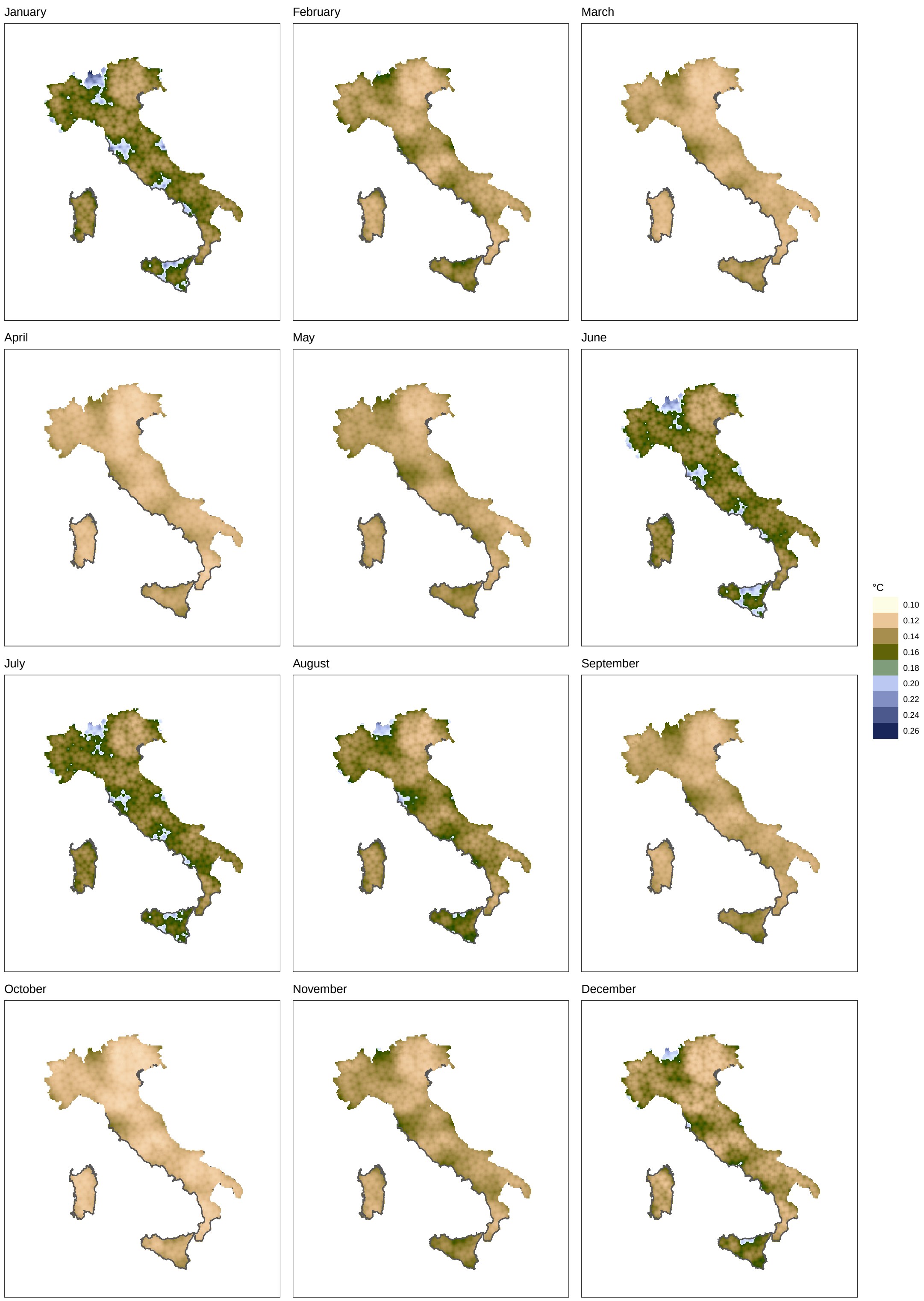

FIGURE 14 Standard deviation maps for the monthly climatologies in Italy for the period 1991–2020 (Tmax) [Colour figure can be viewed at wileyonlinelibrary.com]

Figure 15 displays the mean of such sample distributions. Here, we have used a black contour line to mark those areas were the difference is statistically significant (i.e., the whole 95% credible interval is above or below the zero value). For lack of space, we have illustrated the difference maps only for Tmax and 1 month per season.

