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On the Ratchet Effect in Competitive Markets*

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Abstract

We study a two-period economy with a continuum of perfectly competitive firms, each composed by a principal and an exclusive agent privately informed about his (persistent) production cost. Principals lack commitment power and can only use spot contracts. If players do not discount future profits, so that principals cannot fully screen types in the first period, and the adverse selection problem is sufficiently severe, there exists an equilibrium in which: (i) principals and agents randomize in the first period; (ii) principals' expected profit is an increasing function of the severity of the adverse selection problem; (iii) aggregate output may decrease over time (declining industry). If, instead, full separation is attainable in the first period, then, for intermediate values of the adverse selection problem, the game features a novel type of semi-separating equilibrium in which principals, rather than agents, randomize. In this equilibrium, aggregate production is constant over time, and principals are better off when the adverse selection problem worsens. These qualitative results also hold when considering long-term renegotiation-proof contracts.

JEL Classification: D40, D82, D83, L11.

Keywords: Adverse Selection, Competitive Markets, Ratchet Effect, Spot Contracts.

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1 Introduction

When the contractual relationship between a principal and a privately informed agent is repeated over time and the principal lacks commitment power, so that she can only offer spot contracts, the *ratchet effect* considerably complicates the structure of the optimal dynamic contract when the agent's type is constant (or sufficiently correlated) over time (see, e.g., Freixas et al. 1985, Laffont and Tirole, 1987, 1988, and Hart and Tirole, 1988, among many others). As the relationship evolves, information about the agent's type is revealed and, due to her inability to commit to a long-term contract, the principal cannot refrain from using opportunistically this information to capture the agent's surplus later on in the relationship. Anticipating such an opportunistic behavior, the agent will alter his early choices in order to secure future rents. For this reason, dynamic incentives may be much more complex than static ones. In particular, compared to a static environment, separation is harder to sustain early on in the relationship: information is gradually revealed over time and, as a consequence, repeated relationships typically feature greater efficiency in later stages.

Non-commitment situations of this kind are very common in long run contractual relationships, and in particular between members of long-lasting organizations. Yet, to the best of our knowledge, the extant literature on the ratchet effect deals with these issues abstracting from cross-organizations externalities — i.e., assuming that the payoff obtained by a principal only depends on the output of the activity exerted by her agent, and not by the contractual decisions taken within other, potentially competing, organizations.

How does the presence of payoff externalities (e.g., due to market competition) modify the analysis of dynamic incentive schemes in long-term relationships under non-commitment? The answer to this question is far from being obvious. The benefit that, in a competitive environment, a principal obtains from learning the private information of her agent at any given stage clearly depends on her expectation on current and future market prices. This suggests that coordination problems may emerge among principals, which in turn creates the scope for the existence of mixed strategy equilibria, in which principals, as well as agents, in one or more periods, randomize between offering different contracts and accepting the offered contracts, respectively. Novel kind of equilibria are thus likely to arise as a consequence of the presence of payoff externalities in a model of repeated adverse selection and short-term contracts within competitive environments. The implications of these novel equilibria, especially in terms of industry dynamics, may well be different than those identified by the previous literature in the absence of competition, and somewhat counterintuitive, as we are going to show.

In this paper, we therefore examine the relationship between competition and the ratchet effect. To this purpose, we study a simple two-period economy populated by a continuum of perfectly competitive firms, each composed by a principal and an exclusive agent who is privately informed about his (persistent) production cost. Following the literature (e.g., Hart and Tirole, 1988) we consider a binary type-space and assume that principals lack commitment power and can only use spot contracts — i.e., in every period they simultaneously offer a wage to their agents.

To begin with, we examine the static benchmark and show that, when the adverse selection problem takes intermediate values, the game does not feature neither a symmetric equilibrium in which all principals offer the separating contract — i.e., shut down production by the inefficient type — nor an equilibrium in which they all offer the pooling contract — i.e., all firms produce regardless of the agents' type. In this region of parameters, we find that there exists either a symmetric mixed-strategy equilibrium in which principals randomize between offering the separating and the pooling contract, or an asymmetric, payoff-equivalent, equilibrium in which a fraction of principals offers the separating contract and the rest the pooling one. The reason is that, when the market price is responsive to aggregate supply, if all principals offered the separating (resp. pooling) contract, then the market price would be sufficiently high (resp. low), implying that any principal would find it profitable to deviate by offering the pooling (resp. separating) contract.

Building on this insight, we then turn to characterize the equilibria of the two period game, assuming that players do not discount future profits. Under such assumption, the ratchet effect is magnified, since principals cannot fully screen types in the first period — i.e., information can be learned in the first period only by offering a semi-separating contract, whereby efficient agents reveal their type in the first period with some probability, and inefficient agents never produce (see, e.g., Bolton and Dewatripont, 2005, Ch. 9). We find that, when the adverse selection problem is sufficiently severe, there exists a novel type of mixed strategy equilibrium in which all principals and efficient agents randomize in the first period. Specifically, principals offer the pooling contract with a certain probability and the semi-separating contract otherwise. In the latter case, as in the standard analysis with inelastic demand, efficient agents randomize between accepting or not that contract. Finally, no randomizations occur in the second period, in which all efficient agents produce and obtain no rent, whereas inefficient types are shut down. The considered equilibrium has two peculiar features. First, despite learning occurs (with some probability) in the first period, aggregate output is decreasing over time (*declining industry*) when the adverse selection problem is not too severe. The reason is that, even when the adverse selection problem is sufficiently severe, so that in a static game all principals would offer the separating contract, due to the ratchet effect, it is too costly for the principals to shut down production in the first period, implying that the first period is characterized by extensive pooling, whereas in the second period, in which separation of types is costless for the principals, they find it optimal to let only efficient types produce. Second, in such an equilibrium, the principals' expected profit increases as the adverse selection problem worsens. This is because, as the rent to be paid to the efficient type grows larger, principals shut down production in the first period with a higher probability, which results in a higher price, and in turn in larger principals' profits.

We then analyze the dynamic game under the hypothesis that players discount future profits, which implies that full separation is attainable also in the first period. We show that, for intermediate values of the adverse selection problem, namely in the region of parameters in which the static game features the above outlined mixed strategy equilibrium, the two-period game features a novel type of semi-separating equilibrium in which principals, rather than agents, randomize between offering the pooling and the separating contracts in both periods. Interestingly, in this equilibrium, aggregate production is the same in both periods. Therefore, even though principals can fully screen types in the first period, and learning indeed occurs with some probability in that period, the standard result that (with spot contracting) efficiency increases over time may fail to hold in competitive markets. Moreover, once again, due to the presence of price externalities, principals benefit from facing a more severe adverse selection problem.

Finally, we discuss how our results change in a framework in which principals can commit to long-term contracts in the first period and Pareto-improving renegotiations can occur before the second period. We find that for every equilibrium outcome (i.e., market prices, inter-temporal rents and profits) that can be achieved by means of a sequence of short-term contracts, there exists a long-term renegotiation-proof contract which yields the same outcome. However, in line with literature, we also find that, even when players do not discount future profits, there exists a long-term renegotiation proof contract which allows principals to implement a fully separating outcome in the first period. We then show that, for intermediate values of the adverse selection problem, the game under commitment and renegotiation admits three novel mixed strategy equilibria in which the long-term contract which implements the separating outcome of the model with spot contracts is offered with positive probability. In all these equilibria, aggregate quantity is constant over time and in two of them principals' profit is increasing in the extent of adverse selection. Therefore, the main insights of the spot contracting game remain valid under commitment and renegotiation.

Summing up, our simple model shows that, in an economy with informational asymmetries, the presence of repeated market interactions among vertical hierarchies plays a major role in shaping the optimal dynamic incentive schemes within each organization, in non-commitment situations where long-term principal-agent relationships are ruled by short-term contracts. Specifically, once payoff externalities are taken into account, efficiency may decrease over time (i.e., declining industries may be observed) and principals, as well as agents, may benefit as the adverse selection problem worsens, which instead harms consumers.

The rest of the article is organized as follows. After discussing the related literature, we describe the model in Section 2. Section 3 characterizes the equilibria in a static environment. Section 4 shows and analyzes the equilibria of the two-period game without discounting and presents the main results. In Section 5, we extend our analysis to the case in which players discount profits. Section 6 concludes. Proofs are in Appendix A. The analysis under long-term renegotiation proof contracts is detailed in Appendix B.

Related literature. By analyzing a model of dynamic contracting with competing vertical hierarchies, our paper is related and contributes to two different strands of the literature.

Vertical contracting with competing hierarchies. We contribute to the literature on competing vertical managerial firms, in which, to the best of our knowledge, only static models have been considered so far. Within this literature vein, several contributions examine price and/or quantity competition among managerial firms, each composed by a profit oriented owner (principal) and a self interested manager (agent), in models with hidden information and/or hidden action:

see, among the earliest contributions, Caillaud et al. (1995), Martin (1993) and Raith (2003). More closely similar to our framework, Hart (1983) and Scharfstein (1988), dealing with managerial slack, were among the first to consider competing managerial firms in a perfect competitive market. A model of hidden information with a continuum of managerial firms competing in a perfect competitive market has been recently proposed by Kastl et al. (2018), who contribute to the literature on information intermediaries. By considering a two-period model, we add to this literature relevant insights concerning industry dynamics.

Dynamic adverse selection. Our paper is related to the literature on dynamic adverse selection. More specifically, we contribute to previous work on repeated adverse selection with limited commitment, in dynamic contracting situations with persistent agent's type. The seminal contributions by Stokey (1981), Fudenberg and Tirole (1983) and Gul et al. (1986) focus on a durable good monopoly, showing that Coasian dynamics (Coase, 1972) arises under seller's limited commitment.[1] The durable good model is compared to the rental (or non-durable good) model in Hart and Tirole (1988): for both cases, the authors derive the solution under full-commitment (in which the parties can commit themselves to a mechanism or contract once and for all), the solution with long-term contract and renegotiation (in which the parties can write a long-term contract, but cannot commit themselves not to renegotiate this contract by mutual agreement),[2] and the solution under limited commitment, in which the parties can only write short-term contracts which rule within a period. In the last case, which we consider in our baseline model, the ratchet effect arises, as it was already shown in the two-period principal-agent models by Freixas et al. (1985) and Laffont and Tirole (1987, 1988), which bear a certain resemblance to the renting framework in Hart and Tirole (1998). As we already pointed out, all these works examine a principal-agent relationship taken in isolation, whereas we explicitly model market interactions among vertical hierarchies.

Finally, in examining the impact of competition on the ratchet effect, our work is also related to papers investigating others circumstances under which the ratchet effect is mitigated: see, e.g., Kanemoto and MacLeod (1992), in which an external source of contract enforcement, namely competition for second-hand workers, is taken into account; Carmichael and MacLeod (2000), who allow for non-Markovian strategies, by considering the threat of future punishment to sustain cooperation between an infinitely lived firm and a stream of short lived workers;[3] Acharya and Ortner (2017), who introduce productivity shocks in the principal's benefit from having the agent work; Beccuti and Möller (2018), analyzing a model in which the seller is more patient than the buyer; Gerardi and Maestri (2020), who show that rehiring is another possible remedy to the ratchet effect.

[1] For more recent contributions on the durable good monopoly under limited commitment in a mechanism design framework, with one or more potential buyers, see Skreta (2006, 2015). See also Ortner (2017), who considers a continuum of buyers and stochastic production costs.

[2] See also Laffont and Tirole (1990), who characterize the equilibrium of a two-period procurement model with commitment and renegotiation. The renegotiation issue in an infinite horizon framework has been recently addressed by Strulovici (2017) and Maestri (2017).

[3] For earlier contributions on the ratchet effect in models of labor contracting, see, e.g., Gibbons (1987), Dewatripont (1989) and Hosios and Peters (1993).

2 The model

Market and players. Consider a two-period economy in which, in every period $t = 1, 2$, there is a perfectly competitive market with a of unit mass of risk-neutral firms that produce a homogeneous good. In every period there is a representative consumer with a smooth quasi-linear utility function

$$u(x_t) - p_t x_t,$$

where $x_t \geq 0$ represents the quantity consumed and p_t the market price, with $u'(\cdot) > 0$ and $u''(\cdot) \leq 0$. Since consumers take the price p_t as given, the first-order condition for utility maximization, $u'(x_t) = p_t$, yields a standard differentiable downward-sloping demand function $D(p_t) \triangleq u'^{-1}(p_t)$.

Firms also take the (correctly anticipated) market price as given when choosing how much to produce. Each firm owner (principal) relies on a self-interested and risk-neutral manager (agent) to run the firm. In every period a firm i's production technology depends on the agent's (private) marginal cost of production $\theta_i \in \Theta \triangleq \{\underline{\theta}, \overline{\theta}\}$, with $\Delta \triangleq \overline{\theta} - \underline{\theta} > 0$ and $\Pr[\theta_i = \underline{\theta}] = \nu$, which realizes once and for all at the outset of the game, as, e.g., in Freixas et al. (1985) and Laffont and Tirole (1987). The parameter Δ can be interpreted as a measure of the severity of the adverse selection problem between principals and agents: the larger is Δ, the more relevant is agents' private information.

We assume that each firm either produces 1 unit of the good, or it does not produce at all — i.e., firm i's supply in period t is $y_{it} \in \{0, 1\}$. A binary production technology can be interpreted as an approximation of symmetric firms' production decisions in a perfectly competitive market, where firms are price takers and can either produce zero or a fixed share of the total quantity demanded. This is equivalent to assuming that firms are capacity constrained, so that no firm can supply the whole market, due for example to unmodelled technological constraints that prevent them from arbitrarily increasing the quantity produced (as in the shipping and transportation industries, electricity markets, and the hospitality industry).[4]

Hence, in every period t, aggregate supply is

$$y_t \triangleq \int_0^1 y_{it} di,$$

and the market clearing condition requires

$$p_t \triangleq u'(y_t).$$

We assume that with complete information — i.e., if there is no uncertainty about the agents' costs — it is always profitable for firms to produce, even when the cost is high (which is consistent with the adverse selection literature: see, e.g., Kastl et al., 2018). This requires that the lowest possible market price (when all firm produce) is sufficiently high.

[4] Since firms may be ex post asymmetric and have different marginal cost of production, with unbounded supply a low-cost firm would always be able to reduce the market price up to the point where production is unprofitable for a high-cost firm, thus driving it out of the market.

Assumption 1. $u'(1) \geq \overline{\theta} \quad \Leftrightarrow \quad \Delta \leq u'(1) - \underline{\theta}$.

Moreover, to obtain interesting results, we introduce the following restriction.

Assumption 2. *The difference* $u'(\nu) - u'(1)$ *is not too large.*

This assumption requires that the utility function of the representative consumer is not too concave (more below).

Payoffs and contracts. In every period $t = 1, 2$, if a firm i produces, given a price p_t and a transfer (wage) w_{it} paid by the principal to the agent, the principal obtains Bernoulli utility equal to $p_t - w_{it}$ and the agent obtains Bernoulli utility equal to $w_{it} - \theta_i$. Players' outside option is normalized to zero without loss of generality. For simplicity, in the baseline model, we assume that the discount factor (common to all players) is $\delta = 1$ — i.e., there is no discounting. This assumption magnifies the ratchet effect, which is at the core of our analysis. We consider discounting — i.e., $\delta \in (0, 1)$ — in Section 5.

Following Hart and Tirole (1988), we assume that there is no commitment — i.e., principals cannot commit in the first period to a wage to be offered in the second period (spot contracting) — and, without loss of generality, we posit that principals post a wage in every period — i.e., each agent i receives a wage offer w_{it} in every period t, which he can either accept ($x_{it} = 1$) or reject ($x_{it} = 0$).

Timing and equilibrium. The timing of the game is as follows. Agents learn their marginal costs at the outset of the game. Then, in every period $t = 1, 2$:

- Principals (simultaneously) offer wages and agents choose whether to accept them;
- Firms produce, wages are paid, and goods are traded.

