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Weak Orders for Intersecting Lorenz Curves: Analysis of income distribution in European countries

Tommaso Lando[1], Lucio Bertoli-Barsotti[2], Michaela Staníčková[3]

Abstract. The low growth of the EU performance has increased concerns regarding an increasing wage dispersion and income inequality, which hampers sustainable and inclusive economic development. In this context, the Lorenz curve can be used to define different ranking criteria for comparison of income distributions in terms of inequality, such as the Lorenz dominance and some weaker dominance relations that have been introduced more recently in the literature. In this paper, we review such methods and propose an empirical study with which we analyse and compare the income distributions of the 28 European Union member states (EU-28) plus other five European countries, on the basis of the dominance criteria discussed. The database for empirical analysis has been downloaded from the Eurostat's website – Income and living conditions statistics.

Keywords: Lorenz curve, income inequality, disparity, stochastic dominance, EU-28.

JEL Classification: C44, E24, O52.
AMS Classification: 62C, 91B14.

1 Introduction

In this study, the term "income" is referred to as household disposable income in a particular year, which consists of earnings, self-employment, capital income and public cash transfers; income taxes and social security contributions paid by households are deducted. The income of the household is attributed to each of its members, with an adjustment to reflect differences in needs for households of different sizes.

The Lorenz curve (LC) is a fundamental tool for comparison of income distributions in terms of inequality. The LC gives rise to a preorder, that is, the Lorenz dominance (LD), which is generally used to rank distributions based on their degree of inequality. In an economic framework, the LD is coherent with the Pigou-Dalton condition, i.e. the so called "principle of transfers". According to this principle, the higher of two non-intersecting Lorenz curves can be obtained from the lower one by an iteration of income transfers from "richer" to "poorer" individuals (the so called *elementary transfers* or *T-transforms*, see [8], p. 32), also called *progressive transfers*, [10]). For this reason, the "coherence" with the LD represents a fundamental property for all inequality (or concentration) measures.

However, many empirical studies revealed that Lorenz curves often intersect in the practice. It is not rare to find couples of distributions that cannot be ranked by the LD. In such cases, we can compare the intersecting distributions by relying on weaker orders of inequality, such as the *second-degree Lorenz dominance, 2-LD* [1]. The basic idea of the 2-LD is to move from the (first-degree) LD to a dominance relation of higher degree by cumulating LCs i) from the bottom or, ii), from the top, which gives rise to a Lorenz dominance of second degree. In the first case (i), this idea has been analysed in several works, related to the concept of third-degree inverse stochastic dominance [1,9]. These orderings emphasize the left tail of the distribution. Indeed, many authors (see e.g. [2,10]) agree that an elementary transfer should be more equalizing the "lower" it occurs in the distribution, that is, the principle of *aversion to downside inequality* [4]. On the other hand, in an economic context, a lot of attention is recently given to those variations occurring at the top of the income distribution [7]. Thus, the second idea (ii), introduced by Muliere and Scarsini [9] and more recently developed by Aaberge [1], emphasizes the right tail of the distribution in spite of the left one. More recently, Lando and Bertoli-Barsotti [6] introduced a new weak order, namely *disparity dominance*, which is based on the cumulated difference between the upper and the lower parts of the LC and it is aimed at emphasizing inequality in both tails of the distribution. Indeed, this approach attempts to combine the main features of approaches i) and ii) into a single preorder.

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In this paper, we analyse the income distributions of the EU-28 countries plus other five non-WU countries in the year 2015 with a twofold objective: i) compare the effectiveness of the weak preorders studied in terms of discriminating power and; ii) characterize mathematically the differences between the distributions of income of the EU-28 countries. In section 2, we introduce the notation and present the dominance relations used in our empirical analysis. In section 3 we present the data and describe the empirical results. In section 4 we summarize the results.

2 Methods

We recall that a preorder is a binary relation $\leq$ over a set S that is reflexive and transitive. In particular, observe that a preorder $\leq$ does not generally satisfy the antisymmetric property (that is, $a \leq b$ and $b \leq a$ does not necessarily imply $a = b$) and it is generally not total (that is, each pair a, b in S is not necessarily related by $\leq$).

