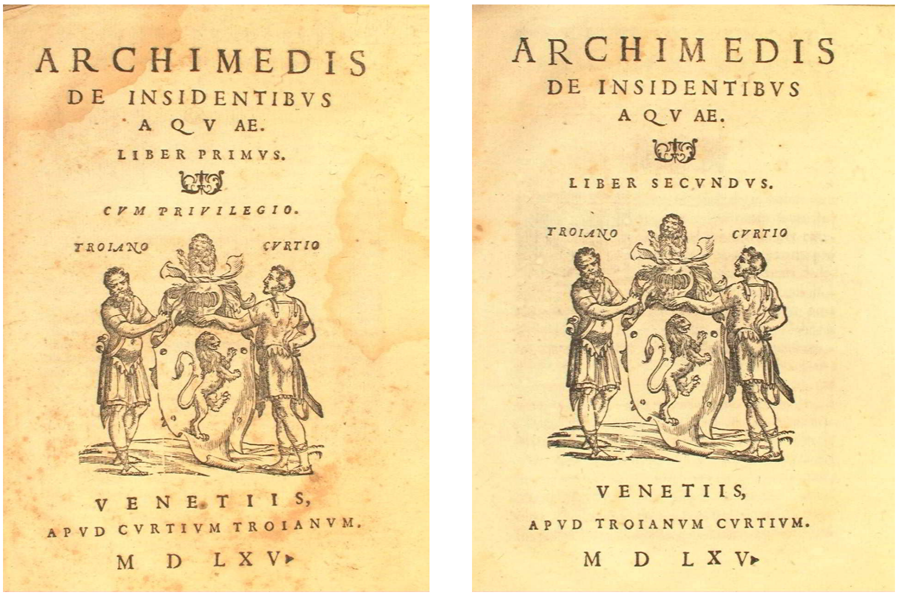
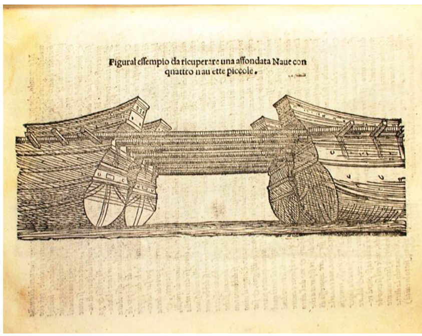


Fig. 1.40 Plates from *Archimedis de insidentibus aquae*, *Books I–II* (1565)[150]

Both Books *Archimedis de insidentibus aquae* contain propositions concerning how water/boats work in relation to the displacement and density of the objects in the water. Particularly, *Book II* seems to be considered a mature work. It presents a study on the stable equilibrium positions of floating right paraboloids of various shapes and densities. The study is restricted to a case–study concerning the base of the geometrical paraboloid figure when it is positioned either entirely above or entirely below the fluid surface, or completely–partially submerged. On this point, Tartaglia adopted an interesting mathematical Archimedean method to bring up a floating boat concerning a recent sunken ship where the sea was somewhat shallow. It was reported in the section *Regola Generale da sulevare con ragione e misura non solamente ogni affondata nave, ma una torre solida di metallo* (Tartaglia 1551b):

[150] Tartgalia 1565–*Insidentibus*.



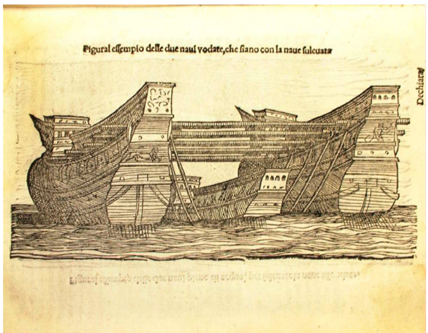


Fig. 1.41 Plates from *Archimedis de insidentibus aquae* on a method for floating boats[151]

1.5 Contents, Former Pupils and Philological Notes

As previously said, *Quesiti et Inventioni diverse* was published in Venice in 1546 and then again in Venice in 1554. A posthumous edition was published, again in Venice in 1562 (Tartaglia 1546, 1554, 1562; see also Chasles 1881, 195; see below chapters 5 and 6). It was re–edited in other languages even though they were partial translations.

The *Quesiti* is a collection of nine books written in Italian (*vulgare*), each of which discusses a specific topic: from the application of mechanics to military arts to (nowadays) topography, from studies of fortifications to those on the equilibrium of bodies. The text was dedicated to Henry VII, King of England (1457–1509). Tartaglia was 45 years old, according to the title page of the *Quesiti*.

[151] Tartaglia, *Regola Generale* with *Ragionamenti I-III* and *Supplimento* 1551b, 7r (left), 6v (right). See See also Tartaglia, *Regola Generale* with *Supplimento* and *Ragionamenti I–II* 1562, 4v (right), 5v (left)).

Figs. 1.42 and 143 Plates from *Quesiti*, 1546 (left) and 1554 (right)[152]

In the proceedings of *the International Congress of historical sciences* of 1904, one of Tartaglia's letters (without a date, signature, or place) was published and discussed, in which the author referred to an imminent publication. This was presumably the 1546 edition of the *Quesiti* since the document was found in the Tartalea pamphlet of 1546 (Tonni–Bazza 1904b, 295–296). As previously expressed, *Quesiti* also contain autobiographical information on Tartaglia's childhood (Tartaglia 1554, *Book VI*, Q 8). Every *Book* has one or more interlocutors with whom, in the form of a dialogue, Tartaglia speaks. At times, these are anonymous characters, "capo dei bombardieri" (Tartaglia 1554, *I*, Qs 20–21) "un fiorentino" (*Ivi*, *Book IX*, Q 5) an "architettore" (*Ibid*, Q 12) but frequently, the name of the character is given: Francesco Maria della Rovere (1490–1538), Duke of Urbino and expert on fortifications, Gabriele Tadino (ca. 1480–1543) Knight of Rodi, Prior of Barletta and artillery expert, Don Diego Hutardo de Mendoza (1503–1575), ambassador to Carlo V in Venice; among the mathematicians Gerolamo Cardano (1501–1576) stands out. Some are also Tartaglia's students: the architect Giovanni Antonio Rusconi (1520–1587), the mathematician Maphio Poveiani and the English gentleman Richard Wentworth.

[152] Tartaglia 1546, 1r; 1554, 1r and see also 4r.

Below, Italian bibliographical notes (see also below Chpaters 5 and 6) are presented:

Table 1.3. *Quesiti et invention diverse*[153] in Cd–Rom, Brescia

Cd 1, Vol I	Cd 2, Vol 2	Cd 3, Vol III
Nova scientia, *Quesiti e invention diverse*, *Cartelli di sfida matematica*, *Travagliata inventione*, *Opera Archimedis*, *Archimedis de insidentibus aquae,* *Jordanus Nemorarius*, *Tutte l'opere d'arithmetica*	Euclide Megarense	*General Trattato* 3 Volumes *Opere del famosissimo Nicolo' Tartaglia*, (Venetia 1606)
Details	Details	Details
Nova Scientia Venezia 1537 Venezia 1550 Venezia 1558 Venezia 1583 *Quesiti et inventioni* Venezia 1546 Venezia 1554 Venezia 1562 Cartelli di sfida matematica (1547–1548), [Enrico Giordani (ed), Milano 1876, op. cit] *Travagliata inventione* *Regola generale*, Venezia 1551 *Ragionamenti I–III e Supplimento*, Venezia 1551 *Regola generale con Supplimento e Ragionamenti I–II*, Venezia 1562 *Opera Archimedis*, Venezia 1543 *Archimedis de insidentibus aquae*, Venezia 1565	*Euclide Megarense* Venezia 1543 Venezia 1565–66 Venezia 1569 Venezia 1585 Brescia, 2007	*General Trattato* TOMO I: *La prima parte*, Venezia 1556 TOMO II: *La seconda parte,* Venezia 1556 TOMO III: *La terza* [*–sesta*] *parte*, Venezia 1650 *Opere del famosissimo Nicolò Tartaglia*, Venezia 1606

[153] See also: *L'Archivio Tartaglia* by Arnaldo Masotti, *Biblioteca Centrale del Politecnico di Milano*. *Documentazione*, Tartaglia's biography; *Riproduzione delle opere*, some of original Tartaglia's pages; *Trascrizioni di opere*, some e–reproductions; *Piano dell'opera*, by Pizzamiglio; *Tutte le opere*, reproduction by Pizzamiglio (4 Cd-Rom).

Jordanus opuscolum Nemorarius, Venezia 1565
Tutte l'opere d'arithmetica, Venezia 1592–93

1.5.1 A Content of *Quesiti et inventioni diverse*

In the following, by means of tables 1.4 and 1.5, we present a list of arguments *a mò* of Content: *La nuova edizione dell'opera* "*Quesiti et inventioni diverse de Nicolo Tartaglia brisciano*, *Riproduzione in facsimile dell'edizione del 1554*, by Masotti, Commentari dell'Ateneo di Brescia, Tippgrafia La Nuova cartografica, Brescia (Tartaglia 1554).

Table 1.4. An Index of the *Quesiti* and most notable interlocutors cited

Book	Number of *Questions*	Argument	Main Notable Interlocutors
I	30	*On artillery shots*	Francesco Maria della Rovere (*Ivi*, Qs 1–3) Gabriele Tadino (*Ivi*, Qs 4–17)
II	12	*On ball dimension artillery*	Gabriele Tadino (*Ivi*, Qs 1–7)
III	10	*On gunpowder*	Gabriele Tadino (*Ivi*, Qs 1–8)
IV	13	*On firearms and tactics of infantry*	Gabriele Tadino (*Ivi*, Qs 5–13)
V	7	*On recording of topographical data*	Richard Wentworth[154] (*Ivi*, Qs 1–7)
VI	8	*On requisites of fortifications*	Gabriele Tadino (*Ivi*, Qs 1–8)
Gionta	7	*On fortifications*	Marc'Antonio Morosini *Ivi*, Qs 1–7
VII	7	*On equilibrium of balances*	Don Diego Hutardo de Mendoza (*Ivi*, Qs 1–7)
VIII	42	*On theory of centres of gravity*	Don Diego Hutardo de Mendoza (*Ivi*, Qs 1–42)
IX	42	*On arithmetic, geometry and algebra*	Gerolamo Cardano (*Ivi*, Qs 31–36; Qs 38–40)

[154] Also cited in the *Book IX*.

	(cubic equation)

- *Book I–II–III.* These involve a series of studies on the ballistics of projectiles already seen in *Nova scientia* (Tartaglia 1537). In these writings, in addition to the interesting theoretical considerations on the speed of projectiles and their range (Tartaglia 1554, *Book* I, Q 1), are the applications of battle machinery and "squadre" of artillerymen.
- *Book IV.* Here, Tartaglia studies the tactics of the "squadre" of infantry from a mathematical point of view, for example, proposing a "[…] square battle of people […]" (Ivi, *Book* IV, Q 1) rather than the construction "in wedge over triangular form" (Ivi, *Book* IV, Q 5).
- *Book V.* Surveying and the problems regarding it is the subject of this book. He is dedicated to finding a solution to such problems, even specifying the instruments (for example a compass) and methods of measurement.
- *Book VI.* Differently from the other *Libri*, here a character of Tartaglia emerges "that appears to us as a *technician*. The *Quesiti* show scholars of various branches of the technology of the time: ballistic technology, practical geometry, military architecture" (Tartaglia 1554, XXXIV). Moreover, he also worries about producing new systems of fortification like the "parianette" (a sort of planks) placed on the curtain for defense against recoil.
- *Book VII. On equilibrium of balances* (see Chapter 3 below).
- *Book VIII. On theory of centres of gravity* (see Chapter 3 below).
- *Book IX.* Tartaglia certainly attained fame for his mathematical procedures (and controversy) and in this *Book* important studies are collected such as the algebraic solution of cubic equations that "[…] at the end of the XV century Luca Pacioli judged 'impossible' with the means of the times –in the first half of the 16th century was achieved independently by Scipione del Ferro and Niccolò Tartaglia […]" (*Ivi*, XXIII).

1.5.2 Scholars, Former Pupils, Correspondence and Commentaries in *Quesiti* and around Tartaglia's science

In the following tables, we present former pupils, scholars, letters cited in the *Quesiti*; further we make quite complete summary of most important works (in context) where the *Quesiti* and Tartaglia are cited.

Table 1.5. The main scholars and Tartaglia's pupils cited[155] in the *Quesiti* (1554)

Scholars cited	Quotation
1 – Signor Iacomo de Achaia	(*Ivi*, *Book I*, 23rv, *Book II*, 35v)
2 – M. Alberghetto di Alberghetti	(*Ivi*, *Book I*, 25r–27v)
3 – Magnifico M. Bernardo Segreo	(*Ivi*, *Book II*, 33v)
4 – Signor Giulio Savorgnano	(*Ivi*, *Book II*, 34r)
5 – M. Zanantonio di Rusconi*	(*Ivi*, *Book II*, 34r–35v)
6 – Hieronimo from isle of Cipro	(*Ivi*, *Book III*, 41v–42v)
7– Conte Hieronimo from Piagnano	(*Ivi*, *Book IV,* 43r–46r)
8 – M. Richard Ventworth*	(*Ivi*, *Book V,* 54v–63v, *Book IX*, 126v)
9 – Maestro Francesco Feliciano	(*Ivi*, *Book IX,* 98r, 99v–100r)
10 – Fra Raphaelle from S. Zorzi in Verona	(*Ivi*, *Book IX,* 98r)
11 – Maestro Maphio from Mantova*	(*Ivi*, *Book IX,* 98v)
12 – Maestro Alovise Pirovano from Milano	(*Ivi*, *Book IX,* 99r)
13 – Maestro Alessandro from Venetia	(*Ivi*, *Book IX,* 100r)
14 – Maestro Antonio Veronese[156]	(*Ivi*, *Book IX,* 101r)
15 – Maestro Zuanne de Tonini da Coi	(*Ivi*, *Book IX,* 101r, 103v, 106r–107r, 110r, 111v)
16 – M. Bernardin Dona from Zano	(*Ivi*, *Book IX,* 101v)
17 – Frate Ambrosio from Ferrara	(*Ivi*, *Book IX,* 102r)
18 – Maestro Alessandro Venetiano	(*Ivi*, *Book IX,* 102r)
19 – Maestro Anton Maria Fior	(*Ivi*, *Book IX,* 102v)
20 – Magnifico Zuanbattista Memo	(*Ivi*, *Book IX,* 103r)
21 – Hieronimo Trivisano	(*Ivi*, *Book IX,* 105r, 109r, 112v)
22 – M. Zuantonio Libraro from Hieronimo Cardano[157]	(*Ivi*, *Book IX,* 113r)
23 – Maestro Maphio Poveiani from Bergamo*[158]	(*Ivi*, *Book IX*, 122r, 126r)

Legenda: *: qualified Tartaglia's former pupils, i.e, in terms of "Honorando", "nostro discepolo"

[155] We only consider the names which have been cited by Tartaglia. For sake of brevity we avoid reporting general quotations like "A head of gunneries", "etc.

[156] Probably from Verona. Tartaglia added: "Zenero de Maestro Francesco Feliciano" (Tartaglia [1554] 1959, 101r).

[157] Here Cardano is called "[…] un messere Hieronimo Cardano, Medico & delle mathematice lettor pubblico inMilano, adi. 2. Genaro.1539"." (*Ibidem*).

[158] It is not historically clear if it is Maphio from Bergamo or Maphio from Mantova. In effect the term "from Bergamo" is never cited in the title of *Quesito* XXXVII. "Maphio Poveiani, already our former pupil […] in Bergamo" is cited, only.

Table 1.6. The letters cited in the *Quesiti*[159]

Destinatary	Date	Source
1 – to Giovanni di Tonini from Venezia	3–3–1537	(*Ivi*, 113v–114v)
2 – to Hieronimo Cardano from Venezia	18–2–1539	(*Ivi*, 118r–121v)
3 – to Hieronimo Cardano from Venezia	23–4–1539	(*Ivi*, 124r–124v)
4 – to Hieronimo Cardano from Venezia	27–5–1539	(*Ivi*, 125r)
5 – to Maphio Poveiani from Venezia	19–7–1539	(*Ivi*, 125v)
6 – to Hieronimo Cardano da Venezia	7–8–1539	(*Ivi*, 126r–127r)
7 – to Maphio Poveiani da Venezia	24–4–1540	(*Ivi*, 129v–130r)

Table 1.7. The main circulations and commentaries around *Quesiti* and Tartaglia's science

Date	Author	Work	Country[160]
1533	Benedetti	*Resolutio omnium Euclidis problematum*[161]	Italy
1567	Nuñez	*Libro de Algebra en artithmetica y geometria*[162]	Flandres
1568	Pérez de Moya	*Obra intitulada fragmentos mathematicos*[163]	Iberia
1572	Bombelli	*L Algebra*[164]	Italy
1573	Pérez de Moya	*Tratado de Geometria*[165]	Iberia

[159] Tartaglia 1554; see also archive at the Biblioteca di Brescia "Carlo Viganò".

[160] The country/region of the city cited in the frontespice is reported, only. Of course, the circulation of Tartaglia's science quoted in the book would be in the author's country, as well, I suppose.

[161] Benedetti 1533, [*In Dedication* (pages without numbers)] 4v.

[162] Nuñez 1567, 324r, 332r, 333v, 334rv.

[163] Particularly the Tartaglia's *General Trattato* (Tartaglia 1556–1560, Part III, 1r, Part IV, 17v–22v, Part V, 22v–23v) was an evident source of inspiration for his *Obra intitulada fragmentos mathematicos* in several parts (Pérez de Moya 1568, 1, 61, 77–79). Tartaglia also made use of a previous reasoning belonging to Ringelberg's *Ad mathematicen* (Ringelberg 1531–1532, 485. Cfr.: Céu Silva 2013, 5–6).

[164] Bombelli 1572 [*A gli Lettori* (pages without numbers)] 3r, 51, 53, 57, p 58, p 65, p 66. See also all indirect quotations to Ferrari(–Tartaglia) controversy.

[165] Pérez de Moya 1573, 5, 28; see also (respectively) Tartaglia 1556–1560, Part III, 1r, Part V, 7v. The *Tratado de Geometria* also includes contents of the *Obra intitulada fragmentos mathematicos*; i.e., *Tratado de Mathematicas* (Pérez de Moya 1573, Libro II, 50, 53, 57, 58–64, 248; see also (respectively) Tartaglia 1556–1560, Part IV, 1r, Part III, 14v, 11v–12r, Part V, 13r–16r, 21r).

1574	Clavius	*Euclidis Elementum,* VI[166]	Italy
1585	Benedetti	*Diversarum speculationum Mathematicarum, & Physicarum liber*[167]	Italy
1613	Gosselin	*L'Arithmétique de Nicolas Tartaglia Brescian, Grand Mathematicien, et Prince des Praticiens*[168] (1578)	France
1634	Stevin	*L'Arithmétique,* II[169] (1585)	Flanders
1634	Stevin	*Nouvelle maniere de Fortification par éscluses*[170] (1594)	Flandres
1634	Stevin	*Livre de la Geométrie. De la section proportionelle*[171]	Flanders
1663	Cardano	*Opera Omnia*[172]	France

[166] Clauvius 1574, Scholion, Problem 8–Proposition 28, Book VI, 219v. On Clavius see also Knobloch 1990, 2002; Giard and Romano 2008, 51–98.

[167] Benedetti 1585, [*Ad Lectorem* (pages without numbers)] 2r, 92, 93–96, 101, 105, 111–112, 114–115, 148–150 (on Tartaglia–de Nemore), 161, 168, 258–259, 271–272, 274, 301, 315 (On Tartaglia–Cardano), 340, 360, 364–365 (On Tartaglia–Ponderis) 380.

[168] Gosselin [1578] 1613. The title of the book is an evident *homage*. Taraglia is very often cited in the whole book.

[169] Stevin 1634, *L'Arithmétique,* II, 30 [1585, II, 125], 62 [1585, II, 268], 70 [1585, II, 302].

[170] Stevin 1634, *Nouvelle maniere de Fortification par éscluses*, VI, 601–678. Stevin's work on fortification copy the period, 1593–1594, of Galileo's fortifications: *Breve instruzione all'architettura militare* (Galilei 1890–1909, II, 15–75) and *Trattato di fortificazione* (Galilei 1890–1909, II, 77–146). The titles were suggested by Favaro since they were without titles and not published–works. Recently we worked on copies of the Ambrosiana Galilean fortifications' manuscripts (Galilei, *Ms A, Ms B*; *Ms m*) in order to show that a) they are Galileo's didactic speeches, b) he never wrote and c) never published them. In *Opere Nazionali* Favaro differently presented them as Galileo's works (Galilei 1890–1909, *Iuvenilia*, I, 7, 9). Stevin cites Tartaglia's name and arguments in his fortifications; but more impressive is the profound similarity between his drawings and up cited Galilean fortifications speech. For example see: Stevin 1634, 609–610, 633–634, pp 650–658, 660, 662, 664–667, 671–674); on scale's (*Ivi*, 659). The latter become of great historical interest if it is also correlated with *idem* argument in Galileo's, Tartaglia's and Lorini's fortifications (Lorini 1596, 1609). On Tartaglia's, Galileo's and Lorini's fortifications and scale, see Pisano 2008, I, Chapters IV–V, 2013c, 2009c, Pisano and Capecchi 2010a, 2012, 2008, 2009.

[171] Stevin 1634, *Livre de la Geométrie. De la section proportionelle*, 401–417. Particularly see when Stevin cites Cardano on equations, as well (*Ivi*, 62, 70–71, 92). On Stevin, see specialist works by Patricia Radelet-de Grave in References section below.

1876	Ferrari	*Cartelli di matematica disfida*[173]	Italy

1.5.3 Philological Notes and an Historical Hypothesis

Favaro (Favaro 1881, pp 32–35) provided an editorial–philological reasoning about Tartaglia's *Part I* and *Part II* of the *General Trattato* (at the end of 1557), because the latter were cited in his testament and edited by Curzio Troiano (dei) Navò (1556). Tartaglia also possessed (as cited in his testament) various copies of Parts III and IV, which are supposed to have been published only in 1560. A question arises: *why in 1560?* According to Favaro (*Ibidem*) the matter was an editorial hindrance typical of the XVI century, in which *Parti* and *Colophon* were replaced, thereby exchanging original dates and parts with those of editions in progress. In this regard, a discussion on the dates of *Quesiti et Inventioni Diverse* was also brought about at the beginning of the last century:[174]

> Tartaglia, as we can see, when responding to Castrioti, is delighted that their single studies on fortifications lead to results that conform; and this, Tartaglia says, will be seen in *Book dei quesiti fatto da me nuovamente nel sesto Book*. *Quesiti et inuentioni diverse* had already been published for the first time i 1546, but in 1554 at reprint occurred […] with the appendix to the sixth book which Tartaglia alludes to […]. Here, other problems that the *Magnifico e Clarissimo sig. Marc'Antonio Morosini dottore e Philosopho Eccellentissimo* suggested to him appear. Castrioti does not appear; even though topics contained in the « discorsi » with him are

[172] Cardano 1663. The quotations and mention of indebetedness are obviously many, i.e., both in *Artis magnae sive de regulis algebraicis* (Cardano 1663, IV, IV; see also IV, V) and *De Vita propria, Liber* (Cardano 1663, I, I). In Cardano's lifetime, the former had two main editions, 1545 and 1570. The latter was posthumously published, 1643 (and in the *Opera Omnia*, 1663); later it was translated into Italian (1821) and into French (1936), as well.

[173] Tartaglia 1876. The quotations are obviously many.

[174] In 1904, in the proceedings of the *Congresso internazionale di scienze storiche* held in Roma (1903) and edited by *Sezione VIII di Storia delle Scienze Fisiche, Matematiche, Naturali e Mediche*, a paper (Tonni–Bazza 1904b, 293–307), reported a discussion on the last results concering Tartaglia's death, the controversy on some content published in 1546 and/or 1554 of *Quesiti et inventioni diverse* and others things around the *Brisciano*. We note that the paper begins with the typical title page of *Quesiti et inventioni diverse*, but without including the date et al., so it is unclear which edition it is.

> involved, and in his letter, Tartaglia promises a *risposta partichulare et generale*.[175] (*particular and general response*).[176]

Problems tied to permits and *nulla osta* from religious institutions should also be considered.

On my side, based on my previous researches since 2005, four observations for reader's convenience are proposed. An historical hypothesis ends this section.

First observation. A 1546–edition is web–published in ECHO–*Cultural Heritage Online* Archive by *Max Planck Institute for the History of Science* (MPIWG) of Berlin. Thanks to an extraordinary digital job provided by MPIWG – and with respect to my researches – I have hunbly recognize that this edition is not complete. In fact, it lacks the following Books: *II–III–IV–V–VI–VII*, and of course, it lacks the *Gionta* to *Book VI.* One, e.g., can only discover the existence of a *Book VI* on fortifications from the Content (Fig. 144a) and at the end of the manuscript, only (Trataglia 1546, 133r).

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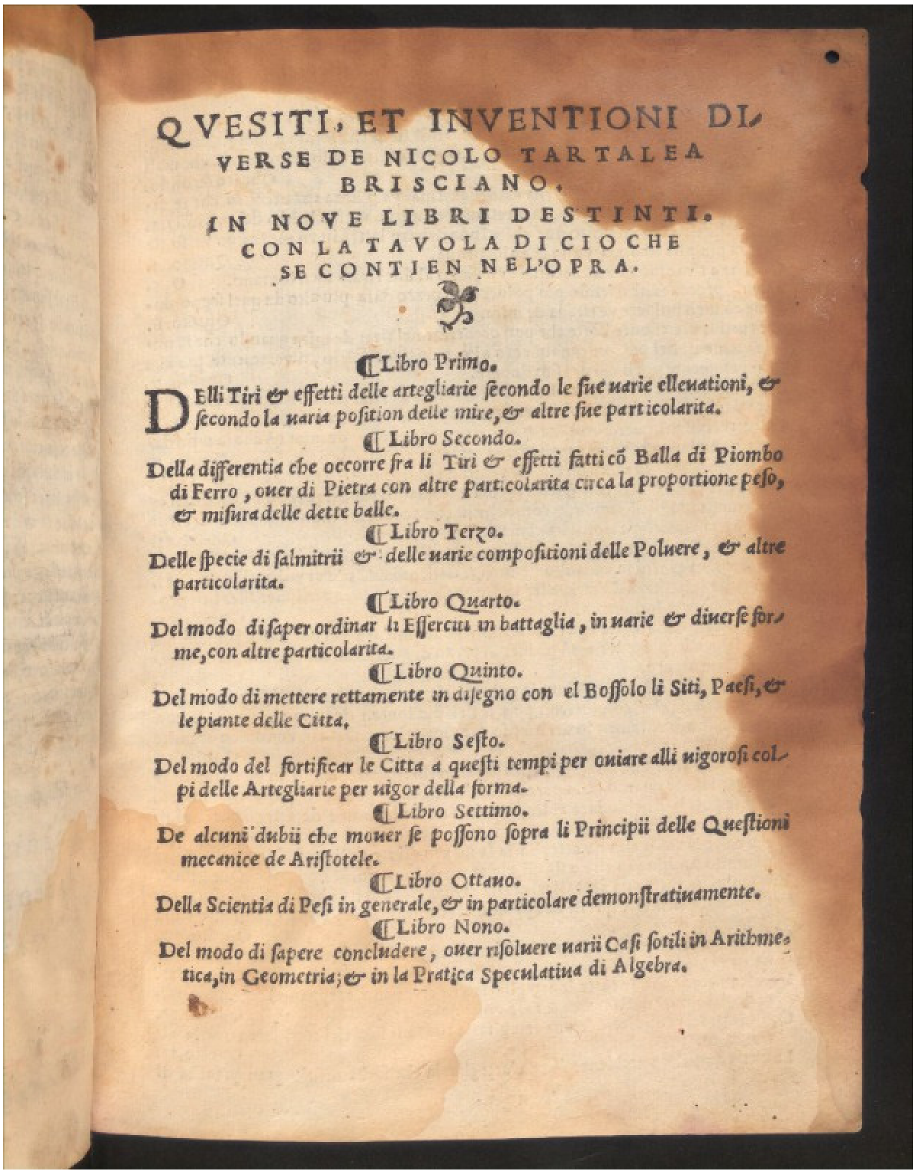
175 Tonni–Bazza 1904b, 303, line 13. (Author's italics and quotations marks).

176 "Il Tartaglia, come si vede, rispondendo al Castrioti, si rallegra che i loro singoli studi sulle fortificazioni conducano a risultati conformi; e ciò, dice il Tartaglia, si vedrà nel *Book dei quesiti fatto da me nuovamente nel sesto Book*. I *Quesiti et inuentioni diverse*, già erano stati pubblicati la prima volta nel 1546; ma nel 1554 sopravvenne la ristampa [...] con la appendice al sesto Book cui allude il Tartaglia [...]. *Ivi* figurano alcuni problemi propostigli dal *Magnifico e Clarissimo sig. Marc' Antonio Morosini dottore e Philosopho Eccellentissimo*. Non figura il Castrioti; sebbene vi si trattino argomenti contenuti nei « discorsi » di lui, e nella sua lettera, il Tartaglia, prometta una *risposta partichulare et generale*". (Tonni–Bazza 1904b, 303, line 13 (Author's italics and quotations marks)).



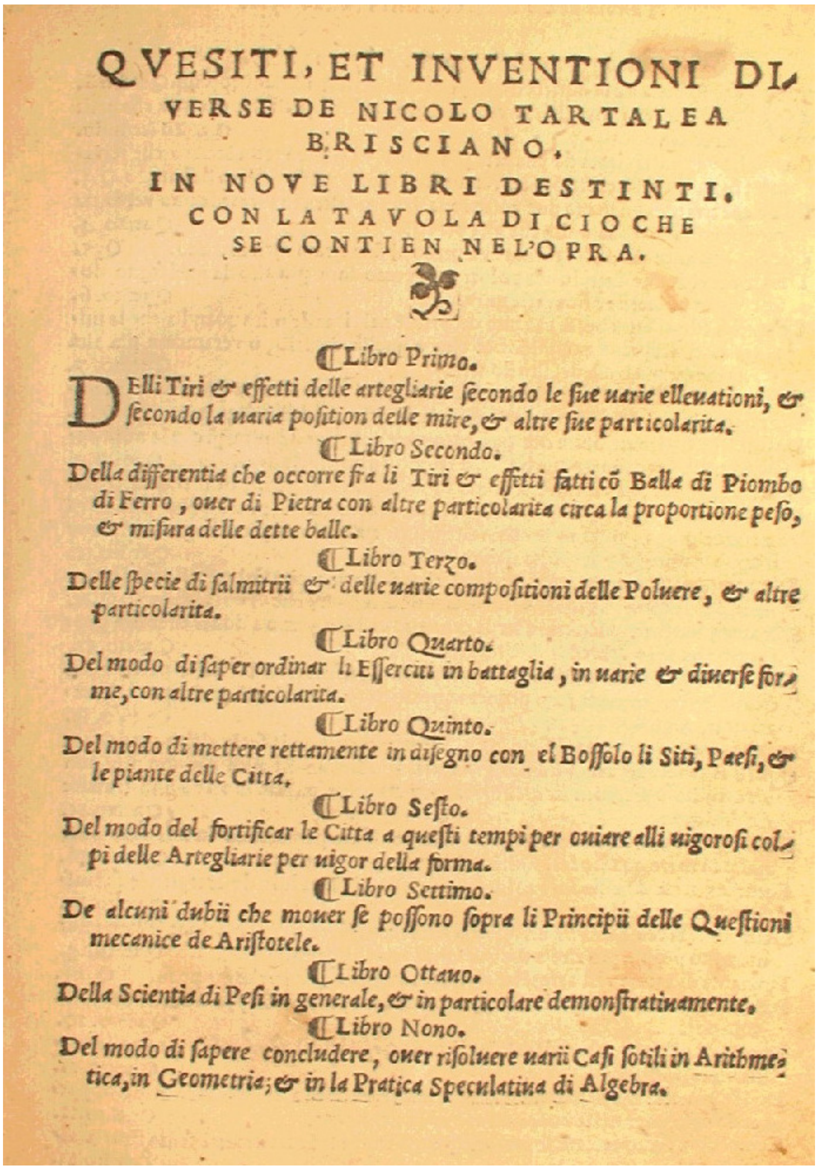
QVESITI, ET INVENTIONI DIVERSE DE NICOLO TARTALEA BRISCIANO.
IN NOVE LIBRI DESTINTI.
CON LA TAVOLA DI CIO CHE SE CONTIEN NEL'OPRA.

¶Libro Primo.
DElli Tiri & effetti delle artegliarie secondo le sue varie ellevationi, & secondo la varia position delle mire, & altre sue particolarita.
¶Libro Secondo.
Della differentia che occorre fra li Tiri & effetti fatti cõ Balla di Piombo di Ferro, over di Pietra con altre particolarita circa la proportione peso, & misura delle dette balle.
¶Libro Terzo.
Delle specie di salmitrii & delle varie compositioni delle Polvere, & altre particolarita.
¶Libro Quarto.
Del modo di saper ordinar li Esserciti in battaglia, in varie & diverse forme, con altre particolarita.
¶Libro Quinto.
Del modo di mettere rettamente in disegno con el Bossolo li Siti, Paesi, & le piante delle Citta.
¶Libro Sesto.
Del modo del fortificar le Citta a questi tempi per oviare alli vigorosi colpi delle Artegliarie per vigor della forma.
¶Libro Settimo.
De alcuni dubii che mover se possono sopra li Principii delle Questioni mecanice de Aristotele.
¶Libro Ottavo.
Della Scientia di Pesi in generale, & in particolare demonstrativamente.
¶Libro Nono.
Del modo di sapere concludere, over risolvere varii Casi sotili in Arithmetica, in Geometria; & in la Pratica Speculativa di Algebra.

Fig. 1.44a Plate from *Quesiti* (1546) from *ECHO–Cultural Heritage Online*[177]

[177] "Tartaglia, Niccolo, *Quesiti et inventioni diverse*: Libro 1, Quesiti 1-7; Libro 8; Libro 9, 1546." The Collection Browser of the *Archimedes Project*. Permanent URI: http://echo.mpiwg-berlin.mpg.de/MPIWG:YFRAG0Z1

The Biblioteca di Brescia "Carlo Viganò" has also 1546–edition. This time the Content (Fig. 1.44b) appears at beginning of the manuscript (*folio* 3r).



QVESITI, ET INVENTIONI DIVERSE DE NICOLO TARTALEA BRISCIANO.

IN NOVE LIBRI DESTINTI.

CON LA TAVOLA DI CIO CHE SE CONTIEN NEL'OPRA.

Libro Primo.

DElli Tiri & effetti delle artegliarie secondo le sue varie ellevationi, & secondo la varia position delle mire, & altre sue particolarita.

Libro Secondo.

Della differentia che occorre fra li Tiri & effetti fatti cō Balla di Piombo di Ferro, over di Pietra con altre particolarita circa la proportione peso, & misura delle dette balle.

Libro Terzo.

Delle specie di salnitrii & delle varie compositioni delle Polvere, & altre particolarita.

Libro Quarto.

Del modo di saper ordinar li Esserciti in battaglia, in varie & diverse forme, con altre particolarita.

Libro Quinto.

Del modo di mettere rettamente in disegno con el Bossolo li Siti, Paesi, & le piante delle Citta.

Libro Sesto.

Del modo del fortificar le Citta a questi tempi per oviare alli vigorosi colpi delle Artegliarie per vigor della forma.

Libro Settimo.

De alcuni dubii che mover se possono sopra li Principii delle Questioni mecanice de Aristotele.

Libro Ottavo.

Della Scientia di Pesi in generale, & in particolare demonstrativamente.

Libro Nono.

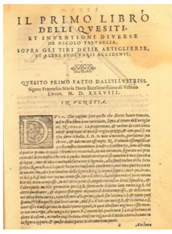
Del modo di sapere concludere, over risolvere varii Casi sotili in Arithmetica, in Geometria; & in la Pratica Speculativa di Algebra.

Fig. 1.44b Plate from *Quesiti* (1546) from Biblioteca "Carlo Viganò" [178]

[178] Cd's Tartaglia edition (Tartaglia 2000, Cd-I).

Second observation.

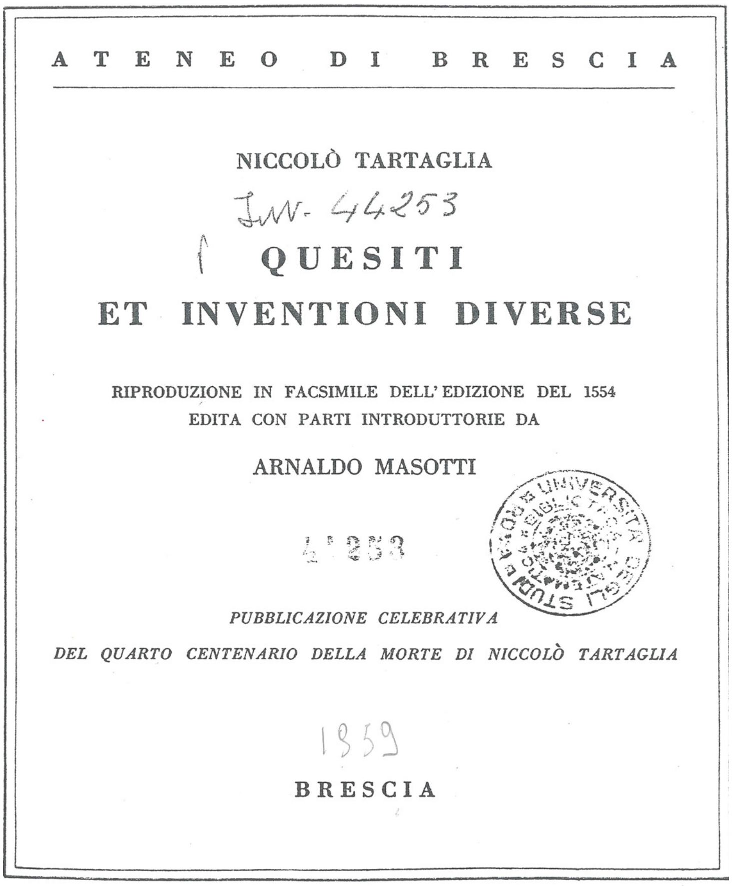
Table 1.8. Tartaglia's *Quesiti* covers (and/or part of) main known editions-reprints

In Venetia Ruffinelli Editor (1st)	In Venetia de Bascarini editor (2nd)	In Venetia Navò editor(?) (3rd)	In Venetia[179] Manassi editor (4th)
 1546	 1554	 ca. 1562 (not >1566)	
			
			 1606

Mariano d'Ayala (1808–1877) in *Dizionario Militare Francese Italiano* cites "1528" as the first date of *Quesiti*'s publication (d'Ayala 1841, 27; see also 1854, 155). Until now, within my researches I have found no historical proof of that.

[179] It appeared in a collection of literary works.

Third observation. A full 1554–edition of *Quesiti* (*Book VI* et *Gionta* included), is archived by Biblioteca "Carlo Viganò" (Tartaglia 2000, Cd-I); and, in 1959, a commentary edition was published by Arnaldo Masotti (Tartaglia [1554] 1959):


ATENEO DI BRESCIA

NICCOLÒ TARTAGLIA

QUESITI
ET INVENTIONI DIVERSE

RIPRODUZIONE IN FACSIMILE DELL'EDIZIONE DEL 1554
EDITA CON PARTI INTRODUTTORIE DA

ARNALDO MASOTTI

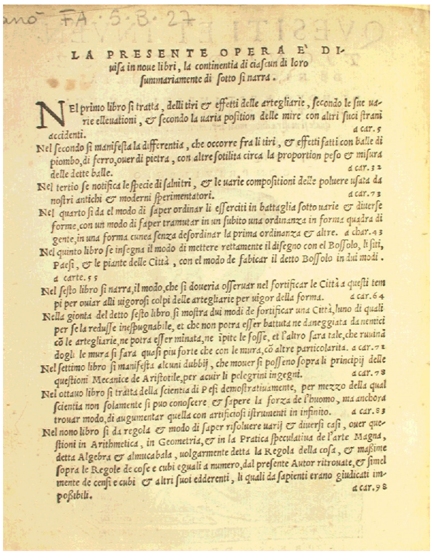
PUBBLICAZIONE CELEBRATIVA
DEL QUARTO CENTENARIO DELLA MORTE DI NICCOLÒ TARTAGLIA

BRESCIA

Fig. 1.45 Plate from the cover of *Quesiti et inventioni diverse de Nicolo Tartaglia brisciano* (1554) edited by Arnaldo Masotti (1959)

A similary full 1554–edition has been web–published in ECHO–*Cultural Heritage Online* Archive by MPIWG[180], as well. I note that the two mentioned editions are different for an overlay image only, *folio* 72v. In the following are the Contents of *Quesiti* 1554–edition:

[180] "Tartaglia, Niccolo, *Quesiti et inventioni diverse*, 1554"
Permanent URI:http://echo.mpiwg-berlin.mpg.de/MPIWG:KQ9TP5T3



LA PRESENTE OPERA E' DI-
uisa in noue libri, la continentia di ciascun di loro
summariamente di sotto si narra.

NEl primo libro si tratta, delli tiri & effetti delle artegliarie, secondo le sue uarie elleuationi, & secondo la uaria position delle mire con altri suoi strani accidenti. a car. 5

Nel secondo si manifesta la differentia, che occorre fra li tiri, & effetti fatti con balle di piombo, di ferro, ouer di pietra, con altre sotilita circa la proportion peso & misura delle dette balle. a car. 32

Nel tertio se notifica le specie di salnitri, & le uarie compositioni delle poluere usata da nostri antichi & moderni sperimentatori. a car. 73

Nel quarto si da el modo di saper ordinar li esserciti in battaglia sotto uarie & diuerse forme, con un modo di saper tramutar in un subito una ordinanza in forma quadra di gente, in una forma cunea senza desordinar la prima ordinanza & altre. a char. 43

Nel quinto libro se insegna il modo di mettere rettamente il disegno con el Bossolo, li siti, Paesi, & le piante delle Città, con el modo de fabicar il detto Bossolo in dui modi. a carte. 55

Nel sesto libro si narra, il modo, che si doueria osseruar nel fortificar le Città a questi tempi per ouiar alli uigorosi colpi delle artegliarie per uigor della forma. a car. 64

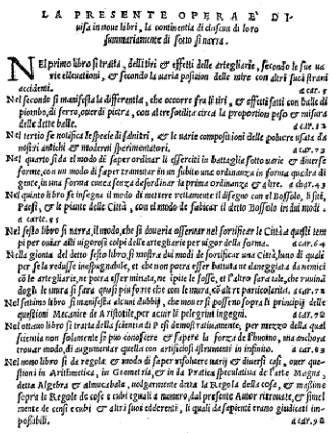
Nella gionta del detto sesto libro si mostra dui modi de fortificar una Città, luno di quali per se la redusse inespugnabile, et che non potra esser battuta ne daneggiata da nemici cō le artegliarie, ne potra esser minata, ne ĩpite le fosse, et l'altro sara tale, che ruinandogli le mura si sara quasi piu forte che con le mura, cō altre particolarita. a car. 71

Nel settimo libro si manifesta alcuni dubbij, che mouer si posseno sopra li principij delle questioni Mecanice de Aristotile, per acuir li pelegrini ingegni. a car. 78

Nel ottauo libro si tratta della scientia di Pesi demostratiuamente, per mezzo della qual scientia non solamente si puo conoscere & sapere la forza de l'huomo, ma anchora trouar modo, di augumentar quella con artificiosi istrumenti in infinito. a car. 83

Nel nono libro si da regola & modo di saper risoluere uarij & diuersi casi, ouer questioni in Arithmetica, in Geometria, & in la Pratica speculatiua de l'arte Magna, detta Algebra & almucabala, uolgarmente detta la Regola della cosa, & maßime sopra le Regole de cose e cubi eguali a numero, dal presente Autor ritrouate, & simelmente de censi e cubi & altri suoi edderenti, li quali da sapienti erano giudicati impoßibili. a car. 98

Fig. 1.46 Plate from the *Quesiti* (1554), *Viganò Library*[181]



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summariamente di sotto si narra.

NEl primo libro si tratta, delli tiri & effetti delle artegliarie, secondo le sue uarie elleuationi, & secondo la uaria position delle mire con altri suoi strani accidenti. a car. 5

Nel secondo si manifesta la differentia, che occorre fra li tiri, & effetti fatti con balle di piombo, di ferro, ouer di pietra, con altre sotilita circa la proportion peso & misura delle dette balle. a car. 32

Nel tertio se notifica le specie di salnitri, & le uarie compositioni delle poluere usata da nostri antichi & moderni sperimentatori. a car. 73

Nel quarto si da el modo di saper ordinar li esserciti in battaglia sotto uarie & diuerse forme, con un modo di saper tramutar in un subito una ordinanza in forma quadra di gente, in una forma cunea senza desordinar la prima ordinanza & altre. a char. 43

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Nel nono libro si da regola & modo di saper risoluere uarij & diuersi casi, ouer questioni in Arithmetica, in Geometria, & in la Pratica speculatiua de l'arte Magna, detta Algebra & almucabala, uolgarmente detta la Regola della cosa, & maßime sopra le Regole de cose e cubi eguali a numero, dal presente Autor ritrouate, & simelmente de censi e cubi & altri suoi edderenti, li quali da sapienti erano giudicati impoßibili. a car. 98

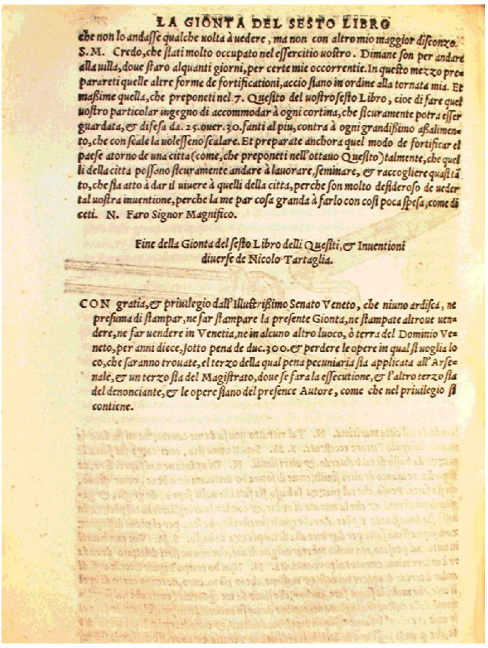
Fig. 1.47 Plate from the *Quesiti* (1554), *Max Planck web edition*

Fourth observation. In *Gionta* to *Book VI,* Tartaglia's interlocutor (Marcantonio Morosini) cited "[…] altre forme de fortificazioni" [that is other kinds of fortifications], which Tartaglia should address "[…] accio siano in ordine alla tornata mia [of Marcantonio Morosini]":

[181] The *Gionta* book is evident in both of the Contents (Figs. 1.46 and 1.47).

[… S.M.]. Dimane son per andare alla uilla, doue staro alquanti giorni, per certe mie occorrentie. In questo mezzo preparareti quelle altre forme de fortificationi, accio siano in ordine alla tornata mia. Et masime quella, che preponeti nel. 7. Quesito del uostro sesto Book, cioe di fare quel uostro particolar ingegno di accommodar à ogni cortina, che sicur amente potra esser guardata, & difesa da. 25. ouer. 30. fanti al piu, contra à ogni grandißimo aßalimento, che con scale la uolesseno scalare.[182]

[…]. CON gratia, & priuilegio dall'Illustrisimo Senato Veneto, che niuno ardisca, ne presuma di stampar, ne far stampare la presente Gionta, ne stampate altroue uendere, ne far uendere in Venetia, ne in alcuno altro luoco, ò terra del Dominio Veneto, per anni diece, sotto pena de duc. 300 & perdere le opere in qual si uoglia lo co, che saranno trouate, el terzo della qual pena pecuniaria sia applicata all'Arsenale, & un terzo sia del Magistrato, doue se fara la essecutione, & l'altro terzo sia del denonciante, & le opere siano del presence Autore, come che nel priuilegio si contiene.[183]


LA GIONTA DEL SESTO LIBRO

che non lo andaſſe qualche uolta à uedere, ma non con altro mio maggior diſconzo. S.M. Credo, che ſiati molto occupato nel eſſercitio uoſtro. Dimane ſon per andare alla uilla, doue ſtaro alquanti giorni, per certe mie occorrentie. In queſto mezzo preparareti quelle altre forme de fortificationi, accio ſiano in ordine alla tornata mia. Et maßime quella, che preponeti nel. 7. Queſito del uoſtro ſeſto Libro, cioe di fare quel uoſtro particolar ingegno di accommodar à ogni cortina, che ſicuramente potra eſſer guardata, & difeſa da. 25. ouer. 30. fanti al piu, contra à ogni grandißimo aßalimento, che con ſcale la uoleſſeno ſcalare. Et preparate anchora quel modo de fortificar el paeſe atorno de una citta (come, che preponeti nell'ottauo Queſito) talmente, che quelli della citta poſſeno ſicuramente andare à lauorare, ſeminare, & raccogliere quaſi tanto, che ſia atto à dar il uiuere à quelli della citta, perche ſon molto deſideroſo de ueder tal uoſtra inuentione, perche la me par coſa granda à farlo con coſi poca ſpeſa, come diceti. N. Faro Signor Magnifico.

Fine della Gionta del ſeſto Libro delli Queſiti, & Inuentioni diuerſe de Nicolo Tartaglia.

CON gratia, & priuilegio dall'Illuſtrißimo Senato Veneto, che niuno ardiſca, ne preſuma di ſtampar, ne far ſtampare la preſente Gionta, ne ſtampate altroue uendere, ne far uendere in Venetia, ne in alcuno altro luoco, ò terra del Dominio Veneto, per anni diece, ſotto pena de duc. 300. & perdere le opere in qual ſi uoglia loco, che ſaranno trouate, el terzo della qual pena pecuniaria ſia applicata all'Arſenale, & un terzo ſia del Magiſtrato, doue ſe fara la eſſecutione, & l'altro terzo ſia del denonciante, & le opere ſiano del preſence Autore, come che nel priuilegio ſi contiene.

Fig. 1.48 Plate from *Quesiti* (1554) end page of *Gionta*[184]

An Historical Hypothesis.

Based on the previous passage, one can hypothesize that the *Gionta* should – or could – have had a sequel. In fact,

a) Tartaglia concludes his *Gionta* by writing down notes regarding its sale for *300 ducati veneti* (Tartaglia 1554, *Gionta*, 77v, line 16).
b) The *Gionta* does not appear in (the previously cited) editions of 1546 of *Quesiti.*
c) In the 1546–edition (both in the above cited archive), one can read a quotation regarding *Book VI* on fortifications, only.

[182] Tartaglia 1554, *Gionta*, Q VII, 77v, line 2.
[183] *Ibidem*, line 16.
[184] Tartaglia 1554.

d) Thus, the editor included *Gionta* in 1554–edition, *at the last minute*, just after *Book VI* on fortifications.
e) He called it "La Gionta del Sesto Libro".
f) But, it was a separted–previous booklet, so for typographical and "requisitione" arrangements he was obliged to report the final notes (at the end of *Gionta*) concerning the cost of the book[185].
g) Finally, by considering Tartaglia's date of death, and by considering that *Gionta* is lacking in the 1546–edition, then the *Gionta* should be a booklet written before the *Quesiti* edition of 1554 and after the *Quesiti* edition of 1546.

From both pure historical and historical epistemology standpoints, this means that Tartaglia – during his lifetime research on arithmetics and geometry – surely wrote about fortifications, as well. Further, by considering his advancements in the science of weights (*Book VII* and *Book VIII*), his correlated–interdisciplinary studies on fortifications (Pisano 2008, 2013c; Pisano and Capecchi 2009, 2010a, 2012) also be of great interest within the history of mechanics, not simply as a separate part. In effect, previous reasoning is based on final notes in the end of the *Gionta*; being in the original text, they can be considered historical sources.

In conclusion, in order to have a general idea of the publication of the *Gionta* and its data, the following list concerns the main works available:

Nova Scientia	Venice 1537, 1550
Quesiti et invenzioni diverse	Venice 1546, 1554
Contro Cartelli di matematica disfida	Venice 1547–1548
Travagliata invention (Regola generale)	Venice 1551
Ragionamenti I – III e Supplimento	Venice 1551
Opera Archimedis	Venice 1543

[185] "[…] ne far vendere in Venetia, ne in alcuno altro luoco, ò terra del Dominio Veneto, per anni diece, sotto pena de duc. 300 & perdere le opere in qual si voglia […]" (Tartaglia 1546, 77v).

Chapter 2

Ancient and Modern Statics in the Renaissance

Forza, dico essere una virtù spirituale, una potenzia invisibile, la quale per accidentale esterna violenza è causata dal moto e collocata e infusa ne' corpi i quali sono dal loro naturale uso retratti e piegati, dando a quelli vita attiva di maravigliosa potenzia; costrigne tutte le create cose a mutazione di forma e di sito; corre con furia alla sua desiderata morte e vassi diversificando mediante le cagioni. (da Vinci, *Ms A*, 34v)

Statics is the science of equilibrium. The term appears in the Latin version (translated by Snel) of Simon Stevin (1548–1620) most famous textbook, *Tomus quartus mathematicorum hypomnematum de statica* (Stevin 1605, p 5). This work can be considered the hinge between ancient and modern statics. Ancient statics was the science of equilibrium of weights; modern statics is the science of equilibrium of forces. In ancient Greece statics was part of mechanics, the science of transportation of bodies by means of machines. In the Middle Ages and first Renaissance statics was known as *scientia de ponderibus* (science of weights); its main object was the study of principles of equilibrium for heavy bodies suspended from a balance. Presently, statics is part of mechanics, which is the general science studying equilibrium and motion of bodies and their assembly, of any kind.

Hereinafter I will use the term *scientia de ponderibus* to indicate ancient statics – more precisely the ancient statics of Middle Ages and Renaissance – and simply statics to indicate modern statics.

2 The Background

Scientia de ponderibus (science of weights) is the name given by the medieval schoolmen to the discipline that treats the equilibrium of heavy bodies with particular reference to those hanging from a balance. The *Scientia de ponderibus* was different from Greek mechanics, both for the scope – Greek mechanics placed transportation of weights, instead of their equilibrium, at the centre – and for the methodology – *Scientia de ponderibus* charged only to theoretical foundations of equilibrium and not applicative aspects. The *Scientia de ponderibus* was also different from the mechanics of the early XVI century, the centrobaric, a discipline developed in the wake of the rediscovery of Archimedes, which was concerned mainly with the mathematical problems of determining the geometric centres of gravity of plane figures and solids.

2.1 The *Scientia de ponderibus* in the Middle Ages

In the western Middle Ages, the science of weights was classified as *subalternate–science*, following the Aristotelian tradition which identified astronomy, optics, and music as the more physical of the mathematical sciences (Aristotle 1984, *Physics*, II, 2, 194a). They are *mixed sciences* (XVII century terminology), i.e. sciences with ranges both in physics and mathematics, and which are subordinate to mathematics. To these three sciences Aristotle had added a fourth, mechanics (Aristotle 1984, *Posterior analytics* I, 9, 76a; Aristotle 1984, *Metaphysics* M, 3, 1078a; Aristotle 1963, *Problemata mechanica*, 847a). Physics – the subalternate science – can demonstrate that things are so (demonstrations *quia*) while mathematics– the subalternate science – can demonstrate that things are so (demonstrations *quia*) while mathematics, geometry to be precise – the subalternating science – demonstrates why (*propter quid*) they are so. As a rule, the subject matters of the subalternating and subalternate science are not the same: if they were exactly the same, one would have a single science and not two separate sciences. Therefore, for example, the subject of geometry is geometrical lines, whereas the subject of optics is visual lines. Since a visual line is naturally associated to a geometrical line, optics falls under geometry. Geometry, then, can be used to study optics, but only the aspects that can modelled by it; a large portion of optics remains, which is the object of physics alone.

Apart from astronomy, the subalternate–sciences that attracted the greater attention by mathematicians were geometrical optics and mechanics. They were structured on the basis of the Euclidean model, based on definitions, suppositions (principles) and propositions (theorems). The main difference with respect to the Euclidean model was that some of the principles rather than being purely geometric related to the physical world. They were the translation into mathematical terms of what belonged to physics. In the Aristotelian circles, this translation appeared unproblematic; mathematicians, instead, did not exhibit the same level of tolerance as the Aristotelian philosophers, and doubted the evidence of the principles often assigning them the status of postulates.

Recent studies (Machamer 1978; Lennox 1985; Biener 2004–2008; Van Dyck 2006) have highlighted the role of the subalternate–sciences matured within Aristotelian scholarship, which provided a mathematical interpretation of the physical world quite similar to that proposed by Archimedes. In truth, these studies remain at a superficial level; for example, they do not explain why the subalternate–sciences once they have passed into the hands of professional mathematicians assume a structure different from what they had in the hands of philosophers. Nevertheless, mainly they do not study in depth what professional mathematicians, and not philosophers, actually did. One of the main concerns of philosophers was to preserve the homogeneity of demonstrations, particularly in mathematics and physics. But in the treatises of science classified as subordinate (including the Archimedean ones, which will see their diffusion in the XVI century), there was no trace of this purism, and statements about the physical aspects, such as the heaviness, were intermixed with statements about the geometry with no concern to maintain the homogeneity of the demonstrations.

2.1.1 The Roots in the Arabic Middle Ages

The scientia de ponderibus saw its birth in the Arabic land; its status of a distinct scientia first appeared in Abū Naṣr Muḥammad ibn Muḥammad Fārābī's (ca. 870–950) Kitab ihsa' *al-'ulum* (*The Book of Enumeration of the sciences*). In particular, he definitively distinguished between science of weights and sciences of devices (or machines). In his classification of knowledge Abū Naṣr Muḥammad ibn Muḥammad Fārābī' (hereafter Al-Farabi) took six distinct sciences: language, logic, mathematics, nature, metaphysics and politics. The mathematics were divided into seven topics: arithmetic, geometry, perspective, music, science of weights and sciences of machines or devices. These last are defined as follows:

> As for the *science of weights* [emphasis added], it deals with the matters of weights from two standpoints: either by examining weights as much as they are measured or are of use to measure, and this is the investigation of the matters of the doctrine of balances (umūr al-qawl fī l-mawāzīn), or by examining weights as much as they move or are of use to move, and this is the investigation of the principles of instruments (uṣūl al-ālāt) by which heavy things are lifted and carried from one place to another.
> As for the *science of devices* [emphasis added], it is the knowledge of the procedures by which one applies to natural bodies all that was proven to exist in the mathematical sciences... in statements and proofs into the natural bodies, and [the act at] locating [all that], and establishing it in actuality. The sciences of devices are therefore those that supply the knowledge of the methods and the procedures by which one can contrive to find this applicability and to demonstrate it in actuality in the natural bodies that are perceptible to the senses.[1]

Al-Farabi's setting was never seriously challenged, although there were different nuances in subsequent classifications. Some scholars divided the science of weights into science of balances and science of weight lifting; for example, Ibn Sina (980–1037). Al-Isfizārī (1048–1116) and al-Khāzinī (1115–1130) singled out the theory of centers of gravity from the science of weight (Abattouy 2008, 103).

The new science of weights was characterized by a strong deductive system, in which components of qualitative physics were formulated *more geometrico*. The most common historical point of view is that the science of weights originated from interplay of Aristotelian physics and the physical–geometrical approaches by Archimedes and probably Euclid, on the equilibrium of bodies. Now we did not find studies on the role played by Aristotelian conception about subalternate–science in the development of Arabic science, to contrast this point of view. Surely an important role should be assigned to Heron's writing which spread throughout the Islamic lands.

From a methodological point of view, the majority of treatises in the science of weights followed what is often called dynamical or more properly kinematical approach, in which the equilibrium is seen as a balance of opposing forces and the movement, virtual or real, has an important role. In these treatments the Aristotelian dichotomy, between the natural and forced, upward and downward, motions, disappears for they are considered on the balance, in which the weight is also the natural cause of lifting other weights. The pure geometrical approach, like the one

[1] Al-Fārābī cited in: Abbatouy 2008, 100.

carried out by Archimedes, is certainly uncommon, so that some historians do not even consider it as part of the science of weights.

The production of Arabic texts developed from the IX to the XII centuries. First, there was a phase of recovery and digestion of the works of Greek origin. Besides the translations of Aristotle's theoretical works, *Physics* and *On the Heaven*, available since the IX century, Islamic scholars surely had access to *Mechanics* by Pappus and Heron written in Greek. Also circulating were two treatises on the balance attributed to Euclid (*Euclid's book on the balance* and *De ponderoso et levi*). It seems instead that of Archimedes' mechanical works, only that on floating bodies was known, while regarding the Aristotelian *Problemata mechanica*, it can be stated with certainty that only a partial epitome was known (Abattouy, 2006a).

The analysis of the general significance of the Arabic medieval science of weights shows that this tradition did not represent a mere continuation of the traditional doctrine of mechanics as inherited from Greeks. Rather, it means the emergence of a new science of weights recognized very early in Arabic learning as a specific branch of mechanics, and embodied in a large scientific and technical corpus. Comprehensive attempts at collecting and systematizing (as well as updating with original contributions) the mainly fragmentary and unorganized Greco–Roman mechanical literature that had been translated into Arabic were highly successful in producing coherent and orderly mechanical systems.

The main Arabic texts on the science of weights are listed below in Table 2.1; for further information see (Abattouy 2008, 94–95).

Table 2.1. Arabic treatises on the science of weights

Kitāb fī il-qarastūm by Thābit ibn Qurra[2]	It is probably the first Arabic text about the steel yard. It exists into four manuscripts in Arabic: one conserved in London, one in Kraköw and another in Beirut. The first manuscript was edited, translated into French and commented on by Khalil Jaouiche (Jaou 1976). The second, while in Berlin, was edited and translated into German by Eilhard Wiedmann (Wiedmann 1911), and subsequently

[2] Al-Ṣābiʾ Thābit ibn Qurra al-Ḥarrānī (836–901) was a native of Harran and a member of the Sabian sect. He was a great scholar in mathematics and astronomy; translated and revised many of the important Greek works: particularly all the works of Archimedes that have not been preserved in the original language and Apolonius' *Conic sections* (Heath 1896; see also Panza 2008, 165–191). He was a founder of the science of weights.

	studied by Mohammed Abattouy (Abattouy 2001). The third one was studied by Knorr (Knorr 1982). A fourth partial copy was recently found in the archives of the Laurentiana Library in Florence (Abattouy 2008, 94).
The treatise on centres of gravity, by al-Qūhī and Ibn al-Haytham, two most important mathematicians of X-XI centuries.	It survived only on *al-Kāzinī's Kitāb mĪzān al-ḥikma* (*The Book of the balance of wisdom*).
Irshād dhawī al-ʿrfān ilā ṣināʿat al-qaffān (Guiding the learned men in the art of steelyard), by al-Isfizari.[3]	A fundamental treatise written about 1050–1110. Here different Arabic and Greek traditions are reported, together with a unified mechanical theory.
al-Kāzinī's Kitāb mĪzān al-ḥikma by *al-Kāzinī's Kitāb mĪzān al-ḥikma*.[4]	An encyclopedia of mechanics completed in 1121–1122, well kown as the *Book of the balance of wisdom*. A source of information about theoretical and practical knowledge of medieval mechanics. It is known in the West by Khanikoff's partial translation (Khanikoff 1857).

2.1.1.1 Thābit 's Kitāb fī il-qarastūm

Thābit's contribution is for sure the most relevant for Arabic mechanics. Moreover, it influenced mostly Latin medieval mechanics; for this reason, it deserves a short account. The *Kitāb fī il-qarastūm* was composed of a prologue followed by eight propositions and finally a comment. They all

[3] Abū Ḥātim al-Muẓaffar ibn Ismāʿīl al-Isfizārī. (ca. 1048–ca. 1116). A mathematician, astronomer and an engineer, he was born in Isfizar, a city near Herat. His study of Archimedes' book helped him in identifying the purity of gold and silver for which purpose he made a hydrostatic scale to determine the weight of alloys in the two metals His main scientific contribution was in the field of weights and mechanical designs.

[4] Ab ar-Raḥmān al-Khāzini (ca. 1115–1130) was a Muslim of Greek origin who was brought to Merv as a slave by the Seljuk king after his victory over the Byzantine Emperor. Al-Khazini was a great physicist, astronomer, mathematician, philosopher and an alchemist. He is better known for his contributions to physics. His treatise; *al-Kāzinī's Kitāb mĪzān al-ḥikma* written in four volumes, remained an important part of physics among the Muslim scientists.

relate to the karaston, that is the steelyard or Roman balance, which is a straight–beam balance with arms of unequal length. It incorporates a counterweight, which slides along the calibrated longer arm to counterbalance the load and indicates its weight. The most important postulate Thābit assumed is the following:

> PROPOSITION I. The ratio of two distances covered by two mobiles in two [equal] times is equal to the ratio of the force of the mobile [passing] the plane distance to the force of the other mobile.[5]

Based on the postulate, Thābit can prove the law of the lever, which is given as follows:

> PROPOSITION III. Since this is manifest now, then I propose [the following with respect to] every line which is divided into two different segments and imagined to be suspended by the dividing point and where there are suspended on the respective extremities of the two segments two weights, and the proportion of the one weight to the other, so far as being drawn downward is concerned, is inversely as the proportion of the lines. [I say that in these circumstances] the line is in horizontal equilibrium.[6]

The proof of proposition III, has to relay on the following comment Thābit makes just before its enunciation:

> *We have already said* [emphasis added] that in the case of two spaces which two moving bodies describe in the same time, the proportion of the power of the motion of one of the body to the power of the motion of the other is as the proportion of the space which the first motion cuts to the other space. And point A with the motion of the line has already cut AT and point B with the motion of the line has already cut arc BD, and this in the same time [See Fig. 2.1]. Therefore, the proportion of the power of the motion of point B to the power of the motion of point A is as the proportion, one to the other, of the two spaces which the two points describe in the same time, evidently the proportion of arc BD to arc AT. This proportion has already been shown to be the same as the proportion of line GB to line AG.[7]

[5] "PROPOSITION I. Le rapport de deux distances parcourues par deux mobiles en deux camps [égaux] est égal au rapport de la force du mobile [qui parcourt] la distance plane a la force de l'autre mobile" (Jaouiche 1976, 147).
[6] Moody and Clagett [1952] 1960, 92, 94.
[7] Moody and Clagett [1952] 1960, 92. English translation is mine.

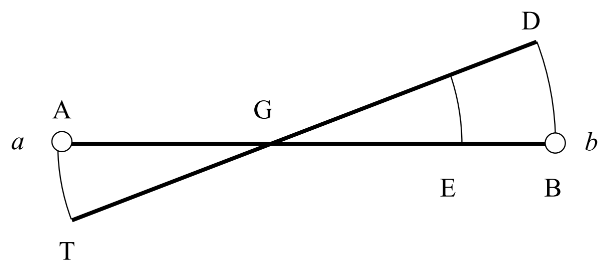


Fig. 2.1 Equilibrium of the balance according to Thābit[8]

Thābit clearly affirms that the 'power of motion' of the point B of the longest arm of the balance is greater than that of the point A, or more generally that the power of motion of a point of a balance is directly proportional to its distance from fulcrum. To note that displacements are measured according to the arcs of circles that weights describe in their motion; this is not peculiar to Thābit, but can be found also in the works by al-Isfizari (Capecchi 2102, 71) and by Galilei himself (Galilei 1649, 164). Thābit justifies his affirmation by saying "We have already said" (Moody and Clagett [1952] 1960, 92) which can only refers to *Proposition I*. Nevertheless this induces, at least for modern readers I think, a serious interpretation problem. Indeed proposition I when adapted to weights seems to make sense only for downward motions, but in the previous passage, Thābit is considering both upward and downward motions. One (Thābit ?) could overcome this difficulty by assuming that if a weight suspended from one side of a balance moves upward it could move downward too the same distance in the same amount of time, when the rotation of balance is imagined to revert and then one can always make reference to a possible downward motion. The same problem occurs in Galileo's demonstrations about equilibrium with the use of the concept of moment (Galilei 1612).

2.1.2 Continuation in the Latin Middle Ages

The very expression *scientia de ponderibus* was derived from the Latin translation of al-Fārābī's *Iḥṣā' al-'ulūm*. Translations of this text were due both to Gerardo da Cremona and Dominicus Gundissalinus in the XIII century. Gundissalinus in his treatise borrowed from al-Fārābī the concept of mechanics as a subalternate science, stemming form Aristotle's division

[8] Redrawn from Moody and Clagett [1952] 1960, 94.

for analogous sciences. He reproduced al-Fārābī's characterization of the sciences of weights and devices, called respectively *scientia de ponderibus* and *sciencia de ingeniis*. The reason for this verbatim acquisition depends on the fact he could not rely on any scientific category in this field in Latin. Even the antique Latin tradition represented by Boece and Isidore of Sevilla (VIII AD) could not furnish any useful data.

In the Latin Middle Ages, various treatises on the *Scientia de ponderibus* circulated. They were Latin translations from Greek or Arabic between XII and XIII centuries, referred to in the following Table 2.2.

Table 2.2. Latin treatises on the science of weights

Liber de canonio	A short treatise on the construction of Roman scale. Translated from a Greek origin (Moody and Clagett [1952] 1960, 64–75). The law of the lever, attributed to Euclid, Archimedes and other is taken for granted (sicut demonstratum est ab Euclide et Archimede et aliis, Moody and Clagett [1952] 1960, 66). Basing on it the laws that regulate the balance of a 'rod equipped with weight divided into unequal parts and loaded at the ends are determined. The problem is to find the position of the point of suspension given a certain tray so that it has equilibrium with no weights added, or vice versa given the point of suspension to find the weight of the tray.
Liber Euclidis de ponderoso et levi	Translated from an Arabic version attributed to Thabit, it would result from a Greek original which with many doubts can be traced back to Euclid. It consists of nine suppositions and some theorems. The version reported in (Moody and Clagett 1952) reports only five suppositions, but it is probably incomplete. Interesting the first theorem, not so much for its demonstration, but for the fact that it was assumed as a principle by Thabit in his Kitāb fī il qarastūm: "Of bodies which traverse unequal places in equal times, that which traverses the greater place is of greater force"[9].

[9] "Corporum que temporibus equalibus loca pertranseunt inequalia, quod maiorem pertransit locum maioris esse virtutis". (Moody and Clagett, [1952] 1960, 26–27).

Liber Archimedis de insidentibus in humidum or *Liber Archimedis de ponderibus*	According to (Moody and Clagett [1952] 1960, 36–37) the text cannot be attributed to Archimedes, despite the medieval claims. It would come for the first part from Latin sources of the eighth century (Isidore of Seivelle), for the second part from Arab sources of the twelfth century. The text is different from the others in content since it is not centred on the equilibrium of the balance but simply arises the problem of assessing the weight of bodies immersed in a medium. Interesting is the revival of the golden crown of the famous problem solved by Archimedes (Moody and Clagett [1952] 1960, 40–53).
Archimedis insidentibus in aquae and *Aequiponderanti*.	This is the translation by William of Moerbeke of the works of Archimedes on the equilibrium of the planar and floating bodies. They had no particular success in the Middle Ages, both for the difficulties intrinsic in the mathematics, and for the inaccurate translation of the concepts by Moerbeke.
Liber karastonis	It is the Latin translation by Gerardo da Cremona of Thābit's Kitāb fī il-qarastūm. None of the Arabic extant copies seem to be the direct model for Gerard's translation (Moody and Clagett [1952] 1960, 88–117). Arabic manuscripts are quite different from the Latin one. The order of propositions, indeed not numbered, in the Arabic versions is different from the Latin one. The texts of propositions are virtually the same as those in the Liber karastonis, except for secondary aspects. The texts of explanations are instead very different; shorter and much less satisfactory than those of the Latin version. The Latin version was repeatedly copied and distributed in the Latin West until the XVII century, as it is documented by several extant manuscript copies. Further, the treatise was used as textbook in the quadrivium, together with works by Jordanus De Nemore and others.
Excerptum de libro Thatbit de ponderibus	It has the same structure as the *Liber karastonis* for the statement of principles and theorems, it is its logical abstract (Brown 1967, 24–40). According to Knorr (Knorr 1982, 42–469), it is not derived from the Latin version but from some Arabic source.

Problemata mechanica	There is no evidence of a Latin translation of Aristotle's text. However, there are indications of its knowledge in the Greek version.

Starting from these treatises, medieval scholars developed their own science of weight. The first texts written directly into Latin are those attributable to various ways to Jordanus de Nemore, a famous mathematician of the XIII century.[10] We report them below with the names that has been attributed (Moody and Clagett [1952] 1960):

Elementa Jordani super demonstratione de ponderibus	Version E	Herein after version E or *Elementa*
Liber Jordani de ponderibus cum commento	Version P	Herein after version P or *Liber de ponderibus*
Liber Jordani de Nemore de ratione ponderis	Version R	Herein after version R or *Liber de ratione ponderis*

Moody and Clagett (1952) with certainty attribute the version E to de Nemore and consider possible the attribution of version R. More uncertainty is the attribution of version P, the less refined. Brown (1976) considers the *Elementa* ascribable to de Nemore but seems to opt for a different assignment for the *Liber de ratione ponderis*.[11]

De Nemore's treatises were the object of comments up to the XII century. Worthy of notice are some commentaries of XIII and XIV centuries, referred below with the name assigned to them by Moody and Clagett (Moody and Clagett [1952] 1960) and Brown (Brown 1976).

Table 2.3. Some commentaries of Jordanus de Nemore tradition

Corpus Christi	It contains a variant reading of the proof of the law of lever, of some interest, though controversial (Brown 1976, 570–647).
Aliud commentum of *Elementa*	Some passages of this text are of particular interest in that they testify a work of research

[10] Practically nothing is known about Jordanus de Nemore's life. He appears at the beginning of the XIII century. Besides writings about mechanics, he is author of many mathematical writings. For some more information see: Klein (Kelin 1964), Høirup (Høirup 1988) and Duhem (Duhem 1905, I, 99–108).

[11] There are various hypotheses about the roots of Jordanus' mechanical works. Quite convincing is the hypothesis of the Arabic roots: Abattouy (Abattouy 2006b, p 17), Folkerts and Lorch (Folkerts and Lorch 2007, 4, 12); Brown (Brown 1967), Clagett (Clagett 1959).

	regarding the principles of mechanics, somewhat distinct from that carried out by de Nemore (Brown 1976, 164–347).
Questiones super tractatum de ponderibus. By Biagio Pelacani of Parma (c. 1316–1465)	End of XIV century. A short work where three questions were raised. Contains comments on various treatises of the science of weights (Moody and Clagett [1952] 1960, 232).
Tractatus Blasi de ponderibus. By Biagio Pelacani of Parma	It is an independent text divided into three parts. The first two mainly refer to *De ponderibus* and *De canonio*, without new arguments. The third part refers to the *Liber Archimedis de insidentibus in humidum* (Moody and Clagett [1952] 1960, 238–279).

2.1.2.1 Jordanus de Nemore's Liber de ratione ponderis

Of the three versions (E, P, R) attributed to Jordanus de Nemore that denoted by R or *Liber de ratione ponderis*, is the most complete. It is quite a complex treatise, ideally divided into four parts with 7 suppositions (principles) and 43 (or 45 according to the manuscripts) propositions (theorems) of the science of weights. The first part has a theoretical aim and collects the suppositions and the most interesting propositions, among which the proof of the laws of the lever and inclined plane; the second and third parts are more technical and concern the solutions of some of the problems of the balance, with arms endowed or not with natural weight. The fourth part is about various issues, among which the fall and breaking of bodies. The version P assumes the same suppositions (7) and 13 propositions, the first seven coinciding with the propositions of the first book of version R, the other with other propositions of the second book. The version E is the shorter; it has the same suppositions but only 9 propositions corresponding to the first nine propositions of version P. All versions use two, not independent, fundamental laws:

1. The first law assumes the concept of gravity position for which the efficiency of a weight to descend or its resistance to be raised depends on the kinematic constraints to which it is subject. The law states that the effectiveness and strength are the greater the closer the path (made possible by constraints) to the vertical.
2. The second law has a precise mathematical expression and says that "what can raise a weight p at height h, can lift a weight p/n at a height nh, or vice versa a weight np to the height h/n". In other

words, the discriminant magnitude is the product *ph*, as requested by the modern principle of virtual displacement.

The first law is presented by de Nemore as a principle, it could have been derived by Aristotle's considerations in his *Problemata mechanica* on the amazing properties of the circle, but could also have origins in everyday experience of practical mechanicians; de Nemore says nothing about it. Weights are considered both as active elements, which push down and as passive elements, which offer resistance to be raised. The second law has a logical status that does not appear clear from the reading of texts. According to mine interpretation, as argued later on, it is a theorem proved from simple principles. The weight in this case is considered only as a passive element.

Jordanus de Nemore only uses the first law to demonstrate propositions of a qualitative nature, such as the demonstration that the lever *ab* of Fig. 2.2 with unequal arms and equal weights tilts on the side of *a*. The rationale is that the path *ag* of *a* is closer to the vertical than that *bf* of *b*.

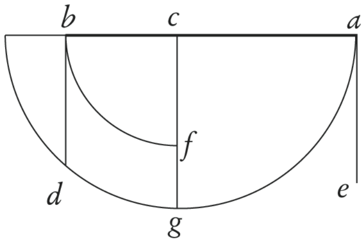


Fig. 2.2 Balance with unequal arms and equal weights[12]

Note that the use of the first law can lead to errors. This occurs in versions P and E when studying the equilibrium of the angular lever of Fig. 2.2. In the version P, de Nemore' reasoning is muddled; in version E the reasoning appears clear and consistent. Unfortunately, the result is wrong (Duhem 1905–1906, 121). In order to show his reasoning, in the following plate (See Fig. 2.2bis) from version E (de Nemore fl. 13th) and a description (See Fig. 2.3) are reported.

[12] Redrawn from de Nemore 1565, 5r.

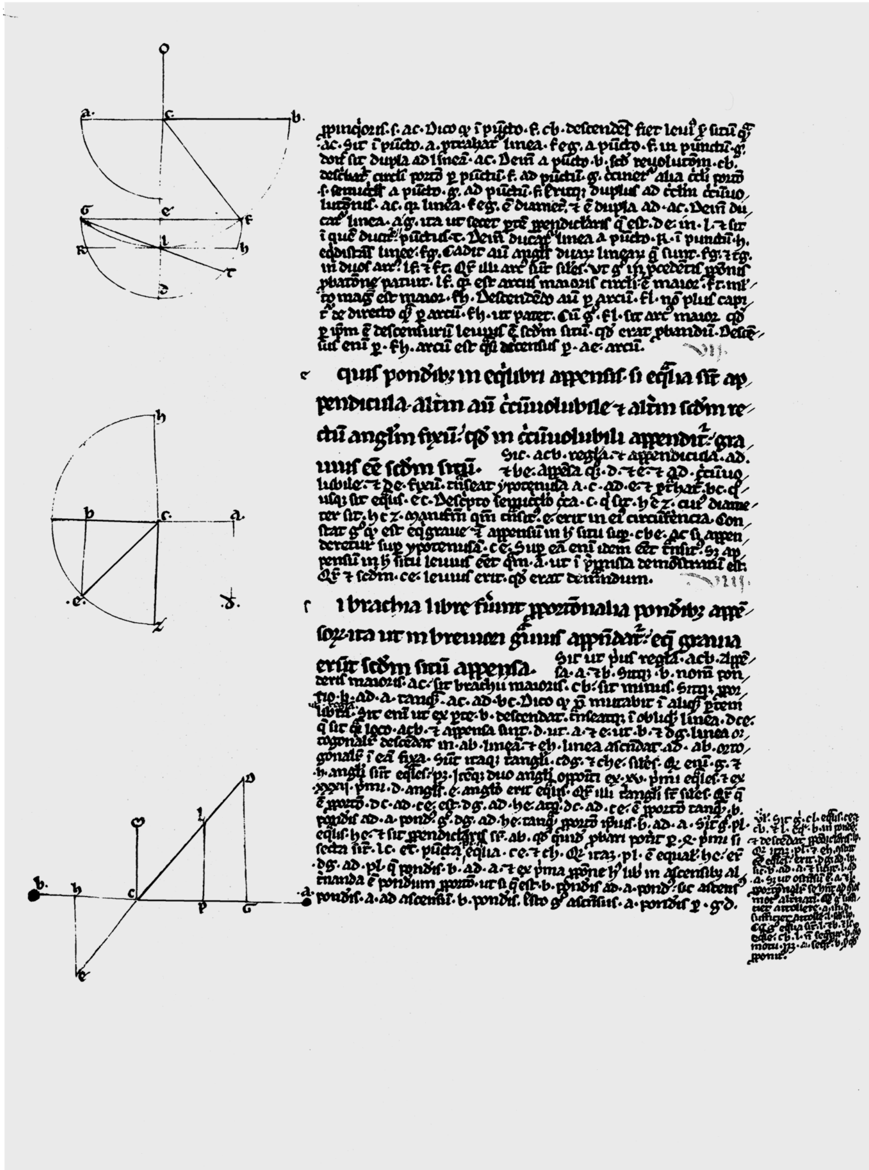


Fig. 2.2bis Plate from Jordanus de Nemore's *Elementa Jordani super demonstratione de ponderibus* (or *De ratione ponderis*, versione E)[13]

[13] de Nemore 13th, 4r. Particularly see the first figure. The manuscript (and with permission) of the *Oxford Bodleian Library* in our possession is not numbered: we proposed an order based on the copy received.

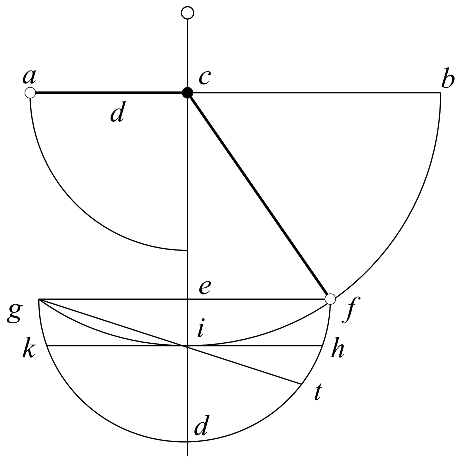


Fig. 2.3 Equilibrium of the angular balance[14]

Consider the angular balance–or–lever *acf* (Fig. 2.3) at whose ends two equal weights *a* and *b* are suspended symmetrically with respect to the vertical *cd*. Fixed a vertical segment *em*, weight *b* passes after covering the arc *fm*; weight *a* instead passes the same vertical *cd* by covering a shorter arc *fh*. It is clear from figure that the path *fh* is closer to the vertical than the path *fm*, and then the gravity position of *a* is greater than that of *b*. As a result, there should be no equilibrium and the angular lever should rotate anticlockwise.[15]

Actually, things do not go this way and the angular balance remains in equilibrium. De Nemore will correctly prove this fact (R version) in which the angular lever is studied with the use of the second law without making any reference to the concept of gravity position (de Nemore 1565, 6rv).

One more case where the concept of gravity of position is used, this time successfully, is in the study of the balance with equal arms and weighs, which is the object of proposition II:

> [PROPOSITION II] When a horizontal position is gained [for a balance of equal arms], then, if equal weights are suspended [from its extremities], the balance will not leave the horizontal position; and if it is moved from the horizontal position, it will revert to the horizontal position. If instead unequal [weights] are suspended, [the balance] will

[14] Redrawn from de Nemore 13th, 4r. See Fig. 2.2bis.

[15] Cfr.: Moody and Clagett [1952] 1960, 136.

> fall on the side of the heavier [weight] until it reaches the vertical position.[16]

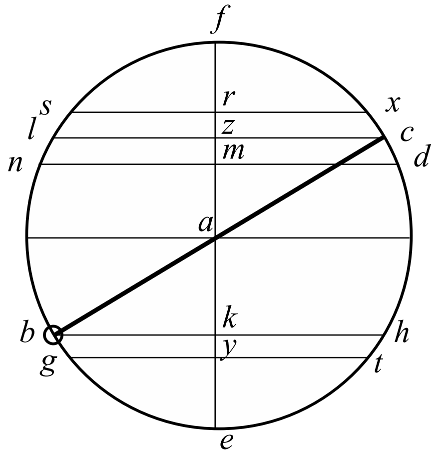


Fig. 2.4 Equilibrium of the balance with equal weights and arms[17]

The first part of the proposition, equal weights hanging from a balance with equal arms are equilibrated in the horizontal position, rather than being taken as a postulate, is demonstrated in the same manner as Thabit did, arguing that the two weights are moving with the same obliquity, so they have the same gravity of position and equilibrate themselves. The second part is proved by showing that when the balance assumes a position different from the horizon, the gravity of position of the weight that is lower (*b* in Fig. 2.4) is less than the weight that is higher (*c* in Fig. 2.4) because in a virtual rotation of each arm of the balance, the higher *c* is lowered more than the lower *b*, when passing equal arcs. So its gravity of position is greater and the balance returns horizontally:

> Let it now be supposed that the balance is tilted down on the side of *b*, and up on the side of *c* [Fig. 2.4]. I say that it will revert to the horizontal position. The descent from *c* toward the horizontal position is indeed less oblique than the descent from *b* toward *e*. Assume indeed equal arcs, *as small as you please* [emphasis added], *cd* and *bg*; and draw the lines parallel to the horizontal *czl* and *dmn*, and also *bkh* and *gyt*, and draw, vertically, the diameter *frzmakye*. Then *zm* will be

[16] "Quum aequilibris [aequilibriis] fuit positio aequalis aequis ponderibus appensis ab aequalitate non discedet: et si a rectitudine separatur, ad aequalitatis situm revertetur. Si vero inaequalia appendantur, ex parte gravioris usque ad directionem declinare cogetur." (de Nemore 1565, 3v). English translation is mine.

[17] Redrawn from de Nemore 1565, 4r.

> greater than *ky*, because if an arc, equal to *cd*, is taken in the direction of *f*, and if the line *xrs* is drawn transversally, then *rz* will be smaller than *zm,* what is easy to show. And since *rz* equals *ky*, *zm w*ill be greater than *ky*. Since because any arc you please, which is beneath *c*, takes more of the vertical than an arc equal to it, taken beneath *b*, the descent from *c* is more direct than the descent from *b*; and then *c* will be heavier in the most elevated position, than *b*. Therefore, [the balance] will revert to the horizontal position.[18]

Note that historians considered usually de Nemore' assumption of arcs "as small as you" (*Ivi*) like the adoption of reasoning about infinitesimals.[19] According to them de Nemore did not make the passage to the limit and then he "failed" (*Ivi*) to notice that in the limit, for infinitesimal arcs, vertical displacements of *c* and *b* are equal, then the gravity of their position are equal, then equilibrium is indifferent. Actually things are not so, as will be explained in section 3.2.4, *Proposition VI*, of the present book.

However, de Nemore's failure can hardly be blamed since his way of reasoning was still maintained long after infinitesimals were introduced. In his criticism to Lagrange, Joseph Louis François Bertrand (1822–1900) and Carl Gustav Jacob Jacobi (1804–1851) two important mathematicians of the XIX century would have subscribed de Nemore's position to assume finite arcs.[20] The error of de Nemore in case would have been to consider

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[18] "Ponatur item quod submittatur ex parte *b*, et ascendat ex parte *c*, dico quoniam redibit ad aequalitatem. est enim minus obliquus descensus *a*, *ad* aequalitatem, quam *a*, *b*, versus *e*. Sumantur enim sursum arcus aequales, quantumlibet parvi qui sint *c*, *d*, *et h*, *b*, et ductis lineis ad aequidistantiam aequalitatis, quae sint, *c*, *h*, *l*, et *d*, *m*, *n*. Item *b*, *k*, *h*, *g*, *y*, *t*, dimittatur orthogonaliter descendens diametrum quae sit *f*, *m*, *a*, *k*, y, *e*, erit quod *z*, *m*, maior *k*, *y*, quia sumpto versus *f*, arcu ex eo quod sit aequalis *c*, *d*, et ducta ex transverso linea *x*, *r*, *s*, erit *r*, z, minor z, *m*, quod facile demonstrabis. Et quia *r*, *z*, est aequalis *k*, *y*, erit *z*, m, maior *k*, *y*. Quia igitur quilibet arcus sub *c*, plus capiat de directo quam ei aequalis sub *b*, directo est descensus *a*, *c*, quam *a*, *b*, et ideo in altiori situ gravius erit c, quam *b*, redibit ergo ad aequalitatem" (de Nemore 1565, 3v). English translation is mine.

[19] Cfr.: Clagett 1959, Chapter 2.

[20] Bertrand's criticism is reported in the third issue of Lagrange's *Mecanique analytique*, first volume edited by Bertrand himself (Lagrange 1853, 22). Jacobi gave profound criticims of Lagrange's mechanics in his *Vorlesungen über analytische Mechanik*, Berlin, 1847–1848, particularly concerning the role of mathematics in the empirical sciences. For details and references, see Pulte 1998. Note that Bertrand and Jacobi, as well as Jordanus, considered infinitesimals as small as you like but always finite quantities.

the gravity of position of the two heavy bodies, *c* and *b*, as both moving downward. If he had assumed a congruent motion according to which when one weight raises the other falls, he would have found equality of gravity of positions for *c* and *b*.

However, the reduction to infinitesimal motion, according to the modern view,[21] would lead to an evaluation of the gravity of position different from that proposed by de Nemore. If the motion on a given circle with infinitesimal displacements is assumed; gravity of position is maximum at the horizontal position of the balance and is zero in the vertical position; in an intermediate position, the gravities of the weights are equal and the balance is in equilibrium. Nevertheless, if circles of different radius are considered, the infinitesimal displacements do not attribute the greater gravity to the weights that are on the larger circle. Considering finite displacements instead enables this attribution. The concept of gravity of position, although interesting and suggestive, seems to take more than a simple infinitesimal reinterpretation in order to be adopted by modern statics.

2.2 Revival during the Age of Humanism

In XV century Italy there was a sparkling situation for economic social and political conditions, on the one hand and cultural achievements on the other hand. A situation, which then would be established in the rest of Europe (Garin 1993; Garin 2008; Tenenti 1990). Regarding cultural aspects, besides emergence of the culture of the middle class which played an important part in accounting calculation, geography, economics and financial technique, the emergence of the humanist movement should be highlighted[22]. This was made possible by the new social and economic conditions, offering new perspectives on the world, which on one hand allowed the members of the middle class to be able to devote time to study and on the other hand allowed the courts to play a more or less disinterested activity of patronage.

[21] In the modern view, infinitesimals are considered in the limit, and the infinitesimal motion is closer to a velocity than a displacement.

[22] Since different intellectual schools of thought are identified with the term *Humanism*, here are just a few words to remark that by this term we mean in particular the Italian humanist group (*human nature*) busy with lecturing, transcription, and studies of the mathematical sciences from Greek and Latin manuscripts.

The XV century records a check on growth in the development of science and the publication of scientific papers. The check existed of course for the science of weights too. In this case it also depended on the fact that the discipline, formulated axiomatically had reached its complete internal maturity and only the proposition of new problems could have lead to an evolution. Although until the early years of the XVI century no new major scientific treatise was written[23], except the *Summa de arithmetica, geometria, proportioni et proportionalità* (Pacioli 1992) and *De divina proportione* (Pacioli 1509)[24] by Luca Pacioli (c. 1445–1517), it must be said that in this period the foundations of a major renovation were laid down, with the breaking of the spirit of the scholasticism system and the repudiation of the principle of authority, particularly that of Aristotle, the rediscovery of Plato and Pythagoras and the valorization of mathematics which was the premise for the new philosophy of nature of the second half of the XVI century.

2.2.1 A Variety of Approaches to Mathematics

At the end of the Middle Age mathematics was taught essentially at universities and at abacus schools. In the university, mathematics was taught in the *quadrivium* (arithmetic, geometry, astronomy and music) of the faculties of arts that while maintaining their autonomy, were instrumental to the training of future physicians and theologians. [25] The medical faculties of the early Renaissance were usually those in which mathematics had more space[26]. Medicine was, in fact, connected to the study of astrology, which required the students to have rudiments of Ptolemaic astronomy and then knowledge of elements of geometry and arithmetic. Professors of these subjects were the masters of liberal arts of the *quadrivium*, whose teaching and research many of the mathematical

[23] The last book of some importance toward the end of the XIV century was *Questiones super tractatum de ponderibus* by Biagio (or Blasius) Pelacani da Parma.

[24] The *De divina proportione* is well known also for the famous Leonardo da Vinci's engravings it contains. (Pisano 2013; Pisano 2014-submitted).

[25] For the role of European universities in the XV century, refer to (Duhem 1988, X; Grant 2001; de Ridder–Symoens 2003). For the Italian universities see the Annals of the history of Italian universities (CLUEB, Bologna) and Grendler's work (Grendler 2002).

[26] This is the case for example of Padova, where the introduction of mathematics into the undergraduate curriculum preceded that of astronomy-astrology related to medicine.

works of the XV century are connected. However, the place occupied by mathematics was still marginal[27] and the level of mathematical knowledge, was, except for some teachers, limited to what, was indispensable for the exercise of astrology. In fact, it did not cover the study of many Greek classics that at the time were already available in Latin translations from Arabic of the XII century. However not to be forgotten is that, for instance, Galileo Galilei was nurtured at a university. The University of Padova in particular was an important centre for training in science. Among its students in the XV and XVI centuries the following people should be noted: Paolo da Pozzo Toscanelli (1397–1482), Leon Battista Alberti, (1404–1472), Francesco della Rovere alias Pope Sixtus IV (1414–1484), Giovanni Pico della Mirandola (1463–1494), Pietro Bembo (1470–1547), Nicolaus Copernicus (1473–1543), Francesco Guicciardini (1483–1540), Girolamo Cardano (1501–1576?), Bernardino Telesio (1509–1588), Torquato Tasso (1544–1595), Roberto Bellarmino (1542–1621), Paolo Sarpi (1552–1623), Giovanni Domenico Campanella called Tommaso Campanella (1568–1693), William Harvey (1578–1657).

Different considerations holds for the schools of abacus. They were born in the XIII century with the spread of *Liber abaci* (Fibonacci 2004; Giusti 2002) by Leonardo Pisano's also called Fibonacci (1170–1250; see Pisano and Bussotti 2013, 2014; Ulivi 2002). Some of these schools were subsidized by the municipalities, some others by private organizations or individuals. The practical mathematics that emerged from the abacus treatises of XIV and XV centuries had so many characteristics that quite clearly distinguished it from the traditional Euclidean axiomatic–deductive mathematics. The main features of the abacus treatises were the use of the vernacular, the mercantile writing, the great amount of examples and the presence of important drawings for illustrative purposes. The treatises on the abacus had different quality levels, which reflected the skills of teachers who had drawn them up: some were very simple and neglected those parts of mathematics (algebra, practical geometry, speculative arithmetic) that were not immediately applicable in the art of the merchant. Others, however, showed a certain organic quality, aesthetically cured, mainly in the miniatures illustrating the drawings, and treatment of some algebraic problems, which involved the solution of quadratic and higher degree equations (Ciocci 2011, 266–271). Even mathematical textbooks used by the artists had characteristics similar to those of the schools of

d to medicine.

[27] Considering the small number of chairs of mathematics in the University of Padua and Bologna compared to those of medicine until the time of Galileo, it can be seen that the academic discipline was marginal (Ciocci 2011, 261).

abacus, where, however, drawings and operational rules prevailed over theoretical aspects.

Piero della Francesca, Michelangelo Buonarroti (1475–1564), Niccolò Machiavelli (1469–1527), Leonardo da Vinci (1452–1519) and Alberti were influenced by the mathematrics of this environment. Most studies of the history of science, including mechanics, focus on the influence of Euclidean and Archimedean mathematics and neglect that of abacaus mathematics, which should not have been small, especially in view of its non–axiomatic approach (Pisano 2013; Pisano 2009a,b,c).

With the Renaissance in the XV century the medieval mathematics is joined by the new mathematics, or rather the rediscovered ancient Greek mathematics to which the humanist movement gave a great contribution. The essential role of Italian humanism in the Renaissance of mathematics during the XV and XVI centuries was well documented in (Rose 1975). Many humanists returned from their travels to Byzantium with codes of Apollonius, Ptolemy, Pappus and Heron written in Greek. In the early XVI century, within a few decades, many revisions and translations of classics were delivered. Some of the most important were: the *De expetendis et fugiendis rebus* (1501) by Giorgio Valla (1447–1500), a rich encyclopaedic anthology of Greek scientific texts[28] a new translation of Euclid (Venezia, 1505) led by Bartolomeo Zamberti (fl. second half XV c.), the first Archimedean texts published (Venezia 1503) by Luca Gaurico (1476–1558), the *editio princeps* of Euclid's *Elements* (Basel, 1533), the translation of Apollonius' *Conic sections* (Venezia, 1537) by Giovanni Battista Memmo (1503/1504–1579), the Italian translation of Euclid and the publication of several works of Archimedes (Venezia 1543) presented by Niccolò Tartaglia (1499/1500–1557), and the *editio princeps* of Archimedes with Greek and Latin text (Basel, 1544). It was however a non-Italian humanist, Johannes Müller von Königsberg, whose Latin toponym was Johannes Regiomontanus (1436–1476), the first to embark on a complete restoration of mathematics and astronomy (Pisano and Bussotti 2012) based on his acquaintance with Italian classicists and humanists related to Basilio Bessarione (1403–1472). In effect, the scientific knowledge spread by humanists during the Renaissance depended on the scientific aptitudes of translators and many other factors related to circulation of information:

[28] Printed for Aldo Manuzio's types, *De rebus expetendis et fugiendis* consisted of 49 books, 30 of which were devoted to sciences. The first book presents a classification of philosophy, within which the mathematical sciences plays a dominant role as given on the basis of the commentary to Euclid's *Elements* made by Proclus. Valla's book contains references to Archimedes' works.

> As we have seen, the starting point for this renaissance of mathematics was the correction of Greek mathematical texts, to be undertaken by those who were expert in both the Greek language and astronomy. To make the refurbished traditions of Greek mathematics available to mathematicians generally, Regiomontanus from at least 1461 was engaged on a series of Latin translations. But by 1471, this means of communication was revolutionised by Regiomontanus' discovery of the new invention of printing. Through printing, an astonishingly rapid and accurate dissemination of texts and translations become possible that had been inconceivable in an age where manuscripts represented the sole means of circulating the written word. In its fusion of mathematics, Greek and printing Regiomontanus' publishing *Programme* of 1474 marks the formal beginning of the renaissance of mathematics.[29]

Thus, the reacquisition of mathematical techniques was rather slow. What the humanist movement had since carried on was of a meta–mathematical character and concerned the new role that mathematics acquired within the philosophy of the Platonic and Pythagorean schools of thought. Important to this purpose was the role played by Luca Pacioli, who was at the same time a teacher of abacus and magister theologiae, which allowed him to mediate the culture of technicians and learned men. The biblical–metaphysical idea inspired[30] Luca Pacioli in his dedicatory letter to Guidobaldo da Montefeltro (1472–1508). It regarded a book of nature that – later resumed by Galileo Galilei (1564–1642) as well – was written in mathematical characters.

[29] Rose 1975, 110.

[30] "[…] Fratris Luca de Burgo Sancti Sepulcri, ordinis minorum, sacre theologiae Magistri [...]" "Ad Illustrissimum principem sui Ubaldum Duces Montis Feretri, Mathematice discipline cultorem serventissimum […]". (Pacioli 1494, *Summa*, 3r).

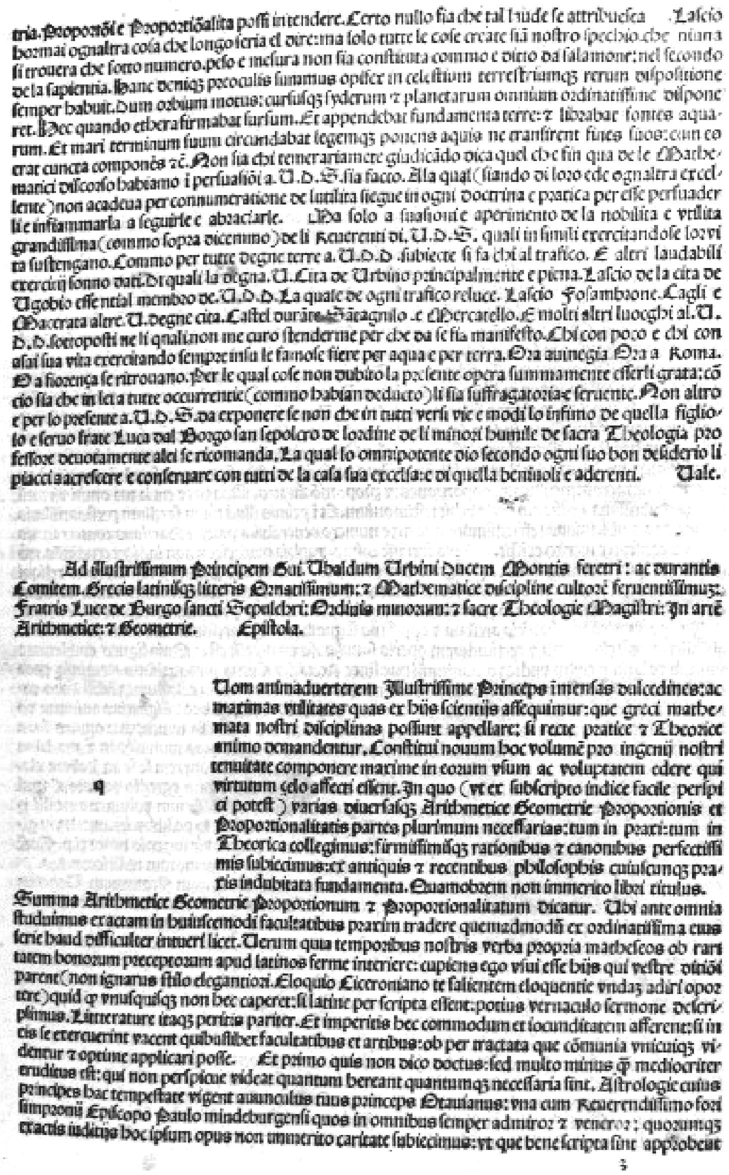

Fig. 2.5 Plate from the initial part of the dedicatory letter by Pacioli.[31]

> Let all create beings be our mirror, as no one will found to be constituted but as number, weight and measure, as said by Salomon in the second book of the Sapientia.[32]

[31] Pacioli 1494, *Summa*, f. 3r; see also 4r. Source: Max Planck Institute for the History of science–Echo/Archimedes Project.

2.2.2 The Emergence of a New type of Intellectual Technician: the Engineer

Regarding economic aspects of the times, the emergence of a middle class of which the merchant was a key element should be emphasized. The middle class had long since conquered a great economic and social weight and had acquired the consciousness of its social role and the possession of a culture, independent of universities and various humanist circles. The evolution of the economy and society was strongly influenced by three fundamental technological discoveries: circumnavigability of the earth, gunpowder, and printing. The possibility to circumnavigate the globe was perhaps the most important discovery leading to a boost in the economy of many nations. It also entailed the development of navigation techniques with invention of the compass, the representations of geographic maps, the improvement of astronomy for navigation using the stars, and the crafting of ships, which no doubt provided a stimulus to the improvement of many applied sciences (Singer 1954, II–III).

The spread of modern artillery based on the propellant effect of gunpowder was important, especially for the development of mechanics. Knowing what causes the beginning of motion, and its sequel, was considered important by commanders of the armies and therefore also by states. This was true especially since the XVI century, when artillery had become extremely effective. The development of artillery had as a natural consequence the development of defensive techniques. This gave birth to the bastioned fortresses, first appearing in Italy and then becoming a real battleground for numerous national and foreign armies. Perhaps even more than artillery, fortress design mobilized engineers and architects, leading to the development of methods of construction and a better understanding of the strength of materials (Pisano 2009, Pisano and Capecchi 2008, 2009, 2010ab, 2012, 2013).

The emergence of the engineer as an intellectual technician, seen as a new kind of technician in some way educated in sciences, is a characteristic feature of the XV century and the first half of the XVI. Indeed this is perhaps the main feature of science, where the reduced creativity (real or apparent) of ‘pure’ scientists, was counterbalanced by the great creativity of “applied” scientists. A short list is sufficient to give an idea of the dimension of the phenomenon: Mariano di Jacopo, called Taccola, (1381–1458), Leon Battista Alberti, Francesco di Giorgio Martini (1439–1501), Leonardo da Vinci, Vannoccio Vincenzio Austino Luca Biringuccio also known as Vannuccio, Biringuccio (1480–1539),

[32] Pacioli 1494, *Summa*, 4r.

Francesco de' Marchi (1504–1576), Giovanni Battista Bellucci (1506–1554), and Daniele Barbaro (1513–1570).

Although there was no public funding to encourage scientists to devote their efforts to the study of technical applications and to improvement of their knowledge, a common ground arose, particularly in Central and Northern Italy. A link between engineers and scientists emerged, at least in part, through the creation of some technical centres in the courts of the principalities which had been set up. This was the case of the Medici's court in Florence, but also, and perhaps more importantly, the court of Milan under Francesco Sforza with its very rich library. Another important centre was Urbino. Here among others the presences of Francesco di Giorgio Martini (1480–1490), who translated Marcus Vitruvius Pollio's (ca. 80–70 BC – after 15 BC) *De Architectura* into Italian[33], which although questionable from a philological point of view, made this author known to all technicians[34] and Piero della Francesca (1415–1492), one of the greatest mathematicians and painters of the time, are to be reported.

2.2.3 Leonardo da Vinci's Science of Weights

It is not easy to understand how the science of weights may have influenced the training of technicians. Certainly, some basic aspects on the working of the lever and the block and tackle needed for the construction of building and industry machinery was available independently of mechanics treatises. There was a long tradition of transmission of technological knowledge from antiquity that found concrete expression in the regular use of construction machinery designed during the Hellenistic era. There is however no doubt that when a certain culture of mathematics and drawing began to spread, a precise knowledge of the basic laws of mechanics, which could be acquired with limited scientific knowledge, gave the opportunity for the design of machines at the work table.

In the hands of technicians, the theoretical medieval science of weights could evolve toward a more mature discipline, in the attempt of its application to situations required by the technology of the time. This possibility of evolution was widely exploited by a man who is today universally regarded as the engineer of the XV century par excellence:

[33] Probably one of the first partial translations from Latin to Italian, which was not published. On our side, no historical documents we know of has claimed that it was really the first.

[34] Francesco di Giorgio Martini added elements of theory of machines and construction in book X already devoted to use and construction of machines. For an English edition, see: Rowland and Howe (Rowland and Howe 1999).

Leonardo da Vinci. In the following, we will expose how the science of weights will be transformed in his hands. The choice of studying Leonardo is partly motivated by the fact that the studies conducted so far on him, not always exhaustive, have shown the great theoretical significance of his writings, but it is also motivated by the fact that now I have access to the complete set of Leonardo's works (Pisano 2013). His many interests were considered in the early 1400 by Taccola who was interested in the writings of mechanics and military technics. In more recent times Giambattista Venturi published, in 1797, a famous essay on the scientific work of Leonardo da Vinci (Venturi 1797). In the years 1880–1940 da Vinci's notebooks were published in facsimile and nearly all the manuscripts were printed with a diplomatic transcription[35] and translation in different languages, resulting in approximately a thousand drawings and propositions. However, an organic edition is still lacking, with the happy exception presented by Arturo Uccelli who edited with a critical transcription[36] nearly all the mechanical writings, ordering them according to a criterion inspired by Leonardo himself (da Vinci 1940).

Between 1482 and 1499 Leonardo da Vinci[37] was in the service of the Duke of Milan. During his service he also advised on architecture, fortifications and military matters and worked as a hydraulic and mechanical engineer and became interested in geometry. He read Leon Battista Alberti's *De re ædificatoria* on architecture (ca. 1450) and Piero della Francesca's *De prospectiva pingendi* on perspectives studies. He illustrated Pacioli's *Divina proportione*[38] (1498) and worked with him.

[35] A transcription that respects the original spelling and punctuation marks, spaces included.

[36] A transcription that is faithful to the original but avoids typos, resolves "u" in "v" according to the modern practice, uses a standard character for "s", unifies the writing of words with the same meaning to the most used form, and so on.

[37] Leonardo da Vinci was born in 1452 in *Vinci*, a small village near Empoli and province of Firenze, in the Toscana department, Italy. He died in 1519 at the *Château du Clos Lucé*, in the *Indre-et-Loire* department, France. He was educated in his father's house receiving thereby usual elementary notions of reading, writing and arithmetic.

[38] Leonardo da Vinci's (written down at an earlier meeting with Pacioli) transcripts of his handful of whole passages of the Summa (Pisano 2013). On 10th November 1494 (Venice) finally released in print in Latin, Luca Pacioli's *Summa arithmetic*, g*eometry, proportions et proportionality*. Luca Pacioli inspired Leonardo da Vinci (Pisano 2014-submitted) and was his counselor, teacher and translator. Da Vinci purchased the Summa (119 soldi) as he himself claimed (da Vinci, *Codex Atlanticu*s, 288r f. 104r, 331r) and noted: "Learn multiplication of the roots by master Luca" (da Vinci, *Codex Atlanticus*, 331r [120r]). From 1496 to

Leonardo studied Euclid and Pacioli's *Summa* and began his own researches on geometry, sometimes giving mechanical solutions. In 1499 Leonardo left Milan together with Pacioli; in 1506 he returned there for a second period. Again his scientific work took precedence over his painting and he was involved in hydrodynamics, anatomy, mechanics, mathematics and optics. In 1513 Leonardo accepted an invitation from King Francis I to enter his service in France (Gillispie 1971–1980, VIII, 199–244).

Leonardo da Vinci is a difficult subject to be confined within a fixed frame and it is difficult to give a full account of the opinions of historians on Leonardo's role in science in general and mechanics in particular. One goes from an enthusiastic vision of the early XIX century, especially on the side of historians of science educated in literature, to a more mature appreciation of Duhem and finally to a fierce criticism by Clifford Ambrose Truesdell (1919–2000) who minimised (Truesdell 1968, 1–29) both the originality and the contribution to the subsequent science development of Leonardo's work and George Sarton (1884–1956) who affirmed (Sarton 1953, 11–22) that the development of mechanics would have been the same without Leonardo. Eduard Jan Dijksterhuis (1892–1965) eventually considered studying Leonardo as being of interest not for his contributions to science, but for the opportunity offered by his copious notes that were written to follow the maturation of various scientific concepts (Dijksterhuis 1961).

A better understanding of the history of mechanics and a different conception of history of science with the trend to greater contextualization of the work of scientists has certainly contributed to this change of opinions. Today there is a phase of stagnation on the studies of Leonardo as a scientist, probably due to the concerns aroused by the latest criticisms and the concern to approach a job seemingly titanic at first glance. It is with reverential awe and humility that we have set about the study.

One of the difficulties in reading Leonardo's texts is that they consist largely of scattered notes, often repeated with slight variations, sometimes with inconsistencies. Although attempts were made to reach a chronologically consistent order, different scholars have not yet obtained results sufficiently shared, also because Leonardo had the habit of putting his own hands to the manuscripts and editing them with continuous

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1504 Leonardo studied Luca Pacioli's works and summarized his theory of proportions (da Vinci, *Codex Madrid*, 8936). Particularly, geometrical figures were presented for the first time in the *Codex Forster* and finally included in the *De divina proportion* (Pisano 2013). For Leonardo's sources ee the *Pinacoteca Ambrosiana* in Milan and *Museo Galileo–Istituto e Museo di Storia della Scienza* in Florence.

additions and deletions. The only valid criterion is the search for logical consistency and the persistence of certain statements over others.

Arturo Uccelli (da Vinci 1940), Roberto Marcolongo (1862–1943; Marcolongo 1937), Pierre Maurice Marie Duhem (1861–1916; Duhem 1906) and others, among which we want to name at least Edmondo Solmi (1874–1912; Solmi 1908), attempted to find the source of the thought of Leonardo da Vinci. The enterprise is difficult because in the XV century they were not particularly generous in quotations; Leonardo specifically names only: Aristotle (384 BC – 322 BC), Archimedes, Euclid (ca. 323 BC-286 BC), Abū l Hasan Thābit ibn Qurra' ibn Marwān al-Sābi' al-Harrānī (826–901), Jordanus de Nemore (fl. XII or XIIIth), Biagio of Parma (c. 1365–1416), Albertus Magnus (1193/1206–1280) also known as Albert the Great and Albert of Cologne, Albert of Saxony (ca. 1316–1390), Alberti and perhaps Richard Swineshead[39] (fl. 1340–1354). Moreover, it is also difficult to understand the influence of Leonardo on posterity because it seems that he had not made his works known, except to a very restricted circle. We have set ourselves an easier task in trying to decipher Leonardo's thought by framing it within his time on the basis of medieval texts of mechanics known to us but maybe not to him. Stating that Leonardo's claims are original with him is perhaps misleading and at best uninteresting, since we are convinced that he was not an isolated genius, but probably a representative engineer with beliefs common to others.

The science of weights in the hands of Leonardo became a discipline very like to modern statics, closer to that of Simon Stevin, a century after, than to that of Guidobaldo del Monte (and even Galileo), who proposed restoring Greek mechanics, limiting the study to simple machines, the lever, an axle with a wheel, the wedge, the screw and the inclined plane.

2.2.3.1 Powers: Gravity and Force

Before moving on to analyse the more technical contributions of Leonardo to mechanics we should make a clarification of the meaning of certain terms, including: power, gravity and weight. The following quotes give a first idea:

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[39] Cfr.: Arturo Uccelli (da Vinci 1940).

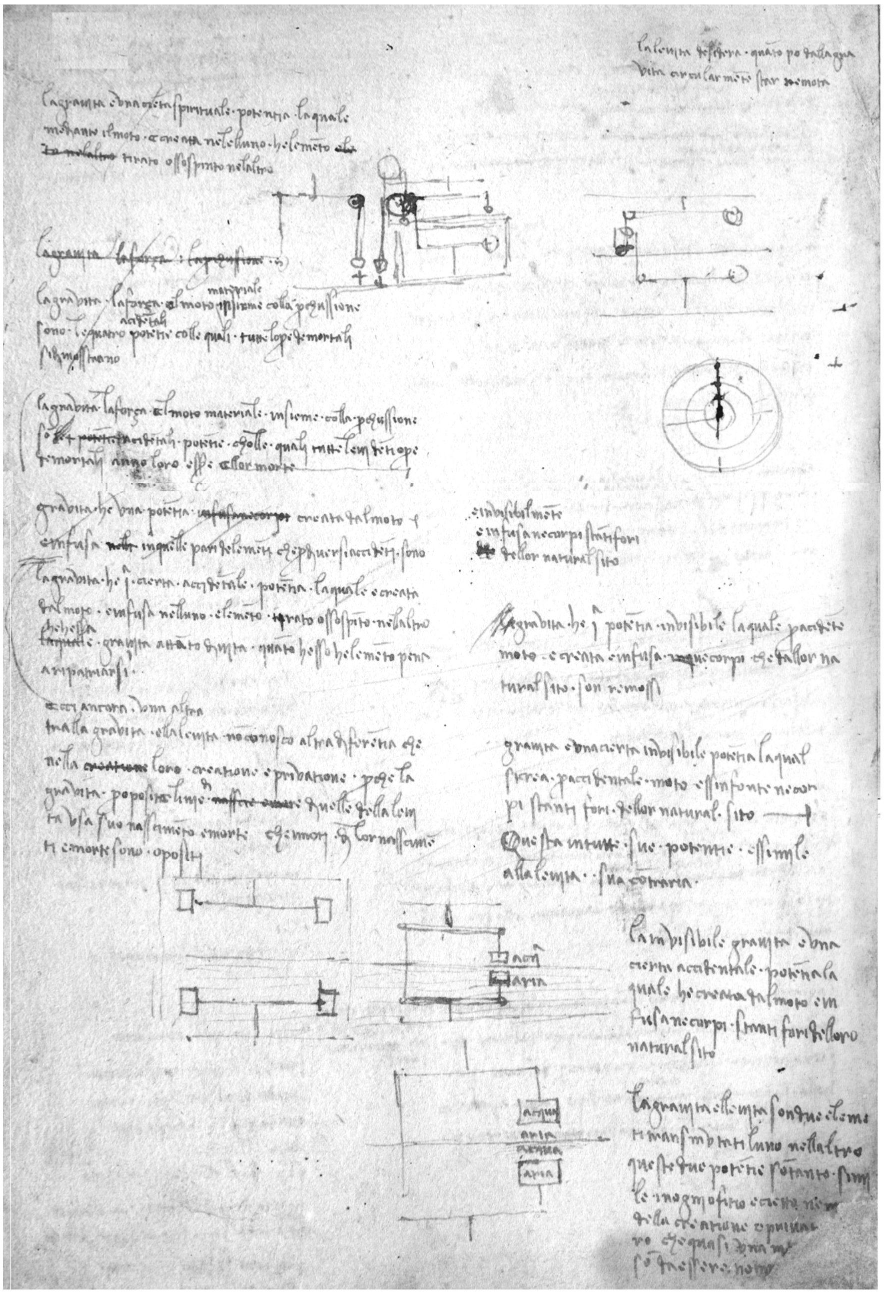

Fig. 2.5bis Plate from the studies on gravity and force[40]

[40] da Vinci, *Codex Arundel,* 37r.

> Gravity is an accidental power, which is created by motion and infused into bodies out of their natural site.[41]
> […] Gravity, force and accidental motion (material motion), together with percussion are the four accidental powers, by which all the evident work of mortal beings have their origin and their death.[42]

In this passage, Leonardo da Vinci refers to the four powers (with a modern language, forces). Regarding the gravity, it can be said that Leonardo married the traditional Aristotelian school thesis considering it as the tendency of bodies to reach their natural place (Duhem, I, 16–17). For Leonardo gravity is caused by motion:

[41] "Gravità è una potentia invisibile la quale per accidente moto è creata, e infusa ne' corpi che dal lor natural sito sono remossi". (da Vinci, *Codex Arundel*, 37r. See also: da Vinci 1940, 31). English translation is mine.

[42] "La gravità, la forza, e'l moto accidentale, insieme colla percussione, son le quattro accidentali potenzia, colle quali tutte le evidenti opere de' mortali hanno loro essere e loro morte". (da Vinci, *Codex Forster II*, 116v. See also: da Vinci 1940, 32). English translation is mine.

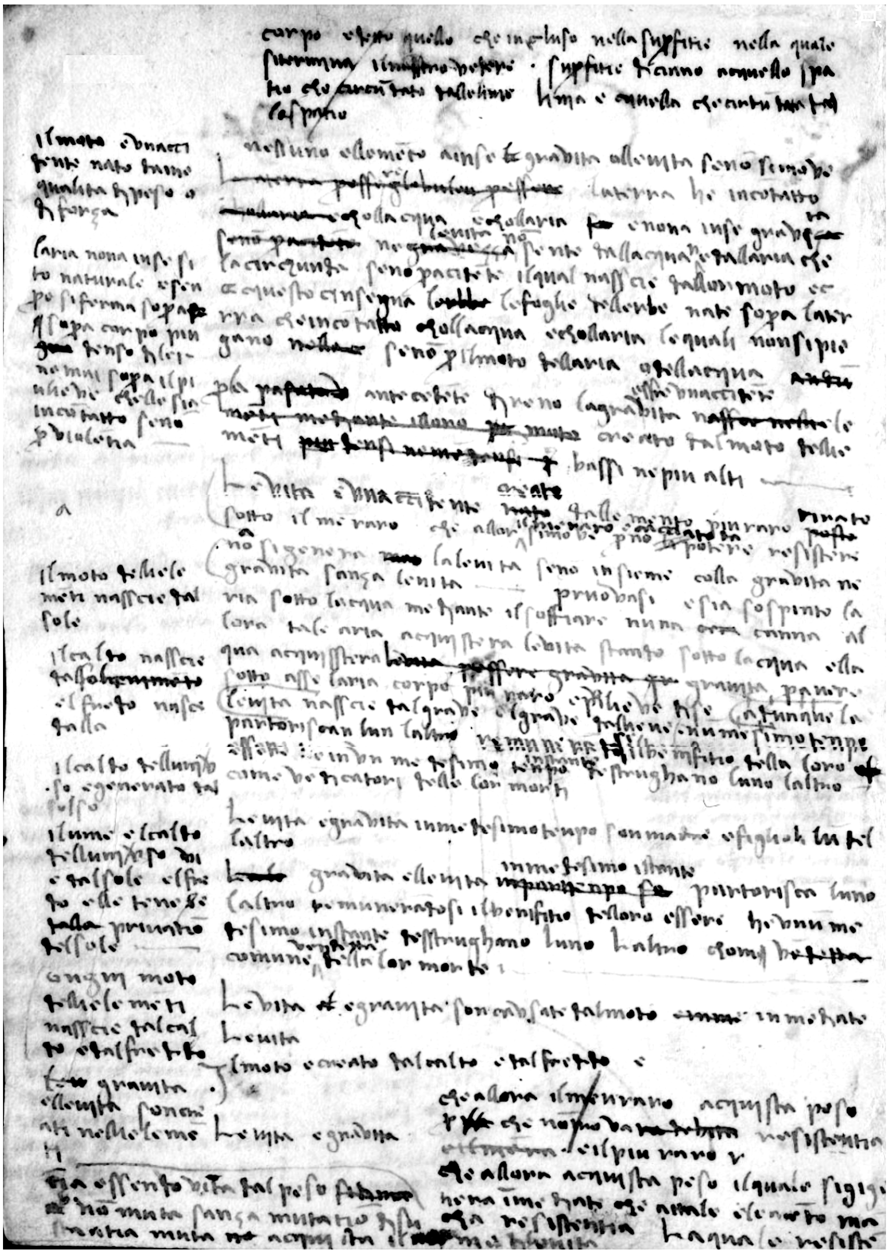

Fig. 2.6 Plate from the studies on gravity[43]

[43] da Vinci, *Codex Arundel,* 205r.

> No element has in itself gravity or levity if it does not move. The earth is in contact with the air and water and has in itself neither gravity nor levity; it has not stimulus neither from the water nor from the surrounding air, unless by accident, which originates by motion. And this teaches us the leaves of herbs, born above the earth, which is in contact with the water and the air, which do not bend if not for the motion of air or water.[44]

To this statement, a bit cryptic for a contemporary, Leonardo adds an explanation:

> Gravity is an accident created by the motion of the lower elements into the upper.[45]

That is, a body shows its gravity if, following an upheaval of the underlying parts, an imbalance of the upper parts is determined. More problematic is the interpretation of the term force. On the purpose, quite clarifying was the following famous quotation, which is interesting from a literary point of view also, as a very effective example of scientific prose, in which someone wanted to see the influence of the neo–Platonic philosophy of universal animation.

[44] "Nessun elemento ha in sè gravità o levità se non si move. La terra è in contatto coll'aria e coll'acqua e non ha in sè gravità nè levità; non sente dall'acqua nè dall'aria che la circunda se non per accidente, il qual nasce dal lor moto. E questo c'insegna le foglie dell'erbe nate sopra la terra ch'è in contatto coll'acqua e coll'aria, le quali non si piegano se non per il moto dell'aria o dell'acqua". (da Vinci, *Codex Arundel*, 205r. See also: da Vinci 1940, 30). English translation is mine.

[45] "La gravità essere un accidente creato dal moto delli elementi bassi ne' più alti." (da vinci, *Codex Arundel,* 205r. See also: da Vinci 1940, 30). English translation is mine.

Fig. 2.7 Plate from the studies of the equilibrium of weights and of impact ("percossa")[46]

[46] da Vinci, *Ms. A* f. 1v.

> Force I say is a strong spiritual virtue, an invisible power, caused by accidental external violence of motion and located and instilled into bodies, which are moved from their natural habit [the rest] and determined by giving them active life of wonderful power: constrains all created things to change form and site, runs with fury to her desired death and comes diversifying through the causes. Slowness makes it great and swiftness weak, it comes into being from violence and dies for freedom and the greater the sooner is it consumed. Drives away in a rage what is opposed to her decay; she wants winning, to kill by its causes any constraints and winning, it kills herself. It becomes stronger where it finds a stronger contrast. Nothing will move without its. The body from which it originates does not change form or weight.[47]

It seems to define the impetus of scholastic conception, which is generated in the bodies by the motion transmitted to it by another body, for example by the hand that launches a stone.

Leonardo distinguishes between natural gravity and accidental gravity. The former is the ordinary one and is invariant; the latter is not clearly defined or at least is not defined in a unique way. According to Duhem (Duhem 1906, I, 114–115), the schoolmen used this term as a synonym of impetus and Leonardo, following the ideas of Albert of Saxony who assumed the natural gravity concentrated in the centre of gravity, would consider also the accidental concentrated in a point, named the centre of accidental gravity:

> Each body has three centres of figure, one of which is a natural centre of gravity, the other of the accidental gravity and the third one of the magnitude.[48]

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[47] "Forza, dico essere una virtù spirituale, una potenzia invisibile, la quale per accidentale esterna violenza è causata dal moto e collocata e infusa ne' corpi i quali sono dal loro naturale uso retratti e piegati, dando a quelli vita attiva di maravigliosa potenzia; costrigne tutte le create cose a mutazione di forma e di sito; corre con furia alla sua desiderata morte e vassi diversificando mediante le cagioni. Tardità la fa grande e prestezza la fa debole; nasce per violenzia more per libertà. E quanto è maggiore, più presto si consuma. Scaccia con furia ciò che si oppone a sua disfazione, desidera vincere, uccidere la sua cagione, il suo contrasto e, vincendo, sè stessa occide. Fassi più potente, dove trova maggior contrasto. Ogni cosa volentieri fugge sua morte. Essendo costretta, ogni cosa costrigne. Nessuna cosa sanza lei si move. Il corpo dove nasce non cresce in peso nè in forma. (da Vinci, *Ms. A*, 34v. See also: da Vinci 1940, 253–254). English translation is mine.

[48] "Ogni corpo di disforme figura ha 3 centri, de' quali l'uno è centro della gravità naturale, l'altro dell'accidentale e l 3° della magnitudine". (da Vinci,

In other cases, Leonardo seems to give a different meaning to the accidental gravity. For instance (cfr. Marcolongo 1937, 64) the centre of accidental gravity coincides with the centroid of a system, composed by accident of many components. This description could be compatible with the other, because in the forced motion, by accident, actions are focused on the accidental centre, so in the case of weights joined by *accident* all motion behaves as if the centre of gravity were a point that is the centre of gravity of no body. As regards the term weight, Leonardo uses it as in the modern Italian, to indicate either a heavy body, or the weight of a heavy body. When a body is constrained, the weight is often understood as power, a measure of the effectiveness of gravity according to site. For example, a weight of three pounds that slides on an inclined plane with a ratio between height and length of 2:3, weighs two lbs. Leonardo speaks of weight also to indicate the tension of ropes, designed as a portion of the weight carried by them, considered as the portion of the weight supported.

2.2.3.2 The Balance and Lever

Leonardo da Vinci, instead of the term *lever* (lieva) prefers *balance* – sometimes *scale* – which for him does not necessarily have equal arms. The *lieva* is thus to indicate the balance arm placed where resistance is located, while the *contro–lieva* is the other arm to which power is applied. Note that Leonardo avoids separate treatments of the lever, balance and wheel and axle, as done by del Monte, considering all of one type, as defined by the balance. Of course, da Vinci knows the law of the lever. He does not report, however, demonstrations of it but merely terms. The applications of Leonardo are of such richness that they have a theoretical value in themselves because they both offer new issues, which could only be imagined by an engineer and not a mathematician or a humanist, and because the proposed solutions, although not supported by experiments, are very stimulating. One of the innovations in the texts of Leonardo da Vinci compared to the traditional science of weights is the use of forces (modern term) applied to the arms of the balance or lever by means of ropes connected to weights with the use of pulleys which modify the direction of application.

In order to understand da Vinci's use of quantitative expressions, the mathematics of time based on proportions must be taken into account. Here the determination of an unknown term was not immediate and instead of writing a simple algebraic equation, as we would do today, it required

Co*dex Atlanticus,* 188v(b); See also: da Vinci 1940, 45). English translation is mine.

algorithms now obsolete, including that of the three simple steps derived by the treatise of the abacus. According to the use of this treatise, Leonardo da Vinci often exposes his results, not with propositions having general character, but with numerical examples. They have the function to exemplify the general laws for it is not difficult to imagine that the chosen numbers could be replaced by other numbers. It would therefore represent the need for Leonardo da Vinci to move from his geometrical language based on arithmetical proportions to an early algebraic language which is not formalized enough because of the difficulties in deposing of efficient algebraic rules.

> Even with the rule of three one can say: in arms *ab* and *bf* that are 2 and 3, who exchanges suspended weights according to the proportions, they will resist to the descent one of; thus the 5, weight placed in the arm of two spaces resists to weight of 2 placed in the 3 spaces. So you will say for rule of 3: if the 2 of *ab* located in *f* would change in 6 and *f*, which would as to change 5 of *bf* placed, it would be 9 and so inversely, knowing the weight *a* and looking for weight *f*.[49]

In the following passage, Leonardo da Vinci proposes a rule much more complex to calculate the counter–weight:

> RULE TO FIND A COUNTERWEIGHT TO A GIVEN WEIGHT IN ONE OF THE ARMS OF THE BALANCE. Multiply the number of times the arm [*b*] of the counterweight contains the other arm [*a*] by the number of the given weight [*p*], then divide the weight [*p*] with this result [*q*], and multiply the result by the number of weight [*p*]. This result will give the searched counterweight [*r*] to the given weight.[50]

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[49] "Ancora colla regola del 3 potrà dire: ne' bracci ab e bf, che son 2 e 5, chi scambia e' pesi attaccati secondo le proporzioni, essi resisteranno al discenso luno dell'altro, onde il 5, peso posto nel braccio di due spazi resiste al peso di 2 posto ne li 5 spazi; onde dirai per la regola del 3: se 'l 2 di ab posto in f trasmutassi in 6 che in f, il che sarebbe a trasmutare il 3 di bf posto in [?] sarebbe 9 e così de converso, sapendo il peso a e cercando del peso f". (da Vinci, *Codex Windsor*, 12602v. See also da Vinci 1940, 76). English translation is mine.

[50] "REGOLA DA TROVARE IL CONTRAPPESO A UN DATO PESO NELL' UN DE' BRACCI DELLA BILANCIA. Multiplica il braccio del contrappeso per tante volte il numero del dato peso, quante sono le volte che esso riceve in sè il suo opposite braccio, e colla somma parti il numero del peso, e quel che ne viene rimultiplica con esso numero del peso, e co' la resultata somma arài fatto il debito contrappeso al già dato peso". (da Vinci, *Codex Atlanticus*, 309r(d). See also da Vinci 1940, 86; Author's capital letters). English translation is mine.

Basically if p is the weight, a the length of the lever, b that of the counter-lever, r the counterweight, Leonardo performs the following calculations: multiply the weight p by the ratio of the lengths of the arms getting the result $q = p \times b / a$; divides then p *by* the result q and multiply again by p: p: $q \times p$ and obtain $a / b \times p$, which is not difficult to verify to be the correct value of the counterweight (r). Marcolongo (Marcolongo 1937, 31–32) argues that the previous quotation was written before 1500, subsequently Leonardo would have given up this complicated rule for the simpler rule of the three. In addition to the relationship between forces in the lever, Leonardo also knows that between displacements:

> That proportions that the length of the lever will have with its counter–lever, this same proportion you will find in their weights and similarly in the slowness of motion and in the path made by each of their ends when they arrive to the permanent height of their pole.[51]

Leonardo also knows how to handle balances with more weight hanging from them (cases also considered by Thābit and de Nemore) and thus address the case of balances whose arms are endowed with weight, by concentrating it in their centre of gravity.

Of interest is Leonardo’s comment on the triangular balance, the *Equilibra*, proposed by Leon Battista Alberti (Alberti 15th, Alberti 1973; Di Pasquale 1992).

[51] “Quella proporzione, che arà in sè la lieva colla sua contralíeva, tale proporzione troverai in nelle qualità de’ pesi, in nella tardità del moto e in nella qualità del cammino fatta da ciascuna loro stremità, quando sieno pervenute alla permanente altezza del loro polo”. (da Vinci, *Codex Atlanticus,* 173r(a). See also da Vinci 1940, 165). English translation is mine.

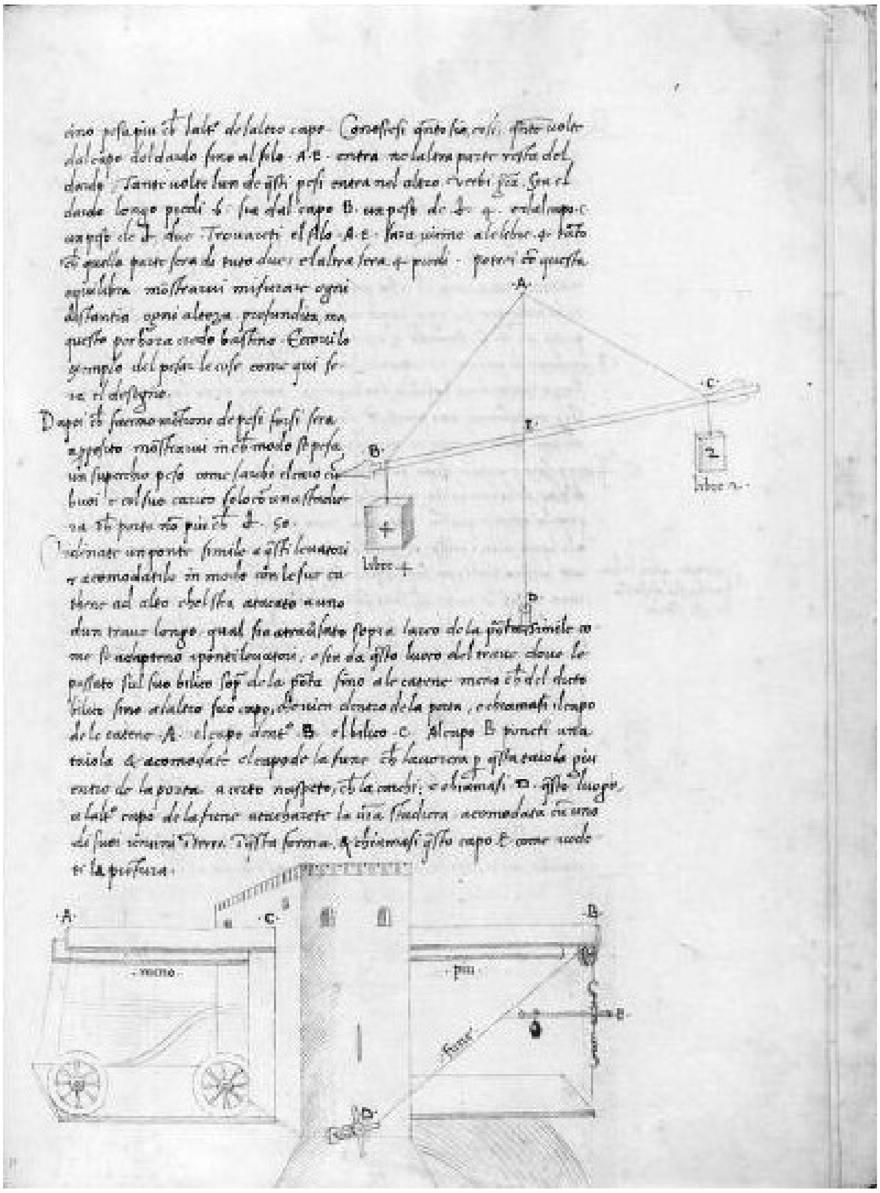

Fig. 2.8a Leon Battista Alberti Alberti *Equilibria*[52]

[52] Alberti 15th, *Ms 422.2*, 10rv. With Permission of the President and Fellows of the Harvard College Copyright. The Houghton Library. The Harvard University Cambridge–MA, U.S.A.