A symmetric equilibrium in pure strategies is such that: (i) each principal offers a wage in the first period w_1^* and a wage in the second period $w_2^*(h_i)$ contingent on the history of the game $h_i \triangleq (p_1, x_{i1})$ observed up to period 1; (ii) given his cost θ_i, each agent decides whether to produce or not — i.e., $x_{it}^*(\theta_i) : \Theta \rightarrow \{0, 1\}$ for every period $t = 1, 2$; (iii) the aggregate supply function (because of the continuum of firms) is almost surely equal to[5,6]

$$y_t^* \triangleq \mathbb{E}\left[y_t^*(\theta)\right],$$

and (iv) the equilibrium price $p_t^* \triangleq u'(y_t^*)$ equalizes demand and aggregate supply (so that the product market clears). Mixed strategies are randomizations over pure strategies (more below).

Although we consider symmetric equilibria, we will argue that, by the law of large numbers, symmetric equilibria in mixed strategies can be interpreted as asymmetric equilibria in pure strategies in which principals offer different contracts.

[5] Throughout the paper, we make the standard convention that the strong law of large numbers holds.

[6] As in Legros and Newman (2013), the aggregate supply should be interpreted as a "short run" supply curve, when there is no exit or entry of new firms in the market.

3 The static benchmark

To gain intuition, we first consider the static version of the game.[7] In this case there are two possible symmetric equilibria in pure strategies. Namely, a separating equilibrium in which every principal offers $w^* = \underline{\theta}$ and only the efficient type produces, and a pooling equilibrium in which every principal offers $w^* = \overline{\theta}$ so that both types produce.

Consider first a separating equilibrium. By the law of large numbers, in this equilibrium aggregate production is $y^* = \nu$, and the market clearing price is $p^* = u'(\nu)$. Therefore, principals' equilibrium profit is $\nu(u'(\nu) - \underline{\theta})$, while (because firms are price takers) a deviation to a pooling contract yields $u'(\nu) - \overline{\theta}$. Hence, the game features a separating equilibrium if and only if

$$\nu\left(u'(\nu) - \underline{\theta}\right) \geq u'(\nu) - \overline{\theta} \quad \Leftrightarrow \quad \Delta \geq \Delta_1 \triangleq (1-\nu)\left(u'(\nu) - \underline{\theta}\right).$$

Essentially, if the adverse selection problem is sufficiently severe — i.e., if the rent Δ that has to be paid by a principal to her efficient agent is sufficiently large — every principal prefers to shut down the inefficient type.

Next, consider a pooling equilibrium in which every principal offers $w^* = \overline{\theta}$. By the law of large numbers, in this equilibrium aggregate production is 1 and the market clearing price is $p^* = u'(1)$. Therefore, principals' equilibrium profit is $u'(1) - \overline{\theta}$, while (because of perfect competition) a deviation to a separating contract yields $\nu(u'(1) - \underline{\theta})$. Hence, the game features a pooling equilibrium if and only if

$$u'(1) - \overline{\theta} \geq \nu\left(u'(1) - \underline{\theta}\right) \quad \Leftrightarrow \quad \Delta \leq \Delta_0 \triangleq (1-\nu)\left(u'(1) - \underline{\theta}\right).$$

Clearly, if the adverse selection problem is not too severe, principals prefer to pay an information rent to the efficient type rather than shutting down the inefficient type.

However, because firms always produce in the pooling equilibrium, the market clearing price is lower than in the separating equilibrium. In other words, in the pooling equilibrium principals exert a (negative) price externality one on each other, which is reflected by the fact that $\Delta_1 > \Delta_0$. As a result, in the region of parameters where $\Delta \in (\Delta_0, \Delta_1)$ there are no symmetric equilibria in pure strategies and, if a symmetric equilibrium exists, it must involve randomizations by the principals. Assuming that every principal offers $w^* = \underline{\theta}$ with probability α^* and $w^* = \overline{\theta}$ with complementary probability, it must be

$$\underbrace{u'\left(\alpha^*\nu + (1-\alpha^*)\right) - \overline{\theta}}_{\text{Profit if } w=\overline{\theta}} = \underbrace{\nu\left(u'\left(\alpha^*\nu + (1-\alpha^*)\right) - \underline{\theta}\right)}_{\text{Profit if } w=\underline{\theta}}.$$

Summing up, we can state the following.

Proposition 1. *The equilibria of the static game are as follows.*

- *For $\Delta \leq \Delta_0$ the game features a unique symmetric equilibrium in which every principal offers $w^* = \overline{\theta}$.*

[7] Time subscripts are therefore omitted throughout this section.

- *For $\Delta \in (\Delta_0, \Delta_1)$ there is a unique symmetric equilibrium in mixed strategy such that each principal offers $w^* = \underline{\theta}$ with probability α^* and $w^* = \overline{\theta}$ with probability $1 - \alpha^*$, with*

$$\alpha^* \triangleq \frac{1}{1-\nu} \left[1 - u'^{-1} \left(\underline{\theta} + \frac{\Delta}{1-\nu} \right) \right] \in (0,1) \,. \tag{1}$$

- *For $\Delta \geq \Delta_1$ the game features a unique symmetric equilibrium in which every principal offers $w^* = \underline{\theta}$.*

The existence of the equilibrium in which principals randomize is a direct consequence of market competition. In particular, the area in which the mixed strategy equilibrium exists expands as the price becomes more responsive to quantity — i.e., $\Delta_1 - \Delta_0 = (1-\nu)(u'(\nu) - u'(1))$. Since neither a pooling nor a separating equilibrium exists for intermediate values of Δ, principals must be indifferent between producing in both states or shutting down inefficient types.

Remark. The law of large numbers implies that, when the game features an equilibrium in mixed strategies, it also features an asymmetric equilibrium in pure strategies such that a measure α^* of principals offers $w^* = \underline{\theta}$ and a measure $1 - \alpha^*$ offers $w^* = \overline{\theta}$. This equilibrium is in fact payoff equivalent to that in mixed strategies. By construction, if a fraction α^* of principals offer $w^* = \underline{\theta}$ while a fraction $1-\alpha^*$ offers $w^* = \overline{\theta}$, each principal is indifferent between offering the separating and the pooling contract. This implies that compared to the case of inelastic demand — i.e., where $u''(\cdot) = 0$ and $\Delta_1 = \Delta_0$ — competition in the product market may induce ex-ante identical principals to choose different contracts in equilibrium. Specifically, some of them shut down the inefficient type, but the market price compensates them for this loss.

4 Equilibrium with repeated interaction

We now turn to examine the two-period model and study how the interplay between competition in the product market and limited commitment alters the principals' equilibrium behavior with respect to the static benchmark. In doing so, for expositional purposes and in light of the results shown in Proposition 1, we divide the space of parameters in three relevant regions:

1. Weak adverse selection: $\Delta \leq \Delta_0$;

2. Moderate adverse selection: $\Delta \in (\Delta_0, \Delta_1)$;

3. Strong adverse selection: $\Delta \geq \Delta_1$.

Before characterizing the equilibrium in each region, it is useful to observe that, regardless of the severity of the adverse selection problem, principals cannot fully screen agents in the first period. Essentially, by assuming $\delta = 1$ we are de facto maximizing the severity of the ratchet effect. This is because agents assign the same weight to current and future profits in their intertemporal utility function. Hence, an efficient agent has the largest incentive to hide his type in the first period in order to gain a rent in the second period.

To formalize the argument, suppose that a separating equilibrium in the first period exists. In this candidate equilibrium only the efficient type produces in the first period and obtains a wage $w_1 \geq \underline{\theta}$, and $w_2 = \underline{\theta}$ if $x_1 = 1$ and $w_2 = \overline{\theta}$ otherwise. Therefore, if a separating equilibrium exists, the efficient type must prefer to produce in the first period and obtain no surplus in the second period rather than producing only in the second period and obtain a rent Δ — i.e., the first-period wage must be such that

$$w_1 - \underline{\theta} \geq \Delta \quad \Leftrightarrow \quad w_1 \geq \overline{\theta},$$

but the inefficient type would then also accept w_1, thereby making a separating outcome in the first period impossible. Hence, the standard equilibrium in which principals screen agents in the first period in order to fully extract their surplus in the second period does not exist.

We can now turn to characterize the equilibrium set of the game in each region of parameters defined above.

Weak adverse selection. As in the static benchmark, when $\Delta \leq \Delta_0$ also the repeated game features a unique pooling equilibrium, in which all agents produce in both periods.

Proposition 2. *With weak adverse selection, the dynamic game features a unique symmetric pooling equilibrium in which* $w_1^* = w_2^* = \overline{\theta}$.

The intuition is straightforward: for Δ sufficiently small, principals prefer to pay the information rent to the efficient type rather than shutting down the inefficient type. In other words, any candidate equilibrium that involves separation of types in either period cannot exist because a deviation to a pooling contract is always profitable. Hence, in equilibrium both types of agents produce in both periods. Notice that such pooling equilibrium maximizes static and dynamic efficiency.

Moderate adverse selection. Suppose now that $\Delta \in (\Delta_0, \Delta_1)$. First, we show that, in this region of parameters, a symmetric semi-separating equilibrium *à la* Hart and Tirole (1988), such that in the first period only the efficient types produces with probability γ (see, e.g., Bolton and Dewatripont, 2005, Ch. 9), does not exist. To see why, notice that the posterior probability that an agent is efficient in the second period given that he has rejected the first-period offer is

$$\Pr\left[\theta = \underline{\theta} | x_1 = 0\right] = \frac{\nu\left(1-\gamma\right)}{\nu\left(1-\gamma\right) + 1 - \nu},$$

which is clearly decreasing in γ: the higher the probability that the efficient type produces in the first period, the lower is the probability that an agent who has not produced in the first period is efficient. In Appendix A we show that, regardless of the market prices, as in the standard analysis without product market competition, in a semi-separating equilibrium, $w_1^* = \underline{\theta}$ and

$w_2^* = \underline{\theta}$ for every $x_1 \in \{0,1\}$, so that inefficient types are always shut down. The efficient type always produces in the second period and accepts the first-period offer with probability γ^* being defined as the probability that makes a principal indifferent between offering $w_2 = \overline{\theta}$ and $w_2 = \underline{\theta}$ in the second period given $x_1 = 0$ — i.e., since $p_2^* = u'(\nu)$,

$$\underbrace{u'(\nu) - \underline{\theta} - \Delta}_{\text{Profit if } w_2=\overline{\theta}} = \underbrace{\frac{\nu(1-\gamma^*)}{\nu(1-\gamma^*)+1-\nu}\left(u'(\nu) - \underline{\theta}\right)}_{\text{Expected profit if } w_2=\underline{\theta} \text{ given } x_1=0} \quad \Leftrightarrow \quad \gamma^* = \frac{\Delta - \Delta_1}{\nu\Delta}. \tag{2}$$

Notice that this probability is increasing in Δ: as the adverse selection problem becomes more severe, the principals' profit from offering $w_2 = \overline{\theta}$ drops. Hence, to satisfy the indifference condition (2), γ^* must increase in order to reduce the profit associated with an offer $w_2 = \underline{\theta}$. Hence, when Δ is not large enough (i.e., $\Delta \leq \Delta_1$) a semi-separating equilibrium *à la* Hart and Tirole (1988) does not exist. Intuitively, this is because, with moderate adverse selection, a principal has no incentive to shut down the inefficient type in the second period given the market price of this candidate equilibrium — i.e., if $x_1 = 0$, it is optimal to offer $w_2 = \overline{\theta}$ for every $\gamma \in [0,1]$ when $p_2 = u'(\nu)$ (see Appendix A). Anticipating this, the efficient type is never willing to produce in the first period, which in turn implies that an equilibrium in which all principals offer $w_1 = \underline{\theta}$ cannot exist.

Next, consider a pooling equilibrium in which all principals offer $w_1^* = \overline{\theta}$. In this case, since no information is learned in the first period, the second period is *de facto* identical to the static game analyzed in Section 3. Therefore, neither $w_2^* = \overline{\theta}$ nor $w_2^* = \underline{\theta}$ offered by each principal can be an equilibrium because the static game does not feature neither a pooling nor a separating equilibrium for $\Delta \in (\Delta_0, \Delta_1)$. Hence, following a first-period offer $w_1^* = \overline{\theta}$, in the region of parameters under consideration, principals must randomize in the second period — i.e., they must offer $w_2^* = \underline{\theta}$ with probability α^* and $w_2^* = \overline{\theta}$ with probability $1-\alpha^*$, with α^* defined in (1). To show that this is indeed an equilibrium, notice that, since principals are indifferent between pooling and separating in the second period, the profit earned in the second period does not matter for a deviation to be profitable. Therefore, we only need to check that a principal does not find it optimal to deviate to $w_1^D = \underline{\theta}$.

However, this deviation cannot be profitable since no agent is willing to accept this offer. To see this, recall that the inefficient type never accepts $w_1^D = \underline{\theta}$, whereas the efficient type accepts this offer with probability γ^D such as to make the deviating principal indifferent between offering $w_2 = \overline{\theta}$ and $w_2 = \underline{\theta}$ in the second period given $x_1 = 0$ — i.e., since $p_2^* = \underline{\theta} + \frac{\Delta}{1-\nu}$,[8]

$$\underbrace{\frac{\Delta}{1-\nu} - \Delta}_{\text{Profit if } w_2=\overline{\theta}} = \underbrace{\frac{\nu\left(1-\gamma^D\right)}{\nu(1-\gamma^D)+1-\nu} \cdot \frac{\Delta}{1-\nu}}_{\text{Expected profit if } w_2=\underline{\theta} \text{ given } x_1=0} \quad \Leftrightarrow \quad \gamma^D = 0.$$

[8]Clearly, the efficient agent must be indifferent between accepting or not the wage $w_1^D = \underline{\theta}$ — i.e., it must be $w_2^D = \underline{\theta}$ for all $x_1 \in \{0,1\}$. If on the contrary, the agent believes that, with some positive probability, the deviating principal will offer $w_2^D = \overline{\theta}$ to an agent who did not produce in the first period, then it would never accept $w_1^D = \underline{\theta}$. Put it another way, throughout this section, principals can find it optimal to offer $w_1 = \underline{\theta}$ only if this offer induces semi-separation — i.e., it leads the efficient types to randomize between accepting or not the first-period offer, and the principal then offers $w_2 = \underline{\theta}$ for all $x_1 \in \{0,1\}$.

In words, if an efficient agent is willing to accept $w_1^D = \underline{\theta}$ with any positive probability, the principal would be better off offering $w_2^D = \overline{\theta}$ in the second period if $x_1 = 0$, but then the efficient agent would never accept the first-period offer. Summing up, with moderate adverse selection, a deviating principal cannot induce a semi-separating outcome. As a result, if she were to offer $w_1^D = \underline{\theta}$, her first-period profit would be zero, and, as she would not learn anything, her second-period profit would be equal to the equilibrium one, implying that the deviation is not profitable.[9] We can thus state the following.

Proposition 3. *With moderate adverse selection the dynamic game features a unique symmetric equilibrium in which principals offer the pooling contract in the first period — i.e., $w_1^* = \overline{\theta}$ — and randomize in the second period — i.e., $w_2^* = \underline{\theta}$ with probability α^*, and $w_2^* = \overline{\theta}$ otherwise.*

Hence, following the logic of Proposition 1, in the region of parameters where Δ takes intermediate values, if a symmetric equilibrium exists, then principals must randomize in the second period. The intuition behind this proposition is simple. Since principals can partially screen agents in the first period only by shutting down production with a high probability, with moderate adverse selection they all prefer to give up a rent to efficient agents. Hence, shut down of inefficient types occurs, with some probability, only in the second period, which, being identical to the static game, features the mixed strategy equilibrium characterized in Proposition 1.

Strong adverse selection. Finally, suppose that $\Delta \geq \Delta_1$. As seen before, in this region of parameters a symmetric semi-separating equilibrium *à la* Hart and Tirole (1988) might exist, since the value for γ^*, given by (2), is strictly positive.[10] However, in this region of parameters, novel types of equilibria may exist as well.

