

PROCEEDINGS OF THE FIFTEENTH
**MARCEL GROSSMANN
MEETING ON
GENERAL RELATIVITY**



Editors

**Elia S. Battistelli
Robert T. Jantzen
Remo Ruffini**

THE FIFTEENTH MARCEL GROSSMANN MEETING

On Recent Developments in Theoretical and Experimental
General Relativity, Astrophysics and Relativistic Field Theories

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**On Recent Developments in Theoretical and Experimental
General Relativity, Astrophysics and Relativistic Field Theories**

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Black hole thermodynamics and Gravity's Rainbow

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We consider the effects of rotations on the calculation of some thermodynamical quantities like the free energy, internal energy and entropy. In ordinary gravity, when we evaluate the density of states of a scalar field close to a black hole horizon, we obtain a divergent result which can be kept under control with the help of some standard regularization and renormalization processes. We show that when we use the Gravity's Rainbow approach such regularization/renormalization processes can be avoided. A comparison between the calculation done in an inertial frame and in a comoving frame is presented.

1. Introduction

Gravity's Rainbow (GRw) is a modification of the space-time close to the Planck scale. It has been introduced for the first time by Magueijo and Smolin¹. Basically one defines two unknown functions $g_1(E/E_P)$ and $g_2(E/E_P)$ having the following property

$$\lim_{E/E_P \rightarrow 0} g_1(E/E_P) = 1 \quad \text{and} \quad \lim_{E/E_P \rightarrow 0} g_2(E/E_P) = 1. \quad (1)$$

This property guarantees the recovery of ordinary General Relativity when sub-planckian physics is involved. In this formalism, the Einstein's field equations are replaced by a one parameter family of equations

$$G_{\mu\nu}(E) = 8\pi G(E) T_{\mu\nu}(E) + g_{\mu\nu} \Lambda(E), \quad (2)$$

where $G(E)$ is an energy dependent Newton's constant and $\Lambda(E)$ is an energy dependent cosmological constant, respectively. They are defined so that $G(0)$ is the physical Newton's constant and $\Lambda(0)$ is the usual cosmological constant. In this context, the Schwarzschild solution of (2) becomes

$$ds^2(E) = - \left(1 - \frac{2MG(0)}{r} \right) \frac{dt^2}{g_1^2(E/E_P)} + \frac{dr^2}{\left(1 - \frac{2MG(0)}{r} \right) g_2^2(E/E_P)} + \frac{r^2}{g_2^2(E/E_P)} d\Omega^2. \quad (3)$$

An immediate generalization of the metric (3) is represented by the following line element

$$ds^2(E) = - \left(1 - \frac{b(r)}{r} \right) \frac{\exp(-2\Phi(r))}{g_1^2(E/E_P)} dt^2 + \frac{dr^2}{\left(1 - \frac{b(r)}{r} \right) g_2^2(E/E_P)} + \frac{r^2}{g_2^2(E/E_P)} d\Omega^2. \quad (4)$$

The function $b(r)$ will be referred to as the "shape function" and it may be thought of as specifying the shape of the spatial slices. The location of the horizon is

determined by the equation $b(r_H) = r_H$. On the other hand, $\Phi(r)$ will be referred to as the “redshift function” and describes how far the total gravitational redshift deviates from that implied by the shape function. The line element (4) describes any spherically symmetric space-time with a horizon: by definition, this is a black hole distorted by GRw. Note that a metric of the form

$$ds^2(E) = -\frac{\exp(-2\Phi(r))}{g_1^2(E/E_P)} dt^2 + \frac{dr^2}{\left(1 - \frac{b(r)}{r}\right) g_2^2(E/E_P)} + \frac{r^2}{g_2^2(E/E_P)} d\Omega^2, \quad (5)$$