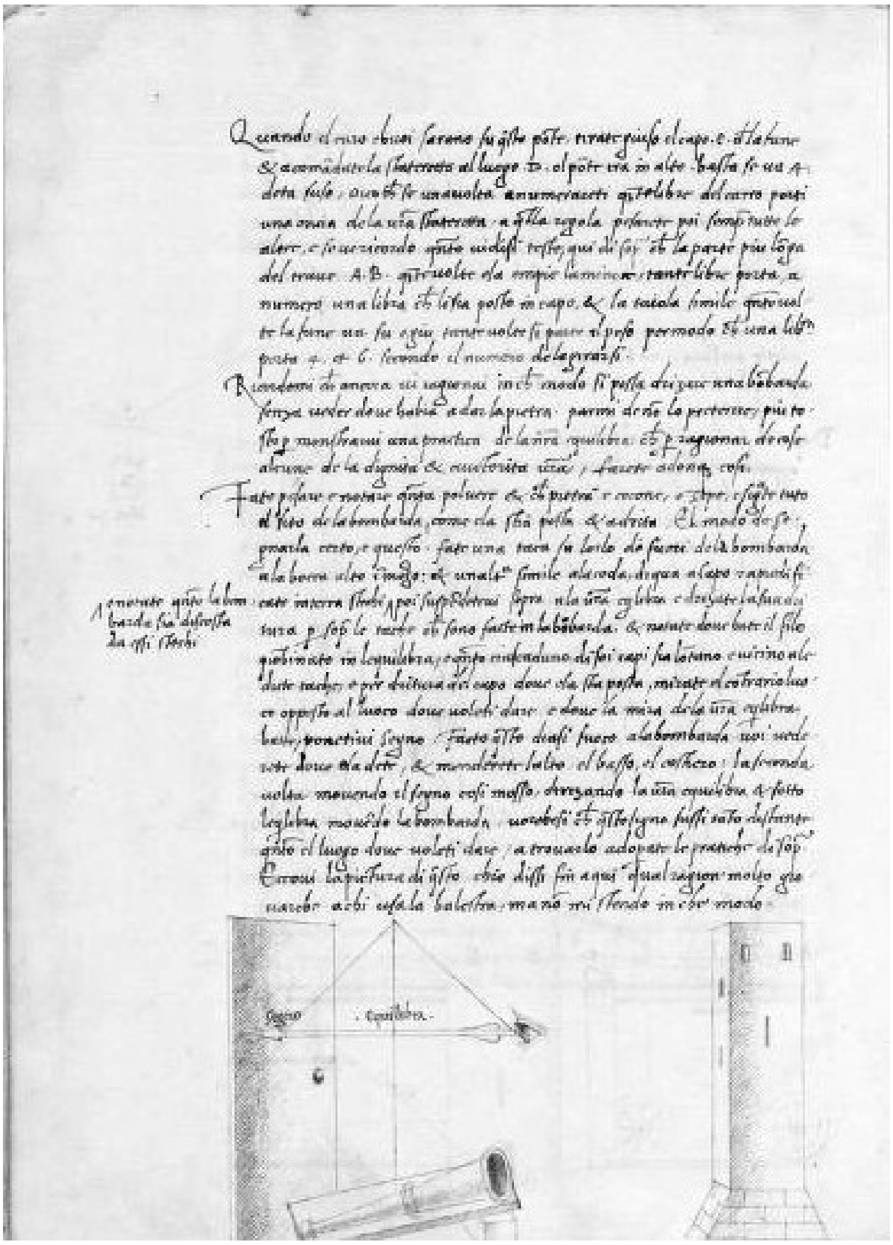

Fig. 2.8b Leon Battista Alberti, *Equilibria*[53]

[53] Alberti 15[th], *Ms 422.2*, 10rv.

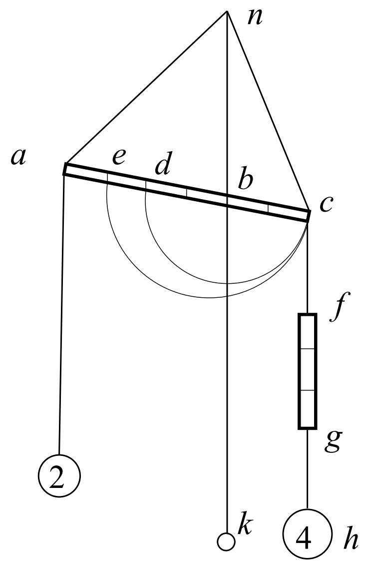

Fig. 2.8 The triangular balance according to Leonardo da Vinci[54]

Although Alberti suggests building the *Equilibra* with a rod connecting the ends of a wire longer than the rod and suspended in the middle point (see Fig. 2.8), Leonardo considers from a theoretical point of view the *Equilibra* as a balance with equal arms with the fulcrum located at the top With this balance one can determine a weight *P* of any one value with a fixed known counterweight *p*. With reference to Fig. 2.8 the following relation of proportionality holds true: $ab : bc = P : p$.

Leonardo da Vinci argues that in reality things do not go that way because of the weight of the rod:

> Battista Alberti says in a work titled *Ex ludi rerum mathematicarum*: that when the balance *abc* will have the arms *ba* and *bc* in double proportion, with weights suspended from its ends, that dispose it such way, they are in the same proportion of arms, but converse, that is, the more the weight the smaller the arm [See Fig. 2.8][55].

[54] Redrawn from da Vinci, *Codex Arundel*, Ludi matematici 66r.

[55] The title should be in Italian: *Ludi matematici*, as the book was in Italian vulgare. Its original dedication was however: "Leonis Baptistae Albertis ad Illustrissimum principem dominum Meliadusium Marchionem Estensem ex Ludis Rerum mathematicarum". From that, it can be deduced that the original title was probably *Ludi rerum mathematicarum*. Indeed a Latin title for a work in vernacular was a quite common use of the time".

> […] Which the experience and reason show to be a false proposition, because he puts the opposite weights 2 vs 4 in a balance, which in itself weighs 6 pounds, it is 7 vs 2, and so the balance will remain at rest with equal resistance of arms. And here he wandered, for not to mention the weight of the beam of the balance which is unequal in weight.[56]

It must be said that Leonardo is not consistent and when he uses Alberti's *Equilibra* he does this without taking into account its own weight. Leonardo da Vinci is not exempted from the examination of the equal arm balance and weights, which had been and would be a key paradigm of the science of weights. His conclusion is the same as de Nemore; when the arms are horizontal, the balance is in a stable equilibrium configuration and resumes its configuration if moved so

> [...] balance with equal arms and weights removed from the site of equality will make unequal arms and weights, so necessity constraints it to acquire again the lost equality of arms and weights.[57]

Here it is not entirely clear why Leonardo speaks of unequal arms, unless he wants to consider, as shown in some of his drawings, and differently from the medieval science of weights, the descents of weights converging toward the centre of the earth.

The circular balance instead is for Leonardo da Vinci in a state of neutral equilibrium, because of polar symmetry. The indifference changes in to stability, however if two consistent weights are added:

[56] "Alla qual cosa la sperienza e la ragion li mostra essere falsa proposizione; perché dove lui mette li pesi oppositi 2 contro 4 nella bilancia che in sé pesa 6 libbre, vole essere 7 contro 2; e così resterà la bilancia ferma con equali resistenzia di braccia. E qui errò esso altore per non far menzione del peso dell'aste della bilancia, che è ineguale di peso". (da Vinci, *Codex Arundel* 66r. See also da Vinci 1940, 101–102). Here Leonardo's calculations do no sound right.

[57] "La bilancia di braccia e pesi uguali, remossa del sito dell'equailità farà braccia e pesi inequali, onde necessità la costrigne a riacquistare la perduta equalità di braccia e di pesi". (da Vinci, *Ms. E*, 59r. See also da Vinci 1940, 74–75). English translation is mine.

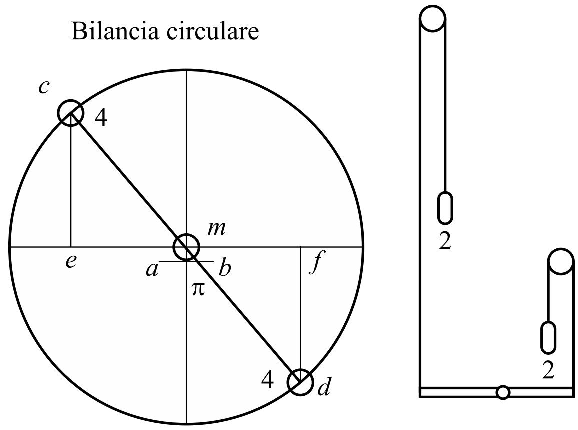

Fig. 2.9 Equilibrium for the circular balance[58]

> CIRCULAR BALANCE. This circular scale [See Fig. 2.9] for it be of uniform gravity, to any lines around its pole, does not completely make the office which would do the common scale, i.e., that which, when moved from the site of equality, it returns there by itself. But this, having heavy weights equally distant from its centre, being removed from the site of equality, it itself does return there. But I think it would return, if the weights attached to it largely overcomes the weight of that wheel.[59]

2.2.3.3 The inclined Plane Law

The equilibrium of weights posed on inclined planes was studied by Heron, Pappus and de Nemore. Only the latter had obtained a *correct* solution.

Leonardo da Vinci, as happens in the school of Jordanus de Nemore, does not typically consider a single inclined plane but two opposing planes, on each of which two weights are arranged connected by a rope that passes over a pulley disposed at the intersection of the planes. He does not always refer to the law of the inclined plane in the same way.

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[58] Redrawn from da Vinci, *Codex Atlanticus* 1018 [new numeration].

[59] "BILANCIA CIRCULARE. Questa bilancia circulare, per essere lei d'uniforme gravità, per qualunque linia intorno al suo polo, essa non fa totalmente tutto l'uffizio che farebbe la bilancia comune, cioè, che quella, essendo mossa del sito della equalità, essa per sè medesima vi ritorna; e questa, avendo e' pesi equahnente pesanti e distantí dal suo centro, essendo remossa del sito della equaütà, essa per sè non vi ritorna" (da Vinci, *Codex Atlanticus*, 365r(a). See also da Vinci 1940, 103). Author's capital letter. English translation is mine.

Generally speaking he correctly states that the effectiveness of the weight decreases with the obliquity, using a term and a concept typical of the school of de Nemore: the term obliquity to mean the inclination of a plane from the vertical and the concept of gravity of position according to which the effectiveness of a weight varies with the obliquity. The problem is that Leonardo does not always measure obliquity in the same way. Sometimes he measures it as the ratio between base and height, sometimes as the ratio between length and height of the plan; this way, as well known today, is the correct one. Leonardo provides an explanation of the different efficacy of weights disposed on an inclined plane, stating that the weight that moves on the more oblique plane undergoes a greater resistance (da Vinci 1940, p 109). Therefore, Leonardo seems to consider the effectiveness of the weight determined by the effectiveness of the constraints and not by the variation of gravity, which often he claims to be invariable.

In the following passage, the obliquity is clearly measured by the ratio between the base and the height, in this way the effectiveness of the weights depends on the cotangent of the angle formed by the inclined plane with the horizontal. Leonardo da Vinci did not realize that in this case, when the plane becomes vertical, one faces a relationship between a finite value and zero.

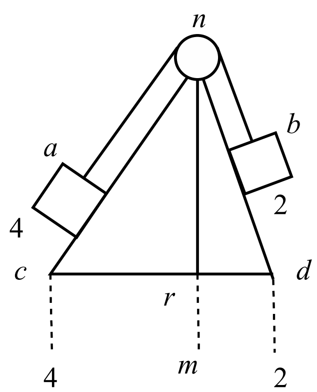

Fig. 2.10 Equilibrium of two weights on an inclined plane by Leonardo da Vinci[60]

> If the weights *a*, *b* [See Fig. 2.10] do not push toward the centre of the world, for they are separated, their combined centre tends to the centre of the world, as the central line *nm* teaches us passing through the proportions of weights 2 and 4 and for the proportions of the basis of triangles 2 and 4; but the site of them has no proportionate spaces,

[60] Redrawn from da Vinci, *Ms G*, 77v.

> because in the same obliquity a weight may be high and the other low and [the obliquity] will not vary in this situation; the double ratio of the weights will vary in height.[61]

The reading of the following passage seems to show that this time obliquity is measured by the ratio between the length and height of the inclined plane, if for *obbliqua* it means the inclined plane.

> The equality of declinations in accord with the equality of weights. If the proportion of weights and the *obbliqua* [enphasis added] where will they stay will be the same but inverse, the said weight will remain the same in gravity and in motion.[62]

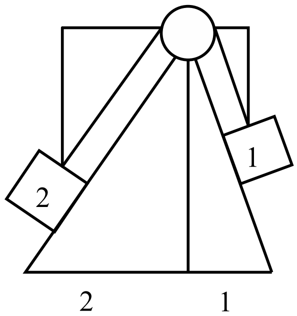

Fig. 2.11 Second case-study concerning the equilibrium of two weights on an inclined plane[63]

Unfortunately, examination of Fig. 2.11 next to the quotation does not allow this interpretation and in this case also the obliquity should be understood as the ratio between the base and height. In the third case-study, Leonardo asserts quite clearly that the obliquity can be measured 'correctly' by the ratio between length and height of the plane, with the

[61] "*Se a b*, pesi, non spingono inverso il centro del mondo, essendo come son separati, il lor congiunto attende a esso centro del mondo, come ci insegna la linia centrale nm che passa per le proporzioni de' pesi 2 e 4 e per le proportioni delle base che hanno li triangoli 2 e 4; ma il sito d'essi pesi non ha spazi proporzionati, perchè nelle medesime obbliquita un peso pò stare alto e l'altro basso e non varierà in tal situazione; varia in altezza, la proporzion de' pesi dupla" (da Vinci, *Ms. G*, 77v. See also da Vinci 1940, 109). English translation is mine.

[62] "La equalità della declinazione osserva la equalità de' pesi. Se le proporzioni de' pesi e dell'obliqua dove si posano saranno equali ma converse, essi pesi resteranno uguali in gravità e in moto" (da Vinci, *Codex Atlanticus*, 981b [new numeration]. See also da Vinci 1940, 110; English translation and is mine.

[63] Redrawn from da Vinci, *Codex Atlanticus*, 981b [new numeration].

following Fig. 2.12 commented with a few words. The balance will be to weight *ab* as weight *cd*[64]

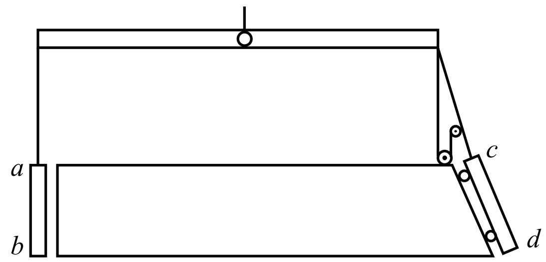

Fig. 2.12 Third case-study: concerning the equilibrium of two heavy prisms on an inclined plane[65]

From Figure 2.12, it is indeed clear how the weights, given by the two prisms of the same thickness, are proportional to the length of the inclined planes. Marcolongo (Marcolongo 1937, 54) saw in this figure, an analogy with Stevin's modelling of weights on the inclined plane by means of a necklace. On the basis of the above and other passages not reported, it can thus be stated with certainty that Leonardo did not possess the law of the inclined plane, except for the observation derived from daily experience that the effectiveness of the weight decreases with the obliquity, and it is also possible that the results he shows be simply uncritical replication of current views of the time.

Finally, one more case-study should be reported that relates to motion rather than the equilibrium of the inclined plane, but which still gives information even for the equilibrium case.

[64] Redrawn from da Vinci, *Ms H*, 81v.
[65] *Ibidem*.

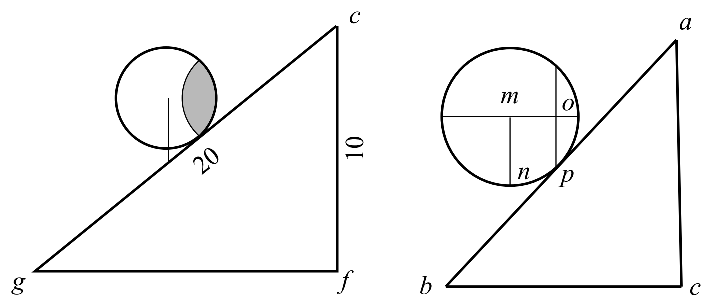

Fig. 2.13 Fourth case-study concerning the motion of a sphere on an inclined plane[66]

> ON MOTION. The spherical body will take by itself a motion the faster the more the contact with the site is farther from the vertical passing through its centre. As much as *ab* is longer than *bc*, so the ball will fall slower for its line *ab*, and as much slower, as the part *o* is less than *m*, because being *p* the pole of the ball, the part *m*, being over *p*, would fall with faster motion, if it there were not but the small resistance which the counterweight *o* makes [Fig.2.13b]. And without this counterweight the ball would descend on the line *bc* the sooner the more *o* is close to *m*, i.e. if the part *o* enters *m* 100 times, [the ball] would descend faster than one hundredth of his time than when the part *o* is missing; *mn* is the line from the centre and *p* is the pole where the ball touches its plane, and the more the space *np*, the faster its way.[67]

In the previous passage, Leonardo asserts that the ball moves with the greater velocity the greater the ratio between the segment *o* and the segment *m* (the sum of which is the diameter of the sphere), and that the part *o* opposes the descent. This analysis seems intermediate between those

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[66] Redrawn from da Vinci, *Ms A*, 52r.

[67] "DEL MOTO. Il corpo sperico e ponderoso piglierà per sè tanto più veloce moto, quanto il contatto suo col loco dove corre fia più lontano dal perpendiculare della sua linia centrica. Tanto quanto ab è più lungo che ac, tanto caderà più tardi la palla per la sua linia che per la linia *ab*, e tanto più tardi, quanto la parte *o* è minore che la parte *m*; perchè essendo *p* il polo della palla, essendo sopra *p* la parte *m*, caderebbe con più veloce moto, se non fussi quel poco della resistenzia che gli fa di contrappeso la parte *o*; e se non fussi detto contrappeso, la palla discenderebbe per la linia af tanto più presto, quanto o entra in *m*; cioè se la parte *o* entra nella parte *m* 100 volte, mancando sempre nel voltare della palla la parte *o*, discenderebbe più presto il centesimo del suo tempo; *mn* è la linia centrica e *p* sia il polo dove la palla tocca il suo piano, e quanto Ha maggiore spazio da *np*, tanto fia più veloce il suo corso" (da Vinci, *Ms. A*, 52r. See also da Vinci 1940, 343). English translation is mine.

by Pappus of Alexandria[68] (ca. 290 AC – ca. 350) and by Heron of Alexandria. The idea that we should consider *p* as a pole is Pappus's, of whom, however, the idea that a force different from zero is necessary to make the ball roll on a horizontal plane is not taken up. The similarity with the analysis of Heron is evident from Fig. 2.13a[69] where it is shown how much the left side exceeds the right one. This is not the only point where Leonardo seems to refer to Heron's *Mechanica*, normally considered to be unknown in the West at least until the XVIII century. One can then make a reasonable guess that the text of Heron was not completely unknown and that Leonardo has become aware of it either directly or indirectly.

2.2.3.4 The Pulley, Block and Tackle

Leonardo considers in depth a subject that was completely ignored by the Middle Ages science of weights; i.e. pulleys and the assembly of pulleys or block and tackles. They were commonly used in machines for lifting weights for military and civil constructions, so it is no wonder that Leonardo considered them. He however knows also the rule that connects power to resistance; this information could have been obtained from his reading regarding traites concerning mechanics, or other available sources.[70]

The pulley is seen by Leonardo da Vinci sometimes as a mere device to divert the action of a tight rope, other times as a circular lever. The following comments are interesting:

> I call circular scale the pulley or the wheel, with which water from wells is drawn, with which it will never be raised more weight than the weight of the drawn water. No heavy body will lift by means of the circular scale with the strength of its sheer weight more weight than its own.[71]

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[68] Cuomo 2004.

[69] Note that this figure will be taken again by Nicola Antonio Stigliola (1546–1623) also known as Colantonio Stelliola (Cfr.: Gatto 1996).

[70] A reasonable conjecture would be that he could have obtained information by some epitome of Heron's text of mechanics (a book intended for architects, containing means by which to lift heavy objects). Nevertheless, even if Heron's *Mechanica* (3 Books) was quite close to the Archimedian ideas circulating in the Renaissance, i.e, shapes, proportion statics problems and balance (Taisbak 1981–1982; Drachmann 1963), it is remarkable that it was preserved only in an Arabic language (Tybjerg 2000). Instead, the idea that theoretical information may be derived also by *Book X* of Vitruvius' *De architectura*, could be less conjectural. In fact, da Vinci could have reasonably known it from the Italian translation due to Francesco di Giorgio Martini.

[71] "Bilancia círculare chiamo la rotella ovver carrucola, colla quale si trae l'acqua de' pozzi, colla quale non si leverà mai più peso che si pesi quello che

> The circular scale, said pulley, being of such relevance in mechanical instruments (maximum in transmutations of forces), is not to be neglected; for with it the power of the motor of said machine is increased, as seen in the block and tackles, where the power grows as much as the number of pulleys. Thus we will define its nature and power, and before will show as the strings without motion support the weight due to the supported heavy bodies, and this we will call natural weight, then we will say of motion, varying the weight supported by the strings and we will name this weight accidental weight, i.e., forces, which grows the more the more the [motion] is faster, but the natural weight never varies. The power of the engine varies with the resistance of moved thing and the air which condenses and resists, as the air in fat of watches.[72]

For assemblies of pulleys, the block and tackles, Leonardo da Vinci refers laws both for forces and displacements:

> THE ROPE, which passes among the pulleys, is named in two ways, the part that gives cause to motion which is fixed to the winch, is named *arganica*, and that which is fixed to the superior pulley and which makes the pulleys neither falling nor slipping is called *ritenente*.
> ON MOTION. The longer the motion of the arganica rope, that moves the weight, which is not the motion of the weight which by means of block and tackles, by this rope is moved, the larger the number of wheels that are in the block and tackle.
> ON TIME. The larger the number of wheels, which forms the block and tackle, the faster the motion of the arganica rope than that of the ritenente rope.

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attigne l'acqua. Nessuno corpo ponderoso leverà in bilancia circulare con forza del suo semplice peso più peso di sè medesimo" (da Vinci, *Ms A*, 62r. See also da Vinci 1940, 104). English translation is mine.

[72] "La bilancia circulare, detta carrucola, essendo di tanta importanzia nelli strumenti machinali (e massime nelle trasmutazioni delle forze), non è da preterire; con ciò sia che mediante quella si multipllca la potenzia al motore delle dette machine, come si vede nelle taglie, dove tanto cresce la potenzia, quanto cresce il numero di tal carrucole; adunque difiniren la sua natura e potenzia, e prima mostreremo come le corde sanza moto sentano equal peso della gravita da lor sostenuto, e questo domanderen peso naturale; poi diren del moto, e che varia il peso che nelle corde si comparte e questo nomineren peso accidentale, cioè forza, la quale tanto si cresce, quanto più si fa veloce; ma il peso naturale mai si varia, variasi la potenzia nel motore insieme colla resistenzia della cosa mossa e della resistenzia dell'aria, che si condensa e resiste, come fa l'aria alla ventola delli orilogi". (da Vinci, *Codex Atlanticus,* 566 [new numeration]. See also da Vinci 1940, 104). English translation is mine.

ON Weight. The larger the number of wheels of block and tackle, the greater the supported weight than that which supports.[73]

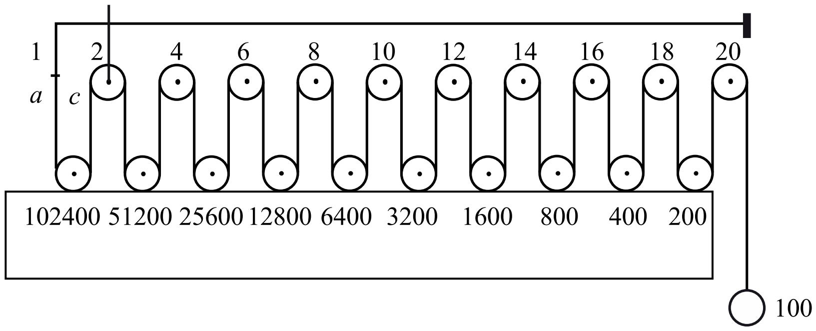

Fig. 2.14 An example of a large block and tackle[74]

If you want to know the weight [the force] of the rope that supports the latest pulley, always multiply the applied weight at the bottom by the number of pulleys, and what this multiplication gives, be the number of pounds that the last rope receives of said weight attached at the bottom. Let thus, may the attached weight be 4, so you say: 4 pounds times 4 pulleys is 16 numbers, and then say: 4 times 16 is 64, and the rope it supports 64 pounds for the 4 applied by at the bottom, and if they were 6 pulleys, you would say: 4 times 6, 24, and 4 times 24, 98,[75] and this the weight that the last rope of 4 pounds attached at the bottom sustains.[76]

[73] "LA CORDA, che passa infra le taglie ai sua stremi, in due modi nominati, quella parte che dà causa al moto che si ferma all'argano, si nomina, arganica; e quella ch'è ferma alla superiore taglia, che non lascia scorrere nè cadere le taglie, è detta ritenente. DEL MOTO. Tante volte fia più lungo il moto della corda arganica che 'l peso move, che non è il moto del peso, che, mediante le taglie, per essa corda è mosso, quanto è il numero delle rote che in esse taglie stanno. DEL TEMPO. Tanto quanto fia il numero delle rote, che nelle taglie stanno, tanto fia più veloce il moto fatto dalla corda atganica, che quello fatto dalla corda ritenente. DEL PESO. Quanto fia il numero delle rote delle taglie, tanto fia maggiore il peso sostenuto, che quello che sostiene". (da Vinci, *Codex Atlanticus*, 882 [new numeration]. See also Vinci 1940, 496. Author's italic). English translation is mine.

[74] Redrawn from da Vinci, *Ms A*, 52r.

[75] It should be 96; 24 times 4.

[76] "Se voi sapere che peso ha la corda che sostiene l'ultima carrucola, multiplica sempre cubicamente il peso appiccato da piè col numero delle carrucole, e quel che di tal multiplicazione resulta, fia il numero delle libbre che tale ultima corda riceve di detto peso attaccato da piè. Diciamo adunque ch'esso peso attaccatto da piè sia 4, onde tu dirai: 4 libbre vie 4 carrucole fa 16 numeri; e poi dì: 4 vie 16 fa 64; ed è multiplicato cubicamente, e essa corda di sopra sostiene 64 libbre per le 4

No explicit rule is proposed but examples sufficiently clear are made, as typical in the mathematics of abacus. The explanation of the operation of the block and tackle sometimes seems that proposed in *Problemata mechanica* which calls for the law of lever (Aristotle 1955c, 852b, 367–370), sometimes that of Heron who assumed a constant stress in the ropes which encircles the pulleys an thus the whole weight lifted is given by the resultant of all the rope forces of the block and tackle. This type of reasoning is reported in the following quotation:

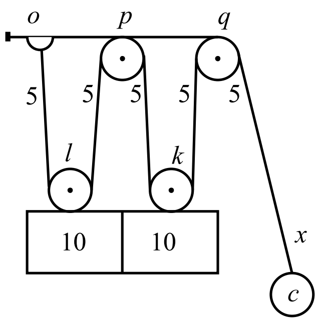

Fig. 2.15 Model concerning the evaluation of the power to necessary to lift a given weight by means of a block and tackle[77]

> If you want to supply the block and tackle of 4 ropes, which block and tackle has to lift 20 pounds [Fig. 2.15]. I say that the wheel *l* will support 10 pounds, and the wheel *k* will support 10, which are transferred to they higher supports, that is, *o* takes 5 pound from *l* and *p* also takes 5 from *l*, and 5 from *k*, and this same *k* will take 5 from *q*. And whoever wanted to win the 5 of *q*, put 6 into the counterweight *x*, and putting the last place 6 against 5 of each of the 4 ropes that support 20 pounds, not supporting itself more than 5 pounds, the one pound more that I put in the rope *qx*, find no resistance in the opposed ropes equal to it, all wins and all moves.[78]

appiccate da piè; e se esse carrucole fussino 6, diresti: 4 via 6, 24, e 4 vie 24, 98; e tanto peso sostiene l'ultima corda delle 4 libbre attaccate da piè". (da Vinci, *Codex Foster II*, 82v. See also da Vinci 1940, 501). English translation is mine.

[77] Redrawn from da Vinci, *Ms A*, 62r.

[78] "Se tu voi incordare le taglie in 4 doppi, le quali taglie abbino a le- vare 2o libbre di peso, dico che la girella l sosterrà 10 libbre, e 10 ne sosterrà la rotella k, le quali si trasferiscano a' sua superiori sostentaculi, cioè o piglia da l 5 libbre, e 5 ne piglia ancora p da l, e 5 da k, e questo medesimo k ne da 5 a q; e chi volessi vincere le 5 di q ne metta 6 nel contrappeso x, e mettendo in l'ultimo loco 6 contra 5 in ciascuna delle 4 corde che sostengono le 20 libbre, non sentendo per sè se non quelle 5 libbre, quella libbra più ch'io metto nella corda gx, non trovando in

Note that Leonardo distinguishes motion from equilibrium and to obtain motion the power should be a little greater than the resistance; in the previous quotation 6 vs 5. Quite interesting is the Fig. 2.16. This is a situation that it actually occurs in practice when the pull of the rope is relatively low compared to the friction.

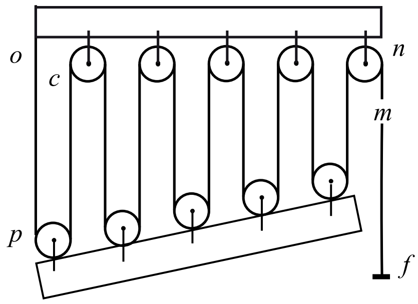

Fig. 2.16 Behaviour of a block and tackle for the effect of friction[79]

Leonardo also poses other problems in block and tackles, such as the way the stress in the rope varies with motion, the location where the rope is more stressed and thus where it breaks more easily, the effect of the diameter of the ropes on the effectiveness of the pulleys, the load carried by the supports of the pulleys. His comments are not always flawless, but are notwithstanding interesting to any readers, and are perhaps the most interesting of Leonardo's contributions to block and tackle theory.

2.2.3.5 The Concept of Momento of a Force

In presenting some of Leonardo's quotations, because of the uncertainty of dating we attempted a *rational reconstruction*. According to this reconstruction Leonardo would have developed the idea of *potential arm* in his study on the equilibrium of levers, introducing the concept, if not the term of *moment of a force*. The potential arm of a lever for Leonardo da Vinci is both the distance between the line of action of a *power* from the fulcrum and the imaginary-material arm, orthogonal to the power, which could replace the real arm. Then he would have extended this concept to the study of the composition of forces. It is however possible, that there were not two distinct phases and the idea of potential arm was driven by the need to solve the

nessuna delle opposite corde pari peso a sè, tutte le vince e tutte le move". (da Vinci, *Ms A*, 62r. See also da Vinci 1940, 499). English translation is mine.

[79] da Vinci, *Codex Arundel*, 96r. On the friction in Leonardo da Vinci's studies see also the *Banco for studies on friction* (da Vinci, *Codice Arundel*, 40v–41r; da Vinci, *Ms L* 11v; see also Pisano 2009, 2013).

problem of the composition of powers. Notice that Leonardo da Vinci to indicate what we commonly call force uses terms like power and weight, so we will do the same in the following.

The first time the idea of potential arm appears, according to our reconstruction is in the study of the balance in which weights are suspended through pendants. In this situation, Leonardo assumes that the weights tend toward the centre of the world and then the pendants are not vertical but convergent (Fig. 2.17):

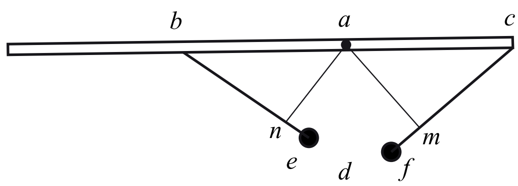

Fig. 2.17 Balance with converging pendants[80]

Leonardo da Vinci is not explicit but everything suggests that the potential arms are those marked with *an* and *am* (See Fig. 2.17). The closer the balance is to the centre of the world *d*, the shorter they are.

> Each of the arm of the balance is double; one of which is real, the other potential and they are located in different places with ends distant from each other.[81]

The real arms are always longer than the potential arms and the longer the closer to the centre of the world.

> The real arms are not in the same proportion between them as the potential arms, but the more different the closer to the centre of the world.[82]

[80] Redrawn from da Vinci, *Ms A*, 62r.

[81] "Ciascuno de' bracci della bilancia è duplo; de' quali l'uno è reale e l'altro potenziale e son posti in diversi siti distanti con l'estremi l'un dall'altro, e son di varie lunghezze" (da Vinci, *Codex Atlanticus*, 338 [new numeration]. See also da Vinci 1940, 70). English translation is mine.

[82] "Sempre le braccia reali della bilancia sono più lunghe di quelle potenziali e tanto più quanto esse sono più vicine al centro del mondo" (da Vinci, *Ms E*, 64r. See also da Vinci 1940, 72). English translation is mine.

Subsequently the idea of potential arm, although not explicitly named, is used in the study of equilibrium of an angular balance. In the following quotation the rule of angular balance is worded clearly enough and makes clear that equilibrium is determined by weights and their distances from the fulcrum measured horizontally.

> RULE OF THE ANGULAR BALANCE. The angular balance is a balance for which the conjunction of its arms is angular; the pole being located in the angle. Arm means where the centre of suspended weight falls. The distances of the opposite ends of the angular balance from the central line of the pole have always the same proportion of the lengths of the arms of the balance, but with inverse order. Let consider the angular balance *cef* [See Fig. 2.18] the pole of which is in the corner *e*; the opposite extremes *f* and *c*, have their distances from the central line ab in the same proportion of the length of the arms *c* and *f*, but converse: i.e. the smallest arm has its end farther from the centre as much as it is smaller than the greatest. And so the distance of the greatest arm from the central line, is as lower as its arm is greater than the lowest. Here the portions of circles are not equal to the motion of the arms, but in the distances from the central line.[83]

[83] "REGOLA DELLA BILANCIA ANGULARE. La equilibra angulare è quella della quale la congiunzione delle sue diritte braccia è angulare; nel quale angulo il suo polo è collocato. Braccio si intende dove cade il centro del peso appiccatovi. Sempre le distanzie che hanno li oppositi stremi della bilancia angulare dalla linia centrale del polo suo han nella medesima proporzione qual'è quella che hanno le lunghezze delle braccia d'essa bilancia infra loro; ma sia proporzione conversa. Come dire della bilancia angulare *cef*, de la quale il polo è nell'angolo *e*, che lo stremo *f* e *c* oppositi hanno nelle loro distanzie dalla linia centrale ab tal proporzione qual'è quella della lunghezza delle sue braccia ec e ef; ma è conversa: cioè che 'l braccio minore ha il suo estremo tanto più discosto dalla centrale quant'egli è minor del suo maggiore. E così lo spazio, che ha il braccio maggiore da tale linia centrale, è tanto minore quanto il suo braccio è maggiore che 'l suo minore. Qui le porzíon de' cerchi non sono equali nel moto de' bracci, ma sì nelle distanzie dalla linia centrale". (da Vinci, *Codex Arundel*, 32v. See also da Vinci 1940, 99). English translation is mine.

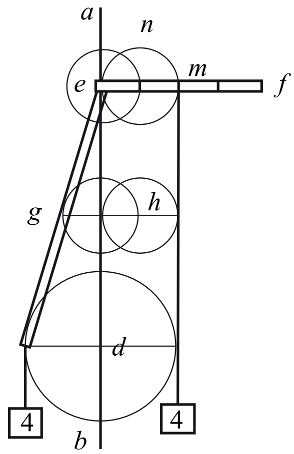

Fig. 2.18 Leonardo da Vinci's angular balance[84]

Notice the particular type of angular balance studied by da Vinci, made up of two arms with uniform section endowed with weight. The weight of the arms is concentrated in their centre of gravity; equilibrium requires inverse proportionality between the weights and the distances of their centres of gravity from the vertical line passing through the fulcrum. The idea for the study of this particular kind of balance is likely to have derived from Leon Battista Alberti's *Equilibra*.

Leonardo da Vinci continues his analysis of the angular balance extending the concept of potential arm to the case of straight levers in which weights are applied obliquely by means of ropes. Probably the most explicit statement regarding the introduction of the term potential arm occurs in some pages from which the drawings (See Fig. 2.19) are obtained. In particular, for the first drawing Leonardo da Vinci writes:

> This is told the true end of the arm of the balance, the connection of which with the line of the rope, loaded by the weight, will be made according to the right angle as you can see in *s* with *ma* and similarly in *pn* with *na* (spiritual arm).[85]

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[84] Redrawn from da Vinci, *Codex Arundel*, 32v.

[85] "Quello è detto vero termine di braccio di bilancia, il quale giungendo la sua retta colla rettitudine della corda, tirata dal peso, questa congiunzione sarà fatta componendo l'angolo retto come si vede in *s* con *ma* e similmente *pn* con *na* (braccio spirituale)" (da Vinci, *Ms M*, 40r. See also da Vinci 1940, 170). English translation is mine.

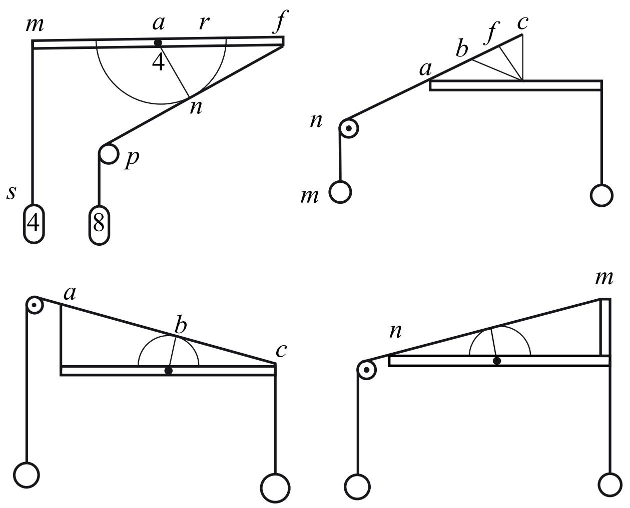

Fig. 2.19 Instances of potential arms in varius kinds of lever[86]

These figures leave no doubt that at least from a certain period and Leonardo considered the effectiveness of a power in order to equilibrate a balance, or more generally of a rigid body constrained to a point, a 'circonvolubile', determined only by the value of the powers and the distance of its line of action from the fulcrum. This is also the opinion of Duhem (Duhem 1905-1906, I, pp 24-25) who however balances his positive opinion with the statement that Leonardo's mechanical writings there are no essential ideas, which were not present in the writings of the mathematicians of the Middle Age.[87] Duhem certainly refers to the fact that the idea of potential arm was contained *de novo* in the writings of Jordanus de Nemore. The latter in *Liber de ratione ponderis*[88] (de Nemore 1565, 6r) studied the case of an angled lever with equal weights, arguing and demonstrating that equilibrium is achieved when the two weights are at the same distance measured horizontally from the fulcrum. In another point, de Nemore also stated that the parameter determining the balance of a

[86] From top to bottom and left to right: da Vinci, redrawn from *Ms H*, 40r, 50v, 39v, 39v.

[87] Cfr.: Duhem 1905–1906, I, 192.

[88] As announced in the Chapter 1, this works belongs to Jordanus de Nemore and generally assigned to be dited by Tartaglia and posthumously published by Curtio Troiano in 1565.

body is given by the horizontal distance measured from the fulcrum (de Nemore 1565, 10v). One could go further back and climb up to Heron and Archimedes who knew the law of angular balances (Capecchi 2012a, 53). However, Leonardo in our opinion has gone much farther. The argumentation of de Nemore on the angular lever, only referred to weights hanging from the balance, was based on the analysis of their descent and ascent. He could hardly have carried out his argumentation in the case of weights suspended from inclined ropes.

In common expositions of the history of mechanics, this discovery of Leonardo is often attributed to Giovanni Battista Benedetti in his *Diversarum speculationum mathematicarum physicarum et liber* (Benedetti 1585). This attribution can find a partial justification in the fact that Benedetti proved his result, albeit not entirely convincingly, and that the text of Benedetti had a wide circulation while Leonardo da Vinci's has remained hidden to most.

2.2.3.6 The Law of Composition of Forces

Probably the most important of Leonardo da Vinci's contribution to statics concerns the rule of composition-decomposition of a force along two given directions. The problem to be solved was to find the tensions of two inclined ropes supporting a weight. To remove any ambiguity, the forces of the ropes also were associated with weights.

Da Vinci besides formulating the rule also correctly proved it. This is normally not recognized by historians and even Duhem suggested only as a possibility that Leonardo understood the rule; only Marcolongo asserted his priority with no doubt.[89] The analysis of texts has however led us to believe that in this case Marcolongo's analysis is correct and actually Leonardo recognized the rule of weight distribution in two ropes supporting a weight. There are of course, as typical in Leonardo, situations in which the rule is loosely worded, and sometimes wrongly. But, although there are no certain dating criteria, the analysis of the manuscripts shows a long series of examples with a lot of correct arguments that can leave no doubt that Leonardo reached a conscious knowledge of the rule of composition of forces (Capecchi 2012b).

The following quotations start from the intuitive finding that the weight distribution depends on the obliquity of the ropes.

> ON WEIGHT. If two ropes converge to support a heavy body, one of which is vertical the other oblique, the oblique one does not sustain any

[89] Note that Duhem did not study the fundamental *Codex Arundel*.

> part of the weight. But if two oblique ropes would support a weight, the proportion of weight to weight would be as the obliquity to obliquity. For ropes that descend with different obliquity from the same height, to support a weight, the proportion of the accidental weight of the ropes is the same as that of the length of these ropes.[90]

From these passages it could be deduced that by the term *obliquity* Leonardo refers to the slope rather than to the length of the ropes – see the final part of the previous quotation – while the accidental weight could be understood as the tension of the ropes. The statement is patently incorrect, but one could think that Leonardo had become confused and meant to speak of the inverse ratio of obliquity, which is still wrong but at least the tendency is correct. The analysis of the following passage shows however, that Leonardo's statement is not a typo, because he clearly states that the weight is divided into proportion of the angles formed by the ropes with the vertical, which is clearly false:

> Let consider two lines concurring in the angle which sustains the weight, if you draw the perpendicular which divides this angle, then the weights [tensions] of the two ropes have the same ratio as that of the two angles generated by the above division. If between the two lines *ac* and *ec*, which form the angle *c*, from which the weight *f* is suspended, the perpendicular dc is drawn that divides this angle into two angles *acd* and *dfe*, we say that these ropes will receive the weight in proportion equal to that of the two angles they form and equal to the proportion of the two triangles. And the perpendicular that divides the angle of this triangle will split the gravity suspended in two equal parts, because passing through the centre of such gravity.[91]

---

[90] "DEL PESO. Se due corde concorrono alla sospensione d'un grave e che l'una sia diritta e l'altra obbliqua, essa obbliqua non sostiene parte alcun d'esso peso. Ma se due corde obblique concorreranno al sostenere d'un peso,tal proporzione fia da peso a peso, qual fia da obbliquità a obbliquità. Delle corde che da una medesima altezza che con diverse obbliquità discendano alla sospensione d'un peso, tal proposizione fia quella che a tal corda del peso accidentale si congiugne, qual'è quella delle lunghezze d'esse corde" (da Vinci, *Ms E*, 70r. See also da Vinci 1940, 142). English translation is mine

[91] "Quando dalla linia equigiacente discenderan due linie concorrenti all'angolo sospensore del grave, caderà la perpendiculare dividitrice di tale angolo, allora sarà diviso il peso alle due corde d'esso sospensore infra li quali pesi fia la medesima proporzione ch'è quella de' due angoli, generata dalla predetta division del primo angolo; come se dalla equigiacente a e discendessi le due linie ac e ec, concorrenti alla composizion dell'angolo c, al quale angolo si sospenda il peso f, cadessi la perpendiculare dc dividitrice d'esso angolo in due altri angoli acd e dfe; dico che tale corde riceveranno il predetto peso in tal proporzione qua1'è quella che hanno

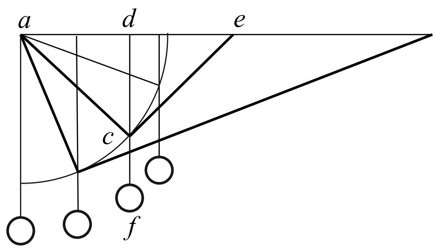

Fig. 2.20 A wrong instance of decomposition of forces[92]

It is difficult to understand how Leonardo could pesent so clearly wrong examples. Perhaps he is thinking of a weight hanging from the middle of a rope in which the greater the obliquity – i.e. the angle they form with the vertical – the larger the tensions in the rope.

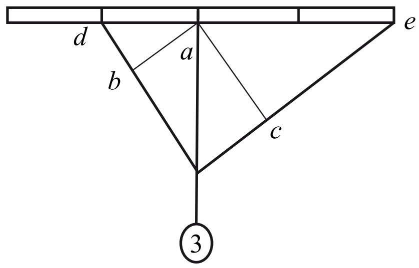

Fig. 2.21 A correct instance of decomposition of forces by Leonardo da Vinci[93]

Marcolongo (Marcolongo 1937) argues, however, that these wrong results date back to the years before 1508, when Leonardo had not yet reached his final idea, which is well expressed in the passage:

> For the 6th and 9th [propositions], the weight 3 [See Fig. 2.21] does not split into the two real arms of the balance in the same proportion of these arms, but in the proportion of the potential arms.[94]

infra loro li due angoli predetti e qual fia la proporzione delle quantità de' due triangoli infra loro. E sempre la perpendiculare dividitrice dell'angolo di tal triangolo sarà dividitrice della gravità sospesa in due parti equali, perché passa per il centro di tal gravità" (da Vinci, *Ms E*, 71r; See also da Vinci 1940, 143). English translation is mine.

[92] Redrawn from da Vinci, *Ms E*, 71r.

[93] Redrawn from da Vinci, *Codex Arundel*, 1v.

Here Leonardo asserts, without proving it, that the suspended weight is supported by tensions *b* (left) and *c* (right) having inverse ratio to the potential arms *ab* and *ac*, i.e.: $b : c = ac : ab$. The relation, correctly, allows us to find the ratio of tensions in the two ropes.

In other passages, Leonardo proves the asserted relation and also indicates the way to evaluate the absolute value of the tension in each rope. He introduces the terms: *potential lever* and *potential counter lever*. The potential lever corresponds somewhat to the potential arm; the potential counter lever is the horizontal segment connecting one support of a rope to the vertical from the suspended weight. The reading of the following quotation is useful to illustrate the use of these terms. The potential lever associated to the arm *fm* is *fe*, the potential counter lever is *fa*.

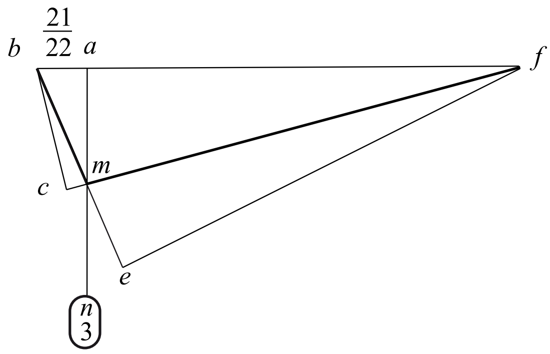

Fig. 2.22 Example of a potential lever and potential counter lever[95]

> Here the weight is sustained by two powers, i.e. *mf* and *mb*. Now we have to find the potential lever and counter lever of the two powers. The lever *fe* and the counter lever *fa* will correspond to the power *mb*. The appendix *eb* is added to the lever *fe*, which is connected with the engine *b*; and the appendix *ab* is added to the counter lever *f*a, which sustains the weight *n*. By having endowed the balance with the power and the resistance of engine and weight, the proportion between the lever *fe* and the counter lever *ab* should be known. Let *fe* be 21/22 of the counter lever *fa*. Then *b* supports 22 when the weight *n* is 21.[96]

[94] "Per la 6° del 9°, il grave 3 non si distribuisce alle braccia reali della bilancia nella medesima proporzione che è quella d'esse braccia, ma in quella proporzione che hanno infra loro le braccia potenziali" (da Vinci, *Codex Arundel,* 1v. See also daVinci 1940, 171). English translation is mine.

[95] Redrawn from da Vinci, *Codex Arundel*, 7v.

[96] "Qui è il peso *n* sostenuto da due potenzie varie, cioè *mf* e *mb*. Ora mi bisogna trovare la lieva e contralieva potenziale d'esse due potenzie *bm* e *fm*. Delle quali

Attention is centred on the rope *bm* with the aim to find its tension. A similar argument can be repeated for the rope *fm*. Basically Leonardo imagines the rope *fm* as 'solidified', i.e. as a rigid beam hinged at *f*. According to his embryonic concept of moment of a force, Leonardo asserts the validity of the following relation: $b : n = fa : fe$, where *b* is the tension of the rope *bm* and *n* is the suspended weight. He gives as an example $fa : fe = 21 : 22$; for $n = 21$ it results $b = 22$.

The previous quotation deserves some comments. First: the idea to solidify the rope anticipates what is commonly called solidification principle, according to which if a body is in equilibrium its state is not perturbed by adding additional constraints. This principle has been used to study deformable bodies by many scientists, including Stevin, Lagrange, Cauchy, Louis Poinsot (1777–1859) and Duhem.

2.3 Tartaglia's Legacy. A Transition between Science of Weights and Modern Statics

At the beginning of the XVI century there was in Italy a broad debate on the role of mathematics in the natural sciences as a result of the increasing use of mathematics in applications and the fact that mathematicians were beginning to give a distinct form of knowledge to their discipline; debate which became even more pressing in the second half of the century. While almost no one denied the fundamental role of mathematics in itself, not everyone agreed on the status of knowledge in regard to the physical world. The importance of the role of mathematics was certainly carried out by supporters of Platonist instances, which in addition to their diffusion through the humanist circles, found their support from a professional mathematician, Luca Pacioli, whose *Summa* (Pacioli 2009; Pisano 2013) was read and appreciated by all the major mathematicians of the early XVI century, Tartaglia, Cardano, Giovanni Battista Benedetti (1530–1590), Federico Commandino (1509–1575). There were, however, even within Aristotelism advocates of the use of mathematics in physics, some who made reference to the Aristotelian theory of subalternate–sciences.

alla potenzia *b* sarà data la lieva *fe* e la contralieva *fa*. Alla lieva *fe* si dà l'appendiculo *eb*, al quale sta appiccato il motore *b*; e alla contralieva *fa* si da l'appendiculo *an*, che sostiene il peso *n*. Avendo ordinata la bilancia della potenzia e resistenzia del motore e peso, è necessario vedere che proporzione ha la lieva *fe* colla contralieva, *fa*. La quale *fe* è li 21/22 della contralieva *fa*. Adunque *b* sente 22, quando il peso *n* fusi 21". (da Vinci, *Codex Arundel,* 7v. See also da Vinci 1940, 179). The translation is mine.

The second half of the XVI century saw the dissemination of Archimedean mathematical (and mechanical) work, which deeply modified the approach to mechanics. Though Archimedes work was influential everywhere, its stimulus was different in different regions. In the Northern school, formed by Benedetti, Tartaglia, Cardano, Archimedes texts received less attention than Jordanus de Nemore or *Problemata mechanica*. The contrary holds for the centre school, formed by Commandino, del Monte, Bernardino Baldi (1553–1617) and the southern formed by Francesco Maurolico (1494–1575), Nicola Antonio Stigliola (1546–1623) and Luca Valerio (1553–1618) (Gatto 1988, 1996, 2006; Galileo 2002; Nastasi 1985).

Table 2.4 Heron, de Nemore, Archimedes' texts published in Italy during the XVI century

	Title	Author
Heronian		
1501	*De expetendis et fugientis rebus*	Valla
1521	*Di Lucio Vitruvio Pollione de architectura libri dece traducti de latino in vulgare affigurati*	Cesariano
1550	*De subtilitate*	Cardano
1575	*Spiritalium liber*	Commandino
1588	*Mathematica collectiones*	Commandino
1589	*Gli artificiosi et curiosi moti spirituali*	Aleotti
1589	*Automata.*	Baldi
1581	*Pneumatica*	Baldi
1592	*Spiritali di Herone Alexandrino, ridotte in lingua volgare*	Giorgi
Nemorean		
1533	*Liber de ponderibus.*	Apianus
1546	*Quesiti et inventioni diverse.*	Tartaglia
1565	*Jordani opuscolorum de ponderositate*	de Nemore
Archimedean		
1543	*Opera Archimedis.*	Tartaglia
1544	*Archimedis Syracusani philosophi ac geometrae excellentissimi Opera*	Cremonensis
1551	*Archimedis de insidentibus aquae* (into Italian)	Tartaglia
1558	*Archimedis opera non nulla*	Commandino
1570?	*Momenta omnia mathematica* (published 1685)	Maurolico
1565	*Archimedis De iis quae vehuntur in aqua libri duo*	Commandino
1588	*In duos Archimedis aequeponderantium libros paraphrasis*	del Monte

2.3.1 Statics in Italy during the XVI century

The medieval science of weights was not sufficient for the needs of the XVI century, because it was confined to a small number of cases, and because it was founded on principles not always shared. In the text of Jordanus de Nemore, *Liber de ratione ponderis*, the most advanced, except for various types of scales, only the inclined plane case is reported. Nothing is said about pulleys or aspects regarding situations of practical interest, such as for example, the horizontal transport of weights, which had also been addressed in the Aristotelian *Problemata mechanica*. Regarding the laws of equilibrium formulated in the *Liber de ratione ponderis*, only those of the lever were unanimously accepted while that of the inclined plane was not known or was not shared. Leonardo da Vinci, Girolamo Cardano, Guidobaldo del Monte, Stigliola, offered alternative solutions, unfortunately not correct.

The reworking of the science of weights carried out by the engineers of the XV century, including that of Leonardo da Vinci, was not sufficient to meet the new requirements of mathematical rigor and development of general laws that would have allowed going beyond the rigid schematism of the medieval statics. This demand was picked up by a new generation of engineer–scientists, with a greater mathematical and philosophical training.

The first representative of this new generation was Niccolò Tartaglia. He gave a clear place to mechanics and introduced many ideas. In ballistics he asserted that the trajectory of a projectile is curved everywhere and nowhere,i.e., there are both straight and circular paths. He also stated that the maximum range of a projectile is obtained by firing with an inclination of 45 degrees and that any intermediate distance may be covered by firing with two different angles. Moreover, he made clear, against the Aristotelian thesis, that the air is an impediment and does not aid motion. He was the ultimate champion of the science of weights adding mathematical rigour to traditional presentations. Starting from a manuscript of de Nemore's *Liber de ratione ponderis* in his possession (de Nemore 1565), he wrote an important section, the book VIII, of his treatise *Quesiti et inventioni diverse* where he revisited in a more organic way Jordanus de Nemore's theory. Nevertheless mainly Tartaglia was the first to use mathematics as the fundamental theoretical tool in the study of mechanical and physical problems, as it will be manifest in the subsequent chapter. Tartaglia, although according to the Aristotelian epistemology conceived mathematical objects as abstracted from matter, assumed that conclusions derived from mathematics are 'true' and should necessarily be verified from an empirical point of view. It that were not the case it would not depend on mathematics but on experience, which was not well exploited.

After Tartaglia, and somehow their heirs, Giovanni Battista Benedetti, Guidobaldo del Monte and Galilei follow. Benedetti made important contributions to the analysis of natural motion of bodies. In statics, he made clear and universally known that the effect of a force depends on the distance of its line of actions from the fulcrum, results that now historians call the law of static moment.

Guidobaldo del Monte attempted the restoration of Greek mechanics in the spirit of Pappus Alexandrinus, whose work was published by Federico Commandino, basing it on an Archimedean approach. He attempted however, a synthesis with the Aristotelian approach of subalternate–science in which physical aspects were clearly present. For example, when studying the balance, he treats of a physical body and not simply a geometrical figure, giving substance also to the fulcrum, which for Archimedes was a simple geometrical point. Del Monte's mechanics was not only a science of the principles of equilibrium of weights on a balance. It was rather a science of machines, Greek meaning; and, even if the equilibrium was crucial as well, the role of the displacement of bodies was examined.

Galileo, well known as the founder of modern dynamics, also made fundamental contributions to statics, somehow managing to reconcile the medieval science of weights, with references to kinematics, with Archimedes' mechanics, purely geometric. However, it was not a true synthesis because he flanked medieval methodologies alongside Archimedean ones without making a decisive choice of field, while expressing a preference for the Archimedean approach.

Below we list a few details about the contribution to statics of the authors mentioned above, highlighting the legacy of Niccolò Tartaglia, whose contribution has been and will be studied in depth in other chapters.

2.3.1.1 Giovanni Battista Benedetti

Giovanni Battista Benedetti received his first and only systematic education in philosophy, music and mathematics from his father. Though never mentioned by Tartaglia Benedetti was nevertheless one of his pupil for a short time. In mechanics, his chief work was the *Diversarum speculationum mathematicarum et physicarum liber* of 1585 (Benedetti 1585). The book deals largely with questions of dynamics; there were however fundamental contributions to statics. Here a concept of static moment of a force, more precisely defined than Leonardo's is referred to. Though the *Diversarum speculationum mathematicarum et physicarum* *liber* may be considered a commentary on *Problemata mechanica*, Benedetti's approach was essentially Archimedean. He criticised both Tartaglia and de Nemore for their kinematic analysis.

The Concept of static moment.

In *Diversarum speculationum mathematicarum et physicarum liber* (Benedetti 1585, Chapter 3, section *De mechanicis*, 141–167) Benedetti made considerations of quantitative character about the effect of a force – associated to a weight attached to a rope or a muscle – on an arm of a balance, however inclined, obtaining the result that it is proportional to the distance of the line of action of the force from the fulcrum and to the force itself. This was at the time an already known result, but Benedetti for the first time formulated this fact as a general law, that now historians call the law of static moment. The main difference between Leonardo and Benedetti's static moment laws does not so much concern the generality of the law but the reference, for Benedetti, to a proof, or at least an intuitive justification.

First Benedetti's argument is developed for the not problematic case of vertical forces:

> From what we have already shown it may easily be understood that the length of B*u* [Fig. 2.23], which is virtually perpendicular from centre B to the line of inclination F*u*, is the quantity that enables us to measure the force of F itself in a position of this kind, i.e., a position in which line F*u* constitutes with arm FB the acute angle BF*u*.[97]

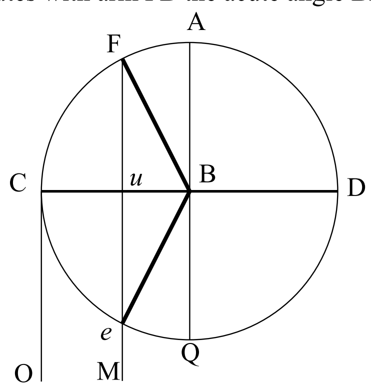

Fig. 2.23 Evaluation of the static moment of a weight[98]

[97] "Ex iis quae nobis hucusque sunt dicta, facile intelligi potest, quantitatis B.u. quae fere perpendicularis es a centro .B. ad lineam .F.u. inclinationis, ea est, quae non ductis in cognitionem quantitatis virtutis ipsius F in huiusmodi situ constituens videlicet linea .F.u. cum brachio .F.B. angulum acutum" (Benedetti 1585, 142–143. See also Drake and Drabkin 1969, 169). Drake and Drabkin's tranlsation.

[98] Redrawn from Benedetti 1585, 142–143.

Then the argument is referred to forces or weights, which act along inclined directions, an argument that even if unequivocal – in substance does not appear entirely convincing.

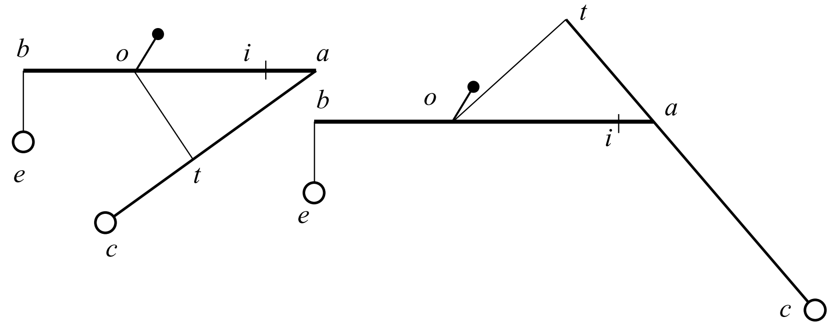

Fig. 2.24 Evaluation of the static moment for inclined forces by Benedetti[99]

> To understand this better, let us imagine [Fig. 2.24] a balance *boa* fixed at its centre *o*, and suppose that at its extremities two weights are attached, or two moving forces, *e* and *c*, in such a way that the line of inclination of *e*, that is *be*, makes a right angle with *ob* at point *b*, but the line of inclination of *c*, that is *ac*, makes an acute angle [Fig. 2.24a] or an obtuse angle [Fig. 2.24b] with *oa* at point *a* Let us imagine, then, a line *ot* perpendicular to the line of inclination *ca*. [...]. Imagine, then, that *oa* is cut at point *i*, so that oi is equal to *ot*, and that a weight is suspended at *i*, equal to *c* and with a line of inclination parallel to that of weight *e*. But we assume that the weight or force *c* is greater than *e* in proportion as *bo* is greater than *ot*. Obviously, then, according to Archimedes, De ponderibus, *boi* will not move from its position. Again, if in place of *oi* we imagine *ot* rigidly connected [in the same line] with *ob* and subjected to force *c* acting along line *tc*, the result will obviously be the same, *bot* will not move from its position.[100]

[99] Redrawn from Benedetti 1585, 143.

[100] "Ut hoc tamen melius intelligamus, imaginemur libram .b.o.a. fixam in centro .o. ad cuius extrema sint appensa duo pondera, aut duae virtutes moventes .e. et .c. ita tamen, linea inclinationis .e. idest .be. faciat angulum rectum cum .o.b. in puncto .b. linea vero inclinationis .c. idest .a.c. faciat angulum acutum, aut obtusum cum .o.a. in puncto .a. Imaginemur ergo lineam .o.t. perpendicularem lineae .c.a. inclinationis […] secetur deinde imaginatione .o.a. in puncto .i. ita ut .o.i. aequalis. sit .o.t. & puncto .i. appensum sit a pondus aequale ipsi .c. cuius inclinationis linea parallela sit linea inclinationis ponderis .e. supponendo tamen pondus aut virtutem .c. ea ratione maiorem esse ea, quae est .e. qua .b.o. maior est .o.t. absque dubio ex 6 lib. primi Archi. de ponderibus .b.o.i. non movebitur situ, sed si loco .o.i. imaginabimur .o.t. consolidatam cum .o.b. & per lineam .t.c.

Benedetti's criticisms of Tartaglia.

Benedetti knew Tartaglia's work very well, considering that for some time he was his pupil, and was surely influenced by him. Though only one generation younger, Benedetti's approach to mechanics is very different from Tartaglia's. As Tartaglia, he assumes mathematics at the foundation of mechanics, but he has a different cultural background; different because he received an education in philosophy, different because he became acquainted with Archimedes' mathematics and physics, also thanks to Tartaglia's editorial work. Benedetti is completely outside Aristotelianism. He fights against Aristotle in all his physical assumptions: on the existence of voids; on the law of fall of heavy bodies, on the nature of forced motion and so on. He is also outside the medieval science of weights, which interests him only for marginal aspects. From Archimedes he derived a greater attention to rigour in mathematical proofs but he also renounced the important resource that Tartaglia had: the algebraic calculus to solve geometrical and mechanical problems.

In Chapter VII of the section *De mechanicis,* Benedetti refers some criticisms toward Tartaglia's consideration on the science of weights (that, he specified, were partially "[...] taken from a certain ancient writer Jordanus [...]" (Benedetti 1585, VII, 148). Benedetti's criticism refers both to general assumptions and to defects in the exposition of the matter. He criticizes[101] in particular the proof of Tartaglia's Propositions III–IIII of the *Quesiti et invention diverse* (Tartaglia 1554, *Book VIII*, Qs XXX–XXXI Propositions III–IIII, 87rv–88rv), commenting that Archimedes had proved it more properly (Archimedes 2002, *Book I*, Propositio VI, 192–194). More important, for us, is the criticism about proposition V (Tartaglia 1554, *Book VIII*, Q XXXII, Propositione V, 88v–89rv) on the equilibrium of the balance with equal arms and weights in the *Quesiti et invention diverse*. Differently from Tartaglia (and de Nemore) who considers the tendency of both the two weight hanging from the opposite sides of the balance to go down, he assumes the congruent situation for which while one weight descends the other ascends. In such a case, he notices, the path along the vertical is the same for the two weights, so the balance is in equilibrium whichever its inclination is:

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attractam virtute .c. similiter quoque contingent ut .b.o.t.; communi quadam scientiam, non moveatur situ" (Benedetti 1585, Chapter 3, p 143. See also Drake and Drabkin 1969, 169–170). Drake and Drabkin's tranlsation.

[101] Benedetti really started his criticisms by with comments on Tartaglia's proposition II concerning his errors in external resistance on motion (Tartaglia 1554, *Book VIII*, Quesito XXIX [in the book "XIX" is wrongly reported], Propositione II, 86rv–87r).

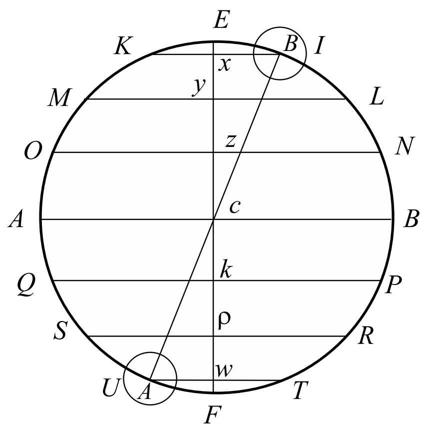


Fig. 2.25 Equilibrium of balance with equal weights and arms according to Tartaglia[102]

> And in the second part of the fifth proposition he [Tartaglia] fails to see that no difference in weight is produced by virtue of position in the way in which he argues. For if body *b* must descend on arc *il*, body *a* must ascend on arc *vs*, equal and similar to arc *il* and placed in the same way. Therefore, just as it is easy for body *a* to ascend on arc *vs* it is easy for body *b* to descend on arc *vs*. And this fifth proposition is the second proposed by Jordanus [de Nemore 1565, Quaestio secunda, 3v–4r].[103]

One more criticisms, that will be made again by Guidobaldo del Monte, concerns the cause for which the tendency to descend of a body suspended from a hinged rod decreases with its inclination. According to Benedetti the cause of this fact is the greater resistance the weight receives from the rod and the fulcrum – mechanical cause[104]. According to Tartaglia (and de Nemore) it depends on a lower facility of descent as a kinematic constraints. Other criticisms seems to us simply a way to quibble to show

[102] Redrawn from Tartaglia 1554, *Book VIII*, Q XXXII, Propositione V, 90v.

[103] "Sed in secunda parte quinte propositionis non videt vigore situs eo modo, quo ipse disputat, nulla elicitur ponderis differentia quia si corpus .B. descendere debet per arcum .IL. corpus .A. ascendere debet per arcum .V.S. Haec autem quinta propositio Tartalea est secunda quaestio a Iordano proposita". (Benedetti 1585, VII, 148. Drake Drabkin's translation 1969, 174–175). The figure in the text belongs to Tartaglia (Tartaglia 1554 *Book VIII*, Q XXXII, Propositione V, 89v; see also de Nemore 1565, 3r–5r) since Benedetti did not report it in his Chapter VII of the section *De mechanicis* (Benedetti 1585, VII, 148–149).

[104] Benedetti 1585, VII, 147–148.

his superiority. As when Benedetti criticizes Tartaglia for having considered as parallel the lines of the descents of heavy bodies, while he himself in some situation does the same, or when he blames Tartaglia for not having considered the resistance due to medium on motion (virtual) of weights hanging from a balance.

2.3.1.2 Guidobaldo del Monte

Guidobaldo (also Guido Ubaldo and other several ways) del Monte attended the university of Padova in 1564 as along with a companion Torquato Tasso (144–1595). He studied mathematics with Federico Commandino and was teacher of Bernardino Baldi. He was one of the greatest mathematicians and mechanician of the late XVI century. In 1577 he published the *Mechanicorum liber* (del Monte 1577; del Monte 2010, 2013), translated into Italian vulgare by Filippo Pigafetta in 1581 as *Le mechaniche* (del Monte 1581). The book had an enormous editorial success and was read throughout the whole XVII century.

Del Monte's criticisms of Tartaglia and Jordanus de Nemore.

Del Monte was one of the major critics of the approach of Jordanus de Nemore and Niccolò Tartaglia. According to him those of de Nemore and Tartaglia, are not valid demonstrations and goes so far as to say that de Nemore should not even be counted among the true mathematicians. Bernardino Baldi went still further and considered as paralogisms the demonstrations of de Nemore (Baldi 1621, 32). Del Monte, like Benedetti, knew Tartaglia's work very well and does not share his position. Like Benedetti, and differently from Tartaglia, he individuates the lower tendency to go down of heavy bodies suspended from a more inclined arm in the greater resistance the weight receives from the rod and the fulcrum (mechanical cause). Other criticisms, very often repeated, concern the approximation adopted by Tartaglia for the lines of descent of heavy bodies.

Criticisms of del Monte must be placed in his time to be understood. As noted in the introduction of this section, scholars of mathematics of the period, particularly those of Centre and South Italy, could not fail to be charmed by the elegance and rigor of geometry as it was revealed by the recently published Greek translations of Euclid and Archimedes. Archimedes, flanking his mathematical theory, developed a consistent mechanical theory with the same standards of rigor.

It was therefore natural to accept the argument of Archimedes in mechanics and reject those by de Nemore. Although to a modern observer, the full refusal of de Nemore seems unjustified because the *Liber de* *ratione ponderis* has an Euclidean approach based on definitions, axioms and theorems: it is certainly the ancient text in which the Euclidean approach is extended further outside geometry, in the wake of subalternate sciences. It is overall a very modern text. Del Monte, however, could hardly accept to reason with concepts such as gravity of position, which remained a bit undefined.

Given that de Nemore's and Tartaglia theses were then quite common in Italy, del Monte somehow felt the need to re–establish the *truth*, by writing the *Mechanicorum liber Archimedis aequeponderantium* and the *Mechanicorum liber* (del Monte 1577) that can be seen as the natural completion of the work of spreading Archimedes' mechanical thought. The hostility towards the approach of de Nemore also led del Monte to refuse the correct proof of the inclined plane for the incorrect one by Pappus of Alexandria. However one can show that del Monte was not as strict an Archimedes' follower as normally accepted. His mechanics is less abstract than Archimedes', and if he refused the concept of gravity of position because of its physical pregnancy, he contaminates the Archimedean approach. For instance he gives a material consistence to the fulcrum of the lever (which for Archimedes was a simple geometrical point), which is also capable of delivering forces; he gives a physical definition to the centre of gravity; he used the concept of muscle force (van Dyck 2006). Although the *Mechanicorum liber* on the one hand had given up the fertility of de Nemore's approach, based on the concept of gravity of position and a law of virtual work, playing in some way a conservative role, it expanded the scope of mechanics. The medieval science of weights, in which attention was focused on demonstrating the law of the lever, is led back to the Greek tradition of mechanics as a science of machines, influenced in this by the *Problemata mechanica*, but especially by Heron's approach, then known only through the work of Pappus of Alexandria just translated by Commandino.

The balance with equal weights and arms.

In order to show del Monte's way of reasoning, below we report a summary of the way he studies the equilibrium of the balance with equal arms and weights. Proving this balance is in a position of indifferent equilibrium is a crucial point for him. In fact if that could not be the case the whole Archimedean building of centrobaric would collapse, because the fourth proposition of Archimedes' *Aequiponderanti* (Archimedes 2002, 191), according to which two equal weights have their centre of gravity in the middle of the segment joining them, would be false. Indeed if and only if the balance with two equal weight is in an indifferent situation of equilibrium its fulcrum – the middle point – coincides with the centre of gravity of the two weights and Archimedes' proposition is verified. Del Monte's strategy to defend the Archimedean centrobaric is two fold. In the first step he 'proves' that Tartaglia and de Nemore's result, for which the balance will revert to its horizontal position when disturbed (stable equilibrium) is false. The proof is carried out by provisionally accepting the concept of gravity of position but assuming the convergence of the lines of descent of heavy bodies (that for de Nemore were parallel to each others). From Fig. 2.25 it is clear that the weight in E ha a greater gravity of position than the weight in D (its descent is less oblique, for the angle between the line of descent ES and the path of E – the tangent in E to the circle FBGA – is less than the angle of LS and the descent of D); thus the balance rotates until it will reach the vertical configuration FS (unstable equilibrium). After having falsified de Nemore's results on his own ground, in the second step del Monte leaves the concept of gravity of position to show that also the last result is false. It indeed could be true if the two weights were isolated, but they are joined and they contrast each other so that their motion will tend to be along the line EK and DK (parallel to CS) which are intermediate between ES and DS. The motion however does not occur because C is constrained and the horizontal configuration is of indifferent equilibrium.

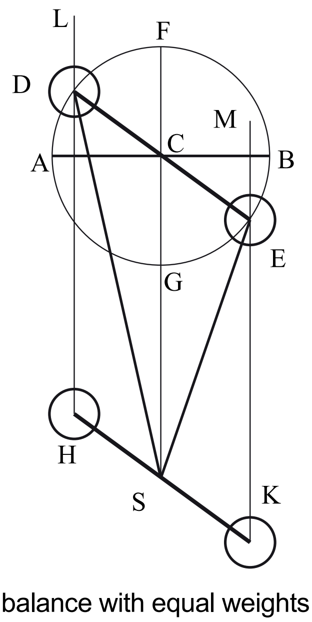


Fig. 2.26 Equilibrium of balance with equal weights and arms according to del Monte[105]

[105] del Monte [1581] 1615, 34.

ON BALANCE. [...]. If the weight placed at E is heavier than the weight placed at D, the balance DE will never remain in that position, as we have undertaken to maintain, but it will move to FG. To which we reply that it makes a great deal of difference whether we consider the weights separately, one at a time, or as joined together; for the theory of the weight placed at E when it is not connected with another weight placed at D is one thing, and it is quite another when the weights are joined in such a way that one cannot move without the other. For the straight and natural descent of the weight placed at E, when it is without connection to another weight, is made along the line ES; but when it is joined with the weight D, its natural descent will no longer be along the line ES, but along a line parallel to CS. For the combined magnitude of the weights E and D and the balance DE has its centre of gravity at C, and, if this were not supported at any place, it would move naturally downward along the straight line drawn from the centre of gravity C to the centre of the world S until C reached S. [...] But if the weights E and D are joined together and we consider them with respect to their conjunction, the natural inclination of the weight placed at E will be along the line MEK, because the weighing down of the other weight at D has the effect that the weight placed at E must weigh down not along the line ES, but along EK. The same is true of the weight at E; that is, the weight at D does not weigh down along the straight line DS, but along DH, both of them being prevented from going to their proper places [...].Thus the descent of the weight at D will be equal to the rise of the weight at E, and the weight at D will not raise the weight at E. From which it follows that the weights at D and E, considered in conjunction, are equally heavy.[106]

[106] " DELLA BILANCIA. [...]. Se dunque il peso posto in E è più grave del peso posto in D, la bilancia DE non starà giamai in questo sito, la qual cosa noi habbiamo proposto di mantenere, ma si moverà in FG. Alle quali cose rispondiamo che importa assai, se noi consideriamo i pesi overo in quanto sono separati l'uno dall'altro, overo in quanto sono tra loro congiunti: perche altra è la ragione del peso posto in E senza il congiungimento del peso posto in D, et altra di lui con l'altro peso congiunto, si fattamente che l'uno senza l'altro non si possa movere. Imperoche la diritta, et naturale discesa dal peso posto in E, in quanto egli è senza altro congiungimento di peso, si fa per la linea ES, ma in quanto egli è congiunto col peso D, la sua naturale discesa non sarà più per la linea ES, ma per una linea egualmente distante da CS percioche la magnitudine comporta de i pesi ED, et della bilancia DE il cui centro della gravezza è C, se in nessun luogo non sarà sostenuta, si muoverà naturalmente in giù nel modo che si trova, secondo la grandezza del centro per la linea diritta tirata dal centro della gravezza C al centro del mondo S, finche il centro C pervenga nel centro S [...] Ma se i pesi posti in ED sono l'un l'altro fra se congiunti, et gli considereremo in quanto sono congiunti, sarà la naturale inclinazione del peso posto in E per la linea MEK, percioche la gravezza dell'altro peso posto in D fa si, che il peso posto in E non gravi sopra la

It is one thing, he says, to consider the weights in D and E separately, in which case they would move toward the centre of the world S along DS or ES respectively, the other is to consider them together, so their centre of gravity would move to S along CS, while the weights in D and E along DH and EK, as shown in Fig. 2.26. But since C cannot sink, the weights remain at their place, D and E.

Del Monte claims to have verified empirically the indifference of equilibrium. And if the result of some scholar does not correspond to his theory it is because the experiment was not well executed and there were differences between the ideal and real situations. The following excerpt from the Italian version of the *Mechanicorum liber*, clearly expresses del Monte's ideas:

> […] that being the balance supported in its center by gravity it still remains wherever it is, which effect in particular has no longer been expressed by anyone, save only by the author. Indeed so far it was considered false, and impossible to put by all our predecessors; who with many reasons have endeavored to prove not only the opposite, but also have said for sure, that experience shows the scale never stops except when it is equally distant from the horizon. This thing is contrary to all reason, first, to be the demonstration of such fourth proposition as clear, simple, and true, and I do not know, how it can be contradicted, and then the experience which the author did with very finely balances, right on purpose to clarify this truth, one of which I have seen in the hands of the Illustrious Mr. Vicenzo Pinello,[107] sent to him by the author himself, which supported from the center of its gravity, moved in any position and then left, stops at every point it comes left. It is true, in making this experience, that we must not striving so to rage, for it is something very difficult, as the author says above, to make a scale, which is supported precisely in the middle of its arms at the center of its own gravity.[108]

linea ES, ma nella EK. Il che fa parimente la gravezza del peso posto in E, cioè, che'l peso posto in D non gravi per la linea reta DS, ma secondo DH impedirsi ambedue l'uno l'altro, che non vadino a propri luoghi [...]. Adunque il peso posto in D non moverà in su il peso posto in E. Dalle quali cose segue che i pesi posti in DE, in quanto tra loro sono congiunti, sono egualmente gravi" (del Monte [1581] 1615, 34–36. See also Drake and Drabkin 1969, 281–282). Drake and Drabkin's translation.

[107] Gian Vincenzo Pinelli (1535–1601), erudite Neapolitan and bibliophile man, was a friend of Galileo.

[108] "[…] che essendo la bilancia sostenuta nel suo centro dalla gravezza sta ferma dovunque el la si trova, il quale effetto in particolare non è piu stato tocco, ne veduto, ne man co da niuno manifestato, fuor che dall'autore: anzi fin hora

2.3.1.3 Galileo Galilei

Galileo Galilei was born in Pisa in 1564 and died in Arcetri (Florence) in 1642. In Pisa he undertook the study of mathematics under the guidance of Ostilio Ricci (1540–1603), a pupil of Niccolò Tartaglia. In 1638 he published *Discorsi e dimostrazioni matematiche sopra due nuove scienze* (Galileo 1638; Koyré 1996; Drake 1999, 2000).

The contribution Galileo provided to statics is far less decisive than that to dynamics, nonetheless it is important. Though there may be doubts on the originality of some of his writings, it is certain that no one before him had formulated and solved his own problems with extraordinary clarity. Differently from Benedetti and del Monte he does no disdain the science of weights and maintains for some important respects its kinematic approach. In his first important writing in statics, *Le mecaniche*,[109] which is related in part to del Monte's *Mechanicorum liber*, Galileo prevalently adopts an Archimedean approach (Pisano 2009a, 2009b) and presents an elegant proof of the law of lever, based on purely geometrical arguments. He then reduces all the simple machines to the lever, including the inclined plane, which escaped to Guidobaldo del Monte. Nevertheless, the Archimedean approach is flanked by the kinematic approach both for the lever and inclined plane laws. Kinematic approach that will become dominant for the problem of equilibrium in the subsequent works: *Discorso intorno alle cose che stanno in su l'acqua e scritture varie*, printed in 1612 (Galilei [1612] 1888–1905, IV) and the already cited

tenuto falso, & impossibile da tutti gli predecessori nostri; i quali con molte ragioni si sono sforzati di provare non solamente il contrario, ma hanno etiandio affermato per certo, che la sperienza mostra la bilancia non dimorare gia mai ferma se non quando ella è egualmente distante dall'orizonte. La qual cosa in tutto è contraria alla ragione prima, per essere la dimostratione della sudetta quarta propositione tanto chiara, facile, & vera, che non sò, come se le possa in modo alcuno contradire: & poi all'esperienza concio sia che l'autore habbia fatto sottilissimamente lavorare bilancie giuste a posta per chiarire questa verità, una delle quali hò io veduto in mano dell'Illustre Signor Gio. Vicenzo Pinello, mandatagli dall'istesso autore, la quale per essere sostenuta nel centro della sua gravezza, mossa dovunque si vuole, & poi lasciata, sta ferma in ogni sito dove ella vien lasciata. Ben è egli vero, che non bisogna, nel fare cotesta esperienza, correr cosi a furia, per essere cosa oltra modo difficile, come dice l'autore di sopra, il fare una bilancia, la quale sia nel mezo del le sue braccia sostenuta à punto, & nel centro proprio della sua gravezza". (del Monte [1581] 1615, 56). Drake and Drabkin's translation.

[109] In the 1593–1594 the early manuscripts was and first printed in a French version by Mersenne (Mersenne 1634). It was published into Italian (1649) after the death of Galileo (Galilei 1649).

Discorsi e dimostrazioni matematiche sopra due nuove scienze.

The Galilean concept of moment.

In *Le mecaniche*[110] Galileo introduced a concept and a term, that of *moment* (*momento*), that will be of great fortune and adopted, at least in Italy, until the early nineteenth century. The concept, formulated in *Le mecaniche*, was taken up and elaborated in the *Discorso intorno alle cose che stanno in su l'acqua*:

> Moment for mechanics, means that virtue, that force, that effectiveness with which the motor moves and the *mobile resists* [emphasis added], virtue which depends not only on the simple gravity, but on the speed of motion, from the different angles of the spaces over which the motion is made, because a heavy body makes more impetus in a very inclined space than in one less inclined. The second principle [the first was that equal weights with equal speed have equal forces and moments] is, that the moment and the force of gravity is increased by the speed of motion so that absolutely equal weights, but combined with unequal velocities, are of force, moment and virtue unequal, and the fastest is more powerful, according to the proportion of its speed to the speed of the other. Of this we have very suitable example in the balance with unequal arms, where absolutely equal weights do not press and are not equally strong, but that which is at the greatest distance from the centre, around which the balance moves, sinks and rises the other, and it is the motion of the ascending fast, the other slow: and such is the force and virtue that the speed of motion gives to the mobile that receives, and it can compensate as much weight is added to the other mobile; so that if one arm of a balance were ten times longer than the other, in order to move the balance around his middle, the end of that passed ten times more space that the end of this, a weight placed at the greater distance can sustain and equilibrate another ten times heavier than it is, and this because, moving the balance, the lower weight will move ten times faster than the other.[111]

[110] In 2014 Galileo's anniversary is celebrated. *1564-2014. Homage to Galileo Galilei. History and Historical Epistemology of Sciences within Iuvenilia-Early Galilean Works*. It is a Special issue of *Philosophia Scientiæ* (21/1: February 2017). Guest editors: Raffaele Pisano (University of Lille 1, France), Paolo Bussotti (The Alexander von Humboldt Foundation, Berlin, Germany).

[111] "Momento, appresso i meccanici, significa quella virtù, quella forza, quella efficacia, con la quale il motor muove e 'l mobile resiste; la qual virtù depende non solo dalla semplice gravità, ma dalla velocità del moto, dalle diverse inclinazioni degli spazii sopra i quali si fa il moto, perché più fa impeto un grave descendente in uno spazio molto declive che in un meno. Il secondo principio è, che il momento e la forza della gravità venga accresciuto dalla velocità del moto: sì che pesi assolutamente eguali, ma congiunti con velocità diseguali, sieno di forza,

From the reading of passages quoted above it is clear as Galileo espoused the view that the downward velocity of a heavy body increases its efficacy or *force* do go down while the upward velocity increases its resistance to be lifted. His conception is rather uncommon in statics and differed from del Monte and Benedetti's who instead believed that there was no increase of `force' due to velocity, but only a greater velocity due to lower resistance of constraints. It also differs from Tartaglia's who equally saw an increase of gravity but justified because of a virtual – determined by the kinematics – a not real velocity. Galileo specified that moment is also the resistance to gain speed. Therefore, the equilibrium is not from the equality of two trends to go down, but from the balance of the impetus to go down and the resistance to go up, both increased by the speed.

In *Le mecaniche*, after having proved the law of the lever according to Archimedes and similarly to what he will do in the first day of the *Discorsi*, Galileo examined the equilibrium of the lever using the concept of moment. The principle he invoked for the equilibrium is the equality of moments. He stated that this principle could be deduced from the *Problemata mechanica* (Galileo [1612] 1888–1905, IV, 275). Nevertheless, that is probably a rhetorical artifice only and he more simply took his inspiration from the science of weights tradition and perhaps from Tartaglia.

About the origin of Galileo's concept of moment many pages have been written; for a historical reconstruction philologically based, reference can be made to (Galluzzi 1979). It seems, however, that a reconstruction based on similarity of concepts is of more interest to me. This obviously can lead only to demonstrate the possibility and not the need – but even an accurate historical reconstruction is not necessarily conclusive. There is no doubt

momento e virtù diseguale, e più potente il più veloce, secondo la proporzione della velocità sua alla velocità dell'altro. Di questo abbiamo accomodatissimo esemplo nella libra o stadera di braccia disuguali, nelle quali posti pesi assolutamente eguali, non premono e fanno forza egualmente, ma quello che è nella maggior distanza dal centro, circa il quale la libra si muove, s'abbassa sollevando l'altro, ed è il moto di questo che ascende, lento e l'altro veloce: e tale è la forza e virtù che dalla velocità del moto vien conferita al mobile che la riceve, che ella può compensare altrettanto peso che all'altro mobile più tardo fosse accresciuto; sì che, se delle braccia della libra uno fosse dieci volte più lungo dell'altro, onde nel muoversi la libra circa il suo centro, l'estremità di quello passasse dieci volte maggiore spazio che l'estremità di questo, un peso posto nella maggiore distanza potrà sostenerne ed equilibrarne un altro dieci volte assolutamente più grave che non egli è; e ciò perché, muovendosi la stadera, il minor peso si moveria dieci volte più velocemente che l'altro". (Galilei [1612] 1888–1905, IV, pp 68–69). The translation is mine.

that the concept of moment in Galileo has some similarities with that of gravity position in de Nemore, and that some of its connotations are also present in the Aristotelian *Problemata mechanica*. It seems, however, that apart from these rather general similarities Galileo could have found some more specific ideas in the writings of Tartaglia and Benedetti. The idea that led Galileo to express the moment as proportional to the (virtual) velocity could have come from the proposition IV of book VIII of *Quesiti et inventioni diverse* (Tartaglia 1554, *Book VIII*, Q XXXI Propositione IIII, 88rv) which says that the gravity of the position of a heavy body, suspended from a lever, grows linearly with its distance from fulcrum. As the speed increases linearly with distance, it is natural for a reader of Tartaglia, imagine the gravity of position (and then the moment) as proportional to speed. From Benedetti Galileo may have drawn the idea that in the study of the equilibrium of heavy bodies one must consider motions congruent with each other. In the case of the balance, the congruence of the motions implies that when a weight drops the other raises. From this Galileo's idea to consider moment proportional to speeds even in the case of upward motion would have come up (see Benedetti above).

Inclined plane law.

Today the inclined plane is seen as a conceptual model different from that represented by the lever and essentially not reducible to it. The inclined plane is representative of virtual displacement laws, it is somehow its geometric representation; the lever is representative of the virtual velocity laws (Capecchi 2004). In the past however things were not seen this way. That the inclined plane had its peculiarities was understood by Aristotle who did not treat it and by Heron who treated it apart from the other machines. However, after Pappus of Alexandria had reduced it to the lever, the difficulties in the study of the inclined plane seemed to vanish. In the Renaissance the problem reappeared because some scholars did not accept Pappus' solution, considering it both logically unconvincing and empirically inadequate. For example, it featured an infinite value of the force required to lift a weight on a vertical plane, and this is patently absurd. Other scholars did not accept it because in contrast with de Nemore' solution, whose demonstration seemed more consistent, though the principles adopted could appear not very obvious.

With Galileo the reductionist project, started with Pappus and strongly supported by Guidobaldo del Monte, to reduce all simple machines including the inclined plane to the lever, was perfected. Note that Galileo's attempt to reduce the inclined plane to the lever was accepted not because verified empirically – with the conceptions of experiment of the times also the results of Heron or Cardano were verified – but because he finally presented a rigorous reasoning and employed reasonable assumptions. Moreover, Galileo's result along with that of de Nemore coincided with that of Stevin more or less of the same period, very elegant and based on different assumptions. Note also that if the reasoning of Galileo was corroborated by the result of de Nemore and Stevin, the reasoning of de Nemore and Stevin was corroborated by that of Galileo and from now on the problem of the inclined plane was considered definitively solved by all mathematicians.

In the section devoted to the mechanics of the screw, Galilei (1649) showed how the inclined plane can be reduced to the lever and furnished a simple mathematical law. The proof reproduces he had reported in *De motu* (Galilei [1590] 1888–1905, I, 297–298), differing mainly for the use of the word *moment* instead of *gravitas*.

> The present Speculation hath been attempted by Pappus Alexandrinus in Lib. 8. de Collection. Mathemat. but, if I be in the right, he hath not hit the mark, and was overseen in the Assumption that he maketh.
> […]. Let us therefore suppose the Circle AIC, and in it the Diameter ABC, and the Centre B, and two Weights of equal Moment in the extreams B and C; so that the Line AC being a Leaver, or Ballance moveable about the Centre B, the Weight C shall come to be sustained by the Weight A. But if we shall imagine the Arm of the Ballance BC to be inclined downwards according to the Line B F, but yet in such a manner that the two Lines AB and BF do continue solidly conjoyned in the point B, in this case the Moment of the Weight C shall not be equal to the Moment of the Weight A, for that the Distance of the point F from the Line of Direction, which goeth accord ing to BI, from the Fulciment B unto the Centre of the Earth, is diminished: But if from the point F we erect a Perpendicular unto BC, as is FK, the Moment of the Weight in F shall be as if it did hang by the Line KF. [112]

[112] "È la presente speculazione stata tentata ancora da Pappo Alessandrino nel'8° libro delle sue Collezioni Matematiche; ma, per mio avviso, non ha toccato lo scopo, e si è abbagliato […]. Intendasi dunque il cerchio AIC, ed in esso il diametro ABC, ed il centro B, e due pesi eguali momenti nelle estremità A, C; sì che, essendo la linea AC un vette o libra mobile intorno al centro B, il peso C verrà sostenuto dal peso A. Ma se c'immagineremo il braccio della libra BC essere inchinato a basso secondo la linea BF, in guisa tale però che le due linee AB, BF restino salde insieme nel punto B, allora il momento del peso C non sarà più eguale al momento del peso A, per esser diminuita la distanza del punto F dalla linea della direzione che dal sostegno B, secondo la BI, va al centro della terra. Ma se tireremo dal punto F una perpendicolare alla BC, quale è la FK, il momento del peso in F sarà come se pendesse dalla linea KF". (Galilei [1649] 1888–1905, II, 181). Salusbury's translation (Salusbury 1661–1665, II, 294).

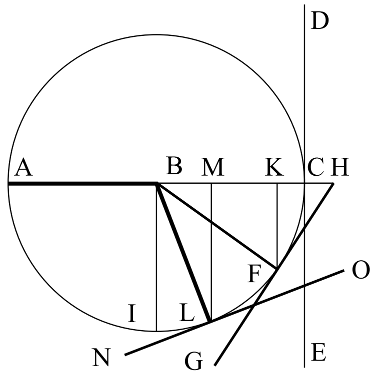


Fig. 2.27 Galilean inclined plane law[113]

See therefore that the Weight placed in the extream of the Leaver B C, in inclining downwards along the Circumference CFLI, cometh to diminish its Moment and Impetus of going downwards from time to time, more and less, as it is more or less sustained by the Lines BF and BL.

[…]. If therefore upon the Plane HG the Moment of the Moveable be diminished by the total Impetus which it hath in its Perpendicular DCE, according to the proportion of the Line K B to the Line BC, and BF, being by the Solicitude of the Triangles KBF and KFH the same proportion betwixt the Lines KF and FH, as betwixt the said KB and BF, we will conclude that the proportion of the entire and absolute Moment, that the Moveable hath in the Perpendicular to the Horizon to that which it hath upon the Inclined Plane HF, hath the same proportion that the Line HF hath to the Line FK; that is, that the Length of the Inclined Plane hath to the Perpendicular which shall fall from it unto the Horizon. So that passing to a more distinct Figure, such as this here present, the Moment of Descending which the Moveable hath upon the inclined Plane CA hath to its total Moment wherewith it gravitates in the Perpendicular to the Horizon CP the same proportion that the said Line PC hath to CA. And if thus it be, it is manifest, that like as the Force that sustaineth the Weight in the Perpendiculation PC ought to be equal to the same, so for sustaining it in the inclined Plane CA, it will suffice that it be so much lesser, by how much the said Perpendicular CP wanteth of the Line CA: and because, as sometimes we sce, it sufficeth, that the Force for moving of the Weight do insensibly superate that which sustaineth it, therefore we will infer this

[113] Redrawn from Galilei [1649] 1888–1905, II, 181.

universal Proposition, that upon an elevated plane the force hath to the weight the same proportion.[114]

The key assumptions to demonstrate the law of the inclined plane are:

a) For static purposes, moving on the inclined planes like to NO or GH is the same as moving on the circumference described by the lever arms BL or BF (see Fig. 2.27)
b) The effectiveness of a heavy body on an angled lever is determined by the horizontal distance from the fulcrum.

The second assumption is an accepted theorem of statics, but the first has a logic status not completely clear. It indeed appears quite intuitive, at least after its formulation, because to study the equilibrium it seems sufficient to verify that also very small displacements cannot occur. In this way the displacements at the extremity of the lever and on the inclined plane are the same, the two kinds of constraints are locally equivalent and can be replaced the one with the other. But this intuitive character stems more from empirical than logical considerations; it would be then a postulate which could even not be accepted. Moreover the first assumption has a

[114] "Vedesi dunque come, nell'inclinare a basso per la circonferenza CFLI il peso posto nell'estremità della linea BC, viene a scemarsi il suo momento ed impeto d'andare a basso di mano in mano più, per esser sostenuto più e più dalle linee BF, BL. […]. Se dunque sopra il piano HG il momento del mobile si diminuisce dal suo totale impeto, quale ha nella perpendicolare DCE, secondo la proporzione della linea KB alla linea BC o BF; essendo, per la similitudine de i triangoli KBF, KFH, la proporzione medesima tra le linee KF, FH che tra le dette KB, BF, concluderemo, il momento integro ed assoluto che ha il mobile nella perpendicolare all'orizzonte, a quello che ha sopra il piano inclinato HF, avere la medesima proporzione che la linea HF alla linea FK, cioè che la lunghezza del piano inclinato alla perpendicolare che da esso cascherà sopra l'orizonte. Sì che, passando a più distinta figura, quale è la presente, il momento di venire al basso che ha il mobile sopra il piano inclinato FH, al suo totale momento, con lo qual gravita nella perpendicolare all'orizonte FK, ha la medesima proporzione che essa linea KF alla FH. E se così è, resta manifesto che, sì come la forza che sostiene il peso nella perpendicolare FK deve essere ad esso eguale, così per sostenerlo nel piano inclinato FH basterà che siano tanto minore, quanto essa perpendicolare FK manca dalla linea FH. E perché, come altre volte s'è avvertito, la forza per muover il peso basta che insensibilmente superi quella che lo sostiene, però concluderemo questa universale proposizione: sopra il piano elevato la forza al peso avere la medesima proporzione, che la perpendicolare dal termine del piano tirata all'orizonte, alla lunghezza d'esso piano". (Galilei [1649] 1888–1905, II, 182–183). Salusbury's translation (Salusbury 1661–1665, II, 294–296).

kinematic connotation, which makes it closer to the science of weights approach than the Archimedean's.

2.3.2 Stevin's Legacy. The Circulation of Statics in Europe

Mechanics in the XVI century developed mainly in Italy. In the XVII century, things began to change and the dominance, for that which concerns the science of balance too, went to France, the Netherlands and England. In this process, is appropriate to mention a very important transitional figure: Simon Stevin. A contemporary of Galileo, and therefore a man of the XVI century, he is not Italian. In an ideal temporal representation of the evolution from the science of weights to the modern statics, fairly regular in truth, the presence of Stevin marks a net–discontinuity. He transformed the science of equilibrium of weights into a science of equilibrium of forces for which he proposed a composition rule. The very Latin word for statics[115] (a neologism from *status*), while giving a unique name to a discipline, also demarcated areas, emphasizing its main reference to equilibrium. Statics is distinct from mechanics, which also deals with motion, but is also separated from the science of weights, as statics is centred because it also deals with forces and not weight only.

Simon Stevin (1548–1620) was for some years book–keeper in a business house at Antwerp; later he secured employment in the administration of the Franc of Bruges. In 1583, he entered the University of Leiden. From 1604 Stevin was an outstanding engineer who advised on

[115] We may say in agreement with Patricia Radelet de-Grave, that the word *statica* appears in the title of the fourth parts of the translation of Stevin's major work in mechanics *Wisconstige Gedachtenissen* (Stevin 1605–1608c). Willebrord Snel van Royen (1580–1626) in his Latin translation as *Hypomnemata mathematica,* published into two volumes (Stevin 1605–1608b), immediately after the original Flemish publication, uses the term "Statica". Particularly, Snel uses the word "Statica" in the volume 2 (Stevin 1605–1608b, II [*Tomus quartus mathematicorum hypomnematum de Statica*] *Liber Primus Staticae, de Staticae Elementis*, 5). Jean Tuning (see next footnote) in his French translation of Stevin's work as *Memoires mathematiques* (Stevin 1605–1608a) uses the word "art pondéraire". Then, in 1634 Albert Girard (1595?–1632) reused – in his French work, as *Les Œuvres Mathematiques de Simon Stevin de Bruges* (Stevin 1634), the term "L'art pondérarire ou de la statique […]". This word was not so much of succesful, at least until to *Nouvelle mécanique* (1725) by Pierre Varignon (1654–1722). Therefore, it seems that the introduction of the notation *Statica* should be attribueted to Snel rather than to Stevin. Nevertheless, as suggested by Radelet de-Grave, since Snel translated in collaboration with Stevin, it is hard to establish the history of genesis of the scientific term "Statica" in Stevin context.

building windmills, locks and ports. Author of many books, he made significant contributions to trigonometry, mechanics, architecture, musical theory, geography, fortification, and navigation. He introduced the use of decimals in mathematics in Europe (Struik 1981).

Stevin wrote important works on mechanics. Mainly dealing with equilibrium they are his books *De Beghinselen der Weegconst* (*The elements of the art of weighing*) (Stevin 1586a) and *De Beghinselen des Waterwicht*s (*The elements on the weight of water*) (Stevin 1586b) both published in 1586 into Flemish language. Although he undertook his mathematical work earlier in his life, Stevin collected together some of his mathematical writings and edited and published them during the years 1605 to 1608 in *Wiskonstighe Ghedachtenissen*[116] – mathematical memoirs, in Latin *Hypomnemata mathematica* – (Stevin 1605–1608b; see also 1955). As a custom of the times, he did not quote his predecessors with the exception of Archimedes, Commandino and Cardano but in the last case only to criticize his (wrong) result for the inclined plane. Assessing Stevin's contribution to the history of mechanics is not simple because his ideas were originally written in Flemish and thus read by few. When they were translated into Latin and later, again into French language (Stevin 1634) by Girard the state of mechanics had already changed. He is indeed, in any case, the founder of statics in the modern sense.

Stevin's major work, *Tomus quartus mathematicorum hypomnematum de statica* (Stevin 1605) is divided into five books, plus an *Appendix* and some *Additions* to the Flemish edition of 1586. The approach is of Euclidean type, in the sense that for every book there is a different topic, first there are definitions, then postulates and finally theorems that are linked together. In the first part of the first book (Stevin 1605, *Book* I) Stevin demonstrates the law of the lever, with an argument similar to that used by Galileo in *Le mecaniche*. Starting from a continuous prismatic body with geometric considerations in the wake of Archimedes, he finds the law of inverse proportionality between weight and arm length. In the second part of the same book, Stevin gives his famous demonstration of the law of the inclined plane, determining the value of the force parallel to the slope enough to maintain a heavy body in balance. Stevin extends his

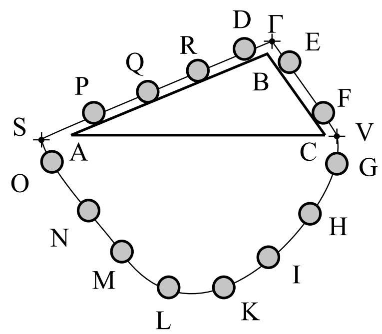
[116] This Stevin's book was immediately – but partially – translated both into French language as *Memoires mathematiques: contenant ce en quoy s'est exercé le très-illustre, très-excellent Prince & Seigneur Maurice, Prince d'Orange, Conte de Nassau, Catzenellenboghen, Vianden, Moers* […] (Stevin 1605–1608a) by Jean Tuning and into Latin language as *Hypomnemata mathematica, hoc est eruditus ille pulvis, in quo se exercuit* […] *Mauritius, princeps Auraïcus* […] *a Simone* into two volumes (Stevin 1605–1608b) by Willebrord Snel van Royen.

result to the case where the uplifting force is not parallel to the inclined plane. Gilles Personne de Roberval (1602–1675) found Stevin's proof not satisfactory and gave a much more convincing proof (de Roberval 1636). Basing on the law of the inclined plane generalized to a force of any direction, with a rather complex argument that is developed with many theorems and corollaries, Stevin puts the groundwork for the proof of the rule of the parallelogram of forces which is satisfactory if the generalized law for the inclined plane is accepted.

In the *Addition*s Stevin considers and devises demonstrations for pulleys and treats with some generality the case of forces applied by means of ropes in a section called *Spartostatica*. In this section statics has already became the science of equilibrium of force – modern meaning – and no longer of weights. It contains the wording of the rule of the parallelogram, which is a rule of composition of forces, even though it is presented as a way to determine the tension of two ropes which sustain a weight (Stevin 1605). This change of attitude is a fundamental Stevin's contribution to modern statics, and it does not matter if the rule of composition of forces is given an imperfect proof; it is however a rule which works. In the final part of the *Spartostatica* Stevin considers for the first time fundamental arguments that can be conceived only in the new frame of reference based on forces, i.e. the funicular polygon, the weight sustained by more than two ropes in the plane and out of plane. It is worth noticing however that Stevin never uses the terms *force* or *power*. This holds good also when it is clear that he is concerning with a muscle force; as well as when in his drawings he shows the images of human hands sustaining or lifting a weight. The reading of Stevin's mechanical work offers a much more modern view than that of Guidobaldo del Monte (Del Monte 1577) and Galileo (Galilei [1649] 1888–1905, II). The approach of Archimedean kind is equally rigorous, but less verbose. Unlike Galilei, Stevin does not bother to set up statics on a single principle, that of lever. He uses the Archimedean geometric proof for the lever, but when he relies on the law of the inclined plane he uses an empirical principle, in part still controversial, that of the impossibility of perpetual motion.

2.3.2.1 The Law of the Inclined Plane

Although he declares his opposition to the kinematic approach for which the equilibrium of a body depends on its possible motion, in the proof of the law of the inclined plane Stevin seems to contradict himself by deducing the equilibrium from the impossibility of motion. He considers a chain that wraps around the inclined plane, as shown in Fig. 2.28, which is accurately described:



PREPARATION. Let consider around the triangle ABC a necklace of fourteen equal globes, like E, F, G, H, I, K, L, M, N, O, P, Q, R, D, so that they ca rotate around their centres and that there are two globes on the side BC and four on the side BA, so that as the line is to the line, the number of globes is to the number of globes. Let S, Γ, V be three fixed points, on which the line, or the lace, could slide. And the two parts above the triangle be parallel to its sides AB, BC; so that the whole should rotate freely and without friction on the said sides AB, BC.[117]

Fig. 2.28 Equilibrium of the necklace wrapped around an inclined plane by Stevin[118]

The proof is conducted by reduction to the absurd. Suppose, says Stevin, the necklace is not in equilibrium and moves to reach equilibrium. Since the relative configuration of the necklace cannot change, if it is not equilibrated in one configuration it is not equilibrated in any other configuration. Thus perpetual motion would occur, which is absurd. The necklace is so in equilibrium:

> It is not possible that a given motion has no end.[119]

---

[117] "PRAEPARATIO. Triangulum A B C quatuordecim globorum pondere et magnitudine æqualium, quasi corona ut E, F, G, H, I, K, L, M, N, O, P, Q, R, D, cunctum fingamus, qui omnes lineâ per centro ipsorum, ut in illis moveri possint, transeunte, colligati aequali inter se spacio distent, ut illorum bini lateri B C, quaterni vero B A accommodentur, hoc est, quemadmodum linea ad lineam; ita globi sint ad globos. Insuper in S, Γ, V tria sint puncta immota ac fixa, quae a linea sive globorum funiculo, cum movetur, raduntur, ac stringuntur: duaeque funiculi partes, quae supra trianguli basin, lateribus A B, B C sint parallelae, ut, quando connexio illa seriesque; globorum adscendit, descenditve, globi pes crura A B, B C volui possint". (Stevin 1605, 34). The translation is mine. See also important works by Radelet–de grave 1996.

[118] Redrawn from Stevin 1605, 34.

Thus a comparison of weights of the necklace that rely on the two opposing inclined planes (see Fig. 2.28) immediately gives the law of the inclined plane according to which two heavy bodies on two inclined planes are equilibrated when their weights are proportional to the length of the planes. Notice that Stevin considers the negation of the perpetual motion as unproblematic, but does not assume it explicitly as a principle of statics, though it is as fundamental for his mechanics at least as the law of the lever. The simple justification for this is that probably Stevin did not want his book to appear too new by introducing since from the beginning a non–standard statement. Stevin pretends to extend the law of the inclined plane to cases where the force to uplift the load is not parallel to the inclined plane. To this purpose, he concentrates his attention on a prism sliding on the plane.

In corollary V to the law of the inclined plane reported in the second half of the first book (Stevin 1605, p 36) it is easy for Stevin to show that the ratio between the weight *M* of the prism (Fig. 2.29), i.e. the force to lift it, called the *direct uplifting*, and the force *E* needed to move it on the inclined plane, called the *oblique uplifting*, is equal to the ratio of the segments LD and DI identified by the intersection of the ropes with the prism (because *M* : *E* = AB : BC = LD : DI).

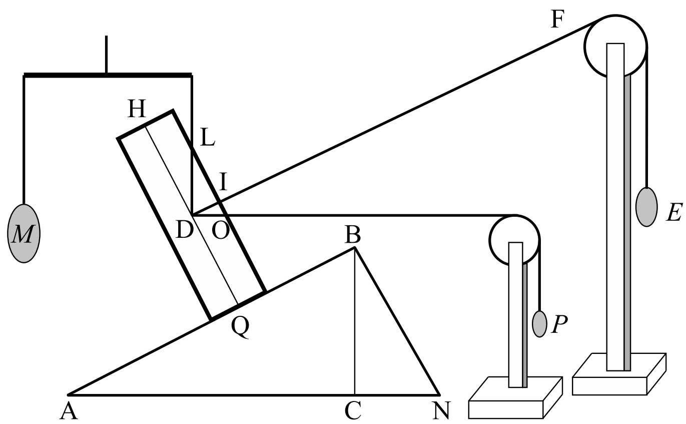


Fig. 2.29 Uplifting forces for various directions[120]

In corollary VI (Stevin 1605, 36–37) Stevin considers a horizontal uplifting measured by weight *P* (see Fig. 2.29). Imagining a rotation of ninety degrees, the horizontal uplifting becomes vertical and the plane

[119] "[...] ipsique globi ex sese continuum et aeternum motum efficient, quod est falsum" (Stevin 1605, 35). The translation is mine.

[120] Redrawn from Stevin 1605, 36.

ABC turns into a tilted plane whose slope is as NL of the triangle NCB. Following this rotation the ratio between direct and oblique upliftings is equal to that between the segments DO and DI. Stevin believes that this relationship is maintained even when the rope carrying the load *P* is effectively horizontal. At this point, he can say that in the vertical, in the inclined and in the horizontal directions the values of the `forces' necessary to keep the prism in balance are proportional to the length of the segments DL, DI, DO, intercepted by the ropes on the prism, to conclude (improperly) that this fact applies to all directions. Stevin's argument is interesting only for its strong rhetorical value, at least for the generalization to the case of any direction. The belief of the reader is made possible by the choice of a prism as the body to be lifted. It should be stressed however, that even if the reasoning cannot convince us the result is correct.

Below Stevin's proof of *Consectarium* (corollary) VI follows, to allow the reader to judge the lawfulness of the reasoning:

> Let BN be conducted cutting AC and extended to N, and the same DO cutting in O the extension of LI, so that the angle IDO be equal to the angle CBN, and then the uplifting *P* be applied along DO, taking the column in its position (with weights *M* and *E* balanced); then as LD is homologous to BA in the triangle BAC and DI with BC, it follows that BA is to BC as the weight on BA is to the weight on BC [...]. And also DL is to DI as the weight belonging to DL is to that to DI, i.e. *M* to *E*. Similarly the three lines LD, DI, DO being homologous to the three segments AB, BC, BN, then BA is to BN as the weights that belong to them, and also LD to DO will be like the weights that belong to them, i.e. *M* to *P*. Because this proportion is not valid only at that elevation as DI perpendicular to the axis, but for all sorts of angles.[121]

[121] "BN ducatur, secans AC continuatam in N, consimiliter D O secans continuatam LI, hoc est, latus columnæ in O, ut angulus IDO aequalis sit angulo CBN. Appendatur quoque ad DO pondus P oblique attollens, quod (amotis M, E ponderibus) columnam in suo situ conservet. Quia vero DL et BA, item DI et BC latera triangulorum DLI et BAC homologa sunt, hujusmodi conclusio inde deducitur. Quemadmodum BA ad BC: ita sacoma lateris B A ad anti sacoma lateris BC (per 2 consectarium) item quemadmodum DL ad DI: ita sacoma lateris DL ad antisacoma lateris DI, hoc estita M ad E. sed homologa latera triangulorum similium ABN, LDO sunt AB et DL, item BN, et DO. Itaque ut supra, quemadmodum BA ad B N: ira sacoma B A ad anti sacoma B N (per 1 consectarium) Et quemadmodum DL ad DO: ita illius sacoma ad hujus anti sacoma, id est, M ad P. si linea BN à puncto B aliovorsum; A scilicet versus, ultra BC fuisset ducta, etiam recta DO à D ultra DI cecidisset, hoc est, ut nunc citra: ita tunc ultra cecidisset, et praecedens demonstratio etiam isti situi accommoda

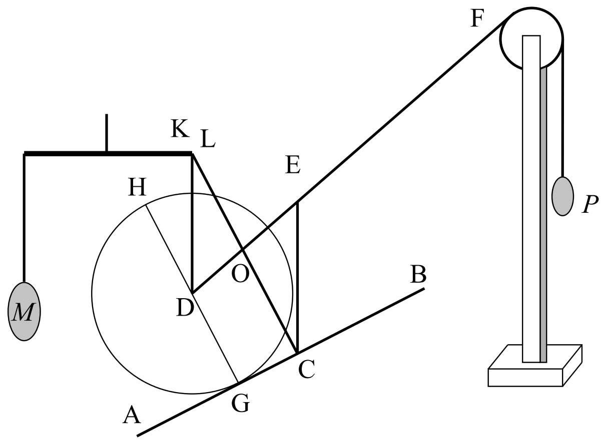


Fig. 2.30 Uplifting forces for a generic direction[122]

Stevin extends his result to non–cylindrical bodies, for example the sphere of Fig. 2.30. Stevin claims that the ratio between the direct uplifting *M* (the weight of the sphere) and the oblique uplifting *P* is as DL to DO, with LC orthogonal to the inclined plane, i.e.:

$$M : P = \mathrm{DL} : \mathrm{DO}$$

For similar triangles, it also holds good the relation:

$$M : P = \mathrm{EC} : \mathrm{OE}$$

EC, OE are the sides of the triangle OEC. A modern reader can easily see that Stevin's result is in fact correct, as the weight of the sphere is balanced by the tension of the rope and the constraint reaction orthogonal to the plane AB.

2.3.2.2 Forces' composition: the Rule of Parallelogram

The demonstration of the rule of the parallelogram for composition of *forces* is carried out by Stevin with a long series of theorems and

fuisset, hoc est, quemadmodum BA ad BN ita sacoma lateris BA, ad anti sacoma lateris BN esset: et quem-admodum DL ad DO: ita sacoma lateris DL, ad anti sacoma lateris DO. hoc est M ad P. Ut ista proportio non tantum in exemplis valeat, in quibus linea attollens, ut DI, perpendicularis est axi, sed etiam in aliis cuiusmodi cunque sint anguli" (Stevin 1605, 36–37). The translation is mine.

[122] Redrawn from Stevin 1605, 37.

corollaries (about twenty) that leave the modern reader a little upset (Capecchi and Drago 2005). This happens also because as each theorem and corollary is proved with rather slender mathematical reasoning, very close to the modern sensibility, it is difficult to understand the reason for Stevin's prolixity. A part of this difficulty might be overcome by assuming that Stevin's objective originally was not to formulate the rule of composition, of which perhaps he did not understand the full extent, but only to make a series of comments on the way weights can be lifted. In fact, the explicit formulation of the rule of the parallelogram is in the section of the *Additions* named spartostatica.

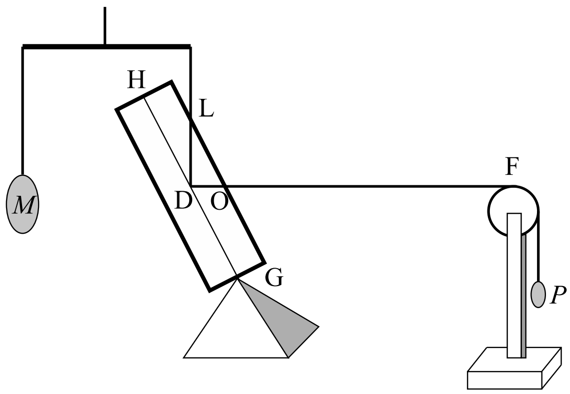


Fig. 2.31 Uplifting forces for a punctiform support by Stevin[123]

Consider the prism (See Fig. 2.31) with direct and horizontal upliftings applied to its centroid. Stevin assumes that the ratio between the direct and horizontal upliftings is the same as that of the segments DL and DO. Styevin does not pause to justify the lawfulness of the replacement of the inclined plane with the fixed point G.

Reading among the lines it can be understood that since for every inclination of the rope the intercept with the side of the prism provides the 'force' necessary to maintain the equilibrium whichever is the inclination of the inclined plane, the inclined plane can be replaced with a constraint that performs its essential function, i.e. to offer a support to the prism.

By means of the following Theorem 12, Propositio 20, Stevin extends his result to the case where the fixed point and the upliftings are applied anywhere in the axis of the prism.

[123] Redrawn from Stevin 1605, 39.

> 12 THEOREM. 20 PROPOSITION. If in the axis of the prism there are a fixed point and a movable point, which could be maintained in any disposition by means of a direct uplifting, the line of direct uplifting is to the line of inclined uplift as the direct uplifting is to the oblique uplifting.[124]

The result of Stevin, namely the determination of the *force* necessary to support the prism constrained to a fixed point could have been extended quite easily to the case of a body of any shape to get a rule of equilibrium as efficient as the vanishing of the static moments. But Stevin does not do it.

The next step – basically the definitive one – consists in the consideration of the situation of following Fig. 2.32 for which Stevin states the following theorem:

> 18 THEOREM. 27 PROPOSITION. If a column is maintained in equilibrium by two oblique uplifting as the line of the oblique uplifting is to the line of the direct uplifting, so each oblique uplifting is to its direct uplifting.[125]

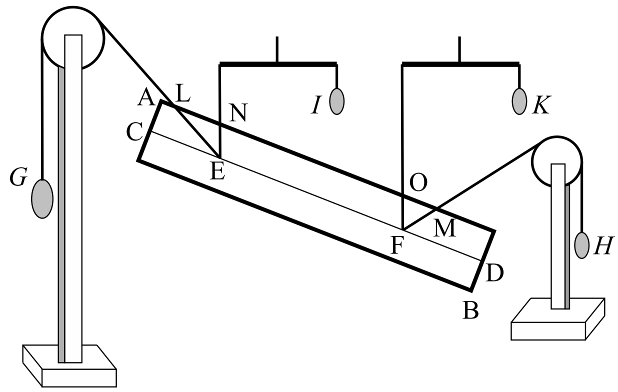


Fig. 2.32 Equilibrium of a column supported into two points[126]

[124] "12 THEOREM. 20 PROPOSITIO. Si axis columnae puncta habeat, firmum, et mobile, et ex isto dependentia pondera, unum rectè, alterum obliquè extollens, in dato situ columnam conservant: erit quemadmodum linea recte extollens ad lineam oblique extollentem; ita illius pondus, ad pondus hujus". (Stevin 1605, 41).

[125] "18 THEOREM. 27 PROPOSITIO. Si columna, et duo pondera oblique extollentia situ aequilibria sunt, erit quemadmodum linea oblique extollans, ad lineam recte extolletem: ita ponderum quodque obliquum ad suum pondus rectum". (Stevin 1605, 48).

[126] Redrawn from Stevin 1605, 49.

Notice that if points E and F have the same distance from the centre of gravity of the prism the vertical upliftings *I* and *K* will be the same, so LE and FM have the ratio of *G* and *H*. Indeed from theorem XVII the relations can be written:

$$\text{LE} : \text{NE} = G : I$$
$$\text{OF} : \text{MF} = K : \text{H}$$

which if $I = K$ can be combined to give:

$$\text{LE} : \text{FM} = G : H$$

From this result, it is very easy to arrive at the parallelogram *Rule of the additions*. To get the rule of the parallelogram from Theorem 18 (Stevin 1605, p 48) it suffices to consider the case where the two points E and F (see Fig. 2.32) coincide with each other and with the centroid of the prism as shown in the following Fig. 2.33:

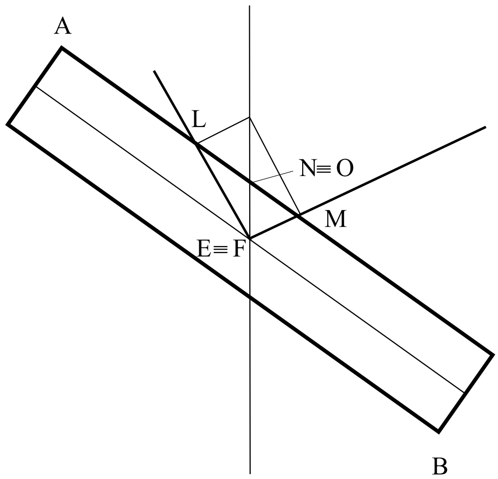


Fig. 2.33 Reconstruction of the application of parallelogram of forces rule according to Stevin

In this case it can be affirmed that the proportion between segments FQ, EL, EM is the same as the weight, and inclined upliftings (Stevin 1605, Corollary III, 35) of the *Additions*; but this is the rule of the parallelogram. In order to prove this it is enough to consider that the whole uplifting, i.e., the weight of the prism, in Fig. 2.33 is given by $I + K = 2\,I = 2\,K$, which is proportional to 2NE = 2OF.

PART II

History & Historical Epistemology Analyses

Chapter 3

The Analysis of Books VII and VIII of *Quesiti et inventioni diverse*

[…] *Signor Clarisimo parte di questa scientia* [of weights] *nasce,*
ouer deriua dalla Geometria, & parte dalla Natur al Philosophia: perche, parte
delle sue conclusioni se dimostrano Geometricamente,
& parte se approuano Physicalmente, cioe naturalmente.
(Tartaglia 1554, *Book VIII*, Q I, 82v)

I analyse Niccolò Tartaglia's Books VII and VIII of the *Quesiti et inventioni diverse*. The discussion is organized both from historical and epistemological points of view. Particularly, I will focus on the reasoning proposed by Tartaglia against the arguments of the Aristotelian *Problemata mechanica* on the accuracy and stability of a balance – with large or small arms, and fulcrum below or above – (Book VII) and concerning the principles of the science of weights (Book VIII). The latter arguments are discussed taking into account Jordanus de Nemore's corpus on the science of weights for exploration of the structure of the shared knowledge of early modern statics aiming to discuss alternative frameworks, and so distinguishing between individual and shared structures in the literature belonging to early modern mechanics. In this sense, this Chapter is devoted to *historical epistemology of science* presenting an integrated history and epistemology of scientific methods, which combine epistemological and historical approaches to identify significant historical hypotheses within the relationship between physics and mathematics (physical observations and theoretical mechanical modeling).

3 A Historical Epistemology Outline on Early Statics in the *Books VII* and *VIII*

Niccolò Tartaglia's studies of the science of weights cannot be understood unless without an exploration of the structure of the shared knowledge of early modern statics. Particularly, it is not possible to know his definitive cultural background with certainty because reliable biographical information about his reading and shared literatures is too weak. Tartaglia's education (see Chapter 1), probably not that of a self–taught man as he would have us believe, was very much influenced by Aristotelian physics and could not be alien to the discussions then in progress on the nature of subalternate–science. It is not certain that he knew the theory of impetus, at least up to 1540. As a teacher of abacus, first in Verona then in Venice –cities where there was a thriving printing industry– Tartaglia was in the ideal situation to come into contact with new scientific publications. Because of his profession, he had a wide experience with algebra and developed application of geometry and algebra to various practical problems.

Tartaglia knew Euclidean geometry, considering that when he wrote the first edition of *Quesiti et invention diverse* in 1546 he had already published an important Italian translation of the *Elements* (Tartaglia 1543), which had an enormous editorial success throughout Europe. It is reasonable that he also knew the *Conic sections* of Apollonius, published in 1534 (Ekhalm, 189) by his friend Giambattista Memo[1] (d. 1536). As for Archimedes's writings, we know that translations of his works had been published already in 1543. About the texts of the science of weights, we cannot be certain of my speculations on his readings. It is likely that he had early access to the Latin edition of the *Problemata mechanica* (Leonico Tomeo 1525; 1530) by Niccolò Leonico Tomeo (1456–1531), professor at the University of Padova. Thus, it reasonable to think that he read *Liber Iordani Nemorarii viri clarissimi, de ponderibus propositiones XIII* (Jordanus1533) edited by Petrus Apianus (1495–1552) who reproduced the *Liber Jordani de ponderibus* and added an interesting commentary. Following this point of view, we think that he probably also knew Biagio Pelecani of Parma's works *Tractatus Blasi de ponderibus* (Moody and Clagett [1952] 1960, IV; see also Crombie 1959, 101) between the science of weights of Northern Europe and Italy. He also knew the two medieval texts: the *Liber Euclidis de ponderoso et levi* – published as an appendix to

[1] Mathematician and Greek translator from the Latin, he lived in Venezia where he obtained the chair of mathematics at the same university (1530).

his *Elements*'s edition (Tartaglia 1569, 316r) – and the *Liber Archimedis de insidentibus in humidum*, or *Liber Archimedis de ponderibus*. Nevertheless mostly he was in possession since 1539 (Laird 2000, 16) of a manuscript of the *Liber de ratione ponderis* that Curtio Troiano Navò published after Tartaglia's death.

Tartaglia's Books VII and VIII on science of weights established the long–term development of mechanical knowledge concerning instruments and conceptual streams built on this theoretical framework, centring on the role of shared knowledge, of physical and mathematical objects.

3.1 The Analysis of *Book VII* (1554)

Book VII of the *Quesiti et invention diverse* was inspired by the Aristotelian *Problemata mechanica*, in particular because of those problems/questions that today are normally known as the first and second and are related to the accuracy and stability of balances. Aristotelian mechanics[2] was of considerable importance in the Renaissance; by its nature it was able to mobilize people of different backgrounds, humanists involved in the physical and philosophical aspects and mathematicians and engineers involved in its theoretical and technological content. However the interest was mainly philosophical for it stimulated discussion about the role of mathematics in physics. There is agreement that the *Problemata mechanica* as such remained without direct influence from the decline of Hellenistic science until the Greek revival of the Renaissance. Latin writers of the Middle Ages who encountered the Greek text were insufficiently impressed by it to continue the discussion. The beginning of the XVI century saw two important Latin translations by two humanists. The first was due to Vittore Fausto (1480–1511), but the most largely

[2] Aristotle 1955bc, 1984; see also Baldi 1621 and Aristotle 2000. In the Aristotelian school, the *Problemata mechanica* remained an argument which was long debated. In this regard, see Drake (Rose and Drake) and, recently, Winter (Winter 2007). See also: Duhem 1905–1906, II, 292, 1906–1913; Clagett 1956, 1959, 1964–1984; Clagett and Moody [1952] 1960; Brown 1967–1968, 1976; Lindberg; Truesdell 1968. During the Middle Ages and Renaissance the attribution of the *Problemata mechanica* to Aristotle was substantially undisputed. Today there is the spread feeling that it was not Aristotle's but of some one of his circle. Main Aristotelian works on mechanical arguments, besides *Problemata mechanica*, are in *Physics* (Aristotle 1999), *On the Heaven* (Aristotle 1984), and in *Problemata mechanica* (Aristotle 1955c). From an epistemological point of view, Aristotle dealt with the organization of science particularly in *The posterior analytics* (Aristotle 1853; see also *Id*., 1996, 1955c, 1949).

circulating copy was the second translation by Tomeo[3] (Leonico Tomeo 1525, Leonici Thomei 1530). When Tartaglia wrote *Quesiti et inventioni diverse* (1546) he had most probably read only Leonico's version because that of Vittore Fausto was practically unknown in Italy. For this reason – as far as we know – the following references to *Problemata mechanica* mostly will came from Leonico Tomeo[4] (1456–1531).

Book VII concerns a dialogue developing in a day between Tartaglia and Diego Hurtado de Mendoza (1503–1575), an aristocrat and humanist who was the Spanish ambassador to Venice from 1539 to 1546, and to Rome from 1547 to 1552 (Drake and Drabkin 1969, 104). Mendoza asks seven questions to which Tartaglia gives answers. The first three questions concern the accuracy of balances, the last four the stability for various positions of the fulcrum. The book was studied in depth in (De Pace 1993) for aspects regarding relations between physics and mathematics.

3.1.1 The Aristotelian Mechanical Problems

3.1.1.1 The Accuracy of Balances

In the first problem Aristotle wants to explain why larger balances are more accurate than smaller ones:

> [Problem 1] First of all then a difficulty will arise as to what happens to the balance; why, that is, larger balance are more accurate than smaller ones?[5]

Tartaglia will question this conclusion but, for the moment, we do not and consider it as a physical truth. Aristotle wants to explain the physical fact he asserts by means of mathematical argumentations; thus assuming mechanics as a subalternate science in which physics is subalternate to geometry. In fact mechanical problems

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[3] Note that in Leonico Tomeo's translation the numbering of problems starts from Heet's second one. Thus, the first problem has no number and the second is Leonico Tomeo's first problem. The English translation is that of (Aristotle 1955bc).

[4] Nicholas Leonicus Thomaeus (or Niccolò Leonico Tomeo, Nikollë Leonik Tomeu, Leonik Tomeu) was born in Albany and worked as professor of philosophy at the University of Padua.

[5] "In primis igitur quae accidunt circa libram dubitare faciunt, quae nam ob causam exactiores minoribus maiores sunt librae". (Aristotle 1525, 25v. See also *Problemata mechanica* (Aristotle 1955c, 848b, 337).

> [...] are not altogether identical with physical problems, nor are they entirely different from them, but they are common both to mathematical and physical speculations, for the *why* is demonstrated by mathematical speculations, but the *object* is demonstrated by physics.[6]

In the following, we report some of Aristotle's reasoning. He refers the balance to the circle, and the problem of accuracy to the fact that forces applied to points of the circle are the more efficient the farther from the centre they are. The geometrical reasoning consists in showing that with the same tangential (vertical) displacement farther points remain closer to the circle; for physical reason it can be conjectured that they suffer less resistance in the motion, and the applied forces are more effective (Capecchi 2009, 2012a):

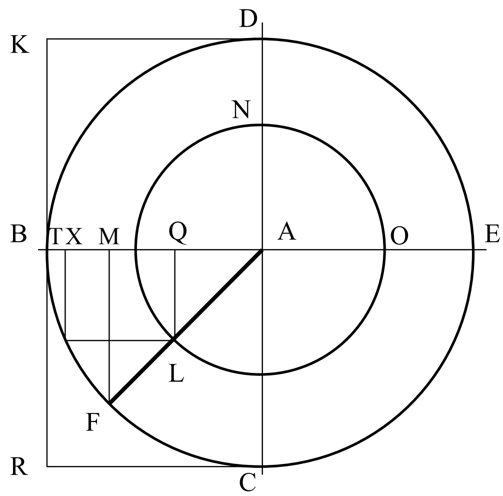

Fig 3.1 Motions according and against nature in the circle[7]

> [Continued from Problem 1] The origin of this is the question why that part of the radius of a circle which is farthest from the centre moves quicker than the smaller radius which is close to the centre, and is moved by the same force. The word quicker is used in two senses; if a point covers the same distance as another in a shorter space of time we call it quicker, and also if it covers a greater distance in an equal time. But in our case the greater radius describes a greater circle in equal

[6] Aristotle 2000, 55. My translation.

[7] Redrawn from Aristotle 1525, 27r. See also *Problemata Mechanica* 848b in Aristotle 1955c, 849ab, 343.

> time; for the circumference outside is greater than the circumference inside. The reason is that the radius describing the circle is performing two movements. Now whenever a body is moved in two directions in a fixed ratio it necessarily travels in a straight line, which is the diagonal of the figure which the lines arranged in this ratio describe[8].
> From what has already been said the reason why the point more distant from the centre travels more quickly than the nearer point, though impelled by the same force, and why the greater radius describe the greater arc, is quite obvious. Why also greater balances are more accurate than smaller ones, is clear from these considerations.[9]

After "proving" the greater accuracy of larger balances, Aristotle comments on some other physical facts, i.e. that a large balance does tilt for a small weight added to one arm while a smaller balance does not.

> [Continued from Problem 1] Now the extremity of the balance scale must move at a greater rate under the influence of the same weight, inasmuch as it is further from the cord, and consequently in small balances some weights must make no impression on the senses, but in large balances the movement must be obvious ; for there is nothing to prevent a quantity from moving too little for it to be observed by the senses. But in a large balance the same weight makes the movement visible. Some movements are obvious in both cases, but are much more obvious in larger balances, because then the extent of the swing is much greater for the same weight. This is how sellers of purple arrange their weighing machines to deceive, by putting the cord out of the true centre, and pouring lead into one arm of the balance, or by employing wood for the side to which they want it to incline taken from the root or

[8] "Huius autem rei principium est quam ob rem in ipso circulo quae plus distat linea, eadem vi commota citius fertur, quam illa quae minus distat. Citius enim bifariam dicitur. Sive enim in minori tempore aequalem pertransit locum, citius fecisse dicimus, seu in aequali maiorem. Maior autem in aequali tempore maiorem describit circulum; qui enim extra est, maior eo qui intus est. Horum autem causa, quoniam duas fertur lationes ea, quae circulum describit" (Aristotle 1525, 25v–26r. See also *Problemata mechanica* in Aristotle 1955c, 848b, 337).

[9] "Omni quidem igitur circulum describenti istuc accidit: ferturque eam quae secundum naturam est lationem secundum circunferentiam; illam vero quae praeter naturam in transversum et secundum centrum, maiorem autem semper eam quae praeter naturam est, ipsa minor fertur, quia enim centro est vicinior, quod retrahit vincitur magis: Quod autem magis quod praeter naturam est movetur ipsa minor quam maior illarum, quae ex centro circulos describunt, ex iis manifestum" (Aristotle 1525, 27r–27v. See also *Problemata mechanica* in Aristotle 1955c, 849b, 347).

> from where there is a knot. For the part of the tree in which the root lies is heavier, and a knot is in a sense a root.[10]

3.1.1.2 The Stability of Balances

The Aristotelian problem 2 is related to what is today known as the problem of stability. It concerns balance having their fulcrum either above or below their beam.

> [Problem 2] Why a balance fixed from above by a cord, when after the beam has inclined the weight is removed, the balance ascends. If, however, it is supported from below, then it does not ascend but rest?[11]

The explanation of the two cases is quite simple and convincing, even though no reference to a declared mechanical law is stated.

> [Continued from Problem 2] It is because, when the support is from above (when the weight is applied) the larger portion of the beam is above the perpendicular. For the cord is the perpendicular. So that the greater weight must swing downwards until the line dividing the beam coincides with the perpendicular, because the greater weight is in the raised part of the beam.[12]

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[10] "Ab eodem igitur pondere citius moveri necesse est extremum librae, quo pus a sparto discesserit. Et nonulla quidem in parvis libris imposita non manifesta sensui sunt pondera; in magnis autem manifesta. Nihil enim prohibet minorem moveri magnitudinem quam ut visioni sit manifesta. In magna autem libra idem pondus visibile efficit magnitudo. Quedam vero vero manifesta sunt in utrisque, sed multo magis in maioribus, quoniam multo maior inclinationis sit magnitudo ab eodem pondere in maioribus. Quam ob rem machinantur ii, qui purpuram vendunt, ut pendendo defraudent, tum ad medium spartum non ponentes, tum plumbum in alterutram librae partem infundentes, aut ligni quod ad radicem vergebat, in eam quam deferri volunt partem constituentes, aut si nodum habuerit (ligni enim gravior illa est pars, in qua est radix; nodus vero radix quaedam est)" (Aristotle 1525, 30r. See also *Problemata mechanica* in Aristotle 1955c, 849b, 347).

[11] "Cur siquidem sursum fuerit spartum, quando deorsum lato pondere quispiam id amovet sursum ascendit libra, si autem deorsum constitutum fuerit non ascendit, sed manet?" (Aristotle 1525, 30v. See also *Problemata mechanica* in Aristotle 1955c, 850a, 347–349).

[12] "An quia sursum quidem sparto existente plus librae extra perpendiculum sit; spartum enim est ad perpendiculum quare necesse est deorsum ferri id quod plus est donec ascendat quam bifariam libram dividit, ad ipsum perpendiculum, cum onus incumbat ad librae partem sursum raptum". (Aristotle 1525, 30v. See also *Problemata mechanica* in Aristotle 1955c, 850a, 347–349).

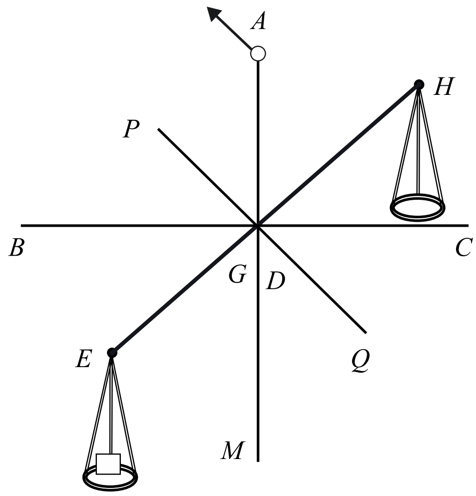

Fig. 3.2 Equilibrium of the balance with fulcrum above[13]

> [Continued from Problem 2] If, however, the support is from below, the opposite result ; for now the portion of the beam which is lower than the perpendicular dividing it is more than half, consequently it does not return to its place, for the part rising above is lighter[14]

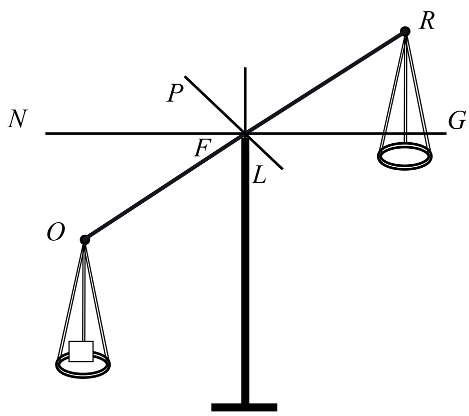

Fig. 3.3 Equilibrium of the balance with fulcrum above[15]

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[13] Redrawn from Aristotle 1525, 30v.

[14] Aristotle 1955b, 353. Here we consider Hett's translation as Leonico Tomeo's is not clear to us.

[15] Redrawn from Aristotle 1525, 331r. See also *Problemata Mechanica* 850ab in Aristotle 1955c, 850ab, 351.

If the balance is supported from above, the horizontal position is a stable equilibrium position for if the balance is removed from the horizontal position it recovers it; while if the balance is supported from below, the horizontal position is an unstable equilibrium position for if the balance is removed from the horizontal position it does not return to its place. The geometry serves to prove that in the two cases the axis cuts the beam of the balance into two different parts. One further physical argument says that the larger part pushes down the smaller part.

