UNIVERSITÀ DEGLI STUDI DI BERGAMO

Scuola di dottorato in Antropologia ed Epistemologia della Complessità

XXVIII Ciclo

Complex Problem Solving and the Theory of Complexity in High School Teaching

PhD candidate

Enrico Giustiniano Micheli

Matricola:1026708

Relatore: Chiar.mo Prof. Enrico Giannetto

Anno Accademico 2014/2015

To my wonderful wife Elisa and my beautiful children Giovanni and Rebecca

Index

Preface

When I started this adventure, I was a very different person both in my personal and professional attitude.

During these Ph.D. years, I have learned many things thanks to what I studied and the people I met. The basic idea of being multidisciplinary -- that really drives the University of Bergamo's Ph.D. in Anthropology and Epistemology of Complexity -- has been a fantastic occasion for meeting, talking, learning from people with similar and very different background compared to mine.

I decided to write a sort of engaged Anthropology dissertation in which I take the side of complexity and try to bring this issue in the Italian High school world, which I know very well as I am a Mathematics and Physics teacher in Italy. I will then tell the story of the project using first person in many cases and explain the way I carried it out. In proposing this possible new topics into the curriculum, I tried to be creative in order to give the deepest sense of being a teacher. While doing it, I found an unexpected interest for this topic among teachers and students. I also confirmed my realization of what an incredible resource, how motivated and very open to challenge the Italian teachers I met are, and how I am sure they are a good representative sample of all the people in the world that each day try to develop knowledge and competence in the minds of new generations. I love my job, and this experience made me love it even more. Nonetheless now, more than ever, I think that Education is not a deterministic process. It is a complex process where exchange among students, teachers and both the school and family environment takes place. Knowledge and competence construction, indeed, emerge when a teacher leads the whole class to the comprehension of a concept through an interconnected teamwork. This approach changes every year in every class, because, like in the Heraclites' concept of πάντα ῥεῖ (panta rhei) "everything flows": school is always in becoming.

Heraclites explains the concept of becoming -- nothing stays the same but always changes -- through the famous metaphor of the river, which is different every time you touch it. If, instead, something does not normally change as the river, then the becoming comes from you, who are different when you touch it. So is school every year, in every class: the program is supposed to be the same, but students and times are different, therefore knowledge transfer and learning process are created every day by teachers and students working together with the emergence of new patterns. This is very unlike universities, where interactions between the lecturers and the class are not as strong (at least in Italy).

Nevertheless, when there is a lack of motivation or effort by one or both sides, the learning process becomes frustrating and even obtaining the smallest result is very hard.

As a teacher, I have also found that you learn from students as much as they learn from you. Sometimes the lack of knowledge, if accompanied by the struggle to understand, can be very helpful, because knowing a lot implies having an opinion that can sometimes prevent you to openly accept and understand new ideas (see Introduction). So maybe, in the change in paradigm about science education, students are a good starting point.

That is because everybody in the world has experienced school, and everybody has an opinion on how learning should take place. When I myself say to anyone that I am a Mathematics and Physics high school teacher, they usually say that they remember their own teacher: it does not matter if s/he was good or bad (in their opinion): s/he had a big impact in this person's life anyway, more than the teacher him/herself would think.

Therefore, I think it is important that students and teachers share a sense of common research, so that the latter can create their own knowledge, the idea that everyone participates in the research for learning development with his/her own contribution.

Initially, my project was different, as I was different. I changed it as much as it has changed me. The experience of designing how the intervention in class would take place and the topic to present has made me want to teach what I learned even more. It was also a great challenge for me, as I had to learn about Psychology, Chemistry, Philosophy, Biology, Computer Science, Game Theory and so on. The most important thing multidisciplinary learning taught me, I think, is that in any field there is an incredibly vast and complex academic literature, accompanied by a genuine sense of trying to bring any topic to the general public in order to build a wider and informed audience.

Even if the general outcome of the project was different from expected, there was the suspense and surprise for the results, the awareness of being able to actually do it and the certainty that what you find in your hands, eventually, will not be what you estimated either in a good and in bad way

I was also impressed by the problem of measuring when the variables are somehow related to people. You cannot repeat the measure; you cannot unknow what is known. This is very different if compared with the measures you take in any scientific experiment, where if you find that data are not sufficient, you can simply retake them -- although with modern development in Physics and other sciences this might be of dispute.

In Physics or Chemistry, for example, you can go back and take new data, apart from those places where the economic effort is so big that the time for your measure is limited by others that are in line after you. I think about time at the Hubble Space Telescope or a beamline in a synchrotron where you use the radiation produced for multipurpose experiments. In those cases, you simply schedule more time.

Instead, dealing with psychometrics or (possible) effects of training becomes complex like anything involving human interaction: if something does not work out, you have to repeat the experiment with another sample. Nevertheless, the sample can learn something and might be changed -- maybe only in a very small way -- by the measures and the project, and this is a characteristic pure science does not have. In other words, pure science experiments teach researchers many things, but experiments with people also reverse the process.

I think I have been the first to bring Complex Problem Solving into high school in Italy and among the few who have tried to deal with this high school application to date. I have researched many papers and books written about this topic and the small contribution I give with this dissertation and project makes me want to develop these ideas even more in the future.

Acknowledgments

I would like to thank all the people and institutions for the help and support given to my project.

Thanks to:

The entire LUCET team at the University of Luxembourg directed by Prof. Romain Martin, and in particular Philipp Sonnleitner for his great support and friendship, Ulrich Keller and Willibrord Koch.

The University of Milan, Department of Mathematics, Prof. Simonetta Di Sieno

The principals of:

Istituto Statale di Istruzione Superiore Giosuè Carducci - Dante Alighieri Trieste --Oliva Quasimodo

Liceo Scientifico "Guglielmo Oberdan" Trieste -- Maria Cristina Rocco

Vienna International School -- Elizabeth Stanners; and her collaborators: Tanja Dawber deputy principal, and Derrick Devenport

The schools:

Convitto Nazionale Paolo Diacono - Liceo Scientifico - Cividale del Friuli, Udine

Istituto Statale di Istruzione Superiore Giosuè Carducci - Dante Alighieri, Trieste

Liceo Scientifico "Guglielmo Oberdan", Trieste

Scuola Media Statale Leonardo Da Vinci, Saronno, Varese

Scuola Secondaria di 1° grado Umberto Saba, Milano.

Scuola Secondaria di 1°grado “Salvatore Quasimodo”, Milano.

Scuola Secondaria di 1°grado “Tommaseo”, Milano

Vienna International School, Vienna

The teachers:

Anna Asti, Consilia Balsamo, Ronny Brusetti, Milvia Corso, Enrica de Giorgi, Barbara di Majo, Raffella Dussich, Pietro Fera, Daniela Fontana, Donatella La Macchia, Gabriella Lodigiani, Renzo Nicolini, Marina Romanelli, Emanuela Scaioli, Giusy Sessa, Armida Turconi.

And all the students who have taken part in the project.

A particular thanks for their not just technical help and support to: Beatrice Biggio, Paola Celli, Giulio Donini, Pietro Giavedoni.

The University of Bergamo’s FYRE project sponsored by Fondazione Cariplo.

Introduction

Nowadays, anyone wanting to investigate in either a Human or a Natural Science research field must face the challenge of multidisciplinary and interconnected knowledge. This change in the way of thinking may affect the general public in many aspects and it may also help challenging the misleading vision of Science shared by a number of people, especially in Italy. I have been teaching Mathematics and Physics in high school even before my graduation in Physics with Astrophysics. During the 10 years between my first experience in teaching and the beginning of my Ph.D. leave, I worked in different schools such as Scientific and Human Sciences high school, Professional high schools in Italy; and Italian Gymnasium in the Republic of Slovenia. I then developed a clear idea of this misleading vision that even younger people have when they deal with events or problems connected with Science and its role in modern life. From the theory of probability --involved in scratch and win, lottery and gambling -- and the role of statistics in Medicine, in my opinion there is a general misunderstanding about what Science can say or not say about anything and how helpful this can be in dealing with problems that occur in everyday life. This lack of scientific culture may lead some to reduce Science to a simple classification of notions. On the contrary, what teachers try to develop every day is that the set of notions Science provides must always be seen in close connection with the very tools of scientific thought development. In this context, anyone who approaches the latter should therefore be prepared to constantly revise their own beliefs, favoring a non-dogmatic attitude towards scientific disciplines. The main task of science teaching is, then, to train younger generations into this approach and to develop an awareness of the enormous potential it has to offer.

This means that Science and the tools it creates and develops are interconnected with what it is discovered with them. Many times in the history of science, a newly found tool has driven to a discovery that was not imagined before. I will give a few examples where indeed people, ideas, experiments and interconnected knowledge between science disciplines have initiated a change in the way of thinking that has not only led to a paradigm shift, but sometimes also to the birth of a new branch of Science.

Superconductors are one good first example to start with. These materials have no resistance to the flow of electricity and represent one of the last great frontiers of scientific discovery. The limits of superconductivity are not yet known and the theories that explain superconductor behavior seem to be constantly under review even in present days. The initial discovery of this phenomenon is due to Dutch Physicist Heike Kamerlingh and may well make the point of a change in the way of thinking. His team and he found out that a super cooled mercury wire at a temperature of 4.2 degrees K (approximately -269°C) could experience a sudden and large drop in its electrical resistance.

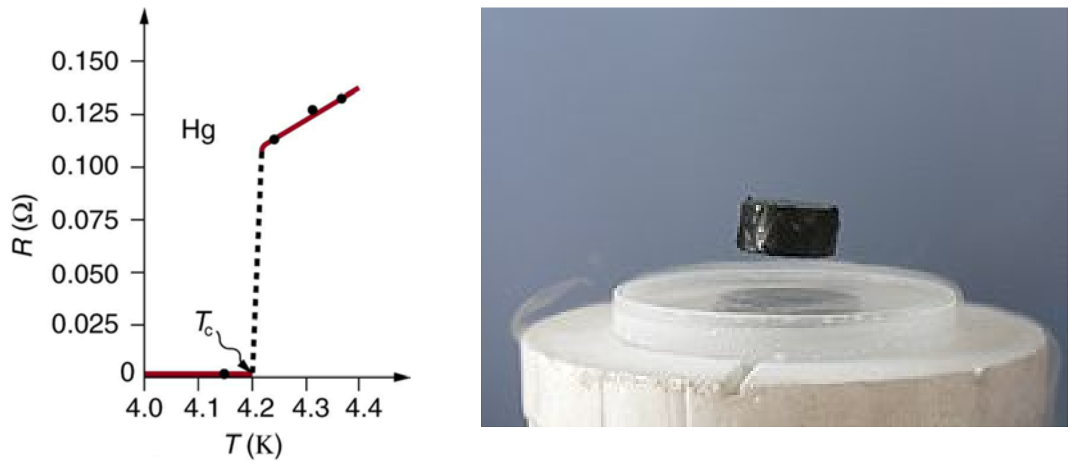

Figure I.1 One of the effects of electric current is to create magnetic field. Since there is no resistance because of superconductivity, the cube could float in the air virtually forever (as long as the circuit under it is kept supercool[1])

[1]First image College Physics. Authored by: OpenStax College. Located at: http://cnx.org/contents/031da8d3-b525-429c-80cf-6c8ed997733a/College_Physics. License: CC; second image Meissner effect: levitation of a magnet above a superconductor, 13 October 2007, Source: https://commons.wikimedia.org/wiki/File:Meissner_effect_p1390048.jpg, Author: Mai-Linh Doan Permission: Own work, copyleft: Multi-license with GFDL and Creative Commons CC BY-SA-2.5 and older versions (2.0 and 1.0).

The drop was enormous. Resistance became at least twenty thousand times smaller. In the beginning, a problem with instruments was thought to be the cause, but eventually they designed an experiment where, once the wire was kept super cool, an electric current could persists in the circuit without the addition of any further power. This seemed a kind of perpetual motion that could exist only because a large amount of power had been used in order to keep the wire at that really low temperature. This, at that time, changed any previous scheme about the behavior and structure of matter and was a starting point to develop an all-new field of Physics. Since then, many other superconducting materials have been discovered and the theory of superconductivity has been developed. These subjects remain presently an active area of study in the field of Condensed Matter Physics, so the practice discovery and the theory developed out of it create that kind of exchange that puts previous models constantly in doubt, trying to keep the development of ideas always in motion.

Another great and widely more famous example is Albert Einstein's special Theory of Relativity. When Einstein wrote his first paper, based on ideas developed by Lorentz, Poincaré and others, he was trying to explain some very important aspects of the famous 1887 Michelson and Morley experiment. In this work, it was proven that the velocity of light is actually independent from any frame of reference. This was really unexpected and caused many problems to general Physics theories.

I will try to give a quick explanation of the problem involved in this discovery. In our macroscopic world, where the speeds measured are really small compared to the speed of light, velocities are measured relatively to a frame of reference. For instance, when you are on a train that is running at 250 km/h, this means that in respect to the station that is fixed (attached to the Earth) you are moving at that speed. If on a parallel track, you find a train that is going at 300 km/h, always compared to the station, its actual velocity compared to you is instead "only" 50 km/h higher because, when travelling on the same direction, you can only measure the motion difference.

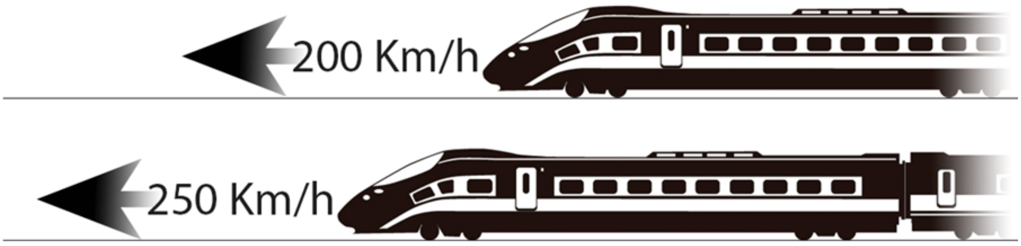

Figure I.2 Two trains travelling in the same direction with different velocites on parallel track.[2]

In another situation, if the two trains were running at the same speed, you could say that they are not actually moving in respect to each other. Instead, when two trains on parallel tracks are running in opposite directions, the velocity that each one measures in respect to the other is actually found out by adding the two velocities. Indeed, if your train is running at 250 Km/h and meets a parallel train that is running at 200 Km/h in opposite direction the velocity that one measures respect to the other is actually 450 Km/h. In fact, the first train is distancing itself from the other at that speed. This is a common sensation that anyone experience when travelling on high-speed trains (always a lot slower than light!!). The behavior of trains in motion is called Galilean relativity and it applies to objects and systems moving a lot slower than light.

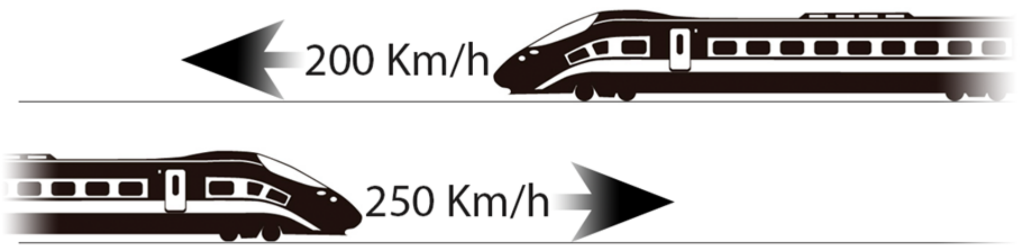

Figure I.3 Two trains travelling in opposite direction with different velocites on parallel tracks[3]

[2] Designed by Giulio Donini from a partial available draft in www.Freepik.com
[3] Designed by Giulio Donini from a partial available draft in www.Freepik.com

The Michelson and Morley experiment, instead, proves that this is not true for light and its velocity is constant measured in any frame of reference. It is somehow common knowledge that the velocity of light is 300.000 km/h. Michelson and Morley used a quite complicated device that could account of differences in such a large velocity magnitude. Thanks to that instrument, the experiment showed that if, theoretically, someone could travel at the speed of 250.000 km/h, he would not measure 50.000 Km/h for the velocity of a parallel ray light travelling in the same direction, as it is with trains. The value is instead always 300.000 Km/h, no matter how fast you are travelling or in which direction towards or backwards in respect to the ray of light. This was and still is astonishing, counterintuitive, and therefore pretty much challenging.

In order to explain this, Einstein, in his 1905 paper “Zur Elektrodynamik bewegter Körper" ("On the Electrodynamics of Moving Bodies"), states that this discovery needs a big epistemological change. The new principle is that light and electromagnetic waves in general have a velocity that must be the same in any frame of reference. Consequently, if you accept this for true, then you have to “simply” transform velocity’s definition. He does that by “simply” changing the notion and concept of space, time and its measure. Since velocity is indeed the ratio between space and time, you must change both of them in order to be consistent with the hypothesis.

This revolution changed Physics’ paradigm in many aspects and moved it to a different way of thinking. Since that moment, a different kind of rules have been accepted: on one hand Newton’s law on motion and the Galilelian notion of relativity were still valid at a “low” speed range; on the other hand, when the speeds involved were comparable to the speed of light, another whole new set of laws applies.

Similarly, but with some differences, this happened with Mathematics in early 19th century with non-Euclidean geometries. In those years, new developments were challenging somehow even the very foundations of this Science. There were ideas, like spaces with more than three-dimensions or the proof of geometry’s independence of

Euclid’s fifth postulate, that have been and still are profoundly connected with how to deal with abstract tools and their relations with reality.

If you focus on the latter, it is possible to understand how tough this challenge must have been. Various mathematicians had indeed proved that, in geometry, once removed Euclid’s fifth postulate while keeping all the others, you could build a perfectly consistent geometry by “simply” changing the perception and sometimes the definition of what is a straight line, an angle or a parallel line. In geometry, indeed, the parallel postulate[4] states that:

> If a line segment intersects two straight lines forming two interior angles on the same side that sum to less than two right angles, then the two lines, if extended indefinitely, meet on that side on which the angles sum to less than two right angles[5].

One of its consequences, that the interior angles of a triangle add to 180° (a flat angle), is widely used in schools to explain this concept.

Euclidean geometry is the study of geometry that satisfies all of Euclid's axioms, including the parallel postulate and it is the one everyone is familiar with. Nevertheless, you could build many different geometries that are consistent with the other postulates, but not with this. They are all perfectly acceptable, once you have proved there is no contradiction within themselves. In order to describe how it works, it is possible to give an example of a non-Euclidean geometry by presenting a simple case of an elliptic geometry. The simplest case can be identified as the surface of a sphere, and for instance the Earth is a good approximation. Therefore, on this surface, that by definition is not plane, a “straight” line is then called “great circle”, and the shortest distance

[4] This is also called Euclid's fifth postulate because it is the fifth postulate in Euclid's Elements.
[5] Euclid's Elements Book I Postulate 5, in Euclide, *Tutte le opere*. Testo greco a fronte a cura di Acerbi F. Milano: Bompiani, 2007.

between two points is therefore an arc of a great circle. In this framework, a metric can be introduced that allows a measurement of lengths and angles.

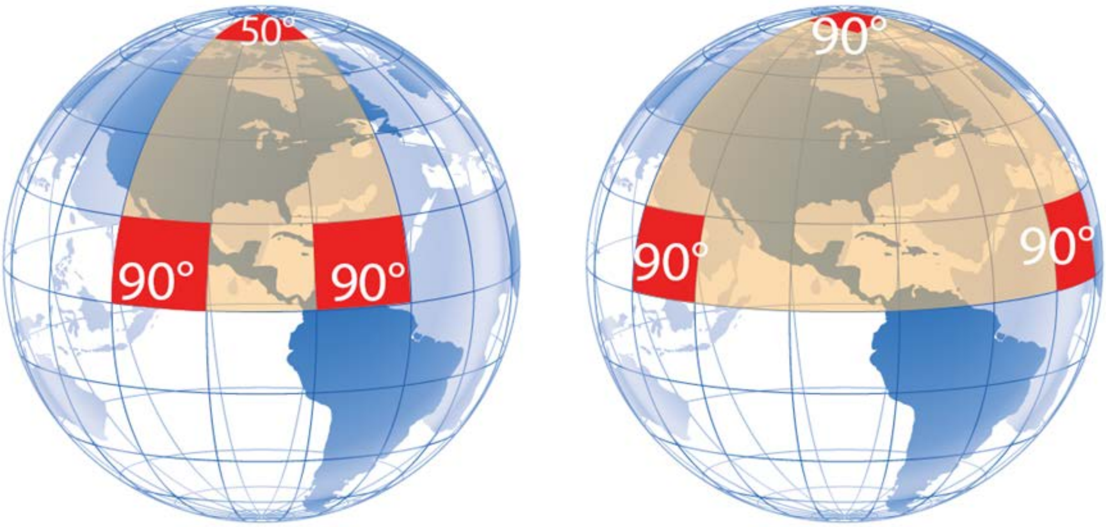

Figure I.4. On the Earth's surface, the interior angles of a triangle do not add to 180° or a flat angle. The sum is always greater than 180°, and it goes up to 270° in case of a maximum triangle. The surface of a sphere is not plane unless you consider a very small part where locally, "normal" rules of Euclidean geometry apply with good approximation. Picture by Giulio Donini

Gauss and Riemann were the first to discover this particular geometry that is actually the one that adapts to the Earth. Because of this debate, Mathematics and its foundations went through a crisis that was partially resolved by Felix Klein and his famous 1872 Erlangen program[6], which defines a way of characterizing geometries based on group theory and projective geometry. Klein worked at the University of Erlangen-Nürnberg, and this is why history remembers it as the Erlangen program. This work provides a frame in which it is possible to treat Euclidean and non-Euclidean

[6] "Vergleichende Betrachtungen über neuere geometrische Forschungen", Nürnberg, 1876.

geometries as subsets of a higher abstract tool called Projective Geometry and therefore creates a new paradigm[7].

There are also many examples in history of Biology with the famous 18th century debate on Spontaneous Generation challenging the idea of what dead and alive meant. It was once believed that life could come from non-living things and many experiments were designed and conducted by Francesco Redi and John Needham in 17th century, and Spallanzani[8] in 18th century to test this theory. The basic idea was that a piece of fresh meat or broth is considered dead when you use different ways to kill everything that is supposed to be inside. These ways include burning, boiling and so on, therefore if spontaneous generation is not permitted out of dead thing, nothing should grow out of that.

Instead, if you put the remaining pieces in two separate jars, one in open air and the other covered by a piece of cloth, after a certain amount of time life is created in the form a various small creatures worm-like. Spallanzani had then the idea of using a recently invented tool called vacuum air pump to remove air from one of the jars before sealing it. The vacuum-sealed jar did not present signs of life even after a reasonable long amount of time, proving that life is generated only by life. Spallanzani was anyway contested by the fact that maybe air is involved in life generation. Finally, Louis Pasteur[9] proved the role of airborne lifeforms in this experiment. Since then, *Omne vivum ex vivo*, Latin for “every living thing comes from a living (thing)”, has been one of Biology’s foundation.

---

[7] Bonotto, Cinzia *The Erlangen program revisited: a didactic perspective*, for the learning of Mathematics Vol. 27 No. 1 (2007).
[8] Spallanzani, Lazzaro, “Saggio di osservazioni microscopiche concernenti il sistema della generazione dei signori di Needham e Buffon (1765),” in *Opere scelte d Lazzaro Spallanzani*. A cura di C. Castellani. Torino: Utet, 1978.
[9] Pasteur, Louis, *Opere* a cura di Onorato Verona, Torino: Utet, 1972.

These are a few examples in the History of Science where experiments, theory, people's ideas combined together define a new way of thinking that in some cases has even changed the vision of the discipline itself.

According to these new themes of interdependency and constantly developing new visions, the world of education is also facing a change in paradigm, and especially in the way some apparatuses could be effective to fit the needs of new generations. Indeed, not only themes but also instruments to deal with them are rapidly changing. On this matter, innovation is what everybody is asking and in present years, for many people, this means introducing the use of computers in school. Therefore, computer-based learning and assessments are growing more and more in educational systems and schools throughout the world. For example, Singapore has been faithfully implementing Master Plans since 1997 for integrating technology into education,

> However, changing pedagogy is a very personal matter. Therefore, other than professional development, we use the strategy of exposing our teachers to the technological possibilities and supporting them in exploring new pedagogies with technology. The focus is not on technology. It is on using technology to enhance teaching and learning.[10]

So says Dr. Pak Tee Ng[11], Associate Dean, Leadership Learning, Office of Graduate Studies and Professional Learning, and Head and Associate Professor, Policy and Leadership Studies Academic Group, at the National Institute of Education, Nanyang Technological University, Republic of Singapore.

Somehow, then, these two aspects of change might be merged or more correctly co-developed to face the needs of future students and schools.

---

[10] http://www.huffingtonpost.com/c-m-rubin/singapore-education_b_2194696.html

[11] Singapore has been in recent years a center for innovation in education and a laboratory for testing computer based tools because of its general diffusion of computers and other technology objects among its population.

This task is complicated and long to be approached in its general form. The idea of this doctoral dissertation is to try to give a partial contribution. The aim is to see if it is possible, with a small experiment applied in few schools, to find some hints and ideas to combine these aspects of a complex and interdisciplinary world with complex problem-solving and computer-based testing.

The concept of how to deal with, to frame and, eventually, to solve multidisciplinary problems is considered very important among high school teachers; and, at the same time, it presents a certain number of difficulties. One of the most interesting ways to approach this idea has been the introduction of the concept of problem-solving in a more specific and detailed way in high school's curricula.

This was one of the reasons why, in recent years, especially after the PISA and INVALSI[12] tests were introduced, a strong interest in Problem Solving has been developing in Italy. The most important aspect consists in its use as a method to develop the students' reasoning skills and increase their motivation to study Mathematics.

In actual fact, the method of learning through Problem Solving can help to make sense of the more and more technically "advanced" calculation procedures, which seems to characterize teaching of Mathematics and Science during the five years of high school in Italy. Otherwise, excluding or not fostering that option enough may reduce these study topics to the assimilation of a sterile list of "recipes", built *ad hoc* to solve particular types of equations, in which students do not find any useful applied aspect.

[12] The Organization for Economic Cooperation and Development (OECD) has developed a testing program called PISA (Program for International Student Assessment). The aim of this project was to test Mathematics, Language and Problem Solving skills of 15 years students through all the participating countries. The tests are translated in all languages and they are administered every three years. The first test was in the year 2000, since then the obtained data are elaborated and published by the organization. This data contribute to create a ranking among countries' education performances. INVALSI (Istituto Nazionale per la Valutazione del Sistema dell'Istruzione) test, instead, is administered every year in Italian 8th grade schools since 2008. The aim of the test is to assess only Math and Language skills, and it is been recently introduced also for 10th grade (2011). There is no ranking for the INVALSI test, nevertheless the 8th grade counts as grade's final exam.

A discrepancy between the abstract tools provided in math teaching and the actual capability to use them in real situations taught by teachers can be found in many types of Italian schools, especially in non-scientific tracks like the Human Sciences Lyceum or the Classical Lyceum.

About this, I can recall a problem that one of my students presented me with a few years ago. It was based on a simple practical problem: the organization of a trip to the mountains.

The difficulty faced by the student, and especially the effort in interpreting a possible correct solution (also from an ethic standpoint), could generate thoughts on the actual effects of teaching on the students' thought mechanisms.

Figure I.5 A figurative explanation of the problem each house represents a day and the number of people that are supposed to sleep in that day[13]

[13] Designed by Giulio Donini from a partial available draft in www.Freepik.com

The problem itself was very simple: a group of nine young people is set to go to the mountains for a week during the New Year's Eve holidays and for this purpose decides to rent an apartment for a certain amount of money. The problem of solving it as a mathematical question was raised when knowing that a number of people would arrive two days later, while some others needed to go away three days before the rest, what would be correct to charge each participant? It was pointed out that the solution should not only be correct from a mathematical point, but also possibly fair to everyone. By fair they meant not only that people arriving late or leaving early should not pay as much as the others, but also that those staying over should accept a higher charge for the remaining days.

The student who asked my opinion proposed a numerical solution obtained by simply dividing the amount by the number of days, and the result further divided by the number of people who would be sleeping in the apartment every day. Each participant, in order to calculate his/her share, should simply multiply the amount by the number of days. "Is the solution correct?", the student asked, "because the numbers do not add up, so where is the problem?"

I then answered: "I am sorry, your solution is not entirely correct. This is a complex problem; because you have also as a condition that the cost should be shared as fair as possible between the participant, and this involves the introduction of more constraints, i.e. variables and equations. In fact, just to begin with, you need to know who sleeps in each night, otherwise the solution is not fair because some nights would cost more than others. In other words, I reckon you are somehow in the situation that politician and diplomats experience at the climate change conferences." The easier and numerically balanced solution on who must reduce its emission is more in contrasts with the idea that it is not fair taking account only of the present and not of the past. Lower industrialized countries say that they are as free as the others were before reaching the same amount of welfare rich countries now possess. So is it fair to divide following the rule that whoever pollutes the most must reduce the most; or is it more reasonable taking into account the past and develop a lot more complex solution that involves many new variables, like history of pollution or absolute population?

The discussion arose and, once again, it was pointed out how useful and sometimes illuminating is discussing practical problems together.

I eventually proposed two alternative solutions for the trip problem, which needed the introduction of one more variable only. The solution along with the numbers is reported as annex 1.

Later, I tried to help the class in trying to turn this into an algebraic problem and provide the equations, together with the unknown quantities. Here the situation had undergone a decisive step; the students had extreme difficulty in setting up the general problem for lack of method in structuring a practical multi-constraint problem. In my opinion, these difficulties were not related (or at least not only) to the understanding of mathematical tools, but, more in general, to an absence of training in asking how to use these tools to solve a very practical problem. I appreciate that this is a problem in Italy more than in other countries, especially the Anglo-Saxon ones, but anyway, using this base it would be possible to develop a general concept of problem-solving teaching in high school.

The goal of this approach could be to see if this process allows to help formation of ideas from inside, in the awareness that, perhaps, it is not possible to teach the students to think, but it may be possible to teach them how to develop and give a structure to their own thoughts.

That is the reason why this Ph.D. research is aimed at understanding and developing strategies for bringing the theory of complex problem solving into the Italian high school world. The main purpose of this research study is to test the potential improvement in learning through the inclusion of Complex Problem Solving (CPS) in different types of schools and different education systems, both Italian and International.

The research was developed as a close collaboration between the University of Bergamo Ph.D. program and the Luxembourg Centre for Educational Testing (LUCET), group of the

University of Luxembourg. The test was performed by introducing some typical complex problems to students of the same age, but with different backgrounds. Within the experiment, they were also asked to answer a questionnaire and in a following session, to use The Genetics Lab: a computer-based test to assess students' complex problem solving abilities. I personally translated it and adapted it into Italian when I spent some time in Luxembourg, thanks to an education grant provided by the University of Bergamo’s FYRE project sponsored by Fondazione Cariplo.

Chapter 1. Framing the Problem: Problem solving and its application in education

1.1. Normal Problem Solving

The meaning of the word problem and a brief history of modeling in education

The transfer of interdisciplinary skills from one context to another through answering practical problems by means of problem solving has always been a main task in the history of general education. Therefore, literature about using problem solving in classroom is immense and it follows and accompanies education during all its history.

Indeed, many pages have been written about the major following topic: the use of problems to develop cognitive capacities and to improve effectiveness and long lasting memory of the different subjects involved. Especially, but not only, in the English-speaking world, people have tried to apply this scheme in many different contexts.

Starting with Math and other Sciences, where the application of problem solving is quite evident, it has always been tried to teach this approach as a tool. The general goal would be to use different topics learned in various areas of different disciplines and combine them together in order to find a solution for a new requested task. That is when the students answer questions that activate not only single areas of theory and practical knowledge, but also different logic aspects, allowing to link disciplines to each other in order to solve the requested problems.

In other words, a problem generally involves one or more subtasks that need to be approached using different tools that are related to knowledge and competences students have acquired in various fields. These both mentally and physically experiences spread from general Arithmetic and History to Physics and Geography.

Before starting to give examples, it could be useful to define the word *problem* in order to better clarify the context in which it can be used for this subject.

"A problem arises when a living being has a goal but does not know how to reach it[14]." This definition, given by Karl Duncker (1903--1940), a Gestalt psychologist, clarifies the fact that this concept has a wide meaning and even a wider field of application. The word comes from the Greek term πρόβλημα (próblēma) "projection, promontory, impediment, obstacle". This derives from the verb προβάλλω (probállō) "to put before" composed by the prefix προ- (pro) "before, in front of" and (+) βάλλω (bállo) "to put, to throw".

In general, you deal with a problem when you are required to find a creative solution in response to a situation never encountered before. This could also be the solution of a simple Mathematics assessment like a sum or a multiplication. In that case, you have to figure out, among the known procedures, the one that can lead to the solution. In the case of Problem Solving, it is necessary to "invent" a procedure. Therefore, there is an initial deductive phase, where the situation in question must be framed; then a second phase in which it is necessary to enucleate the data useful to get to the next step. This is the induction phase, in which the student must make an original contribution, find connections, trace a path, and connect clues, in order to create a solution to reach that goal.

The classical Problem Solving strategy follows these steps, using some small precautions:

In the beginning, sometimes, an intuition might be needed to start with; then, using a reductionist approach, the path is divided into small steps in order to deal with them one by one. There is a story from a wise old man that can be repeated to students to foster this divisive approach. He says: "Everybody can swallow an elephant, it is sufficient to cut it in small pieces"[15].

---

[14] Duncker, Karl, *Zur Psychologie des produktiven Denkens* [Psychology of Productive Thinking]. Berlin: Springer, 1935.

