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Reward and Risk in the Italian Fixed Income Market

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Abstract

In this paper, we discuss and examine the portfolio optimization problems in the Italian fixed income market considering two main sources of risk: prices risk and market risk. To achieve this aim, we propose a two-step optimization problem for two types of bonds. In particular, we manage the price risk implementing the classical immunization method and then, using the ex-post results from the optimal immunization problem, we are able to deal with market risk maximizing the portfolio wealth in a reward-risk framework. Adopting this approach, the paper then explores empirical applications on the Italian fixed income market using data for the period 2005-2015. Empirical results shows that the two-step optimization build efficient portfolios that minimize the price risk and the market risk. This ex-post analysis indicates the usefulness of the proposed methodology, maximizing the investor's wealth and understanding the dynamics of the bonds.

Key words

Portfolio selection, bond market, immunization, reward-risk measure

JEL Classification: G11, G12

1. Introduction

The risk reward measures have a central role in the portfolio theory ever since the pioneering work of Markowitz (1952). It follows that many portfolio optimization models based on reward risk measures have been developed for asset allocation, see Farinelli et al. (2008). In addition, for a survey of recent contributions from operation research and finance to the theory of portfolio selection see Fabozzi et al. (2010). Different from portfolio strategies in the stock market, the portfolios of fixed income securities are classically managed using the concepts of duration, convexity, modified duration (which consider the so-called immunization) and future wealth. In particular, the main target of every portfolio manager is to maximize the future wealth computed as the total rate of return. In this context, the total rate of return maximization is typically solved using risk factor models.

The classical theory of immunization introduced by Redington (1952) and Fisher and Weil (1971) defines the conditions under which the value of fixed income portfolio is protected against changes in the level of interest rates. Thus, portfolio managers reduce the interest rate risk by using the principles of immunization (see, e.g., Vasicek (1977) and Munk (2011)). The main result of this theory is that immunization is achieved if the duration of the portfolio is equal to the length of the horizon, and for this reason, the duration matching constraints usually increases the value of future portfolios. Unfortunately, this approach presents some limitations since the portfolio is protected only against the assumed risk and

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does not consider the historical behaviour of the assets. In a more recent development, Ortobelli et al. (2016) consider this aspect and propose a new typology of immunization for bond portfolios with respect to average term structure changes (called immunization on average).

As in Ortobelli et al. (2016), we will consider the issue of maximizing the future portfolio wealth through a two-step optimization problem. In particular, we first maximize the yield to maturity of a portfolio with constant immunization risk, thus, we create optimal baskets of bonds. Then, we optimize one performance measure (the Sharpe Ratio, see Sharpe (1994)) of these funds of bonds. In essence, we manage the price risk implementing the classical immunization method and then, using the ex-post results from the optimal immunization problem, we are able to deal with market risk maximizing the portfolio wealth in a reward-risk framework. Finally, we empirically analyse two kind of bonds traded on the Italian fixed-income market during the period 2005-2015.

The rest of this paper is organized as follows. Section 2 presents the suggested portfolio selection methodology, discussing the two steps and different risk measures. Section 3 shows an implementation of the portfolio selection model applied to two types of bonds (from the Italian fixed income market). In Section 4 we summarize the results.

2. The portfolio selection problem

In the following section, we present and discuss the two portfolio optimization steps adopted to solve the optimal investment in the fixed income market.

2.1 The first optimization step with immunization measures

The traditional theory of immunization has a central role in the fixed income portfolio ever since the seminal paper of Redington (1952) and pioneering work by Fisher and Weil (1971). In the simplest case, immunization can be defined as follows. Investors wish to construct a portfolio such that, irrespective of rise or a fall in the interest rate, the value of the portfolio at the horizon will be at least as large as the liability. Commonly, to achieve this aim, portfolio managers match asset and liability streams to make them equally sensitive to interest rate changes.

