

Digital guidework for augmented thin-tile vaulting construction

Robin Oval ^{a,b} ^{*}, Vittorio Paris ^c, Rafael Pastrana ^{a,d}, Edvard P.G. Bruun ^{a,e}, Salvador Gomis Aviño ^f, Sigrid Adriaenssens ^a, Wesam Al Asali ^g

^a Form Finding Lab, Department of Civil and Environmental Engineering, Princeton University, Princeton, USA

^b Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, The Netherlands

^c Department of Engineering and Applied Sciences, University of Bergamo, Bergamo, Italy

^d School of Architecture, Princeton University, Princeton, USA

^e Bruun Automation Research Lab, School of Civil and Environmental Engineering, Georgia Institute of Technology, USA

^f Bóveda Tabicada, Valencia, Spain

^g School of Architecture and Design, IE University, Segovia, Spain

ARTICLE INFO

Keywords:

Structures
Shells
Masonry
Digital fabrication
Augmented reality

ABSTRACT

Masonry vaults are mechanically efficient structures but deemed uneconomical because of falsework construction. Even a craft like thin-tile vaulting, which does not require centering to support the vault during construction, needs time-consuming guidework to aid the builders follow the vault's geometry. However, this visual support can be digitized, using augmented reality to create digital guidework. The proposed methodology provides a framework that empowers vault builders to remain in control of their analog craft by providing only the right digital visual information. This methodology was developed through a preliminary prototype that led to a demonstrator built in an uncontrolled outdoor environment. Construction results showed productivity gain around 30% in terms of time, and shape accuracy under 1% of the span. The static holographic projection of the guidework could be extended in future research into an interactive aid, through mixed reality for further construction productivity and accuracy, as well as for training and design.

1. Introduction

1.1. Challenges in thin-tile vaulting

Thin-tile vaulting, also called tile vaulting, timber vaulting, or Catalan vaulting, is a traditional masonry vault construction technique that relies on a fast-setting mortar to fix lightweight thin tiles without using a centering (a timber and/or steel scaffold) to support them during construction (López López et al., 2016). The first layer of tiles then provides support to the additional layers, with the masonry courses usually alternating in different directions. This vaulting technique was widespread in Medieval Spain, with the oldest thin-tile vault dated so far being the Hospital of Santa Maria in Lleida, Catalonia, built in 1352. This technique resembles Ancient Roman vaulting, which used fast-setting mortar with thick bricks (Bergos, 1965). Rafael Guastavino transferred this technique from Spain, where he built the Batlló Factory in Barcelona in 1868, for example, to the USA in the 19th century (Ochsendorf, 2013), where many examples can be found today, including a market under Queensboro Bridge in New York City, built in 1909 (Collins, 1968). This craft has remained popular throughout the

centuries and across the world, with examples throughout Cuba, Syria, Rwanda, Jordan, or Spain (Al Asali, 2020).

The strength of a gypsum mortar, or Paris plaster, and the lightness of fired clay hollow thin tiles allow for the reduction of the centering. However, another type of falsework is necessary: guidework. Guidework is intended to provide direction to the builder to produce the desired form of the vault. For primitive shapes, with piecewise constant curvature, such as spherical domes or pointed domes with two radii of curvature, guidework could be made of a simple inextensible string attached to the center of curvature on one end and the other end tracing in space (Fitchen, 1981). As Computer-Aided Design (CAD) technologies have allowed designers to explore more complex shapes with variable radii of curvature, guidework is necessary to describe the vault's shape to the builder. Indeed, complex forms are hard to describe through traditional means like 2D projections, plans, and elevations, and lead to challenges for achieving construction productivity and accuracy. Construction productivity is essential for economic affordability, as labor is the main cost of the construction of a vault. Construction accuracy is essential for structural safety, as imperfection has a large influence on the buckling of a vault. The guidework is

* Corresponding author at: Faculty of Civil Engineering and Geosciences, Delft University of Technology, Delft, The Netherlands.
E-mail address: robin.oval@enpc.fr (R. Oval).

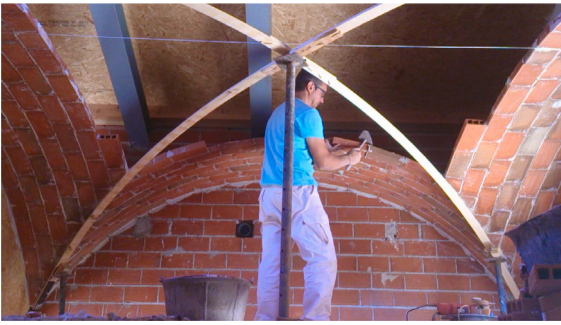


Fig. 1. Standard construction of a thin-tile vault in contemporary Spain using physical guidework — by Salvador Gomis Aviño.

usually made of a set of flexible rods, in metal, wood, or plastic, fixed to a series of poles, or a light timber or cardboard waffle frame. The guidework, though potentially offering some support, is mainly meant to be a visual aid. Yet, designing, producing, positioning, and removing the guidework is a time-consuming task, with a significant impact on the affordability of the construction process, which increases with the complexity of the vault (Ramage et al., 2010; Davis et al., 2011, 2012; López López et al., 2014; Ramage et al., 2019). Fig. 1 shows an example of thin-tile vaulting in contemporary Spain, where strings and splines are positioned to describe the straight lines of the ruled surfaces and the creases of a cross vault. A finer resolution of the guidework for increased accuracy means installing more elements, especially for surfaces with more complex double curvature, which results in lower productivity, as the elements need to be assembled and disassembled, and clutter the workspace during construction.

1.2. Literature review on augmented construction

Extended Reality technologies (XR) have found application in the Architecture/Engineering/Construction (AEC) industry for both design and construction. XR is the umbrella concept where reality and virtuality are fused, where the user can be immersed in a digital environment as Virtual Reality (VR), visualize digital elements in a physical environment as Augmented Reality (AR), or play with the interaction between digital and physical elements as Mixed Reality (MR) (Milgram and Kishino, 1994). The opportunities, challenges, and trends of XR have been studied in several reviews, some focusing on the field of AEC (Wang, 2009; Chi et al., 2013; Rankohi and Waugh, 2013; Wang et al., 2013, 2021). Particularly, VR and MR have the potential to increase productivity in construction (Abraham and Annunziata, 2017), and become more ubiquitous with the digitization of the industry, including the advancement of digital information, Building Information Modeling, Digital Twin technology, and Lean Construction frameworks (Ko and Kuo, 2015). Augmenting thin-tile vaulting through digitization of guidework and leveraging the potential of XR can contribute to making this construction craft more affordable. This technology would then lead to making sustainable vaults, which are material efficient but still labor intensive today, economical again. The development of XR technologies in the construction industry, and in design and manufacturing in general, has been motivated by a seamless transition from digital (design) to physical (fabrication) without the use of analog blueprints. Printed blueprints require the sectioning of the 3D building or object into a set of 2D plans and sections, which can be prone to errors in general, particularly for complex geometries. Indeed, digital design methods for the form finding and optimization of masonry vaults, like Dynamic Relaxation (Barnes, 1999), Force Density Method (Schek, 1974), or Thrust Network Analysis (Block and Ochsendorf, 2007), allow for the generation of efficient structural forms that differ from the canonical shapes of the spherical or parabolic

