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An auction framework for assessing the tendering of subsidised routes in air transportation

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ABSTRACT

Governments offer subsidies along routes that are deemed commercially non-viable but economically and socially essential. Subsidised routes are often criticised for inefficiencies and excessive subsidies, which partly result from restrictions defined by transportation authorities during the tendering process, such as a maximum airfare and minimum number of daily flights. We develop an integrated auction framework—referred to as *Single-round Combinatorial Auction for Subsidised routes (SCAS)*—to provide decision support to transportation authorities when designing tendering processes for subsidised routes. The framework includes two main models as ingredients. First, the Airline Bid Preparation Model (ABPM), which replicates the airline's behaviour when preparing bids for subsidised routes. Second, the Winner Determination Problem (WDP), which is used to select the bids based on a given evaluation criterion. We capture the responsive relationship between passenger demand and supply of air services by including passenger utility as an endogenous variable in the ABPM. Additionally, as input to the ABPM, we estimate the route operating cost for small aircraft that typically operate subsidised routes. The usefulness of the approach is demonstrated with an application to the network of subsidised routes in Sweden, for which we provide policy guidelines. Our analysis suggests that having a restriction on the airfare but not the number of flights is an effective way to design the tendering process, which strikes a good balance between passengers, government and airlines goals. Additionally, we demonstrate that the transportation authorities can compensate not having a requirement on the number of daily flights through ensuring a higher number of passengers, i.e., by including maximisation of the number of passengers in the bid evaluation criterion or using passenger discounts.

1. Introduction

In liberalised air transportation markets, for example, in Europe, United States, Asia, and Australia, governments often adopt subsidy schemes through which they offer compensation to airlines in exchange for air services along routes that are deemed commercially non-viable but economically and socially essential. The objective of these subsidised air transportation routes is to

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ensure a minimal level of air services to outlying communities or remote regions that are difficult to access from the capital, other main cities, or a hub airport (EU Regulation, 2008; DoT U.S., 2009) yet cannot be served commercially due to thin demand. The governments, local authorities, or transportation authorities award contracts to airlines through a public call for tenders respecting specified service requirements. Despite this similarity, the worldwide adoption of subsidised routes varies. Fageda et al. (2018) comprehensively discuss and compare the implementation of subsidised routes worldwide. The majority of the literature (Williams and Pagliari, 2004; Williams, 2005; Bråthen and Halpern, 2012; Calzada and Fageda, 2014) focus on the comparison between the two largest subsidy schemes, i.e., the Essential Air Service (EAS) programme in the United States and the Public Service Obligation (PSO) for air services in the European Economic Area (EEA). They account for about 320 subsidised routes and average annual subsidy payments of up to US\$590 million (Mueller, 2021). All subsidised routes serve the same objective. However, they differ concerning the criteria used to determine their eligibility (e.g., minimum distance to a hub airport as in the United States or minimum travel time to a major city as in EEA), the contract length (2 to 5 years) and the specified service requirements in the call for tenders (e.g., flight frequencies, maximum airfare, flight schedules, and number of seats).

The development and increasing use of subsidised routes has attracted a lot of attention from many scholars. One section of the literature reviews the current subsidised route systems, highlighting two main criticisms—the continuously increasing and potentially unjustified cost of subsidisation (Reynolds-Feighan, 1995; Williams and Pagliari, 2004; Gillies, 2004; Williams, 2005). Another section of the literature provides tools for decision makers to respond to these criticisms by focusing on: (1) identification and evaluation of the routes that actually require subsidies (e.g., Flynn and Ratick (1988), Matisziw et al. (2012), and Kinene et al. (2020)), and (2) setting the appropriate level of service requirements/tender specifications for subsidised routes (e.g., Merkert and O'Fee (2013), Bråthen and Eriksen (2016), Pita et al. (2013a), and Pita et al. (2014)).

The continuously increasing and potentially unjustified cost of subsidisation may be attributed to three main issues. First, the number of subsidised routes has been increasing at a fast pace (Reynolds-Feighan, 1995; Williams and Pagliari, 2004; Williams, 2005)—since their inception (first tender in 1993), the number of PSO routes in Europe has significantly increased from about 67 routes in 1997 to 176 in 2019—, implying a need for decision tools supporting the identification of the routes that actually require subsidies. Notably, Flynn and Ratick (1988), Matisziw et al. (2012), and Kinene et al. (2020) focused on optimisation models that design PSO networks to maximise the accessibility of remote regions while minimising the cost of subsidisation. Bråthen and Eriksen (2016) developed a framework to assess the suitability of the PSO subsidy level with respect to the difference in passengers' generalised travel cost between air and ground transportation.

Second, a realistic explanation, especially in Europe, is the decrease in the number of bids received per route through the years, thus failing to stimulate the competition intended by the tendering processes. For example, in Sweden, the number of bids per route was 4 in the 2002–2004 contract period but reduced to 2 in the 2015–2019 contract period (Dahlström et al., 2018). The incumbent airlines have previous experience in operating subsidised routes and hence have an advantage over new entrants. Additionally, the short time between the submission of the bids and notification of the tender favours the incumbent airlines because they require lower set-up costs (Williams, 2005). Also, incumbent airlines often have more information to estimate the route operating costs and revenues, gaining an information advantage that helps them retain their routes.

Third, a fundamental issue in designing an efficient PSO network is the definition of the tender specifications, that is, the minimum level of service that airlines' bids must meet, and the evaluation criterion (i.e., the criteria used to select the winning bids). On the one hand, loose requirements may harm social welfare by letting the winning airlines exert monopolistic market power, for example by charging high airfares. On the other hand, too restrictive requirements, for example setting a tight maximum airfare or a large minimum number of flights, may lead to higher cost of subsidisation due to an imbalance between passenger demand and supply of air services.

The setting of requirements and evaluation criterion is a difficult task that aims at balancing conflicting goals such as the social need for accessibility with the costs of providing the PSO services, by taking into account the multi-agent and multi-objective nature of the PSO procurement.

The airlines receiving subsidies for air services are profit-oriented and are not obliged by law to minimise the governments' subsidies through their operations. However, if they aim to win the tender, they cannot pursue a pure profit maximisation strategy when preparing the bids as this does not necessarily align with the governments' interest, as captured by the evaluation criterion. In this respect, redesigning and/or relaxing some of the tender requirements stipulated in the call to tender could lead to a more competitive and cost efficient tender process with a balance between profit maximisation and subsidy minimisation. For example, relaxing some of the aircraft specifications in the PSO requirements such as the seating capacity may encourage airlines to select the appropriate aircraft size for a given route demand, which may potentially increase load factors and reduce the airlines' operating cost (Pita et al., 2014), to ultimately achieve a more efficient matching of supply to demand.

Despite the relevance of tender definition on the tender outcome, decision support tools to carry out this task are quite scant in both literature and current practice. Merkert and O'Fee (2013) surveys 20 PSO sponsoring authorities to identify the best practices for setting up an efficient tender for subsidised routes. Concerning prescriptive analytics, the literature focuses on the development of optimisation models that either minimise the total social cost (Pita et al., 2013a, 2014) or minimise subsidies (e.g., Bråthen and Eriksen (2016)). Pita et al. (2013a) developed an integrated flight scheduling and fleet assignment model to determine a network of subsidised routes that minimises the total social cost subject to a target demand; later, Pita et al. (2014) extended this work with a social welfare analysis (including passenger, airline, airport and government surpluses) of the resulting network of subsidised routes. Bråthen and Eriksen (2016) described a framework for assessing the level of service requirements with application to Norway; the framework includes a collection of independent optimisation models for route selection, profit maximisation, operating cost minimisation, and social welfare maximisation. Pita et al. (2013a, 2014) and Bråthen and Eriksen (2016) take a central planner's

perspective and do not specifically take into account the presence of several bidding airlines with different characteristics (e.g., fleet mix). Moreover, these works consider a single evaluation criterion (i.e., minimising total social cost or maximising social welfare) and subsidy policy.

Overall, the existing literature lacks an integrated framework to provide decision support to transportation authorities when designing an efficient tender for subsidised routes, i.e., assess various levels of service requirements, evaluation criterion, and subsidy policies. Additionally, to our knowledge, no published work has modelled this integrated framework as an auction, yet an auction can take into account the multi-agent and multi-objective nature of the procurement for subsidised routes. Previous studies have considered auction frameworks for various modes of transportation, for example, for ferry services (Baird et al., 2012), and regional bus transportation (Baird et al., 2012), but none for subsidised air services.

In addressing this gap, this paper makes four main contributions:

1. We develop an integrated auction framework referred to as the *Single-round Combinatorial Auction for Subsidised routes (SCAS)* for the design of the tendering process of subsidised routes. Such an auction can help authorities analyse the effect of different tender specifications (e.g., the tender requirements and bid evaluation criterion) on the level of subsidies requested by the airlines. The SCAS includes a bid preparation model and a winner determination model used for the determination of the winners by the authorities.
2. We develop a novel *Airline Bid Preparation Model (ABPM)* as an optimisation model specific for subsidised routes. The ABPM is formulated as a multi-objective mixed-integer nonlinear problem. It replicates the airlines' behaviour when preparing the bids in a combinatorial auction framework with an objective of optimising the evaluation criterion subject to the achievement of operating profits. Specifically, we enforce a minimum gross (profit) margin threshold to reflect the airlines' opportunity cost for deploying their assets on the subsidised routes, rather than elsewhere in their networks. We additionally capture the responsive relationship between passenger demand and supply of air services by including passenger utility as an endogenous variable in the ABPM.
3. We develop an empirical route-operating-cost model for small aircraft that typically operate subsidised routes. We take an empirical approach similar to previous studies (e.g., Janic, 2000; Swan and Adler, 2006) but consider turboprops and regional jets with at most 100 seats. This calibration of the cost model provides an accurate representation of operating costs that is specific to regional routes.
4. We demonstrate the application of the auction framework—using the Swedish subsidy scheme as a case study—and suggest policy guidelines. We analyse the impact of various requirements stipulated in the tendering process, bid evaluation criterion, and the subsidy policy on social welfare and accessibility. The results suggest that having a restriction on the airfare but not the number of flights gives a good trade-off in-terms of passenger surplus, airline profits, and government subsidies.

