ORIGINAL PAPER



Identifying types in contest experiments

Francesco Fallucchi¹ • Andrea Mercatanti² • Jan Niederreiter³

Accepted: 24 September 2020 / Published online: 28 October 2020 © The Author(s) 2020

Abstract

We apply the classifier-Lasso (Su et al. 2016) to detect the presence of latent types in two data sets of previous contest experiments, one that keeps the grouping of contestants fixed over the experiment and one that randomly regroups contestants after each round. Our results suggest that there exist three distinct types of players in both contest regimes. The majority of contestants in fixed groups behaves reciprocal to opponents' previous choices. A higher share of reciprocators per group is associated to lower average overspending which hints at cooperative attempts. For experiments in which contestants are regrouped, we find a significantly lower share of 'reciprocators' and no significant association between the share of reciprocators and average efforts.

Keywords Experimetrics · Behavioral types · Experiment · Contest · C-Lasso

Mathematics Subject Classification C38 · C57 · C73

1 Introduction

The characterization of heterogeneous behavior has a long tradition across economic disciplines. Various approaches have been used to group subjects' behavior in parsimo-

Electronic supplementary material The online version of this article (https://doi.org/10.1007/s00182-020-00738-w) contains supplementary material, which is available to authorized users.

Francesco Fallucchi francesco.fallucchi@liser.lu

Andrea Mercatanti @bancaditalia.it

Jan Niederreiter jan.niederreiter@alumni.imtlucca.it

- Luxembourg Institute of Socio-Economic Research (LISER), 11 Porte des Sciences, 4366 Esch-sur-Alzette, Luxembourg
- ² Bank of Italy, Via Nazionale, 91, 00184 Rome, Italy
- ³ IMT Institute for Advanced Studies, Piazza S. Ponziano, 6, 55100 Lucca, Italy



nious, yet tractable ways. One eminent example is the typology in public good settings proposed by Fischbacher et al. (2001) who use a variant of the strategy method (Selten 1965). If we look at competitive settings, however, little progress has been made in defining a meaningful typology.

An early attempt comes from Potters et al. (1998) in the experimental analysis of the Tullock (1967) rent-seeking contest. The authors acknowledge that, from the remarks left by participants, it is possible to classify three types of players: the 'gamesmen', who seem to understand the strategic nature of the game; the 'adapters', who adapt to the outcomes of the previous rounds; and the 'confused', who randomize effort. More recently, Herrmann and Orzen (2008) find support for the existence of different types in a variant of the Tullock contest using the strategy method. Modeling heterogeneity in competitive situations such as contests is an important step to understand conflict resolution, as it facilitates further research on the different motives that drive the behavior of individuals and aids comprehension of group dynamics caused by different type compositions.

In this paper we propose a typology of contestants in the Tullock rent-seeking model using the classifier-Lasso (C-Lasso, Su et al. 2016) that is able to identify and estimate latent group structures in panel data. C-Lasso has been recently applied across economic settings (Lu and Su 2017; Wang et al. 2019). To explore type heterogeneity in competitive environments, we apply the C-Lasso to a data set of six repeated contest experiments that differ in whether the groups of contestants stay fixed (partners) or are randomly re-matched after each round (strangers). Other studies have highlighted differences in average behavior under certain circumstances between partners and strangers (e.g., Baik et al. 2015; Fallucchi and Renner 2016). Instead of focusing on the overall level of rent-seeking, we look at the strategies that players adopt.

Results from our estimations suggest that the optimal number of behavioral types in both partners and strangers contests is three, in line with what has been previously suggested. In partners, we label more than half of contestants as 'reciprocators' since they show an increasing response function with respect to the effort based on the opponents' previous choices. A second group, only formed by a tenth of players, adapts their effort to the past opponents' choices in a concave fashion, although their level of expenditures is the highest among types ('gamesmen'). For the remaining third, the 'others', opponents' effort on current choices seems to matter in a non-standard way. In experiments with strangers matching we also find 'reciprocators'. However they represent a significantly lower fraction of players (around 25%). These findings are coherent with the experimental evidence in other settings that fixed matching protocol induces a higher orientation to reciprocity (e.g. Schmidt et al. 2001). Interestingly, in partners matching, groups that consist of more 'reciprocators' are associated to significantly lower average group efforts. For strangers matching this result does not hold, because of the nature of the game as well as the low fraction of players adopting this strategy reduces their impact on overall effort.

The paper also contributes to the 'experimetrics' literature, suggesting the C-Lasso as a tool to observe heterogeneity in experimental data. Other sophisticated approaches to form taxonomies in experimental settings are based on Bayesian models (Houser et al. 2004; El-Gamal and Grether 1995; Shachat and Wei 2012) or finite mixture models (McLachlan and Peel 2000). The latter have become standard for categoriz-



ing individuals in repeated experiments such as public good games (Bardsley and Moffatt 2007), beauty contests (Bosch-Domènech et al. 2010), lottery choices (Conte et al. 2011), and private information games (Brocas et al. 2014). From an econometric point of view, C-Lasso avoids a series of drawbacks of mixture models. In particular, the likelihood function of a finite mixture model usually shows irregularities such as multimodality (Lehmann and Casella 2006; Spiliopoulos et al. 2018), and therefore introduces a complication in detecting the local maximum point that corresponds to the efficient root. On the contrary, the application of a penalized likelihood maximization of C-Lasso produces a unique solution. One other application of this method is provided by Bordt and Farbmacher (2019) who use experimental data of repeated public good games to replicate the classification of behavioral types proposed by Fischbacher et al. (2001) using the C-Lasso mechanism. Their results are consistent with the re-analysis done using hierarchical clustering by Fallucchi et al. (2019) and suggest a coherence of subject behavior in repeated games with the choices elicited via strategy method.

The remainder of the paper is organized as follows. The next section describes the contest data and presents the C-Lasso mechanism. In Sect. 3 we present the results of the C-Lasso estimations for both contest with fixed groups and randomly re-matched groups. We conclude in Sect. 4 with a discussion of our results.

2 Material and methods

Tullock contests are frequently used to model competitive situations under uncertainty (Konrad 2009) and form an essential part of the experimental economics literature (Dechenaux et al. 2015). In a standard specification of the contest, N players compete for a prize of size P whose assignment is probabilistic. The chance of contestant i to win the prize increases with his own efforts $x_i \in \{0, P\}$ but decreases with the total sum of efforts over all contestants $\sum_{i=1}^{N} x_i$. The individual profit π_i depends thus on all contestants' expenditures, the prize assignment and a homogeneous initial endowment e:

$$\pi_i = \begin{cases} e - x_i + P & \text{with probability } x_i / \sum_{i=1}^{N} x_i \\ e - x_i & \text{otherwise.} \end{cases}$$
 (1)

Our data set consists of six experimental treatments. In two of them (Savikhin and Sheremeta 2013; Mago et al. 2016) contestants repeatedly play the contest against the same three opponents. In the remaining four treatments (Sheremeta 2010; Sheremeta and Zhang 2010; Price and Sheremeta 2011; Chowdhury et al. 2014) contestants are

¹ Alternative approaches to deal with these problems have been proposed in the literature without a general consensus (e.g., McLachlan and Peel 2000; Mercatanti 2013; Feller et al. 2018).



