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ORIGINAL PAPER

Implied volatility smoothing at COVID-19 times

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Abstract
This work aims at studying the impact of the SARS-CoV-2 pandemic on the global financial markets. In particular, such impact is analysed through the changes of the shape of the implied volatility smile of the options written on several equity indexes and on several stocks. The implied volatility function is estimated using the market-based information of liquid options and applying a semi-parametric smoothing technique that exploits a kernel function and no-arbitrage conditions. Such approach is applied to an extensive set of data to study the evolution of the implied volatility functions through the months of the pandemic. We show, in several cases, a sudden and massive change in the shape of the implied volatility functions.

Keywords COVID-19 · Implied volatility · State price density · No-arbitrage conditions · Local polynomial smoothing

1 Introduction

The model of Black and Scholes (1973) and Merton (1973) (BSM model henceforth) represents the standard way to price options. Indeed, together with its extensions, it became famous an widely used in the markets to evaluate various types of options. However, it is well-known that many drawbacks affect the BSM type models, such as assumption of the Gaussianity for the underlying log-returns, their constant and deterministic volatility, and a deterministic risk-free rate. Because of these disadvantages, the empirical observations of the option market prices are usually inconsistent with the results of the BSM type models. The common answer of the practitioners was to invert the BSM formula: consider the market price of liquid options (plain vanilla calls and puts), insert them into inverted BSM type model where all parameters but the volatility are given, and compute a so called implied

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volatility (IV). Following this schema, many traders currently quote the IVs rather than the option prices.

At any time, for a single underlying asset, several call and put options with different strike prices and maturities are traded. Similarly, at any time, we can compute a whole set of IVs. Instead of the strike price, in this contest we adopt the concept of futures moneyness, i.e. the ratio of the strike price and the future value of current underlying asset price. The futures moneyness simplifies the comparision among options having different maturities. When, for a given maturity, we plot IVs against the futures moneyness we obtain a curve similar to a smile (especially for Forex options) or smirk (a non-symmetric smile common for equities). When various values of both futures moneyness and maturity are considered, we obtain a so-called IV surface. Subsequently, the IV curve or surface can be used to construct a local volatility model which can be further adopted for option pricing (see eg. Dupire 1994).

The study of the IV curves and the IV surfaces has been deeply analysed in the last decades. Corrado and Tie (1997) focuses on the relation between the skewness and kurtosis of the underlying and of the IV. Skiadopoulos et al. (2000) applies the principal component analysis to identify those components that better explain the IV smiles. Tompkins (2001) proposes an extensive work on IV surfaces considering several markets and trying to capture similarities among the smile patterns. Hafner and Wallmeier (2007) extracts from IV curves a suggestion to invest or not in volatility-based financial instruments. Ielpo and Guillaume (2010) investigates the mean-reverting behaviour of the IV through time. Silvia (2010) exploits the information inside the IV to forecast the volatility for the underlying and shows the quality of the BSM model. García-Machado and Rybczyński (2017) focuses on the interpretation of the IV curves according to its particular shape in periods of market turmoil. Jang and Lee (2019) uses neural network models to predict the IV of the options. More recently, several papers analyse the option markets to obtain insights about the impact of COVID-19. For instance, Jackwerth (2020) studies the risk-neutral distributions of the S&P500 to understand the reaction of the market between January 2020 and May 2020; while Hanke et al. (2020) compare the evolution of risk-neutral densities for several countries in the first semester of 2020. Both these works highlight the importance of the option to understand the reaction of the financial markets. Partially, we continue these studies trying to understand whether it is possible to achieve similar conclusions focusing on the changes in the IV smiles, rather than on the risk-neutral densities. Thus, it is still of great importance to study the IV curves and be able to smooth the IV function across the quoted values to capture its shape along all the moneyness span. In the last decades, several parametric and semi-parametric models of the volatility surface have been introduced, see Fengler (2012) and Homescu (2011) for a review. Dumas et al. (1998) suggests to model IV surface as a linear function of time-to-maturity and a quadratic function of the moneyness. Similarly, Borovkova and Permana (2009) proposes a semi-parametric representation with quadratic expressions for the oil market. Avellaneda et al. (1997) presents an algorithm that yields an arbitrage-free diffusion process by minimizing the

relative entropy distance to a prior diffusion. Bloch (2010) and Bloch et al. (2011) use a weighted sum of interpolation functions employing a parametric family in order to generate a surface without arbitrage in time and in space, while corresponding as closely as possible to market data. Each function from the family is required to satisfy arbitrage-free constraints. Several families can satisfy these constraints, for instance a sum of log-normal distributions. Unfortunately, there is no continuum of data (with respect to moneyness and maturity) and, thus, we need some kind of interpolation and smoothing before the evaluation of non-traded options. The deficiency is that in some regions the smoothed IV curve or surface can lead to a violation of arbitrage-free conditions.

Benko et al. (2007) combine the IV smoothing with a State Price Density (SPD) estimation in order to avoid these problems. Indeed, the author proposed to use a local polynomial smoothing technique with Epanechnikov (bivariate) kernels. The advantage of this technique is that it provides all quantities needed to calculate the corresponding SPD. Moreover, the approach operates only with the IVs—a major improvement compared to the earlier multi-step approaches moving through the BSM formula from the prices to IVs and vice-versa. For the implied volatility surface arbitrage-free estimation Benko et al. (2007) followed Kahalé (2004) and Fengler (2005) in applying an implied total variance condition to avoid the calendar arbitrage; however, including constraints on both SPD and the implied total variance makes the kernel local quadratic estimation quite computationally demanding. Moreover, a strict application of total variance constraints leads to a semi-infinite programming problem. Therefore, Benko et al. (2007) suggested a discrete approximation that simplifies the algorithm solving a non-linear programming problem.

Kim and Lee (2013) extended and applied the notion of Benko et al. (2007) to KOSPI 200 index options no-arbitrage implied volatility modelling, while Kopa et al. (2017) performed an extensive analysis on DAX index and proposed a measure to capture the magnitude of the arbitrage highlighted by the SPD. Other no-arbitrage IV and SPD estimation procedures were recently presented in e.g. Fengler (2012), Glaser and Heider (2012), Fengler and Hin (2015) and Ludwig (2015).

The aim of this paper is to exploit the techniques proposed in Benko et al. (2007) and the study of García-Machado and Rybczyński (2017) to observe and interpret the impact of the SARS-CoV-2 pandemic on the worldwide stock markets and on the selected representative companies. Indeed, the smoothing methodology has already been applied in periods without market turmoil and we verify its validity also during stressed-periods. In particular, we analyze the changes in time of IVs smiles (smirks) of the European options (with a given maturity) on the most important stock indices and stock prices during the end of 2019 and the first semester of 2020, analyzing the residual effect of the Brexit and the most important period of the pandemic. Both events, especially the second, induce dramatic changes of both location and shape of IVs smiles.

The rest of the paper is structured as follows. Section 2 recalls the theory of implied volatility. Section 3 presents the empirical analysis and Sect. 4 concludes the paper. The Appendix provides further empirical results.

2 Implied volatility estimation

Let's assume that the process of the non-dividend-paying underlying asset of the option evolves as a geometric Brownian motion with some constant drift and volatility, then the value C_t of a call option written on this underlying is expressed as (Black and Scholes 1973):

$$C_t(S_t, K, \tau, r, \sigma) = S_t \Phi(d_1) - Ke^{-r\tau}\Phi(d_2), \quad t < T, \tag{1}$$

$$d_1 = \frac{\log(S_t/K) + (r + \sigma^2/2)\tau}{\sigma\sqrt{\tau}}, \tag{2}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}, \tag{3}$$

where S_t stands for the current price of the underlying asset, K is the strike price of the option, T is the maturity of the option, $\tau = T - t$ is the time-to-maturity, r the risk-free interest rate, σ the volatility of the log-returns of the underlying asset (assumed here to be an unknown and constant parameter), and $\Phi(\cdot)$ denotes the cumulative distribution function of the standard normal distribution. Knowing the market price of an European call (or put) option, inverting the BSM model it is possible to compute the implied volatility. Indeed, the implied volatility $\tilde{\sigma}$ of the underlying asset log-returns is simply defined as the solution of the system (1–3) where the price $\tilde{C}_t$ observed in the market substitutes the value C_t.

Contrary to the assumption of BSM models, the implied volatility is typically not constant, see Brockhaus et al. (2000) and Fengler (2006). Indeed, for a fixed maturity, the implied volatility function is a u-shaped function of the strike price called *implied volatility smile*.

Thus, we assume the following model framework:

$$\tilde{\sigma}(K_i, \tau_i) = \sigma(K_i, \tau_i) + \varepsilon_i, \quad i = 1, \ldots, n, \tag{4}$$

where $\tilde{\sigma}_i = \tilde{\sigma}(K_i, \tau_i)$ is the implied volatility observed on K_i and τ_i, $\sigma_i = \sigma(K_i, \tau_i)$ is its theoretical counterpart, ε_i indicates the model noise, and n is the number of observations. Following Benko et al. (2007), we do not introduce any explicit model expression for σ. Instead, to fit the model, we adopt the local polynomial smoothing presented in Fan and Gijbels (1996), Hardle (1990) and Fan and Yao (2003) which is a semi-parametric regression technique. Local polynomial estimators have been successfully applied to calibrate the volatility surface model (4), see Shimko (1993), Fengler et al. (2003) and Cont and Da Fonseca (2002). Moreover, Benko et al. (2007) extend this modelling scheme imposing constraints which eliminate arbitrage opportunities.

To obtain smoothed IVs that allow an immediate comparison, the implied volatility should be standardized with respect to the underlying value. Thus, we consider the *futures moneyness* $\kappa_t := K/F_t$ with $F_t = S_t e^{r\tau}$, i.e. the relative position of the future value of spot price of the underlying asset with respect to the strike

price. In the following section, we focus on the estimation of the IVs for fixed maturity and then we apply this technique to a large set of data in Sect. 3.

2.1 Implied volatility for fixed maturities

To estimate the implied volatility function, we need a set of observed option prices with a fixed time-to-maturity τ. Then, the general framework of the model (4) can be simplified thanks to the fixed τ and the futures moneyness κ_i:

$$\tilde{\sigma}(\kappa_i) = \sigma(\kappa_i) + \varepsilon_i, \quad i = 1, \dots, n_\tau, \tag{5}$$

where $\tilde{\sigma}_i := \tilde{\sigma}(\kappa_i)$ and $\sigma_i := \sigma(\kappa_i)$ are assumed to be functions of κ_i only, ε_i represents the error term, and n_τ is the number of observed implied volatilities for the fixed time-to-maturity τ. If $\tau = T - t$ is given, the definition of the moneyness becomes: $K_i = \kappa_i S_t e^{r\tau}$, where $F_t = S_t e^{r\tau}$ is supposed to be constant observing daily data, i.e. $S_t := S$, $F_t := F$ and $K_i = \kappa_i F$.

As was previously shown, the model (5) can be calibrated by a local polynomial fitting. Benko et al. (2007) suggests to use a local quadratic estimator. The estimator $\hat{\sigma}(\kappa)$ of the function $\sigma(\kappa)$ in the point κ is then computed as the solution of the following minimization problem:

$$\min_{\alpha_0,\alpha_1,\alpha_2} \sum_{i=1}^{n_\tau} \left[\tilde{\sigma}_i - \alpha_0 - \alpha_1(\kappa_i - \kappa) - \alpha_2(\kappa_i - \kappa)^2\right]^2 \mathcal{K}_h\big(\kappa - \kappa_i\big), \tag{6}$$

where $\mathcal{K}$ denotes a kernel function and the weighting term $\mathcal{K}_h(\kappa - \kappa_i)$ is defined as $\frac{1}{h}\mathcal{K}\left(\frac{\kappa-\kappa_i}{h}\right)$. The kernel function is a weighting function typically used in non-parametric estimation techniques. Specifically, it is a non-negative real-valued integrable function $\mathcal{K}$ satisfying: (i) $\int_{-\infty}^{\infty} \mathcal{K}(x)dx = 1$, and (ii) $\mathcal{K}(x) = \mathcal{K}(-x)$, $\forall x \in \mathbb{R}$. In the literature, several kernel functions have been proposed, see Fan and Gijbels (1996). The parameter $h > 0$ is the so-called bandwidth and it controls the size of the local neighborhood considered for the estimation. As analysed in Kopa et al. (2017), the choice of h has a relevant impact in the computation and in the results of the fitting. On the one hand, a high value of the bandwidth induces over-smoothing, i.e. modelling bias. On the other hand, a small value produces under-smoothing, i.e. noisy estimates. If $h = \infty$, the local polynomial fitting becomes a global quadratic fitting, see Fan and Yao (2003). Moreover, applying the Taylor's expansion of σ in the problem (6), we can explicitly derive: $\alpha_0 = \hat{\sigma}(\kappa)$, $\alpha_1 = \hat{\sigma}'(\kappa)$, and $\alpha_2 = \hat{\sigma}''(\kappa)/2$.

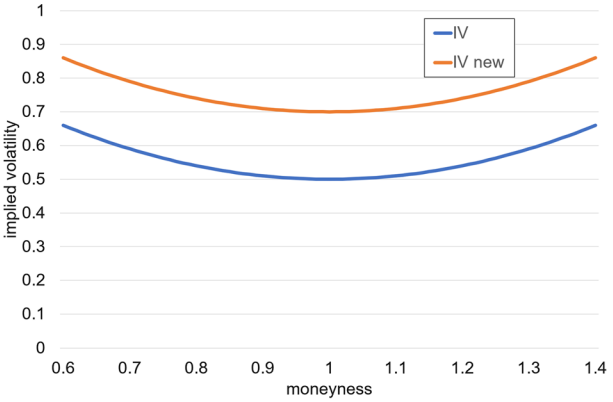
The previous results can be extended for the case in which the option is written on dividend-paying assets. In such case the options is priced as suggested in (Merton 1973). Then, the semi-parametric estimation proposed in Benko et al. (2007) can be adjusted accordingly as it is shown in Kopa et al. (2017) and Vitali et al. (2017). In the empirical analysis, we start from IV raw data that are already dividend-adjusted by the data provider and, thus, we do not need to apply any adjustment for the dividends.

