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Uncertain Multi-Criteria Decision Making: Theoretical and Practical Applications

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Declaration of Authorship

I, Amin Hocine declare that this thesis titled, "Uncertain Multi-Criteria Decision Making: Theoretical and Practical Applications", and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
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- I have acknowledged all main sources of help.
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Absract

This thesis proposes and develops new solutions and ideas to solve multi-criteria decision-making (MCDM) problems in uncertain and imprecise environments. In the first part, the area of uncertain multi-objective decision-making (MODM) is covered. In this respect, four hybrid uncertain techniques-based goal programming (GP) approaches are proposed and examined. First, a novel technique that integrates fuzzy goal programming (FGP) and multi-choice goal programming (MCGP) is developed. Second, a novel technique to solve the interval coefficient in FGP is suggested. Third, a novel technique that integrates interval target rather than coefficient in FGP is proposed. Forth, a novel technique for solving multi-tolerances fuzzy membership function in FGP is developed. In the second part, the area of uncertain multi-attribute decision-making (MADM) is considered. In particular, a new approach for handling uncertainty in MADM using Exclusive-or (XOR) logic is proposed. The underlying philosophy behind the XOR logic and the existing link with uncertainty is motivated. The necessary background information for proceeding with XOR numbers and how ordering and ranking XOR numbers is presented. Ideas to incorporate decision makers' (DMs) preferences are formulated. The concept of "the output mechanism of XOR function" to support the analyst to control uncertainty based on controlled and uncontrolled channels is proposed. Within this framework, new techniques, namely, XOR analytic network process (XOR-ANP), XOR data envelopment analysis (XOR-DEA), and XOR best-worst method (XOR-BWM) are developed. To validate the effectiveness and the feasibility of these techniques, several real-life applications have been conducted such as optimizing renewable energy portfolios under uncertainty, selecting the best location for the wind farm, and measuring the impact of the COVID-19 pandemic, providing valuable decision-making and managerial insights.

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I believe that this work has changed us, our perspectives, and our understanding of decision-making under uncertainty. That, in our opinion, is the most significant achievement. So, if a reader is hesitant to conduct research, we just have one piece of advice: Try it!

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List of Abbreviations

AHP	Analytic hierarchy process
AL	Aspiration level
ANP	Analytic network process
DM	Decision Maker
ELECTRE	Elimination et choix traduisant la realité
FGP	Fuzzy goal programming
FGP-IT	Fuzzy goal programming with interval target
FIGP	Fuzzy interval goal programming
FP	Fuzzy programming
GP	Goal programming
MADM	Multi-attribute decision-making
MAUT	Multi-attribute utility theory
MAVT	Multi-attribute value theory
MCAL	Multi-choice aspiration level
MCDM	Multi-criteria decision-making
MCGP	Multi-choice goal programming
MF	Membership function
MODM	Multi-objective decision-making
MT-FGP	Multi-tolerance fuzzy goal programming
MTL	Multiple target levels
LHS	Light hand side
PROMETHEE	Preference ranking organization methods
RESS	Renewable energy site selection
RMCGP	Revised multi-choice goal programming
RHS	Right hand side
SMART	Simple multiattribute rating technique
TOPSIS	Technique for order preference by similarity to ideal solution
WA-FMCGP	Weighted additive modeling
WGP	Weighted goal programming
WAM	Weighted-additive fuzzy multi-choice goal programming
XOR-ANP	XOR Analytic network process
XOR-BWM	XOR best-worst method
XOR-DEA	XOR data envelopment analysis

Author’s list of publications

1. Hocine, A., Zhuang, Z.-Y., Kouaissah, N., & Li, D.-C. (2020). Weighted-Additive Fuzzy Multi-Choice Goal Programming (WA-FMCGP) for Supporting Renewable Energy Site Selection Decisions. *European Journal of Operational Research*, 285(2), 642–654.

2. Kouaissah, N., & Hocine, A. (2020). Optimizing Sustainable and Renewable Energy Portfolios Using a Fuzzy Interval Goal Programming Approach. *Computers & Industrial Engineering*, 144(1), 106448.

3. Hocine, A., Guellil, M. S., Dogan, E., Ghouali, S., & Kouaissah, N. (2020). A Fuzzy Goal Programming with Interval Target Model and its Application to the Decision Problem of Renewable Energy Planning. *Environmental and Ecological Statistics*, 27(3), 527–547.

4. Kouaissah, N., & Hocine, A. (2021). Forecasting Systemic Risk in Portfolio Selection: The Role of Technical Trading Rules. *Journal of Forecasting*, 285(1), 642–654.

5. Hocine, A., Zhuang, Z.-Y., Kouaissah, N., & Gregory, A., Kiker. (2022). Optimising Sustainable Renewable Energy Portfolio Decisions Using Multi-Tolerance Fuzzy Goal Programming: A Novel Approach. *International journal of geen energy*, (1), 1–16.

6. Kouaissah, N., & Hocine, A. (2022). XOR Data Envelopment Analysis and its Application to Renewable Energy Sector. *Expert System with Application*, 285(1), 118044.

7. Kouaissah, N., Hocine, A. & Ortobili, S. (2022). XOR Analytic Network Process and Assessing the Impact of COVID-19 by Sector. *Computers & Industrial Engineering (Minor revision).*

8. Hocine, A. & Kouaissah, N., (2022). XOR-Best Worst Method for Assessing the Impact of COVID-19 by Sector Under Uncertainty. *Annals of operation research (under review).*

9. Kouaissah, N., & Hocine, A. (2022). Robust Drawdown-Based Performance Measures. *Accepted in Economics Bulletin.*

10. Hocine, A. Kouaissah, N., Ortobili, S. & Aouam , T. Modeling De Novo Programming Within Simon’s Satisficing Theory: Methods and Application in Designing an Optimal Offshore Wind Farm Location System. *Under review in European Journal of Operational Research.*

1 Introduction General

Nothing is certain except the past
-Seneca-

This chapter gives a general introduction to the uncertain multi-criteria decision-making problems and provides the main motivations behind this research project and then and sets out the thesis objectives.

1.1 OVERVIEW

The core idea of this Ph.D. thesis is to model multi-criteria decision-making (MCDM) techniques under uncertain and imprecise environments and solve various real-life problems. MCDM is a relatively new research field that appeared in the 1960s as a sub-branch of operations research and management science. The main aim of this research field is to tackle decision-making problems featuring multiple conflicting criteria, goals, or objectives (Roy, 1985; Zionts, 1992; Keeney & Raiffa, 1993). Modeling uncertainty in MCDM is a very interesting topic and extremely important in many real-life applications. In multi-criteria decision analysis, uncertainty arises from various sources, such as importance weights, thresholds, judgments and preferences, and various related parameters. Providing such information requires significant cognitive effort from the decision maker (DM) (see, e.g., Roy (1989)). For instance, in the MADM context, DMs do not know precisely how to evaluate the *i*-th alternative in terms of the *j*-th criterion or how much they prefer a certain criterion to another criterion. Under these circumstances, DMs generally make judgments about their evaluation that roughly approximate the unknown parameters. The term uncertainty can have many different meanings depending on the system context. Klir & Folger (1988) discuss different definitions of uncertainty based on Webster's Dictionary. Most intersect explicitly or implicitly with the meaning of information availability. In this thesis, the definition provided by the US national research council is adopted with minor edits, as follows (UNRC 2000; Hocine & Kouaissah, 2020):

"Uncertainty is a general concept that reflects our lack of information about something or someone, ranging from just short of complete information to an almost complete lack of conviction about an outcome."

According to Howell (1971) and Kahneman & Tversky (1982), the attribute of uncertainty can be divided into two categories: external uncertainty (also referred to as objective, epistemic, and stochastic) and internal uncertainty (also called subjective, incompleteness, and lack of knowledge). Internal uncertainty can be interpreted as ignorance, which refers to our inability to know the facts surrounding certain events (e.g., "I am not sure whether possums are mammals or not"). With a better knowledge of the problem and the application of appropriate methods, internal uncertainty can be reduced, and sometimes even eliminated. In the literature, the fuzzy theory is one of the most common ways to represent and address this type of uncertainty. External uncertainty, in contrast, can be viewed as the uncontrollable outcomes of an event (e.g., "It is not certain whether it will stop raining by tomorrow morning"). Even if we try many experiments and control all parameters, we still cannot predict precisely what the weather will be tomorrow. External uncertainty will not be eliminated or minimized by gathering more information. This uncertainty is what surprises us and it cannot be easily controlled. In the literature, probability theory is the most common ways to represent and address this type of uncertainty. However, despite this conceptual distinction, there is no clear boundary between internal and external uncertainty, and they may even be handled in the same way in terms of techniques used. Thus, the main challenge today is how to find a formal way to tame this phenomenon while retaining as many features as possible.

Although it is difficult, if not impossible, to eliminate uncertainty, it is important to detect and deal with it in order to avoid its negative consequences in decision making. In the literature, significant effort has been made to address uncertainty, and many new techniques and theories have been proposed, such as probability theory, fuzzy theory, and rough theory (Kolmogoroff, 1933; Zadeh, 1965; Pawlak, 1982). Generally speaking, in recent decades, several papers have been published addressing MCDM using various uncertainty theories, including fuzzy set theory, interval analysis, and probability (or stochastic) theory. Accordingly, three main streams of research have been proposed, namely, interval MCDM, fuzzy MCDM, and probabilistic MCDM. For example, the interval MCDM can be used to deal with rounding and measurement errors.

Consider the following interval number $x = [l, u]$, where l and u are real numbers representing the lower and upper limit of the interval. Determining these two boundary values is very crucial in interval analysis, which contains an infinite number of possibilities with no information whatsoever on the behavior of the data between those two boundary values (Hocine & Kouaissah, 2020). Although it is typically accepted that random variables are captured through stochastic processes and probability theory, consensus fails on whether historical data is sufficient to formulate suitable probabilistic distributions (see Kirkwood, 1992). Unfortunately, under some circumstances such as COVID-19, historical data is lacking or non-existent, and uncertainty may arise from linguistic sources. In these cases, the fuzzy MCDM can be adopted to deal with imprecise numerical quantities. However, the adoption of fuzzy numbers presents difficulties related to the issue of overlapping and the defuzzification procedure may lose a certain amount of information existing in the original data. In general, it can be argued that no approach is superior to the others in all cases; all approaches have their advantages and limitations, and the choice of the right approach should be guided by the analysis. Thus, sometimes, releasing a fair comparison of uncertain MCDM methods is difficult or almost impossible because each uncertain method is based on different assumptions, uses different algorithms and formulas, and requires different information for its application. We believe that these theories intersect with each other in many ways and several studies have confirmed their usefulness in complementing each other. Thus, under some assumptions, conditions, and specific contexts, and through a deep vision gathering them into a unified framework, this is the ultimate aim of generalized information theory (Zadeh, 2005). Even though a great effort has been made in this direction, this goal has not yet been achieved. We recall the following quote from the Chinese philosopher Chuang Tzu:

> *"Nothing is best, nothing is worst. Each thing, seen in its light, stands out in its way. It can seem to be 'better' than what is compared with it. On its terms. But seen in terms of the whole, no one thing stands out as 'better."*

Rare or extreme events which arise from the deeper layers or nucleus of uncertainty, like COVID-19 or stock market crashes, have a strong and widespread impact on the destabilization of any system. This is due to the speed of their emergence, their distinctiveness, their unpredictability, and their ability to control their behavior (impacts). The COVID-19 outbreak has created emergencies across many countries and generated economic slowdowns and shocks that have impacted the global economy. It has been estimated that most financial markets will shrink to their lowest level since the 2008 financial crisis due to widespread fear and aversion to loss among financial market participants. Many specific economic sectors have also been affected by social distancing and stay-at-home orders, such as healthcare, tourism, commerce, energy, and travel. The key challenge is that there are no forecasts, indices, or projections that can help to measure the impact of this extreme event as long as the virus' trajectory is not predictable, and the high uncertainty surrounds it. In such critical issues, we need experts' opinions to guide us and reduce the spread of misinformation. Although experts are valuable sources of information and knowledge, experience and global events have proven that they can also be wrong. Since uncertainty can be viewed as a lack of information, in seeking to reduce its impact, experts often try to provide additional information to enrich discussion and analysis, rather than explaining their opinions in a deterministic way. For instance, when experts are asked about how long it will take the economy to recover from the COVID-19 crisis, the most common answer is something like, "So far, the impact of COVID-19 is still ongoing, if we reach a certain level of control, economic recovery will take 2 or 3 coming years. Since experts do not know the exact right answer, they are forced to express their opinion using exclusive-or (XOR) logic (see Hocine & Kouaissah, 2020; Kouaissah & Hocine, 2022). Because the quality of results is based on how well a mathematical model fits and describes the behavior of a given system, it is better to develop a new paradigm to deal with XOR data than to convert this data or adapt it to work with existing theories, which may affect and reduce the quantity of information that XOR data holds; as noted by Klir (1990):

"... Each mathematical theory is derived from some specific assumptions (axioms). ... If a problem does not conform to them and an applied mathematician trained in the methodology still wants to use it, he has to adjust (reformulate) the problem to make it fit the assumptions. ... The problem adjustment is often not stated explicitly and, as a consequence, an impression is created that the original problem was solved while, in fact, it was not."

1.2 PROBLEM STATEMENT

In practical applications of uncerain MCDM, determining the optimal solution is a complex task. The problem involves meeting the DM's preferences while considering the multiple conflicting objectives. However, one of the main issues that make the formulation of this problem difficult is the different types of uncertainty embedded in DM preferences. For example, in renewable energy (RE) planning, DMs often make conservative initial estimates based on the available information and resource limitations. For instance, a DM might say, "Under our resource limitations, we suggest that the realization time of project 'X' will be approximately 12 or 16 months." Or they might say, "This project will cost around between 350,500 and 400,500 euros." DMs normally make conservative initial estimates because they anticipate problems in achieving these goals (e.g., an architect's reluctance to change specifications or drawings, harsh weather, or errors in executing project activities). However, estimates using language like "approximately ... or ..." and "around between ... and ..." are very difficult. This form of uncertainty is typically called the "hybrid uncertainty." According to our knowledge, the most existing MCDM techniques cannot solve this type of problem. Motivated by these concerns, the main research question is formulated as follows:

How to develop appropriate MCDM methodologies to deal with hybrid uncertainty?

1.3 RESEARCH AIM AND OBJECTIVE

To answer the question of " how to develop appropriate MCDM methodologies to deal with hybrid uncertainty ? " the research aim has been distributed over two main streams of MCDM, namely MODM and MADM.

✠ For MODM paradigms, we aim to develop four novel hybrids uncertain MODM techniques called:

- Weighted-additive fuzzy multi-choice goal programming (WA-FMCGP).
- Fuzzy interval goal programming (FIGP).
- Fuzzy goal programming with interval target (FGP-IT).
- Multi-tolerance fuzzy goal programming (MT-FGP).

✠ For MADM paradigms, we aim to develop three new XOR-MADM techniques called:

- XOR data envelopment analysis (XOR-DEA)
- XOR Analytic network process (XOR-ANP)
- XOR best-worst method (XOR-BWM)

Proceeding towards these aims, the specific objectives of the study are:

✓ To develop a model that quantifies the DM's preferences, the linguistic uncertain term of "approximately ... or ..."

✓ To develop a model that quantifies the DM preferences, the linguistic uncertain term of "around between ... and ..."

✓ To develop a model that quantifies the DM's preferences, the Exclusive-or (XOR) term of "... or ..."

✓ To validate the effectiveness and the feasibility of the proposed techniques, several real-life applications have been adopted.

The first part of this thesis starts by considering the most efficient MODM techniques that are used to deal with uncertainty. In particular, the fuzzy goal programming (FGP) and multi-choice goal programming (MCGP) techniques have been proved their ability to support and provide the DMs under uncertainty with applicable solutions in different research areas. Generally speaking, the main aim of FGP is to deal with linguistic uncertainty related to only one single numerical value by minimizing the distance between the feasible region and fuzzy aspiration level vector using the fuzzy membership functions concept. Initially, Narasimhan (1980) merged the fuzzy set theory to GP and proposed the FGP model. Since then, several works related to FGP have been formulated and proposed in the literature (see, e.g., Hannan (1981); Tiwari et al. (1987); Parra et al. (2001); Chen & Tsai (2001); Turgay & Taskin (2017); and literature therein). For a thorough survey and classification of FGP models, refer to Aouni et al. (2009). However, all these techniques can deal with only one single linguistic uncertainty associated with only one numerical value such as: "approximately 12", "around 350.500". In other words, these techniques cannot solve the problem with multiple linguistic uncertainties with multiple numerical values such as: "approximately 12 or 16", "around between 350.500 and 450.500".

On the other side, as an attempt to deal with the problem of multiple linguistic uncertainties, Chang (2007; 2008) developed a technique called multi-choice goal programming (MCGP). The fundamental concept of MCGP is the multi-choice aspiration level (MCAL), which refers to the fact that a DM can sometimes state several discrete values as the possibly desired aspiration levels for each goal (i.e., 12 or 16). In MCGP modeling, these possible and acceptable alternative target values offer a 'multi-choice' (that is, a one-among-many choice) to find the (more or most) satisfying solution set during the solution determination process. However, this technique can deal with only crisp numerical values. In other words, this technique cannot solve problems in a fuzzy environment. Since MCGP models are thus quite different from the FGP models, which utilize fuzzy set theory or the concept of MFs directly. Given such uniqueness, scholars and researchers have devoted a new hybrid framework to integrating the MCGP concept into FGP models for solving MODM under multiple linguistic uncertainties. This thesis tackles this type of problem by proposing and developing novel models that can merge these important techniques to handle the hybrid uncertainty.

The second part of the thesis concentrates on uncertain MADM method and how the XOR paradigm could be used to model various features that range from uncertainty to integrate DMs preferences as well as modeling the channels of controllability. Within this framework, three widely used MADM techniques have been adopted to show the usefulness of this proposal. In particular, we propose XOR Analytic network process (XOR-ANP), XOR best-worst method (XOR-BWM), and XOR data envelopment analysis (XOR-DEA).

1.4 SUMMARY OF CHAPTERS

The thesis consists of nine chapters. In particular, Chapter 2 presents a general introduction to MCDM techniques and motivate and highlight our contributions. Chapters 3–6 develop and examine four uncertain MODM techniques. Chapters 7–9 propose three uncertain MADM methods and the role of XOR paradigm. In the following, we present a short summary of the main contributions of each chapter.

- **Chapter 2** provides an introduction to the MCDM paradigm, as it is normally pursued and reviews the main mainstream of uncertain MCDM methods.

- **Chapter 3** develops a novel uncertain MODM called weighted-additive fuzzy multi-choice goal programming (WA-FMCGP) to solve the FGP problem with the multiple-choice aspiration levels (MCALs) concept. The main contribution of this model is using an objective function that minimizes the weighted-additive summation of the normalized deviations; thus, the model can adopt any minimization process from any goal programming (GP) variant. The application of the model is verified by solving a renewable energy site selection (RESS) problem.

- **Chapter 4** proposes a novel uncertain MODM called fuzzy interval goal programming (FIGP) that integrates two important GP techniques based on fuzzy set theory and interval analysis. This model contributes to the literature in various ways. First, it generalizes the classic FGP such that each decision variable supports the setting of interval coefficients. Second, it considers the most common types of fuzzy membership functions (i.e., left, right, triangular, and trapezoidal) that we formulate as fuzzy interval membership functions. The application of the model is verified by using a real-world scenario of selecting the optimal RE portfolio for electricity generation in Italy.

- **Chapter 5** presents a novel model called fuzzy goal programming with interval target (FGP-IT), to solve FGP with an interval target (or interval goal). To achieve this aim, it formulates the aspiration levels on the right-hand side (RHS) of the system constraints using the revised MCGP (RMCGP) technique. It also considers the most common types of fuzzy membership functions that are generalized to account for interval target values. To provide a practical perspective, the proposed approach is used to determine the optimal sustainable wind energy portfolio decision in Algeria.

- **Chapter 6** proposes a novel model called multi tolerances fuzzy goal programming (MT-FGP) to overcome the limitations of 'single-tolerance' during FGP modeling (e.g., being unable to fit complicated 'multi-tolerance' decision contexts and running the risk of ignoring solutions that achieve better optimality). An empirical application is considered for an RE portfolio decision that is critical for sustainable RE development in Algeria.

- **Chapter 7** develops a new model called XOR data envelopment analysis (XOR-DEA) to solve MADM problems using XOR logic. The concept of "the output mechanism of XOR function" to support the analyst to control uncertainty based on controlled and uncontrolled channels is proposed. To validate the XOR-DEA modelling, a real-life application of ranking renewable energy technologies has been conducted.

- **Chapter 8** develops a new model called XOR analytic network process (XOR-ANP) to solve MADM problems using XOR logic. The paradigm of XOR multi-criteria decision-making (MCDM) is introduced by discussing the concept of xorness and the main philosophy behind it. Then, it proposes some ideas to incorporate DMs' preferences in the optimization process through three types of preferences: positive (optimistic), negative (pessimistic), and neutral. This contribution enhances the understanding of the role of DMs' preferences in decision-making processes. In particular, it allows us to explore the implications generated by the optimism/pessimism attitude toward a particular decision-making problem. To validate the effectiveness and feasibility of this technique, a real-life application to measure the impact of the COVID-19 pandemic in Morocco has been proposed.

- **Chapter 9** presents a new model called the XOR-best-worst method (XOR-BWM) to solve MADM problems using XOR logic. Ideas to incorporate DMs' preferences in the optimization process through three types of preferences: positive (optimistic), negative (pessimistic), and neutral are given. To validate this uncertain MADM technique, a real-life application to measure the impact of the COVID-19 pandemic in Italy has been adopted.

- **Chapter 10** presents the conclusions, the main contributions, limitations and the future research.

2 The underlying philosophy of Uncertain MCDM Approaches

The significant problems we have cannot be solved at the same level of thinking with which we created them.
-Albert Einstein-

This chapter highlight the underlying philosophy of the uncertain MCDM approaches. The need for taking into account-preference Exclusive-or (XOR) information into the classification method is furthermore intuitively proposed. Conceptually, it motivates and highlights our main contributions.

2.1 INTRODUCTION

Due to the complexity, uncertainty, and multidimensionality of the real life problems, many theorists and mathematicians have recognized the need to develop a new mathematical framework to deal with MCDM problems, and thus many distinguished MCDM methodologies have been proposed; these techniques include, among others, goal programming (GP) (Charnes and Copper, 1961), elimination et choix traduisant la realité (ELECTRE) (Roy, 1968), simple multiattribute rating technique (SMART) (Edwards, 1977), analytic hierarchy process (AHP) (Saaty, 1980), technique for order preference by similarity to ideal solution (TOPSIS) (Hwang & Yoon, 1981), preference ranking organization methods (PROMETHEE) (Brans & Vincke, 1985), multi-attribute utility/value theory (MAUT/MAVT) (Keeney & Raiffa, 1993), analytic network process (ANP) (Saaty, 2001), and many other outranking methods (see Martel & Matarazzo (2016)). Readers interested in a more in-depth treatment of MADM methods should refer to Hwang and Yoon (1981), Triantaphyllo (2000), Tzeng and Huang (2011), and Greco et al. (2016) for the outstanding pioneering survey of this field. Due to its powerful results, MCDM paradigms have become a popular and practical approach for assessing diverse real-world problems in fields such as finance, renewable energy, supply chain management, and many others (see, e.g., Wallenius et al., 2008; Greco et al., 2016). Conceptually, MCDM problems (or techniques) are divided into two main theoretical streams: i) multi-objective decision-making (MODM) and ii) multi-attribute decision-making (MADM). The core of MODM methods is to design the 'best' alternative by considering the trade-offs within the design constraints, where the decision space is continuous (see e.g., Huang & Masud (1979) and Zeleny (1981)). In contrast, MADM methods are discrete and related to problems where the alternatives are predetermined, as shown in Figure 2.1 (Hwang & Yoon, 1981).

Discrete

Criteria		c_1	$\cdots$	c_m
Weights		w_1	$\cdots$	w_m
Alternatives	x_1	a_{11}	$\cdots$	a_{1m}
	$\vdots$	$\vdots$	$\ddots$	$\vdots$
	x_n	a_{n1}	$\cdots$	a_{nm}

Continuous

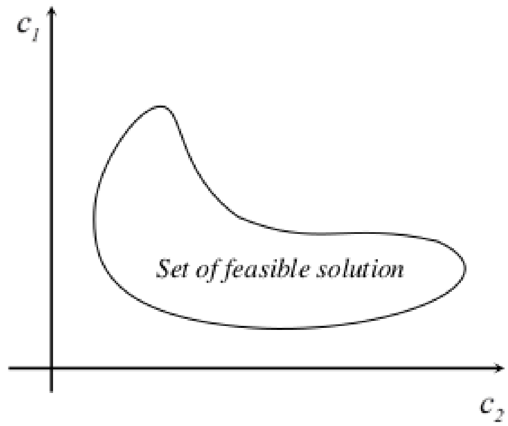


Figure 2.1: Discrete and continuous multi-criteria decision-making problems.

where $A = \{x_1, x_2, \cdots, x_n\}$ is a set of decision alternatives, $C = \{c_1, c_2, \cdots, c_m\}$ represents a set of criteria, and $W = \{w_1, w_2, \cdots, w_m\}$ is a set of the weights (relative importance) of the criteria. The term "select" in MCDM modeling fundamentally indicates the maximization of the multi-criteria value (or utility) function provided by DMs. The evaluation process generates a score a_{ij}, which represents the performance of the alternative with respect to criterion (for $i = 1, 2, \cdots, n$ and $j = 1, 2, \cdots, m$). By mapping the alternatives onto a cardinal scale of value, the best alternative is associated with the highest cardinality. It is easy to realize that uncertainty plays a vital role in MCDM problems. In this framework, the decision process is generally performed based on the selection of the best (optimizing) or in terms of ranking (from the best to the worst) option among a set of conceived alternatives based on multiple criteria taking into account our anticipation and evaluation of the consequences of these actions. The selection mechanism is mainly based on the experts' judgements which is always fraught with ambiguity and uncertainty. Uncertainty is everywhere, in the past, present and more strongly in future. Due to a lack of consistent historical records, uncertainty still exists in the past events such as disasters. We are uncertain about present and future because we do not have enough information to predict the future. These concerns it was and still is the potent and powerful force that motivates scholars to develop new techniques and theories to deal with this phenomenon.

2.2 STOCHASTIC MULTI-CRITERIA DECISION-MAKING

Probability theory was originally founded to study stochastic (or random processes) such as the growth of a bacterial population, an electrical current fluctuating, the movement of a gas molecule, and others. The main idea behind developing this theory is to describes the likelihood of an event taking place. To formalize this framework, the probability theory often uses terminologies such probability space, which allocates a probability measure to a set of outcomes known as the sample space, with values ranging from 0 to 1. Any particular subset of the sample space is defined as an event. The probability distributions are the key stone in probability theory, especially in mathematical modelling. The probability distributions of a random variable can be deduced from statistical regression based on the historical available data. Random variables can be correlated or uncorrelated, and their distributions can take various shapes either discrete or continuous such as Bernoulli, Binomial, Normal, Exponential, Gamma and Beta distributions. Inspired by this line of reasoning, the Operational Research through the mathematical programming field has developed what called the stochastic programming or chance constrained programming to cope with random variables in optimization problems. This technique was first developed by Charnes & Cooper (1959) and Miller & Wagner (1965). The main idea behind this technique is the obtained decision should ensure a certain probability level, where the probability distribution of this random variable should be known a priori. By relaxing the probability model into deterministic equivalent model as an approximation strategy using probability density functions, a linear programming as an example will be adopted to generate the final solutions. This idea is the underlining philosophy and the mathematical core of the most MCDM approaches (for more details about this topic, we refer to Greco et al., 2016). However, there are many real phenomena where it is difficult or even impossible to build a probability distribution for their variables due to the fact that the historical data is insufficient or absent. Think about the extreme events like COVID-19. In this case it is, the use of probability theory is useless and this is one of its main drawbacks.

"*Is Probability Theory Sufficient for Dealing with Uncertainty?*". In recent years, this question has sparked hot debate, particularly in the context of dealing with uncertainty. There are two groups. The first group believe that probability theory is capable to deal with all types of uncertainty, and thus anything that can be done with other techniques can also be done with probability-based techniques. While the second group, criticize the long-standing scientific practice of probability theory in addressing all types of uncertainty, regardless of their nature. More specifically, they argue that the major limitation of probability theory is the two-valued reasoning. This indicates that all concepts in probability theory assume the object X is either true or false. Consider the concept of an event, which is one of the most fundamental concepts in probability theory. E is a measurable subset of the sample space which either occurs or does not occur. More precisely, E cannot occur with a certain degree (see e.g., Buoncristiani, 1980).

2.3 FUZZY MULTI-CRITERIA DECISION MAKING

Fuzzy sets theory is originally introduced to handle the subjective uncertainty (Zedeh, 1965). The subjective uncertainty it is a kind of internal uncertainty that results from using linguistic variables to represent the problem context. The linguistic term refers to a variable that is expressed using verbal words and sentences. To model these linguistic variables, generally triangular fuzzy membership functions (or fuzzy numbers) will assumed such as "very high (very good)," "high (good)," "fair," "low (bad)," and "very low (very awful)," as illustrated in Figure 2.2. Since the appearance of fuzzy set theory, the adoption of linguistic variables has become widespread in decision analysis and MCDM approaches to measure the degree of the performance value of each alternative over each criterion. In the relevant literature, the linguistic variables are usually defined by appropriate membership functions. The concept of membership function (or fuzzy numbers) reflects the degree of belongness of this variable into a certain context. There are many types of membership functions such as triangular, trapezoidal, exponential and others. The main issue with this framework is how can the one chooses or selects the proper membership functions that reflect the linguistic variable.

2.4 INTERVAL MULTI-CRITERIA DECISION MAKING

It is generally known that in order to establish the probability distributions and select the proper membership functions, stochastic MCDM and fuzzy MCDM approaches require sufficient data and information. However, experimental data is frequently inadequate, making it unable to meet the requirements for the existing data to adequately validate the probabilistic dependability model. In this case, the idea of Interval data could be a good tool to overcome these drawbacks. It should be noted that the interval analysis proposed mainly to deal with the errors contained in data. For instance, to acquire a spatial distribution of a temperature in a certain space, the temperature can be monitored with a thermometer at various locations throughout this space. That means different temperatures will be detected e.g., a range of numbers between 19 and 21, which represents the various temperatures in this space. Therefore, the main idea behind the interval analysis is to define two values represent the upper and lower bounds of these errors. This framework has begun with the appearance of Moore's book Interval Analysis in 1966 (Moore, 1966). In interval MCDM paradigms, interval data generally associated with coefficients, performance, weights and aspiration levels. In relevant literature, it is always high recommended to make interval bounds as narrow as possible. This is due to the fact that these intervals contain an infinite number of possibilities with no information whatsoever on the behavior of the data between those two boundary values.

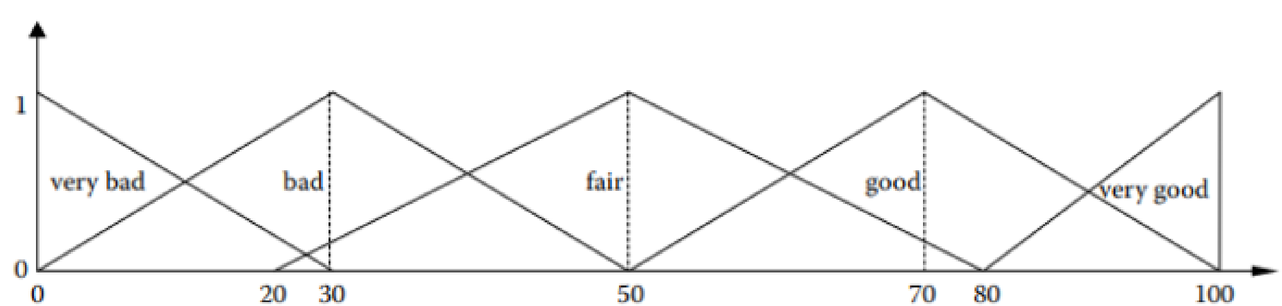


Figure 2.2: Triangular membership function of the five levels of linguistic variables.

2.5 EXCLUSIVE-OR (XOR) MULTI-CRITERIA DECISION MAKING

"*I don't know, maybe today or tomorrow*". We are quite sure that everyone has said this at least once in their private or professional life. But, why do we choose this manner to express our judgments? A possible answer to this question was proposed by Russell (1949):

> *"Hesitation may be observed in animals, but in them, one supposes, it does not find verbal expression. Human beings, seeking to express it, have invented the word 'or'."*

From a psychological perspective, "or" represents a state of hesitation or reluctance. Xorness arises from environments where there are multiple competing options and none is strong enough to overcome the others. In this case, XOR (i.e., "or") is the most natural and commonly used verbal expression to capture the hesitation or reluctance inherent in uncertain choices. According to the Cambridge dictionary, "hesitation" means the act of pausing before doing something, especially because you are nervous or not certain. The etymology or root of "hesitation" is the Latin word haesitationem, which refers to irresolution, reluctance, or uncertainty. Thus, hesitation occurs frequently when we are uncertain or in doubt about choosing a possible course of action (Kouaissah & Hocine (2022)).

According to Hocine & Kouaissah (2020), in many real-life situations, it is challenging for DMs to decide, interpret, or forecast due to a lack of knowledge or insufficient historical data. In this situation, DMs often rely on experts' judgments. Since uncertainty represents incompleteness of information or knowledge, experts usually express their judgments under xorness, rather than explaining their opinions in terms of crisp expressions (e.g., parameters or values), as a way to reduce the effect of this phenomenon. In practical applications, experts try to give a conservative evaluation based on the available information and resources. For instance, financial analysts will say: "Given the circumstances, the oil price will increase by either 10% or 15%," or economic analysts will say: "Given economic conditions, next year the inflation will increase either by 1% or 2%." The main reason that these experts use XOR logic is that they are uncertain about their estimations due to a lack of information or problem complexity (e.g., many factors interact, market conditions, government policy, etc.). Consequently, these experts are forced to express their preferences and judgments using XOR data. Motivated by these concerns, an XOR framework should be developed to deal with these decision-making problems. Distinct from the classic uncertainty theories, the XOR analysis framework provides alternative insights and an intuitive and natural way to model the imprecision and uncertainty existing in real-life problems. XOR analysis, at least in some cases, is the most natural representation of imprecision and uncertainty where no assumptions are required (such as distribution functions, membership functions, or lower and upper bound values). Even though significant effort has been made to tackle uncertainty, no formal paradigm that handles XOR data has yet been developed. This motivates us to ask the following question: how can we introduce a more analytic and practical framework to cope with XOR data? To distinguish the existing theories from the XOR framework, Table 2.1 shows under which conditions it is preferable to adopt one particular approach over another (see Hocine & Kouaissah, 2020).

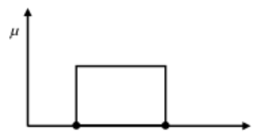
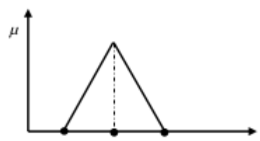
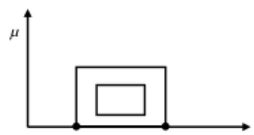
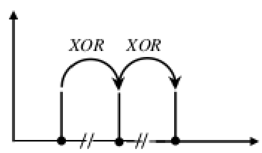
Uncertain theory	Issue	When
Stochastic (or Probability)		Random variables; enough historical data to build a probability distribution.
Interval		Rounding errors and measurement errors; knowing just the upper and lower limits.
Fuzzy		Linguistic variables; uncertainties affect subjective preferences; using the membership function.
Rough		The imperfect knowledge and indiscernibility between objects; using the concept of approximation.
XOR		Exclusive-or logic; multiple preferences; hesitant or irresolute in choosing among them.

Table 2.1: Comparison of different uncertain techniques according to when they are preferable.

Part I

Uncertain multi-objective decision-making (MODM)

3 Weighted Additive Fuzzy Multi Choice GoaL Programming

Know how to solve every problem that has ever been solved.
-Richard Feynman-

This chapter develops a novel MODM model called weighted-additive fuzzy multi-choice goal programming (WA-FMCGP) for the imprecise decision context wherein several conflicting goals are present, but each goal has multiple-choice aspiration levels (MCALs) and, around them, the fuzzinesses are expressed in terms of membership functions (MFs). The main contribution of this model is its use of an objective function that minimizes the weighted-additive summation of the normalized deviations; thus, the model can adopt any minimization process from any goal programming (GP) variant, such as the meta-GP model. This feature makes it more adept at incorporating DM preferences. The application of the model is also verified using real data to support renewable energy site selection (RESS).

3.1 INTRODUCTION

In the practice of MODM, the presence of uncertain goals (i.e., the target value of a goal, or 'goal target', is imprecise) are common. The goal target is usually on the right-hand side of a criterion constraint, and it is commonly referred to as an 'aspiration level' (AL) in goal programming (GP), which is regarded as a major branch of MODM. To deal with the uncertainty around the AL, the fuzzy set theory (Zadeh, 1965; Bellman & Zadeh, 1970) being able to offer 'fuzzy ability' (i.e., the ability to express goal fuzziness) around the goal target, and thus a variety of fuzzy MODM (F-MODM) models have been proposed in the literature (see e.g., Aouni et al., 2009; Jones & Romero, 2016). In this context, another series of studies have been published beginning in 2007, specifically, studies of multi-choice goal programming (MCGP) modelling (Chang 2007; 2008; 2011) to handle the uncertainty around AL. The fundamental concept of MCGP is the multi-choice aspiration level (MCAL), which refers to the fact that a decision-maker (DM) can sometimes state several discrete values as the possibly desired ALs for each goal (i.e., the MCALs); this is again due to the uncertain decision context. In MCGP modelling, these possible and acceptable alternative target values offer a 'multi-choice' (that is, a one-among-many choice) to find the (more or most) satisfying solution set during the solution determination process. So, in a broader sense, MCGP can be regarded as another uncertain MODM approach. MCGP models are thus quite different from the other two series of F-MODM models, which utilize fuzzy set theory or the concept of membership functions (MFs) directly. Given such uniqueness, scholars have devoted themselves to integrating the MCAL concept into the series of FGP models for dealing with uncertainty in MODM.

3.2 RELATED WORKS

The most salient feature of FGP is that it intends to minimize the value of the objective function where there is a summation, in some form, of the deviational variables that are introduced to solve a GP model. These 'forms' include formulating by reference to the weighted-additive modeling (WAM) concept. The concept of WAM can be applied to various MODM models. When the concept is applied to GP, it produces a weighted goal programming (WGP) model. When this concept is then applied to FGP, the resulting model can be referred to as a 'W-FGP' model. For example, some major works using this technique include, but are not limited to, studies by Hannan (1981), Kim & Whang (1998), Kim et al. (2002), and Aouni et al. (2009). The initial model presented by Hannan in 1981 is regarded as the typical (base) FGP model. In contrast, F(L/NL)P maximizes the value of the objective function where there is a summation, in some form, of the utility interpretations of the membership functions (MFs) that are employed to solve a fuzzy programming (FP) model (i.e., it derives the utility value from the functional membership value for each goal and then maximizes the aggregated utility).

Since Hannan's model supported only one type of MF (which limited its ability to express DM preferences), it was improved in 1998 (Kim & Whang, 1998). Two other types of MFs (i.e., left-sided and right-sided) were added to the FGP model. In the objective function, a more flexible weighted-additive style was also offered when aggregating the sum of deviational variables. Later, Kim et al. (2002) addressed the problem that limited the possible value of the deviational variables. In 2008, Yaghoobi et al. (2008) further increased the supporting range of the FGP model by adding another type of common piecewise linear MF, trapezoidal MF. Therefore, their model is used to demonstrate such a 'W-FGP' modeling approach here. Consider the fuzzy goals of a decision, identified by i, each of which can be expressed by a piecewise linear MF. Suppose that there are a number of i0 right-sided MFs, a number of (j0−i0) left-sided MFs, a number of (k0−j0) TMFs, and a number of (K−k0) trapezoidal MFs. Such a W-FGP model can be represented by the following generalized algebraic formulations:

(FGP-Yaghoobi et al)

$$\begin{aligned} \min \quad & \sum_{i=1}^{i_0} w_i \frac{p_i}{\Delta_i^R} + \sum_{i=i_0+1}^{h_0} w_i \frac{n_i}{\Delta^L} + \sum_{i=j_0+1}^{K} w_i \left(\frac{n_i}{\Delta_i^L} + \frac{p_i}{\Delta^R} \right) \\ \text{s.t.} \quad & (AX)_i - p_i \leq b_i \, i = 1, \ldots, i_o \\ & (AX)_i + n_i \geq b_i \, i = i_0 + 1, \ldots, j_o \end{aligned}$$

$$(AX)_i + n_i - p_i = b_i \quad i = j_0 + 1, \ldots, k_o$$
$$(AX)_i - p_i \le b_i^u \quad i = k_0 + 1, \ldots, K$$
$$(AX)_i + n_i \ge b_i^l \quad i = k_0 + 1, \ldots, K$$
$$\mu_i + \frac{n_i}{\Delta_j^L} = 1 \quad i = i_0 + 1, \ldots, j_o$$
$$\mu_i + \frac{p_i}{\Delta_i^R} = 1 \quad i = 1, \ldots, i_o$$
$$\mu_i + \frac{n_i}{4_i^L} + \frac{p_i}{4_i^R} = 1 i = j_0 + 1, \ldots, K$$
$$\mu_i, n_i, p_i \ge 0 \quad i = 1, \ldots, K$$
$$X \in C_s$$

where w_i denotes the weight of the i-th fuzzy goal; μ_i is a model (moderate or intermediate) variable which determines the degree of MFs for the i-th fuzzy goal; ni and pi are the negative and positive (moderating) deviational variables; X is the decision vector including all of the variables that are to be decided numerically; Δ_i^L and Δ_i^R are the MAVs for the left (L, increasing) and right (R, decreasing) MFs, respectively; and define the lower and upper bounds of the interval of total satisfaction for the trapezoidal MF; and Cs is an optional set of hard constraints, as found in the traditional linear programming (LP) approach.

