PART A

PROCEEDINGS OF THE FIFTEENTH

MARCEL GROSSMANN MEETING ON GENERAL RELATIVITY

Editors

Elia S. Battistelli
Robert T. Jantzen
Remo Ruffini

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On Recent Developments in Theoretical and Experimental General Relativity, Astrophysics and Relativistic Field Theories

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On Recent Developments in Theoretical and Experimental General Relativity, Astrophysics and Relativistic Field Theories

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Traversable wormholes and Yukawa potentials

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Traversable Wormhole are amazing astrophysical objects predicted by General Relativity which are able to connect remote region of space-time. Even if their existence has not been proved yet they are object of continuous investigation. From the theoretically point of view, to exist, traversable wormholes need a special form of energy density termed "*exotic*". Since this exotic source must be concentrated on the throat of the wormhole, we discuss the implications of assuming Yukawa-like profiles which could be realize such a configuration.

1. Introduction

Traversable wormholes are solutions of the Einstein's Field Equations (EFE) which have the property of connecting remote space-time regions[1,2]. In Schwarzschild coordinates, the traversable wormhole metric can be cast into the form

$$ds^2 = -\exp\left(-2\phi\left(r\right)\right)dt^2 + \frac{dr^2}{1-\frac{b(r)}{r}} + r^2 d\Omega^2. \tag{1}$$

where $\phi\left(r\right)$ is called the redshift function, while $b\left(r\right)$ is called the shape function and where $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ is the line element of the unit sphere. $\phi(r)$ and $b(r)$ are arbitrary functions of the radial coordinate $r \in [r_0, +\infty)$. A fundamental property of a wormhole is that a flaring out condition of the throat, given by $(b - b'r)/b^2 > 0$, must be satisfied as well as the request that $1 - b(r)/r > 0$. Furthermore, at the throat $b(r_0) = r_0$ and the condition $b'(r_0) < 1$ is imposed to have wormhole solutions. It is also fundamental that there are no horizons present, which are identified as the surfaces with $e^{2\phi} \to 0$, so that $\phi(r)$ must be finite everywhere. The first step to establish if a traversable wormhole exists is given by solving the EFE in an orthonormal reference frame

$$\rho\left(r\right) = \frac{1}{8\pi G}\frac{b'}{r^2}, \tag{2}$$

$$p_r\left(r\right) = \frac{1}{8\pi G}\left[\frac{2}{r}\left(1-\frac{b\left(r\right)}{r}\right)\phi' - \frac{b}{r^3}\right], \tag{3}$$

$$p_t\left(r\right) = \frac{1}{8\pi G}\left(1-\frac{b\left(r\right)}{r}\right)\left[\phi'' + \phi'\left(\phi' + \frac{1}{r}\right)\right] - \frac{b'r-b}{2r^2}\left(\phi' + \frac{1}{r}\right), \tag{4}$$

where $\rho\left(r\right)$ is the energy density, $p_r\left(r\right)$ is the radial pressure, and $p_t\left(r\right)$ is the lateral pressure and using the conservation of the stress-energy tensor, in the same orthonormal reference frame, one gets

$$p_r' = \frac{2}{r}\left(p_t - p_r\right) - \left(\rho + p_r\right)\phi'. \tag{5}$$

One strategy to obtain solutions for the EFE is represented by imposing an Equation of State (EoS) of the following forms:

(1) $p_r(r) = \omega\rho(r)$, with ω constant,
(2) $p_r(r) = \omega\left(r\right)\rho(r)$, with ω function of the radial coordinate r,
(3) $p_r(r) = \omega\rho^\gamma(r)$, with $\gamma \in \mathbb{R}$.