[15] This sentence is normally refereed to General Creighton Abrams during the Vietnam War as "*When eating an elephant take one bite at a time*". This version I think it applies better to the educational task.

A first attempt to define what was known at that time about this topic can be found in Newell and Simon's *Human problem solving* (1972). The authors say: "The aim of the book is to advance our understanding of how humans think. It seeks to do so by putting forth a theory of human problem solving, along with a body of empirical evidence that permits assessment of the theory[16] ."

The theory deals with how humans respond when they are confronted with unfamiliar tasks. In the beginning, all studies were focused on pure theoretical problems like the Tower of Hanoi puzzle or the proof of a mathematical theorem. Later, this framework was used to explain how cognition works. For this matter, the Tower of Hanoi puzzle explains also how then physical world can interact with the brain in order to solve an assigned task. A small explanation about how this game works is helpful to see how many different aspects are involved.

The Tower of Hanoi (also called the Tower of Brahma or Lucas' Tower) is a mathematical game or puzzle invented by E. Lucas[17] in 1883. It consists of three rods, and a number of disks of different sizes, which can slide onto any rod. It is often also used to teach about recursion.

---

[16] Allen Newel and Herbert Simon, *Human Problem solving.* New Jersey: Prentice-Hall, 1972*,* p. 1

[17] François Édouard Anatole Lucas (1842--1891) was a French mathematician, his most important works are related with Fibonacci sequence and series in general. Lucas sequence is named after him. The name "Tower of Hanoi" is due to an old Indian legend set in Brahma. In one of the temples there was a tower made of 64 golden disks placed like the initials three on the image. The disks are placed one on the top of each other in a way that a smaller one is always over a larger. The disks are so fragile that this condition must always apply otherwise they could break. This tower must be moved on another rod using only one possible intermediate stick according to the rules explained over the text. Mathematics says it will take 18446744073709551615 moves to complete the task. Therefore, the legend says that before the priests will be finished the world will fall.

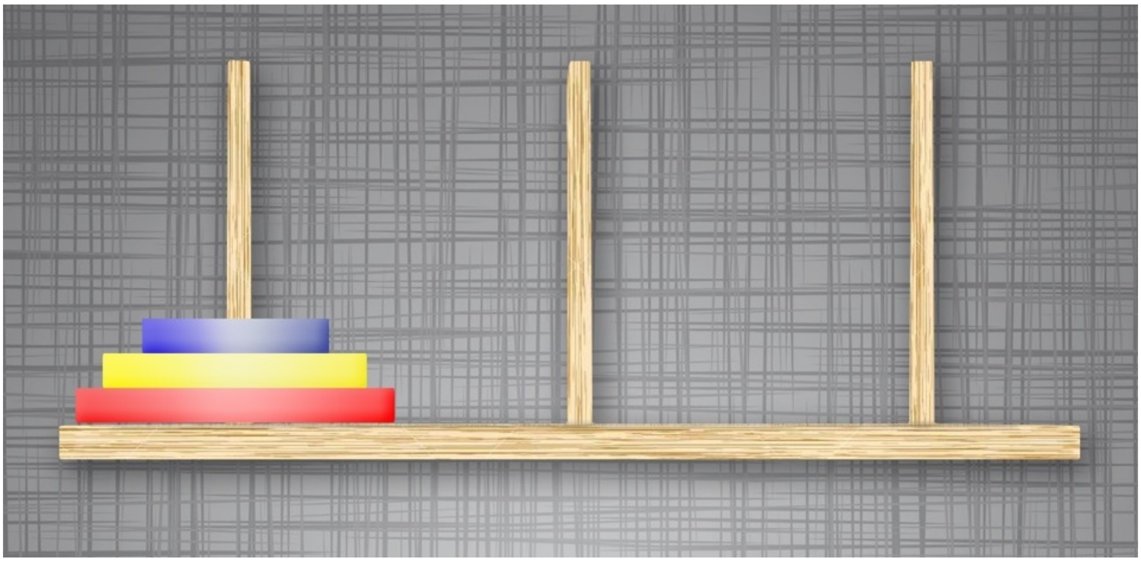

Figure 1.1 The Tower of Hanoi with three disks initial phase Picture by Giulio Donini

The puzzle starts with the disks in a neat stack in ascending order of size on one rod, the smallest at the top, thus making a conical shape. The goal of the puzzle is to move the entire stack to another rod, obeying the following simple rules:

- Only one disk may be moved at a time.
- Each move consists of taking the upper disk from one of the rods and sliding it onto another rod, on top of the other disks that may already be present on that rod.
- No disk may be placed on top of a smaller disk.

With three disks, the puzzle can be solved in seven moves. For example to get to the situation in this image you have moved the smallest disk in the last rod (first move), then the medium in the center rod (second move) and then again the smallest one on the medium in the central rod (third move).

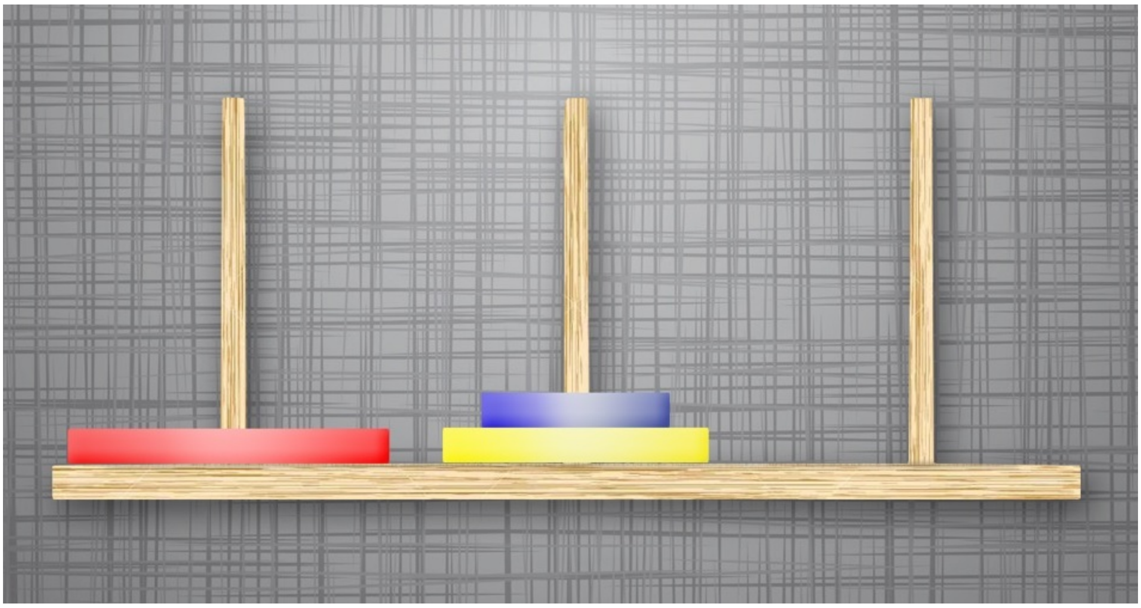

Figure 1.2 the Tower of Hanoi after three moves Picture by Giulio Donini

The fourth move would be to transfer the largest one to the third row, then the smallest one goes again in the first rod (fifth move). Eventually, the last two steps are to put the medium and the small upon the largest that is already on the last rod.

The minimum number of moves required to solve a Tower of Hanoi puzzle is $2^n - 1$, where n is the number of disks.

In their *An Extended Theory of Human Problem Solving* (2005) Langley and Rogers describe the situation: "Newell and Simon (1976) refined this approach into their physical symbol system hypothesis, which states that symbolic processing is a necessary and sufficient condition for intelligent behavior"[18]. Nevertheless, a way to deal with the paces a solver needs to reach the goal is still required, so it was thought that all possible steps are mentally explored, through a sort of a Phase Space of possibilities, and then the one that better suits the task is chosen. Therefore, problem solvers use means-ends analysis (Newell and Simon, 1972). In a way, this can be explained as a theory of perturbations in which you choose the series of actions that lead to desired solutions by

[18] Langley, P., and Rogers, S. *An Extended Theory of Human Problem Solving*. Proceedings of the 27th Annual Meeting of the Cognitive Science Society. Stresa, Italy, 2005.

trying to explore effects of different actions at any step. These can be physical, like when you try to solve a manual problem, or abstract, as the calculations needed in a mathematical problem. This model was used since the 1960s in Artificial Intelligence tests.

Again Langley and Rogers (2005): "Nevertheless, closer analyses of human behavior on novel tasks have suggested that this story is incomplete and that the actual situation is more complicated."

For instance, the role of physical context can be helpful, not only because it allows taking advantage of visual modeling, but also because it introduces new independent aspects that are not related to the original problem. Moreover, Problem Solving is not only a mental-abstract operation, but also rather clearly it interleaves reasoning with execution. In the Tower of Hanoi puzzle, the problem solver may reason backward to select an intended move, but typically makes that move as soon as it is permitted, without waiting to make sure that he can solve the remaining part of the problem from that point. So many studies have proved this is a very powerful way to model human cognitive capacities and it can be an effective way to try to understand the interconnection between knowledge acquisition and its use in different contexts.

An interesting example related to the main task of this dissertation is the generally accepted way to find a solution to a mathematical problem in a classical way. Once the student has obtained the algebra and its formalization tools, it is possible to create an algebraic solution that has the powerful side effect of being abstract and therefore versatile. It can be used in different contexts. A widely used case to practice the model could be the system of linear equations where it is necessary to define a set of variables that exhausts the number of possible degrees of liberty that the system has, then it is required to find relations among these variables and finally to solve the equations.

There are also pre-algebraic methods that are used in early Primary School that do not need to clearly define variables, but only to use a mentally driven guess that leads to the solution. In the end, Problem Solving is a strong and accepted tool to improve the important exchange between theories and practice that in too many cases is missing in the Italian education system.

1.2. Problems and exercises: The exercises train, the problems teach

The first problems presented in school: “The cost of money” and “Counting the hours”

The idea behind introducing complexity tools in high school is related to various aspects and the pressing needs of present day society (Jacobson and Wilensky 2006), but the idea behind this dissertation work relies on what every high school teacher experiences as a starting point. Any Math or Science teacher has been told more than once from a multitude of students that Math is difficult and that the tools developed for Mathematics are too complex or too far from reality. Anybody, especially -- but not only -- in Italy, could say to an audience: “I never understood the real meaning or utility of logarithm and calculus “and not be considered inadequate.

This approach is widespread among the students and sometimes it can even be stretched to less “complicated” mathematical tools like negative numbers. I have heard many colleagues and students challenging the “reality” of some mathematical tools in a sense that their use and applications are open only to an audience of insiders. The idea of too complicated or not real could sometimes also be appointed even to negative numbers because, especially in early years, students have a hard time to familiarize with the concept of missing quantities.

In order to explain the notion of negative, in nearly all school systems, is used the concept of numbers on a straight line where a “zero” point is set. The zero marks a division between positive and negative quantities, which also have a different set of rules. Temperatures below zero or debts are the common “real” concepts used as an example. But the concept may still be found difficult to grasp and disconnected from reality. In order to overcome this difficulty it is indeed necessary to create a bridge between the abstract and the physical world.

I think that to clarify this point the History of Mathematics can help once again: Diophantus of Syracuse[19] considered negative solutions to problems "false" and equations requiring negative solutions were described as absurd. Nevertheless, even if Indian mathematician Brahmagupta[20] had already been using negative quantities in the 7th century, it was not until the 19th century that a theory by mathematicians Peacock and Hamilton laid down the abstract definition of these numbers in Algebra, clarifying the rules and the acceptability of their practical application. This anecdote, I believe, shows how the reality and acceptability of concepts sometimes also goes by its completely abstract and rational definition. If this is true for negative and positive numbers, it can even be true for more complicated tools like logarithm.

As a result, I think that a good balance between practical application and abstract definition of mathematical instruments is somehow required. In order to reach this goal, Problem Solving in practical fields may be a very good clue to understand how to proceed.

Therefore, to begin the experimental part of this dissertation, I decided to submit a question test to a small group of students: an investigating tool without a fully controlled scientific testing, but quite useful to start an approach and see what hitches could be found in the process.

---

[19] Diophantus will have in his solutions no numbers whatever except "rational" numbers; and in pursuance of this restriction he excludes not only surds and imaginary quantities, but also negative quantities. Of a negative quantity per se, i.e. without some positive quantity to subtract it from, Diophantus had apparently no conception. Such equations then as lead to surd, imaginary, or negative roots he regards as useless for his purpose: the solution is these cases ἀδύνατος, impossible. So we find him (v. 2) describing the equation $4 = 4x + 20$ as ἄτοπος, absurd, because it would give $x = -4$. Diophantus makes it his object throughout to obtain solutions in rational numbers, and we find him frequently giving, as a preliminary, the conditions which must be satisfied in order to secure a rational result in his sense of the word. See Heath Thomas, *A History of Greek Mathematics, Volume II: From Aristarchus to Diophantus*, Oxford: Clarendon Press, 1st edition 1921, p. 53.

[20] Boyer Carl B., *Storia della matematica*, Milano: Mondadori, 1990.

The idea behind this test was to remove mathematical difficulties -- or at least reduce them to the minimum -- and propose very simple problems to different age and school groups of students to check their approach. These two problems do not require vast precedent knowledge of mathematical tools, only the four operations or in some cases a few algebra concepts.

As John Dewey, in his 1938 book *Experience and Education*[21], tries to focus on the fact that there is no learning like doing, I have tried to create an occasion of testing the ideas of progressive education in a non-classical form.

I submitted two problem-solving tests to different classes in different schools to see how they could use notions they were supposed to have in different environments.

The two problems are “the cost of money” and “counting the hours”, one of them available as attachments.

In these tests, students were supposed to acquire knowledge of the two topics while trying to solve the problems and discussing the solution with teachers.

The first one is about currency exchange rates and the different money options a tourist has when travelling abroad. The student is given different possibilities of paying in foreign currency: cash before travelling or when already on site, credit card or ATM[22]. These options usually come with some costs, like transaction fees or percent commissions. All the costs and possibilities where displayed on a table and students had to use the information provided to see what option was the most convenient or, after the money exchange, which one was used. This task involves a small algebraic calculation and the well-known problem of reverse solution.

After the testing phase, we discussed solutions and many students said that, while it was clear how to directly change money from one currency to another, the process of

[21] John Dewey, *Experience and Education,* New York: Free Press, 1997.
[22] ATM is an acronym for Automatic Teller Machine, in continental Europe it is commonly known as Bancomat.

calculating how much money there was before the transaction, once the final amount is known, was very hard. Many students, even in 11th grade Scientific Lyceum, said they could not find a way to algebraically model the problem. They needed further explanations to clearly define the unknown and try to find a way to solve the reverse problem.

For the second test, I thought the notion of negative or positive numbers and their use in real problems could be and interesting starting point, so I designed a trial assessment with different features.

The questions involve no difficult mathematical tools whatsoever, only addition, subtraction and the idea of positive and negative numbers on a line. However, this test requires the knowledge of time zones and an idea of what this means from a practical point of view. The practical application of the time zone concept is very interesting. Historically, time zones are measured referring to the meridian passing through Greenwich, in London, where the long famous problem of longitude measuring was solved.[23] This meridian is indicated as the zero on our imaginary line, and time is calculated in GMT, that stands for Greenwich Mean Time, and it can be negative and positive referring to where you are travelling. As an example, people in Paris GMT+1 could say that New York GMT −5 is 6 hours behind, so when the famous Concorde plane was flying there in three hours, you could arrive in New York before the time you were leaving. On the other hand, when Indian software developers in Bengaluru deal with American contractors in San Francisco, they know that a submission deadline on Thursday at 9.00 AM actually means 22.30 PM for them, that is an extension of an entire working day. Bengaluru time zone is indeed GMT +5.30, while San Francisco is GMT −8.00.

[23] For more information about this problem, see Dava Sobel, *Longitude: The True Story of a Lone Genius Who Solved the Greatest Scientific Problem of His Time*, London: Walker Books, 2007.

The test was about flight timetables and the role of time zones in a long route, where a stopover is necessary.

I chose the famous Kangaroo route adapted to Italy: it departs from Rome Fiumicino Airport towards Sydney Kingsford Smith Airport with a technical stopover in Singapore Changi Airport. I gave an approximation of real distance between the cities to facilitate calculations and an actual timetable with departure and arrival given in local time (as it always happens on airplanes).

The students were asked to answer questions like:

- How long is the total flight time for both airlines?
- What date and time will be in Rome when a passenger arrives in Sydney?
- How fast does each aircraft fly?

Distance (km)	City	Emirates Airline	Qantas Airlines
0	ROMA Fiumicino	22.25	22.45
14.000	SINGAPORE Changi	Arrival: 19. 25 l. t. Departure: 21.25 l.t.	Arrival: 19.55 l. t. Departure: 23.00 l.t.
21.000	SYDNEY Kingsford Smith	7.25* l.t.	9.20* l.t.

* Next day

- l.t. means *local time* of the arrival city
- time zones involved are: Roma, GMT +1, Singapore GMT +8, Sydney GMT +10.

I was surprised to discover that students of schools and classes where I proposed this problem found it difficult to solve, even after explanations. They were aware of the notion of time zone and even of how it works, but a significant number of them could not account for a justification of when a plane lands and how much time it will actually take to arrive to destination. The problem was not only the Mathematics involved, but also mainly understanding and comprehending the text and the task required, a clarification of what the terms of the problem are. Therefore, the idea is that they are trained in solving exercises but not, or at least not often enough, in trying to model a solution starting not only from the numbers, but also from the interdisciplinary ideas. Geography, Mathematics and Physics meet in this problem and it is a very interesting challenge for students to eliminate boundaries and to pick whatever you need from each subject. I am not saying that this proves a lack of preparation among the students, who were very well trained in solving Math exercises, but in a way that, when facing a real problem, they had more than one difficulty to solve it. Exercises, which are very important to understand how Mathematics works and are very helpful to avoid mistakes, should be always sustained by problems. The tasks and efforts involved in the process of solving a problem, indeed, can help to reverse the process of knowledge acquisition, starting from the application to the abstract concept.

There was a moment in which all classes were very interested in how important modelling with positive and negative numbers is. I then asked them a simple question, which again raised doubts about how time zones work.

What is local time in the North Pole or South Pole?[24]

1.3. The separation between theory and practice at school: find the answers but also try to ask the appropriate questions

I have a direct experience of Italian high school and I think it has a number of very strong features with some weakness too. On one hand, the Italian high school curriculum, not only but especially in the Liceo, is one of the richest and most complete as far as knowledges and topics (at least in theory). Furthermore, one student that finishes maturity diploma has to face an oral exam, where he or she is asked to firstly present a

[24] For an answer to this question, see annex 2.

brief thesis about a subject linked to as many different school subjects as possible. Could it be more multidisciplinary?

This is a hard question because in my opinion there are some major difficulties that hinder a more complete step towards a more multidisciplinary approach.

Sometimes, especially in a number of Italian high schools, a short circuit occurs between theory and practice during topics based learning. These two aspects, in their broadest sense, should be complementary in the formation of a person, but students in their education experience feel them as separated and non-interacting. As if students were operating a cultural choice: what you learn in school is useful, yes, but only in school!

The difficulties student encountered in both "Counting the hours" and "The cost of money" tests give an idea of how distant they are from the concrete function of what they learn at school.

For school users, in fact, the topics and traditional school subjects by themselves appear not to have any practical application, especially in Italian high school where the themes and subjects are much more complex and sectorial than in many other education systems.

I think this is one of the most interesting problems that new generation science teachers are facing.

Nowadays, Interdisciplinary problems connected with different subjects and disciplines are already a fact and will be more and more part of the everyday life of new generations in years to come. What competences and knowledge will future generations need in order to handle the task that will come? Globalization, climate change, interconnected economy, multicultural societies[25], are already challenging adults now; what will happen

[25] One of the most interesting newly developed concept is the Anthropocene.

in the future, nobody knows. I am confident that there will definitely be more need of understanding complexity.

School should be embracing these challenges and retake its leadership to prepare future citizens of the world.

In Italy, there is also a regimented tradition in education methodologies that is tacitly acquired by students of any age and that is now slowly changing in many schools. “Informal” lessons like those employing PowerPoint presentations, movies, computer based exercises, school trips are not considered full learning moments which could also be part of future testing or assessment. There is a sort of distrust for this approach.

I have seen this key point often during my school experience, for example, when dealing with topics by using PowerPoint presentations or informal teaching: students of different schools and classes always seemed to consider these topics not important, because they did not seem related with grades and exam.

In another way, watching a movie or going to the laboratory to develop an experiment is often not considered by students a consistent and fundamental part of the program. Since these methods do not follow usual school rules like the classic front lecture mediated by the blackboard, students do not feel the need to steer them in structured learning but consider them as a sort of escape, a pause from “normal class teaching“ that does not affect the final result, i.e. grades.

In a direct way, this might be even truer applied to Mathematics and other sciences related subjects. In many 10th or 11th grade classes, a work like “counting the hours” would not be considered important towards the end result, because it only involves simple calculations and there is no algebra.

Both of these aspects represent a brake on the development of multi-disciplinary skill; nevertheless, there is also a problem related to the curriculum itself, which is still very disciplinary and teachers are always asked to treat every point of the school subject.

In order to take a possible different vision of school into account, I believe that teachers should consider the very specific need to have an effective and essential communication in which both the cultural aspect and the technical needs ought to be developed. The informal learning through problem solving activities might be a non-tedious way to improve the students' attention and, therefore, their understanding. I believe that this might be done effectively also by using traditional lecture, computers and active learning-through-doing.

Nowadays more and more Italian teachers feel this need and are moving towards this direction; even the change in ministerial curriculum has taken this fact into account and somehow tries to foster this approach. Nevertheless, this is a long debate in Italian history of education: the problem of which side to take.

One of the most famous historical Italian engineers Giuseppe Colombo (1836--1921), Dean of the Polytechnic of Milan from 1897 to 1921 and author of the most famous engineering manual in Italian history, gave an interesting comment about it. In one paper[26] about the World International Exposition of 1867 held in Paris, France, he noticed that it was common opinion the best locomotives were made by the French, whom technology was born later compared to the English one. He observed that, in his opinion, the reason was due to the introduction in the French University system of the École Polytecniques. Because of that, engineers who have studied in those École had a robust and vast education in pure Mathematics, Physics and Chemistry therefore their mind was more adapt to design new generation's technology for locomotive. English engineers, on the other hand, came from a famous empirical school and they were educated in a different environment focused more on practice than theoretical subject. In that paper, he stated:

[26] Colombo, Giuseppe, *Industria e politica nella storia d'Italia. Scritti scelti 1861-1916*, Milano: Laterza, 1985, pp. 163--181.

> English engineers, if they don't come out directly from the production workshop (and it is most often the case, rather it is the normal rule) they get their education in universities where the theoretical teaching is general. (…) except that in rare cases directors of the English production workshops do not have, as Mr Frankland also says, any knowledge of Chemistry or Physics basis[27].

It is a long debate.

1.4. Jean Lave and the research group in Situated Learning
Mathematics learning in practice

On this point, it is very important to discuss the relationship between cognition and practice of Mathematics itself, not only in the simple sense of its use, but also in the cultural sense of its applications. For example, "many surveys, carried out in ordinary situations in various socio-cultural groups, show that processes such as those related to mathematical activities depend strictly on practical arrangements and the specific objectives for which they are designed" (Ronzon and Grasseni 2004). Therefore, in these tasks they develop some techniques for solving problems that are linked to school experience, but sometimes mature into different ways designed for particular uses. About this topic, there is an evident cross-cultural multidisciplinary meeting point between Mathematics education, Psychology and Anthropology. Many pages were written about Social, Cognitive and Computational Perspectives in different fields; therefore, in order to give a small example of what was done in this subject, it is a good choice to talk about and describe the part of Jean Lave's work that really impressed me and focus on the topics involved in this dissertation

[27] Ibid., pp. 168, 169.

Jean Lave is an anthropologist from the Department of Education at the University of Berkeley, California, and a long-life expert on cognition and practice in Ethnography. One of her most famous works is an analysis of the process of apprenticeship among the Vai and Gola tailors of Liberia[28]. In the second half of the Seventies, she spent some time among a group of apprentice tailors, investigating how they learned and used Mathematics necessary for their work.

For this study, Lave has chosen Arithmetics not because she had a particular interest in Mathematics, but because she thought there was a way to evaluate the different capabilities of people and compare the results (Lave 1996).[29]

The idea was to compare how people used what they had learned in school, where learning takes place outside of any practical context and without any immediate application, with the special skills they learned on every day work[30], when you really need to use what you have learned.[31]

It is always related to the separation between theory and practice I have been referred to earlier: Jean Lave's hypothesis is that the young tailors may have better learnt their Mathematics through their working experience than by studying it in school. And, as "There's no learning like doing", it can be assumed that their mathematical skills have been better internalized because of their daily use. A very good example of the

---

[28] Jean Lave *Apprenticeship in Critical Ethnographic Practice* (Lewis Henry Morgan Lecture Series) Paperback 2011. In this book, Lave resumes her work on Vai and Gola tailors in Liberia in the late 1970s. She elaborates the analysis on process of apprenticeship by presenting new ideas and reflections that have since emerged. This is also an occasion for discussing and questioning what happened since then in ethnographic fieldwork. During the last thirty years, many new theoretical and practical framework emerged in doing an exploration of her past work and she faces the challenging and the good ideas that came out.

[29] "There are various ways to address this problem: Mine involved a turn to mathematics, at first because it simply looked like a common activity in tailor shops and schools and thus a useful medium for comparative experiments on the educational effects of schooling and apprenticeship. After a while, the underlying assumption of the unproblematic universality of mathematics came into question, as practices involving number began to look stickier and stickier. Eventually math took on strategic force in my work, for it provided a worst-case scenario. That is, if math, the pure "universal," is a relational, situated phenomenon, then so, goes the argument, is everything else." Ibid., p.19.

[30] Lave, J., Smith, S. and Butler, M., Problem Solving as an Everyday Practice in The Teaching and Assessing of Mathematical Problem Solving. Research Agenda for Mathematics Education Series. Volume 3. Charles, Randall I., Silver, Edward A., eds. Reston, VA: National Council of Teachers of Mathematics, 1988.

[31] Lave, J. (1996). Teaching, as learning, in practice. *Mind, culture, and activity*, 3(3): 149-164.

complexity of Mathematics applied to tailoring lies in its strong link to Geometry. When tailoring a shirt, for instance, the fabric is cut and sewn in two dimensions, having in mind that it will be then developed on a three-dimensional model. Tailors may either possess knowledge of scale factors and projective geometry or proceed through trial and error by having in mind a precise theoretical model based on years of experience rather than a result of a mathematical process. What Jean Lave's studies have tried to uncovered, though, is that the issue of knowledge and information transfer from one context to another is also relevant when notions are the result of learning through experience. In other words, no substantial difference would emerge between the apprentice tailors and students acquiring the same knowledge in school. Both would find it equally hard to implement their practical or theoretical experience to solve mathematical problems requiring the same knowledge in different contexts. Therefore, it may be assumed that different working contexts will be equally challenging regardless of how knowledge has been acquired.

Since results were from her 1970’s study, these interesting researches were already presented in another book, published in 1988, *Cognition in Practice: Mind, Mathematics and Culture in Everyday Life*. Moreover, in this book she also carries on the same scheme of analysis by reference to the project AMP (Adult Math Project). It is a project carried out in the United States from 1978 to the mid-80s, where Math skills of different categories of adults, from different places, education and social classes, were investigated in relation to the practical problems faced by these people in everyday life and spaces such as the kitchen, the supermarket and so on.

In this study, the interesting observation that emerges is that the attitude taken by the subjects in Mathematics during the school years has almost no effect on their performance computing application contexts. The informal situations, such as shopping at the supermarket or food preparation -- which require application of mathematical concepts such as the price per unit, per kg, or how many milliliters are equivalent to 3 cubic decimeter -- are for adults on a different plane, with different "rules", far from those learned or used in school. Of course, there was a different approach and performance according to age, education and place of living, but the general trend seemed clear.[32]

In this regard, in the book there is a very interesting quotation from a work of Schliemann Carraher of 1982[33]. An example is analyzed where Mathematics and mental patterns are applied to the solution of practical problems. The best summary description and explanation of this citation comes from Keith Devlin's Angle, an internet website where he talks about mathematics facts and books[34] :

> The anecdote cites a survey held in the Brazilian market, featuring a 12-year-old boy who is selling coconuts. If the price for a knob is 35, when asked to sell 10 nuts, he does not immediately calculate 35x10 = 350 to request the total price, but uses a scheme based on the intermediate steps and patterns of thought known to him. In fact, 10 coconuts are a much larger number than the usual purchase, he considers three to be the maximum number, as he knows the total price for that amount is 105. The child, then, first divides the 10 in smaller groups, which he is used to manage, the result of which he knows by heart, namely 3 + 3 + 3 + 1. Arithmetically, he is now faced with the sum 105 + 105 + 105 + 35, which he calculates with little effort: first 105 + 105 = 210, then 210 + 105 = 315 and finally 315 + 35 = 350. He therefore shows some versatility and knowledge of mathematical patterns in spite of his poor education.[35]

This scheme, which seems unnecessarily complex and far from our current reality, is instead found applied in not too different form in many cases, even today. Very often,

---

[32] In another famous book, by Arnold B. Arons, *Teaching Introductory Physics*, from other studies also carried out in the '80s, the same trend is also made clear. There also was interesting criticism on how Physics and reality are perceived in the general public. He tells about a study in which a number of people were reported to be afraid that electric current would come out of a wall socket because it can actually flow in circuits.

[33] Lave, *Cognition in Practice*, 1988, p. 65.

[34]Keith Devlin is the Executive Director of the Human-Sciences and Technologies Advanced Research Institute (H-STAR) at Stanford University and also a Math Blogger.

[35] http://www.maa.org/external_archive/devlin/devlin_7_99.html Devlin's angle July/August 1999 .Accessed June 2013.

when you find yourself in front of a ticket office or a place where tokens are sold for a variety of uses, the cost for one ticket or token is displayed for the public. Then, sometimes in plain sight, sometimes only for the cashier, it is accompanied by a sheet where the price of 2, 3, 4, 5 (I happened to notice up to 20) tickets or tokens is written. For example if the unit costs 4.5 Euros, the table on the sheet says 2 people = 9 Euros, 3 people = 13.5 Euros and so on. In fact, the numbers shown are a multiplication table so that the cashier or the users cannot get it wrong (note that in many places there is no discount for a high number of tickets purchased, but only the simple calculation table).

Ultimately, it seems that the two abilities cannot interact and are perhaps forced to be separated. In this regard, however, further help may come from investigating another interesting aspect of the approach to calculation. Many studies show that this depends not only on the attended school, but also on where the learning takes place, e.g. the country, the language and so on. In fact, there are several different methods to perform calculations and so the matter presents multidisciplinary aspects related to Mathematics, Psychology and Anthropology.

For example, Ethnomathematics is the study of the relationship between Mathematics and culture. The idea behind that is trying to understand how Mathematics is connected to cultural expressions in various ethnics groups and how this can help to better understand both. It goes from the study of the different numeric systems to the mathematical education in various countries. Furthermore, there are many examples of Indian mathematical tools developed in a very different way compared to the western ones. In another aspect, Joseph Needham's monumental work *Science and Civilization in China* gives many examples of Chinese calculation algorithms and also explains why in modern China numbers are expressed with "European digits". For example, in the Arab world, a different set of digits is used, they call it Indian digits.

I think this could also be of a certain interest to the new challenges in Mathematics education.

1.5. New strategies needed

Situated learning, cognitive apprenticeship: are they possible using Complexity?

The main problem in Mathematics Education has always been how to effectively communicate a structured method to think mathematically or analytically.

The main idea behind this dissertation is that new generations might need a different approach in the knowledge transfer and the use of tools compared to other generations. Of course, any generalization and especially the one related to generations is not a scientific sample, but somehow a few features of this model can apply to this study.

The digital natives generation or, as sometimes they are called, the Millennials, presents a number of features similar to previous generations, and others that have never been observed before. Any teacher can notice every day that Millennials are multitasking and possess a technological sophistication which drives their way of life. Italy has one the highest number of mobile phones per person in Europe and young generations use smartphones to do a multitude of things. The use of Facebook and other social network also makes them community conscious, helping them to better perform teamwork-oriented tasks. They are connected 24 hours a day, 7 days a week and use computers as everyday tools.

Sometimes, digital natives are also called the Internet generation because these two aspects are extremely interconnected.

Digital natives are such because they naturally speak the digital language of technology and, through the Internet connection, they communicate with the world in this language. The concept of time, community and in some sense self-learning are therefore brought to a different level.

Video games are also a natural part of their lives, both those played on a console like Xbox or PS4 and those played on smartphones like “Clash of Clans”. This poses new challenges to the digital immigrant’s generation to whom most teachers belong. The digital immigrants are the last generation that experienced the transition to the computer era. They were born in a world were mobile phones did not exist, computers were used in airports, banks or research centers outside the home, encyclopedias were still big and heavy and the library was the main source of widespread information. The previous generation education models are also founded on 19th century Industrial Revolution tools, in a world that can still be modeled in a controlled way. An interesting example of this approach can be found in parts of Scientific Lyceum Mathematics curriculum, where mathematical tools, like trigonometric or irrational inequalities with absolute value, have a very important place.

For digital natives, complexity as a way of thinking might be somehow closer to what they are normally dealing with. There are relatively few studies about complexity and the high school world (Jacobson and Wilensky 2006) and it is possible to see if these concepts can be of any use in everyday education.