In context of MCGP, to cope with the uncertainty, the decision-maker (DM) can set multiple discrete values as the possibly desired ALs for each goal (i.e., the MCALs) to find the most satisfying solution set during the solution determination process. Compared with the previous two series of F-MODM models, both the application context and the logic behind this approach are quite distinct. MCGP allows a DM to set MCALs for each goal, and in its original form, a goal can be associated with several possible discretely spanned ALs to avoid under or overestimating the real level that a left-hand side criterion function can achieve. The right-hand side of each goal criterion function is a summation of several multiplicative terms, each of which is associated with a possible choice of AL, and this involves the introduction of some binary variables to control ‘which choice of AL ought to be (properly) selected (and appear) when the optimal solution is obtained. So, in a broader sense, it is also an F-MODM approach that fits the unique decision context. The general model of MCGP with the WAM concept integrated can be mathematically expressed as follows:

(MCGP)

$$\min \quad \sum_{i=1}^{K} w_i \left(p_i + n_i\right)$$
$$\text{s.t.} \quad (AX)_i + n_i - p_i = \sum_{j=1}^{n} b_{ij} S_{ij}(B) i = 1, \ldots, K$$
$$S_{ij}(B) \in R_i(x) = 1, \ldots, K$$
$$n_i, p_i \ge 0 i = 1, \ldots, K$$

$$X \in C_s$$

where $S_{ij}(B)$ represents a function of binary serial number; b_{ij} is the j-th possible AL (targets) w.r.t. the i-th goal; $R_i(x)$ is the function of resource limitations; n_i and p_i are, respectively, the negative and positive (moderating) deviational variables for the i-th goal; w_i is a model parameter, connoting the priority for the i-th goal, in terms of goal weight; and X is the decision vector including all of the variables that are to be decided numerically.

The rapid development of MCGP has produced many extensions in both modeling and formulation methods. In earlier developments of MCGP, the subject matter to be fuzzified was on the right-hand side of the goal criteria constraints, which is the AL. This description is true for the initial model above (i.e., the MCALs that will possibly be chosen are discrete), the revised MCGP model (Chang, 2008), and the improved model which 'connects the dots' (i.e., several MCALs of a goal) as a continuous span like a utility function (Chang, 2011). Later modeling works have mainly extended the MCAL concept of MCGP to solve or formulate other types of problems. Some of these models are multi-segment GP (MSGP), multi-coefficient GP, and percentage GP (%GP) models (Liao, 2009; Chang et al., 2012a; Chang et al., (2012b)), among others (Jadidi et al., (2015)). As of this thesis's writing, few relevant works have attempted to integrate MCGP with FGP, which is the other main model series of F-MODM.

To the best of the authors' knowledge, the model proposed by Tabrizi et al. (2012) was the first to merge the MCAL concept into F-MODM. This study will call their model '(FP-MCGP-Tabrizi-et-al)'. In their model, the authors adopted the max-min approach proposed by Zimmermann (1978) to tackle the 'multi-choice yet fuzzy goals' (FMCGs), and their study is renowned for its pioneering contribution to merging these models. However, their model supported only one type of MF, the TMF. Before the model was proposed, many studies had voiced doubts about this approach, including those by Martel & Aouni (1998), Chen and Tsai (2001), Pal & Moitra (2003), and Yaghoobi & Tamiz (2007b). Although this model's limited supported types of MFs might be due to the fact it follows the style of the F(L/NL)P series of studies (e.g., the earlier initial study of fuzzy LP, or FLP, by (Tiwari et al., (1987)), such a drawback deterred its use in practical applications because the various fuzzy preferences of the DM could not be expressed comprehensively. More critically, because the (FP-MCGP-Tabrizi-et-al) model originated from and followed the F(L/NL)P model series of F-MODM, it aimed to optimize the utility values of goals interpreted from the MF values, so the objective function of the model maximized the total aggregated utility. Thus, (FP-MCGP-Tabrizi-et-al) should, methodologically speaking, be seen as an 'FP+MCGP' model, in contrast to its given description as an 'FMCGP' model, because a 'true FMCGP' model should be a 'true GP' model whose objective is to minimize the sum of deviations incurred due to the distance between the value of a goal criteria function and the AL (target value) of that goal.

The drawback of having a lack of supported MF types in the (FP-MCGP-Tabrizi-et-al) model was promptly addressed in another model that also took the FP+MCGP approach (Mouslim et al., 2014).

In the present study, that model is called '(FP-MTL-Mouslim-et-al)' or simply '(FP-MTL)', because it integrated the concept of 'multiple target levels' (MTL), which should be equivalent to MCAL. Due to space limitations, only the (FP-MTL) model is discussed in this study, omitting the (FP-MCGP-Tabrizi-et-al) model. For a clear illustration, in the constraints section, only the overlapped function of both models is presented in the constraints section, which are the constraints designed to handle goals that have the TMF type. This also helps in understanding the (FP-MCGP-Tabrizi-et-al) model.

(FP-MTL-Mouslim-et-al)

$$\begin{aligned}
\max \quad & \sum_{i=\beta+1}^{\delta} U_i \\
\text{s.t.} \quad & (AX)_i - p_i + n_i = \sum_{l=1}^{r} \lambda_{il} b_{il}, \forall i = (\beta+1),\ldots,\delta \\
& \sum_{l=1}^{r} \lambda_{ij} = 1, \forall i = (\beta+1),\ldots,\delta \\
& U_i = 1 - \left[\sum_{l=1}^{r} \lambda_{il} \left(p_i/\Delta_{il}^{+} + n_i/\Delta_{il}^{-} \right) \right], \forall i = (\beta+1),\ldots,\delta \\
& U_i \geq 0, \forall i = (\beta+1),\ldots,\delta
\end{aligned}$$

where λ_{il} (l=1,...,r) is a binary variable indicating that the model has decided to choose which MCAL (e.g., biL for the L-th MCAL) from a pool of r MCALs (b_{il} $(l = 1,\ldots,r)$) for the i-th goal; t_{il} is the l-th MTL (MCAL) for goal i (i.e., at the peak of the TMF); U_i is the utility of goal i formed for the DM as determined by some decision vector X; Δ_{il}^{+} and Δ_{il}^{+} are, respectively, the MAVs (in F(L/NL)P, also called 'maximum allowable deviations' rather than 'maximum admissible violations') that the DM has set for the l-th MTL (MCAL) of the i-th goal, each of which represents the fuzziness on the side of the TMF; and p_i and n_i are, respectively, the positive and negative distances between the goal criteria function $(AX)_i$ on the left-hand side and the possibly chosen MTL (b_{iL}) (these are similar to the deviational variables in GP; however, they are not used in the objective function for the minimization process but used for the determination of the utility value U_i).

In the above (FP-MTL) model, when there are several MTLs (MCALs) for a goal, the shape of the TMFs that are formulated near these MTLs is similar to that shown in Figure 3.3, which depicts one of the four types of 'common piecewise linear MFs' as summarized in Yaghoobi et al. (2008). Also, in the above model, $(\beta+1)$ and δ are, respectively, the lower and upper bounds of the goal index, which defines which subset of the entire set of goals in the decision problem has the TMF MF type. Furthermore, when '« constraints for fuzzy multi-target goals in terms of the other three types of MF »' are removed from the model, let $\beta = 0$ and $\delta = K$ (total number of goals) and let $\Delta_{il}^{+} = \Delta_{il}^{-}, \forall i, \forall l$, the (FP-MTL) model will degenerate to a special case, which is exactly the initial (FP-MCGP-Tabrizi-et-al) model of the FP+MCGP model series. However, these observations are not as important as the following points.

First, the (FP-MTL) model is still an FP+MCGP model. This classification is justified based on the form of its objective function, wherein the aggregated total utility is maximized. When modeling in this way, the model's given name 'FGP-MTL' leads to ambiguity, as it is similar to that previously discussed with regard to (FP-MCGP-Tabrizi-et-al). One such true GP model should be developed from taking an 'FGP+MTL' approach, instead of the 'FP+MTL' approach. Since, as discussed, the concept of MTL should be equivalent to that of MCAL, the required model should be 'FGP+MCAL' or 'FGP+MCGP'. This is, once again, reflective of the main aim of this study, which is to formulate a true FMCGP model from 'FGP+MCGP' rather than 'FP+MCGP'.

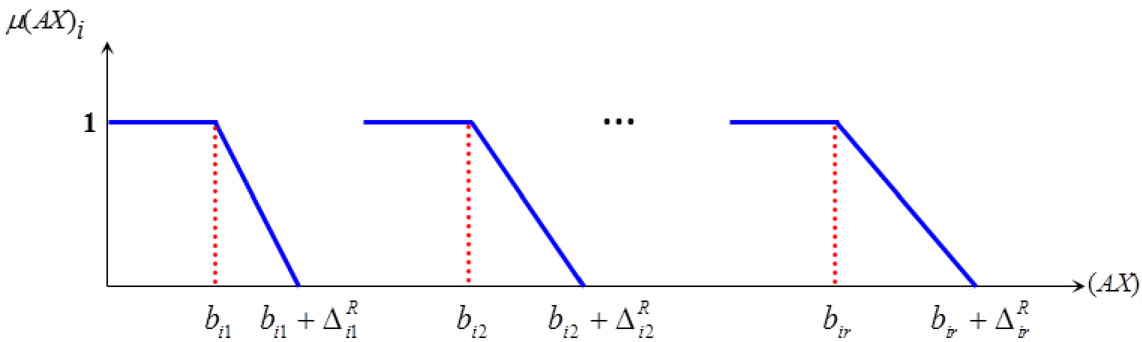
Second, the objective function adopted in (FP-MTL) makes the improvements of this model very difficult, at least in terms of hybridizing it with other existing approaches. In fact, this is one of the main reasons why this field of research has not progressed and gained popularity among researchers. This point has inspired the authors to take the WAM concept for the modeling work of WA-FMCGP into account based on the FGP+MCGP approach and to show how it can easily adopt another GP variant, which is (WGP).

In conclusion, it is notable from past literature that, in order to integrate the concept of MCGP, most of the existing works in F-MODM began with the F(L/NL)P model series. Therefore, methodologically speaking, the resulting models were FP+MCGP models. As relevant model integration studies that begin with the FGP model series (i.e., FGP+MCGP) are rare, the proposed WA-FMCGP model should fill that gap. Furthermore, if the proposed model offers better solution quality (than the FP+MCGP models) for the same F-MODM problem in the encountered highly imprecise decision context, it would be a supplement to F-MODM. The following two sections serve these purposes.

3.3 FUZZY MULI-CHOICE MEMBERSHIP FUNCTIONS

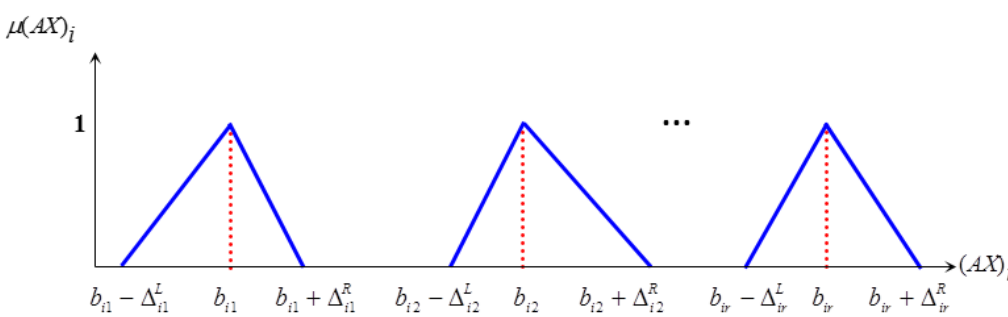
As discussed, in standard GP, the AL for each goal should be predetermined and provided. However, in real-life decision-making, determining precise ALs for goals is a difficult task because there are many situations wherein a DM is unable to specify (or state) those ALs precisely. When the FGP approach was initially developed, Narasimhan (1980) formulated FGP by taking the concept of MF, in which the value of the function is defined on the [0,1] interval, so that it reached '1' when the goal was fully achieved and DMs were totally satisfied; otherwise, it received a value between [0,1). In FGP, the values of these MFs depend on the decision vector (i.e., the values of the decision variables, in the universe of discourse) and are not simply crisps (i.e., either 0 or 1).

As a representative work of F(L/NL)P+MCGP, the study by Mouslim et al. (2014) explored ways to introduce those MTLs to fuzzy MFs. These allowed the fuzziness around the multiple targets (i.e., MCALs) for each goal. In that work, the algebraic structures of the 'four most common linear fuzzy MF types' in Yaghoobi et al. (2008) were adopted and used to formulate the FMCGs. They are outlined in Figures 3.3-3.4, with relevant formulations given. In the equations formulated within Figures 3.3-3.4, Δ^R_{im} and Δ^L_{im} are the MAVs allowed around those MCALs. They are either subjectively chosen by the DM or determined by the tolerances in a technical process (Yaghoobi & Tamiz, 2007a). $S_{im}(B)$ represents a function of the binary serial number, as in the study by Chang (2007); other variables are as defined in FGP.



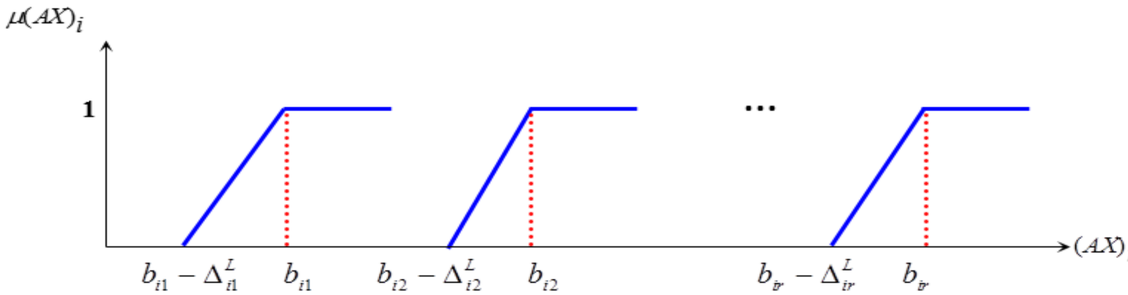
$$\mu(AX)_i = \begin{cases} 1 & (AX)_i \le b_{im1} \\ 1 - \sum_{m=1}^{r} \frac{(AX)_i - b_{imn}}{\Delta^R_{im+1}} S_{imn}(B) & b_{im1} \le (AX)_i \le b_{im+1} + \Delta^R_{im+1} \quad i = 1, \ldots, i_0 \\ 0 & (AX)_i \ge b_{im+} + \Delta^R_{im+1} \end{cases}$$

Figure 3.1: Right-sided multi-target function.



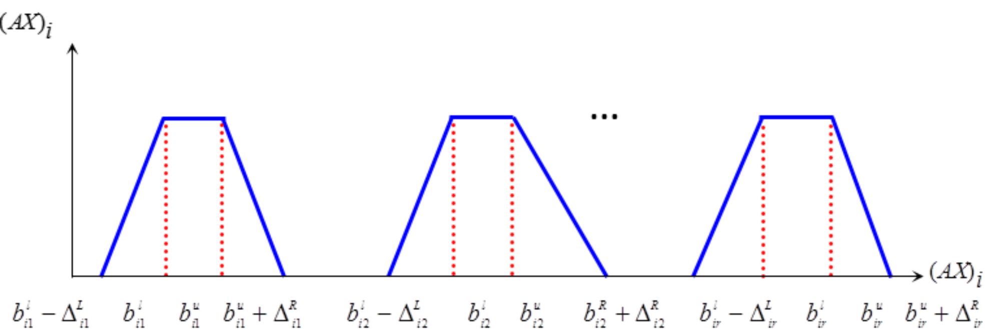
$$\mu(AX)_i \begin{cases} 0 & (AX)_i \le b_{im} - \Delta^L_{im} \\ 1 - \sum_{m=1}^{r} \frac{b_{im} - (AX)_i}{U^L_{im}} S_{im}(B) & b_{im} - \Delta^L_{im} \le (AX)_i \le b_{im} \quad i = j_0 + 1, \ldots, k_0 \\ 1 - \sum_{m=1}^{r} \frac{(AX)_i - b_{im}}{\Delta^R_{im}} S_{im}(B) & b_{im} \le (AX)_i \le b_{im} + \Delta^R_{im} \\ 0 & (AX)_i \ge b_{im} + \Delta^R_{im} \end{cases}$$

Figure 3.3: Triangular multi-target functions.



$$\mu(AX)_i = \begin{cases} 1 & (AX)_i \geq b_{im1} \\ 1 - \sum_{m=1}^{r} \frac{b_{im} - (AX)_i}{\Delta_{imn}^{L}} S_{im}(B) & b_{im} - \Delta_{imn}^{L} \leq (AX)_i \leq b_{im} \\ 0 & (AX)_i \leq b_{imn} - \Delta_{im}^{L} \end{cases}$$

Figure 3.2: Left-sided multi-target function.



$$\mu(AX)_i \begin{cases} 0 & (AX)_i \leq b_{im}^{l} - \Delta_{im}^{L} \\ 1 - \sum_{m=1}^{r} \frac{b_{im}^{l} - (AX)_i}{\Delta_{im}^{L}} S_{im}(B) & b_{im}^{l} - \Delta_{im}^{L} \leq (AX)_i \leq b_{im}^{l} \\ 1 & b_{im}^{l} \leq (AX)_i \leq b_{im}^{u} \\ 1 - \sum_{m=1}^{r} \frac{(AX)_i - b_{im}^{u}}{\Delta_{im}^{R}} S_{im}(B) & b_{im}^{u} \leq (AX)_i \leq b_{im}^{u} + \Delta_{im}^{R} \\ 0 & (AX)_i \geq b_{im}^{u} + \Delta_{im}^{R} \end{cases}$$

Figure 3.4: Trapezoidal multi-target function.

The proposed WA-FMCGP model should also consider at least these four types of MF during modeling.

3.4 MODEL FORMULATION

Following the symbolic conventions, the core concepts of the proposed WA-FMCGP can be sorted by reference to FGP, and MCGP. Doing so entails writing down the following semantic model that optimizes the K multi-choice yet fuzzy goals simultaneously. This serves as a general basis of the problem to be modeled and solved:

(WA-FMCGP-Semantic)

OPTIMISE in terms of $wi, i \in \{1, \ldots, K\}$

$$
\begin{array}{ll}
(AX)_i \lesssim b_{i1} \text{ or } b_{i2} \text{ or, .., or } b_{ir} & i = 1, \ldots, i_0 \\
(AX)_i \gtrsim b_{i1} \text{ or } b_{i2} \text{ or, .., or } b_{ir} & i = i_0 + 1, \ldots, j_0 \\
(AX)_i \cong b_{i1} \text{ or } b_{i2} \text{ or, ,.., or } b_{ir} & i = j_0 + 1, \ldots, k_0 \\
(AX)_i \tilde{\in} \left[b^l_{i1}, b^u_{i1}\right] \text{ or } \left[b^l_{i2}, b^u_{i2}\right] \text{ or, ,..., or } \left[b^l_{ir}, b^u_{ir}\right] & i = k_0 + 1, \ldots, K \\
X \in C_s &
\end{array}
$$

where OPTIMISE means finding the optimal decision vector X such that all fuzzy goals are satisfied (Hannan, 1981; Yaghoobi & Tamiz, 2007a); $(AX)_i$ is the matrix multiplication format (Chang et al., 2012b) of the criterion function for the i-th objective; for i, $i = 1, \cdots, k_0$, b_{im} is the multiple imprecise (possible) ALs for the i-th fuzzy goal, where $m = 1, \ldots, r$; for $i, i = k_0 + 1, \cdots, K$, b^l_{im} and b^u_{im} denote the imprecise lower and upper bounds for the i-th fuzzy goal, respectively, where $m = 1, \ldots, r$; C_S is an optional set of hard constraints, as found in traditional LP; the operator symbols indicate the type of a fuzzy goal, while $\lesssim$, $\gtrsim$, and $\cong$, respectively, denoting that the i-th goal's LHS criterion functional value 'is approximately less than', 'is approximately greater than', and 'is approximately equal to' the MCALs (b_{ir}) of this goal on the RHS; likewise, operator $\tilde{\in}$ denotes that the i-th goal's LHS value 'is approximately between' some intervals given for the MCALs of this goal, while each of the closed intervals is delimited by an upper bound and a lower bound, i.e., $[b^l_{ir}, b^u_{ir}]$.

The general (WA-FMCGP) model is formulated based on (WA-FMCGP-Semantic). It is formulated with reference to the features of FGP and MCGP, while the method for dealing with the goal weights is determined by reference to the work of Yaghoobi, Jones, and Tamiz (Yaghoobi et al., 2008), which strictly followed Romero's (WGP) model in integrating WGP and FGP. In brief, the model aims to minimize the weighted-additive summation of deviations induced by the distance from the criterion function to the possible MCALs of each goal, while there also exists some fuzziness around each AL target. Therefore, the proposed WA-FMCGP model is formulated as follows:

(WA-FMCGP)

$$
\begin{array}{lll}
\min & \sum_{i=1}^{i_0} w_i p_i l^R_i + \sum_{i=i_0+1}^{j_n} w_i n_i t^L_i + \sum_{i=j_0+1}^{K} w_i \left(n_i t^L_i + p_i t^R_i\right) & \\
\text{s.t.} & (AX)_i - p_i \leq \lambda_{i1} b_{i1} \text{ or } \lambda_{i2} b_{i2} \quad \text{or}, \ldots, \text{ or } \lambda_{ir} b_{ir} & i = 1, \ldots, i_o
\end{array}
$$

$$
\begin{aligned}
&(AX)_i + n_i \geq \lambda_{i1} b_{i1} or \lambda_{i2} b_{i2} \quad or, \ldots, or \lambda_{ij} b_{ir} \quad i = i_0 + 1, \ldots, j_o \\
&(AX)_i + n_i - p_i = \lambda_{i1} b_{i1} or \lambda_{i2} b_{i2} or,,.or \lambda_{ir} b_{ir} \quad i = j_0 + 1, \ldots, k_o \\
&(AX)_i - p_i \leq \lambda_{i1} b_a^u or \lambda_{i2} b_i^u or,,.,or \lambda_{ir} b_{ir}^u \quad i = k_0 + 1, \ldots, K \\
&(AX)_i + n_i \geq \lambda_{i1} b_a^l or \lambda_{i2} b_{k2}^l \quad or, \ldots, or \lambda_{ir} b_{ir}^l \quad i = k_0 + 1, \ldots, K \\
&t_i^R = \left(\lambda_{i1} / \Delta_{i1}^R\right) + \left(\lambda_{72} / \Delta_{i2}^R\right) +, \ldots, + \left(\lambda_{ir} / \Delta_{ir}^R\right) \quad i = 1, \ldots, i_0 \\
&t_i^L = \left(\lambda_{i1} / \Delta_{i1}^L\right) + \left(\lambda_{i2} / \Delta_{i2}^L\right) +, \ldots, + \left(\lambda_{ir} / \Delta_{ir}^L\right) \quad i = i_0 + 1, \ldots, \dot{b}_s \\
&t_i^R = \left(\lambda_{i1} / \Delta_{i1}^R\right) + \left(\lambda_{i2} / \Delta_{i2}^R\right) +, \ldots, + \left(\lambda_{ir} / \Delta_{ir}^R\right) \quad i = j_0 + 1, \ldots, K \\
&t_i^L = \left(\lambda_{i1} / \Delta_{i1}^L\right) + \left(\lambda_{i2} / \Delta_{i2}^L\right) +, \ldots, + \left(\lambda_{ir} / \Delta_{ir}^L\right) \quad i = j_0 + 1, \ldots, K \\
&\mu_i + p_i t_i^R = 1 \quad i = 1, \ldots, i_o \\
&\mu_i + n_i t_i^L = 1 \quad i = i_0 + 1, \ldots, j_o \\
&\mu_i + n_i t_i^L + p_i t_i^R = 1 \quad i = j_0 + 1, \ldots, K \\
&\lambda_{i1} + \lambda_{i2} +, \ldots, \lambda_{ir} = 1 \quad i = 1, \ldots, K \\
&\lambda_{ir} \in \{0, 1\} \quad i = 1, \ldots, K \\
&\mu_i, n_i, p_i \geq 0 \quad i = 1, \ldots, K \\
&X \in C_s
\end{aligned}
$$

where w_i denotes the weight of the i-th fuzzy goal and μ_i is a moderate variable which represents the utility level that is derived from the degree of MF for the i-th fuzzy goal; $(AX)_i$ is the matrix multiplication format of the criterion function for the i-th objective; X is the decision vector including all the variables that are to be decided numerically; pi and ni are the positive and negative (moderating) deviational variables, respectively; $\lambda_{i1}, \lambda_{i2}, \cdots, \lambda_{ir}$ are the binary variables which ensure that one AL must be chosen for each goal i; Δ_i^R and Δ_i^L are, respectively, the MAVs that are allowed on the right and left sides of the MF binding to the r-th MCAL of the i-th goal (i.e., b_{ir}); t_i^R and t_i^L are continuous variables which represents a coefficient that can be regarded as the 'equivalent concentration' or 'normality' in e.g., chemistry, that is to be multiplied with the determined w_i prior to the solution time and/or the absolute distance from the criterion function value that is determined for the i-th goal ((AX*)i) to the default (original) AL value (target value) that is selected by using $\lambda_{i1}, \lambda_{i2}, \cdots, \lambda_{ir}$ during solution time (i.e., p_i and/or n_i); and the other variables are as defined previously.

When examining the above model concerning the objective measure, the most salient feature is that the model utilizes the WAM concept from WGP to aggregate its objective measure. More importantly, the objective measure aggregates several forms of deviations, positive (p_i), negative (n_i), or both ($n_i + p_i$), according to the DM preferences, and then minimizes the summation of those deviations, which are both weighted and normalized. This feature distinguishes the (WA-FMCGP) mode from previous FP+MCGP models, which maximize the total utility that is totaled without using the WAM concept. Consequently, the model can adopt any minimization procedure which belongs to other GP variants, such as lexicographic GP and meta-GP (Jones and Jimenez, 2013).

Meta GP, with its original name as "[GP]2", allows a DM to set some meta goals and impose those implicit meta goals on existing explicit goals by introducing additional constraints to a GP model's original deviational variables. From the methodological aspect of GP, a meta goal can be achieved utilizing a lexicographic or a weighted structure, as deemed appropriate by the DM. In terms of the mathematical formulation of GP, there are three types of meta-goals (Rodriguez-Uria et al., 2002):

Type 1 meta-goal: The sum of percentages of the unwanted deviations (against the AL) should not be greater than a certain bound, $Q^{(1)}$:

$$\sum_{i \in s_i^{(1)}} w_i \frac{d_i}{b_i} \leq Q_K^{(1)}$$

Type 2 meta-goal: The maximum percentage of deviation of some goals of concern (i.e., the watched goals) should not be greater than a certain bound, $Q^{(2)}$:

$$w_i \frac{d_i}{b_i} - D_i \leq 0, \quad i \in s_j^{(2)} \quad D_i \leq Q^{(2)}$$

Type 3 meta-goal: Among all or some watched goals, the percentage of the unachieved goals (against the total number of goals) should not be greater than a certain bound, $Q^{(3)}$:

$$\begin{cases} d_i - R_i y_i \leq 0, \quad i \in S_r^{(3)} \\ \dfrac{\sum_{i=1}^{s} y_i}{\text{card}\left(S_r^{(3)}\right)} \leq Q_r^{(3)} \\ y_i \in \{0, 1\}, \quad i \in S_r^{(3)} \end{cases}$$

here w_i is a preferential weight for each imposed explicit goal considered in $s_k^{(1)}$; d_i is every undesired deviation from each imposed explicit goal considered in $s_l^{(2)}$; b_i is the AL (target) of each imposed explicit goal considered in $s_l^{(2)}$ and can be viewed as a normalization constant; D is an extra continuous variable that measures the maximum deviation; y_i is a binary variable for each imposed explicit goal i considered in $s_r^{(3)}$, and R_i is a sufficiently large arbitrary number.

As seen above, introducing meta-GP can improve on the drawbacks of missing goal priority for any GP variant in F-MODM, so too for the proposed (WA-FMCGP) model. In addition to the WAM concept that has been used and formulated to realize the goal priority, the resulting (Meta WA-FMCGP) model can offer more flexibility in manipulating DMs' preferences over FMCGs. Thus, this hybrid model can be formulated as follows:

(Meta WA-FMCGP)

$$\min \quad \left\{\varphi_1^{(1)}, \ldots, \varphi_{r1}^{(1)}, \varphi_1^{(2)}, \ldots, \varphi_{r2}^{(2)}, \varphi_1^{(3)}, \ldots, \varphi_{r3}^{(3)}\right\}$$

$$\text{s.t.} \quad \sum_{i=1}^{i_0} w_i p_i t_i^R + \sum_{i=i_0+1}^{i_0} w_i n_i t_i^L + \sum_{i=j_0+1}^{K} w_i \left(n_i t_i^L + p_i t_i^R\right) + \sigma_s^{(1)} - \varphi_s^{(1)} = Q_s^{(1)}, s = 1, \ldots, r1$$

$$\sum_{i=1}^{i_0} w_i p_i t_i^R + \sum_{i=i_0+1}^{h_0} w_i n_i t_i^L + \sum_{i=j_0+1}^{K} w_i \left(n_i t_i^L + p_i t_i^R\right) - D_l \le 0, i \in s_l^{(2)}, l = 1, \ldots, r2$$

$$D_l + \sigma_l^{(2)} - \varphi_l^{(2)} = Q_l^{(2)}, l = 1, \ldots, r2$$

$$\begin{cases} d_i - R_i y_i \le 0, i \in S_r^{(3)} \quad r = 1, \ldots, r3 \\ \dfrac{\sum_{i=1}^{s} y_i}{\text{card}\left(S_r^{(3)}\right)} + \sigma_k^{(3)} - \varphi_k^{(3)} = Q_r^{(3)} \quad r = 1, \ldots, r3 \\ y_i \in \{0, 1\}, i \in S_r^{(3)} \quad r = 1, \ldots, r3 \end{cases}$$

where $\sigma_s, \sigma_l, \sigma_r, \varphi_s, \varphi_l$ and φ_r are the meta positive and negative deviation variables for the meta goals; all other variables are as defined in previous discussions.

3.5 A REAL APPLICATION IN RENEWABLE ENERGY SITE SELECTION

This section verifies the WA-FMCGP model's applicability and demonstrates its practical effectiveness by tackling a RESS problem. According to the collected data of Algeria's annual wind speed, the southwestern region has the greatest potential for green electricity generation. This information is shown in Figure 3.5 (Chellali et al., 2011; Zhuang and Hocine, 2018). The five places (i.e., the alternative cities that have wind farms in-production or pilot running) under consideration by DMs for the construction of wind farm capacity expansion are as follows: Tindouf, Adrar, Insala, Hassi R'Mel, and Bordj Badji Mokhtar. These cities are shown and marked in Figure 3.6.

A total of four criteria were considered to facilitate the decision of selecting the most suitable location for installing turbines to expand the wind farm capacity. The criteria were as follows: 'electricity generation', 'unit cost energy', 'topography and infrastructure', and 'social acceptability'. Table 3.1 provides the criteria weights and the performance (vector) for each location.

Many criteria are used in RE planning to measure the performance of RE sources. However, since several quantitative and qualitative criteria have been adopted and because DM judgments are not crisp, it is relatively difficult for DMs to provide precise numerical values for their preferences, particularly for qualitative criteria (e.g., social acceptability). Such criteria are generally expressed by linguistic variables (such as "high" or "moderate") and then converted to scale-based numerical values (e.g., 5 and 4 for "very high" and "relatively high"). Therefore, assigning them a single crisp value (or parameter) is unsuitable. While it is generally accepted that random variables (e.g., realization time) should be treated with stochastic methods, consensus fails on whether there is enough data available, which is key to formulating probabilistic distribution (see Kirkwood, 1992). Unfortunately, in the design stage of new systems, the information to have precise probability distributions of these probabilistic variables is not always sufficient due to immeasurability or assumptions. The robust optimization theory has been developed as a supporting tool to mitigate the impact of such uncertainty.

However, robust optimization methodologies have a critical drawback. According to Bertsimas et al. (2013), most robust optimal solutions are highly conservative, aiming to hedge against all possible worst-case realizations of the uncertainty. In some cases, this can lead to overly conservative solutions.

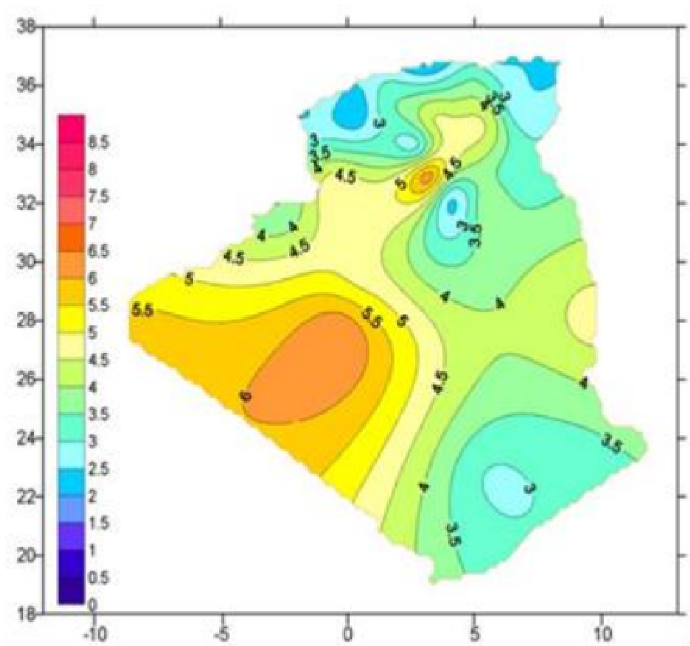


Figure 3.5: Map of annual wind speed in Algeria.

Figure 3.6: Alternative locations for wind farm site expansion construction.

Table 3.1: Evaluating the possible wind-farm sites for expansion construction according to the considered criteria.

Potential locations (decision variables mapping)	Criteria (Criteria Set)			
	Energy generation (a) (GW)	Unit cost energy (a)	Social acceptability	Topography and Infrastructure
	Weights (Criteria Weight Vector, or CWV)			
	0.4	0.3	0.15	0.15
Tindouf (X_1)	42.82	0.1310	6.5	5
Adrar (X_2)	49.01	0.1145	7	6
Insala (X_3)	49.08	0.1143	6	5.5
Hassi R'Mel (X_4)	58.68	0.0956	5	8
Bordj Baji Mokhtar (X_5)	57.56	0.0975	5	7

(a)Obtained from Benmemdejahed and Mouhadjer (2016).

In Table 3.1, the decision variables of the problem are defined as follows:

$$X_j = \begin{cases} 1, & \text{if location } j \text{ is selected,} \\ 0, & \text{otherwise.} \end{cases}$$

where $j = 1$ for Tindouf, $j = 2$ for Adrar, $j = 3$ for Ain Salah, $j = 4$ for Hassi R'Mel and $j = 5$ for Bordj Badji Mokhtar.

For example, if a RE plant is designed too conservatively, then the project may be too expensive or needlessly complex. In such cases, it is assumed that these random variables are controllable and that they are addressed through other uncertainty theories (such as fuzzy set or interval), which are more appropriate than their alternatives because they require less information. Thus, due to limited decision practices, the DM is not entirely sure about the precise values of the ALs (targets) for the goals but has stated them in an FMCG fashion. Specific to the studied RESS decision case here, the possible MCALs for each FMCG and its type of fuzziness (when expressed in terms of the MF) are described in Table 3.2. The MAVs around each MCAL (target value) are listed in Table 3.3.

Table 3.2: Possible aspiration levels (ALs) and the MF type for/of each FMCG.

The FMCGs	Type of goal	Multi-choice aspiration level (MCAL)
FMCG1: Energy generation (GW)	Left	60 **or** 65
FMCG2: Unit cost energy	Right	0.0945 **or** 0.0950
FMCG3: Social acceptability	Triangular	6.5 **or** 7
FMCG4: Topography and infrastructure	Trapezoidal	[4.5,7] **or** [5,8]

Table 3.3: Admissible violations (tolerances) around each possible AL for each fuzzy goal.

Goals	Tolerances (MAVs) around the possible MCALs
Energy generation	15 **and** 10
Social acceptability	(15,10) **and** (15,20)
Topography and infrastructure	(1.1,1.1) **and** (1.2,1.2)

Thus, following the semantic style used to express the (P-Exemplar), the encountered RESS decision problem can be stated as follows:

(P-RESS)

OPTIMISE in terms of $w_i, i \in \{1, \ldots, K\}$

$FMCG1: 0.1310X_1 + 0.1145X_2 + 0.1143X_3 + 0.0956X_4 + 0.0975X_5 \geq 0.0945$ **or** 0.0950

FMCG2 : $42.82X_1 + 49.01X_2 + 49.08X_3 + 58.68X_4 + 57.57X_5 \leq 60$ **or** 65

$FMCG3: 6.5X_1 + 7X_2 + 6X_3 + 5X_4 + 5X_5 \cong 6.5$ **or** 7

$FMCG4: 5X_1 + 6X_2 + 5.5X_3 + 8X_4 + 7X_5 \cong [4.5, 8]$ **or** $[5, 8]$

$s.t., X_1 + X_2 + X_3 + X_4 + X_5 = 1$ and The MAVs around each possible AL in Table 3.3.

The above (P-RESS) problem is modeled using WA-FMCGP, with constraints taking model parameters from Tables 3.1-3.2, and 3.3 under the EWI assumption. The (P-RESS-WA-FMCGP) model was then solved using LINGO, and the optimal solution set obtained is as follows:

(**Solution Set I**)

- Decision vector (variable values): $X^* = (X_1, X_2,_X 3, X_4, X_5) = (0, 0, 0, 1, 0)$;
- Degrees of MFs (also the utilities interpreted): $(\mu_1, \mu_2, \mu_3, \mu_4) = (0.91, 0.98, 0.76, 1)$;
- Deviational variable values: $(n_1, p_2, n_3, p_3, n_4, p_4) = (1.32, 0.0006, 3.5, 0, 0, 0)$.

The obtained solution for the decision vector shows that Hassi R'Mel is the best location for expanding wind farm capacity in Algeria (i.e., $X_4 = 1$). This desired knowledge is the most important empirical implication for both the DM and wind-farm site construction practitioners.

Thus, the solution set, when subject to the constraints given, fully satisfies the DM in terms of only one goal (FMCG4). However, each of the goals can be achieved to a satisfaction level of '≥90%' in terms of the MF-degree measure, except for the third goal (FMCG3). In addition, using this optimal solution achieves energy generation (FMCG1), unit cost energy (FMCG2), topography and infrastructure (FMCG3), and social acceptability (FMCG4) with a pro-rata average of 91%. As in the (WA-FMCGP) model, the degree of MF provides a utility interpretation for each goal; so in terms of total utility, adopting the capacity expansion strategy suggested by the solution set may yield a total utility level of 3.65 (out of 4). A final observation is that when exercising a capacity expansion plan according to this RESS solution, the DM can expect a total deviation of only 4.8206.

All of these results mean that DMs should be highly satisfied with the choice suggested by the (P-RESS-WA-FMCGP) model because the strategic plan closely follows their ideals.

However, the selection of Hassi R'Mel to be the location for wind farm expansion is not an applicable scenario, at least in the short and medium terms. This is because this area is one of the most profitable oil fields in Algeria. Thus, replacing (covering) this area with wind farms will directly affect government revenue (Himri et al., 2009) while simultaneously increasing negative effects regarding social acceptability (Kraft & Kraft, 1978). Therefore, some sensitivity analysis for evaluating whether a solution can further improve is required, and there are numerous possible methods. For example, examining further consequences by either changing the weights of the criteria accordingly or asking DMs to provide additional information about their preferences to control the achievement statuses of goals using the deviation variables is appropriate. This study made the latter sensitivity analysis.

