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An explicit expression of the steady-state error in Smith Predictor applied to linear systems with integral action

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Abstract: Time delays in control systems are often present due to inherent delays of the instrumentation and signal processing computations. Among the proposed solutions for time-delay compensation in linear control systems, an important role is played by the Smith Predictor scheme, as it allows the design of the controller for reference tracking without considering time delay. However, the Smith Predictor presents poor disturbance rejection capabilities, in particular when the system to be controlled has an integral action, e.g. as typical of motion control systems. This paper illustrates the ineffectiveness of the Smith Predictor when applied on an linear time-delay system with integral action and provides an analytical expression of the steady-state error when a step disturbance acts on the control signal.

1. INTRODUCTION

Many industrial control systems present time delays due to different reasons, i.e. data acquisition/analysis processes or sensors and actuators inherent operating times (Eller et al., 1969). It is well known that time delays in control systems cause controllability and potential instability problems (Zhong, 2006). In order to improve the control performance in time-delay systems, dead time compensators have been studied, starting from the introduction of the Smith Predictor (SP) scheme (Smith, 1957). The SP is a well-known dead time compensator for asymptotically stable linear systems: assuming a known time delay and system model, the SP can predict the output of the system, allowing the design of the controller without considering the time delay, see Figure 1 for a classical control block diagram employing the SP.

Nevertheless, the SP presents poor disturbance rejection capabilities (Torrico et al., 2013; Torrico and Normey-Rico, 2005). Consequently, several modified versions of the SP have been studied to improve its disturbance rejection performance (Zhang et al., 2004). Among others, in (Wu, 2010) a modified SP is developed for disturbance rejection of a single sinusoidal signal. However, such SP does not perform satisfactorily when employed with systems having an integral action (Sreevalli et al., 2023).

In the context of motion control, in addition to possible time delays, it is also common to have one stage of integration from the system excitation signal to the sensed feedback one (Ellis, 2004). As an archetypal example, the transfer function of a SISO LTI two-mass drive system with elastic coupling presents an integral action (Sonzogni et al., 2023), and even when it is necessary to control the motion system in position, the open-loop transfer function presents an integrator.

Although the poor disturbance rejection capabilities of the SP scheme are known in literature (Gu, 2008), and

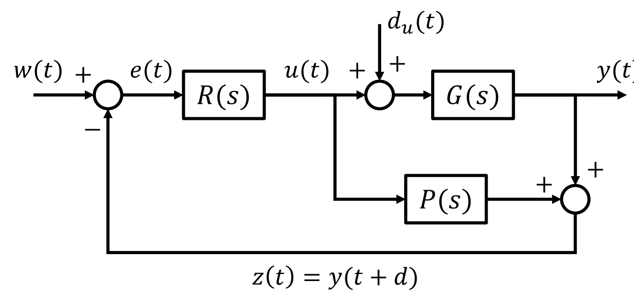

Fig. 1. Block scheme of a SISO LTI system under linear feedback control with the insertion of the Smith Predictor, where $G(s)$ is the transfer function of the system under control, $R(s)$ is the transfer function of the controller and $P(s)$ is the transfer function of the Smith Predictor, $w(t)$ is the reference signal, $e(t)$ is the error signal, $u(t)$ is the control action signal, $d_u(t)$ is the disturbance signal on the control action, $y(t)$ is the output signal, $z(t)$ is the feedback control signal and d is the delay time value.

modified SP schemes exist to deal with integrative systems (Zhang and Sun, 1996), the reason why the insertion of a traditional SP in a feedback control loop with time-delay systems with an integral action causes problems of tracking and disturbance rejection has not been fully studied yet. Specifically, the main contribution of this article is to demonstrate the cancellation of the integral action of the controller $R(s)$ due to the presence of SP in the control loop when $G(s)$ has an integral action. This results in a constant error $e(\infty)$ at steady-state when a step disturbance $d_u(t)$ acts on the control action. Moreover, an explicit analytical expression of the steady-state error $e(\infty)$ is provided in such case.

The structure of the paper is as follows. Section 2 presents the problem statement. Section 3 describes the steady-

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state behaviour of the closed-loop system composed by the open-loop system with time delay, integral action and the SP. Simulation results validating the theoretical ones are given in Section 4 and in Section 5 some concluding remarks are outlined.