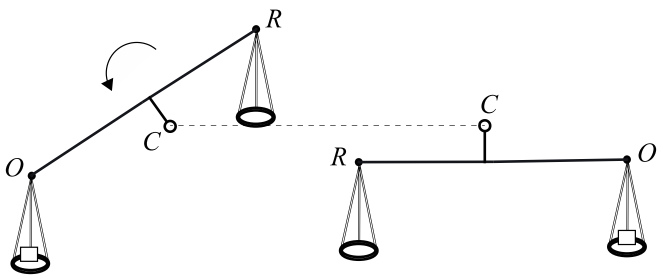

Fig. 3.4 Overturning of the balance with fulcrum below

Notice that the explanation holds good only if the beam of the balance is considered as an heavy body. With a weightless beam stability and instability persist respectively for the two positions of the fulcrum but to prove that calls for more sophisticated theoretical tools than Aristotle's, for example the concept of static moment. When the balance is removed from the horizontal position the weight suspended from the more elevated arm has a greater distance from the fulcrum if it is above and then a greater static moment than that of the other weight and the balance recovers the horizontal position. The contrary occurs when the fulcrum is below. We want to stress that Aristotle is scarcely accurate, or even not correct, in the describing what happens for balances with fulcrum below. He says that if one arm is pressed down it does not recover the horizontal position. Actually, what occurs is that the balance rotates until it is completely reverted and has became a balance with the fulcrum above (see Fig. 3.4).

In order to stress the relevance of the weight of the beam in the Aristotle's discourse, we refer the figures drawn by Walter Stanley Heet to illustrate problem 2, which makes evident the role of the balance beam.

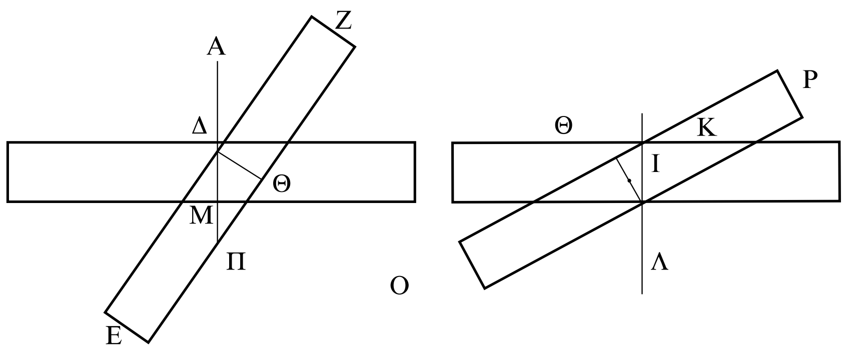

Fig 3.5 Instance of a balances with a two–dimensional beam by Aristotle[16]

> Why is it that small forces can move great weights by means of a lever, as was said at the beginning of the treatise, seeing that one naturally adds the weight of the lever? For surely the smaller weight is easier to move, and it is smaller without the lever. Is the lever the reason, being equivalent to a beam with its cord attached below, and divided into two equal parts? For the fulcrum acts as the attached cord: for both these remain stationary, and act as a centre. But since under the impulse of the same weight the greater radius from the centre moves the more rapidly, and there are three elements in the lever, the fulcrum, that is the cord or centre, and the two weights, the one which causes the movement, and the one that is moved; now the ratio of the weight moved to the weight moving it is the inverse ratio of the distances from the centre. Now the greater the distance from the fulcrum, the more easily it will move. The reason has been given before that the point further from the centre describes the greater circle, so that by the use of the same force, when the motive force is farther from the lever, it will cause a greater motion.[17]

3.1.2 The First Three *Quesiti* on Accuracy

In the first three *Quesiti* of the *Book VII* (Tartaglia 1554, *Book VII*, Qs I–III, 78r–80v) Tartaglia explicitly references to the Aristotelian text and the proof (discussed in three parts[18]) concerning accuracy of the Aristotelian

[16] Redrawn from Aristotle 1955c, 850a, p 349 (left); 850ab, p 351 (right). See also Aristotle 1955, p 349, p 351.

[17] *Problemata mechanica* in Aristotle 1955c, 850a 30, 353.

[18] The first part: Tartaglia 1554, *Book VII*, Q V, 81v. The second part: *Ivi*, Q VI, 81rv–82rv. The third part: *Ivi*, 82v.

balance is in the last fourth *Quesiti* (Tartaglia 1554, *Book VII*, Qs IV–VII, 80v–82r).

In the first *Quesito*, to his interlocutor Don Diego Hurtado de Mendoza Imperial Ambassador in Venezia, who claims to be acquainted with the *Problemata Mechanica* both in Latin and Greek (Tartaglia 1554, *Book VII*, Q I, 78r), Tartaglia replies that

> [*Quesito* I] N. It is quite a while since I saw these [*Problemata Mechanica*], particularly the Latin[19].

Probably – as above noted – he referred to Leonico Tomeo's translation. In his reading he has found some weaknesses that – to be clearly identified – ask for the an understanding of the principles of the science of weights:

> [*Quesito* I] N. Sir, should you wish me to explain them to you properly, many of the problems would require that I first explain to your Excellency the principles of the science of weights.[20]

Immediately after Tartaglia express consideration about the role of mathematics and physics in the Aristotelian text:

> [*Quesito* I] N. It is true that he proves each of his problems partly by physical reasons and arguments and partly by Mathematical. But some of his physical arguments may be opposed by other physical reasoning, and others can even be shown to be false through Mathematical arguments (by means of the said science of weights). And besides that, he omits or remains silent about a problem of no little importance concerning the balance, because (so far as I can judge) one cannot assign the cause for that problem by physical reasoning, but only through the science of weights.[21]

He first notices, though not explicitly[22], that *Problemata mechanica* belongs to the subalternate–science tradition because part of the reasoning is physical (coming from empirical observation of natural facts), part mathematical. Then he asserts that Aristotle makes both wrong references

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[19] "N. Eglie tempo assai che io le vidi, massime Latine" (Tartaglia 1554, *Book VII*, Q I, 78r).

[20] "N. Signore, vi sono dubbii assai, che à volergli à sofficientia delucidare, à me saria necessario prima à dechiarare à vostra Signoria li principii della scientia di pesi" (Tartaglia 1554, *Book VII*, Q I, 78r. Drake and Drabkin's translation).

[21] Tartaglia 1554, *Book VII*, Q I, 78r. Drake and Drabkin's translation.

[22] In the *Book VIII* Tartaglia will use the attribute *subordinate* for mechanics (Tartaglia 1554, *Book VIII,* 82v).

to empirical facts and errors in mathematical reasoning and at least an omission. The wrong references and errors are with respect to the accuracy of balances, the omission to the case of balances with fulcrum centred in the axis. In substance Tartaglia "dares" to contrast some Aristotelian[23] positions "frankly" as Raffaello Caverni (1837–1900) will point up (Caverni 1891–1900, I, 53–54). Actually, we think, more than a question of bravery, it was a self–sponsoring affair. He as a teacher of abacus wanted to show the nobleness of the matter he was skilled on, not against Aristotle himself, but the Aristotelian philosophers of the universities. This would have yielded a larger number of students to him and a greater profit (Cuomo 1998).

To Mendoza who asks how can he distinguishes between physical and mathematical argumentations, Tartaglia replies:

> [*Quesito* I] N. The physicist considers, judges, and determines things according to the senses and material appearances, while the mathematician considers and determines them not according to the senses, but according to reason, all matter being abstracted–as your Excellency knows that Euclid was accustomed to do.[24]

Entering the merit of the accuracy of balances, Tartaglia notices that Aristotle's position would be correct for an ideal balance, deprived of any imperfections. However, for real balances Aristotle's position is generally not true as a matter of fact; indeed normally smaller balances are more accurate than larger ones.

> [*Quesito* I] N. […]. But next, wishing to consider and test that statement materially and with physical arguments, as he does at the end, by the sense of sight and with a material balance. I say that by this sort of argument the problem is not generally verified, and even that the opposite occurs; that is, smaller balances are found to be more sensitive than larger ones. That this is true in material balances, experience makes manifest; for if we have a worn ducat and want to see by how many grains it is too light, using a large balance such as one of those used to weigh spices, sugar, ginger, cinnamon, and such materials, we shall get a poor result; but if we use one of those small balances employed by bankers, goldsmiths, and jewellers, no doubt we can be quite certain of the result. This is just the contrary of that which was concluded in this problem; for here, small balances are more sensitive

[23] On Tartaglia anti–Aristotelian positions, already discussed before the *Quesiti et inventioni diverse* see Bolleti (Bolletti 1958, 45–51).

[24] Tartaglia 1554, *Book VII*, Q I, 78v.

> than large ones because they more thoroughly and more subtly show the difference of weights.[25]

Therefore, Tartaglia opposes to Aristotle a more reliable physical argument than his, and explain why the correct mathematical argumentation worked out by Aristotle may be falsified by experience. It depends on the fact that smaller balances are often made with a greater accuracy and suffer less of the matter impediments.

> [*Quesito* II] N. […] I say, Sir, that the cause that the larger and smaller balances do not behave as the author concludes and proves has its roots in the difference between the material parts or members of which they are composed, which parts or members are the two arms and the pivot (that is, the axis or center on which the arms turn in both cases). For the said arms and pivot in the larger scale or balance are much more gross and bulky than in the smaller. And since the arms of those scales or balances are to be considered mathematically, that is, apart from all material, they are considered and assumed to be as simple lines, without breadth or thickness; and the pivot or axis [of support] is assumed to be a simple indivisible point.[26]

Thus, mathematicians do not accept demonstration made on the strength of the senses and questions which have already been proved with mathematical arguments should not be subject of physical argumentations, which are less certain:

> [*Quesito* I] N. […] And for this and other reasons, the mathematician does not accept or consent to proofs and demonstrations made on the strength and authority of the senses in matter, but only those made by demonstrations and arguments abstracted from all matter. Consequently, the mathematical disciplines are considered by the wise not only to be more certain than the physical, but even to have the highest degree of certainty. And therefore those questions which can be demonstrated with mathematical arguments cannot be suitably proved by physical arguments. Likewise those which have already been demonstrated by mathematical arguments (which are the most certain) should not be subjected to attempts to certify them still better by physical arguments, which are less certain.[27]

[25] Tartaglia 1554, *Book VII*, Q I, 78v.
[26] Tartaglia 1554, *Book VII*, Q II, 79v.
[27] Tartaglia 1554, *Book VII*, Q I, 78v–79r.

Tartaglia, since the beginning, switches to criticize the remarks added by Aristotle to the solution of the first problem, i.e. that it can happen that a small weight makes a large balance to rotate but not a small one:

> [*Quesito* I] N. […]. He [Aristotle] also adds this other conclusion, and in this form: And certainly there are some weights which manifest themselves in both sorts of scales (that is, the large and small), but much more in the larger, a far greater tilting being made there by the same weight.[28]

Tartaglia criticised *open face* ("[...] a viso aperto [...]"[29]) Aristotle's remarks concerning the physical nature which are not generally true because they often are not verified in practice:

> [*Quesito* I] N. […]. Now if we consider, judge, and test this conclusion as physicists that is, by the strength and authority of the sense of sight– then, as was said of the other, it will certainly suffer no less opposition in the said material scales than will the other [conclusion], and for the same reasons.[30]

Nevertheless, Aristotle's remarks are wrong from a mathematical point of view also, because they are not even verified for ideal balances. In such a case large and small balance behave equally: indeed if one adds a weight as small as he likes on one of the arms of a balance with the size one wants, this tilts until it reaches the vertical position:

> [*Quesito* I] N. […]. And similarly if we consider, judge, and test it as mathematicians (that is, apart from any matter), this conclusion will still be false, because every sort of weight placed in any sort of scale will make it tilt continually until it comes to the last and lowest place it can. And all this is demonstrated in the said principles of the science of weights.[31]

A further comment on the role of mathematics follows:

> [*Quesito* I] N. […]. Your Excellency is not mistaken, for in fact all those things that are known by the mind to be true, and particularly by abstraction from all material, should reasonably be verifiable in matter also by the sense of sight; *otherwise mathematics would be wholly vain*

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[28] Tartaglia 1554, *Book VII*, Q I, 79r.
[29] Caverni 1891–1900, I, 3–54.
[30] Tartaglia 1554, *Book VII*, Q I, 79r.
[31] Tartaglia 1554, *Book VII*, Q I, 79r.

> *and useless and devoid of profit to man* [emphasis added]. And if it happens that they are not verified in the aforesaid scales or in large and small balances, as questioned, then it is to be believed and even held for certain that all this proceeds from the disproportionality and inequality of the material parts and members that make one scale differ more than another from balances considered apart from all matter.[32]

The arguments based on mathematics not only are always correct, but the results are also true, otherwise mathematics would be useless, it would be a sterile discipline. When things do not add up, it means that the physical objects that are being studied are too far from the mathematical objects. To get a grip on tying mathematical reasoning to the physical facts Tartaglia proposes to apply mathematical reasoning to physical models very well constructed: he does not pose instead the inverse problem of making richer the geometric model in order to be able to grasp reality in a more satisfactory manner.

> [*Quesito* I] So if we want to defend and save this problem of Aristotle – that is, make it verified in matter and in every kind of balance or scale, large or small – it is necessary to make all the parts or members of each balance uniform, in such a way that all are equally applicable to those considered apart from all material. This done, we shall not only verify sensibly in matter this problem of his for material scales and balances, but will also verify those other two conclusions he adds at the end.[33]

Continuing in the second *Quesito*:

> [*Quesito* II] N. [...]. Since the arms of those scales or balances are to be considered Mathematically, that is, apart from all material, they are considered and assumed to be as simple lines, without breadth or thickness; and the pivot or axis [of support] is assumed to be a simple indivisible point. Such a scale or balance, as much as possible, would be given as in fact despoiled and naked of any sensible material, as is considered by the mind, and would doubtless be agile and responsive far beyond all material scales or balances of the same size, for it would be completely free of any material hindrance.[34]

That is, Tartaglia believes that embodiment in matter can invalidate geometrical reasoning. Clearly, in this passage a conception of matter which resists formal, mathematical treatment is at work. Tartaglia makes

[32] Tartaglia 1554, *Book VII*, Q I, 79v.
[33] Tartaglia 1554, *Book VII*, Q I, 79v.
[34] Tartaglia 1554, *Book VII*, Q II, 79v–80r.

no allusions to the philosophical underpinning of the conception, but it was a basic tenet of the larger framework of scholastic and Renaissance hylomorphism. Although Tartaglia himself was not educated at a university and made sparse contact with the philosophical tradition of his time, a conception of matter similar to the one he invokes can be traced through the philosophical tradition back to the works of Aristotle. It is interesting to note that Tartaglia believed that the mismatch between mathematical arguments and running machines can be minimized by building machines that are as uniform as possible, but he did not believe the mismatch can be entirely eliminated (Biener 2008, 74).

3.1.3 The Last Four *Quesiti* on Stability

The *Quesiti* IV–VII of *Book VII* concern the stability of balances with equal arms and weights. Before beginning to analyse the last four *Quesiti*, brief remarks on the general aims and structure of reasoning proposed by Tartaglia are necessary.

Tartaglia mainly presents his reasoning (Tartaglia 1554, *Book VII*) basing on the following three physical circumstances:

1. Balances with *fulcrum above* the beam for which the horizontal position is asserted to be a stable equilibrium position (*Ivi*, Qs IV–V).
2. Balances with *fulcrum below* the beam for which the horizontal position is asserted to be not stable equilibrium position (*Ivi*, Qs V–VI).
3. Balances with *fulcrum inside* (centred) in the beam for which the horizontal position is asserted to be a stable equilibrium position (*Ivi*, Q VII).

These three physical circumstances appear to be always very important. In fact, at the end of *Book VII*, Tartaglia (by means of his interlocutor Mendoza) remarks two main reasons which moved him to study these cases since: a) – Aristotle omitted the above cited 3rd case concerning the balance with the fulcrum in the centre. In his words:

> [*Quesito* IV] S.A. [...] at the beginning of our reasoning, that Aristotle omitted, or was silent on, a question about balances of great relevance and inquire.[35]

[35] Tartaglia 1554, *Book VII*, Q IV, 80v.

and b) – common sense really would justify the idea that the balances with longer arms are sharper than the balance with shorter arms; an emblematic and anti–Aristotelian situation that

> [*Quesito* VI] S.A. [...] these two parts [cases with fulcrum above or below] almost, our mind grasps for a natural reason [e.g., common sense] without any proof.[36]

Moreover, in order to justify that the Aristotelian subalternate science is not sufficient in itself to the purpose, Tartaglia emphasized the third case–study as the most complex one:

> [*Quesito* VII] S.A. [...] the cause of this seems to me father removed from common sense than for the two usual cases.[37]

But, he claims, it is first necessary to become aware of the science of weights. In his words:

> [*Quesito* VII] N. I have told your Excellency that in order to demonstrate the cause of this effect, it will be necessary for me first to define and explain to your Excellency some of the terms and principles of the Science of Weights.[38]

3.1.3.1 The Balance with Fulcrum Above the Beam

In the following we report the figure and commentaries (*Qs* IV–V) by Tartaglia who discusses the first case of Aristotelian reasoning on balances. In order to justify our previous hypotheses concerning that case, a Latin version of the *Problemata mechanica* was read by Tartaglia. It could have been Leonico Tomeo's formulation; we note how the following Tartaglia's figures are substantially quite similar to two figures reported by Leonico Tomeo (Aristotle 1525).

> [*Quesito* IV] S.A. But if I well remember you also said, at the beginning of our reasoning, that Aristotle omitted, or was silent on, a question about balances of great relevance and inquire. Now tell me what question is this. N. If your Excellency remember his second problem, where he [Aristotle] asks, and consequently proves, the why when the fulcrum is above the scale, and one of his arm is moved by some weight, or pushed downward, removed or taken off the weight,

[36] Tartaglia 1554, *Book VII*, Q VI, 82r.
[37] Tartaglia 1554, *Book VII*, Q VII, 82r.
[38] Tartaglia 1554, *Book VII*, Q VII, 82r.

the scale raises again and returns to his first place. And when that fulcrum is below the scale, and similarly one of his arm is carried by some weight, or pushed downward, when the weight is removed the scale neither raises nor returns to its first place (as it does for the other case) but remains below, i.e. beneath. Now I say, he was silent and mitted one more problem, which here is much more suitable, much more speculative of any of the other problems, which is that. Why when the fulcrum is precisely inside the scale, and one of the arm of is, by any weight, loaded, or pressed down, removed that weigh, the scale again raises to its first place, like as the scale with the fulcrum above. S.A. That looks to me a nice problem, and much farther from our intellect that the two mentioned before and I will appreciate very much to understand the cause of that effect; but I before want you to clarify me a doubt, which persists in my mind about the above cited problems, which is this.[39]

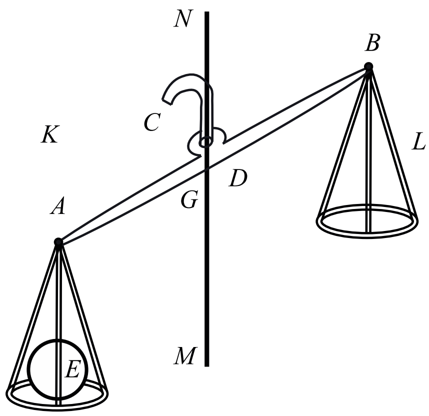
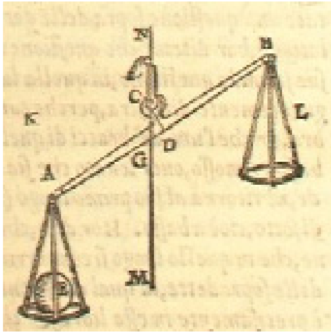


Fig. 3.6 Balances with fulcrum above according to Tartaglia[40]

[39] Tartaglia 1554, *Book VII*, Q IV, 80v.

[40] Redrawn from Tartaglia 1554, *Book VII*, Q V, 81r. It should be compared with Fig. 3.2.



[*Quesito* V] N. To proof the first part of such a question let consider the balance *ab* the cord of which be the point *c* (which is quite above the said balance *ab* as shown in the figure) and its arm *ad* be pressed down by the imposed weight *e*, as shown in the figure. Now I say that if the weight *e* is taken away, the arm *ad* will raise and return to its initial position, i.e. the point *k* and the other arm *db* will descend up to the point *l*. That occurs for in lowering the arm *ad*, more than one half of the beam *ab* is raised, beyond the vertical *nm* through the cord *c* which is called line of direction. That is the raised part *db* becomes the greater the one half of the beam *ab* the lesser the remaining depressed part *ag*. By removing the weight *e* the part *ag* (less strong) is pressed from the grater raised part *db* until the line of direction becomes orthogonal to the beam *ab* and splits it into two equal parts in the point *d*.[41]

Tartaglia's proof is approved by his interlocutor Mendoza who claims that it is similar to that of Aristotle, but better exhibited (Tartaglia 1554, *Book VII*, Q V, 81v). Nevertheless, we should remark that Tartaglia's demonstration, being similar to Aristotle's is only valid if the beam of the balance is weightless; and where *natural motions* were considered proportional to *violent motions*:

> This, happens with any radius which describes a circle; it moves along a curve naturally in the direction of the tangent, but is attracted to the centre contrary to nature. The lesser radius always moves in its unnatural direction; for because it is nearer the centre which attracts it, it is the more influenced. That the lesser radius moves more than the greater in the unnatural direction in the case of radii describing the circles from a fixed centre is obvious from the following considerations.[42]

[41] This is the first part of the proof. Tartaglia 1554, *Book VII*, Q V, 81rv. Figure: *Ivi*.

[42] *Problemata Mechanica* 849a (Aristotle 1955c, 849a, p 343, line 1).

The concept of gravity of position would have been capable of justifying the stability also for a weightless beam, but Tartaglia does not use that.

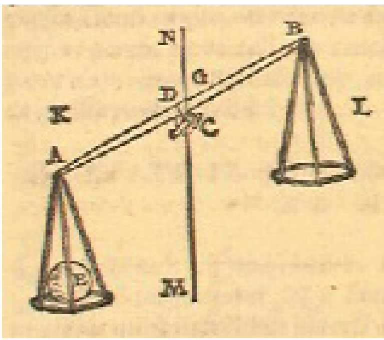
We note that Aristotle (like Tartaglia also will do) reasons around a *disequilibrium state* of the physical system balance-masses: i.e., a mass placed on a side of a lever – with fulcrum in the middle – causes a motion of the beam and a correspondence resistance on the other side. He seeks a general principle by means the rules of geometry (circle):

> Again, no two points on one line drawn as a radius from the centre travel at the same pace, but that which is further from the fixed centre travels more rapidly[43]; it is due to this that many of the remarkable properties in the movement of circles arise […]. [44]

Differently, Tartaglia is more interested to physical properties of the system: in order to re-establish an *equilibrium state* – a so called "luoco della equalità" (*place of the equilibrium*) – he also proposes physical considerations (see above and bottom passages) adding and removing masses:

3.1.3.2 The Balance with Fulcrum Below the Beam

In the following, the second part of the proof is reported where Tartaglia confirms his reasoning around Aristotelian arguments:



> [*Quesito* VI] N. […]. Let *ab* be the scale which has the cord (i.e. that

[43] Senza con questo aver minimamente l'intenzione di scendere nei dettagli del concetto di velocità in Aristotele, preciso solo che con il termine aristotelico "più rapidamente" si possono intendere due casi: a) dati due corpi *A* e *B*, per eguali distanze, se *A* la percorre in un tempo minore, allora $V_A > V_B$. b) dati due corpi *A* e *B*, $V_A > V_B$ continua a valere, se nello stesso intervallo tempo *t, A* ha percorso una distanza maggiore di *B*.

[44] *Problemata Mechanica* 848a (Aristotle 1955c, 848a, 335, line 6).

point, or fulcrum, above which it rotates) rather below, i.e. below the beam *ab* as shown below in point *c* and for the imposition of the weight *e* its arm *ad* is pulled down, as it appears in the figure. I say, that who tooks away the said weight *e* the arm would not return to its original place, i.e. the point *k* (as, in that it does with the fulcrum above) but will remain so inclined at the bottom, and the cause of that depends on the fact that when the said arm *ad* goes down, more than one half of the whole beam, or balance *ab*, is transferred beyond the perpendicular .nm. passing through the cord *c*, so that the whole part *ag* brought down, gets to be much more than one half of all the balance *ab* as *d* is to g and the raised part *gb* becomes lesser of that half, as *d* is to *g* The raised portion *gb* less than the lowered part *ag* is then to be weaker, less powerful of it, and therefore, not sufficient to make it to ascend to its initial position in *k* as in the previous case. Rather it will remain inclined at the bottom, and will keep the other part at the top.[45]

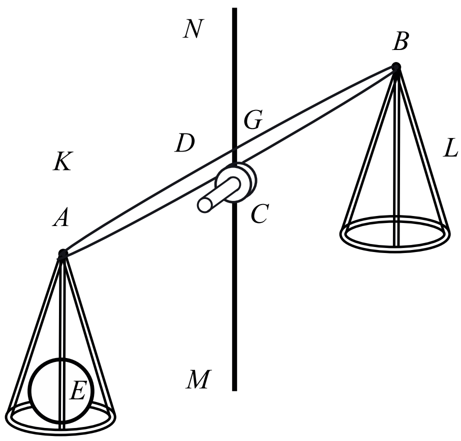


Fig. 3.7 Balances with fulcrum below according to Tartaglia[46]

Note that as Aristotle, Tartaglia also assumes that the balance with the fulcrum below, when removed from the horizontal position remains where it was left; i.e. according to modern nomenclature the horizontal position would be of indifferent equilibrium. We have already noticed that this is not true and the balance makes a complete rotation to assume a stable

[45] The second part of the proof. Tartaglia 1554, *Book VII*, Q VI, 81v–82r. Figure: 81v.

[46] Redrawn from Tartaglia 1554, *Book VII*, Q V, 82v. It is compared with Tomeo's Figure (Leonici Thomei 1530, 30; see also English Translation by Walter Stanley Hett: *Aristotle, Mechanical Problems. Nicolao Leonico Thomaeo interprete, Venise*, 1525).

configuration with the fulcrum that passes from below to above. We do not believe that a clever and practical man, as surely Tartaglia was, did not recognize this fact; more probably, he preferred to not discuss the fact whose explanation would have required more sophisticated theoretical tools than those he had.

Finally, at the end of the second part of the proof (see above) Tartaglia (his interlocutor) claims[47] his anti-Aristotelian thesis on a balance by shorter and longer arms: this geometrical aspect (as valid nowadays, as well) is independent from the sharpness of a balance as scientific device.

3.1.3.3 The Balance with Fulcrum Inside in the Beam

Tartaglia's interlocutor, Mendoza, presents the case with the fulcrum inside the beam (a third part of the proof) for which he has not difficulty to accepting as a matter of fact that the horizontal position is a stable equilibrium position:

> [*Quesito* VII] S.A. Now let us come to the third part, which is still lacking here, that is, how it comes about that, when the support of a scale is precisely in its centre, neither above nor below, but in the centre, as is the case with most of our ordinary scales, and one of the arms is pushed down either by some weight or by our hand, and the weight or hand is then removed, this arm immediately ascends again and returns to its original place, as does the arm of a scale whose support comes from above. For in fact the cause of this seems to me farther removed from common sense than for either of the two usual cases.[48]

Tartaglia had also presented the stability of the balance as a matter of fact. Actually, we doubt that he and Mendoza could think that. Indeed, most experiences with the balance having its fulcrum inside the beam show that it remains where it is left and does not recover the horizontal position unless stimulated to do so. Thus, Tartaglia could not have derived its position from physical facts. More simply he is presenting the position of de Nemore's *Liber de ratione ponderis* (de Nemore 1565).

In the following, some comments on this text will be referred to. Here for the sake of completeness we report what de Nemore says for the balance under consideration:

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[47] "S.A. Queste due parti quasi, che il nostro intelletto le apprende per ragion naturale, senza altra dimostratione. N. [Tartaglia] Così è Signore" (Tartaglia 1554, *Book VII*, Q VI, 82v).

[48] Tartaglia 1554, *Book VII*, Q VII, 82r.

> [Second Question]. When a horizontal position is gained [for a balance of equal arms], then, if equal weights are suspended [from its extremities], the balance will not leave the horizontal position; and if it is moved from the horizontal position, it will revert to the horizontal position. If instead unequal [weights] are suspended, [the balance] will fall on the side of the heavier [weight] until it reaches the vertical position.[49]

Notwithstanding he accepts the matter of fact that Mendoza finds it strange and asks for explanations. This is Tartaglia reply:

> [*Quesito* VII] N. I have told your Excellency that in order to demonstrate the cause of this effect, it will be necessary for me first to define and explain to your Excellency some of the terms and principles of the science of weights. […] N. So far as it concerns simply the demonstration of this particular, it will be quite short; however, if your Excellency wants to learn in an orderly manner all the principles of the science of weights, that will be quite lengthy return, your Excellency.[50]

In addition, with the request of an exposition of the principles of the science of weights book VII ends.

At this stage, it is of some of interest to briefly compare Tartaglia's considerations with Alessandro Piccolomini's reasoning upon the sensitivity of the balances. Tartaglia having reported and approved Aristotelian theses regarding a small mass placed on one arm of a balance (thus being an eventual previous equilibrium/configuration) Piccolomini is not perturbed and comments:

> And if what we have said should seem inconvenient to someone, that is nothing of little weight can be put on a small balance, that not only its motion is not clear, but that really it does not move: we could say against it, and conclude with reason because there was something placed over the balance that before there was not, it is necessary that such a thing, either it is of any weight (and it is false), or that the weight has no tendency to descend, which of course is false. Who doubted thus must be answered, that many things for mathematical demonstration and imagination conclude but actually they do not occur.[51]

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[49] de Nemore 1565, *Quaestio secunda*, 3v.

[50] Tartaglia 1554, 82r.

[51] "E se a alcuno paresse inconveniente quel che habbiam detto ad esso, cioè che alcuna cosa di poco peso si possa metter sopra qualche libra piccola, che non solo il suo moto non sia manifesto, ma che anco veramente non la muova: massime che potremmo dir contra, e concluder con ragione perché s'è posto sopra

In other words Piccolomini suggests that the mathematical reasons make abstracts from natural matter, thus it is no wonder that what is proved by it may not correspond to the real behaviour of bodies.

3.2 The Analysis of *Book VIII*

In the literary form of dialogue adopted by Tartaglia, *Book VIII* contains a discussion between Tartaglia and Mendoza that develops the day after Book VII is registered. It aims to expose the science of weights in an *indisputable* way.

Book VIII is the only book of the *Quesiti et invention diverse* which has a structure quite similar to that of the *Nova scientia* (Tartaglia 1537), since Tartaglia did not dare to break the long tradition of a deductive modelled science typical of Euclid's *Elements*: e.g., an Arabian science of weights during the 10th Century A.D., Jordanus de Nemore writings of the 13th Century and up to Apianus' edition of the 16th Century.

Book VIII strongly stresses the arrangement of the notional elements of the theory and the role played by *Principij primi, Propositioni*, *Suppositioni*, *Petitioni* (Tartaglia 1554, *Book VIII*, 83rv–86rv) considered so important by Tartaglia to be discussed before entering the science of weights.

Tartaglia begins *Book VIII* by stressing the importance of structuring the science of weights by means of (indemonstrable) principles and (demonstrable) propositions.

> [*Quesito* III] N. To proceed in an orderly fashion, we shall today define only some terms and ways of speaking that occur in this science,[52] in order that your Excellency will more easily apprehend the fruit of the understanding of this. Then, tomorrow, we shall proceed to state the principles of that science, that is, those things which cannot be demonstrated in the science;[53] for as your Excellency knows, every science has its indemonstrable first principles, which, being conceded

quelle balance qualcosa che prima non v'era, è necessario, che tal cosa, o sia di nessun peso (il che per quanto si è concesso è falso) o vero che tal peso non abbia alcuna inclinazione al discendere, il che naturalmente è falso. A chi dubitasse in tal modo bisogna rispondere, che molte cose per demonstratione e immaginatione matematica si concluden per vere che non di meno non si danno" (Biringucci 1582, 37–38). The translation is mine.

[52] Tartaglia suggests that the definitions he is introducing are shared definitions. Indeed most of them are.

[53] According to the Aristotelian scientific structure.

> or assumed, afford the means to discuss and sustain the whole science. Then we shall go on by setting forth various propositions or conclusions concerning the science; and part of these we shall demonstrate to your Excellency by geometrical arguments, and part we shall test by physical reasons, as I said before. And after this, your Excellency, you shall put forward those doubts or questions that occur to you concerning things mechanical, and especially the admirable effects of the said material instruments that augment the strength of a man; and by the things said and tested in the science of weights, all will be clarified.[54]

and shortly after specifies the meaning he is giving to *principle*:

> [*Quesito* XXI] N. Some say that the principles of any science should be called dignities ["*dignita*"], because they prove other propositions but cannot be proved from others; some call them suppositions, because they are supposed to be true in the given science; others prefer to call them petitions, because, if we wish to debate such a science and sustain it with demonstrations, we must first request the adversary to concede them. For if he does not concede them but denies them, the entire science would be denied, nor could one debate differently. And since this last opinion pleases me somewhat more than the other two, let us call them petitions and set them forth in the form of requests.[55]

The book was not very innovative, as the many texts on the science of weights of de Nemore's traditions from XIV to XVI centuries were not innovance. Its importance lies in its more precise mathematical formulation and the adoption of an unifying principle to assess equilibrium. Indeed, in de Nemore's tradition, up to Tartaglia, there were two de Nemorian principles in the science of weights, 1) a formulation of the virtual displacement principle and 2) the equality of positional gravity (*gravitas secundm situm*):

1) What can raise a weight p at height h, can lift a weight p/n at a height nh, or vice versa a weight np to the height h/n". A form a virtual displacement principle; the equilibrium is based on the equality of the product ph.
2) The greater the efficacy of a weight (the gravity of position) the more its motion partakes of the vertical. The equilibrium is based in the equality in the gravity of position.[56]

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[54] Tartaglia 1554, *Book VII*, Q III, 83r.
[55] Tartaglia 1554, *Book VIII*, Q XXI, 84v.
[56] See below § 2.1.2.1.

Tartaglia only uses the second one: equality in the gravity of position.

The book VIII was however quite a leading book. Though it was criticized by Benedetti (Benedetti 1585) and del Monte (del Monte 1577, 1615; see also del Monte 2013), its influence could be found in Galileo's *Le mecaniche*, half a century later (Galilei 1649). This influence is evident in the adoption by the two scientists of a similar unifying principle of mechanics: the equality of positional gravity for Tartaglia and the equality of moment for Galileo. Besides being criticized by Benedetti and Cardano from a technique point of view, Tartaglia was accused of plagiarism for having not cited his source, i.e., Jordanus de Nemore.

3.2.1 The *Book VIII* and *Liber de ratione ponderis*. A False Controversy?

Before presenting *Book VIII*, some considerations of mine upon the relationship between the debated controversy Tartaglia–(Jordanus)Ferrari–Cardano and the proof of the inclined plane within *Book VIII* are reported.

In Chapter 1 we already presented the details of the famous *quarrel* between Tartaglia and Cardano on priority for the cubic equation solution. In the developments of the dispute, Ludovico Ferrari, a Cardano pupil, published a series of letters defending his teacher. In one of such letters Ferrari retorts against Tartaglia the accusation of plagiarism, by assuming that he has taken the entire de Nemore's treatise without citing it:

> Since more than a thousand errors of the first books of this your work, you have also placed in the eighth book Jordanus's propositions as your own, without any mention of him: what screaming theft. And making demonstrations of your head, which mostly do not conclude, you make Illustrious Signor Don Diego Mendozza to confess with great shame some things, that I certainly (because I in part know his great doctrine), which he would not say for all the gold in the world.[57]

Here we remark that Ferrari, most surely knew only a part of de Nemore's work, that edited by Apianus in 1533 (de Nemore 1533) and some

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[57] "Atteso che, oltra mille errori de primieri libri di questa vostra opera, havete anchor posto nel libro ottavo le propositioni di Giordano come vostre, senza far mentione alcuna di lui: il che grida furto. E facendovi le dimostrationi di vostra testa, le quali per lo più non conchiudono, fate confessar con gran vostro vituperio all'Illustrissimo Signor Don Diego di Mendozza cose, che io certo (percioche conosco in parte la sua gran dottrina) che egli non le direbbe per tutto l'oro del mondo […]" (Tartaglia 1876, *Ferrari–Primo cartello*, 2).

fragments, but he did not know the *Liber de ratione ponderis* (version R); that we know it was in possession of Tartaglia (see above Chapter 1) which was more complete, containing the proof of the inclined plane law.

> To this I reply that in this case I just have to confess I do the demonstration with my head, and demonstration (as you know) is of much greater consideration, doctrine, and are more scientific and more difficult of pure proposition. Because every mathematical proposition, without its demonstration is deemed worthless for every mathematician, because the offer is easy, and every ignorant may know a proposition, but not prove it.
> If, therefore, you concede me the most learned, most respected, most scientific of these propositions, and confirm that it is mine, as it is, and what it is not dishonest to say these propositions to be mine, and as my order has no relationship with that of Jordanus, and each time one composes a work with a different order than that of another author even if the substance, or the content, were almost the same, without any criticism can he call his this work, because the ability of man to compose depends more on the order than on the difficulty of the subject. Now tell me, how many parts Johannes Regiomontanus removed from the Almagest of Ptolemy, without mentioning the author, but to have exposed them in a way, or order different from that of Ptolemy, it is e permitted to attribute such a thing to him. But how many more particularities took your Lord Hieronimo Cardano from Frate Luca [Pacioli], and Giorgio Valla and inserted them in his practice of Arithmetic [...].
> Secondly for having largely expanded of Definitions, Petitions, and Propositions, and having he purpose to extend it much more in the future if death does not stop my drawings. Third for demonstrations are mine and not of Jordanus, you could say I had to refer to the Author the little part that I borrowed from Jordanus. I answer that if I mentioned him I had to accuse him of no small obscurity in propositions, as in the demonstrations, as any intelligent person can understand, what did not seem useful to me.[58]

[58] "A questo ve rispondo che in questo caso mio basta che voi confessati che faccio le demonstration de mia testa, & la demonstratione (come dovresti sapere) è molto di maggior considerazione, Dottrina, & più scientifica & e di maggior difficultà, della pura Proposizione. Perché ogni propositione Mathematica, senza la sua demonstratione è reputata de niun valore appresso di cadaun mathematico, perche il proponere è cosa facile, & ogni ignorante saperà formar una propositione, ma non dimostrarla. Se adunque la più dottrinata, più istimata, più scientifica parte di tai propositioni me concedeti, & confirmati che la sia mia, come è, en non è cosa inhonestaq a dir tai propositioni esser mie, & tanto più chel mio ordine non ha alcuna convenienza con quello di Giordano, & ogni volta che

Tartaglia's defence consists substantially in sustaining the idea that in a mathematical treatise the manner of exposition is at least as important as the content. Moreover, that it is not sufficient to present a list of theorems; their proof is most important. The first claim is justified with the example of Regiomontanus and Cardano himself, who wrote important treatises working out matter drawn from other authors. The second claim is less convincing because, since the time of the ancient Greeks, exposition of a correct theorem was considered fundamental; its proof was only a painstaking job. It must be confessed however that in Tartaglia's time things were seen differently by some mathematicians, and the proof of a theorem was considered fundamental.

Drake and Drabkin (Drake and Drabkin 1969, 24), in some way, justify Tartaglia's argumentations. They think that Tartaglia cannot be blamed for having not named de Nemore. They think that because the science of weight and the role played by de Nemore were already well known and because in the edition of Euclid's *Elements* of 1543 Tartaglia named Jordanus de Nemore as the founder of the science of weight (Tartaglia 1569, 4v). A controversial[59] argumentation was acceptable only if Tartaglia had hidden the possession of a copy of the *Liber de ratione ponderis* where the theorems are effetely proved with sufficient rigor.

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uno compone una opera con uno ordine diverso di quello d'un Altro autore anchor che la sostatntia, over continentia, fusse quasi quella medesima, senza reprensione la può chiamar sua opera, perché la sufficientia del huomo in el componere più se discerne nel ordine che nella altezza della materia che lui tratta. Mo dittime un poco, qunte particolaritò ha tolte Giovan de monte regio dal Almagesto di Ptolomeo, senza far mentione del Autore, ma per haverle isposte per un modo, over ordine più piano & diverso da quello di Ptolomeo, se ha fatto licito attribuirse tal cosa a se, Ma più quante particolarità ha cavato el vostro Signor Hieronimo Cardano da Frate Luca, & da Giorgio Valla & quelle inserte nella sua pratica di Arithmetica [...]. Secondariamente per haverlo non puoco ampliato de Diffinitioni, Petitioni, & Propositioni, & esser per ampliarlo molto più per l'avvenire se mpre se morte non inetrrompe i miei disegni. Tertio per le mie dimostrationi quale confessati esser mie e non di Giordano, O voi potresti dire quella puoca parte che haveti tolto da Giordano el dover voleva pur che festi mentione di tal Authore. Ve rispondo che voiando io farne mentione a me era necessario a tansarlo di non puoca oscurità nelle propositioni, come nelle demonstrationi, come cadauno inteligente può considerare, la qualcosa non me aparso de fare" (Tartaglia 1876, *Tartaglia–Secondo cartello*, 7–8).

[59] Recently see and interesting work on the *Controversy*: Renn and Damerow 2010b.

In order to allow the reader to judge the controversy himself, we present below the main topics of de Nemore's *Liber de ratione ponderis,* followed by an analysis of Tartaglia's *Book VIII*.

3.2.2 The *Liber de ratione ponderis*

As already argued in previous section 2.1, three texts on the science of weights attributed to de Nemore are:

1. *Elementa Jordani super demonstratione de ponderibus* (hereafter *Elementa*)	version E[60]	1229
2. *Liber Jordani de ponderibus* (cum commento) (hereafter *Liber de ponderibus*)	version P	1533
3. *Liber Jordani de Nemore de ratione ponderis* (hereafter *Liber de ratione ponderis*)	version R	1565

On my side here we only concentrate on the third one making reference to the *Liber Iordani de ratione ponderis* or simply *Liber de ratione ponderis* in the Tartaglia's version posthumously published by Curtio Troiano as *Iordani Opuscolum de ponderositate Nicolai Tartaleae* or simply *Iordani opuscolum* (de Nemore 1565).

The *Liber de ratione ponderis*[61] is quite a complex treatise presenting

- 7 *Suppositions* ("Suppositio")
- 43 *Propositions* ("Quaestio")

Herein after, we present and comment the principles, the main arguments assumed by de Nemore and finally the exposition–and–proof of few propositions as – in my opinion – to be the most representative of the way of arguing within de Nemore's corpus of science of weights. Particularly:

- *Proposition I*, which gives fundaments of the science of weights.
- *Proposition VI*, which refers to the law of lever.
- *Proposition X*, which refers to the law of inclined plane.

[60] The classification *E*, *P* and *R*, nowadays largely adopted, was proposed by Clagett (Moody and Clagett [1952] 1960).

[61] The version edited by Clagett (Moody and Clagett [1952] 1960, 167–227) has 45 propositions and has been divided into four books.

3.2.2.1 The Suppositions of Liber de ratione ponderis

The first part of the *Liber de ratione ponderis* as proposed in Tartaglia's *Iordani opuscolum* version (de Nemore 1565) concerns *Suppositions* and fundamental theorems (*Propositions*) about the science of weights. It starts with seven fundamental *Suppositions* as reported in the following Table 3.1:

Table 3.1. Jordanus de Nemore's *Suppositions*[62]

Number	Proposition
I	The movement of every weight is toward the centre and its strength is a power of tending downward and to resist to the contrary motion, *and we can understand its strength or power from the arm length or from its velocity which is determined by the length of the balance arms*.[63]
II	That which is heavier descends more quickly.
III	It is heavier in descending, to the degree its movement toward the centre is more direct.
IV	It is heavier according to position in that position where its path of descent is less oblique.
V	A more oblique descent is one which, in the same space, partakes less of the vertical.
VI	One weight is less heavy according to position, than another, if it is caused to ascend by the descent of the other.
VII	The position of equality is that of equality of angles to the vertical, or such that these are right angles, or such that the beam is parallel to the plane of the horizon

The logical status of de Nemore's *Suppositions* cannot be framed easily in unique scheme. Some look like principles (contemporary meaning) of empirical character (*Supposition* I, *Supposition* II), some look like definitions (*Supposition* V). The *Supposition* I is the most complex. It contains:

a) A principle in the contemporary meaning, i.e. an assumption about facts (Omnis ponderosi motum esse ad medium).
b) A definition (that of 'virtus') (virtutemque ipsius esse potentia ad inferiora tendendi virtutem ipsius et motui contrario resistendi).

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[62] de Nemore 1565, 3r. The translations are mine. For the Latin original version see Transcription Chapter below. We note that the *Suppositions* are grouped in the first page, while the proposition are presented and discussed in several pages.

[63] According to Clagett (Moody and Clagett [1952] 1960) the emphasized part is due to Tartaglia.

Suppositions III, IV, V and VI introduce the gravity of the position concept.

In *Supposition* III de Nemore makes a generic assertion, for which a body weighs the more, the more directly it goes towards the centre of the world. He implies that 'heaviness' depends not only on the body, but also on its possible, or virtual, motion.

In *Supposition* IV the meaning of *Supposition* III is specified, with introduction of the locution *gravitas secundum situm – gravity according to position* – (de Nemore 1565, 3r; see also arguments on that, 4rv) a body is heavier than another, by position, when its descent is less oblique.

It is then stated precisely when a motion is less or more oblique in *Supposition* V: a direction is more oblique than another when it is closer to the horizon. This is in clear contrast to the modern use of the term obliquity, but which is coherent with de Nemore's ideas for which the reference direction is the vertical one.

The *Supposition* VI on the one hand can be seen as a definition of 'less heavy', on the other hand it describes a factual situation, the rising of a less heavy body caused by a more heavy body. We let note that *Supposition* VI makes it clear that de Nemore would consider a weight to be able to rise another weight and then to act as a motive power. However, in de Nemore's treatise it is never explicitly stated that both weights suspended from the end of a balance tend to go down. It appears that as a body is pushed up it loses its heaviness. It is not clear, if this corresponds to de Nemore's philosophical conception or if it is simply due to his difficulty in quantifying the tendency of bodies to move downwards.

The same holds for *Supposition* VII, which on the one hand can be seen as a definition of equilibrium and on the other hand as a factual situation representing equilibrium.

In de Nemore's *Suppositions* there are some keywords which deserve a special comment because their meaning is not so easy to grasp:

- Gravis
- Ponderosus
- Velocitas
- Virtus
- Gravitas Secundum Situm

For sake of brevity, we only comment the last two keywords *virtus*[64] and *gravitas secundum situm*, which have a particular importance for my aims.

[64] In the Renaissance Latin manuscript traditions we can also read: *virtus promotoria* responsible of the movement, *copia materiae* (mass or volume)

The epistemological interpretation of *virtus* is quite a delicate subject. One is tempted to associate *virtus* with *force*. There are, however, reasons not to do this. The most important is that *virtus*, besides the tendency to go downward, represents the resistance to go upward. In the *De ponderoso et levi*, the term *virtus* is connected to velocity, at least for the motion according to nature:

> Bodies are equal in virtue when their motions are equal in equal times and equal spaces in the same air or water.[65]

Nothing is instead said for the motion against nature.

The *Supposition* I, which explicitly asserts that the weights are not free but are suspended from a balance, proposes a method to evaluate the virtus: virtus is measured [calculated] by velocity.

De Nemore does not explain what causes the *virtus*, but his use of a unique term for both motions against and according to nature, should indicate he is thinking of a unique cause. A modern term to translate *virtus* could be *heaviness*, but this would create ambiguities. For this reason in what follows, *virtus* will often not be translated, or in some cases, it will be translated as strength or force.

Concerning the concept of *gravity of position*, it can be said that there is widespread agreement among historians (Clagett 1952; Duhem 1905) that it is partially derived from *Problemata mechanica*, as evident from the *Suppositions*, particularly from *Supposition* III (Table 3.2). Moreover, this conclusion would be also supported by the preface of *Liber Jordani de ponderibus* (version P). In fact, this preface does not start directly with the *Suppositions* – as the other treatises attributed to de Nemore do – but presents an ample discussion from which we refer to the outstanding points:

> It is therefore clear that there is more violence in the movement over the longer arc, than over the shorter one; otherwise the motion would not become more contrary (in direction) Since it is apparent that in the descent (along the arc) there is more impediment acquired, it is clear that the gravity is diminished on this account. But because this comes about by reason of the position of the heavy bodies, let it be called positional gravity in what follows. For in reasoning in this way about

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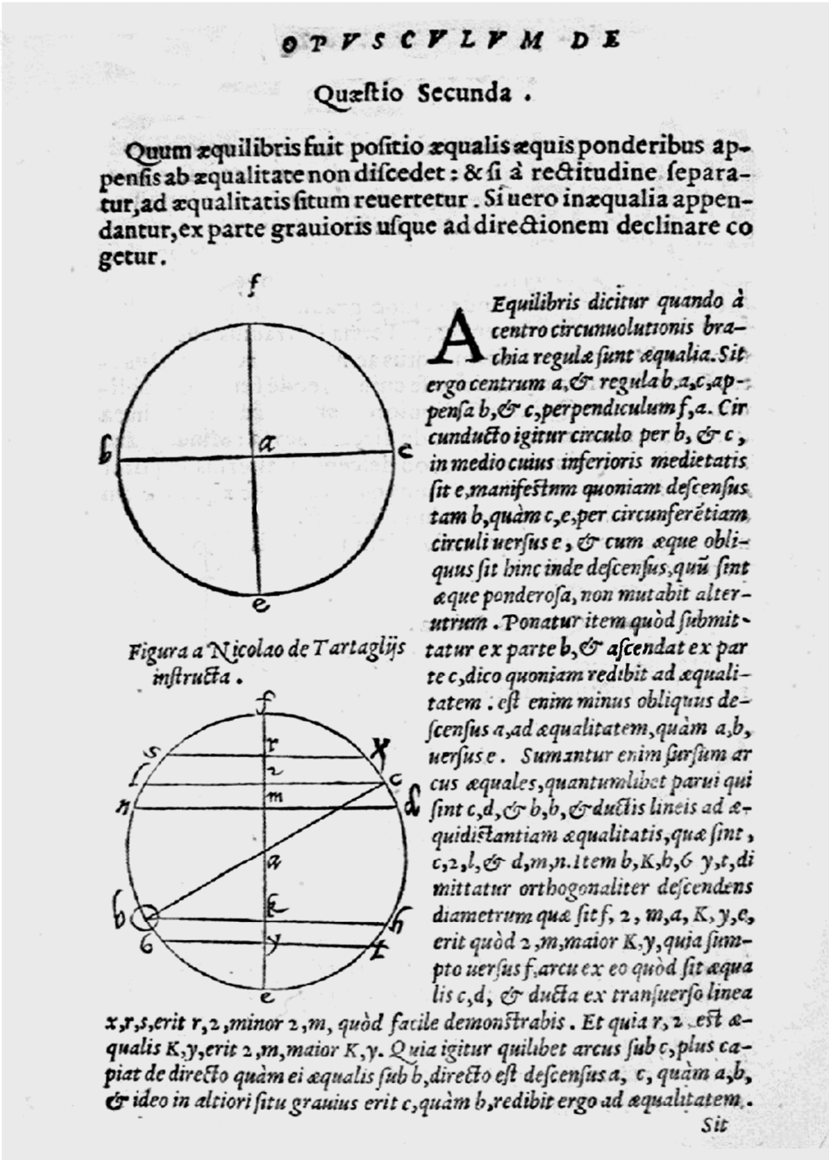
responsible of the gravity, *virtus tractoria* (depending on the mass), *vis*, *gravis*, *anima motrix*, etc. (Pisano and Bussotti 2012, 2013a, 2013b).

[65] "Corpora equalia in virtute sunt quorum motus sunt in temporibus equalibus super loca equalia in eodem aere vel eadem aqua" (Moody and Clagett [1952] 1960, 26).

> motion, as if the motion were the cause of heaviness or lightness, we conclude, from the fact that a motion is more contrary (in direction) that the cause of this contrariety is more contrary - that is, that it contains a greater element of violence. For if a heavy body descends, this occurs by nature; but that its descent is along a curved path, is contrary to its nature, and hence this descent is compounded of the natural and the violent. But since, in the ascent of a weight, there is nothing due to its nature, we have to argue as we do in the case of fire, because nothing ascends by nature. For we reason concerning the ascent of fire, as we do concerning the descent of a heavy body; from which it follows that the more a heavy body ascends, the less positional lightness it has, and therefore the more positional gravity.[66]

Besides the consideration of motion along an arc of a circle with different radii, one should make note of the explicit introduction of the locution *gravitas secundum situm*.

[66] "Patet ergo quod maior est violentia in motu secundum cum maíorem, quam secundum minorem; alias enim non fieret motus magis contrarius. Cum ergo apparet plus in descensu adquirendum impedienti, patet quia minor erit gravitas secundum hoc. Et quia secundum situationem gravium sic fit, dicatur gravitas secundum situm in futuro processo. Ita enim, sillogizando de motu tamquam motus sit causagravitatis vel levitatis, potius per motum magis contrariumconcludimus causam huiusmodi contrarietatis esse plus contrariam, id est, plus habere violentie. Quod quidem grave descendat, hoc est a natura; sed quod per lineam curvam, hoc est contra naturam, et ídeo iste descensus est mixtus ex naturali et violento. In ascensu vero ponderis, cum ibi nihil sit secundum naturam, debet argui sicut de igne, quoniam nihil naturaliter ascendít. De igne enim arguitur in ascensu, sicut de gravi in descensu; ex quo sequitur quod grave, quanto plus sic ascendit, tanto minus habet de levitate secundum situm, et sic plus habet de gravitate secundum situm." (Moody and Clagett [1952] 1960, 151–153).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Quæſtio Secunda.

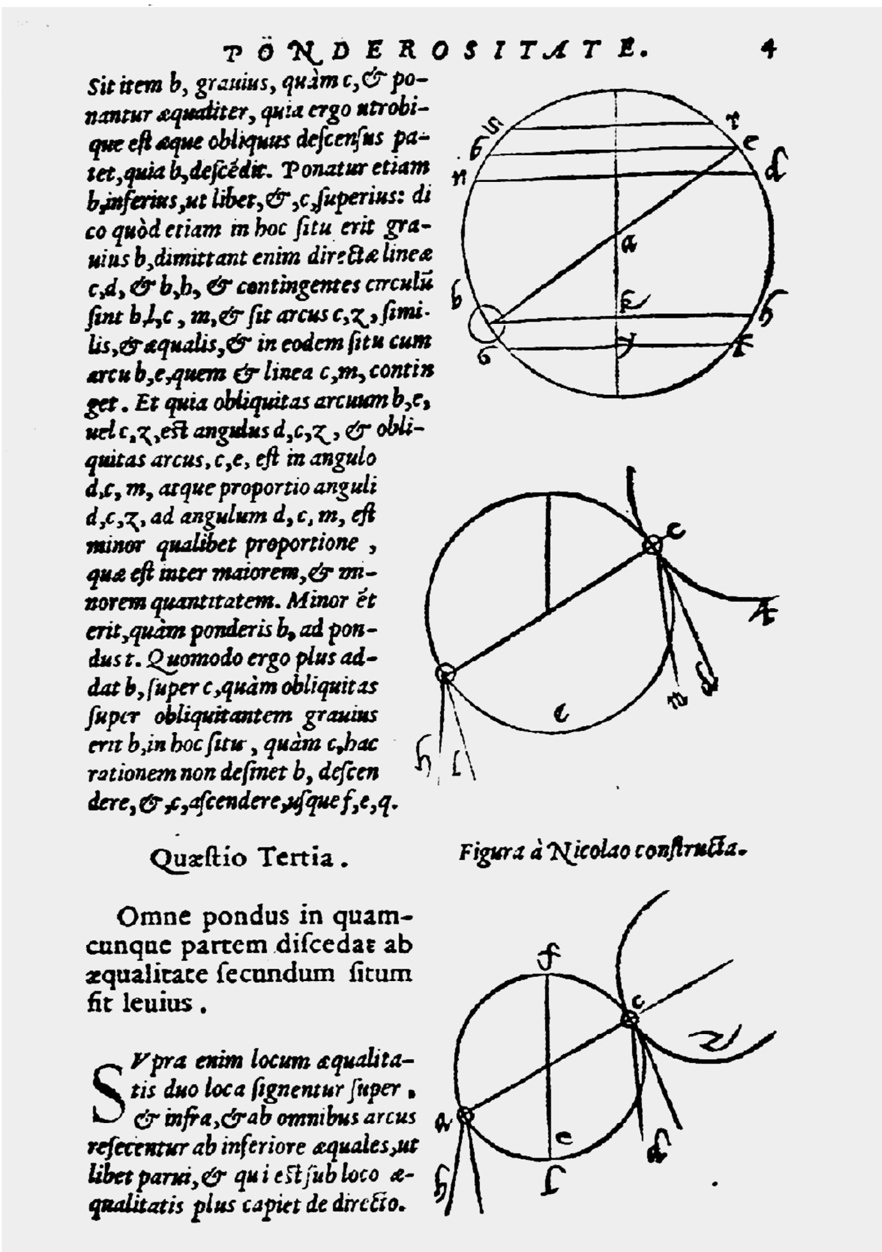
Quum æquilibris fuit poſitio æqualis æquis ponderibus appenſis ab æqualitate non diſcedet: & ſi à rectitudine ſeparatur, ad æqualitatis ſitum reuertetur. Si uero inæqualia appendantur, ex parte grauioris uſque ad directionem declinare cogetur.

Æquilibris dicitur quando à centro circunuolutionis brachia regulæ ſunt æqualia. Sit ergo centrum a, & regula b, a, c, appenſa b, & c, perpendiculum f, a. Circunducto igitur circulo per b, & c, in medio cuius inferioris medietatis ſit e, manifeſtum quoniam deſcenſus tam b, quàm c, e, per circunferêtiam circuli uerſus e, & cum æque obliquus ſit hinc inde deſcenſus, quũ ſint æque ponderoſa, non mutabit alterutrum. Ponatur item quòd ſubmittatur ex parte b, & aſcendat ex parte c, dico quoniam redibit ad æqualitatem: eſt enim minus obliquus deſcenſus a, ad æqualitatem, quàm a, b, uerſus e. Sumantur enim ſurſum arcus æquales, quantumlibet parui qui ſint c, d, & b, b, & ductis lineis ad æquidiſtantiam æqualitatis, quæ ſint, c, 2, l, & d, m, n. Item b, K, b, 6 y, t, dimittatur orthogonaliter deſcendens diametrum quæ ſit f, 2, m, a, K, y, e, erit quòd 2, m, maior K, y, quia ſumpto uerſus f, arcu ex eo quòd ſit æqualis c, d, & ducta ex tranſuerſo linea x, r, s, erit r, 2, minor 2, m, quòd facile demonſtrabis. Et quia r, 2 eſt æqualis K, y, erit 2, m, maior K, y. Quia igitur quilibet arcus ſub c, plus capiat de directo quàm ei æqualis ſub b, directo eſt deſcenſus a, c, quàm a, b, & ideo in altiori ſitu grauius erit c, quàm b, redibit ergo ad æqualitatem.

Sit

Fig. 3.8a Plates from *Iordani opuscolum de ponderosidate* on the *Gravitas secundum situm*.[67]

[67] de Nemore 1565, 3v. Note that a figure is remarked as "Figura à Nicolao constructa [Figures drawn by Niccolò Tartaglia]". See also below transcriptions and translations, Chapter 4.



PONDEROSITATE. 4

Sit item b, grauius, quàm c, & ponantur æqualiter, quia ergo utrobique est æque obliquus descensus patet, quia b, descēdit. Ponatur etiam b, inferius, ut libet, & c, superius: dico quòd etiam in hoc situ erit grauius b, dimittant enim directæ lineæ c, d, & b, h, & contingentes circulū sint b, l, c, m, & sit arcus c, z, similis, & æqualis, & in eodem situ cum arcu b, e, quem & linea c, m, contingget. Et quia obliquitas arcuum b, e, uel c, z, est angulus d, c, z, & obliquitas arcus, c, e, est in angulo d, c, m, atque proportio anguli d, c, z, ad angulum d, c, m, est minor qualibet proportione, quæ est inter maiorem, & minorem quantitatem. Minor ét erit, quàm ponderis b, ad pondus t. Quomodo ergo plus addat b, super c, quàm obliquitas super obliquitantem grauius erit b, in hoc situ, quàm c, hac rationem non desinet b, descendere, & c, ascendere, usque f, e, q.

Quæstio Tertia.

Omne pondus in quamcunque partem discedat ab æqualitate secundum situm fit leuius.

Supra enim locum æqualitatis duo loca signentur super, & infra, & ab omnibus arcus resecentur ab inferiore æquales, ut libet parui, & qui est sub loco æqualitatis plus capiet de directo.

Figura à Nicolao constructa.

Fig. 3.8b Plates from *Iordani opuscolum de ponderosidate* on the *Gravitas secundum situm*.[68]

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[68] de Nemore 1565, 4r.

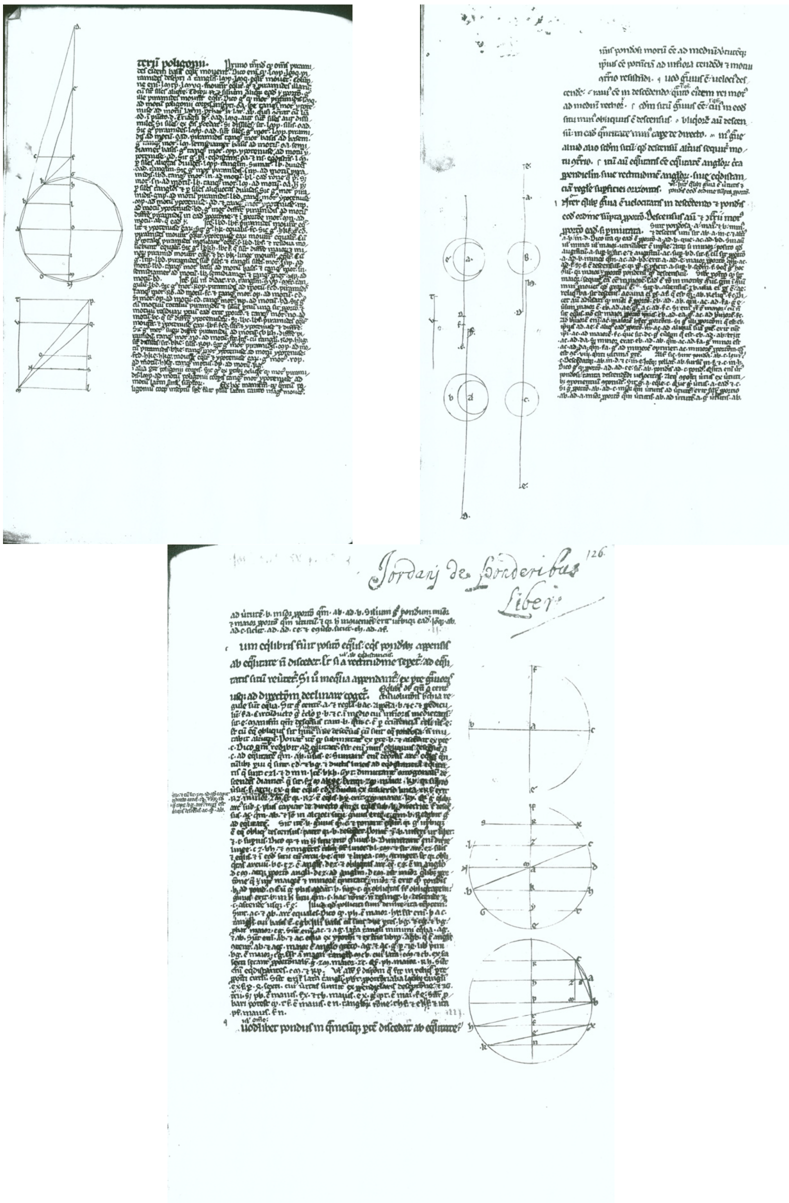


Fig. 3.8c. Plates from *Liber de ratione ponderis* on the *gravitas secundum situm* reasoning by Iordanus de Nemore [69]

[69] *De ratione ponderis*, 1r, 3rv. (Ms. Auct. F.5.28., folios 125v-133r, fl. 13th century. The Oxford Bodleian Library U.K. copyright).

The gravity position concept is a crucial one, but it is not easy to wording. In fact, for downward motion, with a little forcing, the gravity of position can be represented by the product of the weight (p), considered as a force (nowadays mg), and the (virtual) velocity of sinking (v), mathematically pv, that is it is essentially what the Arabic mechanics did (Capecchi 2011). It is difficult to say whether de Nemore would recognize himself in this representation. In effect, he never gives a mathematical expression to gravity of position. For him it remains a qualitative concept, defined by the more or the less, which is useful to prove certain assertions but not to furnish mathematical laws. When he needs a mathematical law, he used a different approach.

3.2.2.2 The Propositions of Liber de ratione ponderis

In the following Table 3.2 we present the propositions of the *Liber de ratione ponderis* in the Tartaglia's *Iordani Opuscolum* version (de Nemore 1565) and, particularly, we comment *Proposition I*:

Table 3.2 Jordanus de Nemore's propositions[70]

Number	Proposition
I	Among any heavy bodies, the strengths are proportional to the weights.
II	When the beam of a balance of equal arms is in the horizontal position, then, if equal weights are suspended from its extremities, it will not leave the horizontal position; and if it is moved from the horizontal position, it will revert to it. But if unequal weights are suspended, the balance will fall on the side of the heavier weight until it reaches the vertical position.
III	In whichever direction a weight is displaced from the position of equality, it becomes lighter in position.
IV	When equal weights are suspended from a balance of equal arms, inequality of the pendants by which they are hung will not disturb their equilibrium.
V	If the arms of the balance are unequal, then, if equal weights are suspended from their extremities, the balance will be depressed on the side of the longer arm.
VI	If the arms of a balance are proportional to the weights suspended, in such manner that the heavier weight is suspended from the shorter arm, the weights will have equal positional gravity.
VII	If two oblong bodies, wholly similar and equal in size and weight, are suspended on a balance in such manner that one is fixed horizontally onto one arm, and the other is hung vertically, and so that the distance

[70] de Nemore 1565, 3r–7r. Translation is mine.

	from the axis of support to the point from which the vertically suspended body hangs, is the same as the distance from the axis to the mid point of the other body then they will be of equal positional gravity.
VIII	If the arms of a balance are unequal, and form an angle at the axis of support, then, if their ends are equidistant from the vertical line passing through the axis of support, equal weights suspended from them will, as so placed, be of equal heaviness.
IX	Equality of the declination conserves the identity of the weight.
X	If two weights descend along diversely inclined planes, then, if the inclinations are directly proportional to the weights, they will be of equal strength in descending.

Proposition I and the *law of virtual displacement.*

Proposition I is the most important proposition of the *Liber de ratione ponderis* because from it nearly all other propositions – as typical of a deductive axiomatic structure – can directly be proved. Its delicacy is highlighted by the fact that different accounts of it are given as shown in Table 3.3. The statements of versions E and version P are substantially the same version (also in the Latin language) but differ from that of version R in two important aspects (*Ivi*):

1. Versions E and P refer to the relation between weight and velocity rather than to weight and *virtus.*
2. Versions E and P explicitly consider both the downward and upward motions.

Table 3.3 The different accounts of *Proposition I*

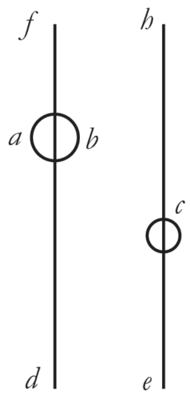
Version E	Version P
The proportion of the velocity of descent, among heavy bodies, is the same as that of weight, taken in the same order, but the proportion of the descent to the contrary ascent is the inverse proportion.[71]	Between any two heavy bodies, the proper velocity of descent is directly proportional to the weight, but the proportion of descent and of the contrary movement of ascent is the inverse.[72]
Version R	
Among any heavy bodies, the strengths are proportional to the weights.[73]	

Considering an epistemological point of view, one could say that the substitution of the term *strength*s in version R with the term *velocity* was made to allow a unitary treatment of upward and downward motions, because the concept of strength is effetely independent from the versus of motion. However, reading of the text does not confirm that point, because, as in versions E and R, the velocity and weight are related directly also here. We conjecture that de Nemore was unsatisfied with the previous versions, but, at the same time, his rephrasing was not completed for unknown reasons. In the following we see the proof of *Proposition I* as proposed in version R:

[71] "Inter quaelibet gravia est velocitas in descendendo et ponderis eodem ordine sumpta proportio, descensus autem, et contrarii motus, proportio eadem sed permutata" (Moody and Clagett [1952] 1960, 128).

[72] "Inter quaelibet duo gravia est velocitas in descendendo proprie, et ponderis eodem ordine sumpta proportio, descensus autem, et contrarii motus, proportio eadem sed permutata" (Moody and Clagett [1952] 1960, 155).

[73] "Inter quaelibet grauia est virtutis et ponderis eodem ordine sumpta proportion" (de Nemore 1565, 3r).



Proposition I
Among any heavy bodies, the strengths are proportional to the weights.
Consider weights *ab*, *c*, of which *c* is the lighter and *ab* descend to *d*, and let *c* descend to *e*. In the same way let *ab* be raised to *f*, and *c* to *h* [Fig. 3.9]. I then say that the proportion of the distance *ad* to the distance *ce*, is as the weight *ab* is to the weight *e*, indeed the velocity of descending is as great as the strength of the heavy body. But the strength of the compound is composed of the strengths of its components. Let *a* then be equal to *c*, so that the strength of *a* is the same as that of *c*. If instead the ratio of *ab* to *c* is less than the ratio of the strength to the strength, the ratio of *ab* to *a* will similarly be less than the ratio of the strength of *ab* to the strength of *a*, and therefore the ratio of the strength of *ab* to that of *b* will likewise be less than that of *ab* to *b*, for (proposition) 30 of fifth book of Euclid (Tartaglia 1543, 104–105), what is absurd. Therefore the ratio of weights will be both greater and less than the ratio of strengths. Since this is absurd, the proportion must be the same in both cases, hence *ab* is to *c*, as [the distance] *ad* is to [the distance] *ce*, and conversely as [the distance] *ch* is to [the distance] *af*.[74]

Fig. 3.9 Displacements of bodies in Jordanus de Nemore's *Proposition I*[75]

[74] "Queastio Prima. Inter quaelibet grauia est uirtutis, et ponderis eodem ordine sumpta proportio.Sint pondera a,b,c, leuius c, descendatque a,b, in d, et c, in e. Itaque ponatur a,b, sursum in f, et c,i,h. Dico ergo quód quae proportio a,d, ad c,e, sicut a,b, ponderis ad c pondus, quanta enim uirtus ponderosi tanta descendendi uelocitas: at quae compositi uirtus ex uirtutibus componentium compununtur. Sit ergo a, aequale c. Quae igitur uirtus a, eadem et, c. Sit igitur proportio a, b, ad c, minor quám uirtutis ad uirtutem. Erit similiter proportio a, b, ad a, minor proportio quám uirtutis a,b, ad uirtutem a, ergo uirtutis a, b, ad uirtutem b, minor proportio quám a, b, ad b. per 30. quinti Euclidis quód est inconueniens. Similium igitur ponderum minor, et maior proportio, quám uirtutum. Et quia hoc inconueniens erit, utrobique eadem ideo a, b, ad c, sicut a, d, ad c, e, et e, contrario sicut c, b, ad a, f." (de Nemore 1565, 3r).

[75] Redrawn from de Nemore 1565, 3r.

The first part of the above passage proves *Proposition I* as formulated in version R; the second part proves what is added in versions E and P. The text makes quite a direct reference to *Suppositions* I and II and an indirect reference to *Supposition* III, by assuming vertical paths of weights instead of circular. According to *Suppositions* I and II de Nemore can assume that *virtus* grows with weight; he goes ahead and assumes also the *additivity* with respect to weight. Additively is assumed explicitly

> But the strength of the compound is composed of the strengths of its components.[76]

It is assumed implicitly when de Nemore affirms that the strength of the portion of *ab* equal to *c* equals that of *c*; this means also that posit $c = a$, the residual part of the *virtus* is that of $ab - c = b$.

The final part:

> [...] hence *ab* is to *c*, as [the distance] *ad* is to [the distance] *ce*, and conversely as [the distance] *ch* is to [the distance] *af*.[77]

is a simple corollary and – by relating strength and velocity – states the proportionality between weight and velocity for the downward motion:

> [...] hence *ab* is to *c*, as [the distance] *ad* is to [the distance] *ce*, [...].[78]

and the inverse proportionality for upward motion:

> [...] *s* [the distance] *ch* is to [the distance] *af* [...].[79]

The proof consists of a *reductio ad absurdum*. If one suppose, says de Nemore (de Nemore 1565, 3r), that the proportionality between strength and weight be not direct but the ratio of weight to weight is less than the ratio of strength to strength. Then, with $p(\dots)$ that means strength, using a modern notation, it follows:

$$\frac{(a+b)}{a} < \frac{[p(a+b)]}{p(a)} = \frac{[p(a)+p(b)]}{p(a)}$$

[76] de Nemore 1565, 3r.
[77] *Ibidem*
[78] *Ibidem*.
[79] *Ibidem*.

De Nemore continues by adding that for the *Proposition 30* of the Vth book of Euclid's *Elements*[80] it is also valid that:

$$\frac{(a+b)}{b} > \frac{[p(a)+p(b)]}{p(b)} = \frac{[p(a+b)]}{p(b)}$$

Shortly, at the same time the ratio of weight to weight is both less and greater than the ratio of strength to strength, which is absurd; then the assumption that the ratio of weight to weight is less than the ratio of strength to strength should be denied.

The proof appears clearly circular to a modern reader and then inconsistent, because it assumes what is to prove (Brown 1967, p 208). The fact that de Nemore did not consider *additivity* and proportionality as equivalent notions, as they would be for modern mathematicians, is probably due to his lack of familiarly with the algebraic calculus.

The conclusion that weight and velocity (space) are proportional, is too hasty, probably because de Nemore had modified the enunciation of *Proposition* I in versions E and P to arrive quickly at R and he may have not finished his work, deferring the discussion of the ratio of strength to velocity to a subsequent (not yet existing) proposition.

Concerning upward motion, de Nemore's text leaves one still more bewildered because of its terseness. Indeed, upward motion is only mentioned in the final sentence: "hence *ab* is to *c*, as [the distance] *ad* is to [the distance] *ce*, and conversely as [the distance] *ch* is to [the distance] *af*" (de Nemore 1565, 3r) where *ch* and *a f* are upward motions.

Now, if the proof of *Proposition I* leaves one unsatisfied, its conclusion is, however, clear. In the downward motion velocities, or equivalently distances, covered in an assigned time *ad* and *ce*, are proportional to weights *ab* and *c* respectively; in the upward motion, distance covered in an assigned time, *ab* and *ch*, are inversely proportional to weights *ab* and *c* respectively. We repeat that these conclusions, particularly the one concerning upward motion, makes sense only when the weights are thought to be suspended from the arms of a balance, where the weight which sinks from one side raises the weight on the other side. Moreover, if the sinking weight which acts as a motive power, is deemed unchanged, at the same distance and with constant velocity, proposition I result can be

[80] "[...] per 30. quinti Euclidis [...]" (*Ibidem*). This proposition states that given four quantities, A, B, H, K, if (A+B)/A > (H+K)/H, then (A+B)/B < (H+K)/K. Therefore, considering modern notation, assumed A = *a*, B = *b*, H = *p*(*a*); K = *p*(*b*), from $a+b)/c < p(a+b)/p(c)$ then $(a+b)/a < [p(a)+p(b)]/p(a)$ it follows $(a+b)/b > [p(a)+p(b)]/p(b) = p(a+b)/p(b)$.

formulated by asserting that what can rise *p* at one eight *h* can rise *p /n* at one eight *n/h*. This is a particular expression of the law of virtual displacements (Pisano 2014b-*forthcoming*).

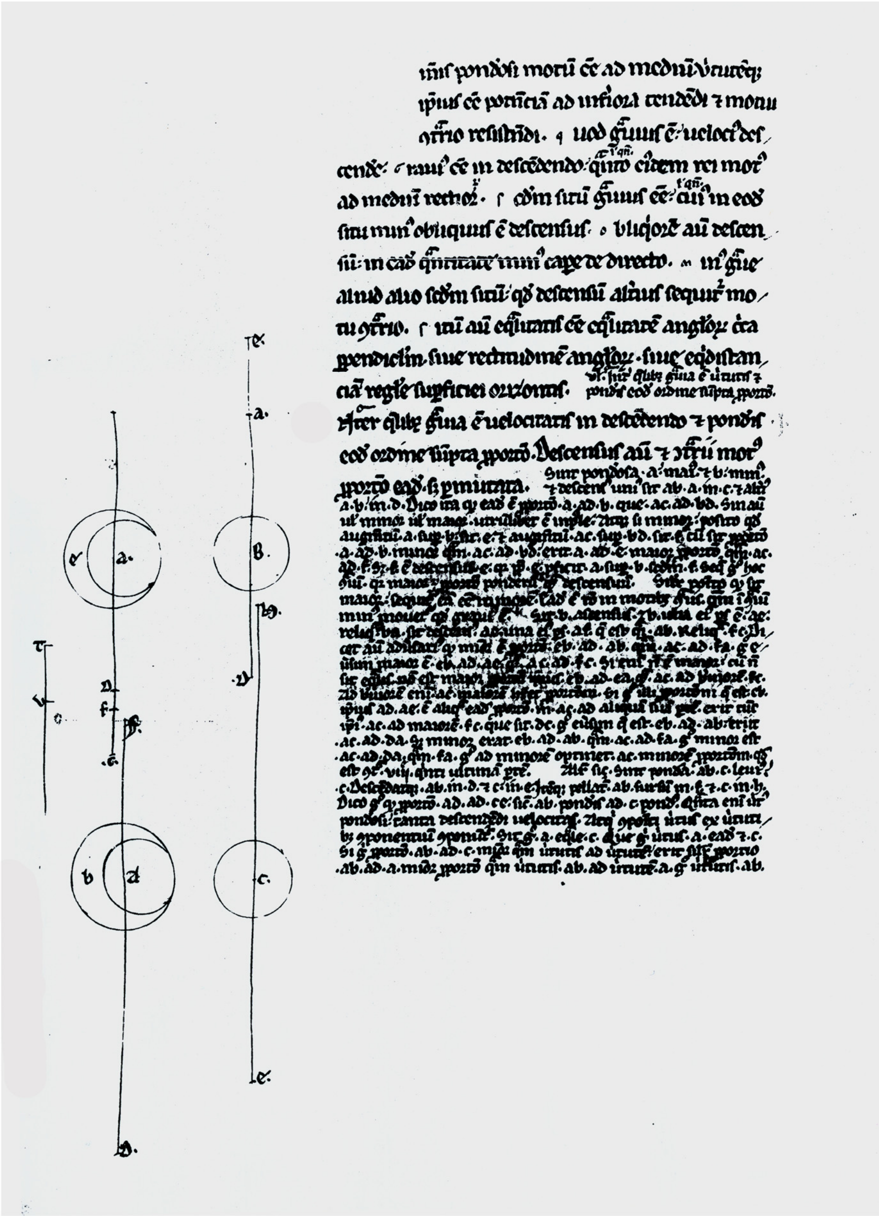


Fig. 3.10 Plate from de Nemore ‘s *Proposition I* in a manuscript of the XIII century[81]

[81] de Nemore, 13th. Ms. Auction F. 5.28, 125v–133r. The Oxford Bodleian Library U.K. copyright.

Based on the virtual work law implicit in *Proposition I*, it was not difficult for de Nemore to give proofs of the law of the lever and of the law of the inclined plane. As they are very similar for the sake of space we report only the proof regarding the lever:

> Proposition VI
> If the arms of a balance are proportional to the weights suspended, in such manner that the heavier weight is suspended from the shorter arm, the weights will have equal positional gravity.
> Let the balance beam be *abc* [See Fig. 3.11], as before, and the suspended weights *a* and *b*; and let the ratio of *b* to *a* be as the ratio of *ac* to *bc*. I say that the balance will not move in either direction. For let it be supposed that it descends on the side of *b*; and let the line *dce* be drawn obliquely to the position of *acb*. If then the weight *d*, equal to *a*, and the weight *e* equal to *a* are suspended, and if the line *dg* is drawn vertically downward and the line *eh* vertically upward, it is evident that the triangles *dcg* and *ehc* are similar, so that the proportion of *dc* to*ce* is the same as that of *dg* to *eh*. But *dc* is to *ce* as b is to *a*; therefore *dg* is to *eh* as *b* is to *a*. Then suppose *cl* to be equal to *cb* and to *ce*, and let *l* be equal in weight to b; and draw the perpendicular *lm*. Since then *lm* and *eh* are shown to be equal, *dg* will be to *lm* as *b* is to *a*, and as *l* is to *a*. But, as has been shown, *a* and *l* are inversely proportional to their contrary (upward), motions. Therefore, what suffices to lift *a* to *d*, will suffice to lift *l* through the distance LM. Since therefore *l* and *b* are equal, and *lc* is equal to *cb*, *l* is not lifted by *b*; and consequently *a* will not be lifted by *b*, which is what is to be proved.[82]

[82] "Questio sexta. Si fuerint brachia librae proportionalia ponderibus appensorum ita, ut in breviori graviter appendatur, aeque gravia erunt secundum situm appensa. Sit ut prius regula a, c, b, appensa a, et b, sitque proportio b, ad a, tam quam a, c, ad bc, dico quod non nutabit in aliqua parte librae, sit enim ut ex parte b, descendat, transeatque in obliquum linea d, c, e, loco a, c, b, et appensa d, ut a, et e, ut b, et d, f, linea orthogonaliter descendat, et e, h, ascendat. palam quoniam trianguli d, c, f, et e, c, h, sunt similes, quia proportio d, c, ad c, e, quam d, b, ad e, h, atque d, c, ad c, e, sicut b, ad a, ergo d, f, ad e, h, sicut b, ad a, sit igitur c, l, aequalis c, b, et c, e, et l, aequatur b, in pondere, et descendat perpendiculum l, m, quia l, m, et e, h, constant esse aequales, erit d, g, ad l, m, sicut b, ad a, est sicut l, ad a, sed ut ostensum est a, et l, proportionaliter se habent ad contrarios motus alternatim. Quod igitur sufficiet attollere a, in d, sufficiet attollere l, secundum l, m. Quum ergo aequalia sint l, et b, et l, c, aequale c, b, l, non sequitur b, contrario motu, neque a, sequitur b, secundum quod proponitur." (de Nemore 1565, Quaestio sexta, 5rv).

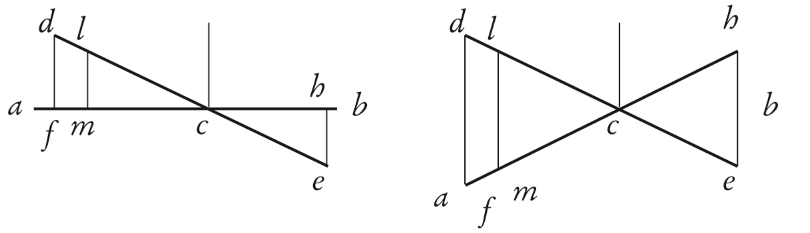


Fig. 3.11 The proof of the law of lever in the *Proposition VI*[83]

The proof is clear enough, except for some prolixity when showing the similitude of triangles. For the sake of brevity, de Nemore substantially claims that,

> *if we suppose the balance is not equilibrated and rises on the left, but this is impossible (absurd) because, for Proposition I, a weight a in d is equivalent to a weight b in l symmetric to b, and the balance should behave as a balance with equal arms and weight, which is in equilibrium because of the symmetry of the configuration.*

Finally let us note that the equilibrium is proved in an indirect way. The weight *a* is not compared directly with weight *b* but is reduced to the weight *l* equivalent to it, hanging from the same side of the balance. At this point we make the comparison between weights on the opposite site of the balance, and the equilibrium is deduced from reduction to the absurd.

3.2.3 The Structure of *Book VIII*

3.2.3.1 Notes of the Roots of Notional Elements in Tartaglia's Corpus

Just before focusing on the chore of studying *Book VIII* and after his criticism of Aristotelian accounts on balances of *Book VII* (Tartaglia 1554, *Book VII*) Tartaglia – on request by his interlocutor Mendoza – arguments on the logical status of his science of weights:

> [Question I] Sir Ambassador [Mendoza]. Now, Tartaglia, I want you to start explaining in due order that Science of Weights of which you spoke to me yesterday. And since I know that it is not a simple science in itself (there being no more than seven liberal arts), but rather that it is

[83] Redrawn from de Nemore 1565, Quaestio sexta, 5r.

> a *subordinate science* [emphasis added] or discipline, I want you first to tell me from which others it is derived.[84]

Tartaglia replies asserting that the science of weights, as well as mechanics, is a mixed science, as he has already argued and more in depth in *Book VII*:

> [Question I] N. Sir, part of this science is derived from geometry and part from natural philosophy; for part of its conclusions are demonstrated geometrically and part are tested physically, that is, through nature.[85]

According to Tartaglia to proceed in an orderly fashion, it is necessary to follow the approach of geometer. The first step is to establish the meaning of some terms and ways of speaking, i.e. to give definitions:

> [Question III] N. To proceed in an orderly fashion, we shall today define only some terms and ways of speaking that occur in this science, in order that your Excellency will more easily apprehend the fruit of the understanding of this.[86]

In this part of *Book VIII*, by introducing definitions Tartaglia is closer to Euclid's approach to science than Aristotle's. Euclid indeed used to distinguish clearly between *definitions*, *petitions*, and *principles*. Aristotle (like de Nemore) considered both definitions and evident assertions as principles.

Moreover, Tartaglia does not distinguish the nature of definitions as typical in the scholasticism between real (which, in the form given to them by Aristotle state the essence of *definendum*) and nominal (whereby the definition of a thing is furnished by already known terms and concepts)[87] and mixes both of them.

After the definitions, the principles of the science should be introduced, i.e.:

> [Question III] N. […]. Then, tomorrow, we shall proceed to state the principles of that science, that is, those things which cannot be demonstrated in the science; or as your Excellency knows, every

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[84] Tartaglia 1554, *Book VIII,* Q I, 82v.

[85] Tartaglia 1554, *Book VIII,* Q I, 82v.

[86] Tartaglia 1554, *Book VIII,* Q III, 83r.

[87] For an Aristotelian distinction between real and nominal definitions see Butlon (Bulton 1976; see also Corbini 2006).

> science has its indemonstrable first principles, which, being conceded or assumed, afford the means to discuss and sustain the whole science.[88]

There are different ways mathematicians assume principles according to Tartaglia (Tartaglia 1554, 84v). We collected them in Tables 3.4a. and 3.4b. Of these ways, Tartaglia declares to prefer the last way and decides to assume (sometime) his principles as *petitioni*.

Table 3.4a. *Definitions*, *Principles, Suppositions*, *Petitions* and *Propositions*

Libro VIII	"Principij primi indemostrabili" (83rv)	"Propositioni ouer conclusion" (83rv)	"Supposition Principij veri" (84rv)	"Petitioni" [as postulates] (84rv)
"Sopra la Scientia di Pesi"	General Principles non demonstrable	Sentences and proofs *Pro* "Scientia di Pesi".	True Principles within "Scientia di Pesi".	Sentences and proofs that can negate "Scientia di Pesi"
Subject	Geometry, "Philosophia Naturale"	Geometry, "Philosophia Naturale"	Geometry, "Philosophia Naturale"	Geometry, "Philosophia Naturale"
Quesiti folia	XXI, 83rv–84rv	XXVIII–XLII, 86rv–97rv	XXI, 83rv–85rv	XXII–XXVII 84rv–86rv

Among them Tartaglia also assumed others assertions to negate prove something within science of weights:

[88] Tartaglia 1554, *Book VIII*, Q III, 83r.

Table 3.4b. Different ways to assess a principle in a science

Dignità (as Greek axiom)	Suppositioni (as Hypotheses)	Petitioni (as Postulates)
"[…] they prove other propositions but cannot be proved from others".[89] Statements which are self–evident and accepted by all for all sciences.	"[…] they are supposed to be true in the given science".[90] Statements which are self–evident.	"[…] if we wish to debate such a science and sustain it with demonstrations, we must first request the adversary to concede them".[91] Statements requested to be accepted by the adversary even if he does not share completely them.

Generally speaking, the argument proposed by Tartaglia was not a novelty. A novelty was its application to science of weights.

When it comes to *Suppositioni*, in the Middle Ages the mathematics and philosophy were considered as necessary foundations. Particularly, they were used in the main way:

1. The first way treats *Suppositioni* as *Propositions*, which are self-evident.
2. The second way, following, i.e., Aristotle in his *Analytica posterior*, qualifies them as *hypotheses*, i.e. sentences which are accepted both by the supporter (*magister*) and the opponent (*disciple*) and could possibly be justified by a superior science (Aristotle 1949, *Posterior analytics*, I, 2, 10).[92]

[89] Tartaglia 1554, *Book VIII*, Q XXI, 84v.
[90] *Ibidem.*
[91] *Ibidem.*
[92] Aristotle, *Posterior analytics*, I, 2, 10. For a comment of the concept of hypothesis in Aristotle, see Upton (Upton 1985) and Gomez–Lobo (Gomez–Lobo 1977).

My epistemological interpretation is based on philological remarks and Tartaglia scientific corpus within Late Middle Age–Renaissance time.

It is known that the term *axiom* goes back to Aristotle (Mathematics) and it denotes a content of sentence as self-evident, that is it does not admit of proof; rather from which a proof proceeds. Aristotle also stressed the distinction between the *general axioms* as sentences common to all sciences (geometry, called κοιναὶ ἔννοιαι by Euclid[93]) and *Proper axioms* to a particular one, i.e., as *postulates* (ἀίτήματα). Aristotle, particularly, considered *Postulates* as well as first propositions in science and having truth and necessity. They are different from *Proper principles* as propositions belong to a particular theory. In his words:

> It is also evident that if the propositions of which a syllogism consists are universal, the conclusion of such a demonstration, and in short of the demonstration of itself, must necessarily be perpetual. There is not then either demonstration, nor in short science of corruptible natures, but so as by accident, because there is not universal belonging to it, but sometimes, and after a certain manner. But when there is such, it is necessary that one proposition should not be universal, and that it should corruptible, corruptible indeed, because the conclusion will be so if the proposition is so, and not universal, because one of those things of which it is predicated will be, and another will not be, hence it is not possible to conclude universally, but that it is now.
> [...] The demonstrations and sciences however of things frequently occurrent, of the eclipse of the moon, evidently always exist, so far as they are such, but so far as they are not always, they are particular, and as in an eclipse, so also is it in other things.[94]
> [...] it is impossible to demonstrate the proper principles of each thing, for they will be the principles of all things, and the science of them the mistress of all (sciences).

93 For example in the *Elements* are: Book I Definitions (23), Postulates (5), Common Notions (5), Propositions (48) ; Book II Definitions (2), Propositions (13); Book III Definitions (11), Propositions (37); Book IV Definitions (7), Propositions (16); Book V Definitions (18), Propositions (25); Book VI Definitions (11), Propositions (37); Book VII; Definitions (22), Propositions (39); Book VIII Propositions (27); Book IX Propositions (36), Book X Definitions I (4), Propositions 1-47, Definitions II (6), Propositions 48-84, Definitions III (6), Propositions 85-115; Book XI Definitions (28), Propositions (39); Book XII Propositions (18); Book XIII Propositions (18).

94 Aristotle 1853. *Things Which are subjects to Change are incapable of Demonstration per se*, *Book I*, Chapter VIII, 263.

> [...] Demonstration however is not suitable to another genus, except as we have said, geometrical to mechanical or optical, and arithmetical to harmonical demonstrations.[95]
> [...] I call those principles in each genus, the existence of which it is impossible to demonstrate. What then first thing, and such as result from these signify, is assumed, but as to principles, we must *assume* that they are, but *demonstrate* the rest, as what unity is, or what the straight and a triangle are. It is necessary however to assume that the unity and magnitude exist, but to demonstrate the other things.[96]
> Proper principles, again, are those which are assumed to be, and about which science consider whatever are inherent per se, as arithmetic assumes unites, and geometry points and lines, for they assume that these are, and that they are this particular thing.[97]

The *intellect* is the instrument used for the *intuition*, *perception*. According Aristotle the experiences have a fundamental role in the acquisition of the *intuition*. Nevertheless, the experience cannot change the validity of the principles within a theory:

> Of those [principles] are employed in demonstrative sciences, some are peculiar to each science, but others are common, and common according to analogy, since each is useful, so far as it in the genus under science. The peculiar indeed are such as, that a line is a thing of this kind, and that the straight is, but the common are, as that if equals be taken from equals the remainders are equal. Now each of these is sufficient, so far as it in the genus, for (a geometrical) will affect the same, thought he should not assume of all, but in magnitudes alone, and the arithmetician in respect of numbers (alone).[98]
> […] geometry what is not proportionate, or what is to be broken, or to incline; but *that* they are demonstrate trough things common and from those which have been demonstrate.[99]

The following Aristotelian distinction between definitions and hypotheses is important for our aim because it is immediately correlated with the *modus demostrandi*.

> Definitions then are not hypothesis, (for they are not asserted to be or not to be) but hypothesis are in propositions [premise]. Now it is only neces-

[95] *Ivi*, Chapter IX, 265.
[96] *Ivi*, “On the Definition and Division of Principles”*, Book I*, Chapter, X, 266. (Author’s italic).
[97] *Ibidem*.
[98] *Ibidem.*
[99] *Ivi*, 266-267. (Author’s italic).

> sary that definitions should be understood, but this is not hypothesis, except someone should say that the verb to hear is hypothesis. But they are hypothesis, from the existence of which, in that they are, the conclusion is produced.[100]
> The things which are immediately obvious and clear to us are usually mixed together; their elements and principles only become intelligible later, when one separates them. That is why we have to progress from the general to the particular; it is because it is whole entities that are more intelligible to the sense, and anything general is a kind of whole, in this sense that it includes a number of things which could call its parts. [...]. And little children initially [also] call all men "father" and all women "mother" and only later distinguish who their fathers and mothers are.[101]

Targalia's proofs within the *Book VIII* (*Propositions*) of the *Quesiti et inventioni diverse* (like most in the previous *Nova scientia*) are argued and presented by deductive and slightly–more–or–less (nowadays called) axiomatically reasoning. Fo sure it is a historical and epistemological error to attribute him an axiomatically process of proof, or reasoning.

Particularly is some of interest to report the following Aristotle's arguments:

> Chap. XXIV. *The superiority of Universal to Particular Demonstration proved.* As one demonstration is universal, but another particular, one also affirmative, but the other negative, it is questioned which is preferable, and likewise also about what is called direct demonstration, and what which leads to the impossible. [...] an isosceles triangle (has two right angle), not because it is isosceles, but it is a triangle, but the particular demonstrates because a thing is what it is, if then the demonstration per se is preferable, and the particular is such rather than the universal, particular demonstration would be better. Besides, if the universal is nothing else than particular, but demonstration produces opinion that this thing is something according to which it demonstrates, and that a certain nature of this kind is in things which subsist, (as of triangle besides particular (triangle), and of figure besides particular (figures), and of number besides particular (numbers), but the demonstration about being is better than that about non-being, and that through which there is no deception than through which there is, but universal demonstration is of this sort, (since men proceeding demonstrate as about the analogous, as that a thing which is of such a kind as to be neither line nor number, nor solid nor superficies, but something besides these, is analogous,) if then this is more

[100] *Ivi*, 268.
[101] *Ibidem.* (Author's quotations).

> universal, but is less conversant with being that particular, and produces false opinion, universal will be inferior to particular demonstration.[102]

With regards the term *dignitas*, it comes from the Latin *dignitas-dignitatis*. It recalls the Greek ἀξίωμα (*axioma, maxima,* or simply *axiom, also "*to deem worth"), which means *to deem worth* (that is *dignity*[103]), but also *to require* (that is *axiom*).

At that time, the term *dignità* (dignity) often meant *common assumptions*, i.e. self–evident sentences common to all sciences. Particularly, it was familiar within philosophical Corpus by Anicius Manlius Severinus Boëthius (c. 480–524/525 AD); and others *schoolmen* at that time.

In the secondary literature, i.e., Drake and Drabkin's choice to translate *dignità* with *axiom* (Drake and Drabkin 1969, 116).

We also remark that in the *Euclide Megarense* Tartaglia uses the term *dignità* as equivalent to *Suppositioni*:

> Before we proceed far away we have to notice that the first principles of each science cannot be known by demonstration, and no science must prove his principles, because this would lead to a process with no end. But such principles are known by the intellect through senses, for the beginning of any our knowledge comes from senses, and by means of them [the first principles] the whole science is proved and sustained; and they are said principles of that science for they prove others and cannot be proved by others in such a science; and these first principles of science are called petitions by some; others say dignities, namely suppositions.[104]

In addition, in the *Nova scientia* Tartaglia also used another term – more common – *Comune Sententie* to define *common assumptions* or *axioms* of the theory (Tartaglia 1537, *Book I*, 11v-12r); as we also announced in the previous Chapter 1.

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[102] *Ivi*, Chapter XXIV, *Book I*, 299-300. See also: *Analytica posteriora*, B 17, 99a16-21.

[103] *Dignita* was written by Tartaglia with final letter "a" without accent as usual in modern Italian language.

[104] "Inanti che procediamo piu oltra, bisogna notare, che li primi principij di ciascaduna scientia non si cognoscono per demostratione: ne etiam alcune scientia è tenuta a provar li suoi principij, perche bisogneria proceder in infinito, Ma quelli tali principij si cognoscono per intelletto, mediante il senso, e pero il principio di ogni nostra cognitione incomincia dal senso, per il che sono supposti nella scientia, et con quelli se dimostra, & sostenta tutta la scientia; & sono detti principij di quella scientia, perche, provano altri, & non essere possono provati da altri, in quella scientia; & questi primi principij delle scientie alcuni li chiamano petitioni, & alcuni di dicono dignità, overo supposition". (Tartaglia 2007, 16).

Based on previous epistemological notes concerning the lacks of a strictly axiomatically organization of the theory in Tartaglia's *Quesiti* (see above Chapter 1) and the Tables 3.4a and 3.4b may be surely questionable, at least for the meanings we have attributed to *dignità* and *supposition* (*Ibidem*).

Finally these previous facts would create embarrassment in to make clear the epistemic concepts adopted by Tartaglia in the *Quesiti et invention diverse*.

3.2.3.2 The Definitions of Book VIII

Table 3.5 reports the *Definitions* of *Book VIII* of *Quesiti et inventioni diverse*, compared with those of the medieval treatises on the science of weights that Tartaglia knew.

Table 3.5 Tartaglia's *Definitions* versus Medieval Tradition

	Tartaglia's *Definitions*	Medieval *Definitions*
I	Bodies are said to be of equal size when they occupy or fill equal spaces.[105]	Bodies equal in volume are those which fill equal places.[106]
II	Similarly the bodies are said to be of different or unequal size when they occupy or fill different or unequal spaces, and greater means that which occupies more spaces.[107]	And those which fill unequal places are said to be of different volume[108]. And what are said to be large, among bodies, are said to be capacious, among places.[109]
III	[…] a heavy body is understood and assumed that power [virtus] which it has to tend or go downward, as also to resist the contrary motion which would draw it upward.[110]	[...] and its virtus is a power of tending downward and to resist to the contrary motion, and we can understand its strength or power from the arm length or from its velocity which is determined by the length of the balance arms.[111]

[105] Tartaglia 1554, *Book VIII,* Q IIII, Definition I, 83r.
[106] *De ponderoso et levi*, Supposition I (Moody and Clagett [1952] 1960, 27).
[107] Tartaglia 1554, *Book VIII,* Q V, Definition II, 83r.
[108] *De ponderoso et levi*, Supposition II (Moody and Clagett [1952] 1960, 27).
[109] *De ponderoso et levi*, Supposition III (Moody and Clagett [1952] 1960, 27).
[110] Tartaglia 1554, *Book VIII,* Q VI, Definition III, 83v.
[111] *Jordani opusculum de ponderositate* (de Nemore 1565, 3r).

IV	Bodies are said to be of equal virtus or power when in equal times they run through equal spaces.[112]	Bodies are equal in strength, whose motions through equal places, in the same air of the same water, are in equal times.[113]
V	Bodies are said to be of different virtus or power when in different times they move through equal spaces, or when in equal times they traverse unequal intervals.[114]	And those which traverse equal places in different times, are said to be of different in virtus.[115]
XII	A body is said to be more or less heavy in descent than another when the straightness, obliquity, or pendency of the place or space where it descends makes it descend more or less heavy than the other, and similarly more or less rapidly than the other, though both are simply equal in heaviness.[116]	
XIII	A body is said to be positionally more or less heavy than another when the quality of the place where it rests and is located makes it heavier [or less heavy] than the other, even though both are simply equal in heaviness.[117]	One weight is less heavy according to position, than another, if it is caused to ascend by the descent of the other.[118]
XIV	The heaviness of a body is said to be known when one knows the number of pounds, or other named weight, that it weighs.[119]	A weight is known when the number of its calculi is known.[120]

[112] Tartaglia 1554, *Book VIII,* Q VII, Definition IIII, 83v.
[113] *De ponderoso et levi*, Supposition IV (Moody and Clagett [1952] 1960, 27).
[114] Tartaglia 1554, *Book VIII,* Q VIII, Definition V, 83v.
[115] *De ponderoso et levi*, supposition V (Moody and Clagett [1952] 1960, 27).
[116] Tartaglia 1554, *Book VIII,* Q XV, Definition XII, 84r.
[117] Tartaglia 1554, *Book VIII*, Q XVI, Definition XIII, 84r.
[118] *Jordani opusculum de ponderositate* (de Nemore 1565, 3r).
[119] Tartaglia 1554, *Book VIII,* Q XVII, Definition XIIII, 84r.
[120] *De insidentibus in humidum*, Definition V (Moody and Clagett [1952] 1960, 41).

XVII	The descent of a heavy body is said to be more oblique when for a given quantity it contains less of the line of direction, or of straight descent toward the centre of the world.[121]	A more oblique descent is one which in the same distance, partakes less of the vertical.[122]

Tartaglia's definitions, as typically at that time, are partly of nominal type and partly of real type (in modern terms). The former ones give a name to the association of other names, and the latter define the essence of the object to be defined. For example, let consider previous *Definition* III (see Table 3.6). It concerns the term/concept *virtus* (power); analyzed according to the modern conception of an axiomatic theory, then it does not appear as a definition of nominal type. In fact, it is composed of three different sentences, 1, 2, and 3, as in the following:

Table 3.6 Tartaglia's *Definition III*

Definitions III	Elementary propositions	Interpretation
[…] a heavy body is understood and assumed that power [virtus] which it has to tend or go downward, as also to resist the contrary motion which would draw it upward.[123]	1. A body tends to go downward.	As postulate
	2. There is a cause for it, a power (*virtus*).	As postulate
	3. I call this cause a power (*virtus*).	As an axiom

Tartaglia certainly did not followed this reasoning. He considered *Definition* III is real type which serves to define *virtus* in its essence, trying to make clear, with the help of *intuition*, its meaning.

Definitions IV and V seems to refer to attributing the modern term *velocity* to the word *virtus*. Thus, it rightly seems a nominal definition like:

$$\textit{velocity} \equiv \textit{virtus}$$

Nevertheless, in this case, for sure we do not want to replace *virtus* with *velocity* since the meaning of the definition changes. The association between velocity and speed is indeed a characterization of *virtus* as defined in *Definition* III. It is a postulate.

[121] Tartaglia 1554, *Book VIII,* Q XX, Definition XVII, 84r–84v.
[122] *Jordani opusculum de ponderositate* (de Nemore 1565, 3r).
[123] Tartaglia 1554, *Book VIII,* Q VI, Definition III, 83v.

3.2.3.3 The Petitions of Book VIII

In the following, we present a comparison between Tartaglia's *Petitions* and de Nemore's *Suppositions* in his *Jordani opusculum de ponderositate* (de Nemore 1565, 3r) as already collected in Table 3.1 (see also Tables 3.4).

Table 3.7. Tartaglia's *Petitions* versus Nemore's *Suppositions*

	Tartaglia's *Petitions*	de Nemore's *Suppositions*	
I	We request that it be conceded that the natural movement of any heavy and ponderable body is straight toward the centre of the world.[124]	The movement of every heavy body is toward the centre [of the world] and its strength is a power of tending downward and to resist to the contrary motion, and we can understand its strength or power from the arm length or from its velocity which is determined by the length of the balance arms.[125]	I
II	Likewise we request that it be conceded that that body which is of greater power should also descend more swiftly; and in the contrary motion, that is, of ascent, it should descend more slowly – I mean in the balance.[126]	What is heavier descends more speedily.[127]	II
III	It still it be conceded that a heavy body in descending is so much the heavier as the motion it makes is straighter toward the centre of the world.[128]	It is heavier in descending, to the degree its movement toward the centre is more direct.[129]	III
IIII	Also we request that it be conceded that those bodies are	It is heavier according to position in that position where its path of	IIII

[124] Tartaglia 1554, *Book VIII*, Q XXII, Petition I, 84v.
[125] de Nemore 1565, 3r.
[126] Tartaglia 1554, *Book VIII,* Q XXIII, Petition II, 85r.
[127] de Nemore 1565, 3r.
[128] Tartaglia 1554, *Book VIII*, Q XXIIII, Petition III, 86r.
[129] de Nemore 1565, 3r.

	equally heavy positionally when their descents in such positions are equally oblique, and that will be the heavier which, in the position or place where it rests or is situated, has the less oblique descent.[130]	descent is less oblique.[131]	
V	Similarly we request that it be conceded that that body is less heavy than another positionally when, by the descent of that other on the arm of the balance, a contrary motion would follow in the first; that is, the first would thereby be elevated toward the sky; and conversely.[132]	A more oblique descent is one which, in the same space, partakes less of the vertical.[133]	V
VI	Also we request that it be conceded that nobody is heavy in itself.[134]	One weight is less heavy according to position, than another, if it is caused to ascend by the descent of the other.[135]	VI
VII		The position of equality is that of equality of angles to the vertical, either these are right angles, or the beam is parallel to the plane of the horizon.[136]	VII

3.2.3.4 The Propositions of Book VIII

Let us now examine the propositions. Those of Tartaglia are fourteen, those of de Nemore ten. They are compared in Table 3.8.

[130] Tartaglia 1554, *Book VIII*, Q XXV, Petition IIII, 86v.
[131] de Nemore 1565, 3r.
[132] Tartaglia 1554, *Book VIII*, Q XXVI, Petition V, 86v.
[133] de Nemore 1565, 3r.
[134] Tartaglia 1554, *Book VIII*, Q XXVII, Petition VI, 86v.
[135] de Nemore 1565, 3r.
[136] de Nemore 1565, 3r.

Table 3.8 Tartaglia propositions versus de Nemore's *Quaestio*

	Tartaglia's *Propositions*	de Nemore's *Quaestio*	
I	The ratio of size of bodies of the same kind is the same as the ratio of their power.[137]	Between any heavy bodies, the strengths are proportional to the weights.[138]	I
II	The ratio of the power of heavy bodies of the same kind and that of their speeds (in descent) is concluded to be the same; also that of their contrary motions (that is, of their ascents) is concluded to be the same, but inversely.[139]		
III	If there are two bodies simply equal in heaviness, but unequal positionally, the ratio of their powers and that of their speeds will necessarily be the same. But in their contrary motions (that is, in ascent) the ratio of their powers and that of their speeds is affirmed to be inversely the same.[140]		
IIII	The ratio of the power of bodies simply equal in heaviness, but unequal in positional force, proves to be equal to that of their distances from the support or centre of the scale.[141]		
V	When a scale of equal arms is in the position of equality, and at the end of each arm there are hung weights simply equal in heaviness, the scale does not leave the said position of	When a horizontal position is gained [for a balance of equal arms], then, if equal weights are suspended [from its extremities], the balance will not leave the horizontal	II

[137] Tartaglia 1554, *Book VIII,* Q XXVIII, Proposition I, 87r.
[138] de Nemore 1565, 3r.
[139] Tartaglia 1554, *Book VIII*, Q XXIX, Proposition II, 87r–88r.
[140] Tartaglia 1554, *Book VIII*, Q XXX, Proposition III, 88r.
[141] Tartaglia 1554, *Book VIII,* Q XXXI, Proposition IIII, 89r.

	equality; and if it happens that by some other weight [or the hand] imposed on one of the arms it departs from the said position of equality, then, that weight or hand removed, the scale necessarily returns to the position of equality.[142]	position; and if it is moved from the horizontal position, it will revert to the horizontal position. If instead unequal [weights] are suspended, [the balance] will fall on the side of the heavier [weight] until it reaches the vertical position.[143]	
VI	Whenever a scale of equal arms is in the position of equality, and at the end of each arm are hung weights simply unequal in heaviness, it will be forced downward to the line of direction on the side where the heavier weight shall be.[144]		
		In whichever direction a weight is displaced from the position of equality, it becomes lighter according to position.[145]	III
		When equal weights are suspended [with wires] from a balance, inequality of the wires will not determine a perturbation of their equilibrium.[146]	IV
VII	If the arms of the scale are unequal, and at the ends of them are hung bodies simply equal in heaviness, the scale will tilt on the side of the longer arm.[147]	If the arms of the balance are unequal, then, equal [weights] suspended [from their extremities], a swinging on the side of the longer [arm] is determined.[148]	V
VIII	If the arms of the balance are	If the [length of the] arms of a	VI

[142] Tartaglia 1554, *Book VIII,* Q XXXII, Proposition V, 89v.
[143] de Nemore 1565, 3v.
[144] Tartaglia 1554, *Book VIII*, Q XXXIII, Proposition VI, 91rv.
[145] de Nemore 1565, 4v.
[146] de Nemore 1565, 4v.
[147] Tartaglia 1554, *Book VIII*, Q XXXIIII, Proposition VII, 92v.
[148] de Nemore 1565, 4v.

	proportional to the weights imposed on them, in such a way that the heavier weight is on the shorter arm, then those bodies or weights will be equally heavy positionally.[149]	balance are proportional to the weights suspended, and the heavier weight is suspended from the shorter, the weights will be equally heavy according to position.[150]	
IX	If there are two solid rods or beams of the same length, breadth, and width, hung on a balance in such a way that one is horizontal and the other vertical, with the distances equal from the centre of the balance to the point of suspension of the latter and the centre of the former, then they will be positionally equally heavy.[151]	If two oblong bodies, wholly similar and equal in size and weight, are hung from a balance so that the one be disposed horizontally and the other comes down vertically, so that the distance from the centre [of the balance] to extremity that descends is the same as the distance to the midpoint of the other, in this position [the two bodies] will be equally heavy.[152]	VII
		If the arms of a balance are unequal, and form an angle at the centre of rotation, then, if their ends are equidistant from the vertical line passing through the centre, equal weights suspended fin this position will weigh equally.[153]	VIII
X	If a solid rod or beam of uniform breadth thickness, substance, and heaviness in every part, and [if] its length is divided into two unequal parts, and at the end of the shorter part there is hung another solid or heavy body which makes the said rod, beam, or balance stay parallel to the	When there is a beam of a balance with uniform weight and thickness and the weight is assigned, by dividing it into unequal parts and an assigned weight suspended from the shorter part maintains the equilibrium, then the portion of the arms of the balance on	XI

[149] Tartaglia 1554, *Book VIII*, Q XXXV, Proposition VIII, 93r.
[150] de Nemore 1565, 5r.
[151] Tartaglia 1554, *Book VIII*, Q XXXVI, Proposition IX, 93v.
[152] de Nemore 1565, 5v.
[153] de Nemore 1565, 6r.

	horizon, then the proportion of the heaviness of that body to the difference between the heaviness of the longer part of the rod (or beam or staff) and the heaviness of the shorter part will be as the ratio of the length of the whole rod, beam, or balance to the double of the length of its shorter part.[154]	each side of the fulcrum will be known.[155]	
XI	If the proportion of the heaviness of a solid suspended at the end of the shorter part of a similar rod (beam or balance) divided into two unequal parts, to the difference between the heaviness of the longer part and that of the shorter, shall be as the ratio of the whole length of the solid rod or staff to the double of the length of its shorter part, such solid rod (beam or staff) will necessarily be horizontal.[156]		
XII	If there is a solid rod, beam, or staff, as in the two preceding [propositions], which is similar and equal in thickness, breadth, substance, and heaviness in every part and of which the heaviness as well as the length is known, and if it be divided into two unequal parts which are also known, it is possible to find a weight which, when suspended at the end of its shorter part, will make the said solid rod, beam, or staff stay horizontal.[157]	But if the lengths of the arms are given the weight will be known.[158]	XII

[154] Tartaglia 1554, *Book VIII*, Q XXXVII, Proposition X, 94v.
[155] de Nemore 1565, 7r.
[156] Tartaglia 1554, *Book VIII,* Q XXXVIII, Proposition XI, 95r.
[157] Tartaglia 1554, *Book VIII,* Q XXXIX, Proposition XII, 95v.
[158] de Nemore 1565, 7v.

XIII	If you shall have a rod, beam, or staff, as often was said above, of which the length, as well as the heaviness, is known, and also a heavy body of which the weight is known, it is possible to determine the place at which the said rod, beam, or staff must be divided in order that the said heavy body, when hung at the end of the shorter part, will cause the rod, beam, or staff to remain parallel to the horizon.[159]		
XIIII	The equality of obliquity [slant] is an equality of weight [according to position].[160]	Equality of declination conserves the identity of weight.[161]	IX
XV	If two heavy bodies descend by paths of different obliquities, and if the proportions of inclinations of the two paths and of the weights of the two bodies be the same, taken in the same order, the power of both the said bodies in descending will also be the same.[162]	If two weights descend along diversely oblique paths, then, if the inclinations are directly proportional to the weights, they will be of equal strength in descending.[163]	X

In the following Table 3.9 we make explicit the correspondences between Tartaglia's and de Nemore propositions.

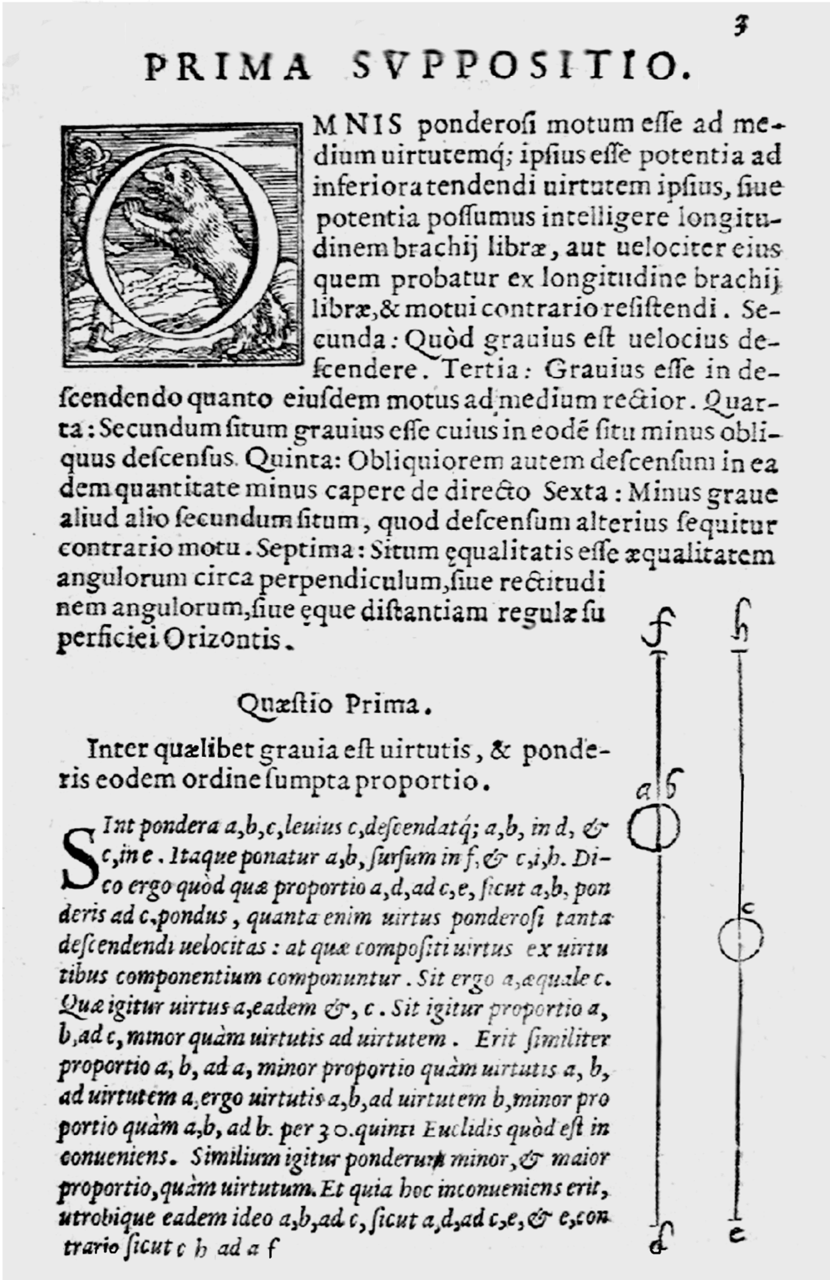
Table 3.9 The correspondence of Tartaglia's propositions and de Nemore's questions

Tartaglia	**I**, II, III, IIII	**V**, VI		**VII**	**VIII**	**IX**		**X**	**XI**	**XII**	XIII	**XIIII**	**XV**
de Nemore	**I**	**II**	III IV	**V**	**VI**	**VII**	VIII	**XI**		**XII**		**IX**	**X**

[159] Tartaglia 1554, *Book VIII,* Q XL, Proposition XIII, 96rv.
[160] Tartaglia 1554, *Book VIII,* Q XLI, Proposition XIIII, 96v.
[161] de Nemore 1565, 6v.
[162] Tartaglia 1554, *Book VIII,* Q XLII, Proposition XV, 97r.
[163] de Nemore 1565, 7r.

Note the replacement by Tartaglia of the first questions with three propositions and elimination of the proposition corresponding to de Nemore's VIII. This absence is not explained by Tartaglia.

Finally, we showed how Tartaglia uses as the only principle the active one based on the concept of gravity of position. However, this, as shown in section 2.1.2.1 leads to erroneous results for the angular lever. Tartaglia, who certainly knew the correct result, avoided facing the problem.



3

PRIMA SVPPOSITIO.

OMNIS ponderoſi motum eſſe ad medium uirtutemq; ipſius eſſe potentia ad inferiora tendendi uirtutem ipſius, ſiue potentia poſſumus intelligere longitudinem brachij libræ, aut uelociter eius quem probatur ex longitudine brachij libræ,& motui contrario reſiſtendi. Secunda: Quòd grauius eſt uelocius deſcendere. Tertia: Grauius eſſe in deſcendendo quanto eiuſdem motus ad medium rectior. Quarta: Secundum ſitum grauius eſſe cuius in eodẽ ſitu minus obliquus deſcenſus. Quinta: Obliquiorem autem deſcenſum in ea dem quantitate minus capere de directo Sexta: Minus graue aliud alio ſecundum ſitum, quod deſcenſum alterius ſequitur contrario motu. Septima: Situm ęqualitatis eſſe æqualitatem angulorum circa perpendiculum,ſiue rectitudi nem angulorum,ſiue ęque diſtantiam regulæ ſu perficiei Orizontis.

Quæſtio Prima.

Inter quælibet grauia eſt uirtutis, & ponderis eodem ordine ſumpta proportio.

SInt pondera a,b,c,leuius c,deſcendatq; a,b, in d, & c,in e. Itaque ponatur a,b, ſurſum in f, & c,i,h. Dico ergo quòd quæ proportio a,d,ad c,e, ſicut a,b, ponderis ad c,pondus, quanta enim uirtus ponderoſi tanta deſcendendi uelocitas: at quæ compoſiti uirtus ex uirtutibus componentium componuntur. Sit ergo a,æquale c. Quæ igitur uirtus a,eadem &, c. Sit igitur proportio a, b,ad c, minor quàm uirtutis ad uirtutem. Erit ſimiliter proportio a, b, ad a, minor proportio quàm uirtutis a, b, ad uirtutem a, ergo uirtutis a,b,ad uirtutem b,minor proportio quàm a,b, ad b. per 30.quinti Euclidis quòd eſt inconueniens. Similium igitur ponderum minor, & maior proportio, quàm uirtutum. Et quia hoc inconueniens erit, utrobique eadem ideo a,b,ad c, ſicut a,d,ad c,e, & e,contrario ſicut c h ad a f

Fig. 3.12a Plate from the initial reasoning around *gravitas secundum situm* by de Nemore[164]

[164] de Nemore 1565, *Quaestio* I, 3r.

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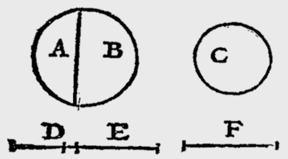
QVESITO. XXVIII. PROPOSITIONE PRIMA.

SIGNOR AMBASCIATORE.. *Hor seguitati Tartaglia queste uostre propositioni, ouer conclusioni consequentemente l'una drieto all'altra, & sotto breuita.* NICOLO.

LA *proportione della grandezza di corpi de un medesimo genere, & quella della lor potentia è una medesima.* S.A. *Datemi uno essempio.* N. *Siano li dui corpi.a.b.&.c. de uno medesimo genere, & sia.a.b. maggiore, & sia la potentia del corpo.a.b.la.d.e.& quella de corpo.c.la.f.* Hor *dico che quella proportione, che è dal corpo.a.b.al.corpo.c.quella medesima è della potentia.d.e.alla potentia.f.* Et *se possibile è esser altramente (per l'auersario) sia che la proportione del corpo.a.b.al corpo.c. sia menore di quella della potentia.d.e.alla potentia.f.* Hor *sia del corpo.a.b. (maggiore) compreso una parte eguale al corpo.c.menore, quale sia la parte.a.& perche la uertu, ouer potentia del composito è composta dalla uertu di componenti. Sia adunque la uertu, ouer potentia della parte.a.la.d.& la uertu, ouer potentia del residuo.b de necessita sara la restante potentia.e.et perche la parte.a.è tolta egual al.c.la potentia.d.(per il conuerso della.7.diffinitione) sara eguale alla potentia.f.& la proportione de tutto il corpo. a.b.alla sua parte.a.(per la seconda parte della.7. del quinto di* Euclide*) sara, si come quella del medesimo corpo.a.b.al corpo.c.(per esser.a.egual al.c.) & similmente la proportione della potentia.d.e.alla potentia.f.sara, si come quella della detta potentia.d.e.alla sua parte.d.(per esser la.d.egual alla.f.)* Adunque *la proportione de tutto il corpo.a.b.alla sua parte.a.sara menore di quella di tutta la potentia.d.e.alla sua parte.d.* Adunque *euersamente (per la.30. del quinto di* Euclide*) la proportione del medesimo corpo.a.b. al residuo corpo.b.sara maggiore di quella di tutta la potentia.d.e. alla restante potentia.e. la qual cosa saria inconueniente, & contra la opinion dell'auersario, il qual uol che la proportione del maggior corpo al menore sia menore, di quella della sua potentia alla potentia del detto menore.* Adunque *destrutto l'opposito rimane il proposito.* S.A. *Sta bene, seguitati.* NIC.



QVESITO. XXIX. PROPOSITIONE SECONDA.

LA *proportione della potentia di corpi graui de uno medesimo genere, & quella della lor uelocita (nelli descensi) se conchiude esser una medesima, anchor quel-*

Fig. 3.12b Plate from the initial reasoning about the *gravitas secundum situm* by Tartaglia[165]

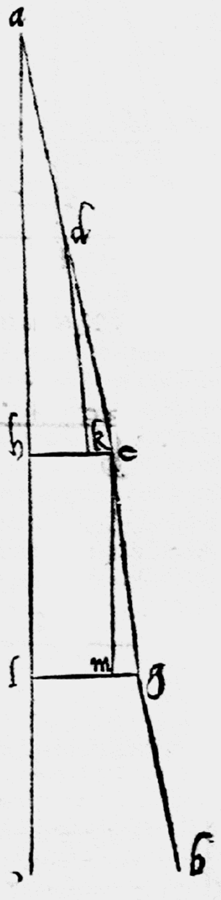
[165] Tartaglia 1554, Book VIII, *Quesito* XXVIII, Proposition I, 87r.

PONDEROSITATE. 7

Quæstio Decima.

Si per diuersarum obliquitatum uias duo pondera descendant, fiantq; declinationum, & ponderum vna proportio eodem ordine sumpta vna erit utriusque uirtus in descendendo.

Sit linea a, b, c, æquedistans orizonti, & super eam orthogonaliter erecta sit b, d, à qua descendant hinc, inde lineæ d, a, d, c, sitq; d, c, maioris obliquitatis proportione igitur declinationum dico non angulorum, sed linearum usque ad æquedistantem resecationem, in qua æqualiter sumunt de directo. Sit ergo e pondus super d, c, & h, super d, a, & sit e, ad h sicut d, c, ad a, d. Dico ea pōdera esse vnius uirtutis in hoc situ, sit enim d, k, linea vnius obliquitatis cum d, c, & pondus super eam. ergo æquale est e, quæ sit 6. Si igitur possibile est, descendat e, in l, & trahat h, in m, sitq; 6, n. æquale h, m, quod etiam æquale est e, l, & transeat per 6. & h, perpēdicularis, super d, b. Sitq; 6, h, y, & ab l, sit l, t, sunt & tunc super 6, h, y, n, z, m, x, & super l, t, erit e, r, quia igitur proportio n, z, ad n, 6, sicut ad d, 6, d, y, propter similitudinem triangulorum, & ideo sicut d, b, ad d, k, & quia similiter m, x, ad m, h, sicut d, b, ad d, a. Erit propter æqualē proportionalitatē perturbatam m, x, ad n z, sicut d, K, ad d, a, & hoc est sicut 6, ad h sed quia r, e, non sufficit attollere 6, in n, nec sufficit attollere m, in m, sic ergo manebunt.



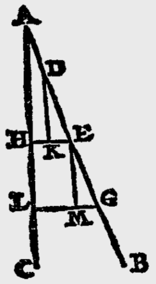
Quæstio Vndecima.

Quum sit responsa libræ vnius ponderis, & grossiciei per totum: & ipsa in pondere data super inæqualia diuidatur, atque ex parte breuiore dependeat æquabiliter pōdus datum, erunt & portiones & regulæ, quæ sunt a centro examinis similiter datæ.

Sit responsa a, b, c, data in pondere, & æqualis in grossicie, & dependeat

Fig. 3.13a Plate from reasoning around *gravitas secundum situm* applied to the inclined plane by de Nemore (Tartaglia)[166]

[166] de Nemore 1565, *Quaestio* X, 7r.



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che ſe pigliaremo ſotto al.d.& al.e.due parti equali nella uia, ouer linea.a.b. Hor poniamo, che l'una ſia la parte.d.e.et laltra la.e.g. Dico, che per le dette parti equali capira equalmente del diretto, cioe della linea.a.c.la qual coſa ſe notificara in queſto modo, dalli dui ponti.e.&.g.ſiano tirate le due linee.e.h.&.g.l. perpendicolare ſopra la linea.a.c.et dalli dui ponti, ouer luochi.d.&.e.le due linee.d.k.&. e.m. perpendicolare ſopra le medeſime.e.h.&.g.l.le qual due perpendicolare, cioe. d.k.&.e.m.ſaranno fra loro equali, perche adunque il detto corpo ponderoſo, ſi eſſendo nel ponto.d.come nel ponto.e.in quantita, ouer deſcenſi equali, capira equalmente del diretto, ſara di una medeſima grauita in qual ſi uoglia de quelli, ſecondo el ſito, ch'è il propoſito. S.A. E ue ho inteſo, ſeguitate pur. N.

QVESITO XLII. PROPOSITIONE XV.

SE dui corpi graui deſcendano per uie de diuerſe obliquita, & che la proportione delle declinationi delle due uie, & della grauita de detti corpi ſia fatta una medeſima, tolta per el medeſimo ordine. Anchora la uirtu de luno, e laltro de detti dui corpi graui, in el deſcendere ſara una medeſima. S.A. Queſta propoſitione mi par bella, e pero datime anchora un eſſempio chiaro, accio che meglio mi piaccia. N. Sia la linea.a.b.c.equidiſtante al orizonte, & ſopra di quella ſia perpendicolarmente eretta la linea.b.d.& dal ponto.d.deſcendano de qua, & de la le due uie, ouer linee.d.a.&.d.c.& ſia la.d.c.di maggior obliquita. Per la proportione adunque delle lor declinationi, non dico delli lor angoli, ma delle linee per fina alla equidiſtante reſecatione, in la quale equalmente ſummemo del diretto. Sia adũque la lettera.e.ſuppoſta per un corpo graue poſto ſopra la linea.d.c.& un'altro la lettera.h.ſopra la linea. d.a.& ſia la proportione della ſimplice grauita del corpo.e.alla ſimplice grauita del corpo.h.ſi come quella della.d.c.alla.d.a. Dico li detti dui corpi graui eſſer in tai ſiti, ouer luochi di una medeſima uirtu, ouer potentia. Et per dimoſtrar queſto, tiro la.d.k. di quella medeſima obliquita, ch'è la.d.c.& imagino un corpo graue ſopra di quella equale a corpo.e.el qual pongo ſia la lettera.g.ma che ſia in diretto con. e. h. cioe equalmente diſtanti dalla.c.k.Hor ſe poßibel è (per lauerſario) che li detti dui corpi e.&.h.non ſiano di una medeſima, & equal uirtu in tai luochi, adunque luno ſara di maggior uirtu, ouer potentia dell'altro, poniamo adunque, che.e. ſia di maggior uirtu, adunque quello ſara atto à diſcendere, & ſimelmente à far aſcendere, cioe à tirare in ſuſo el corpo.h.Hor poniamo (ſe poßibel è) che il detto corpo.e.deſcenda per fina in ponto.l.& che faccia aſcendere il corpo.h.per fin in ponto.m. & faccio, ouer che ſegno la.g.n.equale alla.h.m.la quale anchora lei uien à eſſer equale alla.e.l.Et dal põto.g.tiro la.g.h.e.la qual ſara perpendicolare ſopra la.d.b.per eſſer li detti tre ponti (ouer corpi) g.h.e.ſuppoſti in diretto, & equalmente diſtanti dalla. k. c. & ſimelmente dal ponto.l.ſia tirata la.l.t.equidiſtante alla.c.b.qual ſara pur perpendicolare

BB

Fig. 3.13b Plate from reasoning around *gravitas secundum situm* applied to the inclined plane by Tartaglia[167]

[167] Tartaglia 1554, *Book VIII*, *Quesito* XLII, Proposition XV, 97r.

Before going into the validity of the proof of Tartaglia's fifteen propositions, we want to stress his ideas. Of the two possible principles of statics he found in de Nemore's writings two possible principles of statics, one based on the concept of gravity of position, the other on the capability of a weight to lift another Tartaglia made a choice and decided to base his mechanics only on the gravity of position. This notwithstanding, he maintains trace of de Nemore' ideas and to state the equilibrium of a lever or an inclined plane considers the equivalence of weight disposed on the same side and not on the opposite. This notwithstanding, he maintains trace of de Nemore's ideas, i.e., in order to state the equilibrium of a lever – or an inclined plane – he considered the equivalence of weight disposed on the same side and not on the opposite. Table 3.8 (above) compare Tartaglia's and de Nemore's propositions.

3.2.4 The Proof of *Propositions*

3.2.4.1 Propositions I–IV: Gravitas Secundum Situm

Tartaglia's demonstrations of *gravitas secundum situm* are contained in the first four propositions (Tartaglia 1554, *Book VIII*, 87r–89r) and mainly consisted of clarification of the statement of de Nemore's *Proposition* I (de Nemore 1565, 3r) which, in any case, still remains largely unfulfilled.

In the first four (*Quaestio*) propositions Tartaglia undertakes to 'demonstrate' that the gravity of position of a weight, suspended from the end of the arm of a balance is directly proportional to the length of the arm, as well as the weight itself. Particularly:

I. The first *Proposition*[168] proofs that the power of bodies of the same kind is proportional to their volume (and therefore to their weight).
II. The second *Proposition*[169] proofs that speed is proportional to power for downward motion and inversely proportional to power for upward motion. For the transitive properties we have thus that the speed of ascent or descent is inversely or directly proportional to the weight.[170]
III. The third *Proposition*[171] repeats the second one for weights with different gravity of position.

[168] Tartaglia 1554, *Book VIII*, Q XXVIII, Proposition I, 87r.
[169] Tartaglia 1554, *Book VIII*, Q XXIX, Proposition II, 87r–88v.
[170] Tartaglia 1554, *Book VIII*, Corollary, 88r.
[171] Tartaglia 1554, *Book VIII*, Q XXX, Proposition III, 88rv.

IV. The fourth *Proposition*[172] proofs that the gravity of position of a weight on a scale is proportional to its distance from the fulcrum, and of course to the weight itself.

The proofs of these four propositions follow the same logic. In the following, we report Tartaglia's reasoning on the demonstration of *Proposition I*; we only brief reference the others.

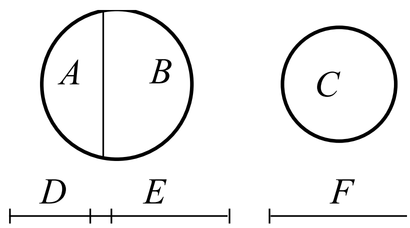

Fig. 3.14 Relation between the ratio of sizes (A, B, C) and powers (D, E, F)[173]

In Tartaglia's words:

> The ratio of volume of bodies of the same kind is the same as the ratio of their power. […] N. Let there be the two bodies *ab* and *c* of the same kind; let *ab* be the greater, and let the power of the body *ab* be [represented by the line] *de*, and that of the body *c* [by the line] *f*. Now I say that that ratio which the body *ab* bears to the body *c* is that of the power *de* to the power *f*. And if possible (for the adversary), let it be otherwise, so that the ratio of the body *ab* to the body *c* is less than the ratio of the power *de* to the power *f*. Now let the greater body *ab* include a part equal to the lesser body *c*, and let this be the part *a*, and since the force or power of the whole is composed of the forces of the parts, the force or power of the part *a* will be *d*, and the force or power of the remainder *b* will necessarily be the remaining power *e*; and since the part *a* is taken equal to *c*, the power *d* (by the converse of Definition 7) will be equal to the power *f*, and the ratio of the whole body *ab* to its part *a* (by Euclid V.7, 2) will be as that of the same body *ab* to the body *c* (*a* being equal to *c*), and similarly the ratio of the power *de* to the power *f* will be as that of the said power *de* to its part *d* (*d* being equal to *f*). Therefore [by the adversary's assumption] the ratio of the whole body *ab* to its part *a* will be less than that of the whole power *de* to its

[172] Tartaglia 1554, *Book VIII*, Q XXXI, Proposition IIII, 89r. See also its corollary (*Ibidem*).

[173] Redrawn from Tartaglia 1554, *Book VIII*, 87r.

> part *d*. Therefore, when inverted (by Euclid V.30) [174], the ratio of the body *ab* to the residual body *b* will be greater than that of the whole power *de* to the remaining power *e*, which will be contradictory and against the opinion of the adversary, who wants the ratio of the greater body to the less to be smaller than that of its power to the power of the lesser body. Thus, the contrary destroyed, the proposition stands.[175]

In *Proposition* I (Tartaglia 1554, *Book VIII*, 87r) Tartaglia assumes bodies of the same material but different size, so there is no doubt on the meaning of the proposition. He takes for granted, even if not explicitly stated in his petitions, that a heavier body has more power than a lighter. Tartaglia essentially reproduces the framework of proof of *Proposition II* by de Nemore in the process making it clearer. Nevertheless, there are still some points not acceptable to a modern reader. Without specifying exactly what it is and how to measure the power of a body, Tartaglia accepts *additivity*: the power of a body is given by the sum of the power of its parts. Like de Nemore, he does not notice, however, that in this way he takes for granted what he wants to prove. A modern reader is baffled by the almost miraculous demonstration such as Tartaglia's, as will that of de Nemore. There is the impression that with this way of reasoning one can prove anything, for example, that beauty is proportional to size.

Tartaglia's proof of his *Proposition II* (Tartaglia 1554, *Book VIII*, 87r–88v) is based on the same reasoning. This time things are slightly clearer because the third and fourth definitions and second petition connect somehow power and speed; in particular they suggest that there is a higher speed if there is a higher power. The first part of this proposition, that bodies fall down with speeds proportional to their size is proved with arguments similar to that used in *Proposition* I. It assumes *additivity* of speed with power and demonstrate proportionality. In order to demonstrate the inverse relationship between power and speed Tartaglia assumes that the resistance to upward motion is proportional to the power of the body. So that power that will barely fit in the other arm to lift the body *ab*, will be sufficient to lift faster the body C and the relationship of speed of *c* to *ab* is that of *ed* to *f* (See Fig. 3.15).

From *Propositions* I and II follows the proportionality (direct or inverse) between weight (size) and speed.