Suppose a DM would like to see the potential solutions when some restrictions to the deviation variables have been imposed because the DM has a proposed meta goal. Based on Solution Set (I), the DM stated that "social acceptability should be fully satisfied". In terms of methodology, this is doubtless a meta goal.

With such a meta goal in mind, and formulating the problem above using the Meta WA-FMCGP, the following optimal solution set can be obtained:

(**Solution Set II**)

- Decision vector (variable values): $X^* = (X_1, X_2, _X3, X_4, X_5) = (0, 1, 0, 0, 0)$;
- Degrees of MFs (also the utilities interpreted): $(\mu_1, \mu_2, \mu_3, \mu_4) = (0.267, 0.35, 1, 1)$;
- Deviational variable values: $(n_1, p_2, n_3, p_3, n_4, p_4) = (10.99, 0.019, 0, 0, 0, 0)$.

The results show that achieving full social acceptability is addressed, and the Meta WA-FMCGP model achieves this watched goal completely. Meanwhile, the potential location for wind farm expansion has moved to Adrar, which is also a viable option if and only if achieving the social acceptability decision criterion (FMCG3) is the only concern. However, in contrast to Solution Set (I), making a new RE construction expansion plan according to this different solution set would strongly erode the goal achievement statuses of both FMCG1 (for energy generation throughput) and FMCG2 (for the unit cost for energy production), the DM should consider the trade-offs.

When dealing with MODM problems, the concept of Pareto (Paretian) efficient solutions should be considered. However, among the various MODM methods, GP was not developed to obtain a single or all non-dominated solutions but as a method for locating satisfactory solutions on the optimal frontier in the objective (goal) space for complex, real-world problems. The extensive study by Zhuang & Hocine (2018) supported just this logic, inspecting how to explain and choose the most satisfactory solution from several optimal solutions that are exactly on, or near, the efficiency frontier. Therefore, obtained GP solutions should be judged solely on how well they meet the goals of the DM and whether they produce a practical solution to the decision problem (Jones & Tamiz, 2010; Romero, 1991). Either (WA-FMCGP) or the (Meta WA-FMCGP) model proposed in this study serves these purposes because they are hybrid GP models in nature and are formulated on a commensurable basis in that all of the integrated 'sub-models' are GP models in the intrinsic. So, based on such properties, it should be noted that some concepts of the so-called 'efficiency detection and restoration techniques' (see Tamiz & Jones, 1996) can be introduced and explored in future works.

3.6 CONCLUSIONS

This chapter proposed a novel F-MODM model for solving a challenging and uncertain MCDM problem with several conflicting goals. In the cases modelled and solved, the decision involved imprecise goal targets whose form are quite complicated (i.e., the FMCGs with MAVs); that is, DMs could not state their preference about goal targets precisely but could specify several MCALs for each goal, the type and 'direction' of goal fuzziness and the MAVs alongside each possible MCAL allowed for the fuzziness. The (WA-FMCGP) model was established and proposed subject to this decision context.

As the (WA-FMCGP) model seamlessly integrates WGP, FGP, and MCGP, it can function as a comprehensive model that can deal with many uncertain decision cases wherein the goal targets are highly imprecise. It is a 'true hybrid GP' model because of the form of its objective function, wherein the unwanted deviations incurred are normalised and aggregated by considering the WAM concept and because all three of the integrated models may share a commensurable basis, which is GP. It is also a 'true FMCGP' model because, unlike the existing models which took the FP+MCGP approach during their formulation (i.e., integrating the MCAL concept with FP, while the objective function is to minimise total utility without the use of the WAM concept), this study takes the FGP+MCGP approach (i.e., integrating the MCAL concept with FGP). As F(L/NL)P and FGP are usually regarded, methodologically speaking, as the two main model series for F-MODM, these two model series are heterogeneous in their origins. As such, these two hybrid approaches (i.e., F(L/NL)P+MCGP and FGP+MCGP) differ. This difference addresses the initial novelty of this study for the fields of FGP (Mirzaee, 2018), GP (Tanino et al., 2013), F-MODM, MODM (de Oliveira et al., 2018; Afrapoli et al., 2019), and thus of multi-objective optimisation (Tsionas, 2019) in the broadest sense, in that while there have been studies of FP+MCGP, studies of FGP+MCGP are rare. The proposed (WA-FMCGP) model fills this gap. Note that some recent developments were intentionally selected and used here for contradistinction.

Solutions to an empirical RESS decision further verified the practical applicability of the proposed model using real data as the model parameters. Unlike the problem of designing a suitable power generation portfolio for the initial construction work undertaken at several known places given a large budget, the problem in question required determining one suitable place for capacity expansion subject to a small annual budget (relative to the initial construction fund deposited). However, as the use of such 'relatively small but still large' budget is usually critical for sustainable development (a wrong decision might lead to unrecoverable consequences), the DM preferences should be treated very carefully every time when one such decision is to be made. The resulting model parameterised based on (WA-FMCGP) was shown to be effective for the encountered RESS decision by suggesting installing additional wind turbines at the most suitable site in Hassi R'Mel (among the five places with in-production or pilot-running turbines). From the perspective of decision-making practices, such an application scenario addresses another niche of the study: a real application of the proposed model was explored (i.e., WA-FMCGP is practically effective) and the application of an approach to a new problem domain (i.e., applying MODM, rather than MADM, to RESS) was provisioned. This result may extend the knowledge base for renewable energy (construction) planning in that the literature is abundant with MADM methods (Ehrgott and Gandibleux, 2002; Figueira et al., 2005) and that these methods have been applied to site selection problems in renewable energy planning (Wang et al., 2009; San Cristóbal, 2012; Reisi et al., 2018). However, the use of MODM on RESS is scant.

Since the effect of randomness was unexplored in the empirical case study, this would be a topic worthy of further exploration (e.g., using and further integrating the chance constraint technique to explore whether a solution can be further improved upon or not). The probabilistic chance constraint programming (CCP) concept initiated in the 1970s (Charnes & Cooper, 1961; Prekopa, 1970) has been applied to GP later and enriched after year 2000 (De et al., 1982; Ballestero, 2001; Aouni et al., 2012). As can be imagined, integrating the chance constraints into the proposed model and replacing the FGP part of the model with stochastic GP are both viable options. This would lead to a 'Stochastic-MCGP' model, which would (still) be a GP model worthy of inquiry.

4 Fuzzy Interval Goal Programming Approach

We should be taught not to wait for inspiration to start a thing. Action always generates inspiration. Inspiration seldom generates action.
-Frank Tibolt-

This chapter proposes a novel MODM model called fuzzy interval goal programming (FIGP) to release the restrictions of FGP with single-coefficient modeling. The proposed model can formulate an interval coefficient for each decision variable in FGP. In doing so, the concept of multi-choice aspiration levels (MCALs) from the revised multi-choice goal programming (RMCGP) technique is adopted. Specifically, the integrated model considers various types of fuzzy membership functions in real-world problems and offers DMs more flexibility to express and formulate their preferences in terms of fuzzy interval goals. The proposed method is illustrated by selecting the optimal RE portfolio for electricity generation in Italy.

4.1 INTRODUCTION

RE planning usually involves dealing with or listening to experts about their opinions on a particular situation, such as future RE markets, potential new technology, energy policy trends, and analyses by world affairs experts. Although experts are valuable sources of information and knowledge, experiences and global events have proven that they can also be wrong. Not only do they bring in a deep knowledge base and thoughts but also biases, which can complicate the evaluation process for DMs. This issue can affect MCDM paradigms in different areas, such as evaluation criteria, performance scores, weights, DM's preferences, objectives, and goals. When fuzzy set theory is employed to fuzzy RE systems (which are viewed as systems of linguistic variables), it is essential to consider all information regarding the linguistic variables related to both perception and cognition. At this stage, fuzzy set theory allows vague human assessments to be included in the analysis process by using membership functions. The membership functions of fuzzy parameters are commonly constructed based on available expert opinions. The RMCGP is another efficient method to express uncertainty. In the RMCGP technique, parameters are defined by a range of interval values with upper and lower limits. In many research fields, the fuzzy method and RMCGP technique are applied separately to conduct uncertainty analysis. However, some complicated planning problems, such as in RE, may consider two or more uncertain values simultaneously. Unfortunately, this gap has not been thoroughly considered by researchers.

The authors often heard opinions from DMs such as, “Under our resource limitations, we suggest that the approximate realization time of project ‘X’ is between 12 and 16 months.” DMs also made statements like, “The approximate cost for this project will be between €307.925 and 467.925.” The reason why DMs initially propose conservative estimations is that they may face multiple challenges achieving their goals (e.g., severe weather or an architect’s reluctance to rework due to errors in executing project activities). These problems cannot be formulated and solved efficiently by current MCDM approaches. The authors developed the proposed FIGP model by hybridizing the FGP and RMCGP techniques to address these limitations.

4.2 RELATED WORKS

To cope with uncertainty in MODM paradigms, the FGP technique has been proved its ability to support and provide the DMs with applicable solutions during the last decades in different research areas (see e.g: Jones & Tamiz (2010), Sasaki et al. (1990), Aouni et al. (2009), Diaz et al. (2018), and Dong & Wan (2018)). Generally speaking, the standard FGP formulations try to minimizes the unwanted deviation between the objective function, and fuzzy aspiration level. The basic FGP models consider for any objective function there exists a real-valued variables defined on the set of feasible regions which the DM associates them with a single, exact and well known value. In other word, each decision variable has only one coefficient or parameter. When we model the conventional FGP, However, in large-scale problems, precisely determining the value of these coefficient demands high cognition effort from the DMs. In practice, there are many decision-making problems in which DMs have only partial information about some parameters. To deal with this problem, i.e., to give the traditional FGP techniques more flexibility to incorporate the uncertainty, one useful tool is the interval analysis techniques proposed by Chang in 2008 (Chang, 2008). Therefore, to give the FGP model more robustness in imprecise environments, this section combines two important techniques—fuzzy set theory and the interval analysis technique—to propose a novel model called fuzzy interval goal programming (FIGP).

It is worth noting that the FIGP model is completely different from the existing literature (e.g., Liao, 2009; 2011); especially the ones that try to merge fuzzy set theory with MCGP (see, Mouslim et al. (2013) and Tabrizi et al. (2012)). Whilst the authors have tried to answer a very interesting question through their approach, this model involves taking the concept of mapping “each fuzzy goal to multiple coefficients.” Table 4.1 summarizes the characteristics of these models and also reports the “supported functions;” so that we can capture explicitly the differences.

Except for Chang’s works, Tabrizi et al. (2012) were the first research to use a fuzzy MCGP (FM-

CGP) formulation. Their model adopted the max-min approach proposed by Zimmermann (1978) to solve an FMCGP problem based on only one type of membership function, the triangular membership function. However, before that model was proposed, such a monotonous treatment of membership functions was criticized by many articles such as Chen and Tsai (2001) and Yaghoobi and Tamiz (2006). The other work to formulate the FMCGP problem was the model proposed by Mouslim et al. (2013), which was called FGP with multi-target level (FGP-MTL). This model obtains a solution by trying to maximize the degree of the membership function as highly as possible.

Table 4.1: FIGP characteristics compared to other models that exist in literature.

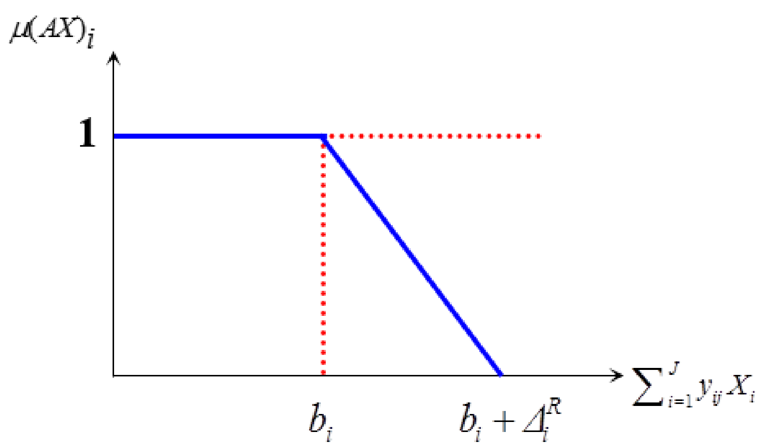
Models	Crisp	Fuzzy	Left-hand-side		Right-hand-side		Type of Membership function used
			Mono	Multiple	Mono	Multiple	
Hannan (1981)		✓	✓				Triangular
Kim & Whang (1998)		✓	✓				Left, right, triangular,
Tiwari et al. (1987)		✓	✓				Left, right, triangular,
Yaghoobi et al. (2008)		✓	✓		✓		Left, right, triangular, trapezoidal
Jimenez et al. (2004; 2018)		✓	✓		✓		Left, right, triangular
Chang (2007; 2008)	✓		✓			✓	
Liao (2009; 2011)	✓			✓	✓		
Mouslim et al.(2014)		✓	✓			✓	Left, right, triangular, trapezoidal
Tabrizi et al. (2012)		✓	✓			✓	Triangular
FIGP model		✓		✓	✓		Left, right, triangular, trapezoidal

Obviously, these two models are constructed based on the initial version of the MCGP model. But unfortunately, the same possible drawbacks of MCGP formulation (as stressed previously) are perhaps inherited (i.e., using the function of a binary serial which is hard to implement) by both models. Therefore, as can be seen from the semantic model, this study incorporates the concepts from the RMCGP model and uses interval numbers to formulate the fuzziness around the fuzzy goal targets. This approach not only provides alternative ways to conduct FMCGP-relevant research but also broadens the application of the concept of RMCGP. In this expanded approach, the fuzzy goal target in an FGP model itself becomes a moveable window that is automatically adjustable at the solution stage, where the base form of the model has already been a fuzzy model that utilizes the membership function concept. In particular, the proposed FIGP model can be viewed as a continuous version; among other FMCGP models, the fuzzy goal targets are not able to be fuzzified (

Compared with both FGP and RMCGP models, the FIGP model is a more general and flexible approach because it considers a range of interval values under uncertain conditions and in imprecise environments. In particular, the proposed FIGP model allows for and supports the setting of an interval coefficient level for each fuzzy goal. In contrast, FGP has only one coefficient value, and RMCGP does not support imprecise values. In general, the FIGP model incorporates the main advantages of both FGP and RMCGP, while integrating them to overcome their relative limitations.

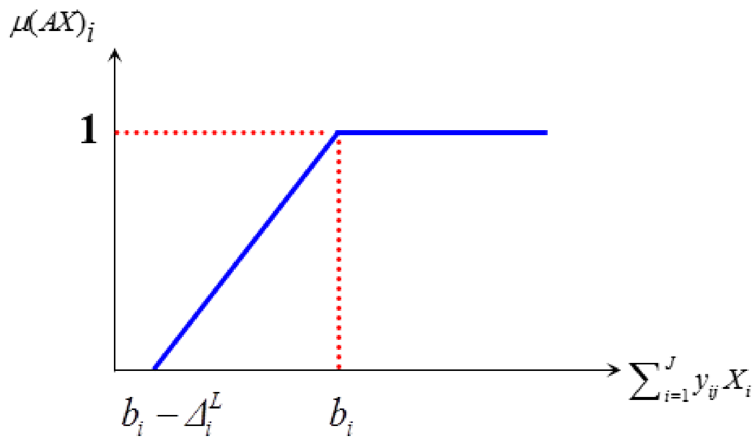
4.3 FUZZY INTERVAL MEMBERSHIP FUNCTION

In real life problems, DMs may face situations of interval coefficients that a LHS criterion function assumes to avoid underestimating the decision. For example, consider the case of the fuzzy interval membership functions shown in Figure 4.1. Given that $y_{ij}X_i \lessapprox b_i$ where $y_{ij} \in [A_{ij,\min}, A_{ij,\max}]$ (the goal's fuzzy target) chooses the most suitable coefficient from the range of interval values that highly satisfies the DM. The authors believe that this case cannot be formulated by current FGP approaches. Inspired by Chang (2008), we propose solving this problem by formulating and proposing four fuzzy interval membership functions as follows (Figure 4.1-4.4):



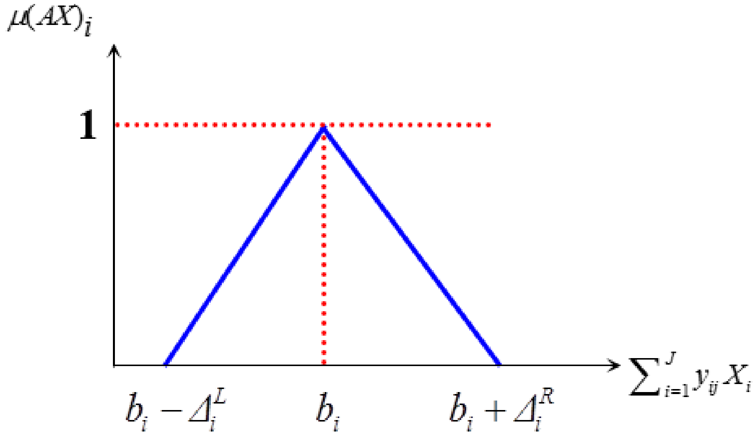
$$\mu_i\left(\sum_{i=1}^{J} y_{ij}X_i\right) = \begin{cases} 1 & \sum_{i=1}^{J} y_{ij}X_i \le b_i \\ 1 - \dfrac{\sum_{i=1}^{J} y_{ij}X_i - b_i}{\Delta_i^R} & b_i \le \sum_{i=1}^{J} y_{ij}X_i \le b_i + \Delta_i^R \quad i = 1,\ldots,i_0 \\ 0 & \sum_{i=1}^{J} y_{ij}X_i \ge b_i + \Delta_i^R \\ \text{where} & y_{ij} \in \left[A_{ij,\min}, A_{ij,\max}\right] \end{cases}$$

Figure 4.1: Right fuzzy interval membership function.



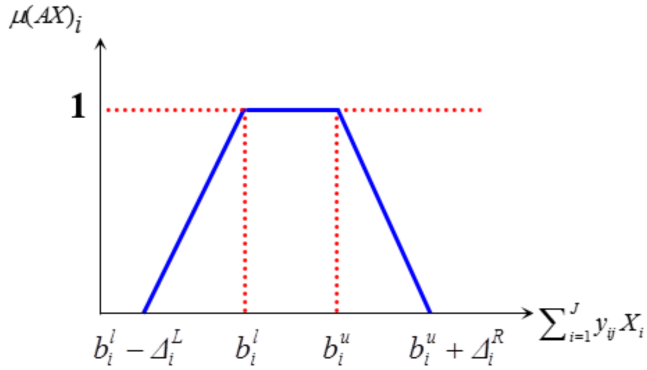
$$\mu_i\left(\sum_{i=1}^{J} y_{ij}X_i\right)_i = \begin{cases} 1 & \sum_{i=1}^{J} y_{ij}X_i \geq b_i \\ 1-\dfrac{b_i-\sum_{i=1}^{J} y_{ij}X_i}{\Delta_i^L} & b_i-\Delta_i^L \leq \sum_{i=1}^{J} y_{ij}X_i \leq b_i \\ 0 & \sum_{i=1}^{J} y_{ij}X_i \leq b_i-\Delta_i^L \\ \text{where} & y_{ij} \in \left[A_{ij,\min}, A_{ij,\max}\right] \end{cases}$$

Figure 4.2: Left fuzzy interval membership function.



$$\mu\left(\sum_{i=1}^{J} y_{ij} X_i\right)_i = \begin{cases} 0 & \sum_{i=1}^{J} y_{ij} X_i \le b_i - \Delta_i^L \\ 1 - \dfrac{b_i - \sum_{i=1}^{J} y_{ij} X_i}{4_i^L} & b_i - \Delta_i^L \le \sum_{i=1}^{J} y_{ij} X_i \le b_i \\ 1 - \dfrac{\sum_{i=1}^{J} y_{ij} X_i - b_i}{4_i^R} & b_i \le \sum_{i=1}^{J} y_{ij} X_i \le b_i + \Delta_i^R \quad i = j_0 + 1, \ldots, k_0 \\ 0 & \sum_{i=1}^{J} y_{ij} X_i \ge b_i + \Delta_i^R \\ \text{where} & y_{ij} \in \left[A_{ij,\min}, A_{ij,\max}\right] \end{cases}$$

Figure 4.3: Triangular fuzzy interval membership function.



$$\mu\left(\sum_{i=1}^{J} y_{ij}X_i\right)_i = \begin{cases} 0 & \sum_{i=1}^{J} y_{ij}X_i \le b_i^l - \Delta_i^L \\ 1 - \dfrac{b_i^l - \sum_{i=1}^{J} y_{ij}X_i}{\Delta_i^L} & b_i^l - \Delta_i^L \le \sum_{i=1}^{J} y_{ij}X_i \le b_i^l \\ 1 & b_i^l \le \sum_{i=1}^{J} y_{ij}X_i \le b_i^u \quad i = k_0+1,\dots,K \\ 1 - \dfrac{\sum_{i=1}^{J} y_{ij}X_i - b_i^u}{\Delta_i^R} & b_i^l \le \sum_{i=1}^{J} y_{ij}X_i \le b_i^u + \Delta_i^R \\ 0 & \sum_{i=1}^{J} y_{ij}X_i \ge b_i^u + \Delta_i^R \\ \text{where} & y_{ij} \in \left[A_{ij,\min}, A_{ij,\max}\right] \end{cases}$$

Figure 4.4: Trapezoidal fuzzy interval membership function.

4.4 MODEL FORMULATION

The MCGP model is among the recent techniques that deal with uncertainties in the GP context (

The RMCGP model conclusively solves many drawbacks of classic GP and presents interesting improvements over the former MCGP (for a deeper discussion see Chang (2008)). However, Chang's models are considered crisp models and are classified as such since they not incorporate imprecise and vague parameters. Notwithstanding, some complex decision-making tasks require aspiration levels that are fuzzy, imprecise, or stochastic. Some attempts have been made to generalize MCGP for fuzzy environments (see Tabrizi et al. (2012) and Mouslim et al. (2013)). But no works known to the authors have been developed to generalize RMCGP for uncertain and imprecise environments. In other words, no work has tried to integrate FGP with RMCGP. According to the concepts discussed above, the FIGP model is formulated and proposed as follows:

(**FIGP**)

$$
\begin{aligned}
\text{Max} \quad & \sum_{i=1}^{K} w_i \mu_i - \sum_{i=1}^{K} \sum_{j=1}^{J} \alpha_{ij} \left(e_{ij}^{+} + e_{ij}^{-} \right) \\
\text{s.t.} \quad & \sum_{j=1}^{J} y_{ij} X_i - p_i \le b_i \quad i = 1, \ldots, i_0 \\
& \sum_{j=1}^{J} y_{ij} X_i + n_i \ge b_i \quad i = i_0 + 1, \ldots, j_0 \\
& \sum_{j=1}^{J} y_{ij} X_i + n_i - p_i = b_i \quad i = j_0 + 1, \ldots, k_0 \\
& \sum_{j=1}^{J} y_{ij} X_i - p_i \le b_i^u \quad i = k_0 + 1, \ldots, K \\
& \sum_{j=1}^{J} y_{ij} X_i + n_i \ge b_i \quad i = k_0 + 1, \ldots, K \\
& y_{ij} - e_{ij}^{+} + e_{ij}^{-} = a_{ij,\max} \text{ or } a_{ij,\min} \quad i = 1, \ldots, K \, j = 1, \ldots, J \\
& a_{ij,\min} \le y_{ij} \le a_{ij,\max} \quad i = 1, \ldots, K \quad j = 1, \ldots, J \\
& \mu_i + \frac{p_i}{\Delta_i^R} = 1 \quad i = 1, \ldots, i_o \\
& \mu_i + \frac{n_i}{\Delta_i^L} = 1 \quad i = i_0 + 1, \ldots, j_o \\
& \mu_i + \frac{n_i}{\Delta_i^L} + \frac{p_i}{\Delta_i^R} = 1 \quad i = j_0 + 1, \ldots, K \\
& \mu_i, n_i, p_i, e_{ij}^{+}, e_{ij}^{-} \ge 0 \quad i = 1, \ldots, K \quad j = 1, \ldots, J \\
& X \in C_s
\end{aligned}
$$

All variables are defined as in the FGP and RMCGP models.

In practice, the FIGP model incorporates the main advantages of both FGP and RMCGP, while integrating them to overcome their relative limitations. Thus, this extension of both FGP and RMCGP allows higher satisfaction levels in the initial stages of the solution process to be reached. The usefulness of this model is demonstrated in the next subsection, which summarizes its utility in RE planning problems.

4.5 A REAL-WORLD APPLICATION

Italy has many potentials RE sources due to its geographical location including, solar, hydropower, biomass, and wind energy. Measures to promote the development of RE have been taken by the Italian government, and ambition is high to adopt the initiatives and directives of the European Union. Recall that the EU target is to increase the energy consumption from RE sources to 20% of total energy needs by 2020 (not easy to achieve but possible attainable). Given this requirement, each

European country has committed itself to reaching a binding target (17% in Italy). In recent years, the development of RE as a clean and promising alternative to conventional energy sources has attracted the interest of numerous international institutions, governments, and local policymakers. In this regard, Italy has engaged in a process that encourages the installation of RE infrastructure, including hydropower, wind, solar, and biomass. According to the National Renewable Energy Action Plan (NREAP (2017)), the Italian government intends to obtain more than 17% of its energy from renewable sources by 2020. All sources of RE have set new records during recent years and many continue to receive generous incentives schemes. In 2014, Italy reached a level of 120.679 GWh from RE sources. As depicted in Figure 4.5, hydropower contributed the most to electricity production (58.545 GWh), accounting for 41.2% of the total production from renewable sources; solar contributed 22.306 GWh, bio-energy contributed 18.732 GWh, wind contributed 15.178 GWh, and geothermal contributed 5.916 GWh (see GSE (2017)).

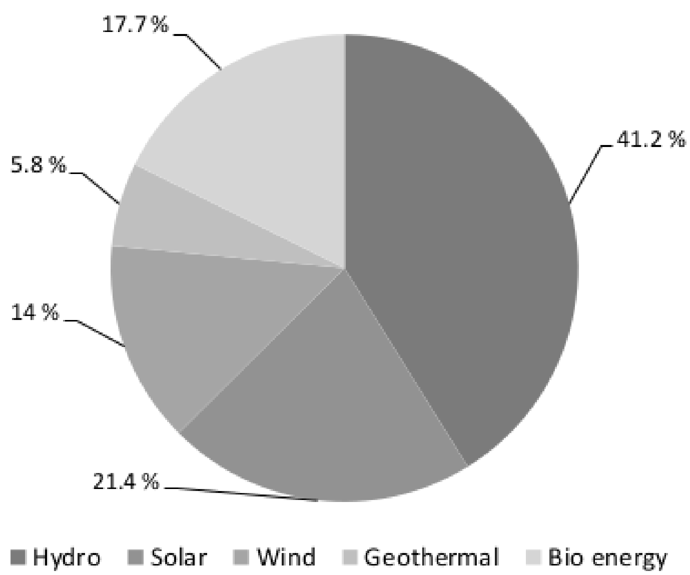


Figure 4.5: Renewable gross electricity production (in GWh) from renewable sources in Italy (2014).

Given these figures and the ambitious target of 17% of RE sources by 2020, many projects of the RE sector will be required in Italy. Thus, optimizing RE portfolios that Italy should guarantee to full its energy policy plan is crucial. This goal is not easily attained in light of the numerous conflicting criteria and inherent uncertainty; therefore, many scholars have devoted themselves to solving these problems. In recent years, several MCDM methods have been utilized to optimize and examine RE selection problems. For instance, Cristóbal (2011) applied the VIKOR (VIšekriterijumsko KOmpromisno Rangiranje) for selecting a renewable energy project in Spain. Streimikiene et al. (2016) used the AHP (Analytical Hierarchy Process) method for the selection of renewable energy sources in Lithuania. For a review of MCDM towards RE development see Wang et al. (2009) and Kumar et al. (2017). However, the MODM methods have been applied to a few problems of RE choices. For example, Chang (2015) applies the MCGP model to select the optimal location of renewable energy facilities in Taiwan. Recently, To optimize sustainable and RE portfolios, the multiple criteria necessary for the DM process should be determined. Criteria selection is key to achieving an optimal decision because DM priorities are generally the crucial step. According to the previous discussion, the constituents involved in evaluating electricity generation are as follows: technical, environmental, societal, and economic (Obviously, other criteria and sub-criteria may be considered and modeled and their choice generally depends on DMs' priorities and preferences). We consider the following five specific criteria. Investment costs (IC) include the costs of purchasing mechanical equipment, plant engineering, and other auxiliary construction. Operating and maintenance costs (OMC) such as workers' salaries and cost of operations. Primary energy saving (PES) refers to the fossil fuel saved from the power plants. Sustainability of climate change (SCC), such as reducing CO2 emissions and greenhouse gases, and Job creation (JC). Policymakers have many reasons to develop RE policies and meet the binding target (e.g., increasing energy security, reducing CO2 emissions, supporting national industry, creating green jobs, and promoting economic and technological development. The five criteria are used to evaluate four different sources of RE— biomass, solar photovoltaic (PV), tidal currents, and wind energy.

In RE planning and management, many criteria are used to measure the performance of RE sources. However, since several types of criteria have been adopted and because DMs' judgments are not crisp, it is quite difficult for DMs to provide precise values for the chosen criteria or attributes, especially for qualitative types (e.g., social acceptability, social benefits, contribution to local development, etc.) which are commonly expressed by linguistic variables (such as "High," "Moderate," etc.) and then converted to scale-based numerical values (e.g., 10, 6, etc.), see Zeshui (2012). For these reasons, this paper evaluates the investment cost, operating and maintenance costs, and job creation criteria as interval values. It is worth mentioning that in the real world, there is generally no way to control how the values of these uncertain criteria are determined and whether there is enough data available to formulate a probabilistic distribution is debatable. In such cases, it is assumed that these random variables (e.g., job creation) are controllable and may address, for instance, by fuzzy or interval theories which require less information.

The aim is to select the optimal RE portfolio for electricity generation considering conflicting criteria and different sources. Therefore, we define the following crucial aspects that are useful in optimizing RE portfolios: i) a description of the problem and its mathematical program formulation, ii) a solution from an alternative model for comparison, iii) a solution obtained using the proposed model, and iv) a discussion of the impact of using different methods, and v) a summary including final conclusions. Thus, we theoretically and empirically compare different methodologies for solving a complex MCDM problem. Italy has committed itself to reach a binding target of 17% by 2020, so policymakers (or DMs) would determine a priori the best "optimal" combination of RE for electricity production. Consider the following four decision variables:

X_1 : Solar PV annual electricity generation (GWh).

X_2 : Wind annual electricity generation (GWh).

X_3 : Biomass annual electricity generation (GWh).

X_4 : Tidal currents annual electricity generation (GWh).

To avoid resulting in a misleading decision or underestimating its impact, the DM sets interval evaluations for some criteria because they cannot be measured exactly and, thus, are expressed as intervals. However, in some real-life applications and especially in creative projects such as new hybrid renewable energy technologies, it could be difficult for the engineers and DMs to give a reasonable approximation for some parameters and try to give a wide range of intervals in the first stage of planning as an initial preventive vision, such as the initial investment cost or the time of realization, where there is no historical data that could help to make a good prediction. In response to practical challenges, where the range is very wide and reflects higher uncertainty or ambiguity, it is commonly recommended going through sensitivity analysis or using some sound prediction models (see, e.g.,

Table 4.2 reports the evaluation of each source according to the five criteria. In particular, if DMs are unsure about what levels of the goals to consider, they approximate them in a fuzzy fashion. The goals' aspiration levels and their possible tolerances are summarized in Table 4.3.

Table 4.2: The evaluations of each location according to the adopted criteria.

	Alternatives			
Criteria (or Goals)	Solar PV	Wind	Biomass	Tidal currents
IC (Euros)	4,048,112≤ y_{11} ≤4,648,112	[2.098.741-3.098.741]	[8.683.567-9.500.000]	[650.000-750.000]
OMC (Euros)	[40.481-46.481]	[82.962-92.962]	[605.571-645.571]	[10.000-15.000]
PES (Kg/per annum)	467.925	1.814.470	9.292.500	375.000
SCC (Kg/per annum)	814.190	3.157.178	16.168.950	652.000
Job creation (persons)	[1500-2000]	[900-1200]	[500-800]	[500-700]

Source: (Amine et al. (2018)).

Table 4.3: The evaluations of each location according to the adopted criteria.

Goals	Aspiration level	Tolerances
IC (Euros)	422.000.000	5.000.000
OMC (Euros)	80.000.000	800.000
PES (Kg/per annum)	450.000.000	4.500.000
SCC (Kg/per annum)	420.000.000	42.000.000
Job creation (persons)	150000	500

The problem can be formulated as a mathematical program as follows:

OPTIMIZE the following objectives:

G1: $[4.048.112 - 4.648.112]X_1 + [2.098.741 - 3.098.741]X_2$
$+[8.683.567 - 9.500.000]X_3 + [650,000 - 750.000]X_4 \lessapprox 422.000.000$ IC

G2 : $[40481 - 46.481]X_1 + [82962 - 92.962]X_2 + [605571 - 645.571]X_3$
$+[10000 - 15.000]X_4 \lessapprox 80.000.000$ OMC

G3 :$467.925X_1 + 1.814.470X_2 + 9.292.500X_3 + 375.000X_4 \lessapprox 450.000.000$ PEC

G4 : $814.190X_1 + 3.157.178X_2 + 16.168.950X_3 + 652.000X_4 \simeq_4 420.000.000$ SCC

G5 : $[1500 - 2000]X_1 + [900 - 1200]X_2 + [500 - 800]X_3 + [500 - 700]X_4 \lessapprox 150000$ JC

s.t

$$\left.\begin{array}{l} X_1 + X_2 + X_3 + X_4 \geq 80 \\ X_1 + X_3 \geq 50 \\ X_2 \geq 25 \\ X_3 \geq 20 \\ X_4 \geq 20 \end{array}\right\} \text{Hard constraints}$$

The hard constraints generally reflect the DM's strategic objectives. Recall that, in constrained optimization, hard constraints usually set conditions for the variables that are required to be satisfied (or true). In order to evaluate the usefulness of the proposed model, we compare its solution with that obtained from the FGP model. For a comparison purpose, Yaghoobi et al.'s FGP model (Section 3.2) has been used to solve the decision-making problem presented in Section 4.4 and the obtained optimal solutions are given below:

(**Solution Set I**)

- Decision variables values: $(X_1, X_2,_X 3, X_4) = (30, 25, 20, 20)$;
- The degree of membership functions:: $(\mu_1, \mu_2, \mu_3, \mu_4, \mu_5) = (0.28, 1, 0.38, 0.52, 1)$;
- Deviational variable values: $(p_1, p_2, p_3, p_4, p_5, n_5) = (35832, 0, 2749500, 1977410, 0, 24000)$.

By applying the proposed FIGP model presented in Section 4.3, the previous problem is formulated as follows:

$$\text{Max} = \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 - e_{11}^+ - e_{11}^- - e_{12}^+ - e_{12}^- - e_{13}^+ - e_{13}^- - e_{14}^+ - e_{14}^- - e_{21}^+ - e_{21}^- - e_{22}^+ - e_{22}^- - e_{23}^+ - e_{23}^- - e_{24}^+ - e_{24}^- - e_{51}^+ - e_{51}^- - e_{52}^+ - e_{52}^- - e_{53}^+ - e_{53}^- - e_{54}^+ - e_{54}^-$$

s.t.

$$y_{11}X_1 + y_{12}X_4 + y_{13}X_1 + y_{14}X_4 - p_1 \leq 422000000$$
$$\mu_1 + p_1/5000000 = 1$$
$$y_{11} - e_{11}^+ + e_{11}^- = 4048112$$
$$y_{12} - e_{12}^+ + e_{12}^- = 2098741$$

$$y_{13} - e^+_{13} + e^-_{13} = 8683567$$
$$y_{14} - e^+_{14} + e^-_{14} = 650000$$
$$4048112 \leq y_{11} \leq 4648112$$
$$2098741 \leq y_{12} \leq 3098741$$
$$8683567 \leq y_{13} \leq 9683567$$
$$650000 \leq y_{14} \leq 750000$$
$$y_{11}X_1 + y_{12}X_4 + y_{13}X_1 + y_{14}X_4 - p_2 \leq 80000000$$
$$\mu_2 + p_2/800000 = 1$$
$$y_{21} - e^+_{21} + e^-_{21} = 40481$$
$$y_{22} - e^+_{22} + e^-_{22} = 82962$$
$$y_{23} - e^+_{32} + e^-_{23} = 605571$$
$$y_{24} - e^+_{24} + e^-_{24} = 10000$$
$$40481 \leq y_{21} \leq 46481$$
$$82962 \leq y_{22} \leq 92962$$
$$605571 \leq y_{23} \leq 645571$$
$$10000 \leq y_{24} \leq 15000$$
$$467.925X_1 + 1.814.470X_2 + 9.292.500X_3 + 375.000X_4 - p_3 \leq 450.000.000$$
$$\mu_3 + p_3/4.500.000 = 1$$
$$814.190X_1 + 3.157.178X_2 + 16.168.950X_3 + 652.000X_4 - p_4 \leq 420.000.000$$
$$\mu_4 + p_4/42.000.000 = 1$$
$$y_{51}X_1 + y_{52}X_2 + y_{53}X_3 + y_{54}X_4 - p_5 + n_5 = 150000$$
$$\mu_5 + p_5/500 + n_5/500 = 1$$
$$y_{51} - e^+_{51} + e^-_{51} = 1500$$
$$y_{52} - e^+_{52} + e^-_{52} = 900$$
$$y_{53} - e^+_{53} + e^-_{53} = 500$$
$$y_{54} - e^+_{54} + e^-_{54} = 500$$
$$1500 \leq y_{51} \leq 2000$$
$$900 \leq y_{52} \leq 1200$$
$$500 \leq y_{53} \leq 800$$
$$500 \leq y_{54} \leq 700$$
$$X_1 + X_2 + X_3 + X_4 \geq 80$$
$$X_1 + X_3 \geq 50$$
$$X_2 \geq 25$$
$$X_3 \geq 20$$
$$X_4 \geq 20$$
$$\mu_i, n_i, p_i, e^+_{ij}, e^-_{ij}\, i = 1, \cdots, 5 \quad j = 1, \cdots, 4$$

Solving the above FIGP problem using LINGO (Schrage, 2009) yields the following optimal solution set:

(Solution Set II)

- Decision variables values: $(X_1, X_2, _X3, X_4) = (30.31157, 25, 19.68843, 20)$;
- The degree of membership functions:: $(\mu_1, \mu_2, \mu_3, \mu_4, \mu_5) = (0.99, 0.99, 0.99, 0.64, 1)$;
- Deviational variable values: $(p_1, p_2, p_3, p_4, p_5, n_5) = (0, 0, 0, 14990020, 0, 29626.17)$.
- Continues variables:
  $(y_{11}, y_{12}, y_{13}, y_{14}, y_{21}, y_{22}, y_{23}, y_{24}, y_{51}, y_{52}, y_{53}, y_{54})$
  $= (4048112, 2098741, 8683567, 650000, 40481, 82962, 605571, 10000, 2000, 1200, 800, 700)$.

Table 4.4 reports the decision variables values, degree of membership functions, and deviational variables values obtained by optimizing both models.

Table 4.4: Comparing the results of the classic FGP and the proposed FIGP model.

Solutions	Item	Yaghoobi et al.'s FGP Model solution set (I)	Proposed FIGP Model solution set (II)	RMCGP Model solution set (II)
Decision Variables Values	X_1	30	30.31157	30.31157
	X_2	25	25	25
	X_3	20	19.68843	19.68843
	X_4	20	20	
Degree of Membership Functions	μ_1	0.28	0.99	
	μ_2	1	0.99	
	μ_3	0.38	0.99	
	μ_4	0.52	0.64	
	μ_5	1	1	
Degree of Membership Functions	p_1	3583225	0	0
	p_2	0	0	0
	p_3	2749500	0	0
	p_4	19774150	14990020	14990020
	p_5	0	0	0
	n_5	24000	29626.17	29626.17
	Total	26130875	15019646.17	15019646.17

Table 4.4 illustrates the superiority of the FIGP model; the advantage of this model is confirmed by the fact that solution set (II) yields a higher number of satisfying goals. When the number of satisfied or unsatisfied goals is the standard, the FIGP model is more efficient than the solution set (I), which was obtained from the classic FGP model. In solution set (II), only the goal of job creation (G4) is not completely met (it reached 64%), while the remaining four goals (investment costs (G1), operating and maintenance costs (G2), primary energy saving (G3), and sustainability of climate change (G5)) are all approximately satisfied and achieved 99.25%.

