Measurement of interfacial stress from interface curvature

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Abstract

The momentum balance of an interface subjected to superficial stresses and of curvature K, if projected in the direction normal to the interface $\mathbf{n}_A$, leads to the capillary equation

$$(\dot{m}'_A \vec{v}_A \cdot \mathbf{n}_A - \dot{m}'_B \vec{v}_B \cdot \mathbf{n}_B) + p_B - p_A + \mathbf{n}_A \cdot \left(\underline{\underline{\mathbf{T}}}_{e,A} - \underline{\underline{\mathbf{T}}}_{e,B} \right) \cdot \mathbf{n}_A + \mathbf{n}_A \cdot \left(\underline{\underline{\mathbf{T}}}_{v,A} - \underline{\underline{\mathbf{T}}}_{v,B} \right) \cdot \mathbf{n}_A = 2\sigma K$$

where the first term is the so-called recoil force (due to momentum flux on both sides of interface), σ is the surface tension, K is the interface curvature, p_A and p_B are the pressures on the two sides of the interface. The deviatoric viscous tensor and the electric stress tensors, $\mathbf{T}_v$, $\mathbf{T}_e$, are finally included.

Generally, this equation is considered in its static version, leading to the celebrated Laplace-Young equation

$$p_A - p_B = 2\sigma K$$

which states that the pressure is always greater on the concave side of the interface, regardless of which fluid (liquid or vapour) occupies that side. So the equation can be equally applied to bubbles and drops.

Later on, Lord Rayleigh extended the equation including the dynamic pressure due to the motion of the fluid on both sides of interface, and Plesset included viscous forces. In recent times, electric stress was added too.

The aim of this work is to illustrate some case studies where, once the curvature is experimentally measured by means of image processing and the hydrostatic pressure is known, the remaining forces, if present, can be determined by equilibrium unbalance and compared with theoretical models.