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174 This Euclidean proposition states that given four quantities, A, B, H, K, if (A+B)/A > (H+K)/H, then (A+B)/B< (H+K)/K (Tartaglia 1543, p 104, 105). So assumed $A = a$, $B=b$, $H = p(a)$; $K = p(b)$, from $(a+b)/c < \mathrm{p}(a+b)/\mathrm{p}(c) \equiv (a+b)/a < [p(a)+p(b)]/p(a)$ it follows $(a+b)/b > [p(a)+p(b)]/p(b) = p(a+b)/p(b)$.

175 Tartaglia 1554, *Book VIII*, Q XXVIII, Proposition I, 87r.

The logical status of the *Proposition* III is not clear; to a modern reader it seems an immediate consequence of *Proposition* II, however, a demonstration is proposed by following exactly the arguments of *Proposition* I.

In *Proposition* IIII (Tartaglia 1554, *Book VIII*, 89r) Tartaglia aims to quantify the concept of gravity of position, at least for bodies connected to the arms of a balance. The proof again follows the same line of argument, with some more difficulty. Tartaglia seems to make the assumption that the sum of distances corresponds to the sum of weights; which at looks very strange to us.

3.2.4.2 Propositions V–VI: Balance with Equal Weights and Arms

Hereinafter we report an epitome of Tartaglia *Proposition* V (Tartaglia 1554, *Book VIII,* 89v–90v) corresponding to *Quaestio* II by de Nemore (de Nemore 1565, 3v–4r) where he proved that a balance with equal weights and arms has the horizon as position of stable equilibrium, i.e. the balance recovers its horizontal position when removed from it for any reasons. This proposition has been carefully considered before and after Tartaglia, and its conclusion, in Thabit's footsteps (Capecchi 2011) that the balance returns to its horizontal position when removed (stable equilibrium) was according, to the various authors, confirmed or denied. For instance:

- Tartaglia agrees with de Nemore.
- Benedetti claims (Benedetti 1585, 148) for unstable equilibrium (balance assumes the vertical position under perturbation of the horizontal one).
- del Monte (del Monte 1615, 36) is for indifferent equilibrium (balance stays where it is left).

This last position is that accepted by modern mechanics.

The problem could not be solved empirically in the Middle Ages and the Renaissance for various reasons: the use of systematic experiments to verify a theory was not estabilished, the presence of imperfection (inequality on masses, friction) made any conclusions difficult, etc.

Tartaglia reasoning reproduces quite exactly that of de Nemore. Below an extended quotation:

> For the second part, let there be also the scale *acb* of equal arms, and at its extremities let there also be hung the two bodies *a* and *b*, simply equal in heaviness, which scale by the reasons adduced above will stand in the position of equality as appears in the figure. Now the arm *ac* having been driven down by hand or by the imposition of some weight

> on the body *a*, if we take away the hand or weight, the arm will rise again and return to its first position of equality.[176]

The proof consists in showing that, in a balance removed from its horizontal position (Fig. 3.15), the weight that is lower than *a* has a gravity of position lower than that of the weight that is higher than *b*. Consequently, as *b* prevails over *a*, the balance rotates to recover the horizontal position.

> And to assign the immediate cause of that effect, let there be described about the centre *c* the circle *aebf* for the journey that the two bodies will make in rising or falling with the arms of the scale; and draw the line of direction *ef*, and divide the arc *af* into as many equal parts as you like (say, into four parts at the three points[177] *q*, *s*, *u*; and into as many parts divide the arc *eb* at the three points *i*, *l*, *n*; and from the said three points *i*, *l*, *n* draw the three lines *no*, *lm*, and *lk* parallel to the position of equality, that is, [parallel] to the diameter or line *ab*, which [three lines] shall cut the line of direction *ef* at the three points *x*, *y*, *z*. Similarly, from the three points *q*, s, *u* are drawn the three lines *qp*, *sr*, and *ut*, also parallel to the same line *ab*, which shall cut the same line of direction *ef* at the three points *w, ρj, kd.* And now let the body *a* be depressed by hand (or by the imposition of some other weight) to the point *u*, and the other body *b* (opposite to that) will be found to be raised with contrary motion to the point *i*. Now with things arranged this way, we have come to divide the whole descent *au* made by the body *a* in descending to the point *u* into three equal descents or parts, which are *aq*, *qs*, and *su*; and similarly the whole descent *ib* which the body *b* would make in descending or returning to its original place (that is, the point *b*) will come to be divided into three equal descents or parts which are *il*, *ln*, and *nb*; and each of these three–plus–three partial descents includes one part of the line of direction; namely, the descent from *a* to *q* partakes of or contains the part *cw* of the line of direction, and the descent *qs* contains the part *wj*, and the descent *su* contains the part *jd*, and the other descent that remains to the said body *a*, that is, the descent *uf* contains the line or part *de*. Likewise the descent of the body *b* from the point i to the point *l* contains the part *xu* of the same line of direction, and in the descent from the point *l* to the point *n* it contains the part *yz*, and from the point *n* to the point *b* it contains the part *zc*, and all these parts are unequal; that is, the part *cz* is greater than *zy*, and *zy* is greater than *yx*, and *yx* than *xe*; and similarly the part *cw* is greater than the part *wj*, and *wj* than *jd*, and *jd* than *df*, and all this can be easily proved

[176] Tartaglia 1554, *Book VIII,* 89v.
[177] Tartaglia 1554, Book VIII, 89v.

geometrically; and also the part *df* can be proved equal to the part *ex*, and *jd* to *xu*, and *wj* to *yz*, and *cw* to *zc*.[178]

LIBRO

detta linea della direttione, cioe, che la parte. ɔ. ɥ. é menore della parte. y. z. Onde per le ragioni di sopra adutte, el detto corpo. b. fara elleuare il detto corpo. a. & ascendere nel ponto. q. & lui descendera nel ponto. n. nel qual ponto. n. el medesimo corpo. b. si trouara pur piu graue anchora, secondo il sito del corpo. a. perche il descenso dal. q. in. s. é piu obliquo del descenso dal ponto. n. nel ponto. b. per esser la parte. z. c. maggiore della parte. &. ɔ. E pero (per le ragioni di sopra adutte) el detto corpo. b. fara reascendere il detto corpo. a. al ponto. a. (suo primo, & condecente luoco) & lui medesimamente descendara nel ponto. b. pur suo primo, & condecente luoco, cioe nel sito della equalita, nel qual sito li detti dui corpi se trouaranno (per le ragioni adutte nella prima parte di questa) egualmente graui secondo el sito, & perche sono anchora simplicemente egualmente graui, se conseruarano nel detto luoco, come di sopra fu detto, & approuato, che é il nostro proposito.

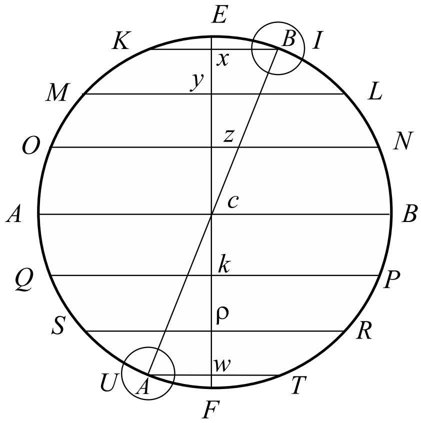
S. A. Questa é stata una bella demostratione, ma se ben me arricordo, uoi dicesti anchor sopra la detta prima question Mechanica de Aristotile, che quelle sue due conclusioni, che lui ui aduce in fine esser false. N. Eglie il uero. S. A. Per che ragione. N. La ragione di tal particolarita, ouer oppositioni se uerificaranno nella sequente propositione, mediante alcuni correlarij, che dalle cose dette, & dimostrate nella precedente si manifestano, delli quali il primo é questo.

CORRELARIO.

DAlle cose detto, et dimostrate di sopra, se manifesta qualmente un corpo graue in qual si uoglia parte, che lui se parta, ouer remoui dal sito della equalita lui si fa piu leue, ouer leggiero secondo el sito, ouer luoco, & tanto piu, quãto piu sara remosso da tal sito, essempi gratia. El corpo. a. si trouara esser piu leue nel ponto. u. chi nel põto. s. et nel põto. s. piu che nel põto. q. & nel ponto. q. che nel ponto. a. sito della equalita, p causa della uarieta di descensi, cioe, che luno é piu obliquo dell'altro, cioe el descenso. u f. uiẽ à esser piu obliquo del descẽso. s. u. perche la parte. f. ɥ. della direttione, é menore della. ɥ. ɔ. et cosi el descẽso. s. u. uiẽ à esser piu obliquo del descẽso. q. s. pche la parte. ɥ. ɔ. é menore della parte. ɔ. &. & lo descẽso. q. s. uiẽ à esser piu obliquo del descenso. a. q. perche la parte. ɔ. &. é menore della parte. &. c. & per le medesime ragioni si manifesta del corpo. b. cioe, che quello sara piu leue nel põto. i. che nel põto. l. & nel põto. l. che nel ponto. n. & nel ponto. n. che nel ponto. b. sito della equalita.

CORRELARIO SECONDO.

ANchora per le cose dette, & dimostrate se manifesta, che remuouendosi li detti dui corpi dal detto sito della equalita, cioe luno ĩ giuso, et laltro in suso, anchor



[Proposition V]. Whenever a scale of equal arms is in the position of equality, and at the end of each arm are hung weights simply equal in heaviness, the scale […] departs from the said position of equality, then […] the scale necessarily returns to the position of equality.[179]

Fig. 3.15 Plate related to an application of the *gravitas secundum situm* concept to the balance with equal weights and arms[180]

Now to resume our proposition, I say that the body *b* standing at the point *i* comes to be positionally heavier than the body *a* standing at the point *u* (as appears in the figure), because the descent of the body *b* from the point *i* to the point *l* is more direct than the descent of the body *a* from the point *e* to the point *f* (by the second part of the fourth petition), because it partakes more of the line of direction. That is, the body *b* in descending from the point *i* to the point *l* partakes the part *xy* of the line of direction, and the body *a* descending from the point *u* to the point *f* partakes the part *df* of the line of direction, and since the part *xy* is greater than the line or part *de*, the descent (by definition 17) from the point *u* to the point *f* will be more oblique than that from the point *i* to the point *l*. Whence (by the second part of the fourth petition) the

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[178] Tartaglia 1554, *Book VIII,* 89v.
[179] Tartaglia 1554, *Book VIII,* Q XXXII, Proposition V, 89v.
[180] Tartaglia 1554, *Book VIII*, 90v.

> body *b* in that position will be positionally heavier than the body *a*. And being thus heavier, when the imposed weight or hand is taken away from the body *a*, it will (by the converse of the fifth petition) make the said body *a* re–ascend with contrary motion from the point *u* to the point *s*, and it will descend from the point *i* to the point *l*; and it will come to be found still positionally heavier than the body *a*, because the said body *a* standing at the point *s* will have the descent *su* more oblique than the descent *ln* of the body *b* because it partakes less of the line of direction; that is, the part *ρw* is smaller than the part *yz*. Whence for the reasons adduced above, the body *b* will raise the body *a* to the point *q*, and *b* will descend to the point *n*, at which point *n* the same body *b* will yet be found appositionally heavier than the body *a* because the descent from *q* to *s* is more oblique than the descent from the point *n* to the point *b*, the part *zc* being greater than the part *k*ρ. And hence (by the reasons adduced above) the body *b* will make the body *a* re–ascend to the point *a* (its first and proper place) and will itself descend to the point *b* (also its first and proper place), that is, to the position of equality, in which position the said two bodies will be found (for the reasons adduced in the first part of this proposition) equally heavy positionally. And since they are also simply equally heavy, they will remain in the said place, as was said and proved above; which is our purpose. [181]

In order to evaluate the gravity of position of the two bodies Tartaglia assumes virtual rotations of the balance from a tilted position, for instance *iu* (See Fig. 3.15) that makes the weight at the ends of the balance arms to descend. In a first clockwise virtual rotation body *b* moves from position *i* to position L; in the vertical direction the body move from *x* to *y*. In a second anti–clockwise virtual rotation body *a* moves from *u* to *f*, in the vertical direction from *w* to *f*, *a* simple geometrical arguments shows that *xy* is greater than *wf* if the arc *il* and *uf* are assumed to be of equal length. This means that *il* partakes more of the vertical than *uf*, consequently gravity of position of *b* is greater than that of *a* and the balance is pressed to rotate clockwise, for example up to *ls*. Repeating the reasoning it can be proved that also in this position the gravity of position of *b* is greater than that of *a* and the balance continues to rotate until it reaches the horizontal position.

De Nemore in his *Quaestio* II (de Nemore 1565, 3v–4r) proved that though the gravity of position of the weight *a* in the lower position is lower than that of the weight *b* in the higher position, this difference is as small as you like and any finite weight added to *a* will cause the balance to assume the vertical position.

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[181] Tartaglia 1554, *Book VIII*, 90rv.

Tartaglia carried out the same argumentation but in a separate proposition (*Proposition* VI) which asserts that a balance with equal arms and different weights will tilt of the side of greater weight to reach the vertical position.

Mendoza argues that this proposition has been proved false for the previous proposition; that is for a balance with equal arms it is possible to achieve equilibrium with different weights:

> [Proposition VI] S.A. If this is true, it is to be believed, or rather thought certain, that, if one should impose on the body *a*, pressed down, another little body which equated in heaviness that difference by which the upper body exceeded positionally the heaviness of the lower, then each would remain in the place where it was.[182]

Tartaglia replies by showing that if it is true that the gravity of position of *b* is smaller than that of *a* (being *a* and *b* equal), then the difference is as small as you like. The proof is carried out, as in de Nemore by showing that the angle that the path *a* and *b* makes with the vertical differs by a quantity as small as you like.

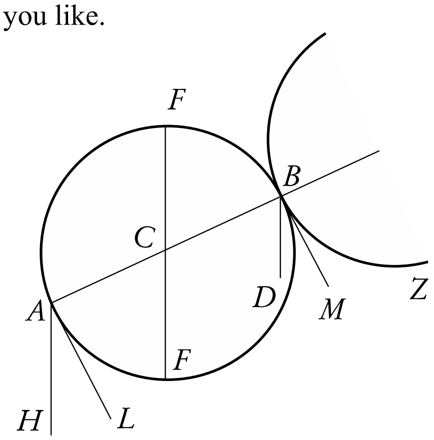


Fig. 3.16 Comparison of contingency angles[183]

The path of *a* and *b* is represented in Fig. 3.16 by the arcs of the circle, respectively *af* and *bf*. They form with the vertical lines from *a* and *b* the angles *haf* and *dbf*, which with a nomenclature of the time are known as mixed–angles. The two mixed angles differ by a quantity as small as you like; then the obliquities and the gravities of position of *a* and *b* differ by a quantity as small as you like. Consequently if a weight as small as you like

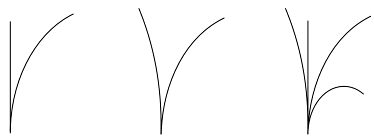
[182] Tartaglia 1554, *Book VIII*, Q XXXIII, Proposition VI, 91r.

[183] Redrawn from Tartaglia 1554, *Book VIII*, Q XXXIII, Proposition VI, 92v.

but of finite value *p* is added to *a,* the gravity of position of *a* + *p* will be greater than that of *b*. For example:

> [Q XXXIII, Proposition VI]. N. Let there be, for example, the same scale *abc* of the preceding proposition, at the ends of which are hung the bodies *a* and *b*, equal in simple heaviness; and let the hand depress the body *a* and lift the body *b* as shown in the next figure. I say that in this position the body *b* is positionally more ponderous or heavy than the body *a*, and that the difference between the heaviness of these two bodies is impossible to give or find between two unequal quantities. And to demonstrate this proposition I draw two straight lines, *ah* and *bd*, perpendicularly to the centre of the world, and I also draw two lines *al* and *bm* tangent to the circle described by the arms of the scale at the points *a* and *b*. I describe also a part of the circumference of a circle touching the same circle *acb* at the point *b*, this being a similar and equal circle, *bz*, such that the arc *bz* is similar and equal to the arc *af* and similarly placed (that is, in position), and the line *bm* which touches or is tangent to this. since the obliquity of the arc *af* (by what was said about the third petition) is measured by means of the angle contained by the perpendicular *ah* and the circumference *af* at the point *a*, and the obliquity of the arc *bf* is measured by the angle contained by the perpendicular *bd* and the circumference *bf* at the point *b*, the body *b* in that position will be as much heavier than the body *a* as the said angle (contained by the perpendicular *bd* and the circumference *bf* at the point *b*) will be less than the angle contained by the perpendicular *ah* and the circumference *af* at the point *a*. And since the angle *haf* is precisely equal to the angle *dbz*, and the said angle *dbz* is as much greater than the angle contained by the said perpendicular *bd* and the circumference *bf* at the point *b* as the angle of contact of the two circles *bz* and *bf* at the point *b*, and since this angle of contingency[184] is more acute than any of the acute angles made by straight lines (as proved by Euclid III.16), then the difference or ratio between the angle *haf* and the angle contained by the perpendicular *bd* and the circumference *bf* at the point *b* is less than any difference or ratio you please which can occur between any large and small quantities. And thus (by the third petition)

[184] The angle of contingency is the angle formed between two curve lines or a curve and straight line in the point where they are tangent to each other. The figure below show different instances of the angle of contingency, between straight lines and curves or between curves.



> the difference of the obliquity of the descent *af* and the descent *bf*, and consequently the difference of positional heaviness of the two bodies *a* and *b*, is less than any you wish between two unequal quantities. therefore any small corporeal quantity that is added, the body *a* will necessarily be heavier in any position than the body *b*, and hence it will not cease to descend continuously as far as the line of direction, that is, to the point *f*; and thus it will continue to raise the body *b* as far as the line of direction, that is, to the point *e*.[185]

At this point Tartaglia and Mendoza take again the discussion of *Book VII* about the difference between mathematical and physical argumentations, to conclude that from a mathematical point of view Aristotle's assertion that large balance are more sensible that the smaller one is simply a nonsense because any balance, whichever is dimension is, will tilt to the vertical position for whichever small weight – a grain of poppy seed – added.

Notice that Tartaglia's reasoning is almost the same as that of de Nemore in the *Liber de ratione ponderis*, but for a modern reader it is perhaps clearer. Not so much for the things that are written in *Proposition* VI, but for those that are not written in *Proposition* V.

Further, when, in *Proposition* V (Tartaglia 1554, *Book VIII,* 89v–91r) Tartaglia considers the circumference of the Fig. 3.15 he merely said that it was divided into arcs of equal length and not also into arcs as small as you like. Therefore, there is no chance of guessing a passage to the limit. To develop his argument Tartaglia just needs the argument that an angle of contingency is always larger than an arbitrary acute angle (Tartaglia 2007, 59r). The measure of the angles of contingency was discussed at length by the pioneers of *Calculus*, among them Gottfried Wilhelm von Leibniz (1646–1716) and Leonhard Euler (1707–1783). The paradox of these angles resided in the fact that, comparing them with angles between straight lines (ordinary angles) they should all be considered equal to each other and zero; while they could be considered different if compared with each other, as it appears intuitive if the angle is interpreted as an extension. It is the same paradox that occurs when the infinitesimals of mathematical analysis are compared with real numbers, in which case they are treated as zeros, while it is possible to establish a hierarchy when comparing among them: infinitesimals of first order, second order, third order, etc.

[185] Tartaglia 1554, *Book VIII*, 91v–92r.

3.2.4.3 Proposition VII: Balance with Equal Weights and Different Arms

Proposition VII, for which a balance with equal weight and different arms tilts on the side of longer arms has no interest in itself. It is however important to understand the role that mathematics plays in mechanics in the Middle Ages: physics is subordinate to mathematics in mechanics; physic explains the *how*, mathematics the *why*. To Mendoza who asserts that proposition VII results from physical argument, Tartaglia replies that he wants to assign the cause of the effect. And the cause is given by the greater gravity of position of the weight hanging from the longer arm; i.e. a mathematical reason.

> QUESTION. XXXIIII. PROPOSITION VII.
> N. Let there be the rod or scale *acb*, with the arm *ac* longer than *cb*. I say that if bodies simply equal in heaviness were hung at the two points *a* and *b*, the scale will tilt on the side of *a*. Because when the perpendicular *cfg* (that is, the line of direction) is drawn, and the two quarter circles, which shall be *ag* and *bf*, are traced on the centre *c*, and when two tangent lines *ae* and *bd* are drawn from the points *a* and *b*, it is manifest that the angle of tangency *eag* is less than the angle *dbf*. Hence the descent made along *ag* is less oblique than the descent made along *bf*. Therefore (by the third petition) the body *a* will be heavier than the body *b* in this position; which is the purpose.[186]

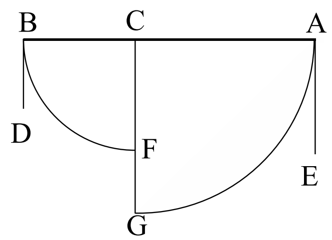


Fig. 3.17 Balance with equal weights and different arms[187]

In effect, physics seems to be subordinate to mathematics in mechanical sciences. Physics collects and explains the phenomena (*how*), mathematics interprets them and gives a result (*why*). In fact, from the previous passage we can read that when Mendoza asserts that *Proposition* VII (Tartaglia 1554, *Book VIII,* 92v) results from physical argument, Tartaglia replies that he wants to assign the cause of the effect. The cause is given by the greater gravity of position of the weight hanging from the longer arm; i.e., a mathematical interpretation.

[186] Tartaglia 1554, *Book VIII,* Q XXXIIII, Proposition VII, 92v–93r.
[187] Redrawn from Tartaglia 1554, *Book VIII,* 92v.

3.2.4.4 Propositions VIII: Law of Lever

With the use of *Proposition* IIII (Tartaglia 1554, *Book VIII*, 89r) the demonstration of the law of lever should be immediate, it would suffice to argue that the two weights hanging from arms of lengths inversely proportional to them are equal in gravity of position and therefore balanced. Tartaglia, however, prefers instead of the equilibrium of opposing tendencies to consider the equivalence of weights that tend to move in the same direction.

> QUESTION. XXXV. PROPOSITION VIII.
> If the arms of the balance are proportional to the weights imposed on them, in such a way that the heavier weight is on the shorter arm, then those bodies or weights will be equally heavy according to such position or site. Let as before the bar or balance *acb* and the weights *a* and *b* hung thereon, and let the ratio of *b* to a be as that of the arm *ac* to the arm *bc*. I say that this balance will not tilt to either side. And if (for the adversary) it is possible for it to tilt, let us assume it to tilt on the side of *b* and to descend obliquely as the line *dce* in place of *acb,* and [let us] take *d* as *a* and *e* as *b*; and the line *df* descends perpendicularly, and the line *eh* rises similarly. Now it is manifest (by Euclid I.16 and I.29) that the two triangles *dfc* and *ehc* have equal angles. Whence (by Euclid VI.4) they will be similar, and consequently will have proportional sides. Therefore the ratio of *dc* to *ce* is as that of *df* to *eh*; and since the weight *b* is to the weight *a* as *dc* is to *ce* (by our assumption), the ratio of *df* to *eh* will be as the weight *b* to the weight *a*. Hence, if we take from cd the part *cl*, equal to *cb* or *ce*, and consider *l* equal in heaviness to *b* and descending along the perpendicular *lm*, then, since it is manifest that *lm* and *eh* are equal, the proportion of *df* to *lm* will be as the simple heaviness of the body *b* to the simple heaviness of the body *a*, or as the simple heaviness of the body *l* to the simple heaviness of the body *d*, because the two bodies are supposed to be the same, and similarly the bodies *b* and *l* (the heaviness of the body l having been assumed equal to that of the body *b*).[188]

[188] Tartaglia 1554, *Book VIII,* 93r.

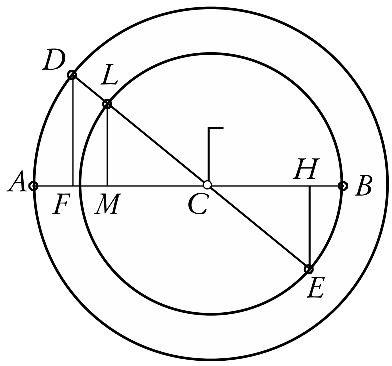


Fig. 3.18 Equilibrium of the lever with different arms by Tartaglia[189]

> Hence I say that the ratio of all *dc* to *lc* will be as the heaviness of the body *l* to that of the body *d*. whence if the said two heavy bodies, that is, *d* and *l* were simply equal in heaviness, standing then in the same positions or places at which they are presently assumed to be, the body *d* would be positionally heavier than the body *l* (by the fourth proposition) in that ratio which holds between the whole arm *dc* and the arm *lc*. And since the body *l* is simply heavier than the body *d* (by our assumption) in the same ratio as that of the arm *dc* to the arm *lc*, then the said two bodies *d* and *l* in position of equality would come to be equally heavy, because by as much as the body *d* is positionally heavier than the body *l*, by so much is the body *l* simply heavier than the body *d*; and therefore in the position of equality they come to be equally heavy. Hence that power or heaviness that will be sufficient to lift the body *a* from the position of equality to the point at which it is at present (that is, to the point *d*) will be sufficient to lift the body *l* from the same position of equality to the place where it is at present. Therefore if the body *b* (for the adversary) is able to lift the body *a* from the position of equality to the point *d,* the same body *b* would also be able and sufficient to lift the body *l* from the same position of equality to the point where it is at present, which consequence is false and contrary to the fifth proposition […]. Thus, the adversary's position destroyed, the thesis stands.[190]

Therefore in *Proposition* VIII, Tartaglia confronts the lever with weights *e* and *d* to the lever in which the weights are *d* and $l = e$, on the same side (See Fig. 3.18). Through his *Proposition* IV he argues that they are equally heavy for position and D may be replaced by *l* arriving at a balance with

[189] Redrawn from Tartaglia 1554, *Book VIII,* 93v.
[190] Tartaglia 1554, *Book VIII*, 93rv.

equal arms ($lc = ec$) and equal weights, and as such, in equilibrium for *Proposition* V (not commented here). Note that Tartaglia like Thābit and de Nemore does not refer to the symmetry. At the end of his *Proposition* VIII Tartaglia refers to the demonstration of Archimedes (Medonza speaks of that as well), stating that since the matter of his treatise is quite different from the Archimedean, he has considered demonstrating the law of lever with other principles as more appropriate. In his words:

> S.A. This is a very pretty proposition, but it seems to me (if I recall correctly) that Archimedes[191] of Syracuse has a similar one, and I believe he does not prove it in this way of yours. N. Your Excellency is right. Indeed, of this proposition he makes two, and these are the fourth and fifth in that book of his wherein he deals with the centres of gravity of heavy bodies.[192]

3.2.4.5 Propositions IX–XIII: Balance with distributed Weights

Propositions IX–XIII (Tartaglia 1554, *Book VIII*, 93v–96v) are essentially of practical nature and mostly take up again de Nemore's considerations. There are however some interesting new statements of Tartaglia's that are worthy of being commented. The object of the propositions is a balance with distributed weight. *Proposition* IX concerns the situations shown in Fig. 3.19.

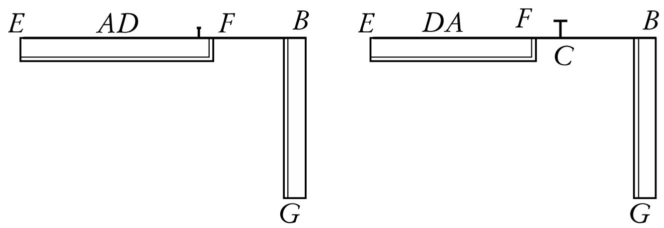


Fig. 3.19 Balances with distributed weigh by Tartaglia[193]

Bodies *AD* or *DA* are such that their centre of gravity is as far as that of body *BG* from the fulcrum *C*; the weights of *AD* and *BG* are equal. The proposition says that this assembly is in equilibrium. Tartaglia proves this proposition in two ways. The first way is in the Archimedean tradition and is the same adopted as that by de Nemore; it is based on the observation

[191] Archimedes' work by Tartgalia was already edited (Tartaglia 1543).
[192] Tartaglia 1554, *Book VIII*, Q XXXV, Proposition VIII, 93v.
[193] Redrawn from Tartaglia 1554, *Book VIII*, Q XXXVI, Proposition IX, 94r.

that the body *ad* is equivalent to a weight equally heavy applied in its centre of gravity. As the centres of gravity of *ad* and *bc* are equally far from the fulcrum *c*, the proposition is proved.

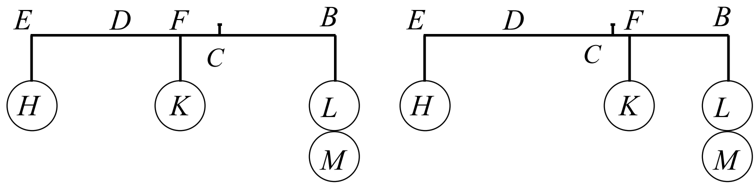


Fig. 3.20 The discrete model of the balance of the previous figure[194]

The second way is Tartaglia's; it uses the result of *Proposition* IV, implicitly assuming additive properties for the gravity of position of heavy bodies located on the same side of the balance. Before carrying over any considerations, Tartaglia changes the system of Fig. 3.19 with that equivalent to Fig. 3.20. As all bodies are equal, their gravity of position is represented by their distance from the fulcrum. So the gravity of position for *h* and *k* are respectively represented (are proportional to) by *ec* + *fc* while the gravity of *l* and *m* are represented by 2*cb*. As for construction *ec* + *fc* = 2*b*, equilibrium is assured. In his words:

> [From Q XXXVI, Proposition IX]
> This proposition can also be demonstrated in another way (which is more suitable because it depends on its own principles rather than imported ones). It is manifest that, when two simply equal bodies, *h* and *k*, are suspended, the one at the point *e* and the other at the point *f*, and two others which shall be *l* and *m*, equal to them, are hung at the point *b*, these weights, I say, will weigh equally at those points, because the ratio of the weight *l* to the weight *k* is as that of the arm *bc* to the arm *fc* (by the fourth proposition); for the body *l* will be positionally as heavy at the point *d* as where it is at present, that is, at the point *b* (since *cd* is equal to *cb* by assumption). Therefore, by the said proposition, this ratio will be that of the positional heaviness of the body *l* to the body *k*, which will be that of the arm *dc* or *bc* to *cf*; and for the same reasons this ratio will be that of the heaviness of the body *m* to the heaviness of the body *h* positionally, that is the ratio of the same arm *cd* or *bc* to the arm *ce*. Therefore the positional heaviness of both the bodies *l* and *m*, together, to the positional heaviness of the other two bodies *h* and *k*, together, will be as the double of the arm *cd* or *bc* to the two arms *ce*

[194] Redrawn from Tartaglia 1554, Book VIII, Q XXXVI, Proposition IX, 94v.

and *cf* together. And since the said two arms *ce* and *cf*, together, are precisely as much as the double of the said arm *cd* or *bc*, it follows also that the heaviness of the said two bodies *l* and *m* is equal to the positional heaviness of the two bodies *h* and *k*; which is the purpose.[195]

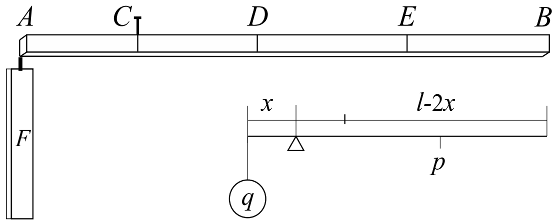


Fig. 3.21 Balance with a beam uniformly heavy[196]

Proposition X (Tartaglia 1554, *Book VIII*, 94v–95r) says that for the situation of Fig. 3.21 of a uniformly heavy rod *ad* suspended from the fulcrum *c* with a weight *f* hanging from *a* if there is equilibrium the proportion holds in modern notation:

$$l : 2x = q : p$$

where *q* is the weight of *f*, *l* the length of AB, *x* = AD and *p* the weight of the part of the rod with length *l*-2*x*. The proposition is proved following Archimedean arguments.

Proposition XI (Tartaglia 1554, *Book VIII*, 95rv) is the converse, i.e., if the previous relation is satisfied then equilibrium follows. The proof is very simple and carried out with reduction to the absurd.

Proposition XII (Tartaglia 1554, *Book VIII*, 95v–96r) is not a theorem but rather a problem. The purpose is to evaluate the weight *f* so that the balance of Fig. 3.21 will be in equilibrium, all other parameters being assigned. The problem is solved by applying the rule of three to proportion 3.1.

Proposition XIII (Tartaglia 1554, *Book VIII*, 96rv) is still in the form of a problem. The purpose is to evaluate the position of the fulcrum for equilibrium. The problem is similar to the others.

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[195] Tartaglia 1554, *Book VIII*, Q XXXVI, Proposition IX, 94r.

[196] Redrawn from Tartaglia 1554, *Book VIII*, Q XXXVII, Proposition X, 95r (above), our modelling (bottom).

> [QUESTION. XL. PROPOSITION XIII]
> [...]. N. To illustrate this problem, let us assume that there is a rod, beam, or staff as the above, 10 feet long and weighing 40 pounds (as in the preceding was assumed). And let us assume also that there is a body weighing 80 pounds. I say that it is possible to determine the place at which the rod may be divided so that the said heavy body, when hung at the end of its shorter part, will make the rod stay parallel to the horizon. And any such problem may be solved by ratios; nevertheless, it may be more easily solved by Algebra, the shorter part of the rod being a matter of feet, a co, whence the longer part is 10 minus co. I double the shorter part (that is one co), which gives 2 co, and subtract these two co from the whole length of 10 feet. There remains 10 minus 2 co, and this will be the difference between the longer part and the shorter. To find the weight of this difference, I multiply it by 4 (because, the whole rod weighing 40 pounds, each foot comes to weigh 4 pounds). Multiplying by 4, as I said, the result is 40 minus 8 co. And since the ratio of the whole rod (which is 10 feet) to the double of its shorter part (which double is 2 co) is as the weight of our heavy body (which is 80 pounds) to the weight of the above difference, which is 40 minus 8 co. Hence by Euclid VII.20 the product of the first [term], or 10 feet, into the fourth, which is 40 minus 8 co (which would be 400 minus 80 co), will equal the product of the third, which is 80 pounds, into the second, which is 2 co (which will be 160 co). Thus we will have 160 co equal to 400 minus 80 co; and restoring the parts by rule we shall find the co to be 1+2/3.[197] Hence 1+2/3 feet will be the shorter part of the said rod or beam, whence the longer will be 8+1/3 feet; which was our problem.[198]

Tartaglia however solves it by using the mathematics of abacus, introducing for the first time in book VIII the use of algebra. This is a quite important subject because for a long time the use of algebra will be substantially proscribed in the name of the purity of Greek geometry. Therefore, Tartaglia represented a sort of cultural bridge between algebra and algebra used in mechanics (at that time).[199]

As typical of the *Abacus school* (Pisano and Bussotti 2014a, 2013b) the problem is solved by means of an example. A rod 10 feet long and weighing 40 pounds, with a weight f of 80 pounds assigned. The quantity to be searched, i.e. the unknown, is the distance from f to c, which

[197] By indicating co with x, the equation Tartaglia is solving is: $160\ x = 400\text{-}80\ x$, which gives $x = 5/3= 1+2/3$.

[198] Tartaglia 1554, *Book VIII*, 96v.

[199] Tartaglia had already used algebra – a second degree equation in the *Nova scientia* (Tartaglia 1537, *Book II*, *Proposition IX*).

following the use of time is named *cosa*,[200] shortened as *co*. The weight of the part of length *l*-2*x* (See Fig. 3.21, bottom one) is:

$$\left(10-2co\right)\times\frac{40}{10}=40-8co$$

Use of the previous proportion gives:

$$10:2co=80:40-8co$$

which according with Euclid VII 20[201] gives:

$$400-80co=160co$$

The equation in *co* has the solution

$$co=400:240=1+\frac{2}{3}$$

3.2.4.6 Propositions XIIII–XV: Law of Inclined Plane

Proposition XIIII (Tartaglia 1554, *Book VIII*, 96v–97r) asserts that the gravity of position does not change if a body moves on an inclined plane. To this proposition, already proposed by de Nemore (de Nemore 1565, *Quaestio* X, 7r), is usually assigned two functions. On the one hand it says that we are considering lines of descent of heavy bodies as parallel to each other. Indeed only in this case will the inclined plane and the lines of descent conserve the same angle, i.e., the same obliquity. On the other hand it asserts that the gravity of position, which is constant along the plane, is determined by the ratio between the length of the plane and the height. Tartaglia does not however make a step that would seem natural, to explicitly state that the gravity of position is inversely proportional to the obliquity. The lack of this step is critical because in the proof of the law of the inclined plane, Tartaglia actually uses that assumption.

[200] The word *thing* (*cos*) to indicate an unknown dates back at least to al-Khwārizmī (Høyrup 1989, 78). Next (ca. 1489) Germany symbols appears as "+" and "–", "*p" (plus)* and "*m" (minus)*. Finally the term "Coss" for "Incognita" (*Arte Cossica*). Adam Riese (1492–1559) wrote his *Die Coss* (1524).

[201] Tartaglia 1543, 144. If one has three proportional numbers, the product of the first by the last will be equal to the product of the second by the third. "Se seranno quattro numeri proportionali quello che vien produtto dal primo in l'ultimo serà equale a quello che vien produtto del second in el terzo."

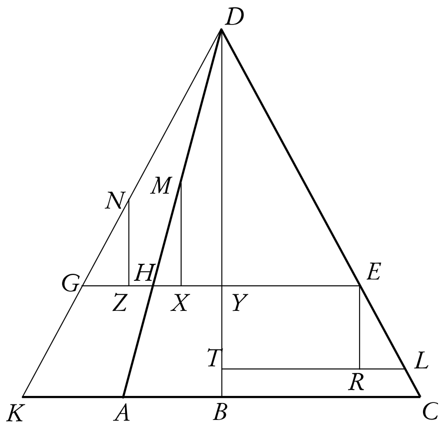

Fig. 3.24 Equilibrium on the inclined plane[202]

The proof of the law of the inclined plane is introduced in proposition XV.

> [QUESTION XLII. PROPOSITION XV]
> If two heavy bodies descend by paths of different obliquities, and if the proportions of inclinations of the two paths and of the weights of the two bodies be the same, taken in the same order, the power of both the said bodies in descending will also be the same. Then let the letter *e* [See Figs. 3.23–3.24] represent a heavy body placed on the line *dc*, and the letter *H* another on the line *da*, and let the ratio of the simple heaviness of the body *e* to that of the body *h* be the ratio of *dc* to *da* I say that the two heavy bodies in those places are of the same power or force. And to demonstrate this, I draw *dk* of the same tilt as dc, and I imagine on that a heavy body, equal to the body e, which I letter g, in a straight line with *eh*, that is, parallel to ck. [...] Also the ratio of *mx* to *nz* will be as that of *dk* to *da*; and (by hypothesis) that is the same as that of the weight of the body *g* to the weight of the body *h*, because *g* is supposed to be simply equal in heaviness with the body *e*. Therefore, by however much the body *g* is simply heavier than the body *h*, by so much does the body *h* become heavier by positional force than the said body *g*, and thus they come to be equal in force or power. And since that same force or power that will be able to make one of the two bodies ascend (that is, to draw it up) will be able or sufficient to make the other ascend also, [then], if (for the adversary) the body *e* is able and sufficient to make the body *h* ascend to *m*, the same body *e* would be sufficient to make ascend also the body *g* equal to it, and equal in inclination. Which is impossible by the preceding proposition.

[202] Redrawn from Tartaglia 1554, *Book VIII,* Q XLII, Proposition XV, 97v.

> Therefore the body *e* will not be of greater force than the body *h* in such place or position; which is the proposition.[203]

The proof is developed as in the case of the lever, bringing the equilibrium to an equivalence of weights located in the same side with respect to the vertical. Nevertheless, the reasoning is less strict, because it asserts without explanation that two heavy bodies *h* and *g* located on planes, *da* and *dk*, with different slope are equally heavy for position when they have weights inversely proportional to their inclinations. In effect, Tartaglia is authorized to affirm that the gravity of position is related to the obliquity, we can also concede he is authorized to say it is inversely proportional to it, but in no place he has justified that the obliquity should be measured by the ratio of the height and the length of the inclined plane, as assumed in proposition XV (it could also and coherently be measured by the ratio between the horizontal projection of the plane and its length). Raffaello Caverni, who seems however to not know the *Liber de ratione ponderis*, considers improperly Tartaglia's demonstration as the first truly exemplary proof, of higher value than that of Jordanus de Nemore (Caverni 1891–1900, IV, 321–232). Appreciation for Tartaglia's proof is found also in Arnaldo Masotti (Tartaglia 1953, XXXV).

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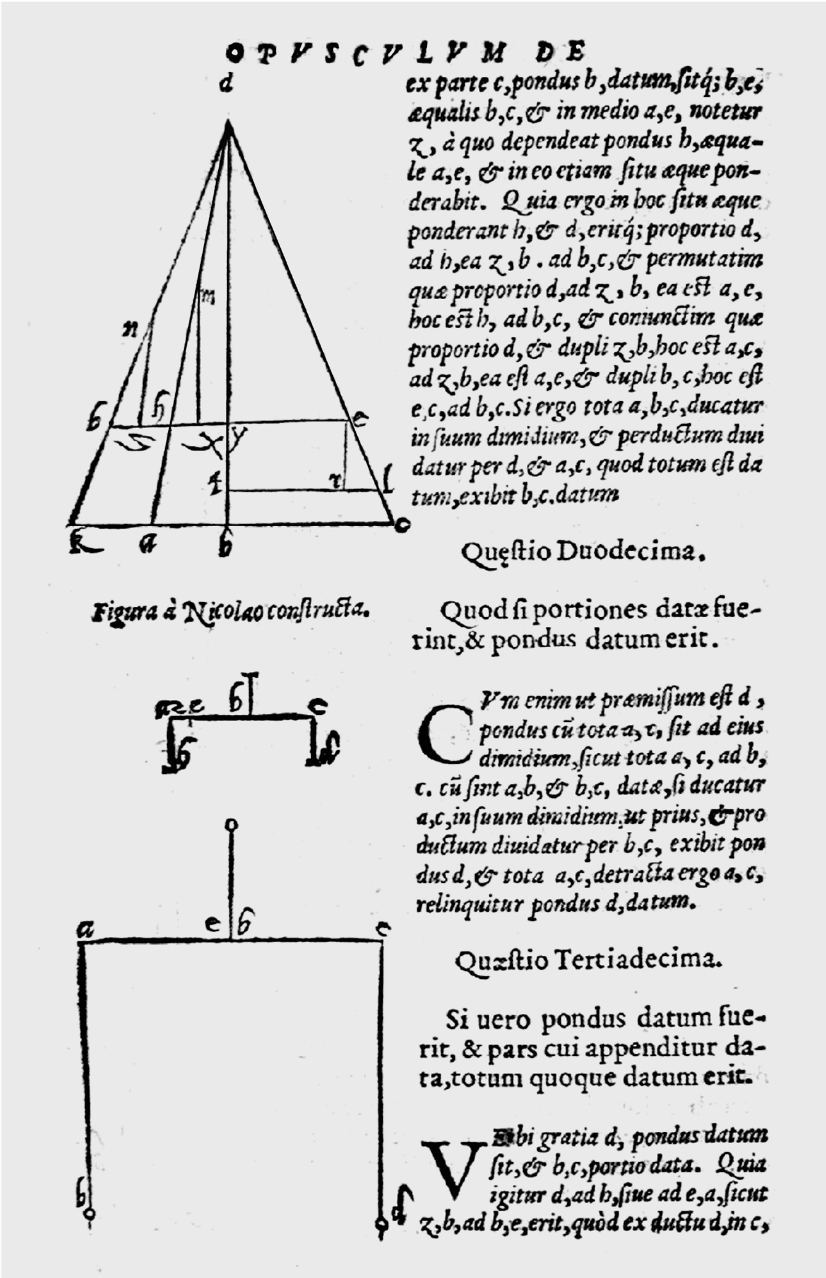
[203] Tartaglia 1554, *Book VIII*, Q XLII, Proposition XV, 97rv.



OPVSCVLVM DE

ex parte c, pondus b, datum, sitq; b, e, æqualis b, c, & in medio a, e, notetur z, à quo dependeat pondus b, æquale a, e, & in eo etiam situ æque ponderabit. Quia ergo in hoc situ æque ponderant b, & d, eritq; proportio d, ad b, ea z, b. ad b, c, & permutatim quæ proportio d, ad z, b, ea est a, e, hoc est b, ad b, c, & coniunctim quæ proportio d, & dupli z, b, hoc est a, c, ad z, b, ea est a, e, & dupli b, c, hoc est e, c, ad b, c. Si ergo tota a, b, c, ducatur in suum dimidium, & perductum diuidatur per d, & a, c, quod totum est datum, exibit b, c. datum

Figura à Nicolao constructa.

Quęstio Duodecima.

Quod si portiones datæ fuerint, & pondus datum erit.

CVm enim ut præmissum est d, pondus cũ tota a, c, sit ad eius dimidium, sicut tota a, c, ad b, c. cũ sint a, b, & b, c, datæ, si ducatur a, c, in suum dimidium, ut prius, & productum diuidatur per b, c, exibit pondus d, & tota a, c, detracta ergo a, c, relinquitur pondus d, datum.

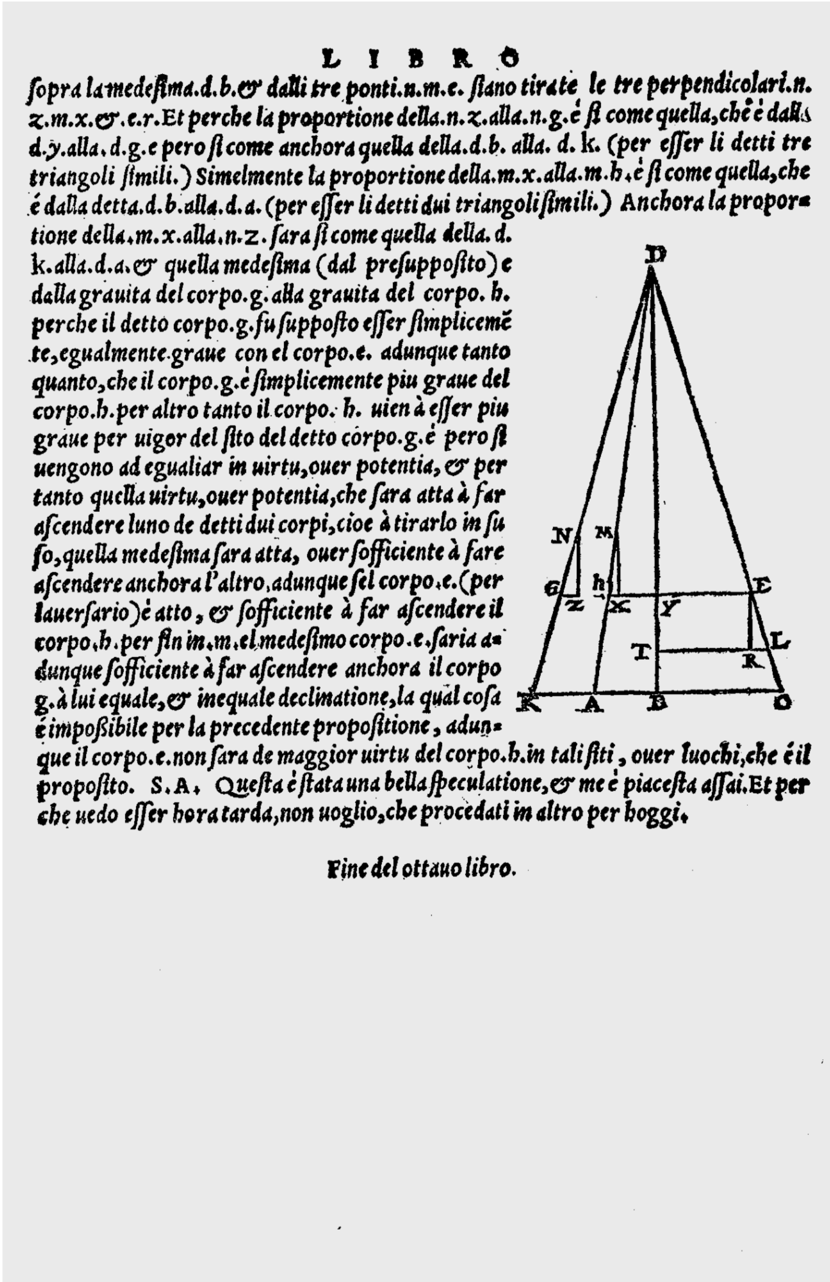
Quæstio Tertiadecima.

Si uero pondus datum fuerit, & pars cui appenditur data, totum quoque datum erit.

VErbi gratia d, pondus datum sit, & b, c, portio data. Quia igitur d, ad b, siue ad e, a, sicut z, b, ad b, e, erit, quòd ex ductu d, in c,

Fig. 3.22 Plate from the proof of inclined plane law by de Nemore[204]

[204] de Nemore 1565, *Quaestio* XI, 7v.



LIBRO

sopra la medesima.d.b.& dalli tre ponti.n.m.e. siano tirate le tre perpendicolari.n.z.m.x.&.e.r.Et perche la proportione della.n.z.alla.n.g.é si come quella,che é dalla d.y.alla.d.g.e pero si come anchora quella della.d.b. alla. d. k. (per esser li detti tre triangoli simili.) Simelmente la proportione della.m.x.alla.m.h.é si come quella,che é dalla detta.d.b.alla.d.a.(per esser li detti dui triangoli simili.) Anchora la proportione della.m.x.alla.n.z.sara si come quella della.d.k.alla.d.a.& quella medesima (dal presupposito) e dalla grauita del corpo.g.alla grauita del corpo.h. perche il detto corpo.g.fu supposto esser simplicemẽte,egualmente graue con el corpo.e. adunque tanto quanto,che il corpo.g.é simplicemente piu graue del corpo.h.per altro tanto il corpo.h. uien à esser piu graue per uigor del sito del detto corpo.g.é pero si uengono ad egualiar in uirtu,ouer potentia,& per tanto quella uirtu,ouer potentia,che sara atta à far ascendere luno de detti dui corpi,cioe à tirarlo in suso,quella medesima sara atta, ouer sofficiente à fare ascendere anchora l'altro,adunque sel corpo.e.(per lauersario)é atto,& sofficiente à far ascendere il corpo.h.per fin in.m.el medesimo corpo.e.saria adunque sofficiente à far ascendere anchora il corpo g.à lui equale,& inequale declinatione,la qual cosa é impoßibile per la precedente propositione, adunque il corpo.e.non sara de maggior uirtu del corpo.h.in tali siti, ouer luochi,che é il proposito. S.A. Questa è stata una bella speculatione,& me è piacesta assai.Et perche uedo esser hora tarda,non uoglio,che procedati in altro per hoggi.

Fine del ottauo libro.

Fig. 3.23 Plate from the proof of inclined plane law by Tartaglia[205]

[205] Tartaglia 1554, *Book VIII*, 97v.

3.2.5 *Book VIII* of *Quesiti* dates back to *Liber de ratione ponderis–Iordani opuscolvum*

Based on previous historical and epistemological analyses of the *Books VII* and *VIII*, we note that de Nemore and Tartaglia's reasoning around *gravitas secundum situm* are quite similar and quite different at the same time, but for sure based on the fundamental assumptions as reported in the following recapitulative Table 3.10:

Table 3.10 The fundamental assumptions assumed by de Nemore and by Tartaglia around *gravitas secundum situm* conceptualization

Basic Concepts	*Liber de ratione ponderis & Iordani opvscvlvm de ponderositate*	*Quesiti et invenzioni diverse*
Inspired by *Problemata mechanica.*	Yes	Yes
Intuitive idea of force within physical system subjected to vincula.	Yes	Yes
Principle of virtual laws.	Yes	Yes
Euclidean geometry.	Yes	Yes
Ad absurdum proofs.	Yes	Yes
Aristotelian–Dynamical conception of centre of gravity.	Yes	Yes (partially)
Main reasoning.	(Tartaglia 1565, 3v–5rv)	(Tartaglia [1546] 1554, *Book VIII*, 89v–90v

See also Appendix below.

PART III

Translations & Transcriptions

Chapter 4

Translation and Transcription

In this first section I provide some information and background on a selection of writings by and about Niccolò Tartaglio. I have focused on translations into English and Italian, as well as Latin transcriptions of Books VII and VIII of the *Quesiti*, and *Iordani opusolum* by de Nemore. Further facsimile texts are added for readers and my critical comments can be found as endnotes to the chapter.

4 General Considerations

For English translations I assumed as a model that of Drake (Drake and Drabkin 1969). The language is however adjusted in many places and portion neglected there have been translated here, as well.

For Italian critical transcriptions we made a few changes from the original text; most of them are simply typographical adjustments, such as the resolution of "s", and the substitution of "u" with "v" when appropriate. I also corrected some misprinting, which mostly derived from a difficult reproduction of the 1546 edition. I avoided reporting italic style as in the original text, when not necessary. Further, I unify the spelling of words, by adopting the most used form. For example, of the two forms "lun" and "l'un" (the one) I changed everywhere the first with the second, because it is more often used.

The editions by Masotti (Tartaglia [1554] 1959) and Drake (Drake and Drabkin 1969) were of some help.

4.1 The *Quesiti et inventioni diverse* (1554)

As an opening anthology, an English translation, a critical Italian transcription and a facsimile are reported for *Books* VII and VIII of the *Quesiti et inventioni diverse*, 1554 edition (Tartaglia 1554), the first containing the *Gionta* to *Book* VI. The text of *Books* VII and VIII of the first *Quesiti* edition of 1546 (Tartaglia 1546) is essentially similar to that of 1554. It mainly differs in typographical adjustments, as for example "horizonte" (1546) versus "Orizonte" (1554). Moreover, the 1546 edition uses full names for Tartaglia and his interlocutor's while in the 1554 edition the initial only are appended before the corresponding dialogues.

4.1.1 Tartaglia's Language

Tartaglia's writings have always been accused of crudeness. A typical example is the following sentence by Bernardino Baldi:

> He paid so little attention to the goodness of the language that he sometimes moves to laughter the reader of his things. (Baldi 1707, 133; my translation).

The assessment changes a little over centuries, with appreciations by some scholars. For example, Durante writes:

> His [Tartaglia] language is full of *lombardismi* [from Lombardy], even if it's a thousand miles from the dialect. But he lags in the choice of language, because in the mid–sixteenth century the Court language [that which refers to Tartaglia] was out–dated by the Florentine model. (Durante 1981, 157; my translation).

In addition:

> *Le scritture dei semicolti*. Tartaglia uses with security a robust northern Italian. (Trovato 1994, 32; my translation).

In a detailed study on Tartaglia's language, Mario Piotti concludes:

> The choice of the vernacular by the sixteenth–century mathematician Tartaglia is not due to his ignorance of Latin, but to precise theoretical reasons. The language of Tartaglia, accused of dialectal tendencies since the sixteenth century, by the analysis conducted on his works (the *Nova scientia* and *Quesiti et invention diverse*), is proved to be a strong northern Italian of middle level that cannot be attributed to semi educated experiences. The scientific specialization of the vernacular is just incipient and appears, besides the lexicon, from which Tartaglia tends to eliminate the more popular terminology in favour of the model Greek Latin, in some textual and syntactic choices. (Piotti 1998, cover; my translation).

Tartaglia's language is not always the same however; it shows an evolution and refinement at least up to the *Quesiti et invention diverse*, so much so that some have speculated the advice of lettered men, which was not uncommon at the time (Piotti 1998, 34–35).

Tartaglia wrote his first work, *Nova scienti*a (Tartaglia 1537) in the form of a treatise; forms of writing scientific texts were more widespread at the time, thus the choices not seeming to have been objects of reflection. Very different is the situation of the *Quesiti et invention diverse* for which he chooses the form of a dialogue, less common, even though it is rich in tradition (i.e., Platonic dialogues). Usually Tartaglia's dialogue is cold, with a distinction of roles: on one hand the other, the scholar, on the other hand the teacher, Tartaglia. However, there is a disconnect of pieces of that dialogue that are not strictly relevant from the technical aspect, which makes the discussion a little less rigid; they continually remind us that we are not in an academic setting. Moreover the controversies, referred to in *Book* IX, with some opponents, such as the mathematicians Antonio Maria de Fiore (or Florido, 16th century), Giovanni de Tonini da Collio (fl. 16th) and especially Cardano, inserts his science into a social context.

Tartaglia introduces new terms, in part derived from the Latin in the completely original between them:

> *Lexical neologisms*: altimetric scale, alternate angle, angle of contingency, outer angle, square battle of people, square battle of land, bi–angle, calculation, to become congruent, coastal, to raise to cube, curve, diopter, fundamental, granite, isoperimetric, line of direction, line of sight, levelled, place of equality, great merlon, right shadow, oblique shadow, horizontal, at white point (point–blank), cube root, square root, residual, to bevel, bevel, fulcrum, to sight, sight, triplication.
> *Semantic neologisms*: opening, ell, concave, design, to contribute, contribution, contingency, curtain, demonstratively, dependence, dissimilar, to lift, lifting, flask, fortifier, fraction, thrower, to trigger, intermediate, irrationality, irresolvable, hand, mechanics, minute, rear sight, obliquely, petition, place, power, principle, quadrant, rule, reflect, retreat, scale, transit, speed.
> [“*Lexical neologisms*: scala altimetria, angolo alterno, angolo della contingenza, angolo esteriore, battaglia quadra di gente, battaglia quadra di terreno, biangolo, calcolazione, congruire, costiero, cubicazione, curva, diottra, fondamentale, granito, isoperimetro, linea della direzione, linea visuale, livellato, luogo dell’egualità, merlone, ombra retta, ombra versa, orizzontale, di punto in bianco, radice cuba, radice quadrata, residuale, smussare, smussatura, sopravanzare, sparto, traguardare, traguardo, triplicazione. *Semantic neologisms*: apritura, braccio, concavo, concezione, concorrere, concorso, contingenza, cortina, dimostrativamente, dipendenza, dissimile, elevare, elevazione, fiasca, fortificatore, frazione, gettatore, innescare, intermedio, irrazionalità, irresolubile, lancetta, meccanico, minuto, mira, obliquamente, petizione, piazza, potenza, principio, quadrante, regola, riflettere, ritirata, scala, transito, velocità.” (Piotti 1998, 174–175; my translation)].

4.2 Philological Notes on *Iordani opusculum de ponderositate* (1565)

The *Iordani opusculum de ponderositate* derives from a witness of a manuscript currently referred to as the *Liber de ratione ponderis* (called version R and) attributed to Jordanus de Nemore; it was the first printed edition. Some considerations about existing manuscripts of Jordanus’ text can be found in Moody and Clagett (Moody and Clagett 1952), Clagett (Clagett 1959) and Brown (Brown 1967–1968).

According to Moody and Clagett (Moody and Clagett 1952, 175–190), *Iordani opusculum de ponderositate* reproduces a good enough version, but there are printer's errors and some figures are not very good. It was printed by Curtio Troiano on Tartaglia's behalf after his death, with the addition of part of the *Liber Archimedis de ponderibus* and some determinations of specific weights. Duhem said he saw the manuscript owned by Tartaglia and that Tartaglia had made very few corrections to it (Duhem 1905–1906, I, 135). The main difference between the manuscript and the printed version was disappearance of the subdivision into four books. A part from Tartaglia's adding of some figures, the manuscript was simply reproduced by the printer, who was not a technician; he explains typos both for the text and figures. The complete title of the book: *Iordani opusculum de ponderositate, Nicolai Tartaleae studio correctum, novisque figuris auctum*, makes explicit reference to the addition of figures by Tartaglia. They are indicated by Curtio Troiano (or Tartaglia) as "Figura à Nicolao constructa" and represent Tartaglia's attempts to make his manuscript readable.

In the following a partial (until folio 7v useful for my aims) fac–simile and English critical *Iordani opuscolum*'s translation is presented; a complete critical Latin transcription is reported, as well. For the English translation we partially drew inspiration, where possible, from (Moody and Clagett 1952, 175–227), though a more faithful translation has been carried out. In the critical Latin translation – as above cited – we resolved some shortenings, modified "u" in "v" and vice versa, "ij" in "ii", where necessary, following the contemporary standard rule of transcription, as well. Both in the English translation and in the Latin transcription the page number of the original printed version is reported in braces. Please pay attention that in order to present unproblematic reading, only for English transcripts, we replace minuscule letters with capitals concerning demonstrations and technical arguments.

4.3 *Book VII* of *Quesiti et inventioni diverse* (1554)

4.3.1 The Fac–simile and English Translation

LIBRO SETTIMO DELLI QVESITI, ET INVENTIONI DIVERSE, DE NICOLO TARTAGLIA.

Sopra gli principij delle Questioni Mechanice di Aristotile.

QVESITO PRIMO FATTO DAL ILLVSTRISS. *Signor Don Diego Hurtado di Mendozza, Ambasciator Cesareo in Venetia.*

SIGNOR AMBASCIATORE. *Tartaglia, dapoi, che noi dessimo uacatione alle lettioni di Euclide, ho ritrouato cose nuoue sopra le Mathematice.* N. *Che cosa ha ritrouato uostra Signoria.* S.A. *Le Questioni Mechanice di Aristotile, Grece, & Latine.* N. *Eglie tempo assai, che io le uidi, massime Latine.* S.A. *Che ue ne pare.* N. *Benissimo, & certamente le sono cose suttilissime, & di profonda dottrina.* S.A. *Anchora io le ho scorse, & inteso di quelle la maggior parte, nondimeno me resta molti dubbij sopra di quelle, li quali uoglio, che me li dichiarati.* N. *Signore, ui sono dubbij assai, che à uolergli à sofficienza delucidare, à me saria necessario prima à dechiarare à uostra Signoria li principij della scientia di pesi.* S.A. *A me mi pare, che Aristotile dimostri il tutto, senza procedere, ouer intendere altramente la scientia di pesi.* N. *Eglie ben uero, che lui approua cadauna de dette questioni, parte con ragioni, & argomenti naturali, & parte con ragioni, & argomenti Mathematici. Ma alcuni di quelli suoi argomenti naturali, con altri argomenti naturali ui si puol opponere. Et alcuni altri con argomenti Mathematici (mediante la scientia di pesi detta di sopra) se possono reprobar per falsi. Et oltra di questo lui pretermette, ouer tace una questione sopra delle libre, ouer bilanze di non poca importanza, ouer speculatione, & questo è processo (per quanto posso considerare) perche di tal questione, non si puo assignar la causa per ragion naturale, ma solamente con la detta scientia di pesi.* S. A. *Non credo, che questo sia la uerita, cioe, che alcuna sua argumentatione patisca oppositione, perche Aristotile non fu un'ocha, ne manco credo, che lui habbia pretermesso, ouer taciuto questione alcuna sopra delle libre, che sia de importantia.* N. *Anci eglie troppo el uero, pche uolẽdo cõsiderare, giudicare, et dimostrare la causa della sua prima q̃stione, si come naturale, cioe cõ q̃lli ultimi argomẽti naturali, che lui aduce sopra le libre ouer bilãce materiale. Medesimamẽte cõ altri argomẽti naturali (come di sopra dissi) si puo approuare, che seguita tutto al cõtrario di q̃llo, che in tal q̃stione cõclude, ouer suppone. Et uolẽdo poi cõsiderare, & giudicare tal Questione, si come Mathematico, & cõ argomẽti Mathematici si puo medesimamente li detti sui argomenti reprobar per falsi, mediante la scientia di pesi detta di sopra.* S. A. *Come se considerano, & giudicano le cose, si come naturale, & come se considerano, & giudicano, si come Mathematico.*

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THE SEVENTH BOOK OF THE QUESITI, ET INVENTIONI DIVERSE, BY NICOLO TARTAGLIA.

On the principles of the Questions of Mechanics of Aristotle.

FIRST QUESTION RAISED BY EXCELLENCY.

Sir Don Diego Hurtado de Mendoza, Imperial Ambassador in Venice.

SIR AMBASSADOR: Tartaglia, since we took a vacation from the reading of Euclid, I have found some new things relating to mathematics. N.And what has your Excellency found? S.A. Aristotle's *Questions of Mechanics* in Greek and in Latin. N. It is quite a while since I saw these, particularly the Latin.[1] S.A. What did you think of them? N. They are very good, and certainly most subtle and profound in learning. S.A. I, too, have run through them and I understood most of them; yet many questions remained with me, which I should like to have more fully explained. N. Sir, should you wish me to explain them to you properly, many of the problems would require that I first explain to your Excellency the principles of the science of weights. S.A. It appears to me that Aristotle proves everything without using, or so much as knowing about, the science of weights. N. It is true that he proves each of his problems partly by natural reasons and arguments and partly by mathematical.[2] But some of his natural arguments may be opposed by other natural reasoning, and others can even be shown to be false through mathematical arguments by means of the said science of weights. And besides that, he omits or remains silent about a problem of no little importance concerning the balance, because (so far as I can judge) one cannot assign the cause for that problem by natural reasoning, but only through the science of weights. S.A. I do not believe this is true, i.e., that any of his arguments can be contradicted; for Aristotle was not a stupid. Nor do I believe that he omitted anything or was silent on any problem of importance concerning the balance. N. Yet it is only too true; for if, as a natural philosopher, one wishes to consider, judge, and prove the cause of his first problem, using natural arguments that he adduces for the material balance or scale, then one can equally prove with natural arguments (as I said before) that things are quite the opposite of what he concludes or assumes in that problem. And if one wishes then to consider and judge this problem as a mathematician, Aristotle's arguments can similarly be proved false by means of the science of weights. S.A. How are things judged and considered as natural and how as mathematical [?]

V *Ij*

LIBRO

N. El naturale cõsidera, giudica, et determina le cose, secõdo el senso, & apparẽtia di quelle in materia. Ma el Mathematico le considera, giudica, & determina, non secondo el senso, ma secondo la ragione (astrate da ogni materia sensibile) come che V. Sig. sa, che costuma Euclide. S. A. Circa di questo non so che rispondere, perche io non me arricordo cosi all'improuiso il soggetto di tal sua prima questione, e pero ditime, come, che quella parla, & dice. N. La dice, & parla precisamente in questa forma.

Perche causa le maggior libre, ouer bilanze, sono piu diligente delle menore.

S. A. Ben: che uoleti dire sopra di tal questione. N. Voglio dir questo, che sumendola, ouer considerandola, si come Mathematico (cioe astrata da ogni materia) senza alcun dubbio tal questione è uniuersalmente uera, si per le ragioni da lui adutte per auanti, come, che per molte altre, che nella scientia di pesi addur se potria. Perche quel la linea, che con la sua mobile istremita piu se allontana dal centro d'un cerchio, mouesta da una medesima uirtu, ouer potentia (in tal sua istremita) piu facilmente, & con maggior celerita, ouer prestezza sara mossa, spenta, ouer portata, di quella, che cõ la detta sua istremita men se alluntanara dal detto centro, & per tal ragione le libre, ouer bilanze maggiori, se uerificano esser piu diligente delle menore. Ma uolendo poi considerare, & approuare tal questione in materia, & con argomenti naturali, come, che in ultimo lui considera, & approua, cioe per el senso del uedere in esse libre, ouer bilanze materiale. Dico, che con tai sorte de argomenti non se uerifica generalmente tal questione, anzi se trouara seguir tutto al contrario, cioe le libre, ouer bilanze menori esser piu diligente delle maggiori, & che questo sia el uero nelle libre, ouer bilanze materiale, la sperientia lo fa manifesto: perche se de uno ducato scarso uoremo sapere de quanti grani lui sia scarso, con una libra, ouer bilanza granda, cioe con una de quelle, che adoprano li speciali per pesar specie, zuccaro, zenzero, e canella, & altre cose simile, malamente se ne potremo chiarire, ma con una di quelle librette, ouer bilancette piccole, che oprano li bancheri, orefici, & gioieleri, senza dubbio se ne potremo totalmente certificare. Per il che seguitaria tutto al contrario, di quello, che in tal questione se conchiude, & dimostra, cioe, che tai bilancette piu piccole siano piu diligente, delle piu grande, perche piu diligentemente, ouer sottilmente dimostrano la differentia di pesi. Et la causa di questo inconueniente non procede da altro, che dalla materia, perche le cose costrutte, ouer fabricate in quella, mai ponno esser cosi precisamente fatte, come, che con la mente uengono imaginate fuora di essa materia, per il che tal hor se uien à causar in quelle alcuni effetti molto contrarij alla ragione. Et per questo, & altri simili respetti, el Mathematico non accetta, ne consente alle dimostrationi, ouer probationi fatte per uigor, & autorita di sensi in materia, ma solamẽte à quelle fatte p̃ demostrationi, et argomẽti astrati da ogni materia sensibile. Et p̃ questa causa, le discipline Mathematice non solamente sono giudicate dalli sapienti esser piu certe delle naturale, ma quelle esser anchora nel primo grado di certezza. Et pero quelle questioni, che con argomenti Mathematici se possono dimostrare, non é cosa conueniente ad approbarle con argomenti naturali. Et simelmente quelle, che sono gia dimostrate con argomenti Mathematici (che sono piu certi) non é da tentare, ne da persuadersi de certificarle meglio con argomenti naturali, li quali sono

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BOOK

N. The natural philosopher considers, judges, and determines things according to the senses and material appearances,[3] while the mathematician considers and determines them not according to the senses, but according to reason, all matter being abstracted – as your Excellency knows that Euclid was accustomed to do. S.A. On this I can say nothing, because at the moment I do not recall the subject of [Aristotle's] first problem. Please tell me what it says. N. It is worded precisely so: Why large balances or scales are more accurate than small ones.[4]

S.A. Good; what would you say about this problem[?] N. Considering it as a mathematician, in abstraction from all matter, I should say that without doubt the statement is universally true, whether for the many reasons prefaced by Aristotle or for many others that may be brought in from the science of weights. For that line whose moving extremity is farther from the center of a circle, being moved by a given force or power at that extremity, is more easily moved, driven, or carried, and with greater speed, than another at its extremity less distant from the center. And for that reason, larger scales or balances are found to be more accurate than smaller ones. But next, wishing to consider and test that statement materially and with natural arguments (as he does at the end) by the sense of sight and with a material balance, I say that by this sort of argument the problem is not generally verified, and even that the opposite occurs; i.e., smaller balances are found to be more accurate than larger ones. That this is true in material balances, experience makes manifest; for if we have a damaged ducat and want to see by how many grains it is too light, using a large balance such as one of those used to weigh spices, sugar, ginger, cinnamon, and such materials, we shall get a poor result; but if we use one of those small balances employed by bankers, goldsmiths, and jewelers, no doubt we can be quite certain of the result. This is just the contrary of that which was concluded in this problem; for here, small balances are more accurate than large ones because they more thoroughly and more subtly show the difference of weights. And the cause of this contradiction stems simply from matter; for things constructed or fabricated thereof can never be made as perfectly as they can be imagined apart from matter, which sometimes may cause in them effects quite contrary to reason. And for this and other reasons, the mathematician does not accept or consent to proofs and demonstrations made on the strength and authority of the senses in matter, but only those made by demonstrations and arguments spoiled from all matter. Consequently, the mathematical disciplines are considered by the wise not only to be more certain than the natural, but even to have the highest degree of certaintyAnd therefore those questions which can be demonstrated with mathematical arguments cannot be suitably proved by natural arguments.[5] Likewise those which have already been demonstrated by mathematical arguments (which are the most certain) should not be subjected to attempts to certify them still better by natural arguments, which are men certi. S.A. A me mi pare che lui uoglia, in tal prima questione, che quella resti ottimamente chiarita(come è il uero)per le ragioni, & argomenti per auanti adutti, & dimostrati, le quale ragioni, ouer argomenti sono tutti Mathematici, & non naturali; perche parte de quelli se uerificano per la. 23. del Sesto di Euclide, & parte per la quarta del medesimo. N. Vostra Signoria insieme con lui dice la uerita, che tal questione è manifesta per le sue ragioni adutte per auanti, & questo medesimo anchora io di sopra lo affermai, perche tai antecedenti sono stati da lui dimostrati con argomenti Mathematici, ma in fine de tai buone argomentationi, ui sottogionge due altre conclusioni, la prima delle quale dice precisamente in questa forma. Et certamente sono alcuni pesi, li quali posti nelle piccol libre, non sono manifesti al senso, & nelle grande sono manifesti. La qual conclusione, uolendola considerare, giudicare, & approuare, si come naturale, cioe per uigore, & autorita del senso del uedere, nelle libre materiale, senza dubbio tal sua conclusione patisse oppositioni assai, perche nelle dette libre, ouer bilanze materiale, la maggior parte delle uolte se trouara seguir tutto al contrario, cioe che sono alcuni pesi, li quali posti, nelle libre, ouer bilanze grande, non se faranno con alcuna inclinatione manifesti al senso del uedere. Et nelle bilanzette piccole se manifestaranno, cioe che faranno inclinatione uisibile, & tutto questo, la sperientia lo manifesta. Perche se sopra una di quelle sopradette bilanze grande de Speciali, ui sara posto un grano di formento. Eglie cosa chiara, che nella maggior parte di quelle, non fara alcuna uisibil inclinatione. Et nella maggior parte di quelle piccolette che usano li Banchieri, faranno inclinatione molto euidente. Ma uolendo poi considerare, giudicare, & dimostrare tal sua questione, ouer conclusione, si come Mathematico, cioe fuora de ogni materia, senza dubbio tal sua conclusione saria falsa, perche ogni piccol peso posto in qual se uoglia libra fara inclinar quella continuamente per fina all'ultimo, ouer piu basso luoco, che inclinar se possa, & tutto questo nelli principij della scientia di pesi à Vostra Signoria, lo faro manifesto. Dapoi lui sottogionge anchora quest'altra conclusione, & dice in questa forma. Et certamente sono alcuni pesi, li quali sono manifesti nell'una, & l'altra sorte de libre(cioe nelle maggiori, & nelle menori)ma molto piu nelle maggiori, perche molto piu granda inclinatione, uien fatta dal medesimo peso nelle maggiori. La qual conclusione, uolendolo considerare, giudicare, & approuare, si come naturale(come fu detto dell'altra)cioe per uigore, & autorita del senso del uedere, nelle dette libre materiale, certamente questa non patira men oppositioni dell'altra, per le medesime ragioni in quella adutte. Et similmente, uolẽdo poi considerare, giudicare, & dimostrare tal conclusione, come Mathematico, cioe fuora de ogni materia medesimamente tal sua conclusione saria falsa, perche ogni sorte di peso posto in qual si uoglia sorte de libra, fara inclinar quella de continuo per fina à tanto che quella sia gionta all'ultimo, ouer piu basso luoco, che quella inclinar si possa, & tutto questo, nelli detti principij della scientia di pesi dimostratiuamente à quella si fara manifesto. S.A. Anchor che tutte queste uostre oppositioni, & argomenti naturali, habbiano del uerisimile non posso credere, che il non ue sia altre ragioni, & argomenti, si naturali, come Mathematici da poter difendere, & saluare, tal sua questione insieme con quell altre due conclusioni. Anci è ho ferma opinione che chi studiasse con

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less certain. S.A. It seems to me that you wish this first problem [of Aristotle] to be given the greatest clarity of truth by reasons and arguments adduced and demonstrated in advance, which reasons or arguments are all mathematical, and not natural, for part of them are verified by Euclid VI.23[6] and part by the fourth book of Euclid. N. Your Excellency well says, with Aristotle, that that problem is made manifest by the reasons he prefaced [to the problem], and I myself affirmed this before, because such antecedent arguments are proved mathematically by him. But at the end of those good arguments, he adds two other conclusions, the first of which is precisely this: "And certainly there are some weights which, placed in the small balance, are not manifest to the senses, and in the larger balance are manifest."[7] Which conclusion when considered, judged, and tested as natural – i.e., by the strength and authority of the sense of sight in material scales – will doubtless suffer much opposition; for in such material scales or balances the exact opposite will be found to occur most of the time. I.e., there are some weights which, placed in large scales or balances, make no tilting manifest to the sense of sight, but which will do so in little balances (i.e., will make a visible tilting); and all this is shown by experience. For if, on one of those great spice scales mentioned above, there shall be placed a grain of wheat, it is obvious that on most of them it will make no visible tilting, while on most small bankers' balances it will make a quite evident tilting. But since we wish to consider, judge, and demonstrate this problem or conclusion of Aristotle's as mathematicians, i.e., without any material, doubtless the conclusion will be false, since every little weight placed in any scale will make it continually incline to the last or lowest place it can go. And all this I shall make manifest to your Excellency in the principles of the science of weights. Aristotle also adds this other conclusion, and in this form: "And certainly there are some weights which manifest themselves in both sorts of scales (i.e., the large and small), but much more in the larger, a far greater tilting being made there by the same weight".[8] Now if we consider, judge, and test naturally this conclusion, i.e., by the strength and authority of the sense of sight–then, as was said of the other, it will certainly suffer no less opposition in the said material scales than will the other [conclusion], and for the same reasons. And similarly if we consider, judge, and test it as mathematicians (i.e., apart from any matter), this conclusion will still be false, because every sort of weight placed in any sort of scale will make it tilt continually until it comes to the last and lowest place it can. And all this is demonstrated in the said principles of the science of weights. S.A. Although all these objections and natural arguments of yours are probable, I cannot believe that there are not other arguments and reasons, both natural and mathematical, by which [Aristotle's solution of] this problem can be saved and defended together with his two additional conclusions. Indeed, I am of the firm opinion that anyone who would study this diligẽtia sopra à tal materia, ritrouaria tutte qlle particolarita materiale, che sono causa, che tal questione, & cõclusioni nõ se uerificano in materia, come che l'autor cõchiude, et dice. Et dapoi che quelle fussero ritrouate, et conosciute, tẽgo che saria cosa facile à rimediarli, & fare che se uerificassero in materia precisamente, come che l'autor propone. N. Vostra Signoria non e di uana opinione, perche in effetto tutte quelle cose che nella mente sono conosciute uere, & maßime per dimostrationi astratte da ogni materia, ragioneuolmente si debbono anchora uerificare al senso del uedere in materia (altramente le Mathematice sariano in tutto uane, & di nullo giouamento, ouer profitto all'huomo, & se per caso quelle non se uerificano, come che nelle sopradette libre, ouer bilance maggior, & menor, e stato detto, & disputato. Eglie da credere, anci da tener per fermo, che il tutto proceda dalla disproportionalita, & inequalita delle parti, & membri materiali, dalli quali uengono composte, cioe che le dette parti, & membri dell'una piu se discostano, ouer allontanano da quelle considerate fuora de ogni materia, di quello che fanno quelli dell'altra. E per tanto uolendo difendere, & saluare tal questione Aristotelica, cioe far che quella sempre se uerifichi in materia, & in ogni qualita de libre, ouer bilance si grande, come piccole. Bisogna agguagliar le dette parti, ouer membri di cadauna di quelle, talmente che quelli stano egualmente distanti da quelle considerate fuora de ogni materia sensibile. Ilche facẽdo non solamente se uerificara tal sua questione al senso in materia, cioe nelle dette libre, ouer bilance materiale, ma anchora se uerificaranno quelle altre due conclusioni, che sottogionse in fine. S. A. Io ho accaro che la mia opinione se sia uerificata.

QVESITO SECONDO FATTO CONSEQVENtemente dal medesimo Illustrißimo Signor Don Diego Ambasciator Cesareo.

SIGNOR AMBASCIATORE. Ma per non hauer troppo ben inteso le ragioni da uoi allegate, uorria che un'altra uolta, & piu chiaramente me le replicasti. N. Dico Signore, che la causa che le sopradette libre, ouer bilance maggiore, & menore, non rispondeno secondo che l'autor conchiude, & dimostra, non procede d'altro, che dalla inequalita delle parti, ouer membri materiali, dalli quali uengono composte, le quai parti, ouer membri, sono li dui bracci, & anchora il sparto (cioe quel axis, ouer centro, sopra del qual girano li detti bracci in cadauna de loro, perche li detti bracci, & sparto nelle libre, ouer bilance maggiore sono molto piu großi, & corpulenti di quelle delle menore. Et perche li bracci di quelle libre, ouer bilance che uengono considerate, come Mathematico, cioe fuora de ogni materia, sono considerati, et supposti, come simplice linee, cioe senza larghezza, ne grossezza, & il sparto, ouer axis di quelle uien considerato, & supposto un simplice ponto indiuisibile, le qual sorte de libre, ouer bilãce. Quãdo che poßibil fosse à darne una cosi realmente spogliata, & nuda de ogni materia sensibile, come che con la mẽte uengono considerate, senza alcun

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matter diligently would discover all the special properties of matter which give rise to [the effects mentioned in] that problem as well as those conclusions that are not verified materially, as the author [Aristotle] concludes and says. And once these were discovered and known, I think it would be easy to remedy them and to make everything verifiable in material precisely as the author proposes. N. Your Excellency is not mistaken, for in fact all those things that are known by the mind to be true, and particularly by abstraction from all material, should reasonably be verifiable in matter also by the sense of sight; otherwise mathematics would be wholly vain and useless and devoid of profit to man. And if it happens that they are not verified in the aforesaid scales or in large and small balances, as questioned, then it is to be believed and even held for certain that all this proceeds from the disproportionality and inequality of the material parts and members that make one scale differ more than another from balances considered apart from all matter. So if we want to defend and save this problem of Aristotle – i.e., make it verified in matter and in every kind of balance or scale, large or small – it is necessary to make all the parts or members of each balance uniform, in such a way that all are equally applicable to those considered apart from all material. This done, we shall not only verify sensibly in matter this problem of his for material scales and balances, but will also verify those other two conclusions he adds at the end.

SECOND QUESTION CONSEQUENTLY RAISED by the same your Excellency Sir Don Diego Imperial Ambassador.

SIR AMBASSADOR. I am glad to hear my opinion confirmed. But since I did not entirely understand your reasons, I should like them repeated more clearly.

N. I say, Sir, that the cause that the larger and smaller balances do not behave as the author concludes and proves has its roots in the difference between the material parts or members of which they are composed, which parts or members are the two arms and the fulcrum (i.e., the axis or center on which the arms turn in both cases). For the said arms and fulcrum in the larger scale or balance are much more gross and bulky than in the smaller. And since the arms of those scales or balances are to be considered mathematically, i.e., apart from all material, they are considered and assumed to be as simple lines, without breadth or thickness; and the fulcrum or axis [of support] is assumed to be a simple indivisible point. Such a scale or balance, as much as possible, would be given as in fact despoiled and naked of any sensible material, as is considered by the mind, and would dubbio quella faria agilißima, & diligentißima sopra à tutte le libre, ouer bilance materiale, di quella medesima grandezza, perche quella faria totalmente libera da ogni material impedimento. Et per tanto conchiudendo dico, che quanto piu le parti, ouer membri di una libra, ouer bilanza materiale, se accostano, ouer appropinquano alle parti, ouer membri della non materiale (qual è la originale, ouer ideale di tutte le materiale) tanto fara piu agile, & diligente di quelle che men ui se accostaranno, ouer appropinquaranno (di quella medesima grandezza) Et perche le parti, ouer membri di quelle bilancette, che adoprano li Bancheri, & Gioieleri (di sopra allegate) molto piu se accostano, ouer appropinquano alle parti, ouer membri della detta sua ideale, di quello che fanno le parti, ouer membri di quelle libre, ouer bilance maggiori, che adoprano li Speciali (di sopra allegate) perche li brazzetti delle dette bilancette piccole sono sottilißimi, & quelli delle grande sono piu grosßi. Onde li sottili piu se accostano alla simplice linea (quale manca de larghezza, & grossezza) di quello fanno li piu grosßi, & corpulenti, & similmente il sparto, ouer axis delle dette librette, ouer bilancette piccole, è piccolino, & sottile, & quello delle grande, è piu grande, & grosso. Onde il detto sparto delle dette bilancette piccole piu se accosta, ouer appropinqua al sparto della sua ideale (qual è un ponto indiuisibile) di quello fa il sparto delle dette bilance grande per esser piu grande, & grosso. Et questa è la principal causa che le sopra dette librette, ouer bilancette menori, se dimostrano al senso piu diligente delle maggiori, cosa totalmente contraria alla sopra allegata Aristotelica questione.

QVESITO TERZO FATTO CONSEquentemente dal medesmo Illustrißimo Signor Don Diego Ambasciator Cesareo.

SIGNOR AMBASCIATORE. Ben in che modo si puo difendere, & saluare tal sua questione, cioe far che quella se uerifichi al senso in materia secondo che lui propone, ouer conchiude. N. Bisogna sondarse sopra le libre, ouer bilance ideale, cioe sopra quelle che uengono considerate con la mente astratte da ogni materia, & uedere in che cosa le maggiore siano differente dalle menore, la qual cosa essendo osseruata nelle libre, ouer bilance materiale fara difesa, & saluata tal questione Aristotelica, cioe che quella sempre se uerificara al senso nelle dette libre materiale. S. A. Non ue intendo parlatime piu chiaro. N. Dico Signore, che à uoler difendere, & saluare tal questione, bisogna fondarse, ouer reggersi per le libre, ouer bilance ideale, cioe per quelle, che con la mente uengono considerate fuora de ogni materia, & uedere in che cosa le maggiori siano differente dalle menori, sopra la qual cosa considerando, & guardando, se trouara, che le dette libre, ouer bilance maggiori, non sono differente dalle menori, eccetto che nella longhezza di suoi bracci, & in tutte le altre cose se agguagliano, perche anchor che li bracci delle libre maggiori siano piu longhi de quelli delle menori, tamen non sono ne piu grosßi, ne piu sottili de quelli, perche, si nelle maggiori, come nelle menori, sono considerati,

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doubtless be agile and responsive far beyond all material scales or balances of the same size, for it would be completely free of any material hindrance. And thus I say in conclusion that the more the parts or members of a material scale or balance resemble or approach the parts or members of an immaterial one (which is the original or ideal of all material ones), so much the more agile and responsive will it be than those which less resemble or approach this, the sizes being the same. And the parts or members of those small scales used by bankers and jewelers, as mentioned above, much more resemble and approach the parts or members of their said ideal than do the parts or members of those larger scales or balances used by merchants; for the little arms of the smaller balances are very thin, and those of the larger ones are gross. Wherefore the fulcrum of the smaller balance much more resembles and approaches to its ideal fulcrum, which is an indivisible point, than does the fulcrum of the large balance by reason of its gross size. And this is the principal reason why the aforementioned small balances are sensibly more accurate than the large ones, which is completely contrary to the Aristotelian view in the problem under discussion.

THIRD QUESTION CONSEQUENTLY RAISED by the same Excellency Sir Don Diego Imperial Ambassador.

SIR AMBASSADOR. In which way can you defend, and save his question, that states that in nature what he proposes occurs, or concludes? N. One must base on ideal scales, or balances, i.e., on these which are considered by the mind spoiled from any matter, and see in what are the larger differs from the smaller, which being observed in the real scales, or balances, the Aristotelian question will be defended, and saved, i.e., that it always occurs with the senses in those real balances. S.A. I do not understand, explain it to me more clearly. N. I say Sir, who wants to defend, and save that question, must be based, or stand on ideal scales, or balances, i.e. those which are spoiled of any matter with the mind, and see in what are the grater different from the smaller, and considering, and looking upon which, if you find that those greater scales, or balances, are no different from the smaller but in the length of their arms, and all the other things are equal, because even if the arms of the greater balances are longer than those of the smaller, they are neither bigger, nor the more subtle of them, because, both in the greater and smaller, they are considered, come simplice linee, le quale mancano di larghezza, & grossezza, e pero in larghezza, & grossezza non ui è alcuna differentia. Et similmente li sparti, ouer axi delle libre, ouer bilance maggiori sono eguali alli sparti, ouer axi delle menori, perche si nelle maggiori, come nelle menori sono considerati, come simplici ponti, li quali ponti per esser tutti indiuisibili, sono eguali, le qual cose essendo diligentemente osseruate nelle libre, ouer bilance materiale, cioe che le maggiore non stano differente dalle menore, eccetto che nella longhezza di suoi bracci, ma che in larghezza, et grossezza stano eguali, & cosi li lor sparti materiali senza dubbio in quelle, non solamente se uerificara al senso quello, che Aristotile nella detta sua questione conchiude. Ma anchora se uerificaranno, quelle altre due conclusioni che ui sottogionse in fine. (Anchor che in astratto, cioe fuora de ogni materia, ambedue false stano, come che per li principij della scientia di pesi à V.S. faro manifesto.) Et stano le dette libre, ouer bilance di che qualita, materia, & condition si uoglia, pur che osseruino la detta egualita nella grossezza di detti bracci, & sparti loro. S.A. Certamente che questo uostro discorso me piace assai.

QVESITO QVARTO FATTO CONSEquentemente dal medesimo Illustrissimo Signor Don Diego Ambasciator Cesareo.

SIGNOR AMBASCIATORE. Ma se ben me aricordo uoi dicesti anchora nel principio del nostro ragionamento, che Aristotile pretermette, ouer tace una questione sopra delle dette libre di non poca importantia, ouer speculatione, hor ditemi, che questione è questa. N. Se V.S. ben se aricorda della sua seconda questione, in quella lui interrogatiuamente adimanda, & consequentemente dimostra, perche causa quando che il sparto sera di sopra della libra, & che l'uno di bracci di quella da qualche peso sia portato, ouer spinto à basso, remosso che sia, ouer leuato uia quel tal peso, la detta libra di nuouo reascende, & ritorna al suo primo luoco. Et se il detto sparto è di sotto della detta libra, & che medesimamente l'uno di suoi bracci sia da qualche peso pur portato, ouer spinto à basso remosso, ouer leuato che sia uia quel tal peso la detta libra non reascende, ne ritorna al suo primo luoco (come che fa nell'altra positione) ma rimane di sotto, cioe à basso. Hor dico, che lui pretermette, ouer tace un'altra questione, che in questo luoco se conueneria, di molta maggior speculatione di cadauna delle sopradette, la qual questione è questa. Perche causa quando che il sparto è precisamente in essa libra, et che l'un di bracci di quella sia da qualche peso portato, ouer urtato à basso, remosso, ouer leuato che sia uia quel tal peso, la detta libra di nuouo reascende al suo primo luoco, si come che fa anchora quella, che ha il sparto di sopra da lei. S.A. Questa mi pare una bella questione, & molto piu remota dal nostro intelletto naturale che le due sopradette, & molto hauero accaro ad intendere la causa di tal effetto, ma prima uoglio che me chiariti un dubbio, che nella mente me intona sopra delle sopra allegate questioni, il quale è questo.

Questo

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like simple lines, which lack of width, and thickness, and thus there is no difference in width, and thickness. And similarly the fulcra, i.e. axes, of scales, or of the greater scales are equal to the fulcra, i.e. axes, of the smaller, because in the greater as in the smaller they are considered as simple points, which points being indivisible, are equal to each other. If these cares are diligently observed in real scales, or balances, i.e., that the greater are not different from the smaller except for the length of their arms, but that in width, and thickness are the same, and so their real fulcra undoubtedly them, not only will be verified with senses what Aristotle concludes his question, but also those two other conclusions that he added in the end will be verified (Although in the abstract, that out of all matter, are both false, such as that for the principles of the science of weights I will manifest to V.S.). And those scales, or balances of any quality, material, and condition you want, when they comply with the said equality in the thickness of their arms and fulcra. S.A. Certainly, this your speech pleased to me very much.

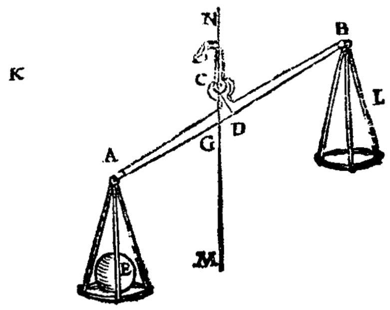
FOURTH QUESTION CONSEQUENTLY RAISED
by the same Excellency Sir
Don Diego, Imperial Ambassador.

SIR AMBASSADOR. But if I well remember you also said, at the beginning of our reasoning, that Aristotle omitted, or was silent on, a question about balances of great relevance and inquire. Now tell me what question is this. N. If your Excellency remember his second problem, where he [Aristotle] asks, and consequently proves, the why when the fulcrum is above the scale, and one of his arm is moved by some weight, or pushed downward, removed or taken off the weight, the scale raises again and returns to his first place. And when that fulcrum is below the scale, and similarly one of his arm is carried by some weight, or pushed downward, when the weight is removed the scale neither raises nor returns to its first place (as it does for the other case) but remains below, i.e. beneath. Now I say, he was silent and mitted one more problem, which here is much more suitable, much more speculative of any of the other problems, which is that. Why when the fulcrum is precisely inside the scale, and one of the arm of is, by any weight, loaded, or pressed down, removed that weigh, the scale again raises to its first place, like as the scale with the fulcrum above. S.A. That looks to me a nice problem, and much farther from our intellect that the two mentioned before and I will appreciate very much to understand the cause of that effect; but I before want you to clarify me a doubt, which persists in my mind about the above cited problems, which is this.

Quesiion

QVESITO QVINTO FATO CONSEQVENTEtemente dal medesimo Illustrißimo Signor Don Diego, Ambasciator Cesareo.

SIGNOR AMBASCIATORE. Doue se troua una libra, ouer bilanza materiale, che il suo sparto sia di sopra, ouer di sotto di quella, anci à me mi pare, che il detto sparto in tutte sia precisamente in esse libre, come, che nella uostra terza question se suppone, & non di sopra, ne manco di sotto. N. Anchor, che di tal sorte bilance non si faccia, ouer si troui el non resta pero, chel non se ne potesse fare. S. A. A me mi pare una materia, à mouer questione sopra à cose, che non si costumano, ne si trouino in eßere. N. Il tutto si fa Signore, perche tutti li artificiosi istromenti, che per augumentare le forze del huomo se oprano, in qual si uoglia arte Mechanica se referiscono à una delle sopradette tre specie de libre, ouer bilance, et cosi in ogni dubbio, ouer questione, che sopra ad alcuno de tai istromenti nascer potesse, uolendone conoscere, ouer aßignare la intrinsica causa. Eglie necessario prima uenir a quella sorte libra, ouer bilanza, alla qual piu se referisse quel tal istromento, & dalla detta libra, ouer bilanza se uien al cerchio, per la mirabil uirtu, & potentia del quale se risolue il tutto, come, che nella scientia di pesi si fara manifesto. S. A. Essendo adunque cose di tanta importantia, uoglio, che me replicati, & dimostrati figuralmente cadauna de dette tre Questioni, ouer parti a una per una: perche le uoglio ben intendere, & cominciati alla prima. N. Per dimostrar in figura la prima parte di tal Questione. Sia la libra. a. b. el sparto della quale sia el ponto. c. (qual sparto sia alquanto di sopra della detta libra. a. b. come nella figura appare) & sia che per la impositione del peso. e. el suo brazzo. a. d. sia da quel tirato a basso, come che di sotto appare in detta figura: hor dico, che chi leuasse uia el detto peso. e. tal brazzo. a. d. reascendaria, &



retornaria al suo primo, & condecente luoco, el qual luoco saria nel ponto, ouer sito. k. & cosi l'altro brazzo. d. b. descendaria per fina al ponto, ouer sito. l. & tutto questo procede: perche nel trasportar el detto brazzo. a. d. a basso, piu della mitta di tutto el fusto della detta libra. a. b. se uien a trasferrirsi in alto, cioe oltra la perpendicolar. n. m. passante per il sparto. c. la qual perpendicolar se chiama

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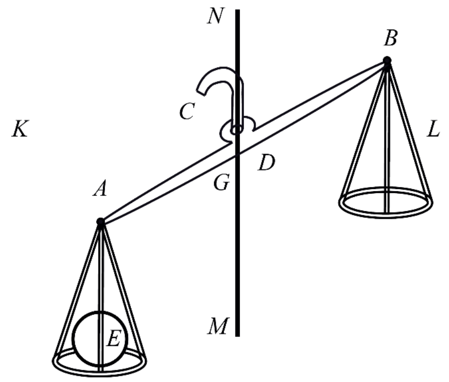
S E V E N T H
FIFTH QUESTION CONSEQUENTLY RAISED
by the same Excellency Sir Don Diego,
Imperial Ambassador.

SIR AMBASSADOR. Where is a scale such that its fulcrum is above or below it? To me it appears that the fulcrum in all the scales be exactly inside them, as supposed in your third problem. Neither above, nor below. N. Though such balances are not used or found its does not mean we cannot speak about the. S.A It looks to me a matter, a problem, over unusual things, which do not exist. N. All is made, Sir, for all the artificial instruments used to increase the force of men, in whichever mechanical art, refer to one of the three named species of scales, or balances. And equally any doubts, or questions, that about these instruments will raise, if one want to known, or to assign, the intrinsic cause, it is necessary to come first to the type of scale or balance to which mainly that instrument is referred to, and from the said balance one comes to the circle, from whose marvellous strength and power all is explained, as in the science of weights will be shown. S.A. For being the things of such relevance, I want that you repeat and demonstrate any of the all three problems, one by one, for I want well understand them, and start by the first. N. To demonstrate with a figure [See Fig. 4.1] the first part of such a problem les us consider the scale *ab* whose fulcrum be *c* (which fulcrum be over enough of the said scale *ab*, as it appears in the figure) and let for the imposition of the weight *e* its arm *ad* be pressed down as it appears in the figure. I now say that if the said weight *e* is removed the arm *ad* will raise and



[Fig. 4.1]

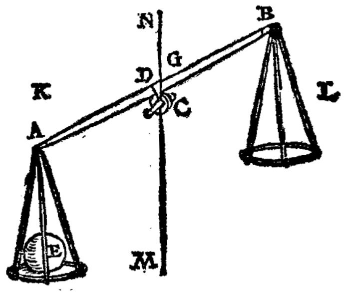
return to its first place, and let such a place be the point, or site, *k* and similarly the other arm *db* will descend up to the point, or site *l*, and all this occurs because to carry the arm *ad* downward, more than one half of the beam of the scale *ab* is transferring upward, i.e. farther from the perpendicular *nm* passing through the fulcrum *c*, which perpendicular is called

LIBRO

la linea della direttione, cioe, che la parte. b. d. g. in alto elleuata uien à esser tanto piu della mita de tutto el fusto. a. b. quanto che è dal. d. al. g. & la restante parte. a. g. ridutta al basso uien à esser tanto manco della mita di tutto el detto fusto. a. b. quanto che è dal detto ponto. g. al ponto. d. perche adunque tal parte. b. d. g. in alto elleuata è molto maggiore del restante brazzo. a. g. al basso trasferto, leuandose uia el detto peso. e. la detta parte. a. g. (piu debole) uien à eßer urtata, & spinta dall'altra maggior parte. b. d. g. in alto elleuata (per esser di lei piu potente) per fin à tanto, che la detta linea della direttione caschi perpendicolarmente sopra el detto fusto, ouer libra. a. b. & che seghi quello in due parti equali in ponto. d. S. A. Questa ragion è quasi simile à quella che aduce Aristotile, ma è alquanto piu chiara, & miglior figura.

QVESITO SESTO FATTO CONSEquentemente dal medesimo Illustrißimo Signor Don Diego Ambasciator Cesareo.

SIGNOR AMBASCIATORE. Hor seguitati la seconda parte. N. Per dimostrare la seconda à uostra Signoria. Pongo sia la libra. a. b. la qual habbia il sparto (cioe quel ponto, ouer polo, sopra del qual lei gira) alquanto di sotto, cioe di sotto dal fusto. a. b. come di sotto appar in ponto. c. & sia anchor, che per la imposition del peso. e. el suo brazzo. a. d. sia da quel tirato à basso, come che di sotto nella figura appar, hor dico, che chi leuasse uia el detto peso. e. tal brazzo non reascenderia ne ritornaria al suo primo luoco, cioe in ponto. k. (come, che fa in quella, che ha il sparto di sopra) ma restaria cosi inclimato à basso, & la causa di questo procede, perche nel trasportarse el detto brazzo. a. d. al basso piu della mitta di tutto el fusto, ouer libra. a. b.



si uien à trasferire drio à quello, oltra la linea della direttione, cioe oltra la perpendicolar. n. m. qual passa per il sparto. c. tal che tutta la parte. a. g. al basso ridutta, uien à esser tanto piu della mitta di tutta la libra. a. b. quanto, che è dal. d. al. g. & la parte. g. b. in alto elleuata uien à restare tanto meno della detta mitta, quanto, che è dal detto. d. al detto. g. per esser adunque la elleuata parte. g. b. di menor quantita della inclinata. a. g. uien à esser piu debole, ouer men potente di lei, e pero, non è atta, ne sofficiete à po-

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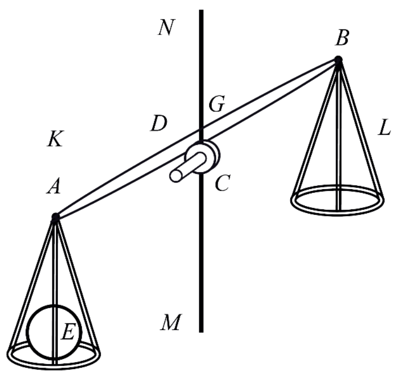
BOOK

the line of direction, i.e that the raised part *bdg* becomes greater than one half of the whole beam *ab* for the part from the point *g* to the point *d*, and the reaming depressed part *ag* becomes less of the one half of the whole beam *ab* for the part form the point *g* to the point *d*. Thus, because the raised part *bgd* is much greater of the remain depressed arm *ag*, by removing the weight *e* the part *ag* (weaker) is hit and pushed by the greater raised part *bdg* (being more powerful of it) until the line of direction falls perpendicularly over the beam, i.e. scale *ab*, and cuts it into two equal parts in the point *d*. S.A. This reason is quite similar to Aristotle's but clearer and better illustrated.

SIXTH QUESTION CONSEQUENTLY RAISED
by the same Excellency Sir Don Diego, Imperial Ambassador.

SIR AMBASSADOR. Now continue for the second part. N. To demonstrate the second part to your Excellency let assume the scale *ab* have its fulcrum (i.e. that point or pole about which it turns) quite below, i.e. beneath the beam *ab*, as shown below, in point *c*, and also let that because of the weight *e* its arm *ad* be pushed down, as it appears in the figure below [See Fig. 4.2].



[Fig. 4.2]

Now I say that if that weight *e* were taken away such arm neither would mount again nor return to its first place, i.e. the point *k* (as that which has the fulcrum above does) but would remain so tilted, below, and the cause of this comes for in carrying below the arm *ad* more than one half of the whole beam, or scale *ab*, is transferred behind, after the line of direction, i.e. after the perpendicular *nm* which passes trough the fulcrum *c*, so that the whole part *ag*, pushed down, becomes more than one half of the whole scale *ab*, according to *dg*, and the raised part *gb* becomes the lesser than of the said half according to *dg*. Thus, because the raised part *gb* becomes lesser that the tilted [part] *ag* it becomes weaker, i.e. less powerful, and as such it is neither able, nor sufficient to stri terla urtare, & sforzare à farla ascendere al suo primo luoco in. k. come fece nella passata, anci quella restara cosi inclinata al basso, & la retenera lei cosi in aere elleuata, che è il proposito. S. A. Queste due parti quasi, che il nostro intelletto le apprende per ragion naturale, senza altra dimostratione. N. Cosi è Signore.

QVESITO SETTIMO FATTO CONSEQVENTEMENte dal medesimo Illustrißimo Signor Don Diego, Ambasciator Cesareo.

SIGNOR AMBASCIATORE. Hor seguitatimo la terza parte, quale diceti, che manca in questo luoco, cioe doue nasce la causa, che quando el sparto de una libra sara precisamente nel mezzo di essa, cioe ne di sotto, ne di sopra, ma nel mezzo di quella, come, che sono tutte le libre, ouer bilance, che communamente se oprano, & che l'uno di brazzi di quella sia da qualche peso (ouer dalla nostra mano) urtato à basso, leuado, che sia uia quel tal peso (ouer mano) immediate tal brazzo riascende, et ritorna al suo primo luoco, si come che anchor fa q̃lla libra, qual tiẽ il sparto di sopra da essa libra. Perche in effetto la causa di questo ultimo effetto mi par molto piu remota dal nostro intelletto de cadauna delle altre due. N. E ho detto à uostra Signoria, che à uoler dimostrare la causa di tal effetto à me è neceßario à diffinire, & dechiarire prima à uostra Signoria alcuni termini, & principij della scientia di pesi. S. A. Sono cosa longa questi principij, che ui bisogna dechiarare. N. Per quãto aspetta à uoler demostrare simplicemente questa particolarita sara cosa breuißima, uero è, che quando, che uostra signoria uolesse intendere ordinariamente tutti li principij di tal scientia, ui saria da dire assai. S. A. Ben sa, che uoglio intendere il tutto ordinariamẽte, come si de. N. L'hora è tarda Signore per far questo effetto. S. A. Ben andati, & ritornati dimane da mattina. N. Ritornaro Signore.

Il fine del settimo Libro.

[82r]

SEVENTH

ke it, and force it to mount up to its first place *k*, as it does in the past case, instead it will remain so tilted and lowered, and [the lowered part] will remain up, which is the purpose. N. That is so, Sir.

SEVENTH QUESTION CONSEQUENTLY RAISED by the same Excellency Sir Don Diego, Imperial Ambassador.

Sir Ambassador. Now let us come to the third part, which is still lacking here; i.e., how it comes about that, when the support of a scale is precisely in its center, neither above nor below, but in the center, as is the case with most of our ordinary scales, and one of the arms is pushed down either by some weight or by our hand, and the weight or hand is then removed, this arm immediately ascends again and returns to its original place, as does the arm of a scale whose support comes from above. For in fact the cause of this seems to me farther removed from common sense than for either of the two usual cases. N. I have told your Excellency that in order to demonstrate the cause of this effect, it will be necessary for me first to define and explain to your Excellency some of the terms and principles of the Science of Weights. S.A. Is this something lengthy, these principles you must explain? N. So far as it concerns simply the demonstration of this particular, it will be quite short; however, if your Excellency wants to learn in an orderly manner all the principles of the Science of Weights, that will be quite lengthy. S.A. You know very well that I should like to learn the whole thing, and in proper order. N. It is getting rather late to accomplish this. S.A. Well, you may go, then, and return tomorrow morning. N. I shall return, your Excellency.

The end of the seventh Book.

X *ij*

4.4 *Book VIII* of *Quesiti et inventioni diverse* (1554)

4.4.1 The Fac–simile and English Translation

LIBRO OTTAVO DELLI QVESITI, ET INVENTIONI DIVERSE, DE NICOLO TARTAGLIA.

Sopra la Scientia di Pesi

QVESITO PRIMO FATTO DAL ILLVSTRISS. *Signor Don Diego Hurtado di Mendozza, Ambasciator Cesareo in Venetia.*

SIGNOR AMBASCIATORE. *Hor uoria Tartaglia, che me incomenciasti à dechiarire ordinariamente quella scientia de pesi, di che me parlasti hieri. Ma, perche conosco tal scientia non esser semplicemente per se (per non esser le arte liberale, saluo che sette) ma subalternata, uoria che prima me dicesti, da che scientia, ouer disciplina quella deriui, & nasci.* N. *Signor Clarissimo parte di questa scientia nasce, ouer deriua dalla Geometria, & parte dalla Natural Philosophia: perche, parte delle sue conclusioni se dimostrano Geometricamente, & parte se approuano Physicalmente, cioe naturalmente.* S.A. *E ue ho inteso circa questa particolarita.*

QVESITO SECONDO FATTO CONSEQVENtemente *dal medesmo Illustrissimo Signor Don Diego Ambasciator Cesareo.*

SIGNOR AMBASCIATORE. *Ma ditime anchora, che costrutto si puo cauar di tal scientia.* N. *Li costrutti, che di tal scientia si potriano cauare, saria quasi impossibile à poterli à uostra Signoria isprimere, ouer connumerare, nondimeno io ue referiro quelli, che per al presente à me sono manifesti. Et per tanto dico, che primamente per uigore di tal scientia, eglie possibile à conoscere, & misurare con ragione la uirtu, & potentia di tutti questi istromenti Mechanici, che da nostri antiqui sono stati ritrouati, per augumentare la forza de l'huomo, nel elleuare, condurre, ouer spingere auanti ogni graue peso cioe in qual si uoglia grandezza, che quelli siano constituidi, ouer fabricati, secondariamente per uirtu di tal scientia, non solamente eglie possibile di poter con ragion conoscere, & misurare simplicemente la forza de l'huomo, ma anchora eglie possibile di trouar el modo di augumentar quella in infinito, & in uarij modi, & cosi in qual si uoglia modo eglie possibile à conoscere l'ordine, & proportione di tal augumentatione, come, che in fine con uarij istromenti Mechanici à Vostra Signoria faro conoscere, & uedere.* S. A. *Questo hauero molto accaro.*

[82v]

THE EIGHTH BOOK OF QUESITI, ET INVENTIONI DIVERSE, OF NICOLO TARTAGLIA.
On the Science of Weights

FIRST QUESTION RAISED BY EXCELLENCY
Sir Don Diego Hurtado of Mendoza, Imperial ambassador in Venice.

SIR AMBASSADOR. Now, Tartaglia, I want you to start explaining in due order that s*cience of weights* [emphasis added] of which you spoke to me yesterday. And since I know that it is not a proper science in itself (there being no more than seven liberal arts),[9] but rather that it is a subordinate science,[10] I want you first to tell me from which other science or discipline it is derived. N. Sir, part of this science is derived from Geometry and part from Natural Philosophy; for part of its conclusions are demonstrated geometrically and part are proved physically, that is, by nature. S.A. I now understand you for this point.

SECOND QUESTION CONSEQUENTLY RAISED
by the same Excellency Don Diego, Imperial Ambassador.

SIR AMBASSADOR. But also tell me what construct can be drawn from that science. N. The constructs which can be drawn from that science would be almost impossible to express to your Excellency, or to enumerate; nevertheless, I shall repeat those which are manifest to me at the moment. Hence I say that first, by the power of this science, it is possible to know and to measure by reason the strength and power of all those mechanical instruments that were discovered by the ancients to augment the strength of a man for raising, carrying, or driving forward all heavy weights, in whatever size they are constituted or fabricated. Second, by virtue of that science it is possible not only to be able to know and measure by reason the strength of a man, but also to find how to augment this infinitely, and in various ways, and thus it is possible to know the order and proportion of such augmentation in any manner, as finally, by means of various mechanical instruments, I shall make your Excellency see and know. S.A. I would like this very much.

QVESITO TERZO FATTO CONSE-quentemente dal medesimo Illustrißimo Signor Don Diego Ambascia tor Cesareo.

SIGNOR AMBASCIATORE. Hor seguitati, come ui pare circa à tal scientia. N. Per procedere regolatamente, hoggi diffiniremo solamente alcuni termini, & modi di parlare occorrenti in questa scientia, accio che il frutto della intelligentia di quella, V.S. piu facilmente apprenda. Dimane poi dichiariremo li principij di tal scientia, cioe quelle cose che in tal scientia non si possono dimostrare, perche (come che V.S. sa) ogni scientia ha li suoi primi principij indemostrabili, li quali essendo concessi, ouer supposti per lor meggio si disputa, & sostenta tutta la scientia, dapoi questo andaremo preponendo uarie propositioni, ouer conclusioni sopra di tal scientia, & parte de quelle dimostraremo à V.S. con argomenti Geometrici, & parte approuaremo con ragioni naturali, come di sopra dißi. Et dapoi questo, V.S. preponera tutti quei dubbij, ouer questioni che à quella gli parera, nelle cose Mecanice, & maßime sopra li mirabili effetti delli sopradetti istromenti materiali che augumentano la forza dell'huomo, che per le cose dette, & approbate, nella detta scientia de pesi, tutte se resolueranno. S.A. Questo uostro procedere cosi regolatamente molto mi piace.

QVESITO QVARTO FATTO CONSE-quentemente dal medesimo Illustrißimo Signor Don Diego Ambasciator Cesareo.

SIGNOR AMBASCIATORE. Hor seguitate adunque le dette diffinitioni consequentemente. N.

QVESITO. IIII. DIFFINITIONE PRIMA.

LI corpi se dicono di grandezza eguali, quando che quelli occupano, ouer empino luochi eguali. S.A. Datemi qualche material essempio. N. Essempi gratia, doi corpi spherici gettati, ouer prontati in una medesima forma, ouer in forme equale, se diriano eguali di grandezza, anchor che fussino di materia diuersa, cioe che l'uno fusse di piombo, & l'altro di ferro, ouer di pietra, & cosi si debbe intendere in qual si uoglia altra diuersita di forma. S.A. E ue ho inteso, seguitati. N.

QVESITO. V. DIFFINITIONE II.

SImilmente li corpi se dicono di grandezza diuersi, ouer ineguali, quando che quelli occupano, ouer empino luochi diuersi, ouer ineguali. Et maggiore se intende quello, che occupa maggior luoco. S. AMBASCIA. E ue ho inteso, seguitati. NIC.

[83r]

EIGHT
THIRD QUESTION CONSEQUENTLY RAISED
by the same Excellency Sir Don Diego, Imperial ambassador.

SIR AMBASSADOR. Now proceed as you wish about this science. N. To proceed in an orderly fashion, we shall today define only some terms and ways of speaking that occur in this science,[11] in order that your Excellency will more easily apprehend the fruit of the understanding of this. Then, tomorrow, we shall proceed to state the principles of that science, that is, those things which cannot be demonstrated in the science;[12] for as your Excellency knows, every science has its indemonstrable first principles, which, being conceded or assumed, afford the means to discuss and sustain the whole science. Then we shall go on by setting forth various propositions or conclusions concerning the science; and part of these we shall demonstrate to your Excellency by geometrical arguments, and part we shall test by physical reasons, as I said before. And after this, your Excellency, you shall put forward those doubts or questions that occur to you concerning things mechanical, and especially the admirable effects of the said material instruments that augment the strength of a man; and by the things said and tested in the science of weights, all will be clarified. S.A. This orderly procedure of yours suits me very well.

FOURTH QUESTION CONSEQUENTLY RAISED
by the same Excellency Sir Don
Diego, Imperial ambassador.

SIR AMBASSADOR. Therefore go on with the said definitions, in order. N.

QUESTION. IIII. FIRST DEFINITION.

Bodies are said to be of equal sizewhen they occupy or fill equal spaces.[13] S.A. Give me some material example. N. For instance, two spherical bodies cast or shaped in the same form, or in equal forms, will be said to be of the same size even though of different materials, as when one were of lead and the other of iron or of stone. And the same is to be understood of any other variety of form. S.A. I understand; go on. N.

QUESTION. V. DEFINITION II.

Similarly bodies are said to be of different size or unequal when they occupy or fill different or unequal spaces, and greater means that which occupies more space.[14] S.AMBASCIA. I understand; proceed. NIC.

LIBRO

QVESITO. VI. DIFFINITIONE TERZA.

LA uertu d'un corpo graue se intende, & piglia per quella potentia, che lui ha da tendere, ouer di andare al basso, & anchora da resistere al moto contrario, cioe à che il uolesse tirar in suso. S.A. Quando che non ui dico altro seguitati, perche col mio tacere, e ue dinoto hauerui inteso, & che debbiati seguitare. N.

QVESITO. VII. DIFFINITIONE QVARTA.

LI corpi se dicono de uertu, ouer potentia, equali, quando che quelli in tempi equali di moto pertransiscono spacij eguali.

QVESITO. VIII. DIFFINITIONE QVINTA.

LI corpi se dicono de uertu, ouer potentia diuersa, quando che quelli in tempi diuersi, pertransiscono di moto, spacij eguali, ouer che in tempi eguali pertransiscono interualli ineguali.

QVESITO. IX. DIFFINITIONE SESTA.

LA uertu, ouer potentia de corpi diuersi, quella se intende esser maggiore, la quale nel pertransire uno medesimo spacio summe manco tempo. Et menor quella che summe piu tempo, oueramente quella che in tempi eguali pertransisse maggior spacio.

QVESITO. X. DIFFINITIONE SETTIMA.

QVelli corpi se dicono essere di uno medesimo genere, quando che sono di egual grandezza, & che sono anchora di egual uertu, ouer potentia.

QVESITO. XI. DIFFINITIONE OTTAVA.

QVelli corpi se dicono essere de diuersi generi, quando che sono di egual grandezza, & che non sono di egual uertu, ouer potentia.

QVESITO. XII. DIFFINITIONE NONA.

QVelli corpi se dicono essere simplicemente eguali in grauita, li quali sono realmente di egual peso, anchor che fusseno di materia diuersa.

QVESITO. XIII. DIFFINITIONE DECIMA.

[83v]

BOOK
QUESTION. VI. THIRD DEFINITION.

The strength of a heavy body is understood and assumed that power which it has to tend or go downward, as also to resist the contrary motion which would draw it upward.[15] S.A. When I say nothing to you, continue, for by my silence I denote that I have understood and wish you to continue. N.

QUESTION. VII. FOURTH DEFINITION.

Bodies are said to be of equal strength or power when in equal times they run through equal spaces.[16]

QUESTION. VIII. FIFTH DEFINITION.

Bodies are said to be of different strength or power when in different times they move through equal spaces, or when in equal times they traverse unequal intervals.[17]

QUESTION. IX. SIXT DEFINITION.

The strength or power of different bodies is assumed to be greater in that which employs less time to traverse the same space, and less in that which employs more time; or [greater in that] which in equal time traverses greater space.[18]

QUESTION. X. SEVENTH DEFINITION.

Those bodies are said to be of the same genus when they are of equal size and also of equal strength or power.[19]

QUESTION. XI. EIGHT DEFINITION.

Those bodies are said to be of different genus when they are of equal size and are not of equal strength or power.

QUESTION. XII. NINTH DEFINITION.

Those bodies are said to be simply equal in heaviness which are actually of equal weight, even though they were of different material.[20]

QUESTION XIII. DEFINITION
TENTH.

VN corpo se dice essere simplicemente piu graue d'un'altro, quando che quello è realmente piu ponderoso di quello, anchor che fusse di materia diuersa.

QVESITO. XIIII. DIFFINITIONE XI.

VN corpo se dice essere piu graue d'un'altro secondo la specie, quando che la sostantia material di quello è piu ponderosa della sostantia material dell'altro, come che è il piombo del ferro, & altri simili.

QVESITO. XV. DIFFINITIONE XII.

VN corpo se dice essere piu, ouer men graue d'un'altro nel descendere, quando che la rettitudine, obliquita, ouer dependentia del luoco, ouer spacio doue descende lo fa descendere piu, ouer men graue dell'altro, & similmente piu, ouer men ueloce dell'altro, anchor che siano ambidui simplicemente eguali in grauita.

QVESITO. XVI. DIFFINITIONE XIII.

VN corpo si dice essere piu graue, ouer men graue d'un'altro, secondo il luoco, ouer sito, quando che la qualita del luoco doue che lui se riposa, & giace, lo fa essere piu graue dell'altro anchor che fusseno simplicemente egualmente graui.

QVESITO. XVII. DIFFINITIONE XIIII.

LA grauita d'un corpo se dice essere nota, quando che il numero delle libre, che lui pesa ne sia noto, ouer altra denomination de peso.

QVESITO. XVIII. DIFFINITIONE XV.

LI bracci de una libra, ouer bilancia se dicono essere nel sito, ouer luoco della equalita, quando che quelli stanno equidistanti al piano dell'Orizonte.

QVESITO. XIX. DIFFINITIONE XVI.

LA linea della diretttione è una linea retta imaginata uenire perpendicolarmente da alto al basso, & passare per il sparto, polo, ouer asis de ogni sorte libra, ouer bilancia.

QVESITO. XX. DIFFINITIONE XVII.

PIu obliquo se dice essere quel descenso, d'un corpo graue, il quale in una medesima quantita, capisse manco della linea della direttione, oueramente del descenso

[84r]
EIGHT

A body is said to be simply heavier than another when it is actually more ponderous, even though it were of different material.

QUESTION XIV. DEFINITION XI.

A body is said to be specifically heavier than another when its material substance is more ponderous than the material substance of the other, as is lead than iron, and other similar materials.[21]

QUESTION. XV. DEFINITION XII.

A body is said to be more or less heavy in descent than another when the straightness, obliquity, or pendency of the place or space where it descends makes it descend more or less heavily than the other, and similarly more or less rapidly than the other, though both were simply equal in heaviness.

QUESTION. XVI. DEFINITION XIII.

A body is said to be positionally more or less heavy than another when the quality of the place where it rests and is located makes it heavier [or less heavy] than the other, even though both are simply equal in heaviness.[22]

QUESTION. XVII. DEFINITION XIV.

The heaviness of a body is said to be known when the number of pounds, or other named measure, that it weighs is known.[23]

QUESTION. XVIII. DEFINITION XV.

The arms of a scale or balance are said to be in the position of equality, or place of equality, when they stand parallel to the plane of the horizon.[24]

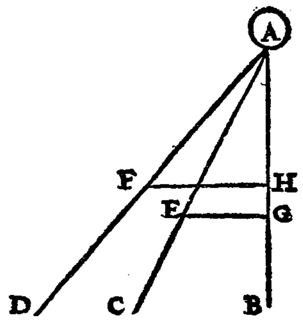
QUESTION. XIX. DEFINITION XVI.

The line of direction is a straight line imagined to come perpendicularly from above to below and to pass through the fulcrum, pole or axis of any kind of scale or balance.

QUESTION. XX. DEFINITION XVII.

The descent of a heavy body is said to be more oblique when for a given quantity it partakes less of the line of vertical direction, or of straight descent.

retto uerso il centro del mondo. S.A. In questa non ue intendo troppo bene. e però datemi uno essempio. N. Per essemplificare questa diffinitione sia il corpo.a.& il retto descenso di quello uerso il centro del mondo sia la linea.a.b. & sia anchora li descensi.a.c.&.a.d.& de questi dui ne sia signati le due quantita, ouer parti.a.e.&.a.f. eguale, & dalli dui ponti.e.&.f.siano tirate le due linee.e.g. &.f.h. equidistanti al piano dell'Orizonte, e perche la parte.a.h.è menore della parte.a.g.il descenso.a.f.d.se dira esser piu obliquo del descenso.a.e.c.perche lui capisse manco del descenso retto, cioe della linea.a.b.in una medesima quantita. Et questo medesimo si debbe intendere in tutti li descensi che potesse fare il detto corpo.a. (ouer altro simile) stante appeso al al braccio di alcuna libra, cioe che quel descenso se dira esser piu obliquo, che per lo medesimo modo capira manco della linea della direttione, in una medesima quantita de descenso. S.A. E ue ho inteso à sofficientia, e pero seguitati se haueti altra cosa da diffinire. N. Signore questa é la ultima cosa che habbiamo da diffinire sopra à questa materia. Dimane poi dichiariremo li principij di questa scientia, secōdo la promessa. S.A. Alla bon'hora.



QVESITO. XXI. FATTO CONSEquentemente dal medesimo Illustrißimo Signor Don Diego Ambasciator Cesareo.

SIGNOR AMBASCIATORE. Hor seguitati Tartaglia questi uostri principij. N. Li principij de qual si uoglia scientia alcuni uogliano che siano detti dignita, perche quelli approuano altri, & loro non ponno essere approuati da altri, alcuni le chiamano suppositioni, perche se suppongono per ueri in detta scientia, altri piacque chiamarli petitioni, perche uolendo disputare tal scientia, & quella sostentare con dimostrationi, bisogna prima adimandar e all'auersario la concessione de quelli, perche se lui non li uolesse concedere (ma negare) saria negata tutta la scientia, ne ui occorreria à disputarla altramente. Et perche questa ultima opinione mi piace alquanto piu delle altre due, petitioni le chiamaremo, & cosi anchora in forma de petitioni li proferiremo.

QVESITO. XXII. PETITIONE PRIMA.

ADimandamo che ne sia concesso, che il mouimento naturale de ogni corpo ponderoso, e graue sia rettamente uerso il centro del mondo. S. AMB. Questo non é da negare.

Questo

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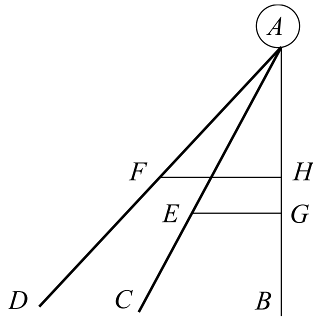
BOOK

toward the centre of the world.[25] S.A. I do not understand this very well; therefore give me an example. N. To exemplify this definition, let there be the body *a*, and its straight descent toward the centre of the world shall be the line *ab*; and let there be also the descents *ac* and *ad*; and of these two, let there be two designated equal quantities, or parts, *ae* and *af* [See Fig. 4.3]. From the points *e* and *f*, draw the two lines *eg* and *fh* parallel to the plane of the horizon. Since the part *ah* is less than the part *ag*, the descent *afd* will be said to be more oblique than the descent *aec*, because it contains less of the straight descent, that is, of the line *ab*, in a equal quantity. And the same is to be said for all descents that could be made by the body *a*, or any similar body, hung from the arm of any balance. That is, that descent will be said to be more oblique which, in the above way, contains less of the line of direction in a equal quantity of descent. S.A. I have sufficiently understood; therefore proceed, if you have anything else to define. N. Sir, this is the last thing that we have to define concerning this subject. Tomorrow we shall explain the principles of this science, according to our promise. S.A. It was time.



[Fig. 4.3]

QUESTION. XXI CONSEQUENTLY RAISED
by the same Excellency Sir Don Diego, Imperial ambassador.

SIR AMBASSADOR. Now, Tartaglia, continue with your principles. N. Some say that the principles of any science should be called dignities [“*dignita*”], because they prove other propositions but cannot be proved from others; some call them suppositions, because they are supposed to be true in the given science; others prefer to call them petitions, because, if we wish to debate such a science and sustain it with demonstrations, we must first request the adversary to concede them. For if he does not concede them but denies them, the entire science would be denied, nor could one debate differently. And since this last opinion pleases me somewhat more than the other two, let us call them petitions and set them forth in the form of requests.

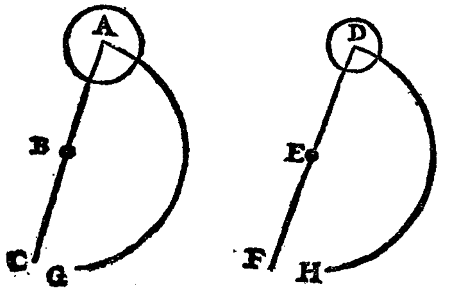
QUESTION. XXII. FIRST PETITION.

We request that it be conceded that the natural movement of any heavy and ponderable body is straight toward the centre of the world.[26] S.A. This is not to be denied.

Question

QVESITO XXIII. PETITIONE II.

Simelmente adimandamo, che ne sia concesso quel corpo, ch'è di maggior potentia debbia anchora discendere piu uelocemente, et nelli moti contrarij, cioe nelli ascẽsi, ascendere piu pigramente, dico nella libra. S. A. Datime uno essempio materiale sopra di questa petitione, se uoleti, che ue intenda. N. Sia, essempi gratia, le due libre. a.b.c. & .d.e.f. equali, cioe, che li dui brazzi. a.b. & .b.c. stano equali alli dui brazzi. d.e. & .e.f. & li lor sparti, ouer centri stano. b. & .e. & nella istremita del brazzo. b.a. ui sia appeso il corpo. a. poniamo de libre due in grauita, & nella istremita de l'altro brazzo, cioe in ponto. c. non ui sia alcuna altra grauita, & cosi nella istremita del brazzo. e.d. ui sia appeso el corpo. d. poniamo di una libra sola in grauita, & nella istremita dell'altro brazzo, cioe in ponto. f. non ui sia alcuna grauita, & stano li detti dui corpi, cosi congionti elleuati con la mano in alto egualmente, come che di sotto appar in figura; hor adimando, che me sia concesso, lasciando andare cadauno de detti dui corpi cosi in alto elleuati, che il corpo. a. (per esser piu graue) discenda piu uelocemente



mente al basso del corpo. d. cioe, che il detto corpo. a. sumara manco tempo à pertransire il curuo spacio. a.g. di quello fara il detto corpo. d o pertransire il curuo spatio. d. h. li quali spacij uengono à esser eguali, perche li brazzi de dette libre sono eguali dal presupposito, e pero li detti dui spacij, ouer descensi curui, uengono à esser circõferentie di cerchij eguali. Et è conuerso, quando, che li detti corpi sarãno discesi nel suo immo, ouer piu basso luoco, cioe l'uno in ponto. g. & l'altro in ponto. h. adimando, che me sia concesso, che quella uirtu, ouer potentia, la qual essendo appesa nell'altro brazzo della libra in ponto. c. sara atta ad elleuare el detto corpo. a. per fin al luoco, doue, che al presente se ritroua nella figura superiore, quella medesima sia atta ad alleuar piu uelocemente il corpo. d. essendo appesa nell'altro brazzo della sua libra, cioe in põto. f. S. A. Questo ui concedo, perche la sperientia ne rende buona testimoniãza. N. Ma uostra Signoria sappia, che quello, che hauemo detto, & adimandato delli detti dui corpi, delli quali l'uno è simplicemente piu potente dell'altro, il medesimo adimandamo de dui corpi simplicemente eguali in potentia ma inequali per uigor della lor positione, ouer sito nel brazzo de una medesima libra, essempi gratia, se nel brazzo. a.b, della

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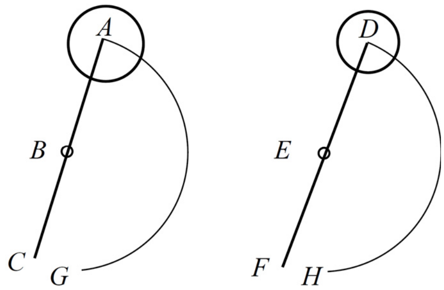
EIGHT
QUESTION XXIII. PETITION II.

Likewise we request that it be conceded that that body which is of greater power should also descend more swiftly; and in the contrary motion, that is, of ascent, it should ascend more slowly–I mean in the balance.[27] S.A. Give me a material example for this petition if you wish me to understand it. N. For example, let there be the two equal scales *abc* and *def* [See Fig. 4.4], with the two arms *ab* and *bc* equal to the two arms *de* and *ef*, and their fulcrums or centres *b* and *e*; and at the extremity of the arm *ba* let there be hung the body *a*, say, of two pounds weight; and at the extremity of the other arm, that is, at the point *c*, let there be no other weight. And at the extremity of the arm *ed* let hang the body *d*, say, of a single pound weight; and at the extremity of the other arm, that is, at the point *f*, let there be no other weight. And let the two said bodies, so conjoined, be elevated by hand to equal heights, as appears below in the figure. Now I request that it be conceded to me that, when both the said two elevated bodies are released, the body *a* (being heavier) will descend more swi[–]

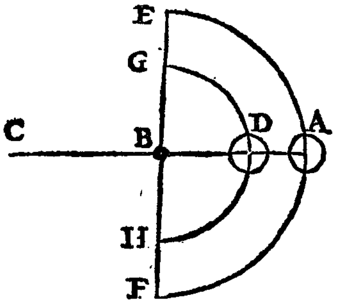


[Fig. 4.4]

ftly than the body *d*; that is, the body *a* will take less time to run through the curved space *ag* than will the body *d* to run through the curved space *dh*, which spaces will be equal because the arms of the scales are assumed equal, whence the said two curved spaces or descents are circumferences of equal circles. And the converse happens when the said bodies shall have descended to their lowest places, that is, one to the point *g* and the other to the point *h*. I ask that it be conceded that the strength or power which shall be hung at the other arm of the scale at the point *c*, in order to elevate the said body *a* to the place where it is presently shown in the figure, will be able to raise the body *d* more swiftly when hung from the other arm of its scale at the point *f*. S.A. This I concede, because experience gives me good evidence of it. N. But your Excellency knows that what we have said and supposed of the two said bodies, of which one is simply more powerful than the other, we suppose of two bodies simply equal in power [in weight] but unequal by strength of their position or placement on the arms of the same balance. For example, on the arm *ab* of the

X

libra.a.b.c. ue sia appeso li dui corpi.a. & .d. eguali simplicemente in potentia, cioe, luno in ponto.a. & l'altro in ponto.d. come di sotto appar in figura, anchor, che stano simplicemente egualmente potenti, nondimeno il corpo.a. in tal positione per la.13. diffinitione se dira esser piu graue del corpo.d. come per lauenire se fara manifesto, perche in questo luoco non si puo aßignar la ragione per le cose dette, ma per lauenire se prouara el corpo.a. in simel sito esser piu graue del corpo.d. e pero essendo quelli elleuati luno in põto.e. & laltro in põto.g. & dapoi eßẽdo ambi dui abandonati, dico, che il corpo.a. discẽdera piu ueloce del corpo.d. & e cõuerso, essendo luno, e l'altro di scesi nelli loro infimi luochi, cioe luno in ponto.f. & laltro in ponto.h. quella potentia che sara atta in ponto.c. ad elleuare il corpo.a. dal ponto.f. per fina al ponto.e. quella medesima sara atta ad elleuare nel medesimo luoco, molto piu uelocemente il corpo.d.



dal ponto.h. per fi al ponto.g. S. A. Anchora questa è cosa chiara, ma uoria intendere due cose da uoi. la prima è, che uoria intendere, perche non fingeti la soprascritta figura de libra, con quelle sue due tazzette appese luna da un capo, & laltra da laltro (come nelle material libre si costuma) per imponerui li pesi, ouer campioni in luna, & nell'altra le cose, che se hanno da ponderare: la seconda è, che uoria sapere se questo essempio de libra si debbe intendere di quelle, che hanno il lor sparto di sopra, ouer di quelle, che l'hanno di sotto, ouer di quelle, che non l'hanno, ne di sopra, ne di sotto, ma in esse libre proprie. N. Circa alla prima, rispondo, che la pura libra se intende per quella pura lõghezza, che forma quelli dui brazzi luno di qua, laltro di la dal sparto, ò stano li detti brazzi equali tra loro, ouer inequali, & quelle due tazzette, che dice V.S. non sono parte della libra, ma ui se aggiongono per commodita del ponderante, per imponerui li campioni, & pesi, che ha da ponderare, si come ch'è anchora la sella dun cauallo, la quale non è parte del cauallo, ma una cosa aggionta per cõmodita di colui, che l'ha da caualcare, e perche meglio si uede, & comprende uno cauallo nudato della sua sella, che cõ la sella, et simelmẽte una libra nudata di quelle sue due tazzette, che con le tazzette senza tazzette la eßemplificamo. Circa alla seconda particolarita, dico, che la presente libra, & simelmente tutte quelle, che per lauenir si proponera (non specificando altro) si debbono intendere di quelle, che hanno il sparto in lor medesime, come nelle materiale si costuma. S.A. E ue ho inteso, seguitati, N.

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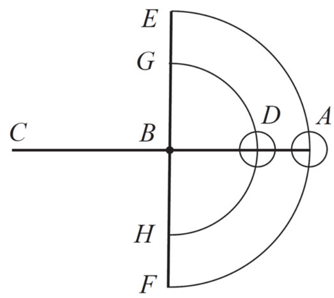
balance *abc* [See Fig. 4.5] let there be hung the two weights *a* and *d*, simply equal in power, that is, one at the point *a* and the other at the point *d*, as appears below in the figure. Although they are equally powerful, nevertheless the body *a* in that position (by the thirteenth definition) will be said to be heavier than the body *d*, as will later be made manifest. For at this time the reason cannot be given for the things said, but later it will be proved that the body *a* in such a position is heavier than the body *d*. Nevertheless, these being raised, one to the point E and the other to the point *g*, and both then released, I say that the body *a* will descend more swiftly than the body *d*; and conversely, if both have descended to their lowest points, that is, one to the point *f* and the other to the point *h*, the power that, at the point *c*, will be able to elevate the body *a* from the point *f* to the point *e* will be able, in the same place, to elevate much more swiftly the body *d*.

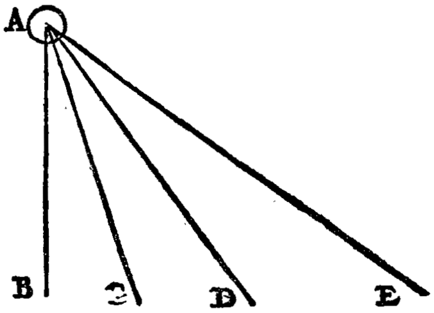


[Fig. 4.5]

from the point *h* to the point *g*. S.A. This is also clear, but I should like to hear from you two things. First, I wish to know why you do not draw the above figure of the scale with its two small cups hung one from one end and one from the other (as is usual in actual scales), where we place weights and samples of things to be weighed. Second, I should like to know if this example of the scale should be understood of those that have their fulcrum above, or of those that have it below, or of those having it neither above nor below, but in the scale itself? N. As to the first, I shall reply that by the ideal scale is intended the mere length that forms the two arms on both sides of the fulcrum, whether such arms are equal or unequal, and that those two small cups of which your Excellency speaks are not part of the scale, but are added for the convenience of the weigher in placing the weights and samples that are to be weighed–just as the saddle is not part of the horse, but something added for the convenience of him who must ride.[28] And just as a horse is better seen and recognized bare of saddle than with saddle, so is a balance denuded of those cups seen better than with them; thus without cups we illustrate it. As to the second matter, I say that the present scale, as well as all those we shall later propose (unless we specify otherwise), should be understood to have the fulcrum within, as is usual with actual balances. S.A. I understand; proceed. N.

