



Procurement design in conflict zones[☆]

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ABSTRACT

This paper studies procurement design in conflict zones when project implementation depends on both contractor performance and protection by a security actor. A government procures from privately informed contractors while relying on an army whose effort cannot be directly contracted upon. When the government faces budget constraints and/or funding frictions, the army can be rewarded only when the project is successfully completed. This creates an efficiency–security trade-off: larger rewards strengthen protection incentives, but reduce the government's net procurement value and make implementation less likely. The model shows that contractor competition supports security by increasing the probability that rewards are paid. It also shows that project selection depends on the interaction between resilience and incentives: resilient projects perform better under tolerated insecurity, whereas fragile but high-value projects may generate stronger incentives for defense. External military support has a non-monotone effect, since limited support can weaken domestic protection incentives, while sufficiently strong support can substitute for them. More broadly, the paper shows that procurement in conflict zones is a joint problem of market design and implementation security.

1. Introduction

Recent geopolitical developments and the re-emergence of conflict across several sensitive regions have not only raised new questions about the future of international stability, but have also made survival, reconstruction, and the continuity of essential public functions an urgent policy concern. Public procurement is central to this challenge. It does not stop when war begins. Even under active conflict, states must continue to buy, repair, and maintain roads, bridges, electricity grids, hospitals, fuel depots, ports, and digital infrastructure. In peaceful environments, procurement is already one of the state's largest economic activities, accounting across the OECD for 12.7% of GDP in 2023 and 29.9% of total government spending (OECD, 2025). In conflict environments, however, its importance becomes even more acute. Governments are not simply purchasing works and services under difficult conditions. They are trying to preserve essential investment and keep public service delivery alive while projects remain exposed to attack, sabotage, severe unrest, and territorial instability.

This perspective is increasingly important because many contemporary conflicts do not end quickly or cleanly. In some cases, open warfare lasts for years. In others, violence persists through insurgency, guerrilla attacks, sabotage, or fragmented territorial control. In these settings, procurement cannot be postponed until a clearly defined post-war phase. It must continue under persistent insecurity. Ukraine offers a clear example. Large-scale public purchasing and reconstruction needs have evolved alongside active hostilities,

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and micro-level evidence shows that wartime procurement became less competitive and more expensive as the invasion disrupted market conditions (Klymak and Vlandas, 2024; World Bank, 2025e, 2026). Similar problems arise in Gaza and the West Bank, where implementation remains vulnerable to renewed violence and severe access constraints (World Bank, 2025d,c). Syria illustrates the same issue over a longer horizon. After more than a decade of conflict, reconstruction needs remain enormous, while minefields, sabotage, damaged transport and water networks, and persistent armed threats continue to shape project feasibility (World Bank, 2025a,b; United Nations Office of the Special Envoy for Syria (OSES), 2025). These cases suggest that, in many parts of the world, the relevant policy problem is not how to procure after conflict ends, but how to procure when conflict persists or is unlikely to end soon.

Once procurement is viewed in this way, a central institutional friction comes into focus. In conflict zones, the state is not simply buying a project from a private contractor. It is buying a project whose value depends on whether implementation can be protected. A road, a power line, or a hospital is valuable only if workers can access the site, logistics can function, and the project can be completed without being destroyed, delayed, or abandoned. Standard procurement models do not capture this complementarity. The government must not only select an efficient contractor, but also induce protection by military or police organizations whose effort is crucial for implementation but is not directly contractible. Procurement and security are therefore complements, and they must be designed jointly.

This paper develops a theory of procurement with endogenous security. A government seeks to procure a project from a privately informed contractor while relying on an army whose protective effort is non-contractible. The project yields value only if the environment is secure. The government can promise the army a reward contingent on successful project completion, a natural assumption in settings where completion is observable and pledgeable but battlefield effort is not. The government therefore faces a joint design problem. It must decide how aggressively to procure from firms and how strongly to incentivize protection.

The analysis begins with a one-contractor benchmark. In that environment, procurement reduces to a reserve-price offer, which makes the main mechanism transparent. The core force is an efficiency-security trade-off. A larger promised reward strengthens the army's incentive to protect the project, but it also lowers the government's net value from successful procurement. This tightens the procurement rule, makes implementation less likely, and in turn weakens the expected value of the promised reward itself. The government therefore cannot strengthen security incentives without also affecting procurement efficiency. To buy more protection, it must become more selective in procurement and risk excluding projects that would otherwise go forward.

The paper then extends the baseline in several directions. First, we allow for competition among contractors, showing that the core efficiency-security trade-off survives when procurement is implemented through a standard auction mechanism rather than a simple reserve-price offer. Competition improves contractor selection and raises the probability that a promised military reward is actually paid, thereby making defense easier to implement. Second, we consider a continuous security technology, in which protection can be increased at the margin rather than chosen as a binary action. This smooth version sharpens the basic mechanism by showing how the government balances the marginal procurement gain from a more permissive rule against the marginal loss in security incentives from a lower success-contingent reward. Third, we allow projects to retain some value even under disruption and show that this weakens the government's incentive to buy protection by improving the payoff from tolerated insecurity. Building on this observation, we then study project selection: when some projects are more resilient than others, the government faces an additional insurance-incentives trade-off between projects that preserve value under unrest and projects that create stronger incentives for defense.

Fourth, we introduce state-dependent contractor costs, so that disruption worsens not only project value but also procurement efficiency by shifting costs upward. This effect is not monotone at the level of the primitive cost distribution. Unlike a pointwise increase in virtual costs, an upward shift in actual costs can lower procurement surplus at a given net value and yet make defense easier to sustain by increasing the probability that successful implementation, and thus military compensation, occurs. In that case, disruptions that worsen the contractor side may partly backfire on the attacker by strengthening the government's incentives for protection. Security then becomes more valuable because it protects both implementation and the supply side of procurement.

Fifth, we study external support. We first consider security-side support, modeled as an exogenous improvement in background security when the army shirks. We then contrast it with procurement-side support, which improves procurement capacity without weakening domestic protection incentives. Security-side support has two opposite effects: it improves the government's outside option under tolerated insecurity, but it also weakens domestic protection incentives by reducing the army's marginal contribution to project survival. As a result, external support is not unambiguously beneficial. When the government initially relies on domestic defense, modest support can reduce welfare, whereas sufficiently strong support can become beneficial by allowing the government to dispense with a distorted domestic incentive scheme.

Finally, we examine an alternative timing in which procurement takes place before the security state is realized. In that environment, insecurity depresses the reserve policy *ex ante*, making induced defense relatively more attractive than in the baseline model.

Taken together, these results deliver clear policy implications. First, procurement policy in conflict environments cannot be designed as if insecurity were merely an exogenous background condition. Rules governing reserve policies, award mechanisms, and project selection interact directly with implementation security. Second, preserving competition has value beyond standard cost savings. Thicker procurement markets not only improve allocation but also strengthen security incentives by making successful procurement more likely. Third, project prioritization should depend not only on social value, but also on resilience to disruption. When protection is difficult to induce, governments may rationally favor projects that retain value even under insecurity. When protection can be induced, more fragile but higher-value projects become more attractive. Fourth, outside military support is not automatically welfare-improving. Small interventions may weaken domestic protection incentives without yet providing an adequate substitute for them, whereas sufficiently strong support can raise welfare by effectively replacing distorted domestic defense

incentives. Procurement-side support, by contrast, is weakly beneficial because it raises procurement surplus and increases the expected value of completion-contingent rewards. More broadly, the model suggests that wartime procurement is not simply standard procurement under stress. It is a joint problem of market design and implementation security.

This perspective helps connect project-level procurement choices to broader questions of state capacity in conflict-affected environments. Governments are often required to maintain infrastructure and basic service provision before territorial control has been consolidated and before violence has subsided. The relevant question is therefore not only how conflict lowers project value, but how it changes the design of procurement itself. This paper argues that the answer lies in recognizing that, in wartime and in prolonged insecurity, procurement must be understood jointly as a problem of efficient purchasing and credible protection. Notably, this logic is not confined to wartime reconstruction but has a wider scope, as it also applies to the procurement of local public services in disadvantaged areas. In peripheral neighborhoods, high-crime areas, or places with weak state presence, service delivery depends not only on selecting an efficient provider, but also on ensuring that schools, transport, waste collection, health services, or utilities can operate safely and continuously. In such settings, the relevant problem is not simply how to buy cheaply, but how to design procurement so that provision is both efficient and credible.

The rest of the paper is organized as follows. Section 2 positions the paper in the literature. Section 3 presents the environment. Section 4 solves the one-contractor benchmark. Section 5 introduces contractor competition. Section 6 discusses the main model assumptions and alternative state-contingent reward regimes. Section 7 develops the extensions. Section 8 concludes. All proofs are collected in the Appendix.

2. Related literature

The paper contributes to three related literatures. It belongs first to the large literature on procurement under private information and incomplete contracting. The benchmark procurement block follows the logic of Myerson (1981). Relative to multidimensional procurement models such as Che (1993) and Asker and Cantillon (2008, 2010), the additional dimension here is not a quality choice made by the contractor, but a protective action supplied by a third party. Relative to Bajari and Tadelis (2001), Lewis and Bajari (2011), and Decarolis (2014), the main distortion does not come from the contractor's post-award effort, renegotiation, or quality shirking, but from the government's need to induce project protection before procurement can generate value. More broadly, recent work emphasizes the importance of procurement institutions, buyer competence, and performance management for procurement outcomes, as shown by Decarolis et al. (2020), Bandiera et al. (2021), Bosio et al. (2022), Lopomo et al. (2023), and Celhay et al. (2024). The present paper adds an external security dimension to that design problem. Procurement performance depends not only on how the government screens and contracts with suppliers, but also on whether it can induce effective protection by military or police organizations.

A second strand of the literature studies procurement under emergencies, discretion, and wartime disruptions. Schmitz (2013) shows how crisis conditions can alter standard procurement choices, while Bandiera et al. (2021) stress the trade-off between speed, discretion, and accountability during emergencies. Klymak and Vlandas (2024) provide direct evidence from Ukraine: wartime procurement became faster and more likely to conclude, but also less competitive and more expensive. In defense procurement more specifically, Carril and Duggan (2020) show how industry consolidation affects competition and contract form in U.S. Department of Defense contracting, while Smith (2022), Pedersen et al. (2025), and Hellberg et al. (2025) highlight the governance, implementation, and supply-chain constraints that arise in defense acquisition. This paper contributes to that literature by offering a theoretical framework in which the government is not merely buying under stress, but is jointly designing procurement and security incentives. In this sense, the paper isolates a microeconomic channel linking reserve policies, market thickness, and security provision.

Finally, the paper speaks to the literature on conflict and state capacity. Besley and Persson (2010) and Gennaioli and Voth (2015) show that conflict shapes incentives to build fiscal and coercive capacity. The present analysis focuses on a narrower, project-level channel. It studies how the need to elicit protective effort changes reserve policies, bidder participation, and the government's choice between induced defense and tolerated disruption. In that sense, the paper connects macro questions about state capacity to the microeconomics of procurement design.

3. The baseline model

This section sets up the model in a standard procurement environment. There is a government, an army, and a single contractor. A public project yields value only if the environment is secure. Let the state be $\theta \in \{s, u\}$, where s denotes a secure environment and u denotes a state of disruption capturing sabotage, severe unrest, or active hostilities that disrupt economic stability. The project's gross value is

$$V(s) = V > 0, \quad V(u) = 0.$$

In the Extensions, we explore alternative mappings between the project's value and the underlying state.

Army. The army chooses an action $e \in \{0, 1\}$, where 1 denotes active defense and 0 denotes shirking. Defense has a cost $\psi > 0$, while shirking is costless. If the army defends, the secure state occurs with probability one. If it shirks, the secure state occurs with probability $\rho \in (0, 1)$. That is,

$$\mathbb{P}[s \mid e = 1] = 1, \quad \mathbb{P}[s \mid e = 0] = \rho.$$

The army's action is not contractible. The baseline contracting restriction is that the government can promise only a non-negative reward t contingent on successful project completion. The assumption is motivated by two institutional features of procurement in conflict and reconstruction settings. First, project completion is verifiable through the procurement, audit, and delivery process, whereas military effort, local security conditions, and battlefield contingencies are not directly contractible. The government may observe whether a project has been delivered, but it may be much harder to verify whether non-completion was due to insufficient military effort, adverse security shocks, contractor failure, or some combination of these factors. Tying the reward to completion therefore relies on a comparatively clear and auditable event.

Second, the relevant pledgeable resources may be project-specific. The assumption is not that the government has no resources in an absolute sense, but rather that it may not have sufficient liquid, discretionary, and pledgeable resources to compensate the army independently of project realization. In many conflict and reconstruction settings, public budgets are under severe fiscal pressure, and funds are often earmarked to particular projects, released against certified milestones, or subject to donor, fiduciary, eligibility, and audit constraints. Some financing arrangements also link disbursement to verified results or project-specific performance indicators. If the project is not completed, the government may therefore have no project-level surplus, disbursement, or authorized budgetary margin from which to compensate the army. The army's reward is then financed out of the residual surplus generated by successful completion, rather than from unrestricted general funds.

For completeness, we also study the opposite polar case in which the secure state is verifiable and the government can compensate the army from general funds independently of project completion. This alternative regime, analyzed in [Section 6](#), clarifies the institutional scope of the baseline model.

Contractor. In the baseline model, there is a single contractor with privately known execution cost $c \in [\underline{c}, \bar{c}]$, where $0 \leq \underline{c} < \bar{c}$. The cost is drawn from a continuously differentiable cdf $F(c)$ with density $f(c) > 0$ on $[\underline{c}, \bar{c}]$. Define the *virtual cost* $H(c)$ as

$$H(c) = c + \frac{F(c)}{f(c)}.$$

The virtual cost is the procurement analogue of the virtual valuation in standard screening problems. The term c is the contractor's real execution cost. The additional term $F(c)/f(c)$ captures the marginal informational-rent cost of expanding the allocation rule to a higher-cost contractor. Hence, the government does not compare the project value only with the contractor's true cost c , but with the virtual cost $H(c)$. We impose the standard regularity assumption that $H(c)$ is strictly increasing. We also assume that $V > \bar{c}$, to guarantee that, under complete information, the project is socially valuable.

Competition among several contractors will be introduced in [Section 5](#).

Timing and the equilibrium concept. The timing is as follows:

1. the government commits to a reward $t \geq 0$ to be paid only upon successful project completion;
2. the army chooses $e \in \{0, 1\}$;
3. the state θ is realized and publicly observed;
4. the contractor privately observes its cost c ;
5. the government makes a procurement offer;
6. if the contractor accepts and the project is completed successfully, transfers are made and the army is paid t .

The timing should be interpreted as follows. The project has already been selected and authorized before the army chooses its action, but the procurement contract has not yet been awarded. The army's protection effort concerns the implementation environment of the project, such as the site, transport routes, access for workers and materials, and the local conditions that determine whether the project can be carried out once a contractor is selected. The secure state s does not mean that a specific contractor is already protected before being chosen. It means that the project environment is sufficiently stable for implementation to be feasible after procurement takes place. [Section 6](#) returns to this timing interpretation, and [Section 7.5](#) studies the alternative case in which procurement must be organized before the security state is realized.

The solution concept is perfect Bayesian equilibrium (PBE), and we restrict attention to pure strategies.

4. Equilibrium analysis

With one contractor, the procurement problem becomes a reserve-price problem rather than a proper auction. This simplification is useful because it makes the interaction between procurement and security transparent.

Fix a promised military reward $t \geq 0$. If the state is secure, successful procurement gives the government net value

$$x = V - t.$$

Given that net value, the government can offer the contractor a take-it-or-leave-it payment r . The contractor accepts if and only if $c \leq r$. The government's expected payoff from offering r is therefore

$$F(r)(x - r).$$

The procurement problem is thus straightforward. The government chooses the reserve payment to trade off a higher probability of acceptance against a lower surplus conditional on acceptance. The first-order condition with respect to r yields

$$x = H(r).$$

The following then holds.

Lemma 1. *For a given net value $x \geq 0$, the optimal procurement contract is a reserve-price offer $r(x)$ given by*

$$r(x) = \begin{cases} 0, & x < H(\underline{c}), \\ H^{-1}(x), & x \in [H(\underline{c}), H(\bar{c})], \\ \bar{c}, & x > H(\bar{c}). \end{cases}$$

Procurement takes place if and only if $c \leq r(x)$. Whenever $x \in [H(\underline{c}), H(\bar{c})]$, the procurement cutoff $r(x)$ is, therefore, the unique c solving $x = H(c)$. At that cutoff, the government's continuation expected payoff is

$$F(r(x))(x - r(x)) = \frac{F(r(x))^2}{f(r(x))}.$$

This payoff is weakly increasing in x .

Lemma 1 already contains the basic trade-off. A higher military reward reduces the net procurement value $x = V - t$. Once x falls, the government's reserve offer falls, so procurement becomes less likely. The promised reward is, therefore, evaluated through the procurement rule that it itself distorts.

For later use, define the one-contractor procurement probability and continuation payoff as

$$q(x) := F(r(x)),$$

$$\Pi(x) := F(r(x))(x - r(x)).$$

On the interior region, $q(x) = F(H^{-1}(x))$ and

$$\Pi(x) = \frac{F(H^{-1}(x))^2}{f(H^{-1}(x))}.$$

Thus $q(\cdot)$ records the probability that the project is implemented, whereas $\Pi(\cdot)$ records the government's expected continuation procurement payoff.

To sharpen the characterization, the rest of this section focuses on the interior case

$$H(\underline{c}) < V < H(\bar{c}).$$

Let c^0 denote the unique cutoff solving

$$V = H(c^0).$$

This is the procurement rule the government would use if it promised no military reward, i.e., if $t = 0$. Now consider any target cutoff $c \in [\underline{c}, c^0]$ for the secure state. Implementing that cutoff requires the government to leave net procurement value

$$t(c) = V - H(c).$$

A lower cutoff, therefore, implies a larger promised reward. This captures the model's fundamental trade-off between efficiency and security. To obtain more protection, the government must tighten procurement.

The army's incentive compatibility constraint guaranteeing that the army de facto prefers to exert effort is then

$$(1 - \rho)F(c)(V - H(c)) \geq \psi, \quad (4.1)$$

This constraint has a clear economic interpretation. Effort is socially valuable only because it removes the residual risk that the project is disrupted. If the army shirks, the good state occurs with probability ρ , and if it exerts effort, the project is completed safely with probability one. Hence, the incremental value of effort is proportional to $(1 - \rho)$. When the environment is already relatively secure, the army's action matters little, whereas in highly fragile environments effort has a first-order effect on project survival. The term $F(c)$ captures the probability that the project is actually carried out. Since the army is rewarded only when procurement takes place and the project is completed, its expected reward is naturally scaled by the likelihood that the government proceeds with the contract. The last term, $V - H(c)$, is the surplus that remains for the government once the contractor has been paid. Under limited liability, this residual surplus is also the largest bonus that can credibly be promised to the army in the good outcome. The product $F(c)(V - H(c))$ is the expected prize attached to effort. It combines the probability that the project is implemented with the size of the reward that successful implementation leaves available for the army.

