



# Mathematics and Geometry in the Nembro Reinforced Concrete Arch Bridge

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## Abstract

Resiliency in concrete infrastructures has become an important issue. Adequate conservation/maintenance plans are essential to ensure their lifespan and preserve their value. Monitoring processes to assess their condition is an essential prerequisite which may benefit from cameras and UAVs. This paper focuses on the experience taken on Nembro arch bridge.

**Keywords** 3D laser scanning · Digital photogrammetry · Multispectral images · Cultural heritage · Conservation

## Introduction

The invention of reinforced concrete is the result of scientific progress that has improved methodologies and techniques and has enabled the creation of increasingly complex and daring architectural works. First used in Europe in the late 19<sup>th</sup> century and then developed throughout the 20<sup>th</sup> century, the material revolutionized the construction industry, so marking a turning point in the growth of the built environment.

Thanks to the intuition and skills of great designers, it was soon converted into buildings and infrastructures, which are evidence of the ability to unifying image with structure and form with function. Constructions able to establish a meaningful

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connection between the various moments of human history, but also culturally relevant (Faccio 2008).

The exponents of the Modern Movement were enthused by this flexible and groundbreaking material, which would finally freeing architects from the constraints of the past. It captivated the engineers of the time for its innovative nature and the great potential of experimentation. The new structures were the result of a combination of mathematics and geometry, the dimensional calculation and verification of which were carried out by integrated methods, both graphical and analytic.

Their trust in the material was unconditional; they were convinced of its durability, seeing it as almost eternal. Unfortunately, the claimed immortality of the concrete was only supposed. Soon, problems arose due to the numerous variables involved in its use – affecting its condition and, therefore, its durability – and the lack of accurate predictions regarding its structural behaviour. As a result, studies on improving performance techniques and preventing premature aging, where possible, were increased (Boriani et al. 2008).

However, reinforced concrete buildings and civilian facilities, like bridges, are still at risk. The development of suitable conservation and maintenance plans is essential to ensure their longevity and to preserve their importance and value. In that sense, the inspection and/or monitoring processes for assessing their condition are an essential prerequisite, although often very difficult to reach. Nevertheless, such activities can today profit from computer-vision technique, used in conjunction with acquisition through remote cameras and unmanned aerial vehicles (UAVs), such as in the experience taken on the arch bridge over the Carso stream in Nembro (Val Seriana) on which this paper focuses (Figs. 1–3).



**Fig. 1** The bridge over the Carso stream after its construction



Fig. 2 The bridge over the Carso stream today (photogrammetric survey with SAPR)

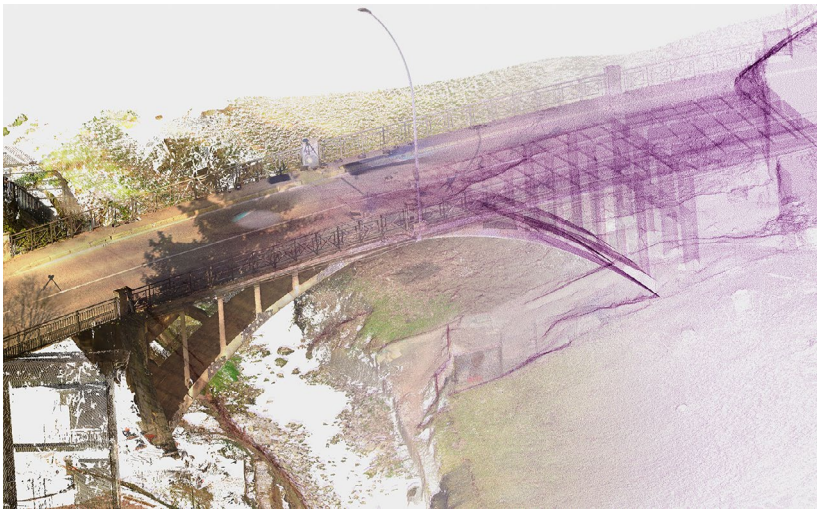


Fig. 3 The model of the bridge (photogrammetric and 3D laser scanning survey)

## The Design of the Reinforced Concrete Archway Bridge on Carso Creek

The development of the infrastructure and systems for mobility, which followed the industrial revolution, as well as the need to open new communication routes for people and goods, led to the huge adoption of reinforced concrete. The art and science of bridge building, in the 19<sup>th</sup> century mainly set in masonry and cast iron (only for

large-span works due to the high construction difficulties and costs), greatly benefited from its advent.