Jean Lave and other anthropologists have been discussing a lot about the importance of situated learning to increase effective knowledge exchange between generations. Cognitive apprenticeship also involves a constructivist approach and its model takes perfectly into account how sometimes teachers can direct students by giving examples and discussing problems. This approach can take into account some of the implicit processes taking place during the progression of learning between a master and his apprentices.

One of the scopes of this dissertation is trying to test if complexity might be a kind of situated learning for the generation of digital natives.

Chapter 2. Complexity in general and its presentation in High School teaching

This chapter will try to give an overview of the basis topics related to complexity through teaching examples[36]: self-organization, emergency, dynamics systems, complex adaptive systems and the concept of computational complexity related to computer science.

Before that, a definition of complexity should be given, but this is a rather impossible task, because of the intrinsic nature of the concept itself. I will, therefore, try not only to clarify what complexity is by giving a number of cases, but I will also consider how this concept can be taught, and I believe this can only be done through examples.

2.1. The general concept of Complexity
A brief presentation

Complexity science and its numerous insights are spread out across a tremendous number of disparate disciplines, fields of study, and areas of research, making the task of assembling this knowledge into a consistent picture rather difficult. Therefore, when I first sat down to write this chapter, I had in mind to write a brief and concise compound on the general concept of complexity. I had the idea of starting with a general introduction and then talking about how complexity affects both natural and human sciences and its applications. So, Complexity in Physics, in Mathematics, in Biology, in Computer science, and so on.

[36] Gregory Bateson in Chapter 3 of his book *Mind and Nature: A Necessary Unity* (Advances in Systems Theory, Complexity, and the Human Sciences) talks about MULTIPLE VERSIONS OF THE WORLD. This narrative takes place through examples of different approaches in order to clarify the point: The case of difference, the case of binocular vision ….and my favorite the case of synonymous languages where he talks about the square of a binomial $(a+b)^2 = a^2 + 2ab + b^2$ and its geometric interpretation.

I realized then that what I was doing was exactly using the paradigm of teaching with separate and independent topics I thought should be revised. Therefore, if I want to give a teaching model of complexity in high school, first of all I must not make the mistake of segregating different disciplines. By saying that, I do not mean that the structured, ordered knowledge, divided into "simple" sub-subjects, classified in single disciplines by the 19th century teaching method, must be thrown away. What I mean is that it should be put in a different, wider and more flexible framework, because these days students experience complexity in everyday life, in a way that is different from that of the role of air in Galileo's experiment on the fallen objects. In that famous experiment, in fact, Galileo thought that the complex and different movements of falling objects were due to their different relation to air, so by simply removing air, all complexity would be gone.

Obviously, Galileo did not say anything about complexity in modern sense, what it meant by that was removing what can take you to error.

Familiarity with peer-to-peer and not broadcast information, like the internet or self-education via YouTube lectures, is a common heritage of present young generations and is not a complex fact that can be removed for education purposes. Nevertheless, it is maybe possible to try and help young minds to understand it and make use of it for learning purposes.

The fact is that in our society, where information and knowledge are so vast and accessible, you can only be an expert (even for educational purposes) in some niche of a particular topic. Therefore, high school education is the last moment in which students can have a bite of every subject, but with a little bit of structure. In my experience, nowadays the point is not only to provide students with information as structured knowledge, but to give them an analysis framework, and that must be, unavoidably, complex. Indeed, what happens in every present day classrooms, is that the teacher does not have to add information; instead, he or she must help them to remove non-important elements and rather focus on those interesting for each goal. School has always been about that since the beginning, but now the need to add a general framework to the discipline you are focusing on is deeply felt by both teachers and students. In every school and class I have been in for the data collection phase of the present study, I was asked why the topic of complexity is not taught in class. In fact, by teaching through complexity I mean that any teacher can point out similarities with other disciplines and cross-cultural references. In this frame, the theory behind, for example, Physics could provide unexpected hints to understand practical, complex problems through Biology (see the following example of the flock of birds). Therefore, discipline, knowledge and complexity framework should work side by side within the education process.

The question is now how to present complexity to students, because these ideas have not yet put foot into the classroom. As early as 1997, Professor Rick Ginsberg, in his article "Complexity Science at the Schoolhouse Gate?" stated: "The failure of schools to evolve with increased knowledge about human brain functioning as 'force of habit,' leaving hallways and classrooms tied to 'their 19th century roots?.'"[37]

The task of teaching complexity is not an easy one, because of its many implications and different patterns. One of the major difficulties to deal with at first is the Mathematics involved, connected with Computer Science, Biology and Economics, the fields in which complexity is closer to its application. Nevertheless, for its important role as a thinking model, it is a very powerful tool to understand such fields as Sociology, Psychology and even Administration of Justice. We will see in Chapter 3 that the latter has many interesting applications in Complex Problem Solving.

Moreover, schools and classes themselves can be considered as complex adaptive systems, because they are open, not closed, systems and they are definitely non-linear.

An interesting experience many teachers have made is how the same lecture can have a different impact on different classes, even when students are the same age and in the

[37] Ginsberg, Rick. Complexity Science at the Schoolhouse Gate? *Complexity*, 2, 4 (March/April 1997): 10.

same school. That is because, being a complex system, they react differently, and solutions and recipes valid for one group prove to be ineffective with another.

For all these reasons and many others, finding a way to adapt an effective description of Complexity is an even more difficult task, and inherently incomplete because of the limits the very concept entails. The various areas of interest are, indeed, spread out in so many areas of research and fields of study, that it is rather challenging to even try to present a comprehensive overview. As Isabelle Stengers says in one of her essays: "it is not possible to define a paradigm (in Kuhn's sense) of complexity because in doing so, you would jeopardize the notion of paradigm itself[38]."

The idea that came to mind was, therefore, to use this chapter as a tool, a small handbook, trying to give ideas about different aspects of complexity in a way designed to create a sort of complexity awareness in school students. In order to apply this approach, I believe it would be useful to talk about complexity in school starting from the ideas involved in it, and giving examples to explain them to students.

You can initially try to explain this concept on the basis of studying the works of some of the great representatives of this field of knowledge. Nevertheless, it is still not possible to give a comprehensive coverage of the countless disciplines and implications that are involved in complexity. Besides, the scope of this chapter is to find useful hints to present this concept to young high school students.

In particular, it may be extremely helpful to start from the two people that defined the cornerstones of this both philosophical and scientific project: Edgar Morin and Ilya Prigogine.

Edgar Morin[39] is a famous French thinker who, with his monumental work *La Méthode*, has developed a project-book about constructivism and reality in six volumes, the first

[38] Stengers, Isabelle. Perché non può esserci un paradigma della complessità, in AA.VV. *La sfida della complessità*, Milano: Bruno Mondadori ed., 2007, p. 38.
[39] See Edgar Morin *La tête bien faite*, 1999; and *Les sept savoirs nécessaires à l'éducation du future,* 2000.

of which was written and published in the late Seventies under the title of “La Nature de la nature” and the last one “L'éthique complexe” was published in 2004.

He defines the role of complex thought in the different disciplines as a true *maître à penser* of the new way of approaching problems.

The problem that Morin is facing is the inadequacy of the experimental method when it involves the analysis of real phenomena in their entirety. He argues that the powerful and effective method that allowed Galileo and Descartes to build a model for the system of the world (i.e. the solar system observational astronomy of the seventeenth century), even with the technical perfection achieved by Lagrange and Laplace, is inappropriate or rather loses effectiveness when viewed through different eyes.

A good metaphor to understand how the idea of this system works is the clock, a repeated and predictable mechanism in which each element interacts with the others, following an established order.

Can this structure be adapted to the whole universe or, in doing so, you could see a distorted and confused representation of it, by mean of framing simplicity into reality?

These are the questions that have always been asked since the birth of Science, matters that epistemologists and scientists are still debating even in our day and age.

Morin, in his writings, provides a possible answer. For example, the three-body problem already poses limitations to the descriptive and predictive power of exact science.

This is the problem of taking an initial set of data that specifies the positions, masses and velocities of three bodies in some particular point in time and then determining their motions, in accordance with the laws of classical mechanics -- Newton's laws of motion and of universal gravitation. When massive bodies subject to their mutually perturbing gravitational attractions are more than two, the problem becomes too dependent on the initial conditions. This was already discovered in the seventeenth century and, in 1887, the mathematicians Heinrich Bruns and Henri Poincaré[40] showed that there is no general analytical solution for the three-body problem given by algebraic expressions and integrals. The motion of three bodies is generally non-repeating, except in special cases.

Morin says that this problem cannot be solved theoretically, not because of our limitations, as we do not know the location and speed of all bodies in the Universe; but because of our intrinsic limitation to model complex behaviors. Otherwise, the *clock model* would perfectly describe the past and accurately predict the future.

What Morin means is that this limitation is not scientific but philosophical. Consequently, a change of approach is needed, or, in his words: "to pass from focusing on the knowledge of nature to the nature of knowledge[41]." This transformation strongly depends on the cognitive tools used to enquire Nature with and, therefore, it needs an analysis on the foundations of both natural and human and social sciences.

This analysis can also represent an opportunity to build a common guideline that unites all sciences, including Human Sciences.

So the new question for Morin becomes: what is reality? Alternatively: what is the best representation of it?

Instead, Ilya Prigogine, Nobel laureate, physicist and chemist, starting from his studies on the Thermodynamic Theory of the Dynamics of Far-from-Equilibrium Systems, tries to form what he calls "the new alliance" between the Physical and Mathematical sciences on one side, and the Biological and Human sciences on the other.

It is evident that, in the schemes of scientific method, simplifying the problem is required to be able to solve it. In other words, it is possible, sometimes, to use the already

---

[40] Wolfram, Stephen. *A New Kind of Science Notes for Chapter 7: Mechanisms in Programs and Nature Section: Chaos Theory and Randomness from Initial Conditions,* Champagne, IL: Wolfram Media, p. 972. Available online.
[41] Mori, E., *La Méthode. I. La Nature de la Nature*, Paris: Seuil, 1977 (trad. it. parz. *Il Metodo. Ordine, disordine, organizzazione*, :Milano Feltrinelli, 1985).

mentioned “how to eat an elephant” scheme. This is obtained, for example, in Physics, by gradually removing the additional constraint and conditions, until you get to the core of the problem, as Galileo did with the falling bodies by eliminating the air!

This timeless world where I can, in theory, run the clock forward or backward without changing the NATURE of reality, is it the only world that I can discern?

Here comes Prigogine who, starting from Thermodynamics, the first science in which you encounter a non-reversible time, prepares the ground for a change in mentality.

If, in fact, the ideal phenomena are reversible, real phenomena they are not!

The second law of Thermodynamics identifies, for the first time, the existence of a preferred direction in the flow of time in which the past affects the future and you cannot reset what happened before even in the simplest phenomena. In his book *The New Alliance*, written with Isabelle Stengers, Ilya Prigogine writes: "it is no longer stable situations or permanency that interest us, but rather evolutions, crises and instabilities.[42]”

In that book, Prigogine and Stengers depict modern science as being "against nature because it denies the complexity and the coming-into-being of the world in the name of a knowable, eternal world that is dictated by a small number of simple, unchanging laws.[43]”

It is true that many phenomena can be described in terms of simple, linear mathematic form. Yet, as Prigogine and Stengers point out, this approach has unfortunately led to a mechanical vision in which Science became an instrument of Nature domination and the scientist shut himself off from Nature with the rest of humanity in the ivory tower of supposed objectiveness.

[42] Ilya Prigogine and Isabelle Stengers, *La nuova alleanza. Metamorfosi della scienza*, 2nd edn, Torino: Einaudi, 1999.
[43] Ibid., p. 48.

Using this framework, he demolishes even the more simple reversible experiment, the pendulum[44].

The time that separates Sciences such as Physics and partially Chemistry, Biology and Cybernetics is no longer a problem; you can build a science of becoming in which chaotic phenomena can also create ordered sub-structures thanks, or through, the laws of Nature.

This opens the door to a method of complexity that is no longer a limitation of knowledge but a guide for the analysis of the most diverse objects and disciplines, even those that are distant from the classical natural sciences.

Building this method means adding the biological field with cybernetics and get to represent a universe whose image is complex as much as the one we see looking within ourselves (Prigogine 1985).

Let us now move to some examples, useful to describe complexity, as well as teaching examples.

2.2. Self-organization and emergence
Different aspects of Complexity

Self-organization is a feature which characterizes open systems; these are typically far from thermodynamic equilibrium and exchange matter, energy and information and the external environment. When related with the environment, a self-organized system reacts and reshapes its own structure based on its experience and on external events. The parts that compose the system normally interact with each other following some simple local rules, they are normally called *agents* because they act through, and within, the system.

[44] Ibid., p. 238

Self-organizing systems share these features[45]:

- Complexity: Complex systems are composed of parts that are interwoven with one another through mutual relationships in permanent change. The parties may change similarly each time. Complexity makes it more difficult to describe and predict the behavior of the systems in their entirety.
- Self-similarity: self-organizing systems are self-referential and show operational unanimity. It says: “every behavior of the system feeds back on itself and becomes the starting point for a new behavior". Closed operating systems do not act as a result of environmental influences, but they are independent and responsible for themselves. Self-reference is a concept also applied in the case of open systems.
- Redundancy: in self-organizing systems, there is no separation, in principle, between the organizing parts driving evolution of the system itself. All these parts are potential creators.
- Autonomy: Self-organizing systems are autonomous if the relations and interactions defining the system as a whole are determined only by the system itself. Autonomy refers only to certain criteria, since there is always a possibility of exchanging matter and energy with the environment.

This behavior could create locally a number of ordered substructures of subroutine that do not possess the knowledge of the system in its entirety. These parts, born within the system, emerge from the chaotic and unpredictable general behavior. This is called *emergence* and it is one of the aftermaths of a self-organized system. A famous way to explain what emergence means is to say that the whole is bigger than the sum of the

[45] Probst, Gilbert J.B., *Selbstorganisation -Ordnungsprozesse in sozialen Systemen aus ganzheitlicher Sicht*, Berlin and Hamburg: Verlag Paul Parey, 1987, p.76-83.

parts. That is to say that the very large number of interacting agents creates a higher hierarchy-ordered structure.

Another characteristic of these systems is that they have a common behavior even within various areas and parts. Therefore, the theory developed to treat one of them can also be applied to another completely different area.

An example, very useful because it is easy to understand and to picture without a specific Mathematical or Physical knowledge, is the behavior of animal species when they interact with each other in respect to an external incentive.

A flock of birds, no matter how big it is, presents the concept of emergence. Migratory birds, like ducks, align in a structure that is perfectly shaped to oppose less resistance to air (the famous V formation)[46]. Therefore, they can save energy. Military aircrafts copy these structures when they fly in groups, saving both fuel and time.

Figure 2.1 a V shaped F4's formation [47]

[46] Portugal, Steven J., Hubel, Tatjana Y., *et al.*, Upwash exploitation and downwash avoidance by flap phasing in ibis formation flight. *Nature* 505, 16 January 2014, pp. 399–402.

[47] A formation of IRIAF F-4s over Bushehr. Watermark info: CC Shahram Sharifi, 16 September 2009, Author: Shahram Sharifi licensed under the Creative Commons Attribution-Share Alike, source: https://commons.wikimedia.org/wiki/File:A_formation_of_IRIAF_F-4s_over_Bushehr.jpg

Figure 2.2 A V shaped Eurasian cranes flock[48]

Obviously, no bird is aware of the law of Aerodynamics, they simply adapt to the environmental conditions, even presenting an optimal flexibility. Indeed, each bird never exits the formation even when it presents a swift motion or a sudden change of course to avoid predators or other possible obstacles. So, collectively, they behave in a way that no single bird is able to understand. Every bird follows simple instinct rules, and yet the network can adapt to different air density, avoid predators and define a migration route.

Another amazing emergence and self-organized phenomenon is the behavior of ants' colonies. No worker has the big picture, nevertheless they can create highly efficient houses, and they can decide[49] what is the best place to build it exploring the

[48] : Eurasian Cranes migrating to Meyghan Salt Lake, Markazi Province of Iran., 12 November 2010. Source: http://www.panoramio.com/photo/43585282, Author: Hamid Hajihusseini licensed under the Creative Commons Attribution-Share Alike.

[49] Portha, Stéphane, Deneubourg, Jean-Louis and Detrain, Claire, Self-organized asymmetries in ant foraging: a functional response to food type and colony needs, *Behavioral Ecology* 13 (6) (2002): 776-781.

surroundings; furthermore, they can build the most efficient road to transport food over any possible paths.

The self-organizing behavior of social animals might allow to extend the concepts of self-organization also to human society and human beings. In fact, there is a line of research in Psychology, which assumes that human being operation is to be read through complex systems theory (see Chapter 3).

In Economics, some say that Capitalism is self-organizing, because it can adapt to the needs of the market by what is called *an invisible hand*. The expected effect would be to regulate the dynamics of an imbalanced system and agents which are not in balance with the system anymore. This would be the periodic crisis of capitalism restoring balance to the agents.

This concept can be applied in simple cases of self-organization encountered in Physics. They are represented by phase transitions (first and second order) and by the spontaneous breakage of symmetry as well as by the Thermodynamics of Far-from-Equilibrium Systems, a type of dynamic equilibrium in which the state of a system is constantly changing with time, due to an external energy (or matter) input.

In Chemistry, self-organization is particularly important in areas such as reaction-diffusion, the oscillating reactions, autocatalysis, supramolecular chemistry and liquid crystal.

In Biology, important examples of self-organizing phenomena are: spontaneous folding of proteins and other bio macromolecules, formation of lipid bilayer membranes.

A big step forward in considering emergence out of self-organized systems is the brain and intelligence.

On this topic Stuart Kaufmann, an American medical doctor, theoretical biologist and complex systems researcher, has tried to understand the complexity of biological systems and organisms as a result of self-organization and far-from-equilibrium dynamics. In his book *Origins of Order* (1993), he states that cell types are network attractors, and therefore Darwinian evolution can be seen as result of mathematical Biology. In January 2010 Kaufmann joined the University of Vermont Complex Systems Center[50] for two years, and what they were trying to do was to eventually create a framework of interpretation that could link all the different aspects of self-organization and emergence in the different branches of science.

Kaufmann and many others (for example Varela, Maturana) have also proposed that intelligence emerges thanks to the vast number of neurons interactions, exactly like an awareness coming out of meaningless small information exchange through brain cells. The several billions of neurons in a brain perform a very simple action: they deliver electrical signals to each other across what is called a synapse. This physical phenomenon is widely known, but what is not yet known is how this phenomenon is linked to the emergence of a conscious mind.

It appears that this is in fact a complex system. It shares many or the features above mentioned, therefore consciousness is more than the sum of the interacting neurons. It is a hierarchically higher structure emerging from this massive number of complex exchanges.

On the other hand, can mind and thought be reduced to physical interactions among neurons? This assumption has a large amount of cognitive and epistemological implications. Francisco Varela and Franz von Foerster started this field of research with the famous Macy conferences[51].

There are many possible consequences deriving from this assumption, but the most important one is the juxtaposition between an intelligence that develops only within the sphere[52] of its abstract information exchange, and the one that deals with the external environment.

---

[50] http://www.uvm.edu/~cmplxsys/
[51] Francisco Varela and Franz von Foerster, in *Bocchi G. and Ceruti M. (eds), La sfida della complessità, Milano: Bruno Mondadori, 2007.*
[52] Ibid., p. 132.

Artificial Intelligence, a branch of computer science, is trying to answer these specific questions by building a functional model of the brain, where synapse activities are mimicked by signal exchange among circuits.

2.3. Experiences in Artificial Intelligence

The workshop on Synthetic Modeling of Life and Cognition

During the summer of 2013, I attended a very illuminating workshop, organized by the University of Bergamo, about these and other subjects, called Synthetic Modeling of Life and Cognition: Open Questions.

Matej Hoffmann, Alessandro Roncone, Giorgio Metta from the Istituto Italiano di Tecnologia based in Genova, Italy, presented their work with iCub, a robot developed as part of the EU project RobotCub and subsequently adopted by more than 20 laboratories worldwide.

The goal of the RobotCub project, fully open source and hardware, is to study cognition through the implementation of a humanoid robot, of the size of a 3.5 year old child.

> The iCub robot has 53 motors that move the head, arms and hands, waist, and legs. It can see and hear; it has the sense of proprioception (body configuration) and movement (using accelerometers and gyroscopes). Improvement goal now is to give to the iCub the sense of touch, and to grade how much force it exerts on the environment.[53]

[53] http://www.icub.org/ presentation of the project.

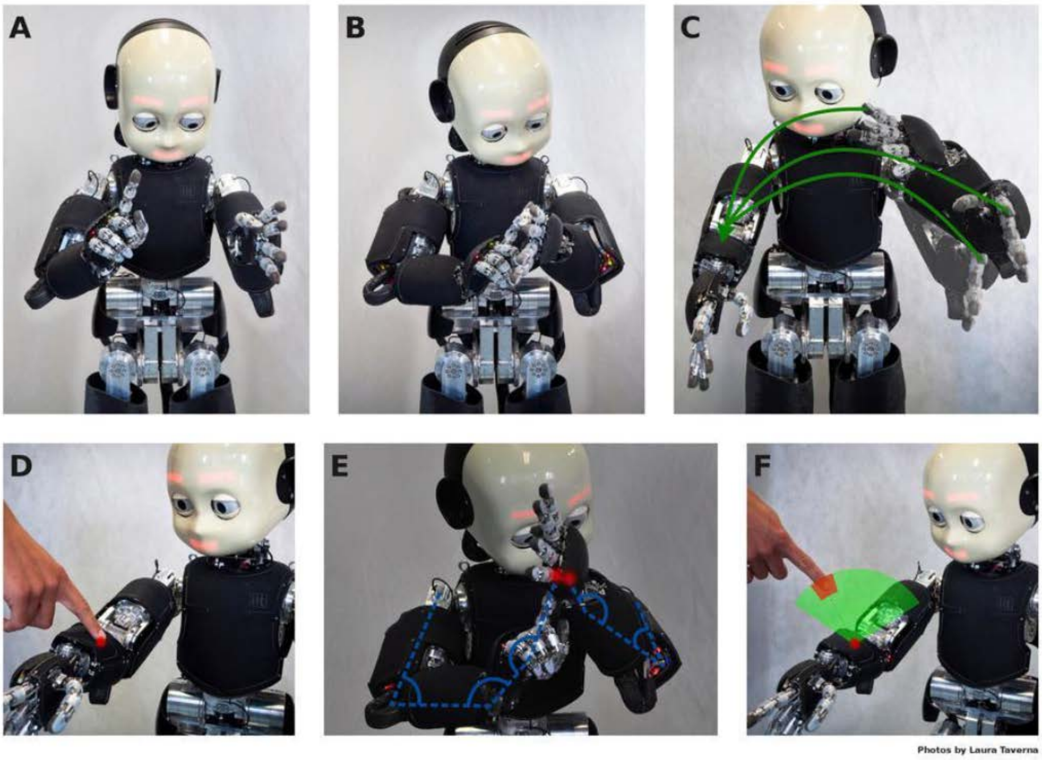

Figure 2.2 ICub during a visual experiment picture available at the workshop[54] Picture by Laura Taverna

The most interesting feature of this robot is the link between internal computer science software and the external senses. The hypothesis behind the theory of this robot, is that while learning kids are evolving -- shaping and forming their brain connections -- through interaction with the outside world. The goal of iCub is to reach the next step in Artificial Intelligence (AI), because if self-learning software is trapped within circuits and does not relate itself with the outside environment, there are some tasks that it will never learn. The iCub research group say that tasks like walking or grasping an object must deal with the concept of *outside* that is a very difficult thing for a computer to "understand". Therefore, this is the next step in building AIs: the possibility to explore another environment, a step that was impossible in early computer times.

[54] Hoffmann, Matej Roncone, Alessandro and Metta, Giorgio. , Modeling the development of human body representations,p. 2. http://www.pt-ai.org/sites/default/files/smlc2013/abstracts/11_Hoffmann_abstract.pdf

This point is where a big debate arises. On one hand, neuroscientists try to model how the human brain works and to describe the role of synapses in creating thoughts, memories and elaborations. On the other hand, computer engineers try to build an artificial brain through software and hardware. They meet and discuss similarities and differences between their approaches in order to understand the common problem they face.

The debate is extremely interesting from an education point of view, because it is an excellent example of the limit and the potentiality offered by multidisciplinary work and its problematic exchange.

Again, in emergence theory, Biologists could apply the same method to the very first topic of their science: life. In the famous Miller–Urey experiment in 1953, they tried to recreate the conditions of early Earth to test the chemical origins of life. Since then, many scientists have been trying to discover how life as self-replicating DNA can come from inorganic material life. Complexity is one of the possible answers; many scientists have built hypothesis on how life could emerge from a chaotic system as a higher hierarchically-ordered self-replicating structure out of fluctuations among early earth energetic, inorganic chemical components.

2.4. Networks and complex adaptive systems

Examples of teaching

One of the very few examples of teaching complexity to undergraduate students (still not in high schools) I found, is given by sociology Professor Brian Castellani of the Kent State University, Ohio, within the SACS[55] (Sociology and Complexity Science) website project.

[55] http://www.personal.kent.edu/~bcastel3/

He says that a very useful kit to explain how complexity in sociology works, is to build a complexity map. This concept is indeed simpler than the application of complexity in other sciences, because it does not involve differential equations, a tool that many students of human faculties do not possess. Castellani introduces what is a primarily visual way to understand complex networks as well as cellular automaton[56]. Moreover, he says that the World Wide Web is already a large model of the complexity map. Here is the explanation provided a paper freely available in his web site:

Once the students have an intuitive and visual understanding of the methods, one can proceed to explore them at greater depth. In fact, an entire series of courses could be devoted just to the methods of the Complexity Map.

Using the students in a classroom, we can build a friendship network, asking a volunteer to tell who he knows in class, and then drawing the student's friendship network on a board. We go on asking those students in the network just drew who they know, and so on. As we develop the ramification, key concepts can be discussed: nodes, links, edges, hubs, informal and formal links, etc. We can also talk about the dynamics inside it: how does gossip spread, the annual flu, etc. This can be the clue to discuss how an epidemiologist might control the spread of the flu in this circle by immunizing the nodes, etc. From here, we can discuss much more complex networks, such as global trading, the small-world phenomenon, etc.

This example may be useful to give an idea of how interactions or even disease adaption can shape people to achieve their goal. The aim might be to affect as many people as possible or to know as many people as possible; the famous theory of six degrees of separation is now becoming redundant, and maybe now any other person on the planet can be reached in only four steps.[57]

---

[56] See Chapter 4.
[57] Teaching Complexity Science to Undergraduates in the Social Sciences, Brian Castellani, Ph.D. http://www.personal.kent.edu/~bcastel3/Teaching%20Complexity%20Science.pdf

2.6 Is it possible to give a mathematical definition of complexity?

The concept of computational complexity and its close relation with computer science

If Complexity, as said before, is finally a possible revolution in method, Computer Science and Mathematics have created tools to successfully work within this new method. In fact, the real problem is how to find an effective way to model Complexity. To solve this problem Chaitin and, independently, Kolmogorov in the 1970s found a way to create a flexible model of complexity that could be adapted in wider fields.

Nowadays, almost everybody who is in school knows what a binary string is and what is a bit. Our whole interconnected world now is more than ever expressed in digital numbers, i.e. a series of 1s and 0s. Every student is familiar with the fact that text, numbers, pictures and sounds are reduced into these sequences and computers treat them indifferently.

Chaitin, in his famous 1974 paper "Information –Theoretic computational complexity", tried to use a metaphor to apply the mathematically defined idea of complexity and its consequences to all other sciences, in particular Biology. From that moment on, this way of framing the problem has been tried to overcome the limits of knowledge of all kinds of sciences, human or natural.

Chaitin says: "The complexity of a binary string is the minimum quantity of information needed to define the string"[58]. In other words, it is the minimum number of instructions for a computer program to recreate the original string. He says also that one of the problems in this definition is to clarify what a computer program is, and if the definition depends on the chosen one. In his paper, he says that, no matter what kind of computer

[58] Chaitin, G.J. (1974), "Information- theoretic Computational Complexity", *IEEE Trans. Inf .Theor.*, IT-20, p. 11.

program you use, you can theoretically apply this definition even to a Turing Machine[59]. Nevertheless, in a way, the problem itself remains.

He gives an example of this idea when he tells a story about his old days as an undergraduate student. He and his friends were challenging each other with the task of who could write the shorter program to solve an exercise proposed by their professor. They realized as a group that by many trial and errors it was not possible to write a computer program with a number of lines shorter than the one published before.

> We realized that in order to be sure, for example, that the shortest program for the IBM 650 that prints the prime numbers has, say, 28 instructions, it would be necessary to prove it, not merely to continue for a long time unsuccessfully trying to discover a program with less than 28 instructions. We could never even sketch a first approach to a proof[60].

I tried to explain this fact to students basing it on the original paper, but yet in another way because they can now understand better the concept of Complexity. This is due to new generation attitude towards computers.

I designed and developed an example that can be used in class to explain Compression and Information-Theoretic Computational Complexity. It is related to what it means and what are the implications of compressing a picture with a computer program in order to reduce its information "weight".

A picture on a computer format is composed by pixels on a screen. This seems complicated (not complex) but, in way, it can be explained like that.

---

[59] An ideal machine that is supposed to compute symbols following a certain number of rules, an abstract mathematical concept of computer.
[60] Chaitin, G.J. (1974), "Information- theoretic Computational Complexity", *IEEE Trans. Inf.Theor.,* IT-20, p. 12.

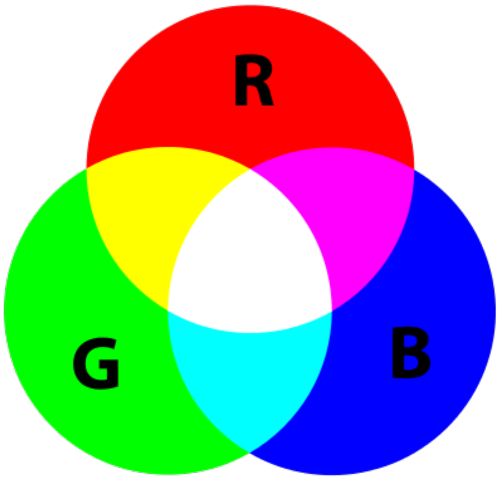

Figure 2.3 The RGB color system when red green and blue are active simultaneously the color white is created[61]

Many modern screens are made by a large number of very small LED (light emitting diodes) forming a grid. Every single LED can emit Red, Green or Blue light (RGB), and every image is created via a mix of these primary additive colours. Imagine the RGB colours on a grid, and each cell on the grid has a number assigned between 0 and 255 to each option (Red, Green and Blue) expressed in a triplet. Each number from 0 to 255 expresses the emission intensity for each channel. This gives a total of 16.777.216 possible colors, equal to 256x256x256 different options[62]. For instance, a completely Blue pixel will be express as (0,0,255). In this way, maximum intensity light can appear only on the last channel, the Blue one. On the other hand, a completely Cyan one will be (0,255,255) (see Figure 2.3). The two extreme cases on this grid will be Black (0,0,0), that means off or no light on each channel; and White (255,255,255), that is full light on all three channels.

---

[61]This RGB image has been released into the public domain by its author Wikipedia user: SHARKD https://commons.wikimedia.org/wiki/File:AdditiveColor.svg

[62] I considered not interesting for the purpose to talk about the actual difference between resolution and actual number LED present on a screen.

Actually, modern LED lamps work exactly in that way by having a series of RGB led all active in order to form white light. It was actually possible to create these lamps only in 1994 when High intensity blue LED was invented [63].

That said, the easier way to depict a picture is to divide it into pixel of a certain resolution (the number of points in which the picture is divided). Therefore, every pixel will have a triplet that expresses how much light any color will need.

The first level of representation is called Bitmap (originally only on Windows operating computers) or with extension .bmp. This actually means giving a color for each point, i.e. a number in the standard 8-bit RGB images. In fact, a computer is limited to exactly this color space when it is manipulating with this standard.

Operating in this way, the file results heavy. In order to reduce its size with a small loss of information, you need a compression algorithm, i.e. a computer program that permits to substitute some parts of it by a calculation.

Indeed, in each picture there are parts in which it is possible to replace a section with its representation. For example, in the picture in Figure 2.4, the area selected can be substituted by the instruction “for every pixel within the rectangle centered in coordinates x, y, z use only color (45,24,125)”. This is how an algorithm works.

[63] Isamu Akasaki, Hiroshi Amano and Shuji Nakamura received the 2014 Nobel Prize in Physics for this invention.

Figure 2.4 An image example. Picture by Giulio Donini

When this operation is applied, the amount of information needed to represent the image is reduced, therefore, the file is now compressed. One of the most widely famous and used compression algorithm is called JPEG, an acronym for the Joint Photographic Experts Group, which created the standard. Every image has, then, a certain amount of information that is uncompressible and a large, but not very large, number of instructions enclosed into the jpg algorithm[64].

The computational complexity deals exactly with this problem. Once you have a string of length *n* bits, following Chaitlin-Kolomogorv, Complexity is the length in bits of the shortest program for calculating the string. The string may be arbitrarily long, so even a picture or a movie is a bit string.

[64] I am aware that this is not a perfectly rigorous explanation because it does not account for the information loss with JPG compression; nevertheless, I think it is useful for the scope.

Somehow, we can then say that a string of length *n* is complex when its complexity is approximately n (confrontable with the length of the string itself). Therefore, it is not possible to find a shorter way to represent it.