2.2 The shape of the implied volatility function

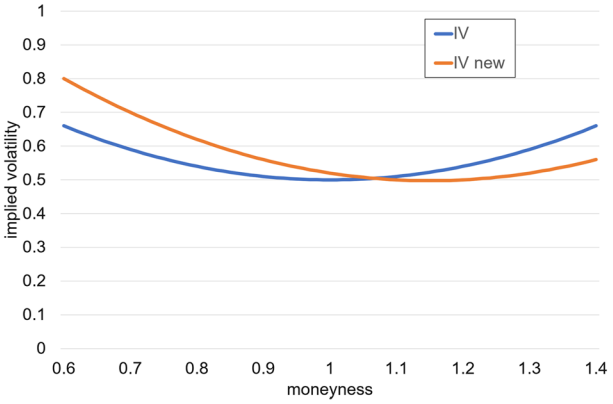
The shape of the IV curve—and its evolution through time—is an indication of the view of the market with respect to the future of the underlying. Specifically, it represents what the market operator think about the movement of the underlying between the current day and the day in which the options will expire. A static interpretation of the IV curve is described in many works, see e.g. García-Machado and Rybczyński (2017). However, the shape of the IV in periods of market turmoil is extremely difficult because of the irregularity of the curve itself. Thus, we focus more on the dynamic variation of the IV curve and we identify three main changes:

- Schema 1: a vertical shift of the IV curve. The minimum point does not move, i.e. the main expectation does not change, but the whole curve moves up, see Fig. 1 (Schema 1), or down. The up movement shows a general increase of the uncertainty, contrary, the down movement a decrease of the uncertainty.
- Schema 2: an horizontal shift of the IV curve. The minimum point moves meaning that the market is changing own main expectation about the future underlying evolution. A shift to the right expresses confidence about a positive growth of the underlying, see Fig. 1 (Schema 2), contrary, a shift to the left a negative return.
- Schema 3: a change of the convexity of the IV curve. The minimum point does not move, thus the main expectation does not change. While an increase of the values of the volatility at high and low moneyness indicates an increase of probability assigned to extreme events, see Fig. 1 (Schema 3), contrary, a reduction of the convexity shows a reduction of the probability of extreme events.

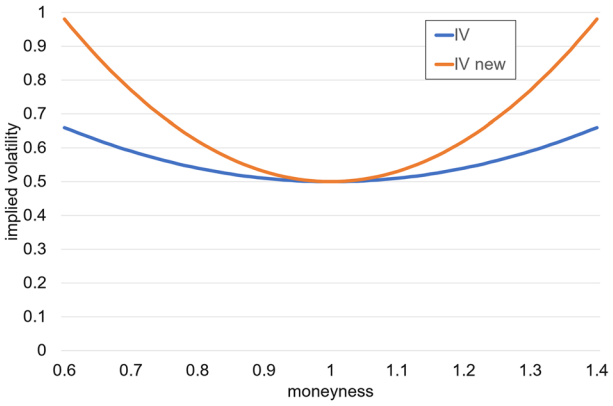
Proposing these three schemas, we have in mind the natural link between the IV and the corresponding SPD. Such relation is impossible to describe mathematically because IV and SPD do not have a mathematical representation since both are point-by-point estimations; however, some relations are well-known and empirically observed in the papers cited above, such as the fact (i) that when the IV is not monotone and presents a minimum point, it corresponds -with some approximation- to the maximum of the SPD, i.e. the moneyness that, in expectation, has the highest probability to be reached by the underlying according to the market sentiment; and (ii) that a more convex IV corresponds typically to a SPD with fatter tails. Differently from other studies, e.g. Jackwerth (2020) and Hanke et al. (2020), we try to reach conclusions on the dynamic of the market without estimating the SPD and relying only on the IV smoothing. We believe that to study only the IV is simpler and, not performing a double-step estimation (IV smoothing plus SPD estimation) it induces less approximation errors; moreover, it requires less assumptions than the SPD estimation. Surely, the above schemas can be combined one with the other and, as we will see in the next section, the real IV curve dynamics can be extremely irregular. Moreover, other changes could still identify some sentiment of the market. For instance, the fact that an option at a specific moneyness is quoted for a given maturity and that in the next month is not quoted anymore indicates that the market stop believing that the underlying can reach that level at all.



Schema 1: vertical shift of the IV curve



Schema 2: horizontal shift of the IV curve



Schema 3: increase of the IV curve convexity

Fig. 1 Three possible changes of the IV curve

3 Empirical results

In this section we show the empirical results of our work. Our aim is to analyse the impact of the pandemic SARS-CoV-2 on the global financial market. For this purpose, we consider options written on five indexes: S&P500 (US index), Hang-Seng (Asian index), EuroStoxx50 (European Index), DAX (German index), and FTSEMib (Italian Index). Moreover, we take into account options written on five large companies representative of sectors particularly affected by the pandemic: Apple, Amazon, Nestlé (Consumer), Boeing (Travel), Bayer (Pharmaceutical). For each underlying, we collect data of the options having maturity December 2019, January 2020, February 2020, March 2020, April 2020, May 2020, and June 2020. We consider the quotation of each option 15 days and 30 days before the maturity. For instance, for the options expiring in December 2020 (the third Friday of the month, i.e. the 20th December 2020) we take the quotation on the 5th December and on the 20th November. The same for the options expiring in subsequent months. In case the day falls in a weekend or in a holiday, we go in the closest trading day. For each option we download from Thomson Reuters Datastream the information about the option price, the implied volatility, the strike, the underlying price, the time-to-maturity (expressed in calendar days), the reference risk-free rate. The estimation of the IV curve implements the procedure presented in Sect. 2.1. Following Kopa et al. (2017) we adopt the Epanechnikov kernel function with bandwidth $h_\kappa = 0.12$ that also in our contest appears to be a well-balanced choice. The results for the 15-days time-to-maturity options are in the subsequent sections, the results for the 30-days time-to-maturity options are in the Appendix.

3.1 Estimating IV curve for index options with 15-day time-to-maturity

Let's start our analysis on the index options. In Figs. 2, 3, 4, 5 and 6, we show the smoothed IV curves for the S&P500, the HangSeng, the EuroStoxx50, the DAX and the FTSEMib, respectively. The figures depict the curves on a large moneyness span, i.e. from 0 to 2; then on a narrower moneyness, i.e. from 0.7 to 1.3, to better observe the behaviour of the IV curves near own minimum; and, finally, the moneyness curves. In Tables 1, 2, 3, 4 and 5, we report some statistics of the returns of the underlying immediately before and after the days in which we observe and study the options.

3.1.1 S&P500

Figure 2 shows the results for the S&P500 considering options expiring in 15 days. We notice that in all considered months the market expected an increase of the underlying value at maturity. In the pre-pandemic months (December, January, February) the minimum point is stable around the moneyness value 1.05. The first news of the virus appear in January and February but we do not observe any impact in the IV curves quoted at the beginning of January and February. The first infections

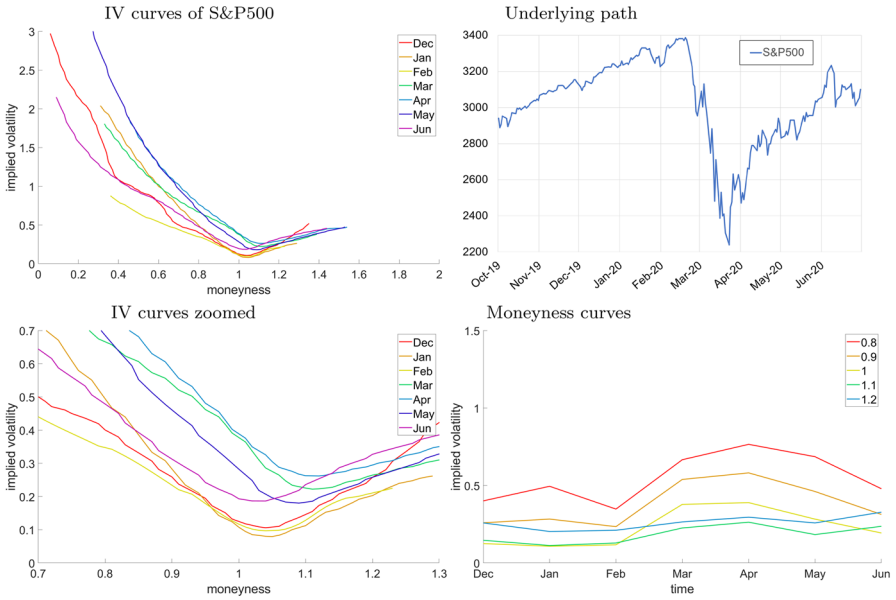


Fig. 2 Estimations of the IV curve of the S&P500 15 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

in Europe at the end of February influence the IV curves quoted for March in terms of increase of the IV in the part below moneyness one, but the minimum increase to 1.1 showing the optimism about the fast recover of the worldwide situation. Even the European lockdown of March and April has an impact on the Index that falls from a peak of the middle of February around 3400 points to 2300 in the middle of April: − 32%, cf. Figure 2 (up right), but the IV curved quoted at the beginning of March, in the middle of the collapse, still show a minimum around 1.12, even if the whole curve significantly shifts up. In general, a common sentiment of a fast recover prevails and the short term options always show a positive attitude of the market operators. April represents a similar contest and the initial signals of a recovery do not influence the IV curve too much. For options expiring in May and in June, the IV curves show a very similar shape and the market expects a rise in the underlying around 8–9% which actually took place. Considering the moneyness curves, cf. Figure 2 (down right), we notice that the IV for moneyness below 1 are always higher than the others.

3.1.2 HangSeng

The options written on HangSeng with 15 days of maturity are analysed in Fig. 3. In general, the IV curves are quite concentrate around moneyness values close to one and the smiles are not very convex. Contrary to the S&P500, since February the IV curves starts to shift up, cf. Figure 3 (down left). And the same

trend continues in the following months. Still, such increase is less relevant than what we observe for the S&P500, and the range of moneyness is always relatively small, even during the hardest months of the pandemic. Even the turmoil of March do not influence the options because we observe them at the beginning of the months when the Index experienced a fall of only -3% in the previous 10 days, cf. Table 2, then, at the end of March, the HangSeng begins an important recovery and the market operators align on quite positive expectation. However, despite such recover, the IV curve of the options of April shows the highest IV for the range of moneyness below one. In the next months the expectations are relatively positive as well and the situation appears very similar to the pre-pandemic months. Figure 3 (down right) shows the moneyness curves that indicates the the IV for moneyness 0.8 increases significantly in April. But, the overall situation appears relatively stable through the first months of 2020.

3.1.3 EuroStoxx50

The options written on EuroStoxx50 are described in Fig. 4. Observing the IV curves, we notice the effect of the Brexit in the options of December 2019 just few days before the election in England and the consequent exit of the United Kingdom from the European Union. Such event generated a great turmoil in the European markets as reflected by the IV curve. Then, in January and February the situation returns to normality with a lower convexity of the IV curve and a smaller range of quoted moneyness. Since March, the options written on

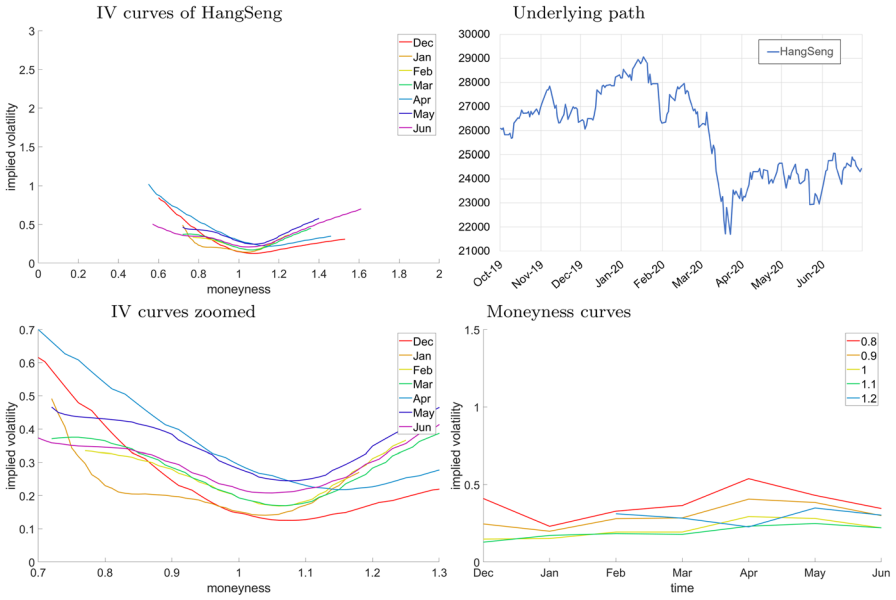


Fig. 3 Estimations of the IV curve of the HangSeng 15 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

EuroStoxx50, one of the main European Index, shows the huge impact of the pandemic that stroke in Europe. The news from China in February do not have a relevant impact, while the first infections in Italy and, then, in the rest of Europe in the very end of February and in the first days of March, induce a relevant change in the options quoted at the beginning of March expiring in March. Indeed, the IV curve shifts up highlighting an increase of the uncertainty in the markets. Still, the minimum point increases to moneyness value 1.1 showing that the market operators were positive about a fast recover of the situation. The effects of the pandemic, and its persistence, appear clearly during March and the IV curve of April is even more shifted up. Between the end of February and mid-March there is a collapse of the value of the Index that falls from around 3800 points to 2400 meaning that the expectation of the market at the beginning of March do not realize at all. From April, the main expectation is a stabilization of the general situation and an increase of the EuroStoxx50 around 5% and the convexity of the IV curves reduces as well. Figure 4 (down left) shows the moneyness curves and we notice that from May the IV of moneyness 1.2 is higher of the moneyness 0.8 highlighting a very positive view of the market for the next future.

