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Managing Risk With Simulated Copula

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Abstract

The aim of this study is verify whether the Average Value at Risk (AVaR) can be a good alternative to Value at Risk (VaR), for estimating great portfolio losses, especially regarding tail events. To do so we use copula framework to estimate dependence between stock returns of a portfolio composed by 94 stock of the S&P100 in order to compute AVaR and VaR and compare the results with respect to a Gaussian Exponentially Weighted Moving Average (EWMA). For computing simulated returns, we use the algorithm presented in [Biglova et all, 2014] and then the model is back-tested with Kupiec's and Christoffersen's tests. The results are coherent with the literature and in particular VaR computed both via copula and EWMA seems to fail to provide an accurate risk measurement while AVaR under copula and EWMA looks more reliable.

Key words

Value at Risk, Average Value at Risk, Copula, EWMA model.

JEL Classification: G17, C15, G12

1. Introduction

The aim of this study is verify whether the Average Value at Risk (AVaR) can be a good alternative to Value at Risk (VaR) for estimating great portfolio losses, especially regarding tail events. To do so we consider the stocks belonging to Standard & Poor's 100. The interval time chosen covers the last 10 years recorded with 2591 daily price observations for each stock from the 1st December 2005 to the 18th March 2016. Since stocks belonging to the index can be substituted by other stocks along time, the sample used is reduced to those which are present in all 10-year period considered: Thus the number of remaining stocks is 94 respect to the original 100.

In portfolio risk management, we have to combine some well known stylized facts: typical features of return series (as asymmetry fat tails and volatility clusters) with asymmetry in dependence structure (lower tail dependence stronger than upper tail dependence) and high dimensionality (see [Cont, 2000] [Papp et al., 2005] [Kondor et al., 2007] [Hong et al. 2007]).

According to [Cont and Tankov, 2003] [Schoutens, 2007], we deal excess kurtosis and skewness using different distributions from the Gaussian law. As for [Cherubini et al., 2003], we examine asymmetry in dependence structure proposing an appropriate copula. Moreover, considering that high dimensionality in portfolio problems might affect the unbiasedness of risk estimators (see [Rachev et al., 2005] [Sun et al., 2008]) and that estimating the impact of tail event requires a high number of observations (see [Papp et al., 2005] [Kondor et al., 2007]), then we model risk of portfolios using proper simulated multivariate scenarios. Finally, we compare the ex post results with those obtained by using the classical RiskMetrics methodology (EWMA model).

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2. The framework

Following [Rachev et al.,2007]. [Biglova et al., 2008] and [Biglova et al., 20014] we tackle all the observed return features by computing VaR and AVaR (often called Conditional Value at Risk - CVaR) on realistic simulated scenarios. The general idea is to assume that each log-return $r_{j,t}$,follows a ARMA(1,1)-GARC(1,1):

$$\boldsymbol{r_{j,t} = a_{j,0} + a_{j,1} r_{j,t-1} + b_{j,1}\varepsilon_{j,t-1} + \varepsilon_{j,t}}$$

$$\varepsilon_{j,t} = \sigma_{j,t} u_{j,t} \tag{1}$$

$$\sigma_{j,t}^2 = c_{j,0} + c_{j,1}\sigma_{j,t-1}^2 + f_{j,1}\varepsilon_{j,t-1}^2$$

and a skewed t-copula with 5 degrees of freedom for the dependence structure, defined as:

$$\boldsymbol{X = \mu + \gamma g(W) + \sqrt{W} Z} \tag{2}$$

with g:[0, ∞)→:[0, ∞),in this work $g(W)=W$, γ vector of skewness parameters, Z~N(0,Σ) with $\Sigma = [\sigma_{ij}]$ and W~Ig(d/2,d/2). The density function the X vector is