The implications for the general theory of science are very interesting, indeed Chaitin continues:

> Now we would like to examine some other possible applications of this viewpoint. In particular, we would like to suggest that the concept of the complexity of a string and the fundamental methodological problems of science are intimately related. (…)
>
> The concept of complexity might make it possible to precisely formulate the situation that a scientist faces when he has made observations and wishes to understand them and make predictions. In order to do this the scientist searches for a theory that is in agreement with all his observations. We consider his observations to be represented by a binary string, and a theory to be a program that calculates this string.(…)
>
> In summary, the value of a scientific theory is that it enables one to compress many observations into a few theoretical hypotheses. There is a theory only when the string of observations is not random, that is to say, when its complexity is appreciably less than its length in bits. (…)
>
> We believe that the concept of complexity that has been presented here may be the tool that von Neumann felt is needed. It is by no means accidental that biological phenomena are considered to be extremely complex. Consider how a human being analyzes what he sees, or uses natural languages to communicate. We cannot carry out these tasks by computer because they are yet too complex for us. The programs would be too long.[65]

[65] Chaitin, G.J. (1974), "Information- theoretic Computational Complexity", *IEEE Trans. Inf. Theor.*, IT-20, p. 16.

I think this work of digital art could usefully explain the difference between a bit string and its representation through a set of instruction.

The picture cannot be reprint due to copyright restrictions

It is available on the author website

http://www.adriansauer.de/arbeiten/16-m

Figure 2.5. Adrian Sauer, 16.777.216 Farben, 125 x 476, Digitaler C-Print, 2010.

This painting is the result of various studies by the artist Adrian Sauer. The idea behind this picture-digital-painting is that, in the standard RGB 8bitperpixel it represents the largest possible image made with all the possible colors appearing only once. The original digital printed image is 125cm x 476cm. I think the relation between this picture and its algorithmic complexity is made clear by Mr. Sauer's own words:

> After manipulating photographs using programs such as Adobe Photoshop for an extended period of time, I increasingly noticed how distinct the division is that the straightforward user interface forms between the basic technology and the user. I finally developed a program that produces images that contain all of these colors exactly once.[66]

So now what is the difference between Mr. Sauer's picture and this one?

[66] http://www.adriansauer.de/arbeiten/16-m accessed November 2015.

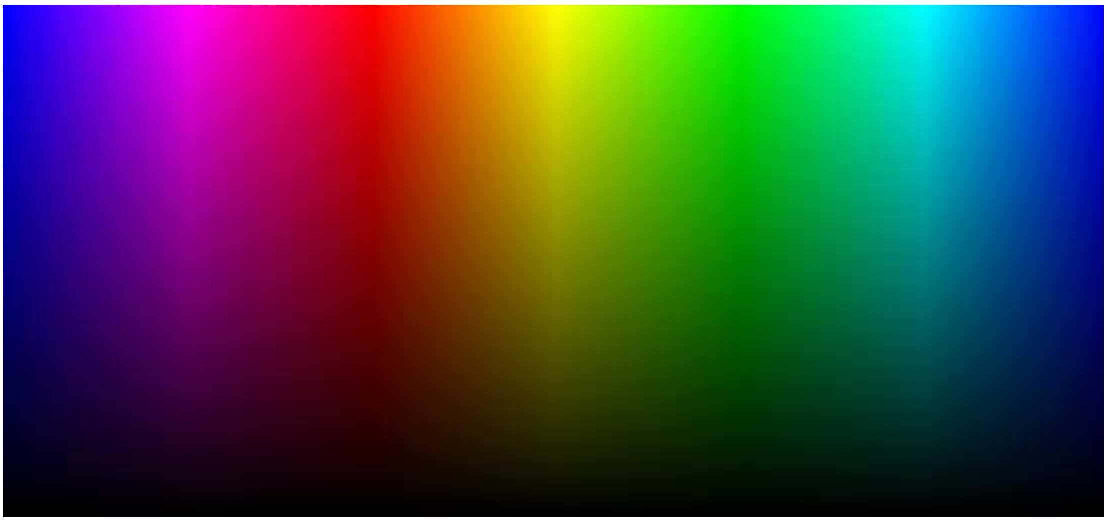

Figure 6 16.777.216 colors gradually disposed [67]

The strings are the same, i.e. all possible triplets between (0,0,0) and (255,255,255) but what is different is their disposition. In other words, the program or set of instructions that gives the location of each triplet is the difference. The first one says to the computer: put all the possible triplets in way that the human eye can distinguish them when they are next to each other. The second one instead simply says: put them in the order that goes (0,0,0), (1,0,0), (2,0,0) (…) (0,0,255)(…) (255,255,255).

I believe in a way it is evident the difference between the two pictures, the length in bit of the string for both them is the same *n*. What changes is their complexity. Mr. Sauer picture is indeed more complex because the set of instructions is much longer than the one necessary for the second picture. It is important to notice also that Mr. Sauer's picture seems completely random but indeed, it is not because there is an important constraint: each pixel must not be near to another one with whom it could be confused.

In my opinion, this explanation could help the students to understand that scientists were always aware that the complexity of the real world is a lot larger than the models

[67] "For the examples below I'll use a different color gradient test map going from blue to blue HSV gradient, with a gradient to black vertically. This represents the entire 8-bit colorspace". GIMP Tutorial - Digital B&W Conversion (text) by Pat David is licensed under a Creative Commons Attribution-ShareAlike 3.0 Unported License. Source: https://www.gimp.org/tutorials/Digital_Black_and_White_Conversion/ image is from GIMP.

used to describe it. Indeed, it was a matter of at what degree of precision is it necessary to do it.

This a very interesting example of multidisciplinary application and a way to see how you can deal with a concept from so many different aspects you could not even imagine.

Kolmogorov, who developed a very similar result independently, proposed to use the concept of complexity also to define a random set. He proposed to call random strings those strings of a complexity close to the length of the initial string n.

This is related with complex problems when all the possible options become so large that it impossible to model it in a simple way. In this moment, you are facing computational complexity due to an incredible number of states or possibilities from the interacting neurons to the solitaire games when the complexity is so big you need to find different strategy.

The travelling salesman problem is a famous and good example of it. Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?[68]

[68] Applegate, D. L., Bixby, R. M., Chvátal, V. and Cook, W. J. *The Traveling Salesman Problem*, 2006.

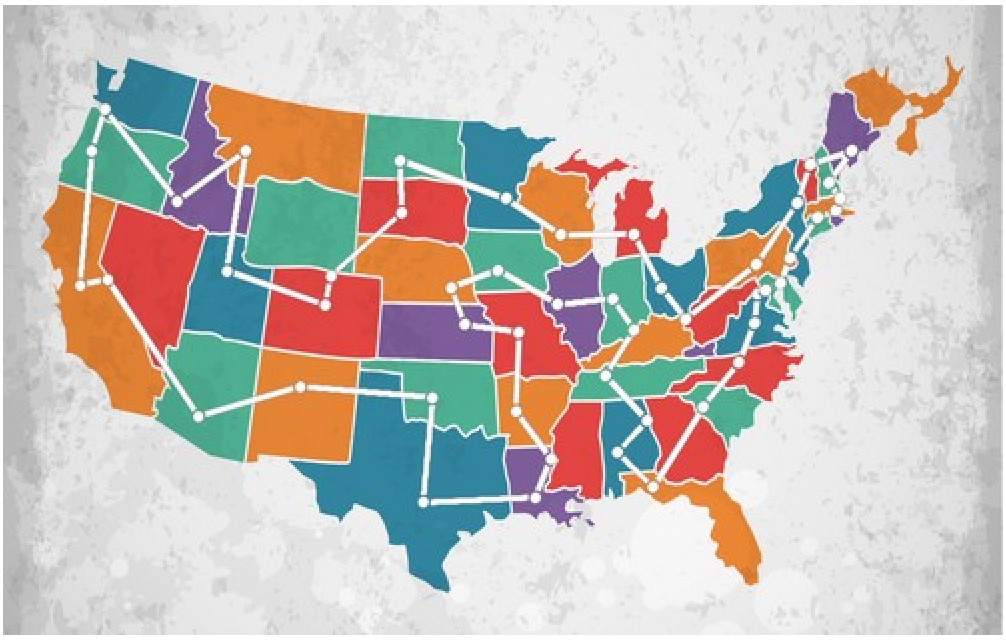

Figure 2.7 Travelling salesman example in the USA. Picture by Giulio Donini

The simple modelling does not work anymore because the instructions you would need to represent them is enormous, so here comes the trial and error approach (exactly as Mr. Sauer has done) to solve the problem. The point is: now, if the possibility space phase is so vast, how could you possibly put a line that separates simple problem solving from complex problem solving?

Here is where complexity comes to your aid for example. The tower of Hanoi puzzle is definitely a long string of moves, but it is not complex. I would say it is complicated to actually make it when the number of disks is sufficiently high, but certainly it is not complex (definitely boring). The salesman problem, instead, is complex, so you cannot find a simple strategy to solve it, i.e. to find the best solution when the number of cities becomes large; an heuristic approach is needed. Therefore, you have to guess, try multiple options, than go back to the previous one: in other words you face a complex problem. A similar, but somehow different, problem that I used in my lecture about complexity is the solitaire example, see Chapter 4.

2.5. Beyond natural Sciences: an example of Language classroom

In their paper: ”Complexity and the Language Classroom”[69], Duane Kindt and other professors of different Japanese universities give an interesting example of classroom activity. Yet, this activity is still designed to be performed in undergraduate classes. Basically, they use complexity as an interpreting grid to understand people’s behavior and communication. They give what they call the “Twelve key characteristics of complex, dynamic systems”:

1. Dynamic: changes over time
2. Complex: having many parts, and/or many interactions.
3. Nonlinear: effect is disproportionate to the cause
4. Chaotic: has a deep, coherent structure within apparent randomness
5. Unpredictable: cannot forecast future states
6. Sensitive to Initial Conditions: small differences lead to vastly different effects
7. Open: energy/information can flow in or out
8. Self-Organizing: a structure or pattern emerges as components interact
9. Feedback Sensitive: feedback is incorporated into behavior
10. Adaptive: optimizes itself according to environment
11. Strange Attractor: deep structural constraints on behavior
12. Fractal: a pattern that repeats itself at different scales

After presenting these concepts to a class, they show different videos about people’s behavior in different environments, and ask students to find which features apply to each case; obviously there can be more than one.

For example in the “ The Nirvana Conversation” (Complex, Dynamic, Sensitive to initial conditions), they show to the class a video clip of a conversation about the famous rock

[69]Kindt, D. Cholewinski, M. Kumai, W. *et al.*, Complexity and the Language Classroom, *Academia: Literature and Language*, 67 (1999):235-258.

group Nirvana in which four people ask each other if they know the band. One of them does and nods, another one has never heard of them and asks: *What is that?* The third one remains silent. They comment:

> This short interaction illustrates the complexity of the environment. The first speakers' brief moment of perplexity regarding this unexpected question was picked up by the larger group, as shown by a small flutter of nervous laughter around the room. An active web of influences becomes evident with the responses and body movements of the inner circle members and the outer circle members' laughter and fidgeting. It is clear that information processing is happening at myriad levels.

It is also interesting how it is explained why it is sensitive to initial condition, because the whole scene would have been really different if no one knew the band, or if all of them were aware of it.

In doing so, they realize that the communication capability is enhanced within class and they:

> see very strong evidence of complex dynamics manifested in the classroom. (…) The language classroom is not a mechanical system. It is made up of individuals who are networked to the outside world, to each other, and to the events as they occur. They are constantly in the process of redefining these connections. The infinite number of possibilities and the highly unpredictable nature of the language classroom can be cause for anxiety--both for teachers and students. Yet as the nature of complexity begins to be understood, these same anxiety-producing factors may be appreciated as a positive, generative force in learning and teaching.

This activity takes place only in class with no relation whatsoever to Mathematics and yet it was found useful in a sort of applied psychology of learning.

2.6. Classical problem solving strategies and complex problem solving

Complex problems may seem more “real” because, in fact, they might be so for new generations. Complexity and complex problems are closer to these days’ effective practice, more than they were before; indeed, young students experience complexity more easily in a field of life and they are familiar with ideas and models connected to it already at a young age. For example, they learn how to use a smartphone by using it, through a trial and error model, and the previous knowledge needed for some usage can be built over practice. There are more examples in the everyday life of new generations. Organizing a self-customized trip managing all parameters like time, money, form of transport and itinerary, or finding a way to measure something like the area of a garden may be other models of complex problems. In other words, you can start striving for a solution and from that point of view you can even arrive to acquire new knowledge.

The first question to arise may be: “Is it possible to use this familiarity to help students modeling a scheme?”

Knowing the situated learning and cognitive apprenticeship theory, is it conceivable to teach complex problem in a learning through doing environment, using software and preparations problems? The transfer of interdisciplinary skills from one context to another and in answering practical problems by means of problem solving might be suitable for this task. In order to do so, the idea is to train students in the concepts of complexity explained in this chapter and see the results of this way of proceeding through a computerized assessment tool.

The next chapter will give a brief description of the history of complex problem solving and its use by psychology in psychometrics explaining the measuring features that are related with it.

Chapter 3. Complexity and Complex problem solving

In this chapter, I clarify the methodology of this research and the tools I decided to use. During the preparation time, I was searching for a tool to both train and measure the ideas of complexity in high school. I then found a very interesting branch of psychology that is related with both complexity and intelligence, the Complex Problem Solving (CPS). In the following section, I will give a small summary of the history of CPS and how it is used in the present study as a tool to measure training in complexity within the high school world.

3.1. Complex Problems and its relation with Complex Problem Solving

In these times, where everyday life is growing more complex and learning how to use even the smallest tools is becoming a different concept, research in problem solving has a shift in interest toward complex problem.

Fischer, Greiff, and Funke (2012) say that "in research on human problem solving CPS is a matter of interest since the 1970s, when there was a shift of emphasis from simple, static, well-defined and academic problems (like the Tower of Hanoi or items of classical intelligence tests), to more complex, dynamic, ill-defined, and realistic problems[70]".

In other words, it is more and more important than in the past to try to give a realistic representation of the difficulties involved in solving real problems, to try to model a theory that explains how to deal with it and, yet more importantly, how to teach it.

Therefore, it is useful to attempt to take these discussions into the high school world in order to prepare new generations to these ideas.

---

[70] Fischer, Andreas, Greiff, Samuel and Funke, Joachim, The Process of Solving Complex Problems, *The Journal of Problem Solving* 4, 1(2012).

The task to give a clear definition of complex problems and therefore of complex problem solving will be discussed in the next paragraph and it involves, once more, a multidisciplinary approach.

3.2. Definition of Complex Problem Solving
The European and American perspective

The first task that comes with CPS is its definition and scope of application. This matter is still under debate, as always when complexity is around. Here, I will summarize this debate and the different perspectives that are born within it.

There is a problem *whenever an organism wants to reach a goal* (Duncker, 1945) and a problem is complex *if a large number of highly interrelated aspects have to be considered in parallel* (Dörner, 1997; Fischer, Greiff, and Funke, 2012). Therefore, both these aspects must fit into the definition of complex problem solving.

Since Newell and Simon, 1972, there was always a debate about where problem solving ends and complexity arises. In other words, the question of when simple problem solving leaves the place to complex problem solving is still in search of a definition (Quesada, Kintsch and Gomez 2005). Nevertheless, some assumption can be made in order to deal with these notions. Quesada, Kintsch and Gomez say:

> At some point in the 1990s, both European and American researchers decided that a line should be drawn between simple and complex problem solving. Both of them seemed to agree about what can be considered classical, 'simple' problem-solving: puzzles that can be solved by simple reasoning programs, using pure logic. The other side of the line is what remains to be clarified.[71]

[71] Ibid., p. 5.

Complex problem solving (Sternberg and Frensc, 1991) and *'Complex problem-solving, the European perspective* (Frensch and Funke, 1995) are the two books that somehow try to draw the separation line within the two concepts. Although they share almost the same title, they define different conceptual maps of what may be intended as Complex Problem Solving in Psychology and various others field of research.

Complex Problem Solving, Principles and Mechanism, edited by Sternberg and Frensch in 1991, is a collection of articles divided in four parts.

I) Reading Writing and Arithmetic

II) Social sciences

III) Natural sciences

IV) Games

It also includes a fifth part meant to draw conclusions from this overview.

The contributors explore various branches of problems and knowledge, trying to give the multiple features of identifying and solving complex tasks. The articles share a common idea of complexity in the field and it comes in a negative form, i.e. complex problem solving, is anything that is not simple in the sense that it cannot be divided and simplified in a lab with a classical approach. The kind of real life problems everyone is dealing with are examples. In this matter, *Cognitive Mechanism and Calculation* by Sokol and Mccloskey is rather interesting. In this paper, the authors draw a picture of the complexity of arithmetic calculation and the effectiveness of previous knowledge to be able to solve this problem in a quick and effective way:

> Calculation provides a very clear if obvious illustration of the general principle that development of problem solving expertise in a domain often involves acquisition of facts and procedures dedicated specifically to solving problems in that domain.(…) In principle, however, arithmetic problems could be solved without these fats and procedure. General reasoning processes, coupled with

> conceptual knowledge of numbers and arithmetic operations would probably suffice. (…) Although these problems could in principle be solved in these ways, the solution process would obviously be extremely slow (and probably error-prone) especially in the case of multidigit problems. From this perspective the calculation mechanisms we have described in this chapter may be seen as a set of specialized facts and procedures well designed for rapid and accurate problem solving. (…) Special-case procedures for processing 0s in multidigit multiplication offer an especially clear example of the role of specialized knowledge in promoting efficient problem solving."[72]

In a way, the methodological approach in dealing with all the examples: it is the expert-novice comparisons. Within this approach, you can somehow group problems such as arithmetic calculations, computer debugging, legal reasoning and international relations because they all share the fact that in order to solve them you need experience. Therefore, the capacity of solving those kind of problems it cannot be measured in a controlled tested environment because it is mostly different in every single case.

The main idea is that experience leads people to find effective solutions to real life complex problems based on previous knowledge and familiarity. Previous experiences are a guideline to deal with new facts and to help solving new problems. The different approach between expert and novice is also quite evident in "Do lawyers reason differently from psychologists? A comparative design for studying expertise"[73] (Amsel *et al.* 1991), where the authors try to examine through different studies whether lawyers or other groups of novices and experts solve the same problems differently.

---

[72] Sternberg, J. R. and Frensch, P. (eds). *Complex Problem Solving.* Hillsdale, NY: Lawrence Erlbaum Associates, 1991. pp. 111--112.

[73]Amsel, Langer, and Loutzenhiser. Do lawyers reason differently from psychologists? A comparative design for studying expertise, in Sternberg, Robert J., Frensch, Peter A. (eds), *Complex Problem Solving: Principles and Mechanisms*, Hove: Psychology Press, 1991, pp. 223--250.

They say that even same problems can be solved differently in different contexts and, therefore, previous knowledge and experience is a key factor in modeling this approach.

On the other hand, Frensch and Funke (1995) examine a completely different approach, confronting a series of tasks in a definite controlled environment, in which standard experiments can be performed. Indeed, the first article is entitled *Definition of Complex Problem Solving*, while the other perspectives almost implicitly deny a possibility to define a general framework, in this one an entire chapter is dedicated to that purpose. In this introductory discussion, the authors explain their way of thinking about the definition of the concept itself:

> Following we offer our own definition of CPS, a definition that is firmly rooted in the European tradition and, in fact, incorporates many aspects of the definitions provided by the contributors to this volume (see previously discussed definitions). According to our definition, CPS occurs to overcome barriers between a given state and a desired goal state by means of behavioral and/or cognitive, multistep activities. The given state, goal state, and barriers between given state and goal state are complex, change dynamically during problem solving, and are intransparent. The exact properties of the given state, goal state, and barriers are unknown to the solver at the outset. CPS implies the efficient interaction between a solver and the situational requirements of the task, and involves a solver's cognitive, emotional, personal, and social abilities and knowledge.
>
> Notice the differences between our definition and the definitions that feature prominent in the North American tradition. Anderson (…) defined problem solving as "any goal-directed sequence of cognitive operations" (p. 257), regardless of whether the task is novel or familiar to the solver, regardless of whether or not the task is complex, and regardless of whether or not a single barrier or multiple barriers exist between given state and goal state. Our

definition, in contrast, constrains potential problems by requiring that they be (a) novel tasks that subjects are unfamiliar with (b) complex, (c) dynamically changing over time, and (d) intransparent. In order to solve these problems, a solver has to be able to anticipate what will happen over time, and has to consider side effects of potential actions.

Also, note that according to our definition CPS is not simply an extension of Simple problem solving (SPS), that is, problem-solving involving relatively simple laboratory problems. CPS and SPS are qualitatively different.
For example, whereas in SPS typically a single barrier needs to be overcome, in CPS a large number of barriers exists. Because there are multiple barriers, a single cognitive or behavioral activity may not be sufficient to reach the goal state. Rather, a well-planned, prioritized, set of cognitions and actions needs to be performed in order to get closer to the goal state.
In addition, note that in contrast to earlier, often implicit views, CPS is not viewed as deterministic in the sense that any problem-solving activity will always lead to the solution of a problem. Rather, CPS may lead to an approximate solution that may advance the solver but may not lead to actually solving the problem. For example, subjects performing the duties of the mayor of a computer-simulated town, may, even after some practice, still not be able to generate the best possible solution to a given problem.[74]

You may notice that this definition applies rather well to the “trip to the mountains” problem discussed in Chapter 1.

---

[74] Frensch P. and Funke J., Definitions, Traditions, and a General Framework for Understanding Complex Problem Solving, in Frensch, P.A. and Funke, J. (eds) *Complex problem solving: the European perspective*. Hillsdale, NY: Lawrence Erlbaum, 1995.

There are two main differences in the above mentioned general approaches. The first one, as we said, is related with the controlled environment in which complex problems may be turned into assessment task, the other one is related to the role of knowledge in the process of solving problems, in particular previous and acquired knowledge.

The European perspective presents a lack of mention to previous knowledge: more or less, the candidate is without experience when dealing with a controlled experiment in which he or she has to deal with unprecedented seen problems. Instead, in the American perspective, knowledge, as it was said before, is a key factor to interpret and find a solution within the range of possible goals space.

In order to explain the necessary steps to solve a complex problem, yet Fischer, Greif and Funke (2012) clarified one of the most important aspects involving any complex problem: the process of information reduction. Indeed, when the number of variables overcharge human processing capabilities, new strategies are needed. Therefore, humans must rely on previous knowledge of the possibilities in order to find a heuristic strategy that could lead to a possible solution (see solitaire in Chapter 4).

Finally, Quesada et Al. stated that defining CPS is still under way.

> In summary, computational, relational and size-based definitions of complexity do not exhaust the meaning of complexity: CPS is still an ill-defined term. The relational complexity measure could be the beginning of a psychological theory of complexity, but, although widely used in developmental psychology, it is still not much used in studies of problem solving and thinking[75].

Eventually the American view on CPS deals more with practical and very complex real problems where the possible solution strongly depends on the ways the problem presents itself. On the other hand, the European perspective lowers the general

---

[75] Quesada, J., Kintsch, W., and Gomez, E. Complex Problem Solving: A field in search of a definition? *Theoretical Issues in Ergonomic Science* 6(1) (2005): p. 16.

complexity of the problem in exchange for the possibility to measure the performance in solving it.

3.3. Microworlds

Within the European perspective, the goal is to prepare, in a small controlled environment, some artificial problems that can be solved in order to try to measure and assess the capacity of solving problems.

These examples are called Microworlds and within them, it is possible to organize psychometric models to assess performances and skills. Microworlds represent a decisive step forwards modeling a testing scheme. In order to clarify this point, it is interesting to explain how Microworlds work. They are based on a series of linear equations centered on the cause - effect model.

One of the fist Microworld with simple linear equation was developed by Berry and Broadbent (1984): it was called **the Sugar Factory.**

In that model, the current production value influences the next according to an equation that is not visible to the user:

$$P_{t+1} = 2W_{t+1} - P_t + e$$

Where P_t is the amount produced at a certain moment unity time t, W_t is the number of workers in the same time, and *e* is a random error term that influences the dynamic environment.

As it can be seen, the production is a direct function of the number of workers and of the previous production.

The dependence is clearly linear, but if the number of variables increases, like in later micro worlds, then complexity arises by dynamic interaction and decision making model.

> Dating back to the 1970s, Dietrich Dörner, whose work is regarded as a pioneering effort on CPS, wanted to improve the assessment of cognitive abilities by introducing his concept of "Operative Intelligence" (Dörner 1980, 1986). By attempting to model and simulate complex everyday problems by using computer-based scenarios, he and his associates tried to overcome limitations of classical cognitive ability tests (Brehmer and Dörner 1993; Brehmer, Leplat, and Rasmussen 1991).[76]

Within this framework, the concept of dynamic decision-making was developed by Dörner, who proposed that situations in real life are complex and solving problems in real life requires managing complex information.

The idea is that standard measures of general intelligence only assess whether individuals perform accurately and quickly in rather simple tasks, but not whether they show intelligent behavior in complex tasks. "Contrary to the impressive consistency of cognitive ability tests, though, measurement of complex problem solving ability underwent substantial change during past decades".[77]

Thanks to this approach, another kind of Microworlds can therefore be created, where a computer-based scenario is the key aspect that can foster the capacity to manage many variables and measure its effects both on the systems and on the person taking the test. In this vision, computers can simulate complex, connected, dynamic, and non-transparent environments.

[76] Sonnleitner, P., The Potential of Complex Problem Solving Scenarios to Assess Students' Cognitive Abilities: Development, Validation, and Fairness of the Genetics Lab Dissertation Dissertation submitted to the Free University of Berlin, Department of Education and Psychology to obtain the degree of doctor philosophiae (Dr. phil) in Psychology Berlin, 2015, p. 21.
[77] Ibid., p. 22.

3.4. The TAYLORSHOP

About the first example of a computer based scenario Fischer *et al.* (2012) say that:

> one famous example for CPS that can be considered complex because the structure of the external problem representation is to be formalized as a complex system—is the TAILORSHOP (see, e.g., Funke, 2003), a computer simulated scenario of a small organization involved in shirt production. Originally programmed by Dörner in the 1980s on his calculator it was implemented on many platforms and used in a variety of contexts.[78]

The model of the TAILORSHOP is the creation of complexity through the managing of a small tailor shop, where the owner must take care of all the possible outcomes and the events that may occur related with the decisions taken to make his shop profitable in the market.

As shown in the figure below, there are various variables at stake and they interact in different ways:

[78] Fischer, Andreas, Greiff, Samuel and Funke, Joachim, The Process of Solving Complex Problems, *The Journal of Problem Solving* 4, 1(2012).

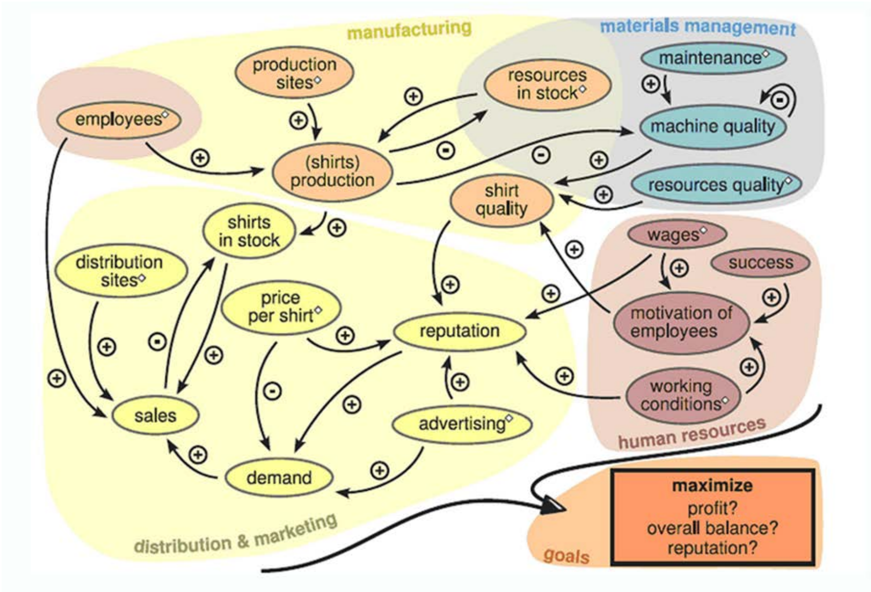

Figure 3.1. The structure of the CPS scenario TAILORSHOP, with the positive and negative dependencies between the influential variables. Diamonds represent the participant's control possibilities. (Engelhart, Funke and Sager, 2011)[79]

The structure of the TAILORSHOP scenario can be seen as a complex dynamic system, consisting of many highly interrelated variables (Funke 2003).

For the purpose of this dissertation, Microworlds are particularly interesting to be presented as examples on a hypothetic complex problem solving course in high school; the Taylorshop represents a useful degree of "simple "complexity, useful to teach the approach needed to face complex problems.

As it was already said, Newell and Simon find a way to model the human problem solving. On this, Fischer Greif and Funke (2012) say:

> 1. Human problem solving starts with constructing an internal representation of the external problem statement, a "problem space" (i.e., a set of possible states of the problem, given the initial state, the applicable operators, and certain goal states). Which operators can be

[79] Image present in Fischer, Andreas, Greiff, Samuel, and Funke, Joachim (2012) "The Process of Solving Complex Problems," *The Journal of Problem Solving* Vol. 4: No. 1. http://dx.doi.org/10.7771/1932-6246.1118 related to Engelhart, M., Funke, J. and Sager, S. (2011). A new test-scenario for optimization-based analysis and training of human decision making. Poster presented at the SIAM Conference on Optimization (SIOPT 2011), May 16-19, Darmstadtium Conference Center, Darmstadt, Germany.

considered applicable might be different for problem solvers of different expertise and intelligence (see Newell and Simon 1972).

2. Given an internal representation of the problem, a method for reaching the current goal is being searched for. General searching algorithms (like "hill-climbing", or "means-end-analysis") are distinguished from more domain specific methods (like "take the hammer to get the nail into the wall")

3. Using a method can change the external problem as well as the internal representation. Of course, changes in the environment or the consequences of a method may lead to new (sub-)problems or new possible solutions. Methods also can be aborted when metacognitive processes do interfere. When a method does not lead to a goal state, (1) another method can be tried, (2) the internal representation may be changed, i.e., the problem may be reformulated, or (3) the attempt of solving the problem may be aborted.[80]

3.5. The MicroDYN

Using computer-based assessment helps to develop a measuring psychometric model. They used the basic ideas of the Microworlds

Greiff and Funke have developed MicroDYN, a computer-based systems that embodies many psychometric possibilities of Microworlds, within a different framework (see Greif e Funke 2009).

> MicroDYN is a generic theoretical and assessment framework for developing CPS tasks, which is based on theories from cognitive psychology, relies on computer-based testing, and has been empirically validated in various studies (e.g., Greiff

[80] Ibid., p. 4.

> *et al.* 2012; Wurstenberg *et al.* 2012). (…) the computer-based MicroDYN tasks, which are based on the formal framework of linear structural equation (LSE) systems (Funke 2001).[81]

Although largely based on the earlier introduced characteristics to define a complex problem (complexity, connectivity, dynamics, opaqueness and multiple goals), these systems used a formalism that offers several other opportunities to measure and evaluate performances and approaches. Apart from having clearly defined optimal solutions (i.e. at any time in the simulation, it is possible to determine what a correct intervention would be), these systems facilitated the systematic manipulation of problem characteristics, allowing, for example, to identify their specific difficulty. Funke, in his 2010 paper, explains quite effectively what are the good aspects of the MicroDYN approach.

> Greiff and Funke choose to work within the formal framework of linear structural equation systems. The MicroDYN approach may be able to overcome some of the shortcomings mentioned earlier: (a) the lack of sound theoretical frameworks calls for a different kind of framework, which MicroDYN systems offer formally (theoretical foundation); (b) MicroDYN systems are easily constructed and can be freely varied in difficulty (scalability); (c) An infinite number of independent items can be presented (unlimited item generation); (d) Many everyday activities can be described by MicroDYN items (ecological validity).[82]

---

[81] Greiff, S *et al.* *Intelligence* 41 (2013) 579--596, p. 582.
[82] Funke, Joachim, Complex problem solving: a case for complex cognition? *Cognitive Processing* 11, 2(2010): 138.

MicroDYN systems use exogenous and endogenous variables and in this model the user can only actively modify the first and see the effects on the latter. On the main structure of this feature again Funke (2010) says:

> The possible effects include main effects, multiple effects, multiple dependencies, autoregressive processes of first order, and side effects, which all can be freely combined. Main effects describe causal relations between exactly one exogenous variable and exactly one endogenous variable. If an exogenous variable is involved in more than one main effect, this is labeled a multiple effect. Effects on an endogenous variable influenced by more than one exogenous variable are labeled multiple dependence. Participants can actively control these three effects as they manipulate the values of exogenous variables within a given range. Effects merely incorporated within endogenous variables are called side effects when endogenous variables influence each other, and autoregressive processes when endogenous variables influence themselves (i.e., growth and shrinkage curves). Participants cannot influence these two effects directly; however, they are detectable by adequate use of strategy. Additionally, all effects may differ in path strength.

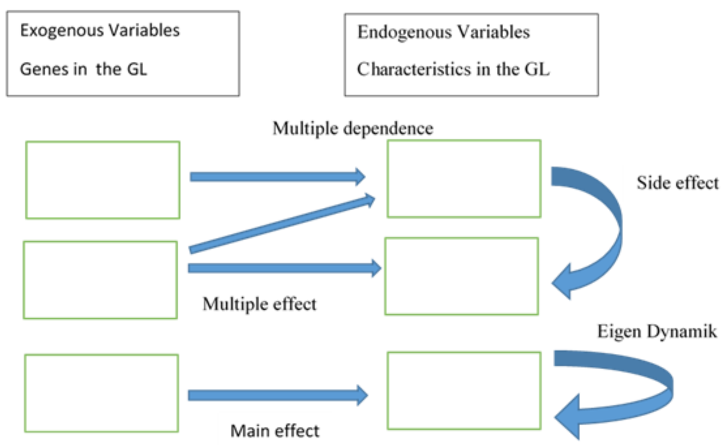

Fig.3.3 elaboration and adaptation for the scope of this dissertation of the original image[83]

---

[83] Ibid. p. 138.