Duration and convexity quantify the variability of prices linked to the changes in the interest rate. Theoretically, the price of a bond is a function of the promised payments and the market rate of return. Assume for simplicity that r_i is a fixed rate of return, then today's price, P_i of bond *i* with *n* coupon payments c_{ik} at times t_k, $k = 1, \dots, n$, is as follows

$$P_i = \sum_{k=1}^{n} \frac{c_{i,k}}{(1+r_i)^{t_k}}. \tag{1}$$

In this representation, r_i is the yield to maturity (in literature also known as internal rate) which is generally not fixed over time.

In the financial literature, immunization theorems have a theoretical justification from Taylor's polynomial approximation. Thus, the return from a change in the interest rate can be approximated by the expression:

$$\frac{\Delta P_i}{P_i} = \frac{\partial P_i}{\partial r_i}\frac{1}{P_i}\Delta r_i + \frac{1}{2}\frac{\partial^2 P_i}{\partial r_i^2}\frac{1}{P_i}(\Delta r_i)^2 + \frac{1}{3!}\frac{\partial^3 P_i}{\partial r_i^3}\frac{1}{P_i}(\Delta r_i)^3 + \cdots +$$

Theoretically, there are an infinite number of orders in this expression. If only the first two terms are considered, the second degree polynomial gives the best approximation of the return. Duration is defined as the coefficient of the first order approximation multiplied by minus one, i.e.

$$D_i = -\frac{1}{P_i}\frac{\partial P_i}{\partial r_i} = \frac{\sum_{k=1}^{n}\left(t_k c_{i,k}(1+r_i)^{-t_k}\right)}{P_i(1+r_i)} \tag{2}$$

In this paper, following Ortobelli et al. (2016) we call D_i modified duration even it is known in literature as duration, while we call $\hat{d}_i = D_i(1+r_i)$ the Macaulay duration (see Macaulay (1951) and Weil (1973)).
While convexity C_i is the coefficient of the second-order approximation:

$$C_i = \frac{1}{P_i}\frac{\partial^2 P_i}{\partial r_i^2} = \frac{\sum_{k=1}^{n}\left(t_k c_{i,k}(1+r\_)^{-t_k}\right)+\sum_{k=1}^{n}\left(t_k^2 c_{i,k}(1+r_i)^{-t_k}\right)}{P_i(1+r_i)^2}. \tag{3}$$

Modified duration and convexity are useful tools to approximate changes in bond prices. For a small change in the interest rate, modified durations considered as close approximation to the actual change in the bond price. However, since the price of a bond is not a linear function to the interest rate, then convexity term gives a closer approximation. Thus, the return from a change in the interest rate r_i is usually approximated by the following relation.

$$\frac{\Delta P_i}{P_i} \approx \left(-D_i \Delta r_i + 1/2\, C_i (\Delta r_i)^2\right). \tag{4}$$

In practice, there exist several formulas for approximating the convexity. In this paper, we approximate the convexity with the following formula (as suggested by DataStream) based on the Macaulay duration $\hat{d}_i$ and the yield to maturity r_i, i.e.

$$\frac{\partial^2 P_i}{\partial r_i^2} \approx \left[\left(\frac{100+r_i}{100+Y_i^{+1}}\right)^{\hat{d}_i} + \left(\frac{100+r_i}{100+Y_i^{-1}}\right)^{\hat{d}_i} - 2\right].10^8, \tag{5}$$

where Y_i^{+1} represents the yield of bond i plus one basis point (0:01%) and Y_i^{-1} the yield minus one basis point (see Fabozzi (2005) and references therein).

2.1.1 The classical immunization approach

In an important article in 1952, Redington proposes immunization concept for infinitesimal shifts in the interest rate, matching the durations of assets and liabilities. For ease of exposition, we refer to this approach as the Redington model. Fisher and Weil (1971) introduce immunization for additive shifts in the yield curve, rather than infinitesimal rate changes, matching the portfolio duration with the maturity of a single liability. Several authors provide different perspective in their reviews of the development of immunization principles (see among others, Fong and Vasicek 1984, Ortobelli et al. 2016 and literature therein).