Collectively, the four contributions of this paper provide practical and integrated decision support for transportation authorities to design the tendering of subsidised routes. Specifically, the direct implementation of the proposed integrated auction framework improves the setting of tender requirements by anticipating (or replicating to some extent) the airlines' behaviour in placing bids, without changing the current functioning of the tendering process. Moreover, the proposed modelling framework provides a valid methodological foundation to investigate collaborative information sharing mechanisms between airlines and authorities, and assess their impact on welfare and expected outcomes.

The remainder of this paper is organised as follows. In Section 2, we present the integrated auction framework (SCAS), and the formulations of its two major ingredients: the ABPM and a Winner Determination Problem (WDP). We also present the models used to estimate passenger demand and route operating cost, which are key inputs to the ABPM. In Section 3, we first describe the Swedish case study, then present the results and give their policy implications. In Section 4, we conclude the paper with a summary of its contents and indicate directions for future research.

2. The single-round combinatorial auction for subsidised routes

In this section, we present our integrated framework for designing the tendering process of subsidised routes. We frame the tendering process as a combinatorial auction. This overcomes major problems of traditional auction mechanisms, wherein each player can only bid on single items/routes. Instead a combinatorial auction allows bidding on bundles of items, thus allowing for complex bid structures that exploit complementary relations among auctioned items and economies of scale (Vohra and Weber, 2000; Pekeč and Rothkopf, 2003; Elmaghraby and Keskinocak, 2004). Note that these are crucial aspects in the procurement of air transport services due to network economies characterising airline networks.

We refer to our model as *Single-round Combinatorial Auction for Subsidised routes (SCAS)*. The SCAS entails three main steps, depicted in Fig. 1:

1. *Auction definition (Section 2.1)*. The auction has the airlines as sellers of air transportation services and the authority as the buyer of the services. At the auction definition phase, the authority specifies the regions that require subsidised air transportation services, the evaluation criterion for the bids, and the tendering requirements for an airline to qualify for bidding, which constitute key inputs to the next phases.
2. *Bid preparation (Section 2.2)*. The second step of the SCAS involves bid preparation by the airlines. Given the set of auctioned regions, the evaluation criterion, and the tendering requirements, airlines formulate their bids for every feasible bundle of regions; we propose an optimisation model to replicate bids from each airline considering their specific fleet, endogenous demand, and operating costs. To obtain reliable inputs, we empirically estimate an operating cost model using historical data from PSO routes' operations and consider reliable demand coefficients obtained from the literature.

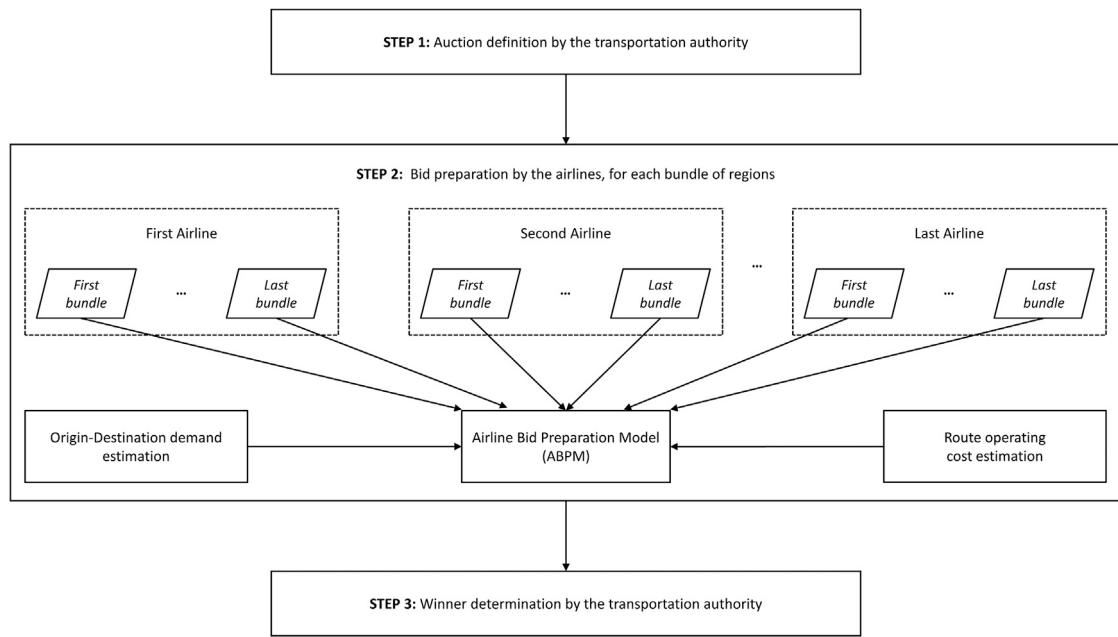


Fig. 1. The steps for the SCAS.

Table 1

Examples of airline bundles. Each bundle b has a price s_b , which is the total subsidy that the airline requires to serve all regions in the bundle.

Airline	Bundle (b)	Total subsidies (s_b)
1	{Region 1, Region 2}	4000
	{Region 1}	3000
	{Region 2}	2000
2	{Region 3}	2500

3. *Winner determination (Section 2.3)*. After the authority collects bids from the airlines, it evaluates them for validity (e.g., whether they meet the tender requirements) and computes the allocations of the regions by solving a winner determination problem that minimises the overall cost of subsidisation.

2.1. Auction definition

In a procurement auction for subsidised routes, the transportation authority defines the originating (remote) regions in need of air transportation services to a given destination for bidding by the airlines (EU Regulation, 2008). The airlines can bid to serve each region either directly or with connecting services and bids can be made for combinations/bundles of regions. Table 1 presents examples of bundle bids by two airlines for three auction items, i.e., Regions 1, 2 and 3. An airline is allowed to submit multiple bids for different combinations of regions. For example, airline 1 could bid for Region 1 (3000) and Region 2 (2000) separately, but also for the bundle {Region 1, Region 2} (4000) with a possibly lower price than the sum of individual bids for Region 1 and Region 2 (5000). This could reflect, for example, cost reductions that the airline might gain by operating both regions through better asset utilisation or leveraging connecting services to pool demand among regions. In turn, allowing airlines to make bids for bundles, and not just single items separately can be beneficial for governments, as it may result in lower costs of subsidisation.

In addition to the regions to be considered in the auction, the transportation authority defines the evaluation criterion for the bids, which is used to select the winning bids. For example, the transportation authority may minimise the subsidies and/or maximise the number of passengers served. The criteria can be combined into a score (or objective function) that is taken into account by the airlines while preparing bids.

Furthermore, the transportation authority specifies the requirements to be met by the bids. In practice, tendering requirements for subsidised (PSO) routes typically involve a minimum service level (e.g. a minimum number of flights) and/or a maximum airfare, to prevent the winning airline from exerting its market power and ensure sufficient and affordable services. Commonly specified tendering requirements include the following: maximum airfare, minimum service frequency, and time-tabling requirements, e.g., in Spain, the United States (U.S.), Italy and Sweden; and minimum seating capacity, e.g., in Sweden and Norway; etc. (EU Regulation,

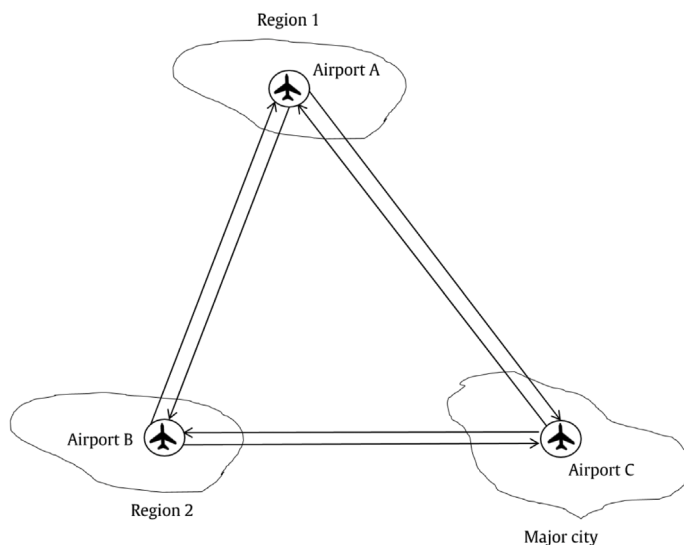


Fig. 2. The relationship between remote regions, airports, markets and flight legs.

2008; DoT U.S., 2009). Tender requirements affect the airlines' flexibility in formulating bids; if not specified properly, they may result in inefficient bids and thus higher costs of subsidisation.

The evaluation criterion and tender requirements are set by governments ex-ante at the auction definition stage, which are then considered by the airlines when preparing the bids. Within the optimisation model for bid preparation, these translate into the objective function and constraints, respectively. In the experimental setup (Section 3), we will test different tendering requirements and evaluation criterion to show how the SCAS can be used by transportation authorities to anticipate how these may affect airlines' bids and ultimately the tender outcome.

2.2. Bid preparation

To obtain the bids, we develop an optimisation model that replicates the airlines' behaviour in preparing bids for a given bundle of regions and solve this for all bundles stated by the transportation authority, for all airlines. We refer to this model as the Airline Bid Preparation Model (ABPM). The airlines naturally aim to maximise profits subject to demand and capacity constraints. This does not necessarily align with the bid evaluation criterion, which traditionally entails some form of minimisation of the cost of subsidisation for the government and/or maximisation of number of passengers. However, if an airline wants to win the auction, it should offer bids that are in line with the given bid evaluation criterion. To capture these two objectives simultaneously, we formulate our bid preparation model with a tailored multi-objective function while enforcing a reasonable gross margin threshold.