> Figure 2 shows examples of the presented effects. Participants see between 8 and 12 of these items, each lasting about 6 minutes, summing to an overall testing time of approximately 1 hour, including instruction and trial time. The MicroDYN items are minimally but sufficiently complex and at the same time sufficient in number. Each item is processed in three stages: (1) Stage 1, exploration phase: participants can freely explore the system. No restrictions or goals are presented at this time. Participants can reset the system or undo their last steps. A history to trace prior steps is provided. Exploration strategies can thus be assessed (2) Stage 2, externalization of the mental model: simultaneous (or subsequent) to their exploration, participants are asked to draw the connections between variables according to their assumptions. This helps to assess acquired causal knowledge. (3) Stage 3, control phase: participants are asked to reach given target values on the endogenous variables by entering adequate values for the exogenous variables. During this phase, the practical application of the acquired knowledge is assessed.
>
> To summarize: two measurement approaches, Tailorshop and MicroDYN, illustrate two different understandings of CPS. Whereas Tailorshop stands for a broad interpretation of CPS but has some weak points from a psychometric point of view, MicroDYN represents the psychometric sound realization of selected but important CPS aspects.

Therefore, you can build a set of causes on the left hand side and a set of effects on the right hand side. Furthermore, the user "plays" with the causes giving a number or setting an activation and, on the other part of the "worlds", effects are displayed. Obviously, it is a complex problem when multi causes procure multi effects as well as when you can modify intensity and positive or negative effects. To see the structure of a multilinear dynamics micro world, see the Genetics Lab in next paragraph.

Many studies have test effectives and usability of the MicroDYN approach (Schweizer, Wüstenberg and Greiff 2013; Greiff and Wüstenberg 2014) with results within the range.

3.6. CPS as measuring tool

I decided to use CPS as a measuring tool, to test if training in complexity and complex problem might have an effect on high school students' performance in CPS scenarios.

The CPS scenario, according to Funke et al (2012), shares these features:

1) Complexity of the structure (calling for information reduction),
2) Interconnectedness of the variables (calling for building a model of the most relevant effects),
3) Polytely of the task (calling for evaluation and for setting priorities),
4) Intransparency of the situation (calling for systematically generating information),
5) Dynamics of the system (calling for Dynamic Decision Making).

These characteristic features of complex problems and the corresponding facets of CPS (see Funke 2001) can be considered a fruitful starting point for measuring operative intelligence, which in turn might be the most important determining factor of CPS performance.

Therefore, what was developed in this work is an investigative method to explore if CPS performance are related not only to intelligence but also to training, in specific subjects or school tracks.

In order to achieve this goal, I have chosen the GENETICS LAB a MicroDYN developed by Philipp Sonnleitner, Ulrich Keller *et al.* since 2012. The reason of using this software as a measuring tool instead of the classical MicroDYN is due to the fact that since its introduction in 2009 MicroDYN was designed for an audience of university students. Instead, the Genetics Lab improved the MicroDYN approach focusing on the idea of framing the whole complexity in a way understandable by high school students. It is well accepted and considered interesting and useful among students (Sonnleitner *et al.* 2012, 2013, 2014) This computer- based assessment tool, founded on the multilinear structure I have described above, was originally developed to test a more effective way to measure intelligence. The idea is to contrast old Intelligence test-items with CPS-scenarios and see what kind of benefits, in terms of improvement in accuracy, can be achieved.

Old-fashioned intelligence tests are static, they have a limited amount of complexity and they can only produce measures of one kind.

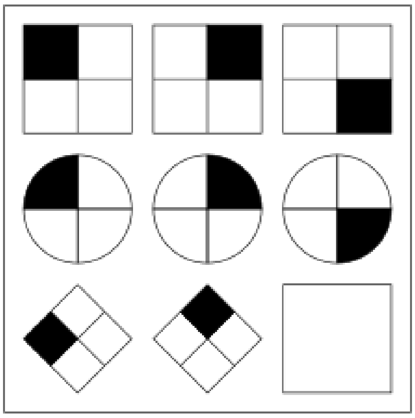

Figure 2.3 An item of classical paper pencil intelligence test[84]t

On the other hand, CPS scenarios are dynamic, they have a large degree of complexity and they share different process measures (e.g. problem solving strategies, information utilization and reduction). Moreover, their game-like appearance can stimulate different abilities and capabilities of new generation's students.

[84] Raven's Progressive Matrices Example, 13 November 2011. Author: Wikipedia User Life of Riley source: https://commons.wikimedia.org/wiki/File:Raven_Matrix.svg

3.7. THE GENETICS LAB

The potential of complex problem solving scenarios to assess students' cognitive abilities

The GENETICS LAB (GL) is a freely available, multilingual, computer-based, and psychometrically sound Microworld that is well accepted among students.

It is available at http://www.assessment.lu/GeneticsLab and for the studies see Sonnleitner *et al.* (2012, 2013, 2014) for further information.

This software was developed by the University of Luxemburg's current LUCET[85] (Luxembourg Centre for Educational Testing) team, with the aim to overcome the severe limitations of typical paper pencil intelligence tests by using computer-based complex problem solving scenarios as an alternative assessment instrument of students' general cognitive ability. The GL was originally available only in a German, French, and English translation. In November 2014, I spent some time with the group thanks to an education grant provided by the University of Bergamo's FYRE project sponsored by Fondazione

[85]The Luxembourg Centre for Educational Testing (LUCET) is a research and transfer center focused on the elaboration and use of assessment and measurement instruments for the educational field.
One of the major LUCET objectives is the setup of a longitudinal database about the evolution of pupils' competency profiles, attitudes and motivations that can be used for evidence-based policy making, for school-quality development and for high-level scientific publications in the field of educational science and educational psychology.
Taking advantage of the multilingual and multi-cultural situation in Luxembourg, LUCET develops new and innovative research-based assessment instruments specifically tailored for the needs of the complex Luxembourgish context, but at the same time highly appealing for an international audience increasingly interested in educational instruments adapted to heterogeneous populations.
LUCET is furthermore specializing in the integration of assessment and learning in technology-rich multilingual learning environments. http://wwwen.uni.lu/recherche/flshase/luxembourg_centre_for_educational_testing_lucet

Cariplo. One of the many outcomes of this collaboration was the Italian version of the software.

The performance scores of the GL come out of a combined problem-solving research grounded in experimental psychology with well-established principles from individual differences research and psychometrics.

The official instructions page says:

> In the GL the task of the students is to examine how the genes of fictitious creatures influence their physical characteristics. The complexity of a creature (i.e., a problem) depends on the number of genes or characteristics, the number of connections between them, the kind of connection (positive or negative), and whether characteristics change without being affected by genes. The examination of each creature is split into two consecutive phases: the exploration phase and the control phase. In the exploration phase, students actively manipulate the creature’s genes. The effects of their genetic manipulations on characteristics are displayed in diagrams. By carefully analyzing this information, students can draw conclusions about the connections and formulate hypotheses that can then be tested. Students document the knowledge they acquire about the relations between genes and characteristics in a database. The resulting causal diagram can be interpreted as the theoretical model developed by the student exploring the creature. In the final control phase, students have to manipulate the genes to alter the characteristics of organisms and reach specified target values. This phase requires the competencies of using a theoretical model to inform concrete actions and controlling the resulting outcomes. Students’ behavior while working on the GL is recorded in a detailed log-file. This information is used to validate whether students work properly on the GL and to derive three performance scores: (a) a process-oriented score reflecting how systematically students explored the creatures, (b) a score that measures students’ level of acquired knowledge about

the relations between genes and characteristics, and (c) another process-oriented score that reflects students' control performance to achieve specified target values.[86]

The results of the University of Luxembourg team empirical studies (Sonnleitner *et al.* 2012) showed that the GL" enjoys a satisfactory level of acceptance among adolescent students and that it may be considered as reliable, valid, and fair alternative assessment instrument of students' general cognitive ability by means of complex problem solving scenarios".

From a student's point of view, the scenario comprises the following steps.

In the beginning, there is an instruction phase in which the user familiarizes with the tools and ideas behind the software. The general instruction phase has a time limit, but the films that show how the system works and the practical examples can be replayed.

There is a total of 12 creatures with several different "genic" characteristics such as height, width intelligence associated with the genes' activation and time through different types of correlations.

The longer you move in the game, the more complex the problem becomes. The first creature has only two genes and two characteristics, while the last ones have three on both sides. Furthermore, time is also a factor affecting the creature's features. This represents the complex solving criteria of Eigendynamik.

[86] Sonnleitner, P., Brunner, M., Greiff, S., Funke, J., Keller, U., Martin, R., Hazotte, C., Mayer, H., and Latour, T Genetics Lab, A Computer-Based Microworld to Assess Complex Problem Solving, *Theoretical Background & Psychometric Evaluation,* p. 6. Summary extract from Sonnleitner, P., Brunner, M., Greiff, S., Funke, J., Keller, U., Martin, R., Hazotte, C., Mayer, H., and Latour, T. The Genetics Lab: Acceptance and psychometric characteristics of a computer-based microworld assessing complex problem solving. *Psychological Test and Assessment Modeling* 54, (2012): 54–72.

In the first tasks, time runs independently with discrete unity and it is not linked with changes in characteristics, while in the last creatures characteristics could freely change in time without any gene's activation.

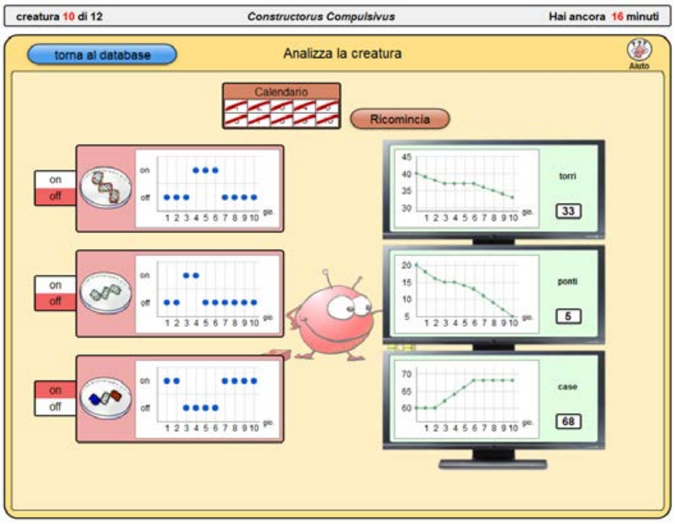

Figure 3.3 Exploration phase screen shot taken from the Italian version translated for this project

The image shows the first phase, each student trying the effect of the genes on the creatures' characteristics by activating and deactivating genes. This is the exploration phase, where students explore the scenarios and take notes about their observations.

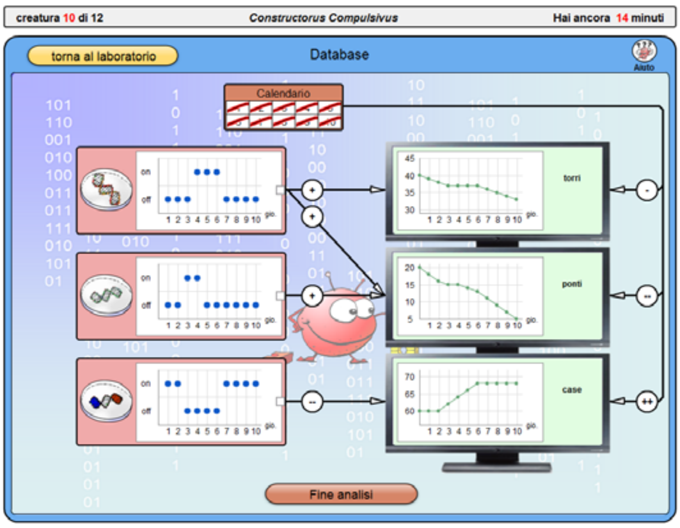

Figure 3.4 The database. In the picture, it is clear where exogenous variables and endogenous variable are

In the second phase, users are asked to store their discovery in a database in order to get a documentation of the knowledge they have acquired.

In Figure 3.4, the different arrows mean that gene A is related to characteristic 1 through a linear negative relation. Therefore, if gene A is active, characteristic 1 decreases.

It is possible to note that in the final stages of the test (like in Figure 3.4) it is also possible to have two different degrees of intensity of increase or decrease. The sign (++) means a higher increase rate of the characteristic, similarly (- -) means a larger decrease rate.

The arrow related to the calendar has a different meaning, while the creatures' characteristics may be affected by time flow. In other words, the creatures' characteristics may change without any involvement by genes, just as a function of time. Therefore, each time a unity of time passes, characteristic A may increase or decrease.

While recording findings in a database, it is possible to note that the relations between variables are linear; the complexity comes from the superposition of the effects (Funke 2001).

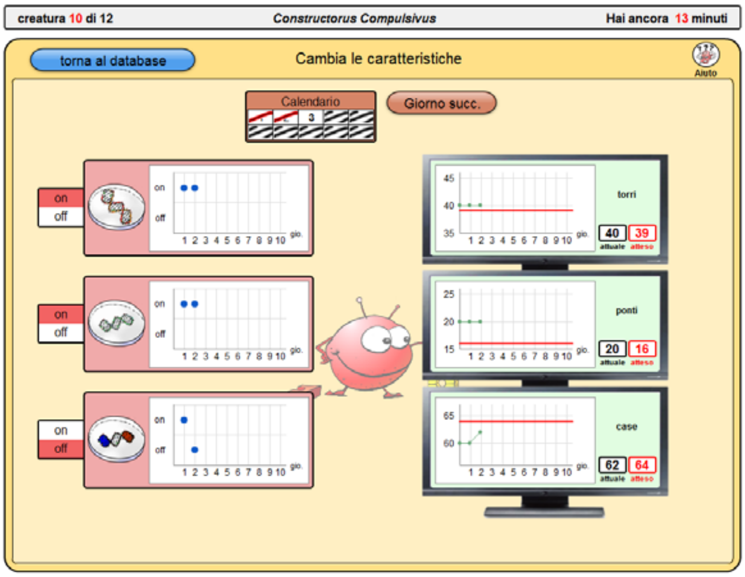

Figure 3.5 The control Phase. Users must reach some target values by activating genes

In the final control phase, the test-taker has to reach certain target values for each creature. The tool to reach these goals is by activating genes and use the change in characteristics of the creature to achieve the prescribed characteristics, therefore the key element to succeed in this last phase is to be able to use the knowledge acquired to control the creatures' characteristics.

The final stage of each step involves feedback about the quality of knowledge acquired and its application in the control performance.

Figure 3.6 Partial result for the single item feedback for the user

Sonnleitner *et al.* in "Students' complex problem-solving abilities: Their structure and relations to reasoning ability and educational success" (2013), tried to find a correlation between CPS performance and intelligence.

They introduce a hierarchical Complex problem-solving model with the following image:

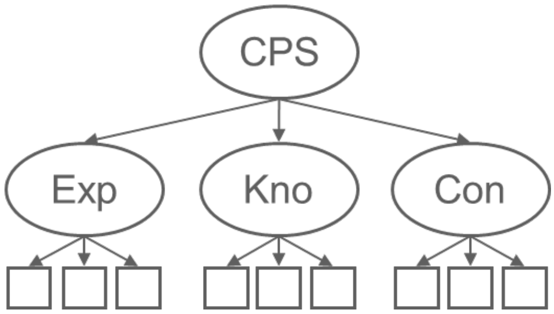

Figure 3.7 The model of CPS separated in the different phase. So all the different aspects Exp = Exploration strategy; Kno = Knowledge; Con = Control performance; concur to the general CPS = general problem solving ability.

In the same paper, the Psychomethric complete model and its relation with the hierarchical model, as well as the g factor, are also explained.

The g factor, shorter for general factor, is a psychometric model that takes into account the different cognitive abilities of humans performing different tasks. It is considered to be partially responsible for different performance in intelligence tests. The existence of the g factor was originally proposed by the English psychologist Charles Spearman in order to find a common possible cause for cognitive effects in human thinking performance. Jensen in *The g factor: The science of mental ability* (1998) argued that the effects of cognitive interventions on abilities can be explained in terms of Carroll's three-stratum hierarchical factor model.

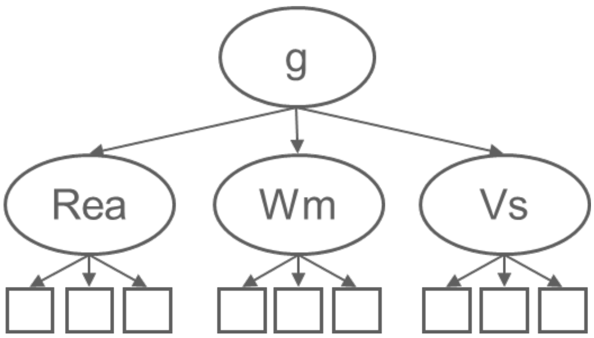

Figure 3.8 The hierarchical model of the g factor

In the current model, as seen in the image, the common g factor typically divides cognitive abilities as a three-level hierarchy, where there are a large number of narrow factors at the bottom of the hierarchy. As in the analysis Rea = reasoning; Wm = working memory; Vs = visuo-spatial ability concur to the g = general intelligence ability.

Nevertheless, in current psychometric models, a debate is still under way about what the effects of the g factor in individual performance in intelligence tests might be. Indeed, in a paper of dynamic decision making measurement, Danner et al. (2011) state:

> The present findings acknowledge the overall approval and usefulness of general intelligence measures. In addition, the results demonstrated that there are significant individual trait differences in cognitive performance beyond IQ. In particular, there was a large proportion of trait variance in implicit learning, which was independent from general intelligence and in addition, dynamic decision making revealed an incremental predictive validity.
>
> These findings make dynamic decision making as well as implicit learning attractive for the research of individual differences.[87]

3.8. Italian translation of the GL

I spent five weeks with the group in Luxemburg during the Autumn of 2014. One of the purposes of my stay was to translate and adapt the software into Italian to make it usable in Italian schools.

Though translation seemed easy at first, I found it to be rather challenging, because of the problems related with the adaptation to Italian reality. One difficult task was to stay

[87] Danner D., Hagemann D., Schankin A. and Hager M., Funke, Beyond IQ: A latent state-trait analysis of general intelligence, dynamic decision making, and implicit learning, *Intelligence* 39 (2011): 332.

within the maximum number of characters for each box, because in Italian you generally need more words than in English to express the same concept.

Another interesting part of that task was to change and adapt the names of the creatures, genes and characteristics and the text in the action buttons. The issue was that some of the text. i.e. names of the creatures, was in “scientific Latin” and could have helped Italian students to get information on those creatures themselves, therefore the need for different names. In the process of renaming, I tried to keep the meaning and spirit of the original names, because the creatures where fictitious. For example, I translated *Lotsus eysus* with *Grandus Visionus* in “Italian” while *Biggus bossus* became *Grandus Capoccius*.

The buttons activating the genes presented me with another challenge: I decided to keep “on” and “off” instead of changing them into “att” and “disatt”. On and off, in fact, can be internationally recognized as signs for the generic idea of active and inactive not only in circuits but also for general purpose.

In order to assess the quality of the translation, I asked many students if they could understand it and the answer was unanimously positive.

3.9. Definition of the Research question

Is it possible to foster motivation or to change the approach of students towards a problem? What if the new generation of students, as it might be argued by their approach to learning in general, actually see their world as already complex, and therefore find it easier to deal with complexity rather than simplicity? Motivation is another element to consider when a student tries to deal with a problem, already at a point in which, in his/her head, the most important idea is: “Do I have the instruments to solve it or is the knowledge I possess sufficient to find a satisfying solution?”

In order to look for the answers to these questions, the following step is to find an example or a problem, or more than one, in which there is no need of previous knowledge to come across the “complex problem solution”. Therefore, the crucial idea here is also to entirely reverse the reasoning process: know in order to be able to try becomes try in order to know. In other words, the simple fact of trying could help you to get the necessary knowledge useful for dealing that specific problem.

In this perspective, there is no need of a general framework of previous knowledge from which to derive the particular solution; rather, each problem has its own specific path to be dealt with, and each attempt of finding a solution potentially provides new specific knowledge.

The project I developed in cooperation with LUCET group University of Luxembourg is generally aimed to test the potential improvement in effectiveness of Science/Math teaching in high school with the inclusion of Complex Problem Solving (CPS), in different types of schools and different educational systems, especially in the Italian schools where the curriculum does not currently include this approach. Because it is the first time that a CPS test is performed in Italy, we have also tried to answer some technical questions like: how well the students perform in CPS skills, and within these results, what differences can be observed between school tracks and age group present?

The idea behind it is to foster motivation to use complexity not only as a framework but rather as an instruction manual in a new developmental sense. Students have always been taught that they must have knowledge in order to face school tasks, and this is obviously very important, but what happens if another learning pattern is developed? They can learn how to do it while doing it, exactly like when they learn how to use a smartphone or a TV or any new technological tool they do not know how to handle yet.

The project was organized in phases: the first step requires an assessment of the students’ abilities in complex problem solving, and the possible relation of such abilities with their school background and the different learning sources that they might have had access to.

This cumulated knowledge can be used to deal with the software GL or other topics that can be developed afterwards and used to foster the idea of *I can do it*.

The basic idea behind fostering motivation is then to create an environment where students do not feel inadequate. When facing a CPS task very little or no previous knowledge is required, and students need to feel reassured about the new idea that they can create their own knowledge, and that such self-produced form of knowledge will render them able to solve problems.

Previous knowledge or modeling capabilities may actually prevent students from trying, but if trying is the only available strategy, then there is nothing they cannot attempt to do. The approach would be the following: first try and then see what you have done because in Microworlds you can try as many times as you want within the problem.

Can they apply the scheme to simple Microworlds like the GL?

Another aspect that goes into the construction of the lecture and the possible use of Complexity as a learning tool in classroom, is the concept of knowledge transfer. The issue of transferring knowledge is one of the most important and traditional parts of any teacher's job, and it is discussed within the lecture part in Chapter 4.

For the purpose of this dissertation, I have had access to many studies showing that there is a lively debate about the possibility of training the g factor, i.e. the possibility of enlarging general intelligence with exercises and lectures (cf. Jensen 1998; Gustafsson and AbergBengtsson, 2010; Hunt 2011).

Nevertheless, also the CPS MicroDYN might be very helpful for the purpose of checking the possible effects of the training, considering that the psychometric model is divided into exploration, knowledge and control phases; with the same software, it is therefore possible to check if any outcome of the training has actually affected the students. Despite the fact that the GL working tool was initially created as a new form of quantitative assessment of intelligence enriched by the CPS scenarios, it might provide a shift of function towards a different application during our research.

Because the GL assemble many difference skills and abilities and they can be decomposed through these various areas, it may be possible to see effects of the training only on a single one.

Therefore, within the Brunswik[88] symmetry, if you want to predict specific effects of a training, a strategy that is within the same range must be defined. In other words, the training is not an introduction to any instruction manual on how to solve CPS, but rather the presentation of a general idea on complexity and problem solving, aimed at fostering motivation and self-awareness. Therefore, since the lesson is not a specific training on how to solve the GL, its effects can be related to the general idea of using complexity in an unexpected framework.

The next chapter will describe the designed training lecture as well as the data collection project.

In the training, my goal has been to verify any effect, after providing and explaining several examples like the Klondike Solitaire, The Underground Treasure hunt and the Prisoner's Dilemma.

I will give examples of complex problems in Chapter 4, and I will describe the way I tried to propose them to students. These problems are in a way familiar to present world students, but despite that, they need a quite small training in order to translate the competences they already master into a much bigger process to understand what they already know they must disclose their implicit competences. Indeed, they might know how to deal with these but they have not discovered it yet. This is the real human power in dealing with the nature of problems itself.

[88] Brunswick, Barrett, P. Symmetry, Complexity, and Non-Quantitative Psychology, NZ Psychology Conference – Hamilton, 2007.

Chapter 4. The testing phase in cooperation with the University of Luxembourg

4.1. Organization and definition of the intervention

The theoretical framework

In order to test the impact of introducing Complex Problem solving and the Theory of Complexity in high school teaching we created a research activity based on the Genetics Lab used as a measuring tool, a preparatory lecture for the students and a scaling questionnaire use for normalization.

The general research questions that we were trying to investigate through the activity in the schools may be summarized in the following way:

• May a lecture and the whole concept of Complexity brought to high school have an impact on students' motivation? The idea that once you have a strategy you must apply to test it indeed may foster the wish of trying to solve a problem even if you have not seen it before and it needs no previous knowledge.

• Knowing that the GL is structured in different phases, the lecture and general concept of Complexity might affect one of the specific phases. Therefore, the use of the GL may help to see in which field the introductory lecture might be more effective.

• Which kind of impact on performance in the GL may the different tracks (scientific, classical of human sciences), the specific teaching method (IB in the VIS) and the language of instruction have?

• How can an introduction on General problem solving strategy affect the Genetics Lab performances on students of different ages and country of education?

Within this approach, the lecture should not be intended as a classical lecture in which a teacher tries to transfer knowledge into students, but more like an instruction manual, a way to present concepts, ideas, models that might help the audience to disclosure the knowledge they already possess and manage.

As most teachers know, it is almost impossible to give a lecture on a topic and expect that the students will immediately be able to manage new ideas and use them. Nevertheless, the main goal of this introductory lecture is to foster motivation, to raise awareness about the fact that the GL or any other CPS does not need previous knowledge and so anyone with a basic education can access it and answer the questions it poses.

Obviously, because the GL was originally developed as a tool to measure intelligence, a higher QI should help to perform better. In this context, intelligence refers to the ability to retrieve information and acquire knowledge as components of the general factor and reasoning. This ability is crucial for solving complex problems and interferes with cognitive processes involved in problem solving. Although intelligence might be a predictor of problem-solving performance, the correlations differ according to the factors of intelligence. In other words, the different aspects contributing to intelligence (see Chapter 3) might play a different role in the capacity to solve the different phases of the GL. Nevertheless, for the purpose of this dissertation it is important to measure the attitude towards solving the problem and not only the ability to complete the assigned tasks.

As a final aim, the results could provide a new way of presenting problems in high school, a possibility to reverse the process of problem solving teaching. Would it be possible to first make the students familiar with Complexity and complex problem and see if they then can model an idea, a tool to be used in the preparation of a scheme to deal with simpler problems?

As Uri and Wilesky (2005) say in their already cited article:

> Students learning to think in terms of complex systems ideas may need to go through a process of "strong" or "radical" conceptual change. Consequently,

> pedagogies and curricula for learning complex systems ideas will need to focus not only on the conceptual aspects of these ideas but also on enriching the cognitive network of beliefs and intuitions students have about the world and about knowledge so as to bridge to a complex systems perspective.[89]

They continue giving an ideal model

> The cognitive construction of a mid-level (solution) thus helped students to form mental models of dynamic complex systems that were used in two main ways related to the students' more general understanding of complex systems principles.(…)
> This work is developing innovative pedagogies and technologies that may help students develop a richer set of cognitive resources that scaffold and support their learning about complex systems ideas and methods. More generally, such research, by pushing the envelope of what kinds of advanced knowledge students can learn ,promises to contribute to the learning sciences discussion of "what is hard" and "why is it hard."[90]

In addition, they give a hint for what this project has been trying to develop:

> Research could explore how pedagogies and representational tools might be developed to help make an organized conceptual perspective based on complex systems ideas salient and explicit to learners and what the learning outcomes might be, such as depth of conceptual understanding and knowledge transfer.[91]

---

[89] Jacobson, Michael J. and Wilensky, Uri. "Complex Systems in Education: Scientific and Educational Importance and Implications for the Learning Sciences" *The Journal of the Learning Sciences* 15, 1 (2006): 15.
[90]Ibid., p. 19.
[91]Ibid., p. 21.

Therefore, in this project we tested students of different age groups and backgrounds in solving an example of CPS (the GL) with or without the help of a complex conceptual framework provided by the lecture. I will discuss results and methods of data analysis in Chapter 5.

4.2. Structure of the testing phase: The GL, the lecture and the Reasoning scale questionnaire

This intervention took place following a scheme divided in three sessions.

The first session is a training lecture in which the students have to deal with examples of problems (one of which is the Solitaire) in a classroom during an open lecture; in a second session, they have to answer a reasoning scale questionnaire. In the third session, they use The Genetics Lab: a computer-based test to assess their complex problem solving abilities.

After many discussions, we decided for an intervention structure as seen in image

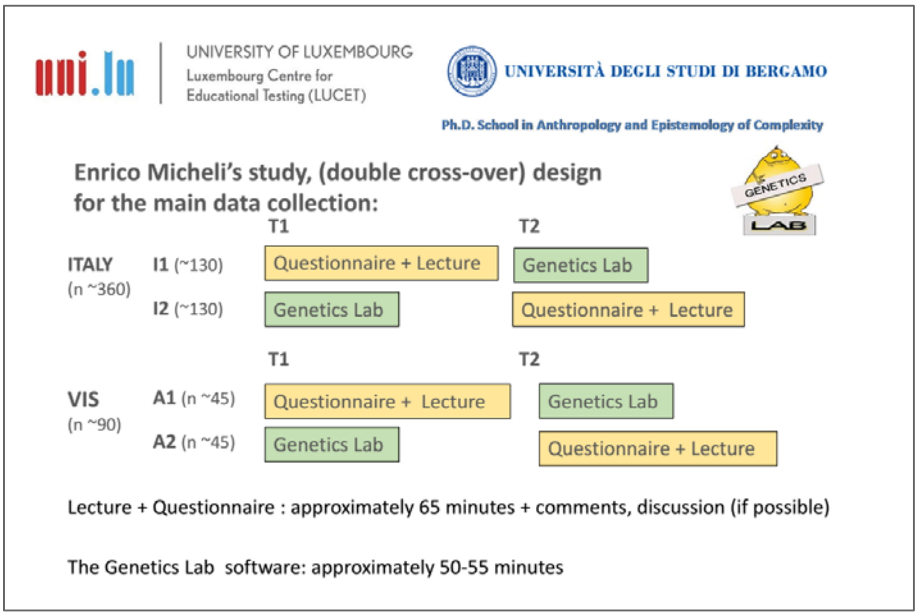

Figure 4.1 The testing phase

Therefore, in every school the groups tested were split into two parts, completely at random. The first group had the reasoning questionnaire, the lecture and then, after a pause, went to the computer room to test the on line version of the GL. The second group had the GL first, then, after a pause, answered the Questionnaire in class and, eventually, had the lecture. According to this scheme, students always answered the questionnaire before the start of any kind of training, so no sample was by any means affected by any kind of human intervention.

The questionnaire is in form of a figure shapes test where the person must find which figure is related with the sample, i.e. to mentally arrange geometric figures. This test was taken from the Intelligence Structure Test IST-2000R (Amthauer, Brocke, Liepmann, and Beauducel, 2001), a reliable and valid measure of intelligence. Every paper had 20 figures and had to be solved in 7 minutes (7 min; score GF available in Sonnleitner *et al.* 2013).

Each paper was marked with a user ID that had to be anonymous. Moreover, each paper had a QR code[92], therefore, it could only be used once as the QR code allowed the computer scanner to uniquely identify each paper. To facilitate the interpretation of the results, all test scores in Chapter 5 will be expressed as the percent of maximum possible score, for which a value of 0 indicates the lowest possible score, and a value of 100 indicates the highest possible score.

The first problem was how to link the paper questionnaire with the Genetic Lab performance, so we decided to assign to each candidate a student ID that they had to write on their paper and then report on the screen at the beginning of the software.

---

[92] A QR code (quick response code) is a type of 2D bar code that is used to provide easy access to information through a computer reader like a scanner or a smartphone camera. In general, this contains more information than a standard bar code due to its 2D features. In our case it was used to identify the paper and linking it to the hardware ID.

For privacy reasons, we asked the students to put only their age and gender on their paper, and no other personal information.

In order to have a different ID for each participant, we decided to give them instructions to create their own ID, and also on how to start the paper test. The instruction paper had the following text:

HELLO, WELCOME TO CPS PROJECT

Now you will be asked to create a password for the hardware ID that will be used during the project

Here are the instructions:

You have to create a unique password consisting of two letters, two numbers and two letters for a total of six (6) characters.

Since you will need it later on, you have to try to build it in a way that will be easy for you to remember. For example, the first two letters may be the initials of someone you know (family, friends, cat, dog). The second pair should be numbers, so you may choose between 00 and 99: a day of the month , a number that you remember for a simple reason (year of birth of your favorite actor or singer , shoe size of an uncle , the graduation year of an aunt , etc. In the third pair of letters, you can use the initials of your favorite singer or band, your favorite brand of clothing or aftershave etc. USE YOUR IMAGINATION.

Example:

I choose the initials of an acquaintance of mine, Antonio Franceschetti, I love the number 32 and I like ABBA. Therefore, my ID will be

A	F	3	2	A	B

My grandfather's name is Jerome, my grandmother's is Rosamund, my birth month is January (01) and I love the clothes of Gino Ginelli. Therefore, my ID will be

J	R	0	1	G	G

Try to be imaginative and to build a unique password, remember that the project is completely anonymous and only you can rebuild your password.

PLEASE WRITE IN CAPITAL LETTERS AND WRITE IT ALSO IN THE NEXT PAGE

MY ID is

The intervention was administered in six different Italian schools and one international school --Trieste, Cividale, Milan, Saronno and VIS- Vienna International School.

We selected different tracks and different ages. In Italy, we tested 8th, 10th and 11th grade students.