describes a traversable wormhole modified by GRw if $\exp(-2\Phi(r))$ never vanishes¹⁸. The line element (5) has been used in a series of papers where to avoid any regularization/renormalization scheme which appear in conventional Quantum Field Theory calculations like one loop corrections to classical quantities². On the other hand, the line element (5) has been considered for the computation of black hole entropy³. In this last case, the idea is to avoid to introduce a cut-off of Planckian size known as “brick wall”⁴. The “brick wall” appears when one uses a statistical mechanical approach to explain the famous Bekenstein-Hawking formula^{5,6}

$$S_{BH} = \frac{1}{4} A/l_P^2, \quad (6)$$

relating the entropy of a black hole and its area. Indeed, when one tries to adopt such an approach, one realizes that the density of energy levels of single-particle excitations is divergent near the horizon. Of course, several attempts have been done to avoid the introduction of the *brick wall*. For instance, without modifying gravity at any scale, it has been suggested that the *brick wall* could be absorbed in a renormalization of Newton’s constant⁷⁻⁹, while other authors approached the problem of the divergent brick wall using Pauli-Villars regularization¹⁰⁻¹². Other than GRw other proposals have been made in the context of modified gravity. For instance, non-commutative geometry introduces a natural thickness of the horizon replacing the ’t Hooft’s brick wall¹⁷ and *Generalized Uncertainty Principle* (GUP) modifies the Liouville measure¹³⁻¹⁶. Nevertheless we can wonder what happens when one introduces rotations. For instance, one could consider the free energy obtained for a real massless scalar field, rotating with an angular velocity Ω_0 around the z axis in Minkowski space

$$F = \frac{1}{\beta} \sum_m \int_0^\infty dg(E, m) \ln \left(1 - e^{-\beta(E - m\Omega_0)}\right). \quad (7)$$

For this case, it is better to work in cylindrical coordinates

$$ds^2 = -dt^2 + dr^2 + r^2 d\phi^2 + dz^2 \quad (8)$$

and with the help of a WKB approximation it is possible to show that a divergence appear close to the speed-of-light (SOL) surface, defined as the surface where $r = \Omega_0^{-1}$ ¹⁹. This divergence can be taken under control with the help of GRw by

modifying the line element (8) in the following way²⁰

$$ds^2 = -\frac{dt^2}{g_1^2(E/E_P)} + \frac{dr^2}{g_2^2(E/E_P)} + \frac{r^2 d\phi^2}{g_2^2(E/E_P)} + \frac{dz^2}{g_2^2(E/E_P)}, \quad (9)$$

where for simplicity we have replaced $ds^2(E)$ with ds^2 . The same above thermodynamical system can be analyzed on a comoving frame rotating with the same angular velocity Ω_0 . By plugging $\phi' = \phi - \Omega_0 t$, from the line element (3) we obtain

$$ds^2 = -\frac{(1 - \Omega_0^2 r^2)}{g_1^2(E/E_P)} dt^2 + \frac{2\Omega_0 r^2 d\phi' dt}{g_1(E/E_P) g_2(E/E_P)} + \frac{dr^2}{g_2^2(E/E_P)} + \frac{r^2 d\phi'^2}{g_2^2(E/E_P)} + \frac{dz^2}{g_2^2(E/E_P)}. \quad (10)$$

It is immediate to see that in this system of coordinates appears a fictitious horizon located at $r = \Omega_0^{-1}$, namely the SOL surface of the rotating heat bath introduced in (7). Note that a mixing between $g_1(E/E_P)$ and $g_2(E/E_P)$ appears. A similar mixing appears also in a Vaidya spacetime for

$$ds^2 = -\left(1 - \frac{2M(v)G}{r}\right) \frac{dv^2}{g_1^2(E/E_P)} + 2\frac{dv dr}{g_1(E/E_P) g_2(E/E_P)} + \frac{r^2 d\Omega^2}{g_2^2(E/E_P)}, \quad (11)$$