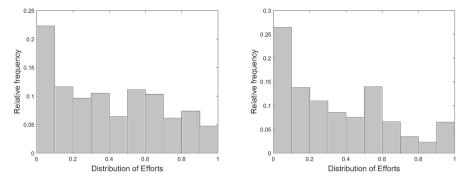


Fig. 1 Distribution of effort choices in FM (left graph) and RM (right graph) studies.

randomly regrouped with three opponents after each round.² We further refer to the two treatment types as fixed matching (FM) and random matching (RM), respectively.³

After each round, they are reminded of their own effort and receive information on their opponents' total effort and whether they have won the prize. At the end of the experiment the accumulated earnings are converted and paid out in cash. We present in Fig. 1 below the distribution of efforts for the studies under the two matching protocols. The distributions look similar, with a modal interval at a higher level in RM treatments than in FM. Comparing the average effort levels to the Nash prediction, we observe substantial rent dissipation across groups, of about 181% of the Nash Equilibirum predictions in FM studies, and 208% in RM studies.

Consistent with evidence from other experimental contests (Dechenaux et al. 2015), the sample shows substantial variation in efforts. Part of the observed heterogeneity may be explained by divergent contestants' reactions to the information provided after each round of the contest. For example, prior effort of opponents could lead to imitative behavior (e.g. Apesteguia et al. 2007) or to adapt own efforts in line with the (myopic) best response function of the contest. Further, how information of the previous round affects effort decisions might depend on whether opponents stay the same or potentially change after each round. We thus are interested in how contestants can be characterized by their reactions to prior information within FM and RM treatments and how contestants differ between fixed and random matching regimes.

We analyze the data of FM treatments, N = 96 contestants observed for T = 19 periods, and of RM treatments, N = 183 contestants observed for T = 29, into two panel data sets $(\omega_{it} = (y_{it}, x_{it}))$. The variables of interest are y_{it} , the effort

⁴ From the original data set we drop the observations of four contestants in FM treatments and nine contestants in RM treatments, whose prize assignments are time-invariant, and the first period due to lag effects in the model.



² In the course of the empirical analysis, we normalize the prize and the range of possible effort values to [0; 1] for all treatments to facilitate the comparison of results.

³ For all treatments, different subjects are recruited from a pool of students (Chowdhury et al. (2014) recruits from University of East Anglia, the other studies from Purdue University). We test for differences in average group efforts within both FM and RM treatments using Kruskal–Wallis rank tests but do not detect significant deviations (p-value FM: 0.222, p-value RM: 0.280).

of contestant i at time t, and the vector of covariates x_{it} : the sum of expenditures of the contestant's opponents at t-1 (l.othereffort), the squared sum of expenditures of the contestant's opponents at t-1 ($l.othereffort^2$), an indicator of the contest outcome for contestant i at t-1 (l.win) and a time indicator (period). We include $l.othereffort^2$, since the contest's (myopic) theoretical best response function is concave ($BR(X) = \sqrt{X_{-i}} - X_{-i}$, where X_{-i} is the sum of opponent expenditures) and thus better approximated by including a square term in the regression. As you can see from Fig. 1, the distribution of effort has a mode at the lowest expenditures interval in both cases. A relevant fraction of choices hit the lower bound of 0 (6% in FM and 13% in RM), while less than 2% in FM and 3% in RM hit the upper bound. Given the censored nature of the outcome, we can plausibly assume a Tobit panel to model the contests. In its standard form, the heterogeneity is captured only by the individual fixed effects μ_i :

$$y_{it} = max(0, \mu_i + x_{it}\boldsymbol{\beta} + \epsilon_{it})$$
 with $\epsilon_{it} \sim N(0, \sigma_{\epsilon}^2)$, (2)

where the vector of coefficients $\boldsymbol{\beta}$ is maintained constant, which implies behavioral pattern homogeneity. The opposite is a model where we impose complete heterogeneity, namely a model that estimates coefficients at individual level $(\boldsymbol{\beta}_i)$. Our interest is to group behavior of contestants into a small number of distinct types. We estimate the optimal number of behavioral types, as well as the types' behavioral patterns, with the C-Lasso (Su et al. 2016). The method allows identifying latent structures in panel data by shrinking the set of N individual coefficients $\boldsymbol{\beta}_i$ to a smaller set of K < N group-specific coefficients $\boldsymbol{\beta}_k$:

$$y_{it} = max(0, \mu_i + \mathbf{x}_{it}\boldsymbol{\beta}_k + \epsilon_{it}) \quad \text{with}$$

$$\boldsymbol{\beta}_k \neq \boldsymbol{\beta}_{k'} \quad \text{when} \quad k \neq k', \quad \text{and} \quad \epsilon_{it} \sim N(0, \sigma_{\epsilon}^2). \tag{3}$$

One advantage of C-Lasso is that the classification of individuals in latent groups is obtained by a statistical learning procedure. This is achieved by adopting the approach proposed by Su et al. (2016), that we extend to the Tobit case, minimizing the penalized nonlinear likelihood (PNL) function:

$$\min_{(\boldsymbol{\beta}_{i},\mu_{i},\boldsymbol{\beta}_{k},\sigma_{\epsilon}^{2})} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \Psi(\boldsymbol{\omega}_{it}; \mu_{i}, \boldsymbol{\beta}_{i}, \sigma_{\epsilon}^{2}) + \frac{\lambda}{N} \sum_{i=1}^{N} \prod_{k=1}^{K} ||\boldsymbol{\beta}_{i} - \boldsymbol{\beta}_{k}||, \quad (4)$$

where the first term denotes the individual log likelihood function of the Tobit model and the second is a penalization term.

Similar to the original version of Lasso (Tibshirani 1996), λ acts as shrinkage parameter on the β coefficients. In Lasso applications, a share of β coefficients are shrunk to zero, whereas in C-Lasso the individual coefficients β_i are shrunk to a smaller set of group-specific β_k via the penalization term. If λ would be posed to 0, the penalization term would disappear and the problem would resolve in optimizing the log likelihood function of the panel Tobit under complete heterogeneity. By contrast, for a given K, the greater the λ , the closer the estimated β_k would be to the case of complete homogeneity.