2.2.1. Optimisation model

We use Fig. 2 to introduce the ABPM. Consider a bundle of two remote regions—Region 1 and Region 2—that require air transportation services to a target destination, for example, a major city. This leads to four origin–destination markets (\mathcal{M}) to be served: Region 1–Major city, Major city–Region 1, Region 2–Major city, and Major city–Region 2. Let A, B, and C be the gateway airport to Region 1, Region 2, and the Major city, respectively.¹ We denote the set of flight legs as \mathcal{F} (e.g., A–B, B–A, B–C, C–B, A–C, C–A). These flights can either carry nonstop passengers or can be combined to provide multi-stop services/routes (e.g. A–C, C–A, A–B–C, C–B–A). We denote the set of routes as \mathcal{I} . We assume that the bidding airline has one or more aircraft types (denoted by \mathcal{K}) that can be differentiated by seating capacity, cruise speed, and operating cost.

We model demand using a discrete choice framework – as commonly done in the aviation industry – explained in detail in Section 2.2.2. This allows to explicitly incorporate the relationship between passenger flows and route attributes, such as travel time, connecting time, and airfare, which are directly optimised by the model. In turn, this introduces a non-linear constraint that makes the ABPM a non-linear optimisation model.

Finally, given these inputs and subject to endogenous demand, the model simultaneously determines the number of flights on flight leg f to be operated by aircraft type k (x_{fk}), the route airfare (p_i) and the number of passengers on route i (q_i), and the total subsidies required to serve the regions in the bundle (s). Table 2 presents a detailed list of notations used in the bid preparation model.

¹ Note that here, we are assuming single gateway airports, but the model could be easily extended to accommodate multi-airport regions.

Table 2
List of notations for the bid preparation model.

Sets	
\mathcal{F}	Set of flight legs indexed by f
\mathcal{I}	Set of routes i
$\mathcal{F}_i \subset \mathcal{F}$	Subset of flights belonging to route i
\mathcal{M}	Set of markets (in the considered bundle) indexed by m
$\mathcal{I}_m \subset \mathcal{I}$	Subset of routes serving market m
$\mathcal{I}_f \subset \mathcal{I}$	Subset of routes using flight leg f
\mathcal{K}	Set of aircraft types indexed by k
Parameters	
t_{fk}	Travel time on flight leg f by aircraft type k
c_{fk}	One-way operating cost for aircraft type k along flight leg f
u_k	Maximum daily utilisation hours of aircraft type k
ρ_k	Number of aircraft of type k
$seat_k$	Seat capacity of aircraft type k
d_m	Potential demand for market m
t_i^{travel}	Travel time for route i
t_i^{conn}	Additional travel time for connecting passengers on route i =1 if route i serves market m ; 0 otherwise
σ_{im}	
p_m^{max}	Maximum one-way airfare for market m
q_m^{min}	Minimum number of daily flights
M	big-M value
ψ	Minimum gross margin threshold
β_b	$b = \{0, 1, 2, 3, 4\}$ parameters of the passenger utility function
Decision variables	
$s \in \mathbb{R}^+$	Total subsidy
$x_{fk} \in \mathbb{Z}^+$	Frequency on flight leg f by aircraft type k
$y_i \in \mathbb{Z}^+$	Frequency on route i
$z_i \in \{0, 1\}$	=1 is route i is operated; 0 otherwise
$p_i \in \mathbb{R}^+$	Airfare on route i
$q_i \in \mathbb{R}^+$	Number of passengers accommodated on route i
$u_i \in \mathbb{R}$	Passenger utility for route i

Next, we formulate the ABPM mathematically.

$$\begin{aligned}
 \max \quad & \Gamma(s, q_i) & (1) \\
 \text{s.t.} \quad & x_{fk} = x_{-f,k} & \forall f \in \mathcal{F}, \forall k \in \mathcal{K} & (2) \\
 & \sum_{f \in \mathcal{F}} x_{fk} t_{fk} \leq u_k \rho_k & \forall k \in \mathcal{K} & (3) \\
 & y_i \leq \sum_{k \in \mathcal{K}} x_{fk} & \forall i \in \mathcal{I}, \forall f \in \mathcal{F}_i & (4) \\
 & u_i = \beta_0 + \beta_1 t_i^{travel} + \beta_2 t_i^{conn} + \beta_3 p_i + \beta_4 y_i & \forall i \in \mathcal{I} & (5) \\
 & q_i \leq \sum_{m \in \mathcal{M}} \sigma_{im} d_m \frac{e^{u_i}}{1 + e^{u_i}} & \forall i \in \mathcal{I} & (6) \\
 & \sum_{i \in \mathcal{I}_f} q_i \leq \sum_{k \in \mathcal{K}} seat_k x_{fk} & \forall f \in \mathcal{F} & (7) \\
 & y_i - M z_i \leq 0 & \forall i \in \mathcal{I} & (8) \\
 & q_i - M z_i \leq 0 & \forall i \in \mathcal{I} & (9) \\
 & \sum_{i \in \mathcal{I}_m} z_i = 1 & \forall m \in \mathcal{M} & (10) \\
 & (1 - \psi) \left(\sum_{i \in \mathcal{I}} q_i p_i + s \right) - \sum_{f \in \mathcal{F}} \sum_{k \in \mathcal{K}} c_{fk} x_{fk} \geq 0 & & (11) \\
 & q_i \in \mathbb{R}^+, p_i \in \mathbb{R}^+, u_i \in \mathbb{R}, y_i \in \mathbb{Z}^+, z_i \in \{0, 1\} & \forall i \in \mathcal{I} & (12) \\
 & x_{fk} \in \mathbb{Z}^+ & \forall f \in \mathcal{F}, \forall k \in \mathcal{K} & (13) \\
 & s \in \mathbb{R}^+ & & (14)
 \end{aligned}$$

In the objective function (1), we maximise the linear function $\Gamma(s, q_i)$ (see Eq. (15))

$$\Gamma(s, q_i) = w_s \left(\frac{s^{max} - s}{s^{max} - s^{min}} \right) + (1 - w_s) \left(\frac{\sum_{i \in \mathcal{I}} q_i - d^{min}}{d^{max} - d^{min}} \right), \tag{15}$$

which is a combination of two important criteria considered by governments (EU Regulation, 2008)—the minimisation of the total subsidies and the maximisation of the number of passengers. Based on a similar technique to the weighted comprehensive criterion method (WCCM) (see Alsyouf and Hamdan (2017) for details), Eq. (15) normalises the two evaluation criterion to ensure an easy and fair comparison. It has two terms: the first term corresponds to the normalisation of the first evaluation criterion that should be minimised, i.e., subsidy, using its maximum (s^{max}) and minimum (s^{min}) values; the second term corresponds to the normalisation of the second evaluation criterion that should be maximised, i.e., number of passengers, using its maximum (d^{max}) and minimum (d^{min}) values. We set parameters s^{min} and d^{min} to zero, $d^{max} = \sum_m d_m$ and s^{max} to the maximum possible amount of subsidies, obtained by only maximising the second objective (or equal to a subsidy budget for the given bundle, if known). The two terms are combined into a single value (score) to be maximised using the weights of importance attributed to each criteria. Here, w_s and $(1 - w_s)$ (where $0 \leq w_s \leq 1$) are the weights attributed to the first and second evaluation criterion, respectively.

Constraints (2) induce symmetry in the network, implying that if a flight leg f is used, then its opposite direction leg $-f$ must also be used. Constraints (3) limit the aircraft type's total utilisation based on the fleet size (number of aircraft) and the expected average utilisation. Constraints (4) compute the frequency on every route, defined as the minimum of the frequencies on its respective flight legs. Constraints (5)–(6) implement the demand calculation. Constraints (5) define the utility of each route (see Section 2.2.2 for further explanation). Constraints (6) set an upper bound on the accommodated demand (q_i) that is equal to the estimated values provided by the right-hand side—based on the empirical specification discussed in Section 2.2.2. Constraints (7) then ensure that the demand accommodated on each flight leg is lower than the allocated seat capacity. The constraints (8) define the binary variables z_i from the frequency variables y_i . Constraints (9) link the demand q_i variables to the route status variables z_i —this prevents having positive demand when the route is not operated. Indeed, under the demand model it is possible to have a demand estimate when frequency is zero. Constraints (10) are specific to PSO routes; they ensure that each market in the bundle is served by one route. Constraint (11) ensures that the airline makes a minimum gross margin threshold, which is specified in relative terms (ψ) as the ratio between the airline's profits and revenues (return on revenues) directly associated with providing air services. Assuming a first-price auction where the airlines will earn the exact amount of subsidy as requested in the bid, total revenues are obtained as the sum of operating revenues and subsidy. Hence, the attainment of minimum gross margin is enforced as follows:

$$\frac{\sum_{i \in I} q_i p_i + s - \sum_{f \in F} \sum_{k \in K} c_{fk} x_{fk}}{\sum_{i \in I} q_i p_i + s} \geq \psi \tag{16}$$

which can be linearised as in constraint (11). The constraints (12)–(14) define the domain of the variables.

In case of specification of a maximum airfare per market and/or a minimum number of daily flights by the transportation authorities, additional constraints (17) and (18) and are added to the bid preparation model.

$$\sum_{i \in I_m} y_i \geq \theta_m^{min} \quad \forall m \in \mathcal{M} \tag{17}$$

$$p_i \leq \sum_{m \in \mathcal{M}} \sigma_{im} p_m^{max} \quad \forall i \in I \tag{18}$$

Constraints (17) enforce a minimum service level by setting the total frequency in a given market greater than or equal to a set threshold θ_m^{min} , and constraints (18) set an upper bound on the route airfare to p_m^{max} (on a market basis).

Given the domain of the decision variables (12) and (13), the objective function (1), the linear constraints (2)–(5), (7)–(10), (11), and non linear constraints (6), the ABPM is a mixed-integer non-linear programming (MINLP) optimisation model (Lee and Leyffer, 2011). Since the first MINLPs in the mid 1970s, the development of solution algorithms unique to them has advanced remarkably. Today, solvers can be used to tackle small to mid-sized (e.g. BARON) and more challenging MINLPs (e.g., SCIP) in reasonable computation times, most commonly using spatial branch-and-bound (Castro, 2016; Vigerske and Gleixner, 2018).