A more permissive procurement rule raises $F(c)$, because the project is awarded more often, but it also reduces $V - H(c)$, because a less selective rule leaves a smaller surplus net of procurement costs. By contrast, a tighter procurement rule preserves more surplus on each realized project, but reduces the probability that the project is undertaken in the first place. The government uses the procurement rule not only to screen contractors, but also to shape the army's incentives. Security can be bought only by distorting

procurement efficiency. The government may need to accept a more selective procurement policy than it would choose absent the army problem in order to generate enough surplus to reward protection.

The constraint also highlights immediately when defense is hard to sustain. For a given procurement rule, incentives are weaker when ρ is high, because effort has little effect on the probability of project survival, and when ψ is large, because the army must be compensated more generously. If even the best procurement rule cannot make the left-hand side of (4.1) reach ψ , then defense cannot be incentivized. If instead (4.1) can be satisfied, the government chooses procurement so as to balance the gain from greater security against the efficiency loss induced by a more distorted allocation rule. The following then holds.

Proposition 1. Assume $H(\underline{c}) < V < H(\bar{c})$. Then,

1. defense is implementable if and only if

$$\psi \leq \max_{c \in [\underline{c}, c^0]} (1 - \rho)F(c)(V - H(c));$$

2. if defense is implementable, then, conditional on inducing defense, the government chooses the largest cutoff $c^d \in [\underline{c}, c^0]$ satisfying the army's incentive constraint (4.1), equivalently the largest c such that (4.1) holds. The associated reward is

$$t^d = V - H(c^d).$$

To induce defense, the government must move from c^0 to some smaller cutoff c , thereby leaving the army a success-contingent reward $t(c)$. The benefit of lowering c is stronger security incentives. The cost of doing so is lower procurement probability and a more selective purchasing rule. The government therefore chooses defense only if it can buy enough protection without pushing the procurement cutoff below the level at which defended procurement ceases to dominate risky procurement. We then have

Proposition 2. Assume $H(\underline{c}) < V < H(\bar{c})$, and define

$$\bar{\psi}(V) := \max_{c \in [\underline{c}, c^0]} F(c)(V - H(c)).$$

Then:

1. if $\psi \geq \bar{\psi}(V)$, the government never induces defense;
2. if $\psi < \bar{\psi}(V)$, there exists a unique threshold $\bar{\rho}(V, \psi) \in (0, 1)$ such that the government induces defense if and only if

$$\rho \leq \bar{\rho}(V, \psi);$$

3. the threshold $\bar{\rho}(V, \psi)$ is decreasing in ψ .

When the survival probability ρ is small, the background environment is fragile, so buying defense yields a first-order gain and the government is willing to distort procurement in order to secure the project. When ρ is large, the project is already likely to survive even if the army shirks. In that region, the efficiency cost of tightening procurement dominates, and the government tolerates residual risk instead of paying for protection. A higher military agency cost ψ shifts the cutoff downward, so defense is chosen only in more fragile environments.

5. Competition among contractors

The one-contractor benchmark isolates the core feedback between procurement and security. This section shows that the same logic survives when procurement becomes genuinely competitive. Suppose that there are $n \geq 1$ potential contractors, each with private cost c_i independently drawn from the same distribution $F(c_i)$ with density $f(c_i)$. We retain the same regularity assumption that the virtual cost function $H(\cdot)$ is strictly increasing.

Conditional on a deterministic net project value x , the optimal procurement mechanism is no longer a posted reserve offer. Instead, it is the familiar second-price procurement auction with a reserve cost (Krishna, 2009; Myerson, 1981).

Lemma 2. For any $x \geq 0$, the government's optimal procurement mechanism with n contractors is a second-price procurement auction with reserve cost

$$\bar{c}(x) := H^{-1}(x)$$

whenever $x \in [H(\underline{c}), H(\bar{c})]$, with the natural truncations at the boundaries. The project is awarded to the contractor with the lowest cost, termed $c_{(1)}$, if and only if $c_{(1)} \leq \bar{c}(x)$, and the winner is paid the minimum of the second-lowest cost and the reserve cost.

The intuition is standard. The government allocates the project efficiently to the lowest-cost contractor, ensures truthful reporting by paying the second-lowest cost, and protects itself from overpaying by setting a reserve cost. Then, the competitive procurement

stage can again be summarized by two objects. First,

$$q_n(x) := \mathbb{P}(c_{(1)} \leq \tilde{c}(x)) = 1 - (1 - F(\tilde{c}(x)))^n$$

is the probability that procurement takes place. Second,

$$\Pi_n(x) := \mathbb{E}[(x - H(c_{(1)}))\mathbb{1}\{c_{(1)} \leq \tilde{c}(x)\}]$$

is the government's expected continuation payoff. These are the direct analogues of $q(x)$ and $\Pi(x)$ from the one-contractor benchmark.

Competition and security. Once $q_n(\cdot)$ and $\Pi_n(\cdot)$ have been characterized, the security problem looks exactly like the one-contractor benchmark. If the government promises reward t , the army defends if and only if

$$(1 - \rho)tq_n(V - t) \geq \psi.$$

Define

$$\tilde{\psi}_n(V, \rho) := \max_{t \in [0, V]} (1 - \rho)tq_n(V - t)$$

and, when defense is implementable,

$$t_n^D := \min \{t \in [0, V] \mid (1 - \rho)tq_n(V - t) \geq \psi\}.$$

The government's payoff under defense is $\Pi_n(V - t_n^D)$, while the payoff under tolerated disruption is $\rho\Pi_n(V)$. The following then holds.

Proposition 3. *Suppose $n' > n$. Then for every $x \geq 0$,*

$$q_{n'}(x) \geq q_n(x), \quad \Pi_{n'}(x) \geq \Pi_n(x).$$

Hence,

1. $\tilde{\psi}_{n'}(V, \rho) \geq \tilde{\psi}_n(V, \rho)$. That is, a larger pool of competing contractors makes defense easier to implement;
2. if defense is implementable under both n and n' , then $t_{n'}^D \leq t_n^D$. That is, with a larger pool of competing contractors defense is cheaper;
3. the government's payoff under induced defense is weakly higher with n' contractors than with n contractors.

The main takeaway of this proposition is that competition helps security. A promised military bonus is only valuable if procurement actually takes place. More contractor competition makes procurement more likely, so it increases the expected value of the same promised military reward. In this model, competition does not change the army's technology of defense. It changes the credibility of the government's promise in expected-value terms. The gain is weakly positive throughout the benchmark because both the procurement probability and the continuation payoff are increasing in the number of contractors. The main force is strongest when the army's incentive constraint is close to binding, because that is precisely when a small increase in procurement probability has the largest effect on security implementability.

6. Discussion of the model assumptions

This section discusses the two assumptions that connect procurement and security in the baseline model. The first is that the army's reward is contingent on successful project completion. The second is that, in the baseline timing, the government observes the security state before choosing the procurement rule. Both assumptions are important from a practical point of view. They describe an institutional environment in which the project has been selected and authorized, completion is the main pledgeable event, and local security conditions are observed before procurement is finalized. We also study the opposite polar case in which the secure state is verifiable and the army can be compensated from general funds independently of project completion.

Completion-contingent rewards. The baseline model is best interpreted as a project-budget or pledgeable-surplus regime. The government cannot freely pledge unrestricted general funds to the army for a project-specific security task. Instead, the pledgeable fiscal event is successful completion of the project. This restriction is natural in conflict and reconstruction settings in which funds are earmarked, released against certified milestones, or subject to donor, procurement, and audit rules. In such environments, the army may not be able to receive a stand-alone payment merely because the environment is secure. The resources from which it can be rewarded are tied to delivery. If the project is not completed, the project-specific surplus or budgetary authorization that supports the reward is not available.

This interpretation is also consistent with the limited-liability structure of the model. When the government implements a cutoff c , the reward that can be promised to the army is $t(c) = V - H(c)$. This is the residual value left after accounting for the procurement cost distortion. Since the reward is paid only when the project is completed, the army's expected prize is $q_n(V - t)t$, or, in the one-contractor benchmark, $F(c)(V - H(c))$. The key mechanism of the paper comes from this project-level pledgeability: increasing the reward improves security incentives, but reduces the government's net procurement value and therefore changes the probability that the reward is paid.

The completion-contingent formulation does not require the security state to be unobserved by the parties. Rather, it requires a distinction between observability and pledgeability. Local security conditions may be observed by the government, the army, and

contractors when procurement takes place, but they may still be difficult to verify, audit, attribute, or pledge as the basis for a stand-alone compensation scheme. Completion, by contrast, is a certified procurement outcome.

Protection before procurement. The baseline timing assumes that the project has already been selected and authorized before the procurement contract is awarded. The army's action is not the protection of a contractor who has already won the contract. It is the protection of the project environment, such as the site, transport corridors, supply access, and local security conditions that determine whether implementation will be feasible once a contractor is selected. This timing fits projects whose location and purpose are known before the identity of the contractor is determined, such as repairing a bridge, restoring a power line, reopening a hospital, or securing a logistics corridor.

The secure state s should be interpreted as an implementation state. If the army defends, the project environment becomes secure with probability one. If it shirks, the environment is secure only with probability ρ . Once the state is realized, the government observes whether implementation conditions are sufficiently stable and then runs procurement.

Baseline timing. The baseline timing is appropriate when procurement can be adjusted to current local security conditions. In conflict zones, governments often postpone, re-scope, or cancel tenders when roads, worksites, or local administration become inaccessible. In such cases, the government learns whether the project environment is secure before making the final procurement offer.

The alternative timing is also relevant. In urgent projects, under centralized procurement, in framework agreements, or in cases with long administrative lead times, the procurement rule may need to be chosen before the security state is known. [Section 7.5](#) studies that case. The main trade-off survives, but the no-defense benchmark becomes less attractive. When procurement is chosen before the state is realized, insecurity lowers the government's reserve policy *ex ante*, and

$$\Pi_n(\rho V) \leq \rho \Pi_n(V).$$

Thus, induced defense is weakly more attractive under *ex ante* procurement than under the baseline timing. The baseline is chosen because it is the cleaner benchmark for projects whose procurement can be conditioned on observed local implementation conditions.

6.1. State-contingent rewards

The previous discussion explains why completion-contingent rewards are natural in a project-budget regime. We now study the opposite polar case. Suppose that the secure state itself is verifiable and that the government can finance a payment to the army from general funds, independently of whether a particular procurement contract is awarded or completed. Let $\alpha \geq 0$ denote a reward paid whenever $\theta = s$. More generally, also allow the government to choose a completion-contingent reward $\beta \geq 0$, paid only if the project is completed successfully in the secure state. The baseline model corresponds to the restriction $\alpha = 0$.

If the army exerts effort, the secure state occurs with probability one. If it shirks, the secure state occurs with probability ρ . Hence, for a given (α, β) , the army's incentive constraint is

$$(1 - \rho) [\alpha + \beta q_n(V - \beta)] \geq \psi.$$

The government's payoff under induced defense is $\Pi_n(V - \beta) - \alpha$. The state-contingent transfer α is sunk at the procurement stage, whereas the completion-contingent transfer β lowers the net project value and therefore affects the procurement rule.

Proposition 4. *Suppose that the secure state is verifiable and that a reward contingent only on $\theta = s$ can be financed independently of project completion. Then, any optimal contract that induces defense can be chosen with*

$$\alpha = \frac{\psi}{1 - \rho} \quad \text{and} \quad \beta = 0.$$

Hence, conditional on induced defense, procurement is undistorted relative to the no-reward procurement benchmark, and the government's payoff is

$$\Pi_n(V) - \frac{\psi}{1 - \rho}.$$

In the baseline case $V(u) = 0$, the government induces defense if and only if

$$\Pi_n(V) - \frac{\psi}{1 - \rho} \geq \rho \Pi_n(V),$$

or, equivalently, if and only if $\psi \leq (1 - \rho)^2 \Pi_n(V)$. More generally, if $V(s) = V_s > V_u = V(u) \geq 0$, the government induces defense if and only if

$$\Pi_n(V_s) - \frac{\psi}{1 - \rho} \geq \rho \Pi_n(V_s) + (1 - \rho) \Pi_n(V_u).$$

Proposition 4 shows that the completion-contingent assumption has bite. When the secure state is itself verifiable and pledgeable, and when the army can be compensated from general funds, the army's reward can be made independent of the procurement outcome. Security incentives no longer depend on the probability that a contractor is selected, and procurement is undistorted at the implementation stage.

Procurement and security do not become completely unrelated. They are still connected through the government's *ex ante* decision on whether defense is worth financing. A higher procurement surplus $\Pi_n(V)$ makes it more attractive to pay the state-contingent

reward. Thus, under state-contingent rewards, procurement affects the value of security, but it does not affect the army's incentive constraint.

This polar case clarifies the scope of the baseline model. The main mechanism applies most directly to environments in which completion is observable, auditable, and pledgeable, while battlefield conditions and military effort are not separately contractible or cannot be compensated from unrestricted general funds. By contrast, the state-contingent case is more appropriate when a centralized government or donor can verify the security state and compensate protection independently of whether a particular project is completed.

6.2. Partial state-contingent budgets

The state-contingent case above is useful because it identifies the polar environment in which procurement and security separate at the procurement stage. That separation requires not only that the secure state be verifiable, but also that the government can pledge enough unrestricted funds to cover the entire incentive cost of military protection. In conflict environments this may be unrealistic. War typically creates severe fiscal stress by weakening revenues, reducing investment, and increasing defense spending (Gupta et al., 2002). At the same time, donor-financed reconstruction and emergency procurement are often governed by project-specific fiduciary and disbursement rules. In project financing, disbursement procedures require documentation of eligible expenditures and specify remedies when funds are used for ineligible purposes (World Bank, 2006). Emergency procurement in contexts of fragility, conflict, and capacity constraints is also subject to specific operational guidance (World Bank, 2019). Some financing instruments go further by linking disbursements directly to verified results or disbursement-linked indicators (World Bank, 2012). Even when the government can make some security-related payment from general funds, the amount that can be pledged independently of project completion may be limited.

This suggests an intermediate case between the baseline model and the fully state-contingent regime. Suppose that the government can promise a state-contingent payment $\alpha \geq 0$, paid whenever the secure state is realized, but that this payment is constrained by a general-fund cap $\alpha \leq \bar{B}$. In addition, the government can promise a completion-contingent reward $\beta \geq 0$, paid only if the project is completed successfully in the secure state. The baseline model corresponds to $\bar{B} = 0$. The fully state-contingent case corresponds to a cap large enough to cover the whole incentive cost of defense.

The state-contingent payment α is sunk at the procurement stage, whereas the completion-contingent payment β reduces the net value of procurement. The following result shows that the government uses the capped state-contingent budget first. If that budget is insufficient, the residual incentive problem is exactly the baseline problem with a lower effective agency cost.

Proposition 5. *Suppose that the government can pay at most $\bar{B} \geq 0$ upon realization of the secure state, independently of project completion, and can also promise a completion-contingent reward in the secure state. If $\bar{B} \geq \psi/(1 - \rho)$, then any optimal contract that induces defense can be chosen as outlined by Proposition 4.*

If instead $\bar{B} < \psi/(1 - \rho)$, then any optimal contract that induces defense can be chosen with

$$\alpha = \bar{B},$$

and the completion-contingent reward solves the same problem as in the baseline model with residual army cost $\psi_{\bar{B}} := \psi - (1 - \rho)\bar{B} > 0$.

Proposition 5 gives a precise sense in which the baseline mechanism is robust. The paper studies the extreme case $\bar{B} = 0$, in which all military incentives must be financed through successful project completion. But the same procurement-security interaction survives whenever $\bar{B} < \psi/(1 - \rho)$. In that region, the general-fund budget relaxes the army's incentive constraint but does not eliminate the residual need for completion-contingent rewards. The model is equivalent to the baseline model with ψ replaced by $\psi_{\bar{B}} = \psi - (1 - \rho)\bar{B}$. As $\bar{B}$ increases, the required completion-contingent reward falls, the procurement distortion becomes smaller, and the procurement rule moves closer to the standard procurement benchmark. The interaction between procurement and security disappears only when the state-contingent budget is large enough to finance the entire incentive payment.

The fully state-contingent case is not the generic implication of verifiability alone. It also requires sufficient unrestricted budgetary capacity. If wartime fiscal constraints prevent the government from pledging enough general funds, then part of the army's incentive payment must still be tied to successful project realization. In that case, the main results of the paper carry over with a weaker but otherwise unchanged incentive channel. Contractor competition still helps security by raising $q_n(\cdot)$. Project selection still affects the surplus from which completion-contingent rewards can be paid. Changes in contractor costs still matter through the probability that the reward is paid. External support still weakens domestic protection incentives by reducing the marginal effect of army effort, although the capped state-contingent budget absorbs part of the required compensation.

7. Extensions

In this section, we extend the baseline model along several dimensions, while preserving the main structure and insights of the original framework.

7.1. Continuous security choice

The binary benchmark is useful because it turns the government's problem into a clean implementability comparison. It is also useful to consider a smooth variant in which protection can be increased at the margin. Return to the one-contractor scenario and maintain the interior assumption $H(\underline{c}) < V < H(\bar{c})$. Instead of choosing $e \in \{0, 1\}$, the army now chooses $\rho \in [0, 1]$, interpreted as the

probability of a secure environment, and bears quadratic cost

$$\frac{\psi}{2} \rho^2.$$

For any target procurement cutoff $c \in [\underline{c}, c^0]$, the implied bonus is

$$t(c) = V - H(c).$$

The army solves

$$\max_{\rho \in [0,1]} \rho F(c)(V - H(c)) - \frac{\psi}{2} \rho^2.$$

To keep the extension focused on the interior trade-off, assume

$$\psi > \max_{c \in [\underline{c}, c^0]} F(c)(V - H(c)).$$

Under this restriction, the army's best response never hits the corner $\rho = 1$. The following then holds.

Proposition 6. *Suppose*

$$\psi > \max_{c \in [\underline{c}, c^0]} F(c)(V - H(c)).$$

Then, for every target cutoff $c \in [\underline{c}, c^0]$, the army's unique best response is

$$\rho^*(c) = \frac{F(c)(V - H(c))}{\psi} \in [0, 1).$$

The government chooses c to maximize

$$\frac{F(c)^3}{\psi f(c)} (V - H(c)).$$

Any interior optimum $c^c \in (\underline{c}, c^0)$ satisfies

$$(V - H(c^c)) \frac{d}{dc} \left(\frac{F(c)^3}{f(c)} \right) \Big|_{c=c^c} = \left(1 + \frac{d}{dc} \left(\frac{F(c)}{f(c)} \right) \Big|_{c=c^c} \right) \frac{F(c^c)^3}{f(c^c)}.$$

A higher ψ lowers the army's security response pointwise and weakly lowers the government's value.