As in Ortobelli et al. (2016), we formulate the Redington model as follows. Consider $r = [r_1, \ldots, r_n]'$ the vector of the yields to maturity of the bonds, $D = [D_1, \ldots, D_n]'$ the vector of the modified durations, $C = [C_1, \ldots, C_n]'$ the vector of the convexities, and $v = [v_1, \ldots, v_n]'$ is the vector of the wealth's invested in the bonds, i.e. $v_i = y_i P_i$, where y_i is the number of the *i*th bond we invest in and P_i its price. Furthermore, in the empirical analysis, we suppose that short sales are not allowed $(y_i \geq 0)$ and that $\frac{y_i P_i}{W}$ $(\forall\, i = 1, \ldots, n)$, where $W = \sum_{i=1}^{n} v_i$ is the interested wealth.

In line with Redington model (1952), we also consider the convexity constraint. Generally, the main goal of any portfolio investor is to maximize the expected future wealth. In this case, we measure the future wealth as $\sum_i v_i (1+r_i) = v'(1+r)$, i.e. the sum of capitalized wealth invested in each asset. Then, consistent with the classical portfolio immunization approach, we suggest a reward/risk portfolio analysis using the expected future wealth $v'(1+r)$ as a return measure and the portfolio modified duration $\frac{v'D}{W}$ as an immunization risk measure. Thus, for some fixed modified duration *d* and an initial wealth *W*, investors want to maximize their final wealth according to this approach, by choosing a solution to the following optimization problem:

$$\begin{aligned} & max_y && v'_{(t)}(1+r_{(t)}) \\ & \text{s.t.} && \sum_{i=1}^{n} y_i P_{i,(t)} = W_{(t)} \end{aligned} \tag{6}$$

$$\frac{v'_{(t)}\, D_{(t)}}{W_{(t)}} = d$$

$$y_i P_{i,(t)} = v_{i,(t)};\ y_i \geq 0;\ \frac{y_i P_{i,(t)}}{W_{(t)}} \leq \theta;\ i = 1, \dots, n$$

$$C_{(t)}(ptf) = \frac{v'_{(t)}\, C_{(t)}}{W_{(t)}} \geq C_{(t-1)}(ptf) = \frac{v'_{(t-1)}\, C_{(t-1)}}{W_{(t-1)}}$$

where $v'_{(t)}$ is the vector of the wealth invested in the bonds at time *t*, $P_{i,(t)}$ is the price of the *i*th bond at time *t*, $D_{(t)}$is the vector of modified durations at time *t* and $C_{(t)}$ is the vector of convexities at time *t*. In particular, we force the convexity at time *t*, given by $C_{(t)}(ptf)$, to be greater than that at the previous time $t-1$. Observe that in this first step we do not need historical observations of bond returns to estimate the risk and return measures, and optimization problem (6) is a linear programming problem.

2.2 The second optimization step with performance measure

The portfolio selection problem, in the equity market, is generally examined in a reward–risk framework, according to which, the portfolio choice is made with respect to two criteria – the expected portfolio return and portfolio risk. In particular, a portfolio is preferred to another one if it has higher reward and lower risk. Markowitz (1952) introduced the first rigorous approximating model to the portfolio selection problem, where the return and risk are modeled in terms of portfolio mean and variance. However, different generalization has been proposed in the literature (see, among others, Bilgova *et al.* 2004, Rachev *et al*. 2008 and the reference therein). Let us briefly formalize the portfolio performance measure (Sharpe ratio) that is used in the empirical analysis section.

Sharpe ratio (1994). The Sharpe ratio is used to characterize how well the return of an asset compensates the investor for the risk taken. The Sharpe ratio computes the price for unity of risk, and calculated by subtracting the risk-free rate from the rate of return of the portfolio and then divide the result by the standard deviation of the portfolio returns. Formally:

$$SR(x'z) = \frac{E(x'z) - z_0}{\sigma_{x'z}}, \tag{7}$$

where, $E(x'z)$ is the portfolio expected returns, z_0 is the risk-free return and $\sigma_{x'z}$ is the portfolio standard deviation.