2.2.2. Demand model

To investigate how relaxing some of the tender requirements may improve the tender outcome, we need a realistic representation of demand that is elastic to changes in the level of airline services. Capturing supply–demand interaction is a key aspect of airline planning. Indeed, supply and demand are strongly interrelated. Earlier studies have been using spill and recapture approaches (e.g., Lohatepanont and Barnhart (2004) and Pita et al. (2013b)), while more recent contributions have used discrete choice modelling to improve the allocation of passengers among competing itineraries and better represent passengers' behaviour (e.g., Yan et al. (2007)). A relevant modelling strategy to suitably link air travel supply and demand is the use of an aggregate nested logit model (Hsiao and Hansen, 2011; Birolini et al., 2021), where potential demand—considered as the theoretical maximum number of passengers expected to travel in a given market if there was no travel impedance—is first split into air and non-air alternatives. Then, the air travel branch is further broken down to capture substitution patterns among air travel alternatives. This modelling approach provides consistent substitution patterns involving diminishing returns to improvements in the level of supply (e.g., lower prices, shorter flight times). Given that PSO routes are operated as monopolies, we can simplify the model formulation using an aggregate multinomial logit that entails only a partition of the potential demand into: (1) those who do not fly (either travel by other modes or do not travel at all), and (2) those who fly.

Let d_m be the potential demand in a given market, q_i be the demand accommodated along route i serving the market, and u_i and u_0 be the deterministic utility component of air travel for route i and non-air alternative, respectively. The (unobserved) non-air utility is normalised to 0, leading to the following formulation:

$$q_i = d_m \frac{e^{u_i}}{1 + e^{u_i}} \tag{19}$$

where u_i is a linear function of relevant service attributes of air travel for route i , i.e., $u_i = \beta^T X_i$, where X_i is the vector of explanatory variables considered, and β are coefficients to be estimated. In this paper, we use a utility function that includes the airfare p_i , the flight time t_i^{travel} , the connection time t_i^{conn} , and the frequency y_i as outlined in Section 3.1.

2.2.3. Route operating cost

The route operating cost comprises of direct and indirect costs. Direct operating costs are categorised into fixed and variable operating cost. The fixed costs mainly include aircraft ownership costs and contribute to approximately 32% of the total operating cost (Swan and Adler, 2006; ICAO, 2017) whereas the variable costs include all flight-related costs such as fuel, crew, and aircraft maintenance and contribute to approximately 50% to 60% of the total route operating costs (ICAO, 2017). The remaining 10% to 20% of the costs are categorised as other costs.

The direct route operating cost can be estimated as a function of aircraft type and the non-stop great circle distance (hereafter called distance) as in Swan and Adler (2006) and Janic (1999). Both models were estimated for aircraft sizes of at least 100 seats, which generally makes them less applicable to subsidised route systems that use regional jets and turbo-props. We use the same model formulation as Swan and Adler (2006) and Janic (1999) but with smaller aircraft to estimate the operating cost:

$$\ln(c_{fk}) = c_0 + c_1 \cdot \ln(seats_k) + c_2 \cdot \ln(distance_f) + \epsilon \tag{20}$$

where c_{fk} is the operating cost for using an aircraft of type k along a flight leg f , $seats_k$ is the aircraft seating capacity, $distance_f$ is the non-stop great circle distance in kilometres, and ϵ is the error term. c_0, c_1, c_2 are the parameters estimated by ordinary least squares (OLS). We discuss the empirical estimation of this function in Section 3.1.

2.3. Winner determination

After the auctioning authority receives the bids from the airlines, it must select the winner(s). In the literature, selection of the winner(s) in an auction framework is known as the Winner Determination Problem (WDP) (De Vries and Vohra, 2003). The WDP in a combinatorial auction can be generally cast as a set covering/packing problem (Pekeć and Rothkopf, 2003; Sheffi, 2004; De Vries and Vohra, 2003).

For our combinatorial auction, the authority aims to maximise the weighted score that either minimises their expenditure and/or maximises the number of passengers, with a condition that every region is allocated to at most one airline (the bidder). With \mathcal{R} , \mathcal{A} , \mathcal{K}_a , and \mathcal{B} as the set of regions in the auction, the set of airlines, the set of aircraft types used by airline a , and the set of bundles, respectively, the WDP can be modelled as a Weighted Set Partitioning Problem (WSPP) (Lehmann et al., 2006; Rothkopf et al., 1998):

$$\max \quad z = \sum_{a \in \mathcal{A}} \sum_{b \in \mathcal{B}_a} \Gamma_{ab} x_{ab} \tag{21}$$

$$\text{s.t.} \quad \sum_{a \in \mathcal{A}} \sum_{b \in \mathcal{B}_r \cap \mathcal{B}_a} x_{ab} = 1 \quad \forall r \in \mathcal{R} \tag{22}$$

$$\sum_{b \in \mathcal{B}_a} w_{bk} x_{ab} \leq \rho_{ak} \quad \forall a \in \mathcal{A}, \forall k \in \mathcal{K}_a \tag{23}$$

$$x_{ab} \in \{0, 1\} \quad \forall a \in \mathcal{A} \forall b \in \mathcal{B}_a \tag{24}$$

where Γ_{ab} is the total score for the bid of bundle b operated by airline a (obtained by implementing the ABPM in Section 2.2), w_{bk} is the number of utilised aircraft of type k in bundle b , \mathcal{B}_a is the subset of bundles with bids from airline a , and \mathcal{B}_r is the subset of bundles that include region r . The binary decision variable x_{ab} is equal to 1 if bundle b operated by airline a is selected, and 0 otherwise. Constraints (22) ensure that every region is allocated to exactly one airline. To avoid inconsistencies between the airline capacity and the required services, we use additional constraints (23) to ensure that the airline wins bundles within its capacity: ρ_{ak} is the number of aircraft in airline a 's fleet.

Given the integer decision variables, the linear objective function (21), and the linear constraints (22) and (23), the WDP is a mixed-integer linear programming (MILP) optimisation model (Wolsey, 1998). Literature has extensively studied MILPs and developed advanced solution algorithms (Jünger et al., 2009). Commercially available solvers such as CPLEX (IBM, 2017) and GUROBI (Gurobi Optimization, 2021) can solve large model instances, with thousands of decision variables and constraints to optimality within reasonable computational time.

Table 3
Swedish PSO data for routes to Stockholm (ARN) in the tender period 2019–2023 (daily values).

Region (Airport IATA code)	Route	Airline	Aircraft	Max. airfare (\$)	Avg. pax	Avg. subsidies (\$)
Vilhelmina(VHM)	VHM-LYC-ARN	Amapola Flyg AB	Fokker 50	117	44	6081.0
Lycksele (LYC)	VHM-LYC-ARN	Amapola Flyg AB	Fokker 50	116	54	6092.0
Hemavan (HMV)	HMV-KRG-ARN	Amapola Flyg AB	Fokker 50	131	43	7299.7
Kramfors (KRF)	HMV-KRG-ARN	Amapola Flyg AB	Fokker 50	109	39	8351.3
Torsby (TYF)	TYF-HFS-ARN	Amapola Flyg AB	Jetstream 32	93	11	3501.2
Hagfors (HFS)	TYF-HFS-ARN	Amapola Flyg AB	Jetstream 32	87	13	3501.2
Sveg (EVG)	EVG-ARN	Jonair Affärsflyg AB	Beech 1900	99	25	6232.6
Gällivare (GEV)	GEV-AJR-ARN	Regional Jet OU	CRJ900	150	78	4778.3
Arvidsjaur (AJR)	GEV-AJR-ARN	Regional Jet OU	CRJ900	129	83	10,730.9

3. Sweden as a case study

In this section, we demonstrate the insights that can be derived from the application of the proposed SCAS framework by applying it to a realistic case study involving the PSOs in Sweden, where Trafikverket (the Swedish Transport Administration) is responsible for the tendering of subsidised routes (PSOs). The authority defines the regions (markets) as per the EC regulations, i.e., by the origin and destination region with a single airport as the gateway (EU Regulation, 2008). To demonstrate the benefits that can be derived from the application of the SCAS, we considered the Swedish PSOs for the tender period 2019–2023. Table 3 presents the nine regions involved in this tender period and the respective routes to Stockholm-Arlanda airport. Currently, Trafikverket states the tender requirements per origin-destination market. They include a maximum airfare, a minimum of two daily return flights on weekdays, and a minimum number of seats per calendar year. Other requirements, such as a maximum number of stops, a maximum flight time, and departure and arrival times can also be specified as additional requirements (Andersson, 2017). In the following analysis, we focus on the two most common requirements in PSO systems. We use the maximum airfares and the minimum frequency from the most recent tendering period, which are typically set based on historical data or similar routes. In Sweden (similar to Norway), the maximum airfares are also set using a linear model calibrated using historical airfares and the direct great-circle distance from the origin to the destination (see Lian et al., 2010; Kinene et al., 2020).

At the time of the study, the PSO network in Sweden is operated by three airlines, that charge a passenger weighted airfare of \$124.0 and provide air services to 389 daily passengers. The government pays \$56,568.3 in subsidies daily. Further details can be found in Table 3, which presents for each region the maximum average airfare (Max. airfare), the operating airline, the aircraft used, the average daily demand (Avg. pax), and the average daily subsidies (Avg. subsidies).

3.1. Model inputs

The SCAS model requires various inputs for implementation. In the auction definition step we consider the nine regions in Table 3. In the bid preparation, we use the five bundles ({Sveg}, {Gällivare, Arvidsjaur}, {Hemavan, Kramfors}, {Vilhelmina, Lycksele}, {Torsby, Hagfors}) stated by Trafikverket in the tender period 2019–2023. For the bundles with two regions, the airlines may submit bids for each of the regions in addition to a bundle bid: this increases the total number of bundles to 13.