Dep. variable: effort	Pooled Tobit	C-Lasso Tobit		
		Type 1 Reciprocators	Type 2 Gamesmen	Type 3 Others
l.othereffort	0.504*** (0.030)	0.113** (0.051)	1.000*** (0.237)	- 0.282*** (0.085)
l.othereffort ²	- 0.151*** (0.015)	- 0.012 (0.020)	- 0.347*** (0.094)	0.101*** (0.030)
l.win	0.126*** (0.016)	0.031** (0.014)	- 0.009 (0.041)	- 0.004 (0.026)
period	0.001 (0.001)	0.000 (0.001)	- 0.005 (0.003)	- 0.003* (0.002)
σ_{ϵ}	0.307*** (0.005)	0.200*** (0.006)	0.263*** (0.012)	0.274*** (0.009)
Obs;N;%	1824; 96; 100%	1064; 56; 58%	171; 9; 9%	589; 31; 32%

Table 1 C-Lasso Tobit regression results for FM treatments

Standard errors (in parenthesis); p-values: $*\le 0.10$, $**\le 0.05$, $***\le 0.01$; Obs. is the total number of observations; N is the number of contestants of each type; % is the relative share of each type with respect to the full sample.

Therefore, the minimization problem (4) has to be preceded by a further optimization to set the λ that balances the two opposite conditions of complete homogeneity and heterogeneity. In particular, when the optimal number of groups K is unknown, as in our case, this preliminary optimization problem simultaneously estimates K and sets λ by minimizing a penalized Information Criterion function. Afterwards, the optimal classification of individuals in groups G_k and estimation of the group-specific coefficients β_k is carried out.

3 Results

Confirming the suggestions of Potters et al. (1998), our estimation results show that the optimal number of latent groups in the FM sample is three. 58% of the contestants form type 1 (56 contestants), 9% fall into type 2 (9 contestants) and the remaining ones account for type 3 (see Table 1). We test whether contestants that belong to the least frequent type would be equally well represented in one of the other two types. However, likelihood ratio tests reveal that assigning the least frequent type to either one of the other types significantly reduces the model fit. We thus turn to the C-Lasso regression results that characterize the differences in types.

Table 1 reports the estimates of a standard Tobit model with pooled observations as well as C-Lasso coefficients for each of the three types. The results of the pooled Tobit regression imply that, over all contestants, the effect of past opponents' effort on own effort is concave given the positive (negative) significant effect of *l.otherefforts* (*l.othereffort*²). Winning the contest in the previous round (*l.win*) significantly increases the efforts of contestants in the subsequent round. Moreover, we do not find a significant time trend (*period*) when pooling observations. Yet, the pooled

⁶ For this procedure, we follow Su et al. (2016) and provide a detailed description in the Appendix. We carry out the C-LASSO estimation using the MATLAB code, which we provide with comments in the supplementary material.



 $^{^{5}}$ In statistical learning this is technically called the overfitting/underfitting tradeoff.

estimation results mask the heterogeneous behaviors observed in the three groups identified by C-Lasso.

For all three contestant types, opponent expenditures in the previous round matter for own effort choices, but how they matter varies across types. For the first type, contestants show an increasing response function with respect to *l.otherefforts*, and further increase (decrease) their effort after a win (loss). Being faced with lower (higher) bidding opponents leads these contestants to bid low (high) themselves, suggesting a 'good for good' and 'evil for evil' reply to previous opponents' choices. We refer to contestants of type 1 as 'reciprocators'. The second type does not significantly respond to previous contest outcome, but own efforts relate positively to other efforts and negatively to its square term. The inverted u-shape that relates *l.othereffort* to current choices, suggests that these players decrease their effort when competition gets fierce, which is line with what would be expected under standard game theoretical considerations. Following the taxonomy of Potters et al. (1998), we label this type as 'gamesmen'. For the third type this relationship appears to be inverted, and moreover, efforts are decreasing over time. We do not find an intuitive explanation for this behavior in the contest literature, which is why we simply label them 'others'.

To visualize how the different player types respond to efforts of opponents from previous rounds, we plot in Fig. 2 the estimated best response (BR) function of each type and the theoretical myopic BR as a function of the average of opponents' effort in the previous round. All estimated best response functions deviate substantially from the theoretical myopic BR. This is not surprising when considering that in the data higher average opponent effort in t-1 is associated to higher own effort in period t. For reciprocators the estimated best response is slowly increasing (its marginal effect is flat and positive), indicating that a marginal increase (decrease) of the average competitor effort in the previous period leads to an increase (decrease) in own current efforts. The best response function of the 'gamesmen' is concave. Yet, when compared to the theoretical BR, their response to prior opponents' expenditures is very aggressive, meaning that 'gamesmen' start to decrease efforts only when the average effort of opponents is already considerably high (around 0.5). For 'others' instead, the response is slightly convex.

Result 1: C-Lasso identifies three contestant types in treatments with fixed matching. We classify them as 'reciprocators' (58%), 'gamesmen' (9%), and 'others' (32%) based on how their choices are explained by opponents' choices in the previous contest round.

For reciprocators, that form the majority of contestants, we want to understand whether their reciprocal bidding behavior reduces average individual efforts or exac-



⁷ Across C-Lasso groups, we reject the null hypotheses that the coefficients of *l.othereffort* and *l.othereffort* are jointly equal to zero $(H_0: \beta_1 = \beta_2 = 0)$ and the hypothesis that the average partial effect of *l.othereffort* on own efforts is zero $(H_0: \beta_1 + 2\beta_2 l.othereffort = 0)$, where $\overline{l.othereffort}$ is fixed to be the mean value of *l.othereffort*) on the 1% significance level using F-tests.

⁸ Since the marginal effect of a regressor on the dependent variable depends in Tobit models on its own level (Wooldridge 2002), we additionally present for each type the marginal effect over the range of average *l.othereffort* in Figs. 8 (FM) and 9 (RM) in the Appendix.

⁹ See Fig. 6, 7 in the Appendix.

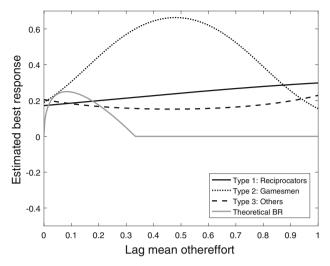


Fig. 2 Estimated myopic best response on average opponents' effort in FM treatments.

