New approaches to modelling heat/mass transfer processes in spherical and non-spherical mono-component droplets

S.S. Sazhin[*1,2,3,4], S. Tonini[4], G.E. Cossali[4], D.V. Antonov[3], P.A. Strizhak[3]
[1]Advanced Engineering Centre, School of Architecture, Technology and Engineering, University of Brighton, Brighton, BN2 4GJ, United Kingdom
[2]Kutateladze Institute of Thermophysics, Siberian Branch of the Russian Academy of Sciences, 1 Lavrentiev Avenue, Novosibirsk 630090, Russian Federation
[3]National Research Tomsk Polytechnic University, 30, Lenin Avenue, Tomsk, 634050, Russian Federation
[4]Department of Engineering and Applied Sciences, Università degli Studi di Bergamo, Viale Marconi 6, 24044 Dalmine (BG), Italy
*Corresponding author: S.Sazhin@brighton.ac.uk

Introduction

The importance of accurately modelling the heating and evaporation of spherical and non-spherical droplets is well known and has been described in numerous papers and books summarised in the monographs [4, 5]. These processes have many industrial applications, including spray drying and combustion, some of which are discussed in [4, 5]. This abstract is focused on a brief analysis of new approaches to modelling heat/mass transfer processes in spherical and non-spherical mono-component droplets, obtained after the publication of [4, 5].

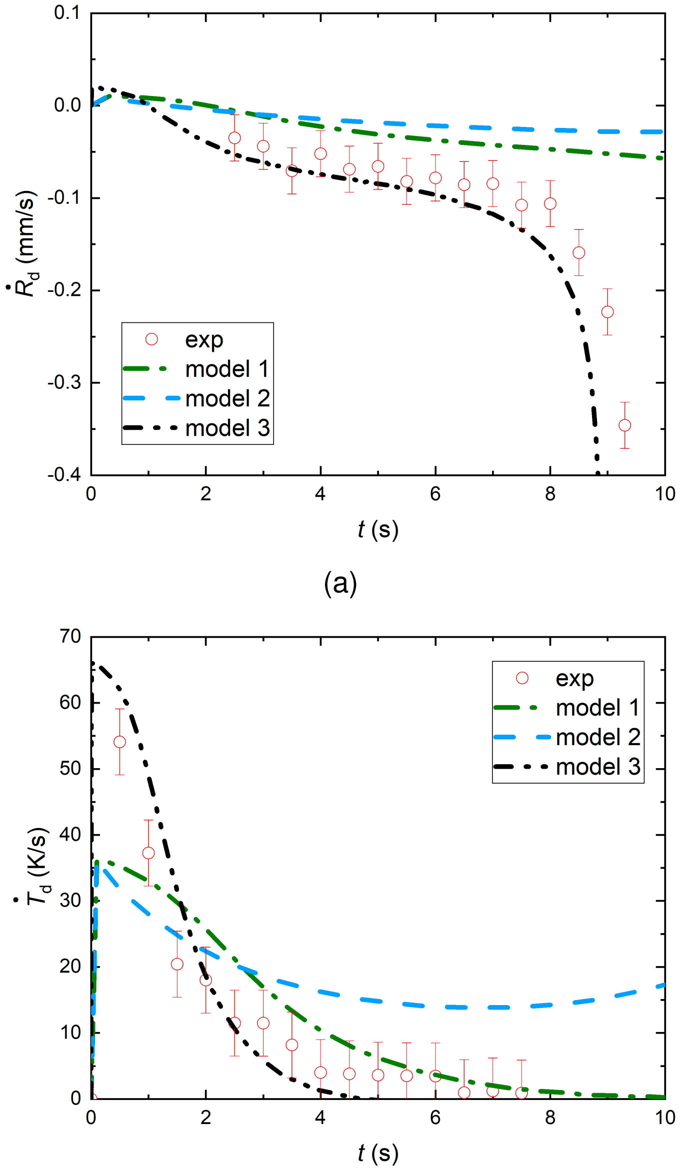
Material and methods

The abstract is focused on the analysis of spherical and spheroidal droplets heated in ambient gas (air), based on a combination of analytical and numerical methods. The analytical solutions of heat and mass transfer equations in the gas and liquid phases were obtained, if possible. Then these solutions in the gas phase were used as boundary conditions for the liquid phase. Thus the new analytical-numerical method was developed and applied to a specific problem of droplet heating and evaporation. In this method the predictions of the analystical or numerical solutions at the end of each time step are used as initial conditions for the following time steps with updated values of input parameters