Due to a lack of detailed and dis-aggregate data for our empirical setup, we adopt the parameters of the utility function presented in Birolini et al. (2020), adjusted on a daily basis. Our specification is as follows:

$$u_i = 4 - 0.678t_i^{travel} - 0.258t_i^{conn} - 0.017p_i + 0.312y_i \quad (25)$$

where t_i^{travel} is the flight time, t_i^{conn} is the connection time, p_i is the airfare, and y_i is the frequency. Notice that we consider these empirical coefficients from the literature as they yield meaningful trade-offs and substitution patterns that are in line with the Sweden context. The parameter estimates in the utility function suggest a value of time of \$39.8/h and \$15.2/h for flight and connecting times, respectively. These values are consistent with previous studies (e.g., Wardman et al., 2016) that suggest a value of time between \$14/h and \$137/h (between \$33/h and \$65/h for Sweden). Also, we adjust the model specification by re-estimating the intercept to ensure adherence with the available aggregated data and improve the fitting to Swedish routes.² For potential demand d_m , we estimated it from historical data using a corrected ordinary least squares (COLS) approach, as a function of gross domestic product, population and travel time by ground transportation.³

To calibrate the route operating cost model, we use data on the sum of fixed and variable aircraft operating cost sourced from Apex—an airline performance analysis platform with a global database of airline/aircraft operating costs (Apex, 2019).⁴ We simulate

² Additionally, we conduct a sensitivity analysis to test the robustness of our results upon variations in the adopted parameter estimates. As detailed in the Appendix, these impact the considered indicators but without changing the key insights and implications.

³ We acknowledge the uncertainty associated with demand estimates, which may affect the model outcomes. As such, we perform a sensitivity analysis for demand uncertainty by varying the potential demand (See the Appendix for details.). This, in turn, highlights how the SCAS can be readily used to account for uncertainties through scenario analysis.

⁴ Apex uses a combination of self reported route operation data from a large group of member airlines and industry estimates for operating (both fixed and variable) costs of various aircraft types on similar routes.

300 route operation cost data points of the typical turboprops (e.g., Fokker 50, Jetstream 31, DHC 8-100, Beech king Air 200, and Dornier 228NG) and regional jets (e.g., CRJ700, ATR 72-600, and ATR 42-320) used for the Swedish domestic routes. The estimated log-linear regression model using OLS (standard errors in parenthesis) is as follows:

$$\ln(C) = 4.9123 + 0.2610 \cdot \ln(\text{seats}) + 0.3633 \cdot \ln(\text{distance}) \quad (26)$$

(0.1618) (0.0264) (0.0231)

The model highlights good fit—reporting an adjusted R^2 of 0.64—and provides a practical means to estimate reliable cost parameters in our model.

The data on the aircraft fleet size and the minimum gross margin threshold are based on the currently operated PSO routes in Sweden. The current PSO routes are operated by three airlines⁵: (1) Amapola Flyg AB with two 50-seat Fokker 50 and one 19-seat Jetstream 32, (2) Jonair Affärsflyg AB with one 19-seat Beech 1900, and (3) Regional Jet with one 64-seat CRJ900. We use a daily maximum aircraft utilisation (u_k) of 10 hours—in line with current practice. Given the current routes and fleet, we estimate an average gross margin threshold (ψ) of 12.5% based on historical airline route revenues and subsidies for the period of October 2019 to March 2020, and the corresponding route operating cost—estimated using Eq. (26) on the currently operated routes. A gross margin threshold of 12.5% is the best simplification of our case study, which also enables a fair *apple-to-apple* comparison with the baseline case (based on the current setting).⁶ Additionally, we performed a sensitivity analysis to investigate how different values of gross margin threshold would affect the results (see [Appendix](#) for details).

3.2. Evaluation of the results

The auction framework was implemented using a python script on a computer with an Intel core i5-6200U CPU. Each Scenario took on average 30 minutes to run. The ABPM for each bundle and the WDP were solved to optimality in reasonable time (at most two minutes) using SCIP ([Gamrath et al., 2020](#)).

The outputs of the SCAS framework are: the winning airlines, the bundles and regions they have won, and the routes they intend to fly. Each route has service characteristics, i.e., the number of flights, the one-way airfare, the number of passengers, travel time, and whether it is non-stop or one-stop.

We use these outputs to form key indicators for the evaluation of alternative tendering processes for subsidised routes. We use *social welfare indicators* for the three main actors (i.e., airlines, government, and passengers). Specifically, the airline surplus is the sum of the operating profits gained by all the winning airlines. The government expenditure is captured by the total amount of subsidies. Finally, we compute passenger surplus based on the notion of consumer surplus. Using a discrete choice setting, the consumer surplus is calculated using the log-sum measure ([McFadden, 1978](#); [De Jong et al., 2007](#)). An individual's consumer surplus is defined as the utility in monetary terms that the individual receives from a given choice. Similar to [De Jong et al. \(2007\)](#), we define each person's consumer surplus along route i as CS_i based on the utility model; $CS_i = (1/\alpha_{inc}) \cdot \ln(e^{u_0} + e^{u_i})$, where u_i is the utility from air travel, u_0 is the utility of the non-air alternative, and α_{inc} is the marginal utility of income. In the cases where airfare or cost variables enter the utility function, and in a linear fashion as in our case, the negative of its coefficient (β_3) gives the α_{inc} by definition. The factor $1/\alpha_{inc}$ translates utility into monetary terms for comparability. The total consumer surplus in an O–D market can be calculated as the sum of each individual's consumer surplus (CS_i). Then, the total consumer surplus in the whole network is calculated by summing over all the O–D markets.

We also consider a set of *accessibility indicators*: the number of flights; the number of passengers; the percentage of detoured passengers as an indicator of the number of passengers inconvenienced by travelling along a one-stop route instead of a non-stop route; the passenger weighted average airfare, calculated as the sum-product of the airfare and number of passengers on all routes divided by the total number of passengers; and the generalised travel cost (GTC), which expresses the monetary cost of travelling and is calculated by converting each route i 's travel time t_i^{travel} , connecting time t_i^{conn} , and number of flights y_i to a monetary value, that is, $GTC_i = \frac{\beta_1}{\beta_3} t_i^{travel} + \frac{\beta_2}{\beta_3} t_i^{conn} + \frac{\beta_4}{\beta_3} y_i + p_i$.

3.3. Policy analysis

We use the auction framework to compare alternative ways of setting up the tenders for subsidised routes. These alternatives are based on:

1. *The service requirements.* These are also referred to as caps. We consider two requirements, i.e., the maximum airfare and a minimum number of daily flights. The combinations of enforcing and relaxing the service caps give four scenarios. *Scenario 1* (the current situation and baseline) has a cap on both maximum airfare and a minimum number of daily flights. In *Scenario 2*, we relax the flight cap but not the airfare cap. In *Scenario 3*, we relax the airfare cap but not the flight cap. *Scenario 4* neither has a flight nor an airfare cap, which gives the airlines total freedom concerning both.

⁵ Note that although we consider these airlines to bid using fixed aircraft fleet, the SCAS can trivially be extended and generalised to accommodate endogenous fleets (instead of fixed).

⁶ Notice, however, that in reality, airlines can have different gross margin thresholds. The SCAS can easily accommodate airline-specific gross margin thresholds, which can be derived from financial reports or ad-hoc network considerations.

Table 4
Minimise subsidies—Annual social welfare results under different requirements (in million \$).

Scenario	Requirements		Gov. subsidies				Airline profits		Pax surplus	
	Max. airfare	Min. flights	Total		Per pax		Value	Var. (%)	Value	Var. (%)
			Value	Var. (%)	Value	Var. (%)				
1 ^a	Yes	Yes	17.9	=	170.2	=	4.3	=	9.9	=
2	Yes	No	5.1	-71.5	50.8	-70.2	3.2	-25.6	8.4	-15.2
3	No	Yes	15.5	-13.7	171.3	0.6	4.2	-2.3	5.4	-45.0
4	No	No	3.8	-78.7	44.9	-73.6	3.3	-23.3	4.7	-52.5

Notes: Var. means variation.

^aIndicates the baseline scenario.

Table 5
Minimise subsidies—Annual number of passengers (# pax), and percentage of detoured passengers (% of detoured pax), generalised travel cost (GTC) in million \$, and passenger weighted average airfare (Avg. Airfare) in \$.

Scenario	Requirements		Avg. Airfare		# pax		% detoured pax		GTC	
	Max. airfare	Min. flights	Value	Var. (%)	Value	Var. (%)	Value	Var. (%)	Value	Var. (%)
1 ^a	Yes	Yes	124.8	=	105,408	=	45.4	=	0.7	=
2	Yes	No	123.9	-0.7	100,800	-4.4	24.0	-21.4	0.8	11.2
3	No	Yes	167.8	34.5	90,432	-14.2	48.4	3.1	1.0	44.6
4	No	No	160.6	28.7	85,248	-19.1	35.8	-9.5	1.1	48.0

Notes: Var. means variation.

^aIndicates the baseline scenario.

2. *The evaluation criterion.* The criterion of determining the winner might affect the outcome of the auction. We begin the analysis with a criterion based solely on minimising subsidies then compare how the results would change when a transportation authority uses a criterion combining minimising subsidies and maximising the number of passengers served.
3. *The subsidy policy.* We compare two subsidy policies, i.e., a lump sum subsidy policy where the airlines bid a total subsidy for serving regions in a given bundle, and a passenger discount policy where the transportation authority gives passengers a discount as a percentage of the airfare charged by airlines.

The service requirements

We begin the analysis by comparing the four scenarios when minimising subsidies under a lump-sum subsidy policy. Scenario 1 captures the main characteristics of the current PSO network in Sweden, which we use as our baseline and compare to the other three scenarios. The main results are presented in Table 4. Table 5 then provides some accessibility indicators to better appraise the solutions and supply configurations obtained under the different scenarios.

First, we note that having no flight cap as in Scenario 2 and Scenario 4, results in less government subsidies than scenarios (Scenario 1 and Scenario 3) where a flight cap is required. When solely minimising subsidies under a lump-sum subsidy policy, airlines tend to serve the minimum number of flights to minimise operating costs and bid for lower subsidies. This leads to 2 daily flights (10,368 annual flights) in case of a floor on the minimum number of return flights (Scenario 1 and Scenario 3) or 1 daily flight (5184 annual flights) in case of no floor on the minimum number of return flights (Scenario 2 and Scenario 4). Enforcing a flight cap of 2 daily return flights increases the airline operating cost without significant incentive for the airlines to capture more demand, for example, by lowering the airfares. The need to satisfy the flight cap thus leads to higher operating costs, which cannot be compensated by an appropriate increase in the revenue due to the low elasticity of demand. Therefore, the airlines require higher subsidies from the government to achieve a reasonable gross margin.