$$f(x) = c\frac{K_{\frac{d+n}{2}}\left(\sqrt{\left(d + (x-\mu)'\Sigma^{-1}(x-\mu)\right)\gamma'^{\Sigma^{-1}}\gamma}\right)\exp((x-\mu)'\Sigma^{-1}\gamma)}{\left(\sqrt{(d + (x-\mu)'\Sigma^{-1}(x-\mu))\gamma'\Sigma^{-1}\gamma}\right)^{-\frac{d+n}{2}}\left(1 + \frac{(x-\mu)'\Sigma^{-1}(x-\mu)}{d}\right)^{\frac{d+n}{n}}}$$

with the normalizing parameter: $c = \frac{2^{\frac{2-(d+n)}{2}}}{\Gamma\left(\frac{d}{2}\right)(\pi d)^{\frac{n}{2}}|\Sigma|^{\frac{1}{2}}}$ where K_λ is a Bessel function of the third kind which is a function that solves the differential equation [Abramowitz and Stegun, 1965] [Demarta and McNeil, 2004]:

$$z^2\frac{d^2y}{dz^2} + z\frac{dy}{dz} + (z^2 - v^2)y = 0.$$

Then we generates the future scenarios according to the following algorithm (see [Biglova et al., 2008] [Biglova et al., 2014][Rachev et al.,2007]).

Step 1: *Margins of the innovation of each log-return series.*

Perform the Maximum likelihood estimation of the model in (1). Then approximate the standardized innovations $\hat{u}_{j,t} = \frac{\hat{\varepsilon}_{j,t}}{\sigma_{j,t}}$ with a α_j-Stable distribution $S_{\alpha_j}(\mu_j, \sigma_j, \beta_j)$. Then simulate N scenario from a Stable distribution for each of the future scenario and compute the sample distribution for each $\hat{u}_{j,T+1}$:

$$F_{\hat{u}_{j,T+1}}(x) = \frac{1}{N}\sum_{i=1}^{N} I_{\left\{\hat{u}_{j,T+1}^{(i)} \le x\right\}}$$

Step 2: *Dependence structure.*

Fit the vector of $\hat{u} = [\hat{u}_1, \dots, \hat{u}_{94}]$ with the asymmetric t-copula defined in (2) and compute the maximum likelihood to obtain the parameters $(d, \hat{\mu}_i, \hat{\sigma}_{ii}, \hat{\gamma}_i)$ which allows to compute the estimator for variance covariance matrix of Z:

$$\hat{\Sigma} = \left(cov(X) - \frac{2d^2}{(d-2)^2(d-4)}\hat{\gamma}\hat{\gamma}'\right)\frac{d-2}{d}.$$

Now generate N scenarios for the vector $\hat{u}$ from (2) and call them: $[\hat{V}_1^i, \dots, \hat{V}_{94}^i]$ for $i = 1, \dots, N$ and let $F_{V_j}(x)$ be the marginal of the estimated asymmetric t-distribution. Now generate

N scenarios for $u_{T+1}^{i} = [u_{T+1}^{1,i}, \dots, u_{T+1}^{94,i}], i = 1, \dots, N,$ where $u_{T+1}^{j,i} = \left(F_{\hat{u}_{j,T+1}}\right)^{-1} U_j^i$ by the probability integral transform[3] and U_j^i is standard uniform taken from the copula $C(t_1, \dots, t_{94}) = F_V\left(F_{V_1}^{-1}(t_1), \dots, F_{V_{94}}^{-1}(t_{94})\right)$[4][5]

