



Investments in transmission lines and storage units considering second-order stochastic dominance constraints

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ABSTRACT

Increasing intermittent renewable generation to meet the climate goals entails a deep transformation of current power systems. The transmission system must adapt to ensure a rapid and flexible response to the changes in the energy flows. Flexibility can be provided by reinforcing the interconnection among all the market agents and/or by installing facilities with fast response to the changes. In this work, we study the investment problem of a central planner that seeks to expand the transmission network and install storage units considering decarbonizing measures. We propose a two-stage stochastic problem with uncertainty on the demand growth and including representative days to characterize the hourly demand and renewable power variability. To obtain better expansion strategies according to a limit on the carbon emissions, second-order stochastic dominance constraints are imposed. Numerical analyses based on the power system of the Canary island (Spain) are provided. The results show that to install storage units is key to efficiently integrate renewable generation. The investments in the transmission system are mostly in lower-voltage lines. The formulation with stochastic dominance constraints results in higher second-stage investments, allowing a better adaptation to the demand growth evolution.

1. Introduction

1.1. Motivation

Experts have stated that July 2023 has been the hottest month ever registered and this is due to human's economic development.¹ To meet the climate goals agreed by most of the countries around the world, it is urgent to keep on increasing the electrification of energy systems and the use of renewable sources to generate such electricity. However, the intermittency of wind and solar energy is one of the major challenges that planners are facing nowadays. Thus, it becomes fundamental to adapt existing power systems to facilitate the increasing penetration of renewable energy in future power systems. This is also a relevant problem from the transmission system operator (TSO) point of view. The transmission systems of current power systems were designed to transport electricity from a reduced set of buses, where large thermal units were located, to demand buses. However, in future decarbonized power systems, most existing large thermal units will be replaced by a large number of small-sized renewable units spread over the network.

This will inevitably modify the values of the power flows in future power systems with respect to the current ones. Then, when designing the network system, the TSO must simultaneously consider that future demand is uncertain and that it must be supplied as much as possible by intermittent renewable production.

In this context, global planners, such as the European Commission (European Commission, 2020), have defined the energy strategy toward flexible energy systems, where the communication between electricity producers and consumers must flow rapidly. This entails a big challenge for TSOs since they must expand the transmission system ensuring the connection among the agents and having available tools to respond quickly to the changes in the energy flows. Storage units represent a valid instrument to provide flexibility to the system operation although the investment cost of building large facilities is still very high, e.g. Luburić et al. (2018).

In this paper, we tackle this problem. We present an investment model in transmission lines and storage units to expand an existing power system in order to accommodate a large intermittent renewable generation considering the uncertainty of the demand growth.

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¹ <https://www.nasa.gov/press-release/nasa-clocks-july-2023-as-hottest-month-on-record-ever-since-1880>

Moreover, to make the best investment decisions toward a decarbonized energy system, we want to achieve a solution that guarantees a level of carbon emissions produced by thermal power plants that is lower than another given decision, typically referred to as a benchmark. Within a stochastic optimization model, a common approach to compare two decisions is to confront their expected value. However, in some problems, this simplification could bring to an extremely risky situation because the optimal solution could have a better expected value of the benchmark but, at the same time, it could generate some extreme effect much worse than the benchmark in some scenarios. In such cases, it would be safer to compare the whole distributions generated on the considered scenarios by the optimal decision and by the benchmark. To tackle this issue the use of Stochastic Dominance (SD) is gaining ground and we also include stochastic dominance constraints in our model to consider a limit to carbon emissions produced by fossil-fuel power plants.

1.2. Literature review

The research landscape in developing investment models for the decarbonization of electric energy systems has witnessed significant contributions. Boffino et al. (2019) proposed a two-stage stochastic programming problem addressing investment decisions in generating, transmission, and storage capacity. Notably, their work includes an exploration of storage capacity expansion. Similarly, Chao and Wilson (2020) focused on coordinating investments in generating and transmission capacity under uncertainty. Notably, their work considers the diverse economic criteria within regional wholesale markets.

Allard et al. (2020) presented an energy model assessing investments in the transmission system to integrate intermittent renewable generation, with a specific application to the European system. Their model determines the necessary enlargement of the transmission network to meet climate goals. Domínguez et al. (2020) investigated the expansion of generating and storage capacity in the European power system towards 2050 using a multi-stage stochastic problem, providing insights into the operation of the resulting power system.

Gonzalez-Romero et al. (2021) introduced a bi-level framework where a central-social planner makes investment decisions in the transmission network, incorporating the perspectives of other market agents. Their study compares outcomes with those obtained from a merchant transmission investor. Grimm et al. (2021) delved into the impact of market design on investments in generation and transmission capacity, evaluating the German market.

Muñoz et al. (2023) analyzed the influence of price control on investments in renewable generating capacity in power systems with strong cross-border constraints. In Xie et al. (2023), a distributionally robust optimization method was proposed to guide expansion decisions in generating, transmission, and storage capacity, albeit without considering short- and long-term uncertainties in the decision plan.

Qiu et al. (2017), Dvorkin et al. (2017), Zhang and Conejo (2018), and Wang et al. (2019) put forth various approaches to determine investment decisions in transmission and storage capacity. Qiu et al. (2017) tackled a multi-stage stochastic problem, considering factors such as planning method, storage units, demand growth, and renewable generation. Dvorkin et al. (2017) analyzed the problem of a merchant storage investor through a tri-level model. Zhang and Conejo (2018) utilized an adaptive robust optimization framework, while Wang et al. (2019) employed robust optimization to address the investment problem. Notably, none of these works incorporated a risk measure to capture and mitigate solution risks concerning the uncertainty of the model.

The notion of SD is well-known in statistics, see Quirk and Saposnik (1962), and was typically used to check ex-post whether a distribution was able to dominate or not another distribution. There exists several types of SD relations, for a complete review see Levy (2006). The most used ones in empirical applications are the First-order Stochastic

Dominance (FSD) and the Second-order Stochastic Dominance (SSD). More recently, SD constraints were used directly within an optimization model to ensure that the optimal solution was also able to dominate a given benchmark. FSD constraints lead to a mixed-integer model, while SSD constraints can be formulated as linear constraints and, thus, their usage spread much more, see Dentcheva and Ruszczyński (2003), Post (2003), Dupačová and Kopa (2012), and Kopa and Post (2015). The SD constraints outperform other approaches to establish preferences between distributions since they consider the whole distributions and not only some statistics or some moments. For instance, the expected value of a distribution does not provide any information about the variance, or about the maximum and the minimum; similarly, the Value-at-Risk ($V@R$) and the Conditional Value-at-Risk ($CV@R$) give information about the tail of the distribution but losing the view about the expected value. SD is able to consider jointly all the mentioned statistics. In particular, SSD is consistent with the $CV@R$ as shown in Ma and Wong (2010) because if a strategy A SSD dominates a strategy B then $CV@R_\alpha(A)$ is better than $CV@R_\alpha(B)$, for any level α . The meaning of “better” relies on the type of distribution (profit vs loss), as it is discussed in Section 3.7. Clearly, in a stochastic optimization model in which the stochasticity is represented by a discrete set of scenarios, it would be possible to impose as many $CV@R$ constraints as the number of scenarios to guarantee the SSD dominance but the computing cost would be much higher.

The SSD has as natural field of application the financial problem where the aim of the model is to define a portfolio that is better than a given benchmark, see e.g. Kopa et al. (2018), Moriggia et al. (2019) and Consigli et al. (2020). However, several works explored the use of SSD constraints also within power system models. Lesser (1990) proposes to use stochastic dominance tests to compare the planning alternatives of an electricity utility. Cheong et al. (2007) analyzes the investment problem of an electricity company and applies SSD constraints to manage the risk within the objective function. Carrión et al. (2009) imposes SSD constraints in the problem of an electricity retailer that wants to determine the forward contracting purchases and the selling prices offered to its potential clients considering as uncertain parameters the future pool prices and the client demands. Jamshidi et al. (2018) focuses on a similar problem, whereas Zarif et al. (2013) studies the problem of the self-scheduling of large consumers. Escudero and Monge (2018) generally discusses modeling approaches to the capacity expansion planning considering SD to control the risk. Domínguez et al. (2021) presents a multi-stage investment model in generating and storage capacity and applies SSD to define the optimal investment strategies able to dominate the decarbonizing pathways proposed by the European Commission toward 2050.

1.3. Contributions

In this study, we consider the decision-making problem of a TSO that seeks to expand the transmission system to allow an efficient integration of intermittent renewable production. To do this, the TSO considers to install new transmission lines and/or storage units. The transmission lines can be built in new corridors, which requires a longer installation period, or in existing ones, which means to increase the capacity of existing lines. The proposed decision-making problem is formulated through a two-stage stochastic problem, where the future demand growth is considered uncertain.

In addition, we suggest to limit the maximum carbon emissions produced by non-renewable units fed by fossil fuels using Second-order Stochastic Dominance constraints (SSD). Imposing SSD constraints allows to obtain a better decision strategy with respect to an acceptable, or just known, strategy considering the whole distributions of both the optimal and the acceptable strategies and not only some scenarios as, for instance, the Conditional Value-at-Risk or some statistics as, for instance, the average.