In this paper, we relax the common assumption of non-negativity of the distribution, because our data in section 3 present also negative incomes. Although this feature does not hamper the methods discussed below, in what follows we shall discuss the effect of negative incomes on the fundamental tools for the measurement of inequality, such as the LC and the Gini index.

Let F be a distribution with positive and finite expectation μ_F (clearly, the mean cannot be zero or less than zero, otherwise the LC cannot be defined). The (generalized) inverse or quantile function of F is given by:

$$F^{-1}(p) = \inf\{z: F(z) \geq p\}, p \in (0,1). \tag{1}$$

The Lorenz curve L_F is defined as follows:

$$L_F(p) = \frac{1}{\mu_F}\int_0^p F^{-1}(t)dt\,, p \in (0,1). \tag{2}$$

The LC is a convex function. However, note that, differently by the usual definition of [5], the LC may be decreasing and negative because of negative incomes.
The Gini index is given by twice the area between the Lorenz curve and the 45° line:

$$\Gamma(F) = 1 - 2\int_0^1 L_F(t)dt. \tag{3}$$

Under the common assumption of non-negative incomes, the Gini index is typically normalized between 0 and 1. However, in presence of negative incomes, the LC can be negative, so that Γ can exceed 1. For this reason, Chen, Tien-Wang and Tong-Shieng [3] propose re-normalizing Γ by dividing it by $\Gamma_{max} = 1 - 2\int_0^h L_F(t)dt$, where $h = \sup\{p: L_F(p) \leq 0\}$. The resulting index is denoted here by $\Gamma^* = \Gamma/\Gamma_{max}$.
Let us also define the complementary curve $\bar{L}_F$, given by:

$$\bar{L}_F(p) = \frac{1}{\mu_F}\int_0^p F^{-1}(1-t)dt = 1 - L_F(1-p), p \in (0,1) \tag{4}$$

Actually, for a given percentage p, $L_F(p)$ represents the percentage of "total" possessed by the low $100p\%$ part of the distribution, while $\bar{L}_F(p)$ represents the percentage of "total" corresponding to the top $100p\%$ part of the distribution. From a geometrical point of view, $\bar{L}_F(p)$, increasing and concave, is the 180° rotation of $L_F(p)$, increasing and convex, with respect to the point $(0.5,0.5)$. Now we can introduce:

$$\Delta_F(t) = \bar{L}_F(t) - L_F(t), t \in [0,1/2]. \tag{5}$$

The interpretation of Δ_F is quite simple. As $\bar{L}_F(t) \geq L_F(t)\ \forall t \in [0,1/2]$, the difference between the Lorenz curves expresses the disparity between the "higher" and the "lower" parts of the distribution. In terms of income distributions, Δ_F equals the difference between the proportion of the society's overall wealth that is held by the society's top (rich) $100t\%$, and the proportion of the society's overall wealth that is held by the society's low (poor) $100t\%$.

It should be stressed, that the definitions below have been originally introduced only for the case of non-decreasing and non-negative LCs. However, there is no loss of generality if we apply them to our definition of the LC, which also account for the possibility of negative incomes.

Definition 1. We write $F \leq_L G$ if and only if $L_F(p) \geq L_G(p), \forall\, p \in (0,1)$.

When the LD is not fulfilled, i.e. when LCs intersect, we need to introduce some weaker criteria in order to obtain unambiguous rankings. Muliere and Scarsini [9] and Aaberge [1] suggest cumulating LCs from the left or from right: that is, attaching more weighting to low or top incomes. Such two parallel approaches give rise to the *upward* and *downward* 2-LD [1], defined as follows.

Definition 2. We say that G second-degree upward Lorenz dominates F, and write $F \leq_L^2 G$ if

$\int_0^t L_F(p)dp \geq \int_0^t L_G(p)dp$, $\forall t \in [0,1]$.