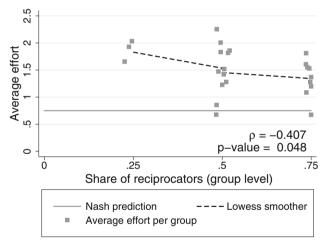


Fig. 3 Relationship between the share of reciprocators and average group effort—FM treatments

erbates competition. We thus examine how the share of reciprocators in a contest group influences the average group effort. Remember that for all treatments each group consists of four contestants. If reciprocal behavior results in collaboration, then groups with a higher fraction of reciprocators should be associated to lower average effort levels. On the contrary, if reciprocity increases competition then we should witness higher average effort levels in groups formed by many reciprocators. Figure 3 shows how the share of reciprocators per group relates to the average group effort. We find that average group effort is decreasing with the fraction of reciprocators per group



Dep. variable: effor	t Pooled Tobit	C-Lasso Tobit		
		Type 1 (Reciprocators)	Type 2	Type 3
l.othereffort	0.350*** (0.017)	0.167*** (0.048)	- 0.058*** (0.021)	- 0.551*** (0.053)
$l.othereffort^2$	- 0.095*** (0.009)	- 0.035* (0.021)	0.033*** (0.009)	0.208*** (0.021)
l.win	0.209*** (0.010)	0.119*** (0.017)	0.020** (0.008)	0.146*** (0.016)
period	0.001* (0.001)	- 0.006*** (0.001)	- 0.006*** (0.000)	- 0.001 (0.001)
σ_{ϵ}	0.321*** (0.003)	0.271*** (0.005)	0.188*** (0.003)	0.309*** (0.007)
Obs;N;%	5307; 183; 100%	1305; 45; 25%	2784; 96; 52%	1218; 42; 23%

Table 2 C-Lasso Tobit regression results for RM treatments

Standard errors (in parenthesis); p-values: $*\le 0.10$, $**\le 0.05$, $***\le 0.01$; Obs. is the total number of observations; N is the number of contestants of each type. % is the relative share of each type with respect to the full sample.

(spearman's $\rho = -0.407$ p-value = 0.048), suggesting that presence of reciprocators tends to reduce efforts over time.¹⁰

Result 2: In FM treatments average group expenditures are lower for contest groups with a higher share of 'reciprocators'.

Contestants of the FM treatments know that their opponents stay the same over the course of the experiments, so that cooperative tendencies seem to occur naturally. Surprisingly, the 'reciprocator' as player type has not received much attention in contest experiments. This might be because in many contest experiments, including Potters et al. (1998), contestants are randomly regrouped after each round, making reciprocal actions less effective. Using data on four different RM contest studies, we explore whether reciprocators also appear in RM treatments.

Result 3: C-Lasso identifies three contestant groups for contests with random matching. Only 25% can be identified as 'reciprocators', significantly less than in fixed matching treatments.

The C-Lasso categorizes the 183 RM contestants into three types. Table 2 reports the C-Lasso estimates and compares them to the pooled Tobit regression estimates, that are similar to the estimates of the FM panel. The first type contains 45 contestants, whose regression estimates and marginal effect of *l.othereffort* on effort resemble the ones of 'reciprocators' in FM treatments. Again we analyze the relationship between the share of 'reciprocators' and average effort. Since in RM treatments the share of reciprocators per group may change every round, we look at the relationship on the session level for which contestants stay fixed. The RM treatments contain in total 16 sessions. The lowess smoother in Fig. 5 suggests no directional relationship between the share of reciprocators in a session and the average session effort. This is in line

reciprocators but also associated with a decrease in the share of gamesmen or others, but we do not find evidence for such an association.



 $^{^{10}}$ A low share of reciprocators implies a high share of contestants assigned to other types. We thus examine if a decrease in average group efforts is not only associated with an increase in the share of Footnote 10 continued

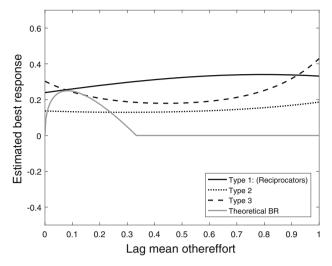


Fig. 4 Estimated myopic best response on average opponents' effort in RM treatments

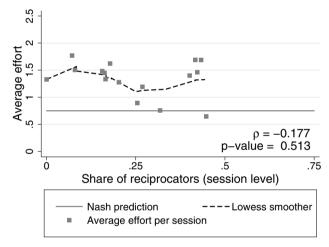


Fig. 5 Relationship between the share of reciprocators and average group effort - RM treatments

with the Spearman correlation coefficient, which we find not significant (spearman's $\rho = -0.177$ p-value = 0.513). Similar to FM treatments, the average session efforts mostly exceed the theoretical equilibrium level.

The share of reciprocators across RM contestants reaches only 25% which is significantly lower than in treatments where opponents remain fixed (about 58%). Presumably we observe less reciprocators in random matched treatments, since there is no direct feedback on the effectiveness of own collaboration attempts. The other

¹¹ A Pearson's chi-squared test, conducted on the contingency table of the empirical distribution of the number of reciprocators in the two samples (FM and RM), rejects the null hypothesis that the samples are drawn from the same super-population (p-value < 0.001).



contestant types two (52%) and three (22%) increase (decrease) their efforts after previously won (lost) rounds and show a u-shaped relationship between *l.othereffort* and current own efforts. The magnitude of the regression coefficients suggests that previous round information affects current efforts to a lesser extent for type two contestants (see also the estimated BR depicted in Fig. 4). Similar to the 'others' type in FM treatments, both types differ from what is expected from standard game theoretical considerations, presumably because contestants do not expect new opponents to behave as their last round opponents did. The convex response of type three to *l.othereffort* suggests that these contestants bid high after a round of especially high or low opponent efforts. Possibly high bids after rounds of high opponent efforts are the result of some sort of spiteful behavior (Herrmann and Orzen 2008), whereas high bids after rounds of low opponent effort might reflect the attempt to increase ones own probability of winning in case opponent efforts remain low.

4 Robustness check: fair and unfair lottery outcome

The previous analysis was based on the assumption that some players would play a myopic best-response strategy. Therefore, we limited the variables that impact players' effort to the information received from the previously played round, including a dummy for prize wins, which is commonly used as a control in experimental contest studies. Nevertheless, the literature suggests other specifications that might offer additional insights. Potentially, one could think about omitting certain variables from the base regression, including opponent efforts of two or three earlier periods, or further separating existing variables, such as *l.win*.

For example, Sheremeta (2011) suggests to separate the effects of fair and unfair wins. A fair (unfair) win is defined as a win for which own effort has been higher (lower) than the average in the group. Sheremeta (2011) finds that fair wins have a positive effect on next round effort, whereas unfair wins decrease own effort.