This comparison confirms the effectiveness of the FIGP model when observing the degrees of membership functions achieved by the final solutions. Given the equal-priority-goals assumption, the total utility brought to the DM by the FGP model is only $0.28 + 1 + 0.38 + 0.52 + 1 = 3.18$, while that brought by the FIGP model is as high as $0.99 + 0.99 + 0.99 + 0.64 + 1 = 4.61$. In other words, if the DM's total utility is the measured goal, FIGP better optimizes the problem. This outcome occurs because FIGP automatically selects the proper parameter level for each evaluation from the interval value (i.e., by the y vector) to determine and achieve the fuzzy goal.

The importance of the FIGP model is also evident when deviations are observed. In solution set (II), the directly aggregated total deviation, expressed by $\sum_{1}^{5}(n_i + p_i)$, is 15019646.17, while in solution set (I), it is 26130875. This salient gap demonstrates the FIGP model's ability to achieve optimality. Moreover, observe that the decision variables from solution set (II) could produce 30.31157, 25, 19.68843, and 20 GWh if the RE projects are undertaken and the five criteria are adopted. Thus, adopting the optimal solution obtained by the FIGP model may generate more total electricity. Finally, as previously stated, fuzzy goals are assumed to have equal importance. Thus, for a fair comparison, the FIGP model operates on an equal-weight assumption. However, because the FIGP model uses a weighted-additive achievement function as its objective, it is possible to assign a different set of weights (relative importance) to the fuzzy goals. That said, obtaining a more suitable solution set is possible.

4.6 CONCLUSIONS

Implementing suitable policies for achieving long-term sustainability is inherently challenging and a complex MCDM problem (it involves several uncertain criteria). Uncertainty is an inherent and intrinsic feature of various decision-making processes. Thus, ascertaining the most sustainable RE source portfolio to meet Italy's target is an uncertain decision-making problem that includes numerous conflicting criteria, goals, and objectives. While Italian policymakers have made important progress towards meeting their RE target of 2020, a complete decision-making process for optimizing a RE source portfolio is required. For this reason, we presented a new method, FIGP, which optimized RE portfolios under uncertain conditions and in imprecise environments. The proposed method presents numerous advantages over traditional MCDM approaches, including flexible and conceptual formulations for resolving some FGP related problems. The proposed decision model (coupled with the possibility of setting interval coefficient level for each fuzzy goal) should support sound decisions. The application and advantages of the model are shown by applying it to solve a real application and comparing the results with those of FGP study. In fact, the suggested model produces the optimal RE portfolios that can achieve sustainable development goals. In general, the decisions made in the energy sector have certain implications (e.g., on the economy, environment, and society); the proposed model supports policymakers' efforts to optimize RE portfolios with acceptable levels of uncertainty.

5 Fuzzy goal programming with interval target

We do not know what we want and yet we are responsible for what we are - that is the fact.
-Jean-Paul Sartre-

This chapter proposes a novel MODM model called fuzzy goal programming with interval target (FGP-IT) based on recent developments of FGP and RMCGP. The model deals with decision-making problems involving a high level of uncertainty by offering decision-makers a more flexible way to formulate and express their preferences in terms of fuzzy interval target goals. The proposed method is used to optimize a hypothetical sustainable wind energy portfolio in Algeria.

5.1 INTRODUCTION

The increasing complexity of today's world has forced us to change our approach to real-life problems to incorporate numerous perspectives—political, environmental, economic, and technological, among others. If decisions are to be made from a multi-dimensional perspective, policymakers will require more mathematical tools to tackle these types of decision problems (see, e.g., Hocine & Kouaissah (2019), Fischer (2019), Hussain et al. (2019), and Hocine et al. (2020)). In this context, determining the optimal sustainable RE portfolio is a key factor in energy policies and deserves study. The problem involves meeting the decision maker's (DM's) preferences while considering the indicators of sustainability. However, one of the main issues that make the formulation of this problem difficult is the uncertainty embedded in DM preferences. In real life, uncertain or fuzzy interval preferences arise in several situations. For example, in RE planning, DMs often make conservative initial estimates based on the available information and resource limitations. For instance, a DM might say, "Under our resource limitations, we suggest that the realization time of project 'X' will be between approximately 12 and 16 months." Or they might say, "This project will cost around 350,500 to 400,500 euros." DMs normally make conservative initial estimates because they anticipate problems in achieving these goals (e.g., an architect's reluctance to change specifications or drawings, harsh weather, or errors in executing project activities). However, estimates using language like "between approximately" and "around … to …" are very difficult to model using the existing fuzzy goal programming (FGP) techniques.

Motivated by these concerns, this chapter tackles the problem by proposing a novel model that can overcome this limitation. The model, named fuzzy goal programming with interval target (FGP-IT), has been formulated to solve fuzzy multi-objective problems with an interval target (or interval goal). In this respect, the novel approaches taken by this paper are as follows. From a theoretical point of view, this paper combines two important techniques—FGP and revised multi-choice goal programming (RMCGP)—to overcome their respective limitations. In particular, it allows FGP to consider interval target values and thereby be applied in more complicated scenarios. Moreover, it makes RMCGP suitable for addressing fuzzy type uncertainty problems. To achieve this aim, it formulates the aspiration levels on the right hand side (RHS) of the system constraints. It also considers the most common types of fuzzy membership functions that are generalized to account for interval target values. This model thus extends the capability of its building blocks to solve a wide range of real-world problems.

5.2 RELATED WORKS

In FGP problems, the goals are viewed as fuzzy sets, and it is assumed that their membership functions are known (Lai & Hwang (1994) and Mirzaee et al. (2018)). However, sometimes the DM may feel it is difficult to specify his/her membership function parameters and, in particular, those aspiration level parameters required in the traditional GP context. This should be true, although each FGP model proved its effectiveness in dealing with uncertainties in real-world decision problems. In some cases, using the MCAL technique may successfully resolve such a problem, and this is one of the main reasons to take the MCGP modeling approach (Patro et al. (2018)). In the initial MCGP formulation, the possible MCALs from which a goal can be chosen are considered to be precise, deterministic, and well known. However, in many decision contexts, it is hard to determine the fixed values for these possible MCALs. Therefore, using interval numbers, the RMCGP model was proposed to 'glue-up' these possible MCALs (Chang (2008)), and, later, the utility-theory-based method involving piece-wised utility function slopes was established (Aalaei and Davoudpour (2016), and Attari et al. (2017)). Aside from Chang's works, Tabrizi et al. (2012) was the first research paper to formulate the fuzzy MCGP (FMCGP) approach. In their model, they adopted the max-min approach proposed by Zimmermann (1978) to solve the FMCGP problem based on only one type of membership function, which is the triangular membership function. However, before this model was proposed, such a monotonous treatment of membership functions was criticized by many articles, such as Chen and Tsai (2001) and Yaghoobi & Tamiz (2007). The second work that formulates the FMCGP problem was the model proposed by Mouslim et al. (2014), named FGP with Multi-Target-Level (FGP-MTL). This model obtains a solution by trying to maximize the degree of membership function.

These two FMCGP models are constructed based on the initial version of the MCGP model (i.e., Tabrizi et al. (2012) and Mouslim et al. (2014)). Unfortunately, that means the possible drawbacks of the MCGP formulation mentioned above are inherited (i.e., using the function of binary serial, making it hard to implement) by both models. Therefore, this study learns the concept from the RMCGP model, while interval numbers are used to formulate the fuzziness around the fuzzy goal targets on the right-hand side. In other words, such an approach not only provides alternative ways of conducting FMCGP-relevant research but also widens the application of the concept of RMCGP. With this modification, the fuzzy goal target in the FGP model becomes a moveable window that is automatically adjustable at the solution stage (when the base form of the model was a fuzzy model taking the membership function concept). This is why the proposed model is called the fuzzy goal programming with interval target (FGP-IT) model.

5.3 FUZZY INTERVAL MEMBERSHIP FUNCTIONS

To express ambiguous or vague information in the decision-making process, the fuzzy sets theory was first introduced by Zadeh (1965). Fuzzy sets have reasonable differences from crisp (classical) sets. Crisp set A in-universe U can be defined by listing all of its elements denoted as x. Alternatively, a $0-1$ valued membership function, $\mu_A(x)$, which is given below, can be used to define on x.

$$\mu_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases} .$$

Unlike crisp sets, a fuzzy set $\tilde{A}$ in-universe U is defined by a membership function, $\mu_{\tilde{A}}(x)$, which takes values on interval $[0,1]$. Therefore, the definition of a fuzzy set can be viewed as an extended version of the crisp set. While the membership function takes a value of 0 or 1 from crisp sets, it can take any possible value on the [0,1] interval from a fuzzy set. In the context of GP, the DM will be asked by a standard GP model to specify a precise aspiration level for each of the objectives. But as known, in real-life problems, there are situations during a decision process wherein a DM is unable to specify all or part of the aspiration levels precisely. Therefore, the FGP method has been developed rightly for this purpose.

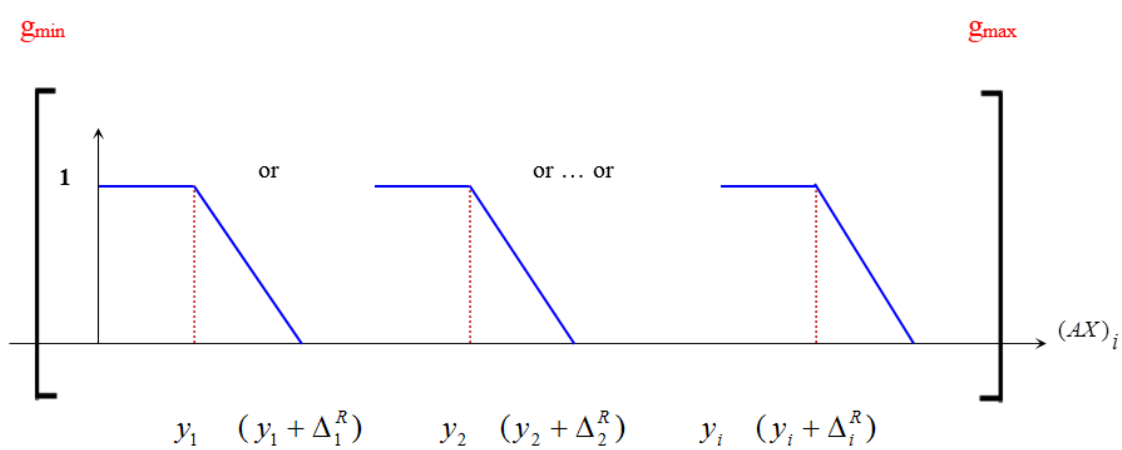
In studies of FGP, there are various ways to express and formulate the base form of goal fuzziness, each of which leads to a different fuzzy membership function. To the best of our knowledge, the first work to formulate the FGP problem by taking the membership function concept was Narasimhan (1981). In subsequent works, including the listed (FGP) model which involves the 'tolerance' concept mentioned in Section 3.2, all these functions are defined on the interval [0,1]. That is, the membership function has a value of 1 when this goal is fully attained and the DMs are totally satisfied, or it has a value in the interval [0,1) otherwise. This should be a main and common property of the FGP models which use the membership function concept. However, in reality, the expression and measurement of the fuzziness of interval goal targets should be another focus. When using FGP models such as (FGP), given his/her preference structure, a DM does not always feel it is easy to specify the parameters of 'the desired attainment' for a membership function, e.g., the b values set on the RHS of each fuzzy goal constraint in the (FGP) model. So, in at least some cases, using an interval value may better serve the purpose (Silva et al. (2013), Mouslim et al. (2014), Umarusman (2018)). In this chapter, we consider the situation in which a DM determines his/her aspiration levels in a fuzzy interval manner and allow him/her a flexible way to express the fuzziness of interval goal targets in constructing a FGP model, as to obtain a better solution for decision making. In order to present this clearly, in the rest of this section, a semantic model of the encountered problem is introduced. The FGP-IT model will then be proposed in Section 5.3. Following symbolic conventions, in this study the following FGP problem with K fuzzy goals is considered, which encounters an extended version of the problem domain considered by the (FGP) model (which enables the 'fuzziness of interval targets'):

(Semantic Model)

OPTIMIZE

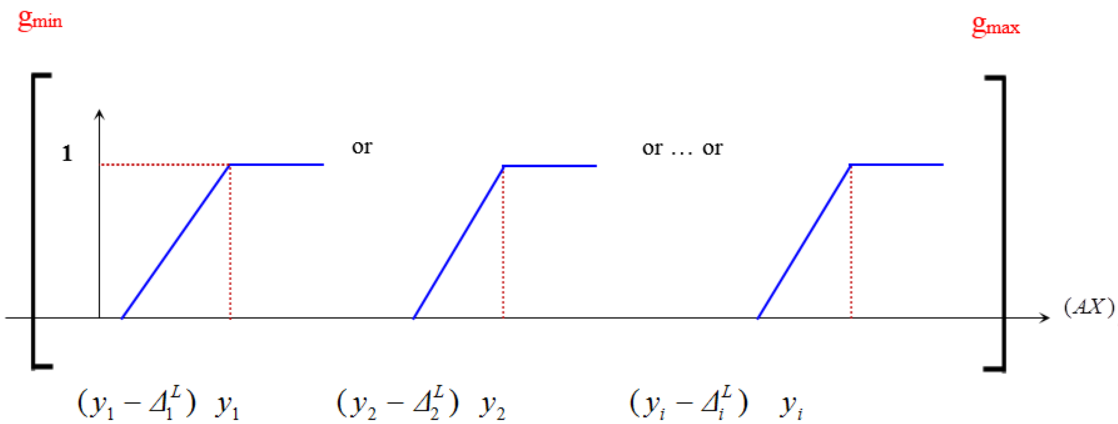
$$\begin{aligned} &(AX)_i \simeq \left[g_{i,\min}, g_{i,\max}\right] \quad i = 1, \ldots, i_0 \\ &(AX)_i \simeq \left[g_{i,\min}, g_{i,\max}\right] \quad i = i_0+1, \ldots, j_0 \\ &(AX)_i \cong \left[g_{i,\min}, g_{i,\max}\right] \quad i = j_0+1, \ldots, K \\ &X \in C_s, \end{aligned}$$

where OPTIMIZE means finding an optimal decision vector X such that all fuzzy goals are satisfied; $(AX)_i$ is the matrix multiplication form of the criterion function for the i-th objective, which can be expanded as: $(AX)_i = \sum_{j=1}^{n} a_{ij} x_j$, $i = 1, \cdots, K$. For $i = 1, \cdots, K$, $[g_{i,\min}, g_{i,\max}]$ represents a continuous span of multiple fuzzy aspiration levels for a goal in terms of a fuzzy interval number; C_s is an optional set of hard constraints as found in traditional LP; and the $\sim$ symbol is the fuzzifier operator, representing the imprecise fashion in which the goals are stated.



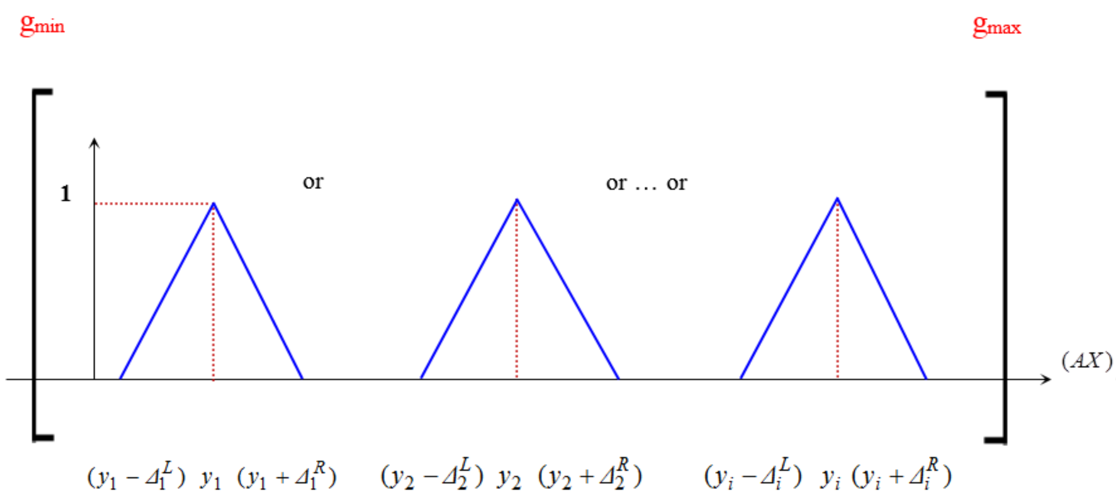
$$\mu(AX)_i = \begin{cases} 1 & (AX)_i \geq y_i \\ 1 - \dfrac{y_i - (AX)_i}{\Delta_i^L} & y_i - \Delta_i^L \leq (AX)_i \leq y_i \quad i = i_0+1, \ldots, j_0 \\ 0 & (AX)_i \leq y_i - \Delta_i^L \\ \text{where} & y_i \in \left[g_{\min}, g_{\max}\right] \end{cases}$$

Figure 5.1: Right fuzzy interval membership function.



$$\mu(AX)_i = \begin{cases} 1 & (AX)_i \geq y_i \\ 1 - \dfrac{y_i - (AX)_i}{\Delta_i^L} & y_i - \Delta_i^L \leq (AX)_i \leq y_i \quad i = i_0 + 1, \ldots, j_0 \\ 0 & (AX)_i \leq y_i - \Delta_i^L \\ \text{where} & y_i \in [g_{\min}, g_{\max}] \end{cases}$$

Figure 5.2: Left fuzzy interval membership function.



$$\mu(AX)_i = \begin{cases} 0 & (AX)_i \leq y_i - \Delta_i^L \\ 1 - \dfrac{y_i - (AX)_i}{\Delta_i^L} & y_i - \Delta_i^L \leq (AX)_i \leq y_i \\ 1 - \dfrac{(AX)_i - y_i}{\Delta_i^R} & y_i \leq (AX)_i \leq y_i + \Delta_i^R \quad i = j_0 + 1, \ldots, K \\ 0 & (AX)_i \geq y_i + \Delta_i^R \\ \text{where} & y_i \in [g_{\min}, g_{\max}] \end{cases}$$

Figure 5.3: Triangular fuzzy interval membership function.

We are interested in the form of the support of membership functions that allows the possibility of setting interval target values. The three types of fuzzy interval membership function involved here are presented and defined in Figures 5.1-5.3 (Yaghoobi and Tamiz (2007)). Observe that in these figures, each y_i is an additional decision variable that plays a key role in providing the flexibility on the RHS of the corresponding fuzzy goal constraint. These formulations generalize the classic membership functions and allow DMs to set multiple target values.

5.4 MODEL FORMULATION

According to the concepts discussed in previous sections, the FGP-IT model is formulated and proposed as follows:

$$
\begin{aligned}
& \textbf{(FGP-IT)} \\
\text{Min} \quad & \sum_{i=1}^{i_0} w_i \frac{p_i}{\Delta_i^R} + \sum_{i=i_0+1}^{j_0} w_i \frac{n_i}{\Delta_i^L} + \sum_{i=j_o+1}^{K} w_i \left(\frac{n_i}{\Delta_i^L} + \frac{p_i}{\Delta_i^R} \right) + \alpha_i \left(e_i^+ + e_i^- \right) \\
\text{s.t.} \quad & (AX)_i - p_i \le y_i i = 1, \ldots, i_0 \\
& (AX)_i + n_i \ge y_i i = i_0 + 1, \ldots, j_0 \\
& (AX)_i + n_i - p_i = y_i i = j_0 + 1, \ldots, K \\
& y_i - e_i^+ + e_i^- = g_{i,\min} \text{ or } g_{i,\max} \quad i = 1, \ldots, K \\
& g_{i,\min} \le y_i \le g_{i,\max} i = 1, \ldots, K \\
& \mu_i + \frac{n_i}{\Delta_i^L} = 1 i = 1, \ldots, i_0 \\
& \mu_i + \frac{n_i}{\Delta_i^L} = 1 i = i_0 + 1, \ldots, j_0 \\
& \mu_i + \frac{n_i}{\Delta_i^L} + \frac{p_i}{\Delta_i^R} = 1 i = j_0 + 1, \ldots, K \\
& \mu_i, n_i, p_i, y_i \ge 0 i = 1, \ldots, K \\
& X \in C_s,
\end{aligned}
$$

where all variables are defined as in FGP and RMCGP.

In summary here, the proposed FGP-IT model can be viewed as a continuous version of other FMCGP models, in which the fuzzy goal targets are not able to be fuzzified (Tabrizi et al. (2012), Mouslim et al. (2013), Ho (2019), and Mirzaee et al. (2018)) and which are thus discontinuous. Another possible advantage of the proposed model is suggested by recent trends in GP scholarship, which is moving to address several types of fuzzy goals for real-world problems. Thus, this extension of classical FGP has more support and resources to accommodate higher satisfaction levels. The following real-life application validates and demonstrates the usefulness of the proposed model.

5.5 A REAL-LIFE APPLICATION: OPTIMAL RENEWABLE ENERGY PORTFOLIO

Returning to the topic of renewable energy (RE), Algeria has recently launched an ambitious program—the National Renewable Energy Action Plan (NREAP (2017))—to develop RE by developing and expanding the use of inexhaustible resources, such as solar and wind power. According to the plan, which was adopted by the government of Algeria in February 2011, the country aims to produce about 27% of its national electricity from renewable sources by 2030. As wind power has been one of the fastest growing RE sources worldwide, the government of Algeria plans to produce a large portion of its electric power from wind by 2030 and will thus operate many wind turbines in order to generate the required electricity. Hence the following question naturally arises: how can the government create the optimal energy portfolio from the three wind farms while satisfying SD restrictions? Wind farms must be built in areas with sufficient wind resources, in other words, where average wind speeds are sufficient. The Sahara Desert is one of the windiest areas on the planet, especially on the western coast, where the Atlantic coastal desert runs through Western Sahara and Mauritania. According to the Algerian RE development center, Algeria's southwestern region has great potential, with speeds exceeding 4 m/s for the site of Timimoun, 5 m/s for the site of Tindouf, and even 6 m/s for the site of Adrar, as shown in Figure 5.4. The set of three locations being considered to build wind farms are as follows: Tindouf, Adrar, and Timimoun, as marked in Figure 5.5.

In Algeria, the electricity market is headed almost entirely by the national company SONELGAZ. Suppose that to produce clean electricity, this company plans to build three hypothetical wind farms of 30 wind turbines, each one with a rated power of 1000 kW (30 MW in total) (Himri et al. (2008)), in three different locations in the southwestern region of Algeria, namely, Adrar (X1), Timimoun (X2), and Tindouf (X3), which means the power generated for each station, as shown in Figure 5.5. Also, assume that the material resources that will be used are the same in each region due to the homogeneous geological nature of the areas (desert nature) selected for the project.

In this study, since the FGP-IT model proposed in Section 5.3 is to be applied to select the optimal sustainable wind energy portfolio under an uncertain and imprecise decision environment, the criteria which will be used in the decision-making process should be first determined. In reference to the previous discussion, the constructs involved in evaluating the energy supply system here are: technical, economic, environmental, and social. Five criteria are considered as follows: Plant capacity factor (PCF) (%), Cost of energy (COE) ($/kWh), Net present value (NPV) ($), Greenhouse gases avoided (GGA) (tons/year), and Social benefits. These, together with the construct to which a criterion belongs, are summarized from the existing relevant literature (i.e., Haddah et al. (2017) and Hocine et al. (2018)).

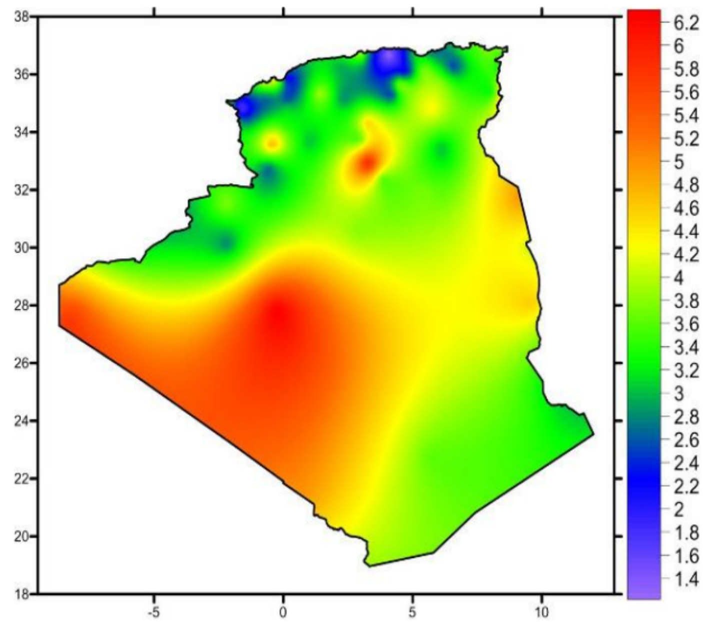


Figure 5.4: Algeria's annual wind speed (https://www.cder.dz/spip.php?article1765).

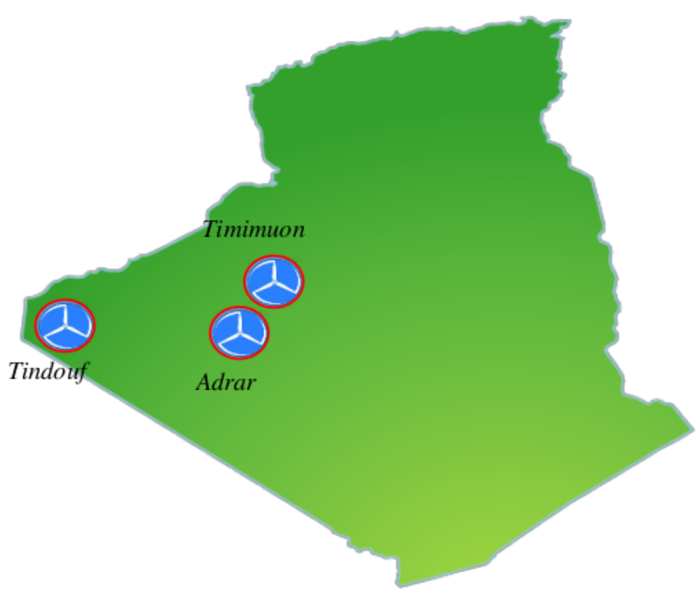


Figure 5.5: Potential locations of wind energy generation.

Therefore, for the decision case, there are five objectives: plant capacity factor (PCF) (%), cost of energy (COE) ($/kWh), greenhouse gases avoided (GHG) (tons/year), net present value (NPV) ($), and social benefits. Each objective's parameters are presented in Table 5.1.

Table 5.1: Data description of objective functions' coefficients.

Objectives (or goals)	Adrar	Timimoun	Tindouf
Plant capacity factor (PCF) (%)	38	30	21
Cost of energy (COE) ($/kWh)	0.0309	0.0430	0.0657
Net present value (NPV) ($)	76,125,725	48,843,997	19,712,439
Greenhouse gases avoided (GGA) (tons/year)	48,577	38,406	27,544
Social benefits	7	6.5	6.5

Source: Himri et al. (2008).

A multi-objective programming model that considers these objectives and the observable resource constraints can be semantically expressed as follows:

$$\begin{aligned}
&\text{Max}\, z_1 = 38X_1 + 30X_2 + 21X_3 && PCF\\
&\text{Min}\, z_2 = 0.0309X_1 + 0.0430X_2 + 0.0657X_3 && COE\\
&\text{Max}\, z_3 = 76125725X_1 + 48843997X_2 + 19712439X_3 && NPV\\
&\text{Max}\, z_4 = 48577X_1 + 38406X_2 + 27544X_3 && GGA\\
&\text{Max}\, z_5 = 7X_1 + 6.5X_2 + 6.5X_3 && SB
\end{aligned}$$

Suppose that the DM has determined his/her preference-related parameters for the five fuzzy goal criteria as summarized in Table 5.1. As suggested by FGP-IT, these data should be given in a fuzzy interval fashion. As such, suppose that the two ends of a continuous MCAL span and the type of each goal as well as its 'tolerance' are described in Table 5.2.

Table 5.2: The interval-based MCALs and goal types are defined on the five fuzzy goal criteria.

Goals	Type of fuzzy interval goals	Tolerance	Target levels
Plant capacity factor (PCF) (%)	Left	5	[40,70]
Cost of energy (COE) ($/kWh)	Right	40	[4000,6000]
Net present value (NPV) ($)	Left	10	[100,200]
Greenhouse gases avoided (GGA)(tons/year)	Left	5000	[50000,120000,30]
Social benefits	Left	2	[8,9]

Therefore, applying the proposed FGP-IT model with the given data set using LINGO (Schrage (2015)), the optimal solution set is obtained in Table 5.3.

As can be seen, the solution set satisfies the DM in terms of the degree of membership functions. All the goals are fully achieved with 100%. In addition, it is observed that according to this optimal solution set, the annual gross energy yield, without losses, from a hypothetical wind farm of 30MW installed capacity at each of the three locations (Adrar, Timimoun, and Tindouf) would be 95, 105, and 85 MWh, respectively. Furthermore, the most important outcome of producing wind energy at these sites would be the avoidance of 48,577 tons of greenhouse gases (GHG) entering the local atmosphere of Adrar each year, and about 1,214,425 tons over the lifetime of the wind power plant. Similarly, at Timimoun and Tindouf, a total of 38,406 and 27,544 tons of GHG emissions would be avoided. This result supports the social acceptability goal to achieve and reach the value of 100% of satisfaction. All of these results mean that the DM should be very satisfied with the choice suggested by the proposed model because the strategic plan should be close to his ideal.

Table 5.3: The results from solving the problem with the FGP-IT model.

Solutions	Item	Electricity generated from each station (MWh/year).
Decision Variable Values	X_1	95
	X_2	105
	X_3	85
Utility in Terms of Degree of MF	μ_1	1
	μ_2	1
	μ_3	1
	μ_4	1
	μ_5	1

The proposed FGP-IT model, which uses the concept of fuzzy interval membership functions, should be more efficient and realistic than previous models that use discontinuous fuzzy membership functions (Tabrizi et al. (2012), Mouslim et al. (2013), Umarusman (2018), and Mirzaee et al. (2018)). According to the modeling results obtained using FGP-IT, the model may make it possible to achieve some goals more completely by adjusting the interval window on the RHS of each goal criterion automatically, which will consequently utilize the resources more appropriately and create a higher satisfaction level for the DM. Based on these observations, the proposed FGP-IT model should offer a superior solution to traditional FGP approaches.

5.6 CONCLUSIONS

The identification of the most suitable RE source for electricity generation is an important issue in the implementation of appropriate RE planning policies. To tackle this problem, in this study, a FGP-IT model is proposed to optimize the creation of a sustainable wind energy portfolio in Algeria when the decision context involves deep uncertainty and the overall aim is SD in the country. This was original motivation for the study. The uncertainty in this situation is deep because, in the decision problem, there are not only fuzzy goals but also fuzzy interval goal targets. Methodologically, a main advantage of the proposed model is its ability to deal with problems having continuous fuzzy MCALs, which cannot be expressed and formulated by current FMCGP techniques. At the very least, it provides an alternative way to formulate the FGP problem when the preferences of the DM are described using complex terms such as "between approximately." Another advantage of the proposed model is that it can be easily applied to handle the complexity of real-world decision making problems, where several main types of membership functions are considered and formulated. At least, this feature was important for the real-life application modelled and solved in Section 6. Furthermore, this has widened the application of FGP, in that this study has applied an MODM model for the optimization of a wind energy portfolio problem. In this sense, as a concluding remark here, we note that the proposed FGP-IT model should be more effective than other FMCGP formulations and that it has broadened the model application context of FGP. These promising results may inspire us to conduct further studies (e.g., sensitivity analysis, comparison of results with those from MADM models, etc.), or to apply it to solve other MODM problems, such as those in the fields of transportation, healthcare planning, supply chain management, and so on.

6 Multi tolerances fuzzy goal programming

Each problem that I solved became a rule which served afterward to solve other problems.
-Rene Descartes-

This chapter proposes a novel FGP model, named an 'FGP-MT' model, to release the restrictions of 'single-tolerance' during FGP modeling. Taking the multi-choice aspiration level (MCAL) concept as a reference, the model can formulate several tolerance levels for a MF while the AL is highly imprecise. An empirical application is further considered for a SREPO decision problem that is critical for sustainable energy development in Algeria. As in the studied case, the model is shown to well support the DM in determining the most sustainable SREPO decision under the given imprecise goal conditions, the general form of tolerance enabled FGP modelling is therefore extended and altered in terms of methodology.

6.1 INTRODUCTION

Due to the importance of energy availability in sustainable development (SD), the United Nations (UN) General Assembly unanimously declared the decade 2014–2024 as the 'Decade of Sustainable Energy for All', setting a goal of ensuring "access to affordable, reliable, socially acceptable and 'environmentally sound' energy resources for the people". Today, it has become clear that energy consumption positively impacts economic growth (Kraft & Kraft, 1978; Lee, 2005; Stern, 2004). Therefore, for any nation to achieve its development goals and support its expanding economy, sufficient energy supplies are always required. However, simply providing reliable and affordable energy to support the country's development can be a challenge, let alone doing so in a way that supports SD.

SD has been defined in many ways over the past decades. To the authors' knowledge, the most widely accepted definition comes from a report called "Our Common Future" (Brundtland et al., 1987) and is: "meeting the current generation's requirements without compromising the ability of future generations". From this perspective, SD can be viewed as achieving the optimal balance between 'current development' and 'future development' by balancing the conflicting factors facing society. With this purpose in mind, governments need to find and adopt viable positions and policymakers should evaluate their decisions with regard to a wide range of decision goals and criteria. Given these requirements, more effective models are required to support the decision process (see, e.g., Zhuang

& Hocine (2018); Hocine et al. (2018); Ezbakhe & Perez-Foguet (2020)).

The demand for SD has spawned many sustainability theories concerning and addressing a wide range of issues. A sustainable plan for energy generation and exploiting and utilising renewable energy (RE) resources meets not only the basic considerations of SD (e.g., social and economic) but also addresses the 'environmentally sound' aspect of the approach. Therefore, the pursuit of an optimal sustainable RE portfolio usually becomes a key country-wide topic, i.e., it aims to determine how the clean RE resources available in a country can be combined for electricity generation and defines the RE-utilisation policies at a national strategic level for SD. This forms the sustainable RE portfolio optimization (SREPO) problem, which involves the various indicators of SD (see Fig. 1) and meeting the decision maker's (DM's) preferences. This problem deserves systematic study.

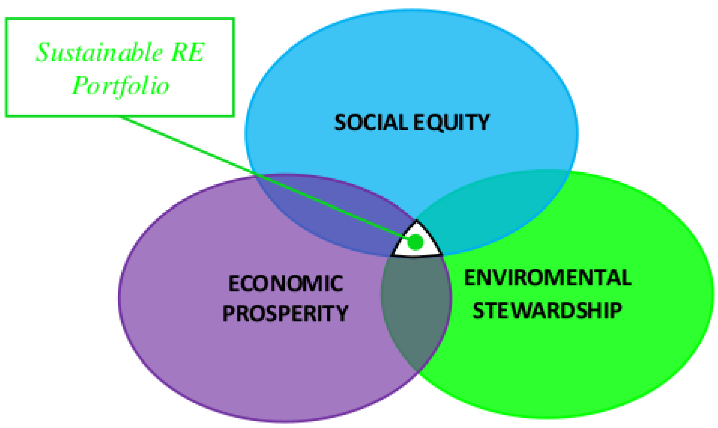


Figure 6.1: Making the best SREPO decision by the principles of SD.

Studies that involve the use of multi-objective decision-making (MODM) methods in making SREPO decisions have started to attract more attention from academic researchers. As examples, recently, Hocine et al. (2018) identified the best SREPO decision for Italy using 'multi-segment fuzzy goal programming' (MS-FGP). This method combined two MODM techniques, fuzzy goal programming (FGP) and multi-segment goal programming (MSGP), which are themselves goal programming (GP) models and thus MODM models. In this context, Kouaissah and Hocine (2020) analysed most existing techniques that tackle the SREPO problem using classic FGP formulations. These models supported a fuzzy decision with only one single tolerance (ST) value (see, e.g., Yaghoobi & Tamiz (2007a; 2007b); Aouni et al. (2009); Jiménez et al. (2018)). However, in many situations, decision makers (DMs) prefer to express their preferences by multi-tolerance (MT, or 'multiple tolerances' or 'multiple admissible violations' alternatively). Since the MT situation is a realistic variant of the SREPO problem (which is a multi-objective fuzzy decision with MTs around the target value), this chapter aims to propose a novel model called 'multi-tolerance FGP' (FGP-MT). The proposed model contributes to the literature in various ways. (i) It generalises the classic FGP models with ST formulation to consider MT situations. This contribution provides more flexibility in dealing with uncertainty; especially when DMs are hesitant or reluctant to choose between two or more competing options of tolerance value (see Hocine & Kouaissah, 2019). In particular, it extends the most common types of fuzzy membership functions (MFs) (i.e., left, right, triangular, and trapezoidal) to allow DMs to set multiple probable admissible violation ranges for each MF. (ii) The proposed FGP-MT model is distinct from other existing works that merge the fuzzy set theory (FST) with the multi-choice goal programming (MCGP) technique (Chang, 2011), which utilises the multi-choice aspiration level (MCAL) concept, see, e.g., Mouslim et al. (2014), Tabrizi et al. (2012), Hocine et al. (2018), Hocine et al. (2020), and Kouaissah and Hocine (2020). These studies aim to fuzzify the MCALs concept and keep only one ST for each MF. More precisely, for each goal, the probable aspiration levels are multiple and fuzzy. Unlike these studies, this work proposes that for each aspiration level the tolerances are multiple, and the shape of the MF will be determined by these tolerances. (iii) The concept of MT plays a significant role in RE planning and many other fields. This feature can lend great support to DMs dealing with SREPO problems, and the proposed model offers a better solution than that obtained using conventional FGP with ST. The results may provide important information to help policymakers make solid electricity generation policies for SD.

6.2 RELATED WORKS

Following the last discussion, the standpoint of the proposed (FGP-MT) model is identified for the entire MODM methodology in general. This section reveals that MT is, in itself, an innovation for FGP modeling. In order not to distort the purpose of the review, only the evidence related to the novelty of the proposed (FGP-MT) model is reviewed and discussed here. Conceptually, the old form of FGP models always allowed only one ST, which does not fit the SREPO problem encountered in Algeria and is likely to be encountered in other countries that share the same needs.

To give a summary of the former (FGP-ST) models and to formulate the (FGP-MT) model in subsequent sections, a general FGP problem with a total number of K fuzzy goals and four possible types of fuzzifier can be written semantically as follows:

$$
\begin{aligned}
&(\textbf{FGP - General})\\
&\text{OPTIMIZE}\\
&(AX)_i \tilde{\leq} b_i \quad i = 1, \ldots, i_0\\
&(AX)_i \tilde{\geq} b_i \quad i = i_0 + 1, \ldots, j_0\\
&(AX)_i \cong b_i \quad i = j_0 + 1, \ldots, k_0\\
&(AX)_i \tilde{\in} \left[b_i^l, b_i^u\right] \quad i = k_0 + 1, \ldots, K\\
&X \in C_S
\end{aligned}
$$

where OPTIMIZE means finding an optimal decision vector X such that all fuzzy goals are satisfied; the symbols that involve ~ are fuzzifier operators, representing the imprecise fashion in which the goals are stated.

Abundant literature has suggested formulating and resolving goal fuzziness in the above (FGP – General) problem using MFs. However, to formulate the required MT-enabled FGP model (FGP-MT), the MT ability for each fuzzifier operator type in the above semantic model should be 'equipped.' This would extend and change the general forms of the MFs that are currently used by FGP. For example, the 'end-point' of the slope of a right-sided fuzzy MF is fixed with the traditional ST setting (e.g., the slope ranges from $(AX)_i = b_i$ where the function value is still 1 to $(AX)_i = b_i + 5$ where the value becomes 0). However, the manifestation of several probable 'end-points' must be allowed (e.g., $(AX)_i = b_i$+(5 or 8 or 12) where the value of the MF becomes 0) when there are MTs in the decision context. This may yield several probable hypotenuses that further determine several probable MF shapes for any fuzzy MF, and this alters the problem. The established (FGP-MT) model can choose the most appropriate MF shape for the associated fuzzy goal. So, accordingly, as the most appropriate MF shapes are chosen for all fuzzy goals, the decision problem is optimized.