3. Ex-post empirical analysis

In this section, we apply the multi-step methodology using two types of bonds traded on the Italian fixed income market. In particular, we discuss the results of the two-step approach to manage immunization risk (Section 3.1) and market risk (Section 3.2).

3.1 Immunization risk management

According to Section 2.2, we control the immunization risk by measuring the sensitivity of bond prices to changes in interest rates. For this aim, we optimize the portfolio yield to maturity for some immunization risk measures and requiring a greater or equal convexity. In this optimization, we consider two different kinds of bonds including Government bonds and corporate bonds. For each type of bonds we obtain a dataset contained in Thomson Reuters DataStream as follows: period July 2005 through June 2015 for Government bonds, period December 2008 through April 2015 for corporate bonds.

For portfolio immunization purpose we determine the modified duration, which we recalibrate every 20 trading days for each type of bonds in order to maintain updated the wealth level, as follows:

- Government bonds: initial duration of 8 years with increasing passes of 0.6 to reach 10 years duration (2005-2015).
- Corporate bonds: initial duration of 5.5 years with increasing passes of 0.375 to reach 7 years duration (2008-2015).

We use a moving average window of 125 working days for the computation of each optimal portfolio and we recalibrate the modified duration every 20 days. We assume that no short sales are allowed $(y_i \geq 0)$ and that it not possible to invest more than 90% in any unique asset ($\frac{v_i}{W} \leq 0.9$). We solve the portfolio optimization problem weekly and then we consider the sample path of the final wealth obtained by solving (6). We assume proportional transaction costs of 20 basis points.

In this empirical analysis, for each category of bonds, we have to compute the optimal portfolio composition every month (twenty trading days). Therefore, at the *k*-th optimization, three steps are performed to compute the ex-post final wealth.

Step 1 Preselect all the liquid and active assets in the last 6 months (125 trading days) for a given dataset. The moving window of 6 months is used to compute the average yield to maturity of each asset for the portfolio problem (6).
Step 2 Determine the optimal portfolio *y* that maximizes the final wealth for a fixed immunization risk measure (i.e. a solution of the optimization problems (6)).
Optimization problems (6) is a linear programming problem and can be solved in a very efficient way.
Step 3 Compute the ex-post final wealth taking into account 20 basis points as proportional transaction costs.

We apply the three steps for each category of bonds until the observations are available. The results of this analysis are reported in Figures 1 and 2 for Government bonds and corporate bonds respectively.

Figure 1: Ex-post wealth obtained in the first step with the Redington model using Government bonds

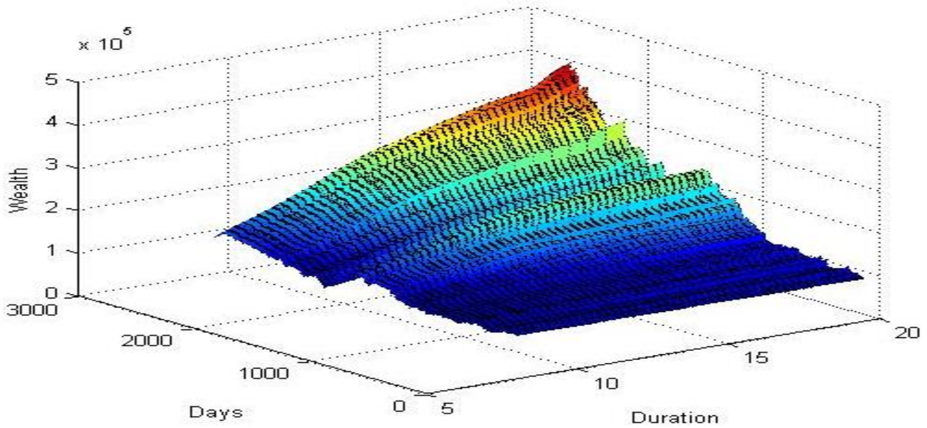

Figure 1 shows the classical results of immunization approach when we use the Italian Government bonds. Generally, we observe that the wealth evolution of Italian sovereign bonds increases expect the impact of subprime crisis where we see some losses. Clearly, for short period we denote more or less a constant evolution of the wealth, which increases slightly in 2008 due to the rise of bond returns. Moreover, as was expected, the higher modified durations provide the best performance in terms of ex-post wealth, which is increased four times initial wealth, but with a significant immunization risk exposure.