Of course, this list does not exhaust the possibilities of constraining the relationship between $p_r(r)$ and $\rho(r)$. For instance, by imposing the first EoS, one finds

$$\phi' = \frac{b\left(r\right) + \omega b\left(r\right)' r}{2r^2\left(1 - \frac{b(r)}{r}\right)} \tag{6}$$

and if we also assume Zero Tidal Forces (ZTF), one gets

$$\phi\left(r\right) = C \qquad \text{and} \qquad b\left(r\right) = r_0\left(\frac{r_0}{r}\right)^{\frac{1}{\omega}}, \tag{7}$$

where the condition $b\left(r_0\right) = r_0$ has been used. The parameter ω is restricted by the following conditions

$$b'\left(r_0\right) < 1; \qquad \frac{b\left(r\right)}{r} \underset{r\to+\infty}{\to 0} \qquad \Longrightarrow \qquad \begin{cases} \omega > 0 \\ \omega < -1 \end{cases} \tag{8}$$

and the metric (1) assumes the particular simple expression

$$ds^2 = -dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{\frac{1}{\omega}+1}} + r^2 d\Omega^2. \tag{9}$$

It is interesting to note that when $\omega \to \infty$, one finds

$$ds^2 = -dt^2 + \frac{dr^2}{1 - \frac{r_0}{r}} + r^2 d\Omega^2, \tag{10}$$

namely a traversable wormhole with zero energy density and ZTF. The corresponding Stress-Energy Tensor (SET) is given by

$$T_{\mu\nu} = \left(\rho\left(r\right), p_r\left(r\right), p_t\left(r\right), p_t\left(r\right)\right) = \frac{r_0}{2\kappa r^3}\left(0,0,1,1\right), \tag{11}$$

that is it is formed by pure transverse pressure. To establish how much negative energy density is necessary, it is useful the computation of the “volume integral quantifier”, which provides information about the “total amount” of averaged null energy condition (ANEC) violating matter in the space-time[3]. This is defined by

$$I_V = \int [\rho(r) + p_r(r)] dV \tag{12}$$

and for the line element (1), one can write

$$I_V = \frac{1}{\kappa}\int_{r_0}^{+\infty} (r - b(r)) \left[\ln\left(\frac{e^{2\phi(r)}}{1-\frac{b(r)}{r}}\right)\right]' dr. \tag{13}$$

For instance, for the metric (10), one finds

$$I_V = -\frac{1}{\kappa}\int_{r_0}^{+\infty} (r - r_0) \left[\ln\left(1-\frac{r_0}{r}\right)\right]' dr = -\frac{r_0}{\kappa}\left[\ln(r)\right]_{r_0}^{+\infty} \to -\infty \tag{14}$$

which means that an infinite amount of negative energy is necessary to build such a wormhole. This was also confirmed in Ref.[1]. For this reason, we are going to explore the possibilities offered by the second EoS. The third EoS requires a careful analysis and it will not presented here. In particular, we would like to consider Yukawa-type profiles for $b(r)$ and $\omega(r)$. The hope is that such a profile concentrates more the energy density to the wormhole throat and thus minimizes the usage of exotic matter.

2. The Inhomogeneous Equation of State and the Volume Integral Quantifier

When we apply the inhomogeneous EoS $p_r = \omega(r)\rho$ to find the corresponding shape function, one finds

$$b(r) = r_0 \exp\left[-\int_{r_0}^{r} \frac{d\bar{r}}{\omega(\bar{r})\bar{r}}\right]. \tag{15}$$

The shape function (15) is obtained by imposing $\phi'(r) = 0$. Since we know the form of the redshift function and of the shape function, the SET can also be easily computed

$$T_{\mu\nu} = \frac{r_0}{\kappa r^3}\left(-\frac{1}{\omega(r)}, -1, \frac{1}{2\omega(r)}+\frac{1}{2}, \frac{1}{2\omega(r)}+\frac{1}{2}\right)\exp\left[-\int_{r_0}^{r} \frac{d\bar{r}}{\omega(\bar{r})\bar{r}}\right]. \tag{16}$$

Note that the SET (16) is traceless. We will examine two specific choices for $\omega(r)$.

2.1. *Two examples for $\omega(r)$*

We are going to focus our attention on two examples. The first one is represented by

$$\omega(r) = \frac{1}{\mu r} \qquad \Longrightarrow \qquad \omega(r_0) = \frac{1}{\mu r_0}, \tag{17}$$

leading to

$$b(r) = r_0 \exp\left[-\mu(r - r_0)\right] \qquad \textit{and} \qquad b'(r) = -\mu r_0 \exp\left[-\mu(r - r_0)\right] \tag{18}$$

satisfying therefore the flare-out condition since $b'(r_0) = -\mu r_0$. The form of the metric (1) therefore becomes

$$ds^2 = -dt^2 + \frac{dr^2}{1-\frac{r_0\exp[-\mu(r-r_0)]}{r}} + r^2 d\Omega^2, \tag{19}$$

which looks like a Yukawa profile, at least on the radial part. From the line element (19), the SET is easily computed and assumes the following form

$$T_{\mu\nu} = \frac{r_0}{2\kappa r^3}\left[diag\left(-2\mu r, -2, \mu r+1, \mu r+1\right)\right]\exp\left[-\mu\left(r-r_0\right)\right]\,. \tag{20}$$