As can be imagined, to migrate from the traditional ST MFs to MT MFs, these probable endpoints must be formulated. For this sake, this study applied the concept of multi-choice aspiration level (MCAL) (for more details see Chang, 2007). Following these discussions, first, a conventional FGP model allows only one ST (or, equivalently, no tolerance) for each MF (of a goal). This is true from the earlier FGP models to the later ones. The (FGP-MT) model that is presented in this study, which allows a DM to set several tolerance levels for a MF of a fuzzy goal, may fill the gap. Therefore, MT would be an initial methodological contribution to FGP in F-MODM.Second, the proposed (FGP-MT) model may improve the decision process by expressing and interpreting more information about the preferences received from a DM in the uncertain decision context: this model starts to allow a DM to set multiple probable admissible violation ranges for each MF. While this is in fact driven by the decision context which contains those highly imprecise goals, MT extends the decision support function of FGP models in decision practice, which is one of the two main approaches used for making fuzzy multi-objective decisions.Third, as the original concept of MCAL was to provide a way for a DM to manifest a discrete set of probable aspiration levels (ALs) for a goal (i.e., each goal's target is discretely fuzzified by its probable ALs), 'what is fuzzified' was the AL on the right-hand side (RHS) of a goal criteria constraint. In another later study, Chang (2011) introduced the 'level-wise achieving' concept, converting the discrete set (of the probable ALs) into a continuous piecewise linear utility function. So, 'what is fuzzified' remained the AL. Therefore, in either study, the situation was 'a changing AL to be determined for a goal'. In contrast, the situation of (FGP-MT) is 'a changing MF to be determined for each fuzzy goal'. So, 'what is fuzzified' is the tolerant level of a MF. Since the subject of fuzzification is different, MT extends the scope of the application of the MCAL concept.Lastly, the proposed (FGP-MT) model is also distinct from other works that merged the FST with MCGP, which proposed the MCAL concept originally. Recent related works include Mouslim et al. (2014), Tabrizi et al. (2012), Hocine et al. (2018), and Hocine et al. (2020). Regardless of the modeling approaches used, these works aim to map each goal to one of the many MCALs and the AL that is to be decided is fuzzified. In these studies, for each goal, the probable ALs are multiple, and the AL associated with this goal is fuzzy. In (FGP-MT), for a goal that has been fuzzy per se, the tolerances are multiple (some tolerances are associated with the MF for that goal) and the shape of the MF (being determined by the determined tolerance) is fuzzy.

To avoid confusion, Table 6.1 summarizes these relevant models. The characteristics of each model and the types of functions it supports are given and can be compared before the subsequent formulation process. It can thus be realized that the proposed (FGP-MT) model is the only model that tackles the problem where there are MTs present, making it different from all other listed works. This is the reason for the main argument that it offers an initial methodological contribution to the field of FGP (see the first point above).

Table 6.1: Characteristics of the proposed model compared to other models that have been proposed.

Models	Crisp	Fuzzy	AL		Tolerence		Base Modelling Technique	Supported MF Types
			Mono	Multiple	Mono	Multiple		
Jiménez et al., 2002; 2018		✓	✓		✓		FGP	Left, right, triangular
Yaghoobi et al., 2008		✓	✓		✓		FGP	Left, right, triangular, trapezoidal
Chang, 2007	✓			✓			MCGP	
Chang, 2011		✓		✓			MCGP	S-function
Mouslim et al., 2014		✓		✓	✓		FP	Left, right, triangular, trapezoidal
Tabrizi et al., 2012		✓		✓	✓		FP	Triangular
Hocine et al., 2018		✓	✓		✓		FGP	Left, right, triangular, trapezoidal
Hocine et al., 2020		✓		✓	✓		FGP	Left, right, triangular, trapezoidal
The proposed model		✓	✓			✓	FGP	Left, right, triangular, trapezoidal

6.3 FUZZY MULTI TOLERANCES MEMBERSHIP FUNCTIONS

In FGP, there are various ways to express the fuzziness around a fuzzy goal target, each of which leads to a different type of fuzzy MF. Narasimhan (1980) and Hannan (1981) were the first to formulate the FGP problem by using the MF concept. All these functions are defined on the interval [0,1], so the MF has a value of 1 when this goal is fully attained and the DMs are totally satisfied. For the opposite case (fully unsatisfied), it has a value of 0.

As can be seen from Figure 4.2, each Δ value was a model parameter in Yaghoobi et al.'s (FGP-ST) model, describing the tolerant level on each 'side' of a MF of a fuzzy goal. But for each 'side' of each MF, only one tolerant level was admitted. Eventually, only one MF shape could exist for any fuzzy goal. Allowing MTs for a MF on each possible 'side' can offer more than one MF shape for any fuzzy goal and a truly flexible manifest in the decision context. Therefore, by reference to the above four common linear MF types, we begin by migrating these MFs from ST to MT.

The migration of MFs from ST to MT is required for establishing the general (FGP-MT) model. However, to illustrate the subsequent formulations straightforwardly, a numerical right-sided MF instance is used at first. The experience learned from this instance can be applied to offer the MT function for the other three MF types. Suppose we have the following decision problem with four fuzzy goals:

OPTIMIZE

$$FG-1: (AX)_i \tilde{\lesssim} 50$$

$$FG-2: (AX)_i \tilde{\gtrsim} 90$$

$$FG-3: (AX)_i \simeq 70$$

$$FG-4: (AX)_i \tilde{\in} [100, 200]$$

Take the fuzzifier operator of fuzzy goal FG-1 as an example; FG-1 should be expressed as a right-sided MF. Suppose that it is the case of a MT MF, and the tolerant levels have been provided by the DM, as follows:

- Tolerances for FG-1: 5 or 8 or 12.

This MT fuzzy MF is illustrated in Figure 6.2. With the standard fully-satisfied target being set at b1, this MF involves the above three probable tolerant levels. As can be observed, it is extended from the ST right-sided fuzzy MF. Here, the goal index i=1 and thus the target value (b_1) for this goal is set as ($b_1 = 50$).

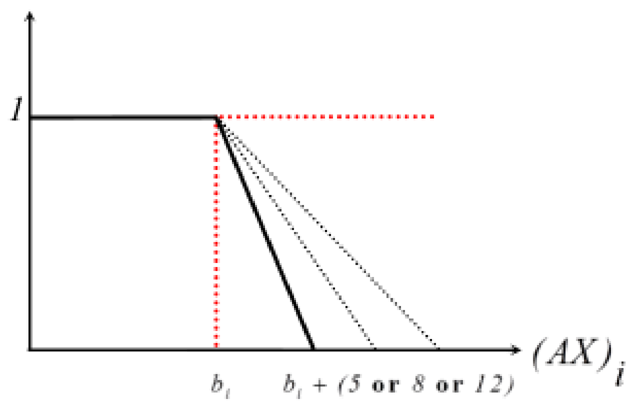


Figure 6.2: Multi-tolerance (MT) right-sided fuzzy MF: an illustration.

To the authors' knowledge, the MT requirement in Figure 6.2 cannot be formulated by any current FGP approach, but by reference to the techniques used to formulate the MCAL concept, for this case, two extra binary variables can be introduced. Thus, the formulation of such a MT fuzzy MF can be defined as:

$$\mu(AX)_i = \begin{cases} 1 & (AX)_i \le 50 \\ 1 - \dfrac{(AX)_i - 50}{5\lambda_1\lambda_2 + 8\lambda_1(1-\lambda_2) + 12(1-\lambda_1)\lambda_2} & b_i \le (AX)_i \le 50 + 5\lambda_1\lambda_2 + 8\lambda_1(1-\lambda_2) + 12(1-\lambda_1)\lambda_2 \\ 0 & (AX)_i \ge 50 + 5\lambda_1\lambda_2 + 8\lambda_1(1-\lambda_2) + 12(1-\lambda_1)\lambda_2 \end{cases}$$

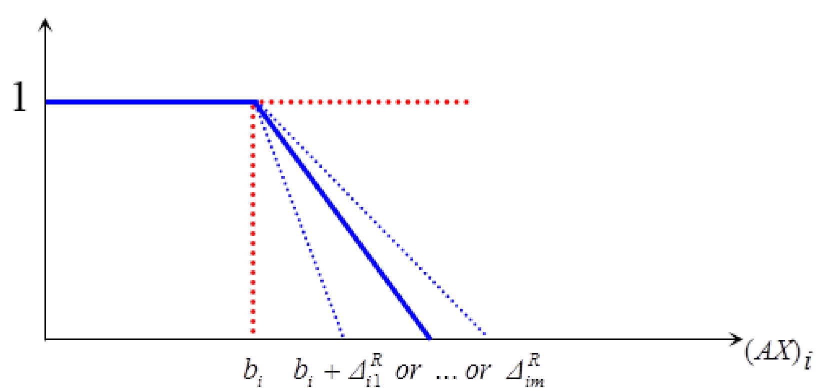
where λ_1 and λ_2 are binary variables; other variables are defined as in FGP. Following this numerical formulation, Figure 6.2 gives the conceptual diagram and the generalized formulae to this type (right-sided) of a MT fuzzy MF.

Therefore, any MT fuzzy goal such as the one shown in Figure 6.2 can be considered in a GP model, (M1), which is as the following:

$$
\begin{array}{ll}
 & \textbf{(M1)} \\
\min & \sum_{i=1}^{i} w_i p_i t_i^R \\
\text{s.t.} & (AX)_i - p_i \leq b_i \, i = 1, \ldots, i_o \\
 & \mu_i + p_i t_i^R = 1 \, i = 1, \ldots, i_0 \\
 & t_i^R = \dfrac{1}{\Delta_{im}^R S_{im}(B)} \, i = 1, \ldots, i_o \\
 & S_{im}(B) \in R_i(X) \, i = 1, \ldots, i_o \\
 & \mu_i, P_i \geq 0 \, i = 1, \ldots, i_o \\
 & X \in C_S
\end{array}
$$

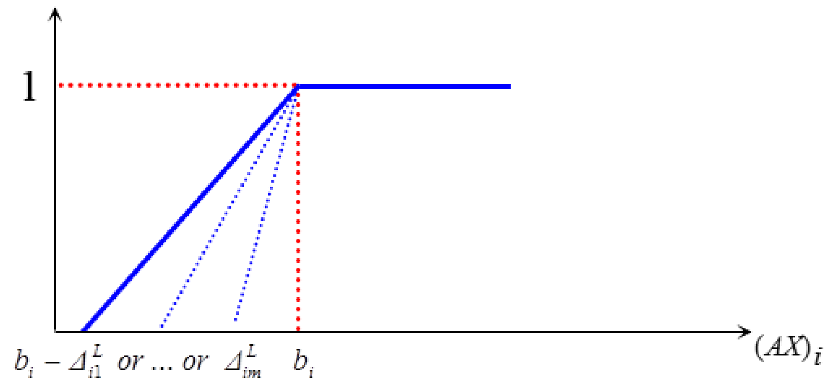
where w_i denotes the weight of the i-th fuzzy goal and μ_i is a model variable that determines the degree of membership function for the i-th fuzzy goal; p_i is positive deviational variable; t_i^R is the continuous variable; $S_{im}(B)$ represents a function of binary serial number; $R_i(X)$ is the function of resource limitations; Δ_{im}^R is the tolerance value, which cannot go under the imprecise aspiration levels b_i; and the other variables are defined as previously.

Following the above process to formulate the MT right-sided MF, the generalized fuzzy MF that supports MT for the other three-goal cases can also be formulated. These are as shown in Figures 6.3-6.5, respectively.



$$
\mu(AX)_i = \begin{cases} 1 & (AX)_i \leq b_i \\ 1 - \dfrac{(AX)_i - b_i}{\Delta_{im}^R S_{im}(B)} & b_i \leq (AX)_i \leq b_i + \Delta_{im}^R S_{im}(B) \quad i = 1, \cdots, i_0 \\ 0 & \end{cases}
$$

Figure 6.3: The right-sided multi-tolerance (MT) fuzzy membership function (MF).



$$\mu(AX)_i = \begin{cases} 1 & (AX)_i \geq b_i \\ 1 - \dfrac{(AX)_i - b_i}{\Delta_{im}^L S_{im}(B)} & b_i \geq (AX)_i \geq b_i + \Delta_{im}^L S_{im}(B) \quad i = i_0 + 1, \cdots, j_0 \\ 0 & \end{cases}$$

Figure 6.4: The left-sided multi-tolerance (MT) fuzzy membership function (MF).

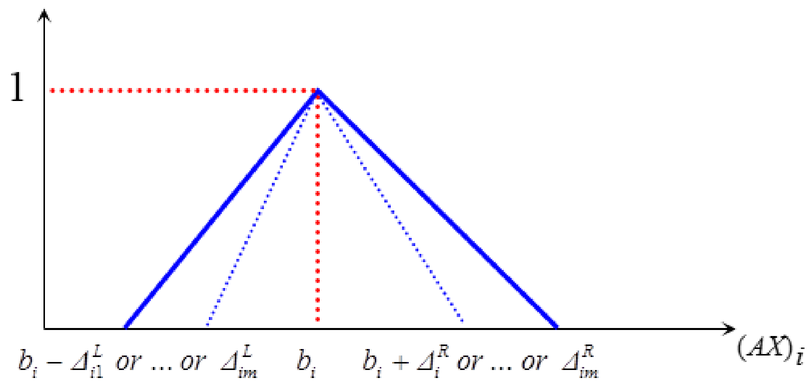
Using FG-2 as an example, the original MF of which should be left-sided (like the one shown Figure **??**), suppose that it is also a case of MT MF and that the MT levels of this fuzzy goal are as follows:

- Tolerances for FG-2: 25 or 35.

Then, based on the target value of FG-2 (b2=90), the formulation aims to choose the most appropriate tolerant level from the two potential levels (either 25 or 35), which might improve the solution and achieve higher satisfaction for the DM. Therefore, any MT fuzzy goal such as the one shown in Figure 6.3 can be generalized in another GP model, (M2), given as the following:

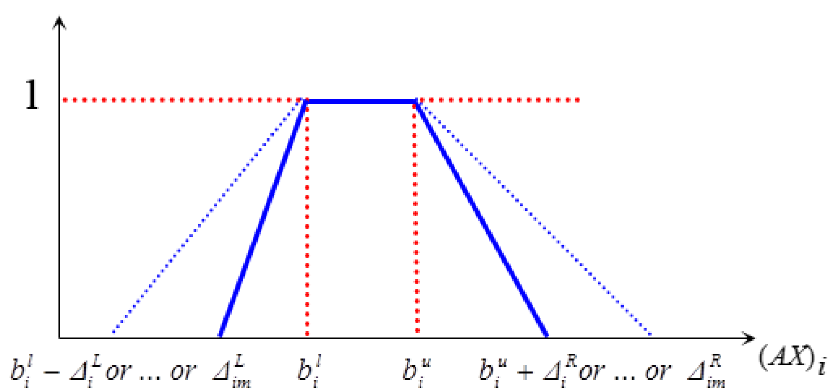
$$\begin{array}{lll} & \textbf{(M2)} & \\ \min & \sum_{i=i_0+1}^{i_n} w_i n_i t_i^L & \\ \text{s.t.} & (AX)_i - n_i \geq b_i & i = i_o + 1, \ldots, j_o \\ & \mu_i + n_i t_i^L = 1 & i = i_0 + 1, \ldots, j_0 \\ & t_i^L = 1/\Delta_{inn}^L S_{imn}(B) & i = i_0 + 1, \ldots, j_0 \\ & S_{inn}(B) \in R_i(X) & i = i_0 + 1, \ldots, j_o \\ & \mu_i, P_i \geq 0 & i = i_o + 1, \ldots, j_o \\ & X \in C_S & \end{array}$$

where n_i is a negative deviational variable; t_i^L is a continuous variable; Δ_{im}^L is the tolerance value, which cannot go beyond the imprecise aspiration levels b_i; and the other variables/symbols are as defined previously. Analogous to the above formulations for the right-sided and left-sided MT fuzzy MFs, the two remaining MFs of the remaining two fuzzy goal types (i.e., triangular and trapezoidal as shown in Figures 6.5 and 6.6) can also be migrated from ST to MT in their general forms.



$$\mu(AX)_i = \begin{cases} 0 & (AX)_i \leq b_i - \Delta^L_{im} Sim(B) \\ 1 - \dfrac{b_i - (AX)_i}{\Delta^L_{im} Sim(B)} & b_i - \Delta^L_{im} Sim(B) \leq (AX)_i \leq b_i \quad i = j_0 + 1, \ldots, k_0 \\ 1 - \dfrac{(AX)_i - b_i}{\Delta^R_{im} Sim(B)} & b_i \leq (AX)_i \leq b_i + \Delta^R_{im} Sim(B) \\ 0 & (AX)_i \geq b_i + \Delta^R_{im} Sim(B) \end{cases}$$

Figure 6.5: Triangular multi-tolerance (MT) fuzzy membership function (MF).



$$\mu(AX)_i = \begin{cases} 0 & (AX)_i \leq b^l_i - \Delta^L_{im} Sim(B) \\ 1 - \dfrac{b^l_i - (AX)_i}{\Delta^L_{im} Sim(B)} & b^l_i - \Delta^L_{im} Sim(B) \leq (AX)_i \leq b^l_i \\ 1 & b^l_i \leq (AX)_i \leq b^u_i \quad i = k_0 + 1, \ldots, K \\ 1 - \dfrac{(AX)_i - b^u_i}{\Delta^R_{im} Sim(B)} & b^u_i \leq (AX)_i \leq b^u_i + \Delta^R_{im} Sim(B) \\ 0 & (AX)_i \geq b^u_i + \Delta^R_{im} Sim(B) \end{cases}$$

Figure 6.6: Trapezoidal multi-tolerance (MT) fuzzy membership function (MF).

The mathematical formulation of triangular and trapezoidal MFs can be presented respectively in (M3) and (M4).

$$
\textbf{(M3)}\quad
\begin{array}{lll}
\min & \sum_{i=j_1+1}^{k_1} w_i \left(n_i t_i^L + p_i t_i^R\right) & \\
\text{s.t.} & (AX)_i + n_i - p_i = b_i & i = j_o + 1, \ldots, k_o \\
& \mu_i + n_i t_i^L + p_i t_i^R = 1 & i = j_0 + 1, \ldots, k_0 \\
& t_i^L = 1/\Delta_{im}^L S_{im}(B) & i = j_0 + 1, \ldots, k_0 \\
& t_i^R = 1/\Delta_{im}^R S_{im}(B) & i = j_0 + 1, \ldots, k_0 \\
& S_{im}(B) \in R_i(X) & i = j_0 + 1, \ldots, k_0 \\
& \mu_i, n_i, P_i \geq \mathrm{O} & i = j_0 + 1, \ldots, k_0 \\
& X \in C_s &
\end{array}
$$

$$
\textbf{(M4)}\quad
\begin{array}{lll}
\min & \sum_{i=k_0+1}^{K} w_i \left(n_i t_i^L + p_i t_i^R\right) & \\
\text{s.t.} & (AX)_i - p_i \leq b_i & i = k_0 + 1, \ldots, K \\
& (AX)_i + n_i \geq b_i & i = k_0 + 1, \ldots, K \\
& \mu_i + n_i t_i^L + p_i t_i^R = 1 & i = k_0 + 1, \ldots, K \\
& t_i^L = 1/\Delta_{imm}^L S_{imm}(B) & i = k_0 + 1, \ldots, K \\
& t_i^R = 1/\Delta_{im}^R S_{imm}(B) & i = k_0 + 1, \ldots, K \\
& S_{im}(B) \in R_i(X) & i = k_0 + 1, \ldots, K \\
& \mu_i, n_i, p_i \geq 0 & i = k_0 + 1, \ldots, K \\
& X \in C_s &
\end{array}
$$

where all variables/symbols are as defined previously.

6.4 MODEL FORMULATION

In consideration of all of the fuzzy goal types, and integrating (M1)–(M4), the general model of (FGP-MT) is then expressed as follows:

$$
\textbf{(FGP-MT)}\quad
\begin{array}{ll}
\min & \sum_{i=1}^{i_0} w_i p_i t_i^R + \sum_{i=i_1+1}^{j_0} w_i n_i t_i^L + \sum_{i=j_1+1}^{K} w_i \left(n_i t_i^L + p_i t_i^R\right) \\
\text{s.t.} & (AX)_i - p_i \leq b_i \quad i = 1, \ldots, i_0 \\
& \mu_i + p_i t_i^R = 1 \quad i = 1, \ldots, i_0 \\
& (AX)_i - n_i \geq b_i \quad i = i_0 + 1, \ldots, j_0 \\
& \mu_i + n_i t_i^L = 1 \quad i = i_0 + 1, \ldots, j_0 \\
& (AX)_i + n_i - p_i = b_i \quad i = j_0 + 1, \ldots, k_0
\end{array}
$$

$$\mu_i + n_i t_i^L + p_i t_i^R = 1 \quad i = j_0 + 1, \ldots, k_0$$
$$t_i^R = 1/\Delta_{im}^R S_{im}(B) \quad i = j_0 + 1, \ldots, k_0$$
$$(AX)_i + n_i \geq b_i \quad i = k_0 + 1, \ldots, K$$
$$\mu_i + n_i i_i^L + p_i i_i^R = 1 \quad i = k_0 + 1, \ldots, K$$
$$t_i^L = 1/\Delta_{im}^L S_{im}(B) \quad i = i_0 + 1, \ldots, K$$
$$t_i^R = 1/\Delta_{im}^R S_{im}(B) \quad i = 1, \ldots, K$$
$$S_{im}(B) \in R_i(X) \quad i = i_0 + 1, \ldots, j_0$$
$$\mu_i, p_i \geq 0 \quad i = i_0 + 1, \ldots, j_0$$
$$X \in C_S$$

where the objective function is a weighted summation of the objective functions of (M1)–(M4).

First, the proposed model, wherein each MF allows several tolerances, is considered more general than the traditional ST fuzzy MF just because a (FGP-ST) problem is a subset of the more generalized (FGP-MT) problem. That is, mathematically, the proposed (FGP-MT) model can degenerate to solve a (FGP-ST) problem, but the situation does not hold vice versa. With this extension to FGP modeling, it can be expected that more appropriate resources can be found in the solution to reach a higher satisfaction level for the DM during decision-making. In other words, by evolving from (FGP-ST) to (FGP-MT), this work might have improved the field of FGP modeling with the new model's ability to deal with MT concerns.

Second, in a conventional MF that allows only one ST (Hannan, 1981; Lai & Hwang, 1994; Kim & Whang, 1998; Tiwari et al., 1987), the DM subjectively determines this ST. However, situations in which the DM expresses his/her preferences about tolerance in a more implicit manner may arise, requiring the use of MT. This inflexibility problem during modeling is now resolved. In addition, providing MT to the DM can also limit the risk of underestimation or overestimation when making decisions. In other words, the (FGP-MT) model can handle a more implicit, uncertain decision context, and it may avoid the under-/overestimation problem to a certain extent during decision-making.

In addition, a situation in which one fuzzy goal maps to one MF that is mapped to multiple tolerances should be a dominant feature of the proposed (FGP-MT) model, and this fits the scope of the SREPO case studied in Section 5. In the next section, the effectiveness of the model is examined using another numerical example.

6.5 A REAL-LIFE APPLICATION

In recent decades, the emergence of RE as a clean and potential alternative to traditional energy sources has attracted the attention of various international institutions as well as government policy-makers. According to growth theories and United Nations studies, investing in RE sources is one of the most effective methods to reach sustainable development (SD) goals and improve national welfare

(Kraft & Kraft, 1978; Stern, 2004). Since the exploitation of RE resources has become a key element in supporting the SD agenda around the world, improving infrastructure and updating RE technology to deliver cleaner and more efficient energy ensures and encourages growth while also benefiting the environment. With this aim in mind, Algeria has recently launched an ambitious program to expand its RE sector by promoting investment in energy infrastructure and clean energy technology.

Reflecting the importance of these resources to SD, the Algerian government is now engaged in a 'SD reengineering' process designed to review and adjust the utilization of RE resources (e.g., hydropower, wind, solar, and biomass) for power generation and also to address RE infrastructure. In an earlier RE plan for 2015-2030 announced in February 2011, the Algerian government aimed to replace 27% of the national electricity production with RE by 2030 and shared the following 'expected' spectrum: 13,575 MW from solar photovoltaic (solar PV), 5,010 MW from wind, 2,000 MW from solar thermal, 1,000 MW from biomass, 400 MW from co-generation, and 15 MW from geothermal (see Figure 6.7). According to recent studies (see, e.g., Hocine et al., 2020), Algeria has revised a program to exploit these inexhaustible RE resources. Thus, to accomplish this plan, the government is launching and developing many projects involving different RE technologies.

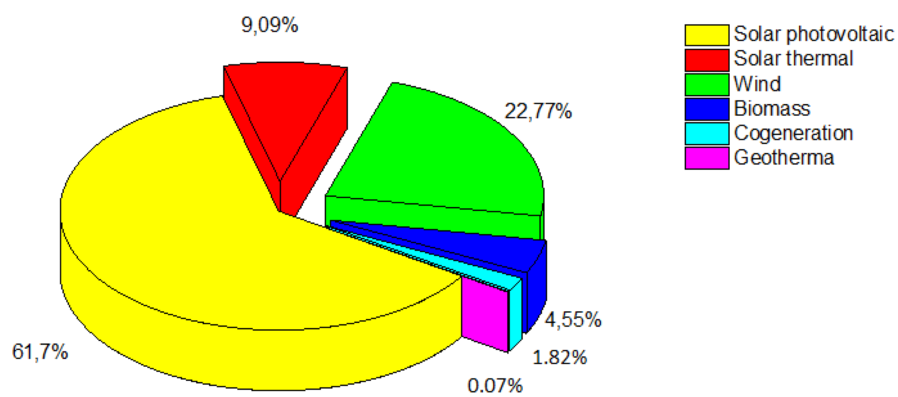


Figure 6.7: National RE Program of Algeria: a Projection (2015–2030).

To optimize sustainable RE portfolio decisions, it is important to consider the factors related to economic and SD theories in the evaluation process. From this perspective, optimising sustainable RE Portfolio decisions becomes a multi-criteria or a multi-dimensional decision problem. The problem involves a search for the best RE technologies that meet SD goals and decision makers' preferences. In particular, as time has gone by and Algeria's plans have progressed, the previous policies should be updated. A question now arises: "Q1: what are the best RE technologies that this country should invest in to achieve its ambitious plans for fulfilling the demand for power generation replacement?" As the RE technology used affects energy supply systems and is naturally determined by the RE resources to be exploited, a second, more strategic, question that should be asked is: "Q2: what is the best portfolio to utilise and exploit the country's RE resources to meet its goals for SD (current and future)?", which is a SREPO problem. Answering Q2 guides the answers to Q1.

To answer Q2, the decision criteria for SREPO must be determined. Following the discussions in introduction, the constructs involved in the evaluation process are as follows: technical, environmental, societal, and economic. The studied case considers six criteria, which are: Investment Ratio (IR) (Euro×10^3/KW), Operating and Maintenance Costs (OMC) (Euro×10^3/KW), Implement Period (IP) (years), Useful Life (UL) (years), Tons of CO2 Avoided (CO2)(tCO2×10^6/year), and Operating Hours (OH) (Hours ×10^3/year). These are applied to evaluate eight types of technologies including solar thermal, wind, biofuel, hydroelectric, and biomass, based on Cristobal's (2011) data.

In the SREPO decision, as shown in Table **??**, the DM assesses and assigns values to each alternative technology according to the five criteria adopted (i.e., creates the 'decision matrix'). If the DM is somewhat unsure about what level a goal should attain, he/she is encouraged to express it in a fuzzy fashion (i.e., a fuzzy goal, including what the type and direction of the fuzzy goal are and an aspirational level at which he/she 'remains fully satisfied'). Eventually, goals relating to four of the five criteria are expressed as fuzzy goals having right-sided MFs, while a goal relating to one of the criteria is expressed as a fuzzy goal having a trapezoidal MF (see Table **??**). Further, if the DM is somewhat unsure about the level at which he/she would become totally disappointed (i.e., the tolerance or the end-point of the MF), he/she is asked to state several probable tolerant levels, in order not to underestimate the potential achievement status of the fuzzy goals during decision-making. These tolerant levels (for each fuzzy goal) are also shown in Table **??**. Note that Table **??** also gives the expected range of electricity production throughput while utilizing each RE resource. These become the hard constraints for the decision variables during modeling.

Table 6.2: Evaluations of each RE resource according to the adopted criteria.

RE technologies	Criteria (FG-)					
	1: IR	2: OMC	3: IP	4: CO2	5: UL	6: OH
X_1: Wind 5–10 Mw	0.937	1.47	1	3.22	20	2.35
X_2: Wind 10–050 Mw	1.500	1.51	1	9.65	20	2.35
X_3: Hydroelectric 10–25 Mw	0.601	0.70	2	0.26	25	2
X_4: Hydroelectric 25–50 Mw	5.000	0.60	2.5	0.26	25	2
X_5: Solar Thermo-electric 10 Mw	1.803	4.2	2	0.48	25	2.59
X_6: Biomass (farming) 5 Mw	1.803	5.42	1	2.52	15	7.5
X_7: Biomass (forest) 5 Mw	1.803	2.81	1	2.52	15	7.5
X_8: Biofuels 2 Mw	1.503	2.51	1.5	5.91	20	7

Table 6.3: Aspirational levels for full attainment of the goals and the stated tolerance levels for each fuzzy goal.

Criteria (and Fuzzy Goal Types)	AL (Default)	MTs
FG1: IR (Euro⋆103/Kw) (Left)	10	0.4 or 0.6 or 0.9
FG2: OMC (Euro⋆/Kwh) (Right)	35	4 or 10
FG3: IP (years) (Triangular)	8	0.2 or 0.8
FG4: CO2 Avoided (tCO2×10^6/year) (Trapezoidal)	[45,85]	10 or 15
FG5: UL (years) (Triangular)	400	10 or 80
FG6: OH (Hours ×10^3/year) (Left)	7	0.3 or 0.7

The above problem is formulated according to FGP as a mathematical programme, as follows:

(**SREPO Case-Semantic**)

$$
\begin{aligned}
&\min \sum_{i\in\{1,3,4,5,6\}} n_i t_i^L + \sum_{i\in\{2,3,4,5\}} p_i t_i^R \\
&s.t. \\
&0.937X_1 + 1.5X_2 + 0.601X_3 + 5X_4 + 1.803X_5 + 1.803X_6 + 1.803X_7 + 1.503X_8 + n_1 \geq 10 \\
&\mu_1 + n_1 t_1^L = 1 \\
&t_1^L = \frac{1}{0.4}\lambda_{11}\lambda_{12} + \frac{1}{0.6}\lambda_{11}(1-\lambda_{12}) + \frac{1}{0.9}(1-\lambda_{11})\lambda_{12} \\
&\lambda_{11} + \lambda_{12} = 1 \\
&1.47X_1 + 1.51X_2 + 0.7X_3 + 0.6X_4 + 4.2X_5 + 5.42X_6 + 2.81X_7 + 2.51X_8 - p_2 \leq 35 \\
&\mu_2 + p_2 t_2^R = 1 \\
&t_2^R = \frac{1}{4}\lambda_{21} + \frac{1}{10}(1-\lambda_{21}) \\
&1X_1 + 1X_2 + 2X_3 + 2.5X_4 + 2X_5 + 1X_6 + 1X_7 + 1.5X_8 - p_3 + n_3 = 8 \\
&\mu_3 + p_3 t_3^R + n_3 t_3^L = 1 \\
&t_3^R = \left(\frac{1}{0.2}\lambda_{31} + \frac{1}{0.8}(1-\lambda_{31})\right) \\
&t_3^L = \left(\frac{1}{0.2}\lambda_{32} + \frac{1}{0.8}(1-\lambda_{32})\right) \\
&3.22X_1 + 9.65X_2 + 0.26X_3 + 0.26X_4 + 0.48X_5 + 2.52X_6 + 2.52X_7 + 5.91X_8 - p_4 \leq 85 \\
&3.22X_1 + 9.65X_2 + 0.26X_3 + 0.26X_4 + 0.48X_5 + 2.52X_6 + 2.52X_7 + 5.91X_8 + n_4 \geq 45 \\
&\mu_4 + p_4 t_4^R + n_4 t_4^L = 1 \\
&t_4^R = \left(\frac{1}{10}\lambda_{41} + \frac{1}{15}(1-\lambda_{41})\right) \\
&t_4^L = \left(\frac{1}{10}\lambda_{42} + \frac{1}{15}(1-\lambda_{42})\right) \\
&20X_1 + 20X_2 + 25X_3 + 25X_4 + 25X_5 + 15X_6 + 15X_7 + 20X_8 - p_5 + n_5 = 400 \\
&\mu_5 + p_5 t_5^R + n_5 t_5^L = 1 \\
&t_5^R = \left(\frac{1}{10}\lambda_{51} + \frac{1}{80}(1-\lambda_{51})\right) \\
&t_5^L = \left(\frac{1}{10}\lambda_{52} + \frac{1}{80}(1-\lambda_{52})\right) \\
&2.35X_1 + 2.35X_2 + 2X_3 + 2X_4 + 2.59X_5 + 7.5X_6 + 7.5X_7 + 7X_8 + n_6 \geq 7 \\
&\mu_6 + n_6 t_6^L = 1 \\
&t_6^L = \frac{1}{0.3}\lambda_{61} + \frac{1}{0.7}(1-\lambda_{61})
\end{aligned}
$$

where all symbols are as defined previously.

Based on the semantic (SREPO Case) model, the problem is modeled and solved with (FGP-ST) and (FGP-MT), respectively. This obtains Solution Set (III) and Solution Set (IV). These two optimal solution sets are first compared in Table 6.4 in terms of energy production portfolios (i.e., using values of the decision variables), goal achievement statuses (i.e., using the degrees of MFs), and how the fuzzy goals deviate from perfect wishes (i.e., using the values of the deviational variables). The optimization process is run using the LINGO global solver.

Table 6.4: Results upon solving the same (SREPO Case) problem with the two FGP models (precision: 10-2).

Solutions	Item	Yaghoobi et al.'s (FGP-ST) Model **Solution Set (III)**	Proposed (FGP-MT) Model **Solution Set (IV)**
Values for the Decision Variables	X_1	1.21	1.42
	X_2	2.85	3.07
	X_3	2.55	3.17
	X_4	4.44	3.82
	X_5	5.51	5.51
	X_6	0.14	0.54
	X_7	0.56	1.26
	X_8	0.29	0.29
Degree of MFs	μ_1	0%	100%
	μ_2	100%	28%
	μ_3	56%	34%
	μ_4	100%	100%
	μ_5	0%	55%
	μ_6	43%	100%
Deviation variables	n_1	0.4	0
	n_3	0	0
	n_4	0	0
	n_5	0	0
	n_6	5.66	0
	p_2	24.41	25.64
	p_3	1.73	6.58
	p_4	10	35.3
	p_5	0	0

Table 6.4 shows the superiority of the (FGP-MT) model in making the encountered SREPO decision. As plotted in Figure 6.7, the advantage of this model is confirmed by the fact that Solution Set (IV) yields more goals that are more satisfied than Solution Set (III). Among the 6 fuzzy goals, (FGP-MT) achieves a greater MF degree than (FGP-ST) for 3 of them (FG-1, FG-5, and FG-6) an equal MF degree to (FGP-ST) for 1 of them (FG-4), while it achieves 2 lower MF degree than (FGP-ST) (FG-2 and FG-3). A closer look reveals that the gains in the MF degrees for FG-1, FG-5, and FG-6 in Solution Set (IV) (solved by (FGP-MT)) compared to those in Solution Set (III) (solved by (FGP-ST)) (gains of 100%, 55%, and 57%, respectively) are much greater than the loss in the MF degree for FG-2 and FG-3,

which are respectively 72% and 22%. Moreover, Solution Set (IV) may perform better in terms of 'the number of totally-satisfied goals' (delivering 3 such goal achievements, $\mu_1 = \mu_4 = \mu_6 = 100\%$ for FG-1, FG-4 and FG-6, vs. only 2, $\mu_2 = \mu_4 = 100\%$ for FG-2 and FG-4, for (FGP-ST)). Thus, (FGP-MT) surpasses (FGP-ST) in 'the number of goals that are more satisfied' with a net value of $3-2=1$ (FGs). Therefore, when either 'the number of more satisfied goals' or 'the degree to which a more satisfied goal is more satisfied' is the standard, the (FGP-MT) model is more efficient than (FGP-ST).

The above result also confirms the effectiveness of the (FGP-MT) model when the total utility achieved by each final solution is observed. With Solution Set (III), the total utility brought by (FGP-ST) to the DM is $0+1+0.56+1+0+0.43 = 2.99$, while that brought by (FGP-MT) is as high as $1+0.28+0.34+1+0.55+1 = 4.17$ with Solution Set (IV). In other words, if the total utility that a model can bring to the DM is the measure, the (FGP-MT) model better optimises the problem by a quite large extent, too. This outcome occurs because with (FGP-MT), a proper tolerant level can be selected by the binary vector, among many, to achieve the fuzzy goals more perfectly while leveraging the achievement statuses among them.

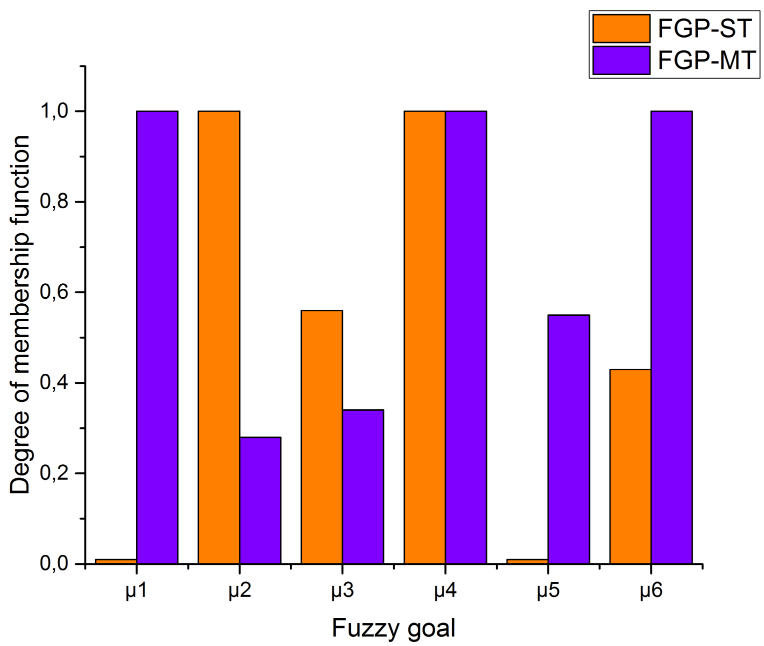


Figure 6.8: Achievements of the fuzzy goals: the MF degrees of the FGs in the two solution sets.

The importance of the (FGP-MT) model is also evident in terms of maximizing electricity throughput, which should be one of the main measures used to justify a SREPO decision. From the determined values for the decision variables in Solution Set (IV), if the RE resources are exploited according to the five criteria and the MT preferences that are set by the DM for each fuzzy goal, they could produce 2.8MW, 3.2MW, and 1.56MW of electricity, respectively, for Algeria. However, without using (FGP-MT), one can only expect to produce 2.6MW, 3.3MW, and 1.56MW of electricity from exploiting these RE resources using Solution Set (III). That is, using the (FGP-MT) model also increases the maximal possible electricity throughput that can be expected by the DM by 0.1MW

(2.8+3.2+1.56=7.56MW > 2.6+3.3+1.56=7.46MW) and this is once again due to the expansion of the goal space after MTs are allowed for each fuzzy goal. Thus, in terms of total electricity throughput, the decision vector of (FGP-MT) is more economically efficient than that of (FGP-ST), as visualized in Figure 6.8.

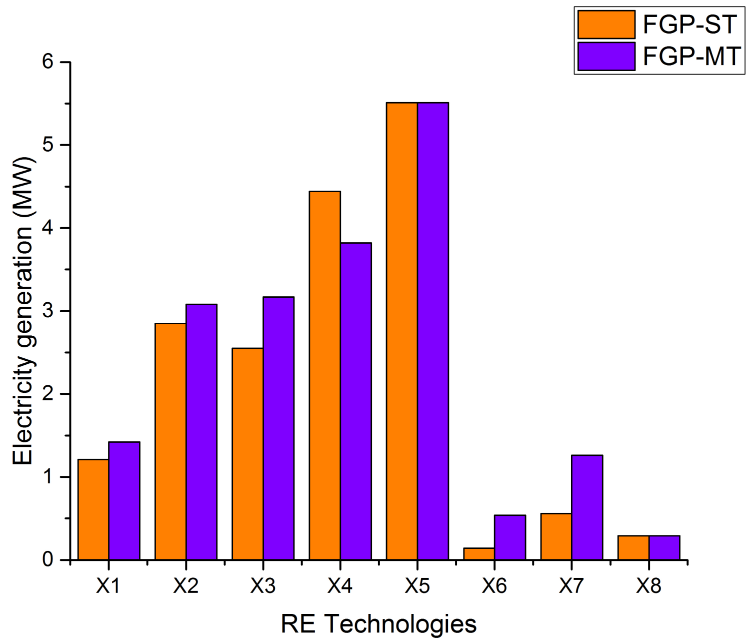


Figure 6.9: Energy generated (MW/year) by utilizing each RE resource: values of the decision variables.