The binary model remains the benchmark because it delivers the central mechanism as a simple implementability problem where either the army is induced to defend or it is not. The continuous specification is not a replacement for that benchmark. It is a robustness check showing that the same procurement–security margin appears when protection can be adjusted smoothly.

The key object is the expected completion-contingent prize. For a given procurement cutoff c , this prize is $F(c)(V - H(c))$. A more permissive procurement rule raises the probability that the project is implemented, but lowers the reward available upon implementation. A tighter rule preserves a larger reward, but makes payment less likely. The government faces the same trade-off as in the binary model, now expressed through the army's marginal security response rather than through an incentive-compatibility threshold.

Appendix B.2 shows formally how this expected-prize channel extends to the other comparative mechanisms studied below.

7.2. Positive project value under unrest

Suppose now that the project retains positive value even in the unrest state. Let

$$V(s) = V_s > V_u = V(u) > 0,$$

and maintain the baseline restriction that military compensation is completion-contingent. Within this class, the army is paid only when completion occurs in the secure state. The Appendix shows that paying for completion in the unrest state is never optimal because it weakens the army's incentive to generate security. If the army defends, the secure state occurs with probability one. Then, a positive unrest-state value does not enter the army's incentive compatibility constraint. The reason is that, once compensation is made only in the secure state, the army's private gain from exerting effort depends only on the increase in the probability of reaching that state. Yet, the possibility of completing the project at a lower, but positive, value in the unrest state changes the government's decision of invest in security.

Proposition 7. *When $V_s > V_u > 0$, defense is chosen weakly less often by the government than in the baseline model.*

The main implication is that a positive unrest-state payoff weakens security incentives only indirectly. It does not make shirking more attractive for the army, because the army is not paid when unrest occurs. Instead, it makes tolerated insecurity more attractive for the government by improving its outside option.

7.2.1. Project selection

The previous setting raises a natural question. In many conflict environments, fiscal constraints, limited administrative capacity, donor conditionality, and the need to restore basic functionality under severe budget pressure often force planners to prioritize some projects before others. In these settings, the relevant policy problem is not only how to procure a given project, but also which type of project should be funded first. This choice is especially important when alternative projects differ in their exposure to insecurity. Some investments generate value only if the local environment remains sufficiently safe, whereas others retain part of their value even when attacks, sabotage, or disruption persist. The extension developed below studies this trade-off by allowing the government to choose between a fragile project, whose payoff is highly state-contingent, and a more resilient project, whose payoff is less sensitive to insecurity.

Suppose then that before setting the military reward, the government must choose between two project types. A fragile project generates value only in the secure state. A resilient project generates a lower payoff in the secure state but retains some value even if the security environment deteriorates.

The extension is useful because it separates two distinct aspects. One aspect is insurance. Resilient projects reduce the cost of tolerated insecurity. The other is incentives. Under completion-contingent rewards paid only in the secure state, the unrest-state value does not directly enter the army's incentive constraint. What matters directly is the value of completion in the secure state, because this determines the surplus from which the army can be rewarded. Thus fragile projects create stronger defense incentives only when fragility is associated with a higher secure-state payoff, as in the equal-expected-value comparison below.

To keep the analysis transparent, suppose there is one contractor and that the contractor's cost distribution is the same across project types. Hence, the procurement objects $q(\cdot)$ and $\Pi(\cdot)$ are common across projects. Let $j \in \{F, R\}$ denote the project type, with secure-state value V_j^s and unrest-state value V_j^u . If project j is selected and the army is rewarded only in the secure state, the incentive constraint is

$$(1 - \rho)tq(V_j^s - t) \geq \psi.$$

Define

$$\psi_j := \max_{t \in [0, V_j^s]} (1 - \rho)tq(V_j^s - t)$$

as the largest army cost that project j can sustain, and let

$$t_j^D := \min \left\{ t \in [0, V_j^s] \mid (1 - \rho)tq(V_j^s - t) \geq \psi \right\}$$

denote the smallest implementing reward whenever defense is feasible. The continuation value associated with project j is therefore

$$W_j = \max \left\{ \mathbb{1}_{\{\psi \leq \psi_j\}} \Pi(V_j^s - t_j^D), \rho \Pi(V_j^s) + (1 - \rho) \Pi(V_j^u) \right\}.$$

In what follows, we impose different structures on the project's value across the states and examine how the government's project choice depends on these assumptions.

Common secure-state value. The first comparison fixes the secure-state payoff and varies only the project's resilience under unrest.

Proposition 8. *Suppose that*

$$V_F^s = V_R^s =: V, \quad V_F^u = 0, \quad V_R^u =: v \in (0, V).$$

Then, the two projects generate identical defense incentives. Conditional on inducing defense, the government obtains the same payoff under the two projects. Yet, conditional on tolerating insecurity, the resilient project yields weakly higher payoff.

This proposition shows that the original insurance–incentives trade-off disappears when secure-state payoffs are equal and military compensation is made only in the secure state. In that case resilience improves the government's outside option without weakening the army's incentives, so the resilient project weakly dominates.

Equal expected value under tolerated insecurity. A more interesting comparison arises when the fragile project has a higher secure-state value, while the resilient project preserves value under unrest.

Proposition 9. *Suppose that*

$$V_F^u = 0, \quad 0 < V_R^u < V_R^s, \quad \rho V_F^s = \rho V_R^s + (1 - \rho)V_R^u.$$

Then, the fragile project provides stronger defense incentives, and induces a lower payment to the army. Hence, conditional on inducing defense, the fragile project yields a weakly higher government payoff.

Once projects are normalized to have the same expected value under tolerated insecurity, fragility can be attractive again. That is, they are in a mean-preserving order. The reason is that the higher secure-state value of the fragile project strengthens defense incentives.

7.3. State-dependent costs

The disruption state may also worsen contractor performance directly through damaged transport networks, more expensive inputs, insurance premia, or the withdrawal of specialized workers. Throughout this subsection we allow

$$V(s) \equiv V_s > V_u \equiv V(u) \geq 0.$$

The baseline case corresponds to $V_u = 0$. The purpose of the subsection is to isolate the possibility that insecurity changes the contractor cost distribution.

Suppose that contractor costs are drawn from state-contingent distributions. Let $F^s(c)$ denote the cost distribution in the secure state and $F^u(c)$ the cost distribution in the disruption state. For each state $\ell \in \{s, u\}$, define the associated virtual-cost schedule

$$H^\ell(c) := c + \frac{F^\ell(c)}{f^\ell(c)}.$$

Let $q_n^\ell(x)$ and $\Pi_n^\ell(x)$ denote, respectively, the procurement probability and the government's continuation payoff when the net project value is x and costs are drawn from $F^\ell(c)$.

To compare procurement conditions across states, it is useful to express virtual costs in quantile space. Let

$$Q^\ell(z) := (F^\ell)^{-1}(z), \quad z \in [0, 1],$$

denote the quantile function in state $\ell \in \{s, u\}$, and define

$$K^\ell(z) := H^\ell(Q^\ell(z)) = Q^\ell(z) + z(Q^\ell)'(z).$$

The function $K^\ell(z)$ is the virtual cost of the contractor located at quantile z of the state- ℓ cost distribution. We assume that disruption worsens procurement conditions in the sense of virtual costs at each quantile, i.e.,

$$K^u(z) \geq K^s(z) \quad \forall z \in [0, 1]. \quad (7.1)$$

This condition ranks the relevant screening objects across states. For each state ℓ , let

$$\tau^\ell(x) := \sup\{z \in [0, 1] \mid K^\ell(z) \leq x\}$$

denote the optimal reserve quantile at net value x , with the usual boundary conventions. Then the probability that at least one contractor is acceptable is

$$q_n^\ell(x) = 1 - (1 - \tau^\ell(x))^n.$$

Condition (7.1) implies

$$\tau^u(x) \leq \tau^s(x) \quad \forall x \geq 0,$$

so disruption induces a weakly tighter procurement rule in quantile space.

We maintain the optimal payment structure characterized in the previous extension. The army is rewarded only when the secure state occurs. Hence, if the government promises a reward t , the army's incentive constraint is

$$(1 - \rho)t q_n^s(V_s - t) \geq \psi.$$

Therefore, disruption-state procurement conditions affect the government's payoff under tolerated insecurity, but do not enter directly into the implementability of defense. The following then holds.

Proposition 10. *Suppose that (7.1) holds. Then, for every $x \geq 0$,*

$$q_n^u(x) \leq q_n^s(x) \quad \text{and} \quad \Pi_n^u(x) \leq \Pi_n^s(x).$$

Moreover,

1. *the implementability of defense is unchanged relative to the case with state-independent disruption-state procurement conditions, and the minimal implementing reward is pinned down by*

$$(1 - \rho)t q_n^s(V_s - t) \geq \psi;$$

2. *the government's payoff under tolerated disruption is*

$$\rho \Pi_n^s(V_s) + (1 - \rho) \Pi_n^u(V_u),$$

which is weakly lower when disruption-state contractor performance worsens in the sense of (7.1);

3. *therefore, a deterioration in disruption-state contractor performance weakly increases the government's incentive to induce defense.*

The key point is that insecurity now distorts procurement on both sides of the problem. It lowers the direct value of successful completion and it also thins the supply side by shifting contractor costs upward. Security is then valuable not only because it protects the project, but also because it preserves procurement efficiency.

7.3.1. Primitive cost shifts and implementation probabilities

The previous subsection imposed an order on virtual costs at common quantiles, which is the object that directly determines reserve quantiles and implementation probabilities. This subsection distinguishes that assumption from a first-order stochastic dominance increase in actual costs. The distinction matters because a worsening of actual costs need not raise virtual costs. It can lower procurement surplus at a fixed net value and yet increase the probability that procurement takes place.

To see the logic, write the one-contractor procurement problem in quantiles. Let $Q(u) = F^{-1}(u)$. Choosing an implementation probability u is equivalent to choosing reserve cost $Q(u)$, so

$$\Pi_F(x) = \max_{u \in [0,1]} u(x - Q(u)).$$

At an interior optimum,

$$x = Q(u) + uQ'(u).$$

The term $Q(u)$ is the actual cost quantile, while $uQ'(u)$ is the informational-rent component embedded in optimal screening. A FOSD worsening raises $Q(u)$, but it need not raise $Q(u) + uQ'(u)$. Hence, the government's continuation payoff at a fixed x falls, but the optimal implementation probability can rise.

Proposition 11. *Suppose that $\hat{F}$ is an FOSD upward shift of F , so that $\hat{Q}(u) \geq Q(u)$ for all $u \in [0, 1]$. Then,*

1. *For every fixed net project value $x \geq 0$,*

$$\Pi_{\hat{F}}(x) \leq \Pi_F(x). \quad (7.2)$$

Hence, a FOSD worsening of actual costs can never raise the government's continuation procurement payoff at a given x .

2. *The implementation probability $q_F(x)$ is not ordered by FOSD in general. There exist regular distributions F and $\hat{F}$ such that $\hat{F}$ is an FOSD upward shift of F , yet*

$$q_{\hat{F}}(x) > q_F(x)$$

for some x .

The first part of the statement is standard, as for each implementation probability u , the government faces a higher actual cost quantile. The second part of the statement is the wartime-relevant one. If the government promises reward t , the army's incentive constraint is

$$\underbrace{(1 - \rho)t q_F(V - t)}_{\text{army's expected reward}} \geq \psi. \quad (7.3)$$

A higher implementation probability can relax the army's incentive constraint even though the government's procurement payoff at a fixed net value is lower.¹

A FOSD upward shift in *actual costs* has two effects at once: it lowers procurement surplus at any given implementation probability, but it may also compress the virtual-cost wedge that governs screening. As a result, a stochastically worse contractor pool can induce the government to procure more often, and can therefore raise the probability that a promised military reward is actually paid. In the context of wartime procurement, this means that a deterioration on the contractor side may strengthen protection incentives without making the government better off.

Proposition 11 implies a sharp negative result at a fixed net project value x . An FOSD worsening of contractor costs can never increase $\Pi_F(x)$. Hence, conditional on a given promised military reward t , the government's continuation procurement payoff weakly falls. What remains ambiguous in the full model is the *ex ante* effect after the government re-optimizes the reward t . The reason is that the same FOSD shift may relax the army's incentive constraint by raising $q_F(V - t)$. The contractor side worsens, but the security side may improve.

The next result shows that a FOSD increase in actual costs changes both the level of costs and the information-rent component embedded in optimal screening. As a result, the procurement margin and the security margin need not move in the same direction. This may raise the expected value of the army's success-contingent reward and can make defense implementable in parameter regions where it was previously infeasible. When this switch from tolerated disruption to induced defense is sufficiently valuable, the gain on the security side of the problem dominates the loss on the procurement side.

Proposition 12. *Consider the one-contractor benchmark. A first-order stochastic dominance upward shift in the contractor's actual cost distribution can never raise the government's continuation procurement payoff at a fixed net value. Nevertheless, after the government re-optimizes the army reward, such a shift can strictly increase the government's ex-ante equilibrium welfare.*

¹ The same logic carries over under contractor competition. With n contractors, implementation is $q_{n,F}(x) = 1 - (1 - q_F(x))^n$. Since the right-hand-side is strictly increasing in $q_F \in [0, 1]$, any comparison of implementation probabilities in the one-contractor benchmark extends immediately to the n -contractor case.

The intuition behind the result above is a regime change. Before the cost shift, defense may be infeasible because the probability of implementation is too low for any completion-contingent reward to satisfy the army's incentive constraint. After the shift, implementation can become more likely at the relevant net value because the virtual-cost wedge falls. If this makes defense feasible, the government may move from tolerated disruption to induced defense. When the gain from that security-regime change is large enough, it can dominate the lower procurement surplus generated by worse actual costs. This result should not be read as saying that higher costs are good. At any fixed net value they are bad for procurement. However, once procurement and security incentives are jointly determined, a shock that weakens the contractor side can also strengthen the security side of the problem.

7.4. External support

So far, we have considered a closed economy in which project implementation depends only on domestic procurement and domestic security provision. In many conflict and reconstruction settings, however, governments may also receive external support. Such support can take different forms. It may directly improve security conditions, for instance through foreign military assistance or outside protection, or it may strengthen procurement capacity without substituting for the domestic army's effort. This section studies how these different forms of external support affect the government's problem. We first consider security-side support, modeled as an exogenous increase in the probability that the environment is secure when the domestic army shirks. We then contrast this with procurement-side support, which improves procurement capacity without weakening the army's marginal contribution to security.

External military support is naturally modeled as an exogenous increase in the probability that the environment is secure when the army shirks (i.e., ρ). This increment captures outside protection, foreign military aid, or other interventions that make disruption less likely even without domestic military effort.

External support has two opposite effects. On the one hand, it improves the government's outside option under tolerated insecurity, because projects are more likely to survive even if the army is not induced to defend. On the other hand, it weakens the army's incentives, because effort now has a smaller marginal effect on the probability that the project succeeds. This tension makes it unclear whether support helps the government by making insecurity less damaging, or hurts it by making protection harder to induce. The answer is non-monotone, as shown by Fig. 1.

We proceed by characterizing the effect of external support. If the government does not induce defense, it sets no military reward, the army shirks, the secure state occurs with probability ρ , and the government's ex-ante payoff is

$$W_n^N(\rho) := \rho \Pi_n(V). \quad (7.4)$$

If instead the government induces defense, it chooses the smallest reward that satisfies the army's incentive constraint,

$$t_n^D(\rho) := \min \left\{ t \in [0, V] \mid (1 - \rho) t q_n(V - t) \geq \psi \right\}, \quad (7.5)$$

whenever the set on the right-hand side is nonempty. The corresponding ex-ante payoff is

$$W_n^D(\rho) := \Pi_n(V - t_n^D(\rho)).$$

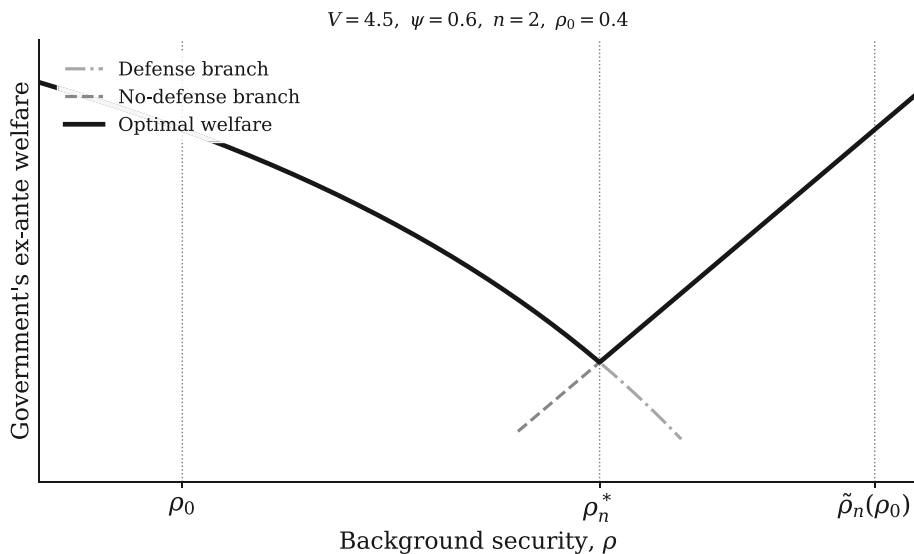


Fig. 1. Equilibrium welfare and external military support in the uniform benchmark. The figure plots the payoff under induced defense $W_n^D(\rho)$, the payoff when defense is not induced $W_n^N(\rho)$, and the equilibrium welfare envelope $W_n(\rho)$. Welfare is non-monotone in ρ : it initially falls as support weakens domestic protection incentives, and rises once tolerated insecurity becomes sufficiently attractive.

Equilibrium welfare is

$$W_n(\rho) := \max \{ W_n^N(\rho), W_n^D(\rho) \},$$

with the convention that $W_n^D(\rho)$ is unavailable when (7.5) has no solution.

Proposition 13. Fix $n \geq 1$.

1. $W_n^N(\rho)$ is strictly increasing in ρ .
2. On the set of ρ for which defense is feasible, $t_n^D(\rho)$ is strictly increasing in ρ , and $W_n^D(\rho)$ is strictly decreasing in ρ .