Therefore, the contributions of this work are threefold: (i) to propose an investment model in transmission lines and storage units for TSOs considering the uncertainty of the demand growth and a limit to the carbon emissions produced by fossil-fuel units; (ii) to include SSD constraints in the optimization model to obtain better investment strategies in terms of carbon emissions; (iii) to apply the proposed model to a realistic case study based on the transmission system of Canary island and to compare the numerical results with those of a model that just limits the average carbon emissions.

2. Decision framework

The objective of this study is to obtain the optimal transmission and storage investment decisions made by the TSO to satisfy the demand. Additionally, one of the objectives of the TSO is to expand the network reducing carbon emissions from non-renewable production under a given generation capacity scenario. Observe that the TSO cannot decide the investments in new power plants, but can decide which lines and storage units can be installed to favor the procurement of the demand using as much renewable production as possible. This problem is of utmost importance for TSOs considering the increasing capacity of new renewable units that are being installed in most power systems worldwide.

This is a long-term planning problem modeled using a target year. The target year is represented by a set of representative days divided into hourly periods. Note that using an hourly characterization allows for proper capture of variabilities in demand and renewable energy throughout the year. Days are denoted by index $d \in D$, whereas hourly periods are represented by index $t \in T$.

Transmission and storage investment decisions were made under different degrees of uncertainty. The construction of new transmission lines in new corridors spans several years. We refer to lines installed in new corridors as new lines that link two buses that are not currently linked by a single line. The bureaucracy and necessary preliminary technical studies for carrying out the installation of this type of lines usually require significant time. However, building new lines using existing corridors is comparatively faster. In addition, the installation of energy storage systems based on battery systems is also rapid. Investments in new lines using existing corridors and in storage units can be made as required by the necessity of the system operation.

The main uncertain parameter in the problem faced by the TSO is the future demand in the target year. Considering the high construction times of transmission lines in new corridors, the TSO must decide which lines should be installed before knowing the actual demand of the system. Subsequently, we propose to formulate the problem faced by the TSO as a two-stage stochastic programming problem. The uncertain demand for each day and hour is modeled as a stochastic process and characterized by a set of scenarios. Observe that the investment decisions in lines in new corridors must be unique for all possible realizations of uncertain demand. Therefore, in this study, we propose to consider the installation of transmission lines in new corridors as here-and-now decisions made before uncertainty is disclosed, whereas investments in transmission lines located in existing corridors and storage units are assumed to be wait-and-see decisions made after uncertainty is revealed.

Fig. 1 illustrates the decision-making approach graphically. The operation of the resulting system was simulated in the second stage to model the influence of investment decisions on system operation.

3. Transmission and storage expansion formulation using second-order stochastic dominance constraints

The SD approach of the transmission and storage expansion problem considering SSD constraints is provided in this section. For the sake of clarity, the notation used in this section is provided in Appendix A.

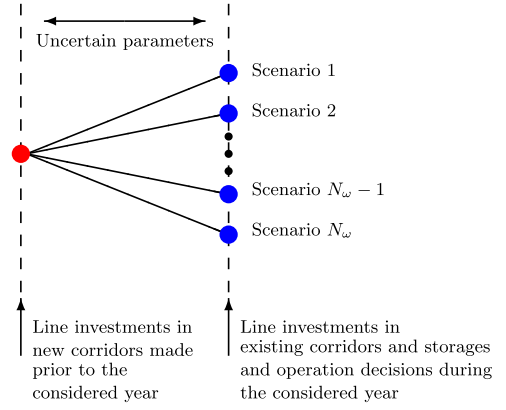


Fig. 1. Decision-making tree.

3.1. Objective function

The objective function of the transmission and storage investment problem faced by TSOs is formulated as follows:

$$\text{Minimize}_{\theta} \sum_{\ell \in L^{C,N}} C_{\ell}^{L,N} y_{\ell}^{L,N} + \sum_{\omega \in \Omega} \pi_{\omega} \left(\sum_{\ell \in L^{C,E}} C_{\ell}^{L,E} y_{\ell}^{L,E} + \sum_{s \in S} (C_s^{L,SP} p_{s\omega}^{L,S} + C_s^{L,SE} e_{s\omega}^{L,S}) \right). \quad (1)$$

This objective function aims to minimize the total investment cost incurred by the TSO, and is equal to the sum of the investment costs in the new transmission lines and storage units. Set θ contains the optimization variables. As described in Section 2, investments in transmission lines installed in new corridors, $y_{\ell}^{L,N}$, are first-stage binary variables independent of the scenario index, whereas decisions on investments in lines placed in existing corridors, denoted by binary variables $y_{\ell}^{L,E}$, can be delayed in time and are second-stage variables dependent on the scenario index, ω . Note that each value of index ℓ represents a specific line (length, voltage, power capacity, origin and destiny) that can be built. On the other hand, investments in energy storage systems are second-stage variables and comprise two terms, power and energy components, which are denoted by continuous and non-negative variables $p_{s\omega}^{L,S}$ and $e_{s\omega}^{L,S}$, respectively. The power component limits the maximum power that can be charged to and discharged from storage units, whereas the energy component refers to the maximum amount of energy stored in a given time period.

3.2. Technical constraints of generating units

The technical constraints of generating units are formulated by expressions (2)–(6):

$$p_{gdt\omega}^G \leq P_g^{L,G}, \quad \forall g \in G^D, \forall d \in D, \forall t \in T, \forall \omega \in \Omega, \quad (2)$$

$$p_{gdt\omega}^G + p_{gdt\omega}^{G,S} = A_{gdt}^D P_g^{L,G}, \quad \forall g \in G^I, \forall d \in D, \forall t \in T, \forall \omega \in \Omega, \quad (3)$$

$$p_{gdt\omega}^G - p_{gdt-1,\omega}^G \leq P_{up,g}^G P_g^{L,G}, \quad \forall g \in G^D, \forall d \in D, \forall t \in T, \forall \omega \in \Omega, \quad (4)$$

$$p_{gdt-1,\omega}^G - p_{gdt\omega}^G \leq P_{dw,g}^G P_g^{L,G}, \quad \forall g \in G^D, \forall d \in D, \forall t \in T, \forall \omega \in \Omega, \quad (5)$$

$$\{p_{gdt\omega}^G, p_{gdt\omega}^{G,S}\} \geq 0, \quad \forall g \in G, \forall d \in D, \forall t \in T, \forall \omega \in \Omega. \quad (6)$$

Constraints (2) and (3) establish the limits of the power generated by the thermal and renewable generating units, respectively. The maximum power production of each generator must be lower than its installed capacity. It is also considered that the power output of renewable units is bounded by the availability of renewable resources, which are characterized by parameter $A_{gdt}^D \in [0, 1]$. Observe that the available renewable production, $A_{gdt}^D P_g^{L,G}$, is equal to the power

generated, $p_{gd\omega}^G$, plus surplus energy, $p_{gd\omega}^{G,S}$. Surplus energy is a part of the available production that cannot be used to satisfy the demand because of the congestion of the transmission network or the lack of flexibility in the operation of thermal units. One of the objectives of the TSO is to design the transmission system in such a way that renewable surpluses are as low as possible to minimize the production of non-renewable units. The power ramps of the thermal units are formulated using (4) and (5). The positive nature of the power output and surplus is established by (6).

3.3. Technical constraints of storage units

The investment and operation of storage units are formulated as follows:

$$0 \leq p_{s\omega}^{L,S} \leq P_{\max,s}^{L,S}, \quad \forall s \in S, \forall \omega \in \Omega, \quad (7)$$

$$0 \leq e_{s\omega}^{L,S} \leq E_{\max,s}^{L,S}, \quad \forall s \in S, \forall \omega \in \Omega, \quad (8)$$

$$e_{s\omega}^{L,S} = \gamma_s^S p_{s\omega}^{L,S}, \quad \forall s \in S, \forall \omega \in \Omega, \quad (9)$$

$$0 \leq p_{sd\omega}^{S,D} \leq p_{s\omega}^{L,S}, \quad \forall s \in S, \forall d \in D, \forall t \in T, \forall \omega \in \Omega, \quad (10)$$

$$0 \leq p_{sd\omega}^{S,C} \leq p_{s\omega}^{L,S}, \quad \forall s \in S, \forall d \in D, \forall t \in T, \forall \omega \in \Omega, \quad (11)$$

$$\gamma_s^{\min} e_s^{L,S} \leq e_{sd\omega}^{L,S} \leq e_{s\omega}^{L,S}, \quad \forall s \in S, \forall d \in D, \forall t \in T, \forall \omega \in \Omega, \quad (12)$$

$$e_{sd\omega}^{L,S} = \gamma_{sd}^{S,D} e_{s\omega}^{L,S}, \quad \forall s \in S, t = 0, \forall d \in D, \forall \omega \in \Omega, \quad (13)$$

$$e_{sd\omega}^{L,S} \geq \gamma_{sd}^{S,F} e_{s\omega}^{L,S}, \quad \forall s \in S, t = T_d, \forall d \in D, \forall \omega \in \Omega, \quad (14)$$

$$e_{sd\omega}^{L,S} = e_{sd-1,\omega}^{L,S} + \eta^S p_{sd\omega}^{S,C} - \frac{1}{\eta^S} p_{sd\omega}^{S,D}, \quad \forall s \in S, \forall d \in D, \forall t \in T, \forall \omega \in \Omega. \quad (15)$$

As mentioned earlier, the investments in the power and energy components of the storage units are characterized by variables $p_{s\omega}^{L,S}$ and $e_{s\omega}^{L,S}$. The limits on investments in the power and energy components are established by constraints (7) and (8). The relationship between energy and power components is defined in (9). The charged and discharged power are bounded by the installed power component in constraints (10) and (11). Constraint (12) establishes the limits of the energy stored in each day, time period, and scenario. The initial and final daily states of storage units are enforced by constraints (13) and (14). Finally, the energy balance of the storage units is formulated using (15).