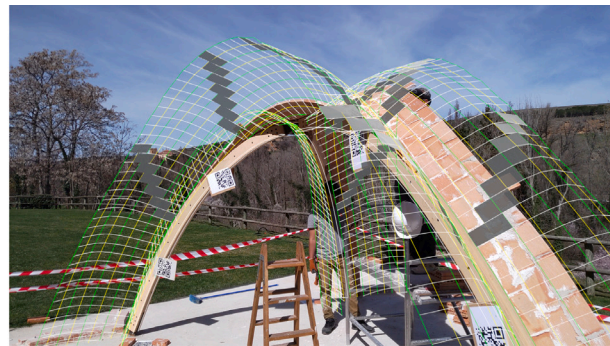
domes and the barrel or cross vaults, for instance. These efficient structures however can become unaffordable to build as the cost of the falsework, including the centering and the guidework, rises. AR enables the overlap of a digital world with the physical world, seen from a phone, a tablet, or a headset. Applications range from gaming and education to industrial applications in AEC. More specifically, AR has been applied to different tasks for augmented construction, as a suitable digital technology for the assembly of discrete structures made of blocks and beams. Assembly applications include stacking, bending, connecting, gluing, welding, printing, and milling different materials like timber, steel, plastic, foam, and masonry. AR lends itself to discrete assembly construction, particularly masonry, due to the complexity of this composite material, made of bricks and mortar. Automation of masonry construction using robotics has been explored for walls using a mobile robotic arm (Dörfler et al., 2016), vaults using a cooperative robotic set-up (Parascho et al., 2021; Bruun et al., 2024), or modular masonry structures, walls, and vaults, using drones (Goessens et al., 2018). However, vaults can feature a changing double curvature that requires a complex tessellation with variable mortar joints, which challenges full automation and highlights the value of human craft, intelligence, and adaptation. Therefore, AR is a promising technology to leverage augmented craft for masonry construction. Researchers have explored craft-oriented interactive augmented construction of complex masonry walls (Fologram and All Brick, 2020; Mitterberger et al., 2020), as well as combining AR and robotics for collaborative construction of vertical masonry structures, with humans laying glue using AR and robots positioning bricks (Fazel and Izadi, 2018; Fazel et al., 2022; Song et al., 2022b). Vertical structures like walls show the assembly sequence, step by step, layer by layer, to avoid cluttering the visual space, for readability and safety. More specifically to masonry vaults, researchers at the Universidad Nacional Autónoma de México (UNAM) built a vault using AR, visualizing the shape of the vault through a tablet using the software Fologram for the positioning of the light but complex falsework, centering and guidework (Oliva Salinas, 2022). Furthermore, Pittet Artisans Sarl used AR for the construction of a state-of-the-art helicoidal tile-vault staircase with a headset in a real-world context (Pittet Artisans Sarl, 2023). Static AR has mostly been the focus among applications of AR to masonry construction so far, although interactive AR has shown interesting results, adding the possibility to inform the builder during the construction process (Mitterberger et al., 2020). Overall, AR has proven to increase productivity and accuracy in the construction of complex brickwork (Jahn et al., 2020).

1.3. Problem statement and research objectives

Thin-tile vaulting is an efficient construction craft that needs little centering to support the bricks during the construction. However, guidework is still needed to inform the builder about the shape of the vault. As physical guidework is unfavorable to construction affordability, this research proposes the development of a *digital guidework* using AR to visualize both the shape and the tessellation of the entire vault. How can AR contribute to the digitization of *shape guidework* and add *tessellation guidework* for masonry vault construction? The representation of the digital guidework is essential for intuitive understanding by the builder with the just-needed visual data, describing the whole vault, as opposed to sparse physical guidework. AR as a tool should not replace but empower the builder and the craft, as continuous on-site construction problem-solving occurs thanks to the builder's manual and cognitive skills, to deal with imperfections, tolerances, and unforeseen events. Such a new approach should enable an improvement in both construction productivity and accuracy compared to the state-of-the-art, as augmented reality has not been leveraged yet as a head-mounted device for the masons to build tile vaults with a complex shape.



(a) Small vault built in lab environment



(b) Large vault built in outdoor environment

Fig. 2. Digital guidework for the vaults built for methodology development and demonstration of augmented vault construction.



Fig. 3. Augmented thin-tile vault construction using digital guidework.

1.4. Contributions and outline

This paper explores the potential of digital guidework as a static holographic projection through AR to reduce falsework for thin-tile vault construction. Overall, this paper demonstrates how augmented reality can substitute physical guidework and make vault construction more economical. Nevertheless, physical centering, which acts as a temporary support, not just a visual guide, is still required for structural shapes with free-standing boundaries. State-of-the-art hardware and software technologies are integrated into a workflow for a novel application, masonry vaults with complex geometries. The methodology is presented in Section 2, including the development of the digital model of guidework, informing the builder about both the shape and the tessellation of the masonry vault. The results are detailed in Section 3 for a preliminary prototype, the 1 m² shallow sail vault in Fig. 2(a) built in an indoor controlled environment, and a demonstrator, the 17 m² vault in Fig. 2(b) with a complex shape built in an outdoor uncontrolled environment. The results highlight the construction productivity and accuracy. The design methodology, so-called 4D funicular design, and the falsework economy of the demonstrator are reported separately in Oval et al. (2024). The potential of this methodology outlined in Section 4 provides suggestions for improvement and development based on the construction experience drawn about current limitations by using XR technologies for the construction of masonry vaults.

2. Methodology

This section presents the methods developed for AR in thin-tile vault construction, as shown in Fig. 3, including workflow, registration of the physical world with markers, digital model for guidework, and post-construction shape control through photogrammetry. The methodology described in this section was developed and refined through building the two vaults presented further in Section 3.

2.1. General workflow

The general workflow is shown in Fig. 4 along with the software used at the different stages:

1. the vault was modeled with CAD before extracting site reference points and the intrados geometry of the vault, using the CAD software Rhino3D, its parametric design environment Grasshopper3D, and Python;
2. once designed, the reference points from the model were extracted and materialized as markers (Section 2.2) for on-site registration between the physical space and the digital guidework (Section 2.3), which stems from the intrados geometry, with data conversion using Twinbuild (Twinbuild, 2021) exported to a Microsoft HoloLens 2;
3. the vault was constructed using tile-vault craft; and
4. after the vault was completed, a digital scan of the vault was made through numerous photos processed with photogrammetry using the open-source framework AliceVision Meshroom (Griwodz et al., 2021). Then, the accuracy of the as-built geometry was measured with respect to the as-designed one, processed using Rhino3D, Grasshopper3D, and Python (Section 2.4).

Thin-tile vaulting requires the builder to use both hands, one holding and positioning the tile and the other holding the trowel to put mortar on the edges of the tile and tap the tile to adjust its position. Therefore, an external system like a handheld device, a tablet, or a phone, was not compatible. A wearable like a head-mounted display (HMD) was required to continuously visualize the guidework. Two HMDs were used on site, the builder continuously using one while the other was being charged but also to use them simultaneously for discussion and planning between two builders. Although only one builder used AR to lay tiles, multiple builders could collectively construct the vault (Atanasova et al., 2022). The model of the digital guidework was uploaded onto the HMD only once as a static object.

2.2. Physical registration

The markers for calibration between the physical and digital worlds were positioned to be as easily accessible to the builder as possible. For the demonstrator, the markers were placed on the centering, facing the vaulting direction, as close as possible to the vault, to always be within the field of view to minimize the movements of the builder. Therefore, the centering provides both support and guidance for the positioning of the tiles. To prevent conflicts due to pairs of markers at the same location but facing opposite directions, the main digital guidework model was split. The main model was copied into separate models with

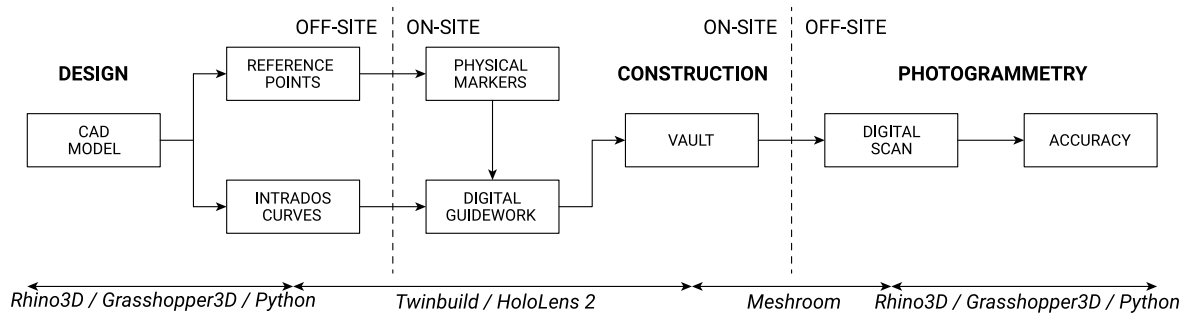


Fig. 4. Workflow for augmented vault construction, from design to assessment.