Results and Discussion

Recent developments in the modelling of heating and evaporation of spherical and non-spherical mono-component droplets ([6, 2, 1, 3]) are reviewed. These include the model linking the previously developed liquid phase model, using the analytical solution to the heat transfer equation at each time step, and the gas phase model, using the solution to the equations of the conservation of mass, momentum, and energy leading to an explicit expression for the Nusselt number and implicit expression for evaporation rate of the droplet [2] (called Model 3). The predictions of the numerical algorithms for this and the previously developed approaches are compared with experimentally observed time dependencies of the rates of change of radii and average temperatures of n-decane droplets at initial temperatures and radii equal to 300 K and 0.85 mm, respectively, placed in a gas at temperature 760 K. The results of the comparison are shown in Figure 1. In Model 1, to which Figure 1 refers, the heat rate, supplied to the droplets to raise their internal energy, is calculated based on the observation that steady-state equations for heat and mass balance in the gas phase should lead to the same droplet evaporation rates. The direct calculation of the above-mentioned heat rate is used in Model 2; the value of this rate is then used for the estimation of the droplet evaporation rate using the Spalding heat transfer number [1].

It can be seen from Figure 1 that the experimentally observed $\dot{R}_d$ and $\dot{T}_d$ are reasonably close to those predicted by Model 3. The deviations between the observed values of $\dot{R}_d$ and $\dot{T}_d$



(a)

(b)

Figure 1. Plots of rate of change of radius ($\dot{R}_d$) and average temperature ($\dot{T}_d$) of an n-decane droplet versus time predicted by Models 1, 2 and 3 and observed experimentally (exp). Gas temperature T_g was 760 ± 10 K; initial droplet temperature (T_{d0}) and radius (R_{d0}) were 300 ± 10 K and 0.85 ± 0.05 mm, respectively. Copyright Elsevier (2024)

and those predicted by Models 1 and 2 are are quite large. These models even predict different trends of change for these parameters, which limits their suitability for studying the time evolution of these parameters during droplet heating/evaporation. The trends similar to those illustrated in Figure 1 were also shown for for the ambient gas temperature equal to 500 ± 10 K. A mathematical model for spheroidal droplet heating and evaporation takes into account the effect of liquid finite thermal conductivity and is based on the previously obtained analytical solution for the vapour mass fraction at the droplet surface and a new correlation for the convective heat transfer coefficient incorporated into the numerical code. The heat transfer equation in the liquid phase is solved numerically using the finite element heat transfer module of COMSOL Multiphysics [3].

It is shown that the lifetimes of prolate and oblate droplets is shorter than that of spherical droplets of the same volume. The difference in these lifetimes is shown to increase with increasing aspect ratios for prolate droplets and decreasing aspect ratios for oblate droplets. The predictions of the model are shown to agree with experimental data. The maximal surface temperatures are predicted near the areas with the maximal surface curvatures. The aspect ratios were shown to be weakly dependent on time, which is consistent with experimental data.

Conclusion

A recently developed model of heating and evaporation of spherical droplets is described. This model links the previously developed liquid phase model, using the analytical solution to the heat transfer equation at each time step, and the gas phase model, based on the solution to the equations of the conservation of mass, momentum, and energy which leads to an explicit formula for the Nusselt number and a realtively simple equation for the evaporation rate of the droplet. It is shown that this model predicts the results closer to experimental data than the previously developed models.

A recently developed model for spheroidal droplet heating and evaporation is described. This model considered the effect of liquid finite thermal conductivity and is based on the analytical solution for the vapour mass fraction at the droplet surface and a correlation for the convective heat transfer coefficient. Both were incorporated into the numerical code.

Nomenclature

R	droplet radius [m]
t	time [s]
T	temperature [T]

Subscripts

d	droplet
g	ambient gas
0	initial

Acknowledgments

Financial support received from the Tomsk Polytechnic University development program, Priority 2030 (Priority-EB-018-202-2024), Ministry of Science and Higher Education of the Russian Federation (Grant 075-15-2024-620) (DA and SS) and the University of Bergamo (SS's visiting professorship) is gratefully acknowledged. The research presented in the paper was initiated during work on a project supported by the Royal Society (UK) (Grant no. IEC 192007).

References

[1] D. V. Antonov, S. Tonini, G. E. Cossali, M. Al Qubeissi, P. A. Strizhak, and S. S. Sazhin. *International Journal of Multiphase Flow*, 179:104922, 2024.
[2] D. V. Antonov, S. Tonini, G. E. Cossali, V. V. Dolgikh, P. A. Strizhak, and S. S. Sazhin. *Physics of Fluids*, 35:073311, 2023.
[3] D. V. Antonov, S. Tonini, G. E. Cossali, P. A. Strizhak, and S. S. Sazhin. *Applied Mathematical Modelling*, 125:687–703, 2024.
[4] G. E. Cossali and S. Tonini. *Drop Heating and Evaporation: Analytical Solutions in Curvilinear Coordinate Systems*. Springer, 2021.
[5] S. S. Sazhin. *Droplets and Sprays: Simple Models of Complex Processes*. Springer, 2022.
[6] S. Tonini, G. E. Cossali, E. A. Shchepakina, V. A. Sobolev, and S. S. Sazhin. *Physics of Fluids*, 34:073312, 2022.