The remaining deviations from the goal achievement status towards the default target of each fuzzy goal when the optimal solution is obtained for each model can also be compared. In Solution Set (IV), the aggregated total deviation, expressed by $\sum_{i\in\{1,2,\cdots,5\}} (n_i + p_i)$, is 10.829. Meanwhile, in Solution Set (III) obtained by the classical (FGP-ST) model, the aggregated total deviation is 12.082. The salient gap is dissected component by component in Figure 6.9 (see also Table 6.4). It, once again, demonstrates the ability of the (FGP-MT) model to pursue optimality, as the deviations towards the fuzzy goals are narrowed (i.e., the goals are 'aligned' to a greater extent).

In a summary, according to the insightful findings mentioned above, it can be claimed that Solution Set (IV) would be closer to the DM's wishes than Solution Set (III). This has been justified in terms of the number of goals that are more satisfied, the degree to which each more satisfying goal is more satisfied, the total utility that can be brought to the DM, the practical expectation of maximizing electricity in any SREPO decision, and the remaining deviations from the goal achievement status towards the default target of each fuzzy goal. Therefore, adopting the optimal solution obtained by the (FGP-MT) model may offer an 'economically better' solution that satisfies the DM to a greater extent than the solution offered by (FGP-ST) when making a SREPO decision.

The numerical model solved for the empirical decision case is constructed based on the semantic (SREPO Case) model (see Section 5), which is further established based on (FGP-MT). Other criteria or sub-criteria, other types of fuzzy goals, and other alternatives could be employed and considered without affecting the model formulations of (FGP-MT) in general. In other words, (FGP-MT) can not only be generalized to solve other SREPO problems, but also to solve any other similar problem where considerations for MT are present and required.

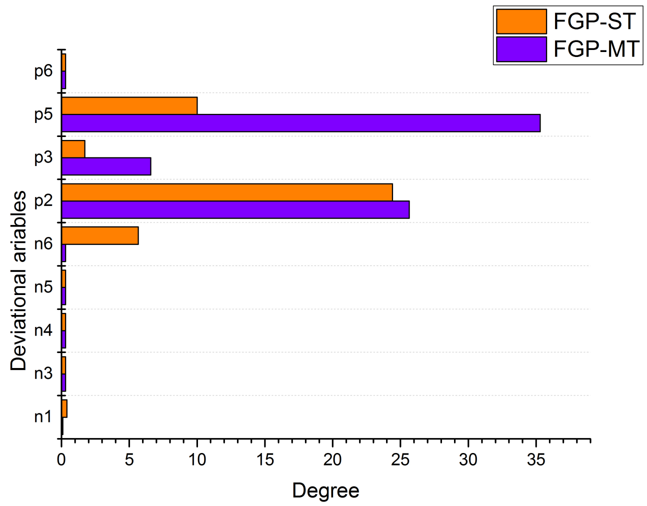


Figure 6.10: Deviations from the default (fully-satisfying) goal targets.

A boundary condition for the SREPO case studied in this section is that the fuzzy goals have equal importance (i.e., the equal-weight index or EWI assumption). And because the (FGP-MT) model operates on this EWI assumption, the former comparisons can be made on a fair basis. However, as (FGP-MT) also formulates a weighted-additive objective function, it is thus capable of receiving a different set of weights (relative importance or goal priority) of the fuzzy goals based on the DM's observable preferences, and the resulting model can lead to a different solution. That is, if an empirically meaningful goal priority was investigated, a more suitable solution set that better meets the preferences of the DM would thus be possible. Moreover, if a variety of pseudo settings for the goal weights are tried (i.e., a sensitivity analysis), the best outcome (e.g., in terms of throughput) could be selected from many probable ones and the best goal priority could thus be identified based on this application scenario.

Finally, the results have some implications for SD policymaking as well. The criteria which are taken into account in the analysis imply that the sustainability of RE portfolios is very sensitive to microeconomic and macroeconomic policies. Any short-run fluctuations in the economic environment, for example, a global shock such as COVID-19, may increase uncertainties, which in turn may have undesirable transitory impacts on related factors such as R&D expenditures, payments to employees, dividends, the number of social projects undertaken, and the number of people employed. On the other hand, the studied case indicates that energy efficiency can contribute to long-term SD if the considered criteria are respected.

6.6 CONCLUSIONS

The traditional FGP model allows each MF to have only one tolerant level (i.e., ST), with all outside levels connoting 'total disappointment', to define the range of the hypotenuse of the MF (where the degree of MF falls from 1 to 0). However, real-world decision-making such as the studied SREPO decision case often requires the flexibility to associate multiple tolerances (MTs) with each MF for a fuzzy goal. In the literature, it is found that there is no existing work which has addressed the formulation or modelling of these MTs. As a remedy to this issue, the (FGP-MT) model is established and proposed. It allows a DM to manifest MTs to admit several probable violations on any possible side of a MF that is set for each fuzzy decision goal. The allowance and flexibility of MTs are the salient features of the proposed (FGP-MT) model. However, the idea originates from the observable limitations of applying the (FGPST) model in real practice. The formulation process of (FGP-MT) is also a successful application of the MCAL concept, i.e., the MTs are controlled by the use of binary variables and thus the most suitable tolerance level can be chosen for a fuzzy goal.

With the confidence gained from applying the (FGP-MT) model, it is then applied to solve the studied SREPO decision problem in Algeria, which is relevant to the SD policy of the country. According to the insights explored, the solution set of (FGP-MT) is better than that of (FGP-ST) for the SREPO case and is closer to the DM's wishes. This is shown in terms of the number of goals that are more satisfied, the degree to which each more satisfied goal is more satisfied, the total utility that the model can bring to the DM, the expected electricity throughput, and the deviations remaining. Therefore, the optimal solution obtained using (FGP-MT) may generate an alternative, 'economically better' solution that satisfies the DM to a greater extent than that obtained using (FGP-ST) in making a SREPO decision. In addition, as the proposed (FGP-MT) model can be generalized, its future applications to SREPO problems in other countries, and to other similar decisions that also involve fuzzy goals and MTs, are both expected. Apart from these matters, the SREPO case studied also has implications for making, re-making, or adjusting the long-term SD policy of a country.

Future studies may include deeper analyses, practical applications of (FGP-MT) in other countries, and possible formulations of model variants, e.g., a more advanced hybrid model taking into account the concept of meta-GP (Uria, Caballero, Ruiz & Romero, 2002; Zhuang & Hocine, 2018). It is hoped that the future developments of the (FGP-MT) model may contribute to the study of FGP extension models, as FGP is one of the two main series of F-MODM studies

Part II

Uncertain Multi-Attribute Decision-Making (MADM) Methods: XOR-MADM

7 XOR-Data Envelopment Analysis

Not everything that can be counted counts, and not everything that counts can be counted.
-Albert Einstein-

This chapter proposes a new research thread of the DEA paradigm, named XOR-DEA, is proposed to deal with decision-making problems under xorness (or XOR input/output data). To incorporate decision-maker preferences in the optimization process, three types of preferences are proposed: positive, negative, and neutral. To cope deeply with uncertainty, a new concept of "the output mechanism of the XOR function" is developed to support the analyst in controlling this phenomenon based on two channels: controlled and uncontrolled. Moreover, to enrich the analysis of practical applications, a new visual analytic material is designed to detect the behavior of the XOR functions during the optimization process. Finally, an illustrative example of ranking renewable energy technologies is presented to evaluate the applicability of the proposed models.

7.1 INTRODUCTION

As a result of globalization, technological advances, and extreme events, today's world system is more complex, volatile, and uncertain than ever before. It has become an ever-larger, increasingly interconnected, heterogeneous, and more dynamic system. Consequently, business in this globalized world is different than it was in the past. Competitiveness is an increasingly significant challenge for international companies, their management, and their boards. In such an environment, the ability of many stakeholders to gain from increased efficiency in their business activities is key to international competitiveness. Under these circumstances, it is therefore of vital importance that managers and board members are aware of the concept of efficiency and how to measure it. As Farrell wrote in his seminal paper on the topic (Farrell, 1957):

> *"The problem of measuring the productive efficiency of an industry is important to both the economic theorist and the economic policymaker. ... Equally, if economic planning is to concern itself with particular industries, it is important to know how far a given industry can be expected to increase its output by simply increasing its efficiency, without absorbing further resources."*

Following this line, the issue of measuring efficiency has received the attention of many researchers during the last decades, and among various approaches that have been developed to tackle this issue the data envelopment analysis (DEA). Recognizing its usefulness, and supported by a worldwide community of researchers, DEA has been (and still is) the prominent methodological tool for measuring the efficiency of a certain system. DEA (or occasionally called frontier analysis) was proposed originally by Charnes et al. (1978, 1981) and developed further in several distinct works such as Banker et al. (1984), Charnes et al. (1987), Petersen (1990), Seiford and Thrall (1990), Cook et al. (1993), Zhu (1996), Tone (2001), Cook and Zhu (2008), Cook and Seiford (2010), Chen and Zhu (2020), and others (see, e.g., Cook & Seiford (2009)). Readers who are not familiar with DEA models are referred to Cooper et al. (2000; 2011) for an up-to-date introduction and a comprehensive bibliography. The main aim of DEA is to optimize the relative efficiencies of a set of decision-making units (DMUs) using multiple inputs and outputs. Although, it has developed within the economics theory, it has enough flexibility for benchmarking any system such as social, economic, industrial, ecological, or environmental. A DMU is considered efficient when it is on the efficient production frontier but is considered inefficient otherwise. The reference point of each inefficient DMU can be obtained by increasing outputs, by reducing inputs or by increasing outputs and reducing inputs simultaneously. Due to its simplicity, solid theoretical foundation, and user support software packages, DEA has become one of the most widely used methods in operations research/management science for assessing diverse real-world problems.

Since the seminal paper of Charnes et al. (1978), thousands of papers have been published, wherein 2019 alone, more than 4000 DEA papers were published. Up through the year 2020, the field has accumulated more than 60000 papers according to ScienceDirect database. This is means that a lot of DEA real-world problems have been solved in different areas such as renewable energy, banking, project selection, management production, supply chain management, and many other fields. Cooper et al. (1999), Cook & Seiford (2009), and Kaffash et al. (2020) provide a thorough survey and classification of DEA extensions. The most prevalent DEA variants include the Charnes, Cooper, and Rhodes (CCR) model of Charnes et al. (1978), the Banker, Charnes, and Cooper (BCC) model of Banker et al. (1984), the Additive DEA model (Charnes et al. (1985)), the Multiplicative DEA models (Charnes et al. (1983)), network DEA models (e.g., Chen, 2009; Cook, Liang, & Zhu, 2010; Tone & Tsutsui, 2014), hierarchical DEA Models (e.g., Kao, 2015) and their respective extensions. Methodologically, any DEA model seeks to optimize which DMUs determine an efficient frontier. The main differences between the basic models are the type of the envelopment surface and the projection path from inefficient DMUs to the efficient frontier (Charnes et al. (1994)). Utilizing classic DEA models, a decision-maker (DM) must choose between the piecewise constant returns-to-scale surface (of the CCR type model) and the variable returns-to-scale surface (of the BCC or Additive models). While the traditional DEA approach has meaningful advantages, can be converted to linear programming and classified as a nonparametric approach, it has some limitations. Among these, its sensitivity to the data utilized (Jahanshahloo et al. (2006)). This is because, as its name refers to, the DEA approach is data-based; and based on the available input and output data the empirical efficient frontier is determined. If a DMU lies on this frontier, it is considered as an efficient unit; otherwise, it is inefficient. Thus, any inherent uncertainties in input and output data can easily cause non-applicability or insignificant results. Thus, to successfully apply DEA, input and output data should be tackled properly to obtain applicable results. There are many situations, such as in management production or renewable energy planning, inputs and outputs are so volatile and complex that they are difficult to measure accurately. In response to these challenges, three main uncertain techniques are developed in the last decades, namely fuzzy DEA (FDEA), interval DEA (IDEA), and stochastic DEA (SDEA) (see, e.g., Sengupta (1982), Cooper et al. (1999), and Olesen & Petersen (2016)).

Typically, in uncertain DEA environments, analysts try to use or select the technique that most effectively addresses their problem. In this regard, the IDEA model is often applied to deal with rounding and measurement errors of input and output data. Cooper et al. (1999) were the first to study how to deal with imprecise (interval) data. Since then, a considerable effort has been paid to tackle this problem (see, e.g., Entani et al. (2002), Zhu (2003), and Yang et al. (2012)). Most papers try to convert the IDEA problem into a linear program system through a series of mathematical transformations to determine the feasible region. Moreover, to address randomness, SDEA is a commonly chosen approach. The first SDEA model was proposed by Sengupta (1982) using the expected value. Since then several studies have been published to develop this research trend (see, e.g., Cooper et al. (2002, 2004), Olesen & Petersen (2016), and Jradi & Ruggiero (2019)). In the context of SDEA, the problem converted into a chance-constrained model by introducing the variability to outputs that are conditional on inputs, which means that only outputs were taken as normally distributed random variables. Then, these stochastic optimization problems are transformed into a deterministic equivalent, which allows determining the efficient DMUs. In this approach, the probability distribution function of the parameters is already assumed; for more details about this research topic, see Chen and Zhu (2018).

In many cases, the uncertainty comes from linguistic sources. In FDEA, the input and output data of DMUs are fuzzy numbers. The first FDEA model was introduced by Sengupta (1992). Since then, different approaches have been proposed for converting FDEA problems to equivalent deterministic models; the most important ones include the tolerance approach, the α-level-based approach, the fuzzy ranking approach, the possibility approach, and the fuzzy arithmetic approach (see, Guo & Tanaka (2001), Emrouznejad et al. (2014)). The α-level approach is perhaps the most popular FDEA model (Saati et al. (2005)). In this approach, the main idea is to convert the FDEA model into a pair of parametric programs to find the lower and upper bounds of the α-level of the membership functions of the efficiency scores (see Angiz et al. (2010)). We recognize and argue that each method has proved effective in addressing uncertainties in DEA-related problems. In general, newer techniques consider traditional methods and either modify or integrate them into hybrid models to develop more powerful solution tools for decision problems where traditional methods present limitations. These techniques are modeled based on different assumptions and theories, require different information for their applications and, thus, may provide incomparable results (see Hocine & Kouaissah 2019).

This chapter contributes to standard uncertain techniques (FDEA, IDEA, SDEA, and RDEA) by proposing a new research thread for dealing with DEA problems under xorness. More specifically, the proposed XOR-DEA method integrates classic DEA techniques and the logic of XOR ("exclusive-or"), which allows an expert to set XOR input/output data. XOR is an uncertain logic that describes a situation in which there is only one choice between two or more competitive actions and neither is strong enough to overcome the others. Unlike classical logic, which requires a comprehensive understanding of a system, exact equations, and precise numeric values, XOR logic offers greater flexibility of thinking and allows complex systems to be modeled using a higher level of abstraction drawn from human knowledge and experience (Hocine & Kouaissah, 2020). Therefore, evaluating the efficiency of a set of DMUs under xorness is a problem worth studying. We do so in this chapter, first presenting i) the necessary background information for proceeding with XOR numbers through new arithmetic operations that use XOR numbers. This contribution helps us to understand the main philosophy behind xorness. To enhance understanding of the importance of DMs' preferences in decision-making processes ii) we suggest some ideas to integrate DMs' preferences in the optimization process through three types of preferences: positive (optimistic), negative (pessimistic), and neutral. This allows us to explore the implications generated by the optimistic/pessimistic attitude toward decision-making choices. Since the concept of uncertainty is essential in this paper, iii) we propose a new concept of "the output mechanism of XOR function" to support analysts in controlling this phenomenon based on two channels: a controlled channel and an uncontrolled channel. These concepts allow us to develop two new DEA models to solve problems under XOR inputs/outputs based on uncontrolled and controlled channels. Finally, to enrich the analysis of real-life applications, a new visual analytic tool is designed to monitor the behavior of XOR functions during the optimization process. How the proposed models offer alternative ways to widen DEA applicability and support DMs under uncertain environments is confirmed by considering a decision-making problem (ranking renewable energy (RE) technologies).

7.2 XOR FUNCTION AND PREFERENCES

Many real-life decision-making problems involve statements that can be expressed as preferences. Preference modeling is an important topic in decision science that takes into account the desires of DM. Let us consider a four-day scientific workshop example, for which each speaker is asked to express a preferred time slot for scheduling his talk. We assume that talks can be given either on Monday, or Tuesday, or Wednesday, or Thursday, and each day the talk can be either scheduled in the morning or the afternoon. A speaker may provide two kinds of preferences. For instance, she/he does not want to present on Monday, while she/he prefers to present on Thursday and does not have any problem in giving a talk either in the morning or afternoon. Given this example, XOR analysis can be employed to model multiple preferences towards particular decisions; especially whenever a DM is hesitant or irresolute in choosing among them. For preference modeling, XOR analysis allows more flexibility and consistency in dealing with multiple human judgments in the context of decision-making process. Therefore, incorporating DM's preferences through XOR analysis is of great interest in many practical applications. Thus, to serve the DM's preferences, the XOR function, corresponding to real-world scenarios, assumes three directions: i) positive, ii) negative, and iii) neutral (no direction is provided) (Hocine & Kouaissah (2019)). Let ϕ be the output of the XOR function.

Definition 1. *ϕ is either a positive output if $\phi \in \Lambda$ or a negative output if $\phi \in \Upsilon$ or a neutral output if $\phi \in \Psi$ where $\Lambda = \{\phi \in XOR(\mathbb{R}) | \phi = \max_i [XOR(x_i)]; i = 1, \ldots, k\}$ is called the positive output set, and each element x of Λ is called a positive output solution. $\Upsilon = \{\phi \in XOR(\mathbb{R}) | \phi = \min_i [XOR(x_i)]; i = 1, \ldots, k\}$ is called the negative output set, and each element x of Υ is called a negative output solution. $\Psi = \{\phi \in XOR(\mathbb{R}) | \phi \notin \{\Lambda, \Upsilon\}\}$ is called the neutral output set, and each element x of Ψ is called a neutral output solution. (i.e., does not follow any mandatory direction (max or min)).*

The results above, and especially the definition 1, are useful for modeling and integrating DMs' preferences. Particularly, we show that the directions of the XOR function allow us to incorporate behavioral information from the DM (e.g., optimism and pessimism) towards a particular choice or decision as suggested by Hocine & Kouaissah (2019). To model the DMs' preferences through the XOR function directions, we adopt the following formulation:

$$\frac{\underset{i=1,\ldots,k}{XOR}(x_i) - \min_i \left[\underset{i=1,\ldots,k}{XOR}(x_i) \right]}{\max_i \left[\underset{i=1,\ldots,k}{XOR}(x_i) \right] - \min_i \left[\underset{i=1,\ldots,k}{XOR}(x_i) \right]} = \delta$$

where $\underset{i=1,\ldots,k}{XOR}(x_i)$ is an XOR function, $\min_i \left[\underset{i=1,\ldots,k}{XOR}(x_i) \right]$ and $\max_i \left[\underset{i=1,\ldots,k}{XOR}(x_i) \right]$ are the minimum and maximum values of its elements, respectively, and δ is a psychological parameter that gauges optimism/pessimism assessment and ranges from 0 to 1 as shown in Table 7.1.

Table 7.1: Scales for optimism/pessimism assessment.

Scale		Definition
Cost Criteria	Benefit Criteria	
0	1	Optimistic
0.2	0.8	Moderately Optimistic
0.5	0.5	Neutral
0.8	0.2	Moderately Pessimistic
1	0	Pessimistic

The scales of assessment inherently represent psychometric measures that quantify and capture to certain extent perceptions, opinion, and, most importantly, psychological behavioral preferences. A scale encompasses and reflects fundamental dimensions of mood and personality to classify DM's preferences that could be for example fully "optimistic" or just "moderately optimistic." In many application, these scales allow us to represent the degrees of DMs' psychological states towards a determined decision or a particular choice. We can observe that the DMs' preference form is nonlinear (as formulated as a fractional form). However, we can easily linearize it as follows:

$$\psi_j \underset{i=1,\ldots,k}{XOR}(x_i) - \psi_j \min_i \left[\underset{i=1,\ldots,k}{XOR}(x_i) \right] = \delta$$

where

$$\psi_j = \frac{1}{\max\limits_i \left[\underset{i=1,\ldots,k}{XOR}(x_i) \right] - \min\limits_i \left[\underset{i=1,\ldots,k}{XOR}(x_i) \right]}.$$

7.3 XOR FUNCTION AND THE OUTPUT MECHANISM

One of the main purposes of XOR analysis is to determine the appropriate element among multiple options. In this respect, we study the behavior of the XOR function output mechanism. The output mechanism of the XOR real-valued function can be performed through two channels: i) The uncontrolled channel and ii) The controlled channel as shown in Figure 7.1.

Definition 2. *Let ϕ be the output of the XOR function. ϕ is an uncontrolled output if and only if $\phi \in \Xi$ and the output process is performed based on the bilateral of model philosophy (P) and algorithm (A) structure, where $\Xi = \{\phi \in XOR(\mathbb{R}) | \phi = f_i(P, A), i = 1, \ldots, k\}$ is called the uncontrolled output set, and each element x of Ξ is called an uncontrolled output solution.*

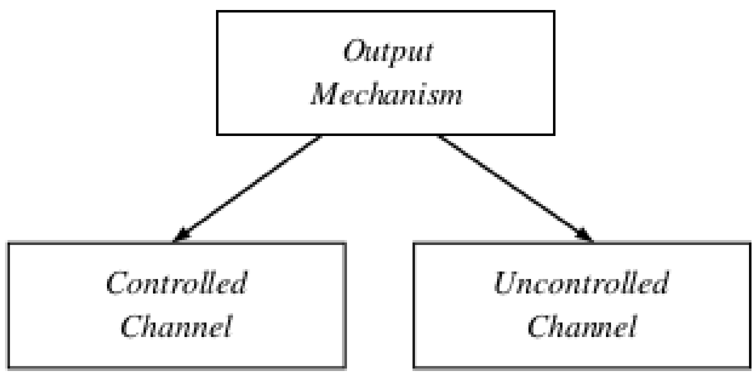
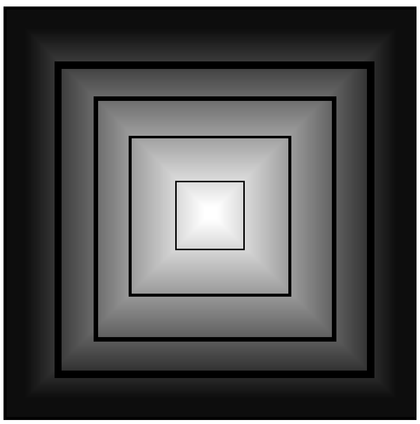
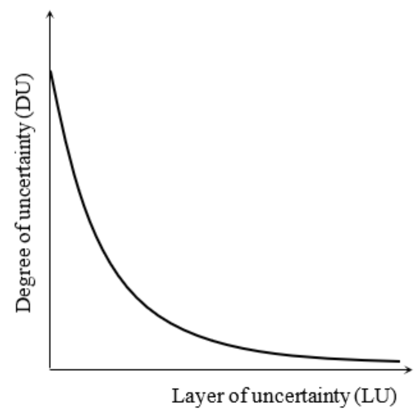


Figure 7.1: Output mechanism of XOR function.

It is clear that the main philosophy of the model is to optimize the objective function under some linear constraints which define the algorithm of the model. In this framework, the visible region forces the output mechanism of theXOR function to choose the appropriate element that optimizes the objective function. However, uncertainty is a complex entity with countless intervening components and confounding factors, which interact with each other in a certain way. Given this complexity, our main motivation is to detect the implications of uncertain layers on the output mechanism of the XOR function. To illustrate this vision, uncertainty can be represented as stacked boxes as shown in Figure 7.2(2a). We call these boxes the layers of uncertainty (LU) (see Figure 7.2(2a)) that complicate and camouflage the reality of the decision-making problem. Fundamentally, layers have a strong relationship with the degree of uncertainty (DU); this means that the DU decreases as much as we penetrate deeply into the LU. Therefore, one of the most useful strategies in dealing with such phenomena is to attempt to dismantle theses boxes; conceptually this could be expressed as follows: $\lim_{LU \to \infty} DU = 0$. Therefore, the main idea here is to try to detect layers that enable us to reduce the DU. It should be noted that the relationship between DU and LU could be represented by a monotone decreasing function $DU(LU)$ defined on the interval from 0 to infinity, as shown in Figure 7.2(2b). This function may satisfy certain properties, such as i) $DU(0) = 1$, which represents the total uncertainty case; ii) $DU(\infty) \approx 0$ when there are complete certainty; and iii) $DU(b) > DU(a)$ for $0 < b < a$. Based on these concepts, we develop the so-called controlled channel.



(2a)



(2b)

Figure 7.2: Layer of uncertainty (LU).

Definition 3. *ϕ is a controlled output if and only if $\phi \in \Theta$ and the output process is performed based on the detection of LU, where $\Theta = \{\phi \in XOR(\mathbb{R}) | \phi = f_i(LU), i = 1 \ldots, k\}$ is called the controlled output set, and each element x of Θ is called a controlled output solution.*

To explain these concepts, let us discuss why the experts and DMs are hesitant or, in other words, what the main concerns that render them uncertain about their judgements are. After a certain amount of analysis, DMs determine some related layers that affect their preferences. For instance, in the context of measuring renewable energy technologies efficiencies, one possible question that may arise is what are the main layers that affect the investment cost criterion. These layers could be, for example, the future prices, market conditions, regulation changes, and technology development. These ideas have been integrated and clarified in the context of DEA models from both theoretical and empirical perspectives. In particular, we formulate the so-called XOR-DEA under controlled and uncontrolled channels and provide a motivating example. Finally, the usefulness of the proposed models is illustrated through a problem of larger dimension.

7.4 DATA ENVELOPMENT ANALYSIS (DEA) MODELING AND XORNESS

7.4.1 The traditional DEA model

DEA is a linear programming-based approach for assessing the relative efficiency of a group of DMUs with multiple inputs and outputs (Cooper et al., 2011). In estimating the efficiency frontier, two main types of DEA models are used: a constant returns-to-scale (CRS) model and a variable returns-to-scale (VRS) model. The first model proposed by Charnes, Cooper, and Rhodes (CCR; Charnes et al., 1978) assumed production as CRS, which indicates that every increase in inputs will result in a proportional rise in output. In a later paper by Banker, Charnes, and Cooper (BCC; Banker et al., 1984), a more general model was presented that allows for VRS, which means an increase in input level will either increase or reduce output level. Methodologically, the BCC model is considered an extension of the CCR model, where the efficient frontiers set is represented by a convex curve that encompasses all efficient DMUs. Moreover, DEA can be either input- or output-oriented.

Mathematically, consider a set of n DMUs (DMUj, $j = 1, \dots, n$) where $X_j = (x_{1j}, \dots, x_{sj})$ is the vector of inputs and $Y_j = (y_{1j}, \dots, y_{mj})$ is the vector of outputs. The production possibility set Ω is the set $\{(X,Y) | the\, outputs\, Y \geq 0\, can\, be\, produced\, from\, the\, inputs\, X \geq 0\}$. Conceptually, the traditional CCR DEA model generalizes the so-called Farrell model (Farrell, 1957) to the multiple-input/multiple-output setting; for more details and discussion of Farrell's efficiency measures refer to Cooper et al. (2011). However, this class of models opts inputs and outputs under the CRS assumption, which does not reveal whether operations were conducted in regions of VRS. In response to these challenges, Banker et al. (1984) propose the BCC model, which allows for a VRS. The corresponding primal linear programming models of the BCC approach can be expressed as follows:

Output-oriented (7.2)

$$
\begin{aligned}
\min \quad & \sum_{i=1}^{s} v_i x_{ik} - c_k \\
\text{s.t.} \quad & \sum_{i=1}^{s} v_i x_{ij} - \sum_{r=1}^{m} u_r y_{rj} - c_k \geq 0 \quad j = 1, \dots, n \\
& \sum_{r=1}^{m} u_r y_{rk} = 1 \\
& u_r,\, v_i \geq 0 \,\forall\, r = 1, \dots, m;\, i = 1, \dots, s
\end{aligned}
$$

Input-oriented (7.3)

$$
\begin{aligned}
\max \quad & \sum_{r=1}^{m} u_r y_{rk} + c_k \\
\text{s.t.} \quad & \sum_{i=1}^{s} v_i x_{ij} - \sum_{r=1}^{m} u_r y_{rj} - c_k \geq 0 \quad j = 1, \dots, n \\
& \sum_{i=1}^{s} v_i x_{ik} = 1 \\
& u_r,\, v_i \geq 0 \,\forall\, r = 1, \dots, m;\, i = 1, \dots, s
\end{aligned}
$$

where u_r is the weight of output r, v_i is the weight of input i, c_k is a measure of returns to scale for DMU k. Since the number of DMUs is usually much larger than the number of inputs, it is usually preferred to express the linear programming models (7.2) and (7.3) in their duality forms. Thus, the duality forms of (7.4) and (7.5) can be formulated as follows:

Output-oriented (7.4)

$$\begin{aligned}
\max \quad & \phi_k \\
\text{s.t.} \quad & \phi_k y_{rk} - \sum_{j=1}^{n} \lambda_j y_{rj} \le 0 \quad r = 1,\dots,m \\
& x_{ik} - \sum_{j=1}^{n} \lambda_j x_{ij} \ge 0 \quad i = 1,\dots,s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \ge 0 \,\forall\, j = 1,\dots,n
\end{aligned}$$

Input-oriented (7.5)

$$\begin{aligned}
\min \quad & \theta_k \\
\text{s.t.} \quad & y_{rk} - \sum_{j=1}^{n} \lambda_j y_{rj} \le 0 \quad r = 1,\dots,m \\
& \theta_k x_{ik} - \sum_{j=1}^{n} \lambda_j x_{ij} \ge 0 \quad i = 1,\dots,s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \ge 0 \,\forall\, j = 1,\dots,n
\end{aligned}$$

As mentioned previously, one of the most critical limitations of conventional DEA models is the assumption of precise values for input and output data. However, the data and information in real-world problems sometimes comes in an XOR-nature format. The following section aims to extend the conventional DEA models to deal with XOR input and output data. This allows the models to capture uncertainty that may impact the relative efficiency of classic DEA models. Moreover, it endeavors to consider DM's preferences with regard to psychological factors such as optimism and pessimism toward particular decisions.

7.4.2 Incorporating decision maker's preferences in the DEA framework

It is widely recognized that optimism and pessimism have strong effects on our decisions. According to Carver and Scheier (2014) studies, optimism and pessimism scientific definitions focus respectively on positive and negative expectations of the future. In a world defined by certainty, optimism and pessimism would have no scope, since everyone knows precisely the outcomes of certain events. In contrast, optimism and pessimism exist in an uncertain world, thus, uncertainty is a sine qua non for their existence (Hey, 1984). By nature, optimistic and pessimistic DMs have different visions in confronting real-world problems and in how well they overcome adversity; these differences influence their choices to a significant extent and, therefore, the consequences of them. Consider, for example, the last subprime financial crisis, which triggered a major global economic crisis. Before the financial crisis, most worldwide financial markets displayed great optimism and that economic prosperity would continue. Immediately upon the outbreak of the crisis, however, most distressed financial markets showed extreme pessimism, translating into severe impacts on the global economy and leading to the bankruptcy of some financial institutions. We utilize the concept of optimism and pessimism as a theoretical framework that is not only useful but also important in understanding, to a certain extent, DM choices.

Let us consider an XOR-DEA problem with the following XOR data, which cannot be solved by current DEA techniques. Suppose DMs want to evaluate the efficiency of three renewable energy projects, wind (W), solar (S), and hydropower (H), by using the XOR input/output data reported in Table 7.2. For example, the second row of Table 7.2 shows that the primary investment costs (IC) for the solar plant is estimated to be 1.4 XOR 1.55 XOR 1.6 ($Euro \times 10^3/Kw$) and the power production (PP) is expected to be 2.6 XOR 2.7 XOR 3 ($MW \times 10^3$).

Table 7.2: XOR input and output data.

Power Plant	Input	Output
	Investment costs (IC)($Euro \times 10^3/Kw$)	Power production (PP) ($MW \times 10^3$)
Wind	1.275 XOR 1.5	2.5 XOR 2.8
Solar	1.4 XOR 1.55 XOR 1.6	2.6 XOR 2.7 XOR 3
Hydropower	2.25	2.9 XOR 3.5

Now, suppose that the DMs have preferences relating to the investment costs of the solar plant, which are 1.4 XOR 1.55 XOR 1.6 ($Euro \times 10^3/Kw$). Besides the DMs' knowledge and some optimistic views upon economic indicators, the forecasting studies confirm that the general trend costs of solar technology will decrease and could reach 1.4 ($Euro \times 10^3/Kw$). Thus, if DMs have an optimistic view about this vision, they will aim for 1.4 ($Euro \times 10^3/Kw$). However, to maintain prudent risk taking, they are not willing to proceed with their decision based on only one value (i.e., 1.4 $Euro \times 10^3/Kw$); they prefer the best selection to be determined through sound decision making processes that trade-off various interests and policies. Therefore, by incorporating the DM's preferences, we evaluate the solar efficiency by solving the following XOR-DEA problem:

(7.6)

$$
\begin{aligned}
\min \quad & \theta_S \\
\text{s.t.} \quad & (2.5XOR2.8)\lambda_{W/S} + (2.6XOR2.7XOR3)\lambda_{S/S} + (2.9XOR3.5)\lambda_{H/S} \geq (2.6XOR2.7XOR3) \\
& (1.275XOR1.5)\lambda_{W/S} + (1.4XOR1.55XOR1.6)\lambda_{S/S} + 2.25\lambda_{H/S} \leq (1.4XOR1.55XOR1.6)\theta_S \\
& (1.4XOR1.55XOR1.6) \in \Lambda \\
& \lambda_{W/S} + \lambda_{S/S} + \lambda_{H/S} = 1 \\
& \lambda_{W/S}, \lambda_{S/S}, \lambda_{H/S} \geq 0 \\
& \Lambda \, \text{is the positive direction set}
\end{aligned}
$$

We can model the above program (7.6) reflecting the DM optimistic view upon the solar investment costs as follows:

(7.7)

$$
\begin{aligned}
\min \quad & \theta_S + \sigma_{12}^- + \sigma_{12}^+ \\
\text{s.t.} \quad & (2.5\eta_{11} + 2.8\eta_{12})\lambda_{W/S} + (2.6\eta_{21} + 2.7\eta_{22} + 3\eta_{23})\lambda_{S/S} + (2.9\eta_{31} + 3.5\eta_{32})\lambda_{H/S} \geq (2.6\eta_{21} + 2.7\eta_{22} + 3\eta_{23}) \\
& (1.275\eta'_{11} + 1.5\eta'_{12})\lambda_{W/S} + (1.4\eta'_{21} + 1.55\eta'_{22} + 1.6\eta'_{23})\lambda_{S/S} + 2.25\lambda_{H/S} \leq (1.4\eta'_{21} + 1.55\eta'_{22} + 1.6\eta'_{23})\theta_S \\
& \left[5 \times (1.4\eta'_{21} + 1.55\eta'_{22} + 1.6\eta'_{23}) - (5 \times 1.4)\right] + \sigma_{12}^- - \sigma_{12}^+ = 0 \\
& \lambda_{W/S} + \lambda_{S/S} + \lambda_{H/S} = 1 \\
& \eta_{11} + \eta_{12} = 1; \eta_{21} + \eta_{22} + \eta_{23} = 1; \eta_{31} + \eta_{32} = 1 \\
& \eta'_{11} + \eta'_{12} = 1; \eta'_{21} + \eta'_{22} + \eta'_{23} = 1 \\
& \lambda_{W/S}, \lambda_{S/S}, \lambda_{H/S} \geq 0; \eta_{lh}, \eta'_{lh} = \{0,1\} \; l = 1,2,3; \; h = 1,2,3
\end{aligned}
$$

To show the usefulness of incorporating DM preferences in the optimization process, we solve the XOR-DEA model (7.7) with and without DM's preferences. Similarly, the rest of renewable energy projects have been evaluated while keeping the optimistic view on the solar investment costs. Using LINGO (Schrage, 2015), the optimal solution of this problem is reported in Table 7.3. Moreover, based on the binary variable set of Table 7.3, the output mechanism of the XOR function outputs of XOR-DEA with and without DM's preferences are reported in Table 7.4. In particular, for simplicity, we denote $XOR(v_{W/PP})$ for the output XOR function of the wind project with respect to power production and similarly denote $XOR(u_{S/IC})$ for the input XOR function of the solar project with respect to investment costs.

Table 7.3: XOR-DEA results with and without DM's preferences.

Power Plant	Efficiency	Binary variables sets											
		η_{11}	η_{12}	η_{21}	η_{22}	η_{23}	η_{31}	η_{32}	η'_{11}	η'_{12}	η'_{21}	η'_{22}	η'_{23}
	XOR-DEA with preferences												
Wind	0.933	1	0	0	1	0	0	1	0	1	1	0	0
Solar	0.910	0	1	0	1	0	1	0	1	0	1	0	0
Hydropower	0.594	0	1	0	0	1	1	0	1	0	1	0	0
	XOR-DEA without preferences												
Wind	0.933	1	0	0	1	0	0	1	0	1	1	0	0
Solar	1	0	1	0	0	1	0	1	1	0	0	1	0
Hydropower	0.594	0	1	0	0	1	1	0	1	0	1	0	0

Table 7.4: XOR function outputs.

Power plants	XOR function				
	$XOR(v_{W/PP})$	$XOR(v_{S/PP})$	$XOR(v_{H/PP})$	$XOR(u_{W/IC})$	$XOR(u_{S/IC})$
			XOR-DEA with preferences		
Wind	2.5	2.7	3.5	1.5	1.4
Solar	2.8	2.7	2.9	1.275	1.4
Hydropower	2.8	3	2.9	1.275	1.4
			XOR-DEA without preferences		
Wind	2.5	2.7	3.5	1.5	1.4
Solar	2.8	3	3.5	1.275	1.55
Hydropower	2.8	3	2.9	1.275	1.4

From Table 7.3 and 7.4, we clearly observe that there are some differences between solutions obtained from XOR-DEA with and that without DM's preferences. In particular, we can check whether the DM's preferences about the solar investment costs have been considered in the optimization problem. In this case, the XOR function of solar investment costs provides different values depending on whether the DMs' preferences are incorporated or not; 1.4 with preferences and 1.55 without preferences. This means that the DM's optimistic expectations have been realized and taken into account in the optimization process of solar energy project. While for the other renewable energy projects, it is not necessarily that the optimistic view upon solar investment costs will affect the output mechanism of the XOR function. This means that the preferences constraint serves as a soft constraint by means of deviation variables (σ_{12}^- and σ_{12}^+) of goal programming approach which provides the related XOR function the ability of selecting the appropriate value that optimizes the efficiency. This way of modelling is inspired from the multi-objective optimization framework that is essentially developed to tackle the multiple conflicting objectives and provide compromise or the so-called "pareto solution" (see, e.g., Miettinen, 1999; Ehrgott, 2005).

7.4.3 The behavior of the XOR-DEA model under the controlled and uncontrolled channel

Following previous definitions, 2–4, only one *element* (value) will be produced from the XOR function. Hence, the following question naturally arises: what is the behavior of XOR-DEA under the controlled and uncontrolled channel? To clarify these concepts, let us reconsider our previous example. In particular, we evaluate the efficiency of solar by solving two optimization programs based on: i) the uncontrolled channel, and ii) the controlled channel. The first following program formulates the model based on the uncontrolled channel.