3.4. Constraints of existing transmission lines

The power flows in existing transmission lines are formulated using the DC model as follows:

$$p_{\ell d\omega}^L = \frac{1}{X_\ell} (\theta_{O(\ell)d\omega} - \theta_{F(\ell)d\omega}), \quad \forall \ell \in L^E, \forall d \in D, \forall t \in T, \forall \omega \in \Omega \quad (16)$$

$$-P_{\max,\ell}^L \leq p_{\ell d\omega}^L \leq P_{\max,\ell}^L, \quad \forall \ell \in L^E, \forall d \in D, \forall t \in T, \forall \omega \in \Omega. \quad (17)$$

Constraint (16) computes the power flows as a function of the difference in the voltage angles at both ends of the transmission lines, whereas the limits of the power flows are established by (17).

3.5. Constraints of candidate transmission lines

Next, we present the mathematical formulation of power flows in candidate lines. It is important to note that the formulation for power flows in candidate transmission lines differs from that of existing lines because it must avoid imposing unrealistic limitations on voltage angles when these transmission lines are not installed. To address this, the proposed formulation is the following:

$$\frac{1}{X_\ell} (\theta_{O(\ell)d\omega} - \theta_{F(\ell)d\omega}) - (1 - y_\ell^{L,N}) M \leq p_{\ell d\omega}^L \leq$$

$$\frac{1}{X_\ell} (\theta_{O(\ell)d\omega} - \theta_{F(\ell)d\omega}) + (1 - y_\ell^{L,N}) M,$$

$$\forall \ell \in L^{C,N}, \forall d \in D, \forall t \in T, \forall \omega \in \Omega, \quad (18)$$

$$-P_{\max,\ell}^L y_\ell^{L,N} \leq p_{\ell d\omega}^L \leq P_{\max,\ell}^L y_\ell^{L,N}, \quad \forall \ell \in L^{C,N}, \forall d \in D, \forall t \in T, \forall \omega \in \Omega, \quad (19)$$

$$\frac{1}{X_\ell} (\theta_{O(\ell)d\omega} - \theta_{F(\ell)d\omega}) - (1 - y_\ell^{L,E}) M \leq p_{\ell d\omega}^L \leq$$

$$\frac{1}{X_\ell} (\theta_{O(\ell)d\omega} - \theta_{F(\ell)d\omega}) + (1 - y_\ell^{L,E}) M,$$

$$\forall \ell \in L^{C,E}, \forall d \in D, \forall t \in T, \forall \omega \in \Omega, \quad (20)$$

$$-P_{\max,\ell}^L y_\ell^{L,E} \leq p_{\ell d\omega}^L \leq P_{\max,\ell}^L y_\ell^{L,E}, \quad \forall \ell \in L^{C,E}, \forall d \in D, \forall t \in T, \forall \omega \in \Omega. \quad (21)$$

Constraint (18) computes the power flow through newly installed lines in the new corridors. In this constraint, parameter M is a sufficiently large value, which is greater than the maximum line power capacity, and is used to prevent the enforcement of limits on voltage angle differences if candidate lines are not installed. Constraint (19) states the upper and lower limits of the power flow in the candidate lines in the existing corridors. In this manner, if a line ℓ is installed, then binary variable $y_\ell^{L,N}$ is equal to 1, constraint (18) states that $p_{\ell d\omega}^L = \frac{1}{X_\ell} (\theta_{O(\ell)d\omega} - \theta_{F(\ell)d\omega})$ and constraint (19) limits the power flow in the line, $-P_{\max,\ell}^L \leq p_{\ell d\omega}^L \leq P_{\max,\ell}^L$. However, if the transmission lines is not installed, $y_\ell^{L,N} = 0$, constraint (18) does not bind the value of the voltage angles because it forces the power flow $p_{\ell d\omega}^L$ to be greater than a small negative value and less than a large positive value, whereas constraint (19) fixes the value of $p_{\ell d\omega}^L$ to zero, $0 \leq p_{\ell d\omega}^L \leq 0$. Constraints (20) and (21) are equivalent to constraints (18) and (19) for candidate lines in the existing corridors. Considering that the investments in lines placed in existing corridors are wait-and-see variables, constraints (20) and (21) are defined for each possible scenario.

3.6. Energy balance

The energy balance is enforced by constraint (22). This constraint ensures for each bus, time period, and scenario that the summation of the production of the generation units located in the considered bus, the energy discharged from the storage, and the energy coming from the set of lines connected to other buses must be equal to the sum of the bus demand, the energy charged to the batteries, and the energy injected to other buses connected by lines to the considered bus. The only parameter in this constraint is demand, $P_{bd\omega}^D$. Demand is an uncertain parameter modeled using a set of scenarios $\omega \in \Omega$. Because the demand must be satisfied in every considered scenario, the power production of the generating units, the power discharged and charged from batteries, and the power flows through transmission lines are second-stage variables that depend on the scenario index, ω . Note also that the power flows simultaneously consider existing and candidate lines. As previously mentioned in Section 3.5, the power flows of the non-built candidate lines are equal to zero by constraints (18)–(21).

$$\sum_{g \in G_b} P_{gd\omega}^G + \sum_{s \in S_b} p_{sd\omega}^{S,D} + \sum_{\ell \in L_b^F} p_{\ell d\omega}^L = P_{bd\omega}^D + \sum_{s \in S_b} p_{sd\omega}^{S,C} + \sum_{\ell \in L_b^O} p_{\ell d\omega}^L, \quad (22)$$

$$\forall b \in B, \forall d \in D, \forall t \in T, \forall \omega \in \Omega.$$

3.7. Maximum carbon emissions produced by fossil-fuel units using second-order stochastic dominance constraints

As previously stated, in this study SSD constraints are used to determine the best investment strategies according to the maximum carbon emissions produced by non-renewable units fed by fossil fuels. Because future demand is uncertain, the carbon emissions produced by non-renewable units is also an uncertain variable that depends on the investment decisions made by the TSO. Note that insufficient development of the transmission network may lead to a situation where a portion of the energy generated by renewable units cannot be injected into the network due to line congestion, potentially resulting in undesirable higher-than-necessary production from fossil-fuel units and increased carbon emissions.

To represent the carbon emissions from non-renewable production, the following auxiliary variable is used:

$$p_{\omega}^{\text{FF,Tot}} = \sum_{d \in D} W_d \sum_{g \in G^D} \sum_{t \in T} B_g^G p_{gd\omega}^G, \quad \forall \omega \in \Omega, \quad (23)$$

where variable $p_{\omega}^{\text{FF,Tot}}$ represents the total carbon emissions generated by non-renewable units in each scenario $\omega \in \Omega$, and parameter B_g^G denotes the amount of carbon emissions per MWh produced by unit g . Therefore, our objective is to determine the optimal investment decisions for TSOs limiting the carbon emissions from non-renewable production considering a benchmark that represents the maximum acceptable quantity of carbon emissions produced by the pollutant units. This benchmark is also a random parameter that can be denoted as $P_{\xi}^{\text{FF,Ben}}$, $\forall \xi \in \Xi^{\text{Ben}}$. Then for each scenario ξ , $P_{\xi}^{\text{FF,Ben}}$ represents the maximum carbon emissions linked to that scenario, with an assigned probability of τ_{ξ} .

Observe that the number of scenarios $\xi \in \Xi^{\text{Ben}}$ selected to characterize the benchmark can be different to the number of scenarios used to represent the demand. In order to reduce the computational size of the resulting optimization problem it would be desirable to keep the number of benchmark scenarios as low as possible.

Considering that the carbon emissions from non-renewable units is a random variable that depends on the considered scenario, the limitation of this production can be performed in terms of expected values as follows:

$$\sum_{\omega \in \Omega} \pi_{\omega} p_{\omega}^{\text{FF,Tot}} \leq \sum_{\xi \in \Xi^{\text{Ben}}} \tau_{\xi} P_{\xi}^{\text{FF,Ben}}. \quad (24)$$

Then, constraint (24) limits the expected carbon emissions from non-renewable units to the expected value of the benchmark $P_{\xi}^{\text{FF,Ben}}$, $\forall \xi \in \Xi^{\text{Ben}}$. Observe that $p_{\omega}^{\text{FF,Tot}}$ is a variable depending on the TSO investment decisions that represents the carbon emissions from non-renewable production in scenario ω . Considering constraint (24), observe that undesirable carbon emission values might occur in some scenarios, even though the expected carbon emissions are acceptable. One alternative manner to limit the carbon emissions from non-renewable units is by means of stochastic dominance theory. As described in Section 1, stochastic dominance allows the comparison of two random variables in terms of a desired acceptability (big or small outcomes). In the case of carbon emissions, where small values are preferred, the SSD can be formulated as follows:

$$\sum_{\omega \in \Omega} \pi_{\omega} s_{\omega \xi'} \leq \sum_{\xi \in \Xi^{\text{Ben}}} \tau_{\xi} S_{\xi \xi'}^{\text{Ben}}, \quad \forall \xi' \in \Xi^{\text{Ben}}, \quad (25)$$

$$s_{\omega \xi} \geq p_{\omega}^{\text{FF,Tot}} - P_{\xi}^{\text{FF,Ben}}, \quad \forall \omega \in \Omega, \forall \xi \in \Xi^{\text{Ben}}, \quad (26)$$

$$s_{\omega \xi} \geq 0, \quad \forall \omega \in \Omega, \forall \xi \in \Xi^{\text{Ben}}. \quad (27)$$