2. PROBLEM STATEMENT

2.1 Review of Smith Predictor theory for linear systems

The applicability of the SP assumes an asymptotically stable SISO LTI system under control $G(s)$ as follows

$$G(s) := G'(s)e^{-sd}, \tag{1}$$

where $G'(s)$ is defined as the transfer function of the system without the time delay and d is the value of the time delay. The SP is a type of predictive control implemented as in Figure 1, where the SP transfer function is defined as follows

$$P(s) := (1 - e^{-sd})G'(s). \tag{2}$$

As a result, the $z(t)$ variable, in Figure 1, is the prediction of the system output $y(t)$ at time $t + d$, as shown in (3)

$$\begin{aligned} Z(s) &= [G(s) + P(s)]U(s) = G'(s)U(s) \\ &= e^{sd}G(s)U(s) = e^{sd}Y(s). \end{aligned} \tag{3}$$

The open loop transfer function results in

$$L(s) := (G(s) + P(s))\, R(s) = G'(s)R(s). \tag{4}$$

So that, the controller $R(s)$ can be designed neglecting the delay issues.

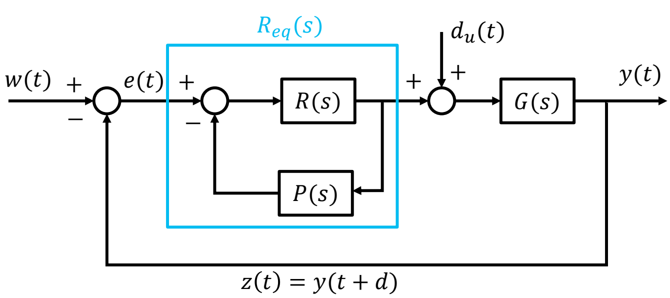

Fig. 2. Smith predictor model equivalent block diagram.

Figure 2 shows an equivalent control diagram of the SP, where the equivalent controller $R_{eq}(s)$ is highlighted and reads as

$$R_{eq}(s) := \frac{R(s)}{1 + R(s)P(s)}. \tag{5}$$

Defining $F'(s)$ as the closed-loop transfer function without the delay d and the predictor $P(s)$

$$F'(s) := \frac{R(s)G'(s)}{1 + R(s)G'(s)}, \tag{6}$$

it is possible to verify that the transfer function of the closed-loop system between the reference $w(t)$ and the output $y(t)$ is

$$F(s) := \frac{R(s)G(s)}{1 + R(s)G(s) + R(s)P(s)} = F'(s)e^{-sd}. \tag{7}$$

Notice that the time delay is still present on the output signal but it does not interfere with the control dynamics.

If the system $G(s)$ is asymptotically stable, the static gain of the Smith Predictor transfer function $P(s)$ is null, i.e.

$$P(0) = 0. \tag{8}$$

On the assumption of asymptotic stability of the closed-loop control system $F'(s)$, the equivalent controller $R_{eq}(s)$ is asymptotically equivalent to the controller $R(s)$. Therefore, the steady-state performance of the closed-loop system $F(s)$ (i.e. the steady-state error in response to references or disturbances on the control action) depends exclusively on the structure of the controller $R(s)$. It is relevant to point out that the SP model is highly sensitive to modelling errors. In particular the equation (3), (4), (7) and the SP model theory are only valid if the model of the system under control is exactly $G(s)$.

2.2 Applicative context

Mechatronic systems are often time-delay systems with integral action, due to the system structure, controllability decisions and the available sensors used for their motion control (Sonzogni et al., 2023). The following assumption defines an archetypal system and controller transfer functions for mechatronic motion control systems. However, the outlined results are generally applicable to systems with structure defined in the following.

Assumption 1: $G(s)$ is a SISO LTI system with integral action and time delay d, as in (9), that presents m zeros with time constants T_{i_1} with $i_1 = \{1, \dots, m\}$, n poles with time constants τ_{i_2} with $i_2 = \{1, \dots, n\}$ and system gain μ_1. The time constants T_{i_1} and τ_{i_2} may be real or conjugated complexes.

$$G(s) = \frac{\mu_1}{s} \frac{\prod_{i_1}^{m}(1 + sT_{i_1})}{\prod_{i_2}^{n}(1 + s\tau_{i_2})} e^{-sd} = G'(s)e^{-sd} \tag{9}$$

The transfer function of the controller $R(s)$ is referred to as

$$R(s) = \frac{\mu_2}{s^g} \frac{\prod_{j_1}^{l}(1 + sR_{j_1})}{\prod_{j_2}^{h}(1 + sr_{j_2})}, \tag{10}$$

where μ_2 is the controller's gain, g is the number of poles in zero, R_{j_1} are the time constants of the l zeros with $j_1 = \{1, \dots, l\}$, and r_{j_2} are the time constants of the h poles with $j_2 = \{1, \dots, h\}$. The time constants R_{j_1} and r_{j_2} may be real or conjugated complexes.