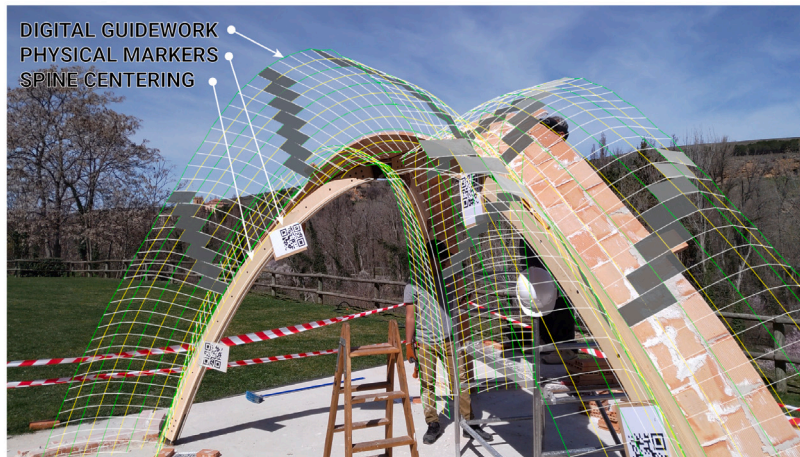
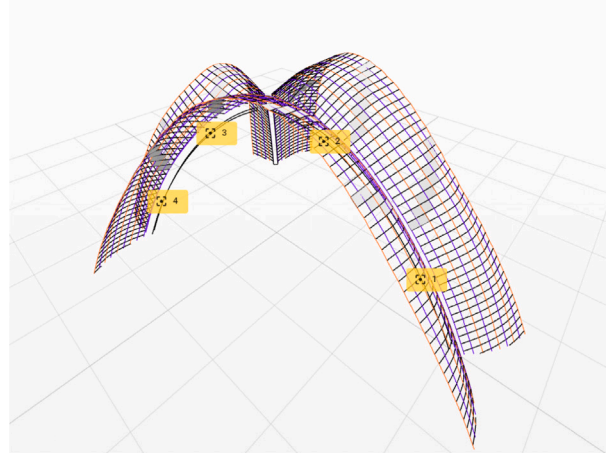
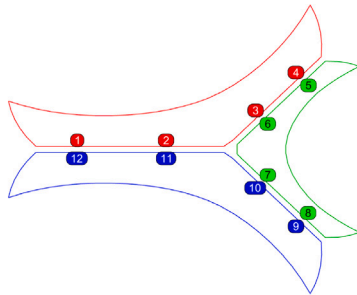


Fig. 5. Registration with physical markers mapped along the centering facing the builder for convenient registration.

the complete geometry but different sets of markers, corresponding to the different sides of the centering, shown in red, blue, and green in Fig. 5(a). The model was selected by the builder based on the current work location to visualize the full digital centering with only a subset of markers, as shown in the model in Fig. 5(b). The markers were spaced by about 2 m and materialized as large QR codes printed at a size around 20 cm × 20 cm, in black and white for high contrast, with a 1 cm white border, on a non-reflective paper glued on a stiff board, as shown in Fig. 5(c).

2.3. Digital guidework

The main challenge lied in providing just the right amount of data to the builder. Enough to understand the shape and the tessellation of the vault, while allowing enough freedom for on-site decisions to deal with tolerances, and without polluting the model with unnecessary and distracting information that obstruct the visual space. The digital guidework was positioned at the intrados of the vault, to intuitively mimic physical guidework, located under the bricks. The vault consisted mostly of a regular staggered tessellation, with courses visualized

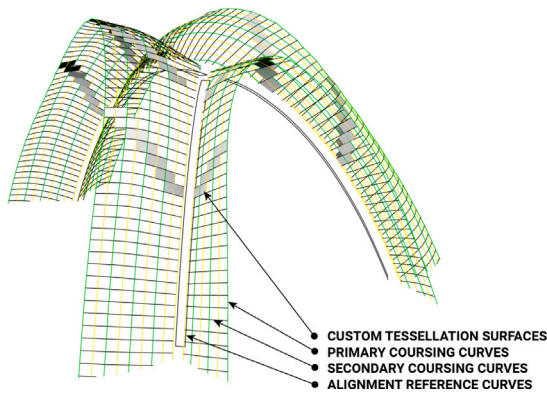


Fig. 6. Digital model for augmented vault construction.

with curves in the digital guidework. At regular intervals, these regular courses were interrupted by loxodromes consisting of transverse tiles. These special tiles required a specific emphasis, as opaque surfaces, in the digital guidework. Hence the hierarchy between the geometrical objects: curves for regular courses and surfaces for singular tiles.

The digital model, shown in Fig. 6, consisted of:

- a primary set of curves, spaced by the width of a tile showing the interface between the main courses in a regular staggered masonry pattern, in alternating colors for easier visualization (in green and yellow);
- a secondary set of curves (in black) transverse to the primary ones, spaced by half the length of a tile. These curves were not meant to show the exact position of the tiles along each course, which was defined by the builder, but were meant to help better visualize the double curvature of the surface, like graph paper;
- a set of rectangular surfaces (in gray) mapped on the mesh formed by the two sets of curves to highlight the position of the tiles that derive from a custom tessellation that differs from the regular staggered pattern, here the loxodromes;
- the outline of fixed elements like the centering of the vault to judge alignment accuracy and notice digital model drifts suggesting the need to register the markers again.

Unlike work on augmented construction of walls (Mitterberger et al., 2020; Jahn et al., 2020; Song et al., 2022b) where the stacked layers are shown step by step to avoid cluttering the visual space, the entire vault is shown at once, as it does not distract nor obstruct the vision of the builder. Moreover, it helps visualize and prepare for the next steps, foreseeing challenges, as the shape of the vault and its tessellation highly depend on the previous steps.

2.4. Photogrammetry

To assess the accuracy of the as-built geometry of the vault against the as-designed one, photogrammetry was performed to obtain a digital scan through numerous photos taken of the vault from different positions, automatically matching image features between overlapping images to recreate a depth map of the vault. The resulting point cloud was then converted into a mesh, which was post-processed, simplified, and remeshed to obtain a final mesh that strikes a compromise in terms of density to reproduce faithfully the geometry using a reasonable number of elements. The methods and results are detailed per application in the following section.

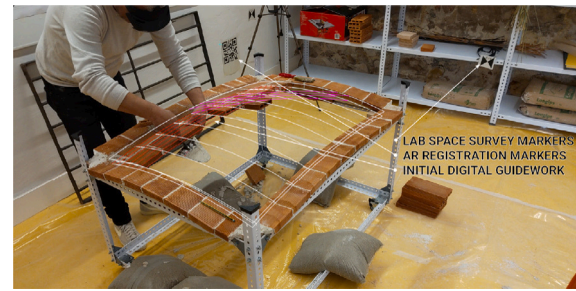


Fig. 7. Construction set-up of the preliminary prototype with digital guidework.

3. Application

This section presents the application of the methodology through the construction of two thin-tile vaults using AR to visualize digital guidework. The first vault is a preliminary small-scale test in a controlled lab environment and the second is a real-scale demonstrator in an uncontrolled outdoor environment. As both these vaults have free-standing boundaries (unlike a dome, for instance), a centering is needed to build the first set of arches and span from there using guidework only.

3.1. Preliminary prototype

This preliminary prototype was used for the development and testing of the methodology, for introducing a new technology to the vault master builder, and for tuning the type and amount of data shown with digital guidework.

This small-scale prototype had the simple form of a shallow sail vault with a width of 0.9 m, a length of 1.2 m, and a rise of 0.1 m. The single-layer regular staggered tessellation was complemented by a loxodrome, a series of 5 tiles transverse to the regular masonry courses. Construction started with building the four boundary arches on a cardboard centering before building the interior part of the vault following a digital guidework. Construction occurred indoors, in the controlled environment of a lab, with uniform artificial lighting conditions, as shown in Fig. 7. A survey of the lab space was initially done to obtain the position of the QR codes as AR markers. The survey was carried out based on a grid of survey markers spaced vertically by about 0.8 m and horizontally by about 1.2 m. A smartphone (12 MP and 20 MP $f/2.2$) was used to take 148 pictures and the digital survey was performed using 3DF Zephyr (Anon, 2024). The resulting survey had an accuracy below 9 mm. With this survey, the position of the AR markers was known both in the physical world and the digital model to register the digital guidework.

After construction, the vault was scanned to compare the as-built extrados with the as-designed extrados. The extrados was used because the vault was too low to obtain high-quality images of the intrados. The mesh was produced using photogrammetry and scaled based on reference points in the scene, fixed with tape on the vault. Finally, the design model and the mesh scan were aligned using their horizontal bounding boxes and the vertical distance between the two models was minimized through a best-fit step. This adjustment was made by minimizing the average distance between a set of 76 reference mesh vertices from the scan with the design surface. This optimization step was performed with Goat's implementation of NLOpt's COBYLA (Powell, 1998) for Grasshopper3D. The resulting average best-fit distance for these reference mesh vertices was 4.0 mm. Measuring the distance of each mesh vertex of the as-built extrados to the as-designed one resulted in an average distance of 3.6 mm with a standard deviation of 2.8 mm. This average distance of 3.6 mm represents 0.3% of the 1200 mm maximum span of the vault. The maximum deviation of

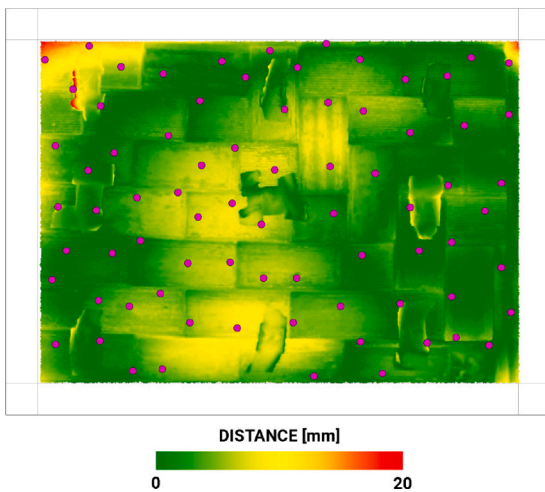


Fig. 8. Accuracy assessment between the as-built and as-designed vault extrados geometries using photogrammetry. The points in pink show the 76 reference mesh vertices used for registration.