We evaluate alternative sets of regressors, by pairwisely comparing the log likelihood of the Tobit base regression with the log likelihood of nine alternative models using likelihood ratio tests. We observe a significant drop in the likelihood of the models that exclude variables from the base model while we detect no significant increase when including additional lagged regressors. Only when replacing the dummy *l.win* with the fair and unfair wins (*l.fairwin* and *l.unfairwin*) the model's likelihood increases significantly. We therefore select the alternative set of regressors: *l.othereffort*, *l.othereffort*², *l.fairwin*, *l.unfairwin* and *period*, and repeat the C-Lasso routine as previously described. The newly formed player types mostly correspond

 $^{^{13}}$ As before, the optimal number of types in RM treatments is three. For FM treatments, we find that K=4 results in a lower IC value, yet one type contains only 3 out of 96 contestants (of which none was previously classified as reciprocator). Hence, we restrict the model to three types to simplify the comparison with the previous specifications.



¹² We report the different specifications and the respective p-values of the likelihood ratio tests in Table 5 in Appendix.

to the already established types (89 out of 96 FM contestants and 171 out of 183 RM contestants do not change type classification).¹⁴

The effects of *l.fairwin* and *l.unfairwin* in the pooled regression of FM and RM treatments are significant and show the same direction than reported by Sheremeta (2011), namely positive for prize wins when the winner's bid is higher than the average bid in the group *l.fairwin* and negative otherwise *l.unfairwin*. ¹⁵ Yet, the C-Lasso estimates uncover that types react heterogenously to a fair and unfair lottery outcome. In FM treatments, *l.fairwin* significantly increases efforts of reciprocators, while *l.unfairwin* significantly decreases the efforts of type others. In RM treatments, *l.fairwin* seems to increase efforts significantly across player types, however l.unfairwin significantly decreases only the efforts of type 2. These two groups represent a high share of contestants (30% in FM and 50% in RM). Compared to the base regression, the coefficients of the remaining regressors are robust with respect to their significance and direction of the effects. Similar to our main results we observe that groups with more reciprocators are associated to lower average efforts in FM treatments which is not the case for RM treatments (see Figs. 10 and 11 in the Appendix). In summary, type-specific effects do not necessarily coincide with average effects in the population. However, this robustness check helped to better characterize the behavior of groups that was not previously well understood. Type heterogeneity is therefore important to keep in mind when drawing general conclusions about contestants behavior.

5 Discussion and conclusion

In many competitive situations the development of a contestant taxonomy is impeded because one cannot directly assess the motivation behind individual choices. We use the C-Lasso methodology that identifies latent group structures in panel data and thus categorizes contestants based on how their effort choices relate to information they received of the previous round.

Using data from six different studies, we find that contestants react heterogeneously to information of the previous contest round. C-Lasso identifies three types of contestants for both contest matching protocols. The majority of contestants in FM treatments can be classified as reciprocators that attempt to cooperate by lowering own efforts when previous opponent efforts were low. As a result, contest groups that contain more reciprocators show lower average effort levels and thus increase group profits. For RM treatments the number of reciprocators appears to be significantly lower, in line with the idea that cooperative attempts in the current round cannot be observed by ones next rounds opponents.

Our exploration on what types of contestants are out there hopefully releases a pulse on how we think about competitive situations such as conflict resolution. Once we acknowledge that choice rules of competing individuals are heterogeneous, we can develop a better understanding for the tools that are needed to mitigate conflicts

¹⁵ We report the regression Tables 6 and 7 in Appendix.



 $^{^{14}}$ We show in Tables 8 and 9 in Appendix the transition of players between the types defined by the base specification and the fairwin specification respectively for FM and RM treatments.

and to reduce overspending in competitive situations. For example, reciprocators are willing to conditionally cooperate over the course of the contest, which is most fruitful whenever the number of them in a group is high and thus cooperation can reinforce itself. Knowing the type composition of groups can possibly deliver valuable insights on what group dynamics can be expected, across applications.

We suggest two directions for future research to understand the main drivers of behavioral types. The first direction is to check whether different types react differently to other changes in the contest structure. For example, excessive over-expenditures observed in larger groups (Lim et al. 2014) may be driven by less cooperative activity, that has been demonstrated for different experimental settings (Nosenzo et al. 2015). We also believe that the extension of such analysis to other market settings could help to better link counterintuitive results with individual behavior (e.g., Orzen 2008). The second direction concerns the use of methods that reveal latent group structures in panel data to shed light on different topics, such as framing (e.g., Chowdhury et al. 2019) or impulsive behavior (e.g., Rubinstein 2016; Sheremeta 2018).

Acknowledgements We thank Subasish Chowdhury, Daniele Nosenzo, Simone Quercia, Roman Sheremeta and participants of the 'Young Economists of Tuscan Institutions 2019' meeting and the '2nd Reading Experimental and Behavioural Economics Workshop'. We also thank the authors of the cited studies for generously providing their data and two anonymous referees for their valuable feedback. The views expressed herein are those of the authors and do not necessarily reflect those of the Bank of Italy.

Funding Open access funding provided by Scuola IMT Alti Studi Lucca within the CRUI-CARE Agreement.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

Appendix A

A.1 C-Lasso procedure for the selection of the optimal number of behavioral types

Recall the penalized nonlinear likelihood (PNL) function from Eq. (4):

$$\min_{(\boldsymbol{\beta}_{i},\mu_{i},\boldsymbol{\beta}_{k},\sigma_{\epsilon}^{2})} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} \Psi(\boldsymbol{\omega}_{it}; \mu_{i}, \boldsymbol{\beta}_{i}, \sigma_{\epsilon}^{2}) + \frac{\lambda}{N} \sum_{i=1}^{N} \prod_{k=1}^{K} ||\boldsymbol{\beta}_{i} - \boldsymbol{\beta}_{k}||, \quad (4)$$

where $\Psi(\boldsymbol{\omega}_{it}; \mu_i, \boldsymbol{\beta}_i, \sigma_{\epsilon}^2)$, the first term, denotes the individual log likelihood function of the Tobit model given the data $\boldsymbol{\omega}_{it}$, the individual fixed effect μ_i , the vector of individual coefficients $\boldsymbol{\beta}_i$, and the variance σ_{ϵ}^2 of the normal i.i.d. idiosyncratic



Table 3 FM panel - values of the information criteria (IC) function for alternative number of groups K, and tuning parameter c_{λ} selection

Tuning parameter	K: Numb	K: Number of groups				
c_{λ}	1	2	3	4		
0.100	0.048	0.031	0.038	Inf		
0.129	0.048	0.035	0.044	0.038		
0.170	0.048	0.034	0.046	0.035		
0.215	0.048	0.039	0.043	0.035		
0.278	0.048	0.043	0.052	0.033		
0.359	0.048	0.042	0.039	0.033		
0.464	0.048	0.049	0.037	0.051		
0.600	0.048	0.050	0.021	0.041		
0.774	0.048	0.053	0.021	0.027		
1.000	0.048	0.055	0.028	0.040		