The system $G(s)$ may be stable, if the time constants have positive real part $\text{Re}(\tau_i) > 0$, but it is not asymptotically stable, due to the pole in zero provided by the integral action. So, the theoretical analyses presented in Section 2.1 are not fully applicable. In this paper, a SP is applied on $G(s)$ in (9) as shown in Figure 1, with transfer function $P(s)$ as in (2). The SP equivalent scheme shown in Figure 2 is still considered with an equivalent controller $R_{eq}(s)$ as in (5). The transfer functions of the closed-loop system $F'(s)$ and $F(s)$ are computed as in (6) and (7), respectively.

3. STATIC PERFORMANCE OF SMITH PREDICTOR SCHEMES ON SYSTEMS WITH INTEGRAL ACTION

The aim of this paper is to analyse the effect of the Smith Predictor (SP) when applied to the control of time-delay systems with integral action (9)-(10).

Assumption 2: The time constants of the poles of $G(s)$ in (9) have positive real part $\text{Re}(\tau_i) > 0$. The controller $R(s)$ in (10) asymptotically stabilises the closed-loop system without time delay $F'(s)$ as in (6).

Considering that the output $y(t)$ is affected by the reference $w(t)$ and the disturbances $d_u(t)$, in the following $y_w(t)$ is defined as the component of the output $y(t)$ affected by the reference $w(t)$ and $y_d(t)$ is defined as the component of the output $y(t)$ affected by the disturbances $d_u(t)$, as follow

$$y(t) = y_d(t) + y_w(t). \tag{11}$$

In (Gu, 2008) it has been proved that when the system $G(s)$ in (9) presents an integral action, the static gain of the SP transfer function $P(s)$ is not null and so it is no longer guaranteed that the steady-state performance of $R_{eq}(s)$ in (5) is equal to the steady-state performance of $R(s)$ in (10).

Nevertheless, it is possible to test and verify that, without disturbances, the closed-loop system $F(s)$ is able to correctly track a step reference due to the presence of an integrator in $G(s)$.

Proposition 1. When a step reference $w(t) = A \cdot step(t)$ is applied to the closed-loop system $F(s)$, in (7), the steady-state value of the output response $y_w(t)$ converges to

$$y_w(\infty) = A. \tag{12}$$

Proof. The proof is in the appendix.

However, for other types of reference $w(t)$, the closed-loop system $F(s)$ does not exhibit the same static performance of $F'(s)$, where the SP and the time delay are not present, as expected from standard linear control theory.

When the Smith Predictor is applied to systems with integral action, the negative feedback loop involving $P(s)$ on the controller $R(s)$, as in Figure 2, erases the integral actions of the controller $R(s)$. In conclusion, the equivalent controller $R_{eq}(s)$ has no integral action and the steady-state tracking and disturbance rejection performance of the feedback system $F(s)$ depends also on the static gain of the SP.

Proposition 2. When the Smith Predictor $P(s)$ is applied to a time-delay system with an integrator as in (9), the equivalent controller $R_{eq}(s)$ in (5) has no integral actions, even if the controller $R(s)$ has one or more poles in zero.

Proof. The proof is in the appendix.

It is important to clarify that the closed-loop system is able to track a step signal reference just because the system $G(s)$ presents an integrator and not because of the integral action of the controller $R(s)$. So, for instance, even if the open-loop transfer function $L(s)$ in (4) has two integrators, the closed-loop system is not able to obtain a zero tracking error when the reference is a ramp signal $w(t) = A \cdot ramp(t)$.

Regarding the disturbance rejection capabilities, the SP applied on a time-delay system with integral actions does not present good performance as shown in the following proposition.

Proposition 3. When a step disturbance $d_u(t) = A \cdot step(t)$ acts on the control action of a system $G(s)$ in (9) with integral action as shown in Figure 1, and the controller $R(s)$ in (10) is designed without poles in zero (i.e. $g = 0$), the steady-state output response $y_d(t)$ converges to

$$y_d(\infty) = A \cdot \frac{1 + \mu_1\mu_2 \cdot d}{\mu_2}. \tag{13}$$

Proof. The proof is in the appendix.

Proposition 4. When a step disturbance $d_u(t) = A \cdot step(t)$ acts on the control action of a system $G(s)$ in (9) with integral action as shown in Figure 1, and the controller $R(s)$ in (10) is designed with poles in zero (i.e. $g \geq 1$), the steady-state output response $y_d(t)$ converges to

$$y_d(\infty) = A \cdot \frac{\mu_2\mu_1^2 \cdot d}{\mu_2\mu_1} = \mu_1 dA. \tag{14}$$

Proof. The proof is in the appendix.