22 mm was located at the corners where the tiles must meet at the narrow supports, inducing construction deviations. Fig. 8 shows the colored scanned mesh, between the boundary arches built on centering with their outlines in black, and the 76 reference mesh vertices for height adjustment in pink.

3.2. Demonstrator

After prototyping at a small scale, the methodology was then demonstrated on a larger vault, with a more complex geometry and built outdoors in an uncontrolled environment.

The shape of the vault was based on 3 boundary arches, the two long ones spanning about 5 m and the short one spanning about 3 m, for a maximum height of 2.8 m and a surface area of 17 m². The complex geometry was form-found based on structural and construction principles to achieve a state of compression in the vault during the entire construction process, therefore minimizing the need for falsework, in an approach called 4D funicular design, detailed in Oval et al. (2024). The masonry tessellation was based on a regular staggered pattern combined with 12 loxodromes of tiles crossing the tessellation transversely and creating a support between successive arches of the vault. The first layer of thin tiles was completed by a second layer of thick bricks following a regular tessellation. Construction started from a centering shaped as a tripod from which the vault was built outwards with digital guidework. By the end of the construction process, two extra tile arches were built on the long sides of the vault, whereas two tile arches that were initially planned were not built on the short side. The centering was fabricated accurately using CNC and its known geometry was used as a reference, discarding the need for a survey of the construction site as done for the preliminary prototype for registration.

3.2.1. Augmented construction

AR was used for several applications when building the vault: first for marking the position of the foundation bases of the vault; then mainly as digital guidework for the construction of the vault itself; and additionally for testing the installation of a local physical guidework.

Marking foundation base. The supports of the vault needed to be positioned accurately for its structural integrity. First, the base triangle of the vault was drawn manually on the slab as a reference for scaling and positioning. The vault was built on a concrete slab, thrusting against a reinforced masonry base anchored to the foundation thanks to vertical

steel reinforcement drilled into the slab. The masonry base at each of the three supports consisted of two rows and two levels of bricks that followed the curved V shapes of the geometry of the vault. A model consisting of the interface between the two rows of bricks and the position of the holes to drill for the reinforcement bars was generated and marked on the slab using AR. The extremities of the V shapes were eventually manually checked by measuring their relative locations, which was within a 10 mm precision. Fig. 9(a) shows the marking of the axis of the masonry bases (as lines) with the position of the rebars (as circles).

Vaulting with digital guidework. AR was mainly used as digital guidework for the entire vault to discard the need for physical guidework. The AR model provided a guide for both the shape of the vault intrados and its tessellation, the main courses, and the transverse loxodromes, as shown in Fig. 9(b). AR was not needed for the second layer of the vault, which consisted of a regular staggered tessellation only, following the tessellation of the first layer below with an in-plane offset of half the width of a brick.

Positioning physical guidework. To increase productivity, by using one HMD at a time but building the vault from several locations, an attempt was made to install physical guidework in a specific location using AR before building this part of the vault without HMD. This guidework was added at the apex of the short span of the vault, where curvature was the strongest and higher accuracy was needed. This physical guidework consisted of a pair of orthogonal metallic rods that were manually bent in place to follow the intrados of the vault, as shown in Fig. 9(c).

Beyond accurate positioning of the tiles during vaulting itself, AR was also used to plan the work, discuss problems and solutions, and perform tiling tests, collaboratively with one or two HMDs. Fig. 9(d) shows such an example, when the builders discussed the angle of the tile at the critical area where the course must blend with the centering.

3.2.2. Construction productivity

In practice, two HMDs were available on site, one being used for vaulting while the other one being charged or used for various checks. An assistant would take care of charging and swapping the headsets for the vault builder, including loading the model and registering the digital and physical worlds. Working with an HMD was a more intense task for the builder, particularly during the uptake of this technology in the first days, as reported through conversations. Therefore, the assistant would sometimes wear the headset and guide the builder to adjust the position of the tiles, which would usually be fine-tuning only. Having two headsets also allowed for two builders to simultaneously visualize the design and discuss practical solutions in advance, instead of last-minute time-consuming corrections, to allow for a more coherent brickwork. The time spent visualizing digital guidework also allowed the builder to obtain a spatial memory of the vault's shape. This spatial memory allowed the builder to understand how the vault curved and to plan for the detailing of the brickwork, tile cuts, and mortar joints, foreseeing challenges ahead. While the builder visually extrapolated a surface from the network of curves when working faster on the regular parts of the tessellation of the vault, it was important to highlight the irregular parts. Here, the loxodrome tiles were shown as gray-colored opaque surfaces, in contrast to the surrounding tessellation shown as wireframe objects. This difference in object types helped the builder to slow down and help the mind shift from *surface thinking* to *tessellation thinking*, which required cognitive effort because of the switch and the focus.

Due to the variable sunlight throughout the day on the different sides of the structure, visualizing the digital model and registering the markers could be hard, therefore slowing down the process. A sunshade accessory like Trimble HoloTint, or external lightweight shading systems, could prevent this challenge for sunlit outdoor application (Mitterberger et al., 2020).

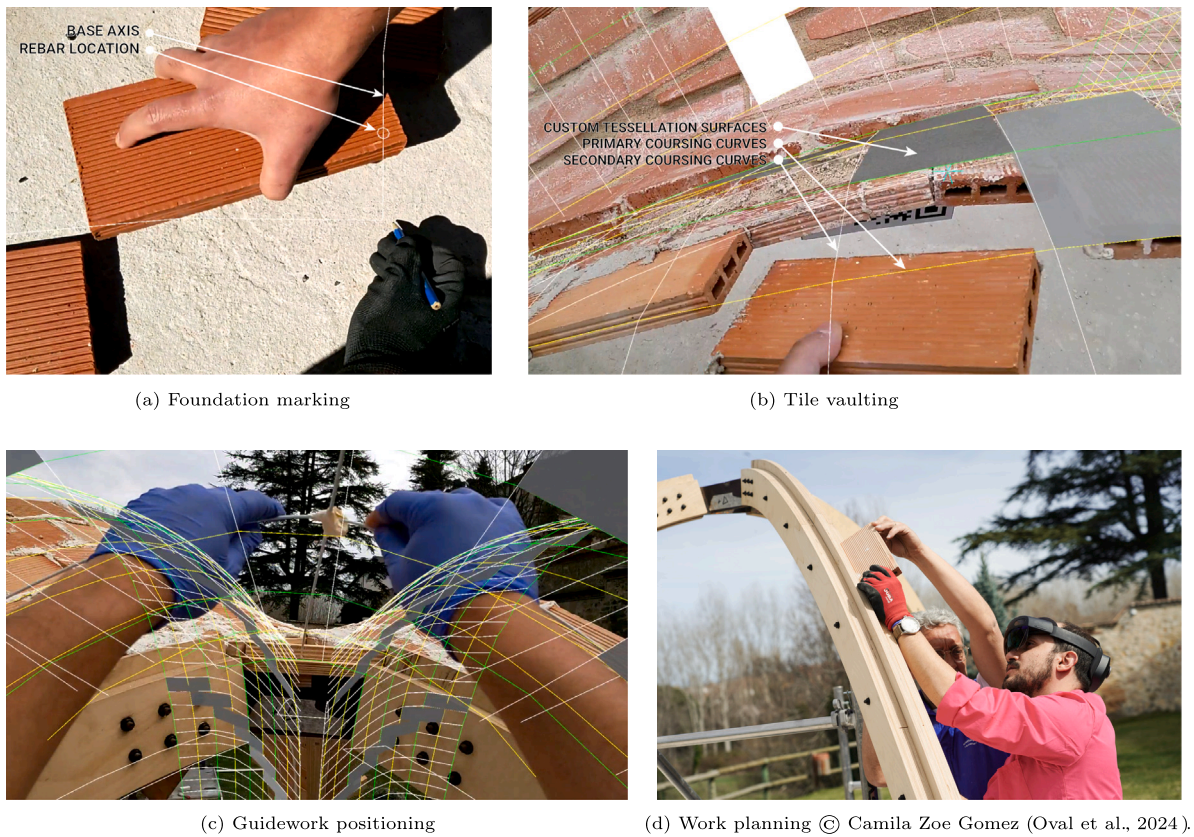


Fig. 9. Applications of AR in thin-tile vault construction.