3.1.4 DAX

Figure 5 depicts the results considering the DAX options with 15 days maturity. Before the pandemic spread, the minimum point of the smile of the IVs is around values of moneyness between 1 and 1.1 meaning that the market had an

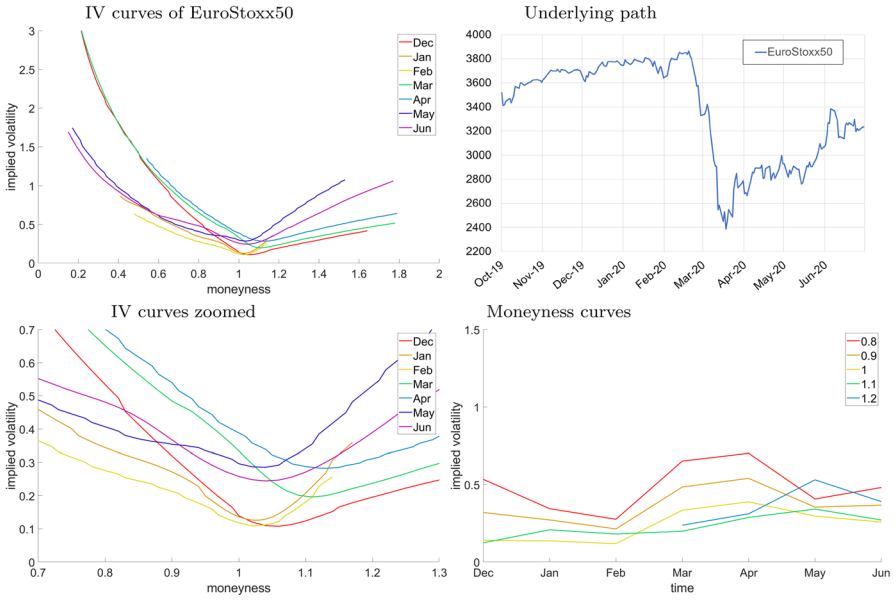


Fig. 4 Estimations of the IV curve of the EuroStoxx50 15 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

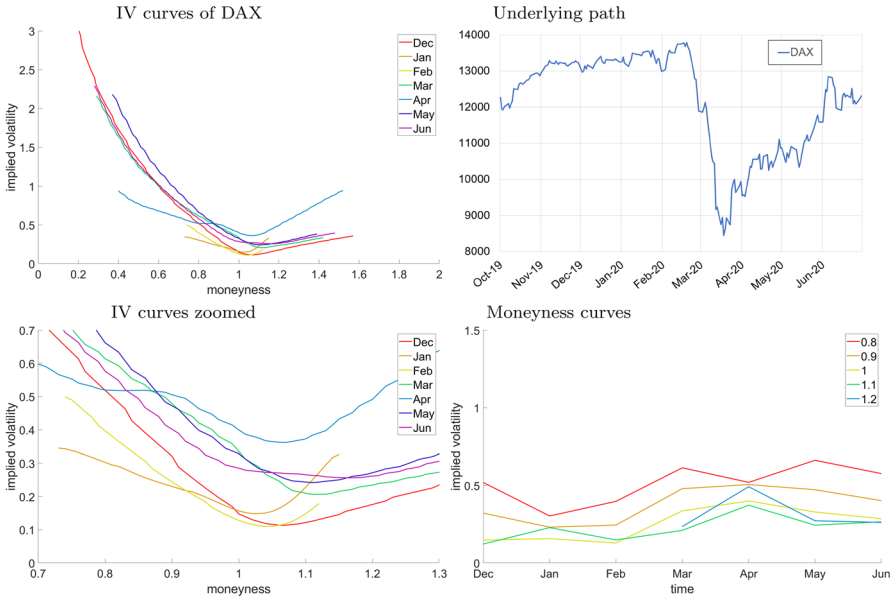


Fig. 5 Estimations of the IV curve of the DAX 15 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

expectation that the market will grow in the next month. Observing December, January and February in Fig. 5 (up left), we note that December shows a relevant convexity of the smile because of the uncertainty induced in Europe by Brexit as it happened for the EuroStoxx50, then, the convexity of January and February is similar although for January the IV for moneyness values around 0.8 is significantly lower than February and December. Moreover, for the options expiring in December, the range of quoted moneyness is very wide compared to the following two months highlighting that Brexit was en event that could generate either huge losses or huge gains. At the beginning of March the pandemic spreads in Europe and the options quoted for the end of March highlight the uncertainty with a shift up of the IV curve, cf. Figure 5 (down left). Still, the main expectation is positive because the minimum point is around moneyness 1.1. During March the DAX suffers a turmoil period, but in the last weeks of March the market recovers and the quotation of the options at the beginning of April shows a great uncertainty: the IV curve shifts up in the central and right part but decrease on the left. This can be interpret saying that the whole uncertainty increase, but the market bets more on a possible increase rather than on another collapse. Moreover, the IV curve shows two local minimum points: the market operators expect either another positive jump or another collapse of − 20. In May and June the situation stabilize but showing a quite high concern for a worsening of the situation. Looking at the moneyness curves in Fig. 5 (down right), we notice that the moneyness of 0.8 have a higher IV in the months of March and May. Moreover, only since March there are options listed with moneyness of 1.2.

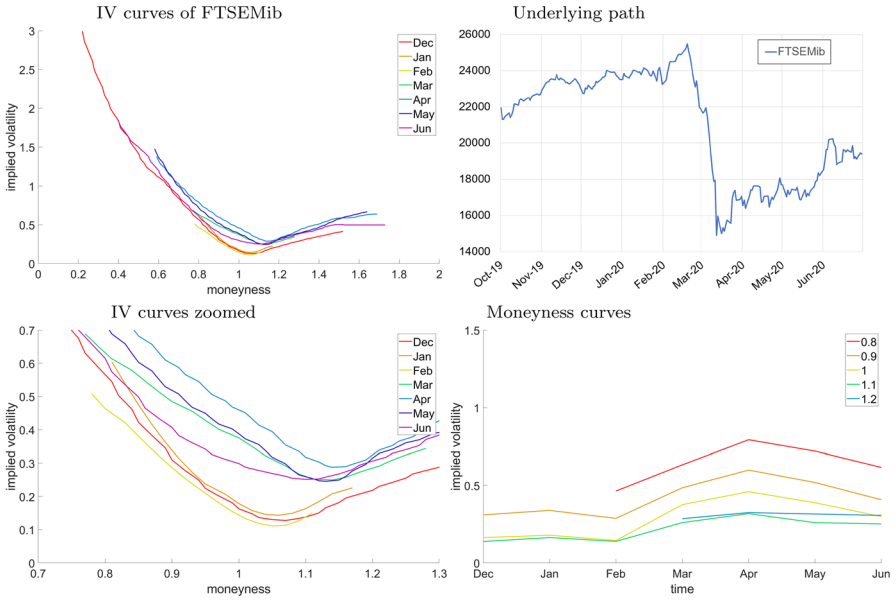


Fig. 6 Estimations of the IV curve of the FTSEMib 15 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

3.1.5 FTSEMib

The market of the options written on FTSEMib, the Italian Equity Index, are not so liquid, but still we focus on them because Italy was the first country of Europe where the virus stroke and the first European country to apply a complete lockdown. The options with 15 days of maturity written on FTSEMib are analysed in Fig. 6. The first infections in the northern part of Italy appears in the beginning of February but the lockdown is declared the 9th of March. The IV curves of the options of

Table 1 Average return of the S&P500 in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 15 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	5-Dec	0.45	2.82	0.46	0.44	1.89
Jan-20	6-Jan	0.69	2.57	0.43	− 0.13	1.60
Feb-20	5-Feb	0.39	1.54	0.64	0.18	2.68
Mar-20	5-Mar	− 10.35	− 20.32	122.39	0.34	2.17
Apr-20	6-Apr	19.05	5.99	19.89	0.86	3.73
May-20	5-May	4.82	1.90	6.61	0.15	2.62
Jun-20	5-Jun	8.07	− 3.01	13.61	− 1.87	7.93

The observed days are 15 calendar days before the corresponding maturity

Table 2 Average return of the HangSeng in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 15 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	5-Dec	− 0.94	6.04	2.34	0.76	3.58
Jan-20	6-Jan	1.15	2.02	1.87	0.48	2.53
Feb-20	5-Feb	− 5.48	3.24	3.17	0.51	4.46
Mar-20	5-Mar	− 3.05	− 18.90	14.86	0.44	1.63
Apr-20	6-Apr	9.46	2.45	4.09	0.21	1.65
May-20	5-May	0.32	2.18	3.65	− 0.34	1.80
Jun-20	5-Jun	8.03	− 0.51	5.62	− 0.37	2.78

The observed days are 15 calendar days before the corresponding maturity

Table 3 Average return of the EuroStoxx50 in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 15 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	5-Dec	− 0.86	2.50	1.12	0.26	2.01
Jan-20	6-Jan	− 0.64	1.24	0.49	0.73	3.05
Feb-20	5-Feb	0.21	2.31	0.85	0.15	1.39
Mar-20	5-Mar	− 12.02	27.04	64.16	− 0.58	2.83
Apr-20	6-Apr	12.49	4.06	12.72	− 0.21	5.12
May-20	5-May	3.03	0.93	11.88	1.08	4.84
Jun-20	5-Jun	16.48	− 3.40	13.62	− 0.11	4.29

The observed days are 15 calendar days before the corresponding maturity

December, January and February are aligned in terms of convexity and location of the minimum point. Also here we notice the effect of Brexit since December, but the convexity remains remarkable in January and February probabily due to the fragility of the Italian economy that was expected to suffer more from Brexit with respect to Germany. Then, the IV curve of the options expiring in March observed the 5th of March (before the lockdown), highlights the very uncertain situation of the financial markets and follow the collapse of the FTSEMib index that started the third week of February, cf. Figure 6 (up right). The IV curve shifts up but the IV on the left part of the IV curve are not sensibly different from the quotation of December, meaning that the market is betting for a relatively quick recover. During March the value of the Index decrease even more, but in the last week of March it seems that the situation is stabilizing. Thus, the IV curve of April is more convex than March and slightly above but the minimum point is shifted to the right, around a moneyness value of 1.13, showing a very positive expectation of the market operator for the next 15

Table 4 Average return of the DAX in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 15 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	5-Dec	− 0.63	1.20	1.14	0.03	1.34
Jan-20	6-Jan	− 1.31	3.21	0.91	0.42	2.71
Feb-20	5-Feb	− 0.28	2.30	1.26	0.02	2.10
Mar-20	5-Mar	− 12.58	− 27.91	54.11	− 0.83	3.25
Apr-20	6-Apr	15.26	5.96	16.34	− 0.07	4.50
May-20	5-May	4.68	3.22	14.30	0.93	4.41
Jun-20	5-Jun	16.02	− 4.02	12.68	− 0.12	4.34

The observed days are 15 calendar days before the corresponding maturity

Table 5 Average return of the FTSEMib in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 15 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	5-Dec	− 1.33	3.22	1.02	− 0.64	2.92
Jan-20	6-Jan	− 1.33	1.79	0.98	− 0.16	1.12
Feb-20	5-Feb	2.24	5.12	0.83	0.34	2.34
Mar-20	5-Mar	− 14.06	− 28.24	119.83	− 0.83	3.95
Apr-20	6-Apr	9.51	0.15	12.70	− 1.00	6.29
May-20	5-May	5.69	− 2.03	8.56	0.37	2.26
Jun-20	5-Jun	16.58	− 2.82	12.76	− 0.64	5.01

The observed days are 15 calendar days before the corresponding maturity

days, cf. Figure 6 (down left). During April the Index rises, even if below lower than expected. Therefore, the IV curve of May is shifted down but show still the same uncertainty as in March. The situation in June is basically the same but the convexity slightly reduces. Observing Fig. 6 (down right), we notice that the moneyness 0.8 was not even quoted in January, while in the next months it becomes the moneyness with the higher IV. In general, during the pandemic the lower values of moneyness have the highest IV, but in May and June the increasing of the IV associated with moneyness 1.2 shows a positive sentiment among the market operators.

3.2 Estimating IV curve for stock options with 15-day time-to-maturity

In Figs. 7, 8, 9, 10 and 11, we show the results for Apple, Amazon, Nestlé, Boeing and Bayer, respectively. The IV curves are smoothed on the options quoted 15 days before own maturity. The figures depict the curves on a large moneyness span, i.e.