8th grade students in Italy do not have a specific academic track, it is a undifferentiated "Compulsory education".

10th and 11th grade students came instead from Liceo Classico, Liceo Musicale, Liceo delle Scienze Umane and Liceo Scientifico. At the Vienna International school, students were from 10th and 11th grade. There were two different waves of testing during the 2014--2015 school year. The first one in autumn 2014 (27 October–3 November) and the second one in Spring 2015 (March and April).

Grade	*Age group*	*Italy*
8th grade	10-11 years old	Terza media
10th grade	14-15 years old	Seconda superiore
11th grade	15-16 years old	Terza superiore

Each test was completely anonymous: it was stated many times that there would be no marks and that the test would not affect students' final school performance in any way. This was done in order to protect the students' privacy and to make them feel free to try without worries of any possible outcome on their school report.

4.3. The training lecture

Structure and subjects of the lecture

The lecture was given with the same PowerPoint presentation in all groups and developed through different points.

1) What is Complexity? Chaitin complexity through the Jpeg example
2) Virtual Worlds and Micro worlds
3) The Game of Life and the cellular automata
4) The changing of approach: Salvador Dalì's portrait of Lincoln and Gala.
5) Situated learning and cognitive apprenticeship with complex problems.
6) The Underground Game

7) The Klondike Solitaire

8) The Prisoner's Dilemma

9) Octopus Learning

The first topic is about what complexity is. Indeed, even among scientists, there is no unique definition of complexity as I said in Chapter 2; therefore I could only give examples of complexity, and I also gave a small hint: "The study of the phenomena emerging from a collection of interacting objects".

I then explained the Chaitin "mathematical definition" of complexity with the Jpeg image example (see Chapter 2).

Successively, I continued by talking about Dynamics worlds and Virtual Worlds. I pointed out to students that their everyday life example with these worlds is amazing and it goes under noticed. Present-day games, like "The Sims", "Clash of Clans", "Terraria", "Funny Farm", are all virtual worlds, in which laws are both mathematical and random through computer science development but are grounded in the idea of free expansion.

4.4. John Horton Conway 's Game of Life

The free evolution system

One of the most important among these "games" is the Game of Life. The mathematician John Horton Conway invented it in 1970, but it was Martin Gardner[93] who first presented it to the general public on *Scientific American*. This was a very interesting example of outreaching to a wider audience of the discoveries of those times by both mathematics and computer science researchers.

[93] Gardner, Martin, Mathematical Games – The fantastic combinations of John Conway's new solitaire game "life". *Scientific American* 223 (October 1970): 120–123.

It was the first of what are now called cellular automatons, simple programs obtained through a computer following a few simple evolution rules. The game set is a mathematical universe. This universe is an infinite two-dimensional orthogonal grid of square cells. Every cell interacts with its eight neighbors, which are the cells that are horizontally, vertically, or diagonally adjacent.

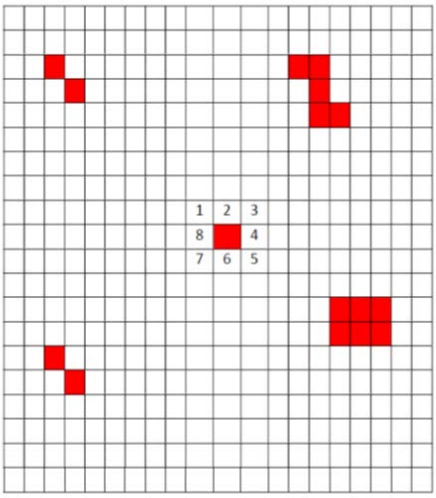

Figure 4.4 The game of life world[94]

The interaction rules are very simple: they follow the concepts of life and death.

Death

- Each cell with one or no neighbors dies, as if by loneliness.
- Each cell with four or more neighbors dies, as if by overpopulation.

Life

- Each cell with two or three neighbors survives.
- Each cell with three neighbors becomes populated.

[94] Image taken from https://fiftyexamples.readthedocs.org/en/latest/life.html © Copyright 2012, A.M. Kuchling. accessed September 2014.

They also follow some operating rules like:

- Births and deaths occur simultaneously.

Time is discrete so every generation follows the others and there is no overlapping.

I showed the audience a film of me playing, defining the initial population and setting the system to evolve through time following the specific rules.

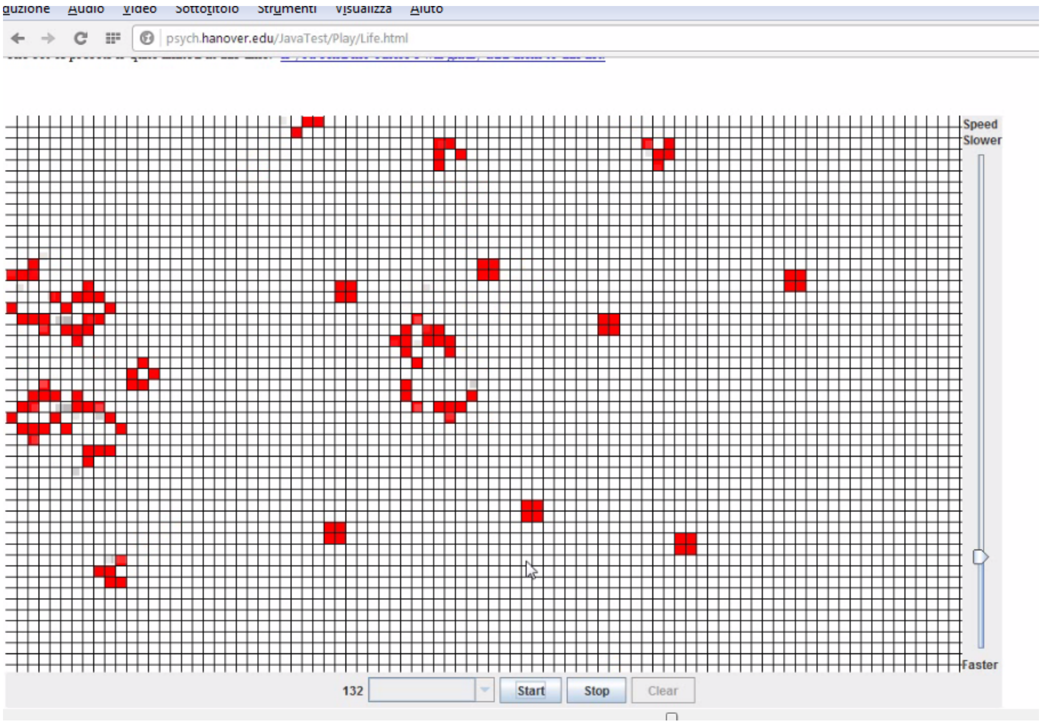

Figure 4.5 Game of life after 132 generations screen shot taken from http://psych.hanover.edu/JavaTest/Play/Life.html developed by John Krantz PhD reprinted with permission.

After 132 generations, the situation was as the one showed in Figure 4.5. There were some moving sets and some equilibrium squares. I then explained that what appeared to be still, was in fact recreating at every generation until some moving items hit it and changed the features even at times destroying it. The free evolution system is in a way similar to the simple mathematical infrastructure on which today's games are grounded.

4.5. The changing of approach: Salvador Dalì's portrait of Lincoln and Gala
Try to see the same things differently

Moving on, I went on to give an example of how important motivation and the approach towards solving problems is. I tried to give students a slogan like: Let's try to see the same things differently in order to change our approach on personal attitude towards problems and obstacles. I gave two examples, a "physical" one and a personal one, to clarify this idea.

I said to the students: "Take a look at the famous Salvador Dalì painting from two different positions, what differences can you see?

The picture cannot be reprint due to copyright restrictions

It is available on Salvador Dalì foundation website

www.salvador-dali.org see note

Figure 4.6 pictures of the painting taken at Theatre Museum Salvador Dalì in Figueres near Barcelona, Spain[95]

You can see both US President Abraham Lincoln and the portrait of a naked woman on the same painting, but it does not only depend on the distance you look at it from.

Indeed, this famous painting is called "Lincoln in Dalivision, Gala Nude Looking at the Sea which at 18 Meters Appears the President Lincoln". It was created on the basis of

---

[95] Dalí i Domènech, Salvador, *Gala Nude Looking at the Sea Which at 18 Metres Appears the President Lincoln*. 1975. Dimensions: 420 x 318 cm, Tecnique: Oil on photographic paper on wood, Location: Dalí Theatre-Museum © Salvador Dalí, Fundació Gala-Salvador Dalí, Figueres, 2014. https://www.salvador-dali.org/museus/teatre-museu-dali/the-collection/119/gala-nude-looking-at-the-sea-which-at-18-metres-appears-the-president-lincoln

the painting “Gala Contemplating the Mediterranean Sea”. Dalí painted two original versions of this painting spanning from 1974–1976, which are similar but not exactly the same. The first resides in the Theatre Museum Salvador Dalì in Figueres near Barcelona, Spain, and the other in the Dali Museum, San Petersburg, Florida.

For the first version painted in 1975 the Salvador Dalì Foundation provides the following historical explanation:

> Dalí produced this work as a tribute to the painter Mark Rothko, from a digital interpretation of Lincoln’s face obtained by the North American cybernetic Leon D. Harmon. Once again, Dalí appears as an initiator and presents us his concept of the double image in a new way. Lincoln’s face is seen in small format in the lower left-hand side of the work, where the figure of Gala is also repeated in a changed pose in the upper section; in a magnificent twilight, Dalí refers to his well-known oil painting “Christ of Saint John of the Cross.” [96]

Looking at this picture you can see once again the visionary genius of Dalí the official website of the Dali Museum in Florida provides the story[97]. It reports that it seems that this portrait was inspired by Dali in an article on *Scientific American* journal, as we saw in the case of the Game of Life. In this article:

> Dali read about visual perception which investigated the minimum number of pixels needed to describe a unique human face. Dali was challenged by that question and set about making this portrait of Lincoln using 121 pixels. In his canvas he pushes this concept of perception and external sight. This double image painting provides a meditation on the dual nature of things.[98]

[96] https://www.salvador-dali.org/museus/teatre-museu-dali/the-collection/119/gala-nude-looking-at-the-sea-which-at-18-metres-appears-the-president-lincoln © Salvador Dalí, Fundació Gala-Salvador Dalí, Figueres, 2014. Accessed November 2015.
[97] http://thedali.org/exhibit/gala-contemplating-mediterranean-sea/ accessed November 2015.
[98] http://thedali.org/exhibit/gala-contemplating-mediterranean-sea/ second paragraph.

Dali with this work lays the foundation for one of the most famous photographic inventions of modern times, the Photomosaic. It is a recognizable image typically constructed through a mosaic of smaller images sometimes themed and sometimes not. In the last twenty years, this type of image has occupied the front pages of many magazines; perhaps the most famous are those that portray President Obama and Steve Jobs.

What matters about the Lincoln Painting is that it does not actually only depend on the distance, but also on the viewer's eyes. If you stretch your eyes, as seen in Figure 4.7, you can see Lincoln even when close to the picture.

Figure 4.7

That is because, in doing so, you somehow flatten you vision in a way that I will briefly explain below.

When I first saw the painting in the museum, I tried with my eyes stretched, and what I saw is that if you put the smartphone between you and the portrait, the effect is the same as when you see Lincoln. Many things concur to produce that effect, but one of the most important is that a camera sees in 2 dimensions while the eye sees in 3 dimensions.

This means that when we see with our eyes we see height, width and depth, a camera has only height and width. There is no way to have the depth in the picture, as a photograph is a flat medium. This is mainly achieved by the stereoscopic vision of the eye. A simple demonstration of this can be obtained by trying to bring the forefingers of both hands to meet from the sides. This is much simpler to do with both eyes open than with one single eye and almost impossible with a camera.

Therefore, in a way, the camera flattens the image and you get an effect similar to the one you get when you watch it from afar. The main idea is that you may be looking at exactly the same object and thinking two different things.

To trigger this approach I then told the students a personal story. What happened to me may really be interesting to see how approaches and ideas can affect performance in different tests.

Once I was working in a school and the IT assistant came to me and told me he would need some help to solve a problem. I then thought it was a sort of intelligence test or one of those tricks that are not immediately clear -- like the well-known one on how to create four equilateral triangle out of six toothpicks.

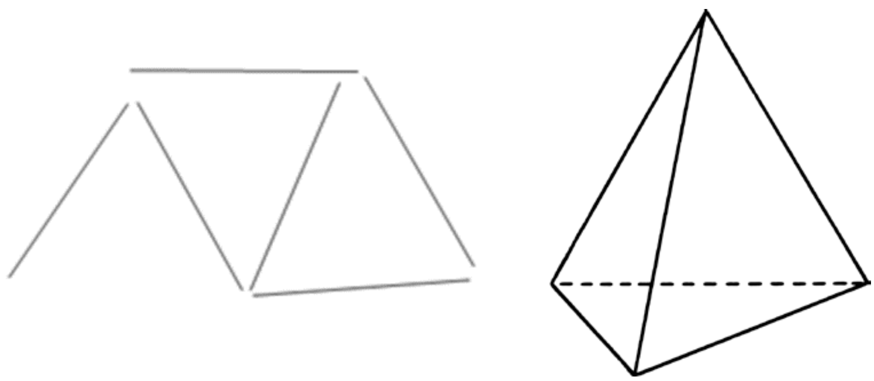

Figure 4.8 How to create four equilateral triangle with only 6 toothpicks. If you work in the 2d space this is not possible but if you move to 3d space then the tetrahedrons[99] is made out of 6 six straight edges which compose 6 equilateral triangles.

The assistant showed me a paper, which I can still clearly remember after many years: it was a calculus exercise related with derivative of series. I said that I would be more

[99] This Tetrahedron image has been released into the public domain by its author, Pearson Scott Foresman.

than happy to solve it, only not right at that moment: it would take me some time to do it. He replied that it was not an intelligence test, but simply one of the exercises given at the faculty of engineering in that month's test for the Calculus Exam. I then realized that it had to be very easy and I solved it in 10 minutes.

The question I asked the audience was: "What did it change in that moment? Was it the exercise that changed?" The students replied that what changed was not the exercise itself but my attitude towards solving it. I knew I could easily solve it because it was back into my knowledge scheme.

All this said, the idea of complexity I was trying to express with that example is: the first step is always trying with an awareness that sometimes the hardest step is to begin. In other words: you cannot know before trying if you are able or not to complete the task. Sometimes students prevent the event of failing simply not taking the test, while it is always a good strategy to try it any way. Nobody knows what you can find while attempting or learn doing.

4.6. Situated learning and cognitive apprenticeship with complex problems

In the next step, I talked about situated learning and cognitive apprenticeship with complex problems. I choose to explain this presenting a classical problem solving strategy, like Algebraic Solving.

The steps in this procedure are mainly three:

- Define a set of variables.
- Find relations.
- Solve equations.

Therefore, first you must define what you need to know, then the relations between variables and, once you find the equations, finally the resolution using the classical algebraic method. An example is the classical problem of two linear equations with two unknown variables.

The power of this method is that when you have found an abstract solution you can apply it to a multitude of different problems. To name one, the initial solution of the problem “trip to the mountains” is a simple case. You need to know how much each participant must pay, the variables are the number of days, the number of people and the total amount of money due to the house owner. The resolution of the problem posed in this way is the initial one given by the students.

Nevertheless, they can also manage pre-algebraic methods where you can define a solution by guided trial and error, so, for example, you try to apply a solution, you test it and then see if it works. Like when you have to split a restaurant bill among payers where the separate bill option is not available. Only the mathematicians define a model to split the bill fairly, usually the people divide bills in equal parts or find a solution by which those who have eaten or drunk less pay a smaller amount.

This strategy may help when applied to complex knowledge seen in different problems, for example, young generations try to discover how to use a smartphone without any sort of instruction manual because they interpret the nature of different apps easily. Their approach is then “try to know” instead of “know to try”. This mode is a sort of learning by using the trial and error model because no previous knowledge is needed for that use. This knowledge instead is built through the different attempts.

In my explanation to the class, I then carried on talking about the fact that classical problem-solving strategies might not suit the initial learning stages of the Millennials perfectly.

In this context, new strategies might be useful in dealing with the complexity environment. For example, situated learning and cognitive apprenticeship have acquired a different meaning into the complex knowledge framework.

Situated learning is a theory stating that knowing is inseparable from doing by arguing that all knowledge is situated in activity bound to social, cultural and physical contexts. Therefore, the process of acquiring knowledge is deeply connected with how it is presented and the tools that are used to develop it.

Cognitive apprenticeship is instead a theory based on the process where a master teaches his skill to an apprentice. This theory holds that masters of a skill often fail to take into account the implicit processes involved in carrying out complex skills when they are teaching novices. Within this theory, teaching complexity in high school might trigger the different levels of learning that are involved in this process.

In my opinion, these two concepts might combine and help students in the process of solving complex problems. Following this idea, I presented the definition in this way:

> Complex Problems require the management of a significant number of variables with a solution depending heavily on the chosen path; moreover, in order to progress in these, it is necessary to make subsequent decisions that affect the result. Result that often presents more than one answer.

Organizing a trip, a conference, or finding a way to measure the area of a garden are examples of complex problems.

Another good point I believe is important in Complex Problems is that there is not one single right answer: it is more or less a case of seeing what effects different causes produce and choosing which one to use.

Therefore, when a student is facing a complex problem he or she must situate his/her knowledge in a specific environment that might even be different from the one in which he/she was educated. Nevertheless, the situations share some common features: in particular, the approach by which you try to see if a solution works and here is where the apprenticeship comes into play. Indeed, a student can learn from someone who has already tried from a theoretical point of view as it happens these days with the Millennials when they refer to the YouTube tutorials. Watching a YouTube video where someone else plays, uses a software or explains how to solve a problem by doing might clear the meaning. This is different from self-education, it is a sort of mix between apprenticeship and situated learning. This sometimes may also be applied to real life complex problems.

4.7. The Underground Game

I tried to clarify with an example of complex problem that might be useful to explain this concept: a problem related to large city urban mobility. I asked the students to imagine someone who needs to go from one place to another by public transport in a large city like London or Vienna, as in the picture I gave to students.

Figure 4.9 Vienna Underground and Overground rail network (U-Bahn and S Bahn)[100]

Going from one place to another using the map is not only a problem of time; it could also be a matter of space, money and other issues. For example in London every time

---

[100] Source: http://homepage.univie.ac.at/horst.prillinger/ubahn/m/largemap-s-wien.html author: Horst Prillinger. Netzplan Schnellverbindungen in Wien von Horst Prillinger ist lizenziert unter einer Creative Commons Namensnennung - Keine Bearbeitungen 4.0 International Lizenz.

you take the underground you pay more than when taking the bus. Urban trains crossing the city can be a lot quicker but more expensive or less frequent. Therefore, a tourist or a traveler must build an itinerary based on the map provided and on experience, create his or her own criteria and then try. Indeed, you never know if you are able to catch the exact moment to change from one line to the other, in this way the task requires management of a significant number of variables with a solution depending heavily on the chosen path. Moreover, in order to progress, it is necessary to make subsequent decisions that affect the final result: time of the day, position within the city or even the day of the year might play an important role in the solution. Time of the day may affect the traffic, if you are in rush hour, so even if a bus is faster it might not be a wise choice; at night, instead, when no one is around, buses may be faster than the underground. A holiday in one country might be different from another: take for example National Day. An effective solution for one city might be completely different for another. For example rush hour in Vienna starts around 6.45 to 7.00 am and ends around 9.00am: would it be the same in Madrid?

This perfectly fits the complex problem definition.

4.8. The Klondike Solitaire

Then we moved to the Klondike Solitaire and complex strategies.

Klondike Solitaire was originally part of the Windows game package since version 3.1; starting from Windows 8, it is not preinstalled anymore, but it can be downloaded for free.

The history of solitaire goes back to the early 19th century and the use with the layouts of tarot cards, long used for divination and fortune telling and other application, is the origin of the current use of it for amusement. Here is an explanation provided by a programmer’s web site

The game is played with a single pack of 52 playing cards. After thoroughly shuffling the deck, 28 cards are dealt face down to form the tableau which consists of 7 columns of 1, 2, 3, 4, 5, 6 and 7 overlapping cards, from left to right respectively. The exposed card at the end of each tableau column is turned face up. The remaining cards are placed face down to form the stock. The game then begins.

Figure 4.10 Klondike Solitaire[101]

The goal of the game is to build the four foundations up in ascending suit sequence from Ace to King: e.g. A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K with cards of identical suit.

As each Ace becomes available, it may be transferred to a row above the tableau to start one of the four foundations. The foundations are built up in ascending suit sequence to the King. The exposed card of a tableau column may be transferred to a foundation of the same suit, if it follows the ascending

[101] A GNOME version of Klondike (solitaire), 25 April 2008, Source: OpenSuSE using the GNOME desktop manager. Author User: Andreas Rosdal.
Source: https://commons.wikimedia.org/wiki/File:GNOME_Aisleriot_Solitaire.png

> sequence; or to the exposed card of another column, if it forms a descending sequence of alternating colors, e.g. 6 on 7 or Q on K. A complete packed column of face up cards may also be transferred to the exposed card of another tableau column, if the join follows the same descending sequence of alternating colors. If the movement of a tableau card exposes a face down card, then it is turned face up. When a tableau column is completely cleared out, the space may only be filled by a King or a packed column headed by a King. When no more moves are available from the tableau, the top three cards from the stock are dealt face up, without upsetting their order, to a single waste pile. The top exposed card of the waste pile is always available for play to the foundations or tableau. When the stock has been exhausted, the waste pile is picked up and turned over to form a new stock and the game continues. This procedure is continued until the game eventually blocks or is won. A variation of the game allows a single card to be dealt to the waste pile instead of three. There is usually a limit to the number of times the waste pile can be re-used to prevent the game coming out too often"[102].

There are three main papers about the mathematics and strategy of solving the game of Solitaire: "Solitaire: Man Versus Machine" by Yan *et al.* (2005); "Searching Solitaire in Real Time" (2007) and "Lower Bounding *Klondike Solitaire* with Monte-Carlo Planning" (2009) both by Bjarnason, Tadepalli and Fern.

In the first one Yan *et al.* (2005) try to test a rollout method for policy improvement to analyze a version of Klondike solitaire. This version, sometimes called Thoughtful Solitaire, has all cards revealed to the player, but then follows the usual Klondike rules. They established a strategy, using iterated rollouts, "wins about twice as many games on average as an expert human player does". In the same paper, they say:

---

[102] http://www.solitairecity.com/Help/Klondike.shtml Copyright Peter Wiseman / Digital Smoke.

> "It is one of the embarrassments of applied mathematics that we cannot determine the odds of winning the common game of solitaire. Many people play this game every day, yet simple questions such as: What is the chance of winning? How does this chance depend on the version I play? What is a good strategy?, remain beyond mathematical analysis.[103]"

Indeed, complexity of the problem comes from dealing with 52![104] possible shuffles or games. Therefore, the number of states is so large that it cannot be successfully solved by following specific, step-by-step instructions that is, by using an algorithm. In the absence of an algorithm, learners or in this case computer programmers must instead use a heuristic strategy, a general problem-solving strategy that proceeds by guided trial and errors.

> This, unlike an algorithm, offers no guarantee of solving any problem. More precisely, heuristics are strategies derived from experience with similar problems, using readily accessible, though loosely applicable, information to control problem solving in human beings, machines, and abstract issues[105].

[103] Yan, X., Diaconis, P., Rusmevichientong, P., and Van Roy, B. (2005). *Solitaire: Man Versus Machine. Advances in Neural Information Processing Systems 17* (eds. L. K. Saul, Y. Weiss, and L. Bottou), pp. 1553–1560, Cambridge, MA: MIT Press, p. 2.

[104] 52! Is the number identifying all the possible shuffles of the solitaire. Because when you choose a card, you have all the others 51 remaining therefore 52 possible cards in the first hand then 51 in the second and so on until only one remains. 52 possible cards times 51 times 50 ….times 1. Evaluating this number it means to calculate the factorial function symbol (!) i.e. to multiply a series of descending natural numbers. In this example 52! = 52x51x50…..3x2x1. Actually, this is a very large number it is equal to 80658175170943878571660636856403766975289505440883277824000000000000 or to say it in words it is an 8 followed by 67 zeros!.

[105] Pearl, J., *Heuristics: Intelligent Search Strategies for Computer Problem Solving*, Reading, MA: Addison-Wesley, 1984.

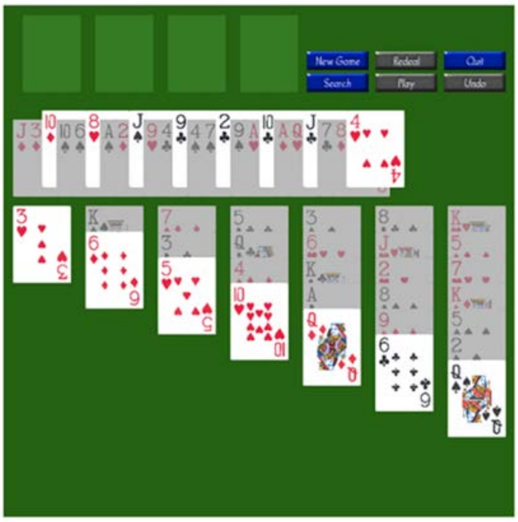

Figure 4.11 Thoughtful solitaire image[106]

What is the best strategy for solving solitaire? As Yan says, there is no definite winning set of moves although you can define guidelines, therefore they tried an heuristic strategy made by rollouts. Bjarnason, Tadepalli and Fern (2007) state:

> This article presents a new real-time heuristic search method for planning problems with distinct stages. Our multistage nested rollout algorithm allows the user to apply separate heuristics at each stage of the search process and tune the search magnitude for each stage. We propose a searchtree compression that reveals a new state representation for the games of Klondike Solitaire and Thoughtful Solitaire (…) we present a Thoughtful Solitaire solver based on these methods that can determine over 80% of Thoughtful Solitaire games in less than 4 seconds. Finally, we demonstrate empirically that no less than 82% and no more than 91.44% of Klondike Solitaire games have winning solutions, leaving less than 10% of games unresolved.[107]

[106] Image from Bjarnason, Ronald, Tadepalli, Prasad and Fern, Alan. Searching Solitaire in Real Time. *International Computer Games Association Journal*, 30(3) (2007): p. 134.
[107] Bjarnason, Ronald, Tadepalli, Prasad and Fern, Alan, Searching Solitaire in Real Time. International Computer Games *Association Journal*, 30(3) (2007): p. 131.

Thanks to these studies, I presented the Solitaire as a "simple" example of a Complex Problem.

4.9. The Prisoner's Dilemma

I presented them the Prisoner's dilemma as a game showing the scheme and presenting the following story:

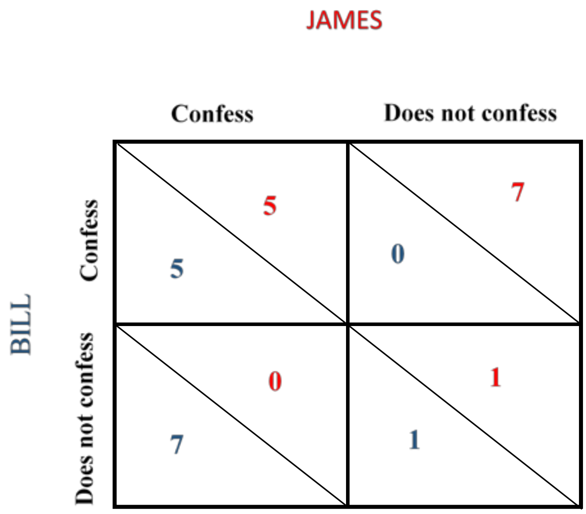

Figure 4.12 Prisoner's Dilemma simplified working scheme

James and Bill (in Figure 4.12) are suspected of having committed a robbery together, they are caught by the police and put in different cells so they cannot speak to each other. The detective investigating their case offers both of them different options

If one of them confesses, he will avoid the punishment, while the other will serve seven years in prison.

If they both confess, both of them will be sentenced to 5 years.

If they both remain silent, both of them will serve 1 year in prison.

If they agree not to confess, then each will be sentenced to one year in prison, but they will not be allowed to communicate, and even if they could, would they trust each other?

In the end? They both confess.

Later I told the students the story of the origin of the Prisoner's Dilemma.

It was initially developed by Merrill Flood and Melvin Dresher, two mathematician working at RAND, a nonprofit American think tank in 1950, studying the Cold War and the winning strategies[108].

I quote from the Stanford Encyclopedia of Philosophy:

> Puzzles with the structure of the Prisoner's Dilemma were devised and discussed by Merrill Flood and Melvin Dresher in 1950, as part of the Rand Corporation's investigations into Game Theory (which Rand pursued because of possible applications to global nuclear strategy). The title "prisoner's dilemma" and the version with prison sentences as payoffs are due to Albert Tucker, who wanted to make Flood and Dresher's ideas more accessible to an audience of Stanford psychologists. Although Flood and Dresher did not themselves rush to publicize their ideas in external journal articles, the puzzle has since attracted widespread and increasing attention in a variety of disciplines[109]

In a way:

---

[108] For more information about this really interesting story see Poundstone, William, *Prisoner's Dilemma: John von Neumann, Game Theory, and the Puzzle of the Bomb*. 1992.

[109]http://plato.stanford.edu/entries/prisoner-dilemma/ First published Thu Sep 4, 1997; substantive revision Fri Aug 29, 2014.

> This peculiar parable serves as a model of cooperation between two or more individuals (or corporations or countries) in ordinary life in that in many cases each individual would be personally better off not cooperating (defecting) on the other[110]

The Prisoners' Dilemma has applications in Economics and Business. For example, if you take two companies selling products of the same kind cars for example. Each one must choose a pricing strategy. When they both charge a high price than each one makes a profit of 100 million euros per trimester. What happens if one decide to lower the price? It will probably drain many customers away from its competitor. Then first company profit will rise up to 120 million dollars while the other's will falls to 70 million. Therefore, the other follows price reduction and when they both have lowered their prices, the profit of each one drops to 90 million euros. In this model the options related to the prisoner's dilemma are three. If they both remain silent, they both charge higher prices and this option is the cooperative strategy. The second where one company increases its prices counting on the fact that it will gain market share against the other is the one in which a prisoner confesses and the other remain silent. The third option and final is when they both confess and so the companies see their profits lowering. The optimal strategy would then be for companies to agree on high prices but, in theory, fair competition should foster the other option and force them to drop both their prices.

Arms races between superpowers or local rival nations offer another important example of the dilemma. Both countries are better off when they cooperate and avoid an arms race. Yet the dominant strategy for each is to arm itself heavily: this applies perfectly to the Cold War.

---

[110] http://perspicuity.net/sd/pd-brf.html, The Prisoner's Dilemma By Leon Felkins, written 9/5/95 Revised on 3/12/01.

However, I love the example used in a Christopher Nolan's 2008 second Batman movie *The Dark Knight* and I use it with students.

The Joker (Batman's worst enemy) has forced Gotham city Police to load two ferries with a trick. One ferry was full of convicted criminals and the other one with normal citizens. While they are both in water they receive the information that both ferries are loaded with explosives connected to a detonator that is on the other boat. The Joker says to all the occupants that they have to activate the other's boat explosives because if no boat has yet exploded by midnight, then they both will be destroyed.

This is exactly a Prisoner's Dilemma and director Cristopher Nolan, who is a trained psychologist, uses a fantastic script trick to solve it. One of the seemingly worst criminals says to the guard holding the detonator: "I will do what you don't have the courage to do with that!" and then, surprisingly for the spectator, throws it out of the window and does not let the other boat explode as the spectator would expect. When midnight comes, no ferry explodes and the Joker is very impressed on how normal people could trust each other.

4.10. A few hints on how to deal with a complex problem.

At this moment, I gave a few hints on how to deal with a complex problem. I showed the students the following list and we discussed what kind of strategy is better suited to a complex problem. All answers were very interesting and surprisingly different in every group, although the idea often expressed was how to deal with the fact that "I feel I cannot solve the problem".

Dynamic Changing Environment.

Need to adapt and to think of different strategy.

There is not always one single right answer.

One has to call previous knowledge and try new solutions.

Once you have the strategy, you must apply to test it.

You must try to find new ways to solve problems.

You can learn by doing.

Next time will be better.

Improve knowledge by mistakes.

Maybe you can better understand a problem after you solved it.

4.11. The final idea octopus learning

Eventually, I concluded the lecture with the famous video of how an octopus can learn to open a jar containing some food. The video shows how an octopus in a controlled environment like the aquarium can learn how to open a container by guided trial and errors. Katherine Harmon Courage[111] has written a book about Octopus and, in her blog on the *Scientific American*, she says:

> It isn't every day in the ocean that an octopus comes across a jar to open—especially one that contains a tasty live crab. Which is why it is particularly impressive that these invertebrates can quickly figure out how to twist off a cap in captivity.
> The treat-in-a-jar trick has long been a favorite activity to give octopuses in aquariums. Just like humans, octopuses get faster at these manipulation tasks with practice. (…)
> These trails aren't just for the amusement of the human keepers. This sort of higher-level stimulation is necessary for octopuses in captivity. Without enough

[111] Katherine Harmon Courage, *Octopus!: The Most Mysterious Creature in the Sea*, London: Penguin, 2013.

to keep these intelligent animals occupied, octopuses can easily become bored and despondent—and in some cases even start eating their own arms.[112]

[112] http://blogs.scientificamerican.com/octopus-chronicles/speedy-octopus-sets-record-for-jar-opening/ By Katherine Harmon Courage on January 13, 2014.