Step 3: *Log-return generation*

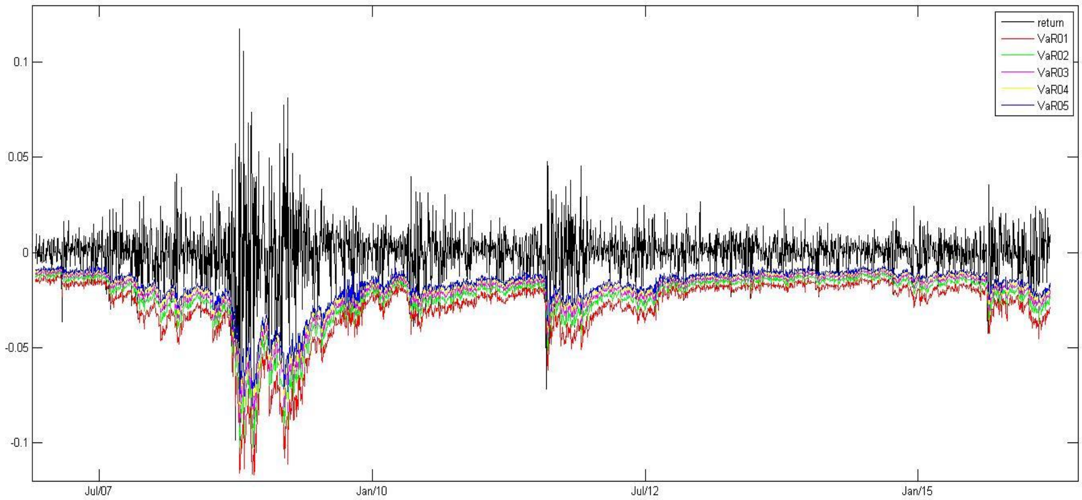
By (1) generate the vector of model's residuals as: $\varepsilon_{T+1}^{i} = [\varepsilon_{T+1}^{1,i}, \dots, \varepsilon_{T+1}^{94,i}] = [\sigma_{1,T+1} u_{T+1}^{1,i}, \dots, \sigma_{94,T+1} u_{T+1}^{94,i}]$. And then finally by (1) generate the vector of future returns: $r_{T+1}^{i} = [r_{T+1}^{1,i}, \dots, r_{T+1}^{94,i}]$.

The simulated returns are multiplied to 9400 portfolios computed as follows:

- Weight of the *i* stock is 0.0*i* while all the other stocks are weighted (1-0.0*i*)/93 , for *i=1,…,100.*

The process is repeated for the 2nd,..,94th stock in order get all the 94*100=9400 portfolios in a 94x9400 matrix. The product between the simulated returns (in a 5000x94 matrix) and the portfolios created define the evolution of the 9400 portfolios for each of the 5000 scenarios, then the outputs are sorted in ascending order for each portfolio, from the worst scenario to the best one. The VaR at α=1%, 2%, 3%, 4% and 5% is computed by selecting the 50th, 100th, 150th, 200th and 250th worst values for each portfolio. The AVaR at α=1%, 2%, 3%, 4% and 5% is computed by making the mean of the worst 50, 100, 150, 200 and 250 values for each portfolio.

Figure 1: Comparison between portfolio and its related VaRs



3. Results

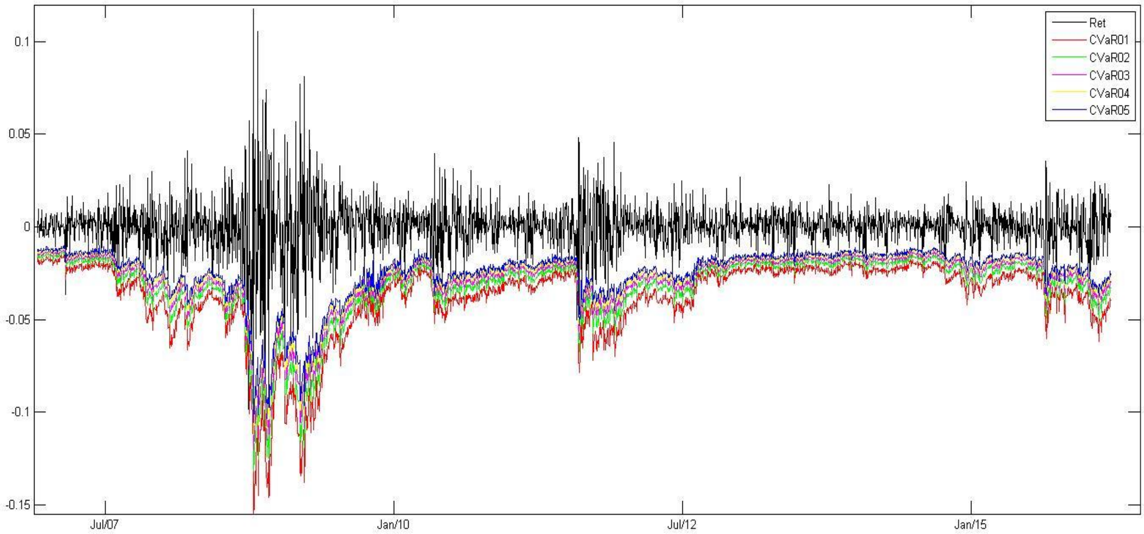
We confront the performances of VaR and AVaR in copula based framework described in the previous section with the EMWA. Here we report the results of a portfolio composed by asset: MONDELEZ INTERNATIONAL CL.A for the 4% and the other stocks equally-weighted. As we can see from Figures 1 and 2 there is a significant number of times for which the portfolio return go under the limits provided by VaRs. Differently AVaR violations are considerably less.