Fix an initial value $\rho_0 \in (0, 1)$ such that the defense is feasible and optimal, namely

$$W_n(\rho_0) = W_n^D(\rho_0) > W_n^N(\rho_0). \quad (7.6)$$

Define

$$\tilde{\rho}_n(\rho_0) := \frac{W_n^D(\rho_0)}{\Pi_n(V)} = \frac{\Pi_n(V - t_n^D(\rho_0))}{\Pi_n(V)}.$$

Then,

$$\rho_0 < \tilde{\rho}_n(\rho_0) < 1. \quad (7.7)$$

Moreover,

1. if external support raises ρ from ρ_0 to some $\rho' \in (\rho_0, \tilde{\rho}_n(\rho_0))$, then

$$W_n(\rho') < W_n(\rho_0); \quad (7.8)$$

2. if external support raises ρ from ρ_0 to some $\rho' \in [\tilde{\rho}_n(\rho_0), 1)$, then

$$W_n(\rho') \geq W_n(\rho_0). \quad (7.9)$$

Proposition 13 gives a simple interpretation. When the government initially relies on domestic defense, a modest improvement in background security is harmful. The reason is that it weakens the army's incentives before it improves the government's outside option by enough to compensate. By contrast, sufficiently strong support is beneficial because it raises the value of tolerated insecurity enough that the government can stop relying on a distorted domestic incentive scheme.

In that sense, the model supports a *go-big-or-stay-out* logic. Conditional on support being evaluated by its effect on government welfare, small interventions may be counterproductive, whereas large interventions that effectively substitute for domestic protection become beneficial. One way to think about this is through interventions such as limited corridor protection, sporadic escort services, or partial air-defense coverage. These may improve background security somewhat, but not enough to replace domestic protection, and so they can fall in the harmful region identified above. Broader interventions that reliably secure implementation, by contrast, lie in the beneficial region.

The general results above are useful because they do not require closed-form solutions. It is useful to state explicit formulas obtained under a standard uniform benchmark. Assume that contractors' costs are i.i.d. uniform on $[0, 1]$, and that V is large enough that the reserve cost never binds along the relevant equilibrium path. In particular, assume

$$V \geq 2 + \frac{\psi}{1 - \rho}. \quad (7.10)$$

Then, procurement always takes place whenever the state is secure, so $q_n(V - t) = 1$ for all relevant t , and the government's continuation procurement payoff is

$$\Pi_n(V - t) = V - t - \frac{2}{n + 1}, \quad (7.11)$$

where for $n \geq 2$ the term $2/(n + 1)$ is the expected second-lowest cost, while for $n = 1$ it coincides with the one-contractor posted-payment benchmark.

Corollary 1. Under (7.10) and (7.11),

$$W_n^N(\rho) = \rho \left(V - \frac{2}{n + 1} \right), \quad (7.12)$$

and

$$W_n^D(\rho) = V - \frac{2}{n + 1} - \frac{\psi}{1 - \rho}. \quad (7.13)$$

The government induces defense if and only if

$$\rho \leq \rho_n^* := 1 - \sqrt{\frac{\psi}{V - \frac{2}{n+1}}}. \quad (7.14)$$

Moreover, if $\rho_0 < \rho_n^*$, then the support level that makes the government exactly indifferent between its initial welfare and tolerated insecurity is

$$\tilde{\rho}_n(\rho_0) = 1 - \frac{\psi}{(1 - \rho_0) \left(V - \frac{2}{n+1} \right)}. \quad (7.15)$$

Hence, support lowers welfare for $\rho' \in (\rho_0, \tilde{\rho}_n(\rho_0))$ and weakly raises welfare for $\rho' \in [\tilde{\rho}_n(\rho_0), 1)$.

Finally, both ρ_n^* and $\tilde{\rho}_n(\rho_0)$ are increasing in n .

The corollary sharpens the economic intuition behind the extension. In the uniform benchmark, greater competition raises procurement surplus and makes domestically induced defense more attractive. For exactly that reason, it also implies that a larger increase in background security is needed before external support becomes beneficial. Contractor competition and outside military support are therefore partial substitutes. The more effective procurement already is, the more support is required before the government prefers to dispense with domestic protection incentives.

This delivers a more subtle implication. In the same benchmark, stronger competition enlarges the range in which modest external support is harmful. When procurement already performs relatively well, a small improvement in background security weakens domestic protection incentives without yet providing a sufficient substitute for them. In this sense, competition is not unambiguously beneficial once external military support is taken into account. This conclusion should, however, be interpreted with care. It is a property of the closed-form uniform benchmark, where the threshold $\tilde{\rho}_n(\rho_0)$ is increasing in n . In the general model, that monotonicity need not hold automatically, because competition improves both the government's payoff under tolerated insecurity and its payoff under induced defense, so the overall effect on the support threshold is not generally signed.

The analysis above treats external support as security-side support. Yet, external assistance can also operate on the procurement side. Donors or allied governments may provide procurement expertise, expand the set of qualified suppliers, improve logistics, reduce administrative frictions, or help certify and monitor delivery. These interventions do not directly change the army's marginal contribution to security. Instead, they improve the procurement environment in which completion-contingent rewards are paid.

To capture this channel in a reduced-form way, let η denote procurement-side external support. For each η , write

$$q_n(x | \eta) \quad \text{and} \quad \Pi_n(x | \eta)$$

for the procurement probability and continuation payoff at net value x . A higher value of η represents a procurement improvement if, for every $x \geq 0$,

$$q_n(x | \eta') \geq q_n(x | \eta) \quad \text{and} \quad \Pi_n(x | \eta') \geq \Pi_n(x | \eta)$$

whenever $\eta' \geq \eta$. This condition is satisfied, for example, by an increase in the number of qualified contractors or by an improvement in procurement conditions that lowers virtual costs.

For a given η , defense is implementable if and only if

$$\psi \leq \bar{\psi}_n(V, \rho | \eta) := \max_{t \in [0, V]} (1 - \rho) t q_n(V - t | \eta).$$

When defense is implementable, let

$$t_n^D(\rho | \eta) := \min \{ t \in [0, V] \mid (1 - \rho) t q_n(V - t | \eta) \geq \psi \}$$

denote the smallest implementing reward. The government's payoff under induced defense is then

$$W_n^D(\rho | \eta) := \Pi_n(V - t_n^D(\rho | \eta) | \eta),$$

whereas the payoff when defense is not induced is

$$W_n^N(\rho | \eta) := \rho \Pi_n(V | \eta).$$

Equilibrium welfare is

$$W_n(\rho | \eta) := \max \{ W_n^N(\rho | \eta), W_n^D(\rho | \eta) \},$$

with the convention that $W_n^D(\rho | \eta)$ is unavailable when defense is not implementable.

Proposition 14. Suppose that $\eta' \geq \eta$ is a procurement improvement in the sense defined above. Then,

1. defense is weakly easier to implement. That is, $\bar{\psi}_n(V, \rho | \eta') \geq \bar{\psi}_n(V, \rho | \eta)$;
2. if the defense is implementable under both η and η' , then $t_n^D(\rho | \eta') \leq t_n^D(\rho | \eta)$;
3. the payoff from induced defense is weakly higher whenever defense is implementable under both environments, i.e., $W_n^D(\rho | \eta') \geq W_n^D(\rho | \eta)$;
4. the payoff when defense is not induced is weakly higher, i.e., $W_n^N(\rho | \eta') \geq W_n^N(\rho | \eta)$;
5. equilibrium welfare is weakly higher, i.e., $W_n(\rho | \eta') \geq W_n(\rho | \eta)$.

Proposition 14 contrasts procurement-side support with the security-side support studied above. A higher ρ improves background security but weakens the marginal contribution of domestic defense, so its welfare effect can be non-monotone. Procurement-side support has no such adverse incentive effect. It makes completion more likely and raises procurement surplus. It both improves the government's procurement payoff and increases the expected value of the completion-contingent reward promised to the army. In this sense, procurement-side assistance and security-side assistance are not equivalent. Limited security-side support may dilute domestic incentives, whereas procurement-side support is weakly beneficial.

This distinction is useful for policy. If the main bottleneck is residual insecurity, then security-side support is most valuable when it is strong enough to substitute for domestic protection. If the main bottleneck is a thin supplier market, weak procurement capacity, or costly implementation, then procurement-side support is especially effective because it improves both contractor selection and military incentives. Budgetary external support can be understood through **Proposition 5**. By relaxing the cap on state-contingent payments, it reduces the residual incentive cost that must be financed through completion-contingent rewards.

7.5. Procurement before the state is realized

Up until now, we assumed that the government observes the security state before running procurement. This is appropriate for projects that can be postponed until local security conditions are known. In many wartime settings, however, procurement must be organized before the state is resolved. This extension considers that alternative timing. Accordingly, suppose that after the government promises t and the army chooses e , procurement takes place *before* the state is realized. The state becomes known only after project completion. Since the procurement rule must now be chosen under uncertainty, the relevant net project value is the expected value conditional on the equilibrium security probability. If equilibrium induces action e , let

$$\mu(e) = \mathbb{P}[s | e] = \begin{cases} 1, & e = 1, \\ \rho, & e = 0. \end{cases}$$

The government's ex ante net value from successful procurement is then

$$\bar{V}(t, e) = \mu(e)(V - t).$$

The following holds.

Proposition 15. *Suppose that procurement occurs before the state is realized. Then, the optimal procurement mechanism with n contractors is again the conditional Myerson procurement auction, with reserve cost. That is,*

$$\bar{c}(\bar{V}(t, e)) = H^{-1}(\bar{V}(t, e)).$$

If the government induces defense, its continuation payoff is unchanged relative to the baseline and equals

$$\bar{W}_n^d(t) = \Pi_n(V - t).$$

If the government sets $t = 0$ and does not induce, its continuation payoff is

$$\bar{W}_n^e = \Pi_n(\rho V).$$

Moreover,

$$\Pi_n(\rho V) \leq \rho \Pi_n(V), \tag{7.16}$$

with strict inequality whenever $q_n(x)$ is non-constant on an interval contained in $(0, V)$.

Inequality (7.16) captures the distinctive wartime distortion under ex ante procurement. In the main model, insecurity acts as a multiplicative factor on the government's procurement payoff, because procurement is skipped whenever disruption is realized. In this alternative timing, insecurity also depresses the reserve policy itself. The government behaves as if the project were intrinsically less valuable, which lowers procurement probability and procurement rents ex ante. The following holds.

Corollary 2. *If defense is implementable, the gain from inducing defense is weakly larger under ex ante procurement than under the main timing.*

The previous result can be stated more generally as a robustness property of the model. Let

$$D_n(V, \rho, \psi) := \{t \in [0, V] \mid (1 - \rho)tq_n(V - t) \geq \psi\}$$

denote the set of completion-contingent rewards that induce defense. This set is the same under the main timing and under ex ante procurement. When the government induces defense, the secure state occurs with probability one, so the procurement rule and the army's deviation payoff are unchanged. What changes is only the payoff from not inducing defense, because under ex ante procurement the government cannot condition the procurement rule on the realized state.

Proposition 16. *Suppose that procurement occurs before the state is realized. Then,*

1. *defense is implementable under ex ante procurement if and only if it is implementable under the baseline timing;*
2. *for every $t \in D_n(V, \rho, \psi)$, the government's payoff from inducing defense is the same under the two timings, and equals $\Pi_n(V - t)$;*

3. the government's optimal payoff when it does not induce defense is weakly lower under ex ante procurement than under the baseline timing, i.e., $\Pi_n(\rho V) \leq \rho \Pi_n(V)$.

Whenever inducing defense is optimal under the baseline timing, it is also optimal under ex ante procurement. Ex ante procurement weakly expands the set of parameters for which the government chooses to induce defense.

Proposition 16 explains why the main mechanism does not rely on the government observing the security state before procurement. Under induced defense, the secure state is reached with probability one, so the procurement rule, the army's incentive constraint, and the payoff from defense are unchanged. Ex ante procurement instead affects the alternative in which the government does not induce defense. Because the procurement rule must be chosen before the state is known, insecurity lowers the effective project value and makes tolerated disruption less attractive. Thus, the timing variation reinforces the case for inducing security.

8. Conclusion

This paper studies procurement design in environments where implementation depends not only on selecting an efficient contractor, but also on inducing military protection under insecurity. The main message is that procurement and security cannot be designed separately. When military effort is essential for project completion but cannot be directly contracted upon, the government must leave enough surplus to reward protection. Yet the very rewards that strengthen security incentives also reduce the government's net procurement value and distort procurement toward more selective implementation. Wartime procurement is therefore shaped by a fundamental trade-off between efficiency and security.

The analysis shows that this tension survives across a range of richer environments. Competition among contractors improves procurement performance and, by making implementation more likely, also makes protection easier to induce. Project selection is likewise shaped by security incentives. Resilient projects perform better when insecurity is tolerated, but more fragile projects may sometimes be preferred because they create stronger incentives for defense. When disruption also worsens contractor performance, security becomes more valuable not only because it protects project completion, but also because it preserves the functioning of the procurement side itself.

A further implication is that disruptions affecting the contractor side need not have monotone effects on government welfare. In particular, a worsening of contractors' actual cost conditions can reduce procurement surplus at a given net value and yet raise the government's ex-ante equilibrium welfare. The reason is that higher actual costs may also increase procurement at the relevant margin, thereby raising the expected value of success-contingent military rewards and making defense easier to induce. This result highlights that, in conflict environments, weakening procurement markets and weakening the government are not the same thing once procurement and security incentives are jointly determined.

The extension on external military support adds a further lesson. Outside support improves background security when domestic forces shirk, but for that very reason it also weakens domestic protection incentives. As a result, support is not unambiguously beneficial. Limited interventions may reduce welfare by undermining domestic defense without adequately replacing it, whereas sufficiently strong support can become beneficial by allowing the government to rely less on a distorted domestic incentive scheme. This suggests that external assistance and domestic security incentives are intertwined rather than independent policy margins.

The timing of procurement also matters. When procurement must be organized before the security state is known, insecurity depresses procurement not only by threatening implementation ex post, but also by lowering the government's reserve policy ex ante. In that environment, induced defense becomes relatively more attractive. More broadly, the results imply that in conflict environments governments should not treat procurement rules, security policy, project selection, and external support as separate problems. Better outcomes require joint design: choosing projects that can realistically be protected, preserving enough surplus to sustain security, maintaining competition where possible, and recognizing that even well-intentioned outside support may be harmful when it is too limited to substitute for domestic protection.

This perspective speaks to a broader question of state capacity under insecurity. In conflict-affected environments, the challenge is not only that violence lowers the value of public investment. It is also that insecurity changes the way procurement itself must be designed. Without minimizing the broader consequences of conflict, our focus here is on the specific ways in which insecurity reshapes procurement design. The broader implication is that procurement in conflict zones is not simply standard procurement under stress, but a joint problem of market design, implementation security, and state capacity.

CRedit authorship contribution statement

Maria Rosa Battaglion: Writing – review & editing, Writing – original draft, Methodology, Investigation, Formal analysis, Conceptualization. **Salvatore Piccolo:** Writing – review & editing, Writing – original draft, Methodology, Investigation, Formal analysis, Conceptualization. **Federico Vaccari:** Writing – review & editing, Writing – original draft, Methodology, Investigation, Formal analysis, Conceptualization.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the authors used ChatGPT, provided by OpenAI, for language revision and general-purpose research assistance. After using this service, the authors reviewed, verified, and edited the content as needed and take full responsibility for the content of the publication.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Proofs

Proof of Lemma 1. Fix x , if the government offers payment r , the contractor accepts the offer if and only if $c \leq r$, and the government's expected payoff is

$$F(r)(x - r).$$

The derivative with respect to r is

$$f(r)(x - r) - F(r).$$

Under regularity, the unique interior optimum solves

$$x = r + \underbrace{\frac{F(r)}{f(r)}}_{H(r)}.$$

Hence, the optimal reserve payment is $r(x) = H^{-1}(x)$ whenever $x \in [H(\underline{c}), H(\bar{c})]$, with the natural truncations at the boundaries. Procurement occurs if and only if $c \leq r(x)$. On the interior region,

$$x - r(x) = \frac{F(r(x))}{f(r(x))},$$

so the government's continuation payoff can be written as

$$F(r(x))(x - r(x)) = \frac{F(r(x))^2}{f(r(x))}.$$

Monotonicity follows from the envelope theorem. Wherever the derivative exists, it is equal to $F(r(x)) \geq 0$. $\square$

Proof of Proposition 1. Fix any target cutoff $c \in [\underline{c}, c^0]$. By Lemma 1, implementing that cutoff in the secure state requires the government to leave a net procurement value

$$V - t(c) = H(c) \quad \Leftrightarrow \quad t(c) = V - H(c).$$

If the army defends, procurement succeeds with probability $F(c)$ and the army obtains

$$U_A(1, c) = F(c)t(c) - \psi.$$

If it shirks, the secure state arrives only with probability ρ , so the army obtains

$$U_A(0, c) = \rho F(c)t(c).$$

Hence, defense is incentive compatible if and only if

$$(1 - \rho)F(c)(V - H(c)) \geq \psi,$$

implying that defense is implementable if and only if there exists at least one cutoff $c \in [\underline{c}, c^0]$ satisfying that inequality, equivalently if and only if

$$\psi \leq \max_{c \in [\underline{c}, c^0]} (1 - \rho)F(c)(V - H(c)).$$

Under defense, the government's payoff at cutoff c is

$$F(c)(V - t(c) - c) = F(c)(H(c) - c) = \frac{F(c)^2}{f(c)}.$$

By Lemma 1, this continuation payoff is weakly increasing in the net value x , and since $x = H(c)$ with H increasing, it is weakly increasing in c as well. Therefore, among all cutoffs that satisfy the army's incentive constraint, the government chooses the largest one. The associated reward is then necessarily

$$t^d = V - H(c^d),$$

which completes the proof. $\square$

Proof of Proposition 2. By Proposition 1, if the government induces defense, it chooses the largest cutoff compatible with the army's incentive constraint. Let

$$W^d(c) := \frac{F(c)^2}{f(c)}$$

denote the government's continuation payoff under defense at cutoff c , and let

$$W^e(\rho) := \rho \frac{F(c^0)^2}{f(c^0)}$$

denote its payoff under tolerated insecurity, where c^0 is defined by $V = H(c^0)$.

We first show that $W^d(c)$ is strictly increasing on $[\underline{c}, c^0]$. By Proposition 1,

$$W^d(c) = F(c)(H(c) - c).$$

Using the definition of the virtual cost,

$$H(c) = c + \frac{F(c)}{f(c)},$$

we obtain

$$W^d(c) = \frac{F(c)^2}{f(c)}.$$

Equivalently, since the defended net value associated with cutoff c is $x = H(c)$, Lemma 1 implies that

$$W^d(c) = \Pi(H(c)),$$

where $\Pi(x)$ is the government's optimized continuation payoff as a function of the net value x . By the envelope theorem,

$$\Pi'(x) = F(r(x)) > 0$$

in the interior region, where $r(x) = H^{-1}(x)$. Since H is strictly increasing by regularity, it follows that $W^d(c) = \Pi(H(c))$ is strictly increasing in c on $[\underline{c}, c^0]$.