Estimating construction productivity on site is complex. Two metrics are provided: the average time needed to position a single tile, which is the key action; and the overall time gained to account for the multiple actions that take place on a construction site, as opposed to a factory. From first-point-of-view recordings, the placement of regular tiles, including plaster making, tile positioning, and joint cleaning, took about 25 seconds per tile, for tiles with a steep slope (about 60–65° with the horizontal). The custom tiles due to the boundary conditions or loxodrome pattern, for instance, needed much more time, as additional marking and cutting can require 1–2 minutes per tile, independently from the nature of the guidework. As a global estimation, a builder produced the first layer of the thin-tile vault of 17 m² in about 4.5 days, or 3.8 m²/day, whereas a contemporary figure for the construction of a thin-tile staircase is about 6 m²/day, as estimated by the master vault builder, which corresponds to an estimated productivity reduction of 40%. For comparison, similar construction productivity was reported in Davis et al. (2012) for the standard tile vault construction of 42.5 work-hours for a vault surface of 28.6 m², or 4.7 m²/day for a 7-hour work day. The benefits of AR lie in the time saved on the installation and removal of physical guidework, which is necessary and challenging for vaults of large scale and/or complex geometry, for which the installation of the physical guidework can take about 50% of the construction time, which was avoided here. Therefore, with an increase in the construction time of the vault of 40% and the removal of the construction of the guidework, an estimated construction productivity gain of $100\% - 50\% \times 140\% = 30\%$ was achieved. However, this number is an estimate that needs to be assessed and refined on a larger number of projects.

3.2.3. Construction accuracy

Construction tolerances. As the centering was produced accurately using a CNC process, with an expected accuracy of a couple of millimeters, the markers were positioned along it. The reference points of the

QR code markers used for AR registration were positioned at the joints between the centering elements, as they were clearly defined physically and digitally. The best achievable accuracy between model and guidework declared as AR hardware and software specifications was 3 mm (Twinbuild, 2021). Imperfections like defects of the bricks were of the same order but higher than the AR accuracy, the tolerance allowed by masonry construction, particularly thin-tile vaulting, through mortar joints being around 5 mm. The estimated accuracy of the tile cuts was in the order of 5 mm. These accuracy values are to be compared with the thickness of the first layer of the vault, 33 mm, and the completed vault, 70 mm, for out-of-plane deviations, as well as the maximum span of the vault, 5000 mm, to take into account the scale of the structure. These shape imperfections can have an important influence on the mechanical behavior of the vault, especially buckling. Regarding in-plane deviations of the tessellation, the relevant values for comparison would be the 240 mm length and the 120 mm width of the tiles. These tessellation imperfections from the exact position of the tiles within the tessellation surface are not as crucial and were not checked, as their influence is secondary to the shape imperfections. Moreover, the tessellation was a soft constraint to the builder, who would cut tiles to better fit the design surface. The discretization of the smooth surface into planar tiles can also contribute to construction inaccuracy. Particularly, areas of high curvature can significantly deviate from the target shape. To reduce this effect, the tiles were shortened to better fit the high curvature at the apex of the short span in this demonstrator. The area where such half tiles were used was assessed visually by the builder using the digital guidework to fit.

Vault photogrammetry. A set of 394 photos of the vault were taken, from different positions and at different orientations, all from ground level, to capture the intrados surface of the vault, at the location of the digital guidework. This set of photos was processed using AliceVision's Meshroom (Grwodz et al., 2021) for photogrammetry to obtain a

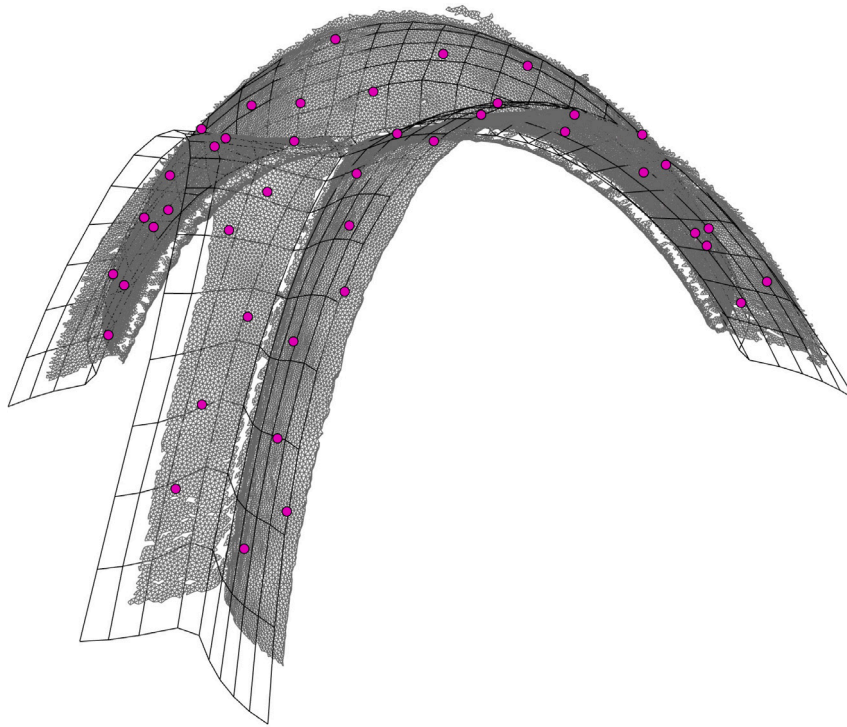


Fig. 10. Registration of the as-built vault intrados mesh (in gray) with the as-designed one (in black) through the best fit of a set of points (in pink).

3D mesh acting as a scan of the vault. The mesh was simplified, noise removed, density reduced, and isotropic remeshing applied. Then, the intrados was automatically extracted from the scanned mesh by selecting only the mesh faces with their normals pointing downward, as opposed to the vertical upward direction. These post-processing steps resulted in a homogeneous triangulated mesh of 29k vertices.

The intrados of this scan mesh (as-built geometry) had to be aligned with the intrados of the form-found vault (as-designed geometry). This registration process was performed using Galapagos (Rutten, 2013), a plugin of Grasshopper3D for Rhino3D, for minimizing the distance between the as-built and as-designed intrados meshes using evolutionary optimization. First, the scan mesh was leveled by using the concrete slab, obtained from the photogrammetry process, and using the best-fit plane interpolating this ground level, to align the scan mesh with the Z direction. Then, the dense scan mesh was populated with 45 points, evenly distributed over the surface, whose average distance to the coarse design mesh had to be minimized. A reduced set of points instead of the entire set of vertices of the dense mesh was used to fit the coarse mesh for computation efficiency. Indeed, the base operation used to measure the distance between a point and a mesh would be computationally too costly during an optimization process where it needs to be performed numerous times. The set of points of the scan mesh was allowed to translate in the three XYZ directions (T_x , T_y , and T_z), to rotate around the Z direction (R_z), and to be scaled uniformly (S). Rotations around the X and Y directions (R_x and R_y) were not necessary due to the previous leveling step based on the ground level. Once the set of points was best fitted to the design mesh, the same transformation was applied to the scan mesh.

The photos were taken a couple of days after the construction of the vault. Therefore, they did not capture long-term deformation but immediate deformation were included. Nevertheless, a vault behaves mainly in compression, including its arches built in sequence. This behavior limits deflection, as shown in Oval et al. (2023) where the deflection during construction of the timber spine of the large scale demonstrator is reported. The average deflection of each leg of the spine ranged between 1 mm and 10 mm, and mostly occurred after the first day of construction. Such deflection was not due to the increased

loading of the vault on the spine but the initial loading leading to joint movement of the spine connections. The vault was expected to be stiffer and therefore showed limited deflection in the order of a couple millimeters.

Result. The registration process of the two meshes optimized the 5 geometric parameters (T_x , T_y , T_z , R_z , and S) and resulted in an alignment with an average distance of 34 mm for the 45 points. The registered meshes are shown in Fig. 10, with the as-designed intrados in black and the as-built intrados in gray with its 45 registration points in pink. As seen, some extra tile arches were built on the long spans, and some were not built on the short span by the end of the construction process. The distance of each mesh vertex of the as-built intrados to the as-designed one was measured and results are plotted in Fig. 11, with perfect fits in green and maximum deviations in red. The intrados at the level of the centering and the additional arches that were added to the vault and were not part of the initial design are shown but not accounted for in the accuracy estimation. The distance between the meshes averaged at 42 mm, with a standard deviation of 36 mm. The highest difference was localized at the lower part of the short span of the vault, up to 100 mm. Compared to the characteristic geometrical dimensions of the vault, the average distance of 42 mm corresponds to 168% of the 25 mm thickness of the first layer of the vault, 60% of the 70 mm full thickness of the completed vault, and 0.8% of the 5000 mm maximum span of the vault.