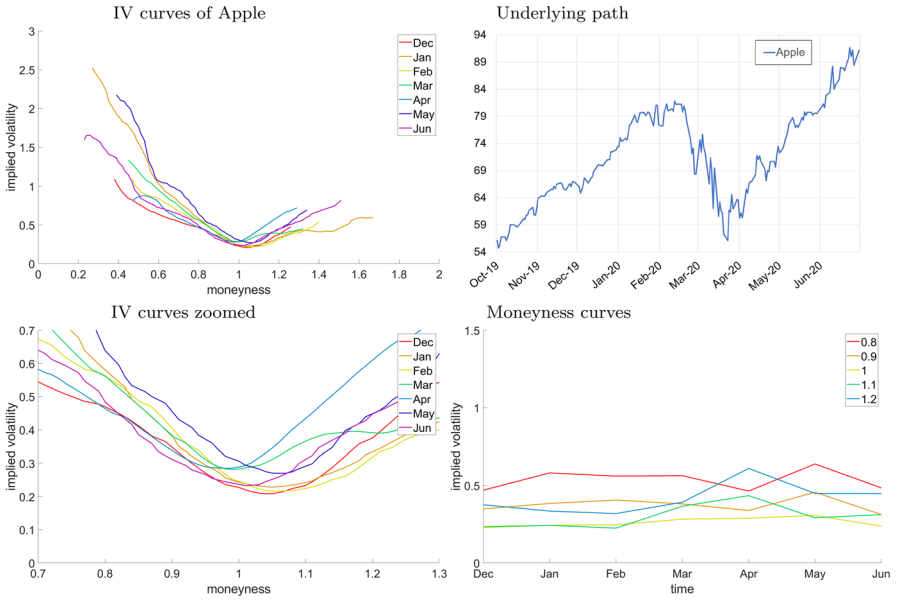


Fig. 7 Estimations of the IV curve of the Apple 15 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

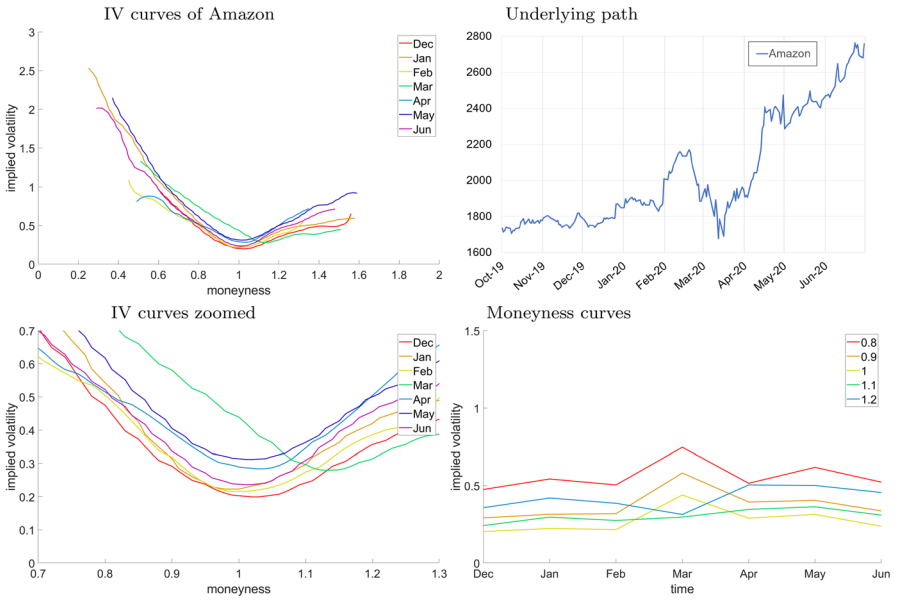


Fig. 8 Estimations of the IV curve of the Amazon 15 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

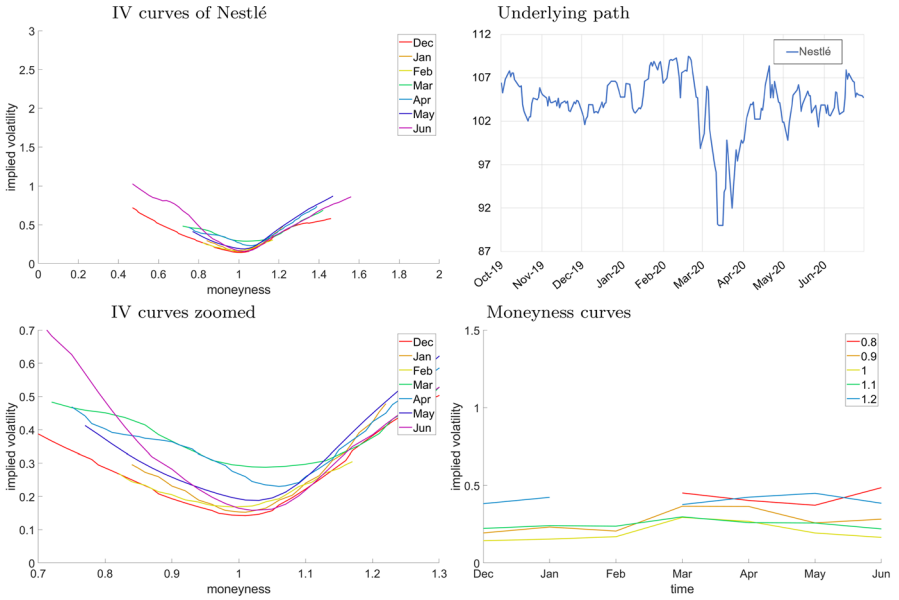


Fig. 9 Estimations of the IV curve of the Nestlé 15 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

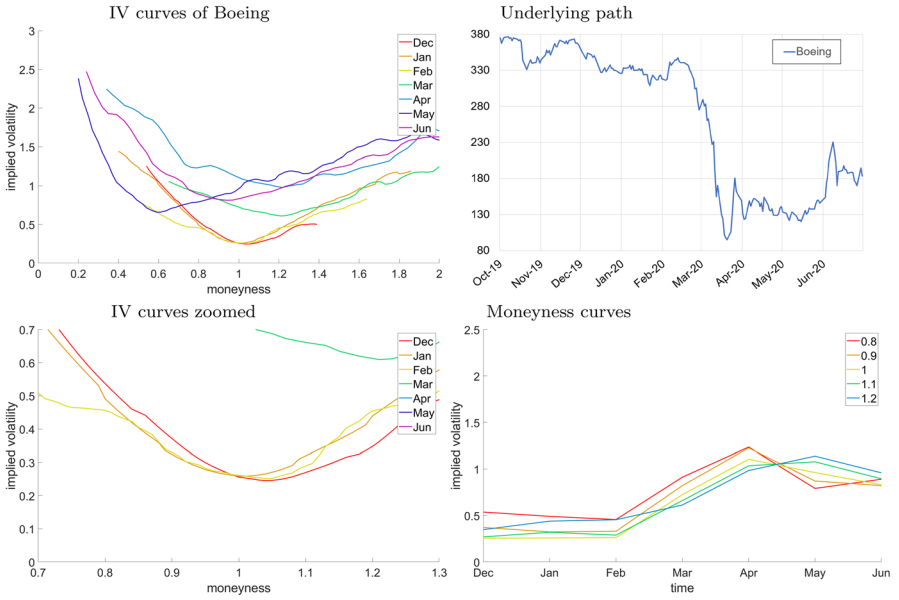


Fig. 10 Estimations of the IV curve of the Boeing 15 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

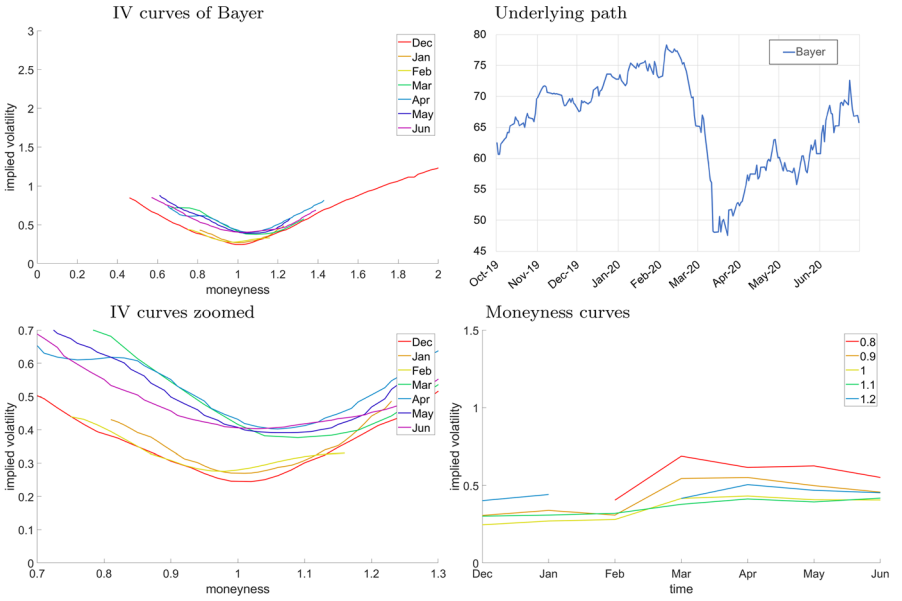


Fig. 11 Estimations of the IV curve of the Bayer 15 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

from 0 to 2; then on a narrower moneyness, i.e. from 0.7 to 1.3, to better observe the behaviour of the IV curves near own minimum; and, finally, the moneyness curves. In Tables 6, 7, 8, 9 and 10, we report some statistics of the returns of the underlying immediately before and after the days in which we observe and study the options.

Table 6 Average return of Apple in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 15 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	5-Dec	1.36	5.44	2.60	− 0.59	3.28
Jan-20	6-Jan	5.56	6.31	3.49	− 0.23	2.09
Feb-20	5-Feb	1.18	0.68	4.46	0.05	2.46
Mar-20	5-Mar	− 8.55	− 16.43	145.99	0.38	2.85
Apr-20	6-Apr	16.98	5.51	27.78	1.40	4.86
May-20	5-May	10.88	5.24	4.48	− 0.25	1.51
Jun-20	5-Jun	3.95	5.50	13.84	− 1.48	6.09

The observed days are 15 calendar days before the corresponding maturity

Table 7 Average return of Amazon in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 15 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	5-Dec	0.33	2.98	1.23	− 0.54	2.98
Jan-20	6-Jan	6.13	− 2.01	1.85	0.39	2.33
Feb-20	5-Feb	8.07	6.39	2.51	0.84	5.05
Mar-20	5-Mar	− 10.64	− 2.24	72.04	0.14	1.60
Apr-20	6-Apr	4.98	19.82	17.53	0.44	1.63
May-20	5-May	− 0.44	5.67	2.50	− 2.45	10.04
Jun-20	5-Jun	1.89	7.73	7.25	− 1.66	7.20

The observed days are 15 calendar days before the corresponding maturity

Table 8 Average return of Nestlé in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 15 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	5-Dec	− 0.66	1.54	1.20	0.45	3.20
Jan-20	6-Jan	− 0.32	0.60	2.16	− 0.18	3.36
Feb-20	5-Feb	0.20	0.42	4.77	0.27	3.86
Mar-20	5-Mar	− 3.55	− 5.38	31.98	0.34	3.41
Apr-20	6-Apr	13.02	4.23	3.49	− 0.67	3.77
May-20	5-May	− 2.75	3.56	3.26	− 1.45	5.72
Jun-20	5-Jun	− 0.35	4.78	5.07	0.49	3.26

The observed days are 15 calendar days before the corresponding maturity

3.2.1 Apple

Figure 7 shows how the smile of the IV of options written on Apple is similar in the three pre-pandemic months (December, January and February) all 3 months, although the IV curve of January is slightly more convex than those of December and February. Market expectations about the underlying 15 days before expiration in December and January are a 5% increase in the underlying, while for February they are around 8%. For the months in which the pandemic begins to spread in Europe, we observe that the trend of the smile is similar to the previous months both in terms of the price range of the options and in the convexity of the smile itself. However, market expectations for March stand for a value of the underlying positioned at the ATM on a maturity, indicating an expectation of stability although there were the first hints of a potential global pandemic. In April the expectations of the underlying at maturity are a loss of 5%, as at the time of

Table 9 Average return of Boeing in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 15 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	5-Dec	− 5.67	− 3.52	8.18	− 0.74	4.93
Jan-20	6-Jan	− 1.13	− 2.87	4.49	− 0.52	1.96
Feb-20	5-Feb	6.65	2.66	8.99	0.88	2.38
Mar-20	5-Mar	− 22.57	− 62.47	297.04	0.26	2.11
Apr-20	6-Apr	40.85	− 3.47	206.64	1.07	3.46
May-20	5-May	− 8.02	4.02	77.42	1.62	5.32
Jun-20	5-Jun	49.37	− 8.96	210.75	− 0.21	2.86

The observed days are 15 calendar days before the corresponding maturity

Table 10 Average return of Bayer in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 15 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	5-Dec	0.70	2.94	2.92	− 1.00	4.80
Jan-20	6-Jan	− 2.53	5.24	3.11	0.48	3.55
Feb-20	5-Feb	3.48	− 2.33	3.83	0.65	3.85
Mar-20	5-Mar	− 11.16	− 24.90	77.70	− 0.82	4.05
Apr-20	6-Apr	17.45	5.04	15.90	− 0.27	3.58
May-20	5-May	2.19	1.82	17.09	1.28	5.38
Jun-20	5-Jun	14.74	4.94	24.93	0.07	2.84

The observed days are 15 calendar days before the corresponding maturity

observation the underlying price stood around 74 Dollars. However, in the most acute period of the crisis linked to the pandemic there was a loss in the value of the stock by as much as 27%. Once the pandemic in Europe became again under control with a slow recovery in production activities, the options listed in May show a slight increase in general uncertainty, i.e. a parity of moneyness and greater volatility due to extreme events linked to losses of 20%, characterized by an expectation an 8% maturity increase which is confirmed by Fig. 7 (up right) where we observe that the value of the underlying increases. A completely different situation is recorded in June, where market expectations are as in the pre-pandemic period, characterized by a wide range of prices compared to previous months and an increase in the underlying of 2–3% as can be seen from Fig. 7 (down left). In Fig. 7 (down right), we notice that the IV of the lower moneyness have constantly a higher value but in the months of April and May when the market sees a fast and important recovery.