where v is the advanced (ingoing) null coordinate and finally also for the Kerr metric which, in the context of GRw, becomes²²

$$ds^2 = \frac{g_{tt} dt^2}{g_1^2(E/E_P)} + \frac{2g_{t\phi} dt d\phi}{g_1(E/E_P) g_2(E/E_P)} + \frac{g_{\phi\phi} d\phi^2}{g_2^2(E/E_P)} + \frac{g_{rr} dr^2}{g_2^2(E/E_P)} + \frac{g_{\theta\theta} d\theta^2}{g_2^2(E/E_P)}, \quad (12)$$

where

$$g_{tt} = -\frac{\Delta - a^2 \sin^2 \theta}{\Sigma}, \quad g_{t\phi} = -\frac{a \sin^2 \theta (r^2 + a^2 - \Delta)}{\Sigma},$$

$$g_{\phi\phi} = \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta}{\Sigma} \sin^2 \theta, \quad g_{rr} = \frac{\Sigma}{\Delta}, \quad g_{\theta\theta} = \Sigma, \quad (13)$$

and

$$\Delta = r^2 - 2MGr + a^2, \quad \Sigma = r^2 + a^2 \cos^2 \theta. \quad (14)$$

Here M and a are mass and angular momentum per unit mass of the black hole, respectively. Δ vanishes when $r = r_{\pm} = MG \pm \sqrt{(MG)^2 - a^2}$, while g_{tt} vanishes when $r = r_{S\pm} = MG \pm \sqrt{(MG)^2 - a^2 \cos^2 \theta}$: they are not modified by GRw and the outer horizon or simply horizon is located at $r_+ = r_H$. Units in which $\hbar = c = k = 1$ are used throughout the paper.

2. GRw Entropy for the Kerr Black Hole

To discuss the entropy for a Kerr black hole we have two options: we can use a rest observer at infinity (ROI) or we can use a Zero Angular Momentum Observer

(ZAMO)^{19,23}. The ROI frame is described by the line element (12) and the appropriate form of the free energy is the following

$$F = \frac{1}{\beta} \int_0^\infty dn(E) \ln \left(1 - e^{-\beta(E-m\Omega_0)} \right). \tag{15}$$

It is immediate to see that when we use a ROI, the problem of superradiance appears when the free energy (15) is computed in the range $0 < E < m\Omega_0$. On the other hand when a ZAMO is considered, the free energy (15) becomes similar to the one used for a Schwarzschild black hole. Basically this happens because near the horizon the metric becomes

$$ds^2 = -\frac{N^2 dt^2}{g_1^2(E/E_P)} + g_{\phi\phi} \frac{d\phi^2}{g_2^2(E/E_P)} + g_{rr} \frac{dr^2}{g_2^2(E/E_P)} + g_{\theta\theta} \frac{d\theta^2}{g_2^2(E/E_P)} \tag{16}$$

and the mixing between t and ϕ disappears. Moreover when we use a ZAMO frame, the superradiance does not come into play because there is no ergoregion. Indeed since we have defined

$$N^2 = g_{tt} - \frac{g_{t\phi}^2}{g_{\phi\phi}} = -\frac{1}{g^{tt}} = -\frac{\Delta \sin^2 \theta}{g_{\phi\phi}}, \tag{17}$$

N^2 vanishes when $r \rightarrow r_H$. The number of modes with frequency less than E is given approximately by

$$n(E) = \frac{1}{\pi} \int_0^{l_{max}} (2l+1) \int_{r_H}^R \sqrt{k^2(r, l, E)} dr dl. \tag{18}$$

Here it is understood that the integration with respect to r and l is taken over those values which satisfy $r_H \leq r \leq R$ and $k^2(r, l, E) \geq 0$. Thus one finds

$$\frac{dn(E)}{dE} = \frac{1}{8\pi^2} \int d\theta d\bar{\phi} \int_{r_H}^R dr (-g^{tt})^{\frac{3}{2}} \sqrt{g_{rr}g_{\theta\theta}g_{\phi\phi}} \frac{1}{3} \frac{d}{dE} (h^3(E/E_P) E^3). \tag{19}$$