The formulation (25)–(27) is based on that used, for instance, in Carrión et al. (2009). Variables $s_{\omega \xi}$ formulate equivalently the function $\max\{p_{\omega}^{\text{FF,Tot}} - P_{\xi}^{\text{FF,Ben}}, 0\}$ using linear expressions. Similarly, parameter $S_{\xi \xi'}^{\text{Ben}}$ is computed as:

$$S_{\xi \xi'}^{\text{Ben}} = \max\{P_{\xi}^{\text{FF,Ben}} - P_{\xi'}^{\text{FF,Ben}}, 0\}, \quad \forall \xi, \xi' \in \Xi^{\text{Ben}}. \quad (28)$$

Based on the values of $s_{\omega \xi'}$ and $S_{\xi \xi'}^{\text{Ben}}$, constraint (25) establishes, for each benchmark scenario $\xi' \in \Xi$, that the expected carbon emissions greater than the benchmark scenario ξ' , $\left(\sum_{\omega \in \Omega} \pi_{\omega} \max\{p_{\omega}^{\text{FF,Tot}} - P_{\xi'}^{\text{FF,Ben}}, 0\}\right)$, has to be less than the acceptable carbon emissions greater than benchmark scenario ξ' , $\left(\sum_{\xi \in \Xi^{\text{Ben}}} \tau_{\xi} \max\{P_{\xi}^{\text{FF,Ben}} - P_{\xi'}^{\text{FF,Ben}}, 0\}\right)$. Then, this constraint ensures for all benchmark scenarios that the expected carbon emissions greater than them is acceptable.

Therefore, the main advantage of using stochastic dominance constraints with respect to considering only the expected value of the carbon emissions is that the entire distribution function determines the

acceptability of investment solutions, not only its expected value. In this way, we avoid the possibility that the solution generates scenarios with carbon emissions remarkably higher than the benchmark, i.e., a situation that can happen to impose the expected value constraint.

Finally, we would like to highlight that the application of stochastic dominance constraints in the transmission expansion problem may not be limited solely to addressing carbon emissions. Instead, if desired, these constraints can be flexibly employed to ensure the acceptability of transmission expansion plans in the presence of other stochastic variables. For instance, a critical aspect of managing the transmission network is preventing unserved demand resulting from line congestion. While our proposed approach takes a conservative stance by disallowing unserved demand, alternative and less conservative strategies may be explored. For example, in the context of the Spanish power system, a maximum interruption time of 15 min per year is permissible, ensuring 99.997% satisfaction of demand. By incorporating stochastic dominance constraints, we can guarantee that the resulting investment plan is robust enough to meet 99.997% of uncertain demand. It is also interesting to enhance this approach by considering unserved demand caused by failures of generating units and transmission lines. This would necessitate expanding the set of uncertain parameters to include scenarios involving failures of generation and transmission assets. Furthermore, stochastic dominance constraints can also be applied to other variables, such as specifying the minimum or maximum acceptable production levels for a given technology, driven by strategic or security considerations. Additionally, constraints may be imposed on the maximum power flow through specific lines in the system. In summary, the use of stochastic dominance constraints can significantly enhance the adaptability of the transmission expansion plan to diverse uncertainties.

4. Case study

The SD approach is tested on a realistic case study based on the Gran Canaria power system in Spain. This is an isolated power system with a high renewable-energy potential that procures the electricity demand of a population of around 850 thousand people. The objective of this case study is to determine transmission-line and storage investments for 2050. Then, the transmission planner must decide which transmission lines must be built from a set of candidate lines and storage units to ensure the procurement of future demand with acceptable carbon emissions. The expected configuration of the generation mix in 2050 is input data for this problem and does not depend on transmission planner decisions. Based on this system, a base case has been solved and two additional sensitivity analyses have been performed to analyze the impact on the investment decisions of (i) the number of scenarios of the benchmark used to limit the carbon emissions and (ii) the congestion of the transmission network.

4.1. Input data

The characterization of the Gran Canaria power system was based on the representation presented in Cañas-Carretón et al. (2021). The considered representation of this power system comprised 29 buses and 47 generating units. The technologies for generating units are Open-Cycle Gas Turbines (OCGT), wind, and solar PV. Fig. 2 shows the power system topology. As observed in this figure, the transmission network is operated at two different voltage levels: 66 and 132 kV. The existing transmission lines are represented in blue, whereas the candidate lines are depicted using dashed lines in red.

A set of 36 candidate lines are considered. The parameters that characterize the set of candidate lines are included in Tables B.12 and B.13 in Appendix B. The power base is 100 MVA, and the voltage bases are 66 and 132 kV. Capital costs are annualized using the capital recovery factor, $\text{CRF} = \frac{r(1+r)^x}{(1+r)^x - 1}$, where x is the plant life of the unit,

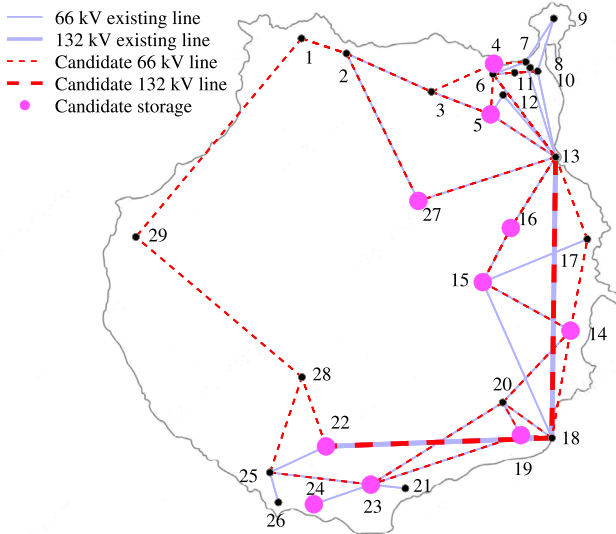


Fig. 2. Single-line diagram of Gran Canaria power system.

Table 1
Demand growth scenarios.

Scenario	Annual demand growth (%)	Probability
1	0.091	0.007
2	0.229	0.066
3	0.365	0.241
4	0.500	0.373
5	0.635	0.241
6	0.771	0.066
7	0.908	0.007

and r the interest rate. In this paper, the considered interest rate is 9% and the life of the assets is equal to 25 years.

The target year was represented by fifteen characteristic days. These days were selected using the procedure proposed in Gröwe-Kuska et al. (2003) considering the series of demand and wind and solar PV availability. Therefore, a total of $15 \cdot 24 = 360$ hourly periods were considered. The following data were obtained from actual values of the Gran Canaria power system in 2018.² Fig. 3 shows the hourly values of wind and solar PV availability.

The future demand of the system in 2050 is an uncertain parameter that is characterized as a stochastic process. For this, it is considered that the annual growth is normally distributed with a mean of 0.5% and a standard deviation equal to 0.15%. These values are consistent with the expected annual demand growth as described in IRENA (2019). This probability distribution was then discretized using seven scenarios. Note that the usage of a normal distribution is not mandatory for generating scenarios, and a different probability distribution may be used if desired. Table 1 includes the annual demand growth and probability per scenario. Then, considering the annual demand of 2018 and the demand growth of Table 1, the resulting demand values per period and scenario are shown in Fig. 4. The total demand per bus is allocated using the share factors listed in Table 2.

Table 3 lists the nominal capacities of the generating units that are scheduled to be in operation during the target year. These capacities are based on the current power capacities and renewable energy potentials described by Gils and Simon (2017). The operational cost of the OCGT units is 288.2 €/MWh, whereas it is equal to 0 for the wind and solar PV units. The OCGT units emit 510 gCO₂/kWh in average.

A total of 10 candidate Li-ion storage facilities are considered, which can be located in buses 4, 5, 14, 15, 16, 19, 22, 23, 24 and 27. A typical ratio between power and energy of a storage unit of 6 h was considered. The total investment cost of this type of storage unit, including the energy and power terms, is 148 €/MWh. The maximum energy storage capacity that could be installed in each candidate location was 1800 MWh. The costs of charging and discharging energy in the day-ahead market are null.

Considering that demand is an uncertain parameter, the maximum acceptable value of carbon emissions is also uncertain. In this case, the maximum carbon emissions were modeled using two benchmarks comprising five scenarios each. Applying the same methodology outlined for demand characterization, both benchmarks are derived with the assumption that the annual demand growth follows a normal distribution $N(0.5, 0.15)$, and the acceptable carbon emissions are determined based on supplying 30% and 25% of the future demand through OCGT units. These percentages were chosen considering the specific renewable potential of the analyzed power system. In Table 4, the probabilities of the scenarios defining the benchmarks are presented. The carbon emissions associated to each scenario are computed considering that the amount of carbon emissions per kWh produced by OCGTs is equal to 510 gCO₂/kWh and that the average annual demand is equal to 4.04 TWh. Notably, the benchmark scenarios allowing emissions associated with 30% of non-renewable production exhibit higher values than those generated for the benchmark involving only 25% non-renewable production. This implies that the benchmark allowing carbon emissions related to 25% of non-renewable production is more restrictive compared to the other benchmark. Please note that defining benchmarks is a subjective decision for the decision-maker, and as such, there exist numerous ways to establish benchmarks for specific problems.