[3] See [Casella and Berger, 2002]

[4] See [Sklar, 1959].

[5] For further details see [Biglova et al., 2014]

Figure 2: Comparison between portfolio and its related AVaRs



In order to check if VaRs and AVaRs computed with the copula model are accurate we compute Kupiec's[6] and Christoffersen's[7] test for all the 9400 portfolios. The results are reported in Table 1.

Table 5.1: Percentages of VaRs and AVaRs accepted by Kupiec's and Christoffersen's test

Copula model	Value-at-Risk			Conditional Value-at-Risk		
α	1%	2%	5%	1%	2%	5%
Kupiec	0,0225532	0,001383	0,0011702	0,8389362	0,4073404	0
Christoffersen	0,0582979	0,0053191	0,0068085	0,927234	0,5787234	0

As we can see from this table VaRs most of the times fail to provide an acceptably accurate measure of risk. The percentage of successfully tested AVaRs at 1% and 2% is greater but only AVaR at 1% seems to provide an acceptable measure of risk given its 83.89% success from Kupiec's test and 92.72% from Christoffersen's test. For AVaR at 5% both tests are always rejected.

Similar observations can be taken regarding EMWA VaRs and AVaRs from Figure 3 and 4. In particular there is a significant number of times for which portfolio a) returns go under the limits provided by EWMA VaRs. Differently, EWMA AVaR violations are fewer than EWMA VaR.

The results of Kupiec's and Christoffersen's test are reported in Table 2. Even in this case most of VaRs provided by the model are not acceptably accurate for measuring risk. The percentage of successfully tested AVaRs at 1% and 2% is considerably greater. In particular, AVaR at 2% provide an acceptable measure of risk given its 98.83% success from Kupiec's test and 99.31% from Christoffersen's test.

[6] See [Kupiec, 1995]

[7] See [Christoffersen, 1998].