Table 3 contains for each (K, c_{λ}) combination the result of the IC function in Eq. (6) using the FM panel

error term. To make the convex optimization methods feasible, $\Psi(\omega_{it}; \mu_i, \beta_i, \sigma_{\epsilon}^2)$ is transformed to (Olsen 1978):

$$-\Psi(\boldsymbol{\omega}_{it}; \boldsymbol{\beta}_{i}, \mu_{i}, \sigma_{\epsilon}^{2}) = \sum_{y_{it}>0} \frac{1}{2} \left[ln(2\pi) - ln\theta^{2} + (\theta y_{it} - \boldsymbol{x}'_{it}\boldsymbol{\delta}_{i} - c'_{it}\eta_{i})^{2} \right]$$
$$+ \sum_{y_{it}=0} ln \left[1 - \Phi(\boldsymbol{x}'_{it}\boldsymbol{\delta}_{i} + c'_{it}\eta_{i}) \right]$$
(5)

Once the minimization procedure has been carried out, the original parameters are retrieved by $\sigma_{\epsilon} = 1/\theta$, $\beta_i = \delta_i/\theta$, and $\mu_i = \eta_i/\theta$.

The second term of the PNL allows the assignment of individuals to groups as well as the estimation of the group coefficients β_k . The value of λ is data dependent and has to be chosen, jointly with the number of groups K, by minimizing the following penalized information criterion (IC) function:

$$\min_{(K,\lambda)} \frac{2}{NT} \sum_{k=1}^{K} \sum_{i \in \hat{G}_k(K,\lambda)} \sum_{t=1}^{T} \Psi(\boldsymbol{\omega}_{it}; \hat{\mu}_{i\hat{G}_k(K,\lambda)}, \hat{\boldsymbol{\beta}}_{\hat{G}_k(K,\lambda)}, \hat{\sigma}_{\epsilon(K,\lambda)}^2) + \nu(NT)^{-0.5} qK$$
(6)

For any couple (K, λ) , the first term in Eq. (6) is evaluated by a two-step procedure. In the first step, a classification of individuals in K groups is obtained by minimizing Eq. (4). These groups are denoted by $\hat{G}_k(K, \lambda)$, k=1,...,K (Table 4). In the second step, the individuals classified in each group $\hat{G}_k(K, \lambda)$ are pooled to estimate the group-specific parameters $\mu_{i\hat{G}_k(K,\lambda)}$, $\beta_{\hat{G}_k(K,\lambda)}$, $\sigma_{\epsilon(K,\lambda)}^2$ by Tobit panel analyses (so called post-Lasso analyses). This two-step procedure is repeated over a grid of values for the couple (K, λ) . For the choice of the values of λ , we follow (Su et al. 2016) who tune λ over a finite set $\lambda(c_\lambda) = var(y)T^{(-1/3)}$, where var(y) is the variance of the dependent



Table 4 RM panel - values of the information criteria (IC) function for alternative number of groups K, and tuning parameter c_{λ} selection

Tuning parameter	K: Numb	er of groups		
c_{λ}	1	2	3	4
0.100	0.197	0.191	0.155	0.166
0.129	0.197	0.190	0.159	0.178
0.170	0.197	0.194	0.158	0.172
0.215	0.197	0.195	0.153	0.175
0.278	0.197	0.194	0.146	0.179
0.359	0.197	0.193	0.120	0.179
0.464	0.197	0.190	0.120	0.179
0.600	0.197	0.187	0.122	0.174
0.774	0.197	0.185	0.132	0.145
1.000	0.197	0.180	0.140	0.155

Table 3 contains for each (K, c_{λ}) combination the result of the IC function in equation (6) using the RM panel.

variable, T the time dimension of the panel, and c_{λ} is a scalar. For our specific case, we choose c_{λ} over a geometrically increasing sequence $0.1 \cdot 10^{(l-1)/9}$ (l=1,...,10). The minimization problem limited to the first term in (6) resolves in a maximum likelihood estimation under complete heterogeneity. To counterbalance, the first term is penalized by the second term $\nu(NT)^{-0.5}qK$, where q is the number of explanatory variables and $\nu \in (0,1)$ a penalization parameter which is set to $\nu=0.22$. 17

We evaluate Eq. (6) for all combinations of the c_{λ} sequence and K=1,...,4 and consequently choose the combination (K,c_{λ}) that minimizes the IC function. We report in Tables 3 and 4 the resulting values for the IC function for the FM and RM studies. The minimum value of the IC function is equal to 0.021 in the FM panel and equal to 0.120 in the RM panel, corresponding to the optimal number of groups K=3 in both cases. With the optimal combination of tuning parameter and number of groups we estimate Eq. (4) and retrieve the C-Lasso group estimates. ¹⁸ Lastly, we do not detect signs of model misspecification using RESET tests (Ramsey 1969; Peters 2000). ¹⁹

Appendix B Supplementary Figures and Tables

See Figs. 6, 7, 8, 9, 10, 11, Tables 5, 6, 7, 8, 9.

¹⁹ We use RESET specifications that include quadratic and cubic terms of linear predictions, as they are found to have high statistical power in Monte Carlo simulations (Ramalho and Ramalho 2012; Lechner 1995). P-values for the quadratic and cubic terms in FM (RM) treatments: 0.667, 0.427 (0.480, 0.196).



 $^{^{16}}$ Su et al. (2016) use a geometrically increasing sequence of ten c_{λ} from 0.01,...,0.1 and from 0.2,...,2 for a linear model and a Probit model, respectively. Since the Tobit model resembles a combination of these two models, our sequence overlaps with both original sequences proposed.

¹⁷ This value is obtained using simulated data for which we fix a-priori the number of groups ($K_{\text{known}} = 1, 2, 3, 4$), and assess the correctly specified number of groups over all K_{known} and a sequence of λ .

¹⁸ Estimates are unbiased using a half-panel jackknife procedure (Dhaene and Jochmans 2015).