Propositions 3-4 show that, when the system $G(s)$ presents an integral action, and the closed-loop system $F'(s)$ is asymptotically stable thanks to the controller $R(s)$, the steady-state output in response to step disturbances $d_u(t)$ acting on the control action converges to a non-zero constant value, that is proportional to the delay d and the system gain μ_1, and does not depend on the values of the controller parameters.

It is important to note that, considering the control diagram of Figure 2 and under Assumption 1 and Assumption 2, the transfer function from the disturbances on the control action $d_u(t)$ to the output $y_d(t)$

$$S_e(s) := \frac{Y(s)}{D_u(s)} := \frac{G(s)}{1 + G(s)R_{eq}(s)}, \tag{15}$$

and the transfer function from the disturbances $d_u(t)$ to the tracking error $e(t)$, defined as

$$\frac{E(s)}{D_u(s)} := \frac{-G(s)}{1 + G(s)R_{eq}(s)} = -S_e(s) \tag{16}$$

are equal, neglecting the sign. So, the absolute value of the steady-state error $e(\infty)$ in response to step disturbances $d_u(t)$ is equal to the absolute value of the steady-state output response $y_d(t)$, that is

$$|e(\infty)| = |y_d(\infty)|. \tag{17}$$

4. SIMULATIONS AND RESULTS

In order to test the validity of Propositions 1-4, different tests have been performed in simulation. For all tests the system $G(s)$ as in (18) is considered. The system $G(s)$ presents positive gain $\mu_1 = 5$, one zero with time constant $T_1 = 0.015[s]$, one pole in zero and two conjugated complex poles with resonant frequency $\omega_p = 73[\frac{rad}{s}]$ and damping coefficient $\xi_p = 0.1$, and time delay $d = 0.5[s]$, as follows

$$G(s) = \frac{\mu_1}{s} \cdot \frac{1 + T_1 s}{1 + \dfrac{2\xi_p s}{\omega_p} + \dfrac{s^2}{\omega_p^2}} e^{-sd}. \tag{18}$$

For the first test, the controller $R(s)$, as in (19), is chosen. The controller $R(s)$ has no poles in zero ($g = 0$), gain $\mu_2 = 0.5$ and a stable real pole with time constant $r_1 = 0.05[s]$. The transfer function of the controller $R(s)$ is then

$$R(s) = \frac{\mu_2}{r_1 s + 1}. \tag{19}$$

All poles of the closed-loop system $F'(s)$ without the time delay in (6) have negative real part and so $F'(s)$ is asymptotically stable.

Figure 3 shows the output $y(t)$, the reference $w(t)$, the error $e(t)$ and the disturbance $d_u(t)$ of the closed-loop system $F(s)$ when a step reference with unit amplitude, as in (20a), is applied at time $1[s]$ and a step disturbance with unit amplitude, as in (20b), is applied on the control action at time $4[s]$, that is

$$w(t) = step(t), \tag{20a}$$
$$d_u(t) = step(t-4). \tag{20b}$$

As it can be seen, according to Proposition 1 the closed-loop system tracks correctly the step reference but it is not able to reject the step disturbance, as argued by Proposition 2. Moreover, the steady-state error $e(\infty)$, in response to step disturbances $d_u(t)$, converges to a constant value computed as in (13), (17) and as claimed by the Proposition 3, that is

$$|e(\infty)| = y_d(\infty) = 1 \cdot \frac{1 + 5 \cdot 0.5 \cdot 0.5}{0.5} = 4.5. \tag{21}$$

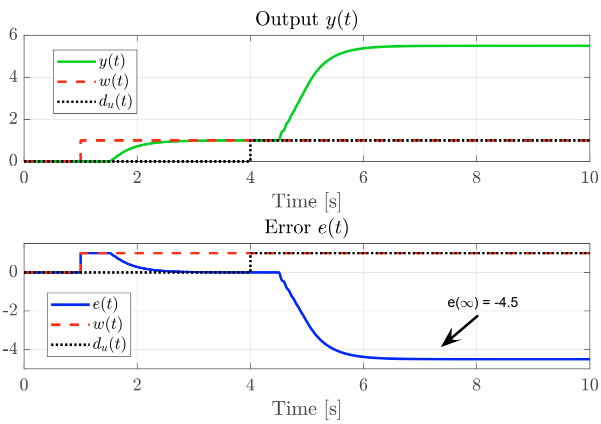

Fig. 3. Tracking performance of $F(s)$ using a Smith Predictor and a controller without integral action.