All simulations were performed with CPLEX 12.6.3, using a server with four 3.0 GHz processors and 250 GB of RAM and a HPE ProLiant DL560 Gen11 server, equipped with four 2.2 GHz 18-core processors and 512 GB of RAM. The number of constraints and continuous variables in the base case are 2.6 and 1 millions, respectively. The number of binary variables is 379, and the maximum solution time attained is 30 h.

4.2. Results: base case

The case described above was solved considering benchmarks assuming maximum acceptable carbon emissions related to non-renewable productions of 30 and 25% as well as stochastic dominance (SD) and expected value (Exp) formulations. The stochastic dominance formulation corresponds to constraints (1)–(22) and (25)–(27); while, the expected value formulation comprises (1)–(22) and (24).

The main results for the different cases are listed in Tables 5 and 6. These tables include the objective function value, transmission and storage investment costs, and the solution time in each case. Note that the transmission costs are divided into first- and second-stage costs. The second-stage costs have been already discounted.

First, Table 5 shows that the objective functions of Exp and SD formulations are identical. This result is a consequence of the fact that the constraints limiting the maximum carbon emissions of non-renewable units are not active when the maximum non-renewable production is 30% or higher. Therefore, the resulting solution times are significantly low. By contrast, Table 6 shows that the investment decisions obtained for the case with the maximum carbon emissions of 25% production of non-renewable units are different for the Exp and SD cases. In particular, we observe that the total investment cost in the SD case is 2.2% greater than that in Exp. This result indicates that the investment decisions obtained in the SD case are more conservative than those in the Exp case, which yields higher investment costs. Note that the benchmark used in the SD case contains scenarios with smaller maximum carbon emissions than the Exp formulation, which further constrains the operation of the system. Additionally, observe that the

² <https://www.ree.es/es>

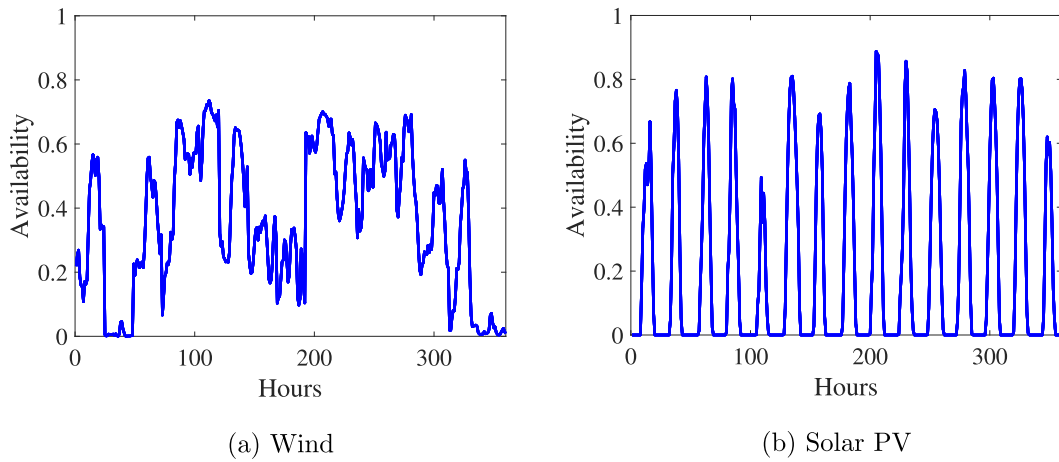


Fig. 3. Renewable availabilities.

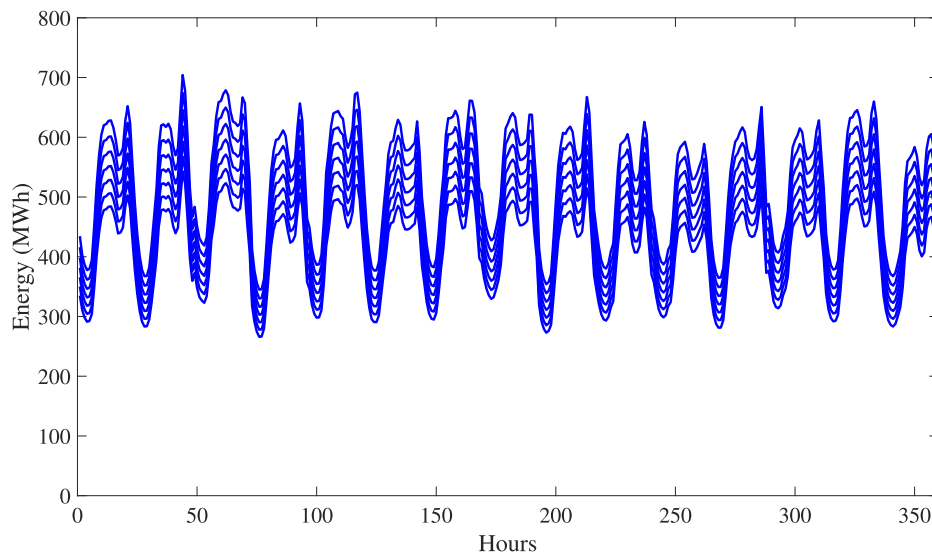


Fig. 4. Demand scenarios.

Table 2
Demand share factor (%).

Bus																			
1	2	3	4	5	6	7	8	9	10	11	12	14	15	16	18	23	27	28	29
3.5	3.4	4.5	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	4.9	3.7	3.7	12.1	8.6	6.3	6.4	2.4	0.9

investment decisions in the *Exp* and *SD* cases are significantly different. For example, investments in storage and second-stage transmission lines are much higher in the *SD* case than in the *Exp* case. The opposite result was observed for first-stage investments in transmission lines. Note that investment in second-stage assets allows the TSO to adapt to each demand scenario. Finally, we observe that the investments in storage in cases with a maximum carbon emissions associated with 25% of non-renewable production are significantly greater than those in the case with a maximum production of non-renewable units equal to 30%. This result reveals that investments in storage are more useful than investments in transmission lines if it is desirable to reduce the production of non-renewable units.

Table 7 provides the transmission and storage costs per scenario for *Exp* and *SD* cases if carbon emissions are less than those in the case with 25% of non-renewable production. Note that the first-stage transmission investments are independent of the scenario index and identical for all scenarios. As mentioned above, the first investments

in transmission lines placed in the new corridors are higher in the *Exp* case. However, the second-stage investment costs were higher in *SD* for most scenarios. Note that because the investment decisions of the *SD* case rely more on the second stage, it is possible to adapt the investment decisions to each particular demand scenario. For instance, observe that the total costs in scenarios 1–3, which are associated with low demand, are lower in *SD* than in *Exp*, whereas they are higher in the rest of the scenarios. Then, the solution obtained by the *SD* case may lead to a low cost in favorable scenarios at the expense of experiencing larger costs in high-demand scenarios, where the limitation of low carbon emissions is more difficult to satisfy. In fact, the total investment cost in the scenario with the highest demand is 6.2 times higher than that in the lowest demand scenario.

Table 8 lists the number of installed transmission lines per type and its length, as well as the storage capacity installed in each bus, for each scenario and for each case. In this table, FS refers to first-stage investments. In accordance with the results provided in Table 7, the

Table 3
Technical characteristics of generating units.

Techn.	Unit	Node	Nominal capacity (MW)	Techn.	Unit	Node	Nominal capacity (MW)
OCGT	1–5	13	32.0		32	12	23.2
	6–10	13	32.0		33	14	235.6
	11–15	18	32.0		34	15	190.2
	16–20	18	32.0		35	16	302.9
Wind	21	1	22.8	Solar PV	36	1	22.3
	22	6	165.8		37	2	6.0
	23	7	2.4		38	3	38.4
	24	8	1.8		39	4	23.2
	25	10	111.1		40	5	23.2
	26	15	143.6		41	6	23.2
	27	21	0.6		42	7	23.2
Solar PV	28	8	23.2	43	18	12.0	
	29	9	23.2	44	23	50.8	
	30	10	23.2	45	27	8.7	
	31	11	23.2	46	28	18.5	
				47	29	5.4	

Table 4
Benchmark scenarios.

Scenario	Carbon emissions related to maximum non-renewable production of 30% (kTon)	Carbon emissions related to maximum non-renewable production of 25% (kTon)	Probability
1	550.8	459.0	0.020
2	586.5	489.6	0.227
3	617.1	515.1	0.504
4	652.8	545.7	0.227
5	693.6	576.3	0.020

Table 5
Base case: Results (30% non-renewable).

Case	Obj. function (M€)	Transmission cost (M€)		Sto. Cost (M€)	Sol. time (h)
		1st stage	2nd stage		
<i>Exp</i>	2.871	2.187	0.343	0.341	0.9
<i>SD</i>	2.871	2.187	0.343	0.341	0.9

Table 6
Base case: Results (25% non-renewable).

Case	Obj. function (M€)	Transmission cost (M€)		Sto. Cost (M€)	Sol. time (h)
		1st stage	2nd stage		
<i>Exp</i>	6.801	2.420	0.051	4.330	6.1
<i>SD</i>	6.950	2.187	0.292	4.471	4.0

Table 7
Base case: Investment costs per scenario (25% non-renewable).