In proximity of the horizon, the free energy can be approximated by

$$F_{r_H} = \frac{1}{8\pi^2\beta} \int d\theta d\bar{\phi} \int_0^\infty \ln(1 - e^{-\beta E}) \frac{d}{dE} \left(\frac{1}{3} h^3(E/E_P) E^3 \right) H(r_H, r_1) dE \tag{20}$$

where we have defined

$$H(r_H, r_1) = \int_{r_H}^{r_1} dr (-g^{tt})^{\frac{3}{2}} \sqrt{g_{rr}g_{\theta\theta}g_{\phi\phi}}. \tag{21}$$

F_{r_H} can be further reduced to

$$F_{r_H} \simeq \frac{C(r_H, \theta)}{8\pi^2\beta} \int_0^\infty \frac{\ln(1 - e^{-\beta E})}{\sigma(E/E_P)} \frac{d}{dE} \left(\frac{1}{3} h^3(E/E_P) E^3 \right) dE, \tag{22}$$

where

$$C(r_H, \theta) = \int d\theta d\bar{\phi} \left[\frac{(r_H^2 + a^2)^4 \sin \theta}{r_H (r_H - r_-)^2 \Sigma_H} \right] \tag{23}$$

and where we have assumed that, in proximity of the throat the brick wall can be written as $r_0(E/E_P) = r_H \sigma(E/E_P)$ with

$$\sigma(E/E_P) \rightarrow 0, \quad E/E_P \rightarrow 0. \quad (24)$$

With an integration by parts one finds

$$F_{r_H} = -\frac{C(r_H, \theta)}{24\pi^2\beta} \int_0^\infty \frac{E^3 h^3(E/E_P)}{\sigma(E/E_P)} \times \left[\frac{\beta}{(\exp(\beta E) - 1)} - \frac{\ln(1 - e^{-\beta E})}{E_P \sigma(E/E_P)} \sigma'(E/E_P) \right] dE. \quad (25)$$

It is possible to show that

$$F_{r_H} = -\frac{C(r_H, \theta)}{24\pi^2\beta} \int_0^\infty \left[\frac{\beta E e^{-3E/E_P}}{(\exp(\beta E) - 1)} - 2e^{-3E/E_P} \ln(1 - e^{-\beta E}) \right] dE \\ = -\frac{C(r_H, \theta)}{24\pi^2\beta} \left[\zeta\left(2, 1 + \frac{3}{\beta E_P}\right) + \frac{\beta E_P}{3} \left(\gamma + \Psi\left(1 + \frac{3}{\beta E_P}\right) \right) \right], \quad (26)$$

where $\zeta(s, \nu)$ is the Hurwitz zeta function, $\Gamma(x)$ is the gamma function and $\Psi(x)$ is the digamma function. In the limit where $\beta E_P \gg 1$, at the leading order, one finds that the entropy can be approximated by

$$S = \beta^2 \frac{\partial F_{r_w}}{\partial \beta} = \frac{E_P^2}{36\beta} \int d\theta d\bar{\phi} \left[\frac{(r_H^2 + a^2)^4 \sin \theta}{r_H (r_H - r_-)^2 \Sigma_H} \right] \quad (27)$$

and even when rotation is included, the “brick wall” does not appear. Of course the entropy (27) can always be cast in the familiar form

$$S = \frac{A_H}{4G}, \quad (28)$$

where A_H is the horizon area. To summarize, we have shown that the ability of Gravity’s Rainbow to keep under control the UV divergences applies also to rotations. However the connection between a ROI and a ZAMO has to be investigated with care²⁴. Indeed in the ROI frame, the superradiance phenomenon appears, while in the ZAMO frame does not. Once the connection is established nothing forbids to extend this result to other rotating configuration like, for example, Kerr-Newman or Kerr-Newman-De Sitter (Anti-De Sitter).