Chapter 5. Results

5.1. Description of the sample

The sample has been collected by running the test in all the selected schools and with teachers that volunteered to participate in the project, compatibly to the technical possibility of my presence in every school to explain the project and give the lecture. The two waves of data collection, one in Autumn (end of October--early November 2014) and the other around Spring (end of March-- early April 2015), were chosen, in accordance to school times as well as specific requirements. As a teacher, I was able to pick the best times during the school year for each test to take place. Because these two intervals are right in the middle of the first and second term of the school year, classes and teachers are free to experiment new strategies and methods without feeling under pressure.

The sample comprises different Italian types of schools: a common curriculum for all students in grade 8 (Scuola Media) as well as different kinds of high schools, namely the Liceo Classico; Liceo Musicale, Liceo delle Scienze Umane. An additional school from a foreign country, implementing the International Baccalaureate Program[113] has also been included: the IB Vienna International School.

After data cleaning, necessary to discard incomplete data and other randomly occurred errors (i.e. lack of hardware ID, other difficulties involved in the paper pencil test and the Genetics Lab performance), the sample included 442 people, with a complete set of data: personal info, figural shapes and the complete Genetics Lab performance.

[113] "Founded in 1968, the International Baccalaureate® (IB) is a non-profit educational foundation offering four highly respected programmes of international education that develop the intellectual, personal, emotional and social skills needed to live, learn and work in a rapidly globalizing world". http://www.ibo.org/about-the-ib/ accessed November 2015.

5.2. Categorization of the sample

The different tracks included in our study can be divided into four groups

Country	**Italy**			**Austria**
Tracks	**Scientific**	**Human**	**Middle School**	**International Baccalaureate**
School Curricula	Liceo Scientifico	Liceo Classico, Liceo Musicale, Liceo delle Scienze Umane	Scuola Media	IB
Number of schools	2 schools	1 school with multiple options	4 schools	1 school
Number of classes	5 in 10th grade 5 in 11th grade	1 Classical 10^{th} grade 1 Music 10th grade 6 Human sciences classes 10th grade	6 classes	1 portion of 10th grade 1 portion of 11^{th} grade
Tot students	169	99	122	72

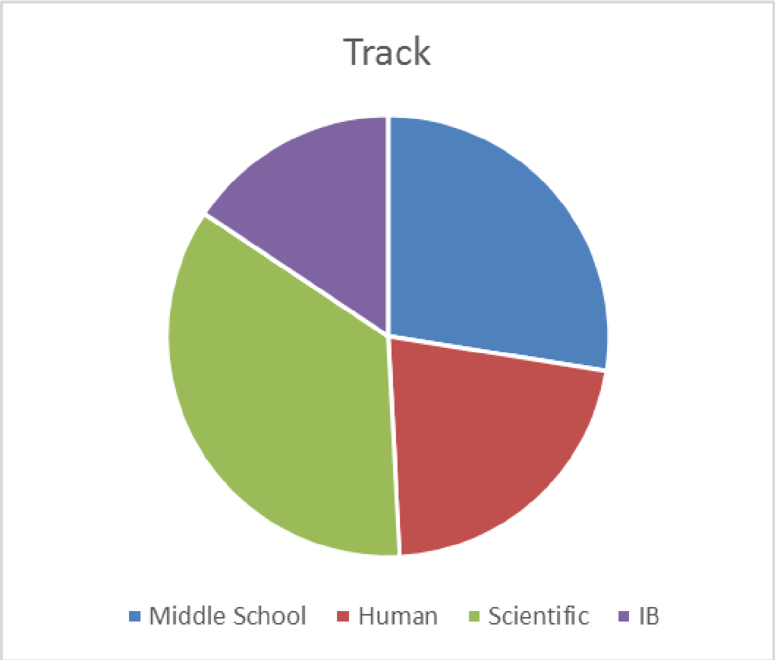

Figure 5.1 Sample composition: Distribution over tracks

Distribution over other variables like grades, sex and age.

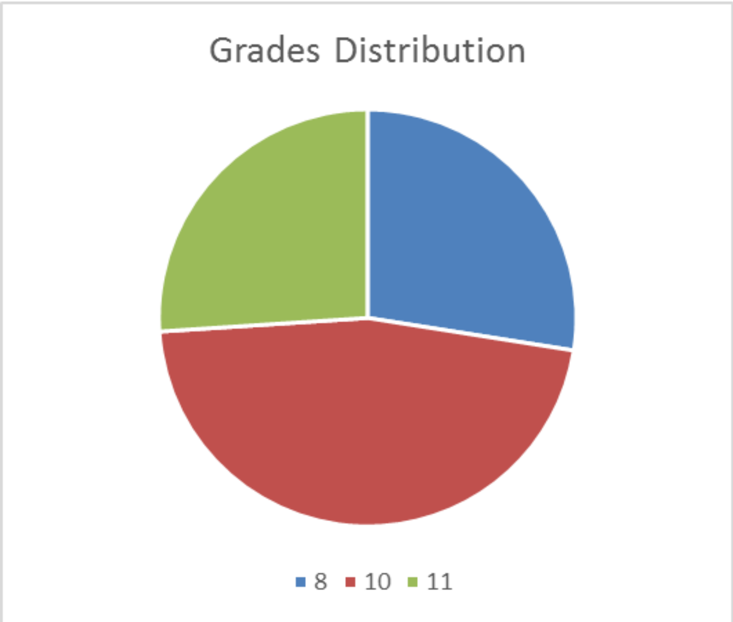

Figure 5.2 Sample composition: Distribution over grades

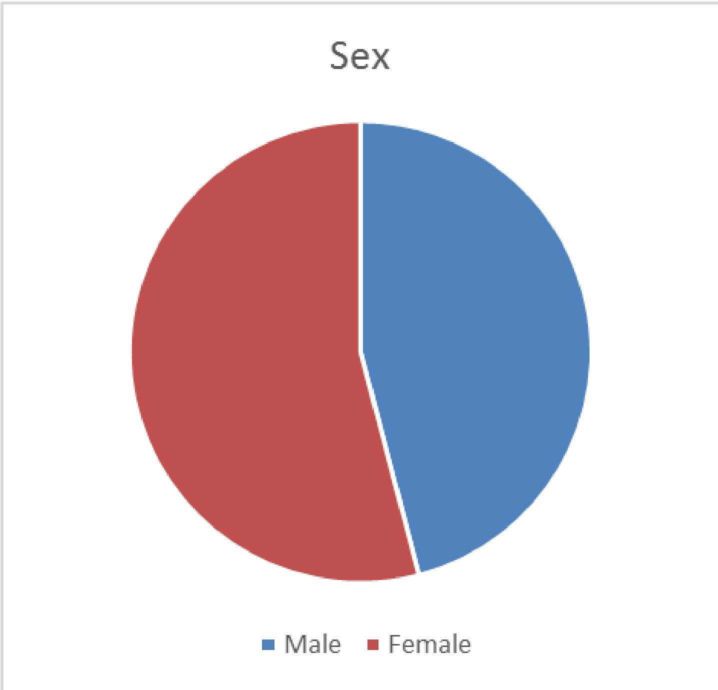

Figure 5.3 Sample composition: Distribution over sex

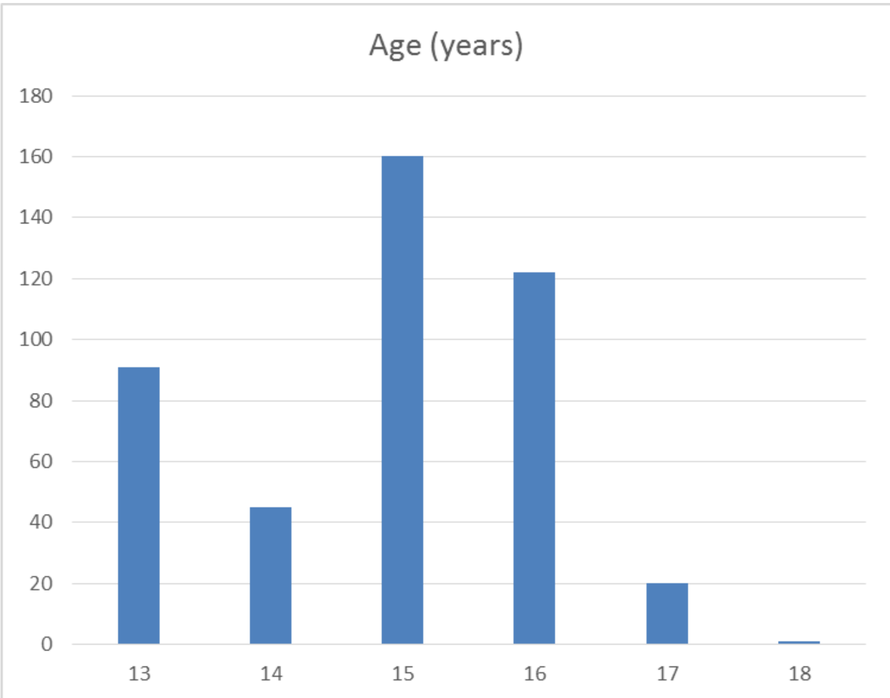

Figure 5.4 Sample composition: Distribution over age

5.3. Splitting of the sample

For the purpose of the study, it was necessary to split the overall sample in two groups, as homogeneous as possible: one to undergo the GL[114] without previously receiving the lecture on Complexity and Complex problems, and one after receiving it. This way, it was possible to investigate the impact of the lecture on the performances in GL. The criteria for sample split are described in the graphic scheme feaured in Chapter 4.2. Nevertheless, during this stage a variety of difficulties were encountered. Ideally, the perfect division would result from randomly dividing each class in two groups. Unfortunately, because of multiple problems such as classes and schools timetables, computer and classrooms availability, in a number of cases a different arrangement was required, and entire classes were included into one of the study groups without splitting. However, as shown in the graph in Figure 5.5, the distribution appears to be reasonably homogeneous and compatible with the designed study.

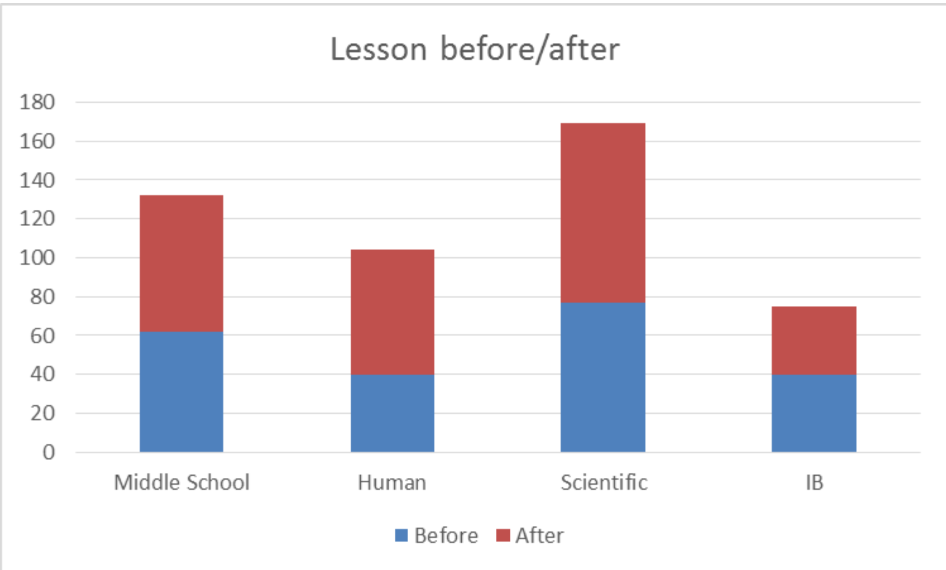

Figure 5.5 Distribution of the two study groups

[114] Hereinafter, the Genetics Lab software will be referred to as GL

5.4. GL performances

The overall sample size of n=442 was similar to the size of the sample considered in the parent Luxembourg study (Sonnleitner *et al.* 2013) [115]with n= 563.

As a starting point, the performances of the samples in GL were compared to those observed and reported by Sonnleitner *et al.* in their study. That study considers students of similar grades (9th and 11th grade) and of different tracks. It has been possible to compare different countries (Luxembourg, Italy and IB), different tracks and similar grades (8th, 10th, 11th present study; 9th and 11th grade Luxembourg main study).

The correlation between the reasoning scale and the performance in the GL is confirmed to be high also in the current study, nevertheless the new data present a rather interesting difference between the different tracks.

The GL performance parameters used for both the current and the Sonnleitner's study are defined here:

Exploration Score 1: how much percent of the problem has been explored/ how many of the possible informative steps have been made.

Exploration Score 2: percentage of informative steps related to the total amount of steps, (Rule Identification).

Knowledge : the score on global knowledge acquired (Rule Knowledge).

Control Phase: how many of the applied steps were optimal (maximum 3 for each item).

The reasoning scale test selecting figures: figural shapes selection expressed percentage of maximum possible score.

[115] Sonnleitner, P., Keller, U., Martin, R. and Brunner, M. Students' complex problem-solving abilities: Their structure and relations to reasoning ability and educational success. *Intelligence* 41 (5) (2013), pp. 289—305..

In the Luxembourg main study, exploration score 1 is not present and the other three are indicated as explained in *Sonnleitner e al.* (2013):

> Performance across scenarios was summarized by three scores that reflected students' proficiency in the three main facets of Complex Problem Solving (the scoring algorithms can be found in Keller and Sonnleitner, 2012): (a) Each student's exploration strategy was scored on the basis of a detailed log-file in which every interaction with the microworld is stored. Thus, it was possible to derive a process-oriented measure (Rule Identification) indicating how efficiently a student explored a scenario by relating the number of informative exploration steps to the total number of steps applied (Kröner *et al.*, 2005). Note that an exploration step is most informative if students manipulate the genes in a way that any changes in characteristics can be unambiguously attributed to a certain gene (see Vollmeyer, Burns, and Holyoak, 1996). (b) Students' Rule Knowledge was assessed by scoring their database records (see Fig. 1B) by adapting an established scoring algorithm (see Funke, 1992, 1993). The resulting Rule Knowledge score thus reflects knowledge about how a gene affects a certain characteristic of a creature and knowledge about the strength of such an effect. (c) Finally, the actions that students took to achieve certain target values on the creature's characteristics during the control phase (see Fig. 1C) were used to compute a process-oriented Rule Application score. Only if a step was optimal in the sense that the difference from the target values was maximally decreased was the step considered to indicate good control performance. Given that all target values must be achieved within three steps, a maximum score of three was possible for each scenario. [116]

[116] Sonnleitner et al., ibid. p. 298.

All the performances are represented in percentages of maximum possible score going from 0 to 100%. SD means Standard Deviation of the sample, p25 is the first quartile (Q1), p75 is the third quartile (Q3).

8th grade total sample of 122 collected in Lombardy

Measure	Mean	SD	P25	Median	P75
Reasoning					
Selecting figures	36,00	15,20	25	35,00	45
Complex problem Solving					
Exploration 1	60	20	47	62	78
Exploration 2	25	16	13	20	32
Knowledge	55,69	14,32	14,32	40,94	45,71
Control	40,82	16,48	27,78	38,89	52,08

9th grade total sample of 300 collected in Luxembourg

Measure	Mean	SD	P25	Median	P75
Reasoning					
Selecting figures	36,92	17,45	25,00	35,00	50,00
Complex problem Solving					
Exploration 2	26,19	13,88	15,42	23,90	35,37
Knowledge	62,43	16,04	50.30	59.48	71.94
Control	50	17,53	36.11	47.22	61.11

You can see from the selecting figures that the performance of the two samples have a rather similar mean, but the SD is a little bit higher in Luxembourg -- probably due to the fact that they were 9th grade students and were divided in two different tracks: academic and non-academic. The academic track is the one leading to the Gymnasium and later possibly to university; instead, the non-academic course is directed towards more practical and job-oriented education. Focusing on the CPS performance, one can see a slightly higher performance in Luxemburg in all three phases of the GL. This might be due to various reasons, probably the age (the test was taken in 9th grade spring time, so the Luxembourg students had one entire additional year of education compared to the Italian students); or furthermore, the fact that the Luxembourg curriculum better exploits CPS skills. The sample is definitely too small to draw conclusions, but it is still quite meaningful to create a guideline and provide a number of hints (according to the scope of this thesis) on what to focus on for further developing these matters.

Now we move to 10th grade performance, considering the Human and Scientific tracks in addition to the IB curriculum.

10th grade total sample of 99 collected in FVG Human Sciences, Music and Classical Liceo

Measure	Mean	SD	P25	Median	P75
Reasoning					
Selecting figures	37,61	13,67	30	37,50	45
Complex problem Solving					
Exploration 1	66	19,63	49,50	70	81
Exploration 2	23	11,63	15,08	21	26,76
Knowledge	56,32	14,09	46,57	52,77	61,76
Control	42,15	15,15	33,33	38,89	50

10th grade total sample of 84 collected in FVG Scientific Track (liceo)

Measure	Mean	SD	P25	Median	P75
Reasoning					
Selecting figures	43,85	16,523	30	40	55
Complex problem Solving					
Exploration 1	72	18,35	61,12	78	83,85
Exploration 2	34	14,86	23,92	31	44,36
Knowledge	67,70	16,13	53,97	65,25	77,42
Control	55,59	17,20	41,67	52,78	66,67

As one might expect, the 10th grade Scientific track performed better in all CPS task than all other Italian tracks, probably because they were trained more in Science. Nevertheless, you must also take into account the fact that the average selecting figures had a general lower performance in the Human track, so this might also have been a reason. Moreover, it is interesting to notice that the increase in CPS performance for 10th grade students between Human and Scientific tracks follow the same relative increase in selecting figures performance.

International school's students performed generally better than the remaining sample, possibly because the IB program, despite being a non-selective program, is more directed toward increasing problem solving abilities.

10th grade International school random chosen sample of 34 in the whole grade.

Measure	Mean	SD	P25	Median	P75
Reasoning					
Selecting figures	50,71	18,79	40	50,00	60
Complex problem Solving					
Exploration 1	72	23,04	47,92	78	92,36
Exploration 2	27	14,93	13,93	26	36,11
Knowledge	72,32	19,69	55,27	75,16	91,67
Control	62,06	21,29	44,44	63,89	77,78

When going to the 11th grade

11th grade total sample of 84 collected in FVG Liceo Scientifico.

Measure	Mean	SD	P25	Median	P75
Reasoning					
Selecting figures	44,625	16,332	30	45	55
Complex problem Solving					
Exploration 1	79,01		71,53	81,94	90
Exploration 2	34,43	15,56	21,21	32,19	43,94
Knowledge	67,04	18,29	53,17	60,40	79,71
Control	58,10	18,56	43,06	55,56	75

11th grade total sample of 263 collected in Luxembourg					
Measure	Mean	SD	P25	Median	P75
Reasoning					
Selecting figures	47,98	19,38	35,00	50,00	65,00
Complex problem Solving					
Exploration 2	31,96	16	20,42	29,39	44,16
Knowledge	73,62	16,38	61.07	74.10	85.61
Control	59,29	19,17	44.44	58.33	75.00

11th grade International school random chosen sample of 38 in the whole grade.					
Measure	Mean	SD	P25	Median	P75
Reasoning					
Selecting figures	50,56	18,96	40	50	63,75
Complex problem Solving					
Exploration 1	82,77	14,66	75,87	87,69	93,75
Exploration 2	34,16	12,73	23,58	30,23	43,76
Knowledge	73,19	16,35	61,07	72,14	87,60
Control	59,51	20,33	41,67	59,72	79,86

Comparing the 10th and 11th grade in the International School, you can see that, as well as in the Italian scientific track, age and grade do not seem to have a statistically significant impact on the GL performances.

These general considerations may lead to respond to the first research questions:

Which kind of impact on performance in the GL may the different tracks (scientific, classical or human sciences), the specific teaching method (IB in the VIS) and the language of instruction have?

From the above described data, it is possible to conclude that generally the scientific track performs better in CPS tasks than other tracks considered, although they have in this data a higher general reasoning ability.

How effective are different academic tracks in order to foster problem solving?

It was already made clear with the Sonnleitner study and it has been confirmed here that more clever students generally perform better. These data also indicate that there is a general increase of performance with age from 8th to 10th grade, which seems to stop afterwards. Indeed, both the international school and the Italian scientific track do not seem to present a significant difference in performances between 10th and 11th grade. This interesting result might lead to a new project designed to further investigate this effect on a larger scale.

5.5. How was the lecture perceived?

The lecture was received very well among both students and teachers. The students expressed genuine interest in the topic of Complexity and Complex Problem Solving. Because the sample was so varied in age, place and type of schools, sample testing might prove effective to mark an initial experiment of the general aim for further developments of teaching Complexity in high school. I have spoken about my project to many teachers, often in the context of conferences, and almost everyone has expressed interest in being involved in the project in his/her school. Unfortunately, repeating the experience in other classes has so far been impossible because of lack of available resources, and the amount of effort required. Indeed, preparing and administering the test, collecting, scanning and processing the data of the two waves has proven to be a substantial effort in terms of time and work for the two of us involved -- me and my counterpart in Luxembourg, Philipp Sonnleitner. I hope that further funding can be made available in the future so that the use of the GL as a measuring and training device along with complex problems and complexity may be brought in other Italian, European or International schools. The GL has already been extensively tested in China and in the US, with a sample of undergraduate University students.

5.6. The effect of the lecture

As mentioned in Chapter 4, during my time in school either each class was split in two groups (if practically possible), or different classes of the same grade and school were randomly chosen to undergo GL first, then lecture, or the other way round.

The process of collecting data was itself quite challenging, as I mentioned. A sufficient number of computers had to be available for the students, so that each one of them would be connected to the online version of the Genetics Lab. The University of Luxembourg server would then start collecting the data, creating a match between time and Hardware ID and store the data correctly. Meanwhile, in schools I personally handed the paper pencil tests, explained the instructions and the Hardware ID, then let the students take the questionnaire.

Data analysis was performed mainly through Excel and SPSS version 21.

The variables used in SPSS to measure different complex problem solving performances were defined as follows:

Sysex.mean	Exploration score 1 (represented related to 1 so 1 is the maximum possible score)
Sysex.rel	exploration score 2 (represented related to 1 so 1 is the maximum possible score)
Gdk.lob	global knowledge
Sc.optim	the control phase
Fspomp	figural shapes selecting figure Percentage of maximum possible score

As a first step to assess the possible impact of the lecture, average and median performances for each group (GL-Lecture, Lecture-GL) were calculated for all these variables, and compared for each student's category.

Here, reported as an example, are the data for the entire 10th grade sample:

Statistics

training			sysex.mean	sysex.rel	gdk.glo.p	sc.optim.p	fs_pomp
GL-Lecture	N	Valid	87	85	87	87	76
		Missing	0	2	0	0	11
	Mean		.6462	.2729	63.2520	51.9796	43.3553
	Median		.6875	.2417	59.8264	50.0000	40.0000
	Std. Deviation		.17290	.14012	18.19459	18.74866	16.72101
Lecture-GL	N	Valid	116	114	116	116	105
		Missing	0	2	0	0	11
	Mean		.6420	.2589	60.7710	47.8927	39.2381
	Median		.6771	.2383	57.5732	44.4444	40.0000
	Std. Deviation		.18073	.13358	14.71035	16.45445	15.12418

What can be observed (in all considered samples) is a difference in mean and median for the all parameters, but the difference is rather small: for each of them it is smaller than the standard deviation.

In addition, and to add statistical value to the assessment, we run and *Independent Samples T-Test*[117] between the two groups sample.

This test is used when you have two independent and separate sets of identically distributed samples, one from each of the two populations being compared. The values coming out from the t-test expresses the probability of finding a statistically significant difference between the two data sets, in this specific case, in performances for the four general scores of the genetics lab defined above.

t is defined as follows:

$$t = \frac{\overline{x_1} - \overline{x_2}}{\sqrt{\frac{(n_1 - 1)\sigma_{x_1}^2 + (n_2 - 1)\sigma_{x_2}^2}{n_1 + n_2 - 2}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

[117] https://statistics.laerd.com/spss-tutorials/independent-t-test-using-spss-statistics.php accessed November 2015.

Where $\overline{x_1}$ and $\overline{x_2}$ are the means of the two samples, n_1 and n_2 are samples sizes, $\sigma_{x_1}^2$ and $\sigma_{x_2}^2$ are the two variances.

Therefore, *t* is a sort of estimate of the difference between the weighted means over a common average standard deviation.

This kind of test is used to assess if a difference in mean between two samples is only statistical fluctuation. The value of *t* obtained to be compared with one tabled value referring to a selected probability is a function of the degree of freedom of the sample.

For example, if the calculated *t* value for a sample with n degrees of freedom is lower than the tabled value corresponding to 95%, it means that the two samples can be considered as different with 95% confidence.

This table exemplifies the results of the *t* test performed on the general sample of 10th grade:

Group Statistics

	training	N	Mean	Std. Deviation	Std. Error Mean
sysex.mean	GL-Lecture	87	.6462	.17290	.01854
	Lecture-GL	116	.6420	.18073	.01678
sysex.rel	GL-Lecture	85	.2729	.14012	.01520
	Lecture-GL	114	.2589	.13358	.01251
gdk.glo.p	GL-Lecture	87	63.2520	18.19459	1.95066
	Lecture-GL	116	60.7710	14.71035	1.36582
sc.optim.p	GL-Lecture	87	51.9796	18.74866	2.01007
	Lecture-GL	116	47.8927	16.45445	1.52776

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means	
		F	Sig.	t	df
sysex.mean	Equal variances assumed	.173	.678	.168	201
	Equal variances not assumed			.169	189.524
sysex.rel	Equal variances assumed	.476	.491	.715	197
	Equal variances not assumed			.710	176.251
gdk.glo.p	Equal variances assumed	6.083	.014	1.074	201
	Equal variances not assumed			1.042	161.895
sc.optim.p	Equal variances assumed	2.765	.098	1.649	201
	Equal variances not assumed			1.619	171.309

For this specific sample, the value corresponding to a 95% confidence is 1.98. Therefore, we can conclude that the observed difference in the means is statistically significant if the calculated *t* value is equal or higher than 1.98.

None of the calculated t-values for the parameters measuring the different GL performances equals nor exceeds 1.98, the highest one (sc.optim.p) being 1.649; while the highest *t* value is the one calculated for the figural shapes performances.

The *t* test was performed for each grade on the overall sample, as well as dividing the sample per track and per sex.

The results were all similar, leading to the conclusion that the observed different performances between the two groups (GL-Lecture, Lecture-GL) are not statistically significant.

Furthermore, in order to test if any effect of the lecture might be connected with the part of CPS performances not related with the reasoning ability (Sonnleitner *et al.* 2012, 2013, 2014), a complete regression model between the performances of the sample in all four GL performance scores against the performance in figural shapes test (fs pomp) was applied.

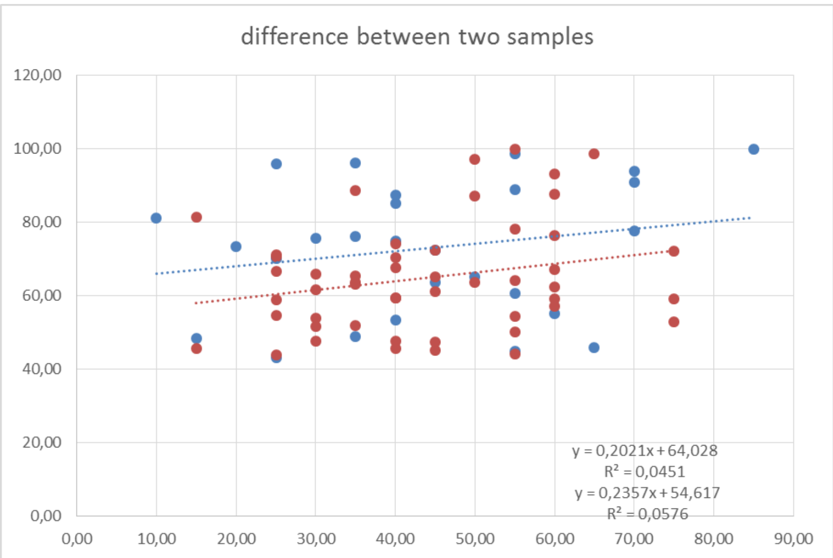

Fig. 5.6 an example of regression line compared between the two samples.

Once this regression for each of the four parameters was performed, the residuals were calculated. The residuals are defined as the difference between the observed and the predicted value of the dipendent variable. In other words, the residuals measure how far from the regression line the observed data are. As the line represents the expected behavior of each parameter against reasoning ability, the residuals should give account of ability not related with the reasoning.

In order to test if there was an effect of the lecture on the abilities not connected with reasoning, an additional t-test was performed on the residuals calculated for the two groups (GL-Lecture, Lecture-GL).

Once more, below as an example, the results for the grade 10 students:

Group Statistics

	training	N	Mean	Std. Deviation	Std. Error Mean
Unstandardized Residual	GL-Lecture	76	-.0017249	.17832733	.02045555
	Lecture-GL	105	.0012485	.17879232	.01744834
Unstandardized Residual	GL-Lecture	74	.0041344	.13448073	.01563307
	Lecture-GL	103	-.0029704	.13204157	.01301044
Unstandardized Residual	GL-Lecture	76	1.4050301	17.51535098	2.00914855
	Lecture-GL	105	-1.0169742	14.15497230	1.38138385
Unstandardized Residual	GL-Lecture	76	2.1811414	17.82963695	2.04519962
	Lecture-GL	105	-1.5787309	15.66437514	1.52868648

Independent Samples Test

		t-test for Equality of Means		
		t	df	Sig. (2-tailed)
Unstandardized Residual	Equal variances assumed	-.111	179	.912
	Equal variances not assumed	-.111	161.997	.912
Unstandardized Residual	Equal variances assumed	.350	175	.726
	Equal variances not assumed	.349	155.689	.727
Unstandardized Residual	Equal variances assumed	1.028	179	.306
	Equal variances not assumed	.993	140.092	.322
Unstandardized Residual	Equal variances assumed	1.503	179	.135
	Equal variances not assumed	1.473	148.733	.143

In the case of the residuals, too, results of the t-test indicated that the different performances between the two groups are not statistically significant. The same behavior appeared for all different data subsets considered.

The conclusion of the data analysis is that the lecture did not have a noticeable impact on the overall students' performance.

5.7. Possible answers to the research questions

The answer to various research questions may be provided by the present work:

First of all, we planned to extend the research performed by Sonnleitner *et al.* including a different national school system (Italian) and an International School (IB) to analyze the possible differences and similarities.

The current study confirmed the conclusions of the parent study, that *CPS "is a construct that is strongly associated with but still distinct from intelligence"*[118].

Differences in performances between different national systems, different school tracks and different grades were analyzed. Within Italian schools, it appears that scientific tracks display better CPS skills than other tracks, and this can be partially related with the overall higher average scoring in the reasoning skills. The IB school, despite being a nonselective track, presented a better performance than all other schools considered. The reason is possibly the better focus IB programs have on Complex Problem Solving, together with higher average reasoning abilities of the population.

The comparison of data for different grades indicates an increasing trend in the considered skills with increasing school grades up to grade 10. Such performances seems not to be different when comparing grade 10 and 11 students.

---

[118] Sonnleitner *et al.* (2013) p. 303,

This matter needs further investigation, i.e. by considering other classes, from grade 9 up to grade 12.

Furthermore, we planned to use the GL capability to evaluate specific problem solving skills to investigate which of them was possibly most influenced by the introductory lecture on CPS.

The other questions were:

May an introduction to the whole concept of Complexity, in the form of a single lecture brought to school, have an impact on the students' motivation or, in general, on their performances?

In order to answer this question and knowing that the GL is structured in different phases, each of them individually scored, the effect of the lecture could have been assessed by observing how it might affect one of the specific phases.

However, the comprehensive data analysis performed indicates that no observable statistically significant difference appeared between the two sample groups GL-lecture vs Lecture-GL.

The present study indicates that also with this kind of topic, even a lecture designed to be an instruction manual, specifically oriented to complex problems, is not sufficient to enhance general performances or motivation to try new schemes.

We expected to find that the impact of explaining to students that previous knowledge is not relevant for CPS performance would be an increase in their motivation to try their hand at it, and that we would be able to measure it through one of the GL parameters, the exploration phase. Unexpectedly, in fact, not even this impact appeared to be measurable.

We still consider the implant of the study to be adequate and its purpose deserving of further consideration.

In order to improve this study, introducing new control parameters (i.e. student's school performances, different teachers, …) in order to narrow down explainable and therefore eliminable differences would also prove to be useful. When introducing additional variables, the size of the sample shall be adequate to ensure a significant conclusion is drawn, therefore it would be advisable to enlarge the sample.

It would also be important to improve the logistics of the study in order to be able to divide each class in the two necessary subgroups. This would further improve the quality of the sample, leading possibly to a higher statistical significance of the conclusions.

Last, but not least, the introduction of a full section of lectures instead than a single one shall be considered. Indeed a single lecture, even if interesting and well perceived, proved not to be really effective in training students.

All this would require a structured project, with the involvement of different schools and teachers and adequate funding.

With a similar structure, it would also be possible to expand the study scope: Jacobson and Wilensky (2005) stated two important ideas that I intended to take into practice:

> Whereas empirical demonstrations of far transfer would be of great near-term importance to the learning sciences research community (see Goldstone and Wilensky. 2005) with important long-term applied implications, it will also be important to explore the potential value of learning complex systems perspectives to enhance students' understanding of "traditional" content in the physical and social sciences (e.g., thermodynamics, evolution, homeostasis, feedback) as current content standards and assessments are based on these areas.[119]

[119] Jacobson, Michael J. and Wilensky Uri. Complex systems in education: Scientific and educational importance and implications for the learning sciences. *The Journal of the learning science* 15.1 (2006): p. 23.