QVESITO. XXIIII. PETITIONE III.

ANchora adimandamo, che ne sia concesso un corpo graue esser in el discendere tanto piu graue, quanto che il moto di quello è piu retto al centro del mondo. S.A. Datime anchora uno qualche material essempio sopra à quest'altra petitione se uoleti, che ui intenda. N. Sia, eßempi gratia, il corpo graue. a. & poniamo, che le quattro linee. a. b. a. c. a. d. a. e. stano quattro luochi, ouer spacij da poter descendere el detto corpo. a. & poniamo anchora, che la linea. a. b. sia il rettißimo, & perpendicolar descenso uerso il cẽtro del mondo, onde la linea. a. d. ueneria ad eßer piu retta uerso il detto centro del mondo della linea. a. e. & per tanto in questo caso adimandamo, che ne sia concesso il detto corpo. a. esser piu graue nel discendere per la linea. a. d. che per la linea. a. e. per esser (come detto) piu retta di quella al centro del mondo, & similmente per la linea. a. c. descendere piu graue, che per la linea. a. d. per esser tal linea a. c. piu retta al centro del mondo della detta linea. a. d. & cosi quanto piu el detto corpo. a. se andara accostando alla detta linea. a. b. nel suo descendere se suppone tanto piu graue descendere, perche quel transito, ouer descenso, che forma piu acuto angolo con la linea. b. a. in ponto. a. se intende eßer piu retto al centro del mondo, di quello, che lo forma men acuto. Onde per la linea. a. b. uien à discendere piu graue, che per qual si uoglia altro uerso.



Et questo, che hauemo detto, & adimandato dal sopradetto corpo. a. separato da ogni libra, il medesimo adimandamo de quelli, che descendono appesi al brazzo di qualche libra. Essempi gratia, sia anc hora el detto corpo. a. appeso al brazzo della libra. a. b. c. girante sopra al sparto, o uer centro. b. oueramente al brazzo della libra a. d. e. girante sopra al sparto, ouer centro. d. & sia el perpendicolar descenso uerso il centro del mõdo la linea retta. a. f. & el descenso, che faria el detto corpo. a. cõ el brazzo. a. b. della libra. a. b. c. sopra el centro. b. la linea curua. a. g. Et el descenso, che faria el medesimo corpo. a. con el brazzo. a. d. della libra. a. d. e. sopra el centro. d. la linea curua. a. h. Hor dico, & adimando, che ne sia concesso il detto corpo. a. esser piu graue nel descendere per il descenso. a. h. che p el descenso. a g. per essere el detto descenso a. h. piu retto al centro del mondo del descenso. a. g. perche el detto desenso. a. h.

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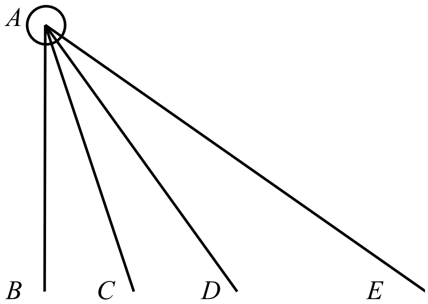
EIGHT
QUESTION. XXIIII. PETITION III.

It still it be conceded that a heavy body in descending is so much the heavier as the motion it makes is straighter toward the centre of the world.[29] S.A. Give me some material example of this new petition, if you want me to understand it. N. For example, let there be the heavy body *a*, and assume that the four lines *ab*, *ac*, *ad*, *ae* are four places or spaces by which the said body *a* can descend [See Fig. 4.6], and let us also assume that the line *ab* is the straightest and perpendicular descent toward the centre of the world. So that *ad* will be more direct toward the centre of the world than the line *ae*, and hence in this case we request that it be conceded that the said body *a* is heavier in descending by the line *ad* than by the line *ae* (because as said, the former goes more directly than the latter to the centre of the world), and similarly is heavier in descending by the line *ac* than by the line *ad*, because the line *ac* is more direct to the centre of the world than the line *ad*. And thus the more the said body shall approach the line *ab* in its descent, it is assumed so much the heavier in descent, because that trajectory or descent which forms the more acute angle with the line *ab* at the point *a* is understood to be more direct toward the centre of the world than one which forms a less acute angle. Whence it comes to descend most heavily along the line *ab* of any direction.[30]

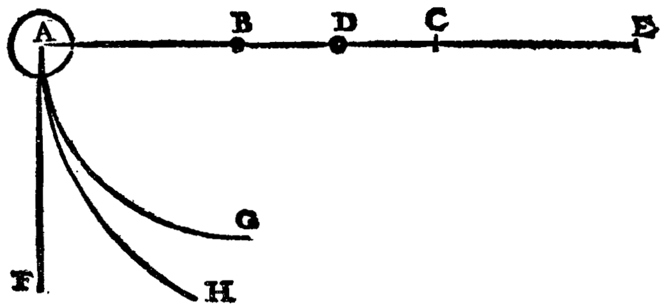


[Fig. 4.6]

And what we have said and requested of the said body *a* separated from any balance, we request of those [bodies] which descend when hung from the arm of any scale. For example [Fig. 4.7], let there be also the body *a* hung onto the arm of scale *abc* that turns on the fulcrum or centre *b* or onto the arm of scale *ade* that turns on the fulcrum or centre *d*; and let the perpendicular descent toward the centre of the world be the straight line *af*; and the descent which the said body *a* would make with the arm *ab* of the scale *abc* on the centre *b* will be the curved line *ag*. And let be the curve *ah* the descent which the same body *a* will make with the arm *ad* of the scale *ade* on the centre *d*. Now I request it to be conceded that the said body *a* is heavier in descending by the descent *ah* than by the descent *ag*, because the said descent *ah* is more direct toward the centre of the world than the descent *ag*, the descent *ah*

X ij

forma piu acuto angolo con la linea.a.f. (qual è l'angolo.h.a.f. della contingentia) di quello fa lo decenso.a.g.



S.A. *E ue ho inteso benißimo,* & *tal petitione non è da negare, e pero seguitati nel l'altra.* N.

QVESITO. XXV. PETITIONE IIII.

ANchora adimandamo, che ne sia concesso quelli corpi esser egualmente graui, secondo el sito, ouer positione, quando che li lor descensi in tai siti sono egualmẽte obliqui, & piu graue esser quello, che nel suo sito, ouer luoco doue se riposa, ouer gia ce ha il descenso manco obliquo. S.A. Anchora questa uiẽ a esser manifesta per quello fu detto nella precedente, & anchora sopra la seconda petitione, e pero seguitati. N.

QVESITO. XXVI. PETITIONE V.

SImelmente adimandamo, che ne sia concesso quel corpo esser men graue dun altro secondo el sito, ouer luoco, quãdo che per el descẽso di quello altro, nell'altro brazzo della libra in lui seguita il moto contrario, cioe, che da lui uien elleuato in suso uerso il cielo, & è conuerso. S.A. Questa è cosa troppo chiara da concedere. N.

QVESITO. XXVII. PETITIONE VI.

ANchora adimandamo, che ne sia concesso, niun corpo esser graue in se medesimo. S.A. Questa uostra petitione non intendo. N. Cioe, che l'acqua nella acqua, il uino nel uino, l'olio nel olio, & l'aere nel aere non essere di alcuna grauita. S.A. E ue ho inteso, & è cosa concesibile, perche la sperientia nel manifesta, si che, seguitati. N. Non ci è altra cosa da adimandare à. V. S. diman, piacendo à Iddio, intraremo nelle propositioni. S.A. Saranno propositioni assai. N. Non troppo signore. S.A. Credeti, che le spediremo dimane. N. Nõ credo Signore, che le spediremo nãche fra diman, e l'altro. S.A. Bẽ andate, ritornate da mattina à bon'hora.

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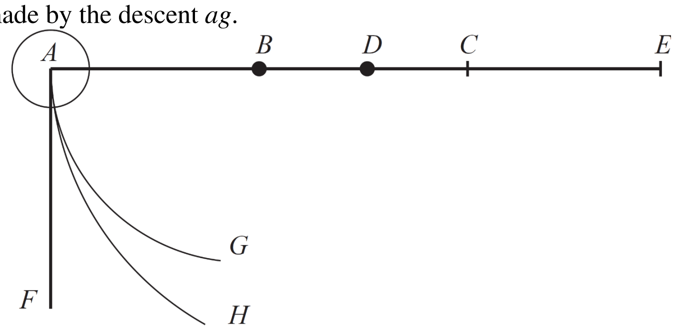
forming a more acute angle with the line *af* (which is the angle *baf* of tangency) than that made by the descent *ag*.



[Fig. 4.7]

S.A. I understand you very well, and that petition is not to be denied. Now go on to the next. N.

QUESTION. XXV. PETITION IIII.

Also we request that it be conceded that those bodies are equally heavy positionally when their descents in such positions are equally oblique,[31] and that is the heavier which, in the position or place where it rests or is situated, has the less oblique descent. S.A. This also is manifest by what was said of the foregoing, and also of the second, petition; therefore proceed. N.

QUESTION XXVI. PETITION V.

Similarly we request that it be conceded that that body is less heavy than another positionally when, by the descent of that other on the arm of the balance, a contrary motion follows in the first; that is, the first is thereby elevated toward the sky; and conversely.[32]

S.A. This is quite clearly to be conceded. N.

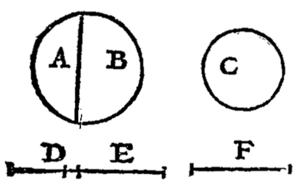
QUESTION. XXVII. PETITION VI.

Also we request that it be conceded that nobody is heavy in itself. S.A. I do not understand this petition of yours. N. I mean, that water in water, wine in wine, oil in oil, and the air in air have no heaviness. S.A. I understand, and this is something that may be conceded because experience makes it manifest; hence go on. N. There are no more petitions to be requested to your Excellency. Tomorrow, God willing, we are going to enter the propositions. S.A. There will be propositions enough. N. Not too many, Sir. S.A. Do you think we can get through them tomorrow? N. I doubt, Sir, that we can finish them tomorrow and the next day. S.A. Well, you may go, and return early tomorrow.

OTTAVO QVESITO. XXVIII. PROPOSITIONE PRIMA.

SIGNOR AMBASCIATORE. Hor seguitati Tartaglia queste uostre propositioni, ouer conclusioni consequentemente l'una drieto all'altra, & sotto breuita. NICOLO.

LA proportione della grandezza di corpi de un medesimo genere, & quella della lor potentia è una medesima. S.A. Datemi uno essempio. N. Siano li dui corpi.a.b.&.c. de uno medesimo genere, & sia.a.b. maggiore, & sia la potentia del corpo.a.b.la.d.e.& quella de corpo.c.la.f. Hor dico che quella proportione, che è dal corpo.a.b.al.corpo.c.quella medesima è della potentia.d.e.alla potentia.f. Et se possibile è esser altramente (per l'auersario) sia che la proportione del corpo.a.b.al corpo. c. sia menore di quella della potentia.d.e.alla potentia.f. Hor sia del corpo.a.b. (maggiore) compreso una parte eguale al corpo.c.menore, quale sia la parte.a.& perche la uertu, ouer potentia del composito è composta dalla uertu di componenti. Sia adunque la uertu, ouer potentia della parte.a.la.d.& la uertu, ouer potentia del residuo.b de necessita sara la restante potentia.e.et perche la parte.a.è tolta equal al.c.la potentia.d.(per il conuerso della.7.diffinitione) sara eguale alla potentia.f.& la proportione de tutto il corpo. a.b.alla sua parte.a.(per la seconda parte della. 7. del quinto di Euclide) sara, si come quella del medesimo corpo.a.b.al corpo.c.(per esser.a. equal al.c.) & similmente la proportione della potentia.d.e.alla potentia.f.sara, si come quella della detta potentia.d.e.alla sua parte.d.(per esser la.d.equal alla.f.) Adunque la proportione de tutto il corpo.a.b.alla sua parte. a.sara menore di quella di tutta la potentia.d.e.alla sua parte.d. Adunque euersamente (per la.30.del quinto di Euclide) la proportione del medesimo corpo.a.b. al residuo corpo.b.sara maggiore di quella di tutta la potentia.d.e. alla restante potentia.e. la qual cosa saria inconueniente, & contra la opinion dell'auersario, il qual uol che la proportione del maggior corpo al menore sia menore, di quella della sua potentia alla potentia del detto menore. Adunque destrutto l'opposito rimane il proposito. S.A. Sta bene, seguitati. NIC.



QVESITO. XXIX. PROPOSITIONE SECONDA.

LA proportione della potentia di corpi graui de uno medesimo genere, & quella della lor uelocita (nelli descensi) se conchiude esser una medesima, anchor quel-

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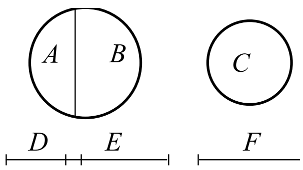
QUESTION. XXVIII. FIRST PROPOSITION.

SIR AMBASSADOR. Now continue, Tartaglia, with your propositions or conclusions in order, one after another, and briefly. NICOLO.

The ratio of volume of bodies of the same kind is the same as the ratio of their power.[33] S.A. Give me an example. N. Let there be the two bodies *ab* and *c* of the same kind; let *ab* be the greater, and let the power of the body *ab* be *be*, and that of the body *c* be *f* [See Fig. 4.8]. Now I say that that ratio which the body *ab* bears to the body *c* is that of the power *de* to the power *f*. And if possible (for the adversary), let it be otherwise, so that the ratio of the body *ab* to the body *c* is less than the ratio of the power *de* to the power *f*. Now let the greater body *ab* include a part equal to the lesser body *c*, and let this be the part *a*; *and since the strength or power of the whole is composed of the strengths of the parts* [emphasis added],[34] the strength or power of the part *a* will be *d*, and the strength or power of the remainder *b* will necessarily be the remaining power *e*; and since the part *a* is taken equal to *c*, the power *d* (by the converse of definition 7) will be equal to the power *f*, and the ratio of the whole body *ab* to its part *a* (by Euclid V.7)[35] will be as that of the same body *ab* to the body *c* (*a* being equal to *c*), and similarly the ratio of the power *de* to the power *f* will be as that of the said power *de* to its part *d* (*d* being equal to *f*). Therefore the ratio of the whole body *ab* to its part a will be less than that of the whole power *de* to its part *d*. Therefore, when inverted (by Euclid V.30),[36] the ratio of the body *ab* to the residual body *b* will be greater than that of the whole power *de* to the remaining power *e*, which will be contradictory and against the opinion of the adversary, who wants the ratio of the greater body to the less to be smaller than that of its power to the power of the lesser body. Thus, the contrary destroyed, the purpose stands. S.A. Very good; continue. NIC.

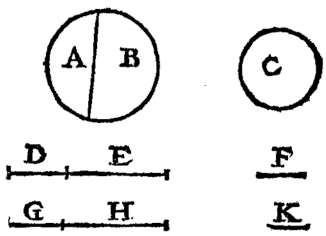


[Fig. 4.8]

QUESTION. XXIX. SECOND PROPOSITION.

The ratio of the power of heavy bodies of the same kind and that of their speeds (in descent) is concluded to be the same; also that *la delli lor moti contrarij (cioe delli lor ascensi) se conchiude esser la medesima, ma trasmutatiuamente.* S.A. *Essemplificatemi tal propositione.* NIC.

*Sia anchora li dui corpi.a.b.& .c.de uno medesimo genere, & di grandezza diuersa, & sia lo.a.b.maggiore, & sia la potentia del.a.b.la.d.e.& del.c.la.f. & perche il corpo di potentia, ouer grauita maggiore (per la seconda petitione) descende piu uelocemente, sia adunque la uelocita nel descender del corpo.a.b.la.g.h. & quella del corpo.c.la.*k.*hor dico, che la proportione della potentia.d.e.alla potentia.f.& quella della uelocita.g.h.alla uelocita.*k.*esser una medesima, & quella delli lor moti contrarij esser quella medesima, ma trasmutatiuamente, cioe che la proportione della uelocita del corpo.a.b.alla uelocita del corpo.c.nel moto contrario (cioe nell'ascendere) esser, si come quella della potentia.f.alla potentia.d.e.ouer, come del corpo.c.al corpo.a.b. la qual cosa se dimostra per il medesimo modo, che fu dimostrata la precedente, cioe se la proportione della potentia.d.e.alla potentia.f.non è (per l'auersario) si come quella della uelocita.g.h.alla uelocita.*k. *necessariamente la sara maggiore, ouer menore, hor poniamo che la sia menore, della potentia.d.e.ne assignaremo la parte.d.eguale alla.f.& cosi della uelocita.g.h.ne assignaremo la parte.g.eguale alla.*k. *& arguiremo, come nella precedēte, dicēdo che la pportione di tutta la potentia.d.e.alla sua parte.d. sara (per la seconda parte della.7.del quinto di* Euclide*) si come quella della medesima potentia.d.e.alla potentia.f. (per esser la d.& .f.eguale) & similmente la proportione de tutta la uelocita.g.h.alla sua parte.g. esser, si come quella della medesima.g.h.alla.*k. *Adunque la proportione di tutta la potentia.d.e.alla sua parte.d.sara menore di quella di tutta la uelocita.g.h.alla sua parte.g.* Onde *(per la.30.del quinto di* Euclide*) la proportione di tutta la medesima potentia.d.e.al suo residuo.e.hauera maggior proportione, che tutta la uelocita.g.h.al suo residuo.h.la qual cosa saria contra la opinione dell'auersario qual supponē, che la proportione della maggior potentia alla menore esser menore di quella della maggior uelocita alla menore. Et con li medesimi argomenti se procederia quando che quel supponesse che la proportione della maggior potentia alla menore fusse maggiore di quella della maggior uelocita alla menore, distrutto adunque l'opposito rimane il proposito, hor per la seconda parte della nostra conclusione, dico, che la proportione della uelocita delli descensi, & delli contrari moti, cioe delli ascensi de detti corpi è una medesima, ma trasmutatiuamente, cioe che la proportione della uelocita del corpo.a.b. essendo da qualche altra uertu imposta nell'altro braccio della libra in alto elleuato (poniamo per fin alla linea della direttione) alla uelocita del corpo.c.dalla medesima uertu, pur in alto elleuato per fin alla medesima linea della direttione sara, si come quella della uelocita.k.alla uelocita.g.h.ouer della potentia.f.alla potentia.d.e.ouer del cor-*



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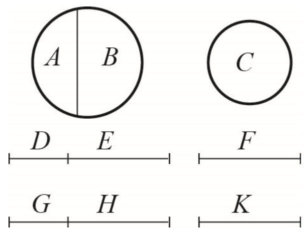
that of their contrary motions (that is, of their ascents) is concluded to be the same, but inversely. S.A. Illustrate this proposition for me. NIC.

Let there be, again, the two bodies *ab* and *c* of the same kind but different size, and let *ab* be the larger, and let the power of *ab* be *de*, and that of *c* be *f* [See Fig. 4.9]; and since the body of greater power or heaviness descends more swiftly (by the second petition), let the speed in descent of the body *ab* be *gh* and that of *c* be *k*. Now I say that the ratio of the power *de* to the power *f* is the same as that of the speed *gh* to the speed *k*, while that of their contrary motions is the same but inversely; that is, the ratio of the speed of the body *ab* to the speed of the body *c* in contrary motion (that is, in ascending) is as that of the power *f* to the power *de*, or as that of the body *c* to the body *a*. This is demonstrated in the same way as the foregoing, that is if the ratio of the power *de* to the power *f* is not (for the adversary) as the ratio of the speed *gh* to the speed *k*, it will necessarily be greater or less; assume it be less. Of the power *de* assume the part *d* equal to *f*, and similarly of the speed *gh* assume the part *g* equal to *k*; and as in the preceding we will argue that the ratio of the whole power *de* to its part *d* will necessarily be (by Euclid V.7,)[37] as the ratio of the same power *de* to the power *f* (because *d* is equal to *f*) and similarly the ratio of the whole velocity *gh* is to its part *g* as that of *gh* to *k*. Therefore the ratio of the whole power *de* to its part *d* will be less than that of the whole velocity *gh* to its part *g*. Therefore (by Euclid V.30),[38] the ratio of the whole power *de* to the residual *e* will be greater than that of the whole speed *gh* to the remaining *h*, which will be against the opinion of the adversary, who wants the ratio of the greater power to the less to be smaller than that of the greater speed to less. Thus, the contrary destroyed, the purpose stands.[39] Now for the second part of our conclusion, I say that the ratio of the speeds of the descents and of the contrary motions (that is, of the ascents) of the said bodies is the same, but inversely; that is, the ratio of the speed of the body *ab* in being raised by some other strength imposed on the other arm of the balance (say, to the line of direction) to the speed of the body *c* raised also by the same strength to the same line of direction will be as that of the speed *k* to the speed *gh*, or of the power *f* to the power *de*, or of the bo[–]



[Fig. 4.9]

po.c.al corpo.a.b.perche quanta uertu, ouer potentia ha un corpo graue per descendere al basso, tanta ne ha anchora per resistere al moto contrario, cioe à che il uolesse tirare, ouer à leuare in alto adunque la potentia del corpo.a.b. per resistere à che il uolesse elleuare in alto, sara tanto quanto la sopradetta.d.e. & quella del corpo.c.sara tanto quanto la sopradetta.f. Adunque quella uertu che nell'altro braccio della libra sara atta ad elleuare cosi à pena il detto corpo.a.b.per fin alla linea della direttione, quella medesima sara atta ad elleuare il detto corpo.c.tanto piu uelocemente (per fin alla detta linea della direttione) quanto che la sua resistentia sara proportionalmente menore di quella del corpo.a.b.& perche la detta resistentia del detto corpo.c. e tanto menore della resistentia del corpo.a.b.quanto che la sua potentia.f.della potentia.d.e. Adunque la uelocita del corpo.c. (nel moto contrario) alla uelocita del corpo.a.b.sara, si come la potentia.e.d.alla potentia.f.ouer come che il corpo.a.b.al corpo.c. che il proposito.

CORRELARIO.

DA qui se manifesta qualmente la proportione della grandezza di corpi di uno medesimo genere, & quella della lor potenzia, & quella della lor uelocita nelli lor descensi esser una medesima. Et similmente quella della lor uelocita nelli moti contrarij, ma trasmutatiuamente. S. AMBASCIATORE. E ue ho inteso, seguitati pur. NICOLO.

QVESITO XXX. PROPOSITIONE III.

SE saranno dui corpi simplicemente eguali di grauita, ma ineguali per uigor del sito, ouer positione, la proportione della lor potentia, & quella della lor uelocita necessariamente sara una medesima. Ma nelli lor moti contrarij, cioe nelli asceosi, la proportione della lor potentia, & quella della lor uelocita se afferma esser la medesima, ma trasmutatiuamente. S. AMBASCIA. Fatemi la dimostratione di questo. NICOLO.

SIANO Li dui corpi.a.&.b. simplicemente eguali di grauita, & sia la libra.c.d. il cui centro, ouer sparto il ponto. e. & sia nella strema parte del brazzo.e.c.cioe in ponto.c.appeso, & sostentato il corpo.a.& in uno altro luoco piu propinquo al sparto nel medesimo brazzo, hor sia in ponto.f.ui sia sostentato il corpo.b. Et à ben che questi dui corpi siano simplicemente eguali di grauita, nondimeno (per la quarta petitione) il corpo.a.sara (per uigor del luogo) piu graue del corpo.b. perche il descenso di quello qual sia lo.c.h. e manco obliquo del descenso del corpo.b, qual sia lo.f.g. (per la terza, & quarta petitione) essendo adunque il corpo.a. piu graue, secondo il sito del corpo.b.sara etiam piu potente, & essendo piu potente (per la seconda petitione) nelli descensi descendera piu uelocemente del corpo.b. & nelli moti contrarij, cioe nelli ascensi piu tardamente. Dico adunque che la proportione della lor uelocita nelli descensi esser simile à quella della loro potentia, & quella delli lo-

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dy *c* to the body *ab*. For that strength or power that a heavy body has by descending, it has also for resisting the contrary motion against anyone who wants to draw it or lift it up. Therefore the strength of the body *ab* to resist whatever would raise it will be as much as the said *de*, and that of the body *c* will be as much as the said f. hence that strength which, on the other arm of the scale, will be barely able thus to elevate the said body *ab* to the line of direction will be able to raise the said body *c* so much the more swiftly to the line of direction as its resistance shall be proportionately less than that of the body *ab*. And since the said resistance of the body *c* is as much less than the resistance of the body *ab* as the power *f* than the power *de*, thus the speed of the body *c* in contrary motion will be to the speed of the body *ab* as the power *de* to the power *f*, or as the body *ab* to the body *c*; which is the purpose.

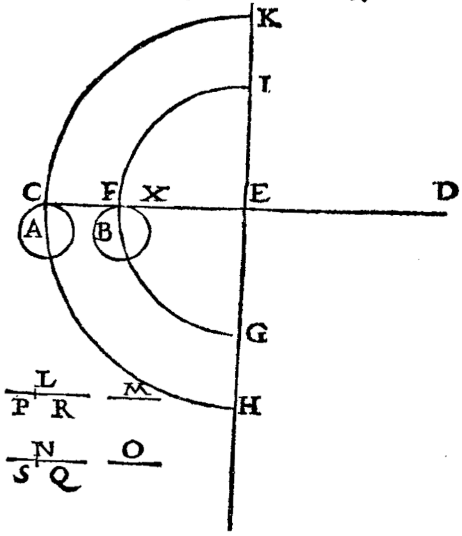
COROLLARY.

From this it is manifest how the ratio of the volumes of bodies of the same kind, and that of their powers, and that of their speeds in descent, is one and the same ratio. And similarly that of their speeds in contrary motion is the inverse ratio.[40] S.A. I understood this; continue. NICOLO.

QUESTION XXX. PROPOSITION III.

If there are two bodies simply equal in heaviness, but unequal positionally, the ratio of their powers and that of their speeds will necessarily be the same. But in their contrary motions, that is in ascent, the ratio of their powers and that of their speeds is affirmed to be the same but inverse. S.A. Give me the proof of this. NICOLO.

Let there be the two bodies *a* and *b* [See Fig. 4.10], simply equal in heaviness, and the balance *cd*, whose centre of fulcrum is the point *e*; and at the end of the arm *ec*, that is, at the point *c*, let there be hung and sustained the body *a*, and at another place closer to the fulcrum on the same arm, say, at *f*, the body *b* is sustained. And though these two bodies are simply equal in heaviness, nevertheless (by the fourth petition) the body *a* will be positionally heavier than the body *b*, because its descent will be *ch*, less oblique than the descent of the body *b*, which is *fg* (by the third and fourth petitions). Hence the body *a*, being positionally heavier than the body *b*, will also be more powerful; and being more powerful, it will (by the second petition) fall more swiftly than the body *b* in descents, and in the contrary motion, of ascents, it will rise more slowly. I say therefore that the ratio of their speeds in descents is similar to that of their powers, and that of their ro ascensi esser pur la medesima, ma trasmutatiuamente, et per dimostrar la prima parte, sia la potentia del corpo.a.la.l.& quella del corpo.b.la.m. & la uelocita del corpo a.(nelli descensi) la.n.& quella del corpo.b.la.o. Dico che la proportione della uelocita.n.alla uelocita.o.esser, si come quella della potentia.l.alla potentia.m.la qual cosa se dimostra, si come la precedente, cioe se poßibil fusse, che la proportione della potentia.l.alla potentia.m.(per l'auersario) potesse esser menore di quella della uelocita.n. alla uelocita.o.sumendo della potentia.l.la parte.p.eguale alla.m.& della uelocita.n.la parte.q.eguale alla.o.& arguendo, come nella precedente, cioe che la proportione di tutta la potentia.l. alla sua parte.p.(per la.7.del quinto di Euclide) sara menore di quella di tutta la uelocita.n. alla sua parte.q.Onde (per la.30.del quinto di Euclide) la proportione della medesima potentia.l.all'altra sua parte, ouer residuo.r.hauera maggior proportione di quello, che hauera tutta la uelocita.n.all'altra sua parte, ouer residuo.s.la qual cosa saria inconueniẽte, et contra la opinione dell'auersario, qual suppone che la proportione della maggior potentia alla menore, esser menore di quella della maggior uelocita, alla menore, & il medesimo inconueniente seguiria quando che l'auersario, supponesse che la proportione della potentia.l.alla potentia.m.fusse maggiore di quella della uelocita.n.alla uelocita.o.distrutto adunque l'opposito rimane il proposito. La seconda parte se risolue, ouer arguisse, si come nella precedente, cioe che quella potentia, che nell'altro brazzo della libra (poniamo in ponto.d.) sara atta ad elleuare il corpo.a.per fin alla linea della direttione, cioe in ponto.k.quella medesima sara atta ad elleuare tanto piu uelocemente il corpo.b.per fina al ponto.i.quanto che la potentia del detto corpo.b.(qual'è la.m.) è menore della potentia del corpo.b.(qual'è la.l.) perche quanto che la potentia d'un corpo è menore tanto men resiste al moto contrario, & econuerso, adunque la uelocita del corpo.b.à quella del corpo.a.(nelli ascensi) sara, si come quella della potentia.l.alla potentia.m. che è il secondo proposito. S. AMB. Questa è stata assai bella propositione, ma seguitati pur. NIC.



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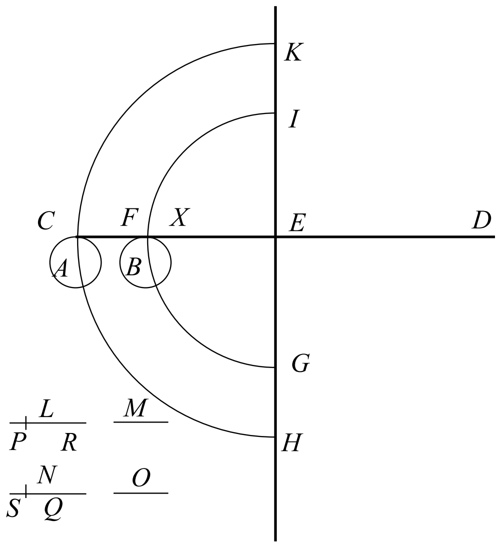
ascents is also the same, but inversely. And to demonstrate the first part, let the power of the body *a* be *l* and that of the body *b* be *m*, and let the speed of the body *a* in descents be *n*, and that of the body *b* be *o*. I say that the ratio of the speed *n* to the speed *o* is as that of the power *l* to the power *m*, which is demonstrated as in the preceding, that is if the ratio of the power *m* (for the adversary) to the power *f* is less (for the adversary) than the ratio of the speed *n* to the speed *o*, by assuming the part *p* of the power *l* equal to *m*, and s of the speed *n* the part *q* equal to *o*; and arguing as in the preceding that the ratio of the whole power *l* to its part *p* will necessarily be (by Euclid V.7)[41] less than the ratio of the whole velocity *n* to its part *q*. Therefore (by Euclid V.30),[42] the ratio of the same power *l* to the residual *r* will be greater than that of the whole speed *n* to the remaining part or residual *s*, which will be unconvincing and against the opinion of the adversary, who wants the ratio of the greater power to the less to be smaller than that of the greater speed to less. And the same holds true when the adversary would assume the ratio of the power *l* to the power *m* would be grater than the ratio of the speed *n* to the speed *o*. Thus, the contrary destroyed, the purpose stands.[43] The second part is resolved or argued just as before; that is, that that power which in the other arm of the scale (assume at the point *d*) will be able to lift the body *a* to the line of direction, that is, to the point *k*, will be able to raise the body *b* to the point *i* as much more swiftly as the power of the body *b* (which is *m*) is less than the power of the body *a* (which is *l*), because by whatever amount the power of a body is less, by that much less it resists contrary motion, and conversely. Therefore the speed of the body *b* (in ascents) will be to the speed of the body *a* as the power l is to the power *m*; which is the second purpose. S.AMB. This is a very pretty proposition, but proceed. NIC.



[Fig. 4.10]

Question

QVESITO XXXI. PROPOSITIONE IIII.

LA proportione della potentia di corpi simplicemente equali in grauita, ma inequali per uigor del sito, ouer positione, & quella delle lor distantie dal sparto, ouer centro della libra, se approuano esser equali. S.A. Datime uno essempio. N.

SIano li dui corpi.a.&.b.della figura precedente simplicemente equali in grauita & sia la libra.c.e.d.el centro, ouer sparto della quale sia el ponto.e.& sia appeso el corpo.a, in ponto.c.& lo corpo.b.nel ponto.f.come nella figura precedente appare. Dico, che la proportione della potentia del corpo.a. (quale sia la.l.) alla potentia del corpo.b. (quale sia la.m.) esser simile à quella, ch'è dalla distantia, ouer brazzo.e.c.alla distantia, ouer brazzo.e.f.& tutto questo si approua secondo lordine della precedente, cioe, se la proportione della distantia, ouer brazzo.c. e. alla distantia, ouer brazzo.f.e.non è (per lauersario, si come quella, ch'è dalla potentia.l.alla poteutia.m. adunque necessariamente sara maggiore, ouer minore, hor sia prima (se poßibil è) menore sia del brazzo, ouer distantia.c.e.maggiore cauato el brazzo, ouer distantia.e.f.menore dalla banda uerso.c.quale sia la.c.x.& dalla potentia.l.ne sia cauata la parte.p.equal alla.m. Adunque per la.7.del quinto di Euclide) la proportione di tutta la distantia, ouer brazzo.e.c.alla sua parte.c.x.hauera menor proportione, di quello, che hauera tutta la potentia.l.alla sua parte.p. Onde per la.30. del quinto di Euclide) la proportione del brazzo, ouer distantia.c.e.alla restante distantia, ouer brazzo.e.x.hauera maggior proportione di quello hauera la potentia.l.alla restante potētia.r. la qual potentia.r.uerria ad esser la potenza del medesimo corpo.b. stante nel ponto x.la qual cosa saria inconueniente, perche, se la proportione della maggiore distantia dal sparto alla menore (per lauersario) hauera maggior proportione, che la maggior potentia alla menore, questo doueria seguire in ogni positione, & tamen se uede occorrere al contrario, cioe, che la proportione della distantia.c.e.alla distantia.e. x.saria maggiore di quella della potentia.l.alla potentia del corpo.b.nel sito, ouer luoco, doue.x.distrutto adunque lo opposito rimane il proposito.

CORRELARIO.

DAlle cose dette, & dimostrate, se manifesta non solamente la proportione delle distantie dal sparto nel brazzo della libra, & quella delle potētie di corpi simplicementi equali in grauita, in tai siti, ouer luochi, & simelmente la uelocita de quelli nelli descensi esser una medesima, ma anchora li lor descensi, & anchora li loro aßcēsi oßeruano la medesima, perche qual proportione è dal brazzo.e.c.al brazzo.e.f.tale è dal curuo descenso.c.h.al curuo descenso.f.g.& simelmēte del curuo assenso.c.k.al curuo assenso.f.i.pche li dette descensi, & assensi uengono à esser cadauno de loro la quarta parte della circonferentia de dui ceochij.delli quali el semidiametro del maggiore uerria à esser el brazzo, ouer distantia.e.c.et del menore el brazzo, ouer distātia.e.f. S.A. Anchor questa é stata una bella propositione seguitati. N.

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QUESTION XXXI. PROPOSITION IIII.

The ratio of the power of bodies simply equal in heaviness, but unequal in positional strength, proves to be equal to that of their distances from the fulcrum or centre of the scale. S.A. Give me an example. N.

Let there be the two bodies *a* and *b* of the preceding figure, simply equal in heaviness, and let the scale be *ced*, whose centre or fulcrum is at the point e; and let there be hung the body *a* at the point *c* and the body *b* at the point *f*, as shown in the preceding figure. I say that the ratio of the power of the body *a*, which is *l*, to the power of the body *b* (which is *m*) is like that of the distance or arm ec to the distance or arm *ef*; and this is proved according to the order of the preceding, that is if the ratio of the distance, or arm, *ce* to the distance, or arm, *fe* is not (for the adversary) as that of the power *l* to the power *m*, it will necessarily be greater or less; assume it be less. Of the greater arm, or distance *ce*, be subtracted the arm, or distance *ce*, from the side of *c,* and let it be *cx*, and from the power *l* let be subtracted the part *p* equal to *m.* Then (by Euclid V.7),[44] the ratio of the whole distance, or distance, *ec* to its part *cx* will be less that that of the power *l* to its part p. Therefore (by Euclid V.30),[45] the ratio of the arm, or distance *ce* to the remaining distance, or arm *ex* will be greater than that of the power *l* to the remaining power *r*, which power *r* is the power of the same body *b* standing at point *x*. This will be unconvincing because if the ratio of the greater distance from the fulcrum to the less (for the adversary) is greater that the greater power to the less, this could occur in any position, and the same holds true in the contrary case, namely when the ratio of the distance *ce* to the distance *ex* will be greater that that of the power *l* to the power of the body *b*, in the position *x*. Thus, the contrary destroyed, the purpose stands.[46]

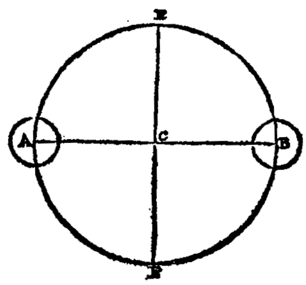
COROLLARY.

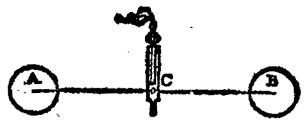
From the things said and demonstrated not only is manifest the sameness of the ratio of the distances from the fulcrum along the arms of the scale, and that of the powers of bodies simply equal in heaviness in such sites or places, and likewise of their speeds in descent; but also both their descents and their ascents observe the same [rule]; for the ratio of the arm *ec* to the arm *ef* is that of the curved descent *ch* to the curved descent *fg*, and likewise of the curved ascent *ck* to the curved ascent *fi*. For the said descents and ascents are in each case one–fourth the circumference of the two [respective] circles, of which the radius of the larger is that of the arm or distance *ec*, and of the smaller, that of the arm or distance *ef*.[47] S.A. This also has been a pretty proposition. Continue. N.

Z

LIBRO
QVESITO XXXII. PROPOSITIONE V.

QVando, che la poſitione de una libra de brazzi equali ſia nel ſito della equalita, & nella iſtremita de l'uno, e l'altro brazzo ui ſiano appeſi corpi ſimplicemente equali in grauita, tal libra non ſe ſeparara dal detto ſito della equalita, & ſe per caſo la ſia da qualche altro peſo in luno de detti brazzi impoſto ſeparata dal detto ſito della equalita, ouer amente con la mano, remoſſo quel tal peſo, ouer mano, tal libra de neceſsita ritornara al detto ſito della equalita. S. A. Queſta è quella Queſtione, della quale uoi dite, che manca Ariſtotile nelle ſue Queſtioni Mechanice. N. Coſi è Signore. S.A. Molto haro à caro à intendere la cauſa di tal effetto, e pero ſeguitate. N. Sia eſſempi gratia la libra.a.c.b. el centro della quale ſia il ponto.c. & ſia el brazzo.a.c. equale al brazzo.b.c. & ſtia nel ſito della equalita, come ſe prepone. Et che nella iſtremita de luno, e laltro brazzo ui ſia appeſo uno corpo (poniamo el corpo.a.&.o.) li quali corpi ſiano ſimplicemente equali in grauita. Dico, che la detta libra (per la impoſitione de detti corpi) non ſe ſeparara dal detto ſito della equalita, & ſe pur quella fuſſe ſeparata dal detto ſito, ò per la impoſitione di qualche altro peſo, ouer con la mano, remoſſo che ſia quel tal impoſto peſo, ouer mano, tal libra de neceſsita ritornara al detto ſito della equalita. La prima parte è manifeſta, perche li detti dui corpi ſono ſimplicemente di equal grauita (dal preſuppoſito) et ſimelmẽte ſono equalmente graui per uigor del ſito, per la quarta petitione (per eſſer li loro deſcenſi equalmente obliqui) e pero eſſendo quelli ſi per uigor del ſito, come che ſimplicemẽte duna equal grauita, e potentia, e pero niun de loro ſara atto à poter elleuar l'altro, cioe à farlo aſcendere di moto contrario, e pero reſtaranno nel medeſimo ſito della equalita. S.A. Queſto ue credo & ue lo haueria largamente conceſſo ſenza altra demonſtratione, per eſſer coſa naturale. Ma ſeguitati la ſeconda parte, la qual me pare molto piu aſtrata, ouer lontana dal noſtro intelletto naturale dell'altra. N. Per la ſecondu parte ſia pur anchora la libra.a.c.b. de brazzi equali, et nella iſtremita de quelli ſtano pur appeſi li dui corpi.a.et.b. ſimplicemente equali in grauita, la qual libra p le ragioni di ſopra adutte ſtara nel ſito della equalita, come di ſotto appar i figura.





HOR eſſendo ſpinto el brazzo.a.c. al baſſo con la mano, ouer per la impoſitione di qualche altro peſo ſopra el corpo.a. remoſſo uia la mano, ouer quel tal peſo, el brazzo di tal libra reaſcendera, & ritornera al ſuo primo luoco della equalita, & per aſsignar la cauſa propinqua di tal effetto, ſia deſcritto ſopra el centro.c. el cerchio.a.e.b.f. per el uiazzo, che fariano li detti dui corpi alzando, ouer arbaſſando li brazzi della detta libra, & ſia tirata la linea della direttione, quale ſia la.e.f. & ſia diuiſo l'arco.a.f. in quanti parti equali ſi uoglia (hor ſia in quattro) nelli trei ponti,

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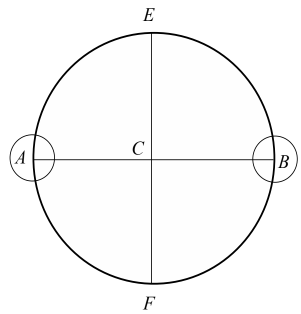
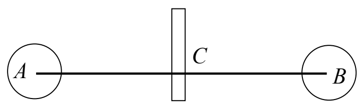
QUESTION XXXII. PROPOSITION V.

When a scale of equal arms is in the position of equality, and at the end of each arm there are hung weights simply equal in heaviness, the scale does not leave the said position of equality; and if it happens that by some other weight [or the hand] imposed on one of the arms it departs from the said position of equality, then, that weight or hand removed, the scale necessarily returns to the position of equality.[48] S.A. This is that problem which you told me Aristotle omitted in his Questions of Mechanics.[49] N. So it is, Sir. S.A. I look forward to hearing the cause of that effect; therefore go on. N. Let there be, for example, the scale *acb*, the centre of which is at the point *c* [See Fig. 4.11], and let the arm *ac* equal the arm *bc*, and let it be in the position of equality as assumed. And at each extremity let there be hung a body (the bodies *a* and *b*) which are simply equal in heaviness. I say that the said scale, by the imposition of the said bodies, will not leave the position of equality; and if it is separated from that position of equality either by the imposition of some other weight or by hand, that imposed weight or hand being removed, the scale will of necessity return to the position of equality. The first part is manifest because the said two bodies are simply equal in heaviness (by assumption), and similarly they are equal positionally heavy by the fourth petition (their descents being equally oblique. Hence, being equal in weight and power both simply and positionally), neither of them will be able to raise the other, that is, to make it ascend with contrary motion; and so they will rest in the same position of equality.[50] S.A. This I believe and would have conceded it freely without any demonstration, it being a natural thing. But go on to the second part, which appears to me much more abstract, or remote from our natural intellect, than the other. N. For the second part, let there be also the scale *acb* of equal arms, and at its extremities let there also be hung the two bodies *a* and *b*, simply equal in heaviness, which scale by the reasons adduced above will stand in the position of equality as appears in the figure.



[Fig. 4.11]



[Fig. 4.12]

Now the arm *ac* having been driven down by hand or by the imposition of some weight on the body *a*, if we take away the hand or that weight, the arm will rise again and return to its first position of equality.[51]And to assign the immediate cause of that effect, let there be described about the centre *c* the circle *aebf* for the journey that the two bodies will make in rising or falling of the arms of the scale [Fig. 4.13]; and draw the line of direction *ef*, and divide the arc *af* into as many equal parts as you like (say, into four parts) at the three point.

q.f.u.& in altre tante sia anchor diuiso l'arco.e.b.nelli trei ponti.i.l.n. & dalli detti trei ponti.n.l.i.siano tirate le tre linee.n.o.l.m.&.i.k.equidistante al sito della equalita,cioe al diametro,ouer linea.a.b.le quale segaranno la linea.e.f. della direttione ne li tre ponti.z y.x.Simelmente dalli tre ponti.q.f.u.siano tirate le tre linee.q.p.f.r.&.u.t.pur equidistante alla medesima linea.a.b.le quale segaranno la medesima linea della direttione.e.f.nelli trei ponti.&.ɔ.ɣ.Et dapoi sia arbassato con la mano il corpo.a.(ouer con la impositione di qualche altro peso)per fin al ponto.u. & laltro corpo.b.(à quel opposito)in tal positione se trouara esser assesо de moto contrario per fin al ponto.i.Onde per queste cose cosi disposite ueniremo ad hauer diuiso tutto el descenso a.u.fatto dal detto corpo.a.nel discendere in ponto.u.in tre descensi,ouer parti equali,le quale sono.a.q.q.f.&.f.u.& simelmente tutto el descenso.i.b. qual faria il detto corpo.b.nel discendere,ouer ritornare al suo primo luoco(cioe in ponto.b.)uerra ad eßer diuiso in trei descensi,ouer in tre parti equali le quali sono.i.l.l. n. &.n.b. & cadauno de questi tre,& tre partiai descensi capisse una parte della linea della direttione,cioe il descenso dal.a.al.q.piglia, ouer capisse della linea della direttione la parte.e.&.& lo descenso.q.f.capisse la parte.&.ɔ. & lo descenso.f.u.capisse la parte. ɔ. ɣ. & laltro descenso,che resta à descendere al detto corpo.a.cioe el descenso.u.f.capiße la linea,ouer parte.ɣ.f.Et simelmente el descenso del corpo.b. dal ponto.i. al ponto.l. capisse della medesima linea della direttione la parte.x.y.& nel descenso dal ponto.l. al ponto.n.capisse la parte.y.z.& dal ponto.n.al ponto.b.capisse la parte.z.c.et tutte queste parti sono fra loro inequale,cioe la parte.c.z.è maggiore della.z.y.& la.z.y.della.y.x.& la.y.x.della.x.e.& simelmente la parte. c. &. è maggiore della parte.&.ɔ.& la parte.&.ɔ.della parte.ɔ.ɣ.& la.ɔ.ɣ.della.ɣ.f.& tutto questo facilmente Geometrice si puo prouare,& simelmente se puo prouare,la parte. ɣ.f. essere equale alla parte.e.x.& la parte.ɣ.ɔ.alla parte.x.y.& la parte.ɔ.&.alla parte.y.z.& la parte.&.c.alla parte.z.c.Hor per tornare al nostro proposito. Dico,che il corpo.b.stante quel nel ponto.i.uien à esser piu graue,secondo il sito del corpo. a. stante quello in ponto.u. (come di sotto appar in figura)perche il descenso del detto corpo b.dal ponto.i.nel ponto.l.è piu retto del descenso del corpo.a. dal ponto. u. nel ponto f.(per la seconda parte della quarta petitione)perche capisse piu della linea della direttione,cioe,che nel descendere il detto corpo.b.dal ponto.i. nel ponto.l. lui capisse, ouer piglia della linea della direttione,la parte.x.y.& il corpo.a. nel discendere dal ponto.u.nel ponto.f.lui caperia della detta linea della direttione,la parte.ɣ.f.& perche la parte.x.y.è maggiore della linea,ouer parte.ɣ.f. (per la.17. diffinitione) piu obliquo sara il descenso dal ponto.u.al ponto.f.di quello dal ponto.i.al ponto.l. Onde (per la seconda parte della quarta petitione)il corpo.b.in tal positione sara piu graue secondo il sito del corpo.a.eßendo adunque piu graue,leuando uia lo imposto peso, ouer la mano dal corpo.a.(per il conuerso della quinta petitione)lui fara reascendere di moto contrario il detto corpo. a.dal ponto.u.al ponto.f. & lui descendera dal ponto.i.nel ponto.l.nel qual ponto.l. lui uenira à trouarse anchora piu graue del detto corpo.a.secondo el sito , perche il detto corpo.a. stante nel ponto. f. hauera il descenso.f.u.piu obliquo del descenso.l. n. del corpo. b. perche capisse men parte della

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q, *s*, *u*; and into as many parts divide the arc *eb* at the three points *i*, *l*, *n*; and from the said three points *i, l*, *n* draw the three lines *no*, *lm*, and *ik* parallel to the position of equality, that is, to the diameter or line *ab*, which [three lines] shall cut the line of direction *ef* at the three points *x*, *y*, *z*. similarly, from the three points *q*, *s*, *u* are drawn the three lines *qp*, *sr*, and *ut*, also parallel to the same line *ab*, which shall cut the same line of direction *ef* at the three points *w, ρ, k*.. And now let the body *a* be depressed by hand (or by the imposition of some other weight) to the point *u*, and the other body *b* (opposite to that) will be found to be raised with contrary motion to the point *i*. Now with things arranged this way, we have come to divide the whole descent *au* made by the body *a* in descending to the point *u* into three equal descents or parts, which are *aq*, *qs*, and *su*; and similarly the whole descent *ib* which the body *b* would make in descending or returning to its original place (that is, the point *b*) will come to be divided into three equal descents or parts which are *il*, *ln*, and *nb*; and each of these three–plus–three partial descents includes one part of the line of direction; namely, the descent from *a* to *q* partakes of or contains the part *cw* of the line of direction, and the descent *qs* contains the part *wj*, and the descent *su* contains the part *jd*, and the other descent that remains to the said body *a*, that is, the descent *uf* contains the line or part *de*. Likewise the descent of the body *b* from the point i to the point *l* contains the part *xu* of the same line of direction, and in the descent from the point *l* to the point *n* it contains the part *yz*, and from the point *n* to the point *b* it contains the part *zc*, and all these parts are unequal; that is, the part *cz* is greater than *zy*, and *zy* is greater than *yx*, and *yx* than *xe*; and similarly the part *cw* is greater than the part *wj*, and *wj* than *jd*, and *jd* than *df*, and all this can be easily proved geometrically; and also the part *df* can be proved equal to the part *ex*, and *jd* to *xu*, and *wj* to *yz*, and *cw* to *zc.* Now to resume our proposition, I say that the body *b* standing at the point *i* comes to be positionally heavier than the body *a* standing at the point *u* (as appears in the figure), because the descent of the body *b* from the point *i* to the point *l* is more direct than the descent of the body *a* from the point *e* to the point *f* (by the second part of the fourth petition), because it partakes more of the line of direction. That is, the body *b* in descending from the point *i* to the point *l* partakes the part *xy* of the line of direction, and the body *a* descending from the point *u* to the point *f* partakes the part *df* of the line of direction, and since the part *xy* is greater than the line or part *de*, the descent (by definition 17) from the point *u* to the point *f* will be more oblique than that from the point *i* to the point *l*. Whence (by the second part of the fourth petition) the body *b* in that position will be positionally heavier than the body *a*. And being thus heavier, when the imposed weight or hand is taken away from the body *a*, it will (by the converse of the fifth petition) make the said body *a* re–ascend with contrary motion from the point *u* to the point *s*, and it will descend from the point *i* to the point *l*; and it will come to be found still positionally heavier than the body *a*, because the said body *a* standing at the point *s* will have the descent *su* more oblique than the descent *ln* of the body *b* because it partakes less of

Z *ij*

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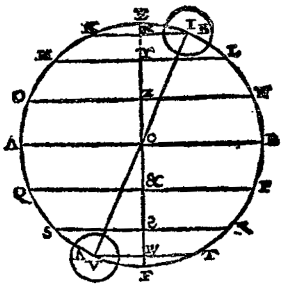
detta linea della direttione, cioe, che la parte.ϡ.ψ.è menore della parte.y.z. Onde per le ragioni di sopra adutte, el detto corpo.b.fara elleuare il detto corpo.a. & ascendere nel ponto.q.& lui descendera nel ponto.n.nel qual ponto.n.el medesimo corpo.b. si trouara pur piu graue anchora, secondo il sito del corpo.a.perche il descenso dal.q. in.s.è piu obliquo del descenso dal ponto.n.nel ponto.b.per esser la parte.z.c.maggiore della parte.&.ϡ.E pero (per le ragioni di sopra adutte) el detto corpo.b.fara reascendere il detto corpo.a.al ponto.a.(suo primo, & condecente luoco) & lui medesimamente descendara nel ponto.b.pur suo primo, & condecente luoco, cioe nel sito della equalita, nel qual sito li detti dui corpi se trouaranno (per le ragioni adutte nella prima parte di questa) egualmente graui secondo el sito, & perche sono anchora simplicemente egualmente graui, se conseruarano nel detto luoco, come di sopra fu detto, & approuato, che è il nostro proposito.



S. A. Questa è stata una bella demostratione, ma se ben me arricordo, uoi dicesti anchor sopra la detta prima question Mechanica de Aristotile, che quelle sue due conclusioni, che lui ui aduce in fine eßer falsе. N. Eglie il uero. S.A. Per che ragione. N. La ragione di tal particolarita, ouer oppositioni se uerificaranno nella sequente propositione, mediante alcuni correlarij, che dalle cose dette, & dimostrate nella precedente si manifestano, delli quali il primo è questo.

CORRELARIO.

DAlle cose dette, et dimostrate di sopra, se manifesta qualmente un corpo graue in qual si uoglia parte, che lui se parta, ouer remoui dal sito della equalita lui si fa piu leue, ouer leggiero secondo el sito, ouer luoco, & tanto piu, quãto piu sara remoßo da tal sito, essempi gratia. El corpo.a.si trouara esser piu leue nel ponto.u.chi nel põto.s.et nel põto.s.piu che nel põto.q.& nel ponto.q.che nel ponto.a.sito della equalita, p causa della uarieta di descensi, cioe, che luno è piu obliquo dell'altro, cioe el descenso.u f.uiẽ à eßer piu obliquo del descẽso.f.u.perche la parte.f.ψ.della direttione, è menore della.ψ.ϡ.et cosi el descẽso.f.u.uiẽ à eßer piu obliquo del descẽso.q.s. pche la parte.ψ.ϡ.è menore della parte.ϡ.&.& lo descẽso.q.f.uiẽ à esser piu obliquo del descenso.a.q.perche la parte.ϡ.&.è menore della parte.&.c.& per le medesime ragioni si manifesta del corpo.b.cioe, che quello sara piu leue nel põto.i.che nel põto.l.& nel põto.l.che nel ponto.n.& nel ponto.n.che nel ponto.b.sito della equalita.

CORRELARIO SECONDO.

ANchora per le cose dette, & dimostrate se manifesta, che remuouendosi li detti dui corpi dal detto sito della equalita, cioe luno ĩ giuso, et laltro in suso, anchor

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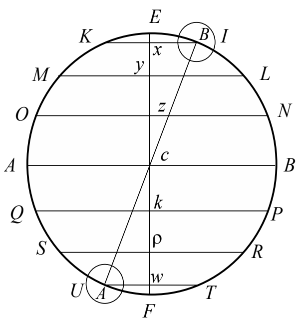
the line of direction; that is, the part *ρw* is smaller than the part *yz*. Whence for the reasons adduced above, the body *b* will raise the body *a* to the point *q*, and *b* will descend to the point *n*, at which point *n* the same body *b* will yet be found appositionally heavier than the body *a* because the descent from *q* to *s* is more oblique than the descent from the point *n* to the point *b*, the part *zc* being greater than the part *k*ρ. And hence (by the reasons adduced above) the body *b* will make the body *a* re–ascend to the point *a* (its first and proper place) and will itself descend to the point *b* (also its first and proper place), that is, to the position of equality, in which position the said two bodies will be found (for the reasons adduced in the first part of this proposition) equally heavy positionally. And since they are also simply equally heavy, they will remain in the said place, as was said and proved above; which is our purpose. S.A. This was a pretty demonstration, but, if I recall correctly, you said also, of the first mechanical problem of Aristotle, that those two conclusions of his that he adduces at the end are false.[52] N. So they are. S.A. For what reason? N. The reason for this objection will be verified in the next proposition, through some corollaries that are manifest from the things said and demonstrated in the above, of which the first is this.



[Fig. 4.13]

COROLLARY.

From the things said and demonstrated above, it is manifest how a heavy body, whenever parted or removed from the position of equality, becomes positionally lighter, and the more the more it is removed from that position. For example, the body *a* will be found lighter at the point *u* than at the point *s*, and more at *s* than at the point *q*, and at *q* than at the point *a*, the position of equality, by reason of the various descents being one more oblique than another. That is, the descent *uf* becomes more oblique than the descent *su* because the part *fw* of the vertical is less than *wf* and so the descent *su* is more oblique than the descent *qs* because the part *wρ* is less than the part *ρk* and the descent *qs* is more oblique than the descent *aq* because the part *ρk* is less than the part *ck* and for the same reasons is manifest for the body *b*, that is, that it will be lighter in the point *i* than in the point *l* and in the point *l* than in the point *n* and in the point *n* than in the point *b* place of the equality.

SECOND COROLLARY.

Also by the things said and demonstrated, it is manifest that the said two bodies being removed from the position of equality, that is, one downward che l'uno, e l'altro sia fatto piu leue secondo il sito, tamen in ogni positione men leue si trouara quello che sara in alto elleuato di quello, che si trouara al basso oppresso, & questo è manifesto per la argomentatione di sopra adutta, cioe che il corpo.b.nel sito, ouer ponto.i.esser piu graue del corpo.a.nel sito, ouer ponto.u. & cosi nelli altri siti superiori si trouara piu graue del corpo.a.nelli siti inferiori, simili. S.A. E ue ho inteso, seguitati. NICOLO.

QVESITO. XXXIII. PROPOSITIONE VI.

QVando che la positione d'una libra de bracci eguali sia nel sito della egualita, & che nella istremita dell'uno è l'altro brazzo ui stano appesi corpi simplicemente ineguali di grauita, dalla parte doue sara il piu graue sara sforzata à declinare per fin alla linea della direttione. S.A. A me non pare che questa uostra propositione possa esser uniuersalmente uera, & questo uoglio che uoi medesimo il confessati, perche uoi sapeti che nel Correlario precedente haueti conchiuso, che remouendosi li detti dui corpi.a.&.b.(dalla figura della precedente propositione) dal sito della egualita, cioe l'uno in giuso, & l'altro insuso, anchor che l'uno è l'altro sia fatto piu leue, ouer leggero, secondo il sito, tamen in ogni positione men leue si trouara quello; che sara in alto elleuato di quello, che si trouara quello, che sara à basso inclinato. N. Eglie il uero Signore. S.A. Se questo è uero, eglie da credere, anci da tener per fermo, che chi imponesse sopra al corpo.a.à basso inclinato, un'altro corpetto qual in grauita fusse eguale à quella differentia, che il corpo elleuato è piu graue, secondo il sito del corpo à basso inclinato, che cadauno de loro restaria nel proprio luoco doue si trouasse, & accio meglio me intendiati, uoi sapeti che il corpo.b.della figura della precedente propositione, stante elleuato per fin al ponto.k.(come in quello appare) & il corpo.a.à basso inclinato per fin al ponto.u.uoi approuasti il detto corpo.b.in tal sito esser piu graue del corpo.a. N. Signore eglie il uero. S.A. Adunque conchiudo che chi imponesse in tal sito un'altro corpetto sopra al corpo.a.qual fusse precisamente di tanta grauita, quanto, che è la differētia, che è fra li detti dui corpi.a.&.b.in tal positione li detti dui corpi restariano fermi, & stabili in tal positione, perche in tal sito se trouariano egualmente potenti, cioe il corpo.b.non saria sofficiente à far reascendere il detto corpo.a.al sito della egualita, per esser il detto corpo.a.(per uigor di quel corpetto aggionto) tanto graue è potente quanto lui, cioe che per quel tanto che il detto corpo.b.è piu potente, ouer graue per uigor del sito del corpo.a.per quel tanto sara piu graue il detto corpo.a.del detto corpo.b.per uigore della grauita di quel simplice corpetto aggiontoui sopra, per ilche il detto corpo.b.non sara atto à far reascendere il detto corpo.a.al sito della egualita, & manco il corpo.a.sara atto à potere piu elleuare il detto corpo.b.del sito.i.e pero l'uno è l'altro de necesita non se potra partire di tal suo luoco, cioe il corpo.a.con la gionta di quell'altro corpo, non potra reascendere al sito della equalita, ne manco potra descendere alla linea della direttione, cioe al ponto.f.come se conchiude nella uostra propositione, & pur il detto corpo.a.insieme con quell'altro corpetto aggionto, saria simplicemente piu graue del corpo.b.e per tanto non poteti ne-

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and the other upward both are made positionally lighter, and yet the one that is lifted up is found to be less light than that which is pressed down; and this is manifest by the argumentation adduced above. That is, the body *b* at the point *i* is heavier than the body *a* at the point *u*, and so at the other higher points it will be heavier than at the corresponding lower points. S.A. I understand; continue. NICOLO.

QUESTION. XXXIII. PROPOSITION VI.

Whenever a scale of equal arms is in the position of equality, and at the end of each arm weights simply unequal in heaviness are hung, it will be pressed downward up to the line of direction on the side where the heavier weight shall be. S.A. To me it does not appear that this proposition of yours can be universally true, and I think you have confessed this to me yourself, since you know that in the preceding corollary you have concluded that if the two bodies *a* and *b* (in the figure for the foregoing proposition) are removed from the position of equality, that is, one downward and the other upward, then, although both are made positionally lighter, yet in every position that one which is lifted up will be less light than that which is pressed down. N. True. S.A. If this is true, it is to be believed, or rather thought certain, that, if one should impose on the body *a*, pressed down, another little body which equated in heaviness that difference by which the upper body exceeded positionally the heaviness of the lower, then each would remain in the place where it was. That you may better understand me, you know that, the body *b* of the figure in the preceding proposition being lifted to the point *i* (as shown there) and the body *a* being depressed to the point *u*, it was proved by you that the body *b* was heavier than the body *a* in that position. N. Sir, this is true. S.A. Therefore I conclude that, if one should add to the body A in that position another small body of precisely as much heaviness as the difference between the said two bodies *a* and *b* in that position, the two bodies would remain fixed and stable in that position; for in that position they would be equally powerful. That is, the body *b* would not be sufficient to cause the body *a* to re–ascend to the position of equality, the said body a being (by the strength of that added little body) as heavy and powerful as it [*b*]. Indeed by the amount that the body *b* is positionally more powerful or heavier than the body *a*, the body *a* is heavier than the body *b* by strength of the simple heaviness of that little body added to it; whence the body *b* will not be able to make the body *a* re–ascend to the position of equality; and still more difficultly will the body gare che tal uostra propositione non sia falsa in quanto al generale, eglie ben uero, che se la grauita di quel corpetto che fusse aggionto sopra al detto corpo.a. fusse maggiore della grauita, nella quale il corpo.b. è piu graue per uigor del sito del corpo.a. seguiria quello che nella detta uostra propositione se conchiude, & se per caso tal grauita di corpetto fusse menore di detta differentia, tal corpo.b. faria ascendere il detto corpo.a. in un'altro sito piu alto del ponto.u. secondo che piu, ouer men scarsezasse la grauita di tal corpetto della detta differentia che è fra loro per uigor del sito. N. Questa oppositione di V.S. certamente è molto speculatiua, & bella, nondimeno auertisco quella, che se ben il corpo.b. in tal sito.i. sia piu graue del corpo.a. nel sito.u. la differentia di queste due grauita ineguale è tanto piccola, ouer minima, ch'eglie impoßibile à potere ritrouare una cosi piccola, ouer minima differentia fra due quantita ineguale. S. A. Questo che haueti detto mi pare una cosa molto absorda da dire, & manco da credere, perche essendo la quantita continua diuisibile in infinito, eglie una materia à uoler dire, che il sia impoßibile à dare un corpettino di tanta poca quantita, & grauita, quanto che è la differentia che è fra la grauita del corpo.b. nel sito.i. & quella del corpo.a. nel sito.u. N. Signore la ragione è quella che ne chiarisse le cose dubbiose, & che ne discerne il uero dal falso. S.A. Eglie il uero. N. S'eglie il uero, nanti che V.S. dia assoluta sententia alla mia propositione quella ascolti prima le mie ragioni. S.A. Seguitati, & dite cio, che ui pare. N. Sia essempi gratia, la medesima libra.a.b.c. della precedente propositione, nelle istremita, della quale stano pur appesi li dui corpi.a. b. eguali simplicemente in grauita, & sia abbassato con la mano il corpo.a. & elleuato il corpo.b. come di sotto appare in figura. Dico che in tal sito, il corpo.b. è piu ponderoso, ouer graue per uigor del sito del corpo.a. & che la differentia che è fra le grauita de questi dui corpi, eglie impoßibile à poterla dar, ouer trouar fra due quantita ineguale, & per dimostrar questa propositione. Tiro le due rette linee.a.h. & .b.d. perpendicolare uerso il centro del mondo, & tiro anchora le due linee.a.l. & .b.m. contingente il detto cerchio, che descriue li brazzi della libra, l'una nel ponto.a. & l'altra nel ponto.b. Et descriuo anchora una parte de una circonferentia d'un cerchio, contingente il medesimo cerchio.a.e.b. in ponto. b. la qual sia pur d'un cerchio simile, & eguale al medesimo cerchio.a. e. b. la qual parte pongo che sia la.b.z. tal che l'arco.b. z. uien à esser simile, & eguale all'arco.a.f. & anchora similmente posto, cioe nel medesimo sito, ouer luoco, & la linea.b.m. che continge, ouer tocca quello, & perche la obliquita dell'arco.a.f. (per quello che fu detto sopra la terza petitione) uien misurata, ouer considerata per meggio dell'angolo contenuto dalla perpendicolar.a.h. & dalla circonferentia.a.f. in ponto.a. & la obliquita dell'arco.b.f. uien misurata, ouer considerata per meggio dell'angolo contenuto dalla perpendicolar.b.d. & dalla circonferentia. b.f. in ponto.b. adunque il corpo.b. in tal sito ueneria ad esser tanto piu graue del corpo.a. quanto che il detto angolo (contenuto dalla perpendicolar.b.d. & dalla circonferentia.b.f. in ponto.b.) sara menore dell'angolo contenuto dalla perpendicolar.a.h. & dalla circonferentia.a.f. in ponto.a. & perche il detto angolo.h.a. f. è precisamente eguale all'angolo.d.b.z. & lo detto angolo.d.b.z. uien ad esser tanto maggiore dell'angolo contenuto dalla detta perpendicolare.b.d. & dalla circonferen-

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a be able to raise the body *b* from the position *i* so neither can leave its place; that is, the body *a* with that other body added cannot re–ascend to the position of equality, nor can it descend to the line of direction, that is, to the point *f*, as concluded in your proposition. yet the said body *a* together with that other little body added to it would be simply heavier than the body *b*, so you cannot deny that your proposition is in general false; though it is true that, if the heaviness of that little body that was added to the body *a* were greater than the heaviness by which the body *b* was positionally heavier than the body *a*, what is concluded in your proposition would follow. And if it happened that the heaviness of that little body were less than the said difference, the body *b* would make the body *a* ascend to another place higher than the point *u*, according to the greater or less deficiency in heaviness of that little body with regard to their said difference in positional strength. N. This objection of yours, Sir, is certainly a very pretty speculation. nevertheless, I note that although the body *b* in that place *i* is heavier than the body *a* in the place *u*, yet the difference of those two unequal heavinesses is so small or minute that it is impossible to find so small or minute a difference between two unequal quantities. S.A. What you have just said seems to me a quite absurd thing to say and not to be believed. Indeed because a continuous quantity being infinitely divisible, it is a quibble to say that it is impossible to have a body of so little quantity and heaviness as is the difference between the heaviness of the body *b* at the place *i* and that of the body *a* at the place *u*. N. Reason, Sir, is the means of clarifying doubts and distinguishing the true from the false. S.A. Very true. N. If this is true, then before your Excellency forms an absolute opinion of my proposition, hear first my reasons. S.A. Go on and say what you like. N. Let there be, for example, the same scale *abc* of the preceding proposition, at the ends of which are hung the bodies *a* and *b*, equal in simple heaviness; and let the hand depress the body *a* and lift the body *b* as shown in the next figure. I say that in this position the body *b* is positionally more ponderous or heavy than the body *a*, and that the difference between the heavinesses of these two bodies is impossible to give or find between two unequal quantities. And to demonstrate this proposition I draw two straight lines, *ah* and *bd*, perpendicularly to the centre of the world,[53] and I also draw two lines *al* and *bm* tangent to the circle described by the arms of the scale at the points *a* and *b*. I describe also a part of the circumference of a circle touching the same circle *acb* at the point *b*, this being a similar and equal circle, *bz*, such that the arc *bz* is similar and equal to the arc *af* and similarly placed (that is, in position), and the line *bm* which touches or is tangent *to this since the obliquity of the arc af (by what was said about the third petition) is measured by means of the angle contained by the perpendicular ah and the circumference af at the point a* [emphasis added],[54] and the obliquity of the arc *bf* is measured by the angle contained by the perpendicular *bd* and the circumference *bf* at the point *b*, the body *b* in that position will be as much heavier than the body *a* as the said angle (contained by the perpendicular *bd* and the circumference *bf* at the point *b*) will be less than the angle contained by the perpendicular *ah* and the circumference *af* at the point *a*. And since the angle *haf* is precisely equal to the angle *dbz*, and the said angle *dbz* is as much greater than the angle contained by the said perpendicular *bd* and the circumference tia.b.f.in ponto.b.quanto che è l'angolo della contingentia delli dui cerchij.b.z.&.b.f. in ponto.b. & perche il detto angolo della detta contingentia è acutissimo de tutti li angoli acuti de linee rette (come per la decimasesta del terzo di Euclide facilmente si puo approuare) adunque la differentia, ouer proportione, che casca fra l'angolo.b.a.f. & l'angolo contenuto dalla perpendicolar.b.d. & della circonferentia.b.f. in ponto.b.è menore di qual si uoglia differentia, ouer proportione, che cascar possa fra qual si uoglia maggiore, & menor quantita, & cosi (per la terza petitione) la differentia della obliquita del descenso.a.f.& del descenso.b.f. & consequentemente la differentia della detta grauita delli detti dui corpi.a. &.b.secondo il sito è menore, del quale si uoglia fra due quantita ineguale, e pero ogni piccola quantita corporea, che sia aggionta sopra il corpo.a.necessariamente in ogni sito sara piu graue del corpo.b. e pero non cessara di descendere continuamente p fin alla linea direttione, cioe p uigor fin al ponto.f. & cosi continuamente quello andara elleuando il corpo.b. per fin alla detta linea della direttione, cioe per fin al ponto.e.& se questo seguiria in tal sito, come che nella sottoscritta figura appare tanto piu seguiria nel sito della egualita, nel qual sito, ouer luoco non ui è, ouer saria alcuna differentia, p uigor del sito, ne p uigor delli lor descensi, cioe che in tal sito sariano egualmente graui, e pero ogni piccola quantita di peso per minima, che sia, che ui sia imposto dall'una delle bande di qual si uoglia libra (cioe granda, ouer piccola de brazzi eguali) immediate sara declinare necessariamente quella da quella medesima banda, ouer brazzo, & continuara tal sua declinatione (per le ragioni di sopra adutte) per fin alla linea della direttione, cioe per fin al ponto.f.la qual cosa saria contra à quelle due conclusioni, che adduce Aristotile sopra la sua prima questione Mecanica, delle quale altra uolta ne parlai con Vostra Signoria, delle quale in l'una dice, che sono alcuni pesi, li quali imposti nelle piccole libre, non se fanno manifesti con alcuna inclinatione al senso, & che nelle grande libre se fanno manifesti, la qual conclusione, sumendola Mathematicamente, cioe astratta da ogni materia, saria falsissima (per le ragioni di sopra adutte) perche si nelle piccole, come nelle grande libre, da quella banda doue sara posto quel tal peso (per piccol che sia) sara sforzata à declinar per fina alla detta linea della direttione, e pero nella declinatione della piccola, & in quella della granda, non sara proportionalmente alcuna differentia, perche in luna, e l'altra la declinatione sara per fina alla linea della direttione, il medesimo seguiria dell'altra sua conclusione, cioe quando dice, che sono alcuni pesi, li quali sono manifesti in luna, & l'altra sorte de libre, cioe nelle maggiori, & nelle menori, ma molto piu nelle maggiori, la qual conclusione (per le ragioni di sopra adutte) saria pur falsa, perche, come detto in luna, & l'altra sara declinare il brazzo della libra per fina alla linea della direttione. S. AMBASCIATORE. Queste uostre ragioni, & argomenti sono ottimi è buoni, nondimeno nelle libre naturale, ouer materiale il si uede pur seguire la maggior parte delle uolte, come che Aristotile conchiude, & dice, perche se sopra qual si uoglia libra (cioe granda, ouer piccola) ui sara posto uno grano, ouer semenza di papauero, o altra simile piccola quantita, rare libre se ritrouara che per si poca grauita, facciano inclinatione sensibile, & se pur ui se ne ri-

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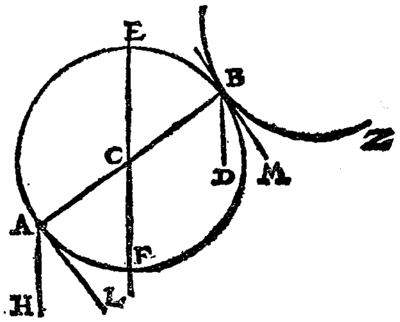
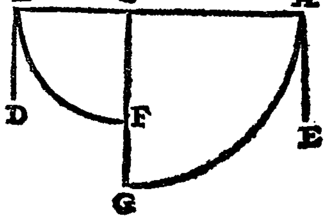
bf at the point *b* as the angle of contingency[55] of the two circles *bz* and *bf* at the point *b*, and since this angle of contingency is more acute than any of the acute angles made by straight lines (as proved by Euclid III.16),[56] then the difference or ratio between the angle *haf* and the angle contained by the perpendicular *bd* and the circumference *bf* at the point *b* is less than any difference or ratio you please which can occur between any large and small quantities. And thus (by the third petition) the difference of the obliquity of the descent *af* and the descent *bf*, and consequently the difference of positional heaviness of the two bodies *a* and *b*, is less than any you wish between two unequal quantities. Therefore any small corporeal quantity that is added, the body *a* will necessarily be heavier in any position than the body *b*, and hence it will not cease to descend continuously as far as the line of direction, that is, to the point *f*; and thus it will continue to raise the body *b* as far as the line of direction, that is, to the point *e* and if this would take place in the position that is shown in the figure, it would happen so much the more at the position of equality, in which position there neither is nor will be any difference of positional heaviness of the descents, that is, in that position they would be equally heavy, and so any small quantity of weight, however minimal, that should be imposed on either side of any scale (that is, with equal arms, whether large or small) will immediately tilt the scale down on that side, and the arm will continue its declination, for the reasons adduced above, as far as the line of direction, that is, to the point *f*. now this would be contrary to those two conclusions which Aristotle adduces concerning the first of his mechanical problems, of which I spoke with your Excellency once before. In one conclusion he says that there are some weights which, imposed on little scales, do not make themselves manifest to our senses by any tilting, while on large scales they do make themselves manifest. This conclusion, looked at Mathematically, that is, abstracted from all matter, would be quite false (for the reasons adduced above), because a small balance as well as a large one will be strength to tilt down on that side where such a weight is placed, however small it be, and to tilt as far as the line of direction. Thus in the tilting of small and large there will be no proportionate difference, and in one as in the other the tilting will continue to the line of direction. The same would follow as to his other conclusion, that is, when he says that there are some weights which are manifest in both sorts of scales, large and small, but much more [manifest] in the larger, that conclusion would also be false (for the reasons adduced above), for, as remarked, in both they will make that arm of the scale decline as far as the line of direction. S. AMBASSADOR These your reasons and arguments are fine and good; nevertheless in actual or material scales it is seen that for the most part things happen as Aristotle says and concludes. For if on any scale you please (large or small) there is placed a grain of poppy seed or some other small quantity, few are the scales that will make a sensible tilting from so little heaviness. And if some trouara alcuna che faccia alcun sensibile segno de declinatione, tamen non procedera per fina alla detta linea della direttione, & non solamente il detto gran de papauero non sara atto à farla declinare per fin alla detta linea della direttione alcuna libra, ma



nanche un gran di formento, qual è molto piu ponderoso, & tutto questo la sperientia lo manifesta. Si che non so che mi dire, perche da una banda per le uostre ragioni, & argomenti, uedo, & comprendo che uoi diceti il uero, & dall'altra trouo per isperientia seguir tutto al contrario. N. Il tutto procede Signor, dalla materia, perche nelle libre considerate con la mente fuora de ogni materia il suo sparto, polo, ouer asis, se suppone un ponto indiuisibile, et nelle libre materiale, tal sparto, ouer asis ha sempre qualche corporal grossezza in se, la qual grossezza, quanto è maggiore tanto men diligẽte redusse la detta libra, & similmente li brazzi delle libre imaginate (cioe ideale) se supponganno linee, cioe senza larghezza, ne grossezza, & nelle libre materiale tai brazzi sono di alcun metallo, ouer di legno, li quali brazzi quanto piu sono corpulenti, è grosi tanto men diligente reducano tal libre. S.A. E ue ho inteso, seguitati se ha ueti altra propositione de adure circa à questa materia. NIC.

QVESITO. XXXIIII. PROPOSITIONE VII.

SE li brazzi della libra saranno ineguali, et che nella istremita di cadauno de quelli ui stano appesi corpi simplicemente eguali in grauita dalla banda del piu lõgo brazzo tal libra fara declinatione. S. A. Questa è cosa naturale. N. Anchor che la sia cosa naturale uolendo procedere rettamente, bisogna asignar la causa di tal effetto. S.A. Seguitati. N. Sia la uerga, ouer libra.a.c.b.et sia il brazzo.a.c.piu longo del.c.b. Dico che essendo appesi corpi simplicemente eguali in grauita, nelli dui ponti.a.&.b.tal libra declinara dalla parte del.a. Perche essendo tirata la perpendicolare.c.f.g.(cioe la linea della direttione) et essen



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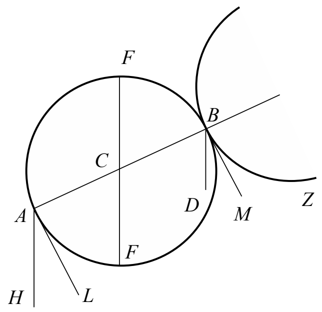
were found which will make some sensible sign of tilting, it does not go so far as the line of direction. And not only will the said grain of poppy seed fail to make any scale tilt as far as the line of direction, but

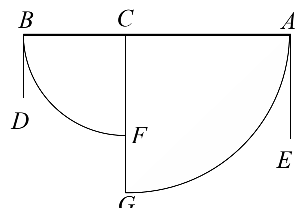


[Fig. 4.14]

so will a grain of wheat that is much more ponderous. And all this is demonstrated by experience. So that I do not know what to say, since on the one side, by your reasons and arguments, I see and understand that you speak the truth, and on the other I find by experience that the opposite happens. N. Sir, all this comes about from matter, because in the scales considered by the mind, apart from all material, the fulcrum or axis is assumed to be an indivisible point. But in material scales that fulcrum or axis has always some corporeal thickness of its own, and the greater that thickness is, the more it reduces the sensitivity of the scale. Likewise the arms of the imagined (that is, ideal) scales are assumed to be lines, without breadth or thickness, but in material scales the arms are of some metal or of wood, and the bigger they are, the more they reduce the sensitivity of the scale.[57] S.A. I understand. Continue if you have further propositions regarding this matter. NIC.

QUESTION. XXXIIII. PROPOSITION VII.

If the arms of the scale are unequal, and at the ends of them are hung bodies simply equal in heaviness, the scale will tilt on the side of the longer arm.[58] S.A. This is a matter of nature [a physical matter]. N. Although it is natural, if we wish to proceed correctly, we must assign the cause of this effect.[59] S.A. Go ahead. N. Let there be the rod or scale *acb*, with the arm *ac* longer than *cb* [See Fig. 4.15]. I say that if bodies simply equal in heaviness were hung at the two points *a* and *b*, the scale will tilt on the side of *a*. Because when the perpendicular *cfg* (that is, the line of direction) is drawn, and



[Fig. 4.15]

do circinate le due quarte parte de circuli, sopra el centro.c.le quale stano.a.g. &.b.f.& essendo dutte dal ponto.a.&.b.due linee contingente, le quale stano.a.e.&.b.d. Eglie manifesto langolo.e.a.g.della detta contingentia, esser menore de langolo.d.b.f.e pero manco obliquo è il descenso fatto per.a.g. del descenso fatto per.b.f, e pero (per la terza petitione) piu graue sara il corpo.a.del corpo.b, in tal sito, ch'è il proposito. S.A. E ue ho inteso, seguitati. N.

QVESITO. XXXV. PROPOSITIONE VIII.

SE li brazzi della libra saranno proportionali alli pesi in quella imposti, talmente, che nel brazzo piu corto sia appeso il corpo piu graue, quelli tai corpi, ouer pesi seranno equalmente graui, secondo tal positione, ouer sito. S.A. Datime uno essempio. N. Sia come prima la regola, ouer libra.a.c.b.& ui stano appesi.a.&.b.et sia la proportione del.b.al.a.si come del brazzo.a.c.al brazzo.b.c. Dico, che tal libra non declinara in alcuna parte di quella, & se poßibil fuße (per lauersario) che declinar potesse, poniamo che quella declini dalla parte del.b.& che quella discenda, & transisca in obliquo, si come sta la linea.d.c.e.in luoco della.a.c.b.& attaccatoui.d.come.a.&.e.come.b.& la linea.d.f.descenda orthogonalmente, & simelmente ascenda la.e.h. Hor eglie manifesto (per la.16.&.29.del primo di Euclide) che li dui trianguli.d.f.c.&.e.h.c.eßer de angoli equali. Onde per la.4.del sesto di Euclide) quelli saranno simili, & consequentemente de lati proportionali; adunque la proportione del d.c.al.c.e.è si come del.d.f.al.e.h.& perche si come del.d.c.al.c.e.cosi è dal peso.b.al peso.a.(dal presupposito) adunque la proportione dal.d.f.al.e.b.sara si come dal peso.b al peso.a.sia adunque dal.c.d.tolto la parte.c.l.equale alla.c.b.ouer alla.c.e.& sia posto.l.equale al.b.in grauita, & descenda el perpendicolo.l.m. Adunque perche eglie manifesto la.l.m.& la.e.h.esser equale, la proportione della.d.f.alla.l.m.sara si come delle simplice grauita del corpo.b.alla simplice grauita del corpo.a.ouer della simplice grauita del corpo.l.alla simplice grauita del corpo d.(perche li dui corpi.a.&.d.sono supposti uno medesimo) & simelmente el corpo.b.&.l.per esser supposta la grauita del.l.equale alla grauita del.b.) e per tanto dico, che la proportione di tutta la.d.c.alla.l.e.sara si come la grauita del corpo.l.alla grauita del corpo.d. Onde se li detti dui corpi graui, cioe.d.&.l.fussero simplicemente equali in grauita, stanti poi in li medesimi siti, ouer luochi, doue, che al presente uengono supposti, el corpo.d.saria piu graue del corpo.l.secondo el sito (per la.4.propositione) in tal proportione, qual è di tutto il brazzo.d.c.al brazzo.l.c.& perche il corpo.l.è simplicemente (dal presupposito) piu graue del corpo.d.secondo la medesima proportione (cioe, si come la proportione del brazzo.d.c.al brazzo.l.c.adunque li detti dui corpi.d.&.l.nel sito della equalita ueneranno ad essere egualmente graui, perche per tanto quanto il corpo.d.è piu graue del corpo.l.per uigor del sito, ouer luoco, per quel medesimo el corpo.l.è simplicemente piu graue del corpo.d.e pero nel detto sito della equalita uengono à restare egualmente graui. Adunque quella potentia, ouer grauita, che sara sufficiente ad elleuare il corpo.a.dal sito della equalita, dal ponto, doue che al presente è (cioe per fin al ponto.d.) quella medesima sara suf-

[93r]

EIGHT

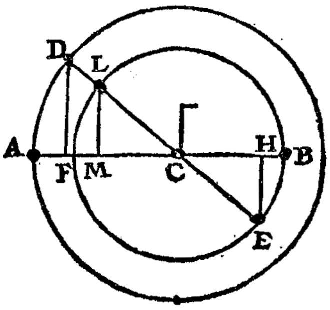
the two quarter circles, which shall be *ag* and bf, are traced on the centre *c*, and when two tangent lines *ae* and *bd* are drawn from the points *a* and *b*, it is manifest that the angle of tangency *eag* is less than the angle *dbf*. Hence the descent made along *ag* is less oblique than the descent made along *bf*. Therefore (by the third petition) the body *a* will be heavier than the body *b* in this position; which is the purpose. S.A. This I understand; continue. N.

QUESTION. XXXV. PROPOSITION VIII.

If the arms of the balance are proportional to the weights imposed on them, in such a way that the heavier weight is on the shorter arm, then those bodies or weights will be equally heavy according to such position or site.[60] S.A. Give me an example. N. Let as before the bar or balance *acb* [Fig. 4.16] and the weights *a* and *b* hung thereon, and let the ratio of *b* to a be as that of the arm *ac* to the arm *bc*. I say that this balance will not tilt to either side. And if (for the adversary) it is possible for it to tilt, let us assume it to tilt on the side of *b* and to descend obliquely as the line *dce* in place of *acb,* and [let us] take *d* as *a* and *e* as *b*; and the line *df* descends perpendicularly, and the line *eh* rises similarly. Now it is manifest (by Euclid I.16 and I.29)[61] that the two triangles *dfc* and *ehc* have equal angles. Whence (by Euclid VI.4)[62] they will be similar, and consequently will have proportional sides. Therefore the ratio of *dc* to *ce* is as that of *df* to *eh*; and since the weight *b* is to the weight *a* as *dc* is to *ce* (by our assumption), the ratio of *df* to *eh* will be as the weight *b* to the weight *a*. Hence, if we take from cd the part *cl*, equal to *cb* or *ce*, and consider *l* equal in heaviness to *b* and descending along the perpendicular *lm*, then, since it is manifest that *lm* and *eh* are equal, the proportion of *df* to *lm* will be as the simple heaviness of the body *b* to the simple heaviness of the body *a*, or as the simple heaviness of the body *l* to the simple heaviness of the body *d*, because the two bodies are supposed to be the same, and similarly the bodies *b* and *l* (the heaviness of the body l having been assumed equal to that of the body *b*). Hence I say that the ratio of all *dc* to *lc* will be as the heaviness of the body *l* to that of the body *d*. whence if the said two heavy bodies, that is, *d* and *l* were simply equal in heaviness, standing then in the same positions or places at which they are presently assumed to be, the body *d* would be positionally heavier than the body *l* (by the fourth proposition) in that ratio which holds between the whole arm *dc* and the arm *lc*. And since the body *l* is simply heavier than the body *d* (by our assumption) in the same ratio as that of the arm *dc* to the arm *lc*, then the said two bodies *d* and *l* in position of equality would come to be equally heavy, because by as much as the body *d* is positionally heavier than the body *l*, by so much is the body *l* simply heavier than the body *d*; and therefore in the position of equality they come to be equally heavy. Hence that power or heaviness that will be sufficient to lift the body *a* from the position of equality to the point at which it is at present (that is, to the point *d*) will be

AA

ficiente ad elleuare il corpo.l.dal medeſimo ſito della equalita al luoco, doue che al preſente è. Adunque ſel corpo.b.(per lauerſario)è atto ad eleuare il corpo.a.dal ſito della equalita per fin al ponto.d.el medeſimo corpo.b.ſaria anchora atto, e ſofficiente ad elleuare il corpo.l.dal medeſimo ſito della equalita per fin al ponto, doue che al preſente è, el qual conſequente é falſo, & contra alla quinta propoſitione, cioe el corpo b.(qual è ſuppoſto equale in grauita al corpo.l.)elleuaria il detto corpo.l.fuora del ſito della equalita, in ſiti equali, cioe equalmente diſtanti dal centro.c.la qual coſa é impoßibile per la detta quinta propoſitione, diſtrutto adunque l'oppoſito, rimane il propoſito. S. A. Queſta è una aſſai bella propoſitione, ma el me pare, ſe bẽ me arricordo, che Archimede Syracuſano



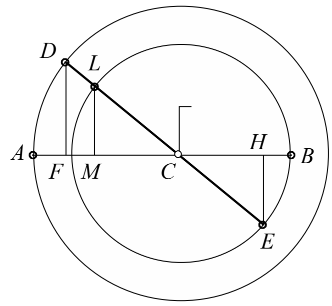
ne ponga una ſimile, ma el non mi pare, che lui la dimoſtri per queſto uoſtro modo. N. Voſtra Signoria dice la uerita, anci di tal propoſitione, lui ne fa due propoſitioni, & queſte ſono la quarta, & quinta di quel libro, doue tratta delli centri delle coſe graue, & in effetto tai due propoſitioni lui le dimoſtra ſuccintamente per li ſuoi principij da lui per auanti poſti, & demoſtrati, & perche tai ſui principij, ouer argomenti.non ſe conuegnariano in queſto trattato, per eßer materia alquãto diuerſa da quella, ne apparſo in queſto luoco de dimoſtrare tal propoſitioni con altri principij, ouer argomenti piu conuenienti in queſto luoco. S.A. E ue ho inteſo ſeguitati. N.

QVESITO XXXVI. PROPOSITIONE IX.

SE ſaranno due ſolide uerghe, traui, ouer baſtoni di una ſimile, & equal longhezza, larghezza, groſſezza, & grauita, & che ſtano appeſi in una libra talmente che luno ſtia equidiſtante al orizonte, & laltro dependi perpendicolarmente, & talmente anchora, che del termine del dependente, & del mezzo dell'altro ſia una medeſima diſtantia dal centro della libra, ſecondo tal ſito, ouer poſitione ueneranno à eßere equalmente graui. S.A. Non ue intendo, e pero datime uno eſſempio. N. Eſſempi gratia. Siano li termini delli brazzi della libra.b.&.d.& il ſparto, ouer centro di quella il ponto.c.& ui ſtano attaccati li dui ſolidi ſimili.& equali, come detto, delli quali luno ui ſta attaccato ſecondo lordine del brazzo della libra, cioe equidiſtantamente al orizonte qual ſia.f.e.del qual il ſuo ponto di mezzo ſia el ponto.d.& laltro ſia attaccato pendente perpendicolarmente qual ſia.b.g. & ſia il termine del ſuo attaccamento il ponto.b.& ſia che la diſtantia del ponto.b.al ponto.c.(centro della libra)ſia tanto, quanto ch'è dal ponto di mezzo de laltro ſolido(cioe dal põto.d.) al medeſimo ponto.c. Dico che li detti dui ſolidi, in tal ſito, ouer poſitione ſono equalmente graui, & queſto ſe puo dimoſtrar in piu modi. El primo di quali è queſto, ch'eglie manifeſto per le coſe dimoſtrate da Archimede in quello del centro della grauita, che

[93v]
B O O K

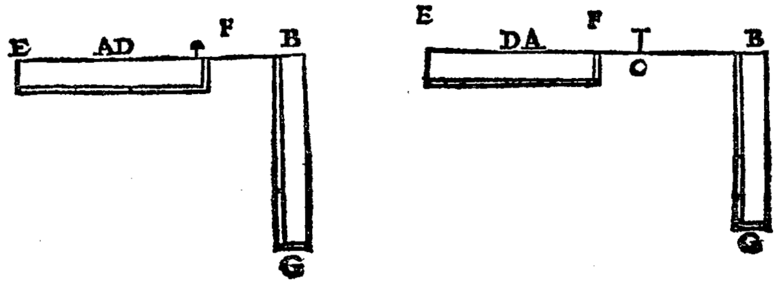
sufficient to lift the body *l* from the same position of equality to the place where it is at present.[63] Therefore if the body *b* (for the adversary) is able to lift the body *a* from the position of equality to the point *d,* the same body *b* would also be able and sufficient to lift the body *l* from the same position of equality to the point where it is at present, which consequence is false and contrary to the fifth proposition; that is, the body *b* (which is supposed equal in heaviness to the body *l*) would lift the said body *l* out of the position of equality [though they are] in equal places, that is, equally distant from the centre *c*, which is impossible by the said fifth proposition. Thus, the adversary's position destroyed, the thesis stands. S.A. This is a very pretty proposition, but it seems to me (if I recall correctly) that Archimedes of Syracuse has a similar one, and I believe he does not prove it in this way of yours. N. Your Excellency is right. Indeed, of this proposition he makes two, and these are the fourth and fifth in that book of his wherein he deals with the centres of gravity of heavy bodies;[64] and in fact he proves those two propositions succinctly by principles of his set forth and demonstrated previously. And since those principles and arguments of his would not be suitable in this treatise, it being of somewhat different subject, it appeared best in this place to prove those propositions with other principles or arguments more appropriate here.[65] S.A. I see. Proceed. N.



[Fig. 4.16]

QUESTION XXXVI. PROPOSITION IX.

If there are two solid rods, beams or staff of the same length, breadth, a width, and weight hung on a balance in such a way that one is horizontal and the other vertical, with the distances equal from the centre of the balance to the point of suspension of the latter and the centre of the former, then they will be equally heavy according to this place or site.[66] S.A. I do not understand you, so give me an example. N. For example, let there be the ends of the balance arms *b* and *e* and the pivot or centre at the point *c* [Fig. 4.17], and let there be attached the two similar equal solids, of which one shall be attached along the balance arm horizontally, called *fe*, whose midpoint is *d*, while the other shall be attached hanging perpendicularly as *bg*, the point of attachment being *b*. And let the distance from the point *b* to the point *c* (centre of the scale) be as much as that from the midpoint of the other solid (that is, the point *d*) to the same point. I say that the two solids in that place or position are equally heavy, and this can be demonstrated in several ways. The first of these is this: it is manifest by the things demonstrated by Archimedes in his centres of gravity that tanto pesa il solido.f.e.in tal positione nella detta libra,quanto che saria se quello fusse anchora lui appeso perpendicolarmente in ponto.d.perche in tal ponto. d. ui sotto giace el centro della grauita de tal solido,& per esser li detti dui solidi equali in grauita dal presupposito,& appesi equalmente distanti dal ponto, ouer centro.c. quelli (per la.5.propositione)non se separano dal sito della equalita,ch'è il proposito.

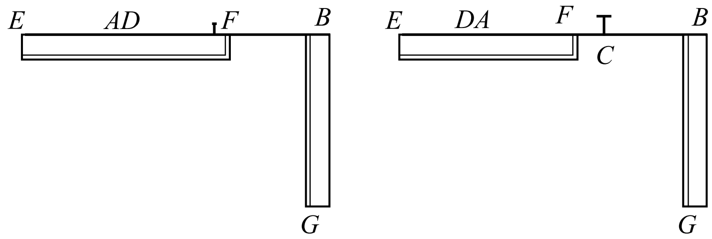


Anchora tal propositione si puo demostrar in questo altro modo (el quale è piu sua conueniente dimostratione,perche se uien à dimostrare per li suoi proprij principij, & non per principij alieni. Eglie manifesto,che essendo suspesi dui pesi simplicemente equali,luno in ponto.f.& laltro in ponto.e.quali poniamo,che siano.h.k. & similmente dui altri equali alli medesimi in ponto.b.quali siano.l.m.nelli quali siti,dico,che tai pesi pesaranno equalmente,perche la proportione del peso. l. al peso. k. è si come del brazzo.b.c.al brazzo.f.c.,per la quarta propositione,perche tanto graue saria el corpo.l.secondo el sito nel ponto.d.quanto che nel ponto,doue si troua al presente, cioe in ponto.b.(per esser.c.d.equale al.c.b.dal presupposito)e pero per la detta propositione,tal proportione sara della grauita del corpo. l. al corpo. k. secondo el sito, quale sara del brazzo.d.c.ouer.b.c.al.c.f.& per le medesime ragioni tal proportione sara della grauita del corpo.m.alla grauita del corpo.h.secondo el sito, quale sara del medesimo brazzo.c.d.ouer.c.b.al brazzo.c.e.adunque la grauita de ambi dui li corpi.l.m.insieme alla grauita de ambi dui li corpi.h.k.insieme secondo il sito sara si come el doppio del brazzo.c.d. ouer del brazzo.c.b.insieme alli dui brazzi.c.f. et.c.e.pur insieme,& perche li detti dui brazzi.c.e.&.c.f.insieme sono precisamente tanto,quanto è il doppio del detto brazzo.c.d.ouer.c.b.seguita anchora, che la grauita delli detti dui corpi.l.m. sia equale alla grauita delli dui corpi.h.&.k.secondo il sito,ch'è il proposito,perche se del sopradetto solido.f.e.ne sara fatto due parti equali, appiccandone una di quelle in ponto.f.& laltra in ponto.e.tanto pesarano cosi separate in tai siti,si come faceuano in longo congionte,come di sopra fu supposto, & similmente facendo del solido.b.g.pur due parti,& appiccarle ambe due in el medesimo ponto.b.tanto pesarano cosi separate,come che congionte, come, che di sopra fu supposto e pero per le cose dette,& allegate seguita il proposito.

[94r]

EIGHT

the solid *fe* weighs as much in that position on the balance as if it were hung perpendicularly at the point *d*, because at that point *d* is situated the centre of gravity of the solid; and the two solids being equal in weight by hypothesis and hung equally distant from the central point *c*, then by the fifth proposition they will not depart from the position of equality; which is the purpose.



[Fig. 4.17]

This proposition can also be demonstrated in another way (which is more suitable because it depends on its own principles rather than imported ones).[67] It is manifest that, when two simply equal bodies, *h* and *k*, are suspended, the one at the point *e* and the other at the point *f*, and two others which shall be *l* and *m*, equal to them, are hung at the point *b* [See Fig. 4.18], these weights, I say, will weigh equally at those points, because the ratio of the weight *l* to the weight *k* is as that of the arm *bc* to the arm *fc* (by the fourth proposition); for the body l will be positionally as heavy at the point *d* as where it is at present, that is, at the point *b* (since *cd* is equal to *cb* by assumption). Therefore, by the said proposition, this ratio will be that of the positional heaviness of the body *l* to the body *k*, which will be that of the arm *dc* or *bc* to *cf*; and for the same reasons this ratio will be that of the heaviness of the body *m* to the heaviness of the body *h* positionally, that is the ratio of the same arm *cd* or *bc* to the arm *ce*. Therefore the positional heaviness of both the bodies *l* and *m*, together, to the positional heaviness of the other two bodies *h* and *k*, together, will be as the double of the arm *cd* or *bc* to the two arms *ce* and *cf* together. And since the said two arms *ce* and *cf*, together, are precisely as much as the double of the said arm *cd* or *bc*, it follows also that the heaviness of the said two bodies *l* and *m* is equal to the positional heaviness of the two bodies *h* and *k*; which is the purpose. For if the said solid *fe* were made into two equal parts, one of those hanging at the point *f* and the other at the point *e*, they would separately weigh as much thus at those points as they were elongated and joined in the manner supposed before. Similarly, if the solid *bg* also were in two parts, both hung at the same point *b*, they would thus weigh as much separated as conjoined (as supposed above); hence from the things said and alleged the purpose follows.

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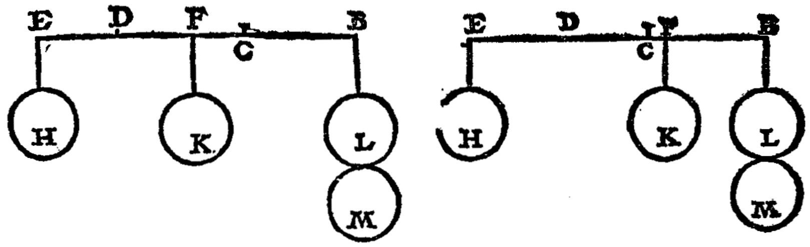


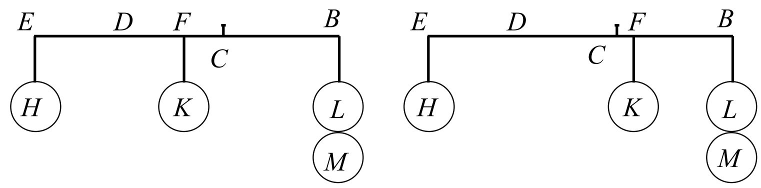
S.A. Voria,che me dimostrasti che il brazzo.c.f.insieme con il.c.e.sia tanto quãto el doppio del brazzo.d.c.ouer.c.b. N. Signor eglie manifesto,che tutto il brazzo c.e.è maggiore del brazzo.c.d.per la parte.e.d.la qual parte.e.d.è equale alla.d.f.di remo adunque,che tutta la.c.e.è equal alla.c.d.& anchora alla sua parte.f.d.alla qual parte.f.d.giontoui el brazzo.f.c.queste due parti insieme se egualiano anchora loro alla medesima.c.d,e pero tutta la.c.e.insieme con la.c.f.sono precisamente il doppio della.c.d & perche la detta.c.d.è equale(dal presupposito)alla.b.c.seguita,che tutta la.c.e.insieme con la.c.f.siano equali al doppio della.c.b.ch'è il proposito. S.A. E ue ho inteso benissimo,e pero seguitati. N.

QVESITO XXXVII. PROPOSITIONE X.

SEl sara una solida uerga,traue,ouer bastone di una simile, & equal larghezza, grossezza,sostantia,& grauita in ogni sua parte,& che la longhezza di quella sia diuisa in due parti inequale,& che nel termine della menor parte ui sia appeso uno altro solido,ouer corpo graue,el quale faccia stare la detta uerga,traue,ouer bastone equidistante al orizonte. La proportione della grauita di tal corpo graue,alla differentia della grauita della maggior parte della detta uerga (traue, ouer bastone) alla grauita della parte menore,sara si come la proportione della lõghezza di tutta la uerga(traue,ouer bastone)al doppio della longhezza della sua menor parte. S.A. Datime un essempio se uoleti,che ui intẽda. N. Sia la solida uerga (traue,ouer bastone) il solido.a.b.di una simile,et equal grossezza,larghezza,sostãtia,et grauita p tutto, cioe p ogni parte,et sia diuiso cõ l'intelletto in due parti inequale in põto.c.et sia signata la.c.d.equal alla.a.c.adunque la.d.b.uiẽ à essere la differẽtia,ch'è fra la parte maggiore.c.b.et la menore.c.a.della qual differẽtia sia trouato il mezzo, qual sia il ponto.e.Hor essẽdo suspeso il detto solido,ouer traue.a.b.nel põto.c.et essẽdoui attaccato, ouer suspeso nel termine della sua menor parte un altro solido (poniamo il solido.f.) qual faccia stare il primo solido,ouer traue.a.b.equidistãte al orizõte. Dico, che tal proportione hauera la grauita del solido.f.alla grauita della differẽtia.d.b. qual hara tutta la lõghezza.a.b.alla.a.d.cioe al doppio della lõghezza della parte menore.a.c. Perche tanto pesa la detta differentia.d.b.in tal positione,come che al presente sta quãto che faria se quella fusse perpendicolarmente sospesa in ponto.e.e pero (per il con-

[94v]

BOOK

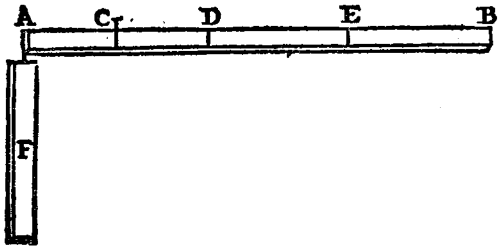


[Fig. 4.18]

S.A. I should like to have you demonstrate to me that the arm *cf* together with *ce* is as much as double the arm *dc* or *bc*. N. Sir, it is manifest that the whole arm *ce* is greater than the arm *cd* by the part *ed*, which part *ed* is equal to *df*. Therefore let us say that the whole of *ce* is equal to *cd* added to its part *fd*, and if to the part *fd* we add the arm fc, these two parts together also equal *cd*. Therefore the whole *ce* together with *cf* are precisely the double of *cd*; and since the said *cd* is equal by hypothesis to *bc*, it follows that the whole *ce* together with *cf* is equal to the double of *cb*; which is the purpose. S.A. I understand very well, so continue. N.

QUESTION XXXVII. PROPOSITION X.

If a solid rod or beam of uniform breadth, thickness, substance, and heaviness is assumed, and [if] its length is divided into two unequal parts, and at the end of the shorter part there is hung another solid or heavy body which makes the said rod, beam, or staff stay parallel to the horizon, *then the proportion of the heaviness of that body to the difference between the heaviness of the longer part of the rod (or beam or staff) and the heaviness of the shorter part will be as the ratio of the length of the whole rod, beam, or staff to the double of the length of its shorter part* [emphasis added].[68] S.A. Give me an example, if you want me to comprehend. N. Let *ab* be a solid rod (beam or staff) of uniform breadth, thickness, substance, and heaviness throughout (that is, at every point), and divide it mentally into two unequal parts at the point *c,* and mark *cd* equal to *ca*; then *db* becomes the difference between the longer part *cb* and the shorter *ca*, of which difference the centre is found, which is the point *e*. Now the said solid beam *ab* being suspended at the point *c*, and there being attached or suspended at the end of the shorter part another solid, which we call *f*, which makes the first solid beam *ab* stand parallel to the horizon, I say that the proportion of the heaviness of the solid *f* to the heaviness of the difference *db* is that of the whole length *ab* to *ad*, the double of the length of the shorter part *ac*. For the said difference *db* weighs as much in that position where it stands at present as it would if it were suspended perpendicularly at the point *e*, and therefore (by the converse uerso della.8.propositione) la proportione della grauità del solido.f.alla grauità del partial solido, ouer traue.d.b. sara, si come la proportione della distantia.c.e.alla distantia.c.a. Et la proportione, che è della distantia.c.e.alla distantia.c.a.(per la.15.del quinto di Euclide) quella medesima sara del doppio della distantia.c.e. al doppio della detta distantia.c.a.& perche il doppio della detta distantia.c.e.è quanto che è tutta la longhezza del solido.a.b.& il doppio della detta distantia.c.a.è quanto che è tutta la a.c.d.seguita(per la.11.del quinto di Euclide)che la proportione della grauita del solido.f.alla grauita della pifferentia.d.b.sia si come la proportione di tutta la longhezza del solido, ouer uerga.a.b.al doppio della longhezza della parte menore.a.c.(qual è la detta.a.c.d.)che è il proposito. S.A. Perche ragione uoleti che il doppio della



distantia.c.e.sia eguale à tutta la longhezza del traue.a.b. N. Perche la detta distantia.c.e.uien à esser precisamente eguale alla mita di tal longhezza.a.b. perche la parte.d.e.è la mita della parte.d.b.& la.d.c.è la mita dell'altra parte.d.a.adunque le due parti.d.e.&.d.c.gionte insieme, uengono à essere la mita delle due parti.d.b.&.d.a. pur gionte insieme. S.A. E ue ho inteso, e pero seguitate in altro. N.

QVESITO. XXXVIII. PROPOSITIONE XI. conuersa della precedente.

SE la proportione della grauita d'un solido sospeso in el termine della menor parte di una simile solida uerga (traue, ouer bastone) diuisa in due parti ineguali, alla differentia, che sara fra la grauita della maggior parte, & quella della menore, sara, si come la proportione di tutta la longhezza della solida uerga, traue, ouer bastone, al doppio della longhezza della sua menor parte. Tal solida uerga, traue, ouer bastone, necessariamente stara equidistante all'Orizonte. S.A. Credo bene che tal precedente propositione se conuertisca, nondimeno non restati da farme la dimostratione. N. Per esser questa il conuerso della precedente, per suo essempio supponeremo la medesima dispositione, ouer figura, cioe supponeremo, che la proportione della grauita del solido.f.alla differentia della grauita della maggior parte alla grauita della menore, cioe della.d.b.esser, si come la proportione di tutta la longhezza della solida uerga. b.al doppio della longhezza della parte menore.a.c.(quale sarta la.a.d.) Dico che stante questo la solida uerga.a.b.de necessita stara equidistante all'Orizonte. Et se pos

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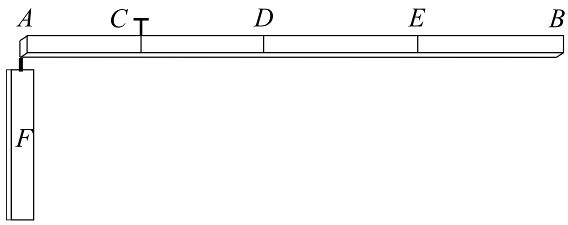
of the eighth proposition) the ratio of the heaviness of the solid *f* to the heaviness of the partial solid beam *db* will be as the ratio of the distance *ce* to the distance *ca*. And that ratio of *ce* to *ca* (by Euclid V.15)[69] will be the same as [the ratio of] the double of the distance *ce* to the double of the distance *ca*. and because the double of the said distance *ce* is the whole length of the solid *ab*, and the double of the distance *ca* is the whole of *acd*, it follows (by Euclid V.11)[70] that the ratio of the heaviness of the solid *f* to the heaviness of the difference *db* is as the ratio of the whole length of the solid rod *ab* to the double of the length of the shorter part *ac* (which is *acd*); which is the purpose. S.A. Why is double the distance CE equal to the whole



[Fig. 4.19]

length of the beam AB. N. Because the distance CE becomes precisely equal to half of that length AB, for the part DE is the half of the part DB, and DC is the half of the other part DA; therefore the two parts DE and DC joined together become the half of the two parts DB and DA joined together. S.A. I understand; therefore go on to the next. N.

QUESTION. XXXVIII. PROPOSITION XI.
opposite of the preceding.

If the proportion of the heaviness of a solid suspended at the end of the shorter part of a similar rod (beam or staff) divided into two unequal parts, to the difference that it will be between the heaviness of the longer part and that of the shorter, shall be as the ratio of the whole length of the solid rod or staff to the double of the length of its shorter part, such solid rod (beam or staff) will necessarily be horizontal. S.A. I well believe that the preceding proposition may have its converse; yet do not fail to give me the demonstration. N. This being the converse of the preceding, for its exemplification let us assume the same arrangement or figure. That is, let us suppose the ratio of the heaviness of the solid *f* to the difference of heaviness between the longer part and the shorter, that is, of *db*, to be as the ratio of the whole length of the solid rod *ab* to the double of the length of the shorter part *ac*, which will be *ad*. I say that this solid rod *ab* will of necessity remain horizontal. If it is

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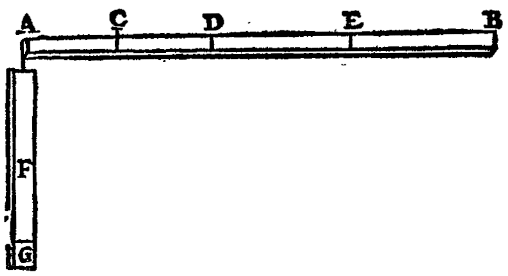
ſibil fuſſe (per l'auerſario) che quella debbia, ouer poſſa declinar da qualche banda, poniamo che declini dalla banda uerſo.b.al ſolido.f. gli aggiongeremo con lo intelletto una tal parte (quale pongo che ſia la parte.g.) che faccia reſtare la detta ſolida uerga, traue, ouer baſtone equidiſtante al detto Orizonte. Adunque, per la precedente, la proportione di tutta la grauita del compoſto delli dui corpi.f.& g.alla differentia, che è fra la grauita della parte maggiore.b.c.& quella della parte menore.a c. (che ſaria quella della.d.b.) ſara, ſi come la proportione di tutta la longhezza.a.b.al doppio della longhezza della ſua parte menor.a.c. il qual doppio, ſaria la.a.d.& perche il ſimplice ſolido.f.ha quella medeſima proportione, alla medeſima differentia (dal preſuppoſito) ſeguitaria (per la.9.del quinto di Euclide) che la grauita del ſimplice ſolido.f.fuſſe eguale alla grauita de tutto il compoſito di dui ſolidi.f.g.la qual coſa è impoſſibile, che la parte ſia eguale al tutto, il medeſimo inconueniente ſeguiria quando che lo auerſario ſupponeſſe che declinaſſe dalla parte.a.perche ſegando uia dal ſolido.f.una tal parte, che il rimanente faceſſe reſtare il detto ſolido.a.b. equidiſtante all'Orizonte, argomentando, come di ſopra fu fatto, ſeguiria pur che la grauita del medeſimo reſiduo fuſſe eguale alla grauita di tutto il ſolido.f. Adunque non potendo declinare ne dalla banda uerſo.b.ne da quella uerſo.a.eglie neceſſario che ſtia equidiſtante all'Orizonte, che è il propoſito. S.A. Sta beniſſimo, hor ſeguitati pur. N.



QVESITO. XXXIX. PROPOSITIONE XII.

SE ſara una ſolida uerga, trauer baſtone, come nelle due precedente è ſtato detto, cioe di una ſimile, & egual groſſezza, larghezza, ſoſtantia, & grauita, in ogni ſua parte, & che di quello ne ſia nota la ſua grauita, & ſimilmente la ſua longhezza, et che quello ſia diuiſo in due parti ineguale pur note. Eglie poßibile di ritrouar un peſo, il quale quando che quello ſara ſoſpeſo al termine della ſua menor parte fara ſtare la detta ſolida uerga, traue, ouer baſtone, equidiſtante all'Orizonte. S.A. Queſto atto operatiuo uoglio che me'l dichiarati con eſſempio materiale, perche lo uoglio intendere bene. N. Sia eſſempi gratia la ſolida uerga (traue, ouer baſtone) a.b. ſecondo che ſe propone, cioe di una ſimile, & equal groſſezza, larghezza, ſoſtantia, & grauita per ogni ſua banda, ouer parte, & poniamo, che la grauita di tal ſolida uerga ne ſia

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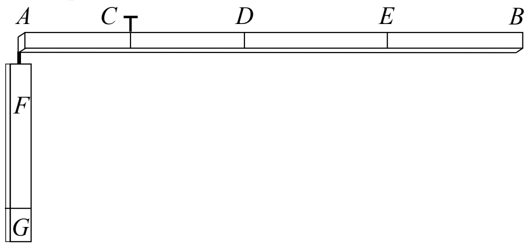
possible (for the adversary) that it must or might tilt from either side, let us assume that it tilts toward *b*. To the solid *f*, we add mentally such a part (which we shall call *g*) which cause the said solid rod or staff to stand parallel to the horizon. Therefore (by the preceding), the proportion of the whole heaviness of the combination of the two bodies *f* and *g* to the difference between the weight of the longer part *bc* and that of the shorter part *ac* (which will be that of *db*) shall be as the ratio of the whole length *ab* to the double of the length of its shorter part *ac*, which double would be *ad*; and since the simple solid *f* has that same ratio to the same difference (by what has gone before), it would follow (by Euclid V.9)[71] that the heaviness of the simple so[–]

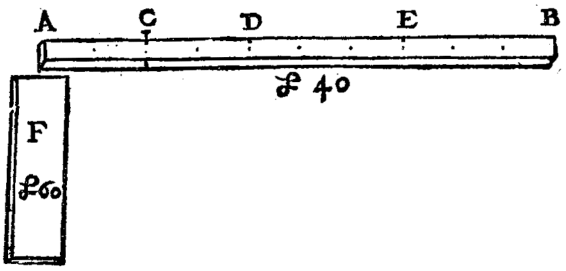


[Fig. 4.20]

lid *f* were equal to he heaviness of the whole combination of the two solids *f* and *g*, which is impossible, for the part would be equal to the whole. The same contradiction would follow if the adversary should assume that it tilted toward *a*, because cutting away from the solid *f* such a part that the remainder would make the solid *ab* rest parallel to the horizon and arguing as above would make it follow that the heaviness of the same remainder was equal to the heaviness of the whole solid *f*. Therefore, being unable to tilt from either side toward *a* or *b*, it necessarily stands parallel to the horizon; which is the purpose. S.A. Very good; now go on. N.

QUESTION. XXXIX. PROPOSITION XII.

If there is a solid rod, beam, or staff, as in the two preceding [propositions], which is similar and equal in thickness, breadth, substance, and heaviness in every part and of which the heaviness as well as the length is known, and if it be divided into two unequal parts which are also known, it is possible to find a weight which, when suspended at the end of its shorter part, will make the said solid rod, beam, or staff stay horizontal. [72] S.A. I should like y better explain to me this operation by means of a material example, for I want to understand it thoroughly. N. For example, let there be the solid rod (beam or staff) *ab* as proposed, that is, equal and similar in breadth, thickness, substance, and heaviness on every side or in every part; and let us assume the heaviness of the said solid rod to be nota,cioe poniamo che tutta pesi lire.40. et che similmente la longhezza di tal uerga, ouer bastone,ne sia nota,cioe poniamo che quella sia longa dui passa,cioe dieci piedi,& poniamo anchora che tal uerga sia diuisa in due parti ineguale in ponto.c.& che le dette parti ne sia note,cioe poniamo che la parte.a.c.menore,sia piedi dui, & che la maggior.c.b.sia piedi.8.Hor dico,che eglie possibile di trouare di quante libre uorra esser quel corpo qual essendo sospeso nel ponto.a.(termine della sua menor parte)faccia stare la detta uerga,ouer traue equidistante all'Orizonte.Perche (per le cose dimostrate nelle due precedente propositioni)eglie manifesto, che la proportione della grauita di quel tal corpo alla grauita di quella differentiā che è fra la parte maggiore.c.b. & la parte menore.a.c.(la qual differentia uerria à esser la.d.b.)sara,si come tutta la longhezza della uerga,ouer traue.a.b.(qual è piedi.10.)al doppio della longhezza della parte menor.a.c. (qual è piedi dui) il doppio della quale uerria à esser piedi.4. qual pongo sia la.a.d.adunque la grauita di quel tal corpo,alla grauita della partial uerga.d.b.sara,si come la longhezza de tutta la.a.b.(qual è piedi.10.)alla longhezza della.a.d.(qual è piedi.4.)Onde arguendo alcontrario,diremo,che la proportione della.a.d.(qual è piedi.4.)à tutta la.a.b.(qual è piedi.10) sara,si come la grauita della partial uerga.d.b.qual(alla ratta di tutta la.a.b.che libre.40.)uerria ad esser libre.24.alla grauita del corpo che recercamo, cioe di quello, che appeso nel ponto.a.debbia mantenere la detta uerga,ouer traue equidistante all'Orizonte.Onde per ritrouarlo procederemo secondo l'ordine della regola uolgarmente detta del tre,fondata sopra la.20.



propositione del.7.di Euclide,moltiplicando.10.fia.24.fa.240.& questo lo partiremo per.4.ne uenira.60.& libre.60.dico che pesara,ouer che douera pesare quel tal corpo,qual pongo sia il corpo.f.che è il proposito. S.A. Questo problema me è piacesto assai,& l'ho inteso benissimo,e pero seguitati se ci è altro da dire. N.

QVESITO. XL. PROPOSITIONE XIII.

SE il se hauera una uerga,traue,ouer bastone,come piu uolte è stato detto, del qual ne sia nota la sua longhezza,& anchora la sua grauita,& anchora un corpo ponderoso, del quale ne sia nota sua grauita, eglie possibile à determinare il luoco doue se hauera da diuidere la data uerga,traue,ouer bastone,talmente che appendendo il det-

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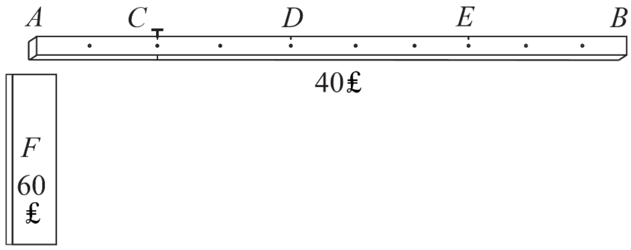
known, that is, assume that it weigh 40 pounds and similarly its length be two paces or ten feet, and let us also assume that the rod is divided into two unequal parts at the point *c* and that the [lengths of] said parts are known, it being assumed that the shorter part *ac* is two feet and the longer *cb* is 8 feet. Now I say that it is possible to find how many pounds that body must be which, suspended at the point *a* (end of the shorter part), will make the said rod or beam stand parallel to the horizon. For (by the things demonstrated in the two previous propositions) it is manifest that the ratio of the heaviness of that body to the heaviness of that difference which exists between the longer part *cb* and the shorter *ac* (which difference becomes *db*) will be as the length of the whole rod or beam *ab* (which is 10 feet) to the double of the shorter part *ac* (which is two feet), and this double comes to be 4 feet. Let us call this *ad*. Then the heaviness of that body [at *a*] will he to the heaviness of the partial rod *db* as the whole length of *ab* (which is 10 feet) is to the length of *ad* (which is 4 feet). Whereby, arguing conversely, let us say that the ratio of *ad* (which is 4 feet) to the whole *ab* (which is 10 feet) will be as the heaviness of the partial rod *db*, which (at the rate of 40 pounds to all *ab*) is 24 pounds to the heaviness of the body we seek that is that which, hung at the point *a*, should main[–]



[Fig. 4.21]

tain the rod or beam parallel to the horizon. Whence in order to find this, we shall proceed by the rule ordinarily called the rule of three, founded on Euclid VII.20;[73] multiplying ten by 24 gives 240, and this we shall divide by four, obtaining 60. I say that that weight which I called the body *f* will be 60 pounds; and this is the purpose. S.A. This problem pleased me very much and I understood it well; therefore go on to the next. N.

QUESTION. XL. PROPOSITION XIII.

If you shall have a rod, beam, or staff, as often was said above, of which the length, as well as the heaviness, be known, and also a heavy body of which the weight be known, it is possible to determine the place at which the rod, beam, or staff must be divided in order that the cit[–]

to corpo ponderoso al termine della sua menor parte faccia stare la detta uerga, traue, uer bastone, equidistante all'Orizonte. S.A. Essemplificatemi questa propositione. N. Per essemplificar questa propositione, supponeremo che il sia pur una uerga, traue, ouer bastone, come fu la precedente, cioe longa piedi.10. & che la grauita di quella sia pur libre.40. (come che nella detta precedente fu supposto.) Et poniamo anchora che il sia un corpo che la grauita di quello sia libre.80. Dico ch'eglie poßibile à determinare il luoco doue se debbe diuidere la detta uerga, talmente che appendendo il detto corpo graue al termine della sua menor parte, faccia star quella equidistante all'Orizonte. Et quantunque tal problema, si possa risoluere per uia di proportioni, nondimeno piu leggiadramente, se risolue per Algebra, ponendo che la parte menore della detta uerga sia una cosa de pie, onde la parte maggiore ueneria à restare piedi.10. men.1.co. Duplico la menor parte, cioe.1 co.fa.2.co, & queste.2.cose le sottro da tutta la uerga qual è piedi.10. resta piedi.10 men.2.cose, & questo sara la differentia, che è fra la parte maggiore, & la menore della detta uerga, onde per trouar la grauita di tal differentia, la moltiplico per.4. (perche pesando tutta la uerga libre.40. ueneria ogni pie di quella à pesar lire.4.) e pero moltiplicando quella per.4. come detto ne uenira libre.40.men.8.cose. Et perche la proportione di tutta la uerga (qual è piedi.10. al doppio della sua menor parte) il qual doppio saria.2.cose (è si come che la grauita del nostro corpo graue (qual è libre.80.) alla grauita della sopradetta differentia, qual fu libre.40.men.8.co. Onde per la.20.del.7.di Euclide (la moltiplicatione della prima) che.10.piedi (fia la quarta che è.40.men.8.cose) qual fara.400.men.80.cose (sara eguale alla moltiplicatione della terza qual è libre.80.fia la seconda, qual è.2.cose (qual fara.160.co.) e pero haueremo.160.cose eguale à.400.men.80.cose, onde ristorando le parti, & seguendo il capitolo, trouaremo la cosa ualer.1. e dui terzi, & de piedi.1. e dui terzi, se douera signar la menor parte della detta uerga, ouer traue, onde la maggiore uenira à restare de piedi.8. e un terzo, che è il proposito. S.A. Questa è stata una bella resolutione, ma seguitati pur, perche uorria che tra hoggi & dimane uedeßimo de ispedire tutto quello, che haueti da proponere sopra di questa scientia, perche uorro poi che me aßignati la causa de alcune questioni, che ho da dirui. N. Non credo di potermene ispedire fra diman, e l'altro, perche continuamente me nasce nuoue materie da proponere circa à tal scientia. S.A. Se non se ne potremo ispedire cosi dimane non importa, non perdemo tempo, seguitati. N.

QVESITO. XLI. PROPOSITIONE XIIII.

LA egualita della declinatione è una medesima egualita de peso. S.A. Datemi un essempio. N. La egualita della declinatione uien conseruata solamente in uia retta. Hor poniamo adunque che la detta uia retta sia la linea.a.b. & dal ponto.a. sia anchor tirata la perpendicolare.a.c. & supponamo anchor nella detta declinata linea.a.b. dui diuersi luochi. Hor poniamo che l'uno sia il ponto.d. & l'altro il ponto.e. Hor dico che discendendo, qualunque corpo ponderoso, ouer dal ponto.d. ouer dal ponto.e. sara de uno medesimo peso, secondo il sito in qual si uoglia de detti luochi. Perche

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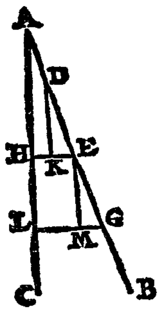
ed heavy body, when hung at the end of the shorter part, will cause the rod, beam, or staff to remain parallel to the horizon. S.A. Give me an example of this problem. N. To illustrate this problem, let us assume that there is a rod, beam, or staff as the above, 10 feet long and weighing 40 pounds (as in the preceding was assumed). And let us assume also that there is a body weighing 80 pounds. I say that it is possible to determine the place at which the rod may be divided so that the said heavy body, when hung at the end of its shorter part, will make the rod stay parallel to the horizon. And any such problem may be solved by ratios; nevertheless, it may be more easily solved by Algebra, the shorter part of the rod being a matter of feet, a co,[74] whence the longer part is 10 minus co. I double the shorter part (that is one co), which gives 2 co, and subtract these two co from the whole length of 10 feet. There remains 10 minus 2 co, and this will be the difference between the longer part and the shorter. To find the weight of this difference, I multiply it by 4 (because, the whole rod weighing 40 pounds, each foot comes to weigh 4 pounds). Multiplying by 4, as I said, the result is 40 minus 8 co. And since the ratio of the whole rod (which is 10 feet) to the double of its shorter part (which double is 2 co) is as the weight of our heavy body (which is 80 pounds) to the weight of the above difference, which is 40 minus 8 co. Hence by Euclid VII.20[75] the product of the first [term], or 10 feet, into the fourth, which is 40 minus 8 co (which would be 400 minus 80 co), will equal the product of the third, which is 80 pounds, into the second, which is 2 co (which will be 160 co). Thus we will have 160 co equal to 400 minus 80 co; and restoring the parts by rule we shall find the co to be 1+2/3.[76] Hence 1+2/3 feet will be the shorter part of the said rod or beam, whence the longer will be 8+1/3 feet; which was our problem. S.A. This was a pretty solution. Now continue, for today and tomorrow I want to finish all that you have to say on this science, after which I should like to have you clear up for me some questions I have for you. N.

QUESTION XLI. PROPOSITION XIIII.

The equality of obliquity [slant] is an equality of weight [according to position]. S.A. Give me an example. N. Equality of obliquity is preserved only in a straight path. Therefore let us assume that the said straight path is the line *ab* [See Fig. 4.22], and let the perpendicular *ac* be drawn from the point *a*, and let also suppose two different places along the said inclined line *ab*. Let one of these be the point *d* and the other the point *e*. Now I say that any heavy body in descending, whether at the point *d* or at the point *e*, will be of the same positional weight as at any of the other said places. For che ſe pigliaremo ſotto al.d.& al.e.due parti equali nella uia,ouer linea.a.b. Hor poniamo,che l'una ſia la parte.d.e.et laltra la.e.g.Dico,che per le dette parti equali capira equalmente del diretto,cioe della linea.a.c.la qual coſa ſe notificara in queſto modo,dalli dui ponti.e.& .g.ſiano tirate le due linee.e.h.& .g.l. perpendicolare ſopra la linea.a.c.et dalli dui ponti,ouer luochi.d.& .e.le due linee.d.k.& . e. m. perpendicolare ſopra le medeſime.e.h.& .g.l.le qual due perpendicolare , cioe. d. k.& .e.m.ſaranno fra loro equali,perche adunque il detto corpo ponderoſo,ſi eſſendo nel ponto.d.come nel ponto.e.in quantita,ouer deſcenſi equali,capira equalmente del diretto,ſara di una medeſima grauita in qual ſi uoglia de quelli,ſecondo el ſito,ch'è il propoſito. S.A. E ue ho inteſo,ſeguitate pur. N.



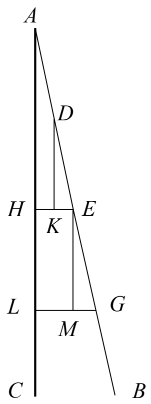
QVESITO XLII. PROPOSITIONE XV.

SE dui corpi graui deſcendano per uie de diuerſe obliquita,& che la proportione delle declinationi delle due uie,& della grauita de detti corpi ſia fatta una medeſima,tolta per el medeſimo ordine. Anchora la uirtu de luno , e laltro de detti dui corpi graui,in el deſcendere ſara una medeſima. S.A. Queſta propoſitione mi par bella,e pero datime anchora un eſſempio chiaro,accio che meglio mi piaccia. N. Sia la linea.a.b.c.equidiſtante al orizonte,& ſopra di quella ſia perpendicolarmente eretta la linea.b.d.& dal ponto.d.deſcendano de qua,& de la le due uie,ouer linee.d.a. & .d.c.& ſia la.d.c.di maggior obliquita. Per la proportione adunque delle lor declinationi,non dico delli lor angoli,ma delle linee per fina alla equidiſtante reſecatione,in la quale equalmente ſummemo del diretto. Sia adũque la lettera.e.ſuppoſta per un corpo graue poſto ſopra la linea.d.c.& un'altro la lettera.h.ſopra la linea. d. a. & ſia la proportione della ſimplice grauita del corpo.e.alla ſimplice grauita del corpo.h.ſi come quella della.d.c.alla.d.a. Dico li detti dui corpi graui eſſer in tai ſiti, ouer luochi di una medeſimo uirtu,ouer potentia.Et per dimoſtrar queſto,tiro la.d.k. di quella medeſima obliquita,ch'è la.d.c.& imagino un corpo graue ſopra di quella equale a corpo.e.el qual pongo ſia la lettera.g.ma che ſia in diretto con. e. h. cioe equalmente diſtanti dalla.c.k.Hor ſe poßibel è(per lauerſario) che li detti dui corpi e.& .h.non ſiano di una medeſima,& equal uirtu in tai luochi , adunque luno ſara di maggior uirtu,ouer potentia dell'altro,poniamo adunque,che.e. ſia di maggior uirtu,adunque quello ſara atto à diſcendere,& ſimelmente à far aſcendere, cioe à tirare in ſuſo el corpo.h.Hor poniamo(ſe poßibel è)che il detto corpo.e.deſcenda per fina in ponto.l.& che faccia aſcendere il corpo.h.per fin in ponto.m. & faccio , ouer che ſegno la.g.n.equale alla.h.m.la quale anchora lei uien à eſſer equale alla.e.l.Et dal põto.g.tiro la.g.h.e.la qual ſara perpendicolare ſopra la.d.b.per eſſer li detti tre ponti(ouer corpi)g.h. e.ſuppoſti in diretto,& equalmente diſtanti dalla. k. c. & ſimelmente dal ponto.l.ſia tirata la.l.t.equidiſtante alla.c.b.qual ſara pur perpendicolare

[97r]

EIGHT

let take under *d* and *e* two equal parts in the path or line *ab*; let one be the part *de* and the other *eg*. I say that the said equal parts partake equally the vertical,[77] that is the line *ac*. This will be proved in the following way; from the two points *e* and *g* let there be drawn the two lines *eh* and *gl*, perpendicular to the line *ac*, and from the two points or places *d* and *e* the two lines *dk* and *em*, perpendicular to the same *eh* and *gl*. Let the two perpendiculars *dk* and *em* be equal, then the said heavy body, at point *d* as at point *e*, in equal quantities or descents [along *ab*] will partake equally the vertical, and hence will be of the same positional heaviness in either of these places; which is the purpose. S.A. I have understood this; therefore continue. N.



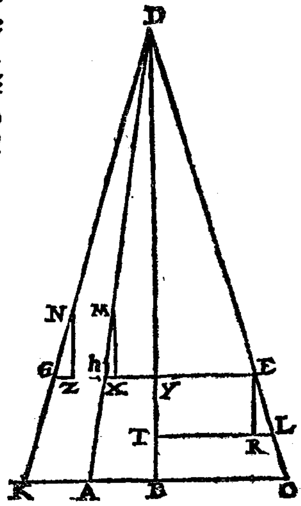
[Fig. 4.22]

QUESTION XLII. PROPOSITION XV.

If two heavy bodies descend by paths of different obliquities, and if the proportions of inclinations of the two paths and of the weights of the two bodies will be the same, taken in the same order, the power of both the said bodies in descending will also be the same. S.A. This proposition seems to me beautiful, and therefore give me a clear example, that I may be better satisfied. N. Let there be the line *abc* parallel to the horizon, and upon this there is perpendicularly erected the line *bd*, and from the point *d* there shall descend on either side the two paths or lines *da* and *dc* [Fig. 4.23]. Let *DC* be the more oblique. *Then by the ratio of their obliquity, I do not mean that of their angles, but of the lines to the parallel cut in which we take an equal part of the vertical* [emphasis added].[78] Then let the letter *e* represent a heavy body placed on the line *dc*, and the letter *h* another on the line *da*, and let the ratio of the simple heaviness of the body *e* to that of the body *h* be the ratio of *dc* to *da*. I say that the two heavy bodies in those places are of the same power or strength. And to demonstrate this, I draw *dk* of the same obliquity as *dc*, and I imagine on that a heavy body, equal to the body *e*, which I letter *g,* in a straight line with *eh*, that is, parallel to *ck*. now if possible (for the adversary) that the said two bodies *e* and *h* are not the same in power and equal in strength, assume that *e* is of greater strength, and hence able to descend and thus to draw up the body *h*. Now let us suppose (if possible) that the said body *e* descends as far as the point *l*, and that it makes the body *h* ascend to the point *m*. Make or draw *gn* equal to *hm*, which becomes also equal to *el*. And from the point *g*, draw *gh*, which will be perpendicular to *db*, the said three points or bodies *g*, *h*, and *e* being assumed in line and parallel to *kc*. And similarly from the point *l*, let *lt* be drawn parallel to *cb*, which will also be perpendicular

BB

ſopra la medeſima.d.b.& dalli tre ponti.n.m.e. ſiano tirate le tre perpendicolari.n.z.m.x.&.e.r.Et perche la proportione della.n.z.alla.n.g.é ſi come quella,che é dalla d.y.alla.d.g.e pero ſi come anchora quella della.d.b. alla. d. k. (per eſſer li detti tre triangoli ſimili.) Simelmente la proportione della.m.x.alla.m.h.é ſi come quella,che é dalla detta.d.b.alla.d.a.(per eſſer li detti dui triangoli ſimili.) Anchora la proportione della.m.x.alla.n.z.ſara ſi come quella della.d.k.alla.d.a.& quella medeſima (dal preſuppoſito) e dalla grauita del corpo.g.alla grauita del corpo. h. perche il detto corpo.g.fu ſuppoſto eſſer ſimplicemẽte,egualmente graue con el corpo.e. adunque tanto quanto,che il corpo.g.é ſimplicemente piu graue del corpo.h.per altro tanto il corpo. h. uien à eſſer piu graue per uigor del ſito del detto corpo.g.é pero ſi uengono ad egualiar in uirtu,ouer potentia, & per tanto quella uirtu,ouer potentia,che ſara atta à far aſcendere luno de detti dui corpi,cioe à tirarlo in ſuſo,quella medeſima ſara atta, ouer ſofficiente à fare aſcendere anchora l'altro,adunque ſel corpo.e.(per lauerſario)é atto, & ſofficiente à far aſcendere il corpo.h.per fin in.m.el medeſimo corpo.e.ſaria adunque ſofficiente à far aſcendere anchora il corpo g.à lui equale,& inequale declinatione,la qual coſa é impoßibile per la precedente propoſitione, adunque il corpo.e.non ſara de maggior uirtu del corpo.h.in tali ſiti, ouer luochi,che é il propoſito. S.A. Queſta é ſtata una bella ſpeculatione,& me é piaceſta aſſai.Et perche uedo eſſer hora tarda,non uoglio,che procedati in altro per hoggi.

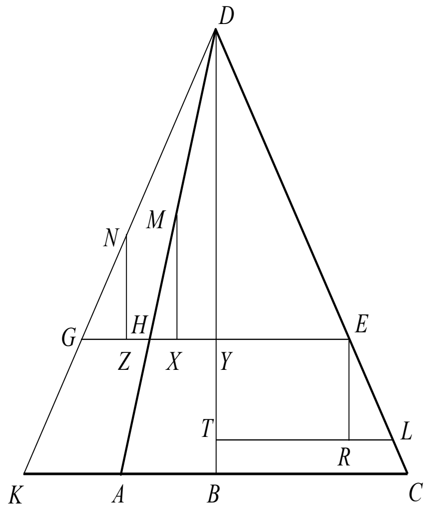


Fine del ottauo libro.