Although having a maximum airfare and a minimum number of flights as in Scenario 1 is in the best interest of the consumers with passenger surplus of \$9.9 million, it is not in the best interest for the government in terms of the amount of subsidies paid. Scenario 1 requires subsidies of \$17.9 million whilst relaxing the flight cap to just the bare minimum of 1 daily return flight as in Scenario 2 would require \$5.1 million, which is lower by 71.5%.

Scenario 4, which has neither a maximum airfare nor a minimum of two daily return flights would require the lowest government subsidies (\$3.8 million). However, this total freedom is bad for the customers because the airlines would exploit their market power and charge high average airfares. Relative to Scenario 1 and Scenario 2, both with an airfare cap, Scenarios 3 and 4 with no airfare cap result in an average of 32.0% higher airfares and 14.8% lower demand; consequently, fewer passengers would be served and the passenger surplus would be the lowest (\$4.7 million in Scenario 4 and \$5.4 in Scenario 3). Note, however, that the increase in airfares is still reduced due to the fact that demand is made elastic in the ABPM and too high airfares would lead to a significant drop in demand (with an overall negative impact on profits).

Scenario 3 is undesirable. Under this scenario, the government enforces a flight cap (thus increasing airline operating cost) but gives the airline freedom in setting the prices. In turn, this leads to an undesirable outcome for both passengers and government, characterised by lower passenger surplus (similar to Scenario 4) and high subsidies (similar to Scenario 1).

Finally, we observe that Scenario 2, with an airfare cap but no cap on the number of flights, provides a compromise between total restriction (Scenario 1) and total freedom (Scenario 4) that suitably trades off the different actors' interests. Scenario 2 increases

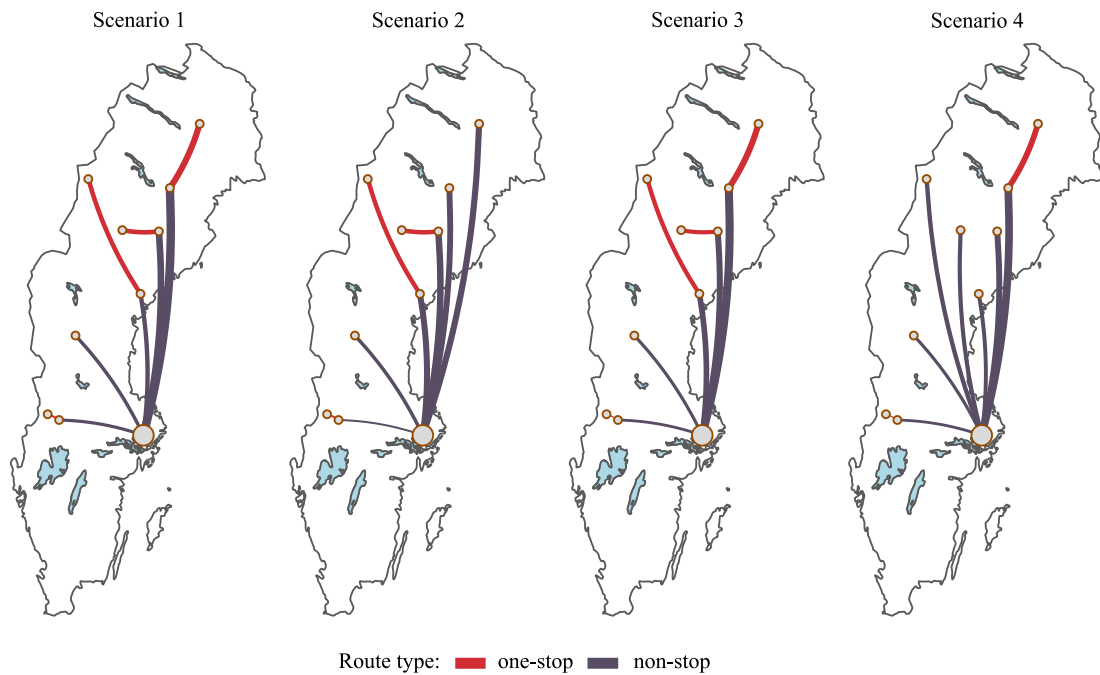


Fig. 3. Non-stop and one-stop routes for each scenario. The lines are sized by the proportion of annual passengers to and from an airport. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

the number of passengers and passenger surplus in Scenario 4 by 18.2% and 78.7%, respectively. At the same time, it would reduce the high subsidies of Scenario 1 by 71.5%. However, from an accessibility perspective, it also means the reduction from two daily return flights, to only one flight, hence a reduction in passenger surplus of 15.2%. Yet, we notice that this drop in accessibility is not too pronounced.

Scenario 2 leads to only -4.4% passengers compared to Scenario 1. More importantly, we observe that having no flight cap (as in Scenario 2 and Scenario 4) results in significant lower percentages of detoured passengers as compared to scenarios with a flight cap—24% and 35.8% in Scenario 2 and 4, while 45.8% and 48.4% under Scenarios 1 and 4—, thus partially compensating for the lower frequencies. For example, in Fig. 3, Scenario 2 has three more non-stop routes (purple links) than one-stop routes (red links) compared to Scenario 1 where airlines use more one-stop routes. Collectively, these results further corroborate the choice of Scenario 2 as a good trade-off solution.

The evaluation criterion

We now analyse the impact of changing the winner determination criterion, from solely minimising subsidies to a combination of minimising subsidies and maximising the number of passengers served. To better capture the goals of minimising subsidies and maximising accessibility, the transportation authority may include the number of passengers explicitly in the evaluation criterion. This leads to a multi-objective function in the ABPM as stated in Eq. (15) for values of ω_s between 0 and 1. Recall that the ω_s denotes the relative weight given to subsidy minimisation at the expense of passenger maximisation. In Fig. 4 we report the Pareto frontiers obtained by varying the ω_s parameter under the different scenarios.

As expected, by plotting the subsidies on the x-axis and the annual passengers on the y-axis, we obtain curves with a regular downward concave shape, denoting that the number of passengers can be increased at the expense of more subsidies. Therefore, any evaluation criterion that minimises subsidies and maximises the number of passengers served results into more passengers served than when the transportation authority solely minimises subsidies (i.e., $\omega_s = 1$).

The effect of the weights varies with the restrictions imposed by the transportation authority. To clarify this, consider a shift from the out-most left point with $\omega_s = 1$ (current situation) to $\omega_s = 0.9$ and the summary presented in Table 6. Between weights of 1 and 0.9, Scenario 3 and 4 reasonably have the most responsive number of passengers due to an increase in government subsidies—that is, 6,511 and 5,583 passengers per an additional \$1 million of subsidies, respectively, compared to 2,658 for Scenario 1 and 2,674 for Scenario 2. This shows how explicitly prioritizing the number of passengers in the evaluation criterion may effectively mitigate the drawback of not enforcing an airfare cap, i.e., having high airfares at the expense of the number of passengers.

If the transportation authority attributes a very low weight to the minimisation of subsidies (e.g., 0.30, 0.20, and 0.10) and consequently a very high weight to the maximisation of the number of passengers, all four Scenarios would result into the same number of passengers. However, having no flight cap as in Scenario 2 and Scenario 4 would result into lower government subsidies than Scenario 1 and Scenario 3 with flight cap.

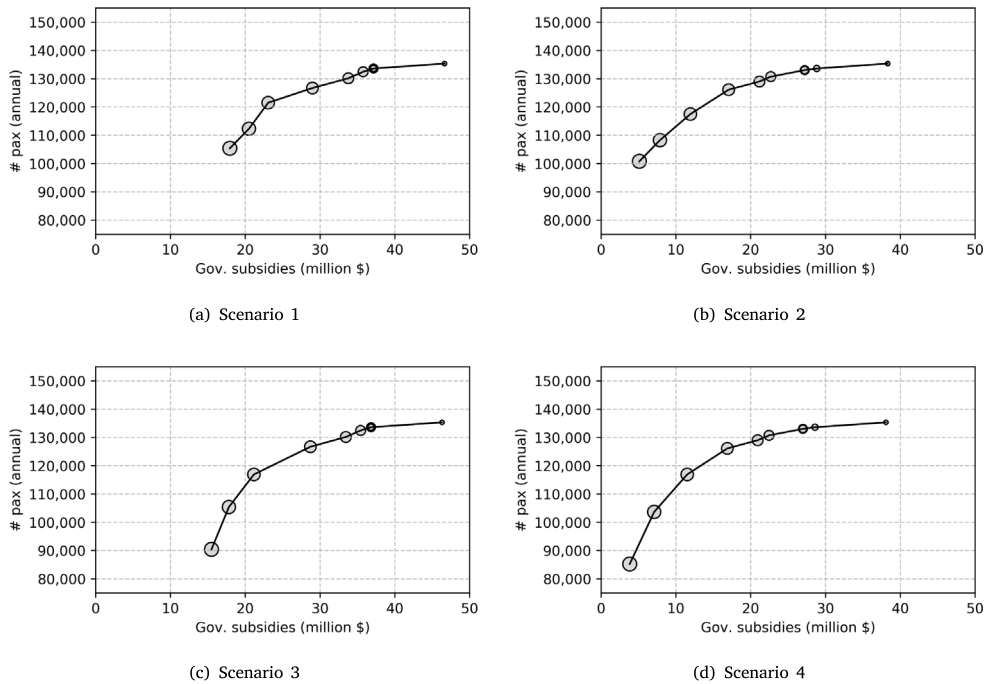


Fig. 4. Change in subsidies and annual passengers for an evaluation criterion combining subsidies and number of passengers. The size of the circle is proportional to the weight attributed to subsidies (the highest weight $w_s = 1$ corresponds to a largest circle) and consequently the number of passengers.