Our results show that the 1991–2020 climatologies are generally warmer than the historical ones in all seasons and regions of Italy. With respect to 1961–1990 and

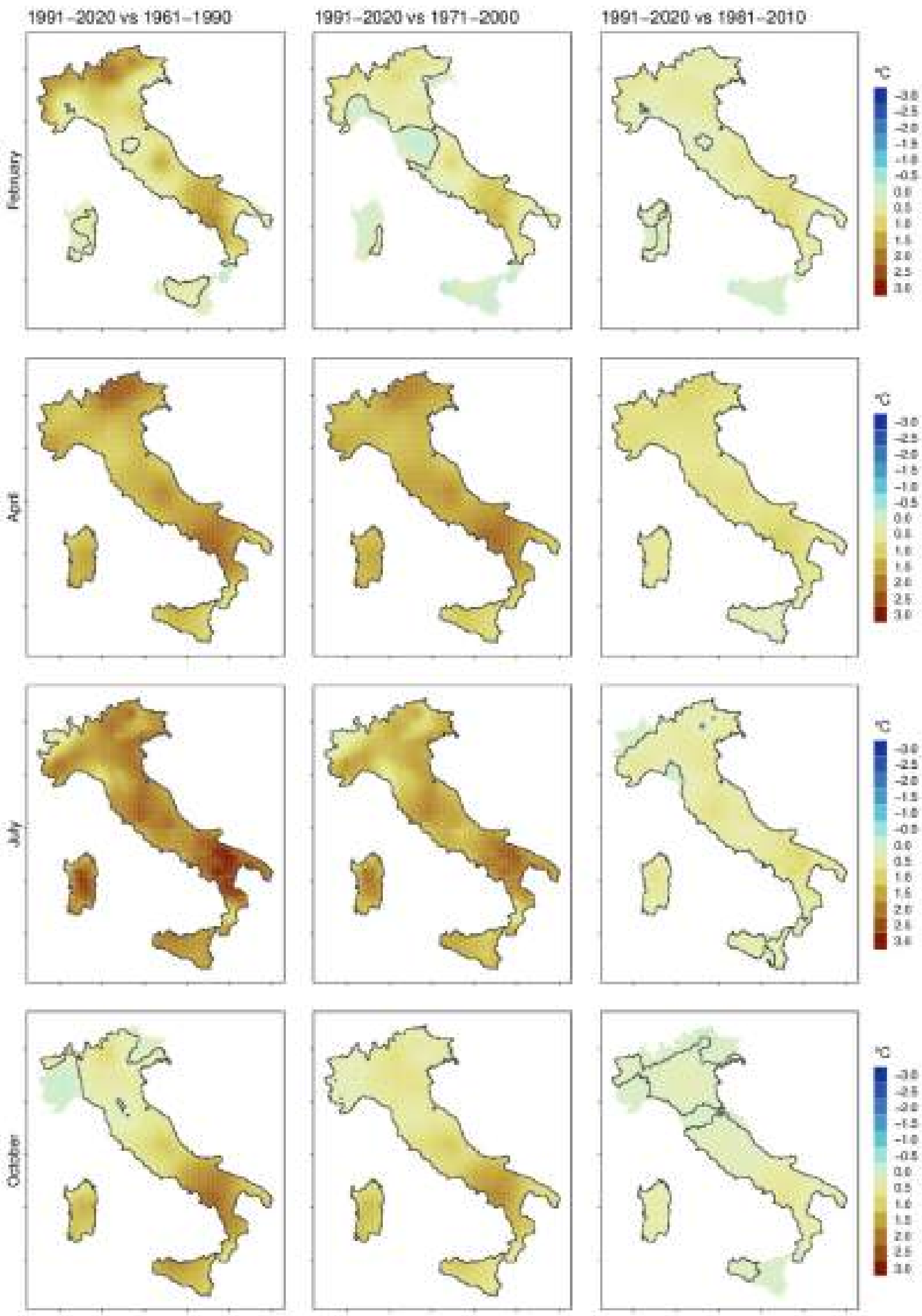

FIGURE 15 Mean of the differences between the monthly 1991–2020 climatology and the historical climatologies: 1961–1990, 1971–2000, 1981–2010 (Tmax). For lack of space, only 1 month per season is represented in the figure. A black contour line marks the areas where the difference is statistically significant from zero [Colour figure can be viewed at wileyonlinelibrary.com]

1971–2000, the warming signal seems to be particularly marked in April and July. Notably, in a few areas the 1991–2020 climatologies are somewhat cooler than those calculated for the previous 30-year standard periods (see, e.g., the negative values which characterize the 1991–2020 vs. 1971–2000 map in February). However, the attached 95% credible interval shows that such negative differences are not statistically significant.

5 | CONCLUSIONS

In this study, we have introduced the INLA-SPDE approach for the interpolation of relevant climate variables. To better illustrate how it works on a real case study, we have implemented a Bayesian regression model for the spatiotemporal interpolation of Tmax and Tmin monthly means during the period 1961–2020. We have used quality controlled and homogenized daily temperature time series from a free and open database. Then, we have generated monthly climate normals of maximum and minimum temperature in Italy for the latest standard 30-year period (1991–2020) and three previous standard ones: 1961–1990, 1971–2000 and 1981–2010.