To begin with, notice that, from the analysis of the static game, it follows that, for every $\Delta > \Delta_1$, if all principals offer the pooling contract in the first period ($w_1 = \overline{\theta}$), then, in the second period, they all find it optimal to offer the separating contract ($w_2 = \underline{\theta}$). Alternatively, principals can offer $w_1 = \underline{\theta}$ in the first period so to induce the efficient type to randomize — i.e., implement the semi-separating outcome, whereby $w_2 = \underline{\theta}$. Therefore, regardless of the contract offered by the principals in the first period, in the second period they all offer $w_2^* = \underline{\theta}$, and obtain a profit $\nu(u'(\nu) - \underline{\theta})$. Thus, to characterize the equilibria of the game for $\Delta > \Delta_1$, we only need to compare the first-period profit that a principal can obtain by offering $w_1 = \overline{\theta}$ (i.e., the pooling contract) or $w_1 = \underline{\theta}$ (i.e., the semi-separating contract).

For given market price p_1, a principal finds it optimal to offer the pooling contract if and only if

$$p_1 - \overline{\theta} > \nu\gamma^*(p_1 - \underline{\theta}),$$

[9] Since second-period profits do not matter for a deviation to be profitable, from the foregoing analysis it immediately follows that a deviation involving a randomization between $w_1^D = \underline{\theta}$ and $w_1^D = \overline{\theta}$ in the first period is never optimal either.

[10] Moreover, under Assumption 1, $\gamma^* < 1$.

with γ^* being given by (2) (because $p_2^* = u'(\nu)$), yielding

$$\Delta < \sqrt{\Delta_1(p_1 - \underline{\theta})}.$$

In Appendix A we prove that, if every other principal offers the pooling contract (i.e., $p_1 = u'(1)$), then any principal finds it optimal to offer the pooling contract as well for every $\Delta < \Delta_2$, where

$$\Delta_2 \triangleq \sqrt{\frac{\Delta_0 \Delta_1}{1-\nu}}.$$

If, on the contrary, every other principal offers the semi-separating contract, so that $p_1 = u'(\frac{\Delta - \Delta_1}{\Delta})$, then a deviation to the pooling contract is profitable if and only if $\Delta < \Delta_3$, with $\Delta_3 > \Delta_2$ being the unique solution of

$$\frac{\Delta^2}{\Delta_1} = u'\left(1 - \frac{\Delta_1}{\Delta}\right) - \underline{\theta}. \tag{3}$$

As a consequence, for every $\Delta \in (\Delta_2, \Delta_3)$, a symmetric equilibrium of the game must involve principals' randomization in the first period. Specifically, we can show the following.

Proposition 4. *With strong adverse selection the game features the following symmetric equilibria:*

- *For $\Delta \in (\Delta_1, \Delta_2]$, there is a unique equilibrium in pure strategies such that every principal offers $w_1^* = \overline{\theta}$ in the first period and $w_2^* = \underline{\theta}$ in the second period.*

- *For $\Delta \in (\Delta_2, \Delta_3]$, there is a unique mixed strategy equilibrium such that every principal offers $w_1^* = \underline{\theta}$ with probability*

$$\rho^* \triangleq \frac{\Delta}{\Delta_1}\left[1 - u'^{-1}\left(\underline{\theta} + \frac{\Delta^2}{\Delta_1}\right)\right], \tag{4}$$

  and $w_1^* = \overline{\theta}$ otherwise. When $w_1^* = \underline{\theta}$ is offered, the efficient types accept the offer with probability γ^*. In the second period, every principal offers $w_2^* = \underline{\theta}$.

- *For $\Delta > \Delta_3$, there is a unique semi-separating equilibrium à la Hart and Tirole (1988) such that $w_1^* = w_2^* = \underline{\theta}$ and the efficient types accept the first-period offer with probability γ^*.*

Recall that in Proposition 3 shut down only occurred with some probability in the second period. By contrast, with strong adverse selection principals are forced to shut down production also in the first period. Indeed, for $\Delta \in (\Delta_1, \Delta_2]$ inefficient types are completely shut down in the second period, as it happens in the static game as well, but not in the first period. The most interesting region of parameters is that in which principals start shutting down production also in the first period. In fact, when $\Delta \geq \Delta_2$ a principal has an incentive to shut down the inefficient type in the first period given that the others are offering the pooling contract in that period, and this deviation clearly destroys the equilibrium where all principals offer the pooling contract in the first period, because the market price is too low relative to Δ. Yet, since it is impossible to fully separate types in the first period, shut down can only occur by implementing the semi-separating outcome, implying that in the first period also efficient agents randomize. Of course, when Δ becomes too large (i.e., $\Delta > \Delta_3$), the semi-separating equilibrium *à la* Hart and Tirole (1988) emerges: the adverse selection problem is extremely severe, so that inefficient types are completely shut down in both periods and even a fraction of efficient types does not produce in the first period.

As in the static model, the existence of an equilibrium in which principals randomize is a direct consequence of market competition (in fact, it can be easily checked that, with inelastic demand, $\Delta_2 = \Delta_3$), and it relates to the presence of price externalities. However, important differences arise with respect to the static benchmark, due to the impossibility of (fully) screening types in the first period. First, this implies that, in the first period, also (efficient) agents randomize. Second, unlike in the static game, in which pooling occurs (with positive probability) only when $\Delta < \Delta_1$, the fact that by offering $w_1 = \underline{\theta}$ also efficient types are shut down with positive probability makes it more attractive for every principal to offer the pooling contract in the first period. Hence, even when the adverse selection problem is relatively pronounced (i.e., for every $\Delta \in (\Delta_1, \Delta_3)$), the dynamic game features, at least to some extent, pooling in the first stage: the ratchet effect.

Dynamics of aggregate output. We are now ready to examine the implications of the foregoing equilibrium analysis for the industry dynamics. This is far from being a trivial question because, in our setting, there are two opposite forces driving the dynamics of aggregate output. On the one hand, information about the agents' (persistent) type can be revealed over time, which, *ceteris paribus*, would lead the aggregate output to increase over time. On the other hand, the ratchet effect makes it impossible to fully separate types at early stages, and costly to achieve a semi-separating outcome: as a consequence, early stages are characterized by extensive pooling, and, other things being equal, this would lead the aggregate output to fall over time. Which one of these two forces dominates clearly depends on the severity of the adverse selection problem. Specifically, we can show what follows.

Proposition 5. *There exists a threshold $\widehat{\Delta} \in (\Delta_2, \Delta_3)$ such that $y_1^* < y_2^*$ if and only if $\Delta > \widehat{\Delta}$.*

When the adverse selection problem is not too severe (i.e., $\Delta \in (\Delta_0, \widehat{\Delta})$),[11] we observe a *declining industry*. The reason is as follows. Even when the adverse selection problem is sufficiently severe such that in a static game all principals would offer the separating contract, due to the ratchet effect, to shut down production in the first period is too costly for the principals, relative to the rents to be paid to efficient types in a pooling contract. Therefore, as we already pointed out, the first period is characterized by extensive pooling. As a consequence, aggregate output is relatively high in the first period, in which, with relatively high probability,

[11]Obviously, for $\Delta \leq \Delta_0$, all principals offer the pooling contract in both periods, hence aggregate output is constant over time.

no information is learned. Hence, in the second period, in which the ratchet effect does no longer play a role, principals are more willing to separate types (since $\Delta > \Delta_0$), which results in a lower aggregate output.

On the contrary, when the adverse selection problem is sufficiently severe (i.e., $\Delta > \widehat{\Delta}$), paying rents to efficient types is extremely costly for the principals, who thus prefer to shut down production with a high probability in the first period, which results in a relatively low aggregate output. In this case, output expands in the second period, in which, as the ratchet effect wipes out, all efficient agents produce.

Profits and welfare. We can finally examine principals' profits and consumer welfare, confining our attention to the most interesting equilibrium — i.e., the novel type of mixed strategy equilibrium defined for $\Delta \in (\Delta_2, \Delta_3]$. In particular, we can show the following comparative statics result.

Proposition 6. *For all $\Delta \in (\Delta_2, \Delta_3]$, principals' expected profit is increasing in Δ, whereas consumer welfare is decreasing in Δ.*

Interestingly, unlike in the game without price externalities, in competitive markets principals may benefit from facing a more severe adverse selection problem. The reason is rather simple. In the mixed strategy equilibrium characterized above, as the rent to be paid to the efficient type grows larger, principals shut down production in the first period with a higher probability (ρ^* being increasing in Δ). The resulting higher price, in terms of principals' expected profit, more than offsets the increased probability of not producing at all (as principals are indifferent between the pooling and the semi-separating contract). However, the lower aggregate output in the first period unambiguously harms consumers.[12]

Remarks. To conclude this section, we briefly discuss some simple extensions of our analysis.

Asymmetric equilibria. Once again, by the law of large numbers, equilibria in mixed strategies can be interpreted as asymmetric equilibria where players play pure strategies. Specifically, for $\Delta \in (\Delta_2, \Delta_3]$, there exists an asymmetric pure strategy equilibrium, which is payoff-equivalent to the mixed strategy equilibrium described above, in which, in the first period:

- A measure ρ^* of principals offers $w_1^* = \underline{\theta}$ and a measure $1 - \rho^*$ offers $w_1^* = \overline{\theta}$;
- Among the agents who have been offered $w_1^* = \underline{\theta}$, only a measure γ^* of the efficient types accepts the proposed contract.

The same reasoning applies to the other mixed strategy equilibria characterized above.

Long-term renegotiation-proof contracts. One may wonder how the equilibrium configuration of the game changes when the possibility of commitment and Pareto-improving renegotiation is admitted — i.e., when each principal can commit to a long-term contract, offered at the

[12]Clearly, the second period plays no role in this analysis, since $w_2^* = \underline{\theta}$ and $y_2^* = \nu$ do not depend on Δ.

beginning of the game and which rules the relationship in both periods, however she cannot commit not to offer a new contract at the beginning of the second period that would replace the initial contract, if her agent finds it acceptable.

It can be easily seen that for every outcome (i.e., market prices, inter-temporal rents and profits) that can be achieved by means of a sequence of short-term contracts, there exists a long-term renegotiation-proof contract which yields the same outcome. However, there also exists a long-term renegotiation proof contract which allows principals to implement a fully separating outcome in the first period, which, in the game without discounting, cannot be achieved by using spot contracts. In fact, principals can commit to a contract specifying that $w_1 = \underline{\theta}$ and $w_2 = \overline{\theta}$ for every $x_1 \in \{0, 1\}$, which is clearly renegotiation-proof. As a consequence, the equilibrium set of the game under long-term renegotiation-proof contracts differs from the one characterized above.

Specifically, in Appendix B we show that, when the adverse selection problem takes extreme values (namely, for $\Delta \leq \Delta_0$ and $\Delta \geq \widehat{\Delta}$), the principals' ability to commit to long-term renegotiation-proof contracts does not alter the equilibrium outcome of the game, whereas for intermediate values of the adverse selection problem (i.e., when $\Delta \in (\Delta_0, \widehat{\Delta})$) the game under commitment and renegotiation admits three novel mixed strategy equilibria in which the long-term contract which implements the separating outcome of the model with spot contracts is offered with positive probability. In all these equilibria, aggregate quantity is constant over time and, in two of them, which only exist with elastic demand, the principals' profit is increasing in Δ.

Thus, the main insights of our model, namely that in competitive markets efficiency does not necessarily improve over time and the principals' profit can be non-monotone with respect to the severity of the adverse selection problem, remain valid when allowing for long-term renegotiation-proof contracts.

Concavity of the utility function. One may wonder how our results would change if the utility function $u(\cdot)$ does not satisfy Assumption 2 — i.e., it is sufficiently concave so that

$$\frac{\Delta_1}{\Delta_0} > \frac{1}{1-\nu}.$$

Since, in these cases, Δ_1 and, *a fortiori*, Δ_2 and Δ_3, do not satisfy Assumption 1, it follows that only the cases with weak and moderate adverse selection survive when Assumption 2 does not hold. It can be easily seen that the results shown in Propositions 2 and 3 apply regardless of the concavity of the utility function.

5 Equilibria with discounting

We now study the interplay between market competition and dynamics under the assumption that players discount second-period profits — i.e., we consider a discount factor $\delta \in (0, 1)$, common to all players. Within this framework, principals can achieve full separation in the first period, by offering a contract $w_1 = \underline{\theta} + \delta\Delta < \overline{\theta}$, and then extract agents' rents in the second period (see, e.g., Bolton and Dewatripont, 2005, Ch. 9).

We first argue that, in this case, the most interesting region of parameters is the one characterized by moderate adverse selection — i.e., $\Delta \in (\Delta_0, \Delta_1)$ — since, under our assumptions,[13] in this region the standard equilibria *à la* Hart and Tirole (1988) — i.e., the pooling, separating and semi-separating equilibria — fail to exist altogether. Moreover, within this region of parameters, we characterize two novel mixed-strategy equilibria in which principals rather than agents randomize in the first and, in some cases, also in the second period.

Accordingly, from now on suppose that $\Delta \in (\Delta_0, \Delta_1)$. Consider first a separating equilibrium in which all principals offer $w_1^* = \underline{\theta} + \delta\Delta$ in the first period and then fully extract the agents' surplus in the second period. It should be clear that, since principals are unable to sustain a separating outcome in the static game, *a fortiori* such an equilibrium must fail to exist in the dynamic game. The intuition is as follows: a deviation to a pooling contract $w_1^D = \overline{\theta}$ allows a principal to gain in the first period not only because the separating behavior of her rivals induces a market price which is larger than the rent obtained by the efficient type in the first period, but also because, by doing so, the deviating principal saves on the inter-temporal rent that has to be granted to an efficient type in order to induce separation.[14],[15]

Next, consider a symmetric semi-separating equilibrium *à la* Hart and Tirole (1988) in which $w_1^* = w_2^* = \underline{\theta}$ so that inefficient types are always shut down, while efficient types randomly accept the first-period offer. Since, in this candidate equilibrium, efficient agents would accept the first-period offer with probability γ^* defined by (2), it is clear that this equilibrium cannot exist for $\Delta < \Delta_1$. The reason is as in the baseline model with no discounting. When Δ is not too large, a deviation to a pooling contract in the second period is always profitable. Formally, this means that there is no mixed strategy that efficient types can play in the first period that makes principals indifferent between offering a pooling and a separating contract in the second period. Essentially, any agents' randomization in the first period tends to increase so much the equilibrium price in the second period that principals no longer find it profitable to shut down the inefficient type. In Appendix A we show that, with moderate adverse selection, this type of logic also rules out equilibria in which the semi-separating contract is offered with positive probability.

Finally, consider an equilibrium in which all principals offer $w_1^* = \overline{\theta}$. Because firms are

[13] Specifically, besides Assumptions 1 and 2, which now requires

$$\frac{\Delta_1}{\Delta_0} < \frac{1}{\sqrt{1-\nu}},$$

we also assume $u'''(\cdot) \leq 0$ — i.e., that the price elasticity of demand is increasing in the market price, or alternatively, that the demand function is concave. However, all our results also hold if $u'''(\cdot)$ is positive but not too large. Details are in Appendix A.

[14] Recall that efficient types anticipate that informed principals achieve full surplus extraction in the second period if they accept a separating contract in the first period. Hence, the rent given up to these types in the dynamic version of the game is larger than the static rent.

[15] Notice that, other things being equal, the deviating principal incurs a second-period (efficiency) loss since in the region of parameters under consideration the inefficient type is shut down. However, this loss is second order relative to the first-period gain(s) since the market price in the second period is kept low by the fact that rivals are informed about their agents' types and always produce.

perfectly competitive, it should be clear that, if the pooling equilibrium does not exist in the static game, it does not exist in the dynamic game either. This is because, in the region of parameters under consideration, as we have seen in the game without discounting, principals randomize in the second period if they all offered the pooling contract in the first period. Hence, they are indifferent between the pooling and the separating contract in the second period. As a consequence, second-period profits do not matter for a deviation to be profitable. Therefore, from the analysis of the static game, we know that each principal has a profitable deviation to a separating contract in the first period.

This in turn implies that the equilibrium characterized in this region of parameters for the game with $\delta = 1$ does no longer exist when principals can perfectly separate types in the first period and achieve full surplus extraction in the second period. Summing up, we can state the following.