Figure 2 Ex-post wealth obtained in the first step with the Redington model using corporate bonds.

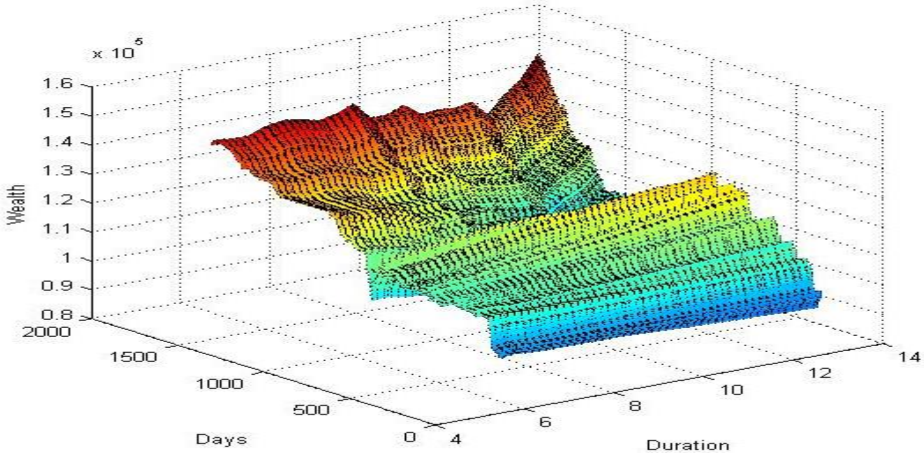

Figure 2 reports the classical results of immunization methodology when we use the corporate bonds for the period December 2008 through April 2015. As it can be seen from the Figure 2, this type of bond presents a fluctuating evolution of the wealth. Indeed we observe that the wealth increases continuously and constantly for the first four years. Then, in the medium term, we denote a massive losses accompanied by a decrease in duration. This results could be explained by the fact that most corporate bonds loses values and liquidity during last crisis that hits the entire financial system. However, at the end of this financial turmoil, we observe an increasing wealth for all durations considered; the wealth passes in short period from 0.8 to reach 1.4.

3.2 Market risk management

After the immunization risk reduction obtained in the previous section, we maximize a performance for each category of bonds, as suggested in the portfolio problems of section 2.2. This step of optimization consider as assets the 20 funds obtained in the immunization risk management step (2.3). Therefore, we proceed with the second optimization model to maximize Sharpe ratio on the 20 historical wealth funds obtained with the Redington model. The portfolio was recalibrated on weekly basis (every five trading days) using a rolling moving windows of 6 months of historical observations (125 trading days).

In the empirical analysis, for each category of bonds, we have to compute the optimal portfolio composition every week (five trading days). Therefore, at the *k*-th optimization, two steps are performed to compute the ex-post final wealth.

Step 1. Determine the market portfolio $x_M^{(k)}$ that maximizes the performance ratio $\rho(x)$ applied to the optimal 20 funds:

$$\begin{gathered} max_x\ \rho(x) \\ \text{s.t. } \textstyle\sum_{i=1}^{n} x_i^{(k)} = 1, \\ 0 \le x_i^{(k)} \le 0.9 \quad i = 1, \dots, n \end{gathered} \tag{14}$$

Here the performance measure $\rho(x)$ is the Sharpe ratio (7). The maximization of the Sharpe Ratio can be solved as a quadratic-type problem and then it possesses a unique solution.