Since we are mainly interested in the long term trends we have run the regression analysis separately for each month (January–December). This allows to tackle the large space–time domain of our study and avoid the need for a cyclic component that accounts for the yearly seasonality. From a computational point of view, this approach is less demanding than a model where the 12×60 months are jointly modelled. Having 12 independent models also allows to easily account for the changing temperature-versus-predictors relationship.

Our model has a simple formulation. There are three relevant spatial predictors and a linear time effect accounting for the temporal trend in the observed monthly temperatures. Furthermore, a Matérn field allows to capture the residual spatiotemporal correlation. Despite its simplicity, this approach provides a useful and flexible model to produce accurate continuous gridded surfaces equipped with model-based uncertainties. Illustrative examples of the successful use of this model (with only minor modifications) are given in Fioravanti et al. (2021), for the interpolation of PM_{10} daily concentrations in Italy during 2015, and in Fioravanti et al. (2022), for the assessment of the spatiotemporal variability of NO_2 in Italy during the COVID-19 lockdown.

Classical kriging-based approaches are very common in climate science. However, they fail to incorporate the uncertainty from the spatial covariance matrix into the variance of predicted values. In this regard, Le and Zidek (1992) observed that uncertainty underestimation can result in unwarranted confidence in the interpolated values and, potentially, to unjustified decisions or regulatory actions. Here, we overcome this issue with the use of Bayesian statistics. Our Bayesian model provides a formal approach to handle and propagate uncertainty in the data and in the fitted model parameters. In this regard, we have shown how, through simulation, we are able to generate multiple plausible gridded surfaces of Tmax and Tmin monthly means, which can be further summarized through measures of central tendency (e.g., posterior mean) or variability (e.g., standard deviation). Following this logic, we have provided examples both of standard deviation maps (to investigate how uncertainty affects our estimates of the 1991–2020 monthly climatologies and where) and 95% credible intervals (to assess the regions where the 1991–2020 period is significantly warmer than the previous 30-year standard periods).

In this work, we use cross-validation as a technique to validate our model. Although this is a very common choice, it is known that cross-validation can be problematic in presence of correlated data (Roberts et al., 2017; Wadoux et al., 2021). Recently, in the INLA framework, a new cross-validation strategy, that takes into account the model structure and the data dependencies, has been proposed by Liu and Rue (2023). This method was not available at the time we prepared this article, but we believe it is an interesting and promising approach that can overcome some of the problems in the common case of cross-validation in presence of correlated data.

Although the mathematics underlying the INLA-SPDE approach may be somewhat intimidating for climate practitioners who are still more familiar with classical geostatistical tools like gstat, the INLA-SDPE approach comes with two user friendly R implementations: the R-INLA and the inlabru package. These packages make the INLA-SPDE methodology a fast, reliable and easy to use tool to scientists with R coding skills, especially if one considers that INLA-SPDE can be also used with non-Gaussian response variables.

The proposed model can be extended in various directions. A first development is to consider a nonlinear time effect in order to accommodate the change-point and the rapid warming in global and European temperature time series from the mid-1970s onward (Toreti & Desiato, 2007). In the current work we have tackled this issue through a random effect $z(t)$ which accounts for the extra temporal variability that is not captured by the linear time trend. An alternative solution offered by R-INLA package is to introduce a smooth component with the use of a random walk model. A more interesting possible development would consist in the use of INLA-SPDE for the integration of in-situ observations and spatially consistent gridded estimates (spatial fusion). A promising solution in this respect has been proposed by Moraga et al. (2017) and, more recently, Wang and Furrer (2021).

6 | CODE AND DATA AVAILABILITY

Our analysis was run using the software package R. The linear model of Equation (1) was fitted with the use of the lm function. The spatiotemporal empirical variograms were calculated using the variogram function of the gstat package. This function requires spatiotemporal objects that can be created using the spacetime package. For the manipulation of the raster maps we used the terra package and the Climate Data Operator (CDO) software (https://code.mpimet.mpg.de/projects/cdo). Our plots use scientifically derived colour maps available through the scico package (Crameri et al., 2020). The inferential analysis was run using the inlabru package (version 2.3.0), an interface for the R-INLA package. To reduce computing time, we enabled the support to the PARDISO library (Alappat et al., 2020; Bollhöfer et al., 2019, 2020). The analysis was run on an workstation with Ubuntu ver. 18.04.6. The running time for each model was of around 20 minutes. Scripts and data used for this study are available on https://github.com/guidofioravanti/climatological_values_inla.

AUTHOR CONTRIBUTIONS

Guido Fioravanti: Conceptualization; methodology; software; data curation; validation; writing – original draft. **Sara Martino:** Conceptualization; methodology; formal analysis; writing – original draft; software; validation. **Michela Cameletti:** Methodology; writing – review and editing. **Andrea Toreti:** Writing – review and editing; methodology.

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