Proposition 7. *With moderate adverse selection, for every* $\delta \in (0, 1)$*, the dynamic game features neither symmetric equilibria in which principals play pure strategies in the first period, nor a mixed strategy equilibrium in which agents randomize.*

Hence, following the logic of Proposition 1, in the region of parameters characterized by moderate adverse selection, if a symmetric equilibrium exists, then principals must randomize, at least in the first period. Indeed, since the game is repeated, randomizations may also occur in the second period. We shall thus consider mixed strategy equilibria in which, in the first period, each principal randomizes between offering the separating and the pooling contract; in the second period, the principals who offered the separating contract learn the agent's type and fully extract his rent, while those who offered the pooling contract can, alternatively: (i) offer a separating contract; (ii) offer a pooling contract; or (iii) randomize again between these two contracts. Clearly, which of these options will prevail in equilibrium depends on the severity of the adverse selection problem. Let

$$\Delta^* \triangleq (1-\nu)\big(u'(\tfrac{1+\nu}{2}) - \underline{\theta}\big),$$

with $\Delta^* \in (\Delta_0, \Delta_1)$ since $u'(\nu) > u'(\frac{1+\nu}{2}) > u'(1)$. We can show the following.

Proposition 8. *With moderate adverse selection, for all* $\delta \in (0, 1)$*, a symmetric equilibrium must induce principals to randomize at least in the first period. Specifically:*

- *If* $\Delta \in (\Delta_0, \Delta^*]$ *there exists only one symmetric equilibrium in which principals randomize in both periods:*
  - *In the first period, each principal offers* $w_1^* = \underline{\theta} + \delta\Delta$ *with probability* α^* *and* $w_1^* = \overline{\theta}$ *with probability* $1 - \alpha^*$*, with* $\alpha^* \in (0, 1/2)$ *being defined in (1).*
  - *In the second period, if she has offered* $w_1^* = \underline{\theta} + \delta\Delta$*, a principal offers* $w_2^* = \underline{\theta}$ *if*

$x_1 = 1$, and $w_2^* = \overline{\theta}$ otherwise. If $w_1^* = \overline{\theta}$, then $w_2^* = \underline{\theta}$ is offered with probability

$$\varepsilon^* \triangleq \frac{1 - u'^{-1}\left(\underline{\theta} + \frac{\Delta}{1-\nu}\right)}{u'^{-1}\left(\underline{\theta} + \frac{\Delta}{1-\nu}\right) - \nu}, \tag{5}$$

and $w_2^* = \overline{\theta}$ with probability $1 - \varepsilon^*$. Moreover, $\varepsilon^* > \alpha^*$.

- *If $\Delta \in (\Delta^*, \Delta_1)$ there exists a unique symmetric equilibrium in which principals randomize in the first period only. That is:*
  - *In the first period each principal offers $w_1^* = \underline{\theta} + \delta\Delta$ with probability β^* and $w_1^* = \overline{\theta}$ with probability $1 - \beta^*$, with $\beta^* \in [1/2, 1]$ being the unique solution of*

$$\frac{1-\delta}{1-\nu}\Delta = u'(\nu\beta^* + 1 - \beta^*) - \underline{\theta} - \delta(u'(\beta^* + (1-\beta^*)\nu) - \underline{\theta}). \tag{6}$$

  - *In the second period, each principal offers $w_2^* = \underline{\theta}$ if in the first period she has offered $w_1^* = \overline{\theta}$ or if she has offered $w_1^* = \underline{\theta} + \delta\Delta$ and $x_1 = 1$. Otherwise, $w_2^* = \overline{\theta}$ is offered.*

With an elastic demand and discounting, two novel equilibria arise in the region of parameters characterized by moderate adverse selection.[16] Indeed, none of these equilibria can exist when demand is inelastic (since principals never randomize in that case), nor when future profits are not discounted (since principals cannot achieve full separation in the first period).

In the first equilibrium, which is defined when Δ is not too large (i.e., $\Delta \leq \Delta^*$), principals play mixed strategies in both periods and randomizations are correlated over time, since, for obvious reasons, a principal randomizes in the second period only when she has not learned the agent's type in the first period (i.e., when the outcome of the randomization in the first period was a pooling contract). By contrast, when Δ is large enough (i.e., $\Delta > \Delta^*$) principals randomize in the first period only, and the indifference condition that pins down β^* equalizes the inter-temporal profits of each principal when she offers a pooling and a separating contract. In the second period each principal fully extracts the agent's surplus if she has learned his type, otherwise the separating contract is offered.

The intuition behind these results is as follows. From the analysis of the static game, we know that, for all $\Delta \in (\Delta_0, \Delta_1)$, all principals randomize in the second period if no learning occurred in the first period. However, when some principals learn their agent's type in the first period, they will always produce in the second one, which induces a drop in the second-period price. This in turn strengthens the incentives of the uninformed principals (i.e., those who offered the pooling contract in the first period) to shut down inefficient types in the second period. Yet, this effect is second order when Δ is small enough, also because in this case principals have still strong incentives to offer the pooling contract in the first period. As a consequence, in this region of parameters, randomizations occur in equilibrium in both periods. By contrast,

[16] Once again, by the law of large numbers, these symmetric mixed strategy equilibria can be alternatively interpreted as asymmetric pure strategy equilibria. Details are omitted for brevity and available upon request.

as Δ grows larger, offering the pooling contract becomes costlier, thereby principals have more incentives to learn their agent's type in the first period.[17] The consequent significant drop in the second-period price implies that, from the viewpoint of the uninformed principals, offering a pooling contract in the second period becomes too costly. Hence, for Δ large enough, principals who offer a pooling contract in the first period then strictly prefer to shut down inefficient types in the second period — i.e., mixed strategies are never optimal in the second period.

Dynamics of aggregate output. Interestingly, when the equilibrium features randomizations in both periods, the market price is constant over time, and coincides with the equilibrium price of the static game — i.e.,

$$p_1^* = p_2^* = \underline{\theta} + \frac{\Delta}{1-\nu}. \tag{7}$$

The reason is as follows. In the second period, in order for the uninformed principals to be indifferent between offering the pooling ($w_2 = \overline{\theta}$) or the separating ($w_2 = \underline{\theta}$) contract, the market price must be such that

$$p_2 - \overline{\theta} = \nu(p_2 - \underline{\theta}),$$

which coincides with the indifference condition of the static game, and yields the equilibrium price p_2^* in (7). To see why the equilibrium price must be the same in the first period, notice that principals who learned their agent's type in the first period obtain an extra-profit in the second period compared to uniformed principal of amount

$$\Pi \triangleq \nu(p_2^* - \underline{\theta}) + (1-\nu)(p_2^* - \overline{\theta}) - (p_2^* - \overline{\theta}) = \nu\Delta.$$

Hence, in order for principals to be indifferent between the pooling ($w_1 = \overline{\theta}$) and the separating ($w_1 = \underline{\theta} + \delta\Delta$) contract in the first period as well it must be

$$p_1 - \overline{\theta} = \nu(p_1 - \underline{\theta} - \delta\Delta) + \delta\Pi.$$

Solving for p_1 then gives $p_1^* = p_2^*$, as the discounted extra-profit obtained in the second period by an informed principal is entirely appropriated by her efficient agent to reveal his type in the first period.

As a result, although randomizations occur in this equilibrium, aggregate quantity is constant over time. This shows that, even though here principals can fully screen types in the first period (given that $\delta < 1$), the standard result that with spot contracting efficiency increases over time may fail to hold in competitive markets, even when learning occurs with some probability in the first period — i.e., when the adverse selection problem is relatively severe such that principals do not always offer the pooling contract.

By contrast, for $\Delta > \Delta^*$, aggregate quantity is increasing over time, as in the standard analysis with inelastic demand. This is because the larger magnitude of the adverse selection problem induces the majority of principals to offer separating contract in the first period (recall

[17] Indeed, the probability of offering the separating contract in the first period is increasing in Δ in both the equilibria under consideration.

that $\beta^* > 1/2$). This extensive learning then translates into a more efficient second-period outcome. To sum up, we can state what follows.

Proposition 9. *If $\Delta \in (\Delta_0, \Delta^*]$ aggregate production is the same in the first and second period — i.e.,*

$$\underbrace{\nu\alpha^* + 1 - \alpha^*}_{y_1^*} = \underbrace{\alpha^* + (1-\alpha^*)(\nu\varepsilon^* + 1 - \varepsilon^*)}_{y_2^*}.$$

If $\Delta \in (\Delta^*, \Delta_1)$ aggregate production is higher in the second period than in the first period — i.e.,

$$\underbrace{\nu\beta^* + 1 - \beta^*}_{y_1^*} < \underbrace{\beta^* + \nu(1-\beta^*)}_{y_2^*}.$$

Hence, in the region of parameters where $\Delta \in (\Delta_0, \Delta^*]$ competition tends to stabilize prices and production over time. This is because, although principals who have not learned the agent's type in the first period tend to be more selective in the second period — i.e., they offer the pooling contract with a lower probability than in the first period ($\varepsilon^* > \alpha^*$) — the other principals have learned in the first period and thus always produce in the second period. The result of Proposition 9 shows that, by the logic described above, in equilibrium, the positive effect of learning on aggregate output exactly offsets the higher probability of shutting down production in second period. Roughly speaking, when the adverse selection problem is relatively weak (i.e., $\Delta \leq \Delta^*$), the possibility of learning the agent's type in the first period is not much attractive for principals, and, as a consequence, efficiency does not improve over time.

However, unlike in the game with $\delta = 1$, aggregate quantity does not (strictly) decrease over time. Thus, when the ratchet effect is weaker, namely when full screening is viable in the first period, the separating contract is offered with positive probability in equilibrium, thereby reducing the extent of pooling in the first period. As a consequence, as seen in the game under principals' commitment to long-term renegotiation proof contracts, the ability to perfectly screen agents in the first period rules out the possibility of observing a declining industry.

Profits and welfare. As for profits and consumer welfare in the considered equilibria, we can show the following comparative statics result.

Proposition 10. *For $\Delta \in (\Delta_0, \Delta^*]$, principals' equilibrium profit is increasing in Δ. This is true also for $\Delta \in (\Delta^*, \Delta_1]$ when δ is relatively small, whereas the opposite holds when δ is sufficiently high. Finally, consumer welfare is decreasing in Δ for all $\Delta \in (\Delta_0, \Delta_1]$.*

The trade-off behind this result is as follows. On the one hand, as Δ grows larger, principals tend to shut down production with higher probability, and, *ceteris paribus*, benefit from the resulting higher prices. On the other hand, they can separate types in the first period only by giving up a rent $\delta\Delta$ to the efficient type. As a matter of fact, when the adverse selection problem is sufficiently severe and δ is relatively high, the latter effect dominates, thereby leading the equilibrium profit to be decreasing in Δ. In the other cases, principals are better off when facing a more severe adverse selection problem.

The result concerning consumer welfare is immediate when $\Delta \in (\Delta_0, \Delta^*]$, since, for the reasons pointed out above, aggregate output is constant over time and decreasing in Δ. In the equilibrium defined for larger values of Δ, it is easy to see that

$$\frac{\partial y_1^*}{\partial \Delta} = -\frac{\partial y_2^*}{\partial \Delta} < 0.$$

This is because principals who let both types produce in the first period then shut down inefficient types in the second period, and viceversa. This of course implies implies that, under the assumption that consumers discount future utility, consumer welfare is strictly decreasing in Δ.

Remarks. Some other results are briefly discussed in the following remarks.

Comparison. It is interesting to study how introducing the possibility of full separation in the first period modifies equilibrium quantities compared to the game without discounting. First, it is immediate to see that, for all $\Delta \in (\Delta_0, \Delta_1)$, aggregate output in the first period is higher when players do not discount future profits — i.e., $y_1^*|_{\delta=1} = 1 > y_1^*|_{\delta<1}$. The reason is simple. As discussed before, when the ratchet effect is stronger (i.e., when $\delta = 1$), the first period is characterized by extensive pooling, which entails a higher aggregate output compared to the game in which principals can fully screen their agents' type in the first period. In the second period, for $\Delta \in (\Delta_0, \Delta^*]$, regardless of the value of the discount factor, the aggregate output in the second period coincides with that of the static game — i.e., $y_2^*|_{\delta=1} = y_2^*|_{\delta<1}$. By contrast, for $\Delta \in (\Delta^*, \Delta_1)$: $y_2^*|_{\delta=1} < y_2^*|_{\delta<1}$.[18] In words, when Δ is sufficiently high, if $\delta < 1$ a relatively high fraction of principals finds it optimal to offer the separating contract in the first period, which yields a higher aggregate output in the second period compared to the static game, whose outcome is achieved in the game without discounting. Taken together, these results also show a discontinuity of the equilibrium outcome at $\delta = 1$.

Weak and strong adverse selection. One may wonder what are the equilibria of the game with discounting in the cases of weak and strong adverse selection. The answer is rather simple. When $\Delta \leq \Delta_0$, it is easy to prove that, as in the game without discounting, in the unique equilibrium of the game, all principals offer the pooling contract in both periods. Thus, even though principals can perfectly screen types in the first period, they all prefer to pay a rent to the efficient type in both periods when the adverse selection problem is weak.

When $\Delta \geq \Delta_1$, within different subregions of parameters, there may be a symmetric separating equilibrium, in which all principals offer $w_1^* = \underline{\theta} + \delta\Delta$ and achieve full surplus extraction in $t = 2$, and a symmetric semi-separating equilibrium *à la* Hart and Tirole (1988). Moreover, there may also be mixed strategy equilibria in which principals offer with positive probability at least two between the pooling, the separating and the semi-separating contract, and in the second period shut down inefficient types unless they learned their agent's type in the first period

[18]Formally, for $\Delta > \Delta^*$, we have $y_2^*|_{\delta=1} < y_2^*|_{\delta<1} \iff \alpha^* + \beta^* > 1$, which is satisfied if $\alpha^* > \beta^*(> \frac{1}{2})$. To see that this holds true in the considered region of parameters, notice that the right-hand side of equation (6), which is increasing in β, is higher than the left-hand side when evaluated at $\beta = \alpha^*$.

and can therefore achieve full surplus extraction.[19] In these equilibria, since the market prices depend on the principals' mixed strategies, their profit may still be non monotonic with respect to the magnitude of the adverse selection problem. However, we expect aggregate output to be non decreasing over time due to the strength of the ratchet effect in this region of parameters.

Long-term renegotiation-proof contracts. It can be proved that, for all $\delta \in (0, 1)$, the equilibrium outcome with spot contracts coincides with the equilibrium outcome with long-term contracts and Pareto-improving renegotiations. This result, which is well known in the two-period model without competition,[20] also holds true in our model with competition, because it can be established a one-to-one correspondence among the outcomes (i.e., market prices and discounted rents and profits) that can be achieved with long-term renegotiation-proof contracts and with short-term contracts. See Appendix B for the details.

6 Conclusion

The analysis of dynamic incentive schemes under limited commitment within competitive markets is a rather complex issue. In this paper, we tackled this problem by studying a two-period economy populated by a continuum of perfectly competitive firms and under the assumption of a discrete characteristics space. This simple set up enabled us to provide a complete characterization of the equilibrium set of the game.

The analysis has revealed that, once market interaction is taken into account, some of the well known results shown by the previous literature on repeated adverse selection with limited commitment and persistent agents' types no longer hold true. Specifically, when the adverse selection problem is relatively severe, despite some principals learn their agents' type early on in the relationship, due to the ratchet effect, efficiency (i.e., aggregate output) may decrease over time. Moreover, a more severe adverse selection problem is likely to be translated into a higher market price, which benefits principals, while harming consumers.

[19] Some details can be found in Appendix A. Additional details are available upon request.

[20] As shown by Hart and Tirole (1988), in general, the sale and rental outcome with long-term contracts and renegotiation coincide both with one another and with the spot contracting sale outcome. Moreover, in the specific case with two periods, under non-commitment, the rental model and the sale model coincide (see, e.g., Bolton and Dewatripont, 2005, Ch. 9). Thus, the outcome of the rental model, which is equivalent to our model in the case of inelastic demand, is the same under non-commitment and under long-term contracts and renegotiation.