Therefore, for every $\rho \in [0, 1]$, there exists a unique cutoff $c_\rho \in [\underline{c}, c^0]$ such that

$$W^d(c_\rho) = W^e(\rho),$$

that is,

$$\frac{F(c_\rho)^2}{f(c_\rho)} = \rho \frac{F(c^0)^2}{f(c^0)}.$$

Moreover, because W^d is strictly increasing and $W^e(\rho)$ is strictly increasing in ρ , the cutoff c_ρ is strictly increasing in ρ .

The government prefers defense if and only if the largest feasible defended cutoff is at least c_ρ . Hence defense is chosen if and only if the army's incentive constraint is satisfied for some $c \in [c_\rho, c^0]$, namely if and only if

$$\psi \leq M(\rho) := \max_{c \in [c_\rho, c^0]} (1 - \rho)F(c)(V - H(c)).$$

Now evaluate this condition at the extremes. When $\rho = 0$, we have $W^e(0) = 0$, so by the strict monotonicity of W^d and the fact that $W^d(\underline{c}) \geq 0$, it follows that $c_\rho = \underline{c}$. Therefore

$$M(0) = \max_{c \in [\underline{c}, c^0]} F(c)(V - H(c)) =: \bar{\psi}(V).$$

Hence, if $\psi \geq \bar{\psi}(V)$, defense is not even implementable in the most fragile environment, and therefore it is never chosen.

Suppose now that $\psi < \bar{\psi}(V)$. The correspondence $\rho \mapsto [c_\rho, c^0]$ is continuous and compact-valued, and the objective $(c, \rho) \mapsto (1 - \rho)F(c)(V - H(c))$ is continuous. By the Maximum Theorem, $M(\rho)$ is continuous in ρ .

We next show that $M(\rho)$ is strictly decreasing. Let $0 \leq \rho' < \rho'' < 1$. Since $c_{\rho'} < c_{\rho''}$, we have $[c_{\rho''}, c^0] \subset [c_{\rho'}, c^0]$. Moreover, for every $c \in [c_{\rho''}, c^0]$,

$$(1 - \rho'')F(c)(V - H(c)) < (1 - \rho')F(c)(V - H(c)),$$

because $F(c)(V - H(c)) > 0$ on $[\underline{c}, c^0]$. Taking maxima yields $M(\rho'') < M(\rho')$. Thus, M is strictly decreasing on $[0, 1]$.

Finally, since the factor $(1 - \rho)$ tends to zero as $\rho \uparrow 1$, we have $M(\rho) \rightarrow 0$ as $\rho \uparrow 1$. Because M is continuous, strictly decreasing, and satisfies

$$M(0) = \bar{\psi}(V) > \psi, \quad \lim_{\rho \uparrow 1} M(\rho) = 0 < \psi,$$

there exists a unique threshold $\bar{\rho}(V, \psi) \in (0, 1)$ such that

$$M(\bar{\rho}(V, \psi)) = \psi.$$

Therefore, the government induces defense if and only if $\rho \leq \bar{\rho}(V, \psi)$.

To prove comparative statics with respect to ψ , fix V and let $0 < \psi' < \psi'' < \bar{\psi}(V)$. By definition of the threshold,

$$M(\bar{\rho}(V, \psi')) = \psi', \quad M(\bar{\rho}(V, \psi'')) = \psi''.$$

Since M is strictly decreasing, it follows that

$$\bar{\rho}(V, \psi') > \bar{\rho}(V, \psi'').$$

Hence, the threshold is decreasing in ψ . $\square$

Proof of Lemma 2. Fix x , by the revelation principle, it is enough to consider direct-revelation mechanisms. Under Bayesian incentive compatibility and interim individual rationality, the procurement analogue of the standard Myerson argument implies that expected government payoff can be written as the expected virtual surplus,

$$\mathbb{E} \left[\sum_{i=1}^n (x - H(c_i)) p_i(c) \right],$$

up to constants pinned down by the zero-rent condition for the highest-cost type. Under regularity, $H(\cdot)$ is increasing, so maximizing expected virtual surplus requires assigning the project to the contractor with the lowest cost whenever the associated virtual cost does not exceed x , and not procuring otherwise. The associated transfer rule is the threshold payment that makes truthful reporting optimal and gives zero rent to the highest-cost type. This is exactly a second-price procurement auction with reserve cost $\tilde{c}(x) = H^{-1}(x)$. $\square$

Proof of Proposition 3. Since

$$q_n(x) = \mathbb{P}(c_{(1)} \leq \tilde{c}(x)),$$

adding contractors weakly lowers the minimum cost in the sense of first-order stochastic dominance, so $q_n(x)$ is weakly increasing in n . Likewise,

$$\Pi_n(x) = \mathbb{E}[(x - H(c_{(1)})) \mathbb{1}\{c_{(1)} \leq \tilde{c}(x)\}]$$

is weakly increasing in n because $H(c_{(1)})$ becomes weakly smaller in distribution.

Since $\bar{\psi}_n(V, \rho)$ is the maximum over $t \in [0, V]$ of $(1 - \rho)tq_n(V - t)$, and q_n is pointwise increasing in n . Hence, the claim. The rest of the proof follows because the set of rewards satisfying the army's incentive constraint expands when n rises, so its minimum weakly falls. Finally, part 3 follows because $\Pi_n(\cdot)$ is pointwise increasing in n while the argument $V - t_n^D$ weakly rises as t_n^D falls. $\square$

Proof of Proposition 4. For a given (α, β) , with $\beta \in [0, V]$, the army's payoff from effort is $\alpha + \beta q_n(V - \beta) - \psi$, whereas its payoff from shirking is $\rho[\alpha + \beta q_n(V - \beta)]$. Thus, defense is incentive compatible if and only if

$$\alpha + \beta q_n(V - \beta) \geq \frac{\psi}{1 - \rho}.$$

Conditional on induced defense, the government's payoff is $\Pi_n(V - \beta) - \alpha$. Hence, for any fixed β , the government optimally chooses

$$\alpha = \max \left\{ \frac{\psi}{1 - \rho} - \beta q_n(V - \beta), 0 \right\}.$$

We show that $\beta = 0$ is optimal. First suppose that $\beta q_n(V - \beta) \leq \frac{\psi}{1 - \rho}$. Then the government's payoff at β is

$$\Pi_n(V - \beta) - \frac{\psi}{1 - \rho} + \beta q_n(V - \beta).$$

Using the envelope property $\Pi'_n(x) = q_n(x)$ and the monotonicity of q_n , we have

$$\Pi_n(V) - \Pi_n(V - \beta) = \int_{V-\beta}^V q_n(z) dz \geq \beta q_n(V - \beta).$$

Therefore,

$$\Pi_n(V - \beta) - \frac{\psi}{1 - \rho} + \beta q_n(V - \beta) \leq \Pi_n(V) - \frac{\psi}{1 - \rho}.$$

Thus, no such β can yield more than the state-contingent contract $(\alpha, \beta) = \left(\frac{\psi}{1 - \rho}, 0 \right)$.

Now suppose that $\beta q_n(V - \beta) > \frac{\psi}{1 - \rho}$. Then $\alpha = 0$, and the government's payoff is $\Pi_n(V - \beta)$. The same envelope argument gives

$$\Pi_n(V) - \Pi_n(V - \beta) = \int_{V-\beta}^V q_n(z) dz \geq \beta q_n(V - \beta) > \frac{\psi}{1 - \rho},$$

so

$$\Pi_n(V - \beta) < \Pi_n(V) - \frac{\psi}{1 - \rho}.$$

Again, β is strictly dominated by the state-contingent contract $(\alpha, \beta) = \left(\frac{\psi}{1 - \rho}, 0 \right)$. Hence, an optimal defended contract can be chosen with $\beta = 0$ and $\alpha = \psi/(1 - \rho)$.

Under this contract, the payment α is sunk at the procurement stage. The government uses the procurement mechanism associated with gross value V , and its defended payoff is

$$\Pi_n(V) - \frac{\psi}{1-\rho}.$$

If defense is not induced, the government sets $\alpha = \beta = 0$, the army shirks, and the secure state occurs with probability ρ . In the baseline case $V(u) = 0$, the no-defense payoff is $\rho\Pi_n(V)$. Thus, defense is chosen if and only if

$$\Pi_n(V) - \frac{\psi}{1-\rho} \geq \rho\Pi_n(V),$$

or equivalently $\psi \leq (1-\rho)^2\Pi_n(V)$.

If $V(s) = V_s > V_u = V(u) \geq 0$, the defended payoff is

$$\Pi_n(V_s) - \frac{\psi}{1-\rho},$$

whereas the no-defense payoff is $\rho\Pi_n(V_s) + (1-\rho)\Pi_n(V_u)$. This gives the final condition in the proposition. $\square$

Proof of Proposition 5. For a given (α, β) , with $\alpha \in [0, \bar{B}]$ and $\beta \in [0, V]$, the army's payoff from effort is $\alpha + \beta q_n(V - \beta) - \psi$, whereas its payoff from shirking is $\rho[\alpha + \beta q_n(V - \beta)]$. Thus, defense is incentive compatible if and only if $\alpha + \beta q_n(V - \beta) \geq \psi/(1-\rho)$. Conditional on induced defense, the government's payoff is $\Pi_n(V - \beta) - \alpha$.

If $\bar{B} \geq \psi/(1-\rho)$, the contract $\alpha = \psi/(1-\rho)$ and $\beta = 0$ satisfies the army's incentive constraint and leaves the procurement rule undistorted. The optimality of setting $\beta = 0$ in this case follows from Proposition 4. A completion-contingent reward lowers the net procurement value, whereas a state-contingent reward provides the same incentive without distorting procurement.

Now suppose that $\bar{B} < \psi/(1-\rho)$. We first show that any optimal defended contract can be chosen with $\alpha = \bar{B}$. Define

$$m(\beta) := \beta q_n(V - \beta)$$

and

$$L(\beta) := \Pi_n(V) - \Pi_n(V - \beta).$$

Using the envelope property $\Pi'_n(x) = q_n(x)$, we can write

$$L(\beta) = \int_{V-\beta}^V q_n(z) dz = \int_0^\beta q_n(V - s) ds.$$

Because $q_n(\cdot)$ is weakly increasing, the function $s \mapsto q_n(V - s)$ is weakly decreasing. Hence, for any $0 \leq \beta' \leq \beta$, we have that

$$L(\beta) - L(\beta') = \int_{\beta'}^\beta q_n(V - s) ds \geq \beta q_n(V - \beta) - \beta' q_n(V - \beta') = m(\beta) - m(\beta').$$

The inequality follows because

$$m(\beta) - m(\beta') = \int_{\beta'}^\beta \frac{d}{ds} [s q_n(V - s)] ds = \int_{\beta'}^\beta [q_n(V - s) - s q'_n(V - s)] ds \leq \int_{\beta'}^\beta q_n(V - s) ds.$$

Consider any feasible defended contract (α, β) with $\alpha < \bar{B}$. Since the contract is feasible, we have $\alpha + m(\beta) \geq \psi/(1-\rho)$. Let

$$R' := \frac{\psi}{1-\rho} - \bar{B} > 0.$$

Because $\bar{B} > \alpha$, we have $R' < \psi/(1-\rho) - \alpha \leq m(\beta)$. Hence, there exists $\beta' \leq \beta$ such that $m(\beta') \geq R'$. Choose the smallest such β' . By continuity, $m(\beta') = R'$. The contract $(\bar{B}, \beta')$ is incentive compatible because

$$\bar{B} + m(\beta') = \bar{B} + R' = \frac{\psi}{1-\rho}.$$

Moreover, its payoff is weakly higher than the payoff from (α, β) . Indeed,

$$\begin{aligned} [\Pi_n(V - \beta') - \bar{B}] - [\Pi_n(V - \beta) - \alpha] &= \Pi_n(V - \beta') - \Pi_n(V - \beta) - (\bar{B} - \alpha) \\ &= L(\beta) - L(\beta') - (\bar{B} - \alpha) \\ &\geq m(\beta) - m(\beta') - (\bar{B} - \alpha) \\ &= m(\beta) - R' - (\bar{B} - \alpha) \\ &= m(\beta) - \left(\frac{\psi}{1-\rho} - \bar{B} \right) - (\bar{B} - \alpha) \\ &= m(\beta) + \alpha - \frac{\psi}{1-\rho} \\ &\geq 0. \end{aligned}$$

Therefore, any defended contract with $\alpha < \bar{B}$ is weakly dominated by one that sets $\alpha = \bar{B}$. It follows that an optimal defended contract can be chosen with $\alpha = \bar{B}$.

With $\alpha = \bar{B}$, the army's incentive constraint becomes

$$(1 - \rho)\beta q_n(V - \beta) \geq \psi - (1 - \rho)\bar{B}.$$

Let

$$\psi_{\bar{B}} := \psi - (1 - \rho)\bar{B} > 0.$$

Defense is implementable if and only if

$$\psi_{\bar{B}} \leq \max_{\beta \in [0, V]} (1 - \rho)\beta q_n(V - \beta).$$

Since $\Pi_n(V - \beta)$ is weakly decreasing in β , the government chooses the smallest completion-contingent reward satisfying the residual incentive constraint, namely

$$\beta_{\bar{B}}^D := \min \{ \beta \in [0, V] \mid (1 - \rho)\beta q_n(V - \beta) \geq \psi_{\bar{B}} \}.$$

The resulting defended payoff is $\Pi_n(V - \beta_{\bar{B}}^D) - \bar{B}$. If defense is not induced, the government sets $\alpha = \beta = 0$, the army shirks, and in the baseline case $V(u) = 0$ the secure state occurs with probability ρ . The no-defense payoff is $\rho\Pi_n(V)$. Thus, defense is chosen if and only if

$$\Pi_n(V - \beta_{\bar{B}}^D) - \bar{B} \geq \rho\Pi_n(V).$$

This completes the proof. □

Proof of Proposition 6. Fix any target cutoff $c \in [\underline{c}, c^0]$. The associated bonus is

$$t(c) = V - H(c).$$

The army solves

$$\max_{\rho \in [0, 1]} \rho F(c)t(c) - \frac{\psi}{2}\rho^2.$$

Under the maintained restriction,

$$F(c)t(c) < \psi \quad \text{for every } c \in [\underline{c}, c^0],$$

so the optimum is interior. The first-order condition gives

$$\rho^*(c) = \frac{F(c)t(c)}{\psi} = \frac{F(c)(V - H(c))}{\psi}.$$

Substituting back into the government's objective yields

$$W(c) = \rho^*(c) \frac{F(c)^2}{f(c)} = \frac{F(c)^3}{\psi f(c)} (V - H(c)).$$

Any interior optimum c^c must therefore satisfy

$$0 = \frac{dW(c)}{dc} \Big|_{c=c^c} = \frac{1}{\psi} \left[(V - H(c^c)) \frac{d}{dc} \left(\frac{F(c)^3}{f(c)} \right) \Big|_{c=c^c} - H'(c) \Big|_{c=c^c} \frac{F(c^c)^3}{f(c^c)} \right].$$

Since

$$H'(c) = 1 + \frac{d}{dc} \left(\frac{F(c)}{f(c)} \right),$$

this is exactly the first-order condition reported in the text. Finally, a higher ψ lowers $\rho^*(c)$ pointwise for every c , and therefore weakly lowers the government's optimized value. □

Proof of Proposition 7. When the army is paid only in the secure state, defense yields the army's payoff

$$U_A^+(1, t) = tq_n(V_s - t) - \psi,$$

whereas shirking yields

$$U_A^+(0, t) = \rho tq_n(V_s - t).$$

Hence the army's incentive constraint is

$$(1 - \rho)tq_n(V_s - t) \geq \psi.$$

This is exactly the same condition as in the baseline case with no value in the unrest state. Therefore a positive unrest-state value does not affect implementability.

What it does affect is the government's outside option under tolerated insecurity. In the baseline case that payoff is

$$\rho \Pi_n(V_s),$$

whereas with positive unrest-state value it becomes

$$\rho \Pi_n(V_s) + (1 - \rho) \Pi_n(V_u).$$

Because $V_u > 0$ and $\Pi_n(\cdot)$ is increasing, the latter expression is strictly larger. Thus, for every pair (ρ, ψ) for which defense is feasible, the government is weakly less willing to induce defense than in the baseline model, and strictly less willing whenever the baseline comparison was not already strictly in favor of insecurity. In this sense defense is chosen less often than in the benchmark. $\square$

Proof of the payment-structure claim in Section 7.2. The text notes that when the project has positive value even in the unrest state, an optimal completion-contingent contract pays the army only upon completion in the secure state. Let (t_s, t_u) denote payments contingent on successful completion in the secure and unrest states, respectively.

Under defense, unrest never occurs. The government's defended payoff therefore depends only on t_s and is equal to

$$\Pi_n(V_s - t_s).$$

The transfer t_u does not affect this payoff.

By contrast, t_u enters the army's incentive constraint negatively. If the army defends, its payoff is

$$U_A(1 | t_s, t_u) = t_s q_n(V_s - t_s) - \psi.$$

If it shirks, its payoff is

$$U_A(0 | t_s, t_u) = \rho t_s q_n(V_s - t_s) + (1 - \rho) t_u q_n(V_u - t_u).$$

Hence defense is incentive compatible if and only if

$$(1 - \rho) [t_s q_n(V_s - t_s) - t_u q_n(V_u - t_u)] \geq \psi.$$

Since limited liability requires $t_u \geq 0$, replacing t_u by zero weakly relaxes the army's incentive constraint and leaves the government's defended payoff unchanged. Therefore any optimal completion-contingent contract can be chosen with $t_u = 0$. $\square$

Proof of Proposition 8. Because the two projects share the same secure-state value V , their defense incentive terms coincide. That is,

$$(1 - \rho) t q(V - t).$$

Hence they support the same maximal army cost and, whenever defense is feasible, they require the same smallest implementing reward. This proves the first claim.

Conditional on defense, the government obtains exactly the same defended payoff under both projects,

$$\Pi(V - t^D).$$

This proves the second claim.

Under tolerated insecurity, the fragile project yields

$$\rho \Pi(V),$$

whereas the resilient project yields

$$\rho \Pi(V) + (1 - \rho) \Pi(v),$$

which is strictly larger because $v > 0$ and $\Pi(\cdot)$ is increasing. This proves the third claim. $\square$

Proof of Proposition 9. The equal-expected-value restriction implies

$$\rho V_F^s = \rho V_R^s + (1 - \rho) V_R^u,$$

so $V_F^s > V_R^s$. Since $q(\cdot)$ is increasing, for every $t \in [0, V_R^s]$,

$$(1 - \rho) t q(V_F^s - t) \geq (1 - \rho) t q(V_R^s - t),$$

with strict inequality whenever $t > 0$. Hence, the fragile project can sustain a weakly larger army cost and, for any given ψ that both projects can implement, it requires a weakly lower payment to the army.