We say that G second-degree downward Lorenz dominates F, and write $F \leq_L^2 G$ iff any of the following equivalent conditions is true:

$\int_0^t \bar{L}_F(p)dp \leq \int_0^t \bar{L}_G(p)dp$, $\forall t \in [0,1]$

$\int_t^1 1 - L_F(p)dp \leq \int_t^1 1 - L_G(p)dp$, $\forall t \in [0,1]$.

Observe that $F \leq_L G$ implies $F \leq_L^2 G$ and $F \leq_{\bar{L}}^2 G$, but the converse is not necessarily true, i.e. $F \leq_L^2 G$ and $F \leq_{\bar{L}}^2 G$ do not imply the LD. Moreover, note that $F \leq_L^2 G$ implies that L_F starts above L_G and presents a lower (or equal) value of the Gini index. Differently, $F \leq_{\bar{L}}^2 G$ implies that L_F starts below L_G but still has a lower (or equal) Gini. Hence it is apparent that in both cases the condition $\Gamma(F) \leq \Gamma(G)$ is necessary to establish a dominance. In particular, if LCs cross only once, the one with higher Gini index dominates the other with respect to upward 2-LD (if the latter starts below, see [11]) or downward 2-LD (if the latter starts above, [6]).

As an alternative approach, Lando and Bertoli-Barsotti [6] combined the basic ideas (preferences) expressed by the $\leq_L^2$ and $\leq_{\bar{L}}^2$ orderings into a single preorder, which emphasizes inequality in both the tails of the distribution. This is done by symmetrically cumulating the Lorenz curve from both sides, through the disparity curve Δ_F, described above. In fact, it can be reasonable to wish that F is preferable to G if Δ_F is (uniformly) as small as possible, compared to Δ_F: this concept can be expressed as follows in terms of an integral inequality.

Definition 3. We say that F second-degree disparity dominates (2-DD) G and write $F \leq_D^2 G$ iff:

$$\int_0^t \Delta_F(p)dp \leq \int_0^t \Delta_G(p)dp \; \forall t \in [0,0.5].$$

Like $F \leq_L^2 G$, also $F \leq_D^2 G$ implies the condition $\int_0^{1/2} \Delta_F(p)dp \leq \int_0^{1/2} \Delta_G(p)dp$, which is equivalent to $\Gamma(F) \leq \Gamma(G)$ ($\int_0^{1/2} \Delta_F(p)dp = \int_0^1 (p - L_F(p))dp$).

Similarly to what has been discussed above, with regard to single-crossing LCs, if the curves are double-crossing, a theorem in [6] determines a sufficient condition for the 2-DD based on the positions of the crossing points and the values of the Gini index. Thus, 2-LD and 2-DD are especially suitable for ranking single-crossing and double-crossing LCs, respectively.

3 Empirical analysis

The main objective of this paper is to investigate the discriminating power of the dominance relations described in section 2. As a general rule, the stronger the preorder, the less it is applicable. For instance, we may find, in practice, many couples of intersecting LCs that are not comparable with LD. Thus, the main advantage of using a weaker criterion is that we can significantly increase the number of pairs of LCs that are comparable. Here we refer to the proportion of couples that are ranked according to a given preorder as the *rate of completeness* of such preorder. By increasing the rate of completeness we reduce the ambiguousness and we provide a normative justification for the use of an index of inequality that is coherent with the preorder considered, such as the Gini index. To perform our empirical analysis, we downloaded the income distributions of the EU-28 countries, plus some other non-EU counties, in the year 2015. The dataset is described as follows.

Data have been retrieved online from Eurostat's website. In particular, Eurostat provides the "distributions of income by quantiles" with two options in terms of income and living conditions indicator, namely: i) *top cut-off point*, which represent the income of the individual at the right end of the given quantile and; ii) *share of national equivalized income*, which is the share of the total income which belongs to a given interval. Eurostat provides i) and ii) for the three quartiles, the four quintiles, the nine deciles and the first (and last) five percentiles. Therefore, by properly cumulating the shares of national equivalized income we can obtain the values of the LC for

$p = 0.01,0.02,0.03,0.04,0.05,0.1,0.2,0.25,0.3,0.4,0.5,0.6,0.7,0.75,0.8,0.9,0.94,0.95,0.96,0.97,0.98,0.99,$

that is, a LC with 22 nodes.