Note that the SET is also traceless as it should be, because it is a particular case of the SET (16). On the throat the SET becomes

$$T_{\mu\nu} = \frac{1}{2\kappa r_0^2}\left[diag\left(-2\mu r_0, -2, \mu r_0+1, \mu r_0+1\right)\right] \tag{21}$$

and the following property is satisfied

$$\lim_{\mu\to 0}\lim_{r\to r_0} T_{\mu\nu} = \lim_{r\to r_0}\lim_{\mu\to 0} T_{\mu\nu}. \tag{22}$$

On the other hand, we find

$$\lim_{\mu\to \infty}\lim_{r\to r_0} T_{\mu\nu} \neq \lim_{r\to r_0}\lim_{\mu\to \infty} T_{\mu\nu}. \tag{23}$$

In particular, $\lim\limits_{\mu\to\infty}\lim\limits_{r-r_0} T_{\mu\nu}$ is not defined, while (27)

$$\lim_{r\to r_0}\lim_{\mu\to \infty} T_{\mu\nu} = \left[diag\left(0,0,0,0\right)\right] \tag{24}$$

corresponding to the Minkowski space written in spherical coordinates. This can be confirmed also by looking at the line element (19). The corresponding I_V for the line element (1) becomes

$$\begin{aligned} I_V &= \frac{1}{\kappa}\int_{r_0}^{+\infty}\left(r - r_0 e^{-\mu(r-r_0)}\right)\left[\ln\left(1-\frac{r_0}{r}e^{-\mu(r-r_0)}\right)\right]' dr \\ &= \frac{r_0}{\kappa}\left(1+e^{\mu r_0}\,\mathrm{Ei}_1\left(\mu r_0\right)\right), \end{aligned} \tag{25}$$

where $\mathrm{Ei}_1(x)$ is the exponential integral. Since the integrand is finite, we can easily evaluate the behavior close to the throat, whose result is

$$I_V \simeq \frac{r_0}{\kappa}\left(1-\exp\left(\mu r_0\right)\mathrm{Ei}_1\left(\mu\left(r_0+\varepsilon\right)\right) + \exp\left(\mu r_0\right)\mathrm{Ei}_1\left(\mu r_0\right) - \exp\left(-\mu\varepsilon\right)\right) \to 0 \quad \text{when } \varepsilon\to 0, \tag{26}$$

namely I_V can be arbitrarily small. The second example we are going to discuss is the following

$$\omega\left(r\right) = \frac{\exp\left[-\mu\left(r-r_0\right)\right]}{\mu r}, \tag{27}$$

which has the following properties

$$\omega\left(r\right)\underset{r\to\infty}{\to} 0 \qquad \textit{and} \qquad \omega\left(r_0\right) = \frac{1}{\mu r_0}. \tag{28}$$

Plugging (27) into the shape function (15), one finds

$$\begin{aligned} b(r) &= r_0\exp\left[1-\exp\left(\mu\left(r-r_0\right)\right)\right] \\ b'(r) &= -\mu r_0\,\exp\left[\mu\,\left(r-r_0\right)+1-\exp\left(\mu\,\left(r-r_0\right)\right)\right], \end{aligned} \tag{29}$$

satisfying the flare-out condition since

$$b'(r_0) = -\mu r_0 \,. \tag{30}$$

Even in this case, the SET is easily computed and assumes the explicit form

$$T_{\mu\nu} = \frac{r_0}{2\kappa r^3}\left[diag\left(-\frac{2}{\omega(r)}, -2, \frac{1}{\omega(r)}+1, \frac{1}{\omega(r)}+1\right)\right]\exp\left[1-\exp\left(\mu\left(r-r_0\right)\right)\right], \tag{31}$$