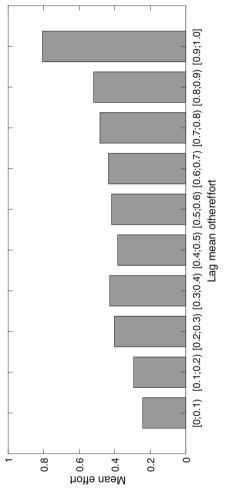


Fig. 6 Relationship between average *l.othereffort* and effort in FM treatments



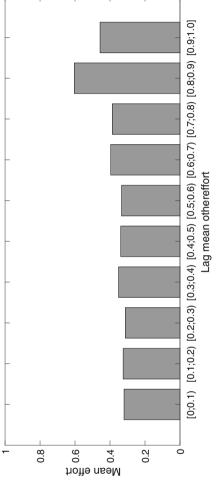


Fig. 7 Relationship between average *l.othereffort* and effort in RM treatments



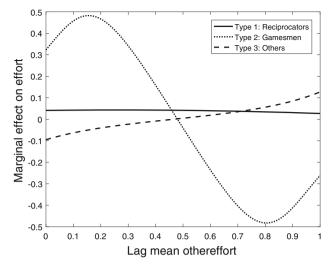


Fig. 8 Marginal effect of mean *l.othereffort* on effort (FM treatments)

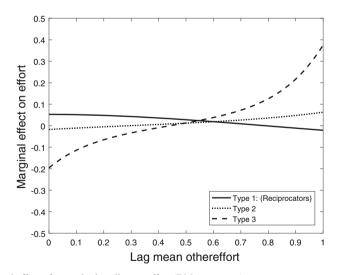


Fig. 9 Marginal effect of mean *l.othereffort* on effort (RM treatments)



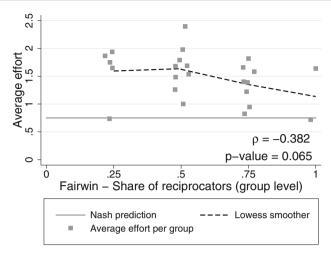
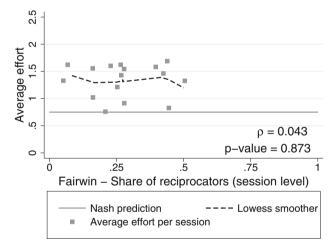


Fig. 10 Relationship between the share of reciprocators and average group effort - FM treatments fairwin specification



 $\textbf{Fig. 11} \ \ Relationship \ between \ the \ share \ of \ reciprocators \ and \ average \ group \ effort \ - \ RM \ treatments \ fairwin \ specification$



Table 5 Likelihood ratio test: p-values for Tobit model selection

Base: l.win, l.othereffort, l	$l.othereffort^2$,	period	FM	RM
Base - period	vs.	Base	0.016	0.000
Base - l.win	vs.	Base	0.000	0.000
Base - l.othereffort ²	vs.	Base	0.022	0.011
Base - l.othereffort - l.othereffort ²	VS.	Base	0.000	0.000
Base	VS.	Base - l.win + l.fairwin + l.unfairwin	0.000	0.000
Base	vs.	Base + l2.othereffort	0.120	0.257
Base	VS.	Base + l2.othereffort + l2.othereffort ²	0.198	0.417
Base	VS.	Base + l2.othereffort + l3.othereffort	0.263	0.113
Base	vs.	Base + l2.othereffort + l3.othereffort	0.442	0.295
		+ l2.othereffort ² + l3.othereffort ²		

Table 5 shows the p-values of the likelihood ratio tests between two Tobit model specifications. For p-values below 0.05, we reject the Null hypothesis that the first model, with less variables, is nested in the second one, implying that including the additional variable(s) of the second model leads to a significant increase its log likelihood.

Table 6 C- Lasso Tobit regression results for FM treatments - fairwin specification

Dep. variable: effort	Pooled Tobit	C-Lasso Tobit			
		Type 1 Reciprocators	Type 2 Gamesmen	Type 3 Others	
l.othereffort	0.496*** (0.029)	0.106** (0.048)	0.744*** (0.186)	-0.280*** (0.095)	
$l.othereffort^2$	-0.143*** (0.015)	-0.007 (0.019)	-0.256*** (0.076)	0.099*** (0.033)	
l.fairwin	0.190*** (0.018)	0.053*** (0.016)	-0.009 (0.044)	0.054* (0.028)	
l.unfairwin	-0.064** (0.029)	-0.037 (0.023)	-0.021 (0.065)	-0.144*** (0.049)	
period	0.000 (0.001)	0.001 (0.001)	-0.006* (0.003)	-0.007*** (0.002)	
σ_{ϵ}	0.302*** (0.005)	0.202*** (0.006)	0.241*** (0.013)	0.280*** (0.008)	
Obs;N;%	1824; 96; 100%	1083; 57; 59%	190; 10; 10%	551; 29; 30%	

Standard errors (in parenthesis); p-values: $*\le 0.10$, $**\le 0.05$, $***\le 0.01$; Obs. is the total number of observations; N is the number of contestants of each type; % is the relative share of each type with respect to the full sample.



Dep. variable: effort	Pooled Tobit	C-Lasso Tobit		
		Type 1 (Reciprocators)	Type 2	Type 3
l.othereffort	0.357*** (0.017)	0.189*** (0.046)	-0.036* (0.021)	-0.529*** (0.057)
$l.othereffort^2$	-0.091*** (0.009)	-0.036* (0.019)	0.029*** (0.008)	0.201*** (0.023)
l.fairwin	0.285*** (0.011)	0.156*** (0.017)	0.052*** (0.010)	0.185*** (0.019)
l.unfairwin	-0.036** (0.018)	-0.062* (0.032)	-0.045*** (0.012)	0.003 (0.041)
period	0.000 (0.001)	-0.007*** (0.001)	-0.005*** (0.000)	-0.001 (0.001)
σ_{ϵ}	0.312*** (0.003)	0.266*** (0.005)	0.182*** (0.003)	0.316*** (0.007)
Obs;N;%	5307; 183; 100%	1479; 51; 28%	2668; 92; 50%	1160; 40; 22%

Table 7 C- Lasso Tobit regression results for RM treatments - fairwin specification

Standard errors (in parenthesis); p-values: $*\le 0.10$, $**\le 0.05$, $***\le 0.01$; Obs. is the total number of observations; N is the number of contestants of each type. % is the relative share of each type with respect to the full sample.