Case	Scenario	Transmission cost (M€)		Sto. Cost (M€)	Total cost (M€)
		1st stage	2nd stage		
<i>Exp</i>	1	2.422	0.000	0.434	2.856
	2	2.422	0.000	1.496	2.918
	3	2.422	0.000	2.937	5.359
	4	2.422	0.000	3.979	6.401
	5	2.422	0.161	6.081	8.664
	6	2.422	0.161	7.699	10.282
	7	2.422	0.161	10.164	12.747
<i>SD</i>	1	2.189	0.344	0.000	2.533
	2	2.189	0.344	0.000	2.533
	3	2.189	0.234	1.887	4.310
	4	2.189	0.234	4.239	6.662
	5	2.189	0.395	6.703	9.287
	6	2.189	0.395	11.185	13.769
	7	2.189	0.395	13.095	15.679

number of lines installed in the first stage is smaller in the *SD* case. In contrast, in scenarios 6 and 7 with higher demand, the total length

of the installed lines is similar in both cases, 63.4 km. However, in scenarios 1 and 2 with small demand, the length of the newly built lines in the *SD* case is 1.2% less than that in *Exp*. It is also interesting to note that all newly built lines correspond to a low-voltage network of 66 kV. Thus, the 132 kV network does not need to be expanded to accommodate the increase in demand and renewable generation. Considering the configuration of the existing power system network represented in Fig. 2 and the location of the generating units provided in Table 3, it can be observed that the main purpose of the existing 132 kV network connecting buses 13, 18, and 22 is to facilitate the evacuation of the energy produced by the OCGT units located in buses 13 and 18. However, by 2050, it is expected that the production of these units will decrease as the installed capacity of new renewable units will increase. For this reason, it is not necessary to increase the transmission capacity of the 132 kV system. In contrast, the expected increase in the production of renewable units that are distributed over most of the power system buses (see Table 3) requires a significant strengthening of the 66 kV transmission system, which is spread over the entire power system.

Additionally, we observe that the storage capacities installed are larger in high-demand scenarios and that the capacity installed in the *SD* case is higher than that in the *Exp* case. Note that storage units are installed only at three particular locations in all cases and scenarios: buses 4, 15, and 19. The selection of the storage locations was based on different reasons. As shown in Fig. 2, bus 4 is located in the northeast of the system where the largest city on the island is placed, and according to Table 2, high values of demand exist in neighboring buses. The purpose of this storage may be to provide the demand for buses located in this part of the system when renewable energy is not available. It can be observed that the installed capacity of this storage is the largest in all the scenarios. Note also that the power capacity installed in the scenario with the highest demand was 45.7% higher in the *SD* case with respect to *Exp*. On the other hand, bus 15 is placed in a strategic location that is connected to four buses of the system and, additionally, two large-sized renewable units are connected to this bus: a wind farm with 143.6 MW and a solar PV farm with 190.2 MW. This bus is an adequate location for installing a storage unit because it can store the surplus production of the renewable units connected to it, and it can easily evacuate the stored energy owing to its good connectivity with the rest of the system. Finally, bus 19 is also a good location for installing storage because it is connected to bus 18, which belongs to the 132 kV network. Then, it can profit from the high-power transfer of this network to withdraw and inject power when needed.

Fig. 5 graphically represents the investments in transmission lines and storage in the *SD* case for scenarios with the lowest and highest demands. The difference between the two systems is that in the scenario with the highest demand, three additional storages of 102/612, 17/102, and 26/156 MW/MWh are installed in buses 4, 15, and 19, respectively, and that lines 14–20 and 18–19 are built.

Table 9 provides the expected energy production of each generation technology for each case and for maximum non-renewable production equal to 30 and 25%. The production of each technology is provided as absolute values and as a percentage of the total demand (system demand plus energy charged by storage). As mentioned above, maximum the carbon emissions associated with a non-renewable production equal to 30% of the demand does not constrain the problem; therefore, the expected energy production of the different technologies is identical for the *Exp* and *SD* cases. We then observed that wind power is a technology that provides most of the system demand. The non-renewable generating technology, OCGT, provides almost 30% of the demand, whereas the participation of storage is rather small. However, when the limit of maximum carbon emissions is associated with 25% of non-renewable production, the expected production of OCGTs is significantly reduced by 16.7%. The reduction in the production of OCGTs was approximately 203 GWh in both cases and was mainly

Table 8
Base case: Investments in transmission lines and storage capacity - (25% non-renewable).

Case	Scenario	Lines			Power capacity storage (MW)		
		132 kV	66 kV	Length (km)	Bus 4	Bus 15	Bus 19
Exp	FS	0	8	57.40	–	–	–
	1	0	0	0.00	2	3	0
	2	0	0	0.00	11	5	0
	3	0	0	0.00	25	8	0
	4	0	0	0.00	32	11	1
	5	0	1	6.00	41	8	18
	6	0	1	6.00	52	12	21
	7	0	1	6.00	70	17	25
SD	FS	0	6	49.00	–	–	–
	1	0	1	7.70	0	0	0
	2	0	1	7.70	0	0	0
	3	0	2	8.40	21	0	0
	4	0	2	8.40	32	13	3
	5	0	2	13.70	41	8	18
	6	0	3	14.40	91	12	21
	7	0	3	14.40	102	17	26

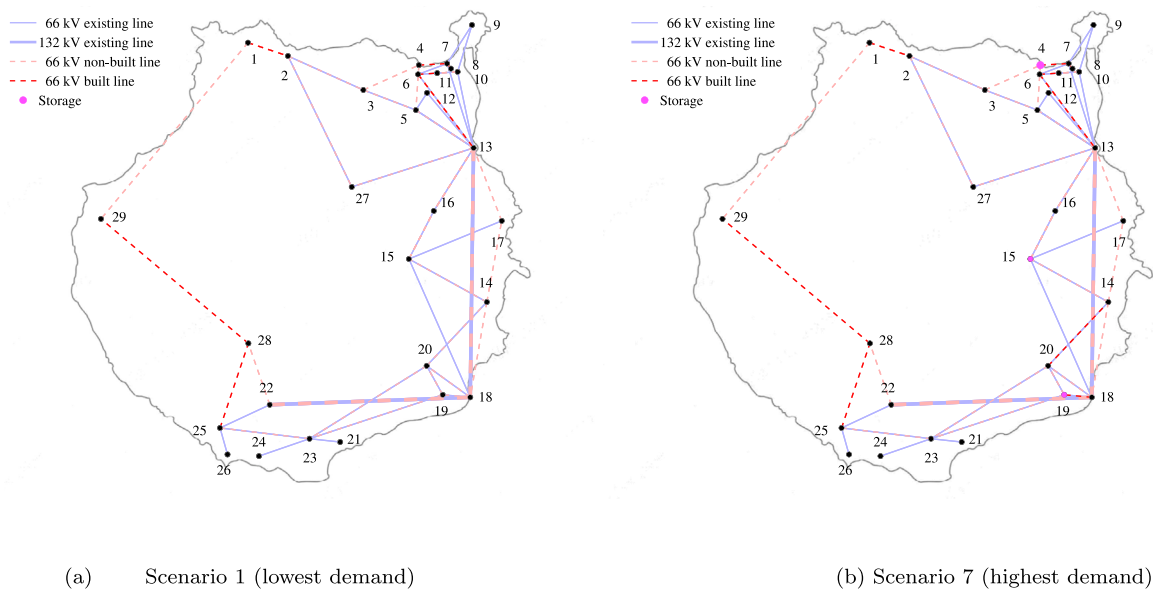


Fig. 5. Transmission and storage investments using the stochastic-dominance formulation.

Table 9
Base case: Expected energy production (GWh).

Percentage non-renewable (%)	Case	Technologies			
		OCGT	Wind	Solar PV	Storage
30	Exp	1215.0 (29.9%)	1542.3 (38.0%)	1294.3 (31.9%)	8.0 (0.2%)
	SD	1215.0 (29.9%)	1542.3 (38.0%)	1294.3 (31.9%)	8.0 (0.2%)
25	Exp	1012.0 (24.3%)	1616.8 (38.8%)	1431.6 (34.4%)	100.6 (2.4%)
	SD	1012.0 (24.3%)	1587.7 (38.1%)	1460.3 (35.1%)	102.8 (2.5%)

compensated by wind power units in the *Exp* case and by solar PV in the *SD* case. It can be observed that the installed capacity of storage is higher in the *SD* case, and the daily charging and discharging cycles of storage coordinate very well with the daily production of the solar PV units. This is illustrated in Fig. 6. This figure represents the expected power production of each generation technology for each hour on the considered set of characteristic days for *SD* case. In this figure, it is observed that for most days, storage is charged during the central hours of the day in which higher solar PV production is obtained. The stored energy is discharged during hours with low solar PV and wind-power production.