[97v]

BOOK

to the same *db*; and from the three points n, m, and e draw the perpendiculars *nz*, *mx*, and *er*. Since the ratio of *nz* to *ng* is as that of *dy* to *dg*, it is as that of *db* to *dk* (for the said three triangles barte similar). Likewise the ratio of *mx* to *mh* is as that of the said *db* to *da* (the two triangles being similar). Also the ratio of *mx* to *nz* will be as that of *dk* to *da*; and (by hypothesis) that is the same as that of the weight of the body *g* to the weight of the body *h*, because *g* is supposed to be simply equal in heaviness with the body *e*. Therefore, by however much the body *G* is simply heavier than the body *h*, by so much does the body *h* become heavier by positional strength than the said body *g*, and thus they come to be equal in strength or power. And since that same strength or power that will be able to make one of the two bodies ascend (that is, to draw it up) will be able or sufficient to make the other ascend. Thus if (for the adversary) the body *e* is able and sufficient to make the body *h* ascend to *m*, the same body *e* would be sufficient to make ascend also the body *g* equal to it, and equal in obliquity, which is impossible by the preceding proposition. Therefore the body *e* will not be of greater strength than the body *h* in such places or positions; which is the purpose.



S.A. This was a beautiful speculation and satisfied me well. And since I see it is now late, I do not want you to proceed further today.

End of the eighth book.

4.5 The Italian Critical Transcriptions

4.5.1 Book VII (1554)

[78r]

LIBRO SETTIMO DELLI
QUESITI, ET INVENTIONI DIVERSE,
DE NICCOLO TARTAGLIA.
Sopra gli principii delle Questioni Mechanice di
Aristotile.

QUESITO PRIMO FATTO DAL ILLUSTRISS.[IMO]
Signor Don Diego Hurtado di Mendozza, Ambasciator
Cesareo in Venetia.

SIGNOR AMBASCIATORE. Tartalea, de poi, che noi dessimo vacatione alle lettioni di Euclide, ho ritrovato cose nuove sopra le Mathematice. N. Che cosa ha ritrovato vostra Signoria. S.A. Le questioni Mechanice di Aristotile, Grece, e Latine. N. Eglie tempo assai che io le vidi, massime Latine. S.A. Che vene pare. N. Benissimo, e certamente le sono cose sutilissime et di profonda dottrina. S. A. Anchora io le ho scorse, e inteso di quelle la maggior parte, nondimeno me resta molti dubbii sopra di quelle, li quali voglio, che me li dichiarati. N. Signore, vi sono dubbii assai, che à volergli à sofficientia delucidare, à me saria necessario prima à dechiarare à vostra Signoria li principii della scientia di pesi. S.A. A me mi pare, che Aristotile dimostri il tutto, senza procedere, over intendere altramente la scientia di pesi. N. Eglie ben vero, che lui approva cadauna de dette questioni, parte con ragioni, e argomenti naturali, e parte con ragioni, e argomenti Mathematici. Ma alcuni di quelli suoi argomenti naturali, con altri argomenti naturali vi si puol opponere. Et alcuni altri con argomenti Mathematici (mediante la scientia di pesi detta disopra) se possono reprobar per falsi. Et oltra di questo lui pretermette, over tace una questione sopra delle libre, over bilanze di non poca importanza, over speculatione, e questo è processo (per quanto posso considerare) perche di tal questione, non si puo assignar la causa per ragion naturale, ma solamente con la detta scientia di pesi. S.A. Non credo, che questo sia la verita, cioe, che alcuna sua argomentatione patisca oppositione, perche Aristotile non fu un'ocha, ne manco credo, che lui habbia pretermesso, over taciuto questione alcuna sopra delle libre, che sia de importantia. N. Anci eglie troppo el vero, perche volendo considerare, giudicare, et dimostrare la causa della sua prima questione, si come naturale, cioe con quelli ultimi argomenti naturali, che lui aduce sopra le libre over bilance materiale. Medesimamente con altri argomenti naturali (come di sopra dissi) si puo approvare, che seguita tutto al contrario di quello, che in tal questione conclude, over suppone. Et volendo poi considerare, e giudicare tal Questione, si come Mathematico, e con argomenti Mathematici si puo medesimamente li detti sui argomenti reprobar per falsi, mediante la scientia di pesi detta di sopra. S.A. Come se considerano, e giudicano le cose, si come natura le, e come se considerano, e giudicano, si come Mathematico[?]

V *ij*

[78v]

LIBRO

N. El naturale considera, giudica, et determina le cose, secondo el senso, e apparentia di quelle in materia. Ma el Mathematico le considera, giudica, e determina, non secondo el senso, ma secondo la ragione (astrate da ogni materia sensibile) come che V. Sig. sa, che costuma Euclide. S.A. Circa di questo non so che rispondere, perche io non me arricordo cosi all'improviso il soggetto di tal sua prima questione, e pero ditime come, che quella parla, e dice. N. La dice, e parla precisamente in questa forma.

Perche causa le maggior libre, over bilanze, sono piu diligente delle menore. S.A. Ben? che voleti dire sopra di tal questione. N. Voglio dir questo, che sumendola, over considerandola, si come Mathematico (cioe astrata da ogni materia.) Senza alcun dubbio tal questione è universalmente vera, si per le ragioni da lui adutte per avanti, come, che per molte altre, che nella scientia di pesi addur se potria. Perche quella linea, che con la sua mobile istremita piu se allontana dal centro d'un cerchio, movesta da una medesima virtu, over potentia (in tal sua istremita) piu facilmente, e con maggior celerita, over prestezza, sara mossa, spenta, over portata, di quella, che con la detta sua istremita men se alontanara dal detto centro, e per tal ragione le libre, over bilanze maggiori, se verificano esser piu diligente delle menore. Ma volendo poi considerare, e approvare tal questione in materia, e con argomenti naturali, come, che in ultimo lui considera, e approva, cioe per el senso del vedere in esse libre, over bilanze materiale. Dico, che con tai sorte de argomenti non se verifica generalmente tal questione, anzi se trovara seguir tutto al contrario, cioe le libre, over bilanze menori esser piu diligente delle maggiori, e che questo sia el vero nelle libre, over bilanze materiale, la sperientia lo fa manifesto: perche se de uno ducato scarso voremo sapere de quanti grani lui sia scarso, con una libra, over bilanza granda, cioe con una de quelle, che adoprano li speciali per pesar specie, zuccaro, zenzero, e canella, e altre cose simile, malamente se ne potremo chiarire, ma con una di quelle librette, over bilancette piccole, che oprano li bancheri, orefici, e gioieleri, senza dubbio se ne potremo totalmente certificare. Per il che seguitaria tutto al contrario, di quello, che in tal questione se conchiude, e dimostra, cioe, che tai bilancette piu piccole siano piu diligente, delle piu grande, perche piu diligentemente, over sottilmente dimostrano la differentia di pesi. Et la causa di questo inconveniente non procede da altro, che dalla materia, perche le cose costrutte, over fabricate in quella, mai ponno esser cosi precisamente fatte, come, che con la mente vengono imaginate fuora di essa materia, per il che tal hor se vien à causar in quelle alcuni effetti molto contrarii alla ragione. Et per questo, e altri simili respetti, el Mathematico non accetta, ne consente alle dimostrationi, over probationi fatte per vigor, e autorita di sensi in materia, ma solamente à quelle fatte per demostrationi, et argomenti astrati da ogni materia sensibile. Et per questa causa, le discipline Mathematice non solamente sono giudicate dalli sapienti esser piu certe delle naturale, ma quelle esser anchora nel primo grado di certezza. Et pero quelle questioni, che con argomenti Mathematici se possono dimostrare, non è cosa conveniente ad approbarle con argomenti naturali. Et simelmente quelle, che sono già dimostrate con argomenti Mathematici (che sono piu certi) non è da tentare, ne da persuader si de certificarle meglio con argomenti naturali, li quali sono

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men certi. S.A. A me mi pare che lui voglia, in tal prima questione, che quella resti ottimamente chiarita (come è il vero) per le ragioni, e argomenti per avanti adutti, e dimostrati, le quale ragioni, over argomenti sono tutti Mathematici, e non naturali, perche parte de quelli se verificano per la .23. del Sesto di Euclide, e parte per la quarta del medesimo. N. Vostra Signoria insieme con lui dice la verita, che tal questione è manifesta per le sue ragioni adutte per avanti, e questo medesimo anchora io di sopra lo affermai, perche tai antecedenti sono stati da lui dimostrati con argomenti Mathematici, ma in fine de tai buone argomentationi, vi sottogionge due altre conclusioni, la prima delle quale dice precisamente in questa forma. Et certamente sono alcuni pesi, li quali posti nelle piccol libre, non sono manifesti al senso, e nelle grande sono manifesti. La qual conclusione, volendola considerare, giudicare, e approvare, si come naturale, cioe per vigore, e autorita del senso del vedere, nelle libre materiale, senza dubbio tal sua conclusione patisse oppositioni assai, perche nelle dette libre, over bilanze materiale, la maggior parte delle volte se trovara seguir tutto al contrario, cioe che sono alcuni pesi, li quali posti, nelle libre, over bilanze grande, non se faranno con alcuna inclinatione manifesti al senso del vedere. Et nelle bilanzette piccole se manifestar anno, cioe che far anno inclinatione visibile, e tutto questo, la sperientia lo manifesta. Perche se sopra una di quelle sopradette bilanze grande de Speciali, vi sara posto un grano di formento. Eglie cosa chiara, che nella maggior parte di quelle, non fara alcuna visibel inclinatione. Et nella maggior parte di quelle piccolette che usano li Banchieri, far anno inclinatione molto evidente. Ma volendo poi considerare, giudicare, e dimostrare tal sua questione, over conclusione, si come Mathematico, cioe fuora de ogni materia, senza dubbio tal sua conclusione saria falsa, perche ogni piccol peso posto in qual se voglia libra fara inclinar quella continuamente per fina all'ultimo, over piu basso luoco, che inclinar se possa, e tutto questo nelli principii della scientia di pesi à Vostra Signoria, lo faro manifesto. Dapoi lui sottogionge anchora quest'altra conclusione, e dice in questa forma. Et certamente sono alcuni pesi, le quali sono manifesti nell'una, e l'altra sorte de libre (cioe nelle maggiori, e nelle menori) ma molto piu nelle maggiori, perche molto piu granda inclinatione, vien fatta dal medesimo peso nelle maggiori. La qual conclusione, volendolo considerare, giudicare, e approvare, si come naturale (come fu detto dell'altra) cioe per vigore, e autorita del senso del vedere, nelle dette libre materiale, certamente questa non patira men oppositioni dell'altra, per le medesime ragioni in quella adutte. Et similmente, volendo poi considerare, giudicare, e dimostrare tal conclusione, come Mathematico, cioe fuora de ogni materia medesimamente tal sua conclusione saria falsa, perche ogni sorte di peso posto in qual si voglia sorte de libra, fara inclinar quella de continuo per fina à tanto che quella sia gionta all'ultimo, over piu basso luoco, che quella inclinar si possa, e tutto questo, nelli detti principii della scientia di pesi dimostrativamente à quella si fara manifesto. S.A. Anchor che tutte queste vostre oppositioni, e argomenti naturali, habbiano del verisimile non posso credere, che il non ve sia altre ragioni, e argomenti, si naturali, come Mathematici da poter difendere, e salvare, tal sua questione insieme con quelle altre due conclusioni. Anci è ho ferma opinione che chi studiasse con

[79v]

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diligentia sopra à tal materia, ritrovaria tutte quelle particolarita materiale, che sono causa, che tal questione, e conclusioni non se verificano in materia, come che l'autor conchiude, et dice. Et dapoi che quelle fusseno ritrovate, et conosciute, tengo che saria cosa facile à rimediarli, e fare che se verificasseno in materia precisamente, come che l'autor propone. N. Vostra Signoria non è di vana opinione, perche in effetto tutte quelle cose che nella mente sono conosciute vere, e massime per dimostrationi astratte da ogni materia, ragionevolmente si debbono anchora verificare al senso del vedere in materia (altramente le Mathematice sariano in tutto vane, e di nullo giovamento, over profitto all'huomo), e se per caso quelle non se verificano, come che nelle sopradette libre, over bilance maggior, e menor, e stato detto, e disputato. Eglie da credere, anci da tener per fermo, che il tutto proceda dalla disproportionalita, e inequalita delle parti, e membri materiali, dalli quali vengono composte, cioe che le dette parti, e membri dell'una piu se discostano, over allontanano da quelle considerate fuora de ogni materia, di quello che fanno quelli dell'altra. E per tanto volendo difendere, e salvare tal questione Aristotelica, cioe far che quella sempre se verifichi in materia, e in ogni qualita de libre, over bilance si grande, come piccole. Bisogna agguagliar le dette parti, over membri di cadauna di quelle, talmente che quelli siano egualmente distanti da quelle considerate fuora de ogni materia sensibile. Il che facendo non solamente se verificara tal sua questione al senso in materia, cioe nelle dette libre, over bilance materiale, ma anchora se verificaranno quelle altre due conclusioni, che sottogionse in fine. S.A. Io ho accaro che la mia opinione se sia verificata.

QUESITO SECONDO FATTO CONSEQUEN– temente dal medesimo Illustrissimo Signor Don Diego Ambasciator Cesareo.

SIGNOR AMBASCIATORE. Ma per non haver troppo ben inteso le ragioni da voi allegate, vorria che un'altra volta, e piu chiaramente me le repli casti. N. Dico Signore, che la causa che le sopradette libre, over bilance maggiore, e menore, non rispondeno secondo che l'autor conchiude, e dimostra, non procede d'altro, che dalla inequalita delle parti, over membri materiali, dalli quali vengono composte, le quai parti, over membri, sono li dui bracci, e anchora il sparto (cioe quel axis over centro, sopra del qual girano li detti bracci in cadauna de loro, perche li detti bracci, e sparto nelle libre, over bilance maggiore sono molto piu grossi, e corpulenti di quelle delle menore. Et perche li bracci di quelle libre, over bilance che vengono considerate, come Mathematico, cioe fuora de ogni materia, sono considerati, et supposti, come simplice linee, cioe senza larghezza, ne grossezza, e il sparto, over axis di quelle vien considerato, e supposto un simplice ponto indivisibile, le qual sorte de libre, over bilance. Quando che possibil fosse à darne una cosi realmente spogliata, e nuda de ogni materia sensibile, come che con la mente vengono considerate, senza alcun

[80r]

SETTIMO

dubbio quella saria agilissima, e diligentissima sopra à tutte le libre, over bilance materiale, di quella medesima grandezza, perche quella saria totalmente libera da ogni material impedimento. Et per tanto conchiudendo dico, che quanto piu le parti, over membri di una libra, over bilanza materiale, se accostano, over appropinquano alle parti, over membri della non materiale (qual è la originale, over ideale di tutte le materiale) tanto sara piu agile, e diligente di quelle che men vi se accostaranno, over appropinquaranno (di quella medesima grandezza). Et perche le parti, over membri di quelle bilancette, che adoprano li Bancheri, e Gioieleri (disopra allegate) molto piu se accostano, over appropinquano alle parti, over membri della detta sua ideale, di quello che fanno le parti, over membri di quelle libre, over bilance maggiori, che adoprano li Speciali (disopra allegate) perche li brazzetti delle dette bilancette piccole sono sottilissimi, e quelli delle grande sono piu grossi. Onde li sottili piu se accostano alla simplice linea (quale manca de larghezza, e grossezza) di quello fanno li piu grossi, e corpulenti, e similmente il sparto, over axis delle dette librette, over bilancette piccole, è piccolino, e sottile, e quello delle grande, è piu grande, e grosso. Onde il detto sparto delle dette bilancette piccole piu se accosta, over appropinqua al sparto della sua ideale (qual è un ponto indivisibile) di quello fa il sparto delle dette bilance grande per esser piu grande, e grosso. Et questa è la principal causa che le sopradette librette, over bilancette menori, se dimostrano al senso piu diligente delle maggiori, cosa totalmente contraria alla sopra allegata Aristotelica questione.

QUESITO TERZO FATTO CONSE–
quentemente dal medesimo Illustrissimo
Signor Don Diego Ambascia–
tor Cesareo.

SIGNOR AMBASCIATORE. Ben in che modo si puo difendere, e salvare tal sua questione, cioe far che quella se verifichi al senso in materia secondo che lui propone, over conchiude. N. Bisogna fondarse sopra le libre, over bilance ideale, cioe sopra quelle che vengono considerate con la mente astratte da ogni materia, e vedere in che cosale maggiore siano differente dalle menore, la qual cosa essendo osservata nelle libre, over bilance materiale sara difesa, e salvata tal questione Aristotelica, cioe che quella sempre se verificara al senso nelle dette libre materiale. S.A. Non ve intendo parlatime piu chiaro. N. Dico Signore, che à voler difendere, e salvare tal questione, bisogna fondarse, over reggersi per le libre, over bilance ideale, cioe per quelle, che con la mente vengono considerate fuora de ogni materia, e vedere in che cosa le maggiori siano differente dalle menori, sopra la qual cosa considerando, e guardando, se trovara, che le dette libre, over bilance maggiori, non sono differente dalle menori, eccetto che nella longhezza di suoi bracci, e in tutte le altre cose se agguagliano, perche anchor che li bracci delle libre maggiori siano piu longhi de quelli delle menori, tamen non sono ne piu grossi, ne piu sottili de quelli, perche, si nelle maggiori, come nelle menori, sono considerati,

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LIBRO

come simplice linee, le quale mancano di larghezza, e grossezza, e pero in larghezza, e grossezza non vi è alcuna differentia. Et similmente li sparti, over axi delle libre, over bilance maggiori sono eguali alli sparti, over axi delle menori, perche si nelle maggiori, come nelle menori sono considerati, come simplici ponti, li quali ponti per esser tutti indivisibili, sono eguali, le qual cose essendo diligentemente osservate nelle libre, over bilance materiale, cioe che le maggiore non siano differente dalle menore, eccetto che nella longhezza di suoi bracci, ma che in larghezza, et grossezza siano eguali, e cosi li lor sparti materiali senza dubbio in quelle, non solamente se verificara al senso quello, che Aristotile nella detta sua questione conchiude. Ma anchora se verificaranno, quelle altre due conclusioni che vi sottogionse in fine. (Anchor che in astratto, cioe fuora de ogni materia, ambedue false siano, come che per li principii della scientia di pesi à V.S. faro manifesto.) Et siano le dette libre, over bilance di che qualita, materia, e condition si voglia, pur che osservino la detta egualita nella grossezza di detti bracci, e sparti loro. S.A. Certamente che questo vostro discorso me piace assai.

QUESITO QUARTO FATTO CONSE– quentemente dal medesimo Illustrissimo Signor Don Diego Ambasciator Cesareo.

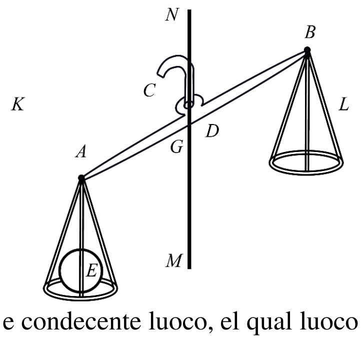
SIGNOR AMBASCIATORE. Ma se ben me aricordo voi dicesti anchora nel principio del nostro ragionamento, che Aristotile pretermette, over ta ce una questione sopra delle dette libre di non poca importantia, over speculatione, hor ditemi, che questione è questa. N. Se vostra Signoria ben se aricorda della sua seconda questione, in quella lui interrogativamente adimanda, e consequentemente dimostra, perche causa quando chel sparto sera disopra della libra, e che l'uno di bracci di quella da qualche peso sia portato, over spinto à basso, remosso che sia, over levato via quel tal peso, la detta libra di nuovo reascende, e ritorna al suo primo luoco. Et se il detto sparto è di sotto della detta libra, e che medesimamente l'uno di suoi bracci sia da qualche peso pur portato, over spinto à basso remosso, over levato che sia via quel tal peso la detta libra non reascende, ne ritorna al suo primo luoco (come che fa nell'altra positione) ma rimane disotto, cioe à basso. Hor dico, che lui pretermette, over tace un'altra questione, che in questo luoco se conveneria, di molta maggior speculatione di cadauna delle sopradette, la qual questione è questa. Perche causa quando che il sparto è precisamente in essa libra, et che l'un di bracci di quella sta da qualche peso portato, over urtato à basso, remosso, over levato che sia sia quel tal peso, la detta libra di nuovo reascende al suo primo luoco, si come che fa anchora quella, che ha il sparto di sopra da lei. S.A. Questa mi pare una bella questione, e molto piu remota dal nostro intelletto naturale che le due sopradette, e molto havero accaro ad intendere la causa di tal effetto, ma prima voglio che me chiariti un dubbio, che nella mente me intona sopra delle sopra allegate questioni, il quale è questo.

Quesito

[81r]

S E T T I M O
QUESITO QUINTO FATO CONSEQUENTE–
emente dal medesimo Illustrissimo Signor Don Diego,
Ambasciator Cesareo.

SIGNOR AMBASCIATORE. Dove se trova una libra, over bilanza materiale, che il suo sparto sia di sopra, over di sotto di quella, anci à me mi pare, che il detto sparto in tutte sia precisamente in esse libre, come, che nella vostra terza question se suppone, e non di sopra, ne manco di sotto. N. Anchor, che di tal sorte bilance non si faccia, over si trovi el non resta pero, chel non se ne potesse fare. S.A. A me mi pare una materia, à mover questione sopra à cose che non si costumano, ne si trovino in essere. N. Il tutto si fa Signore, perche tutti li artificiosi istromenti, che per augumentare le forze del huomo se oprano, in qual si voglia arte Mechanica se referiscono à una delle sopradette tre specie de libre, over bilance, et cosi in ogni dubbio, over questione, che sopra ad alcuno de tai istromenti nascer potesse, volendone conoscere, over assignare la intrinsica causa. Eglie necessario prima venir a quella sorte libra, over bilanza, alla qual piu se referisse quel tal istromento, e dalla detta libra, over bilanza se vien al cerchio, per la mirabil virtu, e potentia del quale se risolve il tutto, come, che nella scientia di pesi si fara manifesto. S.A. Essendo adunque cose di tanta importantia, voglio, che me replicati, e dimostrati figuralmente cadauna de dette tre Questioni, over parti a una per una: perche le voglio ben intendere, e cominciati alla prima. N. Per dimostrar in figura la prima parte di tal questione. Sia la libra .a.b. el sparto della quale sia el ponto. c. (qual sparto sia alquanto di sopra della detta libra .a.b. come nella figura appare) e sia che per la impositione del peso. e. el suo brazzo .a.d. sia da quel tirato a basso, come che di sotto appare in detta figura: hor dico, che chi levasse via el detto peso .e. tal brazzo .a.d. reascendaria, e



retornaria al suo primo, e condecente luoco, el qual luoco saria nel ponto, over sito .k. e cosi l'altro brazzo .d.b. descendaria per fina al ponto, over sito .l. e tutto questo procede: perche nel trasportar el detto brazzo .a.d. a basso, piu della mitta di tutto el fusto della detta libra .a.b. se vien a trasferirsi in alto, cioe oltra la perpendicolar .n.m. passante per il sparto .c. la qual perpendicolar se chiama

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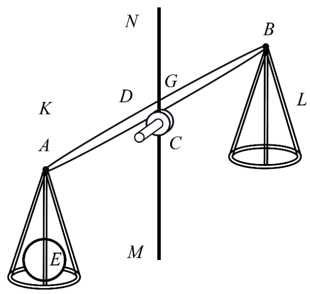
LIBRO

la linea della direttione,[79] cioe, che la parte .b.d.g. in alto ellevata vien à esser tanto piu della mita de tutto el fusto .a.b. quanto che è dal .d. al .g. e la restante parte .a.g. ridutta al basso vien à esser tanto manco della mita di tutto el detto fusto .a.b. quanto che è dal detto ponto.g.al ponto .d. perche adunque tal parte .b.d.g. in alto ellevata è molto maggiore del restante brazzo .a.g. al basso trasferto, levandose via el detto peso .e. la detta parte .a.g. (piu debole) vien à esser urtata, e spinta dall'altra maggior parte .b.d.g. in alto ellevata (per esser di lei piu potente) per fin à tanto, che la detta linea della direttione caschi perpendicolarmente sopra el detto fusto, over libra .a.b. e che seghi quello in due parti equali in ponto .d. S.A. Questa ragion è quasi simile à quella che aduce Aristotile, ma è alquanto piu chiara, e miglior figura.

QUESITO SESTO FATTO CONSE– quentemente dal medesimo Illustrissimo Signor Don Diego Ambasciator Cesareo.

SIGNOR AMBASCIATORE. Hor seguitati la seconda parte. N. Per dimostrare la seconda à vostra Signoria. Pongo sia la libra .a.b. la qual habbia il sparto (cioe quel ponto, over polo, sopra del qual lei gira) alquanto di sotto, cioe disotto dal fusto .a.b. come disotto appar in ponto .c. e sia anchor, che per la imposition del peso .e. el suo brazzo .a.d. sia da quel tirato à basso, come che di sotto nella figura appar, hor dico, che chi levasse via el detto peso .e. tal brazzo non reascenderia ne ritornaria al suo primo luoco, cioe in ponto .k. (come, che fa in quella, che ha il sparto di sopra) ma restaria cosi inclinato à basso, e la causa di questo procede, perche nel trasportarse el detto brazzo .a.d. al basso piu della mitta di tutto el fusto, over libra .a.b.



si vien à trasferire drio à quello, oltra la linea della direttione, cioe oltra la perpendicolar .n.m. qual passa per il sparto .c. tal che tutta la parte .a.g. al basso ridutta, vien à esser tanto piu della mitta di tutta la libra .a.b. quanto, che è dal .d. al .g. e la parte .g.b. in alto ellevata vien à restare tanto meno della detta mitta, quanto, che è dal detto .d. al detto .g. per esser adunque la ellevata parte .g.b. di menor quantita della inclinata .a.g. vien à esser piu debole, over men potente di lei, e pero, non è atta, ne sofficiente à po–

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SETTIMO

terla urtare, e sforzare à farla ascendere al suo primo luoco in .k. come fece nella passata, anci quella restara cosi inclinata al basso, e la retenera lei cosi in aere ellevata, che è il proposito. S.A. Queste due parti quasi, che il nostro intelletto le apprende per ragion naturale, senza altra dimostratione. N. Cosi è Signore.

QUESITO SETTIMO FATTO CONSEQUENTEMEN– te dal medesimo Illustrissimo Signor Don Diego, Ambasciator Cesareo.

SIGNOR AMBASCIATORE. Hor seguitatimo la terza parte, quale diceti, che manca in questo luoco, cioe dove nasce la causa, che quando el sparto de una libra sara precisamente nel mezzo di essa, cioe ne di sotto, ne di sopra, ma nel mezzo di quella, come, che sono tutte le libre, over bilance, che communamente se oprano, e che l'uno di brazzi di quella sia da qualche peso (over dalla nostra mano) urtato à basso, levado, che sia via quel tal peso (over mano) immediate tal brazzo riascende, et ritorna al suo primo luoco si come che anchor fa quella libra qual tien il sparto di sopra da essa libra. Perche in effetto la causa di questo ultimo effetto mi par molto piu remota dal nostro intelletto de cadauna delle altre due. N. E ho detto à vostra Signoria, che à voler dimostrare la causa di tal effetto à me è necessario à diffinire, e dechiarire prima à vostra Signoria alcuni termini, e principii della scientia di pesi. S.A. So no cosa longa questi principii, che vi bisogna dechiarare. N. Per quanto aspetta à voler demostrare simplicemente questa particolarita sara cosa brevissima, vero è che quando, che vostra signoria volesse intendere ordinariamente tutti li principii di tal scientia, vi saria da dire assai. S.A. Bensa, che voglio intendere il tutto ordinariamente, come si de. N. L'hora è tarda Signore per far questo effetto. S.A. Ben andati, e ritornati dimane da mattina. N. Ritornaro Signore.

Il fine del Settimo Libro.

X *ij*

4.5.2 Book VIII (1554)

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LIBRO OTTAVO DELLI
QUESITI, ET INVENTIONI DIVERSE,
DE NICOLO TARTAGLIA BRISCIANO.
Sopra la Scientia di Pesi

QUESITO PRIMO FATTO DAL ILLUSTRISS.[IMO]
Signor Don Diego Hurtado di Mendozza, Ambasciator
Cesareo in Venetia.

SIGNOR AMBASCIATORE. Hor voria Tartaglia, che me incomenciasti à dechiarire ordinariamente quella scientia de pesi, di che me parlasti hieri. Ma, perche conosco tal scientia non esser semplicemente per se (per non esser le arte liberale, salvo che sette) ma subalternata, voria che prima me dicesti, da che scientia, over disciplina quella derivi, e nasci. N. Signor Clarissimo parte di questa scientia nasce, over deriva dalla Geometria, e parte dalla Natural Philosophia: perche, parte delle sue conclusioni se dimostrano Geometricamente, e parte se approvano Physicalmente, cioe naturalmente. S. A. E ve ho inteso circa questa particolarita.

QUESITO SECONDO FATTO CONSEQUEN–
temente dal medesimo Illustrissimo Signor Don
Diego Ambasciator Cesareo.

SIGNOR AMBASCIATORE. Ma ditime anchora, che costrutto si puo cavar di tal scientia. N. Li costrutti, che di tal scientia si potriano cavare, saria quasi impossibile à poterli à vostra Signoria isprimere, over connumerare, nondimeno io ve referiro quelli, che per al presente à me sono manifesti. Et per tanto dico, che primamente per vigore di tal scientia, eglie possibile à conoscere, e misurare con ragione la vertu, e potentia di tutti questi istromenti Mechanici, che da nostri antiqui sono stati ritrovati, per augumentare la forza de l'huomo, nel ellevare, condurre, over spingere avanti ogni grave peso cioe in qual si voglia grandezza, che quelli siano constituidi, over fabricati, secondariamente per vertu di tal scientia, non solamente eglie possibile di poter con ragion conoscere, e misurare simplicemente la forza de l'huomo, ma anchora eglie possibile di trovar el modo di augumentar quella in infinito, e in varii modi, e cosi in qual si voglia modo eglie possibile à conoscere l'ordine, e proportione di tal augumentatione, come, che in fine con varii istromenti Mechanici à V. S. faro conoscere, e vedere. S.A. Questo havero molto accaro.

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OTTAVO

QUESITO TERZO. FATTO CONSE–
quentemente dal medesimo Illustrissimo Signor
Don Diego Ambascia
tor Cesareo.

SIGNOR AMBASCIATORE. Hor seguitati, come vi pare circa à tal scientia. N. Per procedere regolatamente, hoggi diffiniremo solamente alcuni termini, e modi di parlare occorrenti in questa scientia, accio che il frutto della intelligentia di quella, V.S. piu facilmente apprenda. Dimane poi dechiariremo li principii di tal scientia, cioe quelle cose che in tal scientia non si possono dimostrare, perche (come che V.S. sa) ogni scientia ha li suoi primi princpii indemostrabili, li quali essendo concessi, over supposti per lor mezzo si disputa, e sostenta tutta la scientia, dopo questo andaremo preponendo varie propositioni, over conclusioni sopra di tal scientia, e parte de quelle dimostraremo à V.S. con argomenti Geometrici, e parte approvaremo con ragioni naturali, come di sopra dissi. Et dapoi questo, vostra Signoria, preponera tutti quei dubbii, over questioni che à quella gli parera, nelle cose Mechanice, e massime sopra li mirabili effetti delli sopradetti istromenti materiali che augumentano la forza dell'huomo, che per le cose dette, e approbate, nella detta scientia de pesi, tutte se resolveranno. S.A. Questo vostro procedere cosi regolatamente molto mi piace.

QUESITO QUARTO FATTO CONSE–
quentemente dal medesimo Illustrissimo Signor Don
Diego Ambasciator Cesareo.

SIGNOR AMBASCIATORE. Hor seguitate adunque le dette diffinitioni consequentemente. N.

QUESITO. IIII. DIFFINITIONE PRIMA.

Li corpi se dicono di grandezza eguali, quando che quelli occupano, over empino luochi eguali. S.A. Datemi qualche material essempio. N. Essempi gratia, doi corpi spherici gettati, over prontati in una medesima forma, over in forme eguale, se diriano eguali di grandezza, anchor che fusseno di materia diversa, cioe che l'uno fusse di piombo, e l'altro di ferro, over di pietra, e cosi si debbe intendere in qual si voglia altra diversita di forma. S.A. E ve ho inteso, seguitati. N.

QUESITO. V. DIFFINITIONE II.

Similmente li corpi se dicono di grandezza diversi, over ineguali, quando che quelli occupano, over empino luochi diversi, over ineguali. Et maggiore se intende quello, che occupa maggior luoco. S.A. E ve ho inteso, seguitati. NIC.

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QUESITO. VI. DIFFINITIONE TERZA.

La vertu d'un corpo grave se intende, e piglia per quella potentia, che lui ha da tendere, over di andare al basso, e anchora da resistere al moto contrario, cioe à che il volesse tirar in suso. S.A. Quando che non vi dico altro seguitati, perche col mio tacere, e ve dinoto havermi inteso, e che debbiati seguitare. N.

QUESITO. VII. DIFFINITIONE QUARTA.

Li corpi se dicono de vertu, over potentia, equali, quando che quelli in tempi eguali di moto pertransiscono spacii eguali.

QUESITO. VIII. DIFFINITIONE QUINTA.

Li corpi se dicono de vertu, over potentia diversa, quando che quelli in tempi diversi, pertransiscono di moto, spacii eguali, over che in tempi eguali pertransiscono intervalli ineguali.

QUESITO. IX. DIFFINITIONE SESTA.

La vertu, over potentia de corpi diversi, quella se intende esser maggiore, la quale nel pertransire uno medesimo spacio summe manco tempo. Et menor quella che summe piu tempo, overamente quella che in tempi eguali pertransisse maggior spaccio.

QUESITO. X. DIFFINITIONE SETTIMA.

Quelli corpi se dicono essere di uno medesimo genere, quando che sono di egual grandezza, e che sono anchora di egual vertu, over potentia.

QUESITO. XI. DIFFINITIONE OTTAVA.

Quelli corpi se dicono essere de diversi generi, quando che sono di egual grandezza, e che non sono di egual vertu, over potentia.

QUESITO. XII. DIFFINITIONE NONA.

Quelli corpi se dicono essere simplicemente eguali in gravita, li quali sono realmente di egual peso, anchor che fusseno di materia diversa.

QUESITO. XIII. DIFFINITIO
NE DECIMA.

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OTTAVO

Un corpo se dice essere simplicemente piu grave d'un altro, quando che quello è realmente piu ponderoso di quello, anchor che fusse di materia diversa.

QUESITO XIIII. DIFFINITIONE XI.

Un corpo se dice essere piu grave d'un'altro secondo la specie, quando che la sostantia material di quello è piu ponderosa della sostantia material dell'altro, come che è il piombo del ferro, e altri simili.

QUESITO XV. DIFFINITIONE XII.

Un corpo se dice essere piu, over men grave d'un'altro nel descendere, quando che la rettitudine, obliquita, over dependentia del luoco, over spacio dove descende lo fa descendere piu, over men grave dell'altro, e similmente piu, over men veloce dell'altro, anchor che siano ambidui simplicemente eguali in gravita.

QUESITO XVI. DIFFINITIONE XIII.

Un corpo si dice essere piu grave, over men grave d'un'altro, secondo il luoco, over sito, quando che la qualita del luoco dove che lui se riposa, e giace, lo fa essere piu grave dell'altro anchor che fusseno simplicemente egualmente gravi.

QUESITO XVII. DIFFINITIONE XIIII.

La gravita d'un corpo se dice essere nota, quando che il numero delle libre, che lui pesa ne sia noto, over altra denomination de peso.

QUESITO XVIII. DIFFINITIONE XV.

Li brazzi de una libra, over bilancia se dicono essere nel sito, over luoco della equalita, quando che quelli stanno equidistanti al piano dell'Orizonte.

QUESITO XIX. DIFFINITIONE XVI.

La linea della direttione è una linea retta imaginata venire perpendicolarmente da alto al basso, e passare per il sparto, polo, over assis de ogni sorte libra, over bilanza.

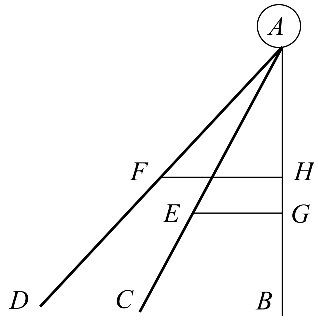
QUESITO XX. DIFFINITIONE XVII.

Piu obliquo se dice essere quel descenso, d'un corpo grave, il quale in una medesima quantita, capisse manco della linea della direttione, overamente del descenso

[84v]

LIBRO

retto verso il centro del mondo. S.A. In questa non ve intendo troppo bene e pero datemi uno essempio. N. Per essemplificare questa diffinitione sia il corpo .a. e il retto descenso di quello verso il centro del mondo sia la linea .a.b. e sia anchora li descensi .a.c. e .a.d. e de questi dui ne sia signati le due quantita, over parti .a.e. e .a.f. eguale, e dalli dui ponti .e. e .f. siano tirate le due linee .e.g. e .f.h. equidistanti al piano dell'Orizonte, e perche la parte .a.b. è menore della parte .a.g. il descenso .a.f.d. se dira esser piu obliquo del descenso .a.e.c. perche lui capisse manco del descenso retto, cioe della linea .a.b. in una medesima quantita. Et questo medesimo si debbe intendere in tutti li descensi che potesse fare il detto corpo .a. (over altro simile) stante appeso al braccio di alcuna libra, cioe che quel descenso se dira esser piu obliquo, che per lo medesimo modo capira manco della linea della direttione, in una medesima quantita de descenso. S.A. E ve ho inteso à sofficientia, e pero seguitati se haveti altra cosa da diffinire. N. Signore questa è la ultima cosa che habbiamo da diffinire sopra à questa materia. Dimane poi dichiariremo li principii di questa scientia, secondo la promessa. S.A. Alla bon'hora.



QUESITO. XXI. FATTO CONSE–
quentemente dal medesimo Illustrissimo Signor
DonDiego Ambasciator Cesareo.

SIGNOR AMBASCIATORE. Hor seguitati Tartaglia questi vostri principii. N. Li principii de qual si voglia scientia alcuni vogliano che siano detti dignita, perche quelli approvano altri, e loro non ponno essere approvati da altri, alcuni le chiamano suppositioni, perche se suppongono per veri in detta scientia, altri piacque chiamarli petitioni, perche volendo disputare tal scientia, e quella sostentare con dimostrationi, bisogna prima adimandare all'aversario la concessione de quelli, perche se lui non li volesse concedere (ma negare) saria negata tutta la scientia, ne vi occorreria à disputarla altramente. Et perche questa ultima opinione mi piace alquanto piu delle altre due, petitioni le chiamaremo, e cosi anchora in forma de petitioni li proferiremo.

QUESITO. XXII. PETITIONE PRIMA.

Adimandamo che ne sia concesso, che il movimento naturale de ogni corpo ponderoso, e grave sia rettamente verso il centro del mondo. S.A. Questo non è da negare.

Quesito

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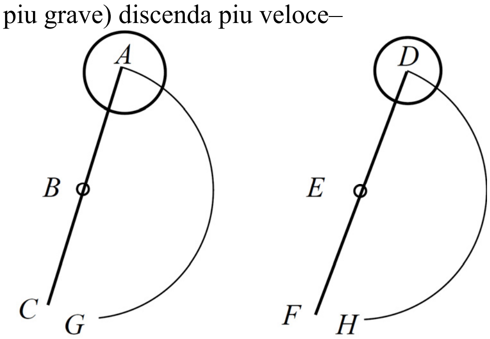
OTTAVO

QUESITO XXIII. PETITIONE II.

Simelmente adimandamo, che ne sia concesso quel corpo, ch'è di maggior potentia debbia anchora discendere piu velocemente, et nelli moti contrarii, cioe nelli ascensi, ascendere piu pigramente, dico nella libra. S.A. Datime uno essempio materiale sopra di questa petitione, se voleti, che ve intenda. N. Sia, essempi gratia, le due libre .a.b.c. e .d.e.f. equali, cioe, che li dui brazzi .a.b. e .b.c. siano equali alli dui brazzi .d.e. e .e.f. e li lor sparti, over centri siano .b. e .e. e nella istremita del brazzo .b.a. vi sia appeso il corpo .a. poniamo de libre due in gravita, e nella istremita de l'altro brazzo, cioe in ponto .c. non vi sia alcuna altra gravita, e cosi nella istremita del brazzo .e.d. vi sia appeso el corpo .d. poniamo di una libra sola in gravita, e nella istremita dell'altro brazzo, cioe in ponto .f. non vi sia alcuna gravita, e siano li detti dui corpi, cosi congionti ellevati con la mano in alto egualmente, come che di sotto appar in figura: hor adimando, che me sia concesso, lasciando andare cadauno de detti dui corpi cosi in alto ellevati, che il corpo .a. (per esser piu grave) discenda piu veloce–



mente al basso del corpo .d. cioe, che il detto corpo .a. sumara manco tempo à pertransire il curvo spacio .a.g. di quello fara il detto corpo .d. pertransire il curvo spatio .d.h. li quali spacii vengono à esser eguali, perche li brazzi de dette libre sono eguali dal presupposito, e pero li detti dui spacii, over descensi curvi, vengono à esser circonferentie di cerchii eguali. Et è converso, quando, che li detti corpi saranno discesi nel suo infimo, over piu basso luoco, cioe l'uno in ponto .g. e l'altro in ponto .h. adimando, che me sia concesso, che quella vertu, over potentia, la qual essendo appesa nell'altro brazzo della libra in ponto .c. sara atta ad ellevare el detto corpo .a .per fin al luoco, dove che al presente se ritrova nella figura superiore quella medesima sia atta ad ellevar piu velocemente il corpo .d. essendo a pesa nell'altro brazzo della sua libra, cioe in ponto .f. S.A. Questo vi concedo, perche la sperientia ne rende buona testimonianza. N. Ma vostra Signoria sappia, che quello, che havemo detto, e adimandato delli detti dui corpi, delli quali l'uno è simplicemente piu potente dell'altro, il medesimo adimandamo de dui corpi simplicemente eguali in potentia ma inequali per vigor della lor positione, over sito nel brazzo de una medesima libra, essempi gratia, se nel brazzo .a.b., della

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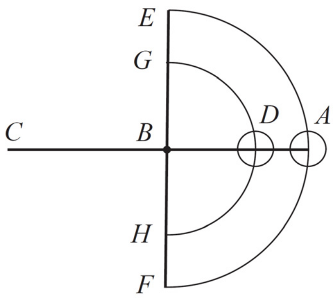
libra .a.b.c. ve sia appeso li dui corpi .a. e .d. eguali simplicemente in potentia, cioe, l'uno in ponto .a. e l'altro in ponto .d. come di sotto appar in figura, anchor, che siano simplicemente egualmente potenti, nondimeno il corpo .a. in tal positione per la .13. diffinitione se dira esser piu grave del corpo .d. come per lavenire se fara manifesto, perche in questo luoco non si puo assignar la ragione per le cose dette, ma per lavenire se provara el corpo .a. in simel sito esser piu grave del corpo .d. e pero essendo quelli ellevati l'uno in ponto .e. e l'altro in ponto .g. e dapoi essendo ambi dui abandonati, dico, che il corpo .a. discendera piu veloce del corpo .d. e è converso, essendo l'uno, e l'altro discesi nelli loro infimi luochi, cioe l'uno in ponto .f. e l'altro in ponto .h. quella potentia che sara atta in ponto .c. ad ellevare il corpo .a .dal ponto .f. per fina al ponto .e. quella medesima sara atta ad ellevare nel medesimo luoco, molto piu velocemente il corpo .d.



dal ponto .h. per fin al ponto .g. S.A. Anchora questa è cosa chiara, ma voria intendere due cose da voi. la prima è, che voria intendere, perche non fingeti la soprascritta figura de libra, con quelle sue due tazzette appese l'una da un capo, e l'altra da l'altro (come nelle material libre si costuma) per imponervi li pesi, over campioni in l'una, e nell'altra le cose, che se hanno da ponderare; la seconda è, che voria sapere se questo essempio de libra si debbe intendere di quelle, che hanno il lor sparto di sopra, over di quelle, che l'hanno di sotto, over di quelle, che non l'hanno, ne di sopra, ne di sotto, ma in esse libre proprie. N. Circa alla prima, rispondo, che la pura libra se intende per quella pura longhezza, che forma quelli dui brazzi l'uno di qua, l'altro di la dal sparto, ò siano li detti brazzi equali tra loro, over inequali, e quelle due tazzette, che dice V.S. non sono parte della libra, ma vi se aggiongono per commodita del ponderante, per imponervi li campioni, e pesi, che ha da ponderare, si come ch'è anchora la sella d'un cavallo, la quale non è parte del cavallo, ma una cosa aggionta per commodita di colui, che l'ha da cavalcare, e perche meglio si vede, e comprende uno cavallo nudato della sua sella, che con la sella, et simelmente una libra nudata di quelle sue due tazzette, che con le tazzette senza tazzette la essemplificamo. Circa alla seconda particolarita, dico, che la presente libra, e simelmente tutte quelle, che per l'avenir si proponera (non specificando altro) si debbono intendere di quelle, che hanno il sparto in lor medesime, come nelle materiale si costuma. S.A. E ve ho inteso, seguitati. N.

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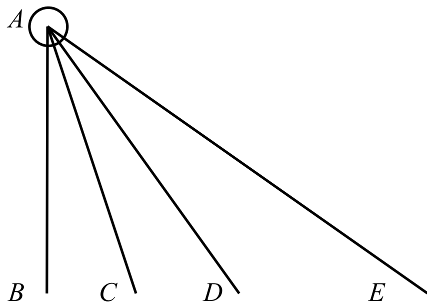
O T T A V O

QUESITO XXIIII. PETITIONE III.

Anchora adimandamo, che ne sia concesso un corpo grave esser in el discendere tanto piu grave, quanto che il moto di quello è piu retto al centro del mondo. S.A. Datime anchora uno qualche material essempio sopra à quest'altra petitione se voleti, che vi intenda. N. Sia, essempi gratia, il corpo grave .a. e poniamo, che le quattro linee .a.b. .a.c. .a.d. a.e. siano quattro luochi, over spacii da poter descendere el detto corpo .a. e poniamo anchora, che la linea .a.b. sia il rettissisimo, e perpendicolar descenso verso il centro del mondo, onde la linea .a.d. veneria ad esser piu retta verso il detto centro del mondo della linea .a.e. e per tanto in questo caso adimandamo, che ne sia concesso il detto corpo .a. esser piu grave nel discendere per la linea .a.d. che per la linea .a.e. (per esser come detto piu retta di quella al centro del mondo), e simelmente per la linea .a.c. descendere piu grave, che per la linea .a.d .per esser tal linea a.c. piu retta al centro del mondo della detta linea .a.d. e così quanto piu el detto corpo .a. se andara accostando alla detta linea .a.b. nel suo descendere se suppone tanto piu grave descendere, perche quel transito, over descenso, che forma piu acuto angolo con la linea .b.a. in ponto .a. se intende esser piu retto al centro del mondo, di quello, che lo forma men acuto. Onde per la linea .a.b. vien à discendere piu grave, che per qual si voglia altro verso.



Et questo, che havemo detto, e adimandato del sopradetto corpo .a. separato da ogni libra, il medesimo adimandamo de quelli, che descendono appesi al brazzo di qualche libra. Essempi gratia, sia anchora el detto corpo .a. appeso al brazzo della libra .a.b.c. girante sopra al sparto, over centro .b. overamente al brazzo della libra a.d.e. girante sopra al sparto, over centro .d. e sia el perpendicolar descenso verso il centro del mondo la linea retta .a.f. e el descenso, che faria el detto corpo .a. con el brazzo .a.b. della libra .a.b.c. sopra el centro .b. la linea curva .a.g. Et el descenso, che faria el medesimo corpo .a. con el brazzo .a.d. della libra .a.d.e. sopra el centro .d. la linea curva .a.h. Hor dico, e adimando, che ne sia concesso il detto corpo .a. esser piu grave nel descendere per il descenso .a.b. che pel descenso .a g. per essere el detto descenso .a.h. piu retto al centro del mondo del descenso .a.g. perche el detto descenso .a.h.

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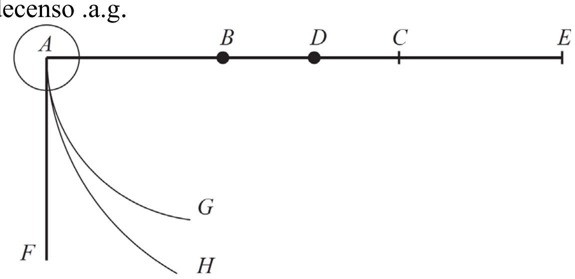
forma piu acuto angolo con la linea .a.f. (qual'è l'angolo .b.a.f. della contingentia) di quello fa lo decenso .a.g.



S.A. E ve ho inteso benissimo, e tal petitione non è da negare, e pero seguitati nell'altra. N.

QUESITO. XXV. PETITIONE IIII.

Anchora adimandamo, che ne sia concesso quelli corpi esser egualmente gravi, secondo el sito, over positione, quando che li lor descensi in tai siti sono egualmente obliqui, e piu grave esser quello, che nel suo sito, over luoco dove se riposa, over giace ha il descenso manco obliquo. S.A. Anchora questa vien a esser manifesta per quello fu detto nella precedente, e anchora sopra la seconda petitione, e pero seguitati. N.

QUESITO. XXVI. PETITIONE V.

Simelmente adimandamo, che ne sia concesso quel corpo esser men grave d'un altro secondo el sito, over luoco, quando che per el descenso di quello altro, nell'altro brazzo della libra in lui seguita il moto contrario, cioe, che da lui vien ellevato insuso verso il cielo, e è converso. S.A. Questa è cosa troppo chiara da concedere. N.

QUESITO. XXVII. PETITIONE SESTA

Anchora adimandamo, che ne sia concesso, niun corpo esser grave in se medesimo. S.A. Questa vostra petitione non intendo. N. Cioe, che l'acqua nella acqua, il vino nel vino, l'olio nel olio, e l'aere nel aere non essere di alcuna gravita. S.A. E ve ho inteso, e è cosa concessibile, perche la sperientia nel manifesta, si che, seguitati. N. Non ci è altra cosa da adimandare à V. S. Diman, piacendo à Iddio, intraremo nelle propositioni. S.A. Saranno propositioni assai. N. Non troppo signore. S.A. Credeti, che le spediremo dimane. N. Non credo Signore, che le spediremo nanche fra diman, e l'altro. S.A. Ben andate ritornati da mattina a bon hora.

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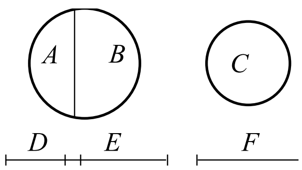
QUESITO. XXVIII. PROPOSITIONE

PRIMA.

SIGNOR AMBASCIATORE Hor seguitati Tartalea queste vostre propositioni, over conclusioni consequentemente l'una drieto all'altra, e sotto brevita. NICOLO.

La proportione della grandezza di corpi de un medesimo genere, e quella della lor potentia è una medesima. S.A. Datemi uno essempio. N. Siano li dui corpi .a.b. e .c. de uno medesimo genere, e sia .a.b. maggiore, e sia la potentia del corpo .a.b. la .d.e. e quella de corpo .c. la .f. Hor dico che quella proportione, che è dal corpo .a.b. al corpo .c. quella medesima è della potentia .d.e. alla potentia .f. Et se possibile è esser altramente (per l'aversario) sia che la proportione del corpo .a.b. al corpo .c. sia menore di quella della potentia .d.e. alla potentia .f. Hor sta del corpo .a.b. (maggiore) compreso una parte eguale al corpo .c. menore, quale sia la parte .a. e perche la vertu, over potentia del composito è composta dalla vertu di componenti. Sia adunque la vertu, over potentia della parte .a. la .d. e la vertu, over potentia del residuo .b. de necessita sara la restante potentia .e. et perche la parte .a. è tolta egual al .c. la potentia .d. (per il converso della .7. diffinitione) sara eguale alla potentia .f. e la proportione de tutto il corpo .a.b. alla sua parte .a. (per la seconda parte della .7. del quinto di Euclide) sara, si come quella del medesimo corpo .a.b. al corpo .c. (per esser .a. egual al .c.) e similmente la proportione della potentia .d.e. alla potentia .f. sara, si come quella della detta potentia .d.e. alla sua parte .d. (per esser la .d. egual alla .f.). Adunque la proportione de tutto il corpo .a.b. alla sua parte .a. sara menore di quella di tutta la potentia .d.e. alla sua parte .d. Adunque eversamente[80] (per la .30. del quinto di Euclide) la proportione del medesimo corpo .a.b. al residuo corpo .b. sara maggiore di quella di tutta la potentia .d.e. alla restante potentia .e. la qual cosa saria inconveniente, e contra la opinion dell'aversario, il qual vol che la proportione del maggior corpo al menore sia menore, di quella della sua potentia alla potentia del detto menore. Adunque destrutto l'opposito rimane il proposito. S.A. Sta bene, seguitati. NIC.



QUESITO. XXIX. PROPOSITIONE
SECONDA.

La proportione della potentia di corpi gravi de uno medesimo genere, e quella della lor velocita (nelli descensi) se conchiude esser una medesima, anchor quel–

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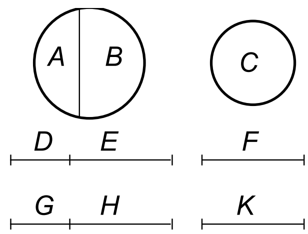
a delli lor moti contrarii (cioe delli lor ascensi) se conchiude esser la medesima, ma trasmutativamente. S.A. Essemplificatime tal propositione. NIC.

Sia anchora li dui corpi .a.b. e .c. de uno medesimo genere, e di grandezza diversa, e sia lo .a.b. maggiore, e sia la potentia del .a.b. la .d.e. e del .c. la .f. e perche il corpo di potentia, over gravita maggiore (per la seconda petitione) descende piu velocemente, sia adunque la velocita nel descender del corpo .a.b. la .g.h. e quella del corpo .c. la .k. hor dico, che la proportione della potentia .d.e. alla potentia .f. e quella della velocita .g.h. alla velocita .k. esser una medesima, e quella delli lor moti contrarii esser quella medesima, ma trasmutativamente, cioe che la proportione della velocita del corpo .a.b. alla velocita del corpo .c. nel moto contrario (cioe nell'ascendere) esser, si come quella della potentia .f. alla potentia .d.e. over, come del corpo .c. al corpo .a.b. la qual cosa se dimostra per il medesimo modo, che fu dimostrata la precedente, cioe se la proportione della potentia .d.e. alla potentia .f. non è (per l'aversario) si come quel la della velocita .g.h. alla velocita .k. necessariamente la sara maggiore, over menore, hor poniamo che la sia menore, della potentia .d.e. ne assignaremo la parte .d. eguale ala .f. e cosi della velocita .g.h. ne assignaremo la parte .g. eguale alla .k. e arguiremo, come nella precedente, dicendo che la proportione di tutta la potentia .d.e. alla sua parte .d. sara (per la seconda



parte della. 7. del quinto di Euclide) si come quella della medesima potentia .d.e. alla potentia .f. (per esser la .d. e .f. eguale) e similmente la proportione de tutta la velocita .g.h. alla sua parte .g. esser, si come quella della medesima .g.h. alla .k. Adunque la proportione di tutta la potentia .d.e. alla sua parte .d. sara menore di quella di tutta la velocita .g.h. alla sua parte .g. Onde (per la 30 del quinto di Euclide) la proportione di tutta la medesima potentia .d.e. al suo residuo .e. havera maggior proportione, che tutta la velocita .g.h. al suo residuo .h. la qual cosa saria contra la opinione dell'aversario qual suppone, che la proportione della maggior potentia alla menore esser menore di quella della maggior velocita alla menore. Et con li medesimi argomenti se procederia quando che quel supponesse che la proportione della maggior potentia alla menore fusse maggiore di quella della maggior velocita alla menore, distrutto adunque l'opposito rimane il proposito, hor per la seconda parte della nostra conclusione, dico, che la proportione della velocita delli descensi, e delli contrari moti, cioe delli ascensi de detti corpi è una medesima, ma trasmutativamente, cioe che la proportione della velocita del corpo .a.b. essendo da qualche altra vertu imposta nell'altro braccio della libra in alto ellevato (poniamo per fin alla linea della direttione) alla velocita del corpo .c. dalla medesima vertu, pur in alto ellevato per fin alla medesima linea della direttione sara, si come quella della velocita .k. alla velocita .g.h. over della potentia .f. alla potentia .d.e. over del cor–

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po .c. al corpo .a.b. perche quanta vertu, over potentia ha un corpo grave per descendere al basso, tanta ne ha anchora per resistere al moto contrario, cioe à che il volesse tirare, over à levare in alto adunque la potentia del corpo .a.b. per resistere à che il volesse ellevare in alto, sara tanto quanto la sopradetta .d.e. e quella del corpo .c. sara tanto quanto la sopradetta .f. Adunque quella vertu che nell'altro braccio della libra sara atta ad ellevare cosi à pena il detto corpo .a.b. per fin alla linea della direttione, quella medesima sara atta ad ellevare il detto corpo .c. tanto piu velocemente (per fin alla detta linea della direttione) quanto che la sua resistentia sara proportionalmente menore di quella del corpo .a.b. e perche la detta resistentia del detto corpo .c. e tanto menore della resistentia del corpo .a.b. quanto che la sua potentia .f. della potentia .d.e. Adunque la velocita del corpo .c. (nel moto contrario) alla velocita del corpo .a.b. sara, si come la potentia .e.d. alla potentia .f. over come che il corpo .a.b. al corpo .c. che il proposito.

CORRELARIO.

Da qui se manifesta qualmente la proportione della grandezza di corpi di uno medesimo genere, e quella della lor potentia, e quella della lor velocita nelli lor descensi esser una medesima. Et similmente quella della lor velocita nelli moti contrarii, ma trasmutativamente. S. AMBASCIA. E ve ho inteso, seguitati pur. NICOLO.

QUESITO XXX. PROPOSITIONE III.

Se saranno dui corpi simplicemente eguali di gravita, ma ineguali per vigor del sito, over positione, la proportione della lor potentia, e quella della lor velocita necessariamente sara una medesima. Ma nelli lor moti contrarii, cioe nelli ascensi, la proportione della lor potentia, e quella della lor velocita se afferma esser la medesima, ma trasmutativamente. S. AMBACIA. Fatemi la dimostratione di questo. NICOLO.

Siano li dui corpi .a. e .b. simplicemente eguali di gravita, e sia la libra .c.d. il cui centro, over sparto il ponto .e. e sia nella strema parte del brazzo .e.c. cioe in ponto .c. appeso, e sostentato il corpo .a. e in uno altro luoco piu propinquo al sparto nel medesimo brazzo, hor sia in ponto .f. vi sia sostentato il corpo .b. Et à ben che questi dui corpi siano simplicemente eguali di gravita, nondimeno (per la quarta petitione) il corpo .a. sara (per vigor del luogo) piu grave del corpo .b. perche il descenso di quello qual sia lo .c.h. e manco obliquo del descenso del corpo .b. qual sia lo .f.g. (per la terza, e quarta petitione) essendo adunque il corpo .a. piu grave, secondo il sito del corpo .b. sara etiam piu potente, e essendo piu potente (per la seconda petitione) nelli descensi descendera piu velocemente del corpo .b. e nelli moti contrarii, cioe nelli ascensi piu tardamente. Dico adunque che la proportione della lor velocita nelli descensi esser simile à quella della loro potentia, e quella delli lo–

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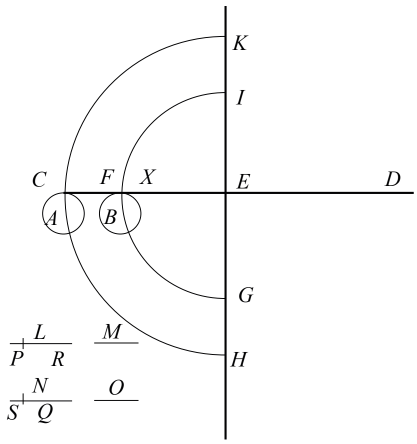
ro ascensi esser pur la medesima, ma trasmutativamente, et per dimostrar la prima parte, sia la potentia del corpo .a. la .l. e quella del corpo .b. la .m. e la velocita del corpo .a. (nelli descensi) la .n. e quella del corpo .b. la .o. Dico che la proportione della velocita .n. alla velocita .o. esser, si come quella della potentia .l. alla potentia .m. la qualcosa se dimostra, si come la precedente, cioe se possibil fusse, che la proportione della potentia .m. (per l'aversario) potesse esser menore di quella della velocita .n. alla velocita. o. sumendo della potentia .l. la parte .p. eguale alla .m. e della velocita .n. la parte .q. eguale alla .o .e arguendo, come nella precedente, cioe che la proportione di tutta la potentia .l. alla sua parte .p. (per la .7. del quinto di Euclide) sara menore di quella di tutta la velocita .n. alla sua parte .q. Onde (per la .30. del quinto di Euclide) la proportione della medesima potentia .l. all'altra sua parte, over residuo .r. havera maggior proportione di quello, che havera tutta la velocita .n. all'altra sua parte, over residuo .s. la qual cosa saria inconveniente, et contra la opinione dell'aversario, qual suppone che la proportione della maggior potentia alla menore, esser menore di quella della maggior velocita, alla menore, e il medesimo inconveniente seguiria quando che l'aversario, supponesse che la proportione della potentia .l. ala potentia .m. fusse maggiore di quella della velocita .n. alla velocita .o. distrutto adunque l'opposito rimane il proposito. La seconda parte se risolve, over arguisse, si come nella precedente, cioe che quella potentia, che nell'altro brazzo della libra (poniamo in ponto .d.) sara atta ad ellevare il corpo .a. per fin alla linea della direttione, cioe in ponto .k. quella medesima sara atta ad ellevare tanto piu velocemente il corpo .b. per fin al ponto .i. quanto che la potentia del detto corpo .b. (qual'è la .m.) è menore della potentia del corpo .b. (qual'è la .l.) perche quanto che la potentia d'un corpo è menore tanto men resiste al moto contrario, e è converso, adunque la velocita del corpo .b. à quella del corpo .a. (nelli ascensi) sara, si come quella della potentia .l. alla potentia .m. che è il secondo proposito. S. AMB. Questa è stata assai bella propositione, ma seguitati pur. NIC.



Quesito

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QUESITO XXXI. PROPOSITIONE IIII.

La proportione della potentia di corpi simplicemente equali in gravita, ma inequali per vigor del sito, over positione, e quella delle lor distantie dal sparto, over centro della libra, se approvano esser equali. S.A. Datime uno essempio. N. Siano li dui corpi .a. e .b. della figura precedente simplicemente equali in gravita e sia la libra .c.e.d. el centro, over sparto della quale sia el ponto .e. e sia appeso el corpo .a. in ponto .c. e lo corpo .b. nel ponto .f. come nella figura precedente appare. Dico, che la proportione della potentia del corpo .a. (quale sia la .l.) alla potentia del corpo .b. (quale sia la .m.) esser simile à quella, ch'è dalla distantia, over brazzo .e.c. alla distantia, over brazzo .e.f. e tutto questo si approva secondo l'ordine della precedente, cioe, se la proportione della distantia, over brazzo .c.e. alla distantia, over brazzo .f.e. non è (per l'aversario), si come quella, ch'è dalla potentia .l. alla potentia .m. adunque necessariamente sara, maggiore, over minore, hor sia prima (se possibil è) menore sia, del brazzo, over distantia .c.e. maggiore cavato el brazzo, over distantia .e.f. menore dalla banda verso .c. quale sia la .c.x. e dalla potentia .l. ne sia cavata la parte .p. equal alla .m. Adunque (per la .7. del quinto di Euclide) la proportione di tutta la distantia, over brazzo .e.c. alla sua parte .c.x. havera menor proportione, di quello, che havera tutta la potentia .l. alla sua parte .p. Onde (per la. 30. del quinto di Euclide) la proportione del brazzo, over distantia .c.e. alla restante distantia, over brazzo .e.x. havera maggior proportione di quello havera la potentia .l. alla restante potentia .r. la qual potentia .r. verria ad esser la potenza del medesimo corpo .b. stante nel ponto .x. la qual cosa saria inconveniente, perche, se la proportione della maggiore distantia dal sparto alla menore (per l'aversario) havera maggior proportione, che la maggior potentia alla menore, questo doveria seguire in ogni positione, e tamen se vede occorrere al contrario, cioe, che la proposizione della distantia .c.e. alla distantia .e.x. saria maggiore di quella della potentia .l. alla potentia del corpo .b. nel sito, over luoco, dove .x. distrutto adunque lo opposito rimane il proposito.

CORRELARIO.

Dalle cose dette, e dimostrate, se manifesta non solamente la proporzione delle distantie dal sparto nel brazzo della libra, e quella delle potentie di corpi simplicemente equali in gravita, in tai siti, over luochi, e simelmente la velocita de quelli nelli descensi esser una medesima, ma anchora li lor descensi, e anchora li loro ascensi osservano la medesima, perche qual proportione è dal brazzo .e.c. al brazzo .e.f. talla è dal curvo descenso .c.h. al curvo descenso .f.g. e simelmente del curvo ascenso .c.k. al curvo ascenso .f.i. perché li dette descensi, e ascensi vengono à esser cadauno de loro la quarta parte della circonferentia de dui cerchii. delli quali el semidiametro del maggiore verria à esser el brazzo, over distantia .e.c. et del menore el brazzo, over distantia .e.f. S.A. Anchor questa è stata una bella propositione seguitati. N.

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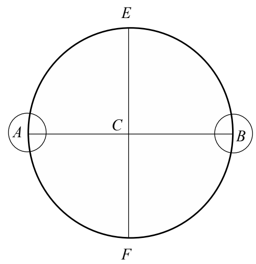
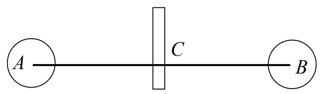
QUESITO XXXII. PROPOSITIONE V.

Quando, che la positione de una libra de brazzi equali sta nel sito della equalita, e nella istremita de l'uno, e l'altro brazzo vi siano appesi corpi simplicemente equali in gravita, tal libra non se separara dal detto sito della equalita, e se per caso la sia da qualche altro peso in l'uno de detti brazzi imposto separata dal detto sito della equalita, overamente con la mano, remosso quel tal peso, over mano, tal libra de necessita ritornara al detto sito della equalita. S.A. Questa è quella Questione, della quale voi dite, che manca Aristotile nelle sue Questioni Mechanice. N. Cosi è Signore. S.A. Molto haro à caro à intendere la causa di tal effetto, e pero seguitate. N. Sia essempi gratia la libra .a.c.b. el centro della quale sia il ponto .c. e sia el brazzo .a.c. equale al brazzo .b.e. e stia nel sito della equalita, come se prepone. Et che nella istremita de l'uno, e l'altro brazzo vi sia appeso uno corpo (poniamo el corpo .a. e .c.) li quali corpi siano simplicemente equali in gravita. Dico che la detta libra (per la impositione de detti corpi) non se separara dal detto sito della equalita, e se pur quella fusse separata dal detto sito, ò per la impositione di qualche altro peso, over con la mano, remosso che sia quel tal imposto peso, over mano, tal libra de necessita ritornara al detto sito della equalita. La prima parte è manifesta, perche li detti dui corpi sono simplicemente di equal gravita (dal presupposito) et simelmente sono equalmente gravi per vigor del sito, per la quarta petitione (per esser li loro descensi equalmente obliqui) e pero essendo quelli si per vigor del sito, come che simplicemente duna equal gravita, e potentia, e pero niun de loro fara atto à poter ellevar l'altro, cioe à farlo ascendere di moto contrario, e pero restaranno nel medesimo sito della equalita. S.A. Questo ve credo e ve lo haveria largamente concesso senza altra demonstratione, per esser cosa naturale.



Ma seguitati la seconda parte, la qual me pare molto piu astrata, over lontana dal nostro intelletto naturale dell'altra. N. Per la seconda parte sia pur anchora la libra .a.c.b. de brazzi equali et nella istremita de quelli siano pur appesi li dui corpi .a. et .b. simplicemente equali in gravita, la qual libra per le ragioni di sopra adutte stara nel sito della equalita, come di sotto appar in figura.



Hor essendo spinto el brazzo. a.c. al basso con la mano, over per la impositione di qualche altro peso sopra el corpo .a. remosso via la mano, over quel tal peso, el brazzo di tal libra reascendera, e ritornera al suo primo luoco della equalita, e per assignar la causa propinqua di tal effetto, sia descritto sopra el centro .c. el cerchio .a.c.b.f. per el viazzo, che fariano li detti dui corpi alzando, over arbassando li brazzi della detta libra, e sia tirata la linea della direttione, quale sia la .e.f. e sia diviso l'arco .a.f. in quanti parti equali si voglia (hor sia in quattro) nelli trei ponti, q.s.u. e in altre tante sia anchor diviso l'arco .e.b. nelli trei ponti

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.i.l.n. e dalli detti trei ponti .n.l.i. siano tirate le tre linee .n.o. l.m. e .i.k. equidistante al sito della equalita, cioe al diametro, over linea .a.b .le quale segaranno la linea .e.f. della direttione ne li tre ponti .z.y.x. Simelmente dalli tre ponti .q.s.u. siano tirate le tre linee .q.p. .s.r. e .u.t. pur equidistante alla medesima linea .a.b. le quale segaranno la medesima linea della direttione .e.f. nelli trei ponti .w.ρ.k. Et dapoi sia arbassato con la mano il corpo .a. (over con la impositione di qualche altro peso) per fin al ponto .u. e l'altro corpo .b. (à quel opposito) in tal positione se trovar a esser asseso de moto contrario per fin al ponto .i. Onde per queste cose cosi disposite veniremo ad haver diviso tutto el descenso .a.u. fatto dal detto corpo .a. nel discendere in ponto .u. in tre descensi, over parti equali, le quale sono. a.q. q.s. e .s.u. e simelmente tutto el descenso .i.b. qual faria il detto corpo .b. nel discendere, over ritornare al suo primo luoco (cioe in ponto. b.) vera ad esser diviso in trei descensi, over in tre parti equali le quali sono .i.l. .l.n. e .n.b. e cadauno de questi tre e tre parti di descensi capisse una parte della linea della direttione, cioe il descenso dal .a. al .q. piglia, over capisse della linea della direttione la parte .c.k. e lo descenso .q.s. capisse la parte .kρ. e lo descenso .s.u. capisse la parte .ρ.w. e l'altro descenso, che resta à descendere al detto corpo .a. cioe el descenso .u.f. capisse la linea, over parte .w.f. Et simelmente el descenso del corpo .b. dal ponto .i. al ponto .l. capisse della medesima linea della direttione la parte .x.y. e nel descenso dal ponto .l. al ponto .n. capisse la parte .y.z. e dal ponto .n. al ponto .b. capisse la parte .z.c. et tutte queste parti sono fra loro inequale, cioe la parte .c.z. è maggiore della .z.y. e la .z. y. della .y.x. e la .y.x. della .x.e. e simelmente la parte .c.k. è maggiore della parte .kρ. e la parte .k.ρ. della parte .ρ.w. e la .ρ.w. della .w.f. e tutto questo facilmente Geometrice si puo provare, e simelmente se puo provare, la parte .w.f. essere equale alla parte .e.x. e la parte .ρw. alla parte .x.y. e la parte .ρ.k. alla parte .y.z. e la parte .k.c. alla parte .z.c. Hor per tornare al nostro proposito. Dico, che il corpo .b. stante quel nel ponto .i. vien à esser piu grave, secondo il sito del corpo .a. stante quello in ponto .u. (come disotto appar in figura) perche il descenso del detto corpo .b. dal ponto .i. nel ponto .l. è piu retto del descenso del corpo .a. dal ponto .u. nel ponto .f. (per la seconda parte della quarta petitione) perche capisse piu della linea della direttione, cioe, che nel descendere il detto corpo .b. dal ponto .i. nel ponto .l. lui capisse, over piglia della linea della direttione, la parte .x.y. e il corpo .a. nel discendere dal ponto .u. nel ponto .f. lui caperia della detta linea della direttione, la parte .w.f. e perche la parte .x.y. è maggiore della linea, over parte .w.f. (per la. 17. diffinitione) piu obliquo sara il descenso dal ponto .u. al ponto .f. di quello dal ponto .i. al ponto .l. Onde (per la seconda parte della quarta petitione) il corpo .b. in tal positione sara piu grave secondo il sito del corpo .a. essendo adunque piu grave, levando via lo imposto peso, over la mano dal corpo .a. (per il converso della quinta petitione) lui fara reascendere di moto contrario il detto corpo .a. dal ponto .u. al ponto .s. e lui descendera dal ponto .i. nel ponto .l. nel qual ponto .l. lui venira à trovarse anchora piu grave del detto corpo .a. secondo el sito, perche il detto corpo .a. stante nel ponto .s. havera il descenso .s.u. piu obliquo del descenso .l.n. del corpo .b. perche capisse men parte della

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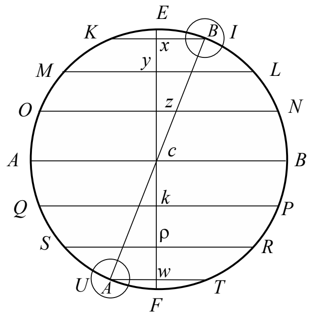
detta linea della direttione, cioe, che la parte .ρ.w. è menore della parte .y.z. Onde per le ragioni di sopra adutte, el detto corpo .b. fara ellevare il detto corpo .a. e ascendere nel ponto .q. e lui descendera nel ponto .n. nel qual ponto .n. el medesimo corpo .b. si trovara pur piu grave anchora, secondo il sito del corpo .a. perche il descenso dal .q. in .s. è piu obliquo del descenso dal ponto .n. nel ponto .b. per esser la parte .z.c. maggiore della parte .k.ρ. E pero (per le ragioni di sopra adutte) el detto corpo .b. fara reascendere il detto corpo .a. al ponto .a. (suo primo, e condecente luoco) e lui medesimamente descendara nel ponto .b. pur suo primo, e condecente luoco, cioe nel sito della equalita, nel qual sito li detti dui corpi se trovaranno (per le ragioni adutte nella prima parte di questa) egualmente gravi secondo el sito, e perche sono anchora simplicemente egualmente gravi, se conservarono nel detto luoco, come di sopra fu detto, e approvato, che è il nostro proposito. S.A. Questa è stata una bella demostratione, ma se ben me arricordo, voi dicesti anchor sopra la detta prima question Mechanica de Aristotile, che quelle sue due conclusioni, che lui vi aduce in fine esser false. N. Eglie è vero. S.A. Per che ragione. N. La ragione di tal particolarita, over oppositioni se verificaranno nella sequente propositione, mediante alcuni correlarii, che dalle cose dette, e dimostrate nella precedente si manifestano, delli quali il primo è questo.



CORRELARIO.

Dalle cose dette, et dimostrate di sopra, se manifesta qualmente un corpo grave in qual si voglia parte, che lui se parta, over removi dal sito della equalita lui si fa piu leve, over leggiero secondo el sito, over luoco, e tanto piu, quanto piu sara remosso da tal sito, essempi gratia. El corpo .a. si trovara esser piu leve nel ponto .u. che nel ponto .s. et nel ponto .s. piu che nel ponto .q. e nel ponto .q. che nel ponto .a. sito della equalita, per causa della varieta di descensi, cioe, che l'uno è piu obliquo dell'altro, cioe el descenso .u.f. vien à esser piu obliquo del descenso .s.u. perche la parte .f.w. della direttione, è menore della .w.ρ. et cosi el descenso .s.u. vien à esser piu obliquo del descenso .q.s. perche la parte .w.ρ. è menore della parte .ρ.k. e lo descenso .q.s. vien à esser piu obliquo del descenso .a.q. perche la parte .ρ.k. è menore della parte .c.k. e per le medesime ragioni si manifesta del corpo .b. cioe, che quello sara piu leve nel ponto .i. che nel ponto .l. e nel ponto .l. che nel ponto .n. e nel ponto .n. che nel ponto .b. sito della equalita.

CORRELARIO SECONDO.

Anchora per le cose dette, e dimostrate se manifesta, che removendosi li detti dui corpi dal detto sito della equalita, cioe l'uno in giuso, et l'altro insuso, anchor

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che l'uno, e l'altro sia fatto piu leve secondo il sito, tamen in ogni positione men leve si trovara quello che sara in alto ellevato di quello, che si trovara al basso oppresso, e questo è manifesto per la argomentatione di sopra adutta, cioe che il corpo .b .nel sito, over ponto .i. esser piu grave del corpo .a. nel sito, over ponto .u. e cosi nelli altri siti superiori si trovara piu grave del corpo .a. nelli siti inferiori, simili. S.A. E ve ho inteso, seguitati. NICOLO.

QUESITO. XXXIII. PROPOSITIONE VI.

Quando che la positione d'una libra de bracci eguali sia nel sito della egualita, e che nella istremita dell'uno e l'altro brazzo vi siano appesi corpi simplicemente ineguali di gravita, dalla parte dove sara il piu grave sara sforzata à declinare per fin alla linea della direttione. S.A. A me non pare che questa vostra propositione possa esser universalmente vera, e questo voglio che voi medesimo il confessati perche voi sapeti che nel Correlario precedente haveti conchiuso, che removendosi li detti dui corpi .a. e .b. (dalla figura della precedente propositione) dal sito della egualita, cioe l'uno in giuso, e l'altro in suso, anchor che l'uno è l'altro sia fatto piu leve, over leggero, secondo il sito, tamen in ogni positione men leve si trovara quello, che sara in alto ellevato di quello, che si trovara quello, che sara à basso inclinato. N. Eglie il vero Signore. S.A. Se questo è vero, eglie da credere, anci da tener per fermo, che chi imponesse sopra al corpo .a. à basso inclinato, un'altro corpetto qual in gravita fusse eguale à quella differentia, che il corpo ellevato è piu grave, secondo il sito del corpo à basso inclinato, che cadauno de loro restaria nel proprio luoco dove si trovasse, e accio meglio me intendiati, voi sapeti che il corpo .b. della figura della precedente propositione, stante ellevato per fin al ponto .i. (come in quello appare) e il corpo .a. à basso inclinato per fin al ponto .u. voi approvasti il detto corpo .b. in tal sito esser piu grave del corpo .a. N. Signore eglie il vero. S.A. Adunque conchiudo che chi imponesse in tal sito un'altro corpetto sopra al corpo .a. qual fusse precisamente di tanta gravita, quanto, che è la differentia, che è fra li detti dui corpi .a. e .b. in tal positione li detti dui corpi restariano fermi, e stabili in tal positione, perche in tal sito se trovariano egualmente potenti, cioe il corpo .b. non saria sofficiente à far reascendere il detto corpo .a. al sito della egualita, per esser il detto corpo .a. (per vigor di quel corpetto aggionto) tanto grave è potente quanto lui, cioe che per quel tanto che il detto corpo .b. è piu potente, over grave per vigor del sito del corpo .a. per quel tanto sara piu grave il detto corpo .a. del detto corpo .b. per vigore della gravita di quel simplice corpetto aggiontovi sopra, per il che il detto corpo .b. non sara atto à far reascendere il detto corpo .a. al sito della egualita, e manco il corpo .a. sara atto à potere piu ellevare il detto corpo .b. del sito .i. e pero l'uno è l'altro de necessita non se potra partire di tal suo luoco, cioe il corpo .a. con la gionta di quell'altro corpo, non potra reascendere al sito della egualita, ne manco potra descendere alla linea della direttione, cioe al ponto .f. come se conchiude nella vostra propositione, e pur il detto corpo .a. insieme con quell'altro corpetto aggionto, saria simplicemente piu grave del corpo .b. e per tanto non poteti ne–

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gare che tal vostra propositione non sia falsa in quanto al generale, eglie ben vero, che se la gravita di quel corpetto che fusse aggionto sopra al detto corpo .a. fusse maggiore della gravita, nella quale il corpo .b. è piu grave per vigor del sito del corpo .a. seguiria quello che nella detta vostra propositione se conchiude, e se per caso tal gravita di corpetto fusse menore di detta differentia, tal corpo .b. faria ascendere il detto corpo .a. in un'altro sito piu alto del ponto .u. secondo che piu, over men scarsezasse la gravita di tal corpetto della detta differentia che è fra loro per vigor del sito. N. Questa oppositione di vostra Signoria certamente è molto speculativa, e bella, nondimeno advertisco quella, che se ben il corpo .b. in tal sito .i. sia piu grave del corpo .a. nel sito .u. la differentia di queste due gravita ineguale è tanto piccola, over minima, ch'eglie impossibile à potere ritrovare una cosi piccola, over minima differentia fra due quantita ineguale. S.A. Questo che haveti detto mi pare una cosa molto absorda da dire, e manco da credere, perche essendo la quantita continua divisibile in infinito, eglie una materia à voler dire, che il sia impossibile à dare un corpettino di tanta poca quantita, e gravita, quanto che è la differentia che è fra la gravita del corpo .b. nel sito .i. e quella del corpo .a. nel sito .u. N. Signore la ragione è quella che ne chiarisse le cose dubbiose, e che ne discerne il vero dal falso. S.A. Eglie il vero. N. S'eglie il vero, nanti che vostra Signoria dia assoluta sententia alla mia propositione quella ascolti prima le mie ragioni. S.A. Seguitati, e dite cio, che vi pare. N. Sia essempi gratia, la medesima libra .a.b.c. della precedente propositione, nelle istremita, della quale siano pur appesi li dui corpi .a. .b. eguali simplicemente in gravita, e sia abbassato con la mano il corpo .a. e ellevato il corpo .b. come di sotto appare in figura. Dico che in tal sito, il corpo .b. è piu ponderoso, over grave per vigor del sito del corpo .a. e che la differentia che è fra le gravita de questi dui corpi, eglie impossibile à poterla dar, over trovar fra due quantita ineguale, e per dimostrar questa propositione. Tiro le due rette linee .a.h. e .b.d. perpendicolare verso il centro del mondo, e tiro anchora le due linee .a.l. e .b.m. contingente il detto cerchio, che descrive li brazzi della libra, l'una nel ponto .a. e l'altra nel ponto .b. Et descrivo anchora una parte de una circonferentia d'un cerchio, contingente il medesimo cerchio .a.e.b. in ponto .b. la qual sia pur d'un cerchio simile, e eguale al medesimo cerchio .a.e.b. la qual parte pongo che sia la .b.z. tal che l'arco .b.z. vien à esser simile, e eguale all'arco .a.f .e anchora similmente posto, cioe nel medesimo sito, over luoco, e la linea .b.m. che continge, over tocca quello, e perche la obliquita dell'arco .a.f. (per quello che fu detto sopra la terza petitione) vien misurata, over considerata per meggio dell'angolo contenuto dalla perpendicolar .a.h. e dal la circonferentia .a.f. in ponto .a. e la obliquita dell'arco .b.f. vien misurata, over considerata per meggio dell'angolo contenuto dalla perpendicolar .b.d. e dalla circonferentia .b.f. in ponto .b. adunque il corpo .b. in tal sito veneria ad esser tanto piu grave del corpo .a. quanto che il detto angolo (contenuto dalla perpendicolar .b.d. e dalla circonferentia .b.f. in ponto .b.) sara menore dell'angolo contenuto dalla perpendicolar .a.b. e dalla circonferentia .a.f. in ponto .a. e perche il detto angolo .h.a.f. è precisamente eguale all'angolo .d.b.z. e lo detto angolo .d.b.z. vien ad esser tanto maggiore dell'angolo contenuto dalla detta perpendicolare .b.d. e dalla circonferenza

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.b.f. in ponto .b. quanto che è l'angolo della contingentia delli dui cerchii .b.z. e .b. f. in ponto .b. e perche il detto angolo della detta contingentia è acutissimo de tutti li angoli acuti de linee rette (come per la decimasesta del terzo di Euclide facilmente si puo approvare) adunque la differentia, over proportione, che casca fra l'angolo .h.a.f. e l'angolo contenuto dalla perpendicolar .b.d. e della circonferentia .b.f. in ponto .b. è menore di qual si voglia differentia, over proportione, che cascar possa fra qual si voglia maggiore, e menor quantita, e cosi (per la terza petitione) la differentia della obliquita del descenso .a.f. e del descenso .b.f. e consequentemente la differentia della detta gravita delli detti dui corpi .a. e .b. secondo il sito è menore, del quale si voglia fra due quantita ineguale, e pero ogni piccola quantita corporea, che sia aggionta sopra il corpo .a. necessariamente in ogni sito sara piu grave del corpo .b. e pero non cessara di descendere continuamente per fin alla linea direttione, cioe per fin al ponto .f. e cosi continuamente quello andara ellevando il corpo .b. per fin alla detta linea della direttione, cioe per fin al ponto .e. e se questo seguiria in tal sito, come che nella sottoscritta figura appare tanto piu seguiria nel sito della egualita, nel qual sito, over luoco non vi è, over saria alcuna differentia, per vigor del sito, ne per vigor delli lor descensi, cioe che in tal sito sariano egualmente gravi, e pero ogni piccola quantita di peso per minima, che sia, che vi sia imposto dall'una delle bande di qual si voglia libra (cioe granda, over piccola de brazzi eguali) immediate fara declinare necessariamente quella da quella medesima banda, over brazzo, e continuara tal sua declinatione (per le ragioni di sopra adutte) per fin alla linea della direttione, cioe per fin al ponto .f. la qual cosa saria contra à quelle due conclusioni, che adduce Aristotile sopra la sua prima questione Mechanica, delle quale altra volta ne parlai con vostra Signoria, delle quale in l'una dice, che sono alcuni pesi, li quali imposti nelle piccole libre, non se fanno manifesti con alcuna inclinatione al senso, e che nelle grande libre se fanno manifesti, la qual conclusione, sumendola Mathematicamente, cioe astrata da ogni materia, saria falsissima (per le ragioni di sopra adutte) perche si nelle piccole, come nelle grande libre, da quella banda dove sara posto quel tal peso (per piccol che sia) sara sforzata à declinar per fina alla detta linea della direttione, e pero nella declinatione della piccola, e in quella della granda, non sara proportionalmente alcuna differentia, perche in l'una, e l'altra la declinatione sara per fin alla linea della direttione, il medesimo seguiria dell'altra sua conclusione, cioe quando dice, che sono alcuni pesi, li quali sono manifesti in l'una, e l'altra sorte de libre, cioe nelle maggiori, e nelle menori, ma molto piu nelle maggiori, la qual conclusione (per le ragioni di sopra adutte) saria pur falsa, perche, come detto in l'una, e l'altra fara declinare il brazzo della libra per fin alla linea della direttione. S.A. Queste vostre ragioni, e argomenti sono ottimi e buoni, nondimeno nelle libre naturale, over materiale il si vede pur seguire la maggior parte delle volte, come che Aristotile conchiude, e dice, perche se sopra qual si voglia libra (cioe granda, over piccola) vi sara posto uno grano, over semenza di papavero, o altra simile piccola quantita, rare libre se ritrovara che per si poca gravita, facciano inclinatione sensibile, e si pur ni se ne ri–

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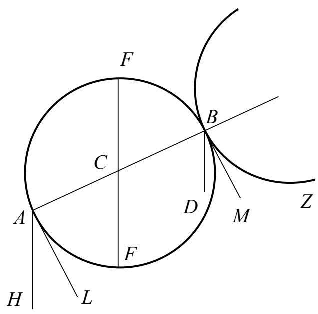
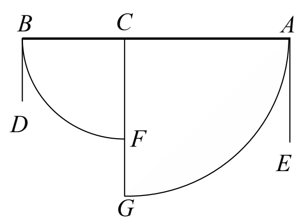
trovara alcuna che faccia alcun sensibile segno de declinatione, tamen non procedera per fina alla detta linea della direttione, e non solamente il detto gran de papavero non sara atto à farla declinare per fin alla detta linea della direttione alcuna libra, ma



nanche un gran di formento, qual è molto piu ponderoso, e tutto questo la sperientia lo manifesta. Si che non so che mi dire, perche da una banda per le vostre ragioni, e argomenti, vedo, e comprendo che voi diceti il vero, e dall'altra trovo per isperientia seguir tutto al contrario. N. Il tutto procede Signor, dalla materia, perche nelle libre considerate con la mente fuora de ogni materia il suo sparto, polo, over assis, se suppone un ponto indivisibile, et nelle libre materiale, tal sparto, over assis ha sempre qualche corporal grossezza in se, la qual grossezza, quanto è maggiore tanto men diligente redusse la detta libra, e similmente li brazzi delle libre imaginate (cioe ideale) se suppongano linee, cioe senza larghezza, ne grossezza, e nelle libre materiale tai brazzi sono di alcun metallo, over di legno, li quali brazzi quanto piu sono corpulenti, e grossi tanto men diligente reducano tal libre. S.A. E ve ho inteso, seguitati se haveti altra propositione de adurre circa à questa materia. NIC.

QUESITO. XXXIIII. PROPOSITIONE VII.

Se li brazzi della libra saranno ineguali, et che nella istremita di cadauno de quelli vi siano appesi corpi simplicemente eguali in gravita dalla banda del piu longo brazzo tal libra fara declinatione. S.A. Questa è cosa naturale. N. Anchor che la sia cosa naturale volendo procedere rettamente, bisogna assignar la causa di tal effetto. S.A. Seguitati. N. Sia la verga, over libra .a.c.b. et sia il brazzo .a.c. piu longo del .c.b. Dico che essendo appesi corpi simplicemente eguali in gravita, nelli dui ponti .a. e .b. tal libra declinara dalla parte del .a. Perche essendo tirata la perpendicolare .c.f.g. (cioe la linea della direttione) et essen[–]



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do circinate[81] le due quarte parte de circuli, sopra el centro .c. le quale siano .a.g. e .b.f. e essendo dutte dal ponto .a. e .b. due linee contingente, le quale siano .a.e. e .b.d. Eglie manifesto l'angolo .e.a.g. della detta contingentia, esser menore del angolo. d. b.f. e pero manco obliquo è il descenso fatto per .a.g. del descenso fatto per .b.f. e pero (per la terza petitione) piu grave sara il corpo .a. del corpo .b. in tal sito, ch'è il proposito. S.A. E ve ho inteso, seguitati. NIC.

QUESITO. XXXV. PROPOSITIONE VIII.

Se li brazzi della libra saranno proportionali alli pesi in quella imposti, talmente, che nel brazzo piu corto sia appeso il corpo piu grave, quelli tai corpi, over pesi seranno equalmente gravi, secondo tal positione, over sito. S.A. Datime uno essempio. N. Sia come prima la regola, over libra .a.c.b. e vi siano appesi .a. e .b. et sia la proportione del .b. al .a. si come del brazzo .a.c. al brazzo .b.c. Dico, che tal libra non declinara in alcuna parte di quella, e se possibil fusse (per l'aversario) che declinar potesse, poniamo che quella declini dalla parte del .b. e che quella discenda, e transisca in obliquo, si come sta la linea .d.c.e. in luoco della .a.c.b. e attaccatovi .d. come .a. e .e. come .b. e la linea .d.f. descenda orthogonalmente, e simelmente ascenda la .e.h. Hor eglie manifesto (per la .16. e. 29. del primo di Euclide) che li dui triangoli .d.f.c. e .e.h.c. esser de angoli equali. Onde (per la .4. del sesto di Euclide) quelli saranno simili, e consequentemente de lati proportionali, adunque la proportione del .d.c. al .c.e. è si come del .d.f. al .e.h. e perche si come del .d.c. al .c.e. cosi è dal peso .b. al peso .a. (dal presupposito) adunque la proportione dal .d.f. al .e.b. sara si come dal peso .b. al peso .a. sia adunque dal .c.d. tolto la parte .c.l. equale alla .c.b. over alla .c. e. e sia posto .l. equale al .b. in gravita, e descenda el perpendicolo .l.m. Adunque perche eglie manifesto la .l.m. e la .e.h. esser equale, la proportione della .d.f. alla .l.m. sara si come delle simplice gravita del corpo .b. alla simplice gravita del corpo .a. over della simplice gravita del corpo .l. alla simplice gravita del corpo .d. (perche li dui corpi .a. e .d. sono supposti uno medesimo) e simelmente el corpo .b. e .l. (per esser supposta la gravita del .l. equale alla gravita del .b.) e per tanto dico, che la proportione di tutta la .d.c. alla .l.c. sara si come la gravita del corpo .l. alla gravita del corpo .d. Onde se li detti dui corpi gravi, cioe .d. e .l. fusseno simplicemente equali in gravita, stanti poi in li medesimi siti, over luochi, dove, che al presente vengono supposti, el corpo .d. saria piu grave del corpo .l. secondo el sito (per la .4. propositione) in tal proportione, qual è di tutto il brazzo .d.c. al brazzo .l.c. e per che il corpo .l. è simplicemente (dal presupposito) piu grave del corpo .d. secondo la medesima proportione (cioe, si come la proportione del brazzo. d.c. al brazzo .l.c. adunque li detti dui corpi .d. e .l. nel sito della equalita veneranno ad essere egualmente gravi, perche per tanto quanto il corpo .d. è piu grave del corpo .l. per vigor del sito, over luoco, per quel medesimo el corpo .l. è simplicemente piu grave del corpo .d. e pero nel detto sito della equalita vengono à restare egualmente gravi. Adunque quella potentia, over gravita, che sara sofficiente ad ellevare il corpo .a. dal sito della equalita, al ponto, dove che al presente è (cioe per fin al ponto .d.) quella medesima sara sof–

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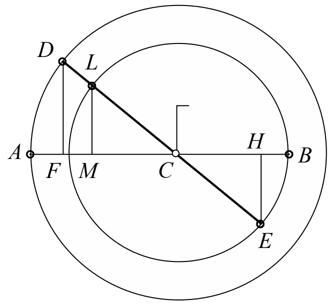
ficiente ad ellevare il corpo .l. dal medesimo sito della equalita al luoco, dove che al presente è. Adunque sel corpo .b. (per l'aversario) è atto ad ellevare il corpo .a. dal sito della equalita per fin al ponto .d. el medesimo corpo .b. saria anchora atto, e sofficiente ad ellevare il corpo .l. dal medesimo sito della equalita per fin al ponto, dove che al presente è, el qual consequente è falso, e contra alla quinta propositione, cioe el corpo .b. (qual è supposto equale in gravita al corpo .l.) ellevaria il detto corpo .l. fuora del sito della equalita, in siti equali, cioe equalmente distanti dal centro .c. la qual cosa è impossibile per la detta quinta propositione, distrutto adunque l'opposito, rimane il proposito. S.A. Questa è una assai bella propositione, ma el me pare, se ben me arricordo, che Archimede Syracusano ne ponga una simile,



ma el non mi pare, che lui la dimostri per questo vostro modo. N. Vostra Signoria dice la verita, anci di tal propositione, lui ne fa due propositioni, e queste sono la quarta, e quinta di quel libro, dove tratta delli centri delle cose grave, e in effetto tai due propositioni lui le dimostra succintamente per li suoi principii da lui per avanti posti, e demostrati, e perche tai sui principii, over argomenti non se convegnariano in questo trattato, per esser materia alquanto diversa da quella, ne apparso in questo luoco de dimostrare tal propositioni con altri principii, over argomenti piu convenienti in questo luoco. S.A. E ve ho inteso seguitati. N.

QUESITO XXXVI. PROPOSITIONE IX.

Se saranno due solide verghe, travi, over bastoni di una simile, e equal longhezza, larghezza, grossezza, e gravita, e che siano appesi in una libra talmente che l'uno stia equidistante al Orizonte, e l'altro dependi perpendicolarmente, e talmente anchora, che del termine del dependente, e del mezzo dell'altro sia una medesima distantia dal centro della libra, secondo tal sito, over positione veneranno à essere equalmente gravi. S.A. Non ve intendo, e pero datime uno essempio. N. Essempi gratia. Siano li termini delli brazzi della libra .b. e .e. e il sparto, over centro di quella il ponto .c. e vi siano attaccati li dui solidi simili, e equali, come detto, delli quali l'uno vi sia attaccato secondo l'ordine del brazzo della libra, cioe equidistantamente al Orizonte qual sia .f.e. del qual il suo ponto di mezzo sia el ponto .d. e l'altro sia attaccato pendente perpendicolarmente qual sia .b.g. e sia il termine del suo attaccamento il ponto .b. e sia che la distantia del ponto .b. al ponto .c. (centro della libra) sia tanto, quanto ch'è dal ponto di mezzo de l'altro solido (cioe dal ponto .d.) al medesimo ponto .c. Dico che li detti dui solidi, in tal sito, over positione sono equalmente gravi, e questo se puo dimostrar in piu modi. El primo di quali è questo, ch'eglie manifesto per le cose dimostrate da Archimede in quello del centro della gravita, che

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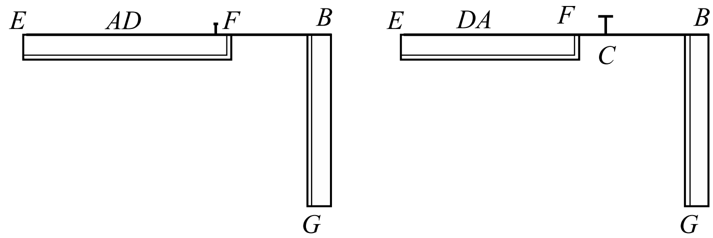
tanto pesa il solido .f.e. in tal positione nella detta libra, quanto che faria se quello fusse anchora lui appeso perpendicolarmente in ponto .d. perche in tal ponto .d. vi sotto giace el centro della gravita de tal solido, e per esser li detti dui solidi equali in gravita dal presupposito, e appesi equalmente distanti dal ponto, over centro .c. quelli (per la .5. propositione) non se separano dal sito della equalita, ch'è il proposito.



Anchora tal propositione si puo demostrar in questo altro modo (el quale è piu sua conveniente dimostratione, perche se vien à dimostrare per li suoi proprii Principii, e non per principii alieni). Eglie manifesto, che essendo sospesi dui pesi simplicemente equali, l'uno in ponto .f. e l'altro in ponto .e. quali poniamo, che siano .h.k. e simelmente dui altri equali alli medesimi in ponto .b. quali siano .l.m. nelli quali siti, dico, che tai pesi pesar anno equalmente, perche la proportione del peso .l. al peso .k. è si come del brazzo .b.c. al brazzo .f.c., per la quarta propositione, perche tanto grave saria el corpo .l. secondo el sito nel ponto .d. quanto che nel ponto, dove si trova al presente, cioe in ponto .b. (per esser .c.d. equale al .c.b. dal presupposito) e pero per la detta propositione, tal proportione sara della gravita del corpo .l. al corpo .k. secondo el sito, quale sara del brazzo .d.c. over .b.c. al .c.f. e per le medesime ragioni tal proportione sara della gravita del corpo .m. alla gravita del corpo .h. secondo el sito, quale sara del medesimo brazzo .c.d. over .c.b. al brazzo .c.e. adunque la gravita de ambidui li corpi .l.m. insieme alla gravita de ambi dui li corpi .h.k. insieme secondo il sito sara si come el doppio del brazzo .c.d. over del brazzo .c.b. insieme alli dui brazzi .c.f. et .c.e. pur insieme, e perche li detti dui brazzi .c.e. e .c.f. insieme sono precisamente tanto, quanto è il doppio del detto brazzo .c.d .over. c.b. seguita anchora, che la gravita delli detti dui corpi .l.m. sia equale alla gravita delli dui corpi .h. e .k. secondo il sito, ch'è il proposito, perche se del sopradetto solido .f.e. ne sara fatto due parti equali, appiccandone una di quelle in ponto .f.e l'altra in ponto .e. tanto pesarano cosi separate in tai siti, si come facevano in longo congionte, come di sopra fu supposto, e simelmente facendo del solido .b.g. pur due parti, e appiccarle ambe due in el medesimo ponto .b. tanto pesarano cosi separate, come che congionte, come, che di sopra fu supposto e pero per le cose detto, e allegate seguita il proposito.

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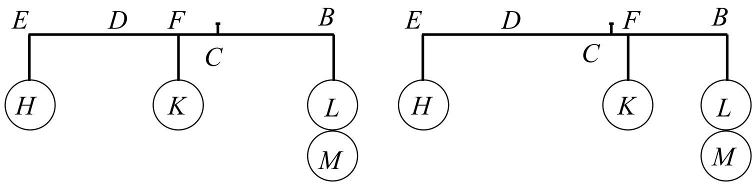

S.A. Voria, che me dimostrasti che il brazzo .c.f. insieme con il .c.e. sia tanto quanto el doppio del brazzo .d.c. over .c.b. N. Signor eglie manifesto, che tutto il brazzo c.e. è maggiore del brazzo .c.d. per la parte .e.d. la qual parte .e.d. è equale alla .d.f. diremo adunque, che tutta la .c.e. è equal alla .c.d. e anchora alla sua parte .f.d. alla qual parte .f.d. giontovi el brazzo .f.c. queste due parti insieme se egualiano anchora loro alla medesima .c.d. e pero tutta la .c.e. insieme con la .c.f. sono precisamente il doppio della .c.d e perche la detta .c.d. è equale (dal presupposito) alla .b.c .seguita, che tutta la .c.e. insieme con la .c.f. siano equali al doppio della .c.b. ch'è il proposito. S.A. E ve ho inteso benissimo, e pero seguitati. N.

QUESITO XXXVII. PROPOSITIONE X.

Sel sara una solida verga, trave, over bastone di una simile, e equal larghezza, grossezza, sostantia, e gravita in ogni sua parte, e che la longhezza di quella sia divisa in due parti inequale, e che nel termine della menor parte vi sia appeso, un altro, solido, over corpo grave, el quale faccia stare la detta verga, trave, over bastone equidistante al Orizonte. La proportione della gravita di tal corpo grave, alla differentia della gravita della maggior parte della detta verga (trave, over bastone) alla gravita della parte menore, sara si come la proportione della longhezza di tutta la verga (trave, over bastone) al doppio della longhezza della sua menor parte. S.A. Datime un essempio se voleti, che vi intenda. N. Sia la solida verga (trave, over bastone) il solido .a.b. di una simile, et equal grossezza, larghezza, sostantia, et gravita per tutto, cioe per ogni parte, et sia diviso con lo intelletto in due parti inequale in ponto .c. et sia signata la .c.d. equal alla .a.c. adunque la .d.b. vien à essere la differentia, ch'è fra la parte maggiore .c.b. et la menore .c.a. della qual differentia sia trovato il mezzo, qual sia il ponto .e. Hor essendo sospeso il detto solido, over trave .a.b. nel ponto .c. et essendovi attaccato, over sospeso nel termine della sua menor parte un altro solido (poniamo il solido .f.) qual faccia stare il primo solido, over trave .a.b. equidistante al Orizonte. Dico, che tal proportione havera la gravita del solido .f. alla gravita della differentia .d.b. qual hara tutta la longhezza .a.b. alla .a.d. cioe al doppio della longhezza della parte menore .a.c. Perche tanto pesa la detta differentia .d.b. in tal positione, come che al presente sta quanto che faria se quella fusse perpendicolarmente sospesa in ponto .e. e pero (per il con–

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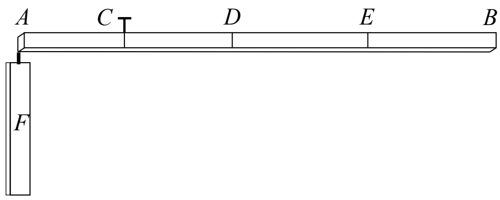
verso della .8. propositione) la proportione della gravità del solido .f. alla gravita del partial solido, over trave .d.b. sara, si come la proportione della distantia .c.e. alla distantia .c.a. Et la proportione, che è della distantia .c.e. alla distantia .c.a. (per la .15. del quinto di Euclide) quella medesima sara del doppio della distantia .c.e. al doppio della detta distantia .c.a. e perche il doppio della detta distantia .c.e. è quanto che è tutta la longhezza del solido .a.b. e il doppio della detta distantia .c.a. è quanto che è tutta la .a.c.d. seguita (per la.11.del quinto di Euclide) che la proportione della gravita del solido .f. alla gravita della differentia .d.b. sia si come la proportione di tutta la longhezza del solido, over verga .a.b. al doppio della longhezza della parte menore .a.c. (qual è la detta .a.c.d.) che è il proposito. S.A. Perche ragione vuoleti che il doppio della



distantia .c.e. sia eguale à tutta la longhezza del trave .a.b. N. Perche la detta distantia .c.e. vien à esser precisamente eguale alla mita di tal longhezza .a.b. perche la parte .d.e. è la mita della parte .d.b. e la .d.c. è la mita dell'altra parte .d.a. adunque le due parti .d.e. e .d.c. gionte insieme, vengono à essere la mita delle due parti .d.b. e .d.a. pur gionte insieme. S.A. E ve ho inteso, e pero seguitate in altro. N.

QUESITO. XXXVIII. PROPOSITIONE XI.
conversa della precedente.

Se la proportione della gravita d'un solido sospeso in el termine della menor parte di una simile solida verga (trave, over bastone) divisa in due parti ineguali, alla differentia, che sara fra la gravita della maggior parte, e quella della menore, sara, si come la proportione di tutta la longhezza della solida verga, trave, over bastone, al doppio della longhezza della sua menor parte. Tal solida verga, trave, over bastone, necessariamente stara equidistante all'Orizonte. S.A. Credo bene che tal precedente propositione se convertisca, nondimeno non restati da farme la dimostratione. N. Per esser questa il converso della precedente, per suo essempio supponeremo la medesima dispositione, over figura, cioe supponeremo, che la proportione della gravita del solido .f. alla differentia della gravita della maggior parte alla gravita della menore, cioe della .d.b. esser, si come la proportione di tutta la longhezza della solida verga .a.b. al doppio della longhezza della parte menore .a.c. (quale saria la .a.d.) Dico che stante questo la solida verga .a.b. de necessita stara equidistante all'Orizonte. Et se pos

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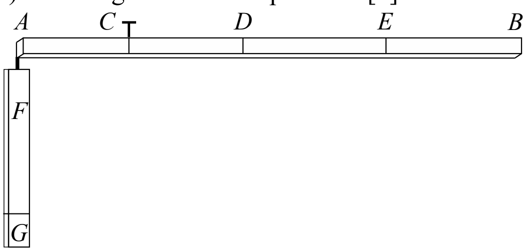
sibil fusse (per l'aversario) che quella debbia, over possa declinar da qualche banda, poniamo che declini dalla banda verso.b.al solido .f. gli aggiongeremo con lo intelletto una tal parte (quale pongo che sia la parte .g.) che faccia restare la detta solida verga, trave, over bastone equidistante al detto Orizonte. Adunque (per la precedente, la proportione di tutta la gravita del composto delli dui corpi .f. e .g. alla differentia, che è fra la gravita della parte maggiore .b.c. e quella della parte menore .a.c. (che saria quella della .d.b.) sara, si come la proportione di tutta la longhezza .a.b. al doppio della longhezza della sua parte menor .a.c. il qual doppio, saria la .a.d. e perche il simplice solido .f. ha quella medesima proportione, alla medesima differentia (dal presupposito) seguitaria (per la .9. del quinto di Euclide)[82] che la gravita del simplice soli[–]



do .f. fusse eguale alla gravita de tutto il composito di dui solidi .f.g. la qual cosa è impossibile, che la parte sia eguale al tutto, il medesimo inconveniente seguiria quando che lo aversario supponesse che declinasse dalla parte .a. perche segando via dal solido .f. una tal parte, che il rimanente facesse restare il detto solido .a.b. equidistante all'Orizonte, argomentando, come di sopra fu fatto, seguiria pur che la gravita del medesimo residuo fusse eguale alla gravita di tutto il solido .f. Adunque non potendo declinare ne dalla banda verso .b. ne da quella verso .a. eglie necessario che stia equidistante all'Orizonte, che è il proposito. S.A. Sta benissimo, hor seguitati pur. N.

QUESITO. XXXIX. PROPOSITIONE XII.

Sel sara una solida verga, trave over bastone, come nelle due precedente è stato detto, cioe di una simile, e egual grossezza, larghezza, sostantia, e gravita, in ogni sua parte, e che di quello ne sia nota la sua gravita, e similmente la sua longhezza, et che quello sia diviso in due parti ineguale pur note. Eglie possibile di ritrovar un peso, il quale quando che quello sara sospeso al termine della sua menor parte fara stare la detta solida verga, trave, over bastone, equidistante all'Orizonte. S.A. Questo atto operativo voglio che mel dichiarati con essempio materiale, perche lo voglio intendere bene. N. Sia essempi gratia la solida verga (trave, over bastone) .a.b. secondo che se propone, cioe di una simile, e equal grossezza, larghezza, sostantia, e gravita per ogni sui banda, over parte, e poniamo, che la gravita di tal solida verga ne sia

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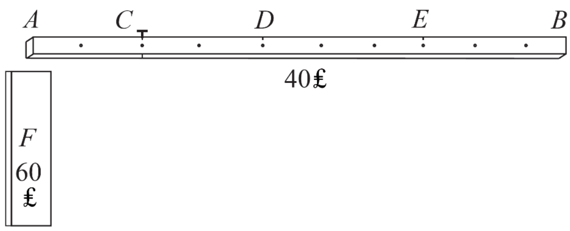
nota, cioe poniamo che tutta pesi lire[83] .40. et che similmente la longhezza di tal verga, over bastone, ne sia nota, cioe poniamo che quella sia longa dui passa, cioe dieci piedi, e poniamo anchora che tal verga sia divisa in due parti ineguale in ponto .c. e che le dette parti ne sia note, cioe poniamo che la parte .a.c. menore, sia piedi dui, e che la maggior .c.b. sia piedi .8. Hor dico, che eglie possibile di trovare di quante libre vorra esser quel corpo qual essendo sospeso nel ponto .a. (termine della sua menor parte) faccia stare la detta verga, over trave equidistante all'Orizonte. Perche (per le cose dimostrate nelle due precedente propositioni) eglie manifesto, che la proportione della gravita di quel tal corpo alla gravita di quella differentia che è fra la parte maggiore .c.b. e la parte menore .a.c. (la qual differentia verria à esser la .d.b.) sara, si come tutta la longhezza della verga, over trave .a.b. (qual è piedi .10.) al doppio della longhezza della parte menor .a.c. (qual è piedi dui) il doppio della quale verria à esser piedi .4. qual pongo sia la .a.d. adunque la gravita di quel tal corpo, alla gravita della partial verga .d.b. sara, si come la longhezza de tutta la .a.b. (qual è piedi .10.) alla longhezza della .a.d. (qual è piedi .4.) Onde arguendo al contrario, diremo, che la proportione della .a.d. (qual è piedi .4.) à tutta la .a.b. (qual è piedi .10.) sara, si come la gravita della partial verga .d.b. qual (alla ratta[84] di tutta la .a.b. che libre .40.) verria ad esser libre .24. alla gravita del corpo che recercamo, cioe di quello, che appeso nel ponto .a. debbia man[–]



tenere la detta verga, over trave equidistante all'Orizonte. Onde per ritrovarlo procederemo secondo l'ordine della regola volgarmente detta del tre, fondata sopra la .20. propositione del .7. di Euclide moltiplicando .10. fia .24. fa .240. e questo lo partiremo per .4. ne venira .60. e libre .60. dico che pesara, over che dovera pesare quel tal corpo, qual pongo sia il corpo .f. che è il proposito. S.A. Questo problema me è piacesto assai, e l'ho inteso benissimo, e pero seguitati se ci è altro da dire. N.

QUESITO. XL. PROPOSITIONE XIII.

Sel se havera una verga, trave, over bastone, come piu volte è stato detto, del qual ne sia nota la sua longhezza, e anchora la sua gravita, e anchora un corpo ponderoso, del quale ne sia nota sua gravita, eglie possibile à determinare il luoco dove se havera da dividere la data verga, trave, over bastone, talmente che appendendo il det–

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to corpo ponderoso al termine della sua menor parte faccia stare la detta verga, trave, over bastone, equidistante all'Orizonte. S.A. Essemplificatime questa propositione. N. Per essemplificar questa propositione, supponeremo che il sia pur una verga, trave, over bastone, come fu la precedente, cioe longa piedi .10. e che la gravita di quella fia pur libre .40. (come che nella detta precedente fu supposto.) Et poniamo anchora che il sia un corpo che la gravita di quello sia libre .80. Dico ch'eglie possibile à determinare il luoco dove se debbe dividere la detta verga, talmente che appendendo il detto corpo grave al termine della sua menor parte, faccia star quella equidistante all'Orizonte. Et quantunque tal problema, si possa risolvere per via di proportioni, nondimeno piu leggiadramente, se risolve per Algebra, ponendo che la parte menore della detta verga sia una cosa de pie,[85] onde la parte maggiore veneria à restare piedi .10. men .1. co. Dupplico la menor parte (cioe .1. co) fa .2. co., e queste .2. cose le sottro da tutta la verga qual è piedi .10. resta piedi .10. men .2. cose, e questo sara la differentia, che è fra la parte maggiore, e la menore della detta verga, onde per trovar la gravita di tal differentia, la moltiplico per .4. (perche pesando tutta la verga libre .40. veneria ogni pie di quella à pesar lire[86] .4.) e pero moltiplicando quella per .4. come detto ne venir a libre .40. men .8. cose. Et perche la proportione di tutta la verga (qual è pie di .10. al doppio della sua menor parte (il qual doppio saria .2. cose) è si come che la gravita del nostro corpo grave (qual è libre.80.) alla gravita della sopradetta differentia, qual fu libre .40. men .8. co. Onde per la .20. del settimo di Euclide (la moltiplicatione della prima) che .10. piedi fia la quarta che è .40. men .8. cose) qual fara .400. men .80. cose (sara eguale alla moltiplicatione della terza qual è libre .80. fia la seconda, qual è .2. cose (qual fara .160. co.) e pero haveremo .160. cose eguale à .400. men .80. cose, onde ristorando le parti, e seguendo il capitolo, trovaremo la cosa valer .1 2/3 e de piedi .1. 2/3 se dovera signar la menor parte della detta verga, over trave, onde la maggiore venira à restare de piedi .8. 1/3, che è il proposito. S.A. Questa è stata una bella resolutione, ma seguitati pur, perche vorria che tra hoggi e dimane vedessimo de ispedire tutto quello, che haveti da proponere sopra di questa scientia, perche vorro poi che me assignati la causa de alcune questioni, che ho da dirvi. N. Non credo di potermene ispedire fra diman, e l'altro, perche continuamente me nasce nuove materie da proponere circa à tal scientia. S.A. Se non se ne potremo ispedire cosi dimane non importa, non perdemo tempo, seguitati. N.

QUESITO. XLI. PROPOSITIONE XIV.

La egualita della declinatione è una medesima egualita de peso. S.A. Datemi un essempio. N. La egualita della declinatione vien conservata solamente in via retta. Hor poniamo adunque che la detta via retta sia la linea .a.b. e dal ponto .a. sia anchor tirata la perpendicolare .a.c. e supponamo anchor nella detta declinata linea .a.b. dui diversi luochi. Hor poniamo che l'uno sia il ponto .d. e l'altro il ponto .e. Hor dico che discendendo, qualunque corpo ponderoso, over dal ponto .d. over dal ponto .e. sara de uno medesimo peso, secondo il sito in qual si voglia de detti luochi. Per[–]

che

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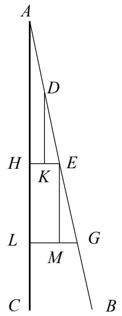
che se pigliaremo sotto al .d. e al .e. due parti equali nella via, over linea .a.b. Hor poniamo, che l'una sia la parte .d.e. et l'altra la .e.g. Dico, che per le dette parti



equali capira equalmente del diretto, cioe della linea .a.c. la qual cosa se notificara in questo modo, dalli dui ponti .e. et .g. siano tirate le due linee .e.h. et .g.l. perpendicolare sopra la linea .a.c. et dalli dui ponti, over luochi .d. et .e. le due linee .d.k. et .e.m. perpendicolare sopra le medesime .e.h. et .g.l. le qual due perpendicolare, cioe .d.k. et .e.m. saranno fra loro equali, perche adunque il detto corpo ponderoso, si essendo nel ponto .d. come nel ponto .e. in quantita, over descensi equali, capira equalmente del diretto, sara di una medesima gravita in qual si voglia de quelli, secondo el sito, ch'è il proposito. S.A. E ve ho inteso, seguitate pur. N.

QUESITO XLII. PROPOSITIONE XV.

Se dui corpi gravi descendano per vie de diverse obliquita, e che la proportione delle declinationi delle due vie, e della gravita de detti corpi sia fatta una medesima, tolta per el medesimo ordine. Anchora la vertu de l'uno, e l'altro de detti dui corpi gravi, in el descendere sara una medesima. S.A. Questa propositione mi par bella, e pero datime anchora un essempio chiaro, accio che meglio mi piaccia. N. Sia la linea .a.b.c. equidistante al Orizonte, e sopra di quella sia perpendicolarmente eretta la linea .b.d. e dal ponto .d. descendano de qua, e de la le due vie, over linee .d.a. e .d.c. e sia la .d.c. di maggior obliquita. Per la proportione adunque delle lor declinationi, non dico delli lor angoli, ma delle linee per fina alla equidistante resecatione, in la quale equalmente summemo del diretto. Sia adunque la lettera .e. supposta per un corpo grave posto sopra la linea .d.c. e un'altro la lettera .h. sopra la linea .d.a. e sia la proportione della simplice gravita del corpo .e. alla simplice gravita del corpo .h. si come quella della .d.c. alla .d.a. Dico li detti dui corpi gravi esser in tai siti, over luochi di una medesime vertu, over potentia. Et per dimostrar questo, tiro la .d.k. di quella medesima obliquita, ch'è la .d.c. e imagino un corpo grave sopra di quella equale al corpo .e. el qual pongo sia la lettera .g. ma che sia in diretto con .e.h. cioe equalmente distanti dalla .c.k. Hor se possibel è (per l'aversario) che li detti dui corpi .e. e .h. non siano di una medesima, e equal vertu in tai luochi, adunque l'uno sara di maggior vertu, over potentia dell'altro, poniamo adunque, che .e. sia di maggior vertu, adunque quello sara atto à discendere, e simelmente à far ascendere, cioe à tirare in suso el corpo .h. Hor poniamo (se possibel è) che il detto corpo .e. descenda per fina in ponto .l. e che faccia ascendere il corpo .h. per fin in ponto .m. e faccio, over che segno la .g.n. equale alla .h.m. la quale anchora lei vien à esser equale alla .e.l. Et dal ponto .g. tiro la .g.h.e. la qual sara perpendicolare sopra la .d.b. per esser li detti tre ponti (over corpi) .g.h.e. supposti in diretto, e equalmente distanti dalla .k.c. e simelmente dal ponto .l. sia tiratala .l.t. equidistante alla .c.b. qual sara pur perpendicolare

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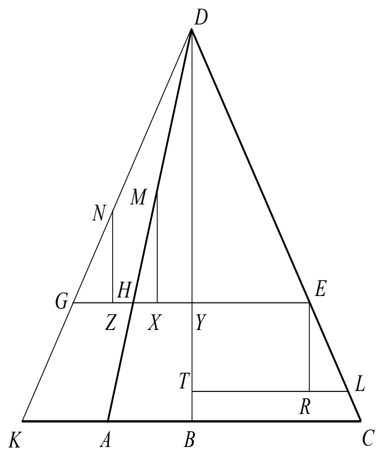
sopra la medesima .d.b. e dalli tre ponti .n.m.e. siano tirate le tre perpendicolari .n.z. .m.x. et .e.r. Et perche la proportione della .n.z. alla .n.g. è si come quella ch'è dalla .d.y. alla .d.g. e pero si come anchora quella della .d.b. alla .d.k. (per esser li detti tre triangoli simili.) Simelmente la proportione della .m.x. alla .m.h. è si come quella, che è dalla detta .d.b. alla .d.a. (per esser li detti dui triangoli simili.) Anchora la proportione della .m.x. alla .n.z. sara si come quella della .d.k. alla .d.a. e quella medesima (dal presupposito) e dalla gravita del corpo .g. alla gravita del corpo .h. perche il detto corpo .g. fu supposto esser simplicemente, egualmente grave con el corpo .e. adunque tanto quanto, che il corpo .g. è simplicemente piu grave del corpo .h. per altro tanto il corpo .h. vien à esser piu grave per vigor del sito del detto corpo .g. e pero si vengono ad egualiar in vertu, over potentia, e per tanto quella vertu, over potentia, che sara atta à far ascendere l'uno de detti dui corpi, cioe à tirarlo in suso, quella medesima sara atta, over sofficiente à fare ascendere anchora l'altro, adunque sel corpo .e. (per l'aversario) è atto, e sofficiente à far ascendere il corpo .h .per fin in .m. el medesimo corpo .e. saria adunque sofficiente à far ascendere anchora il corpo .g. à lui equale, e inequale declinatione, la qual cosa è impossibile per la precedente propositione, adunque il corpo .e. non sara de maggior vertu del corpo .h. in tali siti, over luochi, ch'è il proposito. S.A. Questa è stata una bella speculatione, e me è piacesta assai. Et per che vedo esser hora tarda, non voglio, che procedati in altro per hoggi.



Fine del ottavo libro.

4.6 The *Iordani opusculum de ponderositate* (1565)

4.6.1 The Fac–Simile and Critical English Translation

IORDANI

OPVSCVLVM

DE PONDEROSITATE

NICOLAI TARTALEAE

STVDIO CORRECTVM,

NOVISQVE FIGVRIS AVCTVM.

CVM PRIVILEGIO.

VENETIIS,

APVD CVRTIVM TROIANVM.

M D L X V.

FRANCISCO LABIAE OMNI VIRTVTVM GENERE ORNATO.

CVRTIVS TROIANVS S. D.

NON me fugit ſumma in expectatione te eſſe, cum optimis literarum ſtudijs, qui te uehementius incumbat cognoſcam neminem. nullum profecto doctrinæ genus eſt, in quo non uerſeris, nulla diſciplina, quam non intelligere uelis, tu grammaticorum canones, hiſtorias, & poetarum fabulas mirifice tenes, tu rhetoricis floſculis abundas, dialecticorum argutias ſcrutaris, phyſices arcana, & ſuperiores intelligentias perueſtigas, tu theologorum abdita petquiris, tu mathematicis, & omni denique eruditionis genere delectaris, quamobrem, pro mea in te, & patrem tuum beneuolentia, propter egregiam tuam indolem, iucundiſſimos mores, diuinum inge

[2r]

TO FRANCESCO LABIA
adorned with many good qualities.

Curtio Troiano

I am aware of the great expectations on you for I do not know anyone who applies with more passion than you to the literary studies. Certainly there is not any kind of doctrine you are not versed in; any discipline that you will not understand. You know the rules of grammar very well, the history, the stories of the poets; you excel in rhetoric, you analyse with the keenness of dialecticians, you inquire with superior intelligence about the mysteries of nature. You investigate the secrets of theology, finally you are attracted by mathematics and any kinds of knowledge. For my and your father benevolence, for your egregious nature, joyful customs, divine inge

A 2

nium, ſummam modeſtiam, tibi optimæ ſpei adoleſcenti dicare uolui hunc Iordani ingenioſi, & acuti hominis librum de ponderibus, quem mihi ſuis in fragmentis Nicolaus Tartalea familiaris meus, uir quidem præclaris ornatus ſcientijs excudendum reliquit. Accipias igitur læto vultu hunc in lucem editum, tuoque ſub nomine emiſſum, quandoquidem tibi non modo iucunditati, ſed etiam utilitati fore certo ſcio. Vale: Non. Kalendas Feb.

PRIMA

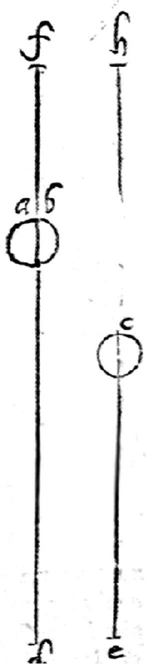
[2v]

nuity, the sum modesty, I want to dedicate excellent youth this book on weights by Jordanus ingenious and acute man, whose fragments Niccolò Tartaglia, my friend, a man of science, left to settle. Receive with pleased face this [book] just published, dedicated to you, because I know for sure that it will be not only entertaining but also useful to you. Greetings. 5th February.

FIRST HOUR

PRIMA SVPPOSITIO.

OMNIS ponderosi motum esse ad medium uirtutemq; ipsius esse potentia ad inferiora tendendi uirtutem ipsius, siue potentia possumus intelligere longitudinem brachij libræ, aut uelociter eius quem probatur ex longitudine brachij libræ, & motui contrario resistendi. Secunda: Quòd grauius est uelocius descendere. Tertia: Grauius esse in descendendo quanto eiusdem motus ad medium rectior. Quarta: Secundum situm grauius esse cuius in eodē situ minus obliquus descensus. Quinta: Obliquiorem autem descensum in eadem quantitate minus capere de directo Sexta: Minus graue aliud alio secundum situm, quod descensum alterius sequitur contrario motu. Septima: Situm ęqualitatis esse æqualitatem angulorum circa perpendiculum, siue rectitudinem angulorum, siue ęque distantiam regulæ superficiei Orizontis.



Quæstio Prima.

Inter qualibet grauia est uirtutis, & ponderis eodem ordine sumpta proportio.

SInt pondera a,b,c, leuius c, descendatq; a,b, in d, & c, in e. Itaque ponatur a,b, sursum in f, & c,i,h. Dico ergo quòd quæ proportio a,d, ad c,e, sicut a,b, ponderis ad c, pondus, quanta enim uirtus ponderosi tanta descendendi uelocitas: at quæ compositi uirtus ex uirtutibus componentium componuntur. Sit ergo a, æquale c. Quæ igitur uirtus a, eadem &, c. Sit igitur proportio a, b, ad c, minor quàm uirtutis ad uirtutem. Erit similiter proportio a, b, ad a, minor proportio quàm uirtutis a, b, ad uirtutem a, ergo uirtutis a,b, ad uirtutem b, minor proportio quàm a,b, ad b. per 30. quinti Euclidis quòd est inconueniens. Similium igitur ponderum minor, & maior proportio, quàm uirtutum. Et quia hoc inconueniens erit, utrobique eadem ideo a,b, ad c, sicut a,d, ad c,e, & e, con

[3r]

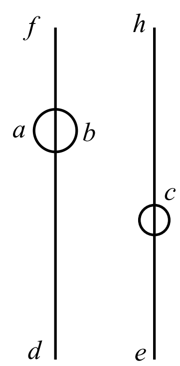
FIRST SUPPOSITION.

The motion of every heavy body is toward the centre [of the world] and its strength is a power of tending downward and to resist to the contrary motion, and we can understand its strength or power from the arm length or from its velocity which is determined by the length of the balance arms. *Second:* What is heavier descends more speedily. *Third:* It is heavier in descending, to the degree its movement toward the centre is more direct. *Fourth:* It is heavier according to position in that position where its path of descent is less oblique. *Fifth:* A more oblique descent is one which, in the same space, partakes less of the vertical. *Sixth;* One weight is less heavy according to position, than another, if it is caused to ascend by the descent of the other. *Seventh:* The position of equality is that of equality of angles to the vertical, either these are right angles, or the beam is parallel to the plane of the horizon.

First Question [Proposition].

Among any heavy bodies, the strength is proportional to the weight.

Consider weights ab, c, of which c is the lighter and ab descend to d, and let c descend to e. In the same way let ab be raised to f, and c to h [Fig. 4.22]. *I then say that the proportion of ad to ce, is as the weight ab is to the weight c, indeed the velocity of descending is as great as the strength of the heavy body. But the strength of the compound is composed by the strengths of its components. Let a then be equal to c, so that the strength of a is the same as that of c. If instead the ratio of ab to c is less than the ratio of the strength to the strength, the ratio of ab to a will similarly be less than the ratio of the strength of ab to the strength of a, and therefore the ratio of the strength of ab to that of b will likewise be less than that of ab to b, for [the proposition] 30 of fifth book of Euclid,*[87] *what is absurd. Therefore the ratio of weights will be both greater and less than the ratio of strengths. Since this is absurd, [the proportion] must be the same in both cases, so ab is to c, as ad is to ce, and conversely as ch is to* [the distance] *af.*

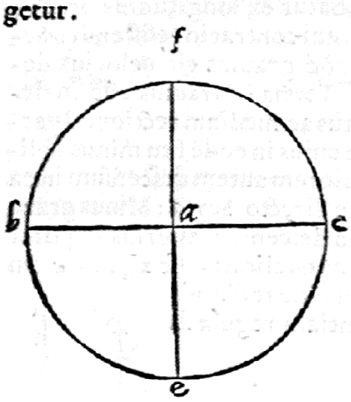


[Fig. 4.22]

OPVSCVLVM DE

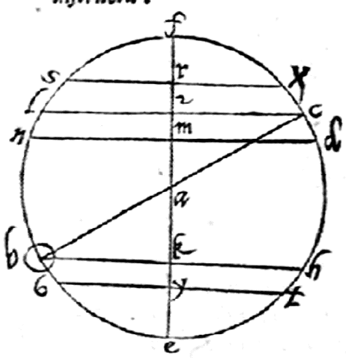
Quæstio Secunda.

Quum æquilibris fuit positio æqualis æquis ponderibus appensis ab æqualitate non discedet: & si à rectitudine separatur, ad æqualitatis situm reuertetur. Si uero inæqualia appendantur, ex parte grauioris usque ad directionem declinare cogetur.



AEquilibris dicitur quando à centro circumuolutionis brachia regulæ sunt æqualia. Sit ergo centrum a, & regula b, a, c, appensa b, & c, perpendiculum f, a. Circunducto igitur circulo per b, & c, in medio cuius inferioris medietatis sit e, manifestum quoniam descensus tam b, quàm c, e, per circunferētiam circuli uersus e, & cum æque obliquus sit hinc inde descensus, quũ sint æque ponderosa, non mutabit alterutrum. Ponatur item quòd submittatur ex parte b, & ascendat ex parte c, dico quoniam redibit ad æqualitatem: est enim minus obliquus descensus a, ad æqualitatem, quàm a, b, uersus e. Sumantur enim sursum arcus æquales, quantumlibet parui qui sint c, d, & b, b, & ductis lineis ad æquidistantiam æqualitatis, quæ sint, c, 2, l, & d, m, n. Item b, K, b, 6 y, t, dimittatur orthogonaliter descendens diametrum quæ sit f, 2, m, a, K, y, e, erit quòd 2, m, maior K, y, quia sumpto uersus f, arcu ex eo quòd sit æqualis c, d, & ducta ex transuerso linea x, r, s, erit r, 2, minor 2, m, quòd facile demonstrabis. Et quia r, 2 est æqualis K, y, erit 2, m, maior K, y. Quia igitur quilibet arcus sub c, plus capiat de directo quàm ei æqualis sub b, directo est descensus a, c, quàm a, b, & ideo in altiori situ grauius erit c, quàm b, redibit ergo ad æqualitatem.

Figura a Nicolao de Tartaglijs instructa.



Sit

[3v]
O P U S C U L U M D E

Second Question [Proposition].

When a horizontal position is gained [for a balance of equal arms], then, if equal weights are suspended [from its extremities], the balance will not leave the horizontal position; and if it is moved from the horizontal position, it will revert to the horizontal position. If instead unequal [weights] are suspended, [the balance] will fall on the side of the heavier [weight] until it reaches the vertical position.A balance is equal, when the arms of the beam, measured from the centre of rotation, are equal.

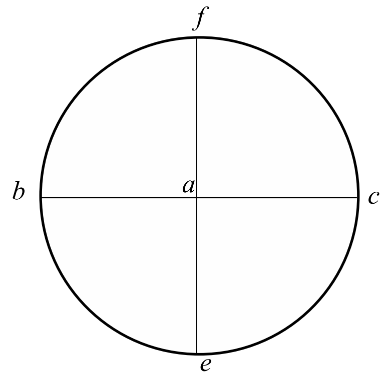
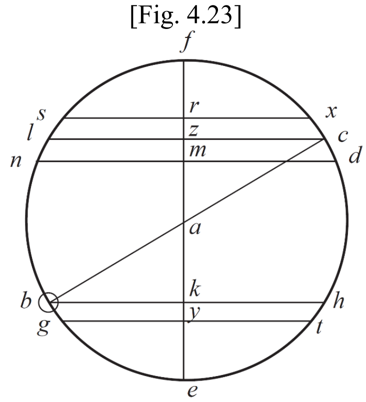


Figure drawn by Niccolò[88]
[Fig. 4.23]



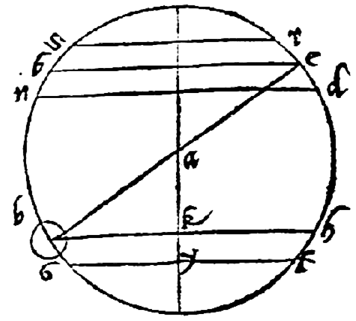
[Fig. 4.24]

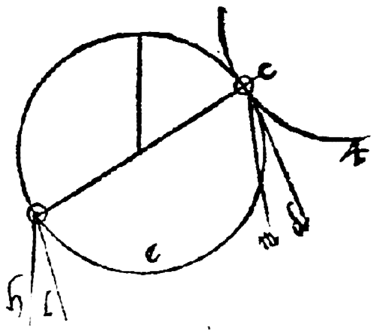
Let the centre, then, be a, and the beam bac; and let b and c be suspended, and fa be the vertical. Drawn a circle through b and c, the mid point of its lower half being e, it is evident that the descent of both b and c will be along the circumference of the circle, toward e. And since the descents along these paths are equally oblique, and [b and e] have equal weight, therefore neither of them will move [See Fig. 4.23]. *Let it now be supposed that the balance is tilted down on the side of b, and up on the side of c* [See Fig. 4.24]. *I say that it will revert to the horizontal position. The descent from c toward the horizontal position is indeed less oblique than the descent from b toward e. Assume indeed equal arcs, as small as you please, cd and bg; and draw the lines parallel to the horizontal czl and dmn, and also bkh and gyt, and draw, vertically, the diameter frzmakye. Then zm will be greater than ky, because if an arc, equal to cd, is taken in the direction of f, and if the line xrs is drawn transversally, then rz will be smaller than zm, what is easy to show.*

And since rz equals ky, zm will be greater than ky. Since because any arc you please, which is beneath c, takes more of the vertical than an arc equal to it, taken beneath b, the descent from c is more direct than the descent from b; and then c will be heavier in the most elevated position, than b. Therefore [the balance] *will revert to the horizontal position.*

Now

Sit item b, grauius, quàm c, & ponantur æqualiter, quia ergo utrobique est æque obliquus descensus patet, quia b, descēdit. Ponatur etiam b, inferius, ut libet, & c, superius: dico quòd etiam in hoc situ erit grauius b, dimittant enim directæ lineæ c,d, & b,h, & contingentes circulū sint b,l, c, m, & sit arcus c,z, similis, & æqualis, & in eodem situ cum arcu b,e, quem & linea c,m, contin get. Et quia obliquitas arcuum b,e, uel c,z, est angulus d,c,z, & obliquitas arcus, c,e, est in angulo d,c, m, atque proportio anguli d,c,z, ad angulum d, c, m, est minor qualibet proportione, quæ est inter maiorem, & minorem quantitatem. Minor ēt erit, quàm ponderis b, ad pondus t. Quomodo ergo plus addat b, super c, quàm obliquitas super obliquitatem grauius erit b, in hoc situ, quàm c, hac rationem non desinet b, descendere, & c, ascendere, usque f, e, q.



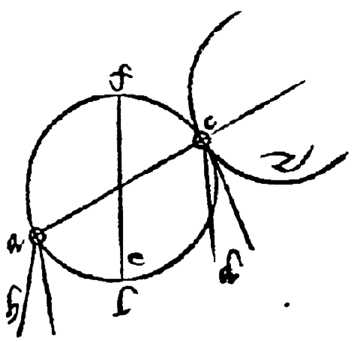


Quæstio Tertia.

Omne pondus in quamcunque partem discedat ab æqualitate secundum situm fit leuius.

Supra enim locum æqualitatis duo loca signentur super, & infra, & ab omnibus arcus resecentur ab inferiore æquales, ut libet parui, & qui est sub loco æqualitatis plus capiet de directo.

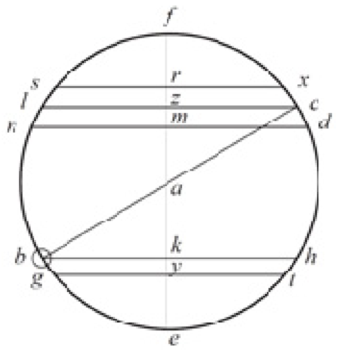
Figura à Nicolao constructa.