Then, it is chosen to test the system $G(s)$ in (18) with a controller that presents a single pole in zero, so that $g = 1$. The controller $R(s)$, shown in (22), is a classical Proportional-Integral (PI) controller, with proportional gain and integral gain equal to $K_p = 0.5$ and $K_i = 3$, respectively. The controller's gain results in $\mu_2 = K_i = 3$ and $R(s)$ presents one stable real zero with time constant $R_1 = \frac{0.5}{3}[s]$. The controller $R(s)$ has been designed in order to obtain a closed-loop system that is asymptotically stable, by setting

$$R(s) = K_p + \frac{K_i}{s} = \mu_2 \frac{1 + sR_1}{s}. \tag{22}$$

Figure 4 shows the results of the closed-loop system $F(s)$ when a step reference signal with unit amplitude, as in (20a), is applied at time $1[s]$ and a step disturbance signal with unit amplitude, as in (20b), is applied on the control action at time $4[s]$.

The closed-loop system $F(s)$ can track the step reference, and it is important to notice that the time delay is still present in the output signal. However, the feedback system

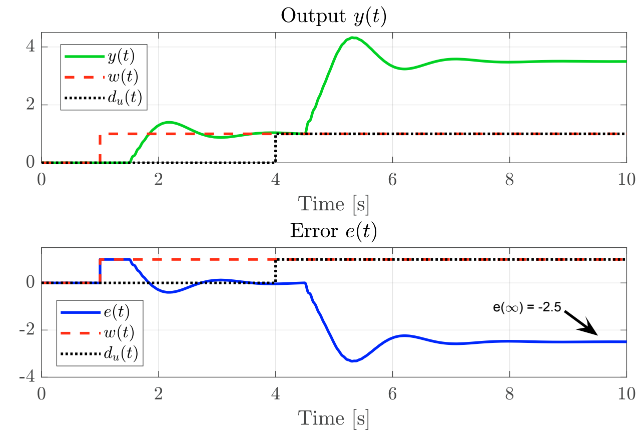

Fig. 4. Tracking performance of $F(s)$ using a Smith Predictor and a controller with integral action.

$F(s)$ can not reject the disturbance on the control action and the absolute value of the steady-state error $e(\infty)$, in response to step disturbances $d_u(t)$, converges to a constant value (23) computed as in (14), (17) and claimed by Proposition 4:

$$|e(\infty)| = y_d(\infty) = \mu_1 dA = 5 \cdot 0.5 \cdot 1 = 2.50. \tag{23}$$

Furthermore, even if the controller $R(s)$ has an integrator as in (22), the closed-loop system is not able to track a ramp reference at steady state (as one would expect when the SP is not necessary, e.g. for a system without time delay, as two integrators are present in the control loop), as shown in Figure 5 and as indicated in Proposition 2.

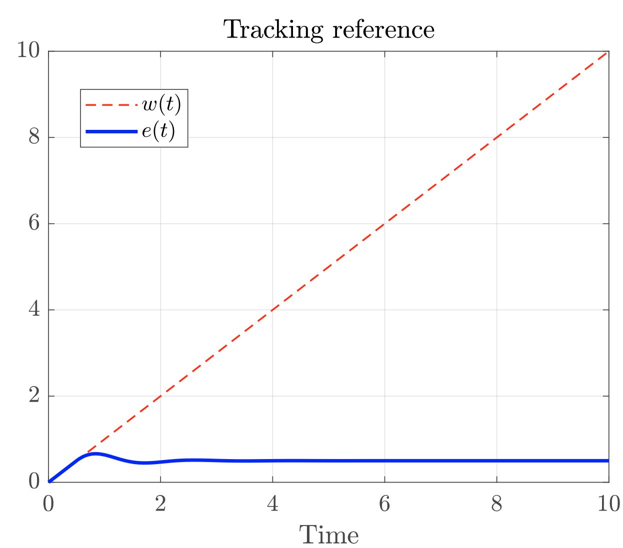

Fig. 5. Inability to track the reference ramp signal by the closed-loop system using a Smith Predictor and a controller with integral action.

5. CONCLUSION

This article presented how the Smith Predictor control scheme behaves when it is applied to time-delay systems with an integral action. As shown in Section 3, the closed-loop system is unable to correctly track all types of reference signals, for example $w(t) = ramp(t)$. In addition, it is also ineffective in rejecting disturbances on the control action. Moreover, it has been proved that the reason is due to the cancellation of the poles in zero of the controller $R(s)$ by the negative feedback loop on the controller made by the Smith Predictor $P(s)$. Additionally, it has been demonstrated that the steady-state error in response to a step disturbance is proportional to the delay d of the system $G(s)$ and it is described by a particular equation presented in (13) and (14). Modified SP schemes able to deal with systems with an integral action are already present in the literature. The aim of the paper was not to devise one such scheme, but rather to analytically study the consequences of applying standard SP schemes to integrative systems, thus providing insights on tracking and disturbance rejection errors. The goal of future works is to study new modifications of the the structure of the Smith Predictor in order to be applicable also on integrative systems.