3.2.2 Amazon

Figure 8 depicts the options written on Amazon stock 15 days before maturity. In the pre-pandemic period (December, January and February) it can be seen from Fig. 8 (down left) how the rising point of the smile is around the value 1 of moneyness, indicating a situation of stability of the market although the news of the first infections in Europe arrived, it deviates slightly for the month of December with an expectation 15 days before the expiry of an increase in the value of the stock of 2–3%, as can be seen from Fig. 8 (up right) inherent to the performance of the underlying. Peculiarities, the higher quotation of OTM call and ITM put for the month of January, therefore possible extreme events were contemplated even with losses of 70% (i.e. moneyness equal to 0.3). In the period in which critical situations caused by the spread of the virus begin to appear, it is noted that in March the smile appears as an ascent point around moneyness values of 1.13, therefore an expectation of upside at maturity as can be seen from the Fig. 8 (up right) the value of the Amazon stock grows over 2150 Dollars. As can be seen in Fig. 8 (down left) for the same moneyness there was greater uncertainty related to possible losses of 10% or 20% therefore more probability of extreme events in March caused by the spread of the virus in Europe. In contrast, the smile that forms for the options listed in April is in line with those of the pre-pandemic months although, at the point where the smile rises, there is greater uncertainty and is positioned around a 4% increase in the value of the share a expiry, as can be seen from Fig. 8 (up right) the value of the Amazon stock stood at around 1800 Dollars 15 days before the expiration followed by a drop of about 20% in mid-April and then at expiration confirm expectations generated by the market. In the months of general economic recovery (May and June) there is a smile of the IV similar both for convexity and for expectations of an increase of the underlying at maturity of about 5%, confirmed by the rising trend of the stock as can be seen from Fig. 8 (up right) which far exceeds expectations by positioning the share value over 2400 Dollars at the end of May, deducing how the e-commerce sector had a huge benefit from the pandemic crisis.

3.2.3 Nestlé

In Fig. 9 we observe the information given by the options written on Nestlé which represents the food and beverage sector, i.e. a type of business that should not have been affected by the crisis given that the food sector has had fewer restrictions, in the pre-pandemic months (December, January and February) the convexity of the smile is almost constant as can be seen from the Fig. 9 (up left), with an expectation on the positioning of the smile at maturity around the 1 value of moneyness. The only difference is noted for the quotation interval for options expiring in December which contemplate greater extreme events with possible losses of 50% (i.e. moneyness equal to 0.5) or gains of approximately 45% (i.e. moneyness equal to 1.45). In the period in which the restrictions come into force and in which the pandemic becomes acute in Europe (March and

April), it can be seen in Fig. 9 (down left) that the smile of the IVs have a rising point around values of 1.05 of moneyness, therefore an expectation 15 days before the expiration of 5% rise in the value of the stock. There are ITM calls and OTM puts with a higher IV than OTM calls and ITM puts, this highlights how extreme events related to an increase in the value of the underlying at maturity are perceived as more likely, given that as can be seen from Fig. 9 (up right) the value of the stock has fallen by 18% in mid-March compared to the same period in March (going from a value of 108 to one of 89 Euros). Then, already in April when Europe was still in lockdown, the value of the stock recovered to the pre-pandemic values and the IV curves shifted above but maintaining a positive expectation. In the months in which the restrictions have been relaxed (May and June), after the collapse of the value in the months of the most acute crisis, there is an expectation of a rise at maturity of around 5%, as can be seen from Fig. 9 (up right) the value of the stock is around a value similar to the pre-pandemic period although the smile that forms is more convex, therefore there is greater general uncertainty. Observing Fig. 9 (down right), we notice that for some months the most negative moneyness were not even quoted. Therefore, we can conclude that the food and beverage sector experienced a fall in March, even if limited, but also a quick recover and the options always showed a quite positive expectation of the market and not so much uncertainty as well.

3.2.4 Boeing

Figure 10 shows the situation for Boeing that represents the sector most affected by the pandemic. Indeed, the evidences are quite the opposite In the pre-pandemic months (December, January and February) the situation was stable both in terms of expectation, with a minimum around moneyness 1, and in terms of convexity. Note that the options we are considering are 15 days before maturity, thus, the options expiring in March are considered the very first days of March when the Boing stock already experience a loss. In the beginning of March the market was extremely positive expecting a jump pf around 25%, cf. the minim point of the green curve in Fig. 10 (up left). Still, the uncertainty was huge because the curve shifted up, but the market expected a fast recovery. Unfortunately, during March the stock value collapsed from around 300 to around 90 Dollars. Still, the market hoped for another recovery because the IV curve of April has a minimum point again around moneyness 1.25 even if the curve shifted even higher. Finally, the market expectation completely invert in May when the pandemic spread hugely also in the USA and the IV curve suddenly move the minimum point around value 0.6. Now the market expect as most probable a further loss of 40%. However, again this expectation does not realize because the stock value during May is relatively stable. In June, despite the first signs of recovery of the stock value, the minimum of the IV curve is around moneyness 0.9 and the uncertainty remains remarkably high. Considering Fig. 10 (down left), we notice that most of the curves are outside the range of the picture, then, comparing with all the other

results showed so far, we can capture the extreme uncertainty that affected the travel and transportation sector.

3.2.5 Bayer

The options related to Bayer stock are shown in Fig. 11. For the period before the outbreak of the pandemic (December, January and February) the smile of the IV has a similar shape in all 3 months, although the expectations 15 days before the expiry date for December and January indicated a stable underlying around ATM values, while for the options listed in February, we observe an expectation of a 5% decline at maturity of the stock, cf. Fig. 11 (down left). It is also noticeable a large quotation range for the month of December, with options beyond the value of moneyness 2. For the period in which the spread of the virus occurs in Europe (March and April) there is a higher level of uncertainty than in the previous months, although the convexity of the generated smile are similar. Market expectations stand for an increase in the underlying at maturity of about 8%, the quotation range is almost identical. Interestingly, for the same moneyness, the options with moneyness 0.8 and 0.9 are more volatile in March, and, therefore, the market expects extreme events with higher probability, while the options with moneyness 1.1 and 1.2 are more volatile to April, since the collapse in the stock's value occurred from mid-February to mid-March, as can be seen from Fig. 11 (up right) when the stock's value dropped from 75 Euro to 47 Euro a month later (i.e. loss of more than 35%). The months after more stringent restrictions for the major countries (April and May) show smile with trends and convexities similar to the crisis period with an expectation for the options listed in May of an increase in the value of the underlying of about 5% at expiration date, as opposed to the June prices with an expectation of a decrease of a few percentage points, although as can be seen from Fig. 11 (up right) the trend after April indicates a strong recovery in the value of the stock, this is also due to the strong demand of the sector to which it belongs. In general, we notice that Bayer, belonging to the pharmaceutical sector, recovers quite quickly and the options written on Bayer stock do not suffer huge turmoil.

4 Conclusion

In this paper, we present an exhaustive analyses on the options written on the most important worldwide indexes and on representative stocks of the main industry sectors. The analysis confirmed that the pandemic of SARS-CoV-2 during the first six months of 2020 had a huge impact on the financial markets. However, the reaction of the various indexes and stocks were very different from each other. In general, we discussed the result with respect to: (i) the index representing the financial situation of a country or of an area or the stock representing an industry sector, (ii) the observed month and the corresponding worldwide situation in terms of spread of the pandemic, (iii) the time-to-maturity of the options: either 15-day or 30-day.

Considering to keep constant (i) and (ii), we can notice that changing (iii) the general level of uncertainty increases going from 15-days to 30-days time-to-maturity options. Indeed, as expected, the markets has a less precise expectation about the evolution till

maturity and the 30-days IV curves are slightly shifted up with respect to the 15-days ones. Moreover, the 15-days IV curves are much more convex indicating that extreme events have a more relevant impact in the short-term. Finally, as expected, the minimum value of the IV curve for the 15-day time-to-maturity options is attained much closer to 1. Clearly, it corresponds to the idea of a smaller expected growth of the underlying in the short-term future.

Considering to keep constant (i) and (iii), we can notice that the evolution of the IV curves in time describes how the markets feel the spread of the pandemic through the world. In December only the European markets show huge turmoils which was caused probably by the Brexit event. The IV curves of January and February are, in general, normal—they assume the classical form without a noticeable convexity or a pronounced shift up. In March, almost all markets and stocks react to the first news of the pandemic spread and the IV curves shift up. Jointly, the minimum of the IV curves moves to higher moneyness indicating that the markets bet on a quick recover from the recent fall. In April, it appears clear to everybody that the pandemic will not be a quick and painless event. Therefore, the IV curves shift up and some very low moneyness are not even quoted anymore, while some very high moneyness option are quoted allowing investors to bet for a recover. In May and June the situation stabilizes meaning that the markets understand that the pandemic will become part of everybody daily life for the next months. Indeed, the general uncertainty remains high but more extreme events are considered less probable.

Considering to keep constant (ii) and (iii), we can notice that the different countries and the different stocks react in distinct ways to the spread of the pandemic. The EuroStoxx50 highlights great turmoil due to both Brexit and the pandemic. The European countries were the first hit by the pandemic and the first to implement restriction to the population that had a relevant impact on the economy and, then, on the financial markets as well. This is highlighted also by the analysis on DAX and, even more, by the FTSEMib. Indeed, both countries suffered the effects of the Brexit and of the pandemic and the IV curves show somehow the difficulties of these economies to face such unexpected and tragic events. S&P500 shows only a marginal impact on the Brexit event while the pandemic spread greatly affects the option IV curves. Still, the impact of the pandemic is not so evident as for the European markets, probably because of the size of the economy itself and for the fact that the US faced the pandemic with a little delay with respect to Europe and, thus, US was able to face and to react to the pandemic following the path showed by the European countries. The HangSeng market shows a remarkable stability throughout the analyzed period. The Asian economies were able to face the pandemic efficiently, probably due to the fact that the populations were already used to personal protection equipment and to social distancing. Indeed, the IV curves of HangSeng show some turbulence only in March, but, generally, they never shift up or change substantially they convexity. As far as the stocks are concerned, we notice great differences between the ones that somehow benefit from the pandemic and the ones that suffer huge losses during the lockdown period. The food and beverage sector, represented by Nestlé, and the consumer sector, represented by Amazon and Apple, actually benefit from the lockdown and from the pandemic. Their IV curves never show a particular concern regarding the next future of these sectors. Contrary, the market bet for an increase of the profit of these sectors and, then, of the values of the underlying

stocks. The same applies to the pharmaceutical sector, represented by Bayer, whose IV curves shift up and jointly to the right, meaning that the uncertainty was only about how much extra profit this sector would do. A completely opposite analysis applies to the travel sector, represented by Boeing, that highlights the most dramatic effect that the pandemic had on the economy. Indeed, the spread of the pandemic completely stopped the economical activity of this sector that did not have the possibility to diversify own business and therefore suffered the most from the pandemic spread. The IV curves made a huge shift up in the first months of the pandemic and they still show the hope for some huge recovery which is definitely abandoned in May and June when it is clear that the movement restriction will stay in place for a long period.

To summarize the main results, we can say that the US index suffered a lot at the beginning of the pandemic but also recovered quickly and almost completely. The Asian index contained the losses but was not able to fully recover. The European and German indexes followed a similar path, while the Italian index (a relatively small economy) suffered a huge loss and did not recover fast. The behaviour and the feeling of the markets as we can read from the options written on these stocks can be summarized as follows.

- The expectations remained mainly positive throughout the whole considered period.
- The uncertainty increased during March, April and May. Later, the effectiveness of the restriction measures implemented by most of the governments of the world somehow reassure the market on the overall stability of the system and in June the shape of the IV curves look like in the pre-pandemic period.
- The convexity of the curves changes during the pandemic but, in general, not relevantly.

The stock options show more clearly the fast reaction of the stock value to the spread of the pandemic. Moreover, such impact is highly differentiated according to the industry sector to which the company belongs. The results of the analysis conducted on the stock options can be summarized as follows.

- Apple, Amazon, Bayer and, partially, Nestlé experienced a fast fall when the pandemic spread in Europe, but the nature of their business (based on online consumption, medicines and basic consumption as food and beverage) made possible a quick recover. The options written on these stocks never shown a pessimistic view of the market and the increase of the uncertainty remained relatively small. On the contrary, especially Amazon, Apple and Bayer seems to benefit from the situation as the number of online transaction increased, the demand for high-tech instruments expanded, and, of course, the expectation of the profit of the pharmaceutical sector grown as well.
- Boeing represents the sector that probably suffered most from the pandemic. The stock value collapsed and did not recover. The options' IV experienced huge shocks, highlighted great uncertainty, and in some period shown a deep pessi-

mism. Moreover, it is probably the only sector that in June did not recover any appearance of stability.

The choice to use and compare the options that expire in all months (even the ones relatively less liquid, i.e. not only the quarterly options) and to observe them 15 and 30 days before the maturity, allows us the following further considerations.

- The option market, even suffering relevant shocks, remained liquid and able to quote a wide range of strikes for all indexes and stocks.
- The comparison of the observation 15 days and 30 days before the maturity appear suitable to catch the intra-month expectation. Indeed, in general, the 30 days observation highlight more optimism and more uncertainty, while the 15 days estimations are typically more pessimistic and show less uncertainty because the negative trend in the months of the pandemic does not induce the idea of a fast recover.