[4r]

PONDEROSITATE.

Now let b be heavier than c, and assume the horizontal position. Then, since the descent on each side is of equal obliquity, it is evident that b will descend. For let b be placed below, in any position, and c above. I say that in this position also, b will be heavier. Indeed let the vertical lines cd and bh[89] *be drawn; and let the lines bl and cm be tangents to the circle* [See Fig. 4.26]; *and let the arc cz be similar and equal and similarly placed as the arc be, so that the line cm is tangent. But because the obliquity of the arcs be or cz is represented by the angle dcz, and the obliquity of the arc ce by the angle dcm, the proportion of the angle dcz to the angle dcm is smaller than any ratio that can be assigned between a greater and a smaller quantity.*[90] *And it will also be less than the ratio of the weight b to the weight c. Since then b exceeds c to a greater extent than the obliquity exceeds the obliquity, b in this position will be heavier than c. For this reason b will not cease to descend, and c to ascend, until the beam is in fe,q.*



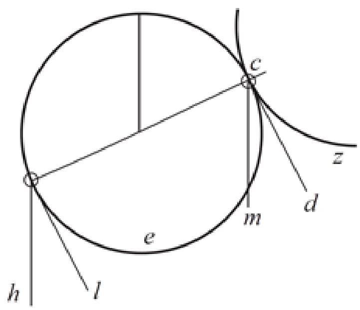
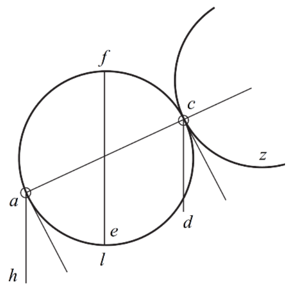


Figure drawn by Niccolò.
[Figs. 4.25]

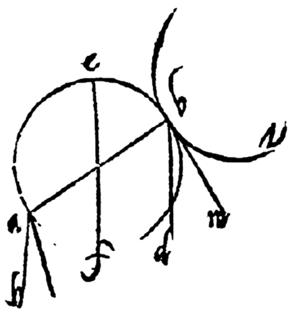
Third Question [Proposition].

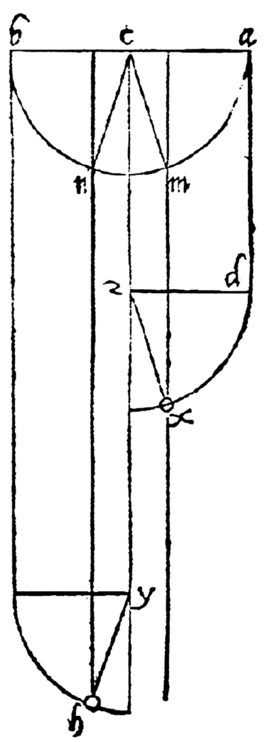
In whichever direction a weight is displaced from the position of equality, it becomes lighter according to position.

Above the horizontal position let there be identified two points, above and below. And from each of these assume equal arcs, as small as you like, on the lower side. Then the arc which is taken below the position of equality will take more of the vertical.



[Fig. 4.26]





Quæſtio Quarta.

Quum fuerint appenſorum pō dera æqualia, non faciet nutum in æquilibri appendiculorum inæqualitas.

Sit reſponſa a, b, c, centrum c, & appendicula a, d, & b, e, longius autem b, e, appenſa b, e, deſcendatq; c, z, y, orthogonaliter quantumlibet, & ductis d, z, & e, y, æque diſtantibus reſpondere, & poſitis centris in z, & y, circunducantur quartæ circulorum per d, &, e. Et quoniam d, z, & e, y, ſunt æquales, erunt & quartæ circulorum æquales. & quia per illorum circunferentias eſt deſcenſus d, & e, quum æque ponderoſa ſint d, & e, & æque obliquus, deſcenſus in hoc ſitu æque grauia erunt. Non ergo nutabit hinc, uel inde reſponſa. Quòd autem per illas ſit illorum deſcenſus, ſic conſtet. Deſcribatur enim ſemicirculus circa centrum c, ſecundum quantitatem b, & a, & dimittatur a, in m, & b, in n, deſcendantq; ab m, & n, ad quartarum circunferentias lineæ m, x, & n, h, æque diſtantes c, x, dico quòd m, x, adæquatur a, d, & n, h, æqualis eſt b, e, quod patet ductis lineis z, x, y, h. Quũ ergo ſemper deſcendant a, & b, per hunc ſemicirculum deſcendunt etiam d, & e, per deſcriptas quartas, & hoc fuit demonſtrandum.

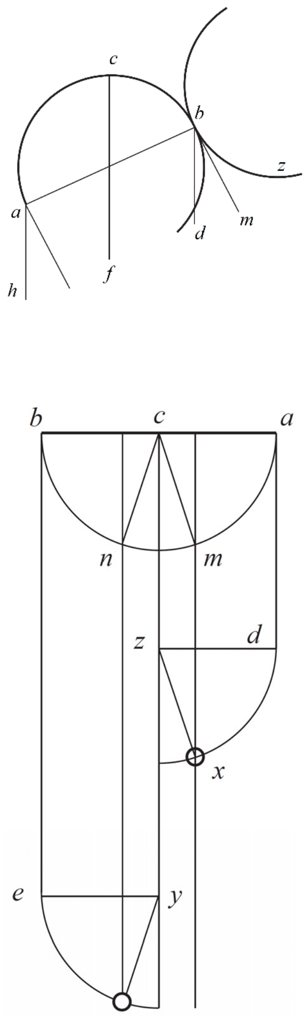
Quæſtio Quinta.

Si brachia libræ fuerint inæqualia, æqualibus appenſis ex parte longiore nutum faciet.

Sit

[4v]

OPUSCULUM DE



[Figs. 4.27]

Fourth Question [Proposition].

When equal weights are suspended [with wires] from a balance, inequality of the wires [pendants] will not determine a perturbation of their equilibrium.

Let the balance be acb, its centre c; the wires ad and be, with be the longer; and the suspended weights d and e. Then let the perpendicular czy go down as long as you like, and draw dz and ey parallel. Then, with centres at z and y, let quarter circles be described through d and e; and since dz and ey are equal, the quarter circles will also be equal. Because d and e descend along the circumferences, and because d and e are of equal weight, and of equal obliquity, they will be equally heavy according to position. Therefore the balance will not move neither here nor there. That their descent is along these paths, is shown as follows. Indeed let a semicircle be drawn around the centre c, through the points a and b; and let a descend to m, and b to n, and from m and n, to the circumferences of the quarter circles, draw the lines mx and nh parallel to cz. I say that mx is equal to ad, and that nh is equal to be: which is evident after the lines zx and yh are drawn. Since therefore a and b descend always along this semicircle, d and e will also descend through the quarter described. And this is what was to be proved.

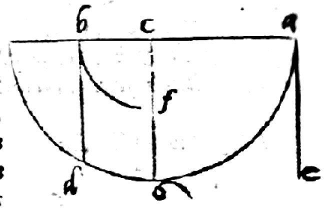
Fifth Question [Proposition].

If the arms of the balance are unequal, equal [weights] suspended [from their extremities], determine a tilting on the side of the longer [arm].

Let

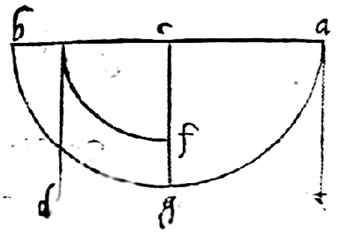
Sit responsa a,c,b, & sit a,c, longior quàm c,b. dico quòd appensis æqualibus ponderibus, quæ sint a, & b. declinabit ex parte a, dimissa enim perpendiculari c,f,b, circinentur duæ quartæ circulorum circa centrum c, quæ sint a b, et b,f, & eductis contingentibus ab a, & b, quæ sint a,e. & b,d, palam est minorem esse angulum e,a,b, contingentiæ, quàm d,b,f, & ideo minor obliquus descensus per a,b, quàm per b,f. grauius ergo a, quàm b, in hoc situ.

A Nicolao constructa.

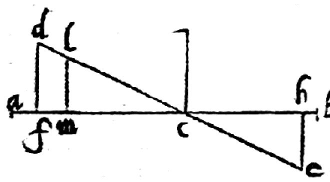


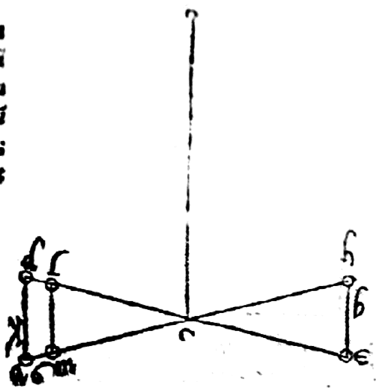
Quæstio Sexta.

Si fuerint brachia libræ proportionalia ponderibus appẽsorum ita, ut in breuiori grauiter appendatur, æque grauia erunt secundum situm appensa.



Sit ut prius regula a,c,b, appensa a, & b, sitq; proportio b, ad a, tã quam a, c, ad b c. dico quòd non nutabit in aliqua parte libræ. sit enĩ ut ex parte b, descendat, transeatq; in obliquum linea d,c,e, loco a,c,b, et





appensa d, ut a, & e, ut b, & d,b, linea orthogonaliter descendat, & e, b, ascendat. palam quoniam trianguli d,c,b, & e,c,b, sunt similes, quia proportio d,c, ad c,e, quàm d,b, ad e,b, atque d,c, ad c,e, sicut b, ad a, ergo d,b, ad e,b, sicut b, ad a, sit igitur c,l, æqualis c,b, & c,e, & l, æquatur b, in pon

B

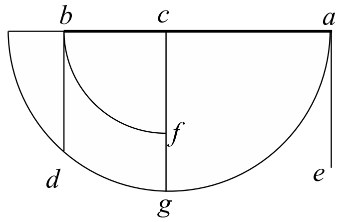
[5r]

PONDEROSIDATE.

Let the balance be acb, and let ac be longer than cb [See Fig. 4.28]. *I say that if equal weights are suspended, as a and b, the balance will decline on the side of a.*

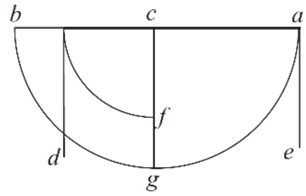
Indeed let the perpendicular cfg be drawn, and let two quarter circles, ag and bf, be described around the centre c; and let the tangents af and bd be drawn from a and b. it is then plain that the angle of contingency eag is smaller than the angle dbf, and that therefore the descent along ag is less oblique than along bf. Then, in this position, a is heavier than b.

Figure drawn by Niccolò[91]



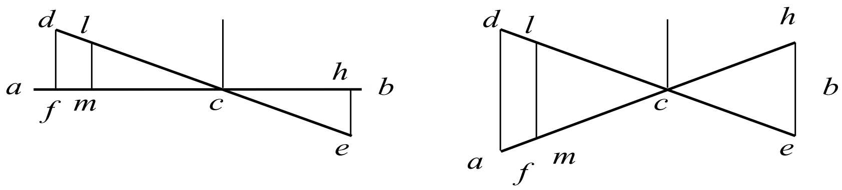
Sixth Question [Proposition].

If the [length of the] arms of a balance are proportional to the weights suspended, and the heavier weight is suspended from the shorter, the weights will be equally heavy according to position.



[Figs. 4.28]

Let consider the beam acb, as before, with suspended [weights] *a and b; and let the ratio of b to a be as the ratio of ac to bc* [See Fig. 4.29]. *I say that the balance will not tilt in any direction.*

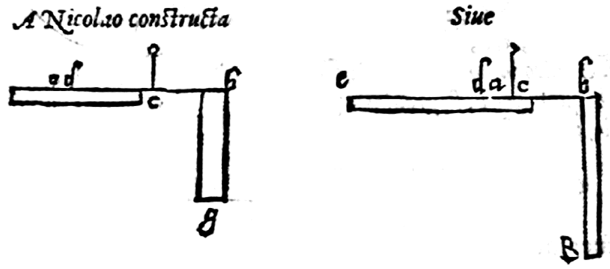


[Figs. 4.29][92]

Suppose it descends on the side of b; and passes to the skew line dce from the position acb. If a weight d, equal to a, and a weight e equal to b, are suspended, and if the line da descends vertically downward and the line eh rises, it is evident that because the triangles dcf and ech are similar, the proportion of dc to ce is the same as that of df to eh. But dc is to ce as b is to a therefore df is to eh as b is to a. Then assume cl equal to cb and to ce, and l equal in weight to b,

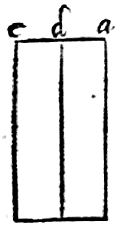
B

dere, & descendat perpendiculum l,m, quia l,m, & e,b, constant esse æquales, erit d,b, ad l,m, sicut b,ad a, & sicut l,ad a, sed ut ostensum est a, & l proportionaliter se habent ad contrarios motus alternatim. Quod igitur sufficiet attollere a, in d, sufficiet attollere l, secundum l,m. Quum ergo æqualia sint l, & b, & l,c, æquale c,b,l, non sequitur b, contrario motu, neque a, sequitur b, secundum quòd proponitur.



Quæstio Settima.

Si duo oblonga per totum similia, & quantitate, & pondere æqualia appendantur ita, ut in alterum dirigatur, alterum orthogonaliter dependeat, ita etiam, ut termini dependentis & medii alterius eadem sit a centro distantia, secundum nunc situm æque grauia fient.



Sint termini regulæ a, & b, centrum c, ut appensa quidem dirigitur secundum situm. Resp. ad æquidistantia orizontis sit, a dde medium eius d, & alterum dependes b,6 fit tūc b,c, sitq; b,c, tanquam c,a,d. Dico quòd a,d,c, & b,6, in hoc situ æque grauiora sunt. Ad huius euidentiam dicimus, quòd si responsa ex parte a, sit ut c, e, & appendantur in a, & e, duo pondera æqualia, sicut z, & y, & duplum utriusque appendatur ad b, quod sit x,l, erit etiam in hoc situ x,l, tanquam z, & y, in pondere. Sint enim x, & l, dimidia eius eritq; pondus eius, x, ad pondus z, tanquam b,c, ad c,e, per præmissam, & commune pondus l, ad pondus y, in hoc situ, sicut ab b,c, ad c,a, itaque erit x,l, ad z, & y, in hoc situ, sicut ad e,c, & a,c, duplum a,b, et quia duplum b,c, est, ut c,a, & c,e, erit x,l, æquale z, & y, in pondere in hoc situ, hac ratione, quoniam omnes partes b,6 pondere sunt equales, & in hoc situ, & qualibet duæ partes a,d,e, æqualiter a,d, distantes sunt in pō

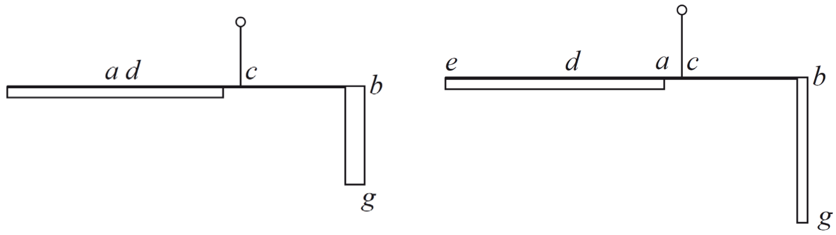
dere

[5v]

OPUSCULUM DE

and draw perpendicularly lm. Since lm and eh are shown to be equal, then df will be to lm as b is to a, and as l is to a. But, as has been shown, a and l are inversely proportional to their contrary [upward] *motions. Therefore, what suffices to lift a to d, will suffice to lift l through lm Since l and b are equal, and lc is equal to cb, l will not follow b; and neither a will follow b in the contrary motion, which is what it is proposed.*

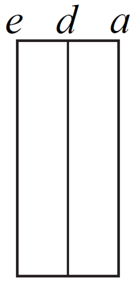
[*Figure*] *drawn by Tartaglia*[93] *Or*



[Fig. 4.30]

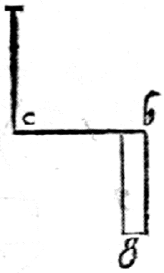
Seventh Question [Proposition].

If two oblong bodies, wholly similar and equal in size and weight, are hung from a balance so that the one be disposed horizontally and the other comes down vertically, so that the distance from the centre [of the balance] to the extremity that descends is the same as the distance to the midpoint of the other, in this position [the two bodies] will be equally heavy.



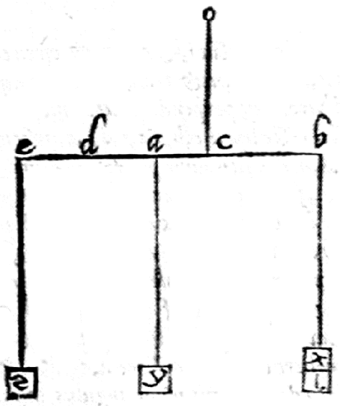
Let a and b be the ends of the beam, c the centre; and be the body disposed horizontally, with d its mid point; and let the other body, which hangs, be bg so that bc be equal to cd [See Fig. 4.30]. *I say that ade and bg, in this position, are equally heavy. To make this evident, we say that if the beam, on the side of a, were equal to ce, and if there were suspended from a and e two equal weights, z and y* [See Fig. 4.31], *and if a weight double of any of these, xl, were suspended from b, then also in this position xl would be equally heavy as z and y. Let indeed x and l the two halves*[94] [of xl] *then the weight x will be to the weight z, as bc is to ce, and the weight l will be to the weight y, in this position, as bc is to ca. Hence xl will be to z plus y, as twice cb is to ec plus ac. And because twice bc is equal to ca plus ce, xl will be equal in weight to z plus y, in this position.*[95] *For this reason, since all the parts of bg are of equal positional gravity, and since the two parts of ade equidistant from d are equal in weight.*

dere æquales duabus æquis partibus b, 6. sequitur ut totum toti.

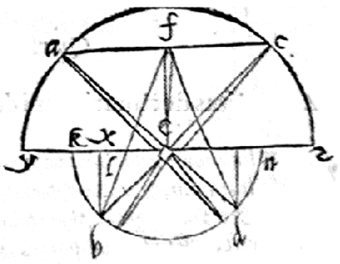


Questio Ottaua.

Si inæqualia fuerint brachia libræ, & in centro motus angulum fecerint: si termini eorum ad directionem hinc inde equaliter accesserint: æqualia appensa in hac dispositione æqualiter ponderabunt.



Sit centrum c, brachia a,c, longius b,c, breuius, & descendat perpendiculariter c,e,6. supra quã perpendiculariter cadant hinc, inde a,6. & b,e, æquales. Quum sint ergo æqualia appensa a,c,b, ab hac positione non mutabũtur, pertranseãt enim æqualiter a,6, & b,e, ad K, & z, & super eas fiant portiones circulorum m,b,h,z, K,x,a,l, & circa centrum c, fiat commune proportio K, y, a, f, similis, & æqualis portionis m,b,h,z, & sint arcus a,x,a,l, æquales sibi atque similes arcubus b,m,b,h. Itemq; a,y,a,f. si ergo ponderosius est a, quã b, in hoc situ descendat a, in x, & ascendat b, in m, ducãtur igitur lineæ z,m, K,x,y,K,f,l, & m,p, super z,b, stet perpendiculariter etiam x,e, & f,d, super K,a,d, & quia m,p, æquatur f,d, & ipsa est maior x,t, per similes triangulos erunt m, p, maior x,t, quia plus ascendit b, ad rectitudinem, quàm a, descendit. quod est impossibile, quum sint æqualia: descẽdat ratione b, in h, & trahat a, in l,



& cadant perpendiculariter b,z, super b,z, & l,n, & y,o, super n, m, fiet l,n, maior y,o, & ideo maior b,r, vnde similiter colligitur impossibile. Ad maiorem autem euidentiam describamus aliam figuram, hoc modo.

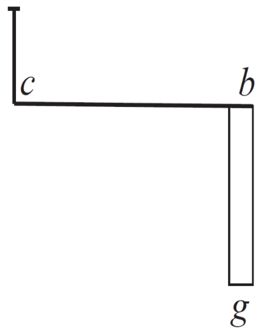
[6r]

P O N D E R O S I D A T E.

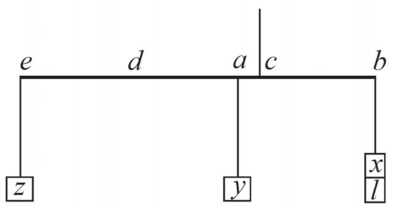
to two equal parts of bg, it follows that the whole is equal to the whole.

Eighth Question [Proposition].

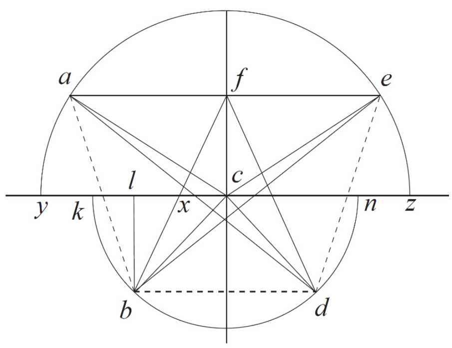
If the arms of a balance are unequal, and form an angle at the centre of rotation, then, if their ends are equidistant from the vertical passing through the centre, equal weights suspended in this configuration will weigh equally



Let the centre be c, the longer arm ac, and the shorter bc and draw the vertical line ceg; and let ag and be be equal lines, perpendicular to this vertical. When equal weights are suspended at a and b, they will not change this position [See Fig. 4.33].[96] *For let ag and be be equally extended to k and z* [See Fig. 4.33]*; and on them draw the arcs of circles, mbhz and kxal and about the centre c let kyaf be similar et equal to mbhz and let the arcs ax and al be equal to each other, and similar to the arcs mb and be and let the arcs ay and af also be equal and similar. If then in this position a is heavier than b, a descends to x and that b raises to m. Then draw the lines zm, kxy, kfl; and mp perpendicular to zbp, and xt and fd on kad. Because mp is equal to fd which is greater than xt, on account of similar triangles,*[97] *mp will also be greater than xt. hence b will be lifted vertically* [of *mp*] *more than a will descend vertically* [of *tx*], *which is impossible since they are of equal weigh*t. *Again, let b descends to h and a lifts to l; and let hr fall perpendicularly on bz, and ln and yo on kon. Then ln will be greater than yo, and consequently greater than hr; so similarly the impossible will result. For a greater evidence, let us draw a different figure, as follows.*[99]



[Figs. 4.31]



[Fig. 4.32[98]]

B 1

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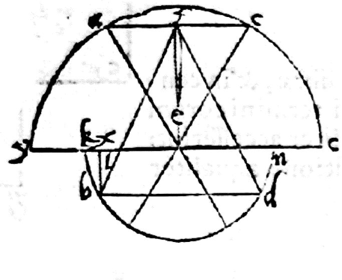
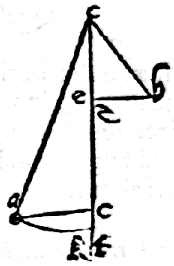
Figura à Nicolao Tartalea costrutta super hanc 8.





Esto linea recta i,k, e,n,z & circa centrum c.hinc inde duo semicirculi y, a,e,z,K,b,d,n.& transeant lineæ æquedistantes à diametro a f,e,& b,l, d,directeq; perpendiculares hinc inde fiant æquales ut b,l, & e,f, pertractis recte lineis e,b,c,a,c,d,f,e,positio quòd pondera sint æqualia m,a,b,d, e,f,in hoc situ æque ponderosa erunt Directæ enim lineæ b,a,b,x,f,b,f, d, a,d,f,d,e, omnes secabuntur per æqualia apud diametrum,ueluti b, x,f, & ita omnes diuisæ erunt per medium . quare ergo in medio omnium sint centra posita sicut sunt pondera posita æqualiter, ergo ponderant : subtilius tamen quædam differentia potest perpendi : ut sit a,ponderosius quàm b,& b,quàm f,& f, quàm d,& d, quàm e,nec tamen potest d . eleuare e, statim enim portio lineæ d,e,uersus e,fieret maior, sed e, potest nutu facto trahere b,& b,similiter a, & d,a,& a,d,& b,f,& f,b . donec circumuoluta dependeant ut fit angulus supra centrum, sub ipso enim motu b,inferius crescet semper pars lineæ b,a,uersus b,& fiat b,grauius.

Quæstio Nona.

Aequalitas declinationis identitatis ponderis.

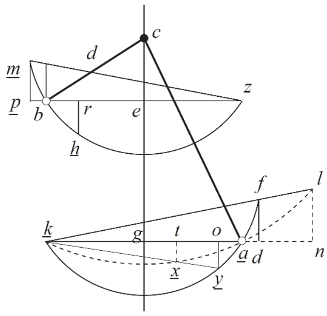
DE linationis æqualitas tantum in uia recta conseruatur, & ipsa sit in linea a,b, & recte descendens linea sit a,c,sintq; in a,b,duo loca d,& e.Siue ergo à d,descendat quodlibet pondus,siue ab e, eiusdem ponderis erit,æquales enim partes sub d, & ,e, sumptæ æqualiter capiunt de directo, quod patet ductis perpendicularibus ad a, c , a, b, eisdem locis quæ sint e,f,h,6,1,& dimissis orthogonaliter super illas d, K, & e, m,lineas,unde siue excedatur pondus supra a,b,siue simul ponatur unius ponderis est.

Quæstio

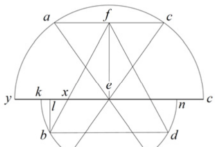
[6v]

OPUSCULUM DE

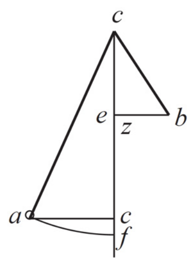
Figure drawn by Niccolò Tartaglia based[100] *on this 8* [Eight Question].



[Fig. 4.33[101]]



[Fig. 4.33*bis* original figure]



[Fig. 4.34]

Let there be a vertical line ykcnz, and around the centre c let there be drawn two semicircles, yaez and kbdn [see Fig. 4.33]*; and let the lines afe and bd be drawn at equal distances from the diameter, and from these let there be drawn the equal perpendiculars bl and cf Then draw the lines cb, ca, cd, and ce and assume that equal weights are suspended at a, b, d, e, and f, they will be of equal weight in this position. For if the lines ba, bxf, be, da, df, and de are drawn, all of them will be bisected by the diameter as for instance bxf. And in the same manner the others will be divided at their mid points. Since weights are placed in the same way they will be of equal weight. A more subtle variant may, however, be determined, if we suppose that a is heavier than b, b heavier than f, f heavier than d, and d heavier than e. Yet d is not able to lift e; for the segment of the line de on the side of e would immediately become greater. But if a is given an impulse downward, it is able to raise b, and similarly b can raise a; and a can raise d; and b can raise f and f can raise b; until they make a complete revolution and hang in such manner that the angle with the axis is beneath them. For when b is moved downward, the segment of the line ba, on the side of b, will become steadily longer, and b will become heavier.*[102]

Ninth Question [Proposition].

Equality of declination conserves identity of weight.

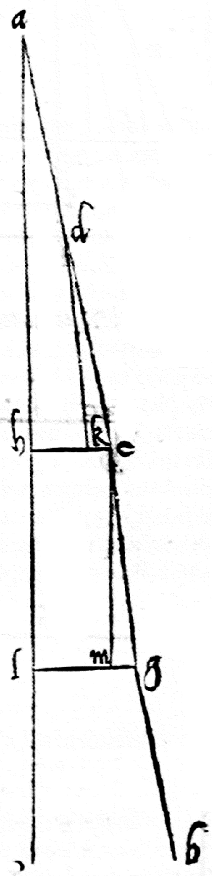
Equality of declination is conserved only on a rectilinear path. Let this [path] *be on the line ad, and let the line ac descend vertically and assume two points, d and e on ab* [Fig. 4.34]. *Any heavy body you like, then, whether it descends from d, or from e, will have the same weight. For equal segments of ad, taken beneath d and e, will have equal components of the vertical. This is clear, if we draw from these points the perpendiculars eh and gl to the line ac, and if we let lines dk and em perpendicularly on them. Thus, whether a heavy body moves along ab, or is placed there, it will be of the same weight.*

Question

Quæstio Decima.

Si per diuersarum obliquitatum uias duo pondera descendant, fiantq; declinationum, & ponderum vna proportio eodem ordine sumpta vna erit utriusque uirtus in descendendo.

Sit linea a, b, c, æquedistans orizonti, & super eam orthogonaliter erecta sit b, d, à qua descendant hinc, inde lineæ d, a, d, c, sitq; d, c, maioris obliquitatis proportione igitur declinationum dico non angulorum, sed linearum usque ad æquedistantem resecationem, in qua æqualiter sumunt de directo. Sit ergo e pondus super d c, & h, super d, a, & sit e, ad h sicut d, c, ad a, d. Dico ea pōdera esse vnius uirtutis in hoc situ, sit enim d, k, linea vnius obliquitatis cum d, c, & pondus super eam. ergo æquale est e, quæ sit 6. Si igitur possibile est, descendat e, in l, & trahat h, in m, sitq; 6, n. æquale h, m, quod etiam æquale est e, l, & transeat per 6. & h, perpēdicularis, super d, b. Sitq; 6. h. y, & ab l, sit l, t, sunt & tunc super 6, h y, n, z, m, x, & super l, t, erit e, r, quia igitur proportio n, z, ad n, 6, sicut ad d, 6, d, y, propter similitudinem triangulorum, & ideo sicut d, b, ad d, k, & quia similiter m, x, ad m, h, si ut d, b, ad d, a. Erit propter æqualē proportionalitatē per turbatam m, x, ad n z. sicut d, K, ad d, a, & hoc est sicut 6, ad h sed quia r, e, non sufficit attollere 6, in n, nec sufficiet attollere m, in m, sic ergo manebunt.



Quæstio Vndecima.

Quum sit responsa libræ vnius ponderis, & grossiciei per totum: & ipsa in pondere data super inæqualia diuidatur, atque ex parte breuiore dependeat æquabiliter pōdus datum, erunt & portiones & regulæ, quæ sunt a centro exaninis similiter datæ.

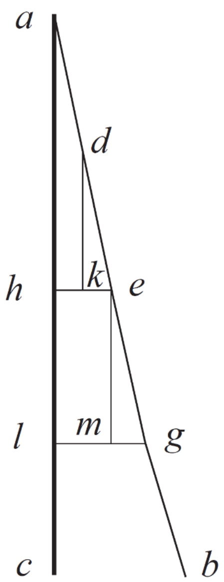
Sit responsa a, b, c, data in pondere, & æqualis in grossicie, & dependeat

[7r]
PONDEROSIDATE.

Tenth Question [Proposition].

If two weights descend along diversely oblique paths, then, if the inclinations are directly proportional to the weights, they will be of equal strength in descending.

Let there be a line abc parallel to the horizon, and let bd be erected vertically on it; and from d draw the lines da and dc, with dc of greater obliquity [Fig. 4.35]. *I then mean by proportion of obliquities not the ratio of the angles, but of the lines measured up to a horizontal line cuts off an equal segment of the vertical. Let the heavy body e, then, be on dc, and the weight h on da; and let e be to h as dc is to da. I say that those weights are of the same strength in this position. For let dk be a line of the same obliquity as dc, and let there be on it a weight g, equal to e. Then let assume possible e descends to l, and lifts h up to m and let gn be equal to hm, which in turn is equal to el. Then draw a perpendicular to db from g to h, which will be ghy; and* [another] *from l, which will be tl. And on ghy, erect the perpendiculars nz and mx; and on lt,* [erect] *the perpendicular er. Since the proportion of nz to ng is as that of dg to dy, for the similitude of triangles, and hence as that of db to dk, and since likewise mx is to mh as db is to da, mx will be to nz as dk is to da, i.e., as g is to h. But because e does not suffice to lift g to n, it does not suffice to lift h to m. Therefore they remain as they are.*



[Fig. 4.35]

Eleventh Question [Proposition]

When there is a balance beam of uniform weight and thickness throughout, and its weight is known, if it is divided into unequal segments and if a body of known weight, suspended from the shorter arm, holds the beam in equilibrium, then the lengths of the arms on each side of the axis of rotation will also be determined.

Let the beam be abc, of a given weight and of uniform thickness. Let a body,

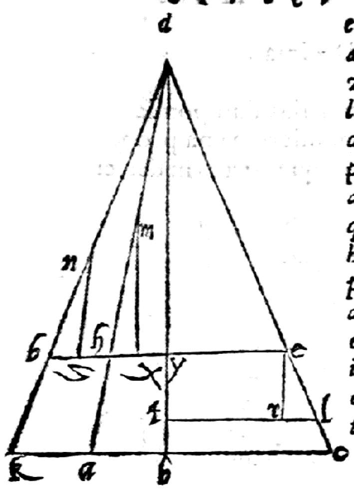
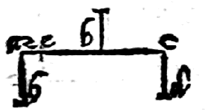
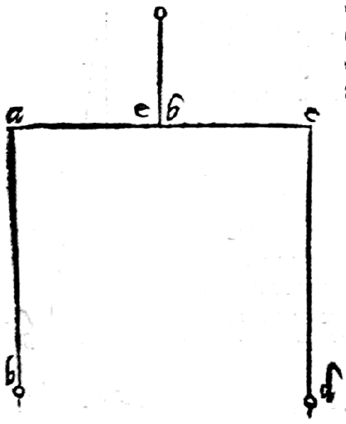


Figura à Nicolao constructa.





ex parte c, pondus b, datum, sitq; b, e, æqualis b, c, & in medio a, e, notetur z, à quo dependeat pondus b, æquale a, e, & in eo etiam situ æque ponderabit. Quia ergo in hoc situ æque ponderant h, & d, eritq; proportio d, ad h, ea z, b. ad b, c, & permutatim quæ proportio d, ad z, b, ea est a, e, hoc est h, ad b, c, & coniunctim quæ proportio d, & dupli z, b, hoc est a, c, ad z, b, ea est a, e, & dupli b, c, hoc est e, c, ad b, c. Si ergo tota a, b, c, ducatur in suum dimidium, & perductum diuidatur per d, & a, c, quod totum est datum, exibit b, c. datum

Quęstio Duodecima.

Quod si portiones datæ fuerint, & pondus datum erit.

CVm enim ut præmissum est d, pondus cũ tota a, c, sit ad eius dimidium, sicut tota a, c, ad b, c. cũ sint a, b, & b, c, datæ, si ducatur a, c, in suum dimidium, ut prius, & productum diuidatur per b, c, exibit pondus d, & tota a, c, detracta ergo a, c, relinquitur pondus d, datum.

Quæstio Tertiadecima.

Si uero pondus datum fuerit, & pars cui appenditur data, totum quoque datum erit.

VBi gratia d, pondus datum sit, & b, c, portio data. Quia igitur d, ad h, siue ad e, a, sicut z, b, ad b, e, erit, quòd ex ductu d, in c,

[7v]

OPUSCULUM DE

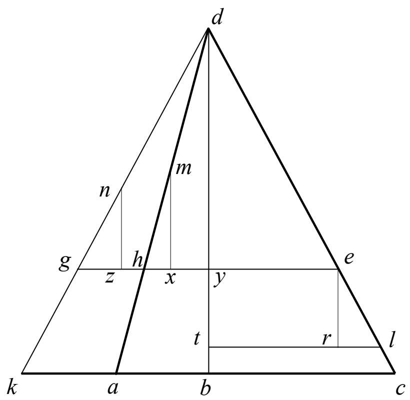


Figure drawn by Tartaglia
[Fig. 4.36]

d, of known weight, hang from the end c, and let be be equale to bc. From the mid point of ae, designated as z, let there be suspended a body, h, equal in weight to the segment of the beam ae; and in this position it will also be of equal heaviness. Since therefore h and d are equally heavy in this position, the proportion of d to h will be that of zb to bc. And by alternation, the proportion of d to zb will be that of ae i.e., of h to bc. And by composition, the proportion of d plus twice zb (i.e., ac) to zb, will be that of AE plus twice bc i.e., ec to bc. If therefore the whole weight abc is multiplied by its half, and the product is divided by the sum of the weights of d and of ac–all these being given–, the weight of the segment bc is thereby determined.

[…]

4.6.2 The Latin Critical Transcription

[2r]

FRANCISCO LABIAE[103]

OMNI VIRTUTUM GENERE ORNATO.

CURTIUS TROIANUS S.D.

Non me fugit summa in expectatione te esse, cum optimi literarum studijs, qui te vehementius incumbat cognoscam neminem. Nullum profecto doctrina genus est, in quo non verseris, nulla disciplina, quam non intelligere velis, tu grammaticum canones, historias, et poetarum fabulas mirifice tenes, tu rhetoricis flosculis abundas, dialecticorum argutia scrutaris, physices arcana, et superior intelligentia pervestigas, tu theologorum abdita perquiris, tu mathematicis, et omni denique eruditionis genere delectaris, quamobrem, pro mea in te; et patrem tuum benevolentia, propter egregiam tuam indolem, iucundissimos more, divinum inge

[2v]

nium, summa modestiam, tibi optimae adolescent dicare volui hunc Iordani ingeniosi, et acuti hominis librum de ponderibus, quem mihi suis in fragmentis Nicolaus Tartalea familiaris meus, vir quidem praeclaris ornatus scientiis excudendum reliquit. Accipias igitur laeto vultu hunc in lucem editu, tuoque sub nomine emissum, quandoquidem tibi non modo iucunditati, sed etiam utilitati fore certo scio. Vale: Nonae Kalendas Februarius.

PRIMA

[3r]

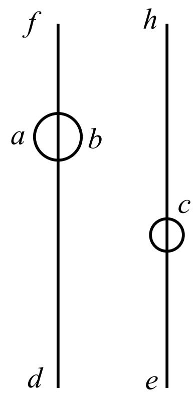
PRIMA SUPPOSITIO.

Omnis ponderosi motum esse ad medium virtutemque ipsius esse potentia ad inferiora tendendi virtutem ipsius, sive potentia possumus intelligere longitudinem brachii librae, aut velociter eius quem probatur ex longitudine brachii librae, et motui contrario resistendi. Secunda: Quod gravius est velocius descendere. Tertia: Gravius esse in descendendo quanto eiusdem motus ad medium rectior. Quarta: Secundum situm gravius esse cuius in eodem situ minus obliquus descensus. Quinta: Obliquiorem autem descensus in eadem quantitate minus capere de directo. Sexta: Minus grave aliud alio secundum situm, quod descensum alterius sequitur contrario motu. Septima: Situm aequalitatis esse aequalitatem angulorum circa perpendiculum, sive rectitudinem angulorum, sive eque [aeque] distantiam regulae superficiei Orizontis [Horizontis].

Quaestio Prima.

Inter quaelibet gravia est virtutis, et ponderis eodem ordine sumpta proportio.

Sint pondera a, b, c, levius c, descendatque a, b, in d, et c, in e. Itaque ponatur a, b, sursum in f, et c in h.[104] *Dico ergo quod quae proportio a, d, ad c, e, sicut a, b, ponderis ad c pondus, quanta enim virtus ponderosi tanta descendendi velocitas: at quae compositi virtus ex virtutibus componentium componuntur. Sit ergo a, aequale c. Quae igitur virtus a, eadem et, c. Sit igitur proportio a, b, ad c, minor quam virtutis ad virtutem. Erit similiter proportio a, b, ad a, minor proportio quam virtutis a, b, ad virtutem a, ergo virtutis a, b, ad virtutem b, minor proportio quam a, b, ad b. per 30. quinti Euclidis quod est inconveniens. Similium igitur ponderum minor, et maior proportio, quam virtutum. Et quia hoc inconveniens erit, utrobique eadem ideo a, b, ad c, sicut a, d, ad c, e, et e, contrario sicut c, h, ad a, f.*



[3v]

O P U S C U L U M D E

Quaestio Secunda .

Quum aequilibris [aequilibriis] fuit positio aequalis aequis ponderibus appensis ab aequalitate non discedet: et si a rectitudine separatur, ad aequalitatis situm revertetur. Si vero inaequalia appendantur, ex parte gravioris usque ad directionem declinare cogetur.

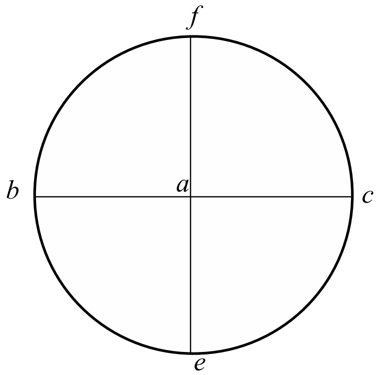
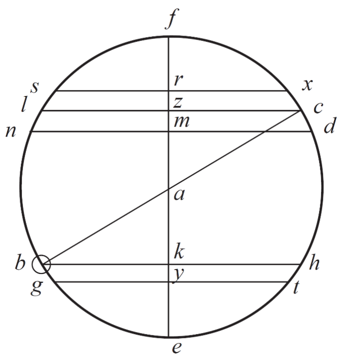


Figura a Nicolao de Tartagliis instructa.

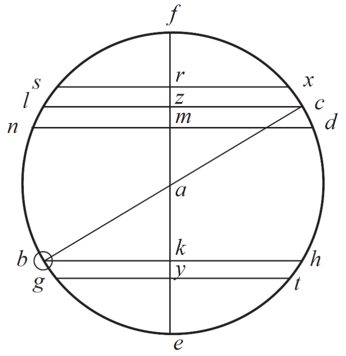


Aequilibris dicitur quando a centro circunvolutionis [circumvolutionis] *brachia regulae sunt aequalia. Sit ergo centrum a, et regula b, a, c, appensa b, et c, perpendiculum f, a. Circunducto* [Circumducto] *igitur circulo per b, et c, in medio cuius inferioris medietatis sit e, manifestum quoniam descensus tam b, quam c, e, per circunferentiam* [circumferentiam] *circuli versus e, et cum aeque obliquus sit hinc inde descensus, quum sint aeque ponderosa, non mutabit alterutrum. Ponatur item quod submittatur ex parte b, et ascendat ex parte c, dico quoniam redibit ad aequalitatem, est enim minus obliquus descensus c,*[105] *ad aequalitatem, quam a, b, versus e. Sumantur enim sursum arcus aequales, quantumlibet parvi qui sint c, d, et b, g,*[106] *et ductis lineis ad aequidistantiam aequalitatis, quae sint, c, h, l, et d, m, n. Item b, k, h, g, y, t, dimittatur orthogonaliter descendens diametrum quae sit f, z, m, a, k, y, e, erit quod z, m, maior k, y, quia sumpto versus f, arcu ex eo quod sit aequalis c, d, et ducta ex transverso linea. x, r, s, erit r, z, minor z, m, quod facile demonstrabis. Et quia r, z, est aequalis k, y, erit z, m, maior k, y. Quia igitur quilibet arcus sub c, plus capiat de directo quam ei aequalis sub b, directo est descensus a, c, quam a, b, et ideo in altiori situ gravius erit c, quam b, redibit ergo ad aequalitatem.*

[4r]

PONDEROSIDATE.

Sit item b, gravius, quam c, et ponantur aequaliter, quia ergo utrobique est aeque obliquus descensus patet, quia b, descendit. Ponatur etiam b, inferius, ut libet, et, c, superius: dico quod etiam in hoc situ erit gravius b, dimittant enim directae lineae c, d, et b, h, et contingentes circulum sint b, l, c, m, et sit arcus c, z, similis, et aequalis, et in eodem situ cum arcu b, e, quem et linea c, m, continget. Et quia obliquitas arcuum b, e, vel c, z, est angulus d, c, z, et obliquitas arcus, c, e, est in angulo d, c, m, atque proportio anguli d, c, z, ad angulum d, c, m, est minor qualibet proportione, quae est inter maiorem, et minorem quantitatem. Minor et erit, quam ponderis b, ad pondus c.[107] *Quomodo ergo plus addat b, super c, quam obliquitas super obliquitatem gravius erit b, in hoc situ, quam c, hac rationem non definet [definiet] b, descendere, et, c, ascendere, usque f, e, q.*



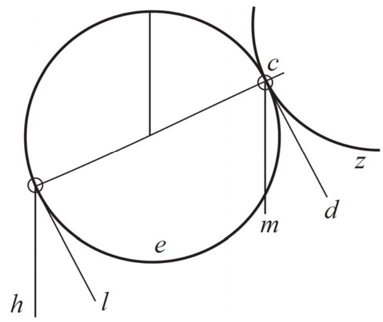
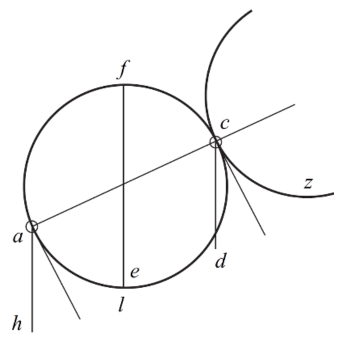


Figura a Nicolao de Tartagliis instructa

Quaestio Tertia.

Omne pondus in quamcunque partem discedat ab aequalitate secundum situm fit levius.

Supra enim locum aequalitatis duo loca signentur super, et infra, et ab omnibus arcus resecentur ab inferiore aequales, ut libet parvi, et qui est sub loco aequalitatis plus capiet de directo.



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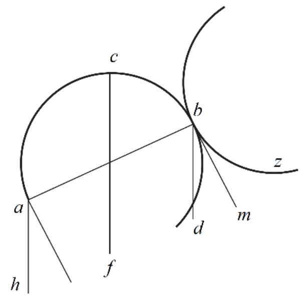
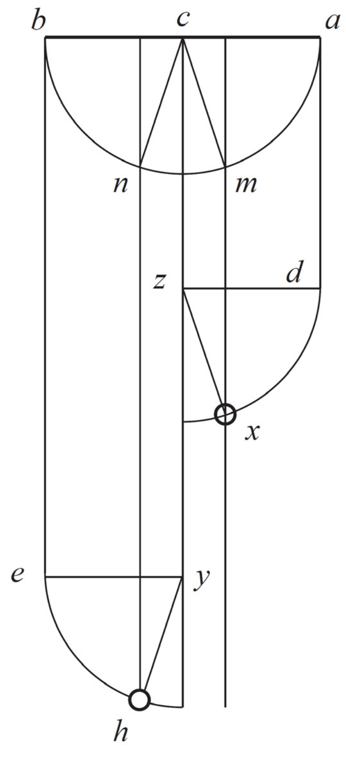


Quaestio quarta.

Quum fuerint appensorum pondera aequalia, non faciet nutum n aequilibri appendiculorum inaequalitas.



Sit responsa [regula] *a, b, c, centrum c, et appendicula a, d, et b, e, longius autem b, e, appensa b, e, descendatque c, z, y, orthogonaliter quantumlibet, et ductis d, z, et e, y, aeque distantibus respondere, et positis centris in z, et y, circunducantur quartae circulorum per d, et, e. Et quoniam d, z, et e, y, sunt aequales, erunt et quartae circulorum aequales. et quia per illorum circunferentias est descensus d, et c, quum aeque ponderosa sint d, et e, et aeque obliquus, descensus in hoc situ aeque gravia erunt. Non ergo nutabit hinc, vel inde responsa* [regula]. *Quod autem per illas sit illorum descensus, sic constet. Describatur enim semicirculus circa centrum c, secundum quantitatem b, et a, et dimittatur a, in m, et b, in n, descendantque ab m, et n, ad quartarum circunferentias lineae m, x, et n, h, aeque distantes c, y*[108]*, dico quod m, x, adaequatur a, d, et n, h, aequalis est b, e, quod patet ductis lineis z, x, y, h. Quum ergo semper descendant a, et b, per hunc semicirculum descendunt etiam d, et e, per descriptas quartas, et hoc fuit demonstrandum.*

Quaestio Quinta.

Si brachia librae fuerint inaequalia, aequalibus appensis ex parte longiore nutum faciet.

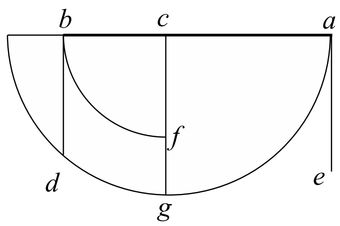
Sit

[5r]

PONDEROSIDATE.

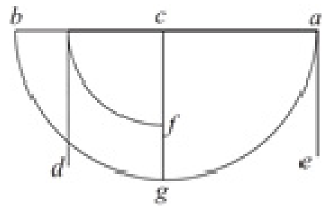
Sit responsa [regula] *a, c, b, et sit a, c, longior quam c, b. dico quod appensis aequalibus ponderibus, quae sint a, et b. declinabit ex parte a, dimissa enim perpendiculari c, f, g*[109]*, circinentur duae quartae circulorum circa centrum c, quae sint a, b, et b, f, et eductis contingentibus ab a, et b, quae sint a, e et b, d, palam est minorem esse angulum e, a, g*[110]*, contingentiae, quam d, b, f, et ideo minor obliquus descensus per a, b, quam per b, f, gravius ergo a, quam b, in hoc situ.*

A Nicolao constructa.

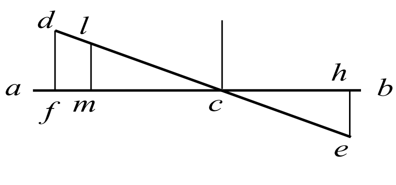


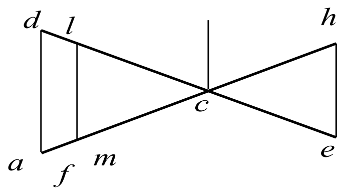
Quaestio sexta.

Si fuerint brachia librae proportionalia ponderibus appensorum ita, ut in breviori graviter appendatur, aeque gravia erunt secundum situm appensa.



Sit ut prius regula a, c, b, appensa a, et b, sitque proportio b, ad a, tam quam a, c, ad bc, dico quod non nutabit in aliqua parte librae, sit enim ut ex parte b, descendat, transeatque in obliquum linea d, c, e, loco a, c, b, et appensa d,





ut a, et e, ut b, et d, f,[111] *linea orthogonaliter descendat, et e, h, ascendat. palam quoniam trianguli d, c, f,* [112] *et e, c, h, sunt similes, quia proportio d, c, ad c, e, quam d, b, ad e, h, atque d, c, ad c, e, sicut b, ad a, ergo d, f,* [113] *ad e, h, sicut b, ad a, sit igitur c, l, aequalis c, b, et c, e, et l, aequatur b, in pon*[–]

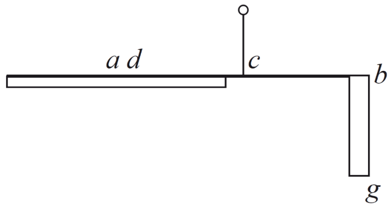
B

[5v]

OPUSCULUM DE

dere, et descendat perpendiculum l, m, quia l, m, et e, h, constant esse aequales, erit d, g,[114] *ad l, m, sicut b, ad a, est sicut l, ad a, sed ut ostensum est a, et l, proportionaliter se habent ad contrarios motus alternatim. Quod igitur sufficiet attollere a, in d, sufficiet attollere l, secundum l, m. Quum ergo aequalia sint l, et b, et l, c, aequale c, b, l, non sequitur b, contrario motu, neque a, sequitur b, secundum quod proponitur.*

A Nicolao constructa



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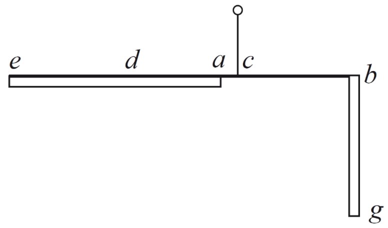
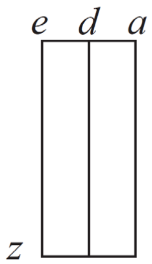


Quaestio Settima.

Si duo oblonga per totum similia, et quantitate, et pondere aequalia appendantur ita, ut in alterum dirigatur, alterum orthogonaliter dependeat, ita etiam, ut termini dependentis et medii alterius eadem sit a centro distantia, secundum nunc situm aeque gravia fient.



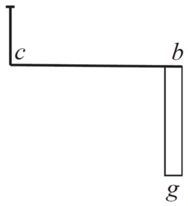
Sint termini regula a, et b, centrum c, ut appensa quidem dirigitur secundum situm. Responsa [regula] ad aequedistantia orizontis sit, adde medium eius d, et alterum dependes b, g, fit tunc b, c,[115] *sitque b, c, tamquam c, a, d. Dico quod a, d, c, et b, g, in hoc situ aeque graviora sunt. Ad huius evidentiam dicimus, quod si responsa* [regula] *ex parte a, sit ut c, e, et appendantur in a, et e, duo pondera aequalia, sicut z, et y, et duplum utriusque appendatur ad b, quod sit x, l, erit etiam in hoc situ x, l, tanquam z, et y, in pondere. Sint enim x, et l, dimidia eius eritque pondus eius, x, ad pondus z, tanquam b, c, ad c, e, per praemissam, et commune pondus l, ad pondus y, in hoc situ, sicut ab b, c, ad c, a, itaque erit x, l, ad z et y, in hoc situ, sicut ad e, c, et a, c, duplum a, b, et quia duplum b, c, est, ut c, a, et c, e, erit x, l, aequale z, et y, in pondere in hoc situ, hac ratione, quoniam omnes partes b, g, pondere sunt aequales, et in hoc situ, et quaelibet duae partes a, d, e, aequaliter a, d, distantes sunt in po*[–]

dere

[6r]

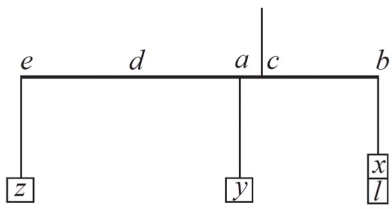
PONDEROSIDATE.

dere aequales duabus aequis partibus b, g. Sequitur ut to–tum toti.

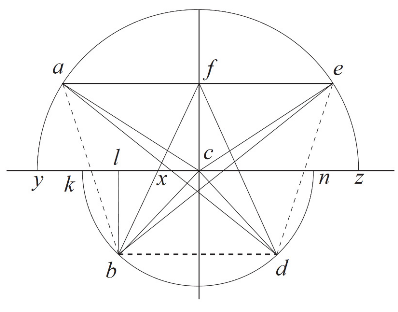


Quaestio Octava.

Si inaequalia fuerint brachia librae, et in centro motus angulum fecerint: si termini eorum ad directionem hinc inde aequaliter accesserint: aequalia appensa in hac dispositione aequaliter ponderabunt.



Sit centrum c, brachia a, c, longius b, c, brevius, et descendat perpendiculariter c, e, g. supra quam perpendiculariter cadant hinc, inde a, g. et b, e, aequales. Quum sint ergo aequalia appensa a, c, b, ab hac positione non mutabuntur, pertranseant enim aequaliter a, g, et b, e, ad k, et z, et super eas fiant portiones circulorum m, b, h, z, k, x, a, l et circa centrum c, fiat commune proportio k, y, a, f, similis, et aequalis portionis m , b, h, z, et sint arcus a, x, a, l, aequales sibi atque similes arcubus b, m, b, h. Itemque a, y, a, f. Si ergo ponderosius est a, quam b, in hoc situ descendat a, in x, et ascendat b, in m, ducantur igitur lineae z, m, k, x, y, k, f, l, et m, p, super z, b, stet perpendiculariter etiam x, e, et f, d, super k, a, d, et quia m, p, aequatur f, d, et ipsa est maior x, t, per similes triangulos erunt m, p, maior x, t, quia plus ascendit b, ad rectitudinem, quam a, descendit. quod est impossibile, quum sint aequalia: descendat ratione b, in h, et trahat a, in l,



et cadant perpendiculariter h, r, super b, z, et l, n, et y, o, super n, k, [116] *fiet l, n, maior y, o, et ideo maior, h, r, unde similiter colligitur impossibile. Ad maiorem autem evidentiam describamus aliam figuram, hoc modo.*

[6v]

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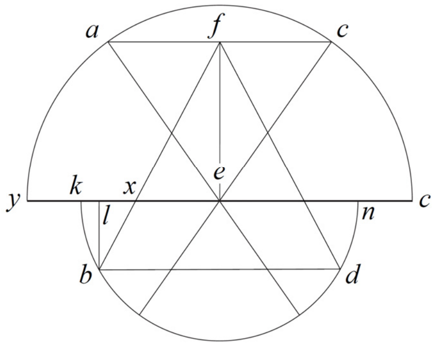
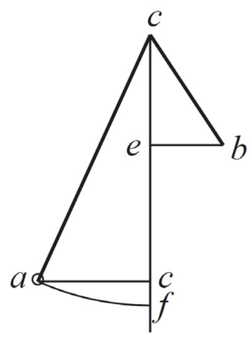
Figura a Nicolao Tartalea constructa super 8.





Esto linea recta y, k, c, n, z,[117] *et circa centrum c, hinc inde duo semicirculi y, a, e, z, k, b, d, n, et transeat lineae aequedistantes a diametro a, f, e, et b, d,* [118] *directeque perpendiculares hinc inde fiant aequales ut b, l, et c, f,* [119] *pertractis recte lineis c b, c, a, d, c, e,* [120] *positio quod pondera sint aequaliam, a, b, d, e, f, in hoc situ aeque ponderosa erunt. Directe enim lineae b, a, b, x, f, b, e, d, a, d, f, d, e, omnes secabuntur per aequalia apud diametrum, veluti b, x, f, et ita omnes divisae erunt per medium. quare ergo in medio omnium sint centra posita, sicut sunt pondera posita aequaliter, ergo ponderant: subtilius tamen quaedam differentia potest perpendi: ut sit a, ponderosius quam b, et b, quam f, et f, quam d, et d, quam e, nec tamen potest d, elevare e, statim enim proportio lineae d, e, versus e, fieret maior, sed e, potest nutu facto trahere b, et b, similiter a, et d, a, et a, d, et b, f, et f, b. donec circumvoluta dependeant ut sit angulus supra centrum, sub ipso enim motu b, inferius crescet semper pars lineae b, a, versus b, et fiat b, gravius.*

Quaestio Nona .

Aequalitas declinationis identitatis ponderis .

Declinationis aequalitas tantum in via recta conservatur, et ipsa sit in linea a, b, et recte descendens linea sit a, c, sintque in a, b, duo loca d, et e. Sive ergo a d, descendat quodlibet pondus, sive ab e, eiusdem ponderis erit, aequales enim partes sub d, et, c, sumptae aequaliter capiunt de directo, quod patet ductis perpendicularibus ad a, c, a, b, eisdem locis quae sint e, k, h, g, l,[121] *et dimissis orthogonaliter super illas d, k, et e, m, lineas, unde sive excedatur pondus supra a, b, sive simul ponatur unius ponderis est.*

Quaestio

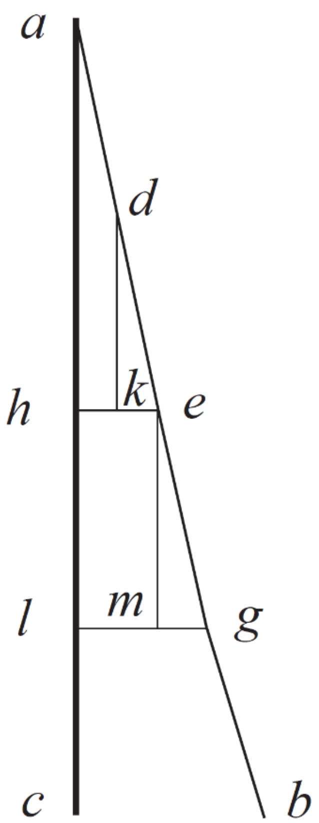
[7r]

P O N D E R O S I D A T E.

Quaestio Decima .

Si per diversarum obliquitatum vias duo pondera descendant, fiantque declinationum, et ponderum una proportio, eodem ordine sumpta una erit utriusque virtus in descendendo.

Sit linea a, b, c, aequedistans orizonti, et super eam orthogonaliter erecta sit b, d, a qua descendant hinc, inde lineae d, a, d, c, sitque d, c, maioris obliquitatis proportione igitur declinationum dico non angulorum, sed linearum usque ad aequedistantem resecationem, in qua aequaliter sumunt de directo. Sit ergo e, pondus super d, c, et h, super d, a, et sit e, ad b, sicut d, c, ad a, d. Dico ea pondera esse unius virtutis in hoc situ, sit enim d, k, linea unius obliquitatis, cum d, c, et pondus super eam. ergo aequale est e, quae sit g. Si igitur possibile est, descendat e, in l, et trahat h, in m, sitque g, n, aequale h, m, quod etiam aequale est e, l, et transeat per g. et h, perpendicularis, super d, b. Sitque g, h, y, et ab 1, sit l, t, sunt et tunc super g, h, y, n, z, m, x, et super l, t, erit e, r, quia igitur proportio n, z, ad n, g, sicut ad d, g, d, y, propter similitudinem triangulorum, et ideo sicut d, b, ad d, k, et quia similiter m, x, ad m, h, sicut d, b, ad d, a. Erit propter aequalem proportionalitatem perturbata m, x, ad n, z, sicut d, k, ad d, a, et hoc est sicut g ad h, sed quia e[122] *non sufficit attollere g, in n, nec sufficiet attollere h*[123] *in m, sic ergo manebunt.*



Quaestio Undecima .

Quum sit responsa libre vnius ponderis,et grossiciei per totum: et ipsa in pondere data super inaequalia diuidatur, atque ex parte breuiore dependeat aequabiliter pondus datum, erunt et portiones, et regulae, quae sunt a centro examinis similiter datae.

Sit responsa a, b, c, data in pondere, et aequalis in grossicie, et dependeat

[7v]

OPUSCULUM DE

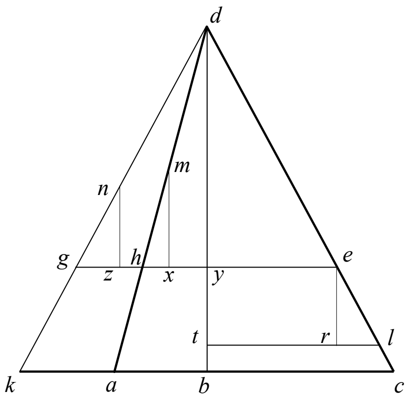


Figura à Nicolao constructa.

ex parte c, pondus b, datum, sitque l aequalis b, c, et in medio a ,e, notetur quo dependeat pondus h, aequale a, e, eo etiam situ aeque ponderabit. Quia erg hoc situ aeque ponderant h, et d, eri proportio d, ad h, ea z, b. ad b, c permutatim quae proportio d, ad z, b, ea a, e, hoc est h, ad b, c, et coniunctim q proportio d, et dupli z, b, hoc est a, c, a b, ea est a, e, et dupli b, c, hoc est e, c, a c. Si ergo tota a, b, c, ducatur in s dimidium, et perductum diuidatur per d, c, quod totum est datum, exibit b, c,. datu

[…].

In the following, for historical completeness, we report Latin transcriptions of all of the others *Quaestio* (de Nemore 1565, 8r–14r), as well. Nevertheless, as announced above, they were not interesting for my research on Tartaglia's Book VII and Book VIII.

[7v]

OPUSCULUM DE

Quaestio Duodecima.

Quod si portiones datae fuerint, et pondus datum erit.

Cum enim ut praemissum est d, pondus cum tota a, c, sit ad eius dimidium, sicut tota a, c, ad b, c. cum sint a, b, et b, c, datae, si ducatur a, c, in suum dimidium, ut prius, et productum diuidatur per b, c, exibit pondus d, et tota a, c, detracta ergo a, c, relinquitur pondus d, datum.

Quaestio Tertiadecima.

Si uero pondus datum fuerit, et pars cui appenditur data, totum quoque datum erit.

Verbi gratia d, pondus datum sit, et b, c, portio data. Quia igitur d, ad h, siue ad e, a, sicut z, b, ad b, e, erit, quód ex ductu d, in c, b, aequale ei, quod ex ductu a, e in b, z. ergo quod ex ductu d, in c,

–

[8r]

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b, bis aequale ei quod ex ductu a, e, in z, b, bis, et hoc est in totum a, c, ergo quod es d, in c, b, bis cum quadrato e, b, est aequale ei, quod ex a, e. in a, c, cum quadrato c, b, sed quod ex a, e, in a, c, cum quadrato c, b, ualent quadratum a, b, per primam, et quartam secundi Euclidis, in materijs igitur quod ex ductu d, in c, b, bis cum quadrato c, b, ualent quadratum, a, b, sed quod ex ductu d, in c, b, bis cum quadrato c, b, est, quoddam datum cum d, et c, b, sint data ergo quadratum a, b, est datum: ergo eius radix, scilicet a, b, est data, cum sit datum quod fit ex d, in b, c, erit et quod ex z, b, in e, a, datum. quare et quod ex z, b, m, z, e, quorum cum sit differentia data, erit utrunque eorum datum: sicque tota a, b, c. data hoc opus est, ut ei quod fit ex d, in b, c, bis addatur quadratum b, c, et compositi radix erit a, b. In hac non ponderandi ratione hic incidunt generalia, scilicet quód quadratum d, c, b, est tanquam quadratum d, et quadratum b, a. Quod enim fit ex d, in c, b, bis est quadratum, quod ex tota c, a, in ea, quare ex d, in c, b, bis cum quadrato c, b, est quantum quadratum b, a. Quadratum ergo d, c, b, ut quadrata d, et b, a, amplius quod fit ex d, c, h, in c, b. bis est, ut quadratum c, b, et quadratum b, a, quod enim fit ex d, in c, b, bis cum quadrato c, b, est, ut quadratum b, a, quare quod est d, in c, b, bis cum quadrato c, b, bis et hoc est quod fit ex d, c, b, in c, b, bis erit, ut quadrata b, a, et b, c. amplius quadratum d, c, b, et quod fit ex d, c, b, in c, b, a, bis est, ut quadrata c, b, a, et d, b, a, erit h, quadratum d, c, b, et quod fit bis ex d, c, b, in c, b, tamquám quadrata d, et b, a, et b, a, et b, e, et tunc fit bis, ex d, c, b, in b, a, est ut quod est, d, atque c, b, in b, a, bis, et sic patet, quod dicitur.

Quaestio Quartadecima.

Quod si pondus datum sit, et pars opposita, data similiter omnia data erunt.

Eadem ubique depositio, et d, atque b, a, data sunt, et quadrata eorum coniuncta data erunt, quae sunt, ut quadratum d, c, b, cuius radix quae est d, c, b, data erit. dempto ergo d, relinquitur c, b, datum, et sic ota a, b, c, data erit.

[8v]

OPUSCULUM DE

Quaestio Quintadecima.

Si responsa dati fuerit ponderis, et pondus appensum cum parte, in qua dependet fecerit quod datum, utrunque eorum datum erit.

Erit enim datum quadratum d, c, b, cum eo quod fit ex ipso in c, b, a, b, a, bis. de quibus dempto quadrato a, b, c, relinquitur quadratum d, b, a, datum erit ergo d, b, a, datur et ipsius ad d, c, b, differentiam data, quae est differentia a, b, ad b, c, sicque utrunque erit datum. Et similiter d, eadem ratione, si data a, b, c, fuerit d, b, a, datur erunt omnia data: quia enim quadrata a, b, c, et d, b, a, sunt, ut quadratum d, b, c, et quod fit ex ipso in a, b, c, bis, erit quadratum d, a, b, cum duplo quadrati a, b, c, tanquam quadratum compositi ex a, b, c, et d, b, c, quod cum sit datum, et a, b, c, datum erit, et d, b, c, datum, sicque ut prius b, a, et b, c, et d, data amplius scilicet d, c, b, et d, b, a, data non autem a, b, c, erit quoque et ipsa data, et singula data, quum sit enim quadratum d, b, c, ut quadratum d, et quadratum b, a, detracto eo de quadrato d, b, a. relinquitur, quod fit ex d, in b, a, bis datum,quare utrunque datum.

Quaestio Sextadecima.

Si brachia librae fuerint data pondere, et breuius in duo secetur similiter data, et a sectione pondus dependeat quod libram inaequalitate componat, ipsum quoque datum esse demonstrabitur.

Sint brachia librae ut prius a, b, longius b, c, breuius quod secetur in e, dependeatque pondus d, quod libram inaequalitate conseruet, dependeat autem et a, quum pondus h, quidem operetur. Quia igitur tam h, quám d, cum c, b, ponderat ut b, a, dempto b, c, aequale erit d, in pondere ad h, in

hoc

[9r]

P O N D E R O S I D A T E.

hoc situ. sicut igitur b, c, ad b, e, et d, ad h. quumque sit h, datum, et d, datum erit. Amplius et si d, datum esset, atque c, e, et c, b, data fierent b, a, et a, c, data. Sicut etiam b, c, ad b, e, et d, ad h, in eadem proportione. quare h, datum ob hoc etiam b, a, data erit. Similiter ratione, si d, pondus fuerit datum, et a, b. et b, c, data erunt b, e, et, c, e, data. quia enim a, b, et b, c, data sunt, erit et h, datum. atque sicut d, ad h, ita c, b, ad b, e, quare b, e, datum erit.

Quaestio Decimaseptima.

Quod si a breuiore duo dependeant pondera, alterum termino, alterum a sectione, quae regulam in aequedistantiam conseruent, compositumque ex ipsis datum sit singulis Responsae sectionibus existentibus datis, utroque appensorum data erunt.

Int ut solent brachia librae data a, b, b, c, et sectiones datae b, e, e, c, et ponderantia h, et d, sitque y. aequale d, ut sit totum h, y, datum. Sit tunc t, pondus, quod dependens a, c, aequalitatem faciat, cuius ad h, y, differentia data sit z, et quia t, est in pondere, ut h, d, h,y, erit maius pondere quam h, et d, quantum est z, ergo y tantum est pondere, quantum d, et z, sed y, ad d, in pondere est, si(−) cut b, c, ad b, e, ergo y, ad z, sicut b, c, ad e, c, et quia z, datum erit, et y, datum similiter. hoc amplius si h, et d, data, atque c, e, et e, b, erit et b, a, datum. quia enim t, ad z. sicut b, e, ad c, e, erit z, datum. Sitque t, atque a, b, data. Amplius si h, et d, data, rationeque a, b, et b, c, erunt b, e, et e,c, data. quia enim a, b, et b, c, data erit t, datum. et ob hoc z, et quia b, c, ad c, e, sic d, ad z, erit c, e, datum. Amplius simili de causa si b, a, et b, c, data atque b, e, et c, e. sitque d, datum, siue h, siue differentia eorum, siue proportio, omnia data erunt.

Quaestio Decimaoctaua.

Si sectiones librae sunt adinuicem datae, pondusque datum in

C

[9v]

OPUSCULUM DE

termine breuioris, siue in sectione dependens, uel etiam duo pondera data alterum in termino, alterum insectione appensa, regulam in aequedistantiam constituant, ipsa quoque in pondere data erit.

Esto ut prius regula a, b, c, sitque a, b, ad c, b, datur in proportione appendaturque pondus d, elatum aequabiliter ex parte c, duo ergo a, b, c, datam esse in pondere. Ponatur enim ipsa alicuius noti ponderis quod diuidatur secundum proportionem a, b, a, d, et c, b, ponaturque maius a, b, et minus e, b, et secundum hoc inuenietur pondus d. sicut ergo se habet pondus d, prius sumptum ad posterius sumptum, ita se habebit pondus a, b, c, ad pondus positum. Si enim maius, uel minus, et t, similiter maius, uel minus quám positum est, erit quód si, d, in e dependeat, et data sit c, b, ad e, b, datum erit, et t, aequaliter pendens a, c, quód si d, et h, data sint, similiter et t, datum erit. quod quoniam datum est, datum erit pondus a, b, c. Commentum respicit prius schema praecedentis propositionis.

Quaestio Decimanona.

Si responsa dati ponderis per inaequalia diuidatur, et alter minus ipsius data pondera appendantur, quae in aequalitate consistant, brachia quoque librae a centro, examinis data erunt.

Verbi gratia, dependeat ex a pondus d, et a, c, pondus utrunque et sit b, z, aequalis b, c, et diui

so

[10r]

PONDEROSIDATE.

so z. a, per aequalia apud t, descendat h, y, quod similiter in pondere respondeat e, sitque y, tanquam a, t, z. eritque proportio e, ad h. y, sicut c, b, ad b, c, et permutatim e, ad c. sicut y, h. siue h, cum a, z, ad b, c. quare sicut e, cum c, b, ad c, b, ita h, cum b, a. ad b, c. Itemque h, ad d, sicut a, b. ad c, h. erit ad a, b, sicut d, ad c, b. Itaque d, et c ,b, ad c, b, sicut h, et a, b. Igitur e, cum c, b, ad d. sicut cum c, b, sicut a, b, ad b, c, et coniunctim sicut e, d, cum a, b, c, aeque quae est dupla c, b, ad d, cum c, b,. Ita tota a, b, c, ad a, b, c. Si ergo a, b, c, ducatur in d, et c, b, perductum diuidatur per d, e, et a, b, c, simul exibit b, c, data. Amplius si data a, b, c, fuerint a, b. et b, c, datae, et totum d, e, datum, et d, et c. erit datum. Amplius si illis datis fuerint, uel d, uel e, datum, erit reliquum datum. Amplius si d, et e, data sint, et proportio a, b, et b, c, data, erit tota a, b, c, data. Quia enim e, cum c, b, est data ad d. cum c, b, quoniam sicut a, b, ad b, c, et quia d, et e. data sunt, erit et c, b. atque a, b, c, tota data. Amplius si datum a, b, et b, c, fuerit proportio e, ad d. data erit, utrunque eorum datum.

Quaestio Vigesima.

Si uero a sectione unius brachii pondus datum appendatur, quod alicui dato, et a termino alterius dependenti in pondere aequentur altera sectionum librae data, reliqua data erit.

Haec habentur ex praemissa, quia mutua est inter pondera, et remotiones proportio. Diuisiones quoque huius plures sunt ueluti in praemissa.

Quaestio Vigesimaprima.

Quod si a termino, et a sectione unius brachii duo pondera data dependeant, quae tertio in termino alterius in aequalitate respondeant sectionibus regulae datis, illud tertium datum erit.

[10v]

OPUSCULUM DE

Ab a, t, quae est sectio a, b. dependeat d, et 3. et a, c, dependeat e, h, 1. penderetque e ut v. et h, ut 3. et b, 1, cum b, e, quantum a, b. eritque singulum eorum datum, quare totum datum. Amplius si e, h, 1. datum est, proportio v. ad 3. data, quodlibet eorum datum erit, dependeat ex a, d, g. quód in pondere respondeat ad e, h, 1. proportio igitur ad 3. data, atque 3. ad d, quare g, ad v. quumque g, s, sit datum, erit utrunque datum, et 3. datum. Aliae quoque plures diuisiones intercidunt.

Quaestio Vigesimasecunda.

Si duo pondera alterum in termino, alterum in sectione longioris brachii suspensa duobus datis ponderibus, et a termino breuioris dimissis in pondere aequentur, locis suis alternatis, singula eorum data erunt.

Vt si d, ab a, et 3. a, t, suspensa sint. dimissum itaque 3. ad a, et d, a, t, respondeant h, in i, pondere tunc sumptis aequalibus d, et 3. quae sint m, et n, pendeat m, cum 3. in t, et n, cum d, in a, ponderabunt simul quanto c, h, quod quum sit datum, et d, n, aequale in 3. erunt ipsa data, sicque et d, et 3. datum erit.

Quaestio Vigesimatertia.

Si supra regulam in perpendiculo centro motus posito quantumlibet pondus utralibet parte dependeat non erit possibile illud usque ad directum centri descendere.

V erbi

[11r]

P O N D E R O S I D A T E.

Verbi gratia. Sit responsa a, b, c, perpendiculum b, u, e, centrum d, et sit a, pondus maius, quám c, ducantur ergo lineae d, a, d, e, et pertranseat d, a, a, 3,. donec sit d, a, 3, ad d, a, tamquam a pondus ad c, sitque , 3, ponderet ut c. Quia igitur tria pondera a, c, 3, sic dependent in a, b, c, atque reuolutio eorum circa centrum d, quare essent in lineis d, a, 3, et d, c, sed positis ita ipsis tantum uellet 3, distare a directo d, quantum , et c, distabit quoque et a, proportionaliter a directo eiusdem non ergo ad directum quum poterit pertingere.

Quaestio Vigesimaquarta.

Quum sit igitur distantia centri a medio. Responsae ad longitudinem ipsius data ponderaque appensa ad pondus regulae data erit perpendiculi declinatio data.

Sit regula, quae directum determinat h, d, l, 3, et c. ut prius, declinetque regula ex parte a, donec linea h, d, l, 3, secet in l, quasi ergo centrum exanimis esset in l, sicut sita est. Responsa quum ergo sine pondera data, et regula, erunt sectiones. Responsae quae sunt a, l, l, c, datae quasi longitudo utriusque ad b, d, data erit similiter et l, b, quia etiam angulus l, d, b, datus erit, et est ut angulus c, u, h, et ipsa est declinatio perpendiculi a directo data.

[11v]

OPUSCULUM DE

Quaestio Uigesimaquinta.

Si uero sub regula centrum designetur, uix continget in hoc situ stabiliri pondera. Sit Responsa ut prius a, b, c, et perpendiculum d, b, e, sitque e, centrum sub Responsa, et pondera a, et c, ductis igitur lineis e, a, e, c, quasi inde ipsis, sint, sic sita sunt pondera. ipsius igitur in hoc situ aeque ponderantibus si fiat qualitercunque nutus in alterutra partium ueluti in a, crescet ex parte a, portio. Responsae usque ad rectitudinem quae signeretur h, l, 3, ut sit communis sectio ipsius, et regulae in l, sicque grauius reddetur continue donec circumuoluatur regula sub e.

Quaestio Uigesimasexta.

Possibile est igitur Responsa aeque distantis collocata quantumlibet pondus in alterutra parte suspendere, quae regular ab aequalitate non separet.

Sic regula a, b, c, centrum b, linea directionis d, b, e, sitque Responsa suo pondere in aequalitate sita. Sumatur igitur alia Responsa aequalis grossiciei, et ponderis, quae sit h, t, 3, posito t, in eius medio, sitque portio regulae h, b, in utralibet parte minor longitudine quam sit h, t, et pendeat regula h, t, 3, ab h, fixa ut t, sit in directo sub b, secta a linea directionis in t, dico ergo ipsa ita dependens non faciet mutare literam, sita est enim quasi si traheretur linea b, 3, et in ipsa linea b, h, dependeret omnesque partes eius aequaliter a, t, distantes aeque ponderarent, distant enim aequaliter a linea directionis, quia t, 3, ponderant, quantum b, t, t, h, non ergo fiet nutus, sed et super hoc si quolibet pondus suspendatur a, t, non faciet, hinc uel inde nutum.

Quaestio

[12r]

P O N D E R O S I D A T E.

Quaestio Vigesimaseptima.

Quolibet ponderoso ab aequalitate ad directionem eleuato secundum mensuram substinentis in omni positione pondus ipsius determinari est possibile.

Sit a, b, ponderosum, et sit ubique aequaliter ponderis situm aequaliter et fixo b, eleuetur in a, donec directum sit c, b, mota a, quae suo describat quartam circuli ab a, in c, sitque situs aequalitatis primus directionis dicatur ultimus, et quando diuidit arcum a, c, per aequalia, sic ipsa b, d, et situs medius, et quum eleuatum fuerit secundum mensurarum substinentis, sit b, e, et perpendicularis e, l, sit pro eleuante, et sit hic situs secundus. In situ uero .3. sit b, f, sitque arcus f, d, aequaliter d, e, dico igitur ipsum semper leuius fieri usque in f, aeque graue ut in e, et inde item semper leuius usque ad c, possibile alius leuius esse in a, quam in d, et grauius, et aeque graue pro quantitate e, l, sit enim g, h, aequaliter e, l, ut orthogonaliter erecta, donec contingat d, b, in h, et dimittatur d, k, recte super a, b. Si igitur g, fuerit in medio a, b, tunc g, h, aequum erit eius dimidio, scilicet dimidio a, b, quia é aequale g, b, quum sit d, b, in d, ad pondus a, b, sicut linea b, k, ad b, a, atque pondus eius in d, ad pondus eius in h, ut b, g, ad b, k, quum sit b, g, ad b, k, sicut b, k, ad b, a, quia sunt consequenter proportionali erit pondus d, b, in h, tanquam pondus a, b, quia habent eadem proportionem ad pondus d, b, in a, quod si g, sit uersus b, erit in h, maius pondus, quam in a, si uero uersus a minus sit, item in u, perpendicularis aequaliter e, l, quia b, k, haberet maior proportio ad b, g, quam ab ad b, k, et

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OPUSCULUM DE

ideo, et pondus in, h, ad pondus in d, contingens b, f, in e, u, m, transeatque linea e, u, p, et ducantur perpendiculares f, r, f, x, ad b, a, b, c. Quia igitur ponderis e, b, ad pondus f, b, ut l, b, ad r, b, siue x, b, ad p, b, a puncta f, et e, aequedistent (ex hypothesi) a punctis c, et a, siue a puncto d, pondusque f, b, in u, ad pondus eius in f, sicut f, b, ad u, b, siue r, b, ad m, b. Et quia x, p, ad p, b, sicut r, b, ad m, b, erit pondus e, b, ad pondus f, b, sicut pondus f, b, in u, pondus eius in f, tantum ergo est pondus e, b, in e, quám f, b, in u, quia figurae, a, b, p, est similis figurae, f, r, b, c, (quod facile probabis) et figura a, u, m, b, p, circa diametrum f, b, (per sextum Euclidis) erit similis eisdem. Ideo sicut b, l, ad b, r, sic b, r, ad b, m, et ideo sicut b, e, in e, ad pondus b, f, m, f, sic erit idem pondus f, b, in u, ad idem pondus f, b, in f, et ideo (per quintam Euclidis) pondera e, b, in e, et b, f, in u, erunt aequalia. Quod autem in e, sit leuius, quám in h, probatur quia d, h, est longior, et est etiam d, r, maior, quám e, z, et angulus b, e, 3, minor angulo u, k, z.

Quaestio Uigesimaoctaua.

Mundus non in medio descendens breuiorem partem secundum proportionem longioris ad ipsam grauitatem redditur.

In, quo suspenditur sit a, b, c, et pondus e. Diuidatur autem e, in d, ac f, ut sit d, ad f, sicut a, b, ad b, c. Si igitur suspenditur d, in c, et f, in a,tanti ponderis quodlibet eorum, quanti e, intellecto quód in opposita, sit quasi centrum librae. substinentibus igitur in a, et c, pondus c, dependens a, b, erit grauitas in a, ad grauitatem c, sicut c, b, ad b, a.

Quaestio

[13r]

PONDEROSIDATE.

Quaestio Vigesimanona.

Omne medium impedit motum.

Esto quód mouetur a, b, quod uero occurit medium sit t, ponaturque c, quasi instantia, quae sit t, e, d. Si igitur c, nullius fuit grauitatis si non impedit motum a, b, descendente quum impellatur ab ipso, cogetur discendere et sic erit ut grauitatem habens, poterit ergo descendens ex parte e, ad pondus ex parte d, attollere, aeque ergo constabat a descensu suo impellere d, quia attollens d, non impedietur a uelocitate sua, quod est impossibile. Quod sic ponderosum finite, si non mouetur quod ipsum impedit, habebit eam ab aqua tenus impedire, si mouetur, quum a, b, ipsum consequetur, erit a, b, grauius quo uelocius sitque 3, aequale a, b, in pondere, possibile igitur est 3, ex parte 3, positum motu c, descendere, et attollere ad pondus ex parte d, fietque tunc 3, in pondere ut c. si igitur a, b, non impeditur impellendo, non impedietur impellendo 3, similiter ergo quum moueantur a, b, et 3. motu naturali, non impediuntur in attollendo d, quod totum est impossibile.

Quaestio Trigesima.

Quo ponderosius est pro quod fit transitus, eo in transeundo difficilior fit descensus.

Huiuscemodi per quod fit transitus sunt aer et aqua, et alia liquida, quod igitur ponderosius est ipsium sit a, b, c, quod leuius sit d, e, f, quodque transit t, transiens autem per illa, offendat in b, et e. Est autem b, grauius, quám e. Quumque ad descendum impedia

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OPUSCULUM DE

ntur, et ipsa quum descendere habeant, stant, pluris est grauitatis quod impedit b, quám quód impedit c, quia autem t, habet, eodem offendendi impedimento, plus offendetur in b, similiter infra b, et e, aequaliter, si sursum pellatur, tardioris erit motus in b.

Quaestio Trigesimaprima.

Quod maius coheret, plus substinet.

Sit quod substinere habet a, b, c, et res descendens t, quae cadens offendat in b, ad hoc ergo, ut per transeat, habet a, b, saeparari a, b, c. Quo ergo cohaeret, uel plus substinebunt t, ut non moueantur ante operationem suam, uel si moueatur, plus habet e, a, secum trahere coniuncta. plus ergo impedient, et ideo prius.

Quaestio Trigesimasecunda.

In profundo magis est descensus tardior.

Sit profundum a, b, g, d, lineis conclusum, et partes, per quas sit descensus sine e, f, k, profundior e, partes collaterales e, b, et g, quanto igitur liquor est profundior, tanto inferiores partes plus comprimuntur, ut e, comprimitur enim et a superioribus et iuxta se positis. Quum enim liquida sint b, g, comprensa a superioribus nituntur undique, euadere. Coarctant ergo e, ita, ut si f, cederet exiret in locum superiorem. Vnde manifestum est, quód non solum e, sustinet f, sed nititur contra e, t, et e, o, magis f, contra k, minusque ideo f, repelleret k si in f, profunditas terminaretur. Tunc enim solidum suppositum substineret tantum f, et non niteretur contra magis igitur, quum impediatur descensus k, in hoc situ quód si minor esset profunditas, et e, magis impedietur.

Quaestio Trigesimatertia.

Altitudo maior minuit grauitatem.

Vt superiorem formam repetamus, dicimus in omni liquido quam libet partem inferiorem a qualibet superiori grauari, ut e non so

lum

[14r]

PONDEROSIDATE.

lum ab f, et k, sed ab a, et d. Quum enim non possit a, descendere i b, tendit et in e, quoniam liquidum est similiter, et f, ab b, omni superiori grauatur, eo quód amplius quanto a, b, latius. quanto igitur plus nititur contra. k, et ideo amplius tardabitur descensus t, tertium grauitatis minuetur.

Quaestio Trigesimaquarta.

Res grauior quo amplius descendit eo fit descendendo uelocior. In aere quidem magis in aqua minus, se habet enim aer ad omnes motus.

Res igitur grauis descendens primo motu trahet posteriora, et mouet proxima inferiora, et ipsa mota mouetur sequentia, ita ut illa mota grauitatem descendentem impediat minus. Vnde grauius efficitur, et cedentia amplius impelli, ita ut iam non impellantur, sed etiam trahant. Sicque fit, ut illius grauitas tractu illorum addiuuatur et motus eorum grauitate ipsius augeatur, unde et uelocitatem illius continue multiplicare constat.

Quaestio Trigesimaquinta.

Forma ponderosi mutat uirtutem ponderis.

Et enim si acutum, et strictum fuit, facilius pertransit, et hoc dicitur leuius enim separat, et sic fit leuius, minori etiam ostendit, minus quidem impeditur, et ob hoc etiam uelocius transit e, contra si obtusum est.

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OPUSCULUM DE

Quaestio Trigesimasexta.

Omne motum plus mouet.

Si quid ex impulsu moueatur, certum est quód impelletur si autem motu proprio descendat, quo plus mouetur, uelocius fit, et eo ponderosius ad quae plus impellit motum, quám sine motu, et quo plus mouetur, eo amplius.

Quaestio Trigesimaseptima.

Quod motum plus impedit plus impellitur.

Sit quod mouetur a, et quod plus impedit c, et quod minus b, sitque libra u, e, f, duoque pondera z, et t, sitque a, quasi in d, suspensum, atque in z, ab f, dependens, quum c, impediat omnino motum a, et t, cum b, patet, ergo quód e, t, quám b, minus, ergo a, t, adiuuat c, quám c, b, substinendum a, plus ergo grauatur c, pondere a, quám b, plus ergo impellitur.

Quaestio Trigesimaoctava.

Et grauius rei motae, et leuitas frustrare uidentur mouentis uirtutem.

Sic mouens a, b, et quod mouetur c, adeo ergo leue potest esse c, respectu uirtutis a, b, ut eam non impediat, et ita uix impelletur. adeo ergo graue, quod uirtuti impellentis non cedat, uel et ideo modicum mouebitur, uel nihil, utrobique ergo uidetur frustrata uirtus impellentis, quia non confert ad motum rei in rapisse uel parum.

Quaestio Trigesimanona.

Virtutem impellentis adiuuat circumactio ipsius, eó amplius, quó fuit longius.

Sit

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PONDEROSIDATE.

Sit quod motum est a, b, c, et motum e, si igitur impellat a, b, c, impellat e, in c, et moueatur a minus impellet, quám si figatur a. Ponderosius est enim c, in situ aequalitatis, quám si dimittatur a, ut ostensum est. Manete item a, plus impelletur e, in c, quám in b, quia grauius in c. Item circumactum c, manete a, plus impellet, quám utroque prius non moto. quia motum plus eó etiam maius, quó longius dicitur. fixo enim a, in centro circumacta b, et, c, describent arcus circulorum, et maiorem e. Quum ergo maius pondus in c, quám in b, et uelocius quoque motum multo amplius impelletur e, in c, quám in b, similiter etiam circumactum e, cum c, magis mouebitur, quám si c, motum prius offendat. Si iterum centrum alterius motus sit in b, ut c, b, t, circa ea: et iterum c, b, moueatur circa b, et augmentabitur uirtus impellendi pro duplici motu, quám aequali tempore multo maiori circumitur, feretur.

Quaestio Quadragesima.

Quod sustentatur in terminis circa medium, citius deprimi tur, et eo amplius si impellatur. et hoc secundum formam impellentis, et quantitatem ipsius fit plurimus.

Sit quod impellatur a, b, c, ipsum quoque si substineatur in a, et, c, plus habebit deprimi circa b, uel omnium substineat b, nisi continuitas ad alia, quam quidem quandoque substinet, quandoque non sufficit. Omnino etiam ex quo incipit descendere b, fit magis ponderosum, quám inimus incipit esse pondus, in a, et c, porro, quanto b, magis distat á terminis, magis ponderabit, quám ipsa sunt in centrum librae, quoniam subste–

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OPUSCULUM DE

ntatur prae longitudine. ergo contingit aggrauari medium, ut rumpatur antequam dirigatur. hoc autem magis contingit etiam b, impellitur, sicque duplicato pondere citius directo continuitatis b, cum a, et, c, soluitur, atque magis sit, si acutum fuerit impellens: magis enim impellet vnum, atque hoc etiam ut e, soliditas continuitatis, et ponderis, et impulsui non cedant, siquae substinent aliquatenus cedant persequutae eo, quod impelli soluatur, quoniam medium semper fit grauius. hoc etiam si inuentus termino substineatur, fit et si in altero, ut in a, quoniam si impellatur in b, quoniam grauius, fiet b, non equetur c, circunuolutionem b, et rumpetur continuitas. alioquin plus transiret c, quám b, quam si leuius esset minima soliditas in c, a.

Quaestio Quadragesimaprima.

Quum medium detinetur facilius extrema curuantur.

Sit ipsum a, b, c ,d, e, medium c, quod quum detineatur, extrema impellantur, quòniam motum eorum in partem, qua impelluntur non potest sequi, oportet curuari, quoniam directam habet solui nisi connexio soliditatis impediat. quae quidem minus perfecit in a, quám in b, et c, quám d, impulsa enim a, et e, quoniam medij connexione detineri habent scilicet b, et d, quum ipsa habilia sint ad sequendum, quum in se non detineantur, minus impedietur a, et e, continuitate ad c, sicque fit, ut quum extrema facilius cedant, in quo illis uiciuiora facilius sequantur, contingat totum curuari in circulum. quanto igitur longius a, c, e, tanto leuius extrema curuantur in eadem ratione, qua et remotiora á centro librae ponderosiora sunt, quoniam maiores arcus describunt eandem quoque: et in omnem partem magis sequentur impellentem, si non pondus ipsum impediat. Notum etiam quód super hoc quidem manente c, non magis impedit pondus a, quám pondus b, impellentem b, quoque ad ipsum pondus.

Quaestio Quadragesimasecunda.

Magis impulsum plus cohaeret.

Haec impulsio sit a posterioribus, quae impulsa habent anteriora perpellere. quae quoniam pondere suo aliquatenus resistunt, habent media constringi. Vnde quando in latus declinantur, hinc etiam contingit, quód inferiora superioribus infixa, uel depulsis infiguntur.

Quaestio

[16r]

P O N D E R O S I D A T E

Quaestio Quadragesimatertia.

Quod partes habet cohaerentes, si motu directe offendantur, redit directe.

Hoc quidem fieri habet per medium, in quo defertur, siue aer, siue aqua, et propter partium raritatem sit in quo defertur b, idest aer, siue aqua, et materiam a, in quo offendit c. Quia ergo a, mouet b, quum recedat a, de e, loco suo, et impellat b, de loco suo, oportet ut ad supplendum

loca posteri. reciperetur b, vnde eodem impulsu et permouetur, et retorquetur eo amplius quum offendat a, in c, quumque b, nequeat procedere pondere imminentis constructum ponderosus refertur, et cum impetus a, refractus sit in c, et ponderet solo iam inuitatur. habet retrahi motum b, nisi pondus eius praeualeat, et directe. quia in omnes partes aequaliter recedit b. Raritas uero partium hoc idem operatur, quoniam priores partes a, quum prius offendantur in e, urgentur mole, et impetu posteriorum, et cedunt in se, sicque deluso impetu redeuntes in locum suum, alias repelluntur recedendo, separabiles sunt partes constrictae, hinc, inde resiliunt.

Si quidem aliquod quo amplius continue demissum descendit, tantum in priori perstrictus efficiatur.

Exitus per quod egreditur a, b, et per prima pars c, quod quum descen

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OPUSCULUM DE

derit ad f, sit e, in exitu. Item quum c, fuerit in u, fit f, e, in 3. quare ergo quo plus descenderit, ponderosius erit c, ponderosius in u, f, quám in a,b. Quia uero dum e, peruenit in u, f, pertingit c, in 3. t, longius erit a, f, quám f, 3. quia gracilius continue, quia partes uelociores, et sic tandem adrumpuntur.

Si res inaequalis ponderis in partem quamcunque impellantur, pars grauior occupabit.

Sit quod impellit a, b, pars grauior a. Si ergo impellatur ex parte a, et b, impellatur, quoniam leuius est, facilius cedet pulsui. quumque facilitatem eius non sequatur a, frustrabitur quidem in se, et grauitate a, adiuuabit; sicque totus uisus reuertetur ad a, habet ergo praecedere in suo impetu trahere b. Si uero b, posterius impellatur, et praecedat a, impulsum quidem b, impellet a, leuitas 3. attrectabitur mouendo a, et ideo prius impelletur a, quia motum ipsius plus impedit, totoque conatu in plurium habebit trahere b, ea finiter liber Ioradam de ratione ponderis.

Et sic finit.

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[1] Tartaglia is very probably referring to Leonico Tomeo's edition 1525 (Aristotle 1525).

[2] Notice the attribution to Aristotle of the use of mathematics. This is coherent with the medieval vision of mixed–sciences for which theoretical mechanics was a mixing of physics and mathematics.

[3] Tartaglia's reference to a "natural philosopher" implies empirical observations only.

[4] "In primis igitur quae accidunt circa libram dubitare faciunt, quae nam ob causam exactiores minoribus maiores sunt librae". (Aristotle 1525, 25. See also *Problemata mechanica* 848b in Aristotle 1955c, 848b, 337).

[5] Facts resulting from empirical evidence cannot disprove theoretical proofs; the discrepancy depends on some aspects of the matter being modelled improperly.

[6] "All the surfaces of equidistant sides which are about the diameter of a parallelogram are similar to the whole parallelogram". "Tutte le superficie de equidistanti lati che stanno intorno al diametro de ogni parallelogrammo sono simile a tutto il parallelogrammo" (Tartaglia 2007, 118r). Tartaglia probably refers to the rule of parallelogram used by Aristotle in problem 1 of *Problemata mechanica*. See above Chapter 3.

[7] Aristotele 1525, 30.

[8] Aristotele 1525, 30.

[9] The liberal arts were those of trivium (grammar, rhetoric, logic) and of quadrivium (arithmetic, geometry, music, astronomy).

[10] A subordinate science was a science that needed another science to explain the phenomena concerning it. Aristotelians of the XVI century considered at least two of the liberal arts, i.e. music and astronomy, as sciences subordinated to mathematics. Consequently, they would not have considered the liberal arts as proper sciences, and Tartaglia's contraposition between them and the science of weights would have no sense.

[11] Tartaglia suggests that the definitions he is introducing are shared definitions. Indeed most of them are. We will indicate in the footnotes the sources.

[12] According to the Aristotelian epistemology.

[13] "Bodies equal in volume are those which fill equal spaces" (*Liber Euclidis de ponderoso et levi* (Moody and Clagett 1952, 27). For example: *grandezza* (*size*) is identified with volume, thus in the following we will translate *grandezza* with volume.

[14] "And those which fill unequal places are said to be of different volume" (*Liber Euclidis de ponderoso et levi* in: Moody and Clagett 1952, 27).

[15] Cfr.: de Nemore1565, *Questio prima*, 3r.

[16] "Bodies are equal in forces, whose motions through equal places, in the same air or the same water, are equal in times" (*Liber Euclidis de ponderoso et levi*, Moody and Clagett 1952, 27).

[17] "And those which traverse equal places in different times, are said to be different in force" (*Liber Euclidis de ponderoso et levi*, Moody and Clagett 1952, 27).

[18] This definition comes both from *Liber Euclidis de ponderoso et levi* (Moody and Clagett 1952, 27): "And that which is greater in its force, it is the lesser in its time" and "What is heavier descends more quickly" (de Nemore 1565, 3r). However, it differs from them because it explicitly refers to different bodies (presumably bodies with different weight). In such a way, the strength of a body (virtus) is identified with its speed and is independent of its weight. Which is in contrast with *Petition* II (See above Chapter 3).

[19] "Bodies are of same kind which, if of equal volume, are of equal force" (*Liber Euclidis de ponderoso et levi*, Moody and Clagett 1952, 27).

[20] We can identify simple heaviness with weight avoiding an additional analyses concerning a modern term, force–weight.

[21] "Of two bodies equal in volume the one whose weight is equal to that of a greater number of *calculi* is of greater specific gravity (gravius esse in specie)" (*Liber archimedis de ponderibus*, Moody and Clagett 1952, 43). We note that a *calculus* is the least measure of weight.

[22] de Nemore 1565, *Questio prima*, 3r. The definition of obliquity is the classical on in the science of weights: a line is more oblique when it makes a greater angle with the line of descent. Tartaglia maintains quite an ambiguity about the directions of lines of descent of heavy bodies. In general, statements (as for example see *Petition* I) he says the lines of descent are toward the centre of the

word; but in the proofs of his entire proposition, he assumes parallel (and vertical) lines of descent.

[23] "A weight is known, when the number of its calculi is known" (*Liber archimedis de ponderibus*, Moody and Clagett 1952, 41). Note: a *calculus* is the least measure of weight.

[24] Cfr.: de Nemore 1565, *Questio prima*, 3r.

[25] Cfr.: de Nemore 1565, *Questio prima*, 3r.

[26] Cfr.: de Nemore 1565, *Questio prima* 3r. I note Tartaglia's reference to lines of descent converging toward the centre of the world.

[27] Cfr.: de Nemore 1565, 3r. Mention of the balance is important because it allows me to frame the problem of descent of weight into a physical and mental model very known and studied, which also make easier possible reference to experience.

[28] Tartaglia is saying that his is a mathematical (*ideal*) model. The presence of small cups to sustain weights has no relevance as all goes with weights hung directly from the scale.

[29] Cfr.: de Nemore 1565, *Questio prima*, 3r.

[30] Note that Tartaglia will give a mathematical definition of obliquity, only at the end of *Book VIII*.

[31] Cfr.: de Nemore 1565, *Questio prima*, 3r.

[32] Cfr.: de Nemore 1565, *Questio prima* 3r. Differently from de Nemore however there is the explicit reference to a balance, where the lowering of a weight causes the raising of the other.

[33] Cfr.: de Nemore 1565, *Questio prima*, 3r.

[34] This is an assumption about the additivity of the power.

[35] "If two equal quantities will be compared to any other quantity, they will have the same ratio. And similarly the ratio of that to those will be the same" ["Se due quantità equale saranno comparate a quale si voglia quantità, di quelle sera una medesima proportione, e simillmente da quella a quelle sera una medesima proportione"] (Tartaglia 2007, 94v).

[36] "Let there be four quantities, of which the ratio of the first plus the second is greater than the ratio of the third plus the fourth to the fourth, then, conversely, the ratio of the first plus the second to the first will be lower than the ratio of the third and fourth to the fourth". ["Se saranno quattro quantità, della quale la prima e seconda alla seconda sia maggiore proportione che della terza e quarta alla quarta, sera eversamente minor proportione che della prima e seconda alla prima che della terza e quarta alla terza"] (Tartaglia 2007, 104r–104v). Let me use letters and assume A, B, C, D the four quantities in the order. The proposition says that if $(A+B) : B > (C+D) : D$, then $(A+B) : A < (C+D) : C$.

[37] Tartaglia 2007, 94v.

[38] Tartaglia 2007, 104rv.

[39] This proof is similar to that of *Proposition* I.

[40] The second part of the corollary, i.e., that speed is proportional to volume, follows from *Proposition* I and *Proposition* II by means the transitive property.

[41] Tartaglia 2007, 94v.

[42] Tartaglia 2007, 104r–104v.

[43] This proof is similar to that of *Proposition* I.

[44] Tartaglia 2007, 94v.

[45] Tartaglia 2007, 104rv.

[46] This proof is similar to that of *Proposition* I.

[47] Here Tartaglia seems to state a trivial theorem of geometry, for which the length of paths of points in a radius of a circle are proportional to the radius. He is probably comparing the path with the power, reconnecting to Jordanus de Nemore's weak form of the virtual work law.

[48] Cfr.: de Nemore 1565, *Quaestio secunda*, 3v.

[49] See above Chapter 3.

[50] Notice that Tartaglia, following Jordanus de Nemore, proves equilibrium for a balance with equal arms and weights and does not assume it as a postulate in the wake of Archimedes's theory of balance.

[51] Tartaglia assumes as a fact of nature that the balance returns to its first position (which is not generally true in effect) and want to explain this fact (the *why*) by means of mathematics.

[52] See above Chapter 3.

[53] Curious expression. In fact, Tartaglia is considering vertical parallel lines.

[54] Notice that Tartaglia is associating an angle – that which the path (curvilinear indeed) of descent forms with the vertical – with obliquity. I also note that the obliquity of the path *bf* is measured by the contingency angle *dbf* though the verical *bd* crosses the path *bf*.

[55] Angle between two curved lines or a curved line and its tangent.

[56] "If from the one end of a diameter of a circumference a line perpendicular to it is drawn, it is necessary that it be external to the circumference and it is impossible that another straight line be contained between the circumference and the perpendicular. And the angle between the said straight line and the circumference is more acute [i.e. lower] than any other acute angle of any two straight lines [contained between the diameter and its perpendicular]; and the angle internal to the diameter is greater of the all acute angle of such lines". ["Se dall'un di termini del diametro de alcun cerchio sera dutta othogonalmente una linea retta le necessario che quella cada di fuora del detto cerchio, e fra quella e il cerchio le impossibile che gli possa capire altra linea retta. E l'angolo contenuto de quella, e dalla circonferentia è più acuto de tutti gli angoli acuti contenuti da line rette e l'angolo fatto di dentro dal diametro, e dalla circonferenza è maggiore de tutti li angoli acuti contenuti da tali rette"] (Tartaglia 2007, 59r).

[57] In this part, considerations about the difference of behaviour of mathematical and real balances developed in *Book VII* are repeated.

[58] Cfr.: de Nemore 1565, *Quaestio quinta*, 4v.

[59] Here Tartaglia is stressing that physics is subalternate to mathematics inmechanics.

[60] Cfr.: de Nemore 1565, *Quaestio sexta*, 5r.

[61] We considered of interest only Euclid I.29: "If a straight line will cross two parallel lines the alternate angles will be equal". ["Se una linea retta caderà sopra

due linee equidistante, li doi angoli coalterni seranno equali […]”] (Tartaglia 2007, 30r).

[62] “Of two triangles with equal angles the sides opposite to the corresponding angles are equal”. [“D’ogni triangoli di quali li angoli dell’un a li angoli di l’altro son equali li lati che riguardano li angoli uguali sono proportionali”] (Tartaglia 2007, 108v).

[63] Here Tartaglia resumes Jordanus de Nemore’s reasoning (de Nemore 1565, 5rv), which is useless for him; he could have finished his proof more clearly by observing that body *d* in *a* is equally as heavy as body *e* in *b*.

[64] When *Quesiti et invention diverse* was published Tartaglia had already edited Archimedes’ work (Tartaglia 1543).

[65] Tartaglia is conscious he is moving in a different tradition than Archimedes’.

[66] Cfr.: de Nemore 1565, *Quaestio settima*, 5v.

[67] The “other way” is to use the concept of the gravity of position. To use the other way Tartaglia changes his model. The horizontal bar is replaced by two equal weights located at its extremity. However he fails to notice that he is using an Archimedean approach to do this; so his method is not fully *other*.

[68] de Nemore 1565 *Quaestio undecima*, 7r. Now Jordanus de Nemore’s proposition does not contain the part in italic. This part is however contained in the body of the proof. See above Chapter 3.

[69] “If some quantities will be divided equally by a multiple, the ratio of the submultiple will be the same” [“Se ad alcune quantità saranno tolti li multipli equalmente, la proportione di multiplici, e quella di submultiplice serà una medesima”] (Tartaglia 2007, 88v).

[70] “Those ratios which will equal a given ratio will be equal” [“Quelle proportioni che a una medesima proportione seranno equali eglie necessario che tra loro siano equale”] (Tartaglia 2007, 96v).

[71] Tartaglia 2007, 96r.

[72] Cfr.: de Nemore 1565 *Quaestio duodecima*, 7v.

[73] “Consider four proportional number [*abcd*], the product of the first with the last [*ad*] will be equal to the product of the second and third [*bc*]. But if the product of the first and last equals that of the second and third the four numbers will be proportional”. [“Se seranno quattro numeri proportionali quello che vien produtto dal primo in l’ultimo, sera uguale a quello che vien produtto dal dutto del secondo in el terzo. Ma se quello che è produtto dal primo in el ultimo è uguale a quello che è produtto dal secondo nel terzo quelli quattro numeri sono proportionali”] (Tartaglia 2007, 144r).

[74] For example, something to be measured in feet; an unknown denoted below as “co”, from the Italian *cose* (things). See above Chapter 3, footnote 186

[75] Tartaglia 2007, 144r.

[76] By indicating co with x, the equation Tartaglia is solving is $160\ x = 400\ -80\ x$, which gives $x = 5/3= 1+2/3$. Note the use of fractions.

[77] Tartaglia considers the vertical in A as parallel to the verticals in D, E, G, etc. I.e. he assumes the lines of descent as parallel to each other.

[78] Here Tartaglia assumes that the obliquity is measured by the ratio of the length to the height of the inclined plane. This actually is the correct choice, but he gives no justification for that.

[79] Neologism.

[80] Inversamente.

[81] From “cercina”: pair of compasses.

[82] The performance is mine.

[83] From “Libre”: pounds.

[84] Ratio.

[85] For example, something to be measured in feet; an unknown denoted below as “cosa”, “cose” (thing, things) or more simply “co”. See above Chapter 3, footnote 186.

[86] Libre.

[87] Tartaglia 1543, 104v, 105r.

[88] Note the label, maybe by the editor Troiano or Tartgalia’s himself, to distinguish figures drawn by Tartaglia.

[89] In the original drawing instead of “*b*” is erroneously reported “*a*”.

[90] They are contingency angles, and as such both of them are different from zero but less then any positive number.

[91] This figure is repeted twice. The second with no reference to Tartaglia, is not reported here.

[92] Of this figure only the left part is commented upon in the text.

[93] This figure, indicated in the text as drawn by Niccolò, is less complete and accurate than that which refers no indication (Fig. 4.30). Consequently the latter has been commented here.

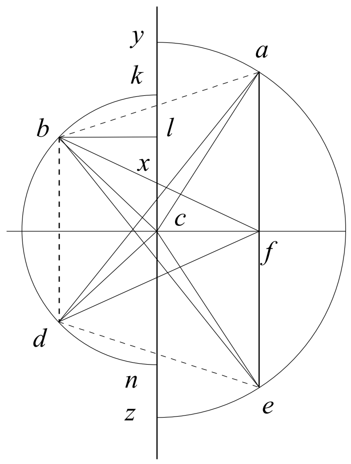
[94] As x and l cannot be equal, as clear from the text, Jordanus de Nemore instead of “halves” would have had to write, more generically, “parts”.

[95] Modern notation. With reference to the figure above, which in Tartaglia’s book follows Figure 8, from the equilibrium of the lever the two proportions can be written: $x : z = bc : ce$ and $l : y = bc : ca$. By adding the two proportions we obtain: $x + l : (z + y) = 2bc : (ca + ce)$ and because $x + l = xl$, $ca + ce = 2cb$ by assumption, it is obtained: $xl = z + y$.

[96] Tartaglia’s (or better probably Curtio Troiano’s) arranging of figures is not very clear. In the body of the text there is only the drawing represented on Fig. 4.31. But at the end of *Opuscoli Jordanus de ponderositate* (de Nemore 1565, 17rv) two drawings like that of Fig. 4.32 are added (probably by Troiano), but with a bad lettering; only the letters underlined in Fig. 4.32 are reported. Also the drawing is incomplete; the dashed lines are missing; see also (Clagett and Moody, 1952, p 186).

[97] There are no similar triangles. The conclusion is however correct.

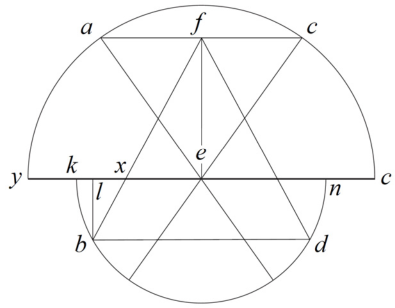
[98] de Nemore 1565, 6r. To make the reading easier, the figure is redrawn below for 90 degrees clockwise rotation:



[99] Jordanus de Nemore's alternative proof is scarcely commented into the literature, probably because it is not very clear being too hasty and probably incorrect. The original system of beams centred about *c* seems to be ideally replaced by some other beams. For example *bxf* is considered as a beam centred in *x*. The weight at *f* and *b* are said to be equilibrated because *x* is in the middle of *bxf*. How can de Nemore make this affirmation? Is he applying the law of moments, which he knows though limited to vertical loads? (He had applied it at least in another part of the *De ratione ponderis* (de Nemore 1565, 10v).

[100] This is an evident case, which shows the role played by Tartaglia within de Nemore's work: he declared what he did with respect to *Iordani opuscolum*'s text.

[101] In order to be choerent with de Nemore's reasoining in the text (de Nemore1565, 17v), the figure in the running text has been corrected in some parts. The original figure is:



[102] I do not believe I have understood de Nemore's reasoning, a part from that he recognized an unstable configuration of equilibrium, which involves, in the case of disturbance, a complete reversal of the system.

[103] Francesco Labia. To him is dedicated also the *Euclide Megarense*, 1565 edition.

[104] In the printed text: "c, i, h" instead of "c in h".

[105] In the printed text “a” instead of “c”.
[106] In the printed text: “b, h” instead of “bg”.
[107] In the printed text: “t” instead of “c”.
[108] In the printed text: “x” instead of “y” or “z”.
[109] In the printed text: “b” instead of “g”.
[110] In the printed text: “b” instead of “g”.
[111] In the printed text: “b” instead of “f”.
[112] In the printed text: “b” instead of “f”.
[113] In the printed text: “b” instead of “f”.
[114] In the printed text: “b” instead of “g”.
[115] “fit tunc bc” has not meaning for us and has been omitted in the traslation.
[116] In the printed text: “m” instead of “k”.
[117] In the printed text: “i, k, e, n, z” instead of “y, k, c, n, z”.
[118] In the printed text: “b, l, d” instead of “b, d”.
[119] In the printed text: “e, f” instead of “c, f”.
[120] In the printed text: “e, b, c, a, d, c, e” instead of “c, b, c, a, c, e”.
[121] In the printed text: “e, f, h, g, l” instead of “e, k, h, g, l”.
[122] In the printed text: “r, e” instead of “e”.
[123] In the printed text: “m” instead of “h”.

PART IV

Circulation of Knowledge & Conclusion

Chapter 5

Foreign Editions of *Quesiti et inventioni diverse*

In this section, I present the results of an historical archive research. It has been finalized to list, as far as possible, the main *Quesiti*'s foreign editions published in the history of science. I also list some uncertain dates and alleged editions cited in the history of science archives. In some cases I do not yet have historical proofs of some quotations. My apologies for any relevant items that may be missing.

5 General Considerations

In between the 16th and 17th centuries, publications concerning scientific works were produced mainly in Latin. Nevertheless, there appeared some in the a kind of Italian (*vulgare*) language, produced by scholars, artists, mechanicists, architects of fortifications, military studies (e.g., Charbonnier 1928; Hall 1952, chapters I–II) etc. Particularly, military engineering (in ca. half of the 16th century) was essentially part of military architecture and thus presented works in architecture, artisanship and military expertise (e.g., Zanchi 1554; Cataneo [1567] 1982; Lantieri 1557; Lupicini 1582a,b; Rusconi 1590, etc.) addressed to *men of war* (Gille 1964). They published compendia, scientific works and tables, the latter being particularly useful and produced by means of images (without previous usual materials errors), as well. Most publications came from France, e,g., de Monluc ([1521–1576] 1964), de Fourquevaux (1548), de la Noue (1587), from Germany, e.g., Fronsperger (1564), from Italy, e.g., Biringucci, della Rovere, de Marchi, Collado, Pigafetta, Lorini, Tadino de Martinengo, Bellucci, Greco, Gromo, Busca, Lupicini, Machiavelli, Peruzzi, Romano, Curtio Troiano, et al, already cited above in Chapter 1, and from England, e.g., Ascham (1545) and Cyprian Lucar (1588).

Tartaglia was one of the Italian mathematicians who were mainly busy with mathematics, geometry, fortifications and science of weights and were translated into *vulgare*. Among his publications, *Quesiti et inventioni diverse* (hereafter *Quesiti*; Tartaglia 1546, 1554) is the most translated work. Generally speaking, Tartaglia's *corpus* underwent a number of translations, some partially and some in full, most of them with regard to *Quesiti*. In my opinion, numerous translations were mainly inspired by the amazing ideas they contained. There also appeared to be a wish, on the one hand, to spread Tartaglia's studies with those of de Nemore's science and, on the other hand, to further developments of the *Nova scientia* within military studies (Webb 1965; Walton 1999). An example of the latter can be found in Cyprian Lucar's (fl. 1590) choice to translate and publish (Tartaglia 1588) the first three books of *Quesiti* (Olivari 2005) only, and to add a special appendix to permit the reader to go into the properties and expertise of gunneries.

5.1 *Quesiti* Foreign Editions

In the following, a list of *Quesiti*'s foreign editions is presented. It includes the main known non–Italian editions from 1546 to 2010. I provide an original orthographical structure within titles and main library accounts.

5.1.1 The Foreign Editions, 1546–2010

1547 Germany	Books I–II–IV–VI–VII–VIII in: *Der furnembsten, notwendigsten, der gantzen Architectur angehörigen Mathematischen und Mechanischen künst, eygentlicher bericht, und haftklare, verstendliche unterrichtung* [...] *in drey furneme Bücher abgetheilet. Als Der newen Perspectiua. I. Buch vom rechten gewissen Geometrischen grund, alle Regulierte und Unregulierte Cörperliche ding* [...] *Weiteren inhalt des II. und III. Buchs der Geometrischen Büxenmeisterei, und geometrischen Messung, sampt der kurtzen summarien des gantzen begriffs der selbigen unterschidnen theil, finden hernach,* Ryff W.H., Nurimberg. [Accounts: Italy[1] and France[2]. Reprint: Holms[3] Verlag, Hildesheim 1981].
1556 French	Book VI in: *Livre VI. Des demandas et inventions diverses de Nicolas Tartalea, Bressan, Sur la maniere de fortifier les Cités, eu esgart à la forme. ET de quelle largeur, espesseur & hauteur doivent etre les Boulleuarts, Platesformes & Cavaliers.* A Rheims de l'imprimerie de Bacquenois, Imprimeur de M. le R. Cardinal de Loraine. [Account: France[4]].

[1] *Catalogue of Milano University*, Italy: Inv. 047 334278. Coll. 3L. 13A.T.068. 001. Note 1 V. Philosophy faculty. 1981–edition is a reproduction of 1547–edition. World biographical Index. Internet-edition. K.G. Saur Electronic Publishing München: www.saur-wbi.de.

[2] *Bibliothèque Nationale de France*: Rés. V 333.

[3] *Catalogue of Genova University*, Italy: CSB di Architettura Fondo: Coll. E.1920. Barcode 00192529.

[4] *Bibliothèque Mériadec Municipal de Bordeaux*. France. Fonds Patrimoniaux, Côte A 5384(2). For *idem* book, Jadart also mentioned the following Archive at the cited Bordeaux bibliotheca: 23, 265A. Section Science et Arts, 8665*. See also: Tonni–Bazza 1901, 1904bc.

1558 Germany	Books I–II–IV–VI–VII–VIII in: *Der furnembsten, notwendigsten, der gantzen Architectur angehörigen Mathematischen und Mechanischen künst, eygentlicher bericht, und haftklare, verstendliche unterrichtung* [...] *in drey furneme Bücher abgetheilet. Als Der newen Perspectiua. I. Buch vom rechten gewissen Geometrischen grund, alle Regulierte und Unregulierte Cörperliche ding* [...] *Weiteren inhalt des II. und III. Buchs der Geometrischen Büxenmeisterei, und geometrischen Messung, sampt der kurtzen summarien des gantzen begriffs der selbigen unterschidnen theil, finden hernach,* Ryff W.H., Nurimberg. [Account: France[5]].
1582 Germany	Books I–II–IV–VI–VII–VIII in: [Der] *Bawkunst oder Architectur aller fürnemmsten, nothwendigsten, Angehörigen mathematischen vnd mechanischen Künsten, eygentlicher Bericht, und verständliche Vnderrichtung, zu rechtem Verstand der Lehr Vitruuij, in drey fürnemme Bücher abgetheilet.* […] *Allen künstlichen Handtwerckern, Werckmeistern,* […] *zu sonderlichem Nutz vnd vielfeltigem Vortheil in truck verordnet, durch Gualtherum H. Riuium medi. & math Getruckt zu Basel, Getruckt, zu Basel: durch Sebastian Henricpetri,* Ryff W.H, Basilea. [Account: Italy[6] and France[7]].
1588 English	Books I–II–III in: *Three books of colloquies concerning the arte of shooting [microform] : in great and small peeces of artillerie, variable randges, measure, and waight of leaden, yron, and marble stone pellets, mineral saltepeeter, gunpowder of diuers sortes, and the cause why some sortes of gunpower are corned, and some sortes of gunpowder are not corned: written in Italian, and dedicated by Nicholas Tartaglia vnto the Royall Prince of most famous memorie Henrie the eight, late King of England, Fraunce, and Ireland, defender of the faith &c. And now translated into English by Cyprian Lucar Gent. who hath also augmented the volume of the saide colloquies with the contents of euery colloquie, and with all the corollaries and tables, that*

[5] *Bibliothèque nationale de France*: N027156-1. I remark that Ryff quoted Tartaglia many times even though he only translated part of his ideas without developing them. Therefore, it is not really a *Quesiti*'s edition. It is a comment on several parts of *Quesiti*. More or less like Drake and Drabkin's made with their *Mechanics in Sixteenth–Century* (Drake and Drabkin 1969).

[6] *Biblioteca Nazionale Centrale di Firenze*, Italy. Coll. MAGL.20.4.14 Inv.: CF005683893. It also includes 3 books of *Nova scientia.*

[7] *Bibliothèque nationale de France*: Loc. N027156-2.

	are in the same volume. Also the said Cyprian Lucar hath annexed vnto the same three books of colloquies a treatise named Lucar Appendix […]. Thomas Dawson for Harrison J, London.[8] [Account: U.K[9], Australia[10] and U.S.A[11]]
1778 Germany	Book VI in: *Das sechste Buch der Fragen und Erfindungen des Nicol. Tartaglia, Von der Befestigung der Städte, so wediesble von der Gestalt der Walle abhänget*, printed for: *Magazin für Ingenieure und Artilleristen*, vol. IV. Bohm A, Universität Giessen.
1845–1846a French	Books I–II–III and *Nova scientia,* in: *Journal des arms specials,* Vol. VI.
1845–1846b French	Books I–II–III and *Nova scientia,* in: *La Balistique de Nicolas Tartaglia, ouvrage publié pour la 1re fois en 1537 sous le titre de "La Science nouvelle", et continué en 1546 dans les deux 1ers livres du recueil du même auteur intitulé. "Questions et inventions diverses", traduit de l'italien* [...] *par François–Xavier–Joseph Rieffel* […]. 1er partie. Correard, Paris.[12] [Account: France[13] and U.K[14]].

[8] The book is also given by Riccardi in his *Biblioteca Matematica Italiana dalla origine della stampa ai primi anni del secolo XIX* (Riccardi 1870–1880, II, 500).

[9] *British Library*, UK. Identifier: System number 003581577. Plates. fol. (Uk) MP1.0003828712. General Reference Collection 62.d.14. [Another issue]. General Reference Collection 62.d.14. UIN: BLL01003581577.

[10] *National Library of Australia*. Bib. ID 1141724 STC (2nd ed.) 23689. Microfilm. Ann Arbor, Mich.: University Microfilms International, 1964. 1 microfilm reel; 35 mm (Early English books, 1475–1640; 1010:15). It is a reproduction of the original archived at the the British Library. I also note: a) the website of the Australian library reports both dates 1587 and 1588, and b) it is also available from UMI 300 N Zeeb Rd., Ann Arbor, MI 48103–1553.

[11] *University of Pennsylvania Library*. 1 microfilm reel, 35 mm. Location: Van Pelt Micro text Call Number: STC I Reel 1010:15. It is a reproduction of the original in the *British Library*. It is also available from UMI, 1964, Ann Arbor, MI 48103–1553.

[12] The book is also given by Riccardi in his *Biblioteca Matematica Italiana dalla origine della stampa ai primi anni del secolo XIX*: "Fu [*Quesiti*] translated into French language, with the title *La Balistique de Nicolas Tartaglia, ouvrage publié pour la 1re fois* […]" and he also cited "*sur la maniere de fortifier les citez* […]" of the 1556 (Riccardi 1870–1880, II, 500).

[13] *Bibliothèque nationale de France*. Two Vols. N°: FRBNF31434939. Loc.: Tolbiac V–53572–3. The translator added a long appendix on ballistic theory.

1845–1846c French	*La balistique de Nicolas Tartaglia, ou, Recueil de tout ce que cet auteur a écrit touchant le mouvement des projectiles et les questions qui s'y rattachent, composé des deux premiers libres de La science nouvelle (ouvrage publié pour la première fois en 1537) et des trois premiers libres des Recherches et inventions nouvelles (ouvrage publié pour la première fois en 1546)*. 2e partie. Corréard J, Paris. [Account: France[15] and U.K[16]].
1969 English	Selections from Quesiti–Books I, VII and VIII In: *Mechanics in Sixteenth–Century Italy: Selections from Tartaglia, Benedetti, Guido Ubaldo, and Galileo*.[17]
1981 Germany	Books I–II–IV–VI–VII–VIII. In: *Der furnembsten, notwendigsten, der gantzen Architectur* […]. Reprint: Holms Verlag, Hildesheim.
2001 Germany	[From 1554–edition] *Die kubischen Gleichungen bei Nicolo Tartaglia: die relevanten Textstellen aus seinen Quesiti et inventioni diverse auf deutsch übersetzt und kommentiert,* in *Veröffentlichungen der Kommission für Geschichte der Mathematik, Naturwissenschaften und Medizin* 53. Wien Verlag der Österreichischen Akademie der Wissenschaften. Translator: Friedrich Katscher. [Account: Austria[18] and Italy[19]].
2010 French	Tartaglia N. 2010. *Niccolo Tartaglia: Questions et inventions diverses, Livre IX* [Book IX only] *ou L'invention de la résolution des équations du troisième degr*é. Hamon G, Degryse L (eds). Hermann, Paris.

[14] *British Library*, UK. Loc.: General Reference Collection 1398.e.9. The books is also cited by Riccardi (Riccardi 1870–1880, 500).

[15] This is the second part of the previous book. The title changes. *Bibliothèque nationale de France*. Two Vols. N°: FRBNF31434939. Loc.: Tolbiac V–53572–3. The translator added a long appendix on ballistic theory.

[16] *British Library*, UK. Loc.: General Reference Collection 1398.e.9.

[17] Drake and Drabkin 1969.

[18] The 2001 edition is part of a book series: *Veröffentlichungen der Kommission für Geschichte der Mathematik, Naturwissenschaften und Medizin*; 53.

[19] *Biblioteca dell'Istituto di Storia della Scienza di Firenze*, Italy. Aritmetica e Algebra Testi, Carteggi. Coll: Misc. 613/16; see also *Istituto Austriaco di Roma*, Coll.: 8.GN.53.

English:	2 editions
French:	5 editions
Germany:	5 editions plus 1 reprint

5.2 Bibliographical Notes

During the latest ten years, my researches on Tartaglia (and correlated history of mechanics from Archimedes to Torricelli) produced numerous results already published (References list). For, this reason, I (sometime in collaboration with co–authors, as well) collected several references concerning alleged *Quesiti* editions. Yet some of them lack historical proofs. Nevertheless, negative results also belong to historical research. In order to make it clear within the international archives programmes, and hoping that they will be of some help, the following are listed as well.

5.2.1 Uncertain Dates around Partial and/or Alleged *Quesiti*'s Editions

Quesiti 1528
An incomplete treatise seems to have first appeared in Venice. Ayala gives 1528 (and 1546, 1550, 1554, 1660, 1562, 1583, 1606) editions in Venetia and another one on 1620 in Carpi.
Cfr: D'Ayala M (1854) *Bibliografia militare italiana e moderna*, Stamperia Reale,Torino, 155.
Cfr.: Ayala M (1841) *Dizionario Militare Francese Italiano.* Tipografia Gaetano Nobile, Napoli, 367.
I do not have historical proves of that. For, I think that it is an error in the Ayala's book.

Delli quesiti 1538
See 1554–edition.
Delli quesiti et inventioni diverse, di Nicolò Tartaglia, stampato a Venezia nel 1538.
It seems to be at the *Biblioteca of the Palazzo dell'Arsenale*[20] Torino, Italy.

Quesiti 1550
De Bascarini.
Cfr.: Riccardi P (1870–1880) *Biblioteca Matematica Italiana dalla origine della stampa ai primi anni del secolo XIX*. Tipografia dell'erede Soliani, Modena, II, 499.
Cfr.: Weiss M (1841) *Biographie universelle ou dictionnaire historique*. Tome VI. Furne & C, Paris, 22, Col. 1.

Quesiti 1551
Ruffinelli, included *Gionta* to Book VI.
Cfr.: Riccardi P (1870–1880) *Biblioteca Matematica Italiana dalla origine della stampa ai primi anni del secolo XIX*, II, 499.
Cfr. Brunet JC (1860–1865) *Manuel du Libraire et de L'Amateur de Livres.* Firmin Didot Frères, Paris, (see also 1878 et succ.).
Cfr.: Graesse JGT (1859–1869) *Trésor de livres rares et precieux*, Vols. I–VII. Dresde, Kuntze.
Cfr.: Weiss M (1841) *Biographie universelle ou dictionnaire historique*. Tome VI. Furne & C, Paris, 22, Col. 1.
CONTRA
Cfr.: Boncompagni B (1881) *Intorno ad un testamento inedito di N. Tartaglia. In memoriam dominici Chelini*. Collectanea Mathematica. Hoepli, Milano, 380–381.
UNCERTAIN
Cfr. Masotti gives it in: Tartaglia N ([1554] 1959) *Quesiti et inventioni diverse de Nicolo Tartaglia brisciano*. Commentari dell'Ateneo di Brescia, Brescia, XXXVIII, fn. 24.
The book in Florence is missing c. 81–132 related to *Books VI–IX* substituted by c. 81–88, 93–128 by 1554–Edition (see Riccardi 1870–1880, II, 499–500).
[Account: Italy[21]].

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[20] The title in the text is exactly that reported by *Biblioteca of the Palazzo dell'Arsenale* in its website. Very probably it should be *Il primo libro delli quesiti, et inventioni diverse de Nicolo Tartaglia, sopra gli tiri delle artiglierie, et altri svoi varii accidenti* (Tartaglia 1538) of the 1554–edition (Tartaglia 1554).

[21] *Biblioteca dell'Istituto di Storia della Scienza*, Firenze, Italy. Old coll.: Antico 1092. Sigla del catalogatore: rl. New coll.: MED 1051/01. It is cited in: *Biblioteca of the Istituto di fisica, Università di Firenze; Laboratorio di Fisica in Arcetri, Università di Firenze Museo di fisica e storia naturale; Istituto di studi*

Quesiti	1558 *Nova Scientia de N. T. con una gionta al terzo Libro. (legato con) Il Primo Libro (-Ottavo) delli quesiti, et inventioni diverse de N. T., sopra gli tiri delle artiglierie, et altri suoi varii accidenti. (legato con) Regola generale di solevare ogni fondata Nave & navilii con Ragione.* Published by s.d. 1562, Vinegia, Curtio Troiano dei Navò[?]. It seems that Riccardi had a copy without the Book IX[22]. Cfr.: Riccardi P (1870–1880) *Biblioteca Matematica Italiana dalla origine della stampa ai primi anni del secolo XIX*, II, 499–500. [Account: Italy[23]].
Quesiti	1562 *Il primo* [-ottavo] *libro delli quesiti, et inventioni diverse de Nicolo Tartaglia, sopra gli tiri delle artiglierie, et altri suoi varii accidenti* […]. *Curtio Troiano dei Nauò*[?]. It seems that Riccardi had a copy without the Book IX.[24] Cfr.: Riccardi P (1870–1880) *Biblioteca Matematica Italiana dalla origine della stampa ai primi anni del secolo XIX*, II, 499–500. [Account: Italy[25]].
Quesiti	Before 1566 *Il primo* [-ottavo] *libro delli quesiti, et inventioni diverse de Nicolo Tartaglia, sopra gli tiri delle artiglierie, et altri suoi varii accidenti* […]. *Curtio Troiano dei Nauò*[?]. [Account: Italy[26]]

superiori, Firenze. Osservatorio meteorologico; Museo strumenti antichi. Università di Firenze.

[22] "Nell'esemplare da me posseduto manca il nono libro e dopo l'ottavo, che termina con la 94° car., vi sono uniti la *Travagliata invenzione* […] e l'Opera di Archimede *de insidentibus aquae dechiarata in volgare* ec. In car. 32 senza num. compresa l'ultima colla impresa e le note di stampa nel recto: IN VINEGIA, Per Curtio Troiano dei Nauò. M.D.LXII.". (Riccardi 1870–1880, II, 499. Author's capital letters and italic style).

[23] *Biblioteca di Storia delle Scienze "Carlo Viganò",* Brescia, Italy.

[24] See 1558 edition (Riccardi 1870–1880, II, 499–500).

[25] *Biblioteca di Storia delle Scienze "Carlo Viganò",* Brescia, Italy.

[26] Cfr.: The *Universal Short Title Catalogue* (USTC) hosted by the University of St Andrews. The date before 1566 is obtained from reading Troiano's publishing activities from 1537 to 1566: 28 works in 36 publications in 3 languages and 115 library holdings. Italian Library copies: *Brescia*, *Biblioteca Ottorino Marcolini dell'Università cattolica del Sacro Cuore;* Cremona, *Biblioteca statale*, Gallarate, *Biobiblioteca Istituto Filosofico Aloi*sianum; L'Aquila, *Biblioteca provinciale Salvatore Tommasi*; Messina, *Biblioteca regionale*

Quesiti 1620
Ayala gives (1528, 1546, 1550, 1554, 1660, 1562, 1583, 1606 in Venetia and) 1620 as an edition in Carpi.
Cfr.: Ayala M (1841) *Dizionario Militare Francese Italiano. Tipografia Gaetano Nobile*, Napoli, 367.
Cfr.: Cockle MJD (1900) *A Bibliography of English foreign and military books. Biography of military books up to 1642.* Simpkin, Marshall. Hamilton, Kent & Co. Ltd, London, 169.

Quesiti 1670
The Book III is translated, only.
Cfr.: Stubbe H (1670) *Legends no histories: or, A specimen of some animadversions upon the History of the Royal Society: Wherein, besides the several errors against common literature, sundry mistakes about the making of salt-petre and gun-powder are detected, and rectified: whereunto are added two discourses, one of Pietro Sardi, and another of Nicolas Tartaglia relating to that subject. Translated out of Italian. With a brief account of those passages of the authors life, which the virtuosi intended most to censure, and expatiate upon: written to save them the trouble of doing any thin besides defending themselves.* […]. Printed at London, London, 110–119.
Cfr.: Cockle MJD (1900) *A Bibliography of English foreign and military books. Biography of military books up to 1642,* 169.

universitaria; Padova, *Biblioteca universitaria*; Roma, *Biblioteca Angelica*; Roma, *Biblioteca dell'Accademia dei Lincei e Corsin*iana; Roma, *Biblioteca nazionale centrale Vittorio Emanuele II*; Roma, *Biblioteca universitaria Alessandrina*; Torino, *Biblioteca Reale*; Trapani, *Biblioteca Fardelliana*; Urbino, *Biblioteca centrale dell'Area umanistica dell'Università degli studi di Urbino*; Firenze, *Biblioteca dell'Osservatorio Ximeniano*, Coll. K.3.38/M.

Chapter 6

Conclusion

Questa è stata una bella speculazione, & me è piaciesta assai. Et perche vedo essere hora tarda, non voglio, che procedati in altro per hoggi.
(Tartaglia 1554, *Book VIII*, Q. XLII, Proposition XV, 98v)

In this section, I present concluding remarks concerning my Ph.D. research.

6 Concluding Remarks

It was 1546 when Italian scholar, Niccolò Tartaglia wrote his first edition of the *Quesiti et inventioni diverse*.

The *Quesiti et inventioni diverse* is an extraordinary and interdisciplinary debate on physics, architecture, statics and mathematics. The science in–common is geometry. The language used is Italian (*vulgare*). Particularly, *Book VII* and *Book VIII* are mainly with regard with *Scientia de ponderibus*, which – with some optional – nowadays we call statics.

The subject of my Ph.D. dissertation at the University of Bergamo is *Niccolò Tartaglia Re–Thinking the Role Played by Science of Weights in the Sixteenth–Century. Selections from Quesiti et inventioni diverse: Books VII–VIII* (619 pages). Therefore, it presents a historical and scientific analysis as historical epistemology of the science of weights and mechanics in the sixteenth century, particularly as developed by Tartaglia in his *Quesiti et inventioni diverse*, *Book VII* and *Book VIII* (1546; 1554).

Mechanics in between the 15th and 16th centuries mainly concerned what largely is now called statics and was referred to as the *Scientia de ponderibus*. Generally, It was pursued with two different approaches; the former, usually referred to as Aristotelian, where the equilibrium of a body stemmed from two contrasting tendencies. The latter, usually referred to as Archimedean, where the study of the equilibrium reduced to the evaluation of the centre of gravity of a body (*centrobaric*). In between the two traditions the Italian scholar, Niccolò Fontana, better known as Tartaglia (1500?–1557), wrote the treatise *Quesiti et inventioni diverse* (1546; 1554). The whole *Quesiti et inventioni diverse* book, is presented in the form of a dialogue; further, in *Book IX* (Tartaglia 1554, Pr. XXVII–XLII) an added method of communication appears, the epistolary. The questions among mathematicians evidently revolved around the problem of solution of the third degree equation; often, the tune echoed mediaeval disputes. Then, a historical epistemology analysis of *Book VII* and *Book VIII* was done. All propositions of *Books VII* and *VIII*, and their relationships with the *Problemata mechanica* by Aristotle and *Iordani opvsculvm de ponderositate* by Jordanus de Nemore were examined.

The Ph.D. volume composed of four parts within six main chapters. At the beginning (Part I, chapter 1) biographical sketches and philological–historical–epistemological reflections are reported. In the Chapter 2 (Part I) an historical account of S*cientia de ponderibus* (*Science of Weights*) and statics during ancient time and the Renaissance is presented. Consequently he extensively analysed Niccolò Tartaglia's *Books VII* and *VIII* of the *Quesiti et inventioni diverse* (Part II, chapter 3) from historical and epistemological standpoints. Particularly, this Chapter is also devoted to *historical epistemology of science* presenting an integrated history and epistemology of scientific methods, which combine epistemological and historical approaches to identify significant historical hypotheses within the relationship between physics and mathematics (physical observations and theoretical mechanical modeling). In Chapter 4 (Part III) I reported on translations into English and transcriptions of the main works studied for his research. Part IV is composed of two chapters. In Chapter 5, I listed foreign editions of *Quesiti et invention diverse* as a component of the history. Bibliographical notes and alleged editions are commented. Finally, in Chapter 6, concluding remarks end the book. After a large and international references section, a list of main *Quesiti* accounts is presented.