To the knowledge of the authors, no data exists on the construction accuracy of contemporary thin-tile vaults for benchmarking. Digitizing vaults' shapes is more common for historical buildings for cultural heritage preservation, monitoring their deformation, and analyzing their design and construction methods. For instance, the Tracing the Past project on English Medieval Vaults (Buchanan et al., 2021) used tracing and modeling techniques to analyze the methods employed, documenting accuracy between models in the order of magnitude of 100 mm. However, such data for historical vaults cannot be used to directly measure construction accuracy itself, as the intended design geometry is inferred and the vaults have experienced displacement over the centuries. Moreover, the material and construction of these Gothic vaults differ from the ones of contemporary thin-tile vaults.

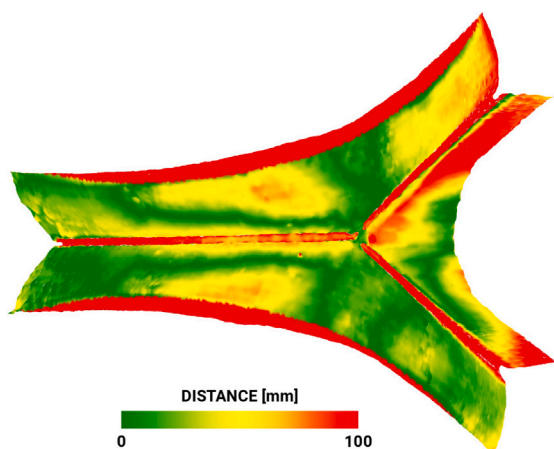


Fig. 11. Accuracy assessment between the as-built and as-designed vault intrados geometries using photogrammetry.

4. Limitation and potential

In this paper, AR is pioneered to provide digital guidework for the construction of complex masonry vaults. Construction is a search for a compromise between efficiency and accuracy. Digital guidework allowed the construction of a demonstrator with a productivity gain (about 30% in time) and a reasonable accuracy (under 1% of the span). However, several improvements could be implemented to tackle current limitations. Mainly, AR was underutilized as a static hologram projection, which has the potential to further enhance construction productivity and accuracy through an interactive MR experience with more contextual information, by exploring different levels of the virtual continuum (Milgram and Kishino, 1994).

4.1. Process robustness

Marker-less AR. Calibration of the physical and digital worlds relied on markers. The registration of the markers was imperfect in a real outdoor environment with varying daylight intensities and orientations. The layout of the markers had to be carefully thought out for the project's specificity, which needs to be adapted for different vault designs and scaled for larger ones. Particularly, relying on an accurate CNC centering simplified the positioning process of the markers in locations known in the digital model. Otherwise, a survey would have been needed to obtain a digital scan of the site, including the position of markers, before producing the vault, especially when it must be inserted into an existing built environment. The development of a marker-less strategy could make this process easier, after further development of this technology to make computation light and fast enough, enabling easier application in larger and more constrained sites (Oufqir et al., 2020). Relying on object tracking, making it more robust to dynamic contexts like construction sites, has been demonstrated through interactive AR where objects and tools are recognized and tracked during fabrication through augmented masonry (Mitterberger et al., 2020) and augmented carpentry (Settimi et al., 2025). Nevertheless, the geometry of the objects, like the bricks, must be known or at least recognizable during assembly to build accurately (Sandy and Buchli, 2018).

4.2. Towards interactive mixed reality

MR provides the potential to make the guidework dynamic and interactive by reacting to the assembly process, instead of being a static object.

Construction accuracy feedback. Positioning and controlling the position of tiles are two different tasks. Control was done directly by the builder and sometimes, at the beginning, through an assistant immersed in the same AR. In the presented work, positioning was assessed visually, not measured. Yet, MR could be leveraged to provide live visual information about the distance between the current position of the tile and digital guidework. Triggered when higher than a given tolerance threshold, this information could consist of colored or scaled arrows to show the magnitude and direction of the deviation, as implemented in Mitterberger et al. (2020) for curved brick walls. Moreover, through measuring the actual imperfections during construction, the design of the vault could be recomputed and updated to compensate for construction errors. This would require an automated robust workflow and increased computation.

Setting time information. Thin-tile vaulting relies on a fast-setting mortar to build up strength to support the next tiles of the vault. When building a vault, the builder must pause to let the mortar cure and gain strength after a certain number of tiles, depending on the quality of the material, the weight of the tiles, the shape of the vault, and other parameters that are based on the builder's experience. The cognitive task of keeping track of which tile was set when could be alleviated by storing this information, and inform the builder when a part of the vault has had enough time to cure to further work at this location. Storing this information is even more important when multiple masons collaboratively work together on a large project and efficient communication is even more crucial.

Digital pencil for tile cut outlining. A bottleneck in building thin-tile vaults in terms of construction productivity is the extra time needed to cut special tiles. Tiles need to be cut either: orthogonally to their edges to change the dimensions of the rectangle to fit between two other tiles for instance; or diagonally to produce a triangle or a pentagon to fit within the boundaries of the vault, the supports, or the centering, or change the orientation of the coursing. Beyond communicating, walking, and cutting the tile, the slowest part consists of accurately marking the tile to cut with a pencil, which can be challenging depending on the type of cut, the obstruction from the other tiles, and working on a scaffolding platform. Inaccurate marking of a tile cut can lead to an error and the need to repeat the process.

Leveraging MR, computer vision, and shape recognition using depth estimation and edge detection could automatically provide the outline of the tile to cut. Previous work includes object detection applied to robotic picking (Song et al., 2022b) and to validation of manual placement of bricks within tolerances (Fazel and Izadi, 2018). Interactively and more robustly, particularly to deal with tolerances, the outline of the tile could be traced by the builder with a pencil. The tip could be automatically tracked, as an extension to hand-gesture recognition (Kyaw et al., 2024), which is a natural element of the craft. The builder could adjust the dimensions of the tile, set tolerances, apply a specific offset, or move the points of the outline polygon. Once the outline is validated, the vault builder could digitally send the model to a cutting workstation. There, another builder could mark the physical tile with the displayed digital outline and cut the tile, or an automated system could perform this task. This digital pencil could provide significant productivity gains, tackling a slow and tedious step, which can lead to errors and delays, particularly for complex brickwork.

4.3. Beyond on-site construction

XR can come into play before the construction process, as a tool for immersive design as well as education and training.

Immersive design. Visualizing a design is a powerful means of communication between different project stakeholders and collaborators (Bouchlaghem et al., 2005). At the design stage, XR could be utilized to immerse designers and builders in the at-scale digital model, as done by Song et al. (2022a) for masonry columns. This immersion would allow the builder to check the reachability of the different parts of the vault, assess feasibility by visualizing the curvature of the vault, and discuss tessellation details using physical or digital bricks. This process of checking brick coursing details happened on site in this project, before construction started. However, integrating it earlier in the design process would allow exploring different tessellations as well as guiding the form finding process.

Education on thin-tile vaulting. Thin-tile vaulting is a traditional craft in Spain, but the number of vault builders has been decreasing, and keeping this craft alive is a challenging education mission when training the next generation. XR has a proven track of education applications, recreating complex and expensive situations, making learning more interactive and gamified (Billinghurst and Dünser, 2012). First-person-point-of-view live or recorded videos provide unique learning material to gain insight into the process of this craft, gesture, and pace, which can be mimicked and learned through the construction of a digital vault, in an immersive environment in collaboration with a trainer, to spread and preserve this cultural heritage.

5. Conclusion

AR can make the construction of structurally efficient vaults more affordable by providing digital guidework to minimize the falsework needed in a traditional craft like thin-tile vaulting. Digital guidework substitutes physical guidework, making vault construction cheaper and safer. Such guidework becomes an obstacle-free and complete visual description of both the shape and the tessellation of the vault, as opposed to physical guidework, which clutters the workspace with physical objects to visualize part of the vault with a set of curves. The proposed methodology was demonstrated on a 17 m² vault with a complex geometry built in an uncontrolled outdoor environment. A compromise was found between productivity and accuracy of vault construction, without having to design, assemble, and disassemble physical guidework. The gain in construction productivity was about 30% in time. The construction accuracy was measured as the average deviation between the as-designed and as-built vault geometries and was under 1% of the span. These numbers are promising estimations from this specific research demonstrator, but need to be validated and improved through additional projects. The limitations and challenges related to the use and development of AR technology are the need to set up markers as references between the digital and physical environments, the robustness of the holograms in an environment subjected to variable light and weather, and the relative comfort when using a head-mounted device for several hours. Nevertheless, this research opens several avenues for further use of XR technologies, empowering vault builders and their craft, augmenting their sense, action, and cognition on the construction site, as well as during design, training, and education.