Figure 3: Comparison between portfolio and its related EWMA VaRs

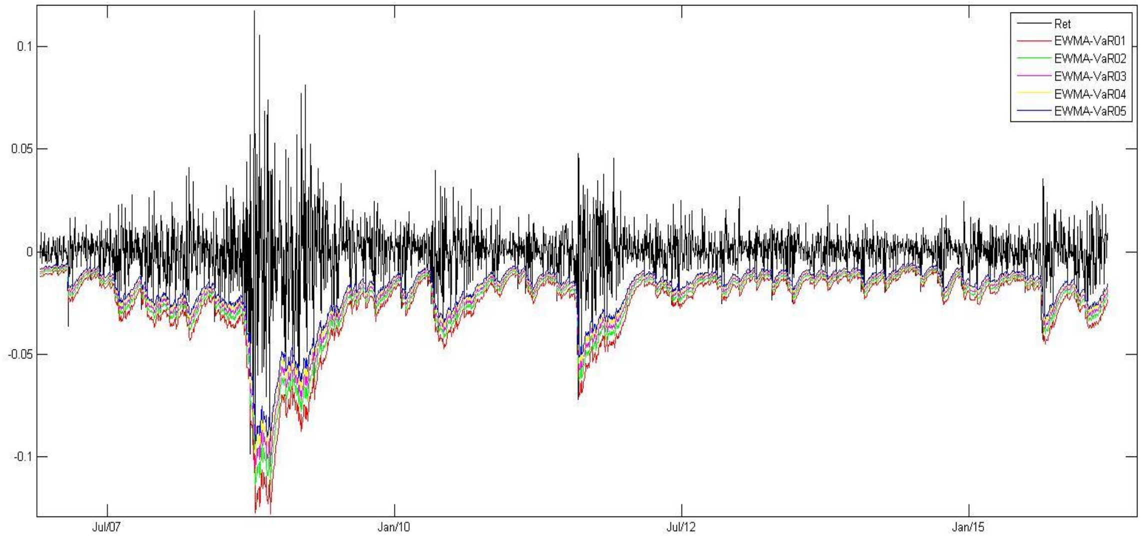


Figure 4: Comparison between portfolio and its related EWMA AVaRs

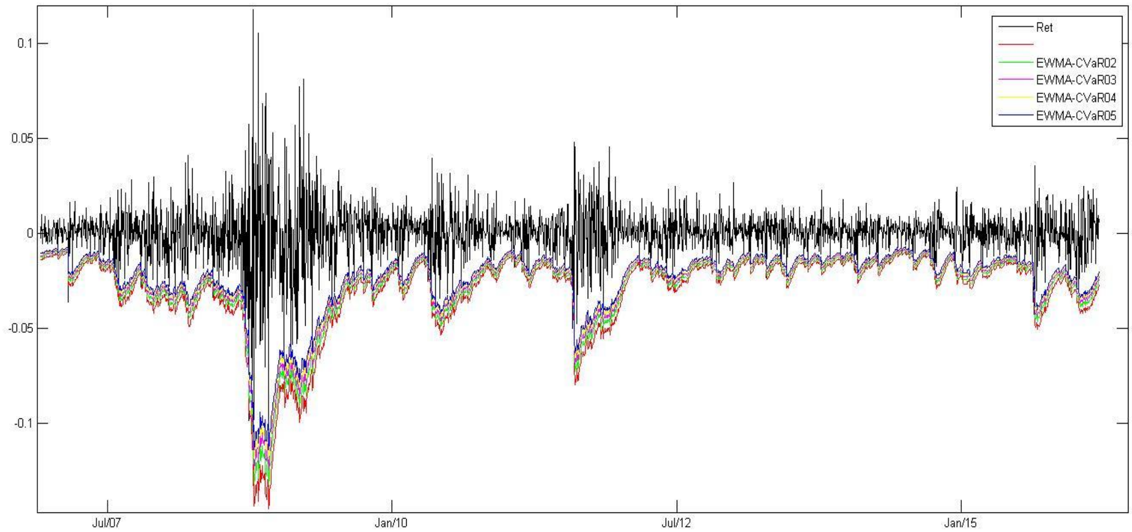
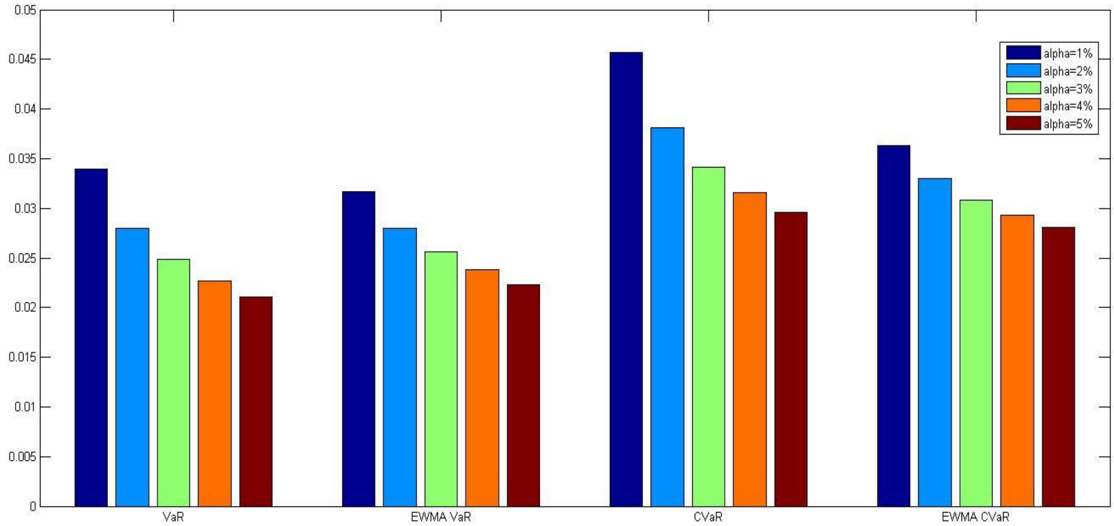