Conditional on defense, the government obtains

$$\Pi(V_j^s - t_j^D)$$

under project $j \in \{F, R\}$. Since $V_F^s \geq V_R^s$ and $t_F^D \leq t_R^D$, it follows that

$$V_F^s - t_F^D \geq V_R^s - t_R^D.$$

Because $\Pi(\cdot)$ is increasing, the fragile project yields weakly higher government payoff under induced defense. $\square$

Quantile representation. Fix a net project value $x \geq 0$, where $x = V - t$ if the government promises the army a reward t . The government chooses a reserve payment r to solve

$$\Pi_F(x) := \max_{r \in [\underline{c}, \bar{c}]} F(r)(x - r). \quad (\text{A1})$$

Let $Q(u) := F^{-1}(u)$ denote the quantile function, with $u \in [0, 1]$. Writing $u = F(r)$ and $r = Q(u)$, problem (A1) becomes

$$\Pi_F(x) = \max_{u \in [0, 1]} u(x - Q(u)).$$

Under regularity, the optimum is interior for $x \in (H(\underline{c}), H(\bar{c}))$, and the first-order condition is

$$x = Q(u) + uQ'(u). \quad (\text{A2})$$

Since $Q'(u) = 1/f(Q(u))$, we have

$$Q(u) + uQ'(u) = Q(u) + \frac{u}{f(Q(u))} = Q(u) + \frac{F(Q(u))}{f(Q(u))} = H(Q(u)),$$

so (A2) is exactly the usual reserve condition $x = H(r)$. Let

$$q_F(x) := F(r_F(x))$$

denote the equilibrium implementation probability, where $r_F(x)$ is the optimal reserve under F . In the quantile representation, $q_F(x) = u_F(x)$, where $u_F(x)$ solves (A2).

Evaluating the objective at the optimum and using (A2),

$$\Pi_F(x) = u_F(x)(x - Q(u_F(x))) = u_F(x)^2 Q'(u_F(x)).$$

Now suppose instead that the primitive cost distribution changes from F to $\hat{F}$, with corresponding quantile function $\hat{Q}$, and that costs become stochastically higher in the FOSD sense. That is, $\hat{F}(c) \leq F(c)$ for all c , or, equivalently,

$$\hat{Q}(u) \geq Q(u) \quad \text{for all } u \in [0, 1].$$

This perturbation does *not* imply an upward shift in virtual costs, because the relevant object for screening is $Q(u) + uQ'(u)$, and FOSD orders Q but does not order $Q + uQ'$. A distribution can have higher actual costs pointwise in the FOSD sense while displaying lower virtual costs on part or all of the support.

Proof of Proposition 10. For each state $\ell \in \{s, u\}$, let

$$Q^\ell(z) := (F^\ell)^{-1}(z)$$

be the quantile function and define

$$K^\ell(z) := H^\ell(Q^\ell(z)) = Q^\ell(z) + z(Q^\ell)'(z).$$

Because H^ℓ is strictly increasing and Q^ℓ is weakly increasing, K^ℓ is increasing. For a given net value x , define the reserve quantile

$$\tau^\ell(x) := \sup\{z \in [0, 1] \mid K^\ell(z) \leq x\}.$$

Procurement takes place if and only if at least one contractor has state- ℓ cost quantile below $\tau^\ell(x)$. Hence,

$$q_n^\ell(x) = 1 - (1 - \tau^\ell(x))^n.$$

By (7.1), $K^u(z) \geq K^s(z)$ for all $z \in [0, 1]$, so

$$\{z \in [0, 1] \mid K^u(z) \leq x\} \subseteq \{z \in [0, 1] \mid K^s(z) \leq x\}.$$

Therefore, $\tau^u(x) \leq \tau^s(x)$ and, as a result,

$$q_n^u(x) = 1 - (1 - \tau^u(x))^n \leq 1 - (1 - \tau^s(x))^n = q_n^s(x).$$

We next compare continuation payoffs. Let $Z_1, \dots, Z_n$ be independent uniform random variables on $[0, 1]$, and let

$$Z_{(1)} := \min\{Z_1, \dots, Z_n\}.$$

For state ℓ , the lowest realized cost can be written as $Q^\ell(Z_{(1)})$. The government's continuation payoff can therefore be written in virtual-surplus form as

$$\Pi_n^\ell(x) = \mathbb{E} \left[(x - K^\ell(Z_{(1)})) \mathbb{1} \{K^\ell(Z_{(1)}) \leq x\} \right].$$

Alternatively,

$$\Pi_n^\ell(x) = \mathbb{E} \left[\max \{x - K^\ell(Z_{(1)}), 0\} \right].$$

Since $K^u(z) \geq K^s(z)$ for every z , we have, for every realization of $Z_{(1)}$,

$$\max \{x - K^u(Z_{(1)}), 0\} \leq \max \{x - K^s(Z_{(1)}), 0\}.$$

Taking expectations gives $\Pi_n^u(x) \leq \Pi_n^s(x)$ for all $x \geq 0$.

We now turn to incentives. Because the army is rewarded only in the secure state, if it exerts effort its payoff is $U_A(1 | t) = tq_n^s(V_s - t) - \psi$, whereas if it shirks its payoff is $U_A(0 | t) = \rho tq_n^s(V_s - t)$. Defense is incentive compatible if and only if $(1 - \rho)tq_n^s(V_s - t) \geq \psi$. This condition depends only on secure-state procurement objects. Hence, for fixed F^s , V_s , ρ , and ψ , a worsening of disruption-state contractor performance does not affect either the implementability of defense or the minimal implementing reward.

Finally, under tolerated disruption the government's payoff is $\rho\Pi_n^s(V_s) + (1 - \rho)\Pi_n^u(V_u)$. Since $\Pi_n^u(V_u) \leq \Pi_n^s(V_u)$, and since a worsening of disruption-state procurement conditions lowers $\Pi_n^u(V_u)$, tolerated disruption becomes weakly less attractive. Therefore, the government is weakly more willing to induce defense. $\square$

Proof of Proposition 11. For part 1, write both procurement problems in quantiles,

$$\Pi_F(x) = \max_{u \in [0,1]} u(x - Q(u)), \quad \Pi_{\hat{F}}(x) = \max_{u \in [0,1]} u(x - \hat{Q}(u)).$$

Since $\hat{Q}(u) \geq Q(u)$ for every u , we have

$$u(x - \hat{Q}(u)) \leq u(x - Q(u)) \quad \text{for all } u \in [0, 1].$$

Taking the maximum over u on both sides yields (7.2).

For part 2, it is enough to provide an explicit regular counterexample. Let

$$F_0(c) = c, \quad F_1(c) = c^2, \quad c \in [0, 1].$$

Then $F_1(c) \leq F_0(c)$ for all $c \in [0, 1]$, so F_1 is a FOSD upward shift of F_0 . The corresponding densities are

$$f_0(c) = 1, \quad f_1(c) = 2c,$$

and the virtual costs are

$$H_0(c) = c + \frac{F_0(c)}{f_0(c)} = 2c, \quad H_1(c) = c + \frac{F_1(c)}{f_1(c)} = c + \frac{c^2}{2c} = \frac{3}{2}c.$$

Hence, the optimal reserve payments at net value x are

$$r_0(x) = H_0^{-1}(x) = \frac{x}{2}, \quad r_1(x) = H_1^{-1}(x) = \frac{2x}{3},$$

as long as x lies in the common interior region $x \in (0, 3/2)$. Therefore, implementation probabilities are

$$q_{F_0}(x) = F_0(r_0(x)) = \frac{x}{2}, \quad q_{F_1}(x) = F_1(r_1(x)) = \left(\frac{2x}{3}\right)^2 = \frac{4x^2}{9}. \quad (\text{A3})$$

Comparing the two,

$$q_{F_1}(x) > q_{F_0}(x) \iff \frac{4x^2}{9} > \frac{x}{2} \iff x > \frac{9}{8}.$$

Thus, for every $x \in (9/8, 3/2)$, the FOSD-worse distribution F_1 yields *more* implementation than F_0 . This proves the claim. $\square$

The counterexample in full detail. Returning to the example contained in the proof of Proposition 11, the continuation payoffs are

$$\Pi_{F_0}(x) = q_{F_0}(x)(x - r_0(x)) = \frac{x}{2} \left(x - \frac{x}{2}\right) = \frac{x^2}{4},$$

and

$$\Pi_{F_1}(x) = q_{F_1}(x)(x - r_1(x)) = \left(\frac{2x}{3}\right)^2 \left(x - \frac{2x}{3}\right) = \frac{4x^3}{27}.$$

On the common interior domain $x \in (0, 3/2)$,

$$\frac{4x^3}{27} < \frac{x^2}{4} \iff 16x < 27,$$

which holds automatically because $x < 3/2 < 27/16$. Therefore, the FOSD-worse distribution F_1 yields

$$q_{F_1}(x) > q_{F_0}(x) \quad \text{for } x > \frac{9}{8},$$

but at the same time

$$\Pi_{F_1}(x) < \Pi_{F_0}(x) \quad \text{for all } x \in (0, 3/2).$$

Thus, higher actual costs can make project implementation more likely while lowering the government's continuation payoff.

Let $V = 1.4$ and consider a promised reward $t = 0.1$, so that the net project value is $x = V - t = 1.3$. Since $1.3 > 9/8$, (A3) implies

$$q_{F_0}(1.3) = \frac{1.3}{2} = 0.65, \quad q_{F_1}(1.3) = \frac{4(1.3)^2}{9} \approx 0.7511.$$

The left-hand side of the army's incentive constraint (7.3) is

$$(1 - \rho) t q_{F_0}(1.3) = 0.065(1 - \rho), \quad (1 - \rho) t q_{F_1}(1.3) \approx 0.0751(1 - \rho).$$

Hence, for any

$$\psi \in (0.065(1 - \rho), 0.0751(1 - \rho)],$$

the promised reward $t = 0.1$ fails to induce defense under F_0 but succeeds under the FOSD-worse distribution F_1 . At the same time, the government's continuation procurement payoff falls,

$$\Pi_{F_0}(1.3) = \frac{1.3^2}{4} = 0.4225, \quad \Pi_{F_1}(1.3) = \frac{4(1.3)^3}{27} \approx 0.3255.$$

Thus, the same perturbation that makes defense easier to sustain also makes procurement less valuable for the government.

This next part proves Proposition 12. The argument uses the one-contractor benchmark, which is enough to establish the possibility result.

Fix a cost distribution F . For any net project value $x \geq 0$, let $\Pi_F(x) := \max_{r \in [0,1]} F(r)(x - r)$ denote the government's continuation procurement payoff, and let $r_F(x)$ be an optimal reserve payment. Define the corresponding implementation probability by $q_F(x) := F(r_F(x))$. Given project value V , army cost ψ , and baseline secure-state probability ρ , the government's ex-ante equilibrium welfare under F is

$$W(F) := \max \left\{ \rho \Pi_F(V), \sup_{t \in [0, V]: (1-\rho)t q_F(V-t) \geq \psi} \Pi_F(V - t) \right\},$$

where the second term is interpreted as $-\infty$ if the set of feasible rewards is empty. The first term is the payoff from tolerated disruption. The second term is the best payoff under induced defense. We first note a general fact.

Lemma 3. *If $\hat{F}$ is an FOSD upward shift of F , then*

$$\Pi_{\hat{F}}(x) \leq \Pi_F(x) \quad \forall x \geq 0.$$

Proof of Lemma 3. Let Q and $\hat{Q}$ denote the quantile functions associated with F and $\hat{F}$, respectively. Since $\hat{F}$ is an FOSD upward shift of F ,

$$\hat{Q}(u) \geq Q(u) \quad \forall u \in [0, 1].$$

Write the one-contractor procurement problem in quantiles. Choosing reserve cost $r = Q(u)$ is equivalent to choosing implementation probability u , so

$$\Pi_F(x) = \max_{u \in [0,1]} u(x - Q(u)).$$

Similarly,

$$\Pi_{\hat{F}}(x) = \max_{u \in [0,1]} u(x - \hat{Q}(u)).$$

For every $u \in [0, 1]$, we have $u(x - \hat{Q}(u)) \leq u(x - Q(u))$, because $u \geq 0$ and $\hat{Q}(u) \geq Q(u)$. Taking maxima over $u \in [0, 1]$ gives

$$\Pi_{\hat{F}}(x) = \max_{u \in [0,1]} u(x - \hat{Q}(u)) \leq \max_{u \in [0,1]} u(x - Q(u)) = \Pi_F(x). \quad \square$$

Lemma 3 shows that an FOSD worsening of actual costs cannot improve the procurement problem at a fixed net value. Proposition 12 states that this does not settle the ex-ante comparison, because the same perturbation may strengthen the army's incentives enough to change the equilibrium regime.

Before giving the example that proves Proposition 12, it is useful to record the underlying mechanism in a general form. Fix two cost distributions F_0 and F_1 , with F_1 a FOSD upward shift of F_0 . For $k \in \{0, 1\}$, define

$$\tilde{\psi}_k := (1 - \rho) \max_{t \in [0, V]} t q_{F_k}(V - t).$$

Suppose that

$$\tilde{\psi}_0 < \psi \leq \tilde{\psi}_1.$$

Then, defense is infeasible under F_0 but feasible under F_1 . Let t_1^D be the smallest reward inducing defense under F_1 . The equilibrium welfare under F_0 is

$$W(F_0) = \rho \Pi_{F_0}(V),$$

whereas under F_1 it is at least

$$\Pi_{F_1}(V - t_1^D).$$

Therefore, a sufficient condition for the FOSD worsening to raise ex-ante welfare is

$$\Pi_{F_1}(V - t_1^D) > \rho \Pi_{F_0}(V).$$

This condition makes it clear that the welfare gain cannot come from a better procurement problem at a fixed net value. It comes from the change in the security regime. That is, the cost shift makes completion-contingent rewards more likely to be paid, thereby making defense feasible.

Proof of Proposition 12. For $c \in [0, 1]$, take as baseline distribution $F_0(c) = c$, and as the alternative distribution

$$F_1(c) = \frac{e^{10c} - 1}{e^{10} - 1}.$$

We first verify that F_1 is a FOSD upward shift of F_0 . The function F_1 is convex on $[0, 1]$, with

$$F_1(0) = 0, \quad F_1(1) = 1.$$

By convexity, and for every $c \in [0, 1]$,

$$F_1(c) \leq (1 - c)F_1(0) + cF_1(1) = c = F_0(c)$$

Hence F_1 FOSD-dominates F_0 upward. We now choose parameters

$$V = \frac{7}{5}, \quad \rho = \frac{1}{10}, \quad \psi = \frac{1}{4}. \quad (\text{A4})$$

Under $F_0(c) = c$, the density is $f_0(c) = 1$, so the virtual cost is

$$H_0(c) = c + \frac{F_0(c)}{f_0(c)} = 2c.$$

For every $x \in [0, 2]$, the optimal reserve payment solves $x = H_0(r)$, that is,

$$r_0(x) = \frac{x}{2}.$$

Therefore,

$$q_{F_0}(x) = F_0(r_0(x)) = \frac{x}{2}, \quad (\text{A5})$$

and

$$\Pi_{F_0}(x) = q_{F_0}(x)(x - r_0(x)) = \frac{x}{2} \left(x - \frac{x}{2} \right) = \frac{x^2}{4}.$$

If the government does not induce defense, its payoff is

$$\rho \Pi_{F_0}(V) = \frac{1}{10} \cdot \frac{(7/5)^2}{4} = \frac{49}{1000}.$$

We next check whether defense can be induced. Under F_0 , the army incentive constraint is

$$(1 - \rho)t q_{F_0}(V - t) \geq \psi.$$

Using (A5) and (A4),

$$(1 - \rho)t q_{F_0}(V - t) = \frac{9}{10}t \cdot \frac{(7/5) - t}{2} = \frac{9}{20}t \left(\frac{7}{5} - t \right).$$

Define

$$g_0(t) := \frac{9}{20}t \left(\frac{7}{5} - t \right), \quad t \in [0, 7/5].$$

This is a strictly concave quadratic. Its derivative is

$$g_0'(t) = \frac{9}{20} \left(\frac{7}{5} - 2t \right),$$

so the unique maximizer is

$$t_0^* = \frac{7}{10}.$$

At that point,

$$g_0\left(\frac{7}{10}\right) = \frac{9}{20} \cdot \frac{7}{10} \cdot \left(\frac{7}{5} - \frac{7}{10}\right) = \frac{441}{2000}.$$

Since

$$\frac{441}{2000} = 0.2205 < \frac{1}{4} = \psi,$$

the incentive constraint cannot be satisfied for any t . Thus, defense is infeasible under F_0 , and therefore

$$W(F_0) = \frac{49}{1000}. \quad (\text{A6})$$

Under F_1 , the density is

$$f_1(c) = \frac{10e^{10c}}{e^{10} - 1} > 0,$$

and the virtual cost is

$$\begin{aligned} H_1(c) &= c + \frac{F_1(c)}{f_1(c)} \\ &= c + \frac{(e^{10c} - 1)/(e^{10} - 1)}{10e^{10c}/(e^{10} - 1)} \\ &= c + \frac{e^{10c} - 1}{10e^{10c}} \\ &= c + \frac{1 - e^{-10c}}{10}. \end{aligned}$$

Since $H_1'^{-10c} > 0$, we have that H_1 is strictly increasing. Moreover,

$$H_1(1) = 1 + \frac{1 - e^{-10}}{10} < 1 + \frac{1}{10} = \frac{11}{10}.$$

Hence, whenever $x \geq 11/10$, we have $x > H_1(1)$. In that case, the optimal reserve payment is at the upper bound, i.e., $r_1(x) = 1$, so procurement occurs surely, as

$$q_{F_1}(x) = F_1(1) = 1, \quad \Pi_{F_1}(x) = x - 1. \quad (\text{A7})$$

If the government does not induce defense, then $x = V = 7/5$, which is greater than $11/10$. Therefore, by (A7),

$$\rho \Pi_{F_1}(V) = \frac{1}{10} \left(\frac{7}{5} - 1 \right) = \frac{1}{10} \cdot \frac{2}{5} = \frac{1}{25}.$$

We next show that defense is feasible under F_1 . Consider the reward

$$t_1^* := \frac{\psi}{1 - \rho} = \frac{1/4}{9/10} = \frac{5}{18}.$$

Because $\frac{5}{18} < \frac{3}{10}$, we have

$$V - t_1^* = \frac{7}{5} - \frac{5}{18} = \frac{101}{90} > \frac{11}{10}.$$

Hence, (A7) applies at $x = V - t_1^*$, so $q_{F_1}(V - t_1^*) = 1$. The army incentive constraint is

$$(1 - \rho)t_1^* q_{F_1}(V - t_1^*) = \frac{9}{10} \cdot \frac{5}{18} \cdot 1 = \frac{1}{4} = \psi.$$

Thus t_1^* induces defense. The government's defended payoff is

$$\Pi_{F_1}(V - t_1^*) = \left(\frac{101}{90} \right) - 1 = \frac{11}{90}.$$

Since $\frac{11}{90} > \frac{1}{25}$, the government strictly prefers induced defense to tolerated disruption under F_1 . Therefore,

$$W(F_1) \geq \frac{11}{90}. \quad (\text{A8})$$

From (A6) and (A8),

$$W(F_1) - W(F_0) \geq \frac{11}{90} - \frac{49}{1000} = \frac{659}{9000} > 0.$$

Hence, $W(F_1) > W(F_0)$. This proves that an FOSD upward shift in actual costs can strictly raise ex-ante government welfare. Finally, Lemma 3 shows that this gain cannot come from an improvement of the procurement problem at a fixed net value. Rather, the gain comes from the security side of the government's problem. $\square$

Lemma 4. Under regularity,

$$\Pi_n(x) = \mathbb{E} \left[(x - H(c_{(1)}))_+ \right], \quad (\text{A9})$$

where $c_{(1)}$ is the lowest cost realization and $(y)_+ := \max\{y, 0\}$. Hence $\Pi_n(x)$ is weakly increasing in x . Moreover, if $x' > x$ and $q_n(x) > 0$, then

$$\Pi_n(x') > \Pi_n(x). \quad (\text{A10})$$

Proof of Lemma 4. By definition,

$$\Pi_n(x) = \mathbb{E} \left[(x - H(c_{(1)})) \mathbb{1} \{c_{(1)} \leq \tilde{c}(x)\} \right],$$

where $\tilde{c}(x) = H^{-1}(x)$ in the interior region, with the usual truncations at the boundaries. Since H is increasing, the event $\{c_{(1)} \leq \tilde{c}(x)\}$ is equivalent to $\{H(c_{(1)}) \leq x\}$. Therefore,

$$\Pi_n(x) = \mathbb{E} \left[(x - H(c_{(1)})) \mathbb{1} \{H(c_{(1)}) \leq x\} \right] = \mathbb{E} \left[(x - H(c_{(1)}))_+ \right],$$

which proves (A9).