CRedit authorship contribution statement

Robin Oval: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Vittorio Paris:** Writing – review & editing, Validation, Software, Methodology, Investigation, Conceptualization. **Rafael Pastana:** Writing – review & editing, Validation, Investigation, Conceptualization. **Edvard P.G. Bruun:** Writing – review & editing, Conceptualization. **Salvador Gomis Aviño:** Validation, Investigation. **Sigrid Adriaenssens:** Writing – review & editing, Supervision, Resources, Project administration, Funding acquisition, Conceptualization. **Wesam Al Asali:** Writing – review & editing, Validation, Supervision, Resources, Project administration, Investigation, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The authors would like to thank Carlo Olivieri and Orsolya Gáspár for their feedback on the project, as well as the technical, media, and administrative teams of IE University for their support. The authors also thank La Paloma Cerámicas for their generous donation of the tiles and bricks, and Twinbuild for their precious technical support. This research was partially funded by the National Science Foundation, United States grant 2122271, Princeton University's Institute for International and Regional Studies (PIIRS), and the Princeton ROBELARCH collaborative network.

Data availability

Data will be made available on request.

References

- Abraham, Magid, Annunziata, Marco, 2017. Augmented reality is already improving worker performance. *Harv. Bus. Rev.* URL <https://hbr.org/2017/03/augmented-reality-is-already-improving-worker-performance>.
- Al Asali, M. Wesam, 2020. Craft-inclusive Construction: Design Strategies for Thin-tile Vaulting. University of Cambridge, <http://dx.doi.org/10.17863/CAM.70426>, URL <https://www.repository.cam.ac.uk/handle/1810/322971>.
- Anon, 2024. 3DF Zephyr: the complete photogrammetry solution. URL <https://www.3dflow.net/>. (Accessed on 16 April 24).
- Atanasova, Lidia, Saral, Begüm, Krakovská, Ema, Schmuck, Joel, Dietrich, Sebastian, Furrer, Fadri, Sandy, Timothy, D'Acunto, Pierluigi, Dörfler, Kathrin, 2022. Collective AR-assisted assembly of interlocking structures. In: *Design Modelling Symposium Berlin*. Springer, pp. 175–187.
- Barnes, M.R., 1999. Form finding and analysis of tension structures by dynamic relaxation. *Int. J. Space Struct.* 14 (2), 89–104. <http://dx.doi.org/10.1260/0266351991494722>.
- Bergos, Joan, 1965. *Tabicados huecos*. *Barc.: Col. de Arquít. de Catalunya Y Balear*. ISSN D.L.B.521 VIII1965.
- Billinghurst, Mark, Dünser, Andreas, 2012. Augmented reality in the classroom. *IEEE Comput. Soc.* <http://dx.doi.org/10.1109/MC.2012.111>.
- Block, Philippe, Ochsendorf, John, 2007. Thrust network analysis: A new methodology for three-dimensional equilibrium. *J. Int. Assoc. Shell Spatial Struct.* 48 (3), URL https://web.mit.edu/masonry/papers/block_ochs_IASS_2007.pdf.
- Bouchlaghem, Dino, Shang, Huiping, Whyte, Jennifer, Ganah, Abdulkadir, 2005. Visualisation in architecture, engineering and construction (AEC). *Autom. Constr.* (ISSN: 09265805) 14 (3), 287–295. <http://dx.doi.org/10.1016/j.autcon.2004.08.012>, URL <https://linkinghub.elsevier.com/retrieve/pii/S0926580504001025>.
- Bruun, Edvard P.G., Oval, Robin, Al Asali, Wesam, Gáspár, Orsolya, Paris, Vittorio, Adriaenssens, Sigrid, 2024. Automating historical centering-minimizing masonry vaulting strategies: Applications to cooperative robotic construction. *Dev. the Built Environ.* 20, 100516.
- Buchanan, Alexandrina, Hillson, James, Webb, Nicholas, 2021. *Digital Analysis of Vaults in English Medieval Architecture*. Routledge, ISBN: 9781138541337.
- Chi, Hung-Lin, Kang, Shih-Chung, Wang, Xiangyu, 2013. Research trends and opportunities of augmented reality applications in architecture, engineering, and construction. *Autom. Constr.* (ISSN: 09265805) 33, 116–122. <http://dx.doi.org/10.1016/j.autcon.2012.12.017>, URL <https://linkinghub.elsevier.com/retrieve/pii/S0926580513000022>.
- Collins, George R., 1968. The transfer of thin masonry vaulting from Spain to America. *J. Soc. Archit. Hist.* 27 (3), 176–201. <http://dx.doi.org/10.2307/988501>.
- Davis, Lara, Rippmann, Matthias, Pawlofsky, Tom, 2012. Innovative funicular tile vaulting: A prototype vault in Switzerland. *Struct. Eng.*
- Davis, Lara, Rippmann, Matthias, Pawlofsky, Tom, Block, Philippe, 2011. Efficient and expressive thin-tile vaulting using cardboard formwork. In: *Proceedings of the IABSE-IASS Symposium 2011*. London, UK, URL <https://brg.ethz.ch/publications/37>.
- Dörfler, Kathrin, Sandy, Timothy, Gifthalder, Markus, Gramazio, Fabio, Kohler, Matthias, Buchli, Jonas, 2016. Mobile robotic brickwork. In: Reinhardt, Dagmar, Saunders, Rob, Burry, Jane (Eds.), *Robotic Fabrication in Architecture, Art and Design 2016*. Springer International Publishing, Cham, pp. 204–217. http://dx.doi.org/10.1007/978-3-319-26378-6_15, ISBN 978-3-319-26376-2 978-3-319-26378-6. URL http://link.springer.com/10.1007/978-3-319-26378-6_15.