Table 8 Type transition between base and fairwin specification - FM treatments

Number of players	Fairwin Type 1	Fairwin Type 2	Fairwin Type 3	Sum
Reciprocators	53	3	0	56
Gamesmen	2	7	0	9
Others	2	0	29	31
Sum	57	10	29	96

Table 9 Type transition between base and fairwin specification - RM treatments

Number of players	Fairwin Type 1	Fairwin Type 2	Fairwin Type 3	Sum
Type 1 (Reciprocators)	43	2	0	45
Type 2	8	88	0	96
Type 3	0	2	40	42
Sum	51	92	40	183

References

Apesteguia J, Huck S, Oechssler J (2007) Imitation—theory and experimental evidence. J Econ Theory 136(1):217–235

Baik KH, Chowdhury SM, Ramalingam A (2015) Group size and matching protocol in contests. Technical report, School of Economics, University of East Anglia, Norwich

Bardsley N, Moffatt PG (2007) The experimetrics of public goods: inferring motivations from contributions. Theory Decis 62(2):161–193

Bordt S, Farbmacher H (2019) Estimating grouped patterns of heterogeneity in repeated public goods experiments. Working paper

Bosch-Domènech A, Montalvo JG, Nagel R, Satorra A (2010) A finite mixture analysis of beauty-contest data using generalized beta distributions. Exp Econ 13(4):461–475



Brocas I, Carrillo JD, Wang SW, Camerer CF (2014) Imperfect choice or imperfect attention? Understanding strategic thinking in private information games. Rev Econ Stud 81(3):944–970

- Chowdhury SM, Sheremeta RM, Turocy TL (2014) Overbidding and overspreading in rent-seeking experiments: cost structure and prize allocation rules. Games Econ Behav 87:224–238
- Chowdhury SM, Mukherjee A, Turocy TL (2019) That's the ticket: explicit lottery randomisation and learning in tullock contests. Theory and decision, pp 1–25
- Conte A, Hey JD, Moffatt PG (2011) Mixture models of choice under risk. J Econometr 162(1):79-88
- Dechenaux E, Kovenock D, Sheremeta RM (2015) A survey of experimental research on contests, all-pay auctions and tournaments. Exp Econ 18(4):609–669
- Dhaene G, Jochmans K (2015) Split-panel jackknife estimation of fixed-effect models. Rev Econ Stud 82(3):991–1030
- El-Gamal MA, Grether DM (1995) Are people Bayesian? Uncovering behavioral strategies. J Am Stat Assoc 90(432):1137–1145
- Fallucchi F, Renner E (2016) Reputational concerns in repeated rent-seeking contests. Technical report, CeDEx Discussion Paper Series
- Fallucchi F, Luccasen RA, Turocy TL (2019) Identifying discrete behavioural types: a re-analysis of public goods game contributions by hierarchical clustering. J Econ Sci Assoc 5(2):238–254
- Feller A, Greif E, Miratrix L, Pillai N (2018) Principal stratification in the twilight zone: weakly separated components in finite mixture models. arXiv:1602.06595
- Fischbacher U, Gächter S, Fehr E (2001) Are people conditionally cooperative? Evidence from a public goods experiment. Econ Lett 71(3):397–404
- Herrmann B, Orzen H (2008) The appearance of homo rivalis: social preferences and the nature of rent seeking. CeDEx discussion paper series
- Houser D, Keane M, McCabe K (2004) Behavior in a dynamic decision problem: an analysis of experimental evidence using a bayesian type classification algorithm. Econometrica 72(3):781–822
- Konrad KA (2009) Strategy and dynamics in contests. Oxford University Press, Oxford
- Lechner M (1995) Some specification tests for probit models estimated on panel data. J Bus Econ Stat 13(4):475–488
- Lehmann EL, Casella G (2006) Theory of point estimation. Springer, New York
- Lim W, Matros A, Turocy TL (2014) Bounded rationality and group size in tullock contests: experimental evidence. J Econ Behav Org 99:155–167
- Lu X, Su L (2017) Determining the number of groups in latent panel structures with an application to income and democracy. Quant Econ 8(3):729–760
- Mago SD, Samak AC, Sheremeta RM (2016) Facing your opponents: social identification and information feedback in contests. J Conflict Resolut 60(3):459–481
- McLachlan G, Peel D (2000) Finite mixture models. Wiley, Amsterdam
- Mercatanti A (2013) A likelihood-based analysis for relaxing the exclusion restriction in randomized experiments with noncompliance. Aust N Z J Stat 55(2):129–153
- Nosenzo D, Quercia S, Sefton M (2015) Cooperation in small groups: the effect of group size. Exp Econ 18(1):4–14
- Olsen RJ (1978) Note on the uniqueness of the maximum likelihood estimator for the tobit model. Econometrica J Econ Soc pp 1211–1215
- Orzen H (2008) Counterintuitive number effects in experimental oligopolies. Exp Econ 11(4):390–401
- Peters S (2000) On the use of the reset test in microeconometric models. Appl Econ Lett 7(6):361–365
- Potters J, De Vries CG, Van Winden F (1998) An experimental examination of rational rent-seeking. Eur J Polit Econ 14(4):783–800
- Price CR, Sheremeta RM (2011) Endowment effects in contests. Econ Lett 111(3):217-219
- Ramalho EA, Ramalho JJ (2012) Alternative versions of the reset test for binary response index models: a comparative study. Oxf Bull Econ Stat 74(1):107–130
- Ramsey JB (1969) Tests for specification errors in classical linear least-squares regression analysis. J R Stat Soc Ser B (Methodol) 31(2):350–371
- Rubinstein A (2016) A typology of players: between instinctive and contemplative. Q J Econ 131(2):859–890
- Savikhin AC, Sheremeta RM (2013) Simultaneous decision-making in competitive and cooperative environments. Econ Inquiry 51(2):1311–1323
- Schmidt D, Shupp R, Walker J, Ahn T, Ostrom E (2001) Dilemma games: game parameters and matching protocols. J Econ Behav Org 46(4):357–377



- Selten R (1965) Die strategiemethode zur erforschung des eingeschränkt rationalen verhaltens im rahmen eines oligopolexperimentes. Seminar für Mathemat. Wirtschaftsforschung u, Ökonometrie
- Shachat J, Wei L (2012) Procuring commodities: first-price sealed-bid or English auctions? Mark Sci 31(2):317–333
- Sheremeta RM (2010) Experimental comparison of multi-stage and one-stage contests. Games Econ Behav 68(2):731–747
- Sheremeta RM (2011) Contest design: an experimental investigation. Econ Inquiry 49(2):573-590
- Sheremeta RM (2018) Impulsive behavior in competition: testing theories of overbidding in rent-seeking contests. Working Papers 18-03
- Sheremeta RM, Zhang J (2010) Can groups solve the problem of over-bidding in contests? Soc Choice Welf 35(2):175–197
- Spiliopoulos L, Ortmann A, Zhang L (2018) Complexity, attention, and choice in games under time constraints: a process analysis. J Exp Psychol Learn Mem Cognit 44(10):1609
- Su L, Shi Z, Phillips PC (2016) Identifying latent structures in panel data. Econometrica 84(6):2215–2264
 Tibshirani R (1996) Regression shrinkage and selection via the lasso. J R Stat Soc Ser B (Methodol) 58(1):267–288
- Tullock G (1967) The welfare costs of tariffs, monopolies, and theft. Econ Inquiry 5(3):224-232
- Wang W, Phillips PC, Su L (2019) The heterogeneous effects of the minimum wage on employment across states. Econ Lett 174:179–185
- Wooldridge JM (2002) Econometric analysis of cross section and panel data, vol 108. MIT Press, Cambridge, MA

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