This analysis could be extended to study the spread of the pandemic more in detail in the US and, therefore, to compare the reactions of the markets to different type of social restrictions. Moreover, it would be possible to analyse the behaviour of the option IV also in the next months, i.e. during the next pandemic waves that shook the world.

A Estimating IV curve for index options with 30-day time-to-maturity

In Figs. 12, 13, 14, 15 and 16, we show the results for the S&P500, the Hang-Seng, the EuroStoxx50, the DAX and the FTSEMib, respectively, considering the options 30 days before own maturity.

In Tables 11, 12, 13, 14, and 15, we report some statistics of the returns of the underlying immediately before and after the days in which we observe and study the options.

A.1 S&P500

Taking into consideration the estimates related to S&P500 with 30-day maturity depicted in Fig. 12, we can notice that in the pre-pandemic period the market expectations on the evolution of the underlying were an increase in the value of the Index by around 5% in all three months (Dec, Jan and Feb) as highlighted by the position of the minimum of the IV curves, cf. Fig. 12 (down right). The market of the options on S&P500 is extremely liquid and, even in December, the options with moneyness of 0.1 (meaning a loss of 90%) were quoted. The pandemic strikes the markets around the middle of February, cf. Fig. 12 (up left) but a slight recovery of the index around the beginning of March makes the operator hope in a positive jump. Indeed, the minimum of the curve of April move to the right, cf. Fig. 12 (down left, light blue curve), but the whole curve shifts up showing

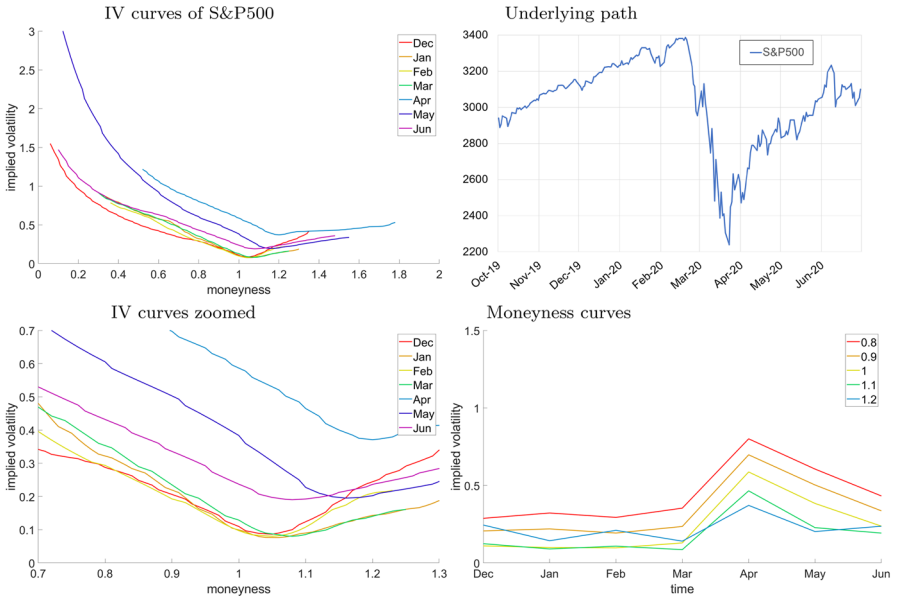


Fig. 12 Estimations of the IV curve of the S&P500 30 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

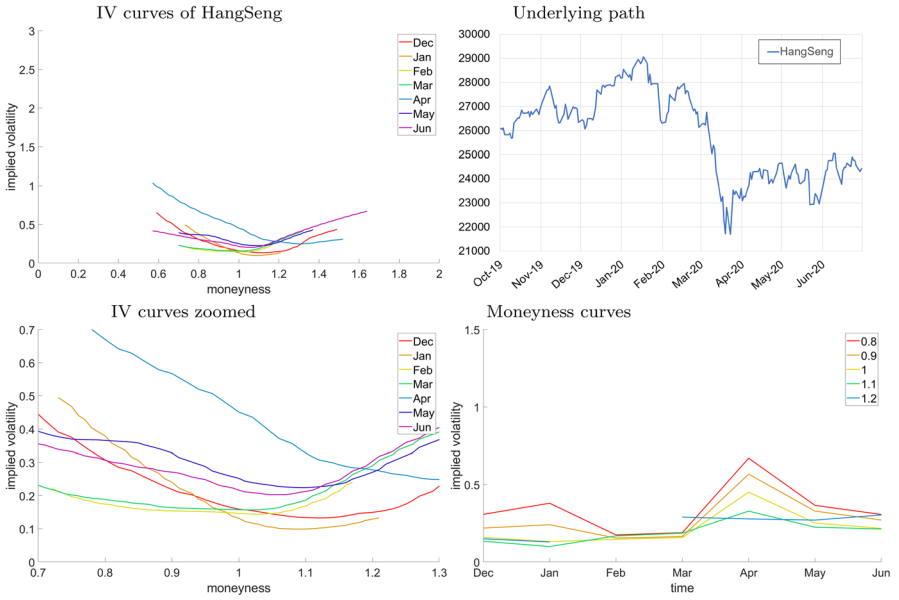


Fig. 13 Estimations of the IV curve of the HangSeng 30 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

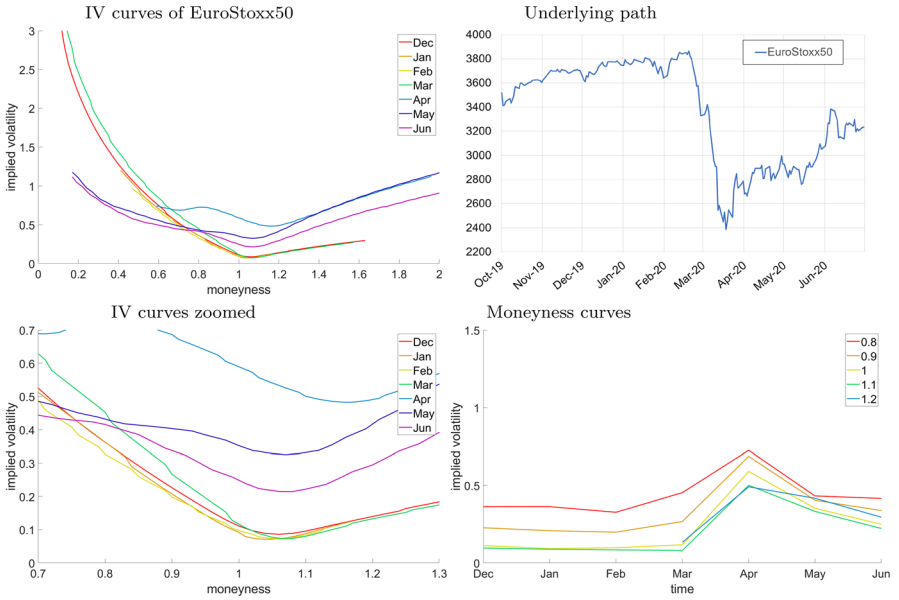


Fig. 14 Estimations of the IV curve of the EuroStoxx50 30 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

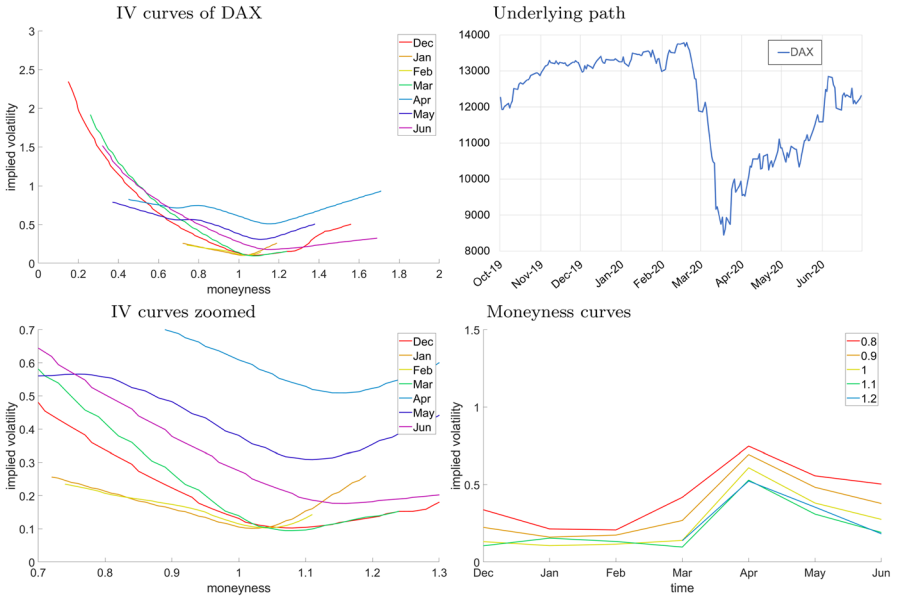


Fig. 15 Estimations of the IV curve of the DAX 30 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

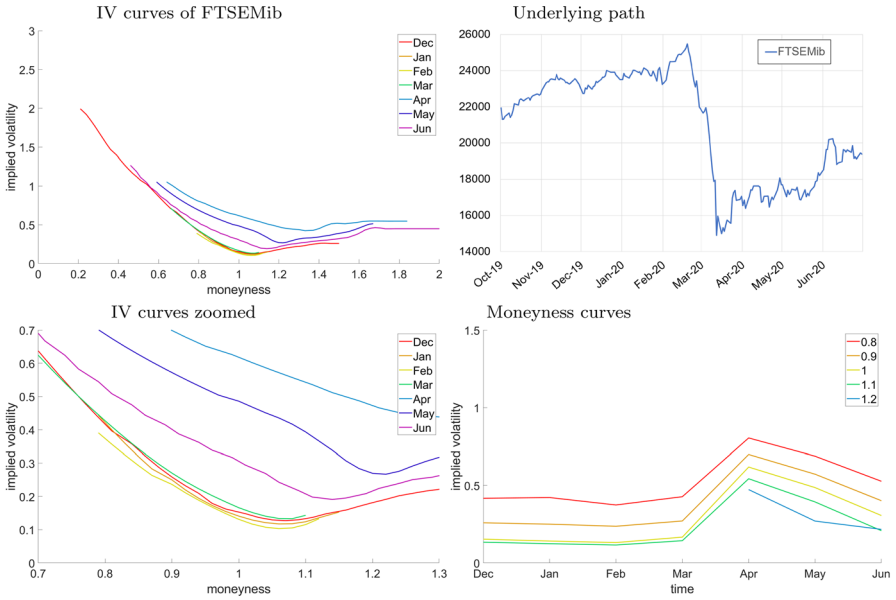


Fig. 16 Estimations of the IV curve of the FTSEMib 30 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

a huge increase of the uncertainty. In the period of the pandemic, the options listed for May have an IV curve much more convex than that of the options listed in June. It can be seen that the smile of May has as a rising point around the value of moneyness of 1.16, therefore an expectation of a rise of 16%, compared to the month of June in which the expectation is around the value of 1.08, i.e. an increase of 8%. In Fig. 12 (down right), we represent the evolution of IVs for different level of moneyness, i.e. from 0.8 to 1.2. We notice that, extreme events of a loss of 10% and 20% were the most probable along

Table 11 Average return of the S&P500 in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 30 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	20-Nov	1.03	3.12	0.63	− 0.14	2.49
Jan-20	20-Dec	2.39	3.37	0.49	− 0.26	2.53
Feb-20	20-Jan	2.57	1.22	1.69	− 0.61	3.78
Mar-20	20-Feb	0.82	− 31.67	71.33	0.19	3.07
Apr-20	20-Mar	− 22.46	22.48	36.43	0.66	3.04
May-20	20-Apr	5.99	3.53	8.14	− 0.14	2.23
Jun-20	20-May	4.32	4.84	7.49	− 2.46	12.07

The observed days are 30 calendar days before the corresponding maturity

Table 12 Average return of the HangSeng in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 30 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	20-Nov	− 2.89	3.39	3.05	0.12	3.41
Jan-20	20-Dec	5.18	3.32	1.32	0.55	2.87
Feb-20	20-Jan	2.02	− 4.40	5.25	− 0.33	3.08
Mar-20	20-Feb	0.42	− 17.40	13.76	0.50	3.83
Apr-20	20-Mar	− 12.78	6.69	13.60	0.00	3.83
May-20	20-Apr	2.45	0.24	5.48	− 1.24	5.33
Jun-20	20-May	1.09	0.27	9.09	− 1.13	6.27

The observed days are 30 calendar days before the corresponding maturity

Table 13 Average return of the EuroStoxx50 in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 30 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	20-Nov	− 0.13	1.50	1.47	− 0.68	5.41
Jan-20	20-Dec	2.28	0.59	0.71	0.72	3.76
Feb-20	20-Jan	1.24	0.99	2.85	− 0.64	4.10
Mar-20	20-Feb	0.46	− 33.34	41.30	− 0.95	4.27
Apr-20	20-Mar	− 21.15	14.17	24.99	0.66	4.78
May-20	20-Apr	4.06	− 0.24	13.02	0.03	3.20
Jun-20	20-May	3.47	10.45	10.35	− 0.39	3.77

The observed days are 30 calendar days before the corresponding maturity

Table 14 Average return of the DAX in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 30 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	20-Nov	− 0.17	0.41	1.39	− 0.88	4.87
Jan-20	20-Dec	1.16	1.73	0.96	− 0.03	3.41
Feb-20	20-Jan	3.21	0.98	3.07	− 0.70	3.93
Mar-20	20-Feb	0.66	− 34.65	36.14	− 1.12	4.80
Apr-20	20-Mar	− 22.64	19.57	29.26	1.09	6.09
May-20	20-Apr	5.96	3.74	14.86	0.16	3.11
Jun-20	20-May	5.82	9.42	11.50	− 0.10	3.29

The observed days are 30 calendar days before the corresponding maturity

Table 15 Average return of the FTSEMib in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 30 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	20-Nov	− 0.09	1.53	1.66	− 1.10	5.71
Jan-20	20-Dec	3.54	0.00	1.18	0.44	2.44
Feb-20	20-Jan	1.79	5.09	4.02	− 0.47	3.35
Mar-20	20-Feb	2.41	− 37.27	65.45	− 1.36	6.49
Apr-20	20-Mar	− 24.37	8.47	20.70	0.96	6.46
May-20	20-Apr	0.15	− 0.17	11.14	− 0.23	2.15
Jun-20	20-May	0.31	13.20	9.61	− 0.85	5.01

The observed days are 30 calendar days before the corresponding maturity

all the months, in particular from March on, while the opposite extreme events, i.e. an increase of 10% and 20%, have more or less the same IV of the negative events, while during the pandemic becomes the ones with the lowest IV.