Vaulted stone or brick bridges were among the most widely used to overcome large natural obstacles, such as large rivers or deep gorges. Those made of reinforced concrete were their direct heirs because they were cheaper and quicker to build. The material was particularly suitable for absorbing compressive stresses; it had the advantage of creating a monolithic element that in shape and size easily adapted to the morphological characteristics of the place. It was usually cast in specifically shaped formworks containing the metal reinforcement. Once removed, they showed the skeleton divided into three main components: the deck (also called superstructure), the vault (raised, lowered, with one or more hinges) and the columns that connected the arch and the superstructure (Nelva 1990).

The first constructions were characterized by a very thick and rigid arch capable of absorbing the greatest pressures and transmitting the loads to the foundation between the impost plane and that of the haunches; the function of the deck was essentially to distribute loads linked to both its weight and crossing (as happened in those in stone or brick). Swiss engineer Robert Maillart developed a new typology of deck-stiffened arch, where the superstructure absorbed most of the effort. His first bridge built in 1901 in Zuoz, on the Swiss Inn river showed the potential of the new material. The work was highly appreciated for the lightness of its natural forms returned through a slender structure. An innovative project resulting from a precise static-geometrical study, which indicated a new aesthetic archetype sought in the interweaving of form and structure. In influencing the philosophy of design, his works have inspired many European designers of the early 20<sup>th</sup> century. The mathematical models for the calculation were closely linked to the geometric choice of the shape and aimed at a realistic ‘visualisation’ of the behaviour of the building in its natural environment.

The Maillart-type arch bridges were therefore particular structures with large wide and thin vaults, with a low moment of inertia and connected at the top to a rigid rod through vertical uprights consisting of simple pillars or partitions (full or with lighting windows). The geometric shape was studied so that all the elements were only compressed; a solution that allowed to both reduce the foundations and create lighter ribs. It was designed to withstand only the weight of the vault, which, once the concrete was hardened, it was used as a new rib for the construction of the superstructure. The bridge, therefore, had to be stiff enough in the bending compared to the arch which was much more elastic.

The calculation was conducted through graphical statics (understood as a tool based on geometry to address issues of balance of forces) with the scale drawing of the structure on which the loads represented by vectors were superimposed. The modification of the shape, to keep the loads constant, caused a different orientation of the vectors and therefore of the distribution of the shares. The system was in equilibrium when the first and last sides of the force polygon (in the case of bridges indicated as pressure curves) were coincident.

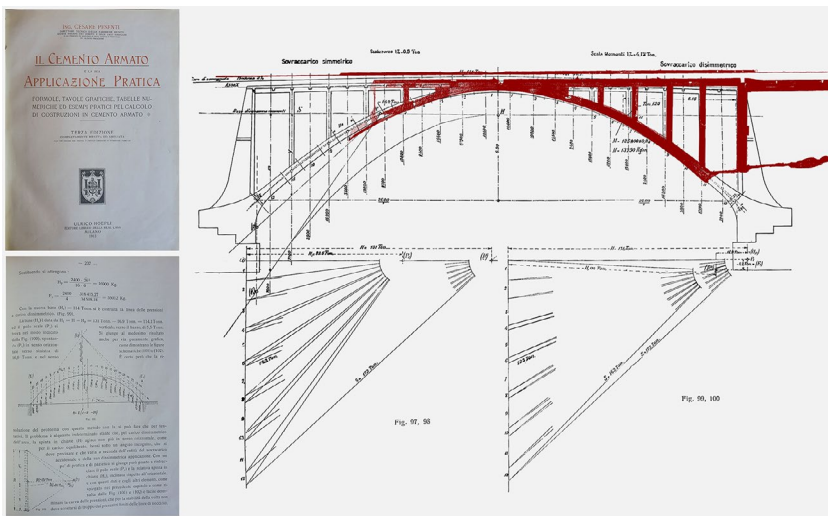
The bridge over the Carso stream, built in 1912, is a Maillart-arch type bridge which has today become an architectural work that characterizes the Serio Valley, both for its unique geometry and for being the historic gateway to the town of Nembro. It was built to enable the passage of the tram that linked the city of Bergamo to