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Appendix

A Proofs

Proof of Proposition 1. The results are proved in Section 3. □

Proof of Proposition 2. Consider a symmetric equilibrium in which each principal offers the pooling contract in the first period ($w_1 = \overline{\theta}$). Given that, in such equilibrium, no information is extracted from the agents in the first period, the analysis of the second period is identical to the analysis of the static game, hence each principal finds it optimal to offer the pooling contract also in the second period for all $\Delta \leq \Delta_0$. In this case, her expected profit is

$$\pi^{PP} \triangleq 2(u'(1) - \overline{\theta}).$$

The considered strategy profile constitutes an equilibrium if and only if there are no profitable deviations from it — i.e., every principal must not want to deviate from the prescribed strategy profile in either period.

Since, as we already noticed, in the considered range of parameters, following a pooling contract in the first period, there are no profitable deviations in the second period, the only possibly profitable deviation for a principal consists in offering $w_1^D = \underline{\theta}$ in the first period.

However, this deviation cannot be profitable since no efficient agent is willing to accept this offer. In fact, if an efficient agent were to accept $w_1^D = \underline{\theta}$ with probability $\gamma > 0$, then the deviating principal would always find it optimal to offer $w_2^D = \overline{\theta}$ — i.e., since $p_2^* = u'(1)$, for all $\gamma > 0$ and $\Delta \leq \Delta_0$:

$$\underbrace{u'(1) - \underline{\theta} - \Delta}_{\text{Profit if } w_2=\overline{\theta}} > \underbrace{\frac{\nu(1-\gamma)}{\nu(1-\gamma)+1-\nu}\left(u'(1) - \underline{\theta}\right)}_{\text{Expected profit if } w_2=\underline{\theta} \text{ given } x_1=0}.$$

This of course implies that, with weak adverse selection, a deviating principal cannot induce a semi-separating outcome. As a consequence, the considered deviation is not profitable, and there cannot exist other equilibria in the region of parameters under consideration. □

Proof of Proposition 3. Within any managerial firm, for any given market prices p_1 and p_2, a semi-separating contract is obtained as follows. The principal offers $w_2^* = \underline{\theta}$, to an agent who has not accepted the wage $w_1 \in [\underline{\theta}, \overline{\theta})$, with a probability σ^* such that the efficient agent is indifferent between accepting or not the wage w_1. Consider the efficient type agent: if he accepts w_1, then he gets $w_1 - \underline{\theta}$ in the first period, and zero in the second period (since the principal, knowing that he is efficient, will offer $w_2 = \underline{\theta}$); if, instead, he refuses the first-period contract, then he will get zero in the first period but a profit Δ with probability $1 - \sigma^*$ in the second period. Equating these payoffs yields

$$w_1 - \underline{\theta} = (1 - \sigma^*)\Delta \quad \Leftrightarrow \quad \sigma^* = 1 - \frac{w_1 - \underline{\theta}}{\Delta}.$$

Next, as detailed in Section 4, each efficient agent accepts w_1 with a probability γ^* such as to make the principal indifferent between offering $w_2 = \underline{\theta}$ and $w_2 = \overline{\theta}$ to an agent who refused w_1 — i.e., for any given market price p_2,

$$\underbrace{p_2 - \underline{\theta} - \Delta}_{\text{Profit if } w_2=\overline{\theta}} = \underbrace{\frac{\nu\,(1-\gamma^*)}{\nu\,(1-\gamma^*)+1-\nu}\,(p_2-\underline{\theta})}_{\text{Expected profit if } w_2=\underline{\theta} \text{ given } x_1=0} \quad \Leftrightarrow \quad \gamma^* = \frac{1}{\nu}\left(1 - \frac{(1-\nu)(p_2-\underline{\theta})}{\Delta}\right). \tag{8}$$

For any given market prices p_1 and p_2, the principal's expected profit is

$$\pi^{SS} \triangleq \nu\gamma^*(p_1 - w_1) + \pi_2,$$

where π_2 is the second-period profit, which she gets from playing according to the mixed strategy σ^* derived above. Notice that, since, by definition, π_2 must be equal to the profit that she can get by setting either $w_2 = \underline{\theta}$ or $w_2 = \overline{\theta}$ with probability one, it does not depend on w_1. Thus, π^{SS} is clearly decreasing in w_1: therefore, for every p_1, $w_1^* = \underline{\theta}$ and, accordingly, $\sigma^* = 1$ — i.e. the principal finds it optimal to offer (with probability one) $w_2^* = \underline{\theta}$ to an agent who refused w_1.

The rest of the proof is provided in Section 4. □

Proof of Proposition 4. For $\Delta > \Delta_1$, the game can admit the following equilibria.

- First consider a pure strategy equilibrium in which all principals offer the pooling contract $w_1^* = \overline{\theta}$ in the first period. In this case, since the second period is identical to the static game, and $\Delta > \Delta_1$, all principals find it optimal to offer the separating contract $w_2^* = \underline{\theta}$ in the second period. Accordingly, in this candidate equilibrium, each principal obtains a profit
$$\pi^{PS} \triangleq u'(1) - \underline{\theta} - \Delta + \nu(u'(\nu) - \underline{\theta}).$$
As we argued in Section 4, a principal can only deviate from the considered strategy profile by offering $w_1^D = \underline{\theta}$, which induces the semi-separating outcome. In this case, the deviating principal obtains a profit
$$\pi^D \triangleq \left(1 - \frac{\Delta_1}{\Delta}\right)\frac{\Delta_0}{1-\nu} + \frac{\nu}{1-\nu}\Delta_1,$$
and it can be easily checked that $\pi^D < \pi^{PS}$ if and only if $\Delta < \Delta_2$.

- Next consider a mixed strategy equilibrium in which, in the first period, principals offer $w_1^* = \underline{\theta}$ (i.e., induce semi-separation) with probability ρ and $w_1^* = \overline{\theta}$ (i.e., the pooling contract) otherwise. As explained in Section 4, in the considered region of parameters, every principal then offers $w_2^* = \underline{\theta}$. In order for such an equilibrium to exists, principals must be indifferent between offering the two considered contracts, which immediately yields
$$\Delta^2 = (1-\nu)(u'(\rho\nu\gamma^* + 1 - \rho) - \underline{\theta})(u'(\nu) - \underline{\theta}),$$
with γ^* being defined by (2), from which we obtain the mixed strategy ρ^* given by (4).

Such value turns out to be positive for $\Delta > \Delta_2$ and lower than one if Δ is *sufficiently small* to satisfy

$$\frac{\Delta^2}{\Delta_1} < u'\left(1 - \frac{\Delta_1}{\Delta}\right) - \underline{\theta} \quad \Leftrightarrow \quad \Delta < \Delta_3.$$

The result $\Delta_3 > \Delta_2$ obtains since it is easy to check that the above inequality is satisfied at $\Delta = \frac{\Delta_1}{\sqrt{1-\nu}} > \Delta_2$. Finally, notice that, since all principals find it optimal to offer $w_2^* = \underline{\theta}$ in the second period and they are indifferent between the two candidate optimal contracts in the first period, no principal can profitably deviate from this equilibrium.

- Finally, consider the symmetric semi-separating equilibrium, in which all principals offer $w_1^* = w_2^* = \underline{\theta}$, efficient agents accept the first-period offer with probability γ^* defined by (2) and they always produce in the first period, whereas inefficient types never produce. In this candidate equilibrium, each principal would obtain

$$\pi^{SS} \triangleq \nu\gamma^* \left(u'(\nu\gamma^*) - \underline{\theta}\right) + \nu \left(u'(\nu) - \underline{\theta}\right) = \left(1 - \frac{\Delta_1}{\Delta}\right)\left(u'\left(1 - \frac{\Delta_1}{\Delta}\right) - \underline{\theta}\right) + \frac{\nu}{1-\nu}\Delta_1.$$

Clearly, the only possibly profitable deviation from this candidate equilibrium consists in offering the pooling contract $w_1^D = \overline{\theta}$ in the first period,[21] and the separating contract $w_2^D = \underline{\theta}$ in the second period.[22]. Accordingly, the deviation profit is

$$\pi^D \triangleq u'(\nu\gamma^*) - \overline{\theta} + \nu(u'(\nu) - \underline{\theta}) = u'\left(1 - \frac{\Delta_1}{\Delta}\right) - \underline{\theta} - \Delta + \frac{\nu}{1-\nu}\Delta_1.$$

It can be immediately checked that $\pi^{SS} = \pi^D$ if and only if Δ solves equation (3). Since the left-hand side and the right-hand side of equation (3) are, respectively, increasing and decreasing in Δ, it follows that the symmetric semi-separating equilibrium exists for all $\Delta \geq \Delta_3$, whenever Δ_3 satisfies Assumption 1 — i.e., if $\Delta_3 \leq \frac{\Delta_0}{1-\nu}$. □

Proof of Proposition 5. Clearly, aggregate output cannot increase over time when all principals offer the pooling contract in the first period — i.e., for all $\Delta < \Delta_2$. On the contrary, for all $\Delta > \Delta_3$: $y_1^* = \nu\gamma^* < y_2^* = \nu$.

Next, it is easy to obtain that, for all $\Delta \in (\Delta_2, \Delta_3)$,

$$y_1^* = u'^{-1}\left(\underline{\theta} + \frac{\Delta^2}{\Delta_1}\right), \qquad y_2^* = \nu. \tag{9}$$

Moreover, y_1^* is clearly decreasing in Δ. Finally, $y_1^*|_{\Delta\to\Delta_2} = 1 > y_2^*$ and $y_1^*|_{\Delta\to\Delta_3} = \nu\gamma^*|_{\Delta\to\Delta_3} = 1 - \frac{\Delta_1}{\Delta} < y_2^*$, which concludes the proof. □

[21] To see this, notice that, since the agents' mixed strategy γ^* does not depend on the wages w_1 and w_2, but only on the equilibrium price $p_2 = u'(\nu)$, it follows that, for every value of the wages, agents do not have any profitable deviation from their mixed strategy. Moreover, since we proved that each principal's optimal choice within the set $w_1 \in [\underline{\theta}, \overline{\theta})$ is $w_1^* = \underline{\theta}$, a principal could profitably deviate from the equilibrium strategy in the first period only by offering the pooling contract $w_1^D = \overline{\theta}$.

[22] Given that $p_2 = u'(\nu)$, and $\Delta > \Delta_1$, the deviating principal (who did not learn the agent's type in the first period) finds it optimal to offer $w_2^D = \underline{\theta}$.

Proof of Proposition 6. It can be easily checked that, for $\Delta \in (\Delta_2, \Delta_3]$, the equilibrium profit is

$$\pi^* \triangleq \frac{\Delta^2}{\Delta_1} - \Delta + \frac{\nu}{1-\nu}\Delta_1.$$

Hence, $\frac{\partial \pi^*}{\partial \Delta} > 0 \Leftrightarrow \Delta > \frac{\Delta_1}{2}$, which is satisfied in the considered range of the parameters.

Consumer welfare, in our simple competitive environment (and no discounting)[23], is defined as

$$W \triangleq u(y_1) - y_1 u'(y_1) + u(y_2) - y_2 u'(y_2),$$

and it is an increasing function of aggregate outputs.[24] Thus, since, from (9), y_1^* is decreasing in Δ, whereas y_2^* does not depend on Δ, it follows $\frac{\partial W}{\partial \Delta} < 0$. □

Proof of Proposition 7. The proof proceeds as follows: through a series of Lemmata, we establish that symmetric pure strategy equilibria and mixed strategy equilibria in which agents randomize do not exist for $\Delta \in (\Delta_0, \Delta_1)$.

Lemma A.1 *A (symmetric) pure strategy equilibrium in which all principals offer the pooling contract in both periods (i.e., $w_1^* = w_2^* = \overline{\theta}$), exists if and only if $\Delta \leq \Delta_0$. Moreover, there are no other equilibria in which all principals offer the pooling contract in the first period.*

Proof. Consider a symmetric equilibrium in which all principals offer the pooling contract in the first period ($w_1^* = \overline{\theta}$). Clearly, in such candidate equilibrium, each principal finds it optimal to offer the pooling contract also in the second period whenever $\Delta \leq \Delta_0$. In this case, her expected profit would be

$$\pi^{PP} \triangleq (1+\delta)(u'(1) - \overline{\theta}).$$

Consider the deviation consisting in offering the separating contract in the first period ($w_1^D = \underline{\theta} + \delta\Delta$). Since the deviating principal then achieves full rent extraction in the second period, she obtains

$$\pi^D \triangleq \nu(u'(1) - \overline{\theta}) + \delta[u'(1) - \nu\underline{\theta} - (1-\nu)\overline{\theta}] = \nu(u'(1) - \underline{\theta}) + \delta(u'(1) - \overline{\theta}),$$

where $\pi^P \geq \pi^D$ if and only if $\Delta \leq \Delta_0$. Notice that there are no other possible deviations from this candidate equilibrium, since, as we already shown in the case with no discounting, the semi-separating outcome cannot be obtained for $\Delta \leq \Delta_0$.

For $\Delta_0 < \Delta < \Delta_1$, in a candidate equilibrium in which each principal offers the pooling contract in the first period, the mixed strategy equilibrium defined by (1) is played in the second period. Accordingly, each principal's profit is

$$\pi^{PM} \triangleq \underbrace{u'(1) - \overline{\theta}}_{w_1 = \overline{\theta}} + \underbrace{\delta\left[\alpha^*\nu\left(u'\left(\nu\alpha^* + 1 - \alpha^*\right) - \underline{\theta}\right) + (1-\alpha^*)\left(u'\left(\nu\alpha^* + 1 - \alpha^*\right) - \overline{\theta}\right)\right]}_{\mathbb{E}[w_2] = \alpha^*\underline{\theta} + (1-\alpha^*)\overline{\theta}}.$$

[23] Clearly, since, in the equilibrium under consideration, the second period output does not depend on Δ, our comparative statics result holds true even assuming that consumers, differently from players, discount future utility.

[24] For all $t = 1, 2 : \frac{\partial W}{\partial y_t} = -y_t u''(y_t) > 0$, by the concavity of the utility function.

A deviation for any principal consists in offering the separating contract in the first period, thus being able to achieve the first-best outcome in the second period, and obtaining

$$\pi^D \triangleq \nu((u'(1) - \underline{\theta} - \delta\Delta) + \delta(u'^* + 1 - \alpha^*) - \underline{\theta})) + (1-\nu)\delta\underbrace{(u'(\nu\alpha^* + 1 - \alpha^*) - \overline{\theta})}_{=\nu(u'(\nu\alpha^*+1-\alpha^*)-\underline{\theta})},$$

It can be easily checked that $\pi^{PM} \geq \pi^D$ if and only if

$$\Delta_0 - (1-\delta\alpha^*)\Delta - \delta\alpha^*(1-\nu)(u'(\alpha^*\nu + 1 - \alpha^*) - \underline{\theta}) > 0,$$

which, substituting α^* from (1), is satisfied for all $\Delta < \Delta_0$: thus, the considered equilibrium does not exist.