A.2 HangSeng

In Fig. 13, we consider options written on the HangSeng. In the pre-pandemic months it can be seen how the IV smiles for December and January assumes a similar shape, with an expectation of an increase in the underlying of about 10%, cf. Fig. 13 (down left). Regarding the quotation range, in December there are options with wider moneyness, the market contemplates more extreme events (possible 40% loss or possible 50% gain at maturity). The IV curves of February and March assume a different shape since the smile is less convex and the minimum shows an expectation of an increase of the Index of around 5%, the range of the moneyness is almost similar to the situation in January. The IV curves of April shows fully the impact of the pandemic. The whole curve shifts up because of the increase of the general uncertainty. However, jointly the market hope in a fast recover because the minimum of the IV curve is at moneyness 1.30, i.e. an increase of about 30% at maturity. From May, the market somehow returns to a pre-pandemic situation: the minimum of the curves of Mat and June is around moneyness 1.05, but the whole curve is shifted up and more convex than before, meaning that the market expects further turmoil in the near future. Considering Fig. 13 (down right), we notice that the moneyness 1.2 was not quoted at all for February when the pandemic started in China.

A.3 EuroStoxx50

The options written on the EuroStoxx50 Index with maturity 30 days are analysed in Fig. 14. We observe that the IV smiles generated in the pre-pandemic months are

similar in terms of convexity and minimum point, i.e. an expectation of 3–4% increase of the Index value at maturity. The moneyness range of December is very large with values up to 1.6 and till 0.2, again because of the Brexit event whose effect was quite unpredictable. When the pandemic strikes in Europe, the market reacts first with the options quoted for April, meaning that the news from China did not spread any alarm until the beginning of March. Moreover, the options quoted for March, i.e. at the beginning of February, show an expected increase of around 7%. The market probably expects a fast recover of the situation and did not expect that the virus would spread so far from China and so fast. The smile of the IV for April is quite another matter, where the whole curve shifts up and generates two smiles, two polarizations around the moneyness values of 0.7 and 1.18. This particular feature illustrates how the market expected as more probable either a loss of 30% or an increase of 18% at maturity. Immediately after the Index will experience a fall from around 3400 points to 2400 points: − 30%, cf. Fig. 14 (up right). The IV curves in the subsequent months assume a similar smile pattern. Indeed, for the months of May and June, the IV smiles are still higher than the pre-pandemic situation but the minimum is always in the region of moneyness greater than one showing a positive expectation around 7–8% at maturity. The moneyness curves, cf. Fig. 14 (down right), shows that for January and February the moneyness 1.2 was not quoted. The general increase of the IV from April is evident, but we notice a quite optimistic view with respect to the previous analysis since the IV after April are quite similar for all moneyness values.

A.4 DAX

In Fig. 15, we analyse the options written on DAX, again with 30 days of maturity. In particular, Fig. 15 (up left) shows that in December the quotation range is very wide with moneyness from 0.2 to more than 1.5 and the IV smile is very convex (because of the Brexit event) with a minimum point around the moneyness 1.05. The smile in January and February is substantially different once the Brexit is not a threat anymore and the markets adjust to the new situation. Indeed, the convexity in these cases is significantly lower and, comparing with December, the moneyness range is much smaller. Market expectations are an increase of 2–3% for the DAX Index. As for the EuroStoxx50, the first news from China at the beginning of February do not influence the quotation of the options expiring in March in terms of average increase, but increases the convexity in particular for the moneyness much lower than one. For April we notice almost the same double smile that we observed for the EuroStoxx50. The IV curve shows a huge uncertainty of the market operators. As a matter of fact, the Index falls in the second half of February from around 13,800 points to 12000, i.e. − 13%, cf. Fig. 15 (up right). However, after the collapse in the first half of March, the Index recovers before the quotation at the end of March of the options expiring in May. Such recover induces some optimism and the IV curve of May is relatively high but with a minimum at 1.12. In June, on the other hand, a situation similar to December re-emerges in terms of convexity and price

range, although over the value of moneyness 1.3 the options for the same moneyness assume a lower IV. The graph representing the IVs associated with the various moneyness shows how the peak occurs in April, coinciding with the crisis situation.

A.5 FTSEMib

The results about the options written on the FTSEMib with a maturity of 30 days are in Fig. 16. We notice that in the pre-pandemic months the shape of the IV smiles are almost the same: both in terms of convexity and in terms of minimum point. Indeed, in December, January and February, the market expected an increase of the value of the FTSEMib at maturity around 6–7%. Again, the Brexit event in December induces a wider range of quoted moneyness. The virus arrived in Italy at the end of February but the IV curve at the end of February for options expiring in March For the month of March, the IV smile that was formed follows the trend and expectations of the previous month. In full lockdown does not show any touchable difference with respect to previous months. Also in these case the news from China and the first infections in the North part of Italy did not influence the option market. The lockdown was declared around the 10th of March and at the end of March the Index fell to 15000 points from the peak of 25000 of the middle of February, i.e. − 40%, cf. Fig. 16 (up right). Such collapse reflects in the IV curve quoted at the end of March for the options expiring in April even if some signs of recovery in the second half of March induces also some optimism. Indeed, the IV curve of April is definitely shifted up but the minimum is around moneyness 1.35 showing an expectation of a huge and fast recover. Also from the options expiring in May we observe that the market expected an increase in the value of the Index around 20% and for the month of June an increase of 14%. Observing Fig. 16 (down right), we notice that moneyness 1.2 was not quoted before April. Moreover, the IV for lower moneyness values are always higher. Indeed, the main expectation was positive, but the overall sentiment was uncertain and prudent.

B Estimating IV curve for stock options with 30-day time-to-maturity

In Figs. 17, 18, 19, 20 and 21, we show the results for Apple, Amazon, Nestlé, Boeing and Bayer, respectively, considering the options 30 days before own maturity.

In Tables 16, 17, 18, 19 and 20, we report some statistics of the returns of the underlying immediately before and after the days in which we observe and study the options.

B.1 Apple

Starting from the analysis of the options referred to Apple, in the pre-pandemic months (December, January and February), Fig. 17 (up left) shows a similar trend of the IV smile, except for the call OTM and put ITM options with lower moneyness characterized by high volatility, this indicates more probability of extreme events

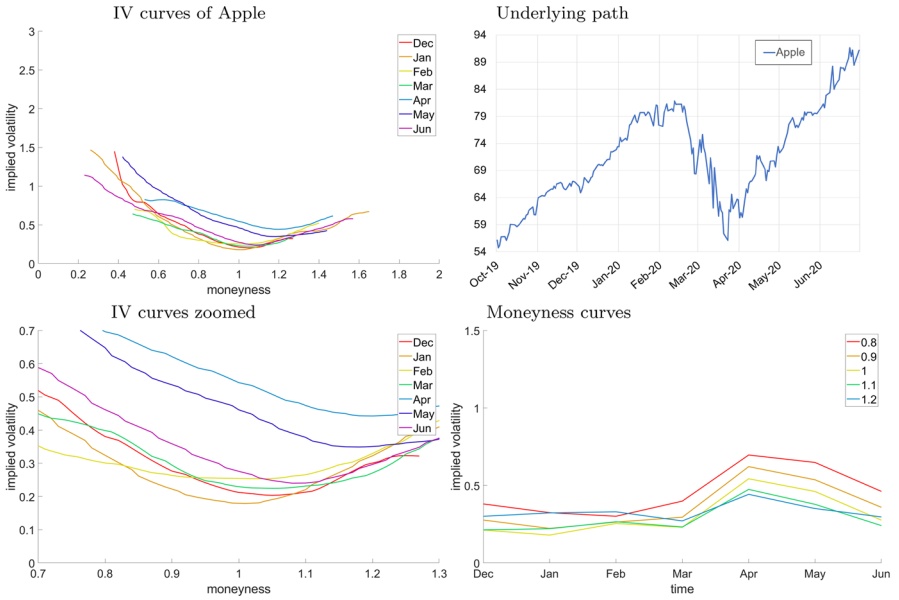


Fig. 17 Estimations of the IV curve of the Apple 30 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

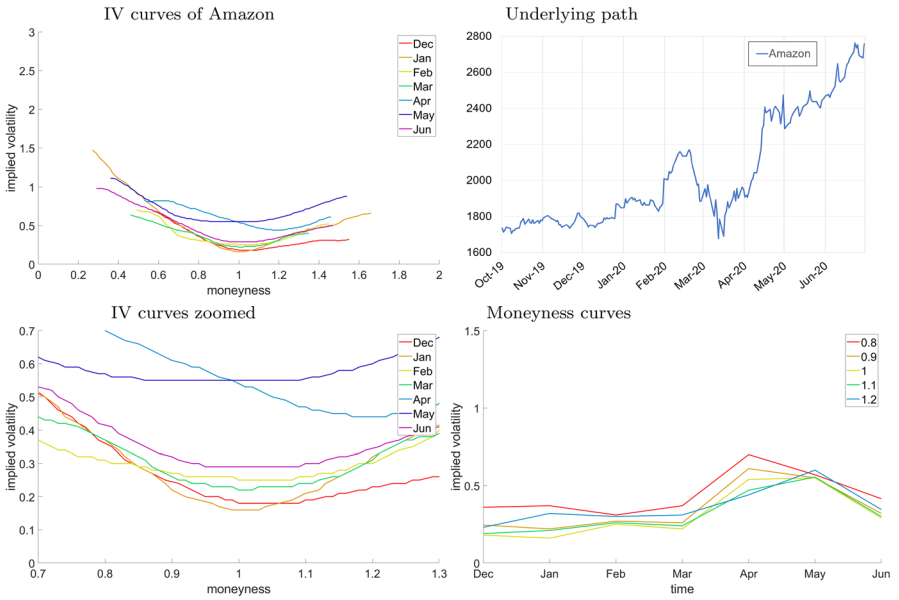


Fig. 18 Estimations of the IV curve of the Amazon 30 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

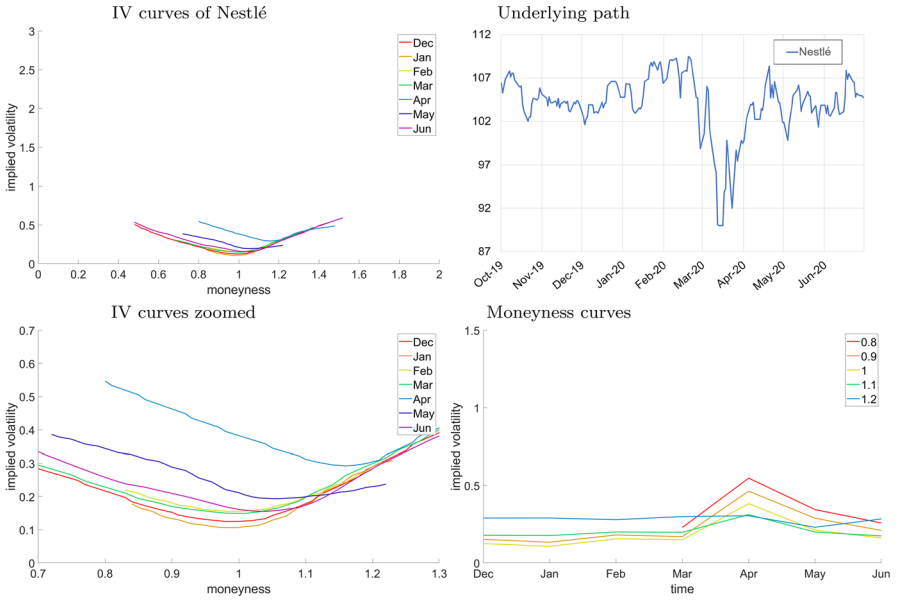


Fig. 19 Estimations of the IV curve of the Nestlé 30 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