(7.8)

$$
\begin{aligned}
\min \quad & \theta_S \\
\text{s.t.} \quad & (2.5XOR2.8)\lambda_{W/S} + (2.6XOR2.7XOR3)\lambda_{S/S} + (2.9XOR3.5)\lambda_{H/S} \geq (2.6XOR2.7XOR3) \\
& (1.275XOR1.5)\lambda_{W/S} + (1.4XOR1.55XOR1.6)\lambda_{S/S} + 2.25\lambda_{H/S} \leq (1.4XOR1.55XOR1.6)\theta_S \\
& (2.5XOR2.8), (2.6XOR2.7XOR3), (2.9XOR3.5) \in \Xi \\
& (1.275XOR1.5), (1.4XOR1.55XOR1.6) \in \Xi \\
& \lambda_{W/S} + \lambda_{S/S} + \lambda_{H/S} = 1 \\
& \lambda_{W/S}, \lambda_{S/S}, \lambda_{H/S} \geq 0 \\
& \Xi \text{ is the uncontrolled set}
\end{aligned}
$$

We can model the above program (7.8) based on the uncontrolled channel as follows:

(7.9)

$$
\begin{aligned}
\min \quad & \theta_S \\
\text{s.t.} \quad & (2.5\eta_{11} + 2.8\eta_{12})\lambda_{W/S} + (2.6\eta_{21} + 2.7\eta_{22} + 3\eta_{23})\lambda_{S/S} + (2.9\eta_{31} + 3.5\eta_{32})\lambda_{H/S} \geq (2.6\eta_{21} + 2.7\eta_{22} + 3\eta_{23}) \\
& (1.275\eta'_{11} + 1.5\eta'_{12})\lambda_{W/S} + (1.4\eta'_{21} + 1.55\eta'_{22} + 1.6\eta'_{23})\lambda_{S/S} + 2.25\lambda_{H/S} \leq (1.4\eta'_{21} + 1.55\eta'_{22} + 1.6\eta'_{23})\theta_S \\
& \lambda_{W/S} + \lambda_{S/S} + \lambda_{H/S} = 1 \\
& \eta_{11} + \eta_{12} = 1; \eta_{21} + \eta_{22} + \eta_{23} = 1; \eta_{31} + \eta_{32} = 1 \\
& \eta'_{11} + \eta'_{12} = 1; \eta'_{21} + \eta'_{22} + \eta'_{23} = 1 \\
& \lambda_{W/S}, \lambda_{S/S}, \lambda_{H/S} \geq 0; \eta_{lh}, \eta'_{lh} = \{0,1\} \; l = 1,2,3; \; h = 1,2,3
\end{aligned}
$$

Using LINGO (Schrage, 2015), the optimal solution of (7.9) is reported in Table 7.

As has been stressed previously (Section 2.3), a key issue in uncertainty analysis is the difficulty in detecting and determining the layers and the challenge of modeling them. Think about the investment cost needed for a new project. Generally, the initial estimate of investment cost could be subjective due to many confounding factors; in addition, the investment cost itself is subject to randomness. Subjective uncertainty refers to the state of mind of the individual making a decision, independent of the status of the objective system on which the decision is being made. What makes the investment cost of project X uncertain? Suppose, after certain analyses, we detected some possible layers that may affect the solar investment costs, as follows: LU1): Time-delay realization; LU2): Infrastructure cost; LU3): Market conditions; and LU4): Environmental issues (severe weather).

Table 7.5 reports the implications of these layers on each investment cost scenario (1.4, 1.55, and 1.6 ($MW \times 10^3$)) using a rating scale of 1 to 10 (from worst to best), which indicates how much these layers contribute to and affect each scenario. For example, due to current renewable energy market conditions, it is estimated that solar investment costs overall will decrease over time. Therefore, LU3, market conditions support the lowest investment cost scenario of 1.4 ($MW \times 10^3$). Accordingly, DMs give it a high rate of "8." The same reasoning holds for the other layers and scenarios. To activate these layers in the optimization process, the concept of a reference point is adopted. Psychologists provide alternative interpretations to the reference point as either an expectation, an aspiration level, an ideal point, or the DM's status quo (see, e.g., Siegel, 1957; Hwang & Yoon, 1981; Tversky & Kahneman, 1991). In this paper, based on the ideal point (or scale) of TOPSIS algorithm (technique for order preference by similarity to ideal solution; Hwang & Yoon, 1981) and expectation theory, we provide an alternative way to compute the reference point. In particular, it is calculated as the average between the ideal scale (e.g., 8 for LU3) and the mean of rating scales of the related layer (e.g., the mean of 8, 6, and 4 of LU3).

Table 7.5: Layer evaluations and their reference points.

Layers/Scenarios	1.4	1.55	1.6	Reference point
LU1	4	7	8	7.16
LU2	5	6	8	7.16
LU3	8	6	4	7
LU4	6	7	8	7.5

To model the problem through the controlled channel, the layers are incorporated into the optimization process. This problem can be reformulated and modeled as follow:

(7.10)

$$\begin{aligned}
\min \quad & \theta_S \\
\text{s.t.} \quad & (2.5XOR2.8)\lambda_{W/S} + (2.6XOR2.7XOR3)\lambda_{S/S} + (2.9XOR3.5)\lambda_{H/S} \geq (2.6XOR2.7XOR3) \\
& (1.275XOR1.5)\lambda_{W/S} + (1.4XOR1.55XOR1.6)\lambda_{S/S} + 2.25\lambda_{H/S} \leq (1.4XOR1.55XOR1.6)\theta_S \\
& (2.5XOR2.8), (2.9XOR3.5), (1.275XOR1.5), (2.6XOR2.7XOR3) \in \Xi \\
& (1.4XOR1.55XOR1.6) \in \Theta \\
& \lambda_{W/S} + \lambda_{S/S} + \lambda_{H/S} = 1
\end{aligned}$$

$\lambda_{W/S}, \lambda_{S/S}, \lambda_{H/S} \geq 0$

Ξ is the uncontrolled set

Θ is the controlled set

We can model the above program (7.10) based on the controlled channel as follows:

(7.11)

$$
\begin{aligned}
\min \quad & \theta_S + \tau^-_{LU1} + \tau^+_{LU1} + \tau^-_{LU2} + \tau^+_{LU2} + \tau^-_{LU3} + \tau^+_{LU3} + \tau^-_{LU4} + \tau^+_{LU4} \\
\text{s.t.} \quad & (2.5\eta_{11} + 2.8\eta_{12})\lambda_{W/S} + (2.6\eta_{21} + 2.7\eta_{22} + 3\eta_{23})\lambda_{S/S} + (2.9\eta_{31} + 3.5\eta_{32})\lambda_{H/S} \geq (2.6\eta_{21} + 2.7\eta_{22} + 3\eta_{23}) \\
& (1.275\eta'_{11} + 1.5\eta'_{12})\lambda_{W/S} + (1.4\eta'_{21} + 1.55\eta'_{22} + 1.6\eta'_{23})\lambda_{S/S} + 2.25\lambda_{H/S} \leq (1.4\eta'_{21} + 1.55\eta'_{22} + 1.6\eta'_{23})\theta_S \\
& 4\eta_{21} + 7\eta_{22} + 8\eta_{23} + \tau^-_{LU1} - \tau^+_{LU1} = 7.16;\ 5\eta_{21} + 6\eta_{22} + 8\eta_{23} + \tau^-_{LU2} - \tau^+_{LU2} = 7.16; \\
& 8\eta_{21} + 6\eta_{22} + 4\eta_{23} + \tau^-_{LU3} - \tau^+_{LU3} = 7;\ 6\eta_{21} + 7\eta_{22} + 8\eta_{23} + \tau^-_{LU4} - \tau^+_{LU4} = 7.5; \\
& \lambda_{W/S} + \lambda_{S/S} + \lambda_{H/S} = 1 \\
& \eta_{11} + \eta_{12} = 1;\ \eta_{21} + \eta_{22} + \eta_{23} = 1;\ \eta_{31} + \eta_{32} = 1 \\
& \eta'_{11} + \eta'_{12} = 1;\ \eta'_{21} + \eta'_{22} + \eta'_{23} = 1 \\
& \lambda_{W/S}, \lambda_{S/S}, \lambda_{H/S} \geq 0;\ \eta_{lh}, \eta'_{lh} = \{0,1\}\ l = 1,2,3;\ h = 1,2,3
\end{aligned}
$$

Using LINGO (Schrage, 2015), the optimal solution of (7.11) is listed in Table 7. Moreover, based on the binary variable set of Table 7, the output mechanism of the XOR function outputs of controlled and uncontrolled channels are reported in Table 8.

Table 7.6: XOR-DEA results

Power Plant	Efficiency	Binary variables sets											
		η_{11}	η_{12}	η_{21}	η_{22}	η_{23}	η_{31}	η_{32}	η'_{11}	η'_{12}	η'_{21}	η'_{22}	η'_{23}
	Controlled channel												
Wind	0.933	0	1	0	0	1	0	1	0	1	1	0	0
Solar	0.910	0	1	1	0	0	0	1	1	0	1	0	0
Hydropower	0.627	0	1	0	0	1	1	0	1	0	0	1	0
	Uncontrolled channel												
Wind	0.933	1	0	0	1	0	0	1	0	1	1	0	0
Solar	1	0	1	0	0	1	0	1	1	0	0	1	0
Hydropower	0.594	0	1	0	0	1	1	0	1	0	1	0	0

From Tables 7–8, we observe that there are some differences between solutions obtained based on controlled and uncontrolled channels. For solar investment costs, the output mechanisms of the uncontrolled and controlled channels provide, respectively, 1.55 and 1.4. This confirms that the layers have impacted the output mechanism of the $XOR(u_{S/IC})$ function. However, for other energy projects, it is not necessarily that the solar investment costs' layers will affect the output mechanism of the $XOR(u_{S/IC})$ function (e.g., 1.55 vs. 1.44 of hydropower). Since there are many interactions between the output mechanism of the XOR functions, the effect of solar investment costs' layers spreads over the power production outputs of wind and solar energy projects. Finally, it is worth noting that the efficiency produced from the controlled channel is not always better than that obtained from the uncontrolled channel.

Table 7.7: XOR function outputs.

Power plants	XOR function				
	$XOR(v_{W/PP})$	$XOR(v_{S/PP})$	$XOR(v_{H/PP})$	$XOR(u_{W/IC})$	$XOR(u_{S/IC})$
Controlled channel					
Wind	2.8	3	3.5	1.5	1.4
Solar	2.8	2.5	3.5	1.275	1.4
Hydropower	2.8	3	2.9	1.275	1.55
Uncontrolled channel					
Wind	2.5	2.7	3.5	1.5	1.4
Solar	2.8	3	3.5	1.275	1.55
Hydropower	2.8	3	2.9	1.275	1.4

7.5 XOR DATA ENVELOPMENT ANALYSIS (XOR-DEA) FORMULATION

Consider a set of n DMUs (DMUj, $j = 1, \ldots, n$) where $XORX_j = (XORx_{1j}, \ldots, XORx_{sj})$ is the vector of XOR inputs and $XORY_j = (XORy_{1j}, \ldots, XORy_{mj})$ is the vector of XOR output. The production possibility set Ω is the set $\{(XORX, XORY) |\, the\, outputs\, XORY \geq 0\, can\, be\, produced\, from\, the\, inputs\, XORX$ 0$\}$. Symbolically, the general XOR-DEA problem following VRS DEA model (7.5) is formulated as follows:

(7.12)

$$
\begin{aligned}
\min \quad & \theta_k \\
\text{s.t.} \quad & XORy_{rk} - \sum_{j=1}^{n} \lambda_j XORy_{rj} \leq 0 \;\; r = 1, \ldots, m \\
& \theta_k XORx_{ik} - \sum_{j=1}^{n} \lambda_j XORx_{ij} \geq 0 \;\; i = 1, \ldots, s \\
& XORy_{rk}, XORx_{ik} \in \Lambda \,(positive\, direction\, set) \\
& XORy_{rk}, XORx_{ik} \in \Upsilon \,(negative\, direction\, set) \\
& XORy_{rk}, XORx_{ik} \in \Psi \,(neutral\, set) \\
& XORy_{rk}, XORx_{ik} \in \Xi \,(uncontrolled\, set) \\
& XORy_{rk}, XORx_{ik} \in \Theta \,(controlled\, set) \\
& XORy_{rk}, XORx_{ik} \in \Omega \,(production\, possibility\, set) \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0 \,\forall\, j = 1, \ldots, n
\end{aligned}
$$

where $XORy_{rj} = y_{rj}^1 XOR y_{rj}^2 XOR \ldots XOR y_{rj}^{l_0}$ and $XORx_{ik} = x_{ik}^1 XOR x_{ik}^2 XOR \ldots XOR x_{ik}^{l_0'}$ are XOR numbers. Based on this canonical form, we are now able to model the XOR-DEA problem (7.12) based on the uncontrolled channel with preferences as follows.

(7.13)

$$\begin{aligned}
\min \quad & \theta_k + \sigma^-_{rk} + \sigma^+_{rk} + \sigma'^-_{ik} + \sigma'^+_{ik} \\
\text{s.t.} \quad & \underset{l=1,\ldots,l_0}{XOR} (y_l, \eta_l)_{rk} - \sum_{j=1}^{n} \lambda_j \underset{l=1,\ldots,l_0}{XOR} (y_l, \eta_l)_{rj} \leq 0 \;\; r = 1,\ldots,m \\
& \theta_k \underset{l'=1,\ldots,l'_0}{XOR} (x_{l'}, \eta'_{l'})_{ik} - \sum_{j=1}^{n} \lambda_j \underset{l'=1,\ldots,l'_0}{XOR} (x_{l'}, \eta'_{l'})_{ij} \geq 0 \;\; i = 1,\ldots,s \\
& \psi_i \underset{l=1,\ldots,l_0}{XOR} (y_l, \eta_l)_{rk} - \psi_i \min_l \left[\underset{l=1,\ldots,l_0}{XOR} (y_l)_{rk} \right] + \sigma^-_{rk} - \sigma^+_{rk} = \delta \\
& \psi'_i \underset{l'=1,\ldots,l'_0}{XOR} (x_{l'}, \eta'_l)_{ik} - \psi'_i \min_{l'} \left[\underset{l'=1,\ldots,l'_0}{XOR} (x_{l'})_{ik} \right] + \sigma'^-_{ik} - \sigma'^+_{ik} = \delta' \\
& \underset{l=1,\ldots,l_0}{XOR} (y_l, \eta_l)_{rk} = y_1\eta_1 + y_2\eta_2 + \ldots + y_{l_0}\eta_{l_0} \\
& \underset{l'=1,\ldots,l'_0}{XOR} (x_{l'}, \eta'_{l'})_{ik} = x_1\eta'_1 + x_2\eta'_2 + \ldots + x_{l'_0}\eta'_{l'_0} = 1 \\
& \sum_{j=1}^{n} \lambda_j = 1, \quad \sum_{l=1,\ldots,l_0} \eta_l = 1, \quad \sum_{l'=1,\ldots,l'_0} \eta'_{l'} = 1 \\
& \sigma^-_{rk}, \sigma^+_{rk}, \sigma'^-_{ik}, \sigma'^+_{ik}, \lambda_j \geq 0; \eta_l \text{ and } \eta'_{l'} = \{0,1\}
\end{aligned}$$

Similarly, the XOR-DEA problem (7.12) based on the controlled channel is formulated as follows:

(7.14)

$$\begin{aligned}
\min \quad & \theta_k + \sigma^-_{rk} + \sigma^+_{rk} + \sigma'^-_{ik} + \sigma'^+_{ik} + \tau^-_{rk} + \tau^+_{rk} + \tau'^-_{ik} + \tau'^+_{ik} \\
\text{s.t.} \quad & \underset{l=1,\ldots,l_0}{XOR} (y_l, \eta_l)_{rk} - \sum_{j=1}^{n} \lambda_j \underset{l=1,\ldots,l_0}{XOR} (y_l, \eta_l)_{rj} \leq 0 \;\; r = 1,\ldots,m \\
& \theta_k \underset{l'=1,\ldots,l'_0}{XOR} (x_{l'}, \eta'_{l'})_{ik} - \sum_{j=1}^{n} \lambda_j \underset{l'=1,\ldots,l'_0}{XOR} (x_{l'}, \eta'_{l'})_{ij} \geq 0 \;\; i = 1,\ldots,s \\
& \psi_i \underset{l=1,\ldots,l_0}{XOR} (y_l, \eta_l)_{rk} - \psi_i \min_l \left[\underset{l=1,\ldots,l_0}{XOR} (y_l)_{rk} \right] + \sigma^-_{rk} - \sigma^+_{rk} = \delta \\
& \psi'_i \underset{l'=1,\ldots,l'_0}{XOR} (x_{l'}, \eta'_{l'})_{ik} - \psi'_i \min_{l'} \left[\underset{l'=1,\ldots,l'_0}{XOR} (x_{l'})_{ik} \right] + \sigma'^-_{ik} - \sigma'^+_{ik} = \delta' \\
& \underset{l=1,\ldots,l_0}{XOR} (y_l, \eta_l)_{rk} = y_1\eta_1 + y_2\eta_2 + \ldots + y_{l_0}\eta_{l_0} \\
& \underset{l'=1,\ldots,l'_0}{XOR} (x_{l'}, \eta'_{l'})_{ik} = x_1\eta'_1 + x_2\eta'_2 + \ldots + x_{l'_0}\eta'_{l'_0} \\
& \underset{l=1,\ldots,l_0}{XOR} (y_l, \zeta_l(\eta_l))_{rk} + \tau^-_{rk} + \tau^+_{rk} = \kappa \\
& \underset{l'=1,\ldots,l'_0}{XOR} (x_{l'}, \zeta'_{l'}(\eta'_{l'}))_{ik} + \tau'^-_{ik} + \tau'^+_{ik} = \kappa' \\
& \underset{l=1,\ldots,l_0}{XOR} (y_l, \zeta_l(\eta_l))_{rk} = \zeta_1\eta_1 + \zeta_2\eta_2 + \ldots + \zeta_{l_0}\eta_{l_0}
\end{aligned}$$

$$\underset{l'=1,\ldots,l'_0}{XOR}(x_{l'},\zeta'_{l'}(\eta'_{l'}))_{ik} = \zeta_1\eta'_1 + \zeta_2\eta'_2 + \ldots + \zeta_{l'_0}\eta'_{l'_0};$$

$$\sum_{j=1}^{n}\lambda_j = 1, \quad \sum_{l=1,\ldots,l_0}\eta_l = 1, \quad \sum_{l'=1,\ldots,l'_0}\eta'_{l'} = 1$$

$$\sigma^-_{rk}, \sigma^+_{rk}, \sigma'^-_{ik}, \sigma'^+_{ik}, \tau^-_{rk}, \tau^+_{rk}, \tau'^-_{ik}, \tau'^+_{ik}, \lambda_j \geq 0; \eta_l \text{ and } \eta'_{l'} = \{0,1\}$$

where $\underset{l=1,\ldots,l_0}{XOR}(y_l,\eta_l)$ and $\underset{l'=1,\ldots,l'_0}{XOR}(x_{l'},\eta'_{l'})$ are the function of XOR input and output; η_l and $\eta'_{l'}$ are binary variables; ζ captures the parameters of LUs. The objective function minimizes θ and the unwanted deviations ($\sigma^-_{ij}, \sigma^+_{rk}, \sigma'^-_{ik}, \sigma'^+_{ik}, \tau^-_{rk}, \tau^+_{rk}, \tau'^-_{ik}$, and τ'^+_{ik}) from the reference points κ and κ' and from the psychological parameter values δ and δ' of input and output respectively. The fifth and sixth constraints guarantee that only one element will be selected from the input and output XOR functions that achieve the highest level of efficiency. The seventh and eighth constraints allow the output mechanism of the XOR function to integrate the layers in the optimization process.

Definition 4. *Unit* $(XORX_o, XORY_o) \in \Omega$ *is called efficient if any optimal solution of* (7.13) *or* (7.14) *satisfies: (a)* $\theta^* = 1$*, and (b) all slack variables are zeroes.*

Definition 5. *Activity* $(XORX, XORY) \in \Omega$ *is weakly Pareto efficient if and only if there is no* $(XORX', XORY') \in \Omega$ *such that* $XORX' < XORX$ *and* $XORY' < XORY$.

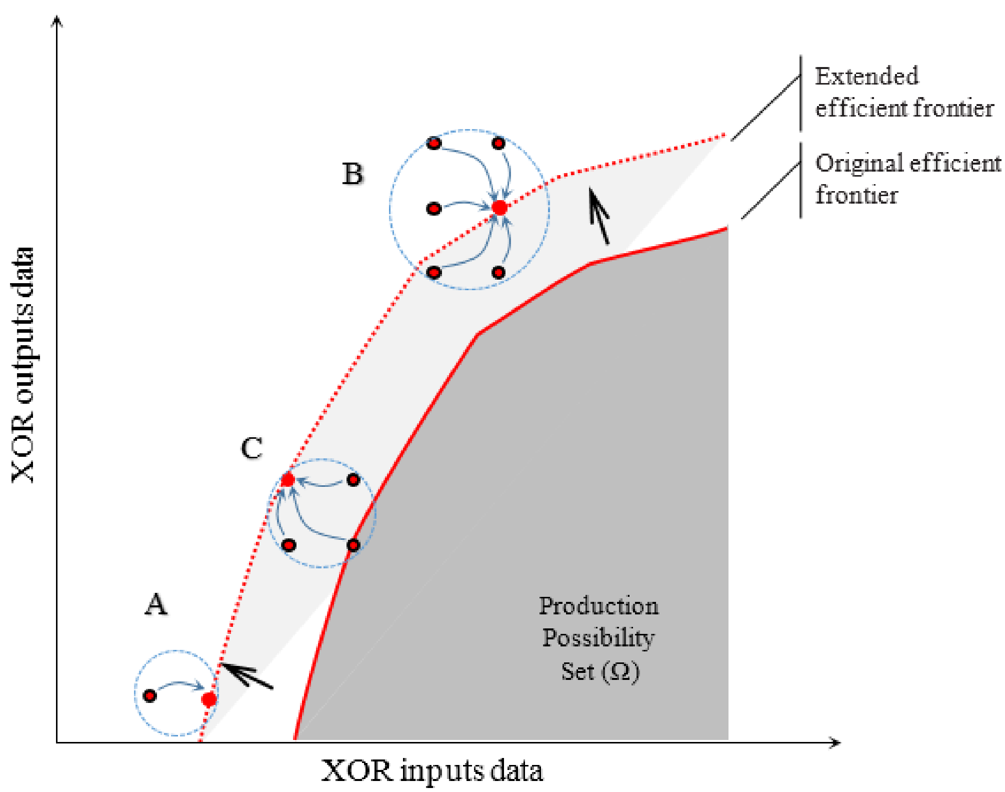


Figure 7.3: XOR DEA optimization process.

Recall that slacks variables can be incorporated easily into both models (controlled and uncontrolled channels) by simply converting the inequalities to equalities, as in classic linear programming techniques. Moreover, as pointed out in Cooper et al. (2011), DEA modeling gives new insights into efficiency measurements that have been tackled by other alternative methods, such as stochastic frontier analysis. In this line, the XOR-DEA modeling provides another advantage by extending its ability to solve problems featuring XOR data. This enables the DMs to extend the original feasible (or production) region obtained from the basic variants of DEA to a larger potential feasible region. In particular, this process allows them to discover further sources that achieve full efficiency, as shown in Figure 7.3.

7.6 REAL-LIFE APPLICATION: RANKS RENEWABLE ENERGY TECHNOLOGIES

According to growth theories and United Nations reports, investment in renewable sources is one of the best ways to attain sustainable development (SD) goals and enhance nations' welfare (Kraft & Kraft, 1978; Stern, 2004). With this vision, Morocco has recently started a large-scale initiative to develop its renewable energy sector by improving infrastructure and upgrading technologies in order to deliver cleaner and more efficient energy. With this program, the government has promised to achieve a valid and enforceable objective of generating 52% of the nation's electricity from renewable sources by 2030. Production is expected to come from a variety of sources: 4560 MW from solar (3560 solar photovoltaic and 1000 MW solar thermal), 4200 MW from wind, 3100 MW from hydropower, and 1000 MW from others (see, e.g., Simelyte, 2020).

To optimize the efficiency of renewable energy resource exploitation, it is necessary to consider the factors relevant to economic and SD theories in the evaluation process. While the meaning of economic growth is clear, the complexity of SD theory has created a plethora of interpretations for the concept. In general, SD can be viewed as a complex process that aims to achieve a desirable balance between different competing factors and limitations, including the environmental, social, and economic dimensions. From this perspective, the renewable energy sources (technologies) evaluation problem becomes a multi-dimensional decision problem. The problem involves a search for the most efficient technologies that meet SD goals and DMs' preferences. This application solves such complex decision-making problems by evaluating the efficiency of and ranking renewable energy technologies under uncertainty that stems from XOR logic.

To capture SD concepts during the process of ranking the alternatives, the selected inputs and outputs should reflect the aspects of SD theory. In related literature, these input/output criteria are typically divided into four categories: technical, economic, environmental, and social. Following this reasoning, we consider three inputs (investment ratio (IR), operating and maintenance costs (OMC), and land requirements (LR)) and three outputs (power production (PP), tons of CO2 avoided (CO2), and useful life (UL)) to evaluate 13 renewable energy technologies in the following areas: solar, wind, hydropower, biomass, and biofuel power. For instance, IR and OMC refer to all costs related to the purchase of mechanical equipment, technological installations, engineering services, wages, and the products and services for the energy system operation. Naturally, criteria such as CO2 emission, LR, and UL are generally used to integrate the environmental and socio-economic dimensions of sustainability (for more details see Wang et al., 2009). Thus, to evaluate the usefulness of the XOR-DEA models using a larger data set, we consider Cristobal's (2011) input/output data under xorness, as reported, respectively, in Tables 7.8-7.9.

To examine the applicability of XOR-DEA models, we focus on two possible scenarios: i) the first scenario covers the DMs' preferences and the controllable channel and ii) the second scenario considers the uncontrollable channel case. For the first scenario, suppose that the DM has an optimistic view of the DMU3. Furthermore, to consider the controllable channel, suppose that, after certain analyses, we detected several layers that may affect the operating and maintenance costs of the DMU7, as follows: LU1): Environmental issues (severe weather); LU2): Skilled labor availability; and LU3): Overhead and monitoring costs.

Table 7.8: XOR input data

Renewable energy technologies	Inputs		
	Investment ratio (IR) (Euro$\times 10^3$/Kw)	Operating and maintenance costs (OMC) (Euro$\times 10^{-3}$/Kwh)	Contribution to regional development (CRD) (Qualitative)
Windpower P < 5 Mw	0.937	1.47	6.5
Windpower 5 < P < 10Mw	0.937	1.47 XOR 1.5	6.5 XOR 7
Windpower 10 < P < 50Mw	1.500 XOR 1.700 XOR 1.900	1.51	7.5
Hydroelectric P < 10Mw	0.700	1.45	5.5
Hydroelectric 10 < P < 25 Mw	0.601	0.70 XOR 0.75	5.5
Hydroelectric 25 < P < 50Mw	5.000	0.60 XOR 0.65	5.5 XOR 6
Solar Thermo-electric P > 10Mw	1.803	4.2	8 XOR 9
Biomass (energetic cultivations) P < 5Mw	1.803	7.11	5
Biomass (forest and agricultural wastes) P < 5Mw	1.803	5.42	5
Biomass (farming industrial wastes) P < 5Mw	1.803	5.42 XOR 5.6	5
Biomass (forest industrial wastes) P < 5 Mw	1.803	2.81	5
Biomass (Co-combustion in conventional central) P > 50Mw	0.856	4.56	5
Biofuels P < 2Mw	1.503	2.51	4.5

Table 7.9: XOR output data

Renewable energy technologies	Outputs	
	Power (Mw $\times 10^3$)	Tons of CO2 avoided (tCO2 * 106/year)
Windpower P < 5 Mw	0.5	1.93
Windpower 5 < P < 10Mw	1.0	3.22
Windpower 10 < P < 50Mw	2.5	9.65
Hydroelectric P < 10Mw	0.5	0.47
Hydroelectric 10 < P < 25 Mw	2.0	0.26
Hydroelectric 25 < P < 50Mw	3.5	0.26 XOR 0.30
Solar Thermo-electric P > 10Mw	5.0	0.48 XOR 0.50 XOR 0.55
Biomass (energetic cultivations) P < 5Mw	0.5	2.52
Biomass (forest and agricultural wastes) P < 5Mw	0.5	2.52
Biomass (farming industrial wastes) P < 5Mw	0.5	2.52 XOR 2.60
Biomass (forest industrial wastes) P < 5 Mw	0.5	2.52 XOR 2.70
Biomass (Co-combustion in conventional central) P > 50Mw	5.6	4.84
Biofuels P < 2Mw	0.2	5.91

Now, let us suppose that the DMs have some preferences about certain input and output as shown in Tables 7.10-7.11.

Table 7.10: DM's preferences

DMU	Renewable energy technologies	Preferences		
		Optimistic	Pessimistic	Neutral
Inputs	Investment ratio	DMU3	-	-
	Operating and maintenance costs	-	-	-
	Contribution to regional development	-	-	-
Outputs	Power	-	-	-
	Tons of CO2 avoided	-	-	-

Table 7.11: XOR function.

DMU	Renewable energy technologies	Output mechanism	
		Controlled channel	Uncontrolled channel
Inputs	Investment ratio	-	-
	Operating and maintenance costs	-	-
	Contribution to regional development	-	-
Outputs	Power	-	-
	Tons of CO2 avoided	-	DMU7

Solving this hypothetical decision-making problem using LINGO (Schrage, 2015), the efficiency score and rank of each DMU evaluated by using the XOR DEA model are shown in Table 7.12.

Table 7.12: XOR DEA results.

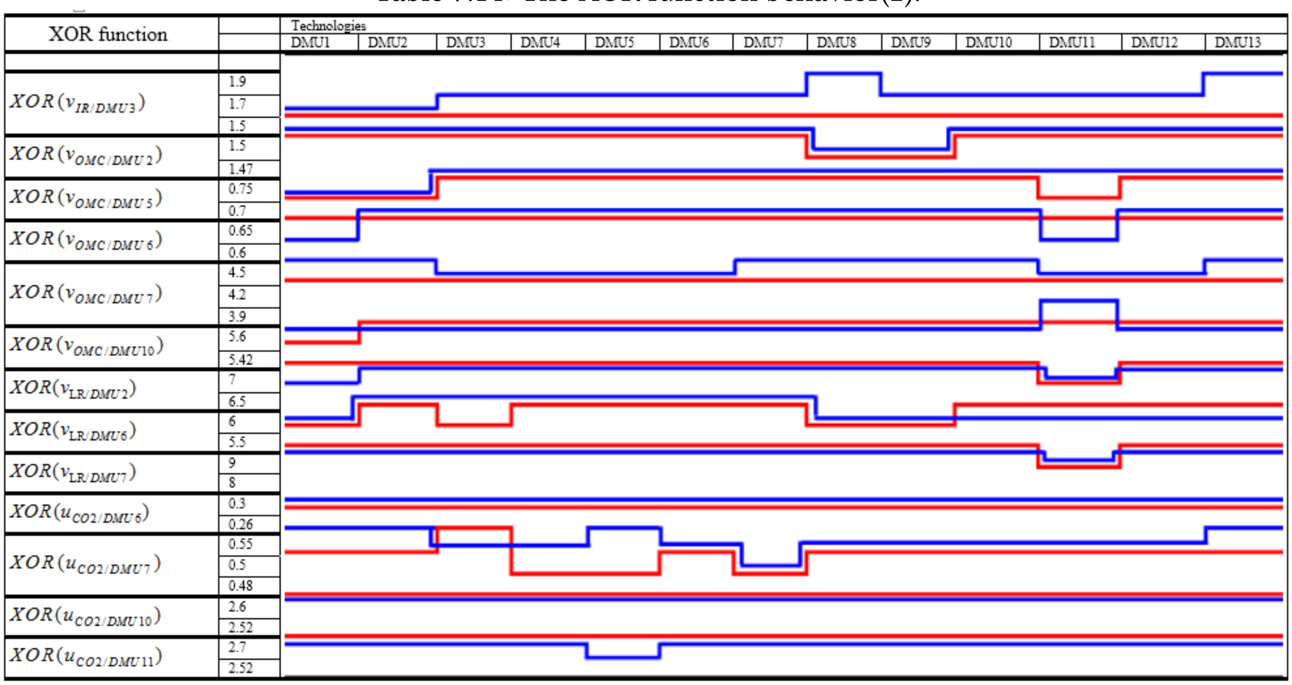
DMU	Renewable energy technologies	Efficiency	Rank
DMU1	Wind-power P < 5 Mw	0.8446	8
DMU2	Wind-power 5 < P < 10Mw	0.9198	4
DMU3	Wind-power 10 < P < 50Mw	1.0000	1
DMU4	Hydroelectric P < 10Mw	0.9768	3
DMU5	Hydroelectric 10 < P < 25 Mw	1.0000	1
DMU6	Hydroelectric 25 < P < 50Mw	0.9999	2
DMU7	Solar Thermo-electric P > 10Mw	0.8623	7
DMU8	Biomass (energetic cultivations) P < 5Mw	0.9055	6
DMU9	Biomass (forest and agricultural wastes) P < 5Mw	0.9055	6
DMU10	Biomass (farming industrial wastes) P < 5Mw	0.9055	6
DMUI1	Biomass (forest industrial wastes) P < 5 Mw	0.9086	5
DMUI2	Biomass (Co-combustion in conventional central) P > 50Mw	1.0000	1
DMUI3	Biofuels P < 2Mw	1.0000	1

Moreover, to enhance and enrich the analysis of the XOR DEA modeling, we provide a visual analytical tool that supports the DMs to have a deep view of this problem. Table 7.13 and Table 7.14 shows the output behavior of the XOR function. From the screen of Table 7.14, we observe that the behavior of the $XOR(v_{IR/DMU3})$ function is stable and stationary along the optimization process. This means that DM preferences have taken into consideration in the optimization process. Moreover, we observe that the behavior of $XOR(u_{CO2/DMU1})$ is also stable and stationary along the optimization process. Tables 7.13-7.14 offer valuable information about the behavior of XOR functions along the optimization process, where we can check and detect any undesired behavior of XOR functions that can affect the efficiency. For instance, the behavior of $XOR(v_{CRD/DMU6})$ and $XOR(u_{CO2/DMU10})$ have changed their behavior simultaneously in the optimization process number 7 ($OP7$), where both of them have tended to produce their maximum value to serve and achieve the maximum potential efficiency to this technology (i.e., 0.8623).

Table 7.13: The XOR function behavior.

XOR function	Technologies												
	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10	DMU11	DMU12	DMU13
$XOR(v_{IR/DMU3})$	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500	1.500
$XOR(v_{OMC/DMU2})$	1.47	1.5	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.47	1.5	1.47	1.47
$XOR(v_{OMC/DMU5})$	0.70	0.70	0.75	0.70	0.75	0.70	0.70	0.75	0.75	0.75	0.75	0.75	0.75
$XOR(v_{OMC/DMU6})$	0.60	0.60	0.60	0.60	0.60	0.65	0.60	0.60	0.60	0.65	0.60	0.60	0.65
$XOR(v_{OMC/DMU10})$	5.42	5.42	5.42	5.42	5.42	5.42	5.6	5.42	5.42	5.42	5.42	5.42	5.42
$XOR(v_{CRD/DMU2})$	7	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5	6.5
$XOR(v_{CRD/DMU6})$	5.5	5.5	5.5	5.5	5.5	5.5	6	5.5	5.5	5.5	5.5	5.5	5.5
$XOR(v_{CRD/DMU7})$	8	8	8	8	8	9	9	8	8	8	9	8	8
$XOR(u_{CO2/DMU6})$	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26
$XOR(u_{CO2/DMU7})$	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55	0.55
$XOR(u_{CO2/DMU10})$	2.52	2.52	2.52	2.52	2.52	2.52	2.56	2.52	2.56	2.56	2.52	2.52	2.52
$XOR(u_{CO2/DMU11})$	2.52	2.52	2.52	2.52	2.70	2.52	2.52	2.52	2.52	2.52	2.70	2.52	2.52

Table 7.14: The XOR function behavior(2).



7.7 CONCLUSION

This chapter extends the conventional DEA methods to cope with uncertainty and incorporate the DM's preferences. To this end, we developed a new model called XOR-DEA, which allows us to measure the efficiency under the xorness environment. The proposed model has several salient features that make it an efficient and practical method: i) it addresses the uncertainty of XOR logic in a natural way; ii) it integrates DM's preferences by taking into consideration their psychological state (e.g., optimistic, pessimistic, and neutral); iii) it offers an ability to cope deeply with uncertainty through the controlled channel; iv) it offers valuable analyses about the behavior of XOR function during the optimization process that helps to understand and detect any undesired solution; and v) it is formulated based on well-known mathematical programming, which makes it easy and straightforward to understand. While this approach has been carried out for the CCR model, the proposed approach can be easily extended to other DEA models. A promising direction for future research would be consider XOR-DEA as a type of group DEA since the XOR numbers could be seen as a collection of data from a group of DMs.

8 XOR-Analytic Network Process

The greatest challenge to any thinker is stating the problem in a way that will allow a solution.
-Bertrand Russell-

This study develops a new technique for dealing with decision problems under uncertainty using exclusive-or (XOR) logic, called the XOR-analytic network process (XOR-ANP). Then, the proposed technique is adopted to assess the impact of COVID-19 in Morocco on seven relevant sectors (tourism, transport, industrial, financial, agriculture, education, and healthcare) by considering social, operational, and economic dimensions.

8.1 INTRODUCTION

Extreme events and their consequences are among the most challenging phenomena of the modern age and demand urgent interest from the scientific community. Albeverio et al. (2010) define extreme events as events that are rare, exceptional, catastrophic, and surprising and which occur within a limited space and time, by chance or necessity (or through a combination of both), and through natural or human-made causes (or a combination of both) and which have a significant impact on the normal functioning of any system. The consequences of any extreme event, such as COVID-19, are severe, unpredictable, and usually facilitate the transmission of financial shocks across the market and multiple sectors. After several months of fighting the pandemic, many countries have started to slowly transition to recovering from COVID-19, and policymakers in these countries require reliable decision support systems that enable them to make wise decisions and to continually improve their decisions and policies. The main challenge is that the literature related to the transition between the response to the pandemic and the recovery phase is relatively scarce (Gupta et al., 2020). By setting efficient policies, governments will be able to plan relevant strategies that go far beyond prevention, such as increasing hospital capacity, strengthening medical staff recruitment, investing in medical equipment, improving social aids, etc. Policymakers must put in place successful social and economic policies to fight against the disease from different perspectives. Recently, many countries and regions with governance mechanisms have announced the development and implementation of various programs aiming to guide economic recovery. In these situations, governments are required to take effective measures that reduce the negative impacts of the pandemic. Thus, their first task is to assess the current situation and determine where support and intervention are needed. From this perspective naturally arise the questions: which sectors have been most affected by the COVID-19 pandemic, and which sectors should receive priority for support and recovery?

The main challenge for decision-makers (DMs) is that there are no forecasts, indices, or projections that can reliably help to answer questions related to the impact of the COVID-19 pandemic. The majority of the existing studies are focused on the earth sciences and natural hazards such as earthquakes and tsunamis (see, e.g., Albeverio et al. (2010)). In this context, the extreme value theory and extreme value analyses remain the most used techniques to estimate the likelihood of the extreme events. This type of analyses mainly rely on statistical methods and probability theory (Benstock & Cegla, 2017). However, one of the most known barriers and limitations of using this statistical techniques is the availability of historical data. As alternatives the qualitative methods such as Delphi and the qualitative risk assessment matrix, which are constructed based on expert judgments, could be very beneficial (Keeney et al., 2011). In many situations, we need experts' opinions to guide us and reduce the spread of misinformation. Expert judgments are an important asset to many decision-making problems and have been studied in various fields (Werner et al., 2017). They are essential and widely used in the design and management of large or complex systems. Such projects are often essentially unique, so there is very limited experience regarding the performance of their components. It is natural then to rely on expert judgments, especially in situations such as COVID-19 where historical data is scarce or absent. In general, the most difficult decisions are those where the consequences are subject to substantial uncertainty. Although experts are valuable sources of information and knowledge, experience and global events have proven that they can also be wrong due to the high level of uncertainty (Ayyub, 2001). Since uncertainty is viewed as a lack of information, when seeking to reduce its impact, experts often try to provide a large amount of information to enrich their analysis, rather than explaining their opinions in a determined or crisp way. For example, when experts are asked how long it will be until an effective vaccine for COVID-19 is available, the most common answer has so far been something along the lines of, "If everything goes well, a vaccine may be available in either the coming months or next year." The experts prefer to use exclusive-or (XOR) logic, "or," to express their judgments about the time of availability, saying: "coming months or next year." Due to a lack of information, experts do not know precisely when an efficient vaccine will be available; consequently, they are forced to express their preferences using XOR logic to reduce uncertainty.