Step 2. Compute the ex-post final wealth (without transaction costs). We apply these two steps until the observations are available for every performance measure and for each type of bonds. The results of this analysis are reported in Figures 3 and 4 for Government bonds and corporate bonds respectively.

Figure 3: Ex-post wealth obtained in the second step maximizing Sharpe Ratio considering the 20 optimal funds of the Government bonds obtained with the classical immunization problem.

Figure 3 reports the results from the second step optimization on Italian Government bonds. We observe, in short period between 2006 and 2008, a fluctuating lower level of the wealth. While, for the period 2008-2010, we not a significant increase that reaches 1.43 due to the rises of Italian Government yields. Unfortunately, with European credit crisis that hits Italy in 2011, we observe a substantial losses. Then, from 2012 we observe a progressive increases of the ex-post wealth. Generally, sovereign debts include a wide range of bonds according to Government needs, for example Italian Government issues the following bonds: BTP, CCT, CTZ and BOT (all considered in this empirical analysis). According to Bertocchi et al (2013), the sovereign issuance segment is still the most important segment of the bond market in the EU representing in September 2012 46% of the total Euro-denominated debt.

Figure 4: Ex-post wealth obtained in the second step maximizing Sharpe Ratio considering the 20 optimal funds of the corporate bonds obtained with the classical immunization problem.

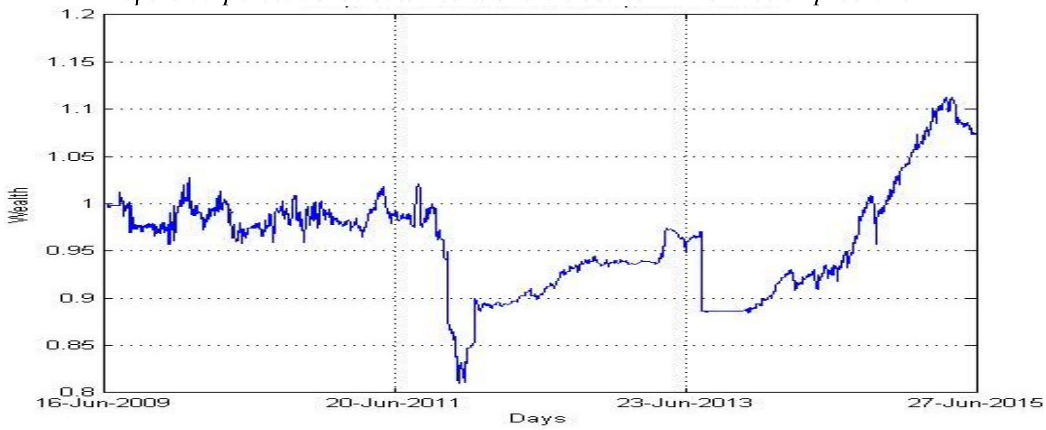

Figure 4 reports the results from the second step optimization on corporate bonds. We observe that for the first two years the wealth fluctuates around 0.97. Then from 2011 we note a substantial losses in this category of bonds, which suffers significantly from European crisis. However, by end of 2013 we observe a progressive upturns.

4. Conclusion

In this paper, we examine and study the portfolio optimization problems in the Italian fixed income market considering two main sources of risk (i.e. prices risk and market risk). In particular, we use two-step optimization problem for two different types of bonds (i.e. Governement and corporate bonds). Essentially, we manage the price risk implementing the classical immunization method and then, using the ex-post results from the optimal immunization problem, we are able to deal with market risk maximizing the portfolio wealth in a reward-risk framework. We evaluate the effectiveness of the proposed approach by an empirical analysis on the Italian fixed income market using data for the period 2000-2015. Empirical results shows that the two-step optimization build efficient portfolios that minimize the price risk and the market risk. This ex-post analysis indicates the usefulness of the proposed methodology, maximizing the investor's wealth and understanding the dynamics of the bonds. Future reasearch will investigate more sophisticated immunization methods and best performance measures.

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