Table 6

Increase in government subsidies (Gov. subsidies) in million \$ and number of passengers (# pax) due to a change in w_s from 1 to 0.9. The percentage increase is indicated in parenthesis.

Scenario	Requirements		Gov. subsidies	# pax
	Max. airfare	Min. flights		
1 ^a	Yes	Yes	2.6 (14.2)	6912 (6.6)
2	Yes	No	2.8 (53.8)	7488 (7.4)
3	No	Yes	2.3 (15.0)	14,976 (16.6)
4	No	No	3.3 (85.3)	18,432 (21.6)

^aIndicates the baseline scenario.

Note that none of the solutions reported here in the pareto frontier are strictly dominated. This analysis has shown how the SCAS can be used to identify a set of optimal candidates under a bi-objective evaluation criterion. Ultimately, the best solutions should be selected according to the utility function of the governments and their budget availability.

The subsidy policy

We now test how a change in subsidy policy affects the outcome. We consider a possibility of the transportation authority offering a percentage discount on the airfare directly to the passengers, still with the possibility of an additional lump-sum subsidy to the airlines. Transportation authorities may use passenger discounts to encourage more passengers. A passenger discount policy is, for example, used in Spain where island residents are offered a 30% to 50% discount on the airfare. Specific to the Swedish case, the subsidy scheme aims to provide air services for commuting purposes and does not differentiate between residents and non-residents. This is in part motivated and justified by the fact that the portion of leisure/non-resident travellers on those routes is small (Andersson, 2017). Therefore, in this paper, we do not differentiate between residents and non-residents.⁷ The ABPM in Section 2.2.1 is for the case of a lump-sum subsidy policy but it can be modified for use in the case of a passenger-discount subsidy policy. In this case, the passengers would receive a percentage discount on the airfare charged by airlines. The utility Constraints (5) in the ABPM are re-written as $u_i = \beta_0 + \beta_1 t_i^{travel} + \beta_2 t_i^{conn} + \beta_3(1 - \delta)p_i + \beta_4 y_i, \forall i \in I$, where $\delta \in [0, 1]$ captures the percentage discount on the airfare p_i .

⁷ Note, however, that our framework can flexibly accommodate the differential application of discount prices, conditional upon the fact that we can discriminate and specify unique demand models for different passenger segments.

Table 7

The number of passengers (# pax), passenger weighted average airfare (Avg. Airfare) in \$, passenger surplus in million \$, and government subsidies (Gov. subsidy) in million \$, under a passenger-discount subsidy policy.

Scenario	Subsidy policy	Avg. Airfare				# pax		Pax surplus		Gov. subsidies			
		Paid by pax		Charged		Value	Var. (%)	Value	Var. (%)	Total		Per pax	
		Value	Var. (%)	Value	Var. (%)					Value	Var. (%)	Value	Var. (%)
1 ^a	lump-sum	124.8	=	124.8	=	105,408	=	9.9	=	17.9	=	170.2	=
	30% discount	88.3	-29.2	126.1	1.1	116,928	10.9	14.3	44.0	20.9	16.3	178.4	4.8
	50% discount	63.5	-49.1	127	1.8	121,536	15.3	17.4	75.5	23.5	30.8	193.1	13.4
2	lump-sum	123.9	=	123.9	=	100,800	=	8.4	=	5.1	=	50.8	=
	30% discount	87.8	-29.1	125.4	1.2	116,928	16.0	12.9	54.2	7.1	39.4	61.0	20.1
	50% discount	63.2	-49.0	126.4	2.0	122,688	21.7	16.1	92.2	9.8	90.8	79.6	56.8
3	lump-sum	167.8	=	167.8	=	90,432	=	5.4	=	15.5	=	171.3	=
	30% discount	167.8	=	239.7	42.9	90,432	=	5.4	=	15.5	=	171.3	=
	50% discount	148.9	-11.3	297.8	77.5	97,344	7.6	7.3	33.4	16.2	4.4	166.1	-3.0
4	lump-sum	160.6	=	160.6	=	85,248	=	4.7	=	3.8	=	44.9	=
	30% discount	153.8	-4.2	219.7	36.8	80,640	-5.4	4.9	3.0	3.8	=	47.5	5.7
	50% discount	137.9	-14.1	275.8	71.7	97,920	14.9	7.2	52.4	4.0	5.6	41.3	-8.1

Notes: Var. means variation.

^aIndicates the baseline scenario.

In [Table 7](#), we present some key metrics to facilitate a comparison of two levels of passenger discounts—30% and 50%—with the results under a pure lump-sum policy.

An introduction of a passenger discount on the airfare attracts more passengers compared to when a lump-sum subsidy policy with 0% passenger discount is used. As a result, the passenger surplus also increases; however, more government subsidies would be required to finance this increase.

For both Scenario 1 and Scenario 2, a 30% passenger discount attracts an equal number of passengers. Although the passengers seem to be better off in Scenario 1 (i.e., higher passenger surplus due to the flight cap), notice that Scenario 2 (with no flight cap) requires lower subsidies (\$7.1 million compared to \$20.9 million).

A higher passenger discount, for example, 50% further increases the number of passengers, increases the passenger surplus, and requires more subsidies. The effect of not having a flight cap can still be seen in [Table 7](#). For a 50% passenger discount, Scenario 2 gives more passengers but slightly lower passenger surplus than in Scenario 1—because the airlines simply increase the number of flights to match the number of passengers—, and requires significantly lower government subsidies.

3.4. Policy guidelines

Using the studied requirements in the PSO tendering process, that is, minimum number of daily return flights and maximum airfare, seems to be an effective way to increase the passenger surplus, but this in-turn requires high subsidies. For the PSO routes under consideration, requiring two daily return flights to accommodate the low demand severely increases the government subsidies because of the additional operating cost for the airlines. The cap on the airfare seems to fulfil its purpose in ensuring that significant demand is not lost due to high airfares, but this also requires higher subsidies. A good trade-off between the current restrictive setup (with both a flight cap and an airfare cap) and total freedom setup of the tendering process (with no restrictions) can be achieved by having an airfare cap and no flight cap. This would increase the number of passengers and the passenger surplus relative to the total freedom setup, and significantly reduce the government subsidies relative to the current restrictive setup.

Transportation authorities can compensate not having a requirement on the number of daily flights through ways of ensuring a higher number of passengers. They may include maximisation of the number of passenger in the bid evaluation criterion or the use of passenger discounts. For both alternatives, more government subsidies would also be required. Ultimately, the best solution should be identified by explicitly taking into account the transportation authority's interests and subsidy budget.

4. Conclusion

In this study, we develop an integrated auction framework—referred to as SCAS—to provide decision support to transportation authorities when designing tendering processes for subsidised routes. We develop two models as ingredients to the integrated framework. First, the ABPM, which replicates the airline's behaviour when preparing bids for subsidised routes. Second, the WDP, which is used to select the bids based on given evaluation criterion.

We have applied the auction framework to the PSO network of subsidised routes in Sweden by providing policy guidelines, and demonstrated its usefulness when designing tendering processes for subsidised routes. Specifically, we analyse the effect of the service requirements, the evaluation criterion, and the subsidy policy on the tender outcomes. We conclude that having a restriction on the airfare but not the number of flights gives a good trade-off—in-terms of passenger surplus and government subsidies—between a restrictive setup (i.e., the current one, with both a flight cap and an airfare cap) and total freedom setup of the tendering process (with no restrictions). Additionally, we demonstrate how the transportation authority may ensure that more passengers are

served by including the maximisation of the number of passengers in the bid evaluation criterion, or the use of passenger discounts. Collectively, these results highlight how the SCAS could be used as a decision support tool for transportation authorities when designing an efficient tender for subsidised routes.

Next, we highlight four directions for future research. First, it should be noted that our modelling framework does not capture time-of-day preferences (although it accounts for schedule delay in aggregate terms through the inclusion of frequency in the utility function). Thus, including a more explicit proxy of time-of-day preferences would be an interesting approach to better capture the impact of departure time preferences on the solutions. The empirical specification of the demand model may also be improved to capture the impact of additional supply attributes, as well as route and passenger heterogeneity. Second, we note that the SCAS is formulated as a single round auction—with many bidding airlines and their characteristics—, which neither explicitly considers collaborative decision making between authorities and airlines nor incorporate collusive/competitive airline dynamics (although it could provide valid methodological basis for it). The consideration of these aspects through a game theoretic setting constitutes an interesting avenue for future research. Third, consideration of the emergence of electric aircraft would be interesting. These are initially expected to be quite small, and with a limited range. Thus, they might be suited for thin PSO routes, which however may have to be designed as multi-stop routes to accommodate the limited range. Fourth, a log-form cost function provided the best fit for our empirical setting, but an extensive exploration of other functional forms and the capturing of heterogeneity regarding the efficiency of individual aircraft types is an interesting consideration for future research.

CRedit authorship contribution statement

Alan Kinene: Conceptualization, Methodology, Formal analysis, Software, Data curation, Writing – original draft, Writing – review & editing. **Tobias Andersson Granberg:** Conceptualization, Supervision, Writing – review & editing, Funding acquisition. **Sebastian Birolini:** Methodology, Formal analysis, Data curation, Writing – original draft, Writing – review & editing. **Nicole Adler:** Conceptualization, Methodology, Validation, Writing – review & editing. **Valentin Polishchuk:** Conceptualization, Funding acquisition. **Jean-Marie Skoglund:** Resources.

Appendix. Sensitivity analysis

To test the robustness and validity of the modelling framework, we perform a sensitivity analysis on three key inputs: the utility parameter estimates, the gross margin threshold and the potential demand—the results are presented in Tables A.8, A.9, and A.10, respectively. For each variation in the three key inputs—taking the parameter values used when minimising subsidies under a lump-sum subsidy policy as reference (results in Tables 4 and 5)—the tables present the annual government subsidies, airline profits, annual passenger surplus, and number of passengers.