Appendix A. PROOFS

A.1 Proof of Proposition 1

Consider the transfer function $G(s)$ of the system under control with an integrator as in (9) and the transfer function of a generic controller $R(s)$ as in (10). The transfer function $F(s)$ from the reference $w(t)$ to the output $y(t)$ is shown in (7). The steady-state output $y_w(\infty)$ of the closed-loop system, when a step reference with amplitude A is applied, i.e. $w(t) = A \cdot step(t)$, is obtained by solving the following limit:

$$\begin{aligned} y_w(\infty) &= \lim_{s\to 0} s \cdot \frac{A}{s} \cdot F(s), \\ &= A \cdot \lim_{s\to 0} \frac{R(s)G'(s)}{1+R(s)G'(s)} e^{-sd}, \\ &= A \cdot \lim_{s\to 0} \frac{\frac{\mu_2\mu_1}{s^{g+1}}}{1+\frac{\mu_2\mu_1}{s^{g+1}}}, \\ &= A \cdot \lim_{s\to 0} \frac{\mu_2\mu_1}{s^{g+1}+\mu_2\mu_1} = A. \end{aligned} \tag{A.1}$$

So, at steady state the closed-loop system $F'(s)$ can follow the value of the step reference signal.

A.2 Proof of Proposition 2

Consider the transfer function $G(s)$ of the system under control with an integrator as in (9), the transfer function of a generic controller $R(s)$ as in (10) and the transfer function of the equivalent controller $R_{eq}(s)$ as in (5) of the Smith Predictor shown in Figure 1. It is possible to prove that at steady state the equivalent controller $R_{eq}(s)$ does not present the integral action even if $R(s)$ has one or more integrators $g \geq 1$. Consider the limit:

$$\begin{aligned} \lim_{s\to 0} R_{eq}(s) &= \lim_{s\to 0} \frac{R(s)}{1+R(s)P(s)}, \\ \lim_{s\to 0} R_{eq}(s) &= \lim_{s\to 0} \frac{\frac{\mu_2}{s^g}}{1+\frac{\mu_2}{s^g}\cdot(1-e^{-sd})\frac{\mu_1}{s}} \end{aligned} \tag{A.2}$$

$$\begin{aligned} \lim_{s\to 0} R_{eq}(s) &= \lim_{s\to 0} \frac{\frac{\mu_2}{s^g}}{1+\frac{\mu_2\mu_1}{s^{g+1}}-\frac{\mu_2\mu_1}{s^{g+1}}e^{-sd}}, \\ &= \lim_{s\to 0} \frac{\mu_2 s}{s^{g+1}+\mu_2\mu_1-\mu_2\mu_1 e^{-sd}}. \end{aligned} \tag{A.3}$$

The limit presents an undetermined form $[\frac{0}{0}]$; it can be solved by applying the de l'Hôpital theorem as follows:

$$\begin{aligned} \lim_{s\to 0} R_{eq}(s) &= \lim_{s\to 0} \frac{\mu_2}{(g+1)s^g+\mu_2\mu_1 d e^{-sd}}, \\ &= \frac{\mu_2}{\mu_2\mu_1 d}, \\ &= \frac{1}{\mu_1 d}. \end{aligned} \tag{A.4}$$

So, since at steady state the equivalent controller $R_{eq}(s)$ assumes a finite real value, it does not present an integral action.

A.3 Proof of Proposition 3

Consider the transfer function $G(s)$ of the system under control with an integrator as in (9) and the transfer function of a generic controller $R(s)$ without poles at zero ($g = 0$) so that

$$R(s) = \mu_2 \frac{\prod_{j_1}^{l}(1+sR_{j_1})}{\prod_{j_2}^{h}(1+sr_{j_2})}. \tag{A.5}$$