Lastly, consider $\Delta \geq \Delta_1$: in this case, in a symmetric equilibrium in which all principals offered the pooling contract in the first period, they find it optimal to offer the separating contract in the second period. Accordingly, each principal's profit is given by

$$\pi^{PS} \triangleq u'(1) - \overline{\theta} + \delta\nu(u'(\nu) - \underline{\theta}).$$

Again, a deviation for any principal consists in offering the separating contract in the first period, thus being able to achieve the first-best outcome in the second period, and obtaining

$$\pi^D \triangleq \nu[u'(1) - \underline{\theta} - \delta\Delta] + \delta[u'(\nu) - \nu\underline{\theta} - (1-\nu)\overline{\theta}] = \nu(u'(1) - \underline{\theta}) + \delta(u'(\nu) - \overline{\theta}).$$

Hence, $\pi^P \geq \pi^D$ if and only if

$$\Delta \leq \frac{\Delta_0 - \delta\Delta_1}{1-\delta} < \Delta_1.$$

Thus, the considered equilibrium does not exist. □

Lemma A.2 *A necessary condition for the existence of a (symmetric) pure strategy equilibrium in which all principals offer the separating contract $w_1^* = \underline{\theta} + \delta\Delta$ in the first period (hence, full surplus extraction is obtained in the second period) is $\Delta \geq \frac{\Delta_1 - \delta\Delta_0}{1-\delta} > \Delta_1$.*

Proof. Consider a symmetric equilibrium in which only efficient types produce in the first period — i.e., each principal offers the separating contract in $t = 1$. In such an equilibrium, $w_1^* = \underline{\theta} + \delta\Delta < \overline{\theta}$, $w_2^* = \overline{\theta}$ if an agent has rejected w_1^* in the first period, and $w_2^* = \underline{\theta}$ otherwise. It is immediate to see that an efficient type has no incentive to reject the first period offer. Hence, there is full surplus extraction in the second period and every principal obtains an equilibrium profit

$$\pi^S \triangleq \nu\left[\left(u'(\nu) - \underline{\theta} - \delta\Delta\right) + \delta\left(u'(1) - \underline{\theta}\right)\right] + (1-\nu)\,\delta\left(u'(1) - \overline{\theta}\right).$$

A first possible deviation from this candidate equilibrium for any principal consists in offering the pooling contract $w_1^D = \overline{\theta}$ in the first period. In the second period, for $\Delta < \Delta_0$, the deviating principal finds it optimal to offer the pooling contract, getting

$$\pi^D \triangleq u'(\nu) - \underline{\theta} - \Delta + \delta\left(u'(1) - \overline{\theta}\right).$$

Comparing the equilibrium and the deviation profits, we find

$$\pi^S \geq \pi^D \quad \Leftrightarrow \quad \Delta \geq \Delta_1.$$

Thus, for $\Delta < \Delta_0$, we have found a profitable deviation: the considered strategy profile does not constitute an equilibrium.

Analogously, for $\Delta \geq \Delta_0$, the profit of a deviating principal who offers the pooling contract in the first period and the separating contract in the second period is

$$\pi^D \triangleq u'(\nu) - \overline{\theta} + \nu\delta\left[u'(1) - \underline{\theta}\right].$$

Comparing the equilibrium and the deviation profits, we find

$$\pi^S \geq \pi^D \quad \Leftrightarrow \quad \Delta \geq \frac{\Delta_1 - \delta\Delta_0}{1-\delta} > \Delta_1,$$

which thus constitutes a necessary condition for the existence of this equilibrium.[25] □

Lemma A.3 *In a (symmetric) semi-separating equilibrium, each principal offers* $w_1^* = w_2^* = \underline{\theta}$*. An efficient agent accepts* w_1^* *with probability*

$$\gamma^* = \frac{\Delta - \Delta_1}{\nu\Delta}$$

and always produces in the second period. Inefficient types are shut down in both periods. A necessary condition for the existence of this equilibrium is $\Delta > \Delta_1$*.*

Proof. In the (symmetric) semi-separating equilibrium, each principal offers $w_2^* = \underline{\theta}$, to an agent who has not accepted the wage $w_1 \in [\underline{\theta}, \underline{\theta} + \delta\Delta)$, with a probability σ^* such that the efficient agent is indifferent between accepting or not the wage w_1 — i.e.,

$$w_1 - \underline{\theta} = \delta(1-\sigma^*)\Delta \quad \Leftrightarrow \quad \sigma^* = 1 - \frac{w_1 - \underline{\theta}}{\delta\Delta}.$$

As in the case without discounting, each efficient agent accepts w_1 with a probability γ^* such that the principal is indifferent between offering $w_2 = \underline{\theta}$ and $w_2 = \overline{\theta}$ to an agent who refused w_1 — i.e., for any given market price p_2, γ^* is given by (8), which is positive as long as

$$\Delta > (1-\nu)(p_2 - \underline{\theta}). \tag{10}$$

Such inequality is thus a necessary condition for the existence of the symmetric semi-separating equilibrium.

Moreover, by the same argument outlined in the case with $\delta = 1$, it can be easily obtained that $w_1^* = \underline{\theta}$ and, as a consequence, $\sigma^* = 1$, hence (with probability one) $w_2^* = \underline{\theta}$. Thus, only the efficient agents produce in the second period, yielding $p_2 = u'(\nu)$. Therefore, the necessary

[25] Such a condition is not in general sufficient for the existence of the considered equilibrium, since for all $\Delta > \Delta_1$ a deviating principal could also offer $w_1 = \underline{\theta}$ so to induce the semi-separating outcome.

condition (10) for the existence of this equilibrium becomes $\Delta > \Delta_1$. □

Lemma A.4 *A necessary condition for the existence of any (symmetric) mixed strategy equilibrium in which the semi-separating contract is offered with positive probability is $\Delta > \Delta_1$.*

Proof. In what follows, we derive the existence conditions for every mixed strategy equilibrium in which principals offer the semi-separating contract with positive probability.

Randomization between pooling and semi-separating contracts. Consider a mixed strategy symmetric equilibrium, in which, in $t = 1$, principals randomize between the semi-separating contract ($w_1 = \underline{\theta}$) and the pooling contract ($w_1 = \underline{\theta} + \Delta$). Then, at time $t = 2$, all principals offer $w_2 = \underline{\theta}$. This is obviously the case for the principals who offered $w_1 = \underline{\theta}$. To see why this holds also for the principals who offered the pooling contract, notice that the semi-separating outcome can be achieved if (10) holds, and under the same condition the principals who offered the pooling contract in $t = 1$ find it optimal to offer the separating contract in $t = 2$. Given that $p_2^* = u'(\nu)$, it follows from (10) that this candidate equilibrium can be defined only for $\Delta > \Delta_1$. □

Randomization between separating and semi-separating contracts. Consider a mixed strategy symmetric equilibrium, in which, in $t = 1$, principals randomize between the semi-separating contract ($w_1 = \underline{\theta}$) and the separating contract ($w_1 = \underline{\theta} + \delta\Delta$). Say that $w_1 = \underline{\theta}$ is offered with probability υ. Clearly, at time $t = 2$, the principals who offered $w_1 = \underline{\theta}$ offer $w_2 = \underline{\theta}$, whereas the principals who offered the separating contract achieve the first-best outcome. It can be easily obtained that principals are indifferent between offering the separating and the semi-separating contract at time $t = 1$ if

$$(1-\nu)(u'(\upsilon\nu\gamma^* + (1-\upsilon)\nu) - \underline{\theta})\left(\frac{u'(\upsilon\nu + 1 - \upsilon) - \underline{\theta}}{\Delta} - 1\right) + \delta(1-\nu)(u'(\upsilon\nu + 1 - \upsilon) - \underline{\theta}) - \delta\Delta = 0,$$

whose left-hand side is increasing in υ: hence, in order for the considered candidate equilibrium to exist, the left-hand side of the above equation, when evaluated at $\upsilon = 0$, must be negative. Such condition leads to

$$\Delta > \frac{\delta\Delta_0 - \Delta_1}{2\delta} + \frac{\sqrt{(1-\nu)(\Delta_1 - \delta\Delta_0)^2 + 4\delta\Delta_0\Delta_1}}{2\delta\sqrt{1-\nu}},$$

this threshold being higher than Δ_1 if

$$\frac{\Delta_1}{\Delta_0} < \frac{\delta(1-\nu) + 1}{(1-\nu)(1+\delta)},$$

which is satisfied under Assumption 2. □

Randomization among pooling, separating and semi-separating contracts. Finally, consider a symmetric equilibrium in which, in $t = 1$, principals offer the semi-separating contract ($w_1 = \underline{\theta}$) with probability μ, the separating contract ($w_1 = \underline{\theta} + \delta\Delta$) with probability κ, and the pooling contract ($w_1 = \underline{\theta} + \Delta$) otherwise (i.e., with probability $1 - \mu - \kappa$). Then, at time $t = 2$, the principals who offered the semi-separating contract offer $w_2 = \underline{\theta}$, the principals who offered the separating contract obtain the first-best outcome, whereas the principals who offered the pooling contract offer the separating contract ($w_2 = \underline{\theta}$). After some algebra, it can be obtained that principals are indifferent among these three alternative strategy profiles if and only if the following system is satisfied

$$\begin{cases} \Delta^2 = (1-\nu)(p_1^* - \underline{\theta})(p_2^* - \underline{\theta}) \\ \Delta = \frac{1-\nu}{1-\delta}(p_1^* - \underline{\theta} - \delta(p_2^* - \underline{\theta})) \end{cases}$$

where equilibrium prices are

$$p_1^* = u'(\mu\nu\gamma^* + \kappa\nu + 1 - \mu - \kappa), \quad p_2^* = u'(\nu + \kappa(1-\nu)),$$

with γ^* being defined by (8). From the second equation we find $p_1^* - \underline{\theta} = \frac{1-\delta}{1-\nu}\Delta + \delta(p_2^* - \underline{\theta})$, which we substitute into the first equation to find

$$\Delta^2 = ((1-\nu)\delta(p_2^* - \underline{\theta}) + (1-\delta)\Delta)(p_2^* - \underline{\theta}),$$

which can be solved for p_2^* yielding the only positive solution:

$$p_2^* = \underline{\theta} + \Delta \frac{\sqrt{(1+\delta)^2 - 4\delta\nu} - 1 + \delta}{2\delta(1-\nu)},$$

from which we easily obtain

$$\kappa^* = \frac{1}{1-\nu}\left(u'^{-1}\left(\overline{\theta} + \Delta \frac{\sqrt{(1+\delta)^2 - 4\delta\nu} - 1 + \delta}{2\delta(1-\nu)}\right) - \nu\right).$$

Hence

$$\kappa^* \in (0,1) \quad \Leftrightarrow \quad \Delta \in \left(\frac{2\delta\Delta_0}{\sqrt{(1+\delta)^2 - 4\delta\nu} - 1 + \delta}, \frac{2\delta\Delta_1}{\sqrt{(1+\delta)^2 - 4\delta\nu} - 1 + \delta}\right),$$

and the lower bound of the interval is higher than Δ_1 under Assumption 2. □

Proof of Proposition 8. To establish the result, we have to characterize the symmetric mixed strategy equilibria in which principals randomize between offering the separating and the pooling contract, given that, from Proposition 7, we already know that no other kind of equilibria can exist in the considered region of parameters. Thus, consider symmetric equilibria in which, in $t = 1$, each principal offers the separating contract with probability β, and the pooling contract otherwise. We thus have $p_1 = u'(\nu\beta + 1 - \beta)$. Clearly, the principals who offered the separating contract in $t = 1$, then achieve the first-best outcome in $t = 2$. As for the principals who proposed the pooling contract in $t = 1$, first notice that they never find it optimal to offer the pooling contract in the second period. In fact, if they offer the pooling contract also in the second period, then the market price will be $p_2 = u'(1)$, which immediately implies that this choice is never optimal for all $\Delta > \Delta_0$. Next consider the case in which the principals who offered the pooling contract in $t = 1$ offer the separating contract in the second period. In this case, the market price in $t = 2$ is $p_2 = u'(\beta + (1-\beta)\nu)$. Consequently, this strategy constitutes an optimal choice for these principals if $\Delta \geq (1-\nu)(u'(\beta + (1-\beta)\nu) - \underline{\theta})$.

In this case, a principal's expected profit if she offers the separating contract in the first period (hence, the first-best solution is achieved in the second period) is

$$\pi^S \triangleq \nu(u'(\beta\nu + 1 - \beta) - \underline{\theta} - \delta\Delta) + \delta(u'(\beta + (1-\beta)\nu) - \underline{\theta} - (1-\nu)\Delta),$$

whereas, if she offers the pooling contract in the first period (and the separating contract in the second period), she obtains

$$\pi^{PS} \triangleq u'(\beta\nu + 1 - \beta) - \underline{\theta} - \Delta + \delta\nu(u'(\beta + (1-\beta)\nu) - \underline{\theta}).$$

It can be easily seen that $\pi^S = \pi^{PS}$ if and only if equation (6) is satisfied. Moreover, the right-hand side of equation (6) is increasing in β^*, from which it follows that such mixed strategy should be an increasing function of Δ. Next notice that, since we are considering $\Delta \geq (1-\nu)(u'(\beta^* + (1-\beta^*)\nu) - \underline{\theta})$, this equilibrium can exists if

$$\frac{1-\nu}{1-\delta}(u'(\beta^*\nu + 1 - \beta^*) - \underline{\theta} - \delta(u'(\beta^* + (1-\beta^*)\nu) - \underline{\theta})) > (1-\nu)(u'(\beta^* + (1-\beta^*)\nu) - \underline{\theta}),$$

which gives $\beta^* > \frac{1}{2}$. Hence, there exists a (unique) equilibrium value β^* satisfying (6) if

$$\Delta - \left(\frac{1-\nu}{1-\delta}(u'(\beta\nu + 1 - \beta) - \underline{\theta} - \delta(u'(\beta + (1-\beta)\nu) - \underline{\theta}))\right)\Big|_{\beta=1} < 0$$

and

$$0 < \Delta - \left(\frac{1-\nu}{1-\delta}(u'(\beta\nu + 1 - \beta) - \underline{\theta} - \delta(u'(\beta + (1-\beta)\nu) - \underline{\theta}))\right)\Big|_{\beta=\frac{1}{2}},$$

yielding

$$\Delta \in \left(\Delta^*, \Delta_0 + \frac{\Delta_1 - \Delta_0}{1-\delta}\right).$$

It is straightforward to see that $\Delta^* \in (\Delta_0, \Delta_1)$, whereas $\Delta_0 + \frac{\Delta_1 - \Delta_0}{1-\delta} > \Delta_1$. However, to establish the existence of the considered candidate equilibrium, we still need to check whether there are profitable deviations from it. Clearly, the only possibly profitable deviation for each principal consists in offering the semi-separating contract.[26] The deviating principal would obtain

$$\pi^D \triangleq \nu\gamma^*(u'(\nu\beta^* + 1 - \beta^*) - \underline{\theta}) + \delta\nu(u'(\beta^* + (1-\beta^*)\nu) - \underline{\theta}),$$

with γ^* being given by (8). After simple algebra, it follows that the equilibrium profit $\pi^{PS} (= \pi^S)$ is higher than π^D if and only if

$$\Delta^2 < (1-\nu)(u'(\nu\beta^* + 1 - \beta^*) - \underline{\theta})(u'(\beta^* + (1-\beta^*)\nu) - \underline{\theta}). \tag{11}$$

[26]From (10), we know that the semi-separating outcome is achievable if $\Delta \geq (1-\nu)(u'(\beta^* + (1-\beta^*)\nu) - \underline{\theta})$, which is always the case in the considered region of parameters. To see this, notice that the right-hand side of this inequality is decreasing in Δ, hence a sufficient condition for this inequality to hold for all Δ in the considered interval is that it is satisfied for $\Delta \to \Delta^*$, which can be immediately verified.