First, we test the impact of the variation in the utility parameter estimates by varying the elasticity associated with the flight time t_i^{travel} (β_1), the connection time t_i^{conn} (β_2), the airfare p_i (β_3), and the frequency y_i (β_4) by +10% and –10% with respect to the baseline values. Reasonably, an increase (decrease) in the utility parameter for airfare (i.e., the demand-price elasticity) and for flight time results in a decrease (increase) in passenger surplus mostly due to lower demand, a decrease (increase) in the airline profits, and an increase (decrease) in government subsidies. Changing the elasticity of connection time by +/-10% has no significant effect on the passenger surplus, the airline profits, and the government subsidies. This follows from the reduced use of connecting routes and limited impact of connecting times on passengers' utility. On the other hand, an increase (decrease) in the parameter estimate for the frequency increases (decreases) the passenger surplus, increases (decreases) the airline profits, and decreases (increases) government subsidies.

Second, we test the impact of demand uncertainty by varying the potential demand by +10% and –10% (see Table A.9). In reference to our empirical setup (and the assumed potential demand), an increase in the potential demand would result into more paying passengers, which would result into higher airline services and the government would pay less subsidies. Indeed, with more demand, the markets would be more self sustaining—from a commercial point of view. Increase in future demand is beneficial for the passengers, airlines and the government. On the other hand, a decrease in the potential demand results into less paying passengers, lower passenger surplus, lower airline profits, and the government would pay more subsidies. These results illustrate how the SCAS can be used to assess and account for uncertainty in the demand via different various implementation scenarios.

Third, we test the impact of a variation in the gross margin threshold by considering values of 15.0%, 12.5% (used for our analysis), 10.0%, and 6.0% (see Table A.10). In reference to our application case (and the used gross margin threshold of 12.5%), considering a higher gross margin threshold would imply more subsidies from the government while a lower gross margin threshold results into airlines requiring less subsidies from the government. Of course, a lower gross margin threshold means a reduction in airline profits. Generally, a variation in the gross margin threshold does not significantly affect the passenger surplus. Interestingly, we notice that having a lower gross margin threshold and no cap on neither the airfare nor the number of flights slightly benefits the passengers as the airlines have more incentive to attract more demand by lowering the airfares.

Overall, despite the effect of these three key inputs, these results remark the applicability of our modelling framework and the highlighted major insights. In particular, they confirm that having a restriction on the airfare but not the number of flights gives a good trade-off between a restrictive setup (i.e., the current one, with both a flight cap and an airfare cap) and total freedom setup of the tendering process (with no restrictions).

Table A.8
Sensitivity analysis for the utility parameter estimates.

Parameters [Δ %]	Scenario	Requirements		Gov. subsidies ^a		Airline profits ^a		Pax surplus ^a		# pax	
		Max. airfare	Min. flights	Value	Var. (%)	Value	Var. (%)	Value	Var. (%)	Value	Var. (%)
β_1 [0]	1	Yes	Yes	17.9	=	4.3	=	9.9	=	105,408	=
	2	Yes	No	5.1	=	3.2	=	8.4	=	100,800	=
	3	No	Yes	15.5	=	4.2	=	5.4	=	90,432	=
	4	No	No	3.8	=	3.3	=	4.7	=	85,248	=
β_1 [+10]	1	Yes	Yes	18.4	2.4	3.9	-8.4	9.5	-3.7	104,832	-0.5
	2	Yes	No	5.2	2.5	3.0	-8.2	7.9	-5.5	99,072	-1.7
	3	No	Yes	16.2	4.5	3.8	-8.6	5.3	-3.4	89,280	-1.3
	4	No	No	4.1	7.1	3.0	-8.7	4.6	-1.7	84,672	-0.7
β_1 [-10]	1	Yes	Yes	17.5	-2.7	4.3	=	10.8	9.3	108,864	3.3
	2	Yes	No	5.0	-2.6	3.4	6.0	8.8	5.1	101,376	0.6
	3	No	Yes	14.7	-5.1	4.2	0.0	5.7	5.2	92,160	1.9
	4	No	No	3.6	-7.1	4.1	23.8	5.2	11.4	89,280	4.7
β_2 [+10]	1	Yes	Yes	18.0	0.1	4.3	=	9.9	-0.1	105,408	=
	2	Yes	No	5.1	0.2	3.2	-0.1	8.4	=	100,800	=
	3	No	Yes	15.5	0.1	4.2	=	5.4	=	90,432	=
	4	No	No	3.8	0.1	3.3	-0.3	4.7	=	85,248	=
β_2 [-10]	1	Yes	Yes	17.9	-0.1	4.3	=	10.0	0.7	105,984	0.5
	2	Yes	No	5.1	-0.2	3.2	=	8.4	=	100,800	=
	3	No	Yes	15.5	-0.1	4.2	=	5.4	=	90,432	=
	4	No	No	3.8	-0.1	3.3	0.3	4.7	=	85,248	=
β_3 [+10]	1	Yes	Yes	18.6	3.8	3.8	-9.7	7.9	-20.1	100,224	-4.9
	2	Yes	No	5.3	3.5	2.6	-20.6	6.4	-23.9	93,312	-7.4
	3	No	Yes	17.0	9.5	3.8	-8.6	4.9	-9.1	90,432	=
	4	No	No	4.5	18.6	2.8	-16.1	4.3	-9.1	85,248	=
β_3 [-10]	1	Yes	Yes	17.0	-5.5	4.3	=	12.6	27.6	110,592	4.9
	2	Yes	No	4.9	-4.0	3.6	11.3	10.5	24.9	102,528	1.7
	3	No	Yes	13.5	-13.0	4.2	=	6.0	11.1	90,432	=
	4	No	No	3.0	-22.7	4.6	39.6	5.7	21.7	88,704	4.1
β_4 [+10]	1	Yes	Yes	17.8	-0.9	4.3	=	10.4	5.0	107,136	1.6
	2	Yes	No	5.1	-1.3	3.3	1.2	8.5	1.7	101,376	0.6
	3	No	Yes	15.2	-2.2	4.2	=	5.6	3.6	91,584	1.3
	4	No	No	3.7	-2.3	3.4	2.0	4.7	=	85,248	=
β_4 [-10]	1	Yes	Yes	18.0	0.5	4.3	=	9.7	-2.4	105,408	=
	2	Yes	No	5.2	1.4	3.2	-1.2	8.5	0.9	100,800	=
	3	No	Yes	15.8	2.1	4.2	=	5.3	-1.9	89,856	-0.6
	4	No	No	3.9	2.3	3.2	-2.0	4.7	=	85,248	=

Notes: Var. means variation wrt baseline scenario; β_1 is the utility coefficient of the flight time, β_2 is the utility coefficient of the connection time, β_3 is the utility coefficient of the airfare, and β_4 is the utility coefficient of the frequency; Δ % is the percentage variation in the estimate of the utility parameter.

^aIndicates values in million \$.

Table A.9
Sensitivity analysis for changes in potential demand.

Δ (%)	Scenario	Requirements		Gov. subsidies ^a		Airline profits ^a		Pax surplus ^a		# pax	
		Max. airfare	Min. flights	Value	Var. (%)	Value	Var. (%)	Value	Var. (%)	Value	Var. (%)
+10	1	Yes	Yes	13.5	-25.0	6.4	49.5	14.7	48.5	155,520	47.5
	2	Yes	No	3.4	-34.1	5.6	73.7	11.4	36.1	139,968	38.9
	3	No	Yes	10.9	-29.5	7.2	71.6	6.7	22.6	122,112	35.0
	4	No	No	2.1	-44.7	8.3	149.4	6.0	27.1	115,776	35.8
0	1	Yes	Yes	17.9	=	4.3	=	9.9	=	105,408	=
	2	Yes	No	5.1	=	3.2	=	8.4	=	100,800	=
	3	No	Yes	15.5	=	4.2	=	5.4	=	90,432	=
	4	No	No	3.8	=	3.3	=	4.7	=	85,248	=
-10	1	Yes	Yes	21.2	18.3	3.8	-9.7	7.6	-23.0	78,336	-25.7
	2	Yes	No	7.2	40.9	2.0	-37.9	6.3	-24.5	73,728	-26.9
	3	No	Yes	19.7	27.1	3.8	-8.6	4.1	-24.8	66,240	-26.8
	4	No	No	5.7	50.0	1.9	-42.0	2.8	-39.8	57,024	-33.1

Notes: Var. means variation; Δ (%) is the percentage change in potential demand.

^aIndicates values in million \$.

Table A.10
Sensitivity analysis for changes in gross margin.

% Margin	Scenario	Requirements		Gov. subsidies ^a		Airline profits ^a		Pax surplus ^a		# pax	
		Max. airfare	Min. flights	Value	Var. (%)	Value	Var. (%)	Value	Var. (%)	Value	Var. (%)
15	1	Yes	Yes	18.9	5.6	5.3	23.5	9.9	=	105,408	0.0
	2	Yes	No	5.5	6.7	3.6	10.6	8.4	=	100,800	0.0
	3	No	Yes	16.5	6.4	5.2	23.5	5.4	=	90,432	0.0
	4	No	No	4.2	9.0	3.7	10.4	4.7	=	85,248	0.0
12.5	1	Yes	Yes	17.9	=	4.3	=	9.9	=	105,408	=
	2	Yes	No	5.1	=	3.2	=	8.4	=	100,800	=
	3	No	Yes	15.5	=	4.2	=	5.4	=	90,432	=
	4	No	No	3.8	=	3.3	=	4.7	=	85,248	=
10	1	Yes	Yes	17.0	-5.3	3.3	-22.2	9.9	=	105,408	=
	2	Yes	No	4.8	-6.3	2.9	-10.0	8.4	=	100,800	=
	3	No	Yes	14.6	-6.0	3.3	-22.2	5.4	=	90,432	=
	4	No	No	3.5	-8.5	3.3	0.1	5.2	9.6	88,704	4.1
6	1	Yes	Yes	15.6	-13.1	1.9	-55.3	9.9	=	105,408	=
	2	Yes	No	4.3	-15.7	2.4	-24.9	8.4	=	100,800	=
	3	No	Yes	13.2	-15.0	1.9	-55.3	5.4	=	90,432	=
	4	No	No	3.0	-21.0	2.8	-14.5	5.2	9.6	88,704	4.1

Notes: Var. means variation.

^aIndicates values in million \$.

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