Finally, Fig. 7 represents the cumulative distribution functions of the carbon emissions for *Exp* and *SD* cases associated with 25% of non-renewable production. For the sake of clearness, the benchmark used to impose stochastic dominance constraints is also depicted. However, note that in *Exp* case it is enforced that the expected carbon emissions be less than the expected value of the benchmark, which is equal to 516.1 kTon. For this reason, Fig. 7(a) shows that the highest carbon emission scenario resulting in the *Exp* case is significantly greater than the maximum benchmark scenario. Specifically, in the scenario with highest demand, the total amount of carbon emissions in *Exp* case is 603.3 kTon, that is 4.1% higher than the maximum carbon emissions in *SD* case. In other words, the solution obtained by the *SD* case for the

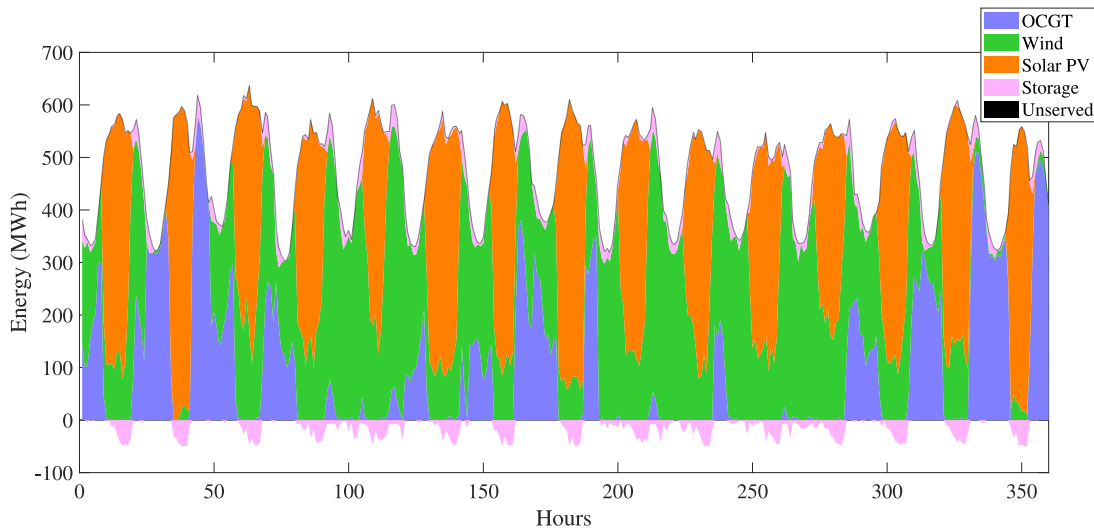


Fig. 6. Base case: Expected energy production in SD case (25% non-renewable).

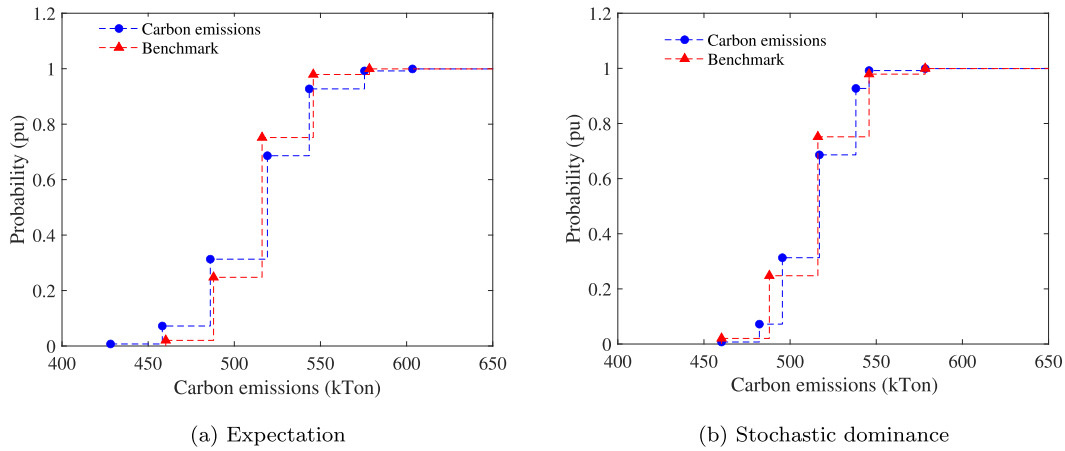


Fig. 7. Base case: Carbon emission distributions (25% non-renewable production).

highest-demand scenario saves the emissions of 25 kTon per year with respect to the solution obtained in *Exp* case.

4.3. Sensitivity analysis: number of benchmark scenarios

This section analyzes the influence of the number of benchmark scenarios on the obtained results. To do so, a different number of scenarios (from 1 to 12) is considered to discretize the benchmark probability distribution of the carbon emissions in the case with 25% non-renewable production of the demand. Demand growth is characterized by a normal distribution $N(0.5, 0.15)$. Table 10 lists the objective function, investment costs, and solution times for different numbers of benchmark scenarios in the *SD* case. Observe that the solution of the *Exp* case only depends on the expected value of the benchmark and not on the particular scenarios considered. The results indicate that first-stage decisions are independent of the number of benchmark scenarios. However, the second-stage decisions depend on the number of benchmark scenarios. We observe that the costs associated with investments in transmission lines and storage decrease significantly as the number of benchmark scenarios increases. The reason for this result is that if the number of benchmark scenarios is low, the probability associated with each benchmark scenario is high, and it is difficult to satisfy the second-order dominance constraints. Therefore, it is necessary to increase

Table 10
Number of benchmark scenarios: Results (25% non-renewable).

# Benchmark scenarios	Obj. function (M€)	Transmission cost (M€)		Sto. Cost (M€)	Sol. time (h)
		1st stage	2nd stage		
1	7.466	2.187	0.319	4.960	2.1
3	6.958	2.187	0.325	4.446	2.6
5	6.950	2.187	0.292	4.471	4.0
9	6.884	2.187	0.292	4.405	4.0
12	6.874	2.187	0.292	4.395	4.0

investments in transmission and storage assets in specific scenarios. The opposite effect occurs when the number of benchmark scenarios is high. In other words, the total investment cost is overestimated if the number of benchmark scenarios is low. For instance, the total investment cost decreases by 7.9% if the number of benchmark scenarios increases from 1 to 12.

4.4. Sensitivity analysis: transmission line congestion

This subsection explores the impact of the configuration of existing transmission networks on investment decisions. Subsequently, the power capacities of the existing lines listed in Table B.12 were

Table 11
Transmission line congestion: Results (25% non-renewable).

Reduction tran. capacity (%)	Case	Obj. function (M€)	Transmission cost (M€)		Sto. Cost (M€)	Sol. time (h)
			1st stage	2nd stage		
-30	<i>Exp</i>	6.416	2.187	0.069	4.159	6.4
	<i>SD</i>	6.593	2.238	0.066	4.289	5.0
0	<i>Exp</i>	6.801	2.420	0.051	4.330	6.1
	<i>SD</i>	6.950	2.187	0.292	4.471	4.0
30	<i>Exp</i>	7.960	3.153	0.188	4.619	78.6
	<i>SD</i>	8.111	3.127	0.217	4.767	58.9

multiplied by 0.7 and 1.3, respectively. In this manner, two new transmission systems are simulated such that the transmission capacity is reduced/increased by 30%, respectively. Table 11 presents the objective function and investment costs for the *Exp* and *SD* cases for the two new transmission systems and the base case. As expected, the main result is that the investment costs increase rapidly as the transmission capacity decreased. If the transmission capacity is increased by 30% with respect to the base case, the total investment costs decrease by 6.2 and 4.1% in the *Exp* and *SD* cases, respectively. However, if the transmission capacity is reduced by 30% with respect to the base case, the total investment costs increase by 16.4 and 18.0% in the *Exp* and *SD* cases, respectively. It is also observed that the solutions obtained by *SD* cases are more conservative than those resulting from *Exp* cases because the total costs in *SD* cases are always higher than those of *Exp* cases. Similarly, investments in storage units are higher in all the *SD* cases.

5. Summary and conclusions

This paper formulates the problem faced by a transmission system planner that desires to determine the optimal investments in transmission lines and storage to satisfy the demand in a future target year. Additionally, the planner intends to enhance the existing network to facilitate demand procurement using renewable energy sources. Considering that future demand is uncertain and that can be characterized as a random variable, the mathematical enforcement of the maximum carbon emissions requires a comparison between two probability distributions. In this situation, the concept of stochastic dominance allows us to mathematically establish the preference of the decision-maker for a distribution function over others. Based on future uncertain demand, a benchmark has been defined to represent the acceptable quantity of carbon emissions. Subsequently, considering that small amounts of carbon emissions are preferred in this problem, using second-order stochastic dominance, it has been ensured that the resulting carbon emissions dominate the carbon emissions of the benchmark.

The proposed formulation is applied to a realistic case study based on an actual isolated power system. The obtained results indicate that the enforcement of second-order stochastic dominance constraints maintains the carbon emissions at an acceptable level compared with the established benchmarks. The performance of the *SD* approach was compared with that of an alternative formulation in which the stochastic dominance constraints were replaced by a condition establishing that the expected carbon emissions must be smaller than a given value. The obtained results indicate that the stochastic dominance formulation is able to maintain carbon emissions under the desired levels in all scenarios used to characterize the demand, and not only at their expected values. Specific conclusions over the application of stochastic dominance constraints to the considered problem are as follows:

- The solution times required to solve the *SD* approach are equivalent to those obtained by the formulation using the expected value of the demand.
- The solution of the *SD* approach is more conservative than that obtained by the *Exp* formulation in the sense that it results in comparatively higher investments, specially in storage units. The

total expected cost resulting from the *SD* approach is 2.2% higher in the analyzed base case.