For the year 2015, we computed the LCs of the EU-28 plus the following five non-EU countries: Macedonia, Iceland, Norway, Serbia and Switzerland, which totalize 33 countries. Then, we compared every pair of LCs (i.e. 33×32/2=528 pairs) based on the four dominance relations studied in section 2, namely LD, upward 2-LD, downward 2-LD and 2-DD. To be noted is that many countries present negative values for the LC, always for $p \leq 0.04$. The rates of completeness are shown in Table 1.

LD	Up. 2-LD	Down. 2-LD	2-DD
0.462	0.634	0.729	0.733

Table 1 Rates of completeness

This results reveal that the LD cannot even rank half of the pairs. This is due to the fact that the countries (especially the EU-28) present similar concentration patterns as well as similar values of the Gini index, thus LCs cross most of the times. Furthermore, we observe that generally the set of countries (at least as it is reported by Eurostat's data) concentrate most of their inequality in the right tails (i.e. high incomes) rather that in the left tails, so that cumulating from the right (downward 2-LD) is more critical (with regard to the discriminating power) than cumulating from the left (upward 2-LD). However, the relevance of the basic idea of downward 2-LD, that is, to emphasize the left tail, is acknowledged my many economists (unlike upward 2-LD). Therefore, it seems to be wiser to adopt 2-DD, so that we can rank the highest possible number of pairs by emphasizing both tails at the same time. However, although 2-DD yields the highest rate of completeness, it is far from reaching a complete ranking of the EU-28. In fact, some of the countries present very similar LCs, which might be ranked only by relying on dominance relations that are even weaker than 2-LD and 2-DD, so that their use would be hardly justifiable.

Further analyses, reported in Table 2, show the effectiveness of the different ranking criteria depending on the number of times a couple of LCs intersect. Downward 2-LD and 2-DD rank 65% of the single-crossing LCs, but 2-DD has slightly more discriminating power when dealing with double-crossing LCs, coherently with the theoretical result of [6]. We also note that the maximum number of times a LC crosses another is 3, but none of the dominance relations analysed is able to rank such pairs (that are just 3 pairs over 528, i.e. the 0.05%).

No. of intersections	Up. 2-LD	Down. 2-LD	2-DD
1	0.35	0.65	0.65
2	0.29	0.30	0.33

Table 2 Proportions of single and double-crossing LCs ranked by 2-LD and 2-DD

Table 3 shows the values of the different values of the Gini index for the EU-28 countries and the other five non-EU countries, and the different rankings yielded. We also computed the normalized Gini index Γ^*, which accounts for the presence of negative incomes, but in our case the differences between the normalized and non-normalized values are negligible (the rankings are identical), thus we do not report the values of Γ^* below (the maximum value of Γ_{max} obtained is 1.0001). However, our results presented above suggest that ranking based on the Gini index (when dealing with the 33 countries considered here) is supported by the LD only in less than 50% of the cases, whilst e.g. the 2-DD supports the Gini index in almost 75% of the cases.

Country	Γ	Country	Γ
Austria	0.541,{24}	**Lithuania**	0.754,{2}
Belgium	0.519,{26}	**Luxembourg**	0.564,{20}
Bulgaria	0.732,{4}	**Malta**	0.561,{21}
Croatia	0.610,{14}	**Macedonia**	0.672,{10}
Cyprus	0.668,{11}	**Netherlands**	0.530,{25}
Czech Rep.	0.502,{27}	**Norway**	0.479,{31}
Denmark	0.547,{23}	**Poland**	0.607,{15}
Estonia	0.689,{6}	**Portugal**	0.675,{9}
Finland	0.501,{28}	**Romania**	0.744,{3}
France	0.581,{19}	**Serbia**	0.759,{1}
Germany	0.597,{16}	**Slovakia**	0.470,{32}
Greece	0.681,{8}	**Slovenia**	0.488,{30}
Hungary	0.561,{22}	**Spain**	0.687,{7}
Iceland	0.469,{33}	**Sweden**	0.499,{29}
Ireland	0.592,{17}	**Switzerland**	0.586,{18}
Italy	0.642,{12}	**U.K.**	0.640,{13}
Latvia	0.705,{5}		