Since the right-hand side is decreasing[27] in Δ, it follows that the above inequality is satisfied for Δ sufficiently small, say $\Delta < \overline{\Delta}$. To prove that $\overline{\Delta} > \Delta_1$, notice that the right-hand side of inequality (11) is higher than $\frac{\Delta_0 \Delta^*}{1-\nu}$, thus, a sufficient condition for (11) to hold at $\Delta = \Delta_1$ is $\Delta_0 > (1-\nu)\frac{\Delta_1^2}{\Delta^*}$, which is the case under Assumption 2.[28]

Finally, we have to consider the case in which the principals who in the first-period offered the pooling contract, then randomize again in the second period between offering the separating and the pooling contract. Denote by ϵ the probability of offering $w_2 = \underline{\theta}$. Then, the market price in $t = 2$ is $p_2 = u'(\beta + (1-\beta)(\epsilon\nu + 1 - \epsilon))$. Thus, a principal who offered the pooling contract in the first period is indifferent between offering the pooling and the separating contract in $t = 2$ if

$$\nu(u'(\beta + (1-\beta)(\epsilon\nu + 1 - \epsilon)) - \underline{\theta}) = u'(\beta + (1-\beta)(\epsilon\nu + 1 - \epsilon)) - \underline{\theta} - \Delta,$$

which yields

$$\varepsilon^*(\beta) = \frac{1}{1-\nu}\left(1 - \frac{u'^{-1}\left(\frac{\Delta}{1-\nu} + \underline{\theta}\right) - \beta}{1-\beta}\right). \tag{12}$$

We have $\frac{\partial \varepsilon^*}{\partial \beta} > 0$ for all $\Delta > \Delta_0$, hence $\epsilon^*(\beta)$ takes its lower value at $\beta = 0$:

$$\epsilon^*|_{\beta=0} = \frac{1}{1-\nu}\left[1 - u'^{-1}\left(\frac{\Delta}{1-\nu} + \underline{\theta}\right)\right] > 0 \quad \Leftrightarrow \quad \Delta > \Delta_0,$$

and $\epsilon^*(\beta) \leq 1$ for

$$\beta \leq \bar{\beta} \triangleq \frac{1}{1-\nu}\left[u'^{-1}\left(\frac{\Delta}{1-\nu} + \underline{\theta}\right) - \nu\right].$$

Notice that, regardless of the equilibrium value of β, the market price in the second period is $p_2 = \frac{\Delta}{1-\nu} + \underline{\theta}$. Thus, if a principal offers the pooling contract in the first period, and plays according to the mixed strategy defined by ϵ^* in the second period, her expected profit is

$$\pi^{PM} \triangleq u'(\beta\nu + 1 - \beta) - \underline{\theta} - \Delta + \delta\left(\epsilon^*\nu\frac{\Delta}{1-\nu} + (1-\epsilon^*)\left(\frac{\Delta}{1-\nu} - \Delta\right)\right),$$

whereas, if she offers the separating contract in the first period (hence, the first-best outcome is achieved in the second period), she obtains

$$\pi^S \triangleq \nu(u'(\beta\nu + 1 - \beta) - \underline{\theta} - \delta\Delta) + \delta\left(\frac{\Delta}{1-\nu} - (1-\nu)\Delta\right).$$

[27] The derivative of the right-hand side with respect to Δ is negative if

$$u''(\nu\beta^* + 1 - \beta^*)(u'(\beta^* + (1-\beta^*)\nu) - \underline{\theta}) > u''(\beta^* + (1-\beta^*)\nu)(u'(\nu\beta^* + 1 - \beta^*) - \underline{\theta}).$$

Since $u'(\beta^* + (1-\beta^*)\nu) \leq u'(\nu\beta^* + 1 - \beta^*)$, a sufficient condition in order for the above inequality to hold is that $u''(\nu\beta^* + 1 - \beta^*) \leq u''(\beta^* + (1-\beta^*)\nu)(< 0)$ — i.e., that $u''(\cdot)$ is a decreasing function, hence $u'''(\cdot) \leq 0$, as we assumed in Section 5.

[28] Specifically, the considered inequality is implied by Assumption 2 if $(1-\nu)\frac{\Delta_1}{\Delta^*} < \sqrt{1-\nu}$, which is in turn implied by Assumption 2. In fact, the considered inequality can be rewritten as $\frac{u'(\nu)-\underline{\theta}}{u'(\frac{1+\nu}{2})-\underline{\theta}} < \frac{1}{\sqrt{1-\nu}}$, whose left-hand side is smaller than $\frac{\Delta_1}{\Delta_0}$.

By equating the two above payoffs, we find that the equilibrium value is given by $\beta = \alpha^*$, as defined by (1), and it can be easily checked that $\alpha^* \in (0, \bar{\beta}) \Leftrightarrow \Delta \in (\Delta_0, \Delta^*)$. Finally, by substituting the equilibrium value α^* into (12), we get the equilibrium value $\epsilon^*(\alpha^*)$ given by (5). Notice that there are not possibly profitable deviations from this equilibrium, since it cannot be optimal to offer $w_1^D = \underline{\theta}$, given that no agent would accept the first-period offer.[29] $\square$

Proof of Proposition 9. It is easy to find that, for $\Delta \in (\Delta_0, \Delta^*)$: $y_1^* = y_2^* = u'^{-1}\left(\underline{\theta} + \frac{\Delta}{1-\nu}\right)$, whereas, for $\Delta \in (\Delta^*, \Delta_1)$: $y_1^* = \beta^*\nu + 1 - \beta^* < y_2^* = \beta^* + (1-\beta^*)\nu$ (since $\beta^* > \frac{1}{2}$). $\square$

Proof of Proposition 10. In the mixed strategy equilibrium defined for $\Delta \in (\Delta_0, \Delta^*]$, principals' equilibrium profit is

$$\pi_1^* \triangleq \frac{\nu(1+\delta)}{1-\nu}\Delta,$$

which, interestingly, does not depend on the demand function, and is clearly increasing in Δ. In the mixed strategy equilibrium defined for $\Delta \in (\Delta^*, \Delta_1]$, the equilibrium profit can be written as

$$\pi_2^* \triangleq \nu(u'(1-(1-\nu)\beta^*) - \underline{\theta} - \delta\Delta) + \delta(u'(\nu + (1-\nu)\beta^*) - \underline{\theta} - (1-\nu)\Delta),$$

hence

$$\frac{\partial \pi_2^*}{\partial \Delta} = (1-\nu)\frac{\partial \beta^*}{\partial \Delta}\left(\delta u''(\nu + (1-\nu)\beta^*)) - \nu u''(1-(1-\nu)\beta^*)\right) - \delta,$$

where, as proved above, $\frac{\partial \beta^*}{\partial \Delta} \geq 0$. It can be immediately seen that

$$\left.\frac{\partial \pi_2^*}{\partial \Delta}\right|_{\delta \to 0} = -\nu(1-\nu)\left.\frac{\partial \beta^*}{\partial \Delta}\right|_{\delta \to 0} u''(1-(1-\nu)\beta^*)) > 0.$$

Moreover, from (6), it can be seen that, as $\delta \to 1$: $\beta^* \to \frac{1}{2}$, implying that

$$\left.\frac{\partial \pi_2^*}{\partial \Delta}\right|_{\delta \to 1} = (1-\nu)^2\left.\frac{\partial \beta^*}{\partial \Delta}\right|_{\delta \to 1} u''\left(\frac{1+\nu}{2}\right) - 1 < 0.$$

We finally consider consumer welfare, defined as[30]

$$W \triangleq u(y_1) - y_1 u'(y_1) + \delta(u(y_2) - y_2 u'(y_2)).$$

Differentiating it with respect to Δ yields

$$\frac{\partial W}{\partial \Delta} = -y_1\frac{\partial y_1}{\partial \Delta}u''(y_1) - \delta y_2 \frac{\partial y_2}{\partial \Delta}u''(y_1).$$

For $\Delta \in (\Delta_0, \Delta^*]$, since $y_1^* = y_2^*$ is decreasing in Δ, we can immediately conclude $\frac{\partial W^*}{\partial \Delta} < 0$. In

[29] In fact, substituting the equilibrium price into (8), it can be easily seen that $\gamma^* = 0$, which implies that the semi-separating outcome cannot be achieved by a deviating principal.

[30] For coherence, we have considered the discount factor δ (used by principals and agent) also for consumers. However, it can be immediately verified that this assumption is immaterial to our results.

the equilibrium defined for $\Delta \in (\Delta^*, \Delta_1]$,

$$\frac{\partial y_1^*}{\partial \Delta} = -(1-\nu)\frac{\partial \beta^*}{\partial \Delta} = -\frac{\partial y_2^*}{\partial \Delta},$$

from which we can write

$$\frac{\partial W^*}{\partial \Delta} = y_1^* u''(y_1^*) - \delta y_2^* u''(y_1^*).$$

Thus, a sufficient condition in order for $\frac{\partial W^*}{\partial \Delta} < 0$ is $y_1^* u''(y_1^*) < y_2^* u''(y_1^*)$, which is in turn always satisfied when $u'''(\cdot) < 0$. □

B Long-term contracts and renegotiation

In this Appendix, we study how the equilibrium configuration of the game changes when we introduce the possibility of commitment and Pareto-improving renegotiation We first consider the model without discounting.

In our model, long-term contracts are as follows:[31]

(1) contracts specifying that $w_1 = \underline{\theta}$ and

- (a) $w_2 = \overline{\theta}$ for every $x_1 \in \{0, 1\}$,
- (b) $w_2 = \underline{\theta}$ for every $x_1 \in \{0, 1\}$,
- (c) $w_2 = \underline{\theta}$ if $x_1 = 1$ and $w_2 = \overline{\theta}$ otherwise,
- (d) $w_2 = \overline{\theta}$ if $x_1 = 1$ and $w_2 = \underline{\theta}$ otherwise;

(2) contracts specifying that $w_1 = \overline{\theta}$ and w_2 is as in the cases (a),(b),(c) and (d) above.

Notice that contracts in which $w_2 = \overline{\theta}$ for every $x_1 \in \{0, 1\}$, namely those labeled with (1a) and (2a), are clearly renegotiation-proof. Importantly, contract (1a) implements the separating outcome of the model with spot contracts, since in the first period only efficient agents produce whereas in the second period all agents produce, and the inter-temporal rent of efficient types is Δ. Thus, unlike in the game with spot contracts, long-term contracting gets rid of the ratchet effect, since the principal can commit to offer $w_2 = \overline{\theta}$ to an agent who produced when paid $w_1 = \underline{\theta}$. By the same token, contract (2a) clearly implements the pooling outcome of the model with spot contracts, since in both periods all agents produce and efficient types obtain a rent Δ in either period. On the contrary, contracts in which $w_2 = \underline{\theta}$, for some x_1, are not renegotiation proof if a principal, after observing the first period outcome, would be better off by offering $w_2 = \overline{\theta}$ — i.e., if $\Delta < (1 - \Pr[\theta = \underline{\theta} | x_1 = 0])(p_2 - \underline{\theta})$. Thus, contract (1b) is renegotiation-proof only when efficient agents randomize — i.e., when the semi-separating

[31] Clearly, each period wage is paid conditional on the agent producing in that period.

outcome is implemented.[32],[33] Contract (2b) is renegotiation proof if $\Delta \geq (1-\nu)(p_2 - \underline{\theta})$. The other contracts can be neglected: contract (1c) is not incentive-compatible;[34] contract (1d) is not renegotiation proof;[35] contract (2c) is equivalent to contract (2b) or to contract (1a);[36] contract (2d) is equivalent to contract (2a).[37]

To sum up, in the game without discounting, for every outcome (i.e., market prices and inter-temporal rents and profits) that can be achieved by means of short-term contracts there exists a long-term renegotiation-proof contract which yields the same outcome. However, there exists a long-term renegotiation proof contract which allows principals to achieve full separation in the first period, which is an outcome that, in the game with no discounting, cannot be achieved by using spot contracts. As a consequence, the equilibrium set of the game under long-term renegotiation proof contracts differs from the one characterized in Section 4. Specifically, we are able to prove the following result.

Proposition B.1 *The equilibria of the game with long-term renegotiation-proof contracts are as follows.*

- *If $\Delta \leq \Delta_0$, in the unique symmetric equilibrium of the game, each principal offers contract (2a).*
- *If $\Delta \in (\Delta_0, \Delta^*]$, there exists only one symmetric equilibrium in which, with probability α^*, defined in (1), each principal offers contract (1a), with the same probability α^*, each principal offers contract (2b), and with complementary probability $1-2\alpha^*$, each principal offers contract (2a).*
- *If $\Delta \in \left(\Delta^*, \frac{\Delta^*}{\sqrt{1-\nu}}\right)$ there exists only one symmetric equilibrium in which principals offer with the same probability ($\frac{1}{2}$) contracts (1a) and (2b).*
- *If $\Delta \in \left(\frac{\Delta^*}{\sqrt{1-\nu}}, \widehat{\Delta}\right]$, where*

$$\widehat{\Delta} \triangleq \frac{\Delta_1}{\sqrt{1-\nu}},$$

there exists only one symmetric equilibrium in which principals offer contract (1a) with

[32] To see this, notice that, if all efficient agents accept to produce in the first period being paid $w_1 = \underline{\theta}$, then $\Pr[\theta = \underline{\theta} | x_1 = 0] = 0$ and, by Assumption 1 contract (1b) is not renegotiation proof. Obviously, if all efficient agents decide not to produce in the first period, we can neglect contract (1b) since principals never find it optimal to offer it.

[33] To be more explicit, contract (1b) can be seen as a contract in which the agent has three options in total: in the first period, he can choose between producing or not; if he produces in the first period, he is to produce in the second period also, and obtains a wage $\underline{\theta}$ each period; if he does not produce in the first period, he then has a second choice in the second period: either he produces, and obtains a wage $\underline{\theta}$, or he does not produce.

[34] In fact, efficient agents would be better off not producing in the first period, which clearly harms principals.

[35] If an agent did not produced in the first period, at the beginning of the second period the principal knows that he is inefficient, hence offering $w_2 = \overline{\theta}$ would be Pareto-improving.

[36] Indeed, under contract (2c), efficient types always accept w_1, whereas inefficient types are indifferent between accepting the first-period offer (thus do not producing in the second period) or declining the first period offer and produce in the second period. In the former case, this contract is equivalent to contract (2b), in the latter case is equivalent to contract (1a).

[37] In fact, all agents produce in the first period, given that $w_2 = \overline{\theta}$ if $x_1 = 1$.

probability

$$\phi^* \triangleq \frac{1}{1-\nu}\left(u'^{-1}\left(\overline{\theta}+\frac{\Delta}{\sqrt{1-\nu}}\right)-\nu\right),$$

contract (1b) with probability

$$\varphi^* \triangleq \frac{1}{\sqrt{1-\nu}}\left(1+\nu-2u'^{-1}\left(\overline{\theta}+\frac{\Delta}{\sqrt{1-\nu}}\right)\right)$$

and contract (2b) otherwise.

- *If* $\Delta \in \left(\widehat{\Delta}, \Delta_3\right]$, *there exists only one symmetric equilibrium in which principals offer contract (1b) with probability* ρ^* *defined by (4), and contract (2b) otherwise.*

- *If* $\Delta > \Delta_3$, *in the unique symmetric equilibrium of the game, each principal offers contract (1b).*

Proof. To begin with, we calculate the profit that, for any market prices p_1 and p_2, a principal obtains offering each one of the four relevant long-term contracts:

$$\pi^{(1a)} \triangleq \nu(p_1-\underline{\theta})+p_2-\underline{\theta}-\Delta,$$

$$\pi^{(1b)} \triangleq \left(1-\frac{(1-\nu)(p_2-\underline{\theta})}{\Delta}\right)(p_1-\underline{\theta})+\nu(p_2-\underline{\theta}),$$

$$\pi^{(2a)} \triangleq p_1-\underline{\theta}-\Delta+p_2-\underline{\theta}-\Delta,$$

$$\pi^{(2b)} \triangleq p_1-\underline{\theta}-\Delta+\nu(p_2-\underline{\theta}).$$

Pure strategy equilibria. We first analyze all possible equilibria in which all principals offer the same contract.

- Consider a candidate equilibrium in which all principals offer contract (2a), obtaining a profit $\pi^{(2a)}$, with $p_1 = p_2 = u'(1)$. From the analysis of the game with spot contracts, we already know that the semi-separating outcome cannot be achieved when $\Delta \leq \Delta_0$, which immediately implies that contract (1b) is not renegotiation-proof, as well as contract (2b), in the considered region of parameters. Thus, to establish the result, it only remains to be check whether a principal does not find it optimal to deviate offering contract (1a) — i.e., to shut down inefficient types only in the first period. The deviating principal would obtain a profit $\pi^{(1a)}$, and it can be immediately verified that, given the equilibrium prices, $\pi^{(1a)} < \pi^{(2a)}$ for all $\Delta \leq \Delta_0$.

- Consider a candidate equilibrium in which all principals offer contract (1a). In this case, each principal would obtain a profit $\pi^{(1a)}$, with $p_1 = u'(\nu)$ and $p_2 = u'(1)$. To see that this candidate equilibrium cannot exist, notice that, for all Δ, since $p_1 > p_2$, each principal has a profitable deviation consisting in offering contract (2b).