Now let $x' > x$. For every realization of $c_{(1)}$,

$$(x' - H(c_{(1)}))_+ \geq (x - H(c_{(1)}))_+.$$

Taking expectations yields $\Pi_n(x') \geq \Pi_n(x)$. To prove strictness, suppose $q_n(x) > 0$. By definition of q_n ,

$$q_n(x) = \Pr(H(c_{(1)}) \leq x) > 0.$$

On the event $\{H(c_{(1)}) \leq x\}$, both positive parts are active, and

$$(x' - H(c_{(1)}))_+ - (x - H(c_{(1)}))_+ = x' - x > 0.$$

Hence,

$$\Pi_n(x') - \Pi_n(x) \geq (x' - x) \Pr(H(c_{(1)}) \leq x) = (x' - x)q_n(x) > 0,$$

which proves (A10). □

Proof of Proposition 13. From (7.4), we have that

$$W_n^N(\rho) = \rho \Pi_n(V).$$

Since $\Pi_n(V)$ does not depend on ρ and is nonnegative, we have

$$\frac{dW_n^N(\rho)}{d\rho} = \Pi_n(V) > 0.$$

Thus, $W_n^N(\rho)$ is strictly increasing in ρ . For each ρ , define the feasible set of implementing rewards

$$\mathcal{A}(\rho) := \left\{ t \in [0, V] \mid (1 - \rho)t q_n(V - t) \geq \psi \right\}.$$

By definition, when $\mathcal{A}(\rho) \neq \emptyset$,

$$t_n^D(\rho) = \min \mathcal{A}(\rho).$$

We first note that whenever $\mathcal{A}(\rho) \neq \emptyset$, the minimal implementing reward binds the army's incentive constraint, i.e.,

$$(1 - \rho)t_n^D(\rho) q_n(V - t_n^D(\rho)) = \psi.$$

To see this, define

$$g_\rho(t) := (1 - \rho)t q_n(V - t).$$

Since $q_n(\cdot)$ is continuous, g_ρ is continuous on $[0, V]$. If instead

$$g_\rho(t_n^D(\rho)) > \psi,$$

then by continuity there would exist $\varepsilon > 0$ such that

$$g_\rho(t) \geq \psi \quad \text{for all } t \in [t_n^D(\rho) - \varepsilon, t_n^D(\rho)].$$

This would imply $t_n^D(\rho) - \varepsilon \in \mathcal{A}(\rho)$, contradicting the minimality of $t_n^D(\rho)$. Hence, the constraint must hold with equality at $t_n^D(\rho)$.

Take $\rho' > \rho$, and suppose defense is feasible at both values. Since $1 - \rho' < 1 - \rho$, for every $t \in [0, V]$,

$$(1 - \rho') t q_n(V - t) < (1 - \rho) t q_n(V - t).$$

If $t \in \mathcal{A}(\rho')$, then $t \in \mathcal{A}(\rho)$. Hence, $\mathcal{A}(\rho') \subseteq \mathcal{A}(\rho)$. Moreover, $t_n^D(\rho) \notin \mathcal{A}(\rho')$, because

$$(1 - \rho') t_n^D(\rho) q_n(V - t_n^D(\rho)) < (1 - \rho) t_n^D(\rho) q_n(V - t_n^D(\rho)).$$

Since $t_n^D(\rho) \in \mathcal{A}(\rho)$, the right-hand side is at least ψ , and strictly larger if the minimum is not attained at equality. In either case, the strict inequality above implies that $t_n^D(\rho)$ cannot be the minimum of $\mathcal{A}(\rho')$. Thus, $t_n^D(\rho') > t_n^D(\rho)$. So $t_n^D(\rho)$ is strictly increasing in ρ on the region where defense is feasible.

Next, take $\rho' > \rho$ such that defense is feasible at both values. Since $t_n^D(\rho') > t_n^D(\rho)$, we have

$$V - t_n^D(\rho') < V - t_n^D(\rho).$$

Because defense is feasible at ρ , we must have

$$q_n(V - t_n^D(\rho)) > 0.$$

Otherwise, the left-hand side of the incentive constraint would be zero and could not cover $\psi > 0$. Lemma 4 implies

$$\Pi_n(V - t_n^D(\rho')) < \Pi_n(V - t_n^D(\rho)),$$

that is,

$$W_n^D(\rho') < W_n^D(\rho).$$

Hence, $W_n^D(\rho)$ is strictly decreasing in ρ wherever defense is feasible. By (7.6),

$$W_n^D(\rho_0) > W_n^N(\rho_0) = \rho_0 \Pi_n(V),$$

so dividing by $\Pi_n(V) > 0$ gives

$$\tilde{\rho}_n(\rho_0) = \frac{W_n^D(\rho_0)}{\Pi_n(V)} > \rho_0.$$

Next, because $\psi > 0$, any implementing reward must satisfy $t_n^D(\rho_0) > 0$. Hence,

$$V - t_n^D(\rho_0) < V.$$

Since defense is feasible at ρ_0 , we have $q_n(V - t_n^D(\rho_0)) > 0$, and Lemma 4 implies

$$\Pi_n(V - t_n^D(\rho_0)) < \Pi_n(V).$$

Therefore,

$$\tilde{\rho}_n(\rho_0) = \frac{\Pi_n(V - t_n^D(\rho_0))}{\Pi_n(V)} < 1.$$

This proves (7.7).

Take any $\rho' \in (\rho_0, \tilde{\rho}_n(\rho_0))$. Because W_n^N is strictly increasing,

$$W_n^N(\rho') = \rho' \Pi_n(V) < \tilde{\rho}_n(\rho_0) \Pi_n(V) = W_n^D(\rho_0) = W_n(\rho_0).$$

If defense is infeasible at ρ' , then $W_n(\rho') = W_n^N(\rho') < W_n(\rho_0)$, and we are done. If defense is feasible at ρ' , then

$$W_n^D(\rho') < W_n^D(\rho_0) = W_n(\rho_0).$$

Thus, in either case

$$W_n(\rho') = \max \{W_n^N(\rho'), W_n^D(\rho')\} < W_n(\rho_0).$$

This proves (7.8).

Now take any $\rho' \in [\tilde{\rho}_n(\rho_0), 1)$. Then

$$W_n^N(\rho') = \rho' \Pi_n(V) \geq \tilde{\rho}_n(\rho_0) \Pi_n(V) = W_n^D(\rho_0) = W_n(\rho_0).$$

Since $W_n(\rho') \geq W_n^N(\rho')$, it follows that

$$W_n(\rho') \geq W_n(\rho_0).$$

This proves (7.9). $\square$

Proof of Corollary 1. Because the reserve does not bind on the relevant path, procurement occurs for sure in the secure state, so $q_n(V - t) = 1$. The army's incentive constraint becomes $(1 - \rho)t \geq \psi$. Hence, the smallest implementing reward is

$$t_n^D(\rho) = \frac{\psi}{1 - \rho}.$$

Substituting this into (7.11) gives

$$W_n^D(\rho) = \Pi_n(V - t_n^D(\rho)) = V - \frac{2}{n+1} - \frac{\psi}{1 - \rho},$$

which proves (7.13). Eq. (7.12) follows directly from $W_n^N(\rho) = \rho \Pi_n(V)$ and $\Pi_n(V) = V - 2/(n+1)$. The government induces defense if and only if $W_n^D(\rho) \geq W_n^N(\rho)$, that is,

$$V - \frac{2}{n+1} - \frac{\psi}{1 - \rho} \geq \rho \left(V - \frac{2}{n+1} \right).$$

Rearranging, multiplying both sides by $1 - \rho$, and taking square roots yields

$$1 - \rho \geq \sqrt{\frac{\psi}{V - \frac{2}{n+1}}},$$

which is equivalent to (7.14).

Now fix $\rho_0 < \rho_n^*$, so that defense is optimal initially. The support level $\tilde{\rho}_n(\rho_0)$ that makes the government exactly indifferent between its initial defended welfare and tolerated insecurity solves

$$\tilde{\rho}_n(\rho_0) \left(V - \frac{2}{n+1} \right) = V - \frac{2}{n+1} - \frac{\psi}{1 - \rho_0}.$$

Solving for $\tilde{\rho}_n(\rho_0)$ gives (7.15). The welfare comparison then follows from Proposition 13.

Since $V - \frac{2}{n+1}$ is strictly increasing in n , both

$$\rho_n^* = 1 - \sqrt{\frac{\psi}{V - \frac{2}{n+1}}} \quad \text{and} \quad \tilde{\rho}_n(\rho_0) = 1 - \frac{\psi}{(1 - \rho_0) \left(V - \frac{2}{n+1} \right)}$$

are strictly increasing in n . This proves the result. $\square$

Proof of Proposition 14. Fix $\eta' \geq \eta$. First, for every reward $t \in [0, V]$,

$$(1 - \rho)tq_n(V - t \mid \eta') \geq (1 - \rho)tq_n(V - t \mid \eta).$$

Taking the maximum over $t \in [0, V]$ yields

$$\tilde{\psi}_n(V, \rho \mid \eta') \geq \tilde{\psi}_n(V, \rho \mid \eta).$$

Hence, defense is weakly easier to implement.

Second, suppose the defense is implementable under both η and η' . Define the implementing sets

$$\mathcal{D}_n(\rho \mid \eta) := \{t \in [0, V] \mid (1 - \rho)tq_n(V - t \mid \eta) \geq \psi\}$$

and

$$\mathcal{D}_n(\rho \mid \eta') := \{t \in [0, V] \mid (1 - \rho)tq_n(V - t \mid \eta') \geq \psi\}.$$

Since $q_n(V - t \mid \eta') \geq q_n(V - t \mid \eta)$ for every t , we have $\mathcal{D}_n(\rho \mid \eta) \subseteq \mathcal{D}_n(\rho \mid \eta')$. Therefore, the smallest implementing reward under η' is weakly lower than the smallest implementing reward under η , i.e., $t_n^D(\rho \mid \eta') \leq t_n^D(\rho \mid \eta)$.

Third, suppose again that defense is implementable under both environments. Since $t_n^D(\rho | \eta') \leq t_n^D(\rho | \eta)$, we have

$$V - t_n^D(\rho | \eta') \geq V - t_n^D(\rho | \eta).$$

Because $\Pi_n(\cdot | \eta')$ is weakly increasing in the net project value,

$$\Pi_n(V - t_n^D(\rho | \eta') | \eta') \geq \Pi_n(V - t_n^D(\rho | \eta) | \eta').$$

Because η' is a procurement improvement,

$$\Pi_n(V - t_n^D(\rho | \eta) | \eta') \geq \Pi_n(V - t_n^D(\rho | \eta) | \eta).$$

Combining the two inequalities gives $W_n^D(\rho | \eta') \geq W_n^D(\rho | \eta)$.

Fourth,

$$W_n^N(\rho | \eta') = \rho \Pi_n(V | \eta') \geq \rho \Pi_n(V | \eta) = W_n^N(\rho | \eta),$$

again by the definition of a procurement improvement.

Finally, we show that equilibrium welfare is weakly higher. If defense is implementable under η , then it is implementable under η' by the first part. In that case,

$$W_n^D(\rho | \eta') \geq W_n^D(\rho | \eta) \quad \text{and} \quad W_n^N(\rho | \eta') \geq W_n^N(\rho | \eta),$$

so $W_n(\rho | \eta') \geq W_n(\rho | \eta)$. If defense is not implementable under η , then $W_n(\rho | \eta) = W_n^N(\rho | \eta)$. Under η' , equilibrium welfare is at least the payoff from not inducing defense, i.e., $W_n(\rho | \eta') \geq W_n^N(\rho | \eta')$. Since $W_n^N(\rho | \eta') \geq W_n^N(\rho | \eta)$, it follows that

$$W_n(\rho | \eta') \geq W_n(\rho | \eta). \quad \square$$

Proof of Proposition 15. When procurement is chosen before the state is realized, the government's expected payoff from successful completion is the scalar

$$\bar{V}(t, e) = \mu(e)(V - t).$$

Conditional on that scalar, the procurement problem is identical to the one in Lemma 2, which implies the optimal mechanism and reserve cost.

If the government induces defense, then $\mu(1) = 1$, so the continuation payoff is

$$\bar{W}_n^d(t) = \Pi_n(V - t).$$

If the government sets $t = 0$ and induces no defense, then $\mu(0) = \rho$, so the continuation payoff is

$$\bar{W}_n^e = \Pi_n(\rho V).$$

Notice that, by the Envelope Theorem, Π_n is convex. Indeed, $\Pi'_n = q_n(x)$, and $q_n(x)$ is weakly increasing in x , so it follows that Π_n is convex.

Finally, since Π_n is convex and $\Pi_n(0) = 0$, for every $\rho \in (0, 1)$ we have

$$\Pi_n(\rho V) = \Pi_n(\rho V + (1 - \rho)0) \leq \rho \Pi_n(V) + (1 - \rho)\Pi_n(0) = \rho \Pi_n(V).$$

Strict inequality holds whenever $\Pi_n(\cdot)$ is strictly convex on an interval intersecting $(0, V)$, which is equivalent to $q_n(\cdot)$ being non-constant there. This proves the proposition. $\square$

Proof of Corollary 2. By Proposition 15,

$$\Pi_n(\rho V) \leq \rho \Pi_n(V).$$

Subtracting both sides from $\Pi_n(V - t_n^D)$ shows that the gain from inducing defense,

$$\Pi_n(V - t_n^D) - \Pi_n(\rho V),$$

is weakly larger under ex ante procurement than the corresponding gain,

$$\Pi_n(V - t_n^D) - \rho \Pi_n(V),$$

under the main timing. This proves the corollary. $\square$

Proof of Proposition 16. Fix a completion-contingent reward $t \in [0, V]$. Suppose first that the government intends to induce defense. If the army exerts effort, the secure state occurs with probability one. Hence, under both timings, the relevant net procurement value is $V - t$, the procurement probability is $q_n(V - t)$, and the government's payoff is $\Pi_n(V - t)$.

The army's incentive constraint is also the same under the two timings. If the army exerts effort, its expected payoff is $tq_n(V - t) - \psi$. If it shirks, the secure state occurs with probability ρ . Since the procurement rule chosen under the intended defense outcome is based on net value $V - t$, the army's expected payoff from shirking is $\rho tq_n(V - t)$. Therefore, defense is incentive compatible if and only if

$$tq_n(V - t) - \psi \geq \rho tq_n(V - t),$$

or, alternatively, when $(1 - \rho)tq_n(V - t) \geq \psi$. Thus, the set of rewards that induce defense is

$$D_n(V, \rho, \psi) = \{t \in [0, V] \mid (1 - \rho)tq_n(V - t) \geq \psi\},$$

under both timings.

Now consider the case in which the government does not choose a reward that satisfies the army's incentive constraint. Then, the army shirks in equilibrium. Any positive completion-contingent reward has no incentive benefit in this case and only lowers the government's net value when the secure state occurs. The government optimally sets $t = 0$.

Under the baseline timing, the government observes the state before procurement. Since $V(u) = 0$, procurement is valuable only in the secure state, which occurs with probability ρ . The optimal no-inducement payoff is $\rho\Pi_n(V)$. Under ex ante procurement, the government must choose the procurement rule before the state is realized. With $t = 0$, the relevant expected project value is ρV , so the optimal no-inducement payoff is $\Pi_n(\rho V)$.

It remains to show that $\Pi_n(\rho V) \leq \rho\Pi_n(V)$. The procurement payoff satisfies $\Pi_n(0) = 0$. Moreover, by the envelope property, we have $\Pi'_n(x) = q_n(x)$, and $q_n(x)$ is weakly increasing in x . Hence, Π_n is convex. Convexity and $\Pi_n(0) = 0$ imply that, for every $\rho \in [0, 1]$,

$$\Pi_n(\rho V) = \Pi_n(\rho V + (1 - \rho)0) \leq \rho\Pi_n(V) + (1 - \rho)\Pi_n(0) = \rho\Pi_n(V).$$

Thus, ex ante procurement leaves both the implementability condition and the payoff from induced defense unchanged, while weakly lowering the government's optimal payoff when defense is not induced. If inducing defense is optimal under the main timing, it is also optimal under ex ante procurement. $\square$

Appendix B. Robustness of the extensions

B.1. Extensions under ex ante procurement

This appendix explains why the binary-effort extensions continue to hold when procurement takes place before the security state is realized. The common logic is the one established in [Proposition 16](#). When defense is induced, the secure state occurs with probability one. Thus, the procurement rule, the army's incentive constraint, and the government's payoff under induced defense are unchanged relative to the main timing. Ex ante procurement affects only the government's payoff when defense is not induced, because the procurement rule must then be chosen before the state is known.