Table 3 Gini indices and rankings for the EU-28 countries and the other non-EU countries

It is also worth mentioning some special cases. The LC of Czech republic LD-dominates 19 of the other 32 countries. However, it can be seen in Figure 1 that the left tail of the LC of Czech republic is above all the others. Indeed, by using upward 2-LD, downward 2-LD and 2-DD, one obtains that Czech Republic dominates 24, 19 and 21 countries, respectively. The reason is that upward 2-LD emphasizes the left tail.

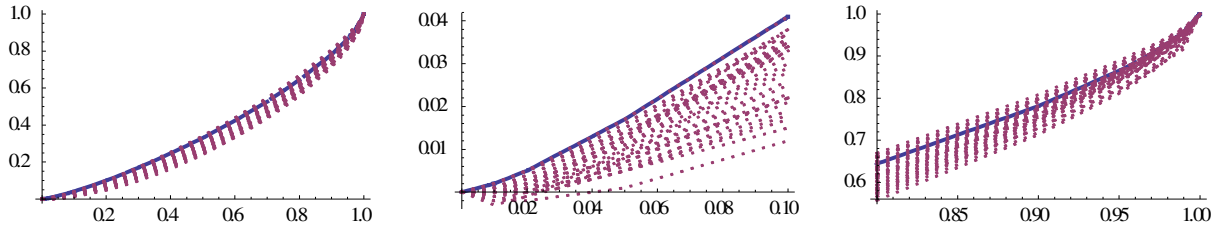

Figure 1 LCs of Czech Republic (solid form) and the other 32 countries (dotted form)

Another interesting case is represented by Iceland, which obtains the minimum value of the Gini index, although, strangely, it does not dominate any of the other countries because it presents also the minimum value of the LC for p=0.01 (see figure 2). Indeed, by using upward 2-LD, downward 2-LD and 2-DD, one obtains that Iceland dominates 0, 27 and 21 countries, respectively. The reason is that downward 2-LD) emphasizes the right tail.

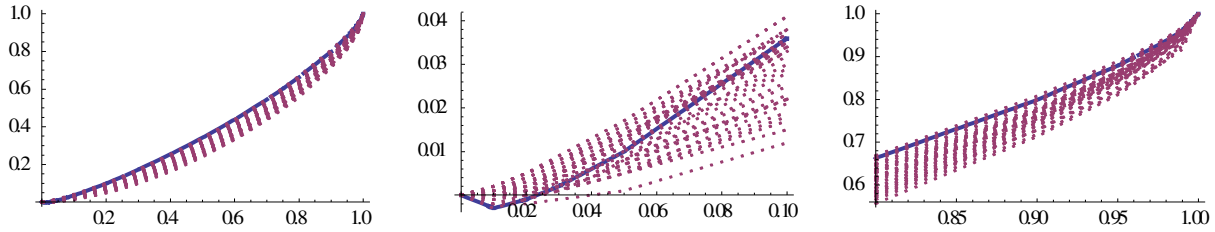

Figure 2 LCs of Iceland (solid form) and the other 32 countries (dotted form)

The examples above let us argue that one should find a compromise between the upward and downward 2-LD approach, in order to obtain more balanced results. This may be represented by the 2-DD, which emphasizes both tails at the same time.