- By the same token, it can be easily checked that a candidate equilibrium in which all principals offer contract (2b) does not exists, since from $p_1 = u'(1) < p_2 = u'(\nu)$ it follows that any principal would find it optimal to deviate offering contract (1a).

- Consider a candidate equilibrium in which all principals offer contract (1b), and assume that this offer induces the semi-separating outcome — i.e., efficient agents accept the first-period offer with probability γ^* defined by (2) and always produce in the second period. In this case, the equilibrium profit is $\pi^{(1b)}$, with $p_1 = u'(\nu\gamma^*)$ and $p_2 = u'(\nu)$. From the analysis of the game with spot contracts, we already know that if a principal deviates to contract (2b),[38] she would obtain a higher profit for $\Delta < \Delta_3$. Thus, to see whether, for every $\Delta \geq \Delta_3$, the considered candidate equilibrium exists, it remains to be checked whether, in this region of parameters, a principal does not find it optimal to deviate offering contract (1a).[39] However, $\pi^{(1a)} < \pi^{(2b)}$ whenever $p_1 > p_2$, which is the case in this equilibrium, implying that, for all $\Delta \geq \Delta_3$, there are no profitable deviations.

Mixed strategy equilibria. We now turn to consider all possible mixed strategy equilibria.

- Consider a candidate equilibrium in which each principal randomizes between offering contract (1a) and contract (2a). It can be immediately seen that, since $p_2 = u'(1)$, each principal would find it optimal to deviate offering contract (2b) for all $\Delta > \Delta_0$, whereas, for every $\Delta \leq \Delta_0$, principals are strictly better off offering contract (2a) than contract (1a) — i.e., there not exists a randomization which makes the principals indifferent between offering the two considered contract. We can thus conclude that the considered equilibrium does not exist.

- Consider a candidate equilibrium in which each principal randomizes between offering contract (1a) and contract (1b). In this candidate equilibrium, $p_1 > p_2$, which implies $\pi^{(2b)} > \pi^{(1a)}$ — i.e., each principal has a profitable deviation consisting in offering contract (2b). This proves that the considered equilibrium does not exist.

- Consider a candidate equilibrium in which each principal offers contract (1a) with probability β and contract (2b) otherwise. In this candidate equilibrium, market prices are $p_1 = u'(\beta\nu + 1 - \beta)$ and $p_2 = u'(\beta + (1-\beta)\nu)$. Imposing $\pi^{(1a)} = \pi^{(2b)}$ immediately yields $\beta^* = \frac{1}{2}$, implying that this equilibrium can exists for $\Delta \geq \Delta^*$, since contract (2b) is renegotiation-proof if $\Delta \geq (1-\nu)(p_2 - \underline{\theta})$. The only possibly profitable deviation for each principal consists in offering contract (1b) — i.e., to induce the semi-separating outcome.[40]. After some algebra, the deviation profit can be written as $\pi^{(1b)} \triangleq \frac{\Delta^*}{1-\nu}\left(1 + \nu - \frac{\Delta^*}{\Delta}\right)$, which turns out to be lower than the equilibrium profit for every $\Delta < \frac{\Delta^*}{\sqrt{1-\nu}}$.[41]

[38] Notice that, under the necessary condition (10) for the attainability of a semi-separating outcome (which, in this case, gives $\Delta \geq \Delta_1$), it follows that contract (2b) is renegotiation-proof.

[39] This is the case since, in the considered region of parameters, it can be immediately checked that the deviation consisting in offering contract (2a) is never profitable.

[40] In fact, it can be immediately verified that, in the considered region of parameters, offering contract (2a) never constitutes a profitable deviation.

[41] To see this, notice that, for $\beta^* = \frac{1}{2}$, the equilibrium profit can be rewritten as $\frac{1+\nu}{1-\nu}\Delta^* - \Delta$.

- Consider a candidate equilibrium in which each principal randomizes between offering contract (2b) and contract (1b), which are renegotiation-proof for $\Delta > \Delta_1$ (since $p_2 = u'(\nu)$). Denoting by ρ the probability with which principals offer contract (1b), and imposing that principals are indifferent between offering these two contracts yields

$$\Delta^2 = \Delta_1(u'(\rho\nu\gamma^*) + 1 - \rho) - \underline{\theta}),$$

where γ^* is given by (2), from which we obtain the equilibrium value ρ^*. Moreover, $\rho^* < 1$ if $\Delta < \Delta_3$. However, in order for this candidate equilibrium to exist, any principal must not find it profitable to deviate offering contract (1a). Imposing this condition yields $\Delta > \widehat{\Delta}$.

- Consider a candidate equilibrium in which each principal randomizes between offering contract (2a) and contract (2b). It can be immediately seen that this equilibrium does not exist. Indeed, from the analysis of the static contract, we know that principals, who all offered $w_1 = \overline{\theta}$, are indifferent between offering $w_2 = \underline{\theta}$ and $w_2 = \overline{\theta}$ if $\Delta \in (\Delta_0, \Delta_1)$. However, in this region of parameters, it can be immediately seen that any principal has incentive to deviate by offering contract (1a).

- Consider a candidate equilibrium in which each principal randomizes between offering contract (2a) and contract (1b). It can be immediately seen that this equilibrium does not exist. In fact, contract (1b) is renegotiation proof if and only if $\Delta > (1-\nu)(p_2 - \underline{\theta})$. However, under the same condition, from the analysis of the static game, it follows that principals are strictly better off offering contract (2b) than contract (2a). For the same reason, also candidate equilibria in which each principal randomizes among offering contracts (1a), (2a) and (1b), or offering the four considered long-term renegotiation-proof contracts cannot exist.

- By a similar reasoning, also an equilibrium in which each principal randomizes among contracts (1b), (2a) and (2b) cannot exist. In fact, in order for this equilibrium to exist, principals who offered $w_1 = \overline{\theta}$ (i.e., contracts (2a) and (2b)) must be indifferent between offering $w_2 = \underline{\theta}$ and $w_2 = \overline{\theta}$, which is the case if $\Delta = (1-\nu)(p_2 - \underline{\theta})$. However, if this equality is fulfilled, then contract (1b) is not renegotiation-proof.

- Consider a candidate equilibrium in which each principal offers contract (1a) with probability β, contract (2a) with probability ϵ and (2b) otherwise. In this candidate equilibrium, market prices are $p_1 = u'(1-(1-\nu)\beta)$ and $p_2 = u'(1-(1-\nu)(1-\beta-\epsilon))$. By imposing $\pi^{(1a)} = \pi^{(2a)}$ and $\pi^{(2a)} = \pi^{(2b)}$, it is easy to see that it must be $p_1 = p_2 \triangleq p$ and $\Delta = (1-\nu)(p - \underline{\theta})$,[42] from which it is trivial to find that $\beta^* = \alpha^*$ defined by (1) and $\epsilon^* = 1 - 2\alpha^*$. This constitutes an equilibrium whenever $\alpha^* \in (0, \frac{1}{2})$, yielding $\Delta \in (\Delta_0, \Delta^*)$.[43]

[42] The latter equality implies that contract (2b) is renegotiation-proof.

[43] In fact, there are no possibly profitable deviations to examine, since contract (1b) can be easily seen to be not renegotiation-proof in the considered region of parameters.

- Finally, consider a candidate equilibrium in which each principal offers contract (1a) with probability ϕ, contract (1b) with probability φ and (2b) otherwise. Hence, $p_1 = u'(\phi\nu + \varphi\nu\gamma^* + 1 - \phi - \varphi)$ and $p_2 = u'(\phi + (1-\phi)\nu)$. By imposing $\pi^{(1a)} = \pi^{(2b)}$ and $\pi^{(1b)} = \pi^{(2b)}$ we obtain $p_1 = p_2 \triangleq p$ and $\Delta^2 = (1-\nu)(p - \underline{\theta})^2$, from which we pin down the equilibrium values ϕ^* and φ^*. In order for this equilibrium to exist, the following conditions must hold:[44]

$$\phi^* \in (0,1) \quad \Leftrightarrow \quad \Delta \in \left(\frac{\Delta_0}{\sqrt{1-\nu}}, \frac{\Delta_1}{\sqrt{1-\nu}} \right),$$

$$\varphi^* \in (0,1) \quad \Leftrightarrow \quad \Delta \in \left(\frac{\Delta^*}{\sqrt{1-\nu}}, \sqrt{1-\nu}\left(u'\left(\frac{1+\nu-\sqrt{1-\nu}}{2} \right) - \underline{\theta} \right) \right),$$

and

$$\varphi^* + \phi^* < 1 \quad \Leftrightarrow \quad \Delta < \sqrt{1-\nu}\left(u'\left(\frac{(1+\nu)\sqrt{1-\nu}-1}{2\sqrt{1-\nu}-1} \right) - \underline{\theta} \right).$$

After some algebra, these conditions yield $\Delta \in \left(\frac{\Delta^*}{\sqrt{1-\nu}}, \widehat{\Delta} \right)$. □

Thus, for every $\Delta \le \Delta_0$ and $\Delta \ge \widehat{\Delta}$, the equilibrium with long-term renegotiation-proof contracts coincides with the equilibrium with spot contracts. The reason is rather simple. When the adverse selection problem is weak ($\Delta \le \Delta_0$), principals never it profitable to shut down inefficient agents in first period — i.e., to offer the long-term renegotiation proof contract (1a) — and, as in the game with spot contracts, all agents produce in both periods. When the adverse selection problem is too severe ($\Delta \ge \widehat{\Delta}$), a positive fraction of principals implement the semi-separating outcome, which implies that the aggregate quantity (resp. the equilibrium price) is increasing (resp. decreasing) over time. As a consequence, offering contract (2b) (i.e., shutting down the inefficient type in the second period) is more appealing than offering contract (1a), in which the inefficient type is shut down in the first period, when the equilibrium price is higher. As a consequence, for all $\Delta \ge \widehat{\Delta}$, the feasibility of a fully separating outcome in the first period does not alter the equilibrium outcome of the game compared to the case with spot contracts.

By contrast, when the severity of the adverse selection problem takes intermediate values (i.e., for all $\Delta \in (\Delta_0, \widehat{\Delta})$), the larger number of available outcomes under long-term renegotiation-proof contracts completely change the equilibrium outcome of the game. Specifically, there are three mixed strategy equilibria and, in all them, contract (1a), which implements the separating outcome of the game with spot contracts, is offered with positive probability. These equilibria have the following features.

Proposition B.2 *For all $\Delta \le \widehat{\Delta}$, aggregate output is constant over time.*
Proof. Aggregate quantities are as follows:

- For $\Delta \le \Delta_0 : y_1^* = y_2^* = 1$;

[44] These conditions are also sufficient for the existence of the considered equilibrium, since there are no possibly profitable deviations to examine, given that offering contract (2a) is never optimal in the region of parameters in which contract (1b) is renegotiation proof.

- For $\Delta \in (\Delta_0, \Delta^*] : y_1^* = y_2^* = u'^{-1}\left(\frac{\Delta}{1-\nu} + \underline{\theta}\right)$;
- For $\Delta \in \left(\Delta^*, \frac{\Delta^*}{\sqrt{1-\nu}}\right] : y_1^* = y_2^* = \frac{1+\nu}{2}$;
- For $\Delta \in \left(\frac{\Delta^*}{\sqrt{1-\nu}}, \widehat{\Delta}\right] : y_1^* = y_2^* = u'^{-1}\left(\frac{\Delta}{\sqrt{1-\nu}} + \underline{\theta}\right)$;
- For $\Delta \in \left(\widehat{\Delta}, \Delta_3\right] : y_1^* = u'^{-1}\left(\frac{\Delta^2}{\Delta_1} + \underline{\theta}\right) < y_2^* = \nu$;
- For $\Delta > \Delta_3 : y_1^* = \nu\gamma^* < y_2^* = \nu$. □

Thus, when full separation is possible in the first period, aggregate quantity does not decline over time. However, as long as the adverse selection problem is not excessively severe, output does not vary across the two periods. On the contrary, for higher values of Δ, the separating and the semi-separating contracts become much more appealing to principals, leading to an increasing aggregate output over time. Notably, these results are qualitatively similar to those we have shown in the model with spot contracts and discounting. This is not surprising, as also in that case full separation of types is viable in the first period.

Moreover, also our results concerning profits and consumer welfare are robust when considering long-term renegotiation-proof contracts, as shown in the following.

Proposition B.3 *When mixed strategy equilibria are played:*

- *Principals' profit is non monotone with respect to Δ: it is decreasing in Δ for $\Delta \in \left(\Delta^*, \frac{\Delta^*}{\sqrt{1-\nu}}\right)$ and increasing otherwise;*
- *Consumer welfare is a non-increasing function of Δ: it is constant for $\Delta \in \left(\Delta^*, \frac{\Delta^*}{\sqrt{1-\nu}}\right)$ and strictly decreasing otherwise.*

Proof. Principals' profits in the mixed strategy equilibria are as follows:

- For $\Delta \in (\Delta_0, \Delta^*] : \pi^* = \frac{2\nu}{1-\nu}\Delta$;
- For $\Delta \in \left(\Delta^*, \frac{\Delta^*}{\sqrt{1-\nu}}\right] : \pi^* = \frac{1+\nu}{1-\nu}\Delta^* - \Delta$;
- For $\Delta \in \left(\frac{\Delta^*}{\sqrt{1-\nu}}, \widehat{\Delta}\right] : \pi^* = \left(\frac{1+\nu}{\sqrt{1-\nu}} - 1\right)\Delta$;
- For $\Delta \in \left(\widehat{\Delta}, \Delta_3\right] : \pi^* = \frac{\Delta^2}{\Delta_1} - \Delta + \frac{\nu}{1-\nu}\Delta_1$.

As for consumer welfare, for any $\Delta \in \left(\Delta_0, \widehat{\Delta}\right)$ aggregate quantity is constant over time and non-increasing in Δ, implying that also consumer welfare is non-increasing in Δ. Finally, for $\Delta \in \left(\widehat{\Delta}, \Delta_3\right]$ the equilibrium outcome is as in the model with spot contracts, and we already proved that consumer welfare is decreasing in Δ. □

Finally, we consider the game with discounted payoffs. The following result holds.

Proposition B.4 *For all $\delta \in (0,1)$, the equilibrium outcome with spot contracts coincides with the equilibrium outcome with long-term contracts and Pareto-improving renegotiations.*

Proof. Long-term renegotiation-proof contracts are as in the game with no discounting. However, when principals discount future profits, a fully separating outcome can be achieved also with spot contracts. Therefore, there is a one-to-one correspondence among the outcomes (i.e., discounted wages and market prices) that can be achieved with long-term renegotiation-proof contracts and with short-term contracts, which establishes the result. Accordingly, the mixed strategy equilibria of the game with spot contracts can be equivalently regarded as randomizations among the corresponding long-term renegotiation-proof contracts. Specifically, the mixed strategy equilibrium for $\Delta \in (\Delta_0, \Delta^*]$ can be equivalently obtained through randomization among three long-term renegotiation-proof contracts:

(i) with probability α^* defined in (1), each principal offers contract (1a).

(ii) with the same probability α^*, each principal offers contract (2b). Such contract is renegotiation-proof if and only if the principals after the first period would not be better off by offering $w_2 = \overline{\theta}$, that is for $\Delta \geq (1-\nu)(p_2^* - \underline{\theta})$, which is satisfied (as an equality).

(iii) with complementary probability $1 - 2\alpha^*$, each principal offers contract (2a).

Analogously, the mixed strategy equilibrium for $\Delta \in (\Delta^*, \Delta_1)$ can be obtained if principal randomize between the following long-term renegotiation-proof contracts:

(i) with probability β^*, defined as the unique solution of equation (6), each principal offers contract (1a).

(ii) with complementary probability $1 - \beta^*$, each principal offers contract (2b).[45] □

[45] Contract (ii) is renegotiation-proof for $\Delta \geq (1-\nu)(p_2^* - \underline{\theta}) = (1-\nu)(u'(\beta^* + \nu(1-\beta^*)) - \underline{\theta})$. Since the right-hand side is increasing in $\beta^* > \frac{1}{2}$ and the inequality is satisfied at $\beta^* = \frac{1}{2}$, we can conclude that the considered contract is renegotiation-proof.