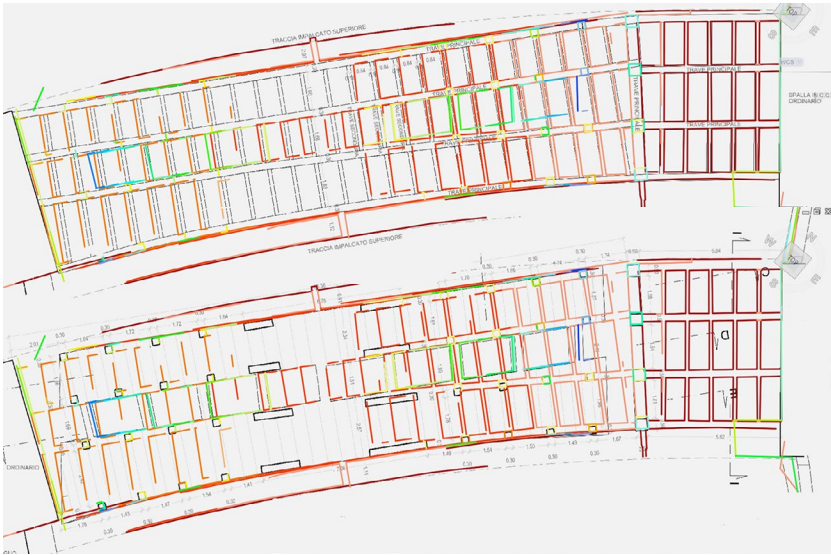
the centre of Albino (Frattini and Ravanelli 2014). The route, designed on the layout of the old road to take advantage of the existing crossings in the Carso Valley due to the steep slope and the winding route, was rethought. The designers wanted to create groundbreaking and technologically innovative work that could be an opportunity for study and experimentation. To this end, the Nembro bridge took inspiration in both the typological choice and the technical solutions from the famous ‘Risorgimento bridge’, built in Rome by the engineer Giovanni Porcheddu for the celebration of the 50<sup>th</sup> anniversary of the unification of Italy (inaugurated on 11 May 1911, it was the first reinforced concrete arch bridge on the peninsula).

The Nembro bridge was thus an opportunity for Lombard designers to apply new theories of calculation for reinforced concrete structures. It was then used as a representative example by the engineer Cesare Pesenti (one of the first in Italy to adopt the construction experimentations of reinforced concrete by developing a new system called ‘alzanese’ and the founder with his five brothers of the Italcementi company of Bergamo) in his famous manual “The reinforced concrete and its practical applications” printed by the Hoepli publishing house in 1913, and a reference text for engineering students even after the Second World War (Pesenti 1913). Cesare Pesenti’s proposed approach relies on the combined use of the graphic method and analytical evaluation (Figs. 4–6). It was based on tables to limit the difficulties linked to the use of complex formulas which were sometimes too generic and abstract.

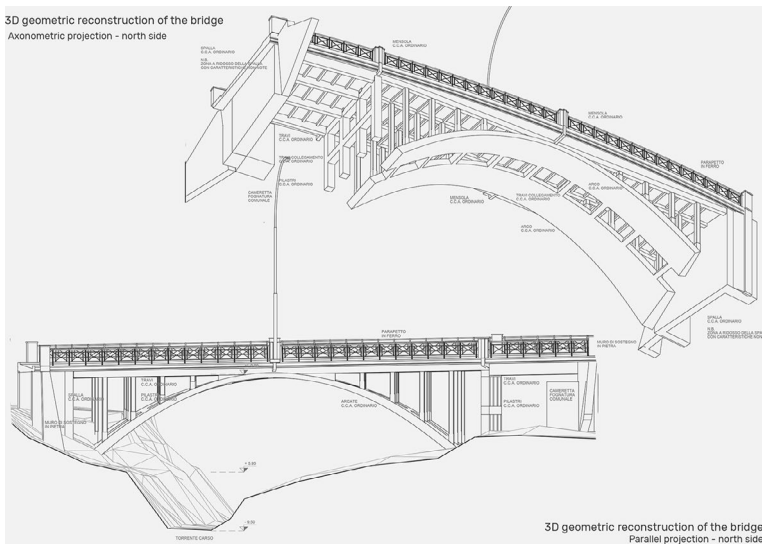
Nowadays, the simplification and the improvement in speed allowed in numerical processing by computers have reduced the importance of calculation based on graphic methods. Nonetheless, the graphical-analytical method allows for a clearer and more immediate understanding of the mechanical problem, consequently leading to the impossibility of gross errors that pure algebra is sometimes unable to avoid (Sinopoli 2012).



**Fig. 4** The bridge in the structural calculation published in Cesare Pesenti’s manual and the as-built conditions as resulted from 3D survey (in red)



**Fig. 5** Comparison between project design and construction



**Fig. 6** 3D reconstruction of the bridge from the project drawings

## Conclusion

Today, the Nembro bridge is a listed good due to its high cultural and historical value. Over a hundred years after its creation, it is however affected by severe decay phenomena and requires conservation interventions and improvements in performance. This paper, through the use of survey methodologies and techniques integrating

image-based e range-based solutions, aims to highlight the differences between the project design and its realization. An as-planned versus as-built analysis which main purpose is that of interpreting conformity with geometrical rules and the balance between the thinking of the designer and the difficulties encountered during construction. An analysis that can be very important for the establishment of repair activities aimed at increasing resiliency of a critical structure.

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## Declarations

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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