- Fazel, Alireza, Fayaz, Rima, Mostaghni, Alireza, Matini, Mohammad Reza, 2022. Optical tool for additive construction of complex brick structures. *Autom. Constr.* (ISSN: 09265805) 140, 104330. <http://dx.doi.org/10.1016/j.autcon.2022.104330>, URL <https://linkinghub.elsevier.com/retrieve/pii/S0926580522002035>.
- Fazel, Alireza, Izadi, Abbasali, 2018. An interactive augmented reality tool for constructing free-form modular surfaces. *Autom. Constr.* (ISSN: 09265805) 85, 135–145. <http://dx.doi.org/10.1016/j.autcon.2017.10.015>, URL <https://linkinghub.elsevier.com/retrieve/pii/S0926580516302497>.
- Fitchen, John, 1981. *The construction of gothic cathedrals: A study of medieval vault erection*. University of Chicago Press, Chicago, ISBN: 978-0-226-25203-2.
- Fologram and All Brick, 2020. Holographic construction of curved brickwork. URL https://docs.fologram.com/0ef4fc03e90e414b9a7eefb6369278d3#Royal_Hobart_Hospital.
- Goessens, Sébastien, Mueller, Caitlin, Lattuer, Pierre, 2018. Feasibility study for drone-based masonry construction of real-scale structures. *Autom. Constr.* (ISSN: 09265805) 94, 458–480. <http://dx.doi.org/10.1016/j.autcon.2018.06.015>, URL <https://linkinghub.elsevier.com/retrieve/pii/S0926580518301961>.
- Griwodz, Carsten, Gasparini, Simone, Calvet, Lilian, Gurdjos, Pierre, Castan, Fabien, Maujean, Benoit, De Lillo, Gregoire, Lanthony, Yann, 2021. AliceVision Meshroom: An open-source 3D reconstruction pipeline. In: *Proceedings of the 12th ACM Multimedia Systems Conference*. pp. 241–247. <http://dx.doi.org/10.1145/3458305.3478443>, URL https://hal.science/hal-03351139v1/file/alice_2021_opensource.pdf.
- Jahn, Gwylim, Newnham, Cameron, Van Den Berg, Nick, Iraheta, Melissa, Wells, Jackson, 2020. Holographic construction. In: *Gengnagel, Christoph, Baverel, Olivier, Burry, Jane, Ramsgaard Thomsen, Mette, Weinzierl, Stefan (Eds.), Impact: Design with All Senses*. Springer International Publishing, Cham, pp. 314–324. http://dx.doi.org/10.1007/978-3-030-29829-6_25, ISBN 978-3-030-29829-9 978-3-030-29829-6. URL http://link.springer.com/10.1007/978-3-030-29829-6_25.
- Ko, Chien-Ho, Kuo, Jiun-De, 2015. Making formwork construction lean. *J. Civ. Eng. Manag.* 21 (4), 444–458. <http://dx.doi.org/10.3846/13923730.2014.890655>.
- Kyaw, Alexander Htet, Spencer, Lawson, Lok, Leslie, 2024. Human–machine collaboration using gesture recognition in mixed reality and robotic fabrication. *Archit. Intell.* (ISSN: 2731-6726) 3 (1), 11. <http://dx.doi.org/10.1007/s44223-024-00053-4>, URL <https://link.springer.com/10.1007/s44223-024-00053-4>.
- López López, David, Domènech Rodríguez, Marta, Palumbo Fernández, Mariana, 2014. “Brick-topia”, the thin-tile vaulted pavilion. *Case Stud. Struct. Eng.* 2, 33–40. <http://dx.doi.org/10.1016/j.csse.2014.09.001>.
- López López, David, Van Mele, Tom, Block, Philippe, 2016. La bóveda tabicada en el siglo XXI. *Inf. de la Constr.* 68 (544), 162. <http://dx.doi.org/10.3989/ic.15.169.m15>.
- Milgram, Paul, Kishino, Fumio, 1994. A taxonomy of mixed reality visual displays. In: *EICE Transactions on Information Systems*. URL <https://www.semanticscholar.org/paper/A-Taxonomy-of-Mixed-Reality-Visual-Displays-Milgram-Kishino/f78a31be8874eda176a5244c645289be9fd4317>.
- Mitterberger, Daniela, Dörfler, Kathrin, Sandy, Timothy, Salveridou, Foteini, Hutter, Marco, Gramazio, Fabio, Kohler, Matthias, 2020. Augmented bricklaying: Human–machine interaction for in situ assembly of complex brickwork using object-aware augmented reality. *Constr. Robot.* 4 (3–4), 151–161. <http://dx.doi.org/10.1007/s41693-020-00035-8>, ISSN 2509-811X, 2509-8780. URL <https://link.springer.com/10.1007/s41693-020-00035-8>.
- Ochsendorf, John Allen, 2013. *Guastavino Vaulting: The Art of Structural Tile*, first ed. Princeton Architectural Press, New York, ISBN: 978-1-61689-244-9.
- Oliva Salinas, Juan Gerardo, 2022. UNAM vault. URL https://docs.fologram.com/d3fb2cda66f49d994395bc08d974865#UNAM_Vault.
- Oufqir, Zainab, El Abderrahmani, Abdellatif, Satori, Khalid, 2020. From marker to markerless in augmented reality. In: *Bhateja, Vikrant, Satapathy, Suresh Chandra, Satori, Hassan (Eds.), In: Embedded Systems and Artificial Intelligence*, vol. 1076, Springer Singapore, Singapore, pp. 599–612. http://dx.doi.org/10.1007/978-981-15-0947-6_57, ISBN 9789811509469 9789811509476. Series Title: *Advances in Intelligent Systems and Computing*. URL http://link.springer.com/10.1007/978-981-15-0947-6_57.
- Oval, Robin, Nuh, Mishael, Costa, Eduardo, Madyan, Omar Abo, Orr, John, Shepherd, Paul, 2023. A prototype low-carbon segmented concrete shell building floor system. *Structures* (ISSN: 23520124) 49, 124–138. <http://dx.doi.org/10.1016/j.istruc.2023.01.063>, URL <https://linkinghub.elsevier.com/retrieve/pii/S2352012423000644>.
- Oval, Robin, Pastrana, Rafael, Bruun, Edvard P.G., Paris, Vittorio, Aviño, Salvador Gomis, Adriaenssens, Sigrid, Al Asali, Wesam, 2024. An integrated digital design and construction approach for falsework-minimal masonry vaults. *Structures* 63, 106428. <http://dx.doi.org/10.1016/j.istruc.2024.106428>.
- Parascho, Stefana, Han, Isla Xi, Beghini, Alessandro, Miki, Masaaki, Walker, Samantha, Bruun, Edvard P.G., Adriaenssens, Sigrid, 2021. LightVault: A design and robotic fabrication method for complex masonry structures. In: *Advances in Architectural Geometry 2020*. Presses des Ponts, Paris, France, ISBN: 978-2-85978-540-6, pp. 350–375. URL <https://oar.princeton.edu/handle/88435/pr1s17ss4d>.
- Pittet Artisans Sàrl, 2023. Escalier sur voûte sarrasine et réalité augmentée. URL https://www.linkedin.com/posts/pittetartisans_rhino3d-grasshopper3d-fologram-activity-7064819361626116096-AVIB?utm_source=share&utm_medium=member_desktop.
- Powell, Michael J.D., 1998. Direct search algorithms for optimization calculations. *Acta Numer.* 7, 287–336. <http://dx.doi.org/10.1017/S0962492900002841>.
- Ramage, Michael, Hall, Timothy J., Gatóo, Ana, Al Asali, M. Wesam, 2019. Rwanda cricket stadium: Seismically stabilised tile vaults. *Structures* 18, 2–9. <http://dx.doi.org/10.1016/j.istruc.2019.02.004>.
- Ramage, Michael H., Ochsendorf, John, Rich, Peter, Bellamy, James K., Block, Philippe, 2010. Design and construction of the mapungubwe national park interpretive centre, south africa. *ATDF J.* 7 (1/2), 14–23, URL <https://www.semanticscholar.org/paper/Design-and-Construction-of-the-Mapungubwe-National-Ramage-Ochsendorf/2afaa92e0387270e4e1959143e20629b67035de9>.
- Rankohi, Sara, Waugh, Lloyd, 2013. Review and analysis of augmented reality literature for construction industry. *Vis. Eng.* (ISSN: 2213-7459) 1 (1), 9. <http://dx.doi.org/10.1186/2213-7459-1-9>, URL <https://link.springer.com/10.1186/2213-7459-1-9>.
- Rutten, David, 2013. Galapagos: On the logic and limitations of generic solvers. *Archit. Des.* 83 (2), 132–135. <http://dx.doi.org/10.1002/ad.1568>.
- Sandy, Timothy, Buchli, Jonas, 2018. Object-based visual-inertial tracking for additive fabrication. *IEEE Robot. Autom. Lett.* 3 (3), 1370–1377.
- Schek, Hans-Jörg, 1974. The force density method for form finding and computation of general networks. *Comput. Methods Appl. Mech. Eng.* (ISSN: 00457825) 3 (1), 115–134. [http://dx.doi.org/10.1016/0045-7825\(74\)90045-0](http://dx.doi.org/10.1016/0045-7825(74)90045-0), URL <https://linkinghub.elsevier.com/retrieve/pii/0045782574900450>.
- Settimi, Andrea, Gamarro, Julien, Weinand, Yves, 2025. Augmented carpentry: Computer vision-assisted framework for manual fabrication. *arXiv preprint arXiv:2503.07473*.
- Song, Yang, Agkathidis, Asterios, Koeck, Richard, 2022a. A design method using augmented reality (AR) for customized bricklaying design algorithms. *Archit. Plan. J.* 28 (3), <http://dx.doi.org/10.54729/2789-8547.1237>.
- Song, Yang, Koeck, Richard, Agkathidis, Asterios, 2022b. Augmented bricklayer: an augmented human-robot collaboration method for the robotic assembly of masonry structures. In: *SiGraDi*. http://dx.doi.org/10.5151/sigradi2022-sigradi2022_30.
- Twinbuild, 2021. Augmented reality for the construction site. URL <https://twinbuild.com/>.
- Wang, Xiangyu, 2009. Augmented reality in architecture and design: Potentials and challenges for application. *Int. J. Archit. Comput.* 7 (2), 309–326. <http://dx.doi.org/10.1260/147807709788921985>, ISSN 1478-0771, 2048-3988. URL <http://journals.sagepub.com/doi/10.1260/147807709788921985>.
- Wang, Peng, Bai, Xiaoliang, Billingham, Mark, Zhang, Shusheng, Zhang, Xiangyu, Wang, Shuxia, He, Weiping, Yan, Yuxiang, Ji, Hongyu, 2021. AR/MR remote collaboration on physical tasks: A review. *Robot. Comput.-Integr. Manuf.* (ISSN: 07365845) 72, 102071. <http://dx.doi.org/10.1016/j.rcim.2020.102071>, URL <https://linkinghub.elsevier.com/retrieve/pii/S0736584520302829>.
- Wang, Xiangyu, Kim, Mi Jeong, Love, Peter E.D., Kang, Shih-Chung, 2013. Augmented reality in built environment: Classification and implications for future research. *Autom. Constr.* (ISSN: 09265805) 32, 1–13. <http://dx.doi.org/10.1016/j.autcon.2012.11.021>, URL <https://linkinghub.elsevier.com/retrieve/pii/S0926580512002166>.