Acknowledgments

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References

1. J. Magueijo and L. Smolin, *Class. Quant. Grav.* **21**, 1725 (2004) [arXiv:gr-qc/0305055].

2. R. Garattini and G. Mandanici, *Phys. Rev. D* **83**, 084021 (2011); arXiv:1102.3803 [gr-qc]. R. Garattini and G. Mandanici, *Phys. Rev. D* **85**, 023507 (2012); arXiv:1109.6563 [gr-qc]. R. Garattini, *JCAP* **017** 1306, (2013); arXiv:1210.7760 [gr-qc]; R. Garattini and B. Majumder, *Nucl. Phys. B* **884** 125, (2014); arXiv:1311.1747 [gr-qc]. R. Garattini and B. Majumder, *Nucl. Phys. B* **883** 598, (2014); arXiv:1305.3390 [gr-qc]. R. Garattini and E. N. Saridakis, *Eur. Phys. J. C* **75**, 343 (2015); ArXiv:1411.7257 [gr-qc].
3. R. Garattini, *Phys. Lett. B* **685**, 329 (2010); arXiv:0902.3927 [gr-qc].
4. G. 't Hooft, *Nucl. Phys. B* **256**, 727 (1985).
5. J. D. Bekenstein, *Phys. Rev. D* **7**, 949 (1973).
6. S. W. Hawking, *Comm. Math. Phys.* **43**, 199 (1975).
7. L. Susskind and J. Uglum, *Phys. Rev. D* **50**, 2700 (1994).
8. J. L. F. Barbon and R. Emparan, *Phys. Rev. D* **52**, 4527 (1995), hep-th/9502155.
9. E. Winstanley, *Phys. Rev. D* **63**, 084013 (2001), hep-th/0011176.
10. J.-G. Demers, R. Lafrance and R. C. Myers, *Phys. Rev. D* **52**, 2245 (1995), gr-qc/9503003.
11. D. V. Fursaev and S. N. Solodukhin, *Phys. Lett. B* **365**, 51 (1996), hep-th/9412020.
12. S. P. Kim, S. K. Kim, K.-S. Soh and Jae Hyung Yee, *Int. J. Mod. Phys. A* **12**, 5223 (1997), gr-qc/9607019.
13. X. Li, *Phys. Lett. B* **540**, 9 (2002), gr-qc/0204029.
14. Z. Ren, W. Yue-Qin and Z. Li-Chun, *Class. Quant. Grav.* **20** (2003), 4885.
15. G. Amelino-Camelia, *Class. Quant. Grav.* **23**, 2585 (2006), gr-qc/0506110. G. Amelino-Camelia, *Gen. Rel. Grav.* **33**, 2101 (2001), gr-qc/0106080.
16. E. C. Vagenas, A. F. Aliha, M. Hemedda and H. Alshal, *Eur. Phys. J. C* **79** 398 (2019) no.5, arXiv:1903.08494 [hep-th].
17. H. Bai and M. L. Yan, *JHEP* **7** 58 (2003), gr-qc/0303006.
18. Visser M 1995 *Lorentzian Wormholes: From Einstein to Hawking* (American Institute of Physics, New York).
19. M. H. Lee and J. K. Kim, *Phys. Rev. D* **54**, 3904 (1996); ArXiv:hep-th/9603055.
20. In preparation.
21. J. L. Cortes and J. Gamboa, *Phys. Rev. D* **71**, 065015 (2005); hep-th/0405285
22. Y. Zhao and X. Liu, arXiv:1606.06285 [gr-qc].
23. E. Chang-Young, D. Lee and M. Yoon, *Class. Quant. Grav.* **26**, 155011 (2009) [arXiv:0811.3294 [hep-th]].
24. In preparation.