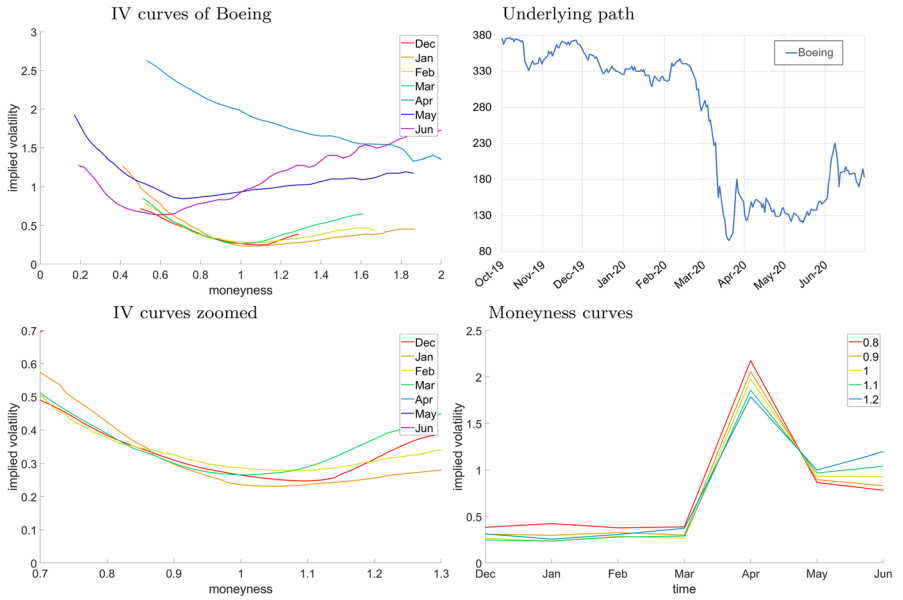


Fig. 20 Estimations of the IV curve of the Boeing 30 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

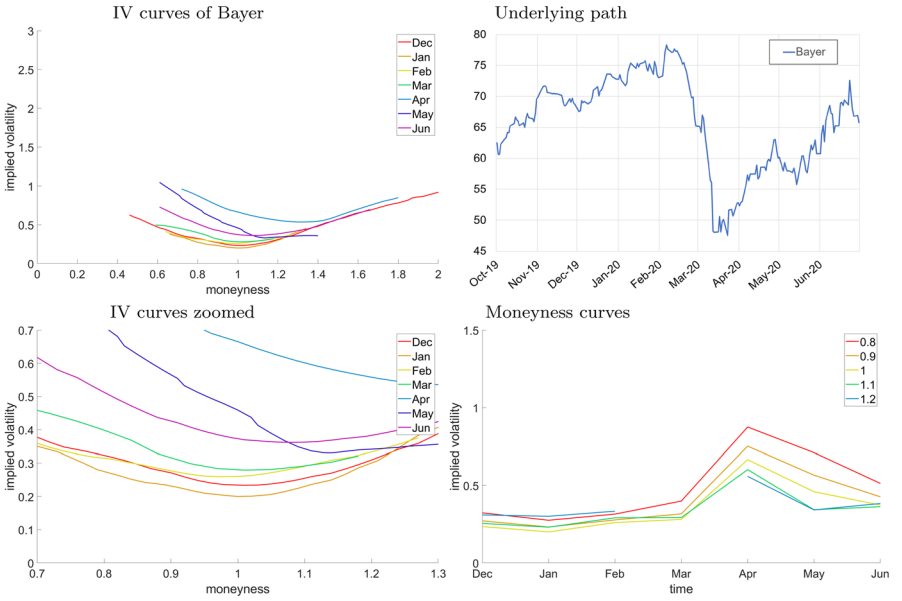


Fig. 21 Estimations of the IV curve of the Bayer 30 days before maturity (up left), underlying's value (up right), focus on IV curves for moneyness around 1 (down left) and moneyness curves (down right)

related to a loss in the value of the security perceived 30 days before the maturity date. Market expectations for December were bullish expecting an increase of around 5% of the underlying, while for January and February the expectations were for an overall stability. Moving to the months when the first signs of the spread of the virus in Europe appeared, for March market expectations reflect the previous months as around February 20, the day of observation, the pandemic had not yet broken out, the smiles assume convexity and range of quotation as can be seen from

Table 16 Average return of Apple in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 30 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	20-Nov	2.31	6.39	3.22	− 0.11	2.05
Jan-20	20-Dec	3.22	14.06	2.99	0.10	2.06
Feb-20	20-Jan	6.31	0.08	8.65	− 0.40	3.97
Mar-20	20-Feb	− 1.51	− 28.43	99.44	0.45	3.20
Apr-20	20-Mar	− 20.69	20.80	44.21	0.66	3.59
May-20	20-Apr	5.51	13.08	8.83	− 0.21	2.01
Jun-20	20-May	6.19	10.18	7.60	− 1.21	7.05

The observed days are 30 calendar days before the corresponding maturity

Table 17 Average return of Amazon in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 30 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	20-Nov	− 2.80	2.68	1.81	0.19	1.955
Jan-20	20-Dec	1.99	4.38	4.70	1.93	7.64
Feb-20	20-Jan	− 2.01	15.60	9.09	2.40	11.25
Mar-20	20-Feb	5.02	− 14.26	44.33	0.35	2.40
Apr-20	20-Mar	− 2.89	29.66	19.20	0.27	2.09
May-20	20-Apr	19.82	2.33	15.09	− 1.73	7.90
Jun-20	20-May	6.24	6.25	5.62	− 0.66	4.04

The observed days are 30 calendar days before the corresponding maturity

Table 18 Average return of Nestlé in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 30 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	20-Nov	− 0.52	− 0.17	1.33	0.09	2.83
Jan-20	20-Dec	2.12	0.75	2.06	0.07	3.54
Feb-20	20-Jan	0.60	0.80	2.99	0.20	4.26
Mar-20	20-Feb	0.24	− 10.41	20.94	0.33	3.55
Apr-20	20-Mar	− 3.12	10.59	10.46	− 1.80	7.74
May-20	20-Apr	4.23	− 2.69	6.39	− 0.55	2.42
Jun-20	20-May	2.08	1.83	4.32	0.55	3.02

The observed days are 30 calendar days before the corresponding maturity

Table 19 Average return of Boeing in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 30 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	20-Nov	4.72	− 10.09	5.88	− 0.63	4.37
Jan-20	20-Dec	− 7.37	− 1.17	4.82	0.46	3.13
Feb-20	20-Jan	− 2.87	4.54	9.03	0.34	2.70
Mar-20	20-Feb	− 1.51	− 71.75	187.34	− 0.69	3.16
Apr-20	20-Mar	− 63.78	51.15	303.72	0.76	2.50
May-20	20-Apr	− 3.47	− 9.17	59.99	1.18	4.78
Jun-20	20-May	9.40	44.23	126.24	− 0.55	4.35

The observed days are 30 calendar days before the corresponding maturity

Table 20 Average return of Bayer in the 10 trading days before the observed day for each maturity, and the average return, variance, skewness, and kurtosis of the daily returns between the observed day and the maturity day

Maturity	Observed day	Previous 10 days	Next 30 days			
		Avg ret (%)	Avg ret (%)	Var (%)	Skew	Kurt
Dec-19	20-Nov	− 3.57	2.69	3.68	0.06	2.59
Jan-20	20-Dec	3.35	5.59	3.02	0.96	4.23
Feb-20	20-Jan	5.24	− 0.38	5.86	0.58	2.68
Mar-20	20-Feb	− 4.64	− 32.90	46.51	− 1.09	5.78
Apr-20	20-Mar	− 21.45	16.97	23.25	0.60	4.94
May-20	20-Apr	5.04	3.05	16.89	1.06	4.33
Jun-20	20-May	1.80	15.21	29.34	0.53	2.71

The observed days are 30 calendar days before the corresponding maturity

Fig. 17 (up left). The options listed in March, given that towards the end of February the collapse in the value of the stock began, going from a value of 81 Dollars to 69 Dollars (i.e. loss of around 15%), tracing a situation of uncertainty, as can be note in Fig. 17 (down right). Still, the market expects a fast recover having expectation for the end of March of a growth of 5%. Such expectation does not realize at the end of March when the value is approximately 64 Dollars, cf. Fig. 17 (up left). At the end of March, the quotation of the options expiring in April is again positive because the market saw the bottom point in the middle of March and bets for a huge positive jump of around 20%. However, the IV smile highlights a relevant uncertainty because the curve of April shifts up, cf. Fig. 17 (up right and down right). During April the recover realizes and the options expiring in May show again a strong optimism. The options expiring in June show that the markets feels that the situation reached and equilibrium and stabilised. The moneyness curves in Fig. 17 (down left) shows that during the pandemic the moneyness below 1 have always a higher associated IV.

B.2 Amazon

Analyzing Amazon, a company whose stock has benefited from the restrictions imposed around the world as can be seen from Fig. 18 (up right), it can be seen that the market expectation in the pre-pandemic months (December, January and February) is that the value of the stock will remain relatively stable. In terms of convexity and quotation range, higher values are recorded in February, this may be caused as a consequence of the first rumours regarding the virus that spread in China in that period. The months related to the pandemic (March and April), the cause of the economic crisis, it is noted that for the prices of March there is no deviation from the expectations of the previous months, given that on the observation day around February 20 the value of the share is it stood at around 2000 Dollars, therefore an increase compared to the previous months as can be seen from Fig. 18 (up left). Quotes for the month of March in which the outbreak of the pandemic causes the

value of the Amazon stock to drop by 15% in the end of February, bringing market expectations to an increase in the value of about 20% at maturity, is quite another case, in fact, as can be seen in Fig. 18 (up left), a further fall in the value of the stock took place and then returned to values similar to the day of observation. As can be seen from Fig. 18 (down left) of moneyness, the most extreme events were considered more likely this month than in other periods. As for the other stocks, the end of March is the most uncertain period and this reflects in the quotation of the options of April. Indeed, the IV curve in April highlights a relevant optimism (the curve shifts right) and also huge uncertainty (the curve shifts up). During April the stock value recovers but at the IV curve of May shows still a lot of uncertainty. In May the situation stabilizes and the options for June makes clear that the markets feel to be as in the pre-pandemic period. It can be said that Amazon has benefited from the restrictions that took place in Europe as the value of the stock from the mid of March has started to grow significantly, even higher than the pre-crisis levels.

B.3 Nestlé

Nestlé, although it is a sector (food and beverage) that should not have been particularly affected by the crisis, noted a stable trend in market expectations in the months before the spread of the virus in Europe, as the rising point of the IV smile stands at ATM values as can be seen from Fig. 19 (up left). The exception concerns the quote range which is wider for the December options where possible losses of more than 50% of the value of the stock on the observation day were contemplated (i.e. value 0.5 of moneyness) or gains of 35% (i.e. 1.35 value of moneyness) as can be seen from Fig. 19 (up left). The month of February, although the period in which the virus begins to spread in Europe and the first restrictions begin to occur, follows the trend of the previous months both for expectations and for the quotation interval given that the observation day turns out to be the February 20. On the other hand, in March, after the crisis has occurred and therefore the value of the security is observed 30 days before the expiry date, there is an expectation of an increase in the underlying at maturity of approximately 18% for the options expiring in April, it can be seen from Fig. 19 (down left) how more extreme events were perceived as more likely in March than in the previous and subsequent months. Expectations at maturity in March are not met, given that more than a rise there is a collapse in the value of the stock going from 107 Euro to 92 within a month (i.e. loss of 14%). In the period following the peak of the pandemic (April and May), the May prices follow the trend of the April prices as can be seen in Fig. 19 (up left), but with the same moneyness there are IV minors and the expectation of the market is bullish, but by 5% about 30 days before the expiry date. The June prices reflect a trend towards stability, as uncertainty is reduced and the expectation on the movement of the underlying reflects the previous month. A peculiarity is the greater range of prices that reflects the December prices, as can be seen from

Fig. 19 (up left), perceiving as a return to pre-crisis levels also inherent to the value of the underlying.

B.4 Boeing

Figure 20 shows the estimates regarding the options on Boeing stock, a sector hugely affected by the pandemic. We can see a stability in the months before the global spread of the virus (December, January and February) and in March, given that the observation day concerned on February 20, the period in which the first infections in Europe began, where the expectation of the underlying almost stable around 1 value of moneyness, except for December when the market expected a rise of 10% 30 days before the date expiry date. After the collapse of the value from mid-March, from a value of 340 points to 90 (i.e. loss of more than 70% of the value), due to the stringent measures to airplanes circulation, the estimation that considers the options expiring in April are characterized by a very strong uncertainty associated with a strong optimism, cf. Fig. 20 (up left). Indeed, the minimum point of the IV curve for April is around 1.9 meaning that the market expectes a jump of 90%. This expectation does not realize. During April and May the value of the stock remains close to the minimum value and the pessimism invades the market as shown by IV curves of May and June. Their minimum points collapse around value 0.5 and the whole curves strongly shift to the left. The moneyness curves n Fig. 20 (down right) confirm the huge turmoil in April and an interesting inversion of the curves in May and June.

B.5 Bayer

The options relating to Bayer, belonging to the pharmaceutical sector that should not have been affected by the crisis, are analysed in Fig. 21. The months before the outbreak of the pandemic (December, January and February) show stability both for the shape of the IV smile and for the ascent point around ATM values. Peculiarity is the high quotation range in December in which possible losses of 50% of the value of the stock were perceived (i.e. moneyness value of 0.5) or possible gains of over 100% (i.e. moneyness values of over 2). A similar trend can be recorded for the March prices, as the observations referred to February 20 and therefore the first infections in Europe were recorded but the strength of the pandemic was not clear yet. For the month of April, after the start of the crisis in mid-February coinciding with the collapse of the stock value from 75 Euro to 47 (i.e. a loss of around 37% of the value), as shown in Fig. 21 (up right), the IV curve shifts up indicating an increase of the uncertainty and shifts right meaning the market remains bullish. The same intuition is confirmed observing Fig. 21 (down right), where in April extreme events linked in particular to losses of 10% or 20% were more likely than in previous months. In the period following the outbreak of the pandemic (April and May) there is a general level of lower

uncertainty, in the month of May the market expectation was positioned on a rise of 12% 30 days before the expiry date as the market expected a rebound in the value of the stock. The options expiring in June reflect trends similar to the pre-crisis months, although the level of uncertainty is globally higher, with an expectation of an increase of around 7–8%, confirmed by the performance of the underlying as can be seen from the Fig. 21 (up right).

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Declarations

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