The main accomplishments concerning my research are:

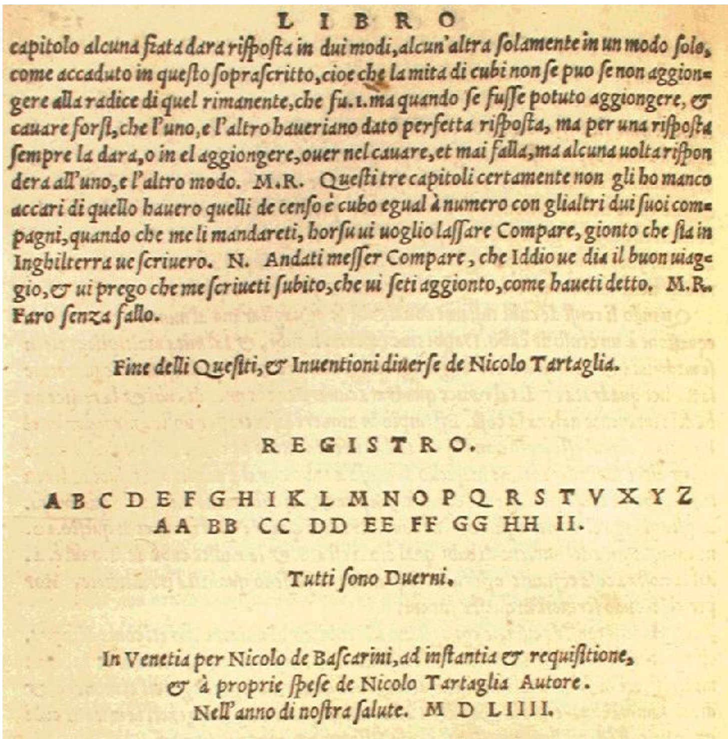
- An historical account of Tartaglia's lifetime, his scientific production and the *Scientia de ponderibus* from the middle Ages to the Renaissance using new documents is assured. The latter are based only on official and original archive research such as Tataglia's last will and testament.
- Tartaglis'a scientific production and the *Scientia de ponderibus* in the Arabic-Islamic culture and with Simon Stevin's science are finally reported in a unique historical account. In fact, the connections of Tartaglia's *science of weights*, not only with the Italian group but also with the Arabic tradition and with Simon Stevin is very well presented.
- A carefully historical epistemology analysis of *Book VII* and *Book VIII* is presented. Here all propositions of *Books VII* and *VIII*, and their relationships with the *Problemata mechanica* by Aristotle and *Iordani opvsculvm de ponderositate* by Jordanus de Nemore are accurately examined for the first time both scientific and historical standpoints with respect the secondary literature of the past century. The reader is then well prepared to read Tartaglia's text, a difficult task indeed, but how fruitful!

- The last part is relative to the original texts and critical transcriptions into Italian and Latin and an English translation. Again, for the first time, the *corpus* entirely is translated from *vulgare* and from Latin in Italian and in English language as well. This part, with its original texts and its translations, with numerous references to other original texts as well as to the secondary literature, will be a useful tool for all those who study this particularly rich period.

I think that the thesis gathers and re-evaluates truthfully the current thinking on this subject. It brings together contributions from my previous an current researchs in the history and historical epistemology of science, within the fields of physics, mathematics and engineering. It also gives much-needed insight into the subject from historical and scientific points of view. The Thesis composition and it original content makes for absorbing reading for historians, epistemologists, philosophers and scientists.

Fine del Tartaglia's Science of Weights and Mechanics in the Sixteen–Century de Raffaele Pisano.

In Bergamo & Lille per le Università Raffaele Pisano, Autore. Nell'anno di nostra Salute. M M XIIII.



LIBRO

capitolo alcuna fiata dara risposta in dui modi, alcun'altra solamente in un modo solo, come accaduto in questo soprascritto, cioe che la mita di cubi non se puo se non aggiongere alla radice di quel rimanente, che fu. 1. ma quando se fusse potuto aggiongere, & cauare forsi, che l'uno, e l'altro haueriano dato perfetta risposta, ma per una risposta sempre la dara, o in el aggiongere, ouer nel cauare, et mai falla, ma alcuna uolta risponderà all'uno, e l'altro modo. M.R. Questi tre capitoli certamente non gli ho manco accari di quello hauero quelli de censo è cubo egual à numero con glialtri dui suoi compagni, quando che me li mandareti, horsu ui uoglio lassare Compare, gionto che sia in Inghilterra ue scriuero. N. Andati messer Compare, che Iddio ue dia il buon uiaggio, & ui prego che me scriueti subito, che ui seti aggionto, come haueti detto. M.R. Faro senza fallo.

Fine delli Quesiti, & Inuentioni diuerse de Nicolo Tartaglia.

REGISTRO.

A B C D E F G H I K L M N O P Q R S T V X Y Z
AA BB CC DD EE FF GG HH II.

Tutti sono Duerni.

In Venetia per Nicolo de Bascarini, ad instantia & requisitione, & à proprie spese de Nicolo Tartaglia Autore.
Nell'anno di nostra salute. M D LIIII.

Tartaglia 1554, *Quesiti et invention diverse, Book IX*, Q XLII, 128v

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In ths section references list is provided. It is divided in Primary and Secondary Literature, Tartaglia's *corpus* and microfilm sources.

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Tartaglia N (1543b) Opera Archimedis Syracusani philosophi et mathematici ingeniosissimi per Nicolaum Tartaleam Brixianum (mathematicarum scientiarum cultorem) multis erroribus emendata, expurgata, ac in luce posita, multisque necessariis additis, quae plurimis locis intellectu difficillima erant, commentariolis sane luculentis & eruditissimis aperta, explicata atque illustrata existunt, appositisque manu propria figuris quae graeco exemplari deformatae ac depravatae erant, ad rectissimam symetriam omnia instaurata reducta & reformata elucent, apud Venturinum Ruffinellum, sumptu & requisizione Nicolai de Tartaleis Brixiani, mense Aprili

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Tartaglia N (1550) La Nova Scientia de Nicolo Tartaglia con una gionta al terzo. Libro. Disciplinae mathematicae loquuntur[.] Stamapata in Venetia per Nicolo de Bascarini a sistantia de l'Autore.

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Tartgalia N (1551b) [Ragionamenti I-III and Supplimento] Regola Generale da Sulevare con Ragione e Misura no[n] solame[n]te ogni affondata Nave: ma una Torre Solida di Mettallo Trovata da Nicolo Tartaglia, delle discipline Mathematice amatore intitolata la Travagliata Inventione. Insieme co un artificioso modo di poter andare, & stare plogo tepo sotto acqua, a ricercare le materie affondate, & in loco profundo. Giontovi anchor un tratttato, di segni della mutationi dell'Aria, over di te[m]pi, material no[n] men utile, che necessaria, a Nauiganti, & altri. Nicolo Bascarini, Venetia

Tartaglia N (1554) Quesiti et inventioni diverse de Nicolo Tartaglia, di novo restampati con una gionta al sesto libro, nella quale si mostra duoi modi di redur una citta inespugnabile. In Venetia per Nicolo de Bascarini, ad istantia & requisistione, & a proprie spese de Nicolo Tartaglia Autore. Nell'anno di nostra Salute. MDLIIII

Tartaglia N (1556–1560) La prima [–sesta] parte del general trattato di numeri, et misure di Nicolo Tartaglia, nella quale in diecisette libri si dichiara tutti gli atti operativi, pratiche, et regole necessarie non solamente in tutta l'arte negotiaria, & mercantile, ma anchorin ogni altra arte, scientia, over disciplina, dove interuenghi il calculo. In Vinegia per Curtio Troiano de i Navò. MDLVI [–MDLX]

Tartaglia N (1558) La Nova Scientia de Nicolo Tartaglia con una gionta al terzo Libro. Disciplinae mathematicae loquuntur[.] Qui cupitis rerum varias cognoscere causas disate nos cunctis hac patet una uia. [Curtio Troiano editor is legible from the text]. In Vinegia. MDLVIII

Tartaglia N (1562) [*Regola Generale* with *Supplimento* and *Ragionamenti I–II*] Regola Generale di solevare ogni fondata Nave & navilli con Ragione. In Vinegia, per Curtio Troiano de i Navò. MDLXII

Tartaglia N (1565) Iordani Opusculvm de Ponderositate, Nicolai Tartaleae Studio Correctum Novisqve Figuris avctum. Cum Privilegio Traiano Curtio, Venetiis, Apud Curtium Troianum. MDLXV

Tartaglia N (1565–*Euclid*) Euclide Megarense acutissimo philosopho, solo introduttore delle scientie mathematice. Diligentemente rassettato, et alla integrita ridotto, per il degno professore di tal Scientie Nicolo Tartalea, Brisciano. Secondo le due tradottioni. Con una ampla esposizione dello istesso tradottore di nuono aggiunta. Talmente chiara, che ogni mediocre ingegno, senza la notitia, over suffragio di alcun'altra scientia con facilità, sera capace a' poterlo intendere. In Venetia, appresso Curtio Troiano, 1565

Tartaglia N (1565–*Insidentibus*) Archimedis de Insidentibus Aquae Liber Primus […] Liber Secundus. Cum Privilegio Traiano Curtio, Venetiis, Apud Curtium Troianum. MDLXV, 1rv–16rv

Tartaglia N (1569) Euclide Megarense acutissimo philosopho, solo introduttore delle scientie mathematice. Diligentemente rassettato, et alla integrita ridotto, per il degno professore di tal Scientie Nicolo Tartalea, Brisciano. Secondo le due tradottioni. Con una ampla esposizione dello istesso tradottore di nuono aggiunta. Talmente chiara, che ogni mediocre ingegno, senza la notitia, over suffragio di alcun'altra scientia con facilità, sera capace a' poterlo intendere. In Venetia, appresso Giovanni Bariletto, 1569

Tartaglia N (1876) I sei scritti di matematica disfida di Lodovico Ferrari coi sei contro–cartelli in risposta di Niccolò Tartaglia, comprendenti le soluzioni de' quesiti dall'una e dall'altra parte proposti. Raccolti, Autografi e Pubblicati da Enrico Giordani, Bolognese. Premesse notizie bibliografiche ed illustrazioni sui Cartelli medesimi, estratte da documenti già a stampa ed altri manoscritti favoriti dal Comm. Prof. Silvestro Gherardi, Preside del'Istit. Tecn. Prov. di Firenze. R. Sabilimento litografico di Luigi Ronchi e tipografia degl'Ingegneri, Milano

Tartaglia N (1583) [La nova scientia de Nicolo Tartaglia con una gionta al terzo libro] Disciplinae mathematicae loquuntur[.] Qui cupitis rerum varias cognoscere causas disate nos cunctis hac patet una uia. In Vinetia, Appresso Camillo Castelli, 1583

Tartaglia N (1585) Euclide Megarense acutissimo philosopho, solo introduttore delle scientie mathematice. Diligentemente rassettato, et alla integrita ridotto, per il degno professore di tal Scientie Nicolo Tartalea, Brisciano. Secondo le due tradottioni. Con una ampla esposizione dello istesso tradottore di nuono aggiunta, talmente chiara, che ogni mediocre ingegno, senza la notitia, over suffragio di alcun'altra scientia con facilità, sera capace a' poterlo intendere. Di nuovo con ogni diligenza ben corretto, e ristampato. In Venetia, appresso gli Eredi di Troian navo, alla libraria dal Lione, MDLXXXV

Tartaglia N (1538) Il primo libro delli Quesiti, et inventioni diverse de Nicolo Tartaglia, sopra gli tiri delle artiglierie, et altri svoi varii accidenti. In: Tartaglia 1554

Tartaglia N (1537) Nova scientia inventa da Nicolo Tartalea. B. Disciplinae mathematicae loquuntur[.] Qui cupitis rerum varias cognoscere causas disate nos cunctis hac patet una via. In Vinegia per Stefano Nicolini da Sabio. Ad instantia di Nicolo Tartalea Brisciano il qual habita a San Saluador. MDXXXVII

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4666 *Tartaglia Niccolò, General trattato*, Vinegia, per Curtio Troiano dei Navò, 1556–1560. (A.3.2–4) [FA.5A.97/1–3]

4670 *Tartaglia Niccolò, Ragionamenti sopra la sua travagliata inventione*, Venetia, per Nicolò Bascarini, a instantia et requisitione et a proprie spese de Nicolo Tartaglia, 1551. [FA.5B.22]

4671 *Tartaglia Niccolò, Regola generale ...intitolata la travagliata inventione*, Venezia, Nicolò Bascarini, c. 1551. [FA.5B.23]

Legenda:

The progressive numbers respectively correspond to the *Fondo Antico* section (printed books) and *Fondo Manoscritti* of the *Catalogo della Biblioteca di Scienze “Carlo Viganò”.*

Fondo antico (1482–1800) e Fondo manoscritti: available both in print (1994. *Vita e pensiero*, 1994, p XIX, p 869) and in CD–Rom.

The final signatures in the brackets (“(...)”) belong to of the *old archive*, the others (“[...]”) refer to the current ne*w cataloging* of the Library.

Main *Quesiti* Accounts

France. Edition of 1554. *Conservatoire nationale des arts et métiers*. *Bibliothèque centrale de Paris,* France. Coll.: 4Y49; and *Bibliothèque nationale de France*. Loc.: EDIT16 CNCE 31875 (from: ETH-Bibliothek Zürich, Rar 5553).

Germany. Edition 1546 and 1554. ECHO – Cultural Heritage Online. Open Access Infrastructure for a Future Web of Culture and Science, *Max Planck Institute for the History of Science*, Berlin.

Italy. Full Tartaglia’s *corpus* both manuscripts and editions. *Biblioteca di Storia delle Scienze “Carlo Viganò”,* Brescia, Italy.

United Kingdom. British Library. Edition of 1546 and 1554. System number 003581581-2. Physical Description: 4º. Holdings Notes: General Reference Collection 8530.c.7.(2) and 52.d.3.(3.) [Another copy.] Shelfmark(s): General Reference Collection 534.g.22.(1.). General Reference Collection 52.d.3.(3.) and 8530.c.7.(2.). UIN: BLL01003581581-2.

APPENDIX

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A Development of the Principle of Virtual Laws and its Framework in Lazare Carnot's Mechanics as Manifest Relationship between Physics and Mathematics

--Manuscript Draft--

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Corresponding Author:	Raffaele Pisano
Corresponding Author Secondary Information:	
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Abstract:	The virtual displacement or work concern a timely idea according to which an actual motion of a certain body is not the unique possible motion, and eventually capable to reduce a particular magnitude; its conceptualization of minimize as hypothesis arises back to Aristotelian school. In the history and Philosophy of science one find various enunciations of the Principle of Virtual Laws and its virtual displacement or work applications, i.e., from Aristotle to Leibniz' vis viva, from Maupertuis' least action to Euler and Lagrange with calculus of variations (statics and dynamics) until to Lazare Carnot' mechanics. In order to show why Carnot's approach is original, I will explain it within the historical context of rival approaches to development of the Principle of Virtual Laws (also known with slights difference as the Principle of Virtual Velocity or of Virtual Work). Carnot's geometric motion as one of the possible, but invertible, movements - applied to virtual displacement as employed in his theories of machines and of collisions - will be discussed as case study, as well.

1 An Outline

A history of the *principle of virtual laws* (velocities, work) states, i.e., that even if one could agree that the *principle of virtual work* could exists prior to all laws of mechanics (and not all scholars agrees with thesis), and could therefore it be derived from the *principle of virtual work*, in the end, the fact that the *principle of virtual work* is self–evident cannot be accepted. In other words, one cannot accept it as a mere principle. Therefore, a proof is necessary; or, a reduction to a theorem of another approach to mechanics, or an attempt to provide a more convincing version. Consequently, the main problem with proving of the *principle of virtual work* sparked a heated debate, especially in France where Lazare Carnot (Carnot L 1786, 1803a), Vittorio Fossombroni (1754–1844; Fossombroni 1794), Fourier (Fourier 1798; 1888–1890, pp. 475–521), Ampère (Ampère 1806) and Poinsot (Poinsot 1838; see also Poinsot 1806) provided major contributions (Capecchi 2004, 2012). In effect, a particular difficulty was linking the problem to Newtonian laws and obtaining its formal validity. Initially, this principle was independent from the Newtonian laws, which concerned generally an isolated particle (or the systems derived from it). The *Principle of virtual work* also deals with extended systems of bodies[1] which include constraints in an essential way[2]. These given forces are constraining reactions that are not included in the classical Newtonian scheme because they are unknown *a priori* (Lagrange 1788, pt II, IV). It was on the principle of least action that young Lagrange concentrated his attention.

Before beginning with the following scientific and historical overviews explanation, just few lines to set that the discussion will focus a) on the history and philosophy of sciences, precisely on the historical epistemology of physics (mechanics) and its relationships with mathematics; b) a mechanical system subject to constraints. Particularly, nowadays the latter is a standardly physical system[3] which deals with movement (kinematics), equilibrium (statics), applied physical forces (dynamics) and in some circumstances (applied

[1] A difference exists with Euler’s reasoning on fluids (Euler 1757, p 286): the partial derivatives in Euler’s equations (Euler’s fluids) are applicable to compressible as well as to incompressible flow. It consists of an application of either an appropriate equation of state or assuming that the divergence of the flow velocity field is zero, respectively.

[2] Some of this historical material is clearly dealt with Darrigol’s *Physics and Necessity* (Darrigol 2014, pp. 80-84).

[3] Generally speaking and nowadays, a physical system is defined as the portion of the physical universe chosen for physical investigations and mathematics modeling.

mechanics) with machines, machineries and micro elements, as well. The constraints (also *vincula*) are a restriction on the freedom of movement of a mechanical system, i.e., of particles. The constraints are presented as mathematical physics relation between coordinates and force. Thus, taking into account these aspects of mechanical system, I am going to develop a historical and epistemological research, which relies upon historical and historiographical (Kragh 1987) sources. My aim is to suggest *history and philosophy of science* as *historical epistemology of science* (Renn 1995; Renn and Damerow 2010).

As to this, Lazare Carnot presented a new mechanics without axioms and metaphysical entities, and when worked on machines, he analyzed the efficiency of machines using the *Principle of virtual laws* and a reasoning about collision model of mechanical interaction. Despite his fundamental works (Carnot L 1786, 1803a,b, 1813) and his notable political career (Dhombres and Dhombres 1997), Lazare Carnot appears as minor figure in comparison with scientists leaders like Newton, D'Alembert, Lagrange and Fourier. This is a general conviction, which deserves to be discussed. An investigation taking into account historical and epistemological aspects of foundations of sciences, particularly focusing on the relationship between physics and mathematics could open new insights.

1.1 The Structure of the Paper

In order to show the originality of the *Principle of virtual work* adopted and used by Lazare Carnot in his mechanics (and geometry) an early historical account upon the development of this principle is necessary. Therefore, the paper is so structured:

- Early approach: *Principle of virtual laws* by Aristotle (*virtual velocities* in *Problemata mechanica*[4]) and applications by de Nemore and Tartaglia (*Science of weights, virtual velocities, virtual displacements*)
- Late approach: mechanics in context, *principle of virtual work* by Lagrange (*analytical mechanics*)
- A case study on physics and mathematics relationship: Lazare Carnot (*equilibrium and movement, machines, collision theory*)

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[4] Attributed to Aristotelian School.

Particularly and for sake of brevity[5] a description of the different mechanical traditions in the seventeenth and eighteenth century are not presented in details.

. On the contrary, taking in to account the object of the paper, the extended part at the beginning on the principle of virtual work/displacements and subsequent parts are largely discussed.

1.2 A Scientific Overview to the *Principle of Virtual Laws*

In order to make clear what scientifically the principle of virtual law is, a short overview follows.

In modern terms, to define the *principle of virtual work,* one can specify that a *displacement* is possible if it is compatible with the fixed constraints. Moreover, it is *virtual* if it is compatible with the constraints even if they are moving. More in detail, a *virtual displacement δs* of a point is any arbitrary infinitesimal change in the position of the point consistent with the constraints imposed on the motion of the point. This displacement can be just imagined. Limiting ourselves to the case of time–independent constraints, we can also derive a *possible displacement* (i.e, a rotation, too). If the *virtual displacement*s are mathematically independent variables, they are also arbitrary.

The *principle of virtual work* – which arises in the application of the *Principle of least action* (hereafter discussed) to the study of forces and movement of a mechanical system – is a law of mechanics for which there is as yet no generally accepted epistemological and ontological account: it can be seen both as a principle and as a theorem to be proven. The *principle of virtual work* is used in statics for solution of a special class of problems involved in system in equilibrium.

Generally speaking:

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The necessary and sufficient condition for equilibrium of a mechanical system without friction is that the virtual work done by the externally applied forces *f* is zero.

$$\sum f_i \cdot dr_i = 0$$

On the whole:

a) The necessary and sufficient condition for the equilibrium of a particle is zero virtual work done by all working forces acting on the body during any virtual displacement δs consistent with the constraints imposed on the particle;

b) The necessary and sufficient condition for the equilibrium of a rigid body is that the virtual work done by all external forces acting on the particle during any virtual displacement δs consistent with the constraints imposed on the body is equal to 0.

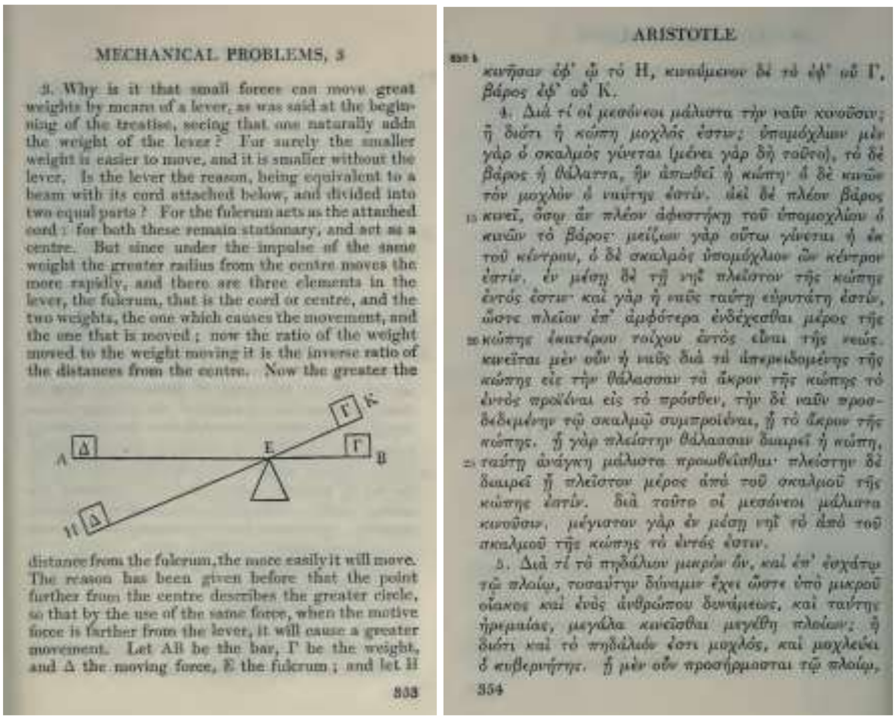
Particularly, if the *principle of virtual work* is applied to a system of rigid bodies (i.e., mechanism) no virtual work is done by internal forces, by reactions in smooth constraints, or by forces normal to the direction of motion. The virtual work is done by reactions when friction is present.

In statics is possible to calculate a force *F* implied in an equilibrium state concerning, i.e., a crank-slider mechanism in the position given by a certain known angle. For example, by knowing essentially a) the position of points of actions of applied forces implied in a given mechanical system, b) the coordinates-position (α, β, γ) of the crank (and distances and others data of the problem) then, according with application of the *Principle of virtual work*, an immediate equilibrium equation can be written using $\delta\alpha$, $\delta\beta$ and $\delta\gamma$.

2 Early Times

Generally speaking, one can consider two main traditions for the formulation of the *principle of virtual work*:

Aristotle (384-322 BC) deals with *virtual velocities* and Jordanus de Nemore (fl. 12th–13th) deals with *virtual displacements*. Particularly in Aristotle:



MECHANICAL PROBLEMS, 3

3. Why is it that small forces can move great weights by means of a lever, as was said at the beginning of the treatise, seeing that one naturally adds the weight of the lever? For surely the smaller weight is easier to move, and it is smaller without the lever. Is the lever the reason, being equivalent to a beam with its cord attached below, and divided into two equal parts? For the fulcrum acts as the attached cord: for both these remain stationary, and act as a centre. But since under the impulse of the same weight the greater radius from the centre moves the more rapidly, and there are three elements in the lever, the fulcrum, that is the cord or centre, and the two weights, the one which causes the movement, and the one that is moved; now the ratio of the weight moved to the weight moving it is the inverse ratio of the distances from the centre. Now the greater the

distance from the fulcrum, the more easily it will move. The reason has been given before that the point further from the centre describes the greater circle, so that by the use of the same force, when the motive force is farther from the lever, it will cause a greater movement. Let AB be the bar, Γ be the weight, and Δ the moving force, E the fulcrum; and let H

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ARISTOTLE

850 b κινῆσαν ἐφ' ᾧ τὸ Η, κινούμενον δὲ τὸ ἐφ' οὗ Γ, βάρος ἐφ' οὗ Κ.

4. Διὰ τί οἱ μεσόνεοι μάλιστα τὴν ναῦν κινοῦσιν; ἢ διότι ἡ κώπη μοχλός ἐστιν; ὑπομόχλιον μὲν γὰρ ὁ σκαλμὸς γίνεται (μένει γὰρ δὴ τοῦτο), τὸ δὲ βάρος ἡ θάλαττα, ἣν ἀπωθεῖ ἡ κώπη· ὁ δὲ κινῶν τὸν μοχλὸν ὁ ναύτης ἐστίν. ἀεὶ δὲ πλέον βάρος κινεῖ, ὅσῳ ἂν πλέον ἀφεστήκῃ τοῦ ὑπομοχλίου ὁ κινῶν τὸ βάρος· μείζων γὰρ οὕτω γίνεται ἡ ἐκ τοῦ κέντρου, ὁ δὲ σκαλμὸς ὑπομόχλιον ὢν κέντρον ἐστίν. ἐν μέσῃ δὲ τῇ νηῒ πλεῖστον τῆς κώπης ἐντός ἐστιν· καὶ γὰρ ἡ ναῦς ταύτῃ εὐρυτάτη ἐστίν, ὥστε πλεῖον ἐπ' ἀμφότερα ἐνδέχεσθαι μέρος τῆς κώπης ἑκατέρου τοίχου ἐντὸς εἶναι τῆς νεώς. κινεῖται μὲν οὖν ἡ ναῦς διὰ τὸ ἀπερειδομένης τῆς κώπης εἰς τὴν θάλασσαν τὸ ἄκρον τῆς κώπης τὸ ἐντὸς προϊέναι εἰς τὸ πρόσθεν, τὴν δὲ ναῦν προσδεδεμένην τῷ σκαλμῷ συμπροϊέναι, ᾗ τὸ ἄκρον τῆς κώπης. ᾗ γὰρ πλείστην θάλασσαν διαιρεῖ ἡ κώπη, ταύτῃ ἀνάγκη μάλιστα προωθεῖσθαι· πλείστην δὲ διαιρεῖ ᾗ πλεῖστον μέρος ἀπὸ τοῦ σκαλμοῦ τῆς κώπης ἐστίν. διὰ τοῦτο οἱ μεσόνεοι μάλιστα κινοῦσιν. μέγιστον γὰρ ἐν μέσῃ νηῒ τὸ ἀπὸ τοῦ σκαλμοῦ τῆς κώπης τὸ ἐντός ἐστιν.

5. Διὰ τί τὸ πηδάλιον μικρὸν ὄν, καὶ ἐπ' ἐσχάτῳ τῷ πλοίῳ, τοσαύτην δύναμιν ἔχει ὥστε ὑπὸ μικροῦ οἴακος καὶ ἑνὸς ἀνθρώπου δυνάμεως, καὶ ταύτης ἠρεμαίας, μεγάλα κινεῖσθαι μεγέθη πλοίων; ἢ διότι καὶ τὸ πηδάλιόν ἐστι μοχλός, καὶ μοχλεύει ὁ κυβερνήτης. ᾗ μὲν οὖν προσήρμοσται τῷ πλοίῳ,

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Fig. 1. Small forces can move great weights[6]

The main Aristotelian *simplicity hypothesis*, concerning virtual work and correlated principle of least action, was based on the actual movement of a body as a natural motion, tending to minimize the motion of a particular material body (i.e., one can see Aristotelian inertia). In Jordanus[7] de Nemore's mechanics (de Nemore [1533]1565a), also known as the principle of virtual displacement, a physical system (e.g., masses subjected to forces) is in equilibrium state if and only if the (forces–)weights are in inversely proportional to their virtual displacements. In his words:

[6] Problemata Mechanica 850a 30 in Aristotle [1936]1955, p. 353. See also: Renn, Damerow and McLaughlin 2003.

[7]See also Jordanus de Nemore in Tartaglia's edition (Tartaglia 1565b; see also Clagett and Moody). Moreover, both *Elementa Jordani super demostrationem ponderum* (1229) and *Liber de ratione ponderis* (fl XIII c.) show an interesting proof of Archimedes' law of the lever, *Quaestio Sexta* (*Liber de ratione ponderis,* in Tartaglia's *Iordani Opvscvlvm de Ponderositate* edition: Tartaglia 1565b, p. 5(–6), line 13), by means of the application of the *principle of virtual work* and where the fall of geometric directions of *displacements* is considered vertical by Jordanus de Nemore. Moreover, one can also see Jordanus de Nemore's *Supposiotio Sexta* (Tartaglia 1565b, p. 1, line 13; see also *Liber de ratione ponderis* edited by Clagett and Moody, pp. 174–175) where one can read that a body is able to raise another lighter body if a lever is utilized, that is, like an embryonic engine.

If two weights descend along diversely inclined planes, then, if the inclinations are directly proportional to the weights, they will be of equal force when descending [idem force – equilibrium][8].

Particularly Jordanus de Nemore[9] studied *principles of virtual displacement* in the following way (Tartaglia 1565b, *Quaestio Sexta*, pp 5–6; see Fig. 1bis):

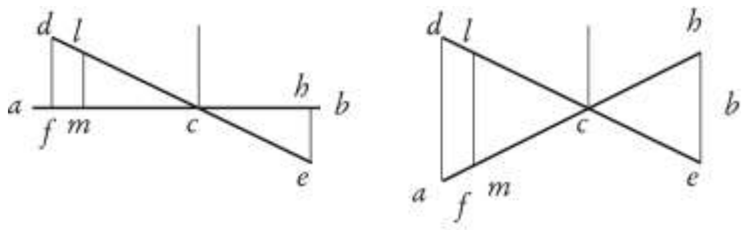

Fig. 1*bis*. Simplified performance model of *Quaestio Sexta* by Jordanus de Nemore[10]

The *principles of virtual displacement*, adapted to force(–weight) and lever, claim that a (virtual) rise *h* of a body *p* placed on an arm of the lever should correspond to a (virtual) lowering *H* of a body *P* placed on the other arm of the lever, so that the relation $ph=PH$ should be valid. In short, two facts related to Jordanus de Nemore's statement (*Ibidem*) should be noted:

a) No reference to the time during *virtual displacement* is considered.

b) De Nemore's principle suggests two trends. If the body goes up, instead going down, then the *virtual work* is negative.

Therefore, the principle proposed by Jordanus de Nemore is a principle of equivalence, or conservation, which is not in a condition of equilibrium. In other words, the equilibrium condition must be obtained for *reductio ad absurdum* (as below discussed). The examination of the proof of the law of the lever reported by

[8] "Quaestio decima. Si per diversarum obliquitatum via duo pondera descendant, fueritque declinationum et ponderum una proportio eodem ordine sumpta, una eritutriusque in discendendo [idem force – equilibrium]" (Tartaglia 1565, p. 7, line 1). See also Tartaglia 1554, Quesito XV–Def. XII; Quesito XVI–Def. XIII, p. 84, line 7 (Pisano and Capecchi 2008, 2010; Capecchi and Pisano 2010a, 2010b; Pisano 2009c, 2008).

[9] Jordanus de Nemore's ideas will have a foundation in René Descartes' (1596-1650) algebra.

[10] Tartaglia 1565b, pp. 5–6.

de Nemore, in his *Liber de ratione ponderis* (*Iordani Opvscvlvm de Ponderositate* edition: Tartaglia 1565b, pp 5–6), addresses this elegant method in order to improve the theory and make it more profound. Nevertheless, we should note that the *principle of virtual displacement* is not mentioned explicitly; it is part of a proof. For instance, the proof of an inclined plane obtained by implementing the *principle of virtual displacement* is an emblematical reasoning and thanks to that principle, the procedure is correctly analysed for the first time. For an application of *Principle of virtual work* within *Science of weights* and related to the lever we have: let us consider a lever where two bodies having mass P and Q are applied at the end points of said lever. These bodies are inversely proportional to the length of the arms.

This statement is commonly expressed by $M=Fd$, where F is, e.g., one of the two force–weights ($F=mg$, related to one of the masses), d is the distance between the two force–weights and the fulcrum, and M is the consequent turning force[11]. Based on the principle of equivalence, mass P can be substituted by a body having mass equal to Q, at distance q from the fulcrum of the lever, and placed on the same side of P, since $Qq=Pp$ should be valid. In this way, thanks to the *principle of sufficient reason*[12]. Nevertheless, this situation logically implies that the lever was in an equilibrium state also before substituting mass P with mass Q. In order to better introduce the *principle of virtual work*, let us see an application of Jordanus de Nemore's reasoning (Pisano and Capecchi 2007; Capecchi and Pisano 2007, 2008, Pisano and Capecchi 2013):

[11] I precise that M is the moment (or rotation). It is not the *Momentum*. The moment generally corresponds to a measure of an effect caused by a physical quantity around a certain axis. The *Momentum* (Galileo 1890–1909, II, pp. 159–161) is a physical property. Nowadays, in classical mechanics is the product of the mass and velocity of an object: $p=mv$. On *Momentum* see also Galluzzi (Galluzzi 1970) and Galluzzi and Torrini (Galluzzi and Torrini 1975–1984).

[12] Where, i.e., anything that happens does so for a reason no state of affairs can be obtained, and no statement can be true unless there is sufficient reason why it should not be otherwise. This principle is implicitly used until the born of the modern statics. For sake of brevity I do not discuss the evident importance of this principle reminding to the reader to so extensive secondary literature. I only precise that the bases of the first Archimedes' law *(Equilibrium Plane*, Archimedes 2002) are mainly two: 1) epistemological one related with two equal weights et equal distances – that is *principle of sufficient reason* – and 2) the two weights are in equilibrium (Capecchi and Pisano 2010a).

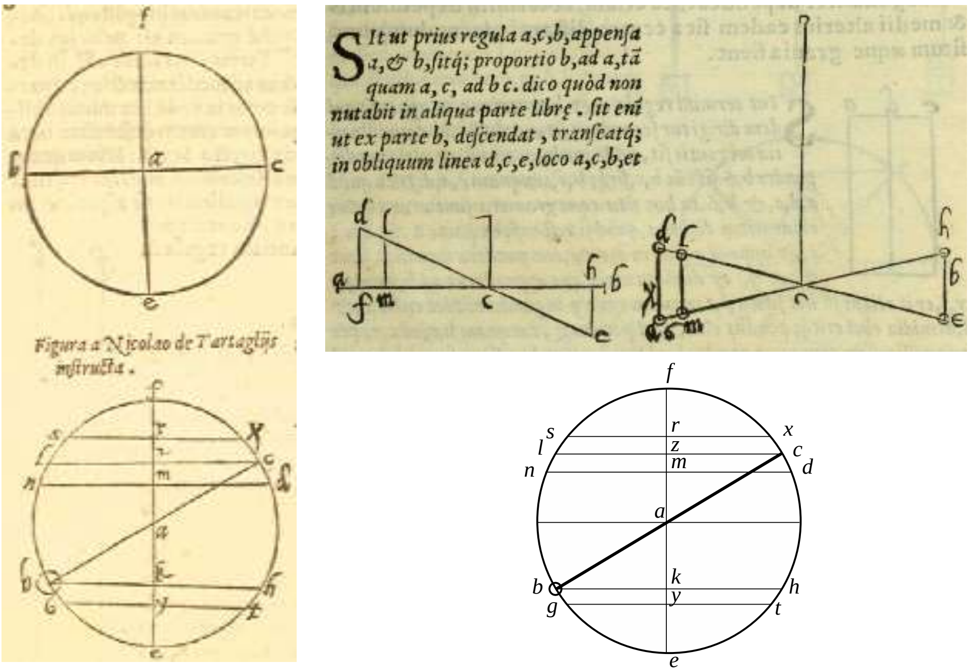

Fig. 2a–2b–2a*bis* Application of the principle of the concept of *gravitas secundum situm* by de Nemore[13]

Let us now consider Nemore (and Tartaglia)'s application to the principle to *gravitas secundum situm* (positional gravity) where

> […] an elementary physical observation: a body acts not only according to its weight, but also to the position in which it is placed.[14]

Let us assume that the body having mass *P* moves down along the arc of the circumference (Fig. 2c) from point *h* to point *z*, thereby describing the arc of circumference *hz*. The path traced by *P*, not along the arc of circumference, but along the diameter of the circle (Fig. 2c), is $h'z'$. Moreover, let us assume, e.g. that the body having mass *P*, in another circumstance moves down along the arc of circumference *ak*=*hz* (Fig. 2c) where the path traced along the diameter of the circle is ok'.

Therefore, since *hz*=*ak*, we obtain that

$$h'z' < o'k'$$

[13] As reported by Tartaglia in *Jordani opuscolvm ponderositate* (published *posthumous*, 1565): Tartaglia 1565b, pp. 3–5, p. 7. The figures should be read from left to right. 2a*bis* is a simplified performance model of concept of *gravitas secundum situm* by Jordanus de Nemore.

[14] Festa and Roux 2008, p. 203.

so the fall of the body having mass *P* along the diameter (Fig. 2c) is more "oblique" because it travels over a shorter segment ("less direct path")**.** Of course *oblique*, means that body has more/less *gravitas secundum situm* with respect another body-position along the circumference; it depends on the geometry of the problem. Here, for my aim related to next Carnot case study it is not necessary longer to deal with it[15]. Just to mention "Ingenious reasoning, but wrong" like Marshall Clagett (1916–2005) claimed in his *The science of mechanics in the middle ages* (Clagett 1959, p. 76 line 18). In other words, Jordanus de Nemore – in *Elementa Jordani super demostrationem ponderum* and partially also in *De ratione ponderis* – improperly applied the concept of *positional gravity* (*gravitas secundum situm*) when he reasoned upon displacements along a circumference (Capecchi and Pisano 2008; Pisano 2011).

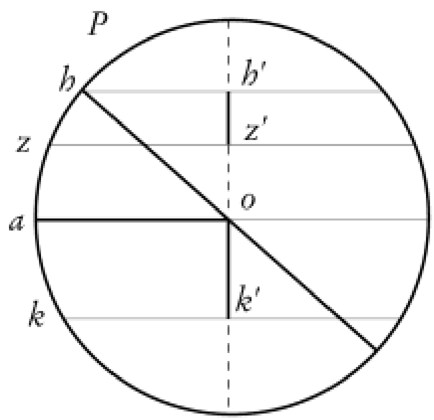

Fig. 2c Simplified performance model of *concept of gravitas secundum situm* by Jordanus de Nemore

On that, very interesting are the reasonings[16] exposed by Jordanus de Nemore in both *Liber de ratione ponderis* and in *Iordani Opvscvlvm de Ponderositate* (Tartaglia 1565b, pp. 3–5, see also p. 7), and by Tartaglia in *Quesiti ed inventioni diverse* (Tartaglia 1554, pp 89–93). Let us see some example.

In the *Suppositio V*[17] in *Iordani Opvscvlvm de Ponderositate* edited by Tartaglia (and in *Liber de ratione ponderis* as well) Jordanus (thus Tartaglia) reasons that a rectilinear segment intercepted along the vertical

[15] In others works I proposed a deep account. See recently Pisano and Capecchi 2014.

[16] Pisano 2007; Pisano and Capecchi 2014.

component of the arc's virtual path suggests the *principle of virtual displacement*. Nevertheless, it is easy to verify that this reasoning is only valid if vertical segments are used. If one considers the entire diameter as the beam of the lever and considers small arcs (ca. infinitesimal) along the circumference, then the difference of their vertical projections (at limit values) is zero. Therefore, the displacements become the circumference and mutually them; therefore vertical components are also equal. Let us see some details of Tartaglia's (or de Nemore's) reasoning in *Quaestio sexta* of *Iordani Opvscvlvm de Ponderositate* (Tartaglia 1565b, p. 5) and in *Propositione* V and VI of *Quesiti ed inventioni diverse* (Tartgalia 1554, XXXII–XXXIII, Props V–VI, pp 89–91, XLI, Prop. XIIII, pp. 96–97). Tartaglia edited Jordanus de Nemore's reasoning on the equilibrium of a lever, using principles of virtual work. Let there be a lever *ACB* (Fig. 3) having fulcrum *C* and, at the end of it, *A* and *B* are applied to two bodies *a* and *b* (Fig. 3). The following relation is therefore valid:

$$b : a = AC : BC \tag{1}$$

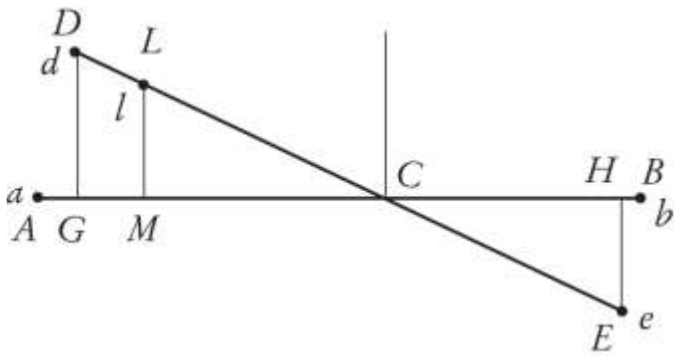

Fig. 3. Simplified performance model of the lever

Therefore, the lever is in an equilibrium state. From an epistemological point of view, with regard to Fig. 3, an *ad absurdum* proof is based on the following two main assumptions:

[17] "Quinta: Obliquiorem autem descensum, in eadem quantitate minus capere de directo [A more oblique descent is one which, in the same distance, partakes less of the vertical]". (Tartaglia1565b, p. 3, line 12). See also *quarta suppositio*, "Quarta: Secundum situm grauius esse cuius in eodem situ minus obliquus descensus" [It is heavier in position when in that position its path of descent is less oblique]. (Tartaglia 1565b, p. 3, line 10).

1) Based on common experience, in a physical lever system the masses (or force–weights) are inversely proportional to the length of the arms. Therefore, it assumes the following configuration: (a) equilibrium or (b) non–equilibrium. For situation (b) a necessary and logical condition follows: one of the two masses at the end of the lever produces an inclination of the lever toward its side.
2) If either of the masses descends, then it raises a mass equal to itself which is distant from the fulcrum, creating a displacement equal to that of its descent.

Let us describe, e.g., by *reductio ad absurdum,* equation (1) assumed in the above-cited configuration (b) at point 1): the lever is not in equilibrium state. This is the ad *absurdum* hypothesis.

Therefore, in that condition of non–equilibrium, the lever should necessarily incline toward one of two sides. Let us suppose that it inclines toward *B* (Fig. 3). In this way, the physical system lever–masses gets the new configuration *DCE* (Fig. 3). If we consider the same masses of previous configuration *ACB*, ($d=a$ in *D* and $e=b$ in *E*), then it is possible to draw the perpendicular *DG* on *AC in descent.* Therefore, by the same method of drawing, the *rise* of the perpendicular *EH* on *CB* is possible. It is clear that both the *descent* and *rise* along the arcs of circumference AB and DE, and rectilinear displacements $\overline{AB}$ and $\overline{DE}$ are considered[18]. In Fig. 3, due to the law of the similitude of triangles *DCG* and *ECH*, one can obtain:

$$DC : CE = DG : EH \tag{2}$$

Moreover, the following relation should also be valid:

$$DC : CE = b : a \tag{3}$$

Therefore

$$DG : EH = b : a \tag{4}$$

At this point, the author considers a point *L* on segment *DC*. The proof assumes an *ad hoc* procedure: point *L* is made to be symmetric to *E* with respect to fulcrum *C*. In *L*, a mass *l* equal to mass *b* is added. Here, it

[18] Let us note that these segments are physically the vertical components of the displacement vector.

seems that Jordanus de Nemore would like to verify what happens to an imagined (virtual) motion of a certain mass in a certain point of beam during the previous configuration. In this sense, rise *e* until *B* corresponds to *descent l* until *M*. In other words, thanks to the geometric construction (and an *ad hoc* procedure) of the problem, one can obtain that *LM=HE*. Again, the law of the similitude of triangles establishes the following relation:

$$\frac{LM}{DG} = \frac{CL}{CD}$$

Therefore

$$DG : LM = b : a\,.$$

Since *l=b*, the following conclusion is obtained:

$$a : l = LM : DG$$

Therefore, masses *a* and *l* are inversely proportional to their vertical (opposite) displacements toward the elevated side. In effect, these displacements concern the reasoning upon the virtual rotation of the lever. At this point, Jordanus de Nemore[19] reasons "Therefore, what suffices to lift *a* to *D*, will suffice to lift [mass] *l* through the distance *LM*"[20]. Since *l=b*, and, also *LC=CE=CB* equilibrium is obtained. In this reasoning, since *b* should not be ("sufficient"– ly) able to displace *l* of a quantity *LM* which is equal to the symmetric of *b*, then *b*, due to a logical consequence, should not even be ("sufficient"– ly) able to displace *a* of a quantity *DG*. Nevertheless, for (Eq. 1) and since it is not possible to presuppose it *a priori*, the lever cannot assume configuration *DCE*. It is evident that the proof is based on an Aristotelian approach, even though certain aspects differ. In fact, with respect to the Aristotelian approach, 1) which considers the displacements along

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[19] Let us remark that Tartaglia (Tartaglia 1554, *Q. XXXII, Prop. V*, pp. 90–91) cited the non–exactness of the Aristotelian rule but he does not cite the previous correction made by Jordanuse de Nemore.

[20] "Quod ergo sufficit attollere a in D, sufficit attollere l secundum LM." (Tartaglia 1565b, p. 6, line 3). In other words: "to displace a mass *p* to height *h* is equivalent to displacing a mass *q=p/k* to height *hk*, whatever is *k*". This statement is, in some way, connected with the principle of the impossibility of perpetual motion.

circular arcs, de Nemore also reasons on *virtual* rectilinear paths; 2) in Supposition IV – "It is heavier in position when in that position its path of descent is less oblique"[21] – he considers the descent along an inclined plane. Thus, according to Giovanni Vailati[22] (Vailati 1896–1897, p 15; see also pp 1–25) Jordanus de Nemore deals with, albeit in an embryonic stage, the problem of the inclined plane (afterwards Tartaglia corrected some aspects). 3) The third instance follows. Initially, it seems that an *ad abusurdum* proof was not necessary –it could have been adopted only for the sake of simplicity. That is to say, *virtual displacement* could be considered a difference of positions, or the mathematical displacements of *a* and of *b* and not as an effect of certain forces (at the time, the concept of force was not well physically and mathematically defined). In order to do so, a more explicit reasoning[23] than Eq. (4) on the virtual displacement is necessary. By using the aforementioned quote: "Therefore, what suffices to lift *a* to *D*, will suffice to lift [mass] *l* through the distance *LM*"[24]. In effect, Eq. (4) effectively establishes a criterion of equivalence applied to the lever. In other words, the work by a certain Force–weights for a body *a* (F_p^a), and considering a certain displacement (*DG=rise*), is equal to work accomplished by the same Force–weights for a body *b* (F_p^b) to displace the same quantity (*LM=rise*), the body *b* symmetrically positioned to *a* with respect to the fulcrum. From a mathematical point of view (Fig. 1) and by considering virtual displacement extended to the principle of virtual work, one can obtain:

$$L^a = (F_p^a \cdot DG) = (F_p^b \cdot LM) \qquad (5)$$

The two works, thanks to the different orientations of the displacements, have opposite signs, and are equal if they are considered in absolute value. Therefore, the total work is null. In this sense, the third cited proof is

[21] "Quarta: Secundum situm gravius esse quando in eodem situ minus obliquus est descensus" (Tartaglia 1565b, p. 3, line 10).

[22] In this regard, Jordanus de Nemore – using some variants on the positional gravity – also applied (*Liber de ratione ponderis*) this concept to the angular lever, reasoning upon the equilibrium of an inclined plane.

[23] This reasoning was criticized by Simon Stevin. The latter was opposed to it, since it was absurd to reason on the fact that a *situation of equilibrium* could be derived from a *situation of motion* (Capecchi and Pisano 2007, 2010; Pisano 2010b; Radelet de Grave 2007, 1996).

[24] Tartaglia 1565b, p 6, line 3.

in conflict with Aristotle's claim that there is a reason to claim the equality in the Equation (5). In fact, since the total work is null, even an engine is not necessary and if an engine is lacking, a motion is also lacking[25]. Therefore, equilibrium is assumed. It should be noted that Tartaglia's reasoning (Tartaglia 1554, pp. 89–91) upon *gravitas secundum situm* in *Quesiti et inventioni diverse* is similar to Jordanus de Nemore's reasonings in *Iordani Opvscvlvm de Ponderositate*.

3. Late Times

Explicit comments regarding the principle of virtual work are also reported by Galileo in *Le Mecaniche* (Galileo 1890–1909, II, pp. 155–191) and in *Discorsi intorno alle cose che stanno in sù l'acqua* (Galileo 1890–1909, IV, pp. 3–141). Particularly, in this latter manuscript, Galileo clearly attributed the law of virtual velocity to Aristotle (Aristotle [1936]1955, 847a 10–15, 847b 10, pp. 329–332) also adding that the idea of the principle of virtual work was born thanks to the observation of the motion of points, which rotate along a circumference. Galileo also dealt with the law of virtual displacement in more than one situation[26].

Recent notable historical studies have considered Isaac Newtonian's contributions (as a paradigm in history), as expounded in his 1687 masterpiece *Philosophiae naturalis principia mathematica*[27] (Newton [1686–7] 1803), as the climax of classical science and mechanical scholars of the Enlightenment added little to it (Blay, et al. 1998). Today the tendency is the realization that this analysis is misleading and that this period, far from being a dark century, was filled with fundamental contributions which helped establish the majority of mechanical concepts. On the other hand, e.g., the change of the role played by practical mathematics in the understanding of the world (Henry 2011, pp. 193–196; Westman 1980, Jardine 1988; Høyrup 1994) is also discussed (Capecchi and Pisano, 2010a, 2010b, 2007, 2008; Pisano and Capecchi 2013).

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[25] On the development of engines, recently: Pisano and Bussotti 2014c.

[26] Galileo 1890–1909 II, pp. 240–242, IV, pp. 68–69; VIII, pp. 310–331, pp. 329–330. See also: Pisano 2010a, 2009a, 2009b, 2009c; Pisano and Bussotti 2012, 2014d.

[27] With regard to Newton's principia, his mechanics and mathematics and science in context I refer to indispensable works by Guicciardini (Guicciardini 1989, 1998, 1999, 2002, 2011). Further a good French edition was published by panza (Panza 2004).

It is known that huge Newtonian mechanics – under certain physical standpoints – only allowed the study of the motion of material points free in space with an incompletely developed mathematical apparatus which was based on a calculus and methods – sometimes – few understood by several scholars at that time, too. That is one of the main reason of the emergence of the next commented editions (Bussotti and Pisano 2014a, 2014b). In fact,

> The mathematization of many problems posed prematurely by Newton in the *Principa* became possible (e.g., in the moon theories of Euler, Clariraut and d'Alembert developed in the 1750s) thanks to calculus which was neither Leibnizian nor Newtonian: is would be better to call it "Eulerian calculus.[28]

Further, problems related to systems of constrained points remained unapproachable, as did the study of continuum bodies both rigid and deformable. Moreover, Newtonian science had to face, mainly on the Continent, people who were reluctant to follow his reasoning; Cartesianism was still dominant and the religious metaphysics behind his work was not well respected (Bussotti and Pisano 2013; Panza 2005; Panza and Malet 2006). This situation promoted a profound innovation in dynamics as formulated by Newton. Newton's main concepts, that of force included, remained dominant among scientists but their interpretation changed and a different approach based on work and energy became a serious contender. Competition was not based on a more or less appealing ontology but on a simpler or more complex mathematical formulation. Before beginning any considerations regarding XVIII century mechanics, it is essential to refer to Newton's own mechanics, which should be ideally located in the XVII century. Only in this way will it be possible to identify the distinctive features of Lagrange's account (Capecchi and Drago 2005; Bussotti 2003) Euler (Panza 2007), Carnot (Gillispie and Pisano 2014) etc.'s mathematics (Alvarez and Dhombres) and mechanics (Capecchi 2011, 2012). It will become clear that in what is referred to today as Newtonian mechanics, only a few aspects can be recognised as Newton's proper mechanics, as defined in his *Philosophiae naturalis principa mathematica*. Newton assumed the following laws of mechanics which he referred to as laws, probably to stress that he considered them to be of an experimental nature (Newton [1686–7] 1803).

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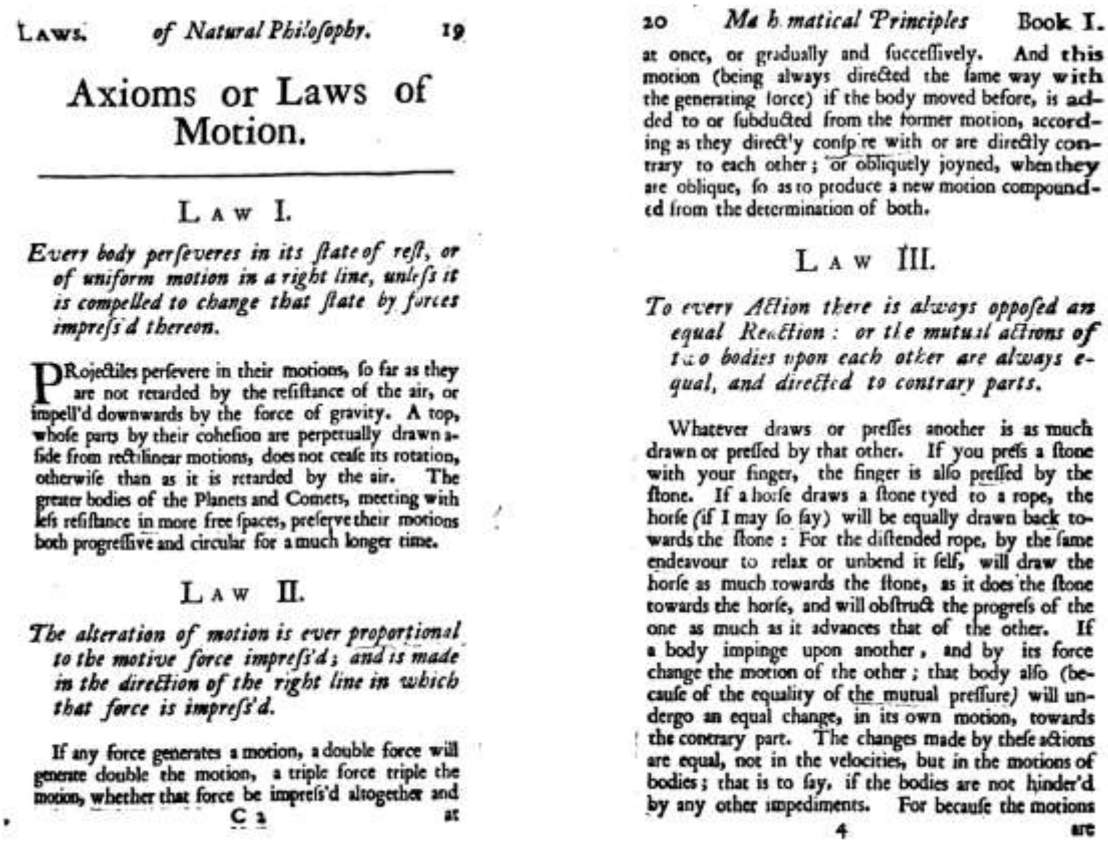
[28] Guicciardini 1999, p. 259. Author's italic.



LAWS. *of Natural Philosophy.* 19

Axioms or Laws of Motion.

LAW I.

Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impress'd thereon.

PRojectiles persevere in their motions, so far as they are not retarded by the resistance of the air, or impell'd downwards by the force of gravity. A top, whose parts by their cohesion are perpetually drawn aside from rectilinear motions, does not cease its rotation, otherwise than as it is retarded by the air. The greater bodies of the Planets and Comets, meeting with less resistance in more free spaces, preserve their motions both progressive and circular for a much longer time.

LAW II.

The alteration of motion is ever proportional to the motive force impress'd; and is made in the direction of the right line in which that force is impress'd.

If any force generates a motion, a double force will generate double the motion, a triple force triple the motion, whether that force be impress'd altogether and

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at once, or gradually and successively. And this motion (being always directed the same way with the generating force) if the body moved before, is added to or subducted from the former motion, according as they directly conspire with or are directly contrary to each other; or obliquely joyned, when they are oblique, so as to produce a new motion compounded from the determination of both.

LAW III.

To every Action there is always opposed an equal Reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

Whatever draws or presses another is as much drawn or pressed by that other. If you press a stone with your finger, the finger is also pressed by the stone. If a horse draws a stone tyed to a rope, the horse (if I may so say) will be equally drawn back towards the stone: For the distended rope, by the same endeavour to relax or unbend it self, will draw the horse as much towards the stone, as it does the stone towards the horse, and will obstruct the progress of the one as much as it advances that of the other. If a body impinge upon another, and by its force change the motion of the other; that body also (because of the equality of the mutual pressure) will undergo an equal change, in its own motion, towards the contrary part. The changes made by these actions are equal, not in the velocities, but in the motions of bodies; that is to say, if the bodies are not hinder'd by any other impediments. For because the motions

4 are

Fig. 4 Newton's laws[29]

DEFINITION III. *The* vis insita, *or innate force of matter is a power of resisting, by which every body, as much as in it lies, endeavours to preserve in its present state, whether it be of rest, or of moving uniformly forward in a right line*.[30]

[29] "Axioms; or Laws of Motion. Law I. *Every body perseveres in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon*; Law II: *The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed*; Law III: *To every action there is always opposed an equal reaction: or the mutual actions of two bodies upon each other are always equal, and directed to contrary parts*." (Newton [1686–7]1803, I, pp. 19-20; *Italic style* and capital letters belong to the author). (Newton [1686–7]1803, I, p 2; author's *italic* style and Capital letters).

[30] Newton [1686–7]1803, I, p. 2; author's *italic* style and Capital letters.

These laws are quite familiar to a modern reader even though some particularities[31], both formal and substantial, do not go unnoticed, mainly for Law II. First, the famous formula $f = ma$, commonly known as Newton's second law, is – with respect nowadays – differently proposed. Mass is not named explicitly but is absorbed in the quantity of motion; in the end no reference is made to acceleration. Scrutiny shows that impressed force, apparently the only element identifiable in the second law, cannot be identified with the modern concept of force. Indeed, integration of the law of motion, considered in the modern mathematical sense as $f = ma$, over a finite interval of time produces

$$m\Delta v = \int f dt$$

where the first part is the variation of the quantity of motion, or according to Newtonian terminology, the alteration of motion. A comparison of the analytical expression just obtained with Law II of motion, shows that what Newton calls force must be equal to $\int f dt$. Newton chose the word *force* to indicate a founding quantity of dynamics, but he did not connect it to any of the concepts known as *force* today. Newton's force is quite far to the whole force introduced by previous scientists such as Descartes (Schuster 2000, 2013a, 2013b; Bussotti and Pisano 2013) and Torricelli (Capecchi and Pisano 2007; Pisano 2009b). This concept today is scarcely used and is not referred to by this name; the most common name for $\int f dt$ is the impulse of force f. In *Scholium*, which followed the three laws of motion, Newton wrote about the force of gravity as an example of a force acting continuously. Stated *verbatim*:

> When a body is falling, the uniform force of its gravity acting equally, impresses, in equally particle of time, equal forces upon that body, and therefore generates equal velocity; and in the whole time impresses a whole velocity proportional to the time.[32]

31

32 Newton [1686–7]1803, I, p. 22.

That is, the whole variation of velocity is proportional to the whole force, which is proportional to time. In *Philosophiae naturalis principia mathematica* the whole force can also represent the intensity of an impulse and the action of continuum force, like gravity, for instance, is usually described as a sequence of impulses, divided by a constant time step Δt, which approaches zero (as the sequence of impulses) goes to infinity. *Nevertheless what kind of foundational and logical problems still remained unsolved?* Leaders of classical mechanics in the Enlightenment age, if we do not consider masters Newton and Leibniz, who were more part of the XVIII century, were Jean d'Alembert (1717–1783), Joseph Louis Lagrange (1736–1813), Leonhard Euler (1707–1783), Lazare Carnot (1753–1823), Pierre–Louis Moreau Maurice de Maupertuis (1698–1759), Jakob Bernoulli (1655–1705), Johann Bernoulli (1667–1748), Daniel Bernoulli (1700–1782), and Jakob Hermann (1678–1733). Each of them provided a contribution which marked the development of mechanical science. There are different, not completely distinct, points of view regarding the historical development of mechanics: the kind of mathematical approach, the kind and logical state of principles, the model of force. In the following section I will briefly present these different points of view.

At the beginning of the XVIII century Newton was surely seen as one of the most prominent mathematicians and physicists but not as the one who established dynamics in its final form. But it is known that Newton's mechanics was considered unsatisfactory by many scholars from both epistemological and ontological points of view, mostly because of his introduction of forces acting at a distance, which were considered occult entities. More fundamentally, for scientists, Newtonian mechanics was considered limited essentially to the motion of material points free in space, unsuitable for solving problems raised by the technology of time. As an example of the opinions of the period, below are some comments by Daniel Bernoulli and Euler:

> Theories for the oscillations of solid bodies that up to now authors furnished presuppose that into the bodies the single point position remains unchanged, so that they are moved by the same angular motion. But bodies suspended at flexible threads call for another theory. Nor it seems that to this purpose the principles commonly used in mechanics are sufficient, because clearly the mutual dispositions of points are continuously changing.[33]

> But as with all writings composed without analysis, and that mainly falls to be the lot of Mechanics, for the reader to be convinced of the very truth of these propositions offered, an examination of these propositions

[33] Bernoulli D 1733, p. 108.

> cannot be followed with sufficient clarity and distinction: thus as the same questions, if changed a little, cannot be resolved from what is given, unless one enquires using analysis, and these same propositions are explained by the analytical method. Thus, I always have the same trouble, when I might chance to glance through Newton's *Principia* or Hermann's *Phoronomiam*, that comes about in using these, that whenever the solutions of problems seem to be sufficiently well understood by me, that yet by making only a small change, I might not be able to solve the new problem using this method.[34]

Therefore, Newtonian laws alone did not seem sufficient for understanding all laws of motion; more fundamental mechanical laws were necessary. Generally speaking, from an epistemological point of view, the problems faced by XVIII century scientists were less demanding than those faced by Newton concerning the search for the general laws. However, this does not mean they were any simpler. They concerned, for example, the search for the oscillation centre of a rigid body and the study of the vibrations of a chain or a thread. The search for the centre of oscillation was quite a relevant and difficult problem. It was equivalent to finding the length of a simple pendulum within the same period. The problem was substantially solved by Christian Huygens (1629–1695) in his *Horologium Oscillatorium sive De motu pendulorum ad horologia aptato demonstrationes geometricae* (Huygens 1673) by means of the first formulation of the theorem of living forces (*vis viva*). Jakob Bernoulli revisited the subject in 1713, with a completely different and promising approach, in the paper *Démonstration générale du centre de balancement et d'oscillation, tirée de la nature du levier* (Bernoulli Jakob 1703). In it one can find the roots of both D'Alembert's principle and the angular moment equation. Johann Bernoulli also faced these problems beginning at the end of the XVII century; his considerations published in *Opera Omnia* (Bernoulli Johann 1742a; 1742b; see also Radelet de-Grave 2009) where he first introduced the concept of angular acceleration are quite relevant. The problem of the vibrating chain was studied by scientists such as Euler, D'Alembert Johann and Daniel Bernoulli. Among the problems that busied the minds of XVIII century scientists, was the study of the motion of bodies on mobile surfaces such as the motion of a heavy body upon an inclined plane which moves on a plane without friction. Johann Bernoulli (Bernoulli Johann 1742b) studied the motion of a material point from a Newtonian approach by introducing constraint reactions among the external forces, referring to them as immaterial

[34] Euler 1736, *Praefatio*, (Author's *italic* style and Capital letters). See also: Euler [1773]1774; 1749; [1750]1752.

forces, since they were outside the bodies in touch. It should be noted that the assimilation of constraint reactions to ordinary forces was quite common in statics, but in dynamics the problem was much more conceptually complex because reactions were endowed with activity. Euler, who developed principles of mechanics which facilitated the introduction of constraint reactions, tried to avoid their explicit use as much as possible. To conclude, in the solution of the various problems, a unique principle was not referred to and it was sought in analogies with problems which had already been solved.

In the mid–eighteenth century, thanks to the brilliant minds of Euler and Johann Bernoulli, some general principles confirmed the living forces theorem and the minimum action principle. On the living forces theorem after Huygens and Leibniz, there were works by Johann and Daniel Bernoulli, D'Alembert and notably Lagrange[35]. The living forces theorem was only correct for problems limited to one degree of freedom, because it gave only a scalar equation. More interesting but equally incomplete was the minimum action principle. It can be attributed to Pierre de Fermat (1601–1665) who in 1662 in his studies on the refraction of light, in a letter addressed to de La Chambre ("Sunday, January 1, 1662") drew on a theological and moral principle, namely, "[…] nature *always acts along the shortest paths* […]"[36] (Fermat 1891–1922, vol II, CXII [D., III, 5r.] p 458). However, Maupertuis gave it the name and extended it to mechanics in various steps. The final step was referred to in his paper *Les loix du mouvement et du repos déduits d'un principe métaphysique* of 1746:

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[35] Johann Bernoulli, *Theoremata selecta pro conservatione virium vivarum demonstranda et esperimenta confirmanda* (Bernoulli Johann 1727), Daniel Bernoulli, *Remarques sur le principe de la conservation des forces vives pris dans un sens général* (Bernoulli D [1748]1750). D'Alembert's *Traité de dynamique* (D'Alembert [1743]1758; see also Hankins 1970; D'Alembert 1751–1780; 1767) and Lagrange's *Recherches sur la libration de la Lune* (Lagrange 1764). On that and a development of mechanics between 17th and 18th century is previously published (Pisano and Capecchi 2013, pp. 97–121). On the history of mechanics (Truesdell 1968a,b) and also history of thermodynamics (Truesdell 1970, 1976, 1980) Truesdell's works (sometime considered by someone *surpassed*) are – in any case – important examples of historical criticism and reflections.

[36] Author's *italic* style.

> Because of so many people who worked about I hardly dare to say I have discovered the principle on which all these laws are based: which extends also to elastic Bodies, from which the movement of all physical corporeal depends.[37]
>
> That is the principle of *least quantity of action* so wise, so worthy of the Supreme Being, et to which Nature looks so constantly joined, that it observes it not only in all its changes, but also in its permanence. *In the impact of Bodies, the Motion distributes so that the quantity of action, which derives from the change, is the least possible*.[38]

Euler tried to give the least action principle a more precise formulation, without however reaching a general formulation, leaving to metaphysics the decision of whether or not it could be assumed as a general principle, or if it could be decided with the laws of mechanics. It was Lagrange in *Application de la méthode exposée dans le mémoire précédent à la solution des différents problèmes de dynamique* (Lagrange [1762]1870–1873, I, pp. 363–468) who provided a proof of the least action principle based only upon the laws of mechanics, without any metaphysical traces. Despite some important successes in the solution of various problems and the existence of some general principles, a general feeling of disappointment prevailed among the scientists of this period which gave rise to an effort to find simpler and more general principles. This effort began to be fruitful in the second half of the XVIII century and was led by Euler and Lagrange to a nearly finished form, respectively, of *vectorial* and *analytical* mechanics (Panza 2009).

In *Application de la méthode exposée dans le mémoire précédent à la solution des différents problèmes de dynamique* Lagrange's results on his studies of the least action principle, the perfecting of its formulation and his convincing proof were published. The use of the principle, however, is possible only for what should be referred today to as conservative systems; in such a case by the addition of the living force theorem any (discrete) dynamical problem could be solved. However, even before *Application de la méthode exposée dans le mémoire précédente à la solution des différents problèmes de dynamique* Lagrange thought of a principle which was more general than the least action principle. In a letter to Euler on November 24th 1759,

[37] Maupertuis, 1746, p. 286. (Author's *italic* style).

[38] *Ibidem.*

Lagrange[39] wrote about having composed elements of differential calculus and mechanics and having developed the true metaphysics of his principle. In *Recherches sur la libration de la Lune* (Lagrange 1764), for the first time, Lagrange obtained dynamical equations of motion with the aid of *a new law of Mechanics*, the principle of virtual work:

[39] Euler's Correspondence with Joseph Louis de Lagrange. In: *Opera Omnia*. Series IV, Vol I, retrieved via: http://eulerarchive.maa.org/correspondence/correspondents/Lagrange.html *Idem* letter was previously edited in Lagrange 1892, Tome XIV, pp 170–174.


RECHERCHES

SUR

LA LIBRATION DE LA LUNE,

Dans lesquelles on tâche de résoudre la Question proposée par l'Académie Royale des Sciences, pour le Prix de l'année 1764.

I.

CET écrit a pour objet d'examiner les différens mouvemens apparens, ou réels que la Lune peut avoir autour de son centre. Je suppose d'abord que cette Planete à une figure quelconque; & je cherche le mouvement qu'elle doit recevoir de l'action de la terre & du soleil. Quoi qu'un très-grand Geomètre ait déja donné des méthodes & des formules générales, qui peuvent aisément s'appliquer à la recherche dont il s'agit ici, néanmoins il m'a paru plus commode de reprendre la question en entier, & de

Prix de l'Acad. Tome IX. A

III.

"There is a principle generally true in Statics, according to which, if a system of whichever bodies or points, each of them subjected to powers [forces], is in equilibrium and if somebody gives the system a small arbitrary motion, for which each point covers an infinitesimal space, the sum of the powers multiplied each of them for the space covered by the point to which it is applied, in the direction of this power, will be always = 0 [zero].[40]

[In the same article "III" he used the following formulas:]

Dans la question présente, si on imagine que les lignes X, Y, Z, R, R', devionnent, en variant infiniment peu la position de la Lune autour de son centre, $X+\delta X$, $Y+\delta Y$, $Z+\delta Z$, $R+\delta R$, $R'+\delta R'$, il est facile de voir que les différences $\delta X, \delta Y, \delta Z, \delta R, \delta R'$, exprimeront les espaces parcourus en même tems par le point α dans des directions opposées à celles des puissances $\alpha\frac{d^2X}{dt}$, $\alpha\frac{d^2Y}{dt^2}$, $\alpha\frac{d^2Z}{dt^2}$, $\alpha\frac{T}{R^2}$, $\alpha\frac{S}{R'^2}$, qui sont censées agir sur ce point; on aura donc, pour les conditions de l'équilibre, l'équation générale.

$$\int\left(\alpha\frac{d^2X}{dt^2}\times-\delta X+\alpha\frac{d^2Y}{dt^2}\times-\delta Y+\alpha\frac{d^2Z}{dt^2}\times-\delta Z+\alpha\frac{T}{R^2}\times-\delta R+\alpha\frac{S}{R'^2}\times-\delta R'\right)=0.$$

$$\left[\sum f_i\cdot\delta u_i=0\right]$$

Fig. 5 Frontispiece of the *Recherches sur la libration de la Lune* and his formulation[41]

[40] Lagrange 1764, p. 5 (Author's Capital letters). See also Lagrange's arguments on the same topic in the *Théorie de la libration de la lune*: "[…] 1. Le principe donné par M. d'Alembert réduit les lois de la Dynamique à celles de la Statique; mais la recherche de ces dernières lois par les principes ordinaires de l'équilibre du levier, ou de la composition des forces, est souvent longue et pénible. Heureusement il y a un autre principe de Statique plus général, et qui a surtout l'avantage de pouvoir être représenté par une équation analytique, laquelle renferme seule les conditions nécessaires pour l'équilibre d'un système quelconque de puissances. Tel est le principe connu sous la dénomination de *loi des vitesses virtuelles* ; on l'énonce ordinairement ainsi : *Quand des puissances se font équilibre, les vitesses des points où elles sont appliquées, estimées suivant la dilection de ces puissances, sont en raison inverse de ces mêmes puissances*. Mais ce. principe peut être rendu très-général de la manière suivante." (Lagrange 1870–3, V, p. 15; Author's *italic* style and Capital letters; see also *idem* arguments in Lagrange 1764, p. 8).

[41] Lagrange 1764. Nowadays Lagrange's equations are called *Symbolic Equation of Dynamics.* In other passages Lagrange try to remark and generalize his conclusions in a note: "IV. Scholie. Le principe de Statique que je viens d'exposer n'est, dans le fond qu'une généralisation de celui qu'on nomme communément le principe des vitesse virtuelles, & qui est reconnu depuis longtem[p]s par les Géomètres pour le principe fondamental de l'équilibre. M. Jean Bernouilli est le premier, que je sache, qui ait envisagé ce principe sous un point de vue général & applicable à toutes

The progress of the Lagrangian formulation (Pisano 2014, 2013a; Panza 2003) of the principle over that of Johann Bernoulli in 1715 (Capecchi 2012) was significant in many ways. Lagrange's formulation was stated in a clearer way for a system of bodies, because only virtual displacements congruent with constraints were used, and also because the principle was embedded in the newly established *variational* calculus. At any rate, the principle of virtual work alone was not sufficient for founding dynamics. It had to be associated with another principle of dynamics, thanks to D'Alembert. The interpretation of D'Alembert's principle as provided by Lagrange has become classic, although it has little do with the original interpretation (Fraser 1983): accelerating forces (ma) with their signs reversed, balance applied forces.

> [IV …] the Principle of Statics I am introducing, combined with the Principle of Dynamics due to M. D'Alembert gives a kind of general formula which contains the solution of all problems relative to the motion of bodies.[42]

In other terms:

$$\sum f_i \cdot \delta u_i - \sum m_i a_i \cdot \delta u_i = 0 .$$

The principle of virtual work (as well the principle of least action) has the peculiarity of being expressed by a unique symbolic equation. This is a *variational* equation and can be laid down without particular attention paid to the choice of reference systems in quite an automatic manner to avoid most of the geometrical problems necessary to solve in writing the equation of motion in the Eulerian style. It should be noted that the virtual work equation allows for the easy solution of the problem of constraints; it is sufficient to use virtual displacements congruent with them. In this way a static, and even, a dynamic problem, is solved by means of a kinematic study. This still implies the need for some geometric considerations. These were avoided by Lagrange with a typical trick in his calculus of variations; the virtual displacements are

les questions de Statique, comme on le peut voir dans la Section IX. de la nouvelle Mécanique de M. Varignon, où cet habile Géomètre, après avoir rapporté , d'après M. Bernouilli, 1e principe dont il s'agit, fait voir par différentes applications, qu'il conduit aux mêmes conclusions que celui de la composition des forces". (Lagrange 1764, p. 6. (Author's Capital letters)).

[42] Lagrange 1764, p. 8. (Author's Capital letters).

considered free from any constraints which are added in their analytical form to the *variational* equations. For instance if $f(u_1, u_2, ..., u_n) = 0$ is a constraint equation, the *variational* problem has the form:

$$\delta W(u_1, u_2, ..., u_n) + \lambda \delta f(u_1, u_2, ..., u_n) = 0$$

where W is the virtual work; is a function of $u_1, u_2, ..., u_n$, now called Lagrangian multipliers, to which the meaning of constraint reactions can be associated.

Lagrange perfected his approach in *Méchanique analitique*[43] (Lagrange 1788), where in the introduction, he emphasized the absence of any geometric considerations.

> One will find no figure in this work. The methods I will expose do not require neither constructions nor reasonings of mechanical or geometrical nature, but only algebraic operations which develop regularly and uniformly.[44]
>
> In what follows, having present the constraint equations among the coordinates of various bodies, which are given by the nature of bodies, the variation of these variables will be reduced to the smallest number, so that the resulting variations are completely independent each other and absolutely arbitrary. We then will equate to zero the summation of all terms concerned with these variations; and we will have all the equations necessary to find the motion of the system.[45]

As a final comment it must be noted that although the Lagrangian principle of virtual work is usually associated with the first edition of *Méchanique analytique* (1788), its elements were provided completely in *Recherches sur la libration de la Lune*. The other question is that of the principle justification. To this end, the two *Mécanique* editions (1787, 1811) tried to offer a satisfactory proof that was considered too weak by many scientists of the time, as will be shown further ahead. Evangelista Torricelli (1608–1647), in his *Opera geometrica* (Torricelli 1644) claimed to have established a rational criterion for equilibrium, playing a fundamental role in mechanics and in the history of mechanics (Capecchi and Pisano 2007). It can undoubtedly be considered the origin of the modern statement of the principle of virtual work.

[43] This is the original title of the 1788 edition. Next editions (1815, 1853) refers to *Mécanique analytique*.

[44] Lagrange 1788, p. vi.

[45] Lagrange 1788, p. 197.

> Two heavy bodies linked together cannot move by themselves unless their common centre of gravity does not descend.[46]

With regard to Torricelli's principle, one can also consider John Wallis's assumptions (Wallis 1693), and Pierre Varignon's (1654–1722) (Varignon 1725) essential and rigorous formulation as a scientific production which aimed at founding all statics upon an easy geometric principle: the composition of forces. In this sense, it is also alternative to the *principle of virtual work*. Let us note that in his letter to Johann Bernoulli (1667–1748), Varignon also dealt with concept of *virtual velocities*, as components of *virtual infinitesimal displacements* towards the direction of the forces (Bernoulli J 1742, II). After Bernoulli, the most significant contribution to the development of the *principle of virtual work* is probably thanks to Vincenzo Riccati (1707–1775) who tried to establish it based on simple principles easily accepted by his contemporaries, introducing *Principles of actions* in *Dialogo di Vincenzo Riccati della compagnia di Gesù* (Riccati 1749) and in *De' principi della meccanica* (Riccati 1772).

3. Details on the Relationships between Physics and Mathematics in Lazare Carnot's Science

3.1 An Approach to Mechanics

Lazare Carnot's mechanics is an operative type of mechanics (Gillispie and Pisano 2013) and presents a strong attitude to Leibniz's ideas[47], *theoretical physics must explain facts with facts.* The mathematics

[46] "*Praemittimus.* Duo gravia simul coniuncta ex se moveri non posse, nisi centrum commune gravitatis ipsorum discenda. (Torricelli 1644, Liber primus De motu gravium naturaliter descendentium, p 99, line 4". Author's italics). [English translation is ours]; Pisano and Capecchi 2010; Capecchi and Pisano 2010, 2009c; Pisano and Capecchi 2007).

[47] In particular, one can see the concept of collision (adopted by Lazare Carnot) presented by Leibniz in his *Dynamica de Potentia et Legibus Naturae Corporeae* (Leibniz 1849–1863, II, sectio III, proposition 1–18, pp 488–507) and the early concept of potential energy (*Ivi*, II, sectio I, p. 435). E.g., Lazare Carnot introduced an advancement of potential energy in his theory of motion applied to machines (Carnot L 1803a, pp 36–38). On the Leibnizian background in Lazare Carnot, one can also see the famous correspondences in 1677 (*Ivi*, VI, pp. 81–106) between Leibniz and Honoré Fabri (also Honoratus Fabrius, 1607–1688). For a first panoramic view on Leibniz and his dynamics, see Pierre Costabel's (1912–1989) works (Costabel 1960). For the most complete (works and letters) series

introduced is that which is absolutely necessary, adapted to represent a previously established physical argument. His theoretical attitudes are independent from the (physical and mathematical) concept of absolute space, typical of previous predictive mechanics (see Table 1). In fact, Carnot's theory (for example on mechanics) is independent from which positions the objects in question assume and which spatial motions (trajectories) they complete. For example, in Lazare Carnot's works[48], the solutions for the equations of motion are *velocity* and *quantity of motion* (Gillispie and Pisano 2014, chaps 2–4, 11). The concept of space for Carnot reveals itself only with the finite volume of a system. Time is also different from typically Newtonian time. For example, for Carnot, time is not absolute and does not have continual variations. It has only one dualistic variation: before and after. In this regard, it can also be noted that by avoiding the use of Newtonian absolute time and space, Carnot's science also omit the use of physical quantities as non–finite mathematical variables, which in common theoretical physics are fundamental for the infinitesimal calculation of the variations of certain physical quantities. Therefore, from the very beginning, their theories did not contain abstract notions such as absolute space or force–cause. Lazare Carnot's mechanical theory was limited to algebraic and trigonometric equations (because, in this theory, the types of equations of the invariants of motion are to be solved with velocity only). Lazare Carnot made this attitude clear in his second book, *Principes fondamentaux de l'équilibre et du mouvement* (1803a) when he stated that all scientific (and mathematical) notions can only come from experiments.

Table 1. Carnot's physical–mathematical–geometrical approach

Main concepts	Lazare Carnot (1786; 1803a; 1813)
Space and time	No absolute and infinite Newtonian space and time
Physics–mathematics relationship	Only physical
Geometry–physics relationship	Independence from position in space

of Leibniz's mathematical writings, see Eberhard Knobloch's VIII edition for "Berlin–Brandenburgische Akademie der Wissenschaften Leibniz–Edition, Reihe VIII" (Leibniz 2009–).

48

Basic concepts	Velocity, Quantity of motion, limited mechanical Work
Mathematical approach	No local and infinitesimal variables
Main mathematical-geometrical technique	Geometric motion

To be precise, in Lazare Carnot's words:

> Following this idea ["to avoid metaphysical notion of force" and… to use "the theory of communications of motions"[49]] we will soon see, as I previously mentioned, the necessity of turning to the experiment, and that is what I did, without neglecting to support myself with reasonings that can confirm it in the most plausible way, using or generalizing the results per induction. At times I even used the name of the force in the vague sense of which I spoke above [...].[50]
>
> […] *Primitive ideas concerning the matter, the space, the time, the rest, the motion, etc*. 7. The first rule to establish in such delicate research on the laws of nature is to only admit notions so clear that they can comprise the bounds of our logic. We must therefore reject the definitions of *matter*, *time*, *space*, *rest*, and *motion* as expressions that are impossible to express with more clear terms, and the ideas that these expressions produce in us primitive ideas outside of which it is impossible to construct. But once these expressions are admitted, we will easily see that which is a body, speed, a motive force, etc. 8. The body is a given part of matter. 9. The apparent space that a body occupies is called its *volume*; the actual space that this same body occupies, or its real quantity of matter, is called its *mass*. When the body is such that equal parts of its volume always correspond to equal parts of its mass, we say that it has a uniform *density*, or that it is equally *dense* in all of its parts; and the relationship from mass to volume, or the quotient of one times the other, is called the *density of* this body. But if unequal masses correspond to equal volumes, we say that the density is variable and for each particle of matter, we call *density* the volume of this particle divided by its mass, or rather, the last reason of these two quantities. The empty parts or gaps lodged between the parts of the matter, and that make the volume or apparent space greater than the actual space are called *pores*.[51]
>
> [On the concept of force in the theory]. […] in my opinion, no rigorous proof of the parallelogram of forces is possible: the mere existence of the *force* in the announcement of the proposition is able to make this demonstration impossible for the nature of things in itself. "It is extremely difficult", as Euler said, "to reason

[49] Carnot L 1803a, p. XVI, line 5.

[50] Carnot L 1803a, p. XVI, line 10.

[51] Carnot L 1803a, pp. 6–7, line 1. (Author's *italics*).

on primary principles of our knowledge […]". This obscurity disappears in the second way [theory of motion] to conceive the mechanics, but another inconvenience appears; that is the fundamental principles that in the first way [theory of forces where cause produces motion] are established such as axioms in favor of the metaphysical expression […] that is to say, […] force, are, in this second case [theory of motion], nothing less than self–evident propositions, and in order to establish them, we need to include the recourse to the experience.[52]

By considering the role played by physics–mathematics in a scientific theory (Pisano and Gaudiello 2009), Newtonian paradigm[53] until Laplace's physics (Fox 1974; see also Locqueneux 2009) in general one can see that by means of mathematical expressions and by using differential equations a mathematical result is obtained[54]. *But, from physical point of view?*

Newton gave, in the second book of his *Mathematical Principles of natural Philosophy* the expression of the speed of sound: how he achieves this is one of the most remarkable features of his genius.[55]

When the temperature of the air is raised, at constant pressure, only part of the heat is used to produce that effect [to raise the temperature]: the other part, which becomes latent, serves to increase its volume. This latter part of the heat is liberated when the air is reduced to its primitive volume by an increase in pressure. When two air molecules come close together in a vibration, the heat released raises their temperature and tends to radiate out into the nearby area; but if this happens very slowly relative to the speed of vibration, we can suppose that the amount of heat remains the same [for the two molecules]. Thus, as the two molecules approach, they meet a resistant force, first, because since their temperature being supposed constant, their [forces of] repulsions augment in inverse proportion to their distances; and second because the latent caloric which develops increase their temperature. Newton only considered the first of these causes of repulsion; but it

[52] Carnot L 1803a, pp. xiij–xiv, line 17.

[53] E.g, the second Newtonian law is not a strictly physical equation. It is – in modern term – a second order differential equation that would interpret (physically) the law of motion. It does so by a mathematical–physical equation, which, of course, one cannot establish experimentally, as instead can be done by dynamometer to measure magnitudes in a static equation, e.g, Hooke's law (Pisano 2007, 2009a, 2009b).

[54] In the1816 Laplace pointed out that the speed of sound in air depended on the heat capacity ratio and corrected Newton's surprising error (Biot 1858, pp. 1–9, 1802, pp. 173–182).

[55] Laplace 1816, p. 238, line 7; author's *italics* and Capital letters. (Pisano and Casolaro 2011, 2012).

is clear that the second cause must increase the speed of sound, since it increases the pressure. By entering it in the calculation, I come to the following theorem: “The real speed of sound is equal to the product of the Newtonian formula times the square root of the ratio of the specific heat of air at constant pressure of the atmosphere and at different temperatures, to its specific heat at constant volume”[56].

Particularly, Newton’s calculation gave 968 (920–1085) English feet per second (Newton [1686-7] 1803, pp. 371–372), which is ca. 20% shorter than the value of the speed of sound, and later 979 English feet per second appeared (Newton 1714, pp. 343–344). It may have been convenient for the experimental data of the time, but it was undoubtedly too low a value[57]. In effect, the adiabatic compression of the air, which results in a local rise in temperature and pressure, has also been taken into account.

Laplace’s investigations in practical physics were confined to those carried out jointly with Lavoisier on the specific heat of various bodies from 1782 to 1784. It should also be noted that this is a similar technique that Émile Clapeyron would use in 1834 to reformulate Sadi Carnot’s theory – but he would not succeed in doing so with his theorem (Clapeyron 1834, pp. 153–190). Lazare Carnot, although he did not believe in caloric (Carnot L 1990), considered the mathematical technique with the differential to be inaccurate. In fact, he considered infinitesimal analysis (Gillispie 1971, ft. 1, p 12, 1979, pp 251–298, § 13, p 256) to be a very clear mathematical apparatus in and of itself, which varies with continuity by means of concrete variables. However, for differentiated variables in the previous technique, the mathematical problem is the opposite: the aim is to determine function Q by using an abstract calculation. Therefore, as Lazare Carnot explains in a footnote in *Principes fondamentaux de l'équilibre et du motion* (Carnot L 1803a, p. 11, ft. 1), that infinitesimal analysis is not suitable in these cases. A different type of mathematics, in which geometry acquires a greater importance[58], is necessary. Lazare Carnot’s mathematics selects geometric motions, which, by definition, admit their opposites.

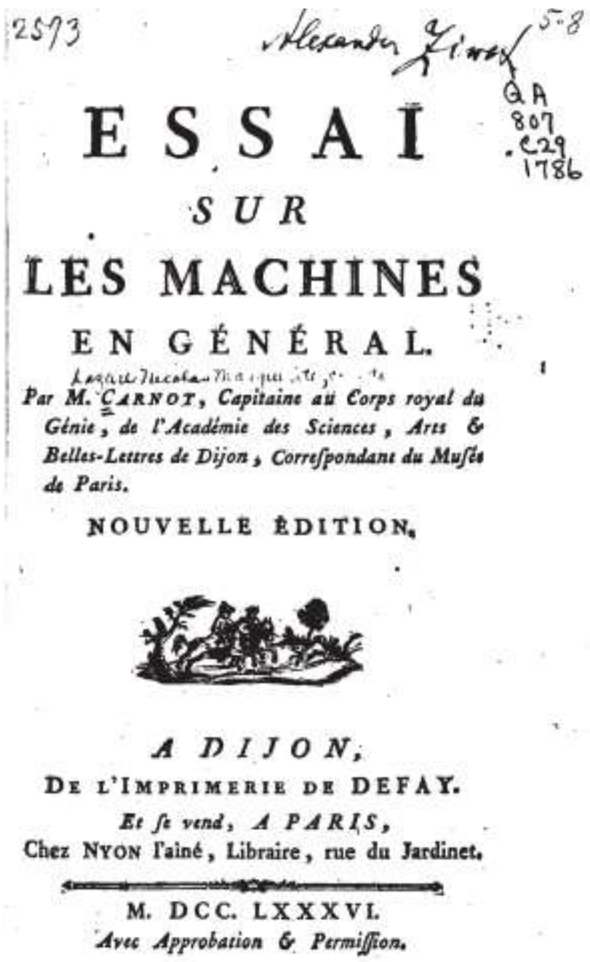
[56] Laplace 1816, pp 238–239, line 24. (Author’s quotation marks).

[57] Finn 1964, ft. 19, p. 8; Newton 1999, pp 772–778.

[58] On a mathematical and geometrical standpoints see: Chemla 1990, 1998; Nabonnand 2010, 2011.

Lazare Carnot[59] is usually considered to be the foremost author who claimed that the empirical nature of mechanics both was theoretical and mechanical (Gillispie and Pisano 2014). Lazare Carnot expressed his view of mechanics in the introductory parts of *Essai sur les machines en général* (Carnot L 1786) and *Principes généraux de l'équilibre et du mouvement* (Carnot L 1803a). This is what Carnot wrote in his *Essai sur les machines en general*:

[59] For the biography see: Gillispie 1970–1980 III, pp. 70–79. On Lazare Carnot's mechanics see the fundamental Gillispie's works: Gillsipie 1971, 1976, Gillispie and Youschkevitch 1979.



ESSAI

SUR

LES MACHINES

EN GÉNÉRAL.

Par M. CARNOT, Capitaine au Corps royal du Génie, de l'Académie des Sciences, Arts & Belles-Lettres de Dijon, Correspondant du Musée de Paris.

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Among philosophers interested in the search of the laws of motion, some makes of mechanics an experimental science, some other makes of it a purely rational science. That is, the former compare phenomena of nature, decompose them, to say, to know what they have in common, and so to reduce them to a small number of main facts which serve in the following to explain all the others, and to anticipate what has to occur in any circumstance. Some others starts from hypotheses, then, by reasoning according to their suppositions, arrive to discover the laws which regulate bodies in their motion; if their hypotheses were conform to nature, they conclude that their hypotheses were exact; that is bodies actually follow the laws that at the beginning they had only supposed.

The former of these two classes of philosophers, start then in their researches from primitive notions which nature has impressed in us, and from the experiences that it offers continuously. the latter starts from definitions and hypotheses. For the former the name of bodies, of powers, of equilibrium, of motion are considered as primitive ideas; they cannot and must not define them; the latter, to the contrary, must attains all from themselves and are obliged to define exactly these terms and to explain clearly all their hypotheses. But if this methods appears more elegant, it is more difficult than the other, because there is nothing more embarrassing in most natural science and especially in this [mechanics] than to assume at the beginning exact definitions deprived of any ambiguity. I would throw myself in metaphysical discussions if I tried to deepen this argument; I will be happy only to examine the first and simpler

[...]

The two fundamental laws from which I started are then purely experimental truths, and I propose them as such. A detailed explanation of these principles is out of the spirit of this work and could serve only but to tangle things: sciences are as a beautiful river whose course is easy to follow, when it has acquired a certain regularity; but if one wants to sail to the source one cannot find it anywhere, because it is far and near; it is diffuse somehow in the whole earth surface. The same if one wants to sail to the origin of science, one finds nothing but darkness and vague ideas, vicious circles; and one loses himself in the primitive ideas.[60]

Fig. 6. Lazare Carnot's *Essai sur les machines en général*[61]

The following a diagram provides a concise reconstruction of Lazare Carnot's main reasonings (Carnot L 1786):

[60] Carnot L 1786, pp. 104–107.

[61] Carnot L 1786.

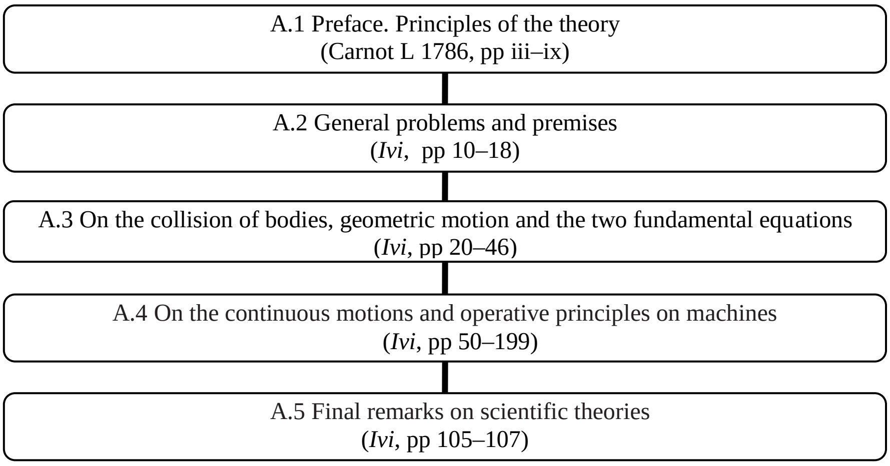

Fig. 7. Lazare Carnot's main reasonings[62]

In the first part Carnot declared his preference for the analytic approach; in the second part he declared the two principles assumed in *Essai sur les machines en général* (the action and reaction and the conservation of *momenta* in the impact) empirical laws. In the introduction of *Principes généraux de l'équilibre et du mouvement* he reasserted his empiric idea:

> Ancients established as an axiom that *all our ideas comes from senses*; and this is no longer object of dispute [....].[63]

But he also expressed the opinion that the laws of mechanics can be considered as either empirical or fully rational:

> 3. This notwithstanding sciences do not derive in the same way their basis from experience. Pure mathematics derive from them less than all the others; then mathematical physic sciences, then physical sciences. [...].
>
> 4. It would be certainly satisfactory, in each science, to be able to decide the point where it breaks off to be experimental and becomes rational; that is to reduce as much as possible the number of possible truths we must obtain from the experience and when accepted are sufficient with the sole reasoning to follows all the branches of the science. But this seems to be too difficult. If one wants to go up too much one will venture to give dark

[62] Carnot L 1786.

[63] Carnot 1803a, p. 2. Author's *italic*.

> definitions ad vague and scarcely clear proofs. There are less drawbacks to obtain from the experience more items of information than those strictly necessary. [...].
>
> It is thus from experience that men derived the first notions of mechanics. This notwithstanding the fundamental laws of equilibrium and motion [...] appears from one hand so natural to reason, and from the other hand they manifest themselves so clearly by means of the most common facts, that it seems difficult to say that is from one instead than from the other that we derive the complete conviction of these laws.[64]

Carnot specified the role analysis plays in the establishment of these laws, which are referred to as hypotheses:

> 72. Now it has to establish upon given facts, and upon other observations which we still could have, *some hypotheses* [italic is ours] which are constantly in accord with these observations and which we can assumes as general laws of nature.[65]

It is not necessary to have concerned phenomena hypotheses which are unrelated to each other:

> 73. My objective is not to reduce them [the hypotheses] to the smallest number; it is enough for me that they were consistent and clear enough [...] but they are the most suitable to confirm the principle [the experimental facts], by showing that they are, as to say, nothing but the same truth which says all the same under different forms.[66]

Carnot assumed seven hypotheses which are summarized in the following Table 2.

Table 2. Some of Carnot's hypotheses (Carnot 1803a)

1	Once at rest a body cannot move by itself and once put in motion it cannot change neither its velocity nor its direction by itself (*Ivi*, p. 49).
3	When many forces, either passive or active, equilibrate themselves, each of this force is

[64] Carnot 1803a, p 3–5.

[65] Carnot L 1803a, pp. 46–47.

[66] Carnot L 1803a, p. 47.

	always equal and opposite to the resultant of all the others (*Ivi*, p. 49).
5	The action that two bodies contiguous exerts each other by impact, pressure or tension, does not depend in any way by their absolute velocity, but only by their relative velocity (*Ivi*, p. 49).
7	When bodies who impact are perfectly hard or perfectly soft, the proceed always together after the impact; that is according to the straight line of their mutual action [...] (*Ivi*, p. 50).

In modern didactic presentations of classical mechanics, force is considered as varying continuously. This was not the case in the XVIII century where the impact of bodies was also considered very important. The relevance of impact was mainly due to the dominant atomistic conception of matter where impact among atoms was the only way to transmit forces. According to Newton's *Opticks* (Newton 1730; Blay 1983) the ultimate constituent of matter was small bodies not completely deformable or hard. Due to this property it was natural to argue, on a rational basis, that when two hard bodies of equal mass and opposite velocity collide they cannot help but stop. When they touch, they must stop because of the impenetrability and they then remain at rest because there is no reason for a rebound. Some scientists however did not accept the hard body model, among them were Johann Bernoulli and Euler who agreed with Leibniz (Bussotti 2014, *forthcoming*). In particular Euler divided bodies in more or less soft (*mollioribus corporibus ut cera vel argilla*) and elastic (*elastica*) (Euler [1730–1]1738, p 161). Independently of its foundation any formulation of mechanics had to address and explain global phenomena where variation of force and motion occurs either continuously or through an impact.

On one hand, Euler, who decided to base mechanics on force acting continuously, considered impact as a continuous process: when two bodies collide they are deformed and exchange forces which continuously vary in time, even if this could happen in a very short time interval. The knowledge of the way the matter warps allows for the discovery of the law of (continuous) force the bodies exchange and for the study of the effect of the impact with the law of a mechanics where force is a continuous quantity, even when bodies are extended and not simple material points. On the other hand, D'Alembert and especially Lazare Carnot, who based their mechanics on impact, considered the continuous variation of force to be due to a sequence of infinitesimal impacts.

3.2 On Lazare Carnot's Concept of Work

Lazare Carnot's approach provided the introduction of force as a physical quantity:

> There are two ways to deduce mechanics from its principle. The first is to consider it as the theory of forces, that is the causes which impress motion. The second is to consider mechanics as the theory of motion in itself.[67]

Lazare Carnot preferred the second way. He was not however against the term *force* which he used quite often; sometimes with a technical meaning, "[...] *will call* moment of activity, *consummated by this force in a given time, the sum of* moments of activity *consummated by it at every instant* […][68]., sometimes following common sense, others even intending the meaning of work. Carnot maintained that as far as the motion of a machine is concerned, force is not the most important concept because the effect it produces also depends on the way it is applied. Carnot used the concept of work to take this way into account (Gillispie and Pisano 2014). He was not the first to do this, but he was the first to emphasize it and give it an operational meaning as a foundation of mechanics, especially for applied mechanics (Gillispie 1976, 1979). The term he used to indicate work was *moment of activity*:

> If a force *P* moves with a velocity *u* and the angle formed by P and u is z, the quantity $P \cos z\, udt$, where *dt* is the element of time, is called *moment of activity* consumed by force *P* during *dt*.[69]

The total moment of activity during a finite interval of time *T* is given by[70]:

$$P\int udt \cos z$$

Lazare Carnot could quite easily formulate, as a corollary, a fundamental result of his mechanics: the conservation of work:

[67] Carnot L 1803a, p. xj.

[68] Carnot L 1786, pp. 65–66. Author's *italic*.

[69] Carnot L 1786, pp. 65–66. Author's *italic*. See also: *Ivi*, pp 96–97.

[70] Carnot L 1786, p. 66.

Corollary V. Particular law concerning the Machines whose motion changes by imperceptible degrees. X LI. In a Machine whose motion changes by imperceptible degrees, the moment of activity in a time given by soliciting forces, is equal to the moment of activity, exerted at the same time by resistant forces.[71]

Lazare Carnot considered the *production of work* to be produced by mechanical machines[72] (Oliveira 2014, Chaps. V-VI, pp. 118–171). The f_i–forces are only important when they are linked to δs_i –displacements of bodies. In his father's mechanical theory, the *production of mechanical work* occurs with the transference of motion from one body to another, both being constrained bodies.

> Preface. Although the theory here presented is applicable to all issues concerning the communication of motions, *Essay on machines in general* was given as title of this pamphlet; first of all, because they are mainly the Machines that are considered the most important argument of mechanics; secondly, because no particular machine is dealt with but we only deal with properties which are common to all of them.[73] […] XXXII. If a force *P* moves having velocity *u*, we call *z* the angle formed by *u* and *P*, the quantity *Pudtcosz*, where *dt* is the element of time, will be called *moment of activity* consummated by the force *P* during *dt*; that is the *moment of activity* consummated by a force *P* in an infinitesimally short time, is the product of this force, orientated such as its velocity, and the path that the point, [*fds*]where this force is applied, does in an infinitesimally short time. I will call *moment of activity,* consummated by this force in a given time, the sum of *moments of activity* consummated by it at every instant […][74]. […] we come back specifically to the second way [theory of motion] of looking at the problem, that is to say, that mechanics are nothing else than the theory of the laws of the communications of the motions.[75] […] The first method [theory of forces where cause produces motion] offers much more ease; so it is, as I mentioned here above, almost generally followed. Nevertheless, I adopted the second [theory of motion] as I already did in the first edition; because I wanted to

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[71] *Ivi*, pp 75–76. (Author's italics).

[72] On the role played by science and technique/machines within history of science see recently Pisano and Bussotti 2014c, 2014d.

[73] Carnot L 1786, p. iij, line 1. Author's *italic*.

[74] Carnot L 1786, pp. 65–66, line 2 Author's *italic*.1; see also pp 96–97. Author's *italic*.

[75] Carnot L 1803a, p. xiij, line 4. Author's *italic*.

avoid the notion of metaphysics of forces, to leave undistinguished the cause and the effect, in short, to bring everything to the only theory of communication of motions.[76]

However, it should be noted that the analogy does not go any further.[77] To be more precise, according to Lazare Carnot, an action of every force with the weight force can be reproduced and, in the end, thanks to the *communication of motion* (Carnot L 1803a, pp. xiij–xvj), one can also theorize on the work it completes as well as the idea *en général* of producing a new physical situation based on the impossibility of perpetual motion. Lazare Carnot studies a mechanical machine in general based on the fundamental affirmation that *perpetual motion is impossible* and the independence from the working substance, bodies and mechanism:

> [...] everyone repeats that in Machines in motion time or speed is always lost when force is gained [...].[78]
>
> [...] The reflections I propose on this law [Ivi, p. vi, op. cit.] lead me to say something about perpetual motion and I will show not only that every machine which is aborted must stop, but I will assign the very instant when this must occur.[79]

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[76] Carnot L 1803a, pp. xv–xvj, line 24. Author's *italic*. See also 1803a.

[77] An analogy between mechanical and heat machines should be noted. If in thermodynamics Q is analogous to f, since neither are state functions f must be substituted by potential $\Delta V = f\Delta s$, while Q must be substituted by entropy, which however has a different formula $\Delta S = \Delta Q/t$. (Thomson 1851, I, pp 175–183; see also Clausius 1850, vol 155, pp. 368–397; pp. 500–524). Moreover, it should be also noted that in the second case, it is not a special physical distance but it is temperature–range, $\Delta t \neq 0$. Sadi Carnot wrote this at the beginning of the discursive part of *Réflexions sur la puissance motrice du feu* and repeats it several times as well as at the end of the demonstration of his celebrated theorem (Carnot S 1978, p 38): *work can be obtained every time there is a difference in temperature between which heat passes*. Thus, it is possible to note a common way of conceiving work in comparison with special and heat motions (Pisano 2010; Gillispie and Pisano 2013).

[78] […] tout le monde répète que dans les Machines en mouvement on perd toujours en temps ou en vitesse ce qu'on gagne en force […]. (Carnot L. 1786, p. vi, line 14; see also *Ivi*, p. viii, line 20).

[79] "Les réflexions que je propose sur cette loi [Carnot L 1786, p. vi], me conduisent à dire un mot du mouvement perpétuel, & je fais voir non–seulement que toute machine abandonnée à elle–même doit s'arrêter, mais j'assigne l'instant même ou cela doit arriver." (Carnot L 1786, p. ix, line 16).

[...] But, I repeat, this Trial only concerns machines in general; each of them have their own particular properties.[80]

[...] we compare these different efforts regarding the agents that produce them, because the nature of the working substance cannot change the forces they must exert to fulfil the different objects for which the Machines are intended.[81]

[...] LVII. what is finally the veritable purpose of moving machines? [...] *the machines in motion, always lose time and velocity, what is they gained in force*.[82]

[...] LXII. We can conclude from that which we have just said regarding friction and other passive forces, that perpetual motion is absolutely impossible, using it to produce only bodies which are not solicited by any motive forces and even heavy bodies [...].[83]

[...] It is therefore evident that we must absolutely give up the hope of producing that which we call perpetual motion if it is true that all of the motive forces that exist in nature [...].[84]

---

[80] "Mais, je répété, cet Essai n'a pour objet que les machines en général ; chacune d'elles à ses propretés particuliers […]." (Carnot L 1786, p. x, line 14). After a re–elaborated work (1781) a publication (nowadays lost) appeared on 1783. The next edition (Carnot L 1786) is the only one consultable.

[81] "[…] l'on compare ces différents efforts dans égard aux agents qui les produisent, parce que la nature des agents ne peut rien changer aux la nature des agents ne peut rien changer aux forces qu'ils font obliges d'exercer pour remplir les différents objets auxquels sont destinées les Machines […]." (Carnot L 1786, p. 62, line 2). After a re–elaborated work (1781) a publication (nowadays lost) appeared on 1783. The next edition (Carnot L 1786) is the only one consultable.

[82] "LVII. Quel est donc enfin le véritable objet des Machines en mouvement ? […] *les Machines en mouvement, on perd toujours en temps ou en vitesse ce qu'on gagne en force*." (Carnot L 1786, pp. 88–89, line 24 (Authors' *Italic* style).

[83] "LXII. On peut conclure de ce que nous venons de dire au fujet du frottement & autres forces passives, que le mouvement perpétuel est une chose absolument impossible, en n'employant, pour le produire, que des corps qui ne seroient sollicites par aucune force motrice, & même des corps pesants […]." (Carnot L 1786, 94, line 16).

[84] "[…] Il est donc évident qu'on doit désespérer absolument de produire ce qu'on appelle un mouvement perpétuel, s'il est vrai que toutes les forces motrices qui existent dans la nature […]." (Carnot L 1786, p. 95, line 33).

Table 2. On the way of conceiving vincula and the production of work

Lazare Carnot (1780; 1786)
The work as a *product* of a mechanical machine; *vincula* bodies.
Mechanical vincula: $M \gg m$
(*Principle of virtual wok). S*ystems of bodies, non–infinitesimal points, but global and with *vincula*.
More than one body having infinite mass cannot be a machine: no work from *vincula*, only.
It is impossible to link (in a direct way) different potential systems to produce work freely (impossibility of perpetual motion).

108. When a body acts on another one it is always directly or through some intermediary body. This intermediate body is in general what we call a machine. The motion that is lost at every moment in each of the bodies applied to this machine is partly absorbed by the machine itself and partly revised by the other bodies of the system but as it may happen that the subject of the matter is only to find the interplay of the bodies applied to the intermediate bodies without the need to know the effect on the intermediate bodies, we have imagined, in order to simplify the question, to ignore the mass of this body, however keeping all the other properties of matter. Hence the science of machines has become a sort of isolated branch of mechanics in which it is to be considered the mutual interplay of different parts of a system of bodies among which there are some that, lacking the inertia as common to all the parts of the matter as it exists in nature, withheld the names of machines. This abstraction might simplify in special cases where circumstances indicating those bodies for whom it was proper to neglect the mass to make it easier for the objective, but we easily know that the theory of machines in general has become much more complicated than before because then this theory was confined in the theory of motion of bodies as they are offered to us by nature, but now it is necessary to consider at the same time two kinds of bodies, one kind as actually existing, the other partially deprived of its natural properties. Now it is clear that the first problem is a special case, since it is more complicated than the other so that by similar hypotheses, we easily find the laws of the equilibrium and of motion in each particular machine such that the lever, the winch, the screw, resulting in a blend of knowledge whose binding can be hardly perceived and only by a kind of analogy; this must necessarily happen as we will resort to the particular figure of each machine to show the property which is common to it and to all the others. Since these properties are the ones we have mainly seen in this first section, it is clear that we will be able to find them only by putting aside the particular forms. So let us start by simplifying the state of the issue by ceasing to consider the system bodies of different natures; finally giving back to machines their inertia it will be easy afterwards to neglect

the mass in the result, we will hold the possibility to consider it or not, and therefore the solution of the problem will be general and easier at the same time.[85]

It should be noted that in Lazare Carnot's theory, it is implicit that many bodies with infinite mass, that is to say the constraints, alone, do not form a machine (Carnot L 1786, pp. 58–59) and therefore never produce work. It can be asserted with the reasoning that once again, otherwise, it would confirm the possibility of perpetual motion. Following this analogy, we can clearly affirm, with the same reasoning as before, that it is impossible for connecting constraints *in a way only directed at different a t thermostats*, that is to say a machine, to run (*produce work)*by letting heat pass without restrictions. In other words, the reflection on the old experiment of the exchange between two bodies inside a calorimeter cannot show how work is produced. In fact, to produce work, other intermediary mechanisms are necessary in addition to thermostats in order to adequately utilize the transference of heat between the two temperatures. This is the *second argument ad absurdum* that unites the (implicit) development of the two theories, according to their common model of theory based on a problem.

The principle states that the total virtual work performed by all the forces acting on a system in static equilibrium is zero for a set of infinitesimal *virtual displacements* from equilibrium. The infinitesimal displacements are virtual because they need not be obtained by a displacement that actually occurs in the physical system. The virtual work is the work performed by the virtual displacements, which can be arbitrary and are consistent with the constraints of the system. Its common mathematical expression is:

$$\delta W = \sum_i F_i^{(a)} \delta s_i = 0$$

.

The theory of mechanical machines may be based on the *principle of virtual work,* and thought of as a consequence of the principle of the impossibility of perpetual motion, e.g., applied to machines and constraints: *it is impossible that the reactions of the constraints on the actions of the bodies, which make up the machine, produce positive work*. In other words it is impossible for forces of bodies of constraints to produce work:

[85] Carnot L 1780, § 108.

$$\sum_i R_i ds_i \leq 0$$

3.3 On Lazare Carnot's *Principle of Virtual Laws*

Lazare Carnot established an approach to science which was different from the common paradigm of his time (Dhombres and Dhombres 1997). The generalization of Lazare Carnot's *Principle of virtual work* is historically very important because it precedes Lagrange's approach in his *Mécanique analytique* (Lagrange 1788, Blay 1992).

Lazare Carnot began by stating his principles, to which he referred also as laws, to underline their empirical content. In this regard, Gillispie observed:

> He [Lazare Carnot] did achieve a greater clarity, most notably in the passages defining geometric motion:
>
> > Any motion that, when imparted to a system of bodies, has no effect on the intensity of the actions that they exert or can exert on each other in the course of any other motions imparted to them, will be named geometric [«DÉFINITIONS» (Carnot 1803a, § 136, p 108). Neither in the 1780 memoir nor in 837 the *Essai sur les machines en général* had Carnot adapted his concept of geometric motions from the principle of virtual velocities. In the *Principes fondamentaux de l'équilibre et du mouvement*, however, he went on to recognize the analogy between such motions and that principle in the use Lagrange made of the latter.[86]

They are only two principles in *Essai sur les machines en général*, which become seven in *Principes généraux de l'équilibre et du mouvement* as we have already shown.

> First law: *the reaction is always equal and contrary to action*.
>
> Second law: *when two hard bodies act on each other, because of an impact and pressure, that is because their impenetrability, their relative velocity, immediately after the impact, is always zero*.[87]

[86] Gillispie and Pisano 2013, p. 72, line 23.

[87] Carnot L 1786, pp 21–22. Author's *italic*.

The first law states that all bodies which change their state of rest or motion always do so due to the action of another body. All bodies resist their change of state; referring to this resistance Carnot uses the term “inertia force”, defending himself, for example, from Euler who considered it to be a confused concept, because of the union of the contrasting ideas of activity (force) and passivity (inertia), which still prevailed in applied mechanics. For Carnot, the inertia force is “the result of the present motion and of a motion equal and opposite with respect to that which it must have in the subsequent instant” (Carnot 1786, pp 60–61).

The second law concerns hard (or completely soft) bodies. Carnot was convinced that this law put aside elastic bodies; he declared this openly and justified it by assuming that the behaviour of elastic bodies can be re–conducted to that of hard bodies considering the former as composed of many small hard bodies connected by springs. It is clear that Carnot’s is a forced justification; the way to quantify elasticity remains to be clarified. By applying his principles to a system of free hard bodies, or to a system of bodies connected by rigid and insensible rods, Carnot obtained a first principle of mechanics which had the following form

$$\sum mVU\cos Z = 0$$

named the “first fundamental equation of mechanics”. Here m is the mass of the corpuscles of the system, *V* the true velocity after the impact, U the lost velocity (such that $W = V + U$ is the velocity the mass would have before the impact) and *Z* the angle between *V* and *U*. At this point Carnot introduced the concept of geometric motion.

> XVI. [...] *if a system of bodies sets out from a given position, with an arbitrary movement, but yet of such* [*a nature*] *that it is possible to make it take another in every respect equal and directly opposite; each of these movements will be named a geometrical movement* [...];[88]

In *Principes généraux de l'équilibre et du mouvement* the definition is slightly different:

[88] Carnot L 1786, p 28. Author’s *italic*. See also *Ivi*, 29–34, pp 41–45. See also Carnot L 1808a, p 212; 1808b.

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de ces résultats à la fin de la première partie. Il ne s'agit ici que de démontrer ces résultats d'une manière rigoureuse par le seul raisonnement, en partant des hypothèses établies ci-dessus, et d'en déduire les conséquences les plus générales. Nous commencerons par le choc des corps, soit immédiat, soit opéré par l'entremise d'une machine. Nous en déduirons ensuite, comme cas particulier, les lois du mouvement d'un système de corps, lorsque ce mouvement change par degrés insensibles. Cette théorie renfermera donc tous les principes fondamentaux de la communication des mouvemens, et par conséquent, de la mécanique elle-même; car, ainsi que nous l'avons déjà observé, on ne considère, en mécanique, aucune force qui ne réside effectivement dans les corps, c'est-à-dire, qui ne soit réellement une quantité de mouvement déjà produite.

DÉFINITIONS.

136. *Tout mouvement, qui imprimé à un systême de corps ne change rien à l'intensité de l'action qu'ils exercent ou pourroient exercer les uns sur les autres si on leur imprimoit d'autres mouvemens quelconques, sera nommé mouvement géométrique.*

La vitesse que prend alors chaque mobile, sera nommée sa vitesse géométrique.

Ainsi, par exemple, si lorsque deux corps sont

DEFINITIONS.

136. *Any motion will be called geometric if, when it is impressed upon a system of bodies, it has no effect on the intensity of the actions that they do or can exert on each other when any other motion is impressed upon them.*[89]

Fig. 8. Geometric motion in *Principes généraux de l'équilibre et du mouvement*[90]

The first definition is purely geometric, that is, geometric motions are reversible motions congruent with constraints; in the second definition there is also a reference to mechanical concepts, because the word "action" calls for concepts like force or work. From the examples Carnot gave in the Essay, it appears that geometric motions can also be infinitesimal (see the example referred to in the note on p. 26). The same holds true for *Principes généraux de l'équilibre et du mouvement* (see theorem IX, p. 130). From an operational point of view the finite or infinitesimal nature of geometrical motion makes no difference because what Carnot used is velocity u associated with the geometric motion, called geometric velocity. It can be said that the modern concept closest to geometric motion is that of virtual velocity, which today is not

[89] Carnot L 1803a, p. 108. Author's *italic*. See also: Gillispie 1971, p. 43.

[90] Carnot L 1803a.

often distinguished from virtual displacement. Carnot gave great emphasis to geometric motions, considering their introduction as one of his major contributions to mechanics:

> The theory of *geometric motions* is very important; it is as I have already noted like a mean science between ordinary geometry and mechanics [...] This science has never been treated in details, it is completely to create, and deserves both for its beauty and utility any care by Savan[t]s.[91]

With the aid of geometric motion, the first fundamental equation of motion can be rewritten in a more meaningful and expressive way. It is easy for Carnot to show that the first fundamental equation remains valid when the true velocity *V* after the impact is substituted by the geometric velocity u, to obtain:

$$\sum muU \cos z = 0$$

where *z* is now the angle between u and *U*. Carnot called this equation the "second fundamental equation of mechanics" and noted that by varying u among all the possible geometric motions one can obtain all the equations needed to find the lost motions *U* of all masses. In this way Carnot has solved what seems to be his main problem: given the initial velocities *V* of a system of masses to find the final velocities *W* after the impact. Indeed when *U* is known, the final velocities are simply given: $W = U + V$. unfortunately, this problem has little practical value, because one rarely has to address the impact of hard bodies, simply because hard bodies do not exist, not even in an approximate way. To pass from the ideal dynamics of impact to the more realistic dynamics of force varying continuously Carnot, in a passage which is difficult for us to understand, passed from the expression of motion lost in the impact to that of motion lost by imperceptible degrees and identifies *mU* (the lost motion) with the force *F*, which can now be read with the modern meaning.

> The tension of treads, or the pressure of a bar, expresses equally both the effort which is exercised on the machine and the quantity of motion who itself loses because of the reaction he tries: if so one call F this force, this quantity F will be the same thing as what is expressed by *mU* in our equation.[92]

[91] Carnot 1803a, p 116. Author's *italic*.

And we can then write the second fundamental equation “F” (Carnot L 1786, p 32) as:

$$\sum Fu\cos z = 0$$

Which is in fact the equation of virtual work as given by Lagrange. Particularly, Lazare Carnot also dealt with the principle of virtual work and then, by means of *geometric motion* (in modern terms, virtual velocities), canonically formulated the principle of virtual velocities in the fundamental theorem (Carnot L 1786, § XXXIV, pp. 68–69). In effect, since his theory of geometric motions coincided with velocities and not with displacements, this allowed Lazare Carnot to avoid, in the formulation of the principle of virtual work, infinitesimal displacements, which could have produced some scientific embarrassment with respect to his assumptions (Carnot L 1813). Furthermore, for the principle of virtual velocity related to any (general) mechanical machine, one can claim that the (forces–) weights that balance each other are reciprocal to their virtual velocities. Incidentally, the two conceptually different approaches/formulations can be mathematical equivalents using the concept of virtual motion as key reasoning.

Lazare Carnot formulated the principle of virtual work by beginning with his law of collisions (Carnot L 1786, 1803a) and without (generally speaking) using classical Newtonian forces. Particularly, Lazare Carnot used the principle of virtual work to discuss and define the conditions of equilibrium of the forces applied to the bodies.

> General principle equilibrium and of motion in machines
>
> XXXIV. *Whatever is the state of repose or of motion in which any given system of forces applied to a Machine, exists, if we take it all at once assume any given geometric motion, without changing these forces in any respect, the sum of the products each of them, by the velocity which the point at which it is applied will have in the first instant, estimated in the direction of this force, will be equal to zero.* That is to say, by calling *F* each of these forces (I), *u* the velocity which the point where it is applied will have at first instant, if we make the Machine assume a geometric motion, and *z* the angle comprehended between the directions of *F* and of *u*, it must prove that we shall have for the whole system [Σ]$Fu\cos z=0$. Now this equation is precisely the

[92] Carnot L 1786, pp. 65–66.

equation (AA) [*ΣFucosZ=0* (Carnot L 1786 p 63, line 15)] found (XXX) [*Ivi*, p 60] which is nothing else in the end but the same [second] fundamental equation (F) [*ΣmuUcosz=0* (*Ivi*, p 32, line 6)] presented under another form. It is easy to perceive that this general principle is, properly speaking, nothing else than that *Descartes*, to which a sufficient extension is to be given, in order that it may contain not only all the conditions of the equilibrium between two forces, but also all those of equilibrium and of motion, in a system composed of any number of powers: thus the first consequence of this theorem will be the principle of *Descartes*, rendered complete by the conditions which we have seen were waiting in it (V)[93].

The aim was to obtain a mathematical expression of its invariant, or the efficiency of a heat machine with respect to all possible kinds of working substances. Therefore, it was necessary to obtain invariants with regard to the efficiency and reversibility of a mechanical machine.

We will now note that in the traditional mechanical theory of hard bodies, the *principle of virtual work* formally defines the condition of *equilibrium* of the forces that act on the bodies in order to produce work:

Corollary II. General principle of equilibrium in weighing Machines. XXXVI. *When several weights applied to any given Machine, mutually form an equilibrium, if we make this Machine assume any geometric motion, the velocity of the centre of gravity of the system, estimated in the vertical direction, will be null at the first instant*.[94]

Lazare Carnot, having the mathematical formula for the *principle of virtual work*, studies the theoretical conditions that translate the practical conditions of equilibrium and also obtains his invariants with regard to the efficiency and reversibility of mechanical machines.

His *first* (Carnot L 1786, p. 32) and *second* equations (*Ivi*, p 33) generalized for multi–body systems are:

$$\sum mVU\cos\left(<\vec{U},\vec{V}>\right)=0 \quad \text{(E)}$$

$$\sum muU\cos\left(<\vec{U},\vec{u}>\right)=0 \quad \text{(F)}$$

m = mass of the body

W = velocity before interaction

V = velocity after interaction

U =*W*–*V*

u = *arbitrary geometric motion*

[93] Carnot L 1786, § XXXIV, pp 68–69 and footnote "(I)". (Author's *italics* and Capital letters).

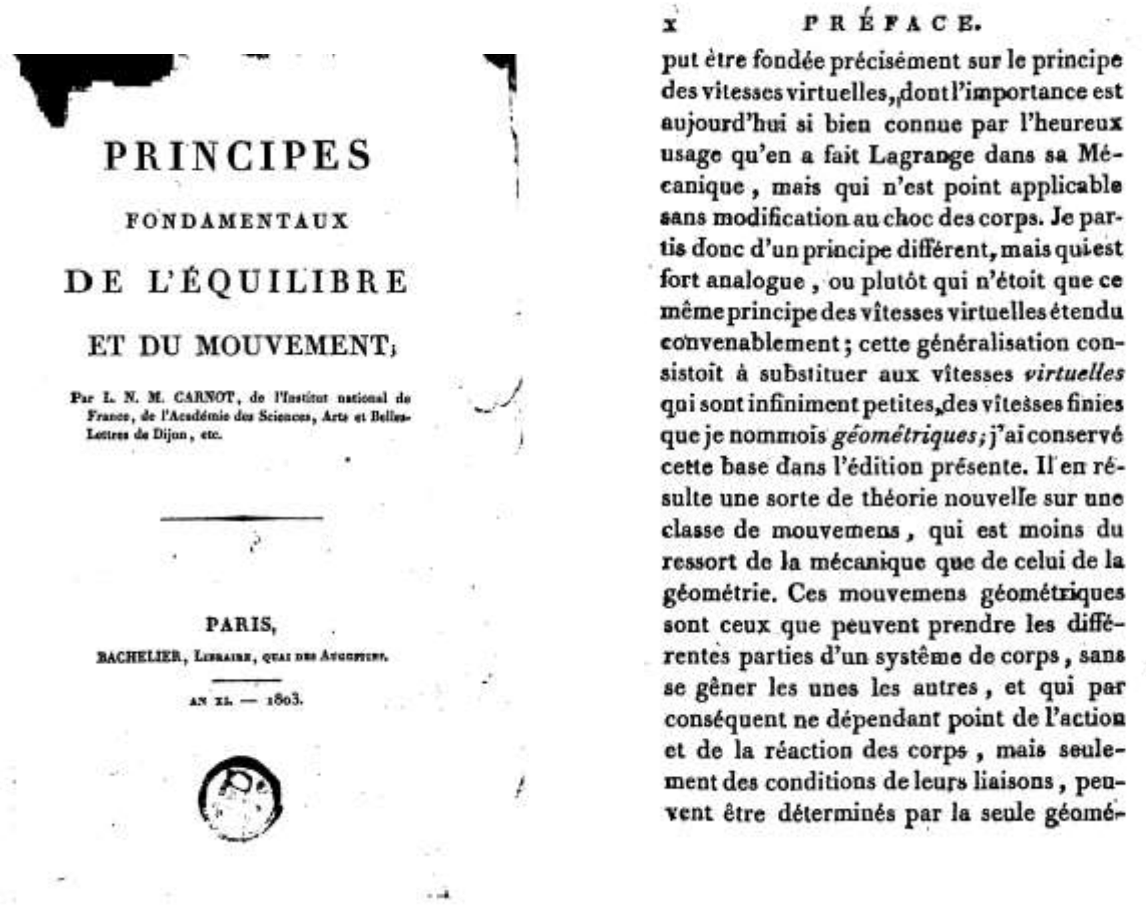
[94] Carnot L 1786, p. 71, line 1. (Author's italics). See also Carnot L 1803a.

In short[95]:

- The mass of the parts of a machine.
- Global magnitudes, abstracting from the mass of the mechanism.
- Kinematics first, then dynamics, and statics is a special case of dynamics.
- A theory of machines concerns a theory of the *communication of motions*.
- A machine is a connected system of (hard) bodies.
- The connections between the bodies constrain the *communication* of motion of the bodies.
- The theory of interaction–collisions by means of insensible degrees (e.g., see Carnot L 1803a, § 293, pp 261–262) as the result of a sequence of infinitesimally small percussions.

In order to complete our discussion on the use of the *principle of virtual work* in the two Carnots' theories, in the following section we summarize Lazare Carnot's reasonings upon his laws of conservation, mainly included in both *Essai sur les machines en général* (Carnot L 1786) and *Principes fondamentaux de l'équilibre et du mouvement* (Carnot L 1803a).

[95] An extensive discussion is in: Gillispie and Pisano 2013.



PRINCIPES
FONDAMENTAUX
DE L'ÉQUILIBRE
ET DU MOUVEMENT;

Par L. N. M. CARNOT, de l'Institut national de France, de l'Académie des Sciences, Arts et Belles-Lettres de Dijon, etc.

PARIS,
BACHELIER, Libraire, quai des Augustins.
AN XI. — 1803.

x PRÉFACE.

put être fondée précisément sur le principe des vitesses virtuelles, dont l'importance est aujourd'hui si bien connue par l'heureux usage qu'en a fait Lagrange dans sa Mécanique, mais qui n'est point applicable sans modification au choc des corps. Je partis donc d'un principe différent, mais qui est fort analogue, ou plutôt qui n'étoit que ce même principe des vîtesses virtuelles étendu convenablement; cette généralisation consistoit à substituer aux vîtesses *virtuelles* qui sont infiniment petites, des vîtesses finies que je nommois *géométriques*; j'ai conservé cette base dans l'édition présente. Il en résulte une sorte de théorie nouvelle sur une classe de mouvemens, qui est moins du ressort de la mécanique que de celui de la géométrie. Ces mouvemens géométriques sont ceux que peuvent prendre les différentes parties d'un système de corps, sans se gêner les unes les autres, et qui par conséquent ne dépendant point de l'action et de la réaction des corps, mais seulement des conditions de leurs liaisons, peuvent être déterminés par la seule géomé-

Fig. 9. Preface of *Principes fondamentaux de l'équilibre et du mouvement* (1803a)

A *law of conservation* for *plastic bodies* – even though he called them *hard bodies* – can be written in the following way:

$$\sum_i m\vec{U}_i = 0$$

m_i = mass of i–th body [for isolate system]

U_i = velocity lost (by that body) during the collision

W_i = velocity before interaction

V_i = velocity after interaction

$$\vec{U}_i = \vec{W}_i - \vec{V}_i$$

Therefore, using the hypotheses of *parfaitement élastiques*[96] bodies (Carnot L 1803a, p 105), one obtains:

[96] Lazare Carnot proposed a generalization from plastic bodies to all bodies by means of an *ad hoc* index. (See below). He presented it in *Essai sur les machines en général* (Carnot L 1786 pp 15–22) and in *Principes fondamentaux*

$$\sum_i m_i \vec{U}_i \vec{V}_i = 0$$

V_i = velocity after interaction is the same for all of them

Generalization for all bodies using a *n*–elasticity index.

Now, by using certain calculations, the *law of conservation of kinetic energy* for soft bodies is obtained:

$$\sum_i m_i \vec{W}_i^2 = \sum_i m_i V_i^2 = 0$$

Following Lazare Carnot's reasonings presented in both his books (Carnot L 1786; 1803a), at this point, by introducing *geometric motions* (Carnot L 1786, pp. 28–30) starting from the previous *law of conservation*, one can write:

$$\sum_i m_i \vec{U}_i \cdot \vec{u}_i = 0$$

m_i = mass of the *i*–th body

U_i = velocity lost (by that body) during the collision

u_i = velocity called "mouvement géométrique"[97]

Here, it is unproblematic to recognize the extension of the *principle of virtual velocity* to the collision of several bodies using $\vec{u}_i = const.$ With following the proof, which we omit for the sake of brevity, we can write:

$$\sum_i m_i \vec{U}_i \cdot \vec{u}_i = 0; \rightarrow \vec{u} \sum_i m_i \vec{U}_i = 0 .$$

By considering the *u*–arbitrariness, we can write:

de l'équilibre et du mouvement (Carnot L 1803a, pp 103–106, pp 131–146). The generalization is differently presented in the two cited books. More specifically, the first reports an inverse procedure with respect to second.

[97] In practice, *u* is physically the velocity of any *geometric motion*. However, it is a mathematically indeterminate variable and each specification produces (in the equation cited in the running text) an equation applicable to the physical system considered.

$$\sum_i m_i \vec{U}_i = 0$$

With

$$\vec{U}_i = \vec{W}_i - \vec{V}_i .$$

In short:

- A theory of interacting bodies by means of collisions.
- A collision is a basic phenomenon. In particular, continuously accelerated motion is obtained as a limiting case of a system driven by a series of pulses.
- Newton's second law is replaced by Lazare Carnot's second fundamental equation for a system of *n*–bodies.
- Due to the arbitrariness of u_i, it can be assumed constant, that is to say the same translation of geometric uniform motions of all bodies is adopted.

By considering another *ad hoc* geometric motion, $\vec{u}_i = \vec{\omega} \times \vec{r}_i$, e.g., the rotation of the system with angular velocity around a fixed axis, and using the properties of the triple product and the arbitrariness of "$\vec{\omega}$", he then wrote his two main *laws of conservation* as *invariants* of motion:

$$\sum_i m_i \vec{W}_i = \sum_i m_i \vec{V}_i$$ Law of conservation of the total–quantity–of–motion

$$\sum_i m_i \vec{r}_i \times \vec{W}_i = \sum_i m_i \vec{r}_i \times \vec{V}_i$$ Law of conservation of the total–angular–momentum

4 Concluding Remarks

The role played by of the *Principle of virtual laws* within *classical mechanics* is not easily definable (Capecchi 2012) and theoretically it does not appear to be crucial (from a technical standpoint). One can think of *rational mechanics* while avoiding problems[98] and not allowing for the theory to take on unresolvable problems. In the mechanics applied to rigid bodies, that is the relationship between physics and its mathematical interpretation, the principle of virtual laws can be powerfully used to solve certain *vincula* problems. Therefore, generally speaking, in modern times it does not appear critical within scientific research. The *vincula* are taken into account by introducing the *constraint reactions* such as *auxiliary unknowns* that are then eliminated by substitution during the solution of the single static problems; and with friction constraints, one can easily provide adequate *constitutive laws*. In *continuum mechanics* its role is very important since the solution is simpler if one adopts particular values of the mathematical expression of the *principle of virtual laws*, such as using the *method of finite elements*. From an historical point of view it is of the utmost importance. It had a long historical process beginning with ancient Greek science (Pisano and Capecchi 2014, chap. 2) until the Aristotelian[99] and Archimedean mechanical approaches.

Based on some of Lazare Carnot's crucial discussions (Carnot L 1786, pp. 28–30), the *geometric motion*[100] essentially expresses a non–mechanical interaction. Lazare Carnot defined these motions as *invertible*: *a motion assigned to a physical system of interacting bodies is geometric if the opposite motion is also possible*. The result is a *possible motion*, but it is not always *invertible* (e.g., the motion of a sliding ring on a rotating rod). Therefore, one should add the hypothesis of *invertibility* for obtaining the concept of *geometric motion*. Conversely, a *geometric motion*, when integrated, gives an *invertible motion*. At this point, for *vincula* independent of time, a *geometric displacement* is equivalent to a *virtual invertible displacement* (but not vice versa). On the contrary, a *possible displacement* only if it is *invertible*, produces, after its derivative, a *geometric motion*. In this sense, we note that initially, the geometric motion is a kind of uniform motion moving on the whole physical system when one considers the equivalence of the state of rest and the state of uniform motion. Consequently, by using double negative sentences (Gillispie and Pisano 2014, chap. 7) one can write:

[98] By the way many assumptions and equations (i.e. Lagrange, Hamilton, Hamilton–Jacobi) derived by it.

[99] In the history of mechanics, if one avoid the theory of centres of gravity (Pisano 2007), a modern interpretation of the principle has its general roots into Aristotelian mechanical school (Pisano and Capecchi 2014, chap. 2).

[100] Carnot L 1786, pp 28–34, pp 41–45; see also Carnot L 1780, § 113; Gillispie 1971, Appendix C, § 113, pp 308–309.

$$\neg[(v = 0) \neq (v \neq 0)].$$

Finally, I should also note that the previous discussion on the *principle of virtual work* in Lazare's work is surely suggestive but it is limited in the two different mathematical approaches with respect to physics adopted. Lazare Carnot himself opened his program of research by explaining the foundation of his mechanics to the reader (Carnot L 1803a, p. x), that is to say, the *principle of virtual work*. In this sense, a new kind mechanics – with respect to Newtonian mechanics – would be born thanks to *l'organisateur de la victoire*.

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Niccolò Tartaglia. Re–Thinking The Role Played By Science of Weights in the Sixteenth–Century. Selections from Quesiti et inventioni diverse: Books VII–VIII

Raffaele Pisano

The Ph.D. thesis is an historical and epistemological analysis of Science of Weights and Mechanics in the Sixteenth–Century, particularly in the Tartaglia's treatise *Quesiti et inventioni diverse* (1546; 1554), *Books VII* and *VIII*.

Mechanics in between the 15th and 16th centuries mainly concerned what largely is now called statics and was referred to as the *Scientia de ponderibus*. Generally, It was pursued with two different approaches; the former, usually referred to as Aristotelian, where the equilibrium of a body stemmed from two contrasting tendencies. The latter, usually referred to as Archimedean, where the study of the equilibrium reduced to the evaluation of the centre of gravity of a body (*centrobaric*). In between the two traditions the Italian scholar, Niccolò Fontana, better known as Tartaglia (1500?–1557), wrote the treatise *Quesiti et inventioni diverse* (1546; 1554).

The volume consists of three main parts. In the first, an historical excursus regarding Tartaglia's lifetime, his scientific production and the *Scientia de ponderibus* from Arabic science, the Middle Ages to the Renaissance, is presented. Secondly, all the propositions of *Books VII* and *VIII*, by relating them with the *Problemata mechanica* by Aristotle and *Iordani opvscvlvm de ponderositate* by Jordanus de Nemore are examined within historical epistemology of science. The last part is relative to the original texts and transcriptions into Italian-Latin languages and an English translation.

This volume aims to gather and re-evaluate the current thinking on this subject. It also brings together current and previous researches of mine in the history and historical epistemology of science, within fields of physics, mathematics, and engineering. It also and gives much-needed insight into the subject from a historical and scientific standpoints.

The volume composition makes for absorbing reading for historians, philosophers, epistemologists and scientists.