where $\omega(r)$ is given by Eq.(27). On the throat, one gets

$$T_{\mu\nu} = \frac{1}{2\kappa r_0^2}\left[diag\left(-2\mu r_0, -2, \mu r_0+1, \mu r_0+1\right)\right] \tag{32}$$

and also in this case we obtain the same behavior of the SET (20). It is interesting to note that, on the throat, the behavior of the SET in (20) and in (31) are equal. The corresponding I_V for the line element (1) becomes

$$I_V = -\frac{1}{\kappa}\int_{r_0}^{+\infty}\left(r - r_0\exp\left[1-\exp\left(\mu\left(r-r_0\right)\right)\right]\right) \times \left[\ln\left(1-\frac{r_0\exp\left[1-\exp\left(\mu\left(r-r_0\right)\right)\right]}{r}\right)\right]' dr = -\frac{1}{\kappa}\int_{r_0}^{+\infty} f(r)\, dr, \tag{33}$$

where

$$f(r) = \frac{r_0\exp\left[1-\exp\left(\mu\left(r-r_0\right)\right)\right]\left(\exp\left(\mu\left(r-r_0\right)\right)\mu r+1\right)}{r}. \tag{34}$$

The exact evaluation of I_V is quite complicated but, an estimate when $r \gg r_0$ is possible to see if the integral is finite for large r. Indeed, one finds that the integrand becomes

$$f(r) \underset{r\to\infty}{\simeq} \frac{\mu r_0\exp\left(\mu\left(r-r_0\right)+1\right)}{\exp\left(\exp\left(\mu\left(r-r_0\right)\right)\right)}$$
$$\Longrightarrow I_V \simeq -r_0\exp\left(1-\exp\left(\mu\left(r-r_0\right)\right)\right) \to 0 \qquad \text{when } r\to\infty \tag{35}$$

and therefore I_V will be finite close to infinity. On the other hand, close to the throat, we can write

$$I_V \simeq \frac{1}{\kappa}\int_{r_0}^{r_0+\varepsilon}\left(\mu r_0+1\right)dr \to 0 \qquad \text{when } \varepsilon\to 0. \tag{36}$$

Note that we might have chosen the following form

$$\omega(r) = \frac{\exp(-\mu r)}{\mu r}, \tag{37}$$

instead of (27). However, while the properties at infinity are equal to the profile (27), close to the throat one finds

$$\omega(r_0) = \frac{\exp(-\mu r_0)}{\mu r_0}. \tag{38}$$

As a consequence, the shape function becomes

$$b\,(r) = r_0 \exp\left(\exp\left(\mu r_0\right) - \exp\left(\mu r\right)\right) \tag{39}$$

while the energy density is

$$\rho(r) = -\frac{\mu r_0}{\kappa r^2} \exp\left(\mu r + \exp\left(\mu r_0\right) - \exp\left(\mu r\right)\right). \tag{40}$$

If we compare the energy density obtained with (27) with the one computed with (37), we can see that, on the throat, the amount of negative energy density is larger for the choice (37) because

$$\rho(r) = -\frac{\mu}{\kappa r_0} \exp\left(\mu r_0\right). \tag{41}$$

This implies that the choice (27) is favored from an energetically point of view. The same conclusion can be reached comparing the energy density obtained with (37) with the one obtained with (17). On the other hand, when we compare the energy density obtained with (17) and the one obtained with (27), we find

$$\frac{\rho_{\omega_2}(r)}{\rho_{\omega_1}(r)} = \frac{\exp\left[\mu\ (r - r_0) + 1 - \exp\left(\mu\ (r - r_0)\right)\right]}{\exp\left[-\mu\,(r - r_0)\right]} \underset{r\to\infty}{\longrightarrow} 0, \tag{42}$$

where $\rho_{\omega_1}(r)$ is the energy density obtained with the help of (17) and represented by the first component of the SET (20), while $\rho_{\omega_2}(r)$ is the energy density obtained with the help of (27) and represented by the first component of the SET (31). The behavior shown in (42) reveals that the negativity of the energy density is really concentrated in the proximity of the throat. A further investigation inspired by Yukawa profiles could be related to the Self Sustained Traversable Wormholes, i.e., traversable wormholes which are sustained by their own quantum fluctuations[4–7].

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