4 Conclusion

Analysis of recent statistics on income inequalities in the European Union is undoubtedly a current topic. Comparisons of standards of living between countries are frequently based on gross domestic product (GDP) per capita, which presents in monetary terms how rich is a country compared to another. However, this headline indicator says very little about the distribution of income within a country and also fails to provide information in relation to non-monetary factors that may play a significant role in determining the quality of life of a particular population. On the one hand, inequalities in income distribution may create incentives for people to improve their situation through work, innovation or acquiring new skills. On the other hand, such income inequalities are often viewed as being linked to crime, poverty and social exclusion. Policymakers in general cannot combat poverty and social exclusion without analysing the inequalities within society, whether they are economic or social in nature.

In the paper, income inequality was measured by the Gini coefficient and represented by Lorenz curve. The shapes of the LCs of the EU-28 are somewhat similar and the main differences can be found in the right tails (high incomes) rather than in the left one. This also implies that downward 2-LD is effective, although most economists recommend ranking LCs on the basis of upward 2-LD. However, 2-DD combines upward and downward 2-LD so that it seems to be a wise compromise between the two approaches, besides providing the highest rate of completeness. The properly measured EU-wide Gini coefficient of disposable income inequality shows that inequality in the EU as whole are still real and not negligible. In Table 3, the Gini index is shown for the whole sample of evaluated countries. In the case of 33 European countries, the value of the Gini index in

2015 ranged from 0.469 (the best results for Iceland, Slovakia and Norway) to 0.759 (the worst results in Serbia followed by Lithuania and Romania). We observe that the most extreme values of the Gini index are obtained for non-EU countries (Serbia and Iceland). Countries at the top of the ranking are Slovenia, Sweden and Finland with the Gini indices from 0.488 to 0.501, whilst those at the bottom are Bulgaria, Latvia and Estonia, having the Gini index from 0.732 to 0.689. Then, the remainder countries can be broadly divided into two groups, with Croatia, Cyprus, Macedonia, Greece, Italy, Poland, Portugal and United Kingdom having the Gini index up to 0.6 value, and others such as Austria, Belgium, Czech Republic, Denmark, France, Germany, Hungary, Ireland, Luxembourg, Malta, Netherlands, Spain and Switzerland having values of the Gini index up to 0.5. Thus, we have two quite large sets with very similar values of the Gini index. Moreover, dominance results show that the Gini index is not sufficient to capture the real differences between the income distributions.

Within the EU, there are differences in income inequality which require policy action. It is important to remember that high levels of income inequality have various negative consequences (including poor health outcomes, weak social mobility and potential swings towards political populism). Therefore, efforts to address income inequalities should be stepped up in a number of countries as well as at the EU level. Many politicians and economists believe that economic growth replaces or diminishes the need for social policies. However, the EU growth over the last decades has been accompanied by increase in inequalities in many countries. Inequalities threaten social cohesion and they threaten growth. Inequality is a key problem facing the EU, and it has significant impacts not only on human well-being, but also on economic performance. If such concerns are correct, it is essential not only to build institutional structures for European social union but also to map social inequalities (including income dispersion) in the EU. Cohesion is about income and employment, but also about other dimensions of well-being.

The recent interest in income inequality is thus simply the recognition of the centrality of the topic to economic theory, policy, and performance. The recent return of the topic of income inequality has been triggered by important contributions to the empirical analysis of inequality, but these empirical analyses must be combined with an economic theory that is adequate to address the macroeconomic and microeconomic effects of income inequality on social welfare. These problems are not always well diagnosed because the empirical measurement of inequality is often unable to take into account the geographical dimension of inequality, which is particularly complex in the EU. To study income inequality in the EU as a whole, one needs adequate statistical tools which can be used in the geographical and political context faced by the EU. Important is also the relevant database as source for required data and indicators. In the case of the EU analysis, the relevant source is Eurostat, i.e. the official statistical office of the EU, but there are also some data particularities in Income and living conditions statistics, e.g. negative incomes. A deeper exploration of the underlying data shows that among those whose income is in the lower tail, negative income components are at work; these negative components can be due to self-employment, tax burden, transfers to other households or loss in property income.

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