Table 2: Percentages of EWMA VaRs and AVaRs accepted by Kupiec's and Christoffersen's test

EWMA model	Value-at-Risk			Conditional Value-at-Risk		
α	1%	2%	5%	1%	2%	5%
Kupiec	0,0041489	0,0343617	0,2191489	0,6754255	0,9882979	0,0431915
Christoffersen	0,0143617	0,065	0,2880851	0,7532979	0,9930851	0,1687234

Then in order to compare the performance between the copula model and the EMWA model. we examine the mean VaR and mean AVaR.(Figure 5)

Figure 5: mean VaRs and CVaRs for copula and EWMA model



Comparing the mean VaRs and AVaRs for both models, it appears that there is no great difference between VaRs and EWMA VaRs, except from the first percentile where we can see that VaR is clearly higher than EWMA VaR, while AVaR are higher than EWMA AVaRs.

Then we perform the Kupiec's and the Christofferen's tests. Results are in table 3.

Table 3: comparison of copula and EWMA test results

	Kupiec		Christoffersen	
	Copula	EWMA	Copula	EWMA
VaR α=1%	0,02255	0,00415	0,0583	0,01436
VaR α=2%	0,00138	0,03436	0,00532	0,065
VaR α=5%	0,00117	0,21915	0,00681	0,28809
CVaR α=1%	0,83894	0,67543	0,92723	0,7533
CVaR α=2%	0,40734	0,9883	0,57872	0,99309
CVaR α=5%	0	0,04319	0	0,16872

The copula model VaR at the first percentile provide a better estimation of risk than the corresponding EWMA one. On the other hand, at greater percentiles the EWMA model performs considerably better than the copula one. Analogous results can be derived from the AVaRs comparisons: at the first percentile the copula AVaR is a more accurate risk measure than EWMA CVaR, but at the second and fifth percentile its accuracy is remarkably lower than EWMA one. Given the results of the tests, it is possible to derive the following suppositions: since all the copula AVaRs are greater than EWMA AVaRs and their accuracy is better for the first percentile but worse for the second percentile, we can presume that EWMA CVaR at α=1% underestimates risk while CVaR at α=2% and more overestimates risk.

4. Conclusions

An accurate measure of risk in portfolio selection should be able to capture extreme values of risk along the distribution tail belonging to the first percentiles. We focus on market risk estimation by applying a skewed t-copula model on portfolios composed by 94 stocks of the S&P100 in order to compute VaR and AVaR and then we compare its results with the outcomes of a Gaussian EWMA model. For computing returns simulations we have made use of ARMA(1,1)-GARCH(1,1) algorithm provided by [Biglova *et al.*, 2014]. Comparing the mean VaRs and AVaRs for both models, it appears that there is no great difference between VaRs and EWMA VaRs, except from the first percentile where we can see that copula VaR is clearly higher than EWMA VaR, while copula AVaRs are higher than EWMA AVaRs. By computing Kupiec's and Christoffersen's tests we discover that, for both copula and EWMA models, the VaRs provided are not acceptably accurate for measuring risk while copula AVaR at α=1 and EWMA AVaR at α=2% are acceptable measure of risk. Moreover, it is possible to presume that EWMA AVaR at α=1% underestimates risk while copula AVaR at α=2% and more overestimates risk. The results of this study should enlighten that VaR is not a good risk measure as believed before the financial crisis, while CVaR is a better estimate of risk, especially for the first two percentiles.

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