The steady-state output $y_d(\infty)$ of the closed-loop system $F(s)$ in (7), when a step disturbance signal $d_u(t)$ with amplitude A is applied on the control action, i.e. $d_u(t) = A \cdot step(t)$, is obtained by solving the following limit:

$$\begin{aligned} y_d(\infty) &= \lim_{s\to 0} s \cdot \frac{A}{s} \cdot \frac{Y(s)}{D_u(s)} \\ &= \lim_{s\to 0} s \cdot \frac{A}{s} \cdot \frac{G(s)}{1+\frac{R(s)}{1+R(s)P(s)}G(s)}, \\ &= A \lim_{s\to 0} \frac{\frac{\mu_1}{s}}{1+\frac{\mu_2}{1+\mu_2\frac{\mu_1}{s}(1-e^{-sd})}\frac{\mu_1}{s}}, \\ &= A \lim_{s\to 0} \frac{\frac{\mu_1}{s}}{1+\frac{\frac{\mu_2\mu_1}{s}}{1+\frac{\mu_2\mu_1}{s}(1-e^{-sd})}}, \\ &= A \lim_{s\to 0} \frac{\frac{\mu_1}{s}+\frac{\mu_1^2\mu_2}{s^2}(1-e^{-sd})}{1+\frac{\mu_1\mu_2}{s}(1-e^{-sd})+\frac{\mu_1\mu_2}{s}}, \\ &= A \lim_{s\to 0} \frac{\frac{1}{s^2}\left(\mu_1 s+\mu_1^2\mu_2(1-e^{-sd})\right)}{\frac{1}{s}(s+\mu_1\mu_2-\mu_1\mu_2 e^{-sd}+\mu_1\mu_2)}, \\ &= A \lim_{s\to 0} \frac{\mu_1 s+\mu_1^2\mu_2(1-e^{-sd})}{s^2+2\mu_1\mu_2 s-\mu_1\mu_2 s e^{-sd}}, \\ &= A \lim_{s\to 0} \frac{\mu_1 s+\mu_1^2\mu_2-\mu_1^2\mu_2 e^{-sd}}{s^2+2\mu_1\mu_2 s-\mu_1\mu_2 s e^{-sd}}. \end{aligned} \tag{A.6}$$

The limit presents an undetermined form $[\frac{0}{0}]$; it can be solved by applying the de l'Hôpital theorem as follows:

$$\begin{aligned} y_d(\infty) &= A \lim_{s\to 0} \frac{\mu_1 + \mu_1^2\mu_2 d e^{-sd}}{2s + 2\mu_1\mu_2 - \mu_1\mu_2 e^{-sd} + \mu_1\mu_2 sd \cdot e^{-sd}}, \\ &= A\frac{\mu_1 + \mu_1^2\mu_2 d}{\mu_1\mu_2}, \\ &= A\frac{1+\mu_1\mu_2 d}{\mu_2}. \end{aligned} \tag{A.7}$$

A.4 Proof of Proposition 4

Consider the transfer function $G(s)$ of the system under control with an integrator as in (9) and the transfer function of a generic controller $R(s)$ with poles at zero ($g \geq 1$) as in (10). The steady-state output $y_d(\infty)$ of the closed-loop system, when a step signal disturbance $d_u(t)$ with amplitude A is applied on the control action, i.e. $d_u(t) = A \cdot step(t)$, is obtained by solving the following limit:

$$\begin{aligned} y_d(\infty) &= \lim_{s\to 0} s \cdot \frac{A}{s} \cdot \frac{G(s)}{1 + \frac{R(s)}{1+R(s)P(s)}G(s)}, \\ &= A \lim_{s\to 0} \frac{\frac{\mu_1}{s}}{1 + \frac{\frac{\mu_2}{s^g}}{1 + \frac{\mu_2}{s^g}\frac{\mu_1}{s}(1-e^{-sd})}\frac{\mu_1}{s}}, \\ &= A \lim_{s\to 0} \frac{\frac{\mu_1}{s}\left(1 + \frac{\mu_2\mu_1}{s^{g+1}}(1-e^{-sd})\right)}{1 + \frac{\mu_2\mu_1}{s^{g+1}}(1-e^{-sd}) + \frac{\mu_2\mu_1}{s^{g+1}}}, \\ &= A \lim_{s\to 0} \frac{\frac{\mu_1}{s} + \frac{\mu_2\mu_1^2}{s^{g+2}}(1-e^{-sd})}{1 + \frac{\mu_2\mu_1}{s^{g+1}} - \frac{\mu_2\mu_1}{s^{g+1}}e^{-sd} + \frac{\mu_2\mu_1}{s^{g+1}}}, \\ &= A \lim_{s\to 0} \frac{\frac{\mu_1}{s} + \frac{\mu_2\mu_1^2}{s^{g+2}} - \frac{\mu_2\mu_1^2}{s^{g+2}}e^{-sd}}{1 + \frac{2\mu_2\mu_1}{s^{g+1}} - \frac{\mu_2\mu_1}{s^{g+1}}e^{-sd}}, \\ &= A \lim_{s\to 0} \frac{\frac{1}{s^{g+2}}(\mu_1 s^{g+1} + \mu_2\mu_1^2 - \mu_2\mu_1^2 e^{-sd})}{\frac{1}{s^{g+1}}(s^{g+1} + 2\mu_2\mu_1 - \mu_2\mu_1 e^{-sd})}, \\ &= A \lim_{s\to 0} \frac{\mu_1 s^{g+1} + \mu_2\mu_1^2 - \mu_2\mu_1^2 e^{-sd}}{s^{g+2} + 2\mu_2\mu_1 s - \mu_2\mu_1 s e^{-sd}}. \end{aligned} \tag{A.8}$$