A new study, performed with the criteria described above, would also fit the needs to investigate the possibility for the students to use complexity knowledge to acquire more familiarity with and increase the knowledge of “normal” topics.

Conclusions

Normally, when designing a PhD project, one tries to find a part of a small specific topic with the idea to enhance the general knowledge of a certain subtopic with his/her work. My Ph.D., on the other hand, was intrinsically complex and multidisciplinary, because the main object of my research was to find hints of the possible change of paradigm in high school teaching towards a more interconnected, multidisciplinary and complex knowledge. During this journey, I tried to acquire familiarity with the specific tools of the disciplines related with the work and also with that really interesting and powerful instrument that is Psychometrics. I encountered a number of difficulties in dealing with Psychology, Philosophy, Epistemology, History of Science, Computer science and other topics not specifically related to my specialization as a Mathematics and Physics high school teacher. In the process, however, I also tried not to forget to find what might be a small specific main goal within the project through which I could enhance general knowledge. I believe that, even if the lecture did not seem to have any measurable effect on students, the examples and strategies I found could be an initial step in the possibility of bringing fascinating and rich concepts like complexity, emergence and the complex problem solving into the minds of the new generation. While it is true that a fully correct answer is available to students working in the Genetics Lab with the psychometric analysis software, in real life these problems are open to a varieties of solutions that need to be customized to the user's needs (see for example the Underground Game of the trip to the mountains problem).

I tried to present complex, various and apparently non-related topics in order to try to revise, or rather, to integrate the paradigm of teaching with separate and independent subjects, through examples of multidisciplinary and multifaceted problems that cannot be solved while remaining within the realm of a single discipline.

The change in paradigm was not only technical but also methodological, as I mentioned when, at the beginning of Chapter 2, I explained why using examples of different applications of complexity theory might be useful to help building the concept itself in the student's mind.

The main problem that came out from the measurements is that even a lecture deigned to be an instruction manual seemed not to be sufficient to foster the students' motivation to trying something new.

What is needed is, maybe, an entire set of lectures and experiments where students can apply their self-constructing knowledge.

The ability to disclose and apply the familiarity acquired in one field to another, or even more than one, remains a difficult task to complete, even nowadays, when, more than ever, the economy of knowledge rules and fast and flexible learning defines the needs for future studies.

On the other hand, I have been in many schools and have met around 26 different classes, and almost everyone has shown great interest in the topics of my study and expressed their need to know more about them. Maybe, as many teachers and students have asked to be involved, appropriate funding would make spreading this idea to Italian schools possible.

In addition, the Genetics Lab was very well accepted among the Italian and the international schools involved, as the 2012 study already proved in Luxembourg.

Indeed, one of the GL project future goals is it to become a training tool effectively available to teachers.

All the teachers I have met have expressed the need to create more usable interdisciplinary tools like the hints topics I gave in my lecture or the GL.

The High School World needs new methods and topics to drive students through difficult and fascinating times when, more than ever, the effort going into creating abilities, competences and knowledge is a complex process involving constant interaction between the teacher and the student, the class and the outside world.

Bibliography

Papers

Amthauer, R., Brocke, B., Liepmann, D. and Beauducel, A. IntelligenzStruktur-Test 2000 R [Intelligence Structure Test 2000 R]. Göttingen, Germany: Hogrefe, 2001.

Barrett, P. Brunswick. Symmetry, Complexity, and Non-Quantitative Psychology, NZ Psychology Conference – Hamilton, (2007).

Beach, K. (1999). Consequential transitions: A sociocultural expedition beyond transfer in education. Review of research in education, 101-139.

Bekebrede, G., Warmelink, H. J. G., & Mayer, I. S. (2011). Reviewing the need for gaming in education to accommodate the net generation. Computers & Education, 57(2), 1521-1529.

Bjarnason, Ronald, Fern, Alan and Tadepalli, Prasad. Lower Bounding Klondike Solitaire with Monte-Carlo Planning. International Conference on Automated Planning and Scheduling (ICAPS-2009).

Bjarnason, Ronald, Tadepalli, Prasad and Alan Fern. Searching Solitaire in Real Time. *International Computer Games Association Journal*, 30(3) (2007): 131-142.

Bonotto Cinzia, The Erlangen program revisited: a didactic perspective, *For the learning of Mathematics* 27 (1) (2007).

Bransford, J.D. and Schwartz, D.L. Rethinking Transfer: A simple proposal with multiple implications. *Review of Research in Education* 24 (1999): 61–100.

Brown, J. S., Collins, A. and Duguid, P. Situated cognition and the culture of learning. *Educational Researcher* 18 (1989) 32--42.

Cañas, J. J., Antolí, A., Fajardo, I., and Salmerón, L. Cognitive inflexibility and the development and use of strategies for solving complex dynamic problems: Effects of different types of training. *Theoretical Issues in Ergonomics Science*, 6(2005): 95-108.

Carroll, J. B. "The three-stratum theory of cognitive abilities" in D. P. Flanagan, J. L. Genshaft et al., *Contemporary intellectual assessment: Theories, tests, and issues*, New York : Guilford Press, 1997, pp. 122--130.

Chaitin, G.J. (1974), "Information- theoretic Computational Complexity", *IEEE Trans. Inf. Theor.*, IT-20, p. 16.

Cheng-Yao Lin, Teaching Multiplication Algorithms from Other Cultures. In: *Teaching Mathematics in the middle school* 13 (5) (December 2007): 298.

Cohen, P., Cohen, J., Aiken, L. S., and West, S. G. The problem of units and the circumstance for POMP. *Multivariate Behavioral Research* 34 (1999): 315–346.

Danner, D., Hagemann, D., Schankin, A., Hager, M. and Funke, J., Beyond IQ: A latent state-trait analysis of general intelligence, dynamic decision-making, and implicit learning, *Intelligence* 39 (2011): 323–334.

Delahaye, J.-P. and Zenil, H. *Towards a stable definition of Kolmogorov-Chaitin complexity*, arXiv:0804.3459, (2007).

Donald E. Broadbent, Levels, hierarchies, and the locus of control, Quarterly *Journal of Experimental Psychology* 29 (2)(1977): 181-201.

Dörner D., Kreuzig, H.W. Problemlose Fähigkeit und Intelligenz. *Psychologische Rundschau* 34(1983): 185–192.

Dörner, D. and Schaub, H. Errors in planning and decision-making and the nature of human information processing. *Appl. Psychol. Int. Rev*. 43(1994): 433–453.

Dörner, D. and Wearing, A.J. Complex problem solving: toward a (computer simulated) theory. In: Frensch, P.A. and Funke, J. (eds) *Complex problem solving: the European perspective*. Hillsdale, NY: Lawrence Erlbaum, 1995, pp. 65–99.

Dörner, D. Diagnostik der operativen Intelligenz. *Diagnostica* 32(1986): 290–308.

Dörner, D. On the difficulty people have in dealing with complexity. *Simul Games* 11(1980): 87–106.

Dörner, D. *The logic of failure. Recognizing and avoiding error in complex situations*. Reading , MA: Addison-Wesley, 1996.

Duit Reinders, Komorek Michael, Wilbers Jens, Studies on Educational Reconstruction of Chaos Theory, *Research in Science Education* 27(3) (1997): 339-357

Fischer, Andreas; Greiff, Samuel; and Funke, Joachim, The Process of Solving Complex Problems, *The Journal of Problem Solving* 4(1) (2012).

Funke J. Dynamic systems as tools for analysing human judgement. *Think Reason* 7(2001): 69–89.

Funke, J. and Frensch, P.A. Complex problem solving: the European perspective—10 years after. In Jonassen, D.H. (ed.) *Learning to solve complex scientific problems*. Hillsdale, NY: Lawrence Erlbaum, 2007, pp. 25–47.

Funke, J. Complex problem solving: A case for complex cognition? *Cognitive Processing* 11(2010): 133–14.

Funke, J. Experimental research on complex problem solving. In Frensch, P.A. and Funke, J. (eds) *Complex problem solving: the European perspective*. Hillsdale, NY: Lawrence Erlbaum, 1995, pp. 243–268.

Funke, J. *Problemlösendes Denken* [Problem Solving Thinking]. Stuttgart: Kohlhammer, 2003.

Gardner, M., Mathematical Games – The fantastic combinations of John Conway's new solitaire game "life". *Scientific American* 223 (October 1970): 120–123.

Ginsberg. R. Complexity Science at the Schoolhouse Gate? *Complexity* 2 (4)(1997): 9-13.

Goldstone, R. L. (2006). The complex systems see-change in education. The Journal of the Learning Sciences, 15(1), 35-43.

Goode, N., & Beckmann, J. F. (2010). You need to know: There is a causal relationship between structural knowledge and control performance in complex problem solving tasks. Intelligence, 38(3), 345-352.

Greiff, S. and Funke, J. Measuring complex problem solving: the MicroDYN approach. In: Scheuermann, F. (ed.) *The Transition to computer-based assessment—lessons learned from large-scale surveys and implications for testing*. Luxembourg : Office for Official Publications of the European Communities, 2009.

Greiff, S. and Wüstenberg, S. Assessment with Microworlds using MicroDYN: Measurement invariance and latent mean comparisons: Psychometric properties across several student samples and blue-collar workers*. European Journal of Psychological Assessment* 30 (4)(2014): 304-314.

Greiff, S., Fischer, A., Wüstenberg, S., Sonnleitner, P., Brunner, M. and Martin, R. A multitrait–multimethod study of assessment instruments for complex problem solving, *Intelligence* 41 (5) (2013): 579-596.

Greiff, S., Wustenberg, S., Holt, D.V., Goldhammer, F. and Funke, J. Computer-Based Assessment of Complex Problem Solving: Concept, Implementation, and Application. *Educational Technology Research and Development* 61(3) (2013): 407-421.

Gustafsson, J. E. and Aberg-Bengtsson, L. Unidimensionality and interpretability of psychological instruments. In S. E. Embretson (ed.) *Measuring Psychological Constructs*. Washington, DC: American Psychological Association, 2010, pp. 97–121.

Haskell, R.E. *Transfer of Learning. Cognition, Instruction, and Reasoning*. New York: Academic Press, 2011.

Jacobson, M. J. (2001). Problem solving, cognition, and complex systems: Differences between experts and novices. Complexity, 6(3), 41-49.

Jacobson, M. J., & Wilensky, U. (2006). Complex systems in education: Scientific and educational importance and implications for the learning sciences. The Journal of the learning sciences, 15(1), 11-34.

Kluge, A., What you train is what you get? Task requirements and training methods in complex problem-solving, *Computers in Human Behavior archive* 24(2) (2008): 284-308.

Kröner, S., Plass, J. L., & Leutner, D. (2005). Intelligence assessment with computer simulations. Intelligence, 33(4), 347-368.

Kuhn, D. (2009). Do students need to be taught how to reason?. Educational Research Review, 4(1), 1-6.

Langley, P. and Rogers, S. An Extended Theory of Human Problem Solving. Proceedings of the 27th Annual Meeting of the Cognitive Science Society. Stresa, Italy, 2005.

Lave J., Smith S. and Butler M., Problem Solving as an Everyday Practice in The Teaching and Assessing of Mathematical Problem Solving. Research Agenda for Series. Volume 3. Charles, Randall I., Silver, Edward A., eds. Reston, VA: National Council of Teachers of Mathematics, 1988.

Lave, J. (1996). Teaching, as learning, in practice. Mind, culture, and activity, 3(3), 149-164.

Lesh, Richard. "Modeling students modeling abilities: The teaching and learning of complex systems in education." *The Journal of the Learning Sciences* 15 (1) (2006): 45--52.

Leutner, D. The fuzzy relationship of intelligence and problem solving in computer simulations. *Computers in Human Behavior*, 18 (2002): 685-697.

Livingston, Eric J. *The Ethnomethodological Foundations of Mathematics (Studies in Ethnomethodology)*, first edn, London: Routledge Kegan & Paul, 1986.

Lynch, Michael, *Scientific practice and ordinary action: Ethnomethodology and social studies of science*, Cambridge: Cambridge University. Press, 1997.

Maani, K. E., & Maharaj, V. (2001, July). Systemic Thinking and Complex Problem Solving. A Theory Building Empirical Study. In Proceedings of the 19 th International Conference of the System Dynamics Society 2001 (p. 101).

May, C. J., Burgard, M., & Abbasi, I. (2011). Teaching complexity theory through student construction of a course wiki: The self-organization of a scale-free network. Complexity, 16(3), 41-48.

Maye,r H., Hazotte, C., Djaghloul, Y., Latour, T., Sonnleitner, P. and Brunner, M. Using Complex Problem Solving Simulations for General Cognitive Ability Assessment: the Genetics Lab Framework, *International Journal of Information Science and Intelligent System* 2 (4) (2013): 71-88.

Portha Stéphane, Deneubourg Jean-Louis, Detrain Claire, Self-organized asymmetries in ant foraging: a functional response to food type and colony needs, *Behavioral Ecology* 13 (6) (2002): 776-781.

Portugal Steven J., Hubel Tatjana Y. *et al.,* Upwash exploitation and downwash avoidance by flap phasing in ibis formation flight. *Nature* 505 (16) (January 2014): 399–402.

Quesada, J., Kintsch, W., and Gomez, E. Complex Problem Solving: A field in search of a definition? *Theoretical Issues in Ergonomic Science* 6(1) (2005): 5–33.

Reiss, Kristina and Törner, Günter. Problem solving in the mathematics classroom: the German perspective ZDM, *Mathematics Education* 39 (5) (October 2007): 431-441.

Sabelli Nora H., Complexity, Technology, Science, and Education, Journal of the Learning Sciences 15(1)(2006): 5-9.

Schweizer F., Wüstenberg F., Greiff S. Validity of the MicroDYN approach: Complex problem solving predicts school grades beyond working memory capacity. *Learning and Individual Differences* 24 (2013): 42–52.

Sonnleitner, P. The Potential of Complex Problem Solving Scenarios to Assess Students' Cognitive Abilities, 2015, Doctoral Dissertation Freie Universität Berlin.

Sonnleitner, P., Brunner, M., Greiff, S., Funke, J., Keller, U., Martin, R. and Hazotte, C. The Genetics Lab. Acceptance and psychometric characteristics of a computer-based microworld to assess Complex Problem Solving. *Psychological Test and Assessment Modeling* 54 (2012): 54-72.

Sonnleitner, P., Brunner, M., Keller, U. and Martin, R. Differential relations between facets of complex problem solving and students' immigration background. *Journal of Educational Psychology* 106 (3) (2014): 681.

Sonnleitner, P., Keller, U., Martin, R. and Brunner, M. Students' complex problem-solving abilities: Their structure and relations to reasoning ability and educational success. *Intelligence* 41 (5) (2013): 289-305.

Stanley, D. (2009). Complex responsive processes: An alternative interpretation of knowledge, knowing, and understanding. Complicity: An International Journal of Complexity and Education, 6(2), 29-39.

Stanley, D. (2009). Complex responsive processes: An alternative interpretation of knowledge, knowing, and understanding. Complicity: An International Journal of Complexity and Education, 6(2), 29-39

Stanley, D. (2009). What complexity science tells us about teaching and learning. What Works? Research into Practice

Van Merriënboer, J.J.G., Perspectives on problem solving and instruction. *Computers & Education* 64 (2013): 153–160.

von Neumann, J. "Method in the physical sciences," in John von Neumann *Collected Works*. New York: Macmillan, 6, 35(1963).

von Neumann, J. and O. Morgenstern, *Theory of Games and Economic Behavior*. Princeton, NJ: Princeton University Press, 1944 (1st edition); 2007 60th anniversary.

von Neumann, J. *Theory of Self-Reproducing Automata*. (Edited and completed by A. W. Burks) Urbana: University of Illinois Press, 1966.

Wenke, D., Frensch, P. A., and Funke, J. Complex problem solving and intelligence—Empirical relation and causal direction. In R. J. Sternberg, and J. Pretz (eds), *Cognition and intelligence: Identifying the mechanics of the mind*. Cambridge: Cambridge University Press, 2005, pp. 160–187.

Wüstenberg, S., Greiff, S., & Funke, J. (2012). Complex problem solving—More than reasoning? Intelligence, 40(1), 1-14.

Yan, X., Diaconis, P., Rusmevichientong, P., and Van Roy, B. Solitaire: Man Versus Machine. *Advances in Neural Information Processing Systems* 17 (2005): 1553–1560.

Books

Applegate, D. L.; Bixby, R. M.; Chvátal, V. and Cook, W. J. The Traveling Salesman Problem: A Computational Study. Princeton, NJ: Princeton University Press, 2006.

Arons Arnold B, Guida all'insegnamento della fisica, Bologna: Zanichelli, 1992

Bateson, G., Mind and Nature: A Necessary Unity (Advances in Systems Theory, Complexity, and the Human Sciences), New York: E. P. Dutton, 1979.

Berveiler, David. *Strategic Solitaire*. McFarland & Company Incorporated Pub, 1987.

Bocchi G. and Ceruti M. (eds), *La sfida della complessità*, Milano: Bruno Mondadori, 2007.

Boero, Paolo. *Theorems in school: from history, epistemology and cognition to classroom practice*, Rotterdam, the Netherlands: Sense Publishers, 2007.

Boyer, Carl Benjamin, *Storia della matematica*, Milano: Mondadori, 1990.

Bruner, Jerome S. *Toward a Theory of Instruction*, Harvard, MA: Belknap-Harvard University Press, 1966. Italian Edition: Milano: Armando editore 1967.

Carroll, J. B. *Human cognitive abilities: A survey of factor-analytic studies*, Cambridge, Cambridge University Press, 1993.

Devlin, Keith, *The Language of Mathematics: Making the Invisible Visible*, New York: Henry Holt and Company, 2000.

Dowker Ann, *Individual Differences in Arithmetical Abilities Psychology Press*, 2005.

Euclide, *Tutte le opere*. Testo greco a fronte a cura di Acerbi F. Milano: Bompiani, 2007.

Frensch, P.A. and Funke, J. (eds). Complex problem solving: The European perspective. Hillsdale, NY: Lawrence Erlbaum Associates, 1995.

Gembillo, Giuseppe, Scienziati e nuove immagini del mondo: A. Einstein, W. K. Heisenberg, M. Planck, N. Bohr, J. Monod, I. Prigogine, Settimo Milanese, Marzorati, 1994.

Giordano, Giuseppe, Da Einstein a Morin. Filosofia e scienza tra due paradigmi. Soveria Mannelli, Rubbettino, 2006.

Giordano, Giuseppe, Storie di concetti: fatti, teorie, metodo, scienza, Firenze, Le lettere, 2012.

Grasseni C., Ronzon F., *Pratiche e cognizione. Note di ecologia della cultura*, Roma: Meltemi, 2004.

Harmon Courage, Katherine, *Octopus!: The Most Mysterious Creature in the Sea*, London: Penguin, 2013.

Heath Thomas, *A History of Greek Mathematics, Volume II: From Aristarchus to Diophantus*. 1st edition. Oxford: Clarendon Press 1921.

Holland, John H. *Complexity: A Very Short Introduction,* Oxford: Oxford University Press, 2014.

Hunt, E. *Human intelligence*. New York: Cambridge University Press, 2011.

Jensen, A. R. *The g factor: The science of mental ability*. London: Praeger, 1998.

Kline, Morris, *La matematica nella cultura occidentale*, Milano: Feltrinelli, 1976.

Lave Jean , *Cognition in Practice: Mind, Mathematics and Culture in Everyday Life*, Cambridge: Cambridge University Press, 1988.

Lave Jean, *Apprenticeship in Critical Ethnographic Practice* , Chicago, IL: University of Chicago Press, 2011.

Lave, J., Wenger, E., *Situated Learning, Legitimate Peripheral Participation*, Cambridge: Cambridge University Press, 1991

Li, Ming and Vitányi Paul, *An Introduction to Kolmogorov Complexity and Its Applications*, Berlin: Springer, 1997.

Livingston, Eric J. The Ethnomethodological Foundations of Mathematics (Studies in Ethnomethodology), first edn, London: Routledge Kegan & Paul, 1986.

Morin E., *La Méthode.* I*. La Nature de la Nature*, Seuil, Paris, 1977 (trad. it. parz. Il *Metodo. Ordine, disordine, organizzazione*, Feltrinelli, Milano 1985).

Morin, Edgar. *I sette saperi necessari all'educazione del futuro*, Milano: Raffaello Cortina, 2001.

Morin, Edgar. *La testa ben fatta. Riforma dell'insegnamento e riforma del pensiero*, Milano: Raffaello Cortina Editore, 2000.

Neumann von, J. The Computer and the Brain. New Haven, CT: Yale University Press, 1958.

Newell, A. and Simon, H.A. *Human problem solving*. Englewood Cliffs, NJ: Prentice-Hall, 1972.

Nunes, Schliemann, and Carraher*, Street Mathematics and School Mathematics*,: Cambridge Cambridge University Press, 1993.

Pearl, J., *Heuristics: Intelligent Search Strategies for Computer Problem Solving*, Reading, MA: Addison-Wesley, 1984.

Pólya, George. *How to Solve It*. Princeton, NJ: Princeton University Press, 1945.

Poundstone, William, *Prisoner's Dilemma: John von Neumann, Game Theory, and the Puzzle of the Bomb*. 1992.

Prigogine Ilya and Stengers Isabelle, *La nuova alleanza. Metamorfosi della scienza*, , Torino: Einaudi1999.

Probst, Gilbert J.B., *Selbstorganisation - Ordnungsprozesse in sozialen Systemen aus ganzheitlicher Sicht*, Berlin and Hamburg: Verlag Paul Parey, 1987.

Ripley, Amanda. *The Smartest Kids in the World: And How They Got That Way*, New York: Simon & Schuster, 2013

Sternberg, Robert J. and Frensch, Peter A. (eds*), Complex Problem Solving: Principles and Mechanisms*, Hove: Psychology Press, 1991.

Suchman Lucy, *Human-Machine Reconfigurations: Plans and Situated Actions*, Cambridge: Cambridge University Press, 2007.

Annex 1 Possible solutions of the trip to the mountains problem

The problem, as it was presented to me, consisted of the following data:

- 1 week stay at the price of 1000€;
- 9 people involved;
- only two people remaining the entire week;
- 4 people arriving 2 days later;
- 3 people leaving 3 days earlier.

Therefore, the problem can be sketched as follows[120]:

[120] Designed by Giulio Donini from a partial available draft in www.Freepik.com

Or in a less graphic and more practical way:

Day	1	2	3	4	5	6	7
people	5	5	9	9	6	6	6

The solution the students initially proposed was to divide the amount of 1000€ in seven parts, so each would amount to 1000€ : 7= 142,86€, and this amount should be divided by the number of people staying that day.

With this data, the table becomes:

Day	1	2	3	4	5	6	7
People sleeping	5	5	9	9	6	6	6
Cost per day	142,86€	142,86€	142,86€	142,86€	142,86€	142,86€	142,86€
Calculation	142,86€:5	142,86€:5	142,86€:9	142,86€:9	142,86€:6	142,86€:6	142,8€:6
Cost per person per day	28,57€	28,57€	15,87€	15,87€	23,81€	23,81€	23,81€

and each person must simply multiply the amount due for the days he/she sleeps in the house; for example, Catherine is staying day 1, 2, 3, 4, so she pays 28,57€ + 28,57€ + 15,87€ + 15,87€ = 88,88€, and Paul stays day 3, 4, 5, 6, 7 and he pays €15,87€ + 15,87€ + 23,81€ + 23,81€ + 23,81€ = 103,17€. Adam, who sleeps the whole week, pays 160,32€.

This system also works with situations that are more complex, as when someone is staying days 2, 3, 4, 5 though in this case the names of exactly who is staying or leaving must be known.

By doing it this way, the solution is mathematically correct but, as reported in the introduction, the students said it is not fair. The two people staying the whole week are also paying for the others. Obviously, when dealing with a real life problem, the absolute and relative value of the money must also be taken into consideration i.e. if a difference of 10 or 20 Euros had been considered irrelevant by the participants, they might not have raised the issue of fairness. Instead, this 10 or 20 Euros difference marks the point of affordable/non-affordable for some of them, and is therefore of primary importance.

Obviously, I am using these amounts to “facilitate” the calculations, but the problem can be represented by using any amount of money, and not only accommodation fees and money, but also any shared value that can be put in numbers, even pollution share rights. Unfortunately, in this case the matter is much more complex.

I will now present the three alternative solutions I have offered my students. They have chosen the first one: nevertheless, many other options are possible, including the easiest one, which is not taking the trip. This would mean a zero cost in money but a high social cost, because one would lose a very good occasion to party.

The first solution is to establish a one-off fee to be paid in equal parts, regardless of the amount of days each person sleeps in the house. This way, everyone pays the cost of the owner’s right not to rent the house for less than one week in equal parts. Subsequently, the remaining amount will be divided using the first option, but with less money.

In this model, if we set 200 € as the one-off fee, then the table becomes:

Day	1	2	3	4	5	6	7
People sleeping	5	5	9	9	6	6	6
One time fee per person	200€:9= 22,2€	200€:9= 22,2€	200€:9= 22,2€	200€:9= 22,2€	200€:9= 22,2€	200€:9= 22,2€	200€:9= 22,2€
Cost per day	800€:7 = 114,29€	800€:7= 114,29€	800€:7 = 114,29€	800€:7 = 114,29€	800€:7 = 114,29€	800€:7 = 114,29€	800€:7 = 114,29€
Calculation	114,29€:5	114,29€:5	114,29€:9	114,29€:9	114,29€:6	114,29€:6	114,29€:6
Cost per person per day	22,86€	22,86€	12,70€	12,70€	19,05€	19,05€	19,05€

Obviously, everyone must agree on what is a fair amount for a one-time fee.

Now Catherine, staying day 1, 2, 3, 4, pays 22,22€ + 22,86€ + 22,86€ + 12,70€ + 12,70€ = 93,34€ and Paul who stays day 3, 4, 5, 6, 7 pays 22,22€ + 12,70€ + 12,70€ + 19,05€ + 19,05€ = 104,77€. Instead, Adam pays 150,49€ for his whole week.

This way, the longer you stay, the closer you pay to the amount of the first model ; vice versa, the shorter you stay the more you pay, always compared to the first model. So Catherine is paying around 4,46 Euros more while Paul around 1,60 Euros. The advantage in this model is that it moves money towards those who stay longer or, in other words, it compensates them for the "inconvenience" of staying longer.

The third option is the one I prefer, because it is easier from my point of view, even if it is somehow slightly less fair than option 2 in terms of money. The solution is somehow related to the one widely used in the economy concept of man-hour -man-day or, in more gender-neutral terms, effort-hour, effort-day.

Man-hour refers to one person working for one hour. For example, two people working on a 10 man-hour task will finish in 5 hours. A man-day refers to one person working for one day. For example, two people working on a 14 man-day task will finish in 7 days.

Obviously, this does not take into account all the variables, but it is nevertheless used in many projects to provide a rough estimate of the amount of work necessary for a specific task.

For the *Trip to the mountains* problem, we can use this framework to model the situation: in order for the problem to become fairer, in this application we need to move to the concept of people staying each day.

Suppose we indicate the problem this way: instead of having 9 people sharing a house for seven days, if they use it in different days we create a new variable. If we sum the numbers of people in each day, it becomes:

Day	1	2	3	4	5	6	7	single day
	5	5	9	9	6	6	6	
people	5 + 5 + 9 + 9 + 6 + 6 + 6 = 46							46 people

Therefore, we imagine having 46 people sleeping in the house for one day.

So each person's contribution for that hypothetical day would be 1000€ : 46= 21,74€. Because you can represent 1 day-person also as 1 person-day (like man-hour can become hour-man), you can say that 46 people sleeping in one day can also be like 1 person sleeping 46 days. In this framework, Catherine sleeps in days 1, 2, 3, 4, therefore she pays 21,74€ x 4 = 86,96€; while Paul pays 21,74€ x 5 = 108,7€. Adam pays 21,74€ x 6= 152,18€.

Finally with the three models:

	Model 1	Model 2	Model 3
Catherine	88,88€	93,34€	86,96
Paul	103,17€.	104,77€	108,7
Adam	160,32	150,49€	152,18.

As you can see, in the third model Adam pays a little bit more compared to model 2 and a lot less than in model 1, while Paul is the more penalized in choosing model 3.

The difference among these amounts of money does not seem very big (to an adult, as to a teenager it very well might), but if you think the real rent may be 2000 or 3000 Euros, then the amounts become more significant. When discussing this problem, the students only agreed on the fact that renting a house for New Year's Eve is very expensive, and that only if you like Mathematics you may have access to so many different solutions to the problem. The debate was very interesting.

Annex 2 What time is it in the North Pole?

This might seem an odd question, nevertheless it is somehow a complex problem because if you find yourself exactly at the North or South Pole, you could simply turn around the axis and go through every time zone of the world.

The problem seems to be less relevant in the case of the North Pole, where floating ice blocks invalidate any point of reference: for example, a weather station would constantly be drifting away from any set point of reference, therefore becoming useless. The problem becomes even more complex as neither the Arctic nor the Antarctic (the latter being a continent while the first is not, technically) belong to any single nation entitled to choose a time zone for itself. Therefore, a simple answer to the question would be; "It's the time anyone brings along with them".

In other words, research bases in Antarctica usually pick they own country time zone. There is also a debate on whether it would be easier to pick up UTC time for the Arctic zone, and therefore UTC +12 hours for the Antarctic zone. UTC is Coordinated Universal Time, the basis for civil time today: this means that no country or territory officially uses UTC as local time. Even if it is practically the same time as GMT, no country is able to claim it. GMT is no longer the universal time standard, but since the seventies it has become “just” a time zone officially used in some European and African countries like the UK, Portugal and Morocco. A number of countries, though, are not always under their time zones: for example, during daylight saving time, the UK moves to BST (British Summer Time) which is one hour later then GMT.

Annex 3 Counting the hours

Ragazzi leggete il brano riportato e poi rispondete alle domande

Confrontare le ore

"Nell'Ottocento, quando il telegrafo già consentiva di scambiare messaggi quasi in tempo reale (e pochi anni dopo la radio avrebbe accelerato i tempi) e i treni viaggiavano a velocità mai raggiunte dall'uomo (e pochi anni dopo gli aerei avrebbero fatto di meglio), emerse il problema di fare i conti con la varietà delle ore locali. Nel 1884 un accordo internazionale stabilì, per prima cosa, di immaginare la superficie della Terra divisa in 24 spicchi ampi ciascuno 15°, cioè un'ora, chiamati fusi orari. Si stabilì, poi, che in tutti i punti compresi entro ciascun fuso orario l'ora 'esatta' fosse quella media, cioè quella del meridiano centrale del fuso: si calcola quando, rispetto al Sole, è mezzogiorno al centro del fuso e si decide che in quel momento è mezzogiorno in tutto il fuso (anche se non è vero: a un'estremità del fuso il mezzogiorno 'locale' è passato da mezz'ora, all'altra estremità arriverà fra mezz'ora). Si stabilì, inoltre, di contare le ore partendo da un meridiano iniziale, convenzionalmente identificato con quello passante per Greenwich (nelle vicinanze di Londra, sede di un celebre osservatorio astronomico): quando sul meridiano di Greenwich (0°) è mezzogiorno, nel fuso subito a est saranno le 13, due fusi più a est le 14 e così via; nei fusi a ovest invece mancherà ancora del tempo a mezzogiorno. I fusi orari, rispetto all'ora di Greenwich, vengono indicati, con il + se a est e con il − se a ovest.

Conoscendo la longitudine di un luogo, si sa a quale fuso appartiene e qual è la differenza di ora rispetto a Greenwich. Una corrispondenza così esatta, però, è vera solo in mare. Infatti, dato che i confini politici terrestri non coincidono con i limiti dei fusi, gli Stati hanno dovuto scegliere l'ora valida per il proprio territorio. L'Italia, che rientra in gran parte nel fuso detto dell'Europa centrale (+1), decise di adottare quell'ora in tutto il paese, anche se tra l'ora locale del Piemonte e quella della Puglia in realtà c'è quasi un'ora di differenza. I paesi molto estesi (come Russia, Stati Uniti, Canada) hanno diviso il loro territorio in più fusi."[121]

Quindi quando è mezzogiorno a Greenwich sono le 13.00 a Roma ed in tutta Italia; quando sono le 13.00 a Londra, in Italia sono le 14.00 e così via.

Consideriamo due aerei, un Boeing 777/200 della Emirates ed un Airbus A340/200 della Qantas, che partono entrambi da Roma in direzione Sydney. L'orario riporta la seguente tabella:

[121] http://www.treccani.it/enciclopedia/fusi-orari_(Enciclopedia-dei-ragazzi)/ di Katia Di Tommaso.

 Km	 Città	EMIRATES	QANTAS
0	RM Fiumicino	22.25	22.45
14.000	SINGAPORE	Arrivo: 19. 25 l. t. Partenza: 21.25 l.t.	Arrivo: 19.55 l. t. Partenza: 23.00 l.t.
21.000	Sydney	7.25* l.t.	9.20* l.t.

* giorno successivo

- l.t. vuol dire *local time* ora della città di arrivo, tutti gli orari sono sempre riferiti all'ora locale
- i fusi orari[122] sono i seguenti : Roma, GMT +1, Singapore GMT +8, Sydney GMT +10

Quanto dura il volo fra Roma e Singapore per i due aerei?

____

____

E fra Singapore e Sydney?

____

____

Qual è la velocità media dei due aerei sulla tratta fra Roma e Singapore?

____

E fra Singapore e Sydney?

____

Le persone che partono hanno delle mamme molto apprensive. Le mamme che rimangono a Roma, a che ora devono aspettarsi di essere chiamate al primo scalo? E a che ora potranno essere chiamate da Sydney?

____

[122] http://24timezones.com/it