- The *SD* approach is able to reduce carbon emissions in high demand scenarios, whereas the formulation that considers the expected value is not able to do that.
- The investments in second-stage assets (new lines in existing corridors and storage units) are higher in the *SD* approach. This facilitates the adaptation of the system to the considered demand scenario.
- First-stage decisions exhibit independence from the number of benchmark scenarios, whereas second-stage decisions are scenario-dependent.
- The usage of a small number of scenarios for characterizing the carbon emissions benchmark overestimates the second-stage investments and the total cost.

The conclusions derived from the resolved case study that could provide valuable insights for transmission system operators and policymakers are as follows:

- Investments in storage and second-stage transmission lines are significantly higher in the proposed formulation. Investing in second-stage assets allows the TSO to dynamically adapt to varying demand scenarios.
- Examining the current power system network configuration and generator unit locations reveals that the existing 132 kV network's primary function is to facilitate energy evacuation from large thermal units in specific buses. However, considering the expected decrease in production from these units by 2050 due to increased renewable capacity suggests no necessity for augmenting the 132 kV system. Conversely, the expected rise in renewable unit production across various buses necessitates substantial reinforcement of the 66 kV transmission system spanning the entire power system.
- Investments in storage prove to be more effective than investments in transmission lines, particularly in scenarios aiming to reduce high carbon emissions.
- Storage units strategically locate near high-consumption buses, buses with significant renewable unit capacity, or robust transportation capabilities.
- Investment costs demonstrate an increase inversely proportional to the decrease in transmission capacity.

Finally, we conclude this paper listing future lines of research:

- To consider investment decisions in generating capacity. This extension aims to cater not only to determining transmission expansion decisions by a TSO but also to encompass a more global approach for a regulator aiming to make informed decisions on generation capacity investments to meet decarbonization goals.
- To solve the capacity expansion problem with stochastic dominance constraints using a bilevel approach so that the market clearing organized by the Power Exchange is represented independently of the TSO expansion decisions.

- To apply stochastic dominance constraints to other variables important for the TSOs, such as, the maximum unserved demand or the production level of specific technologies.

CRedit authorship contribution statement

Ruth Domínguez: Data curation, Formal analysis, Validation, Writing – original draft, Writing – review & editing. **Miguel Carrión:** Conceptualization, Formal analysis, Investigation, Methodology, Supervision, Visualization, Writing – original draft, Writing – review & editing. **Sebastiano Vitali:** Conceptualization, Methodology, Validation, Writing – original draft, Writing – review & editing.

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Appendix A. Notation

The notation used to formulate the problem presented in Section 3 is described below.

Sets and indices

- B Set of buses, indexed by b .
- D Set of characteristic days, indexed by d .
- G Set of generating units, indexed by g .
- G_b Subset of G comprising generating units located in bus b .
- G^D Subset of G comprising dispatchable fossil-fuel generating units.
- G^I Subset of G comprising intermittent and renewable generating units.
- L Set of transmission lines, indexed by ℓ .
- $L^{C,E}$ Subset of L comprising candidate transmission lines located in existing corridors.
- $L^{C,N}$ Subset of L comprising candidate transmission lines located in new corridors.
- L^E Subset of L comprising the existing transmission lines.
- S Set of candidate storage units, indexed by s .
- S_b Subset of S comprising storage units located in bus b .
- T Set of time periods, indexed by t .
- Ω Set of scenarios, indexed by ω .
- Ξ^{Ben} Set of benchmark scenarios, indexed by ξ .

Parameters

- A_{gdt}^D Availability of intermittent unit g in characteristic day d and period t (pu).
- B_g^G Amount of carbon emissions per MWh produced by unit g (kg/MWh).
- $C_\ell^{L,N}$ Annualized capital cost of transmission line ℓ built in a new corridor (€).

- $C_\ell^{L,E}$ Annualized capital cost of transmission line ℓ built in an existing corridor (€).
 - $C_s^{I,SE}$ Annualized capital cost of the energy component of storage unit s (€/MWh).
 - $C_s^{I,SP}$ Annualized capital cost of the power component of storage unit s (€/MW).
 - $E_{\max,s}^{I,SE}$ Maximum energy capacity that can be installed from candidate storage unit s (MWh).
 - M Large enough constant (MW).
 - $P_{bdt\omega}^D$ Demand in bus b , characteristic day d , period t , and scenario ω (MWh).
 - $P_\xi^{\text{FF,Ben}}$ Scenario ξ of the benchmark used to limit the maximum allowable production generated by fossil-fuel units (MWh).
 - $P_{\text{up},g}^G$ Upper ramp factor of generating unit g (pu).
 - $P_{\text{dw},g}^G$ Down ramp factor of generating unit g (pu).
 - $P_g^{I,G}$ Capacity of generating unit g (MW).
 - $P_{\max,s}^{I,SP}$ Maximum power capacity that can be installed from candidate storage unit s (MW).
 - $P_{\max,\ell}^L$ Capacity of transmission line ℓ (MW).
 - $S_{\xi\xi'}^{\text{Ben}}$ Auxiliary parameter used to formulate second-order stochastic dominance constraints (MW).
 - W_d Weight of characteristic day d (h).
 - X_ℓ Reactance of line ℓ (Ω).
 - $\gamma_s^{S,O}$ Factor used to model the initial status of the storage unit s (pu).
 - $\gamma_s^{S,F}$ Factor used to model the final status of the storage unit s (pu).
 - γ_s^S Relationship between energy and power capacities in storage unit s (h).
 - $\gamma_s^{S,\min}$ Factor used to model the minimum energy that must contain storage unit s (pu).
 - η^S Efficiency of charging/discharging storage units (pu).
 - π_ω Probability of scenario ω (pu).
 - τ_ξ Probability of benchmark scenario ξ (pu).
- #### Variables
- p_ω^{FF} Total carbon emissions produced by fossil-fuel units in each scenario ω (kg).
 - $p_{gdt\omega}^G$ Power output of generating unit g in characteristic day d , period t , and scenario ω (MW).
 - $p_{gdt\omega}^{G,S}$ Power spillage of intermittent generating unit g in characteristic day d , period t , and scenario ω (MW).
 - $e_{sd\omega}^S$ Energy stored by storage unit s in characteristic day d , period t , and scenario ω (MWh).
 - $e_s^{I,S}$ Energy capacity built of storage unit s (MWh).
 - $p_s^{I,S}$ Peak power installed of storage unit s (MW).

Table B.12
Candidate transmission line parameters.

Line	Origin bus	Destination bus	Length (km)	Voltage (kV)	Capacity (MW)	# circuits
1	1	2	9.0	66	160	3
2	1	29	17.0	66	160	1
3	2	3	10	66	80	1
4	2	27	17.54	66	80	1
5	3	4	10.0	66	160	1
6	3	5	11	66	80	1
7	4	5	9.0	66	160	1
8	4	7	4.0	66	160	1
9	5	13	6.0	66	80	1
10	6	11	2.0	66	80	1
11	6	13	7.7	66	80	1
12	6	13	7.7	66	160	1
13	6	13	7.7	66	240	1
14	7	8	2.0	66	80	1
15	10	11	2.0	66	160	1
16	10	11	2.0	66	240	1
17	13	16	8.1	66	80	2
18	13	17	8.70	66	80	1
19	13	18	35	220	260	1
20	13	27	16	66	80	1
21	14	15	8.0	66	80	1
22	14	17	7.00	66	80	1
23	14	18	12.8	66	80	1
24	15	16	9.0	66	80	1
25	15	16	9.0	66	160	1
26	18	19	0.7	66	80	1
27	18	20	7.3	66	80	1
28	18	22	33	220	323	1
29	19	20	6.72	66	80	1
30	19	23	15.0	66	80	1
31	20	14	6.0	66	80	1
32	20	23	15.0	66	80	1
33	22	28	20.9	66	80	1
34	23	25	9.6	66	80	1
35	25	28	12.0	66	80	1
36	28	29	20.0	66	80	1

Table B.13
Characteristics of candidate lines.

Capacity (MW)	Voltage (kV)	Reactance (Ω /km)	Existing corridor	Cost (k€/km)
80	66	0.3616	No	373
			Yes	264
160	66	0.1808	No	621
			Yes	439
240	66	0.1205	No	812
			Yes	703
220	132	0.4015	No	714
			Yes	506

- $p_{\ell dt\omega}^L$ Power flow through line ℓ in characteristic day d , period t , and scenario ω (MW).
- $p_{sd t\omega}^{S,C}$ Consumption power of storage unit s in characteristic day d , period t , and scenario ω (MW).
- $p_{sd t\omega}^{S,D}$ Discharged power of storage unit s in characteristic day d , period t , and scenario ω (MW).
- $s_{\omega\zeta t}$ Auxiliary positive variable used to formulate second-order stochastic dominance constraints (MW).
- $y_{\ell\omega}^{L,E}$ Binary variable that is equal to 1 if candidate line ℓ located in an existing corridor is built in scenario ω , being 0 otherwise.
- $y_{\ell}^{L,N}$ Binary variable that is equal to 1 if candidate line ℓ located in a new corridor is built, being 0 otherwise.
- $\theta_{bd t\omega}$ Voltage angle of bus b in characteristic day d , period t , and scenario ω (rad).

Appendix B. Input data

This section includes Tables B.12 and B.13 with technical data used to characterize the case study presented in Section 4.

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