The limit presents an undetermined form $[\frac{0}{0}]$; it can be solved by applying the de l'Hôpital theorem as follows:

$$y_d(\infty) = \lim_{s\to 0} H(s), \tag{A.9}$$

$$H(s) = \frac{(g+1)\mu_1 s^g + \mu_2\mu_1^2 d e^{-sd}}{(g+2)s^{g+1} + 2\mu_2\mu_1 - \mu_2\mu_1 e^{-sd} + \mu_2\mu_1 s d e^{-sd}} \tag{A.10}$$

$$y_d(\infty) = A\frac{\mu_2\mu_1^2 d}{\mu_2\mu_1} = \mu_1 d A. \tag{A.11}$$

Whether the controller presents one or more poles at zero, the steady-state error is proportional to the delay d, to the amplitude of the step disturbance A and the gain of the system μ_1 and so it is controller-independent.

REFERENCES

Eller, D., Aggarwal, J., and Banks, H. (1969). Optimal control of linear time-delay systems. *IEEE Transactions on Automatic Control*, 14(6), 678–687. doi: 10.1109/TAC.1969.1099301.

Ellis, G. (2004). *Control System Design Guide*. Elsevier. doi:10.1016/B978-0-12-237461-6.X5000-0.

Gu, K. (2008). Control of dead-time processes (j.e. normey-rico and e.f. camacho; 2007) [book review]. *IEEE Control Systems Magazine*, 28(5), 136–137. doi: 10.1109/MCS.2008.927324.

Smith, O. (1957). Closer control of loops with dead time. In *Chemistry Engineering Progress*.

Sonzogni, G., Mazzoleni, M., Polver, M., Ferramosca, A., and Previdi, F. (2023). Notch filter design with stability guarantees for mechanical resonance suppression in SISO LTI two-mass drive systems. In *62nd IEEE Conference on Decision and Control, CDC 2023, Singapore, December 13-15, 2023*, 7745–7750. IEEE.

Sreevalli, V., Rathaiah, M., Kiranmayi, R., Bharath Kumar, P., Nagabhushanam, K., and Firdouse Kowsar, A.K. (2023). Enhanced design of pi-pd controller based on smith predictor for integrating processes with time delay. In *2023 Fifth International Conference on Electrical, Computer and Communication Technologies (ICECCT)*, 01–07. doi: 10.1109/ICECCT56650.2023.10179733.

Torrico, B., Cavalcante, M., Braga, A., Normey-Rico, J., and Albuquerque, A. (2013). Simple tuning rules for dead-time compensation of stable, integrative, and unstable first-order dead-time processes. *Industrial & Engineering Chemistry Research*, 52, 11646–11654. doi: 10.1021/ie401395x.

Torrico, B. and Normey-Rico, J. (2005). 2dof discrete dead time compensator for stable and integrative processes with dead time. *Journal of Process Control - J PROCESS CONTROL*, 15, 341–352. doi: 10.1016/j.jprocont.2004.05.006.

Wu, X. (2010). Modified smith predictor for disturbance rejection of single sinusoidal signal. In *IEEE ICCA 2010*, 1648–1653. doi:10.1109/ICCA.2010.5524136.

Zhang, W.D. and Sun, Y.X. (1996). Modified smith predictor for controlling integrator/time delay processes. *Industrial & Engineering Chemistry Research*, 35(8), 2769–2772. doi:10.1021/ie950664v.

Zhang, W., Gu, D., Wang, W., and Xu, X. (2004). Quantitative performance design of a modified smith predictor for unstable processes with time delay. *Industrial & Engineering Chemistry Research*, 43(1), 56–62. doi: 10.1021/ie020732v.

Zhong, Q. (2006). *Robust Control of Time-delay Systems*. Springer London. doi:10.1007/1-84628-265-9.