Competition. With n contractors, the army's incentive constraint under induced defense is

$$(1 - \rho)tq_n(V - t) \geq \psi,$$

exactly as under the main timing. Hence the implementability comparison across n is unchanged. If $n' > n$, then $q_{n'}(x) \geq q_n(x)$ and $\Pi_{n'}(x) \geq \Pi_n(x)$ for every x . Therefore,

$$\max_{t \in [0, V]} (1 - \rho)tq_{n'}(V - t) \geq \max_{t \in [0, V]} (1 - \rho)tq_n(V - t),$$

so competition still makes defense easier to implement. Whenever defense is implementable under both n and n' , the minimal implementing reward is weakly lower with n' contractors, and the payoff from induced defense is weakly higher. Thus, [Proposition 3](#) is unchanged under ex ante procurement.

Positive value amid unrest. Let $V(s) = V_s > V_u = V(u) \geq 0$. Maintain completion-contingent rewards that are paid only upon completion in the secure state. Under induced defense, the secure state occurs with probability one, so the army's incentive constraint remains $(1 - \rho)tq_n(V_s - t) \geq \psi$, and the government's payoff from induced defense is $\Pi_n(V_s - t)$. If defense is not induced, the government sets $t = 0$, the secure state occurs with probability ρ , and ex ante procurement is based on the expected project value $\rho V_s + (1 - \rho)V_u$. Thus, the no-inducement payoff is

$$\Pi_n(\rho V_s + (1 - \rho)V_u).$$

Since Π_n is weakly increasing, this payoff is weakly increasing in V_u . Hence a positive value under unrest still makes not inducing defense more attractive for the government, while leaving the army's incentive constraint unchanged. [Proposition 7](#) continues to hold under ex ante procurement.

Project selection. Let $j \in \{F, R\}$ denote the project type. Under ex ante procurement, the value of project j is

$$\widehat{W}_j = \max \left\{ \mathbb{1}\{\psi \leq \psi_j\} \Pi_n \left(V_j^s - t_j^D \right), \Pi_n \left(\rho V_j^s + (1 - \rho) V_j^u \right) \right\},$$

where

$$\psi_j = \max_{t \in [0, V_j^s]} (1 - \rho) t q_n \left(V_j^s - t \right),$$

and t_j^D is the smallest reward satisfying $(1 - \rho) t q_n (V_j^s - t) \geq \psi$.

If $V_F^s = V_R^s =: V$, $V_F^u = 0$, and $V_R^u = v \in (0, V)$, then the two projects generate the same defense incentives and the same payoff under induced defense. However,

$$\Pi_n(\rho V + (1 - \rho)v) \geq \Pi_n(\rho V),$$

so the resilient project weakly dominates when defense is not induced. As a result, the common-secure-value project-selection result continues to hold.

If instead $V_F^u = 0$, $0 < V_R^u < V_R^s$, and $\rho V_F^s = \rho V_R^s + (1 - \rho)V_R^u$, then the no-inducement payoff is the same for the two projects under ex ante procurement, i.e.,

$$\Pi_n(\rho V_F^s) = \Pi_n(\rho V_R^s + (1 - \rho)V_R^u).$$

Moreover, the equality above implies $V_F^s > V_R^s$. Therefore, for every reward t ,

$$q_n(V_F^s - t) \geq q_n(V_R^s - t),$$

so the fragile project weakly relaxes the army's incentive constraint and requires a weakly lower implementing reward. Conditional on induced defense, it yields a weakly higher government payoff. As a result, the equal-expected-value project-selection result also continues to hold.

State-dependent contractor costs. Suppose contractor costs are state-dependent, with distributions F^s and F^u , and let $\Pi_n^s(\cdot)$ and $q_n^s(\cdot)$ denote the secure-state procurement payoff and implementation probability. Under induced defense, the secure state occurs with probability one. The army's incentive constraint remains $(1 - \rho) t q_n^s(V_s - t) \geq \psi$, and the government's payoff from induced defense is $\Pi_n^s(V_s - t)$. The implementability of defense is unaffected by the disruption-state cost distribution.

If defense is not induced, the government chooses a procurement rule before the state is realized. Let $\widehat{\Pi}_n^\rho(V_s, V_u \mid F^s, F^u)$ denote the value of this ex ante procurement problem. If disruption worsens contractor performance in the sense that, for every procurement rule, the expected payoff in the disruption state weakly falls, then $\widehat{\Pi}_n^\rho(V_s, V_u \mid F^s, F^u)$ weakly decreases. Since the payoff from induced defense and the army's incentive constraint are unchanged, a deterioration in disruption-state contractor performance weakly increases the government's incentive to induce defense. The state-dependent-cost result continues to hold under ex ante procurement.

Primitive FOSD shifts. Consider the one-contractor primitive-cost comparison in [Propositions 11](#) and [12](#). The key object in the army's incentive constraint is still the probability that the completion-contingent reward is paid, i.e., $(1 - \rho) t q_F(V - t)$. This expression is unchanged under ex ante procurement whenever the government induces defense. Therefore, any FOSD upward shift in actual costs that raises $q_F(V - t)$ at the relevant net value continues to relax the army's incentive constraint.

The welfare possibility result also carries over. Let F_0 and F_1 be the two cost distributions in the example, with F_1 a FOSD upward shift of F_0 . Suppose defense is infeasible under F_0 , feasible under F_1 , and let t_1^D be the smallest implementing reward under F_1 . If

$$\Pi_{F_1}(V - t_1^D) > \rho \Pi_{F_0}(V),$$

then, under the main timing, the cost shift raises ex-ante welfare. Under ex ante procurement, the no-inducement payoff under F_0 is $\Pi_{F_0}(\rho V) \leq \rho \Pi_{F_0}(V)$. The same sufficient condition implies $\Pi_{F_1}(V - t_1^D) > \Pi_{F_0}(\rho V)$. Thus, the FOSD welfare possibility result remains valid under ex ante procurement.

External support. Let ρ denote the background security when the army shirks. Under ex ante procurement, if defense is not induced, the government's payoff is

$$\widehat{W}_n^0(\rho) = \Pi_n(\rho V),$$

which is weakly increasing in ρ . If defense is induced, the smallest implementing reward is

$$t_n^D(\rho) := \min \{ t \in [0, V] \mid (1 - \rho) t q_n(V - t) \geq \psi \},$$

whenever this set is nonempty, and the government's payoff is

$$\widehat{W}_n^D(\rho) = \Pi_n(V - t_n^D(\rho)).$$

The reward $t_n^D(\rho)$ is weakly increasing in ρ , because a higher ρ lowers the marginal effect of army effort. Therefore, $\widehat{W}_n^D(\rho)$ is weakly decreasing in ρ .

Fix ρ_0 such that inducing defense is optimal and strictly preferred under ex ante procurement, i.e., $\widehat{W}_n^D(\rho_0) > \widehat{W}_n^0(\rho_0)$. Define

$$\widehat{\rho}_n(\rho_0) := \inf \left\{ \rho \in [\rho_0, 1] \mid \Pi_n(\rho V) \geq \widehat{W}_n^D(\rho_0) \right\}.$$

For every $\rho' \in (\rho_0, \widehat{\rho}_n(\rho_0))$, the payoff from not inducing defense is below the initial payoff, while the payoff from inducing defense is weakly below the initial payoff because $t_n^D(\rho') \geq t_n^D(\rho_0)$. Welfare is lower than that at ρ_0 . For every $\rho' \geq \widehat{\rho}_n(\rho_0)$, not inducing defense yields at least the initial payoff. As a result, the non-monotonic effect of external support continues to hold under ex ante procurement.

B.2. Continuous effort and the expected-prize channel

This appendix explains how the main comparative mechanisms extend to the continuous-effort specification. With binary effort, the government's problem is whether to induce defense or not. With continuous effort, the army's security choice responds smoothly to incentives. The common object across the two formulations is the *expected prize* attached to successful implementation.

To avoid confusion with the background-security parameter ρ , write $p \in [0, 1]$ for the security probability chosen by the army in the continuous-effort model. The army bears cost $\psi p^2/2$. If the army is paid a completion-contingent reward t only when the project is completed in the secure state, and if the secure-state value is V_s , the expected prize generated by reward t is

$$\mathcal{R}_n(t \mid V_s) := tq_n(V_s - t).$$

For a fixed reward t , the army solves

$$\max_{p \in [0, 1]} \left\{ p \mathcal{R}_n(t \mid V_s) - \frac{\psi}{2} p^2 \right\}.$$

The objective is strictly concave, and the army's best response is

$$p_n^*(t \mid V_s) = \min \left\{ \frac{\mathcal{R}_n(t \mid V_s)}{\psi}, 1 \right\}.$$

In the interior region,

$$p_n^*(t \mid V_s) = \frac{tq_n(V_s - t)}{\psi}.$$

The continuous-effort model replaces the binary incentive constraint with a smooth security response. Anything that raises the expected prize $\mathcal{R}_n(t \mid V_s)$ raises the army's security effort.

If the project has value $V_u \geq 0$ in the unrest state, and if the army is paid only upon completion in the secure state, the government's payoff from reward t is

$$G_n(t \mid V_s, V_u) = p_n^*(t \mid V_s) \Pi_n(V_s - t) + (1 - p_n^*(t \mid V_s)) \Pi_n(V_u).$$

In the baseline case $V_u = 0$, this reduces to

$$G_n(t \mid V, 0) = p_n^*(t \mid V) \Pi_n(V - t).$$

This expression makes the continuous-effort trade-off transparent. A larger reward t increases the army's expected prize directly, but lowers the government's net procurement value from V to $V - t$.

Competition. Suppose $n' > n$. Since

$$q_{n'}(x) \geq q_n(x) \quad \text{and} \quad \Pi_{n'}(x) \geq \Pi_n(x)$$

for every x , we have, for every reward t ,

$$\mathcal{R}_{n'}(t \mid V_s) = tq_{n'}(V_s - t) \geq tq_n(V_s - t) = \mathcal{R}_n(t \mid V_s).$$

Hence

$$p_{n'}^*(t \mid V_s) \geq p_n^*(t \mid V_s).$$

In the baseline case $V_u = 0$,

$$G_n(t \mid V_s, 0) = p_n^*(t \mid V_s) \Pi_n(V_s - t),$$

so both the security response and the procurement payoff are weakly higher with n' contractors. Therefore, for every t ,

$$G_{n'}(t \mid V_s, 0) \geq G_n(t \mid V_s, 0),$$

and the optimized payoff is weakly increasing in the number of contractors. Thus, competition continues to complement security under continuous effort.

Positive value amid unrest. Let $V_s > V_u \geq 0$. Since the army is paid only upon completion in the secure state, its expected prize is

$$\mathcal{R}_n(t \mid V_s) = tq_n(V_s - t),$$

which is independent of V_u . Therefore, for every reward t , the security response $p_n^*(t \mid V_s)$ is independent of the value of the project under unrest.

The government's payoff is

$$G_n(t \mid V_s, V_u) = p_n^*(t \mid V_s)\Pi_n(V_s - t) + (1 - p_n^*(t \mid V_s))\Pi_n(V_u).$$

Since $\Pi_n(\cdot)$ is weakly increasing, this payoff is weakly increasing in V_u . However, the payoff gain from increasing the security probability is $\Pi_n(V_s - t) - \Pi_n(V_u)$, which is weakly decreasing in V_u . Positive value under unrest does not directly affect the army's security incentives, but it reduces the government's marginal value of protection. This is the continuous-effort analogue of the positive-unrest-value result.

Project selection. Let $j \in \{F, R\}$ denote a project type, with values V_j^s and V_j^u . Under continuous effort, the value of project j is

$$\max_{t \in [0, V_j^s]} \left\{ p_{n,j}^*(t)\Pi_n(V_j^s - t) + (1 - p_{n,j}^*(t))\Pi_n(V_j^u) \right\},$$

where

$$p_{n,j}^*(t) = \min \left\{ \frac{tq_n(V_j^s - t)}{\psi}, 1 \right\}.$$

If $V_F^s = V_R^s =: V$, $V_F^u = 0$, and $V_R^u = v \in (0, V)$, then the two projects generate the same expected prize and the same security response for every reward t . The resilient project yields weakly higher payoff for every t , because $\Pi_n(v) \geq \Pi_n(0)$. Hence, it also yields weakly higher optimized payoff.

If instead $V_F^s > V_R^s$, then for every $t \in [0, V_R^s]$,

$$q_n(V_F^s - t) \geq q_n(V_R^s - t),$$

and, therefore,

$$\mathcal{R}_n(t \mid V_F^s) \geq \mathcal{R}_n(t \mid V_R^s).$$

The project with the higher secure-state value generates weakly stronger protection incentives. This is the continuous-effort counterpart of the incentive effect behind the fragile-versus-resilient project comparison.

State-dependent contractor costs. Let $q_n^s(\cdot)$ and $\Pi_n^s(\cdot)$ denote the procurement probability and continuation payoff in the secure state, and let $\Pi_n^u(\cdot)$ denote the continuation payoff in the unrest state. If the army is rewarded only upon completion in the secure state, its expected prize is

$$\mathcal{R}_n^s(t \mid V_s) := tq_n^s(V_s - t).$$

Hence, the security response is

$$p_n^*(t \mid V_s) = \min \left\{ \frac{tq_n^s(V_s - t)}{\psi}, 1 \right\}.$$

This response depends on secure-state procurement conditions, but not on the unrest-state cost distribution.

For every reward t , the government's payoff is

$$G_n(t) = p_n^*(t \mid V_s)\Pi_n^s(V_s - t) + (1 - p_n^*(t \mid V_s))\Pi_n^u(V_u).$$

Therefore, any deterioration of unrest-state contractor performance that lowers $\Pi_n^u(V_u)$ leaves the army's security response unchanged and weakly lowers the government's payoff for every t . It also raises the payoff gain from increasing security, i.e., $\Pi_n^s(V_s - t) - \Pi_n^u(V_u)$. As in the binary model, worse contractor conditions under unrest strengthen the government's incentive to rely on protection, because security preserves not only project value but also procurement performance.

Primitive cost shifts. The same expected-prize logic applies to primitive cost shifts. Let F be a cost distribution, and write $q_F(\cdot)$ and $\Pi_F(\cdot)$ for the corresponding one-contractor implementation probability and continuation payoff. Under continuous effort,

$$p_F^*(t) = \min \left\{ \frac{tq_F(V - t)}{\psi}, 1 \right\}.$$

Hence, if two cost distributions F_0 and F_1 satisfy $q_{F_1}(V - t) \geq q_{F_0}(V - t)$ for some reward t , then $p_{F_1}^*(t) \geq p_{F_0}^*(t)$. Any primitive cost shift that raises the probability that the completion-contingent reward is paid also raises the army's continuous security response at that reward.

This shows why the FOSD possibility result has a continuous-effort analogue. A FOSD worsening of primitive costs can lower the government's continuation procurement payoff at a fixed net value, but if it raises the implementation probability $q_F(V - t)$ at the relevant margin, it also raises the expected prize and therefore strengthens security incentives. The overall welfare effect is ambiguous. In particular, if there exists a reward t such that

$$p_{F_1}^*(t)\Pi_{F_1}(V - t) > \max_{\tau \in [0, V]} p_{F_0}^*(\tau)\Pi_{F_0}(V - \tau),$$

then the primitive cost shift strictly raises the government's optimized payoff under continuous effort. As in the binary model, the gain cannot come from better procurement at a fixed net value. It comes from the improvement in the security incentive channel.

External support. To study external support under continuous effort, let $\rho \in [0, 1]$ denote background security when the army supplies no additional protection. Let $e \in [0, 1]$ denote domestic protection effort, with cost $\psi e^2/2$, and suppose that the probability of the secure state is

$$\mu(e, \rho) := \rho + (1 - \rho)e.$$

External support raises background security, but also reduces the marginal effect of domestic protection.

For a given completion-contingent reward t , let

$$\mathcal{R}_n(t) := tq_n(V - t).$$

The army solves

$$\max_{e \in [0, 1]} \left\{ [\rho + (1 - \rho)e] \mathcal{R}_n(t) - \frac{\psi}{2} e^2 \right\}.$$

The objective is strictly concave, and the army's best response is

$$e_n^*(t, \rho) = \min \left\{ \frac{(1 - \rho)\mathcal{R}_n(t)}{\psi}, 1 \right\}.$$

In the interior region,

$$\mu_n^*(t, \rho) = \rho + (1 - \rho)e_n^*(t, \rho) = \rho + \frac{(1 - \rho)^2 \mathcal{R}_n(t)}{\psi}.$$

For a fixed reward t ,

$$\frac{\partial \mu_n^*(t, \rho)}{\partial \rho} = 1 - 2e_n^*(t, \rho)$$

whenever effort is interior. External support lowers the induced security probability at a fixed reward if and only if

$$e_n^*(t, \rho) > \frac{1}{2}.$$

Let

$$\hat{G}_n(\rho) := \max_{t \in [0, V]} \mu_n^*(t, \rho) \Pi_n(V - t)$$

denote the government's optimized payoff in the baseline case $V_u = 0$. If the optimizer $t^*(\rho)$ is interior and the army's effort is interior at that optimizer, the envelope theorem gives

$$\frac{d\hat{G}_n(\rho)}{d\rho} = [1 - 2e_n^*(t^*(\rho), \rho)] \Pi_n(V - t^*(\rho)).$$

External support is locally harmful whenever the induced domestic effort at the optimum is greater than $1/2$, and locally beneficial whenever it is less than $1/2$. This is the continuous-effort analogue of the non-monotonic external-support result. Support improves background security directly, but it also weakens domestic protection incentives by reducing the marginal contribution of the army.

Partial state-contingent budgets. Finally, suppose that the government can pay $\alpha \leq \bar{B}$ whenever the secure state occurs and can also pay a completion-contingent reward β upon successful completion in the secure state. The army's expected prize is $\alpha + \beta q_n(V - \beta)$. Its continuous security response is

$$p_n^*(\alpha, \beta) = \min \left\{ \frac{\alpha + \beta q_n(V - \beta)}{\psi}, 1 \right\}.$$

The state-contingent payment α and the completion-contingent payment β both raise security incentives through the expected prize. However, whenever the completion-contingent component is used, the procurement-security channel remains active: β affects security through the probability $q_n(V - \beta)$ that the reward is paid, and it affects procurement through the net value $V - \beta$. A capped state-contingent budget attenuates the baseline mechanism, but does not eliminate it unless the government can rely entirely on state-contingent compensation.

Data availability

No data was used for the research described in the article.

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