A structure preserving discretization of a unified HTC multiphase model of continuum mechanics

L D. Ferrari*, I. Peshkov, E. Romenski and M. Dumbser
[1]Laboratory of Applied Mathematics, DICAM, University of Trento, Italy
*Corresponding author: davide.ferrari@unitn.it

Abstract
I will present a new structure preserving scheme to mimic at the discrete level some mathematical properties of the multiphase diffuse-interface model that we have recently formulated in the framework of Thermodynamically Compatible Hyperbolic equations, see [1]. The thermodynamic compatibility at the discrete level is achieved through the numerical flux correction recently introduced by Abgrall et al. The consistency with the second law of thermodynamics is satisfied through an appropriate thermodynamically compatible parabolic vanishing viscosity regularization, as well as through an opportune discretization of dissipative processes defined on a continuous level. Moreover, space will be devoted to the derivation of an appropriate discretization that preserves certain nonlinear algebraic constraints of the system of equations, in particular those relating to the physical bounds on density and volume fraction. To this end, a more classical but equivalent form of the PDEs system will be considered, which favours the time evolution of non-dimensional quantities, such as volume fractions. A simple positivity condition can be derived for the numerical solution of these specific non-conservative volume fraction evolution equations.
Then, at the discrete level, an appropriate discretization is achieved, which allows discrete equivalence between the two forms of the PDE system. The validation process includes a wide range of benchmarks and several applications to compressible multiphase problems.

References
[1] D. Ferrari and I. Peshkov and E. Romenski and M. Dumbser, A unified SHTC multiphase model of continuum mechanics, arXiv preprint, arXiv:2403.19298, 2024.