Recently, Hocine & Kouaissah (2020) have developed a new technique for dealing with decision-making problems under uncertainty using XOR logic called XOR analytic hierarchy process (XOR-AHP). Conceptually, XOR is an uncertain logic that describes a situation in which there is only one choice between two or more competitive actions, and neither is strong enough to overcome the others. Unlike classical logic, which requires a deep understanding of a system, exact equations, and precise numeric values, XOR logic offers a more free way of thinking, which allows complex systems to be modeled using a higher level of abstraction originating from our knowledge and experience. In contrast to existing uncertainty theories, the XOR analysis framework provides an intuitive and natural way to model imprecise and uncertain decision-making problems. The use of XOR analysis, at least in some cases, is the natural, simplest, and best representation of imprecision and uncertainty and does not require assumptions such as lower and upper bound values, types of distributions, and fuzzy membership functions. For example, when experts are asked to assess the relative impact of COVID-19 on the tourism and agriculture sectors in terms of operation level, using a numerical scale ranging from 1 (equal) to 9 (extremely more impact), it is very difficult for them to provide an exact estimated value. Experts are forced to express their estimations using XOR logic, for example: "On an operational level, the tourism sector is more impacted than the agriculture sector, by 5 or 6 times." Mathematically, we can express the expert's statement as follows: 5 XOR 6. Thus, problems featuring these characteristics should be modeled using an XOR framework, as suggested in Hocine & Kouaissah (2020). Obviously, in the XOR-AHP model, the criteria are assumed to be independent. However, since many decision problems include the interaction and dependency of top-level elements on down-level elements, they cannot be ordered hierarchically (see, e.g., Saaty, 2006; May et al., 2013; Saaty & Vargas, 2013). It is well known that problems in many fields, such as telecommunications, energy, and economics, have strong correlations and dependencies in their companions and structures that are not compatible with linear modelization. For example, in the context of COVID-19, if we want to evaluate the multisectoral impact of COVID-19 with respect to some dimensions or criteria, such as economic and social, then these dimensions are generally correlated with and dependent on each other. In this regard, the analytic network process (ANP) method (Saaty, 2001) allows the dependencies (or feedbacks) to be modeled realistically, and the results yield more accurate outputs. Dependencies can arise between any of the elements in the decision problem, such as alternatives, criteria, sub-criteria, and the goal (Mu et al., 2020). For instance, dependencies in the alternatives can arise between the different sectors — such as the transport and tourism sectors — where there is a strong correlation between them. This means that the model no longer has a linear structure as in the analytic hierarchy process (AHP) methodology. To tackle this problem, this chapter aims to develop a new model called the XOR-analytic network process (XOR-ANP).

8.2 RELATED WORKS

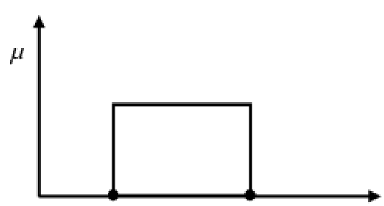
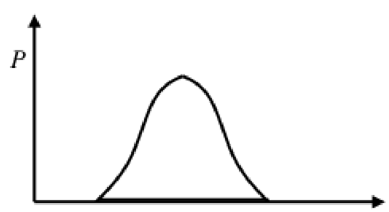
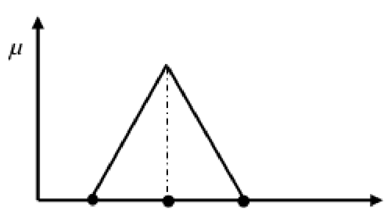
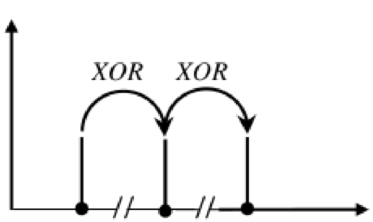
The ANP and its linear version, the AHP, are general techniques of measurement (Saaty & Vargas, 2013). These two methods were developed by Thomas Saaty and his colleagues through a series of studies that began in the early 1970s and are still attracting the attention of many researchers (see, e.g., Harker & Vargas, 1987; Ishizaka et al., 2012; Razaei & Ortt, 2013; Aguaron et al. 2021). Due to their simplicity, solid theoretical foundation, and user support software packages, these two techniques have become one of the most widely used methods in operations research/management science (Saaty & Vargas, 2013; Ishizaka & Nemery, 2013; Ho & Ma, 2018). Chronologically, the AHP method appeared first to deal with linear systems. Later, the ANP was proposed to relieve the fundamental restrictions related to the linear hierarchical structure. Many decision problems cannot be modeled hierarchically due to the interactions and dependence between their elements. Thus, the main idea behind the development of this technique was to avoid the assumption of independence among the decision-making elements of the standard AHP.

Methodologically, the ANP is very similar to the AHP: pairwise comparisons are still key to running the necessary computations to generate the priority vectors, and are carried out for all the components of the network system (i.e., nodes and clusters). Through this process, the DM's preferences are converted from linguistic expressions to numerical values using a numerical 1–9 scale. The priority vectors derived from the pairwise comparisons are used to fill the columns (or blocks) of a new matrix called the supermatrix. The main role of this supermatrix is to synthesize the raw priority vectors obtained from the pairwise comparisons, which is considered as one of the distinguished features of the ANP method. To obtain the weighted supermatrix, the primary (or unweighted) supermatrix must first be transformed into a matrix whose columns each sum to unity, known as a stochastic matrix. Next, to generate the overall priority vectors, the limit supermatrix is created by raising the weighted supermatrix to the power of $2k + 1$, where k is an arbitrarily large number. This step helps to capture the transmission of influence along all possible paths of the supermatrix. Finally, the element with the largest overall priority is the best option. For more details about the theoretical background, mathematical concepts, and best practice of the ANP methodology, see, for example, Saaty & Vargas (2013) and Mu et al. (2020).

"When a DM is uncertain about his preferences, what effect can this uncertainty have on the final decision?" This question was raised by Saaty & Vargas (1987) to highlight the uncertainty issue in the AHP framework. In this study, the authors classify the uncertainty related to AHP into two categories: uncertainty about the occurrence of events, and uncertainty about the range of judgments used to express preferences. The first is related to external uncertainty, which is uncontrollable by the DM, while the second is due to internal uncertainty, which represents a lack of information available to the DM. Many efforts have been made to tackle this issue and three main streams of research have been proposed on fuzzy, interval, and probability theories (see Van Laarhoven & Pedrycz, 1983; Salo & Hämäläinen, 1995; Saaty & Vargas, 1987). In this context, analysts often select the most suitable technique that overcomes their problems. For instance, interval ANP is generally used to cope with rounding and measurement errors. Although it is generally accepted that random variables are handled through stochastic processes and probability theory, consensus fails on whether historical data is sufficient, which is required for formulating probabilistic distribution (see Kirkwood, 1992). Unfortunately, in some situations such as COVID-19, historical data is not sufficiently available or is absent, and uncertainty may come from linguistic sources. Under these circumstances, fuzzy ANP could be adopted, where fuzzy numbers capture the input data of pairwise comparisons. However, since fuzzy numbers are represented by possibility distributions, the issue of overlapping may arise, rendering it difficult to rank these fuzzy numbers. Even when ordering a set of single fuzzy numbers, the defuzzification procedure could result in the loss of some information contained in the original data. It can be argued that no technique is superior to the others in all cases; all techniques have their advantages and limitations, and the choice of the right technique should be directed by the analysis.

The importance of the proper treatment of uncertainties is growing: the consequences of inadequate treatments are very costly in social, economic, and environmental terms. Ongoing global changes create fundamentally new scientific problems that require new concepts, methods, and tools. A key issue concerns a vast variety of practically irreducible uncertainties that challenge most traditional models. In real-life decision problems related to extreme events such as COVID-19, sufficient observations do not exist, it is very difficult to secure necessary data or conduct experiments, and the learning-by-doing process may be very expensive, dangerous, or simply impossible. In this context, XOR logic serves the purpose well and captures experts' hesitancy and uncertainty with the ANP methods. Thus, the main distinguishing characteristic of the XOR-ANP technique is its capability to overcome many limitations of other uncertain AHP methods (e.g., the overlapping problem, an infinite number of possibilities within the interval boundaries, and difficulty in comparison and ranking) and offer many advantages (e.g., incorporating DM preferences). For instance, given the XOR judgment, 3 XOR 4 XOR 5, the XOR logic function, in this case, aims to select the most suitable judgment from among the three potential discrete and discontinuous judgments '3' or '4' or '5' (i.e., there is no chance to consider another scale in the optimization process) that will achieve consistency with the DM's preferences. Another potential advantage of using the XOR logic function is that it can consider more preferences from the DM, i.e., the optimistic and pessimistic situations. Furthermore, XOR could be used to model and represent the preferences of a large number of actors whose preferences may concur or diverge on a particular choice. To the authors' knowledge, these concepts cannot be formulated by the current ANP approaches. Table 8.1 shows conditions under which it is preferable to use one particular technique over another.

Table 8.1: Comparison of different ANP techniques according to when they are preferable.

Uncertain ANP	Issue	When
Interval ANP		Rounding errors and measurement errors knowing just the upper and lower limits
Stochastic (or probability) ANP		Random variables; enough historical data to build a probability distribution
Fuzzy ANP		Linguistic variables; information is linguistic, and uncertainties affect subjective preferences
XOR ANP		Exclusive-or logic; multiple preferences; hesitant or irresolute in choosing among them

8.3 MODEL FORMULATION

According to Saaty & Vergas (2012), the ANP method can be employed with the following three main principles: i) decomposition of the decision problem in a network system; ii) measurement and priority vectors estimation; and iii) synthesis.

Step 1: Decomposition of the decision problem in a network system

This process involves structuring the decision problem into a network system through identifying its main clusters, elements, and dependencies between and within clusters (see, e.g., Figure 8.1). The difference between a hierarchy and a network is illustrated in Figure 8.2). For example, in a network, it is not necessary to specify levels as in a hierarchy model with a linear top-down structure without feedback from lower to higher levels. In general, three types of clusters could exist in the ANP methodology: a source cluster (C1) with no arcs into the cluster, an intermediate cluster (C2) with arcs into and out of the cluster, and a sink cluster (C3) with arcs only into the cluster. These clusters have dependencies and interactions with other clusters, such as the one between C1 and C4 (outer dependence). A cluster of elements also has a loop if its elements depend on each other, resulting in dependence known as inner dependence (see Figure 8.1).

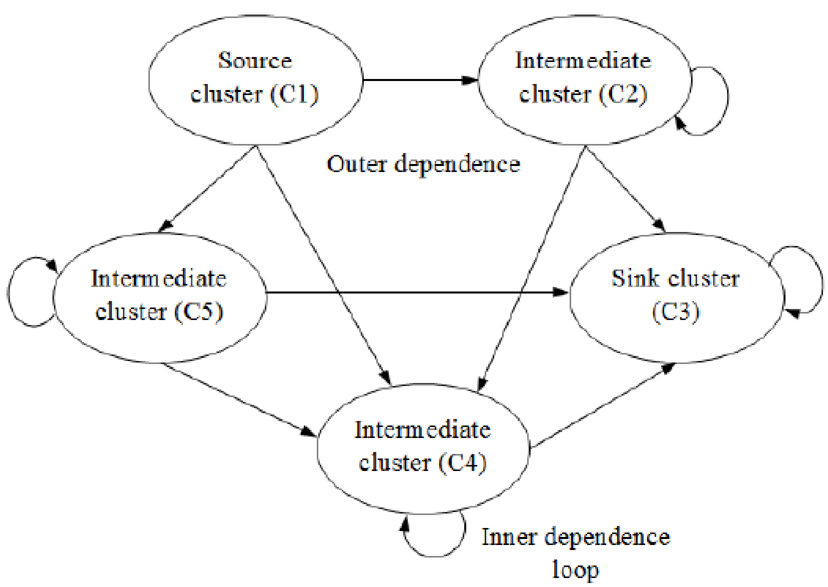


Figure 8.1: Types of components in a network.

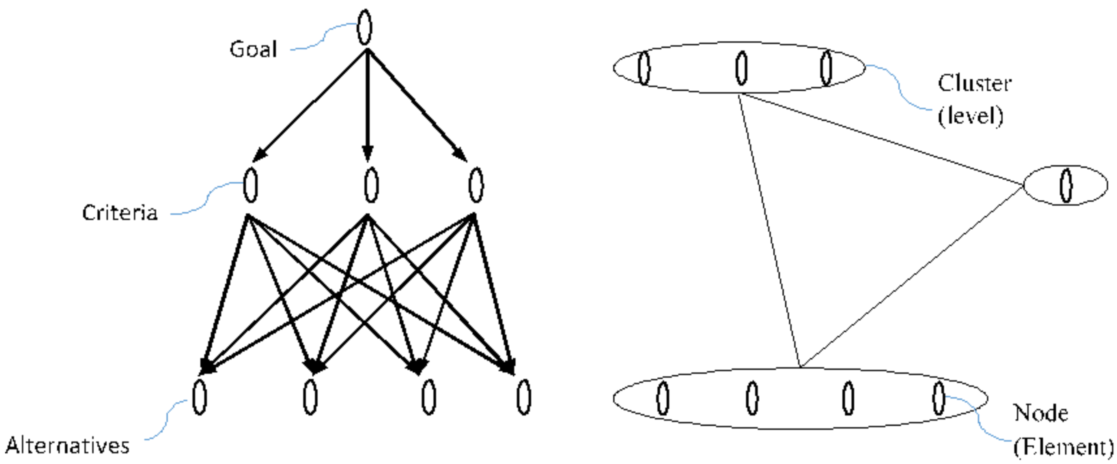


Figure 8.2: Hierarchy and network systems.

Step 2: Measurement and priority vector estimation.
Let C_i be a cluster of the network, with $i = 1, ..., N$, and let N be the total number of clusters in the network. Assume that cluster C_i has n_i elements, denoted by $ei1, ei2, ..., eini$. Following the $XOR - AHP$ methodology, the estimated priority vector of a given set of elements from a cluster on any other element in the network can be generated through the following steps (Hocine & Kouaissah, 2020):

Step 3: Construction of XOR pairwise comparison.
The main idea behind this step is the conversion of the verbal comparisons to numerical values based on a discrete scale ranging from 1 to 9, as reported in Table 8.2. Mathematically, the XOR pairwise matrix should respect some properties as follows.

Property 1. Matrix $XOR - A = (a_{ij})_{n\times n}$ is an XOR comparison matrix if

$$XOR - A = \begin{pmatrix} \underset{k=1,...,m}{XOR}(a^k_{11}) & \underset{k=1,...,m}{XOR}(a^k_{1j}) & \cdots & \underset{k=1,...,m}{XOR}(a^k_{1n}) \\ \underset{k=1,...,m}{XOR}(a^k_{i1}) & \underset{k=1,...,m}{XOR}(a^k_{ij}) & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ \underset{k=1,...,m}{XOR}(a^k_{n1}) & \cdots & \cdots & \underset{k=1,...,m}{XOR}(a^k_{nn}) \end{pmatrix}. \tag{8.1}$$

where $\underset{k=1,...,m}{XOR}(a^k_{11}) = a^1_{ij} XOR a^2_{ij} XOR ... XOR a^k_{ij}$ represents the degree of preference of x_i to x_j.

Property 2. Matrix $XOR - A = (a_{ij})_{n\times n}$ is an XOR comparison matrix if a priority vector $w = (w_1, w2, ..., w_n)$ exists which satisfies $\sum_{i=1}^{n} w_i$ and the following equalities hold:

$$\underset{k=1,...,m}{XOR}(a^k_{11}) = \underset{k=1,...,m}{XOR}(\frac{w^k_i}{w^k_j}) \forall i, j$$

where $\underset{k=1,...,m}{XOR}(\frac{w_i^k}{w_j^k}) = XOR(\frac{w_i^1}{w_j^1})XOR(\frac{w_i^2}{w_j^2})...XOR(\frac{w_i^m}{w_j^m})$. Then

$$XOR-A = \begin{pmatrix} \underset{k=1,...,m}{XOR}(\frac{w_1^k}{w_1^k}) & \underset{k=1,...,m}{XOR}(\frac{w_1^k}{w_j^k}) & \cdots & \underset{k=1,...,m}{XOR}(\frac{w_1^k}{w_n^k}) \\ \underset{k=1,...,m}{XOR}(\frac{w_i^k}{w_1^k}) & \underset{k=1,...,m}{XOR}(\frac{w_i^k}{w_j^k}) & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ \underset{k=1,...,m}{XOR}(\frac{w_n^k}{w_1^k}) & \cdots & \cdots & \underset{k=1,...,m}{XOR}(\frac{w_n^k}{w_n^k}) \end{pmatrix}. \quad (8.2)$$

Property 3. An XOR comparison matrix $XOR-A = (a_{ij})_{n\times n}$ is positive reciprocal if $XOR(a_{ij}^k) > 0$, $\underset{k=1,...,m}{XOR}(a_{ii}^k) = 1$ and $XOR(a_{ij}^k) = 1/\underset{k=1,...,m}{XOR}(a_{ji}^k) \quad \forall i,j \in 1,2,...,n$ Consequently, Matrix (8.1) can be rewritten as follows:

$$XOR-A = \begin{pmatrix} 1 & \underset{k=1,...,m}{XOR}(a_{1j}^k) & \cdots & \underset{k=1,...,m}{XOR}(a_{1n}^k) \\ \frac{1}{\underset{k=1,...,m}{XOR}(a_{1j}^k)} & 1 & \underset{k=1,...,m}{XOR}(a_{ij}^k) & \cdots \\ \vdots & \frac{1}{\underset{k=1,...,m}{XOR}(a_{ij}^k)} & 1 & \vdots \\ \frac{1}{\underset{k=1,...,m}{XOR}(a_{1n}^k)} & \cdots & \cdots & 1 \end{pmatrix}. \quad (8.3)$$

According to Hocine and Kouaissah (2020), the XOR comparison matrix corresponding to real-world scenarios assumes one and only one of three directions: i) optimistic (positive), ii) pessimistic (negative), and iii) neutral (no direction is provided).

Table 8.2: Fundamental ratio scale.

Scales	Definition	Explanation
1	Equal impact	Two factors contribute equally to the objective
3	Moderate impact	Experience and judgment slightly favor activity one over the other
5	Strong impact	Experience and judgment strongly favor one activity over the other
7	Very strong or demonstrated impact	Experience and judgment very strongly favor one activity over the other. Its importance is demonstrated in practice
9	Extreme impact	The evidence favoring one activity over the other is of the highest possible validity
2,4,6,8	Intermediate values	Whenever a compromise is needed

Definition 6. *Matrix $XOR-A = (a_{ij})_{n\times n}$ is an optimistic XOR comparison matrix if $XOR-A = (a_{ij})_{n\times n} = \max_k[XOR-A = (a_{ij})_{n\times n}]$, is a pessimistic XOR comparison matrix if $XOR-A = (a_{ij})_{n\times n} =$*

$\min_k[XOR-A=(a_{ij})_{n\times n}]$ *, or is a neutral XOR comparison matrix if* $XOR-A=(a_{ij})_{n\times n}$ *does not follow any mandatory direction (max or min) for all* $i,j\in 1,2,...,n$

Step 4: Estimate the priority vector

Priority vector derivation is the cornerstone of producing the ranking outputs. To derive the priority vectors from an XOR comparison matrix, a mathematical programming model called the XOR weighting technique (XOR-W) is formulated as follows:

$$\min \quad \pi+\sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{n}(\sigma^{+}_{ij}+\sigma^{-}_{ij})+\sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{n}(\beta^{+}_{ij}+\beta^{-}_{ij}) \tag{8.4}$$

$$\text{s.t.} \quad w_i-\underset{k=1,\ldots,m}{XOR}(x^k_{ij}).w_j\leq\pi \ \ (i,j\in N,\ i\neq j,\ k\in M) \tag{8.5}$$

$$\psi_i\underset{k=1,\ldots,m}{XOR}(x^k_{ij})-\psi_i\min_k\left[\underset{k=1,\ldots,m}{XOR}(x^k_{ij})\right]+\sigma^{-}_{ij}-\sigma^{+}_{ij}=\delta\ (i,j\in N,\ i\neq j,\ k\in M) \tag{8.6}$$

$$\psi_i\underset{k=1,\ldots,m}{XOR}(a^k_{jW})-\psi_i\min_k\left[\underset{k=1,\ldots,m}{XOR}(a^k_{jW})\right]+\sigma^{-}_{jW}-\sigma^{+}_{jW}=\delta\ (i,j\in N,\ i\neq j,\ k\in M) \tag{8.7}$$

$$XOR(x^k_{il})\times XOR(x^k_{lj})+\beta^{-}_{ij}-\beta^{+}_{ij}=XOR(x^k_{ij})\ \ (i,j\in N,\ i\neq j,\ k\in M) \tag{8.8}$$

$$\underset{k=1,\ldots,m}{XOR}(x^k_{ij})=x^1_{ij}\lambda_1+x^2_{ij}\lambda_2+\ldots+x^m_{ij}\lambda_m\ \ (i,j\in N,\ i\neq j,\ k\in M) \tag{8.9}$$

$$\sum_{i=1}^{n}w_i=1,\ \ w>0\ \ (i\in I) \tag{8.10}$$

$$\sum_{k=1}^{m}\lambda_i=1,\ \ (k\in M) \tag{8.11}$$

$$\lambda_k=\{0,1\}\ \ (k\in M) \tag{8.12}$$

$$\sigma^{+}_{ij},\sigma^{-}_{ij},\beta^{+}_{ij}\text{ and }\beta^{-}_{ij}\geq 0\ \ (i,j,B,W\in N,\ i\neq j)$$

where $N=\{1,2,\ldots,n\}$ and $M=\{1,2,\ldots,m\}$, π is the maximum deviation, $XORa_{ij}$ is the element of a $(n\times n)$ pairwise comparison matrix A giving the relative import of the i-th criterion considered against the j-th criterion, w_i is the derived weight given to the ith criterion, $\underset{k=1,\ldots,m}{XOR}(x^k_{ij},\lambda_k)$ is the function of XOR judgment logic, and λ_k is a binary variable. The objective function (Equation (8.4)) ensures that the maximum absolute deviations of the inconsistencies of the XOR judgment comparison matrix and the unwanted deviations (σ^+ and σ^-) from the aspiration level are minimized. Equation (8.6) defines a hyperplane in the n-dimensional priority space. Equation (8.7) represents the DM's preferences, where is a psychological parameter that gauges optimism/pessimism assessment and ranges from 0 to 1, as shown in Table 8.3. Equation (8.8) allows inconsistency to be controlled. Equation (8.11) guarantees that only one evaluation will be selected for the ith and jth pairwise comparisons, which is the most appropriate evaluation among the potential evaluations that will achieve the highest level of DM consistency. Equation (8.10) satisfies the normalization condition and ensures the derivation of a positive priority vector.

Scale		Definition
Cost Criteria	Benefit Criteria	
0	1	Optimistic
0.2	0.8	Moderately Optimistic
0.5	0.5	Neutral
0.8	0.2	Moderately Pessimistic
1	0	Pessimistic

Table 8.3: Scales for optimism/pessimism.

Step 5: Check the consistency
Once judgments have been entered, it is necessary to check that they are consistent. The consistency ratio (CR) is an important indicator to check the consistency degree of pairwise comparison.
Property 4. An XOR pairwise matrix is consistent if $\underset{k=1,\ldots,m}{XOR}(x_{il}^k) \times \underset{k=1,\ldots,m}{XOR}(x_{il}^k) = \underset{k=1,\ldots,m}{XOR}(x_{ij}^k)$ transitivity and reciprocity $\underset{k=1,\ldots,m}{XOR}(x_{ii}^k) = 1$ and $\underset{k=1,\ldots,m}{XOR}(x_{ij}^k) = \dfrac{1}{\underset{k=1,\ldots,m}{XOR}(x_{ji}^k)}$ for all $i, j \in \{1, 2, \ldots, n\}$ rules are respected. Consequently, Matrix (8.1) can be rewritten as follows:

$$XOR - A = \begin{pmatrix} 1 & \underset{k=1,\ldots,m}{XOR}(x_{1j}^k) & \cdots & \underset{k=1,\ldots,m}{XOR}(x_{1n}^k) \\ \dfrac{1}{\underset{k=1,\ldots,m}{XOR}(x_{1j}^k)} & 1 & \underset{k=1,\ldots,m}{XOR}(x_{ij}^k) & \cdots \\ \vdots & \dfrac{1}{\underset{k=1,\ldots,m}{XOR}(x_{ij}^k)} & 1 & \vdots \\ \dfrac{1}{\underset{k=1,\ldots,m}{XOR}(x_{1n}^k)} & \cdots & \cdots & 1 \end{pmatrix}. \tag{8.13}$$

The consistency level of pairwise judgments is measured by calculating a consistency ratio (CR). The following formula is used in this calculation:

$$CR = \frac{\pi^*}{CI} \tag{8.14}$$

where π^* is is the optimal solution of the XOR-W program and CI is the consistency index, determined based on Table 8.3 (Razei, 2015).

Table 8.4: Consistency Index (CI) table.

	1	2	3	4	5	6	7	8	9
Consistency Index (CI)	0.00	0.44	1.00	1.63	2.30	3.00	3.73	4.47	5.23

It is clear that CR takes values between interval $[0, 1]$. Therefore, the most consistent is the nearest to 0.

Step 6: Synthesis After accomplishing all the necessary pairwise compressions, the obtained priority vectors will organize into a special matrix, called the supermatrix. This supermatrix represents the flow of influence either from a cluster to itself or from a cluster to another cluster in the network.

Since the dependencies and influences in real-life decision problems come in different varieties and combinations, Saaty and Vergas (2013) present some potential structures and their supermatrix, including hierarchy, holarchy, suparchy, intarchy, etc. The general form of the supermatrix can be described as follows:

$$W = \begin{array}{cc} & \\ & \\ C_1 & \begin{array}{c} e_{11} \\ e_{12} \\ \vdots \\ e_{1n_1} \end{array} \\ C_2 & \begin{array}{c} e_{21} \\ e_{22} \\ \vdots \\ e_{2n_2} \end{array} \\ \vdots & \\ C_N & \begin{array}{c} e_{N1} \\ e_{12} \\ \vdots \\ e_{Nn_N} \end{array} \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_N \\ e_{11}e_{12}\cdots e_{1n_1} & e_{21}e_{22}\cdots e_{2n_2} & & e_{N1}e_{N2}\cdots e_{Nn_N} \end{array} \\ \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{1N} \\ W_{21} & W_{22} & \cdots & W_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ W_{N1} & W_{N2} & \cdots & W_{NN} \end{bmatrix} \end{array}$$

where Wij is called a block of the supermatrix which represents a principal priority vector of the influence (or importance) of the elements in the i-th component of the network on an element in the jth component. Some of its entries may be zero corresponding to those elements that have no influence. The generic format of the block can be written as follow:

$$W_{ij} = \begin{bmatrix} \overset{(j_1)}{\underset{i1}{W}} & \overset{(j_2)}{\underset{i1}{W}} & \cdots & \overset{(j_{n_j})}{\underset{i1}{W}} \\ \overset{(j_1)}{\underset{i2}{W}} & \overset{(j_2)}{\underset{i2}{W}} & \cdots & \overset{(j_{n_j}))}{\underset{i2}{W}} \\ \vdots & \vdots & \vdots & \vdots \\ \overset{(j_1)}{\underset{in_i}{W}} & \overset{(j_2)}{\underset{in_i}{W}} & \cdots & \overset{(j_{n_j}))}{\underset{in_i}{W}} \end{bmatrix}.$$

Then, the weighted supermatrix is derived by transforming all column sums to unity exactly. This step is very similar to the concept of a Markov chain for ensuring the sum of these probabilities of all states is equal to 1. Next, we raise the weighted supermatrix to limit powers to get the global priority vectors as follows:

$$\lim_{k\longrightarrow\infty} W^k$$

In case the supermatrix has the effect of cyclicity, the Cesaro sum can be used to obtain the average priority as follows:

$$\lim \lim_{k\longleftarrow\infty} \frac{1}{N}\sum_{k=1}^{N} W^k$$

8.4 REAL-LIFE APPLICATION: SECTORIAL ASSESSMENT OF COVID-19 IMPACT THROUGH XOR-AHP TECHNIQUE

Viruses have significant potential to become dangerous and life-threatening and cause irreparable losses to human beings. Hardly does the world learn to cope with one strain of virus when another emerges and poses a threat to the future of humanity. Such a situation has occurred with the emergence of a new strain of severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) that has not been previously known in human history (see Amine et al., 2000). On March 11, 2020, the World Health Organization (WHO) declared COVID-19 a public health emergency and pandemic. By end of March 2020, COVID-19 had spread quickly around the globe; at the time of this writing, over 60 million people had been infected with the virus and it had caused over 1.5 million deaths. One of the most dangerous aspects of COVID-19 is that people carrying the virus may spread the virus to others before or without showing symptoms; its basic reproduction number has been estimated between 1.4 and 5.7, i.e., each infected person in turn infects an average of up to 5.7 people. Figure 8.5 shows the worldwide spread of COVID-19. Like the rest of the world, Morroco has been affected by the COVID-19 pandemic. The virus was confirmed to have spread to Morocco on March 2, 2020, when the first COVID-19 case was confirmed in Casablanca. As the outbreak widened in Morocco, the government called for an emergency, stay-in-home order and intensified various restrictions such as closing borders and social distancing. By end of November 2020, Morocco had reported more than 335,000 confirmed cases and more than 5,500 deaths. Note that due to the rapid spread and evolution of COVID-19, this representation corresponds to today's data and may not include the most recent cases (visit the reports of the World Health Organization (WHO) for updated information on the state of the situation).

As COVID-19 spread across Morocco, its impact increased and began to appear in all aspects of life. For example, the Moroccan office of the high commissioner for planning claims that the growth forecast for 2020 will be dramatically reduced and that the pandemic may cause one the most severe recessions to hit Morocco over the last three decades. Ample evidence suggests that the pandemic has had a significant impact on financial planning and the budget deficit and, consequently, may force the government to adopt austerity policies. It is estimated that the overall budget deficit will deteriorate by approximately 7% of GDP by the end of 2020. As a result, Morocco's external debt could reach 73% of GDP, its highest level in decades. As COVID-19 has spread around the country, policymakers' concerns have shifted from supply-side manufacturing issues to decreased business in the services sector. For instance, in the tourism sector, which is crucial to the Moroccan economy (generating around 500,000 direct jobs and around 2.5 million total jobs including indirect jobs), the national tourism confederation estimates significant losses of $3.6 billion in terms of tourism turnover and $1.5 billion of lost turnover for the hotels industry. The same confederation concludes that more than 500,000 jobs and 8,500 businesses are at risk, including accommodation businesses, catering businesses, travel agencies, tourist transportation companies, and car rental companies.

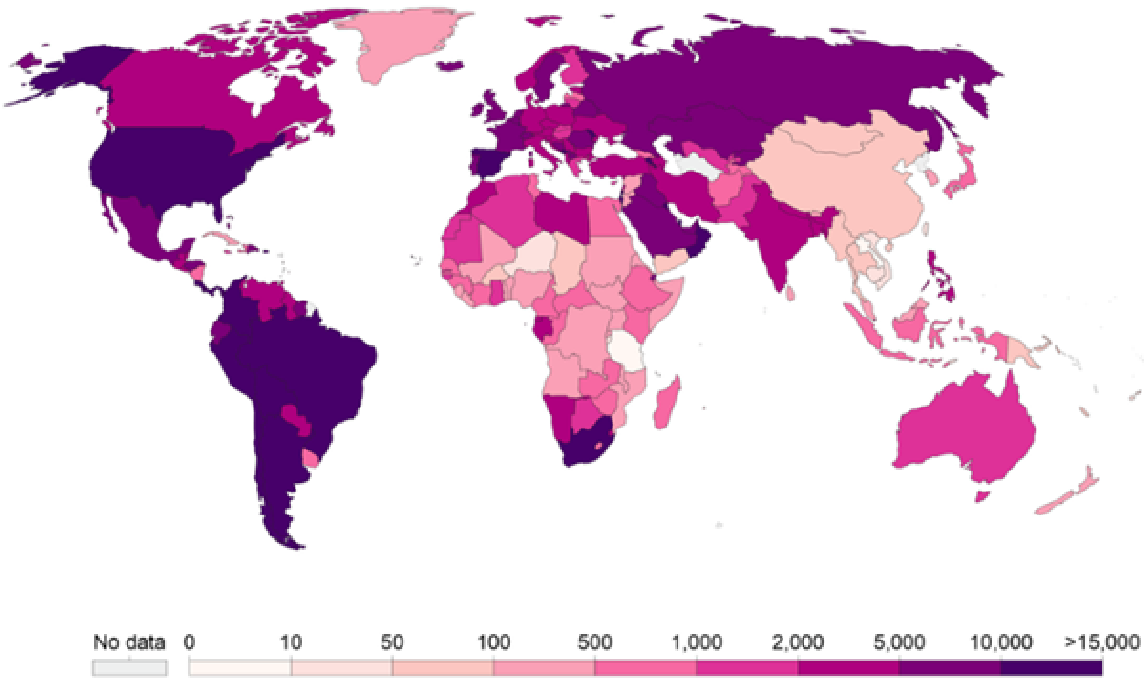


Figure 8.3: World map of total confirmed COVID-19 cases per million people.
Source: European CDC - Situation Update Worldwide - Last updated 19 September, 10:35 (London time) OurWorldlnData.org/coronavirus • CC BY

Another sector that has experienced a negative impact is transportation; according to the international air transport association, the pandemic could trigger losses of around 4.9 million passengers and a shortfall of $728 million in Morocco. Given the fact that the healthcare sector is at front-line or the epicenter of this pandemic, most hospitals, private and public, have witnessed a remarkable increase in demand and even reached their capacity. In the education sector, the Moroccan government announced the closure of schools and universities, leading to the organization of online classes via internet platforms or television channels. And to support the financial sector and facilitate access to credit, the Central Bank lowered the key interest rate by 25 basis points, to 2%. There have also been minor interventions at the governmental and institutional levels aimed at supporting and relaunching the economy. The impact of the pandemic has left goverments and policymakers counting mounting costs and wondering what recovery could look like. In this context, the design of an efficient policy for recovery and economic relaunch is a high priority and requires taking all economic sectors into consideration.

A salient feature of extreme event problems is the complexity and interdependence of their components. Generally, this complexity arises from the interaction and interdependence of one or several elements with the other elements. This feature is clear from the impact of COVID-19. For example, the tourism sector depends on the transport sector and other resources. Education likewise depends on transport. The result is a complex network with varying dependencies. In an attempt to determine the effect of COVID-19 on the economy, in this study, we focus on seven specific primary economic sectors, as defined by the high commission for planning: (1) industrial, (2) tourism, (3) agriculture, (4) transport, (5) financial, (6) healthcare, and (7) education. After determining the most important specific economic sectors that have been impacted by COVID-19, we proceed with the evaluation process by acertaining the different aspects of impact. The social, operational, and economic dimensions are the main criteria chosen for measuring the level of this impact.

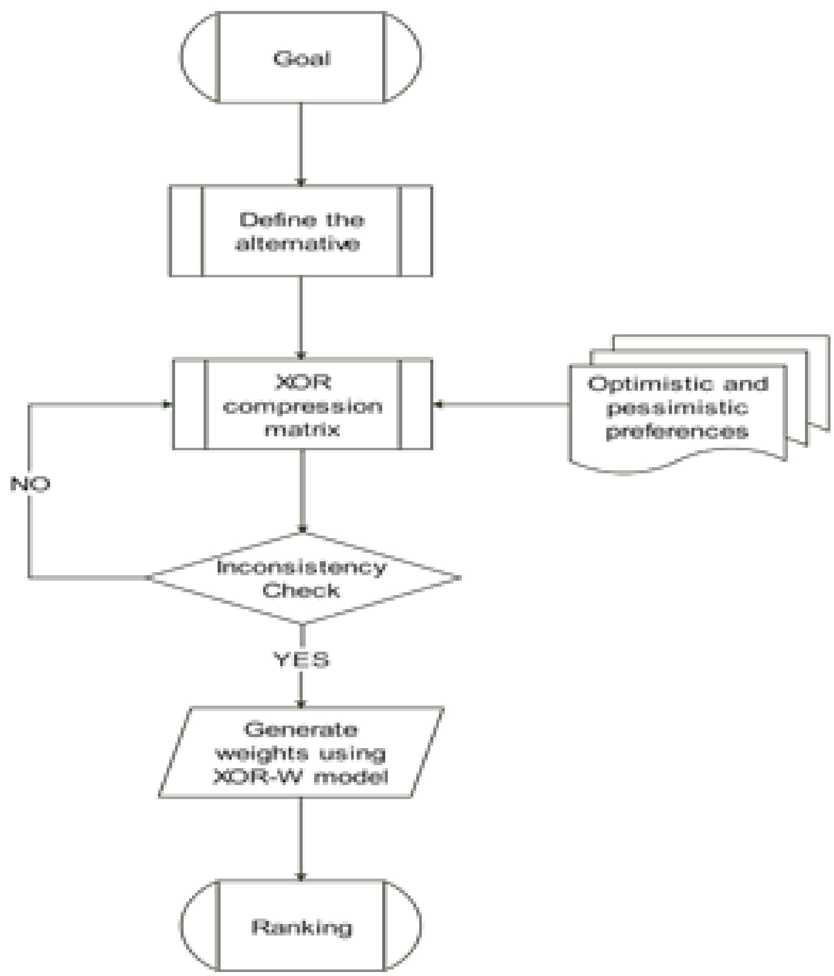


Figure 8.4: The XOR-ANP model.

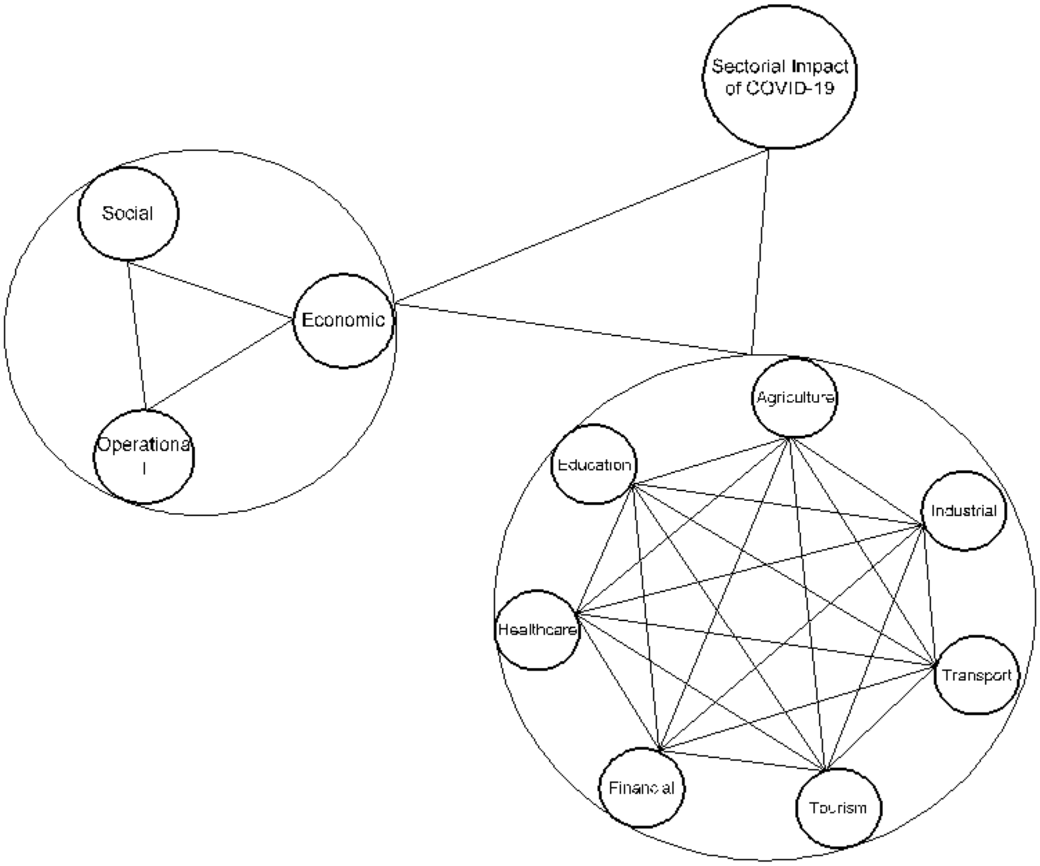


Figure 8.5: COVID-19 XOR-ANP diagram.

The chart in Figure 8.5 illustrates the main steps of the XOR-ANP model, as follows: define the clusters and elements, incorporate the DM's preferences (optimistic, pessimistic, and control the consistency preferences), build the XOR comparison matrix, generate priority vectors, construct the supermatrices (unweighted, weighted, and limited) and, finally, obtain the overall ranking. A typical ANP structure is shown in Figure 8.7, where the goal of the decision problem is to detect the most impacted sector among seven sectors based on three dimensions or criteria. This means that we will have three clusters: (1) cluster of goal with node G; (2) cluster of alternatives with seven nodes (i.e., agriculture, industrial, transport, tourism, financial, healthcare, and education), and (3) cluster of criteria with three nodes (social, economic, and operational).

To proceed with the XOR-ANP technique, the supermatrix should be determined. This means a series of pairwise comparison matrices are required. These matrices reflect the evaluation process of each option over another option based on a fundamental scale ranging from 1 to 9 that captures the verbal expressions of experts and DMs. The necessary pairwise comparisons required to fill the supermatrix are: i) comparisons of the criteria with respect to goals, ii) criteria with respect to other criteria, iii) alternatives with respect to each criterion, and iv) the criteria with respect to the alternatives. First, we start with comparisons of the criteria with respect to the goal. To accomplish this, the following question is asked: "With respect to the goal of selecting the most impacted sector, which criterion is more important in detecting the impact and by how much?" The related pairwise comparison is illustrated in Table 8.4. The estimated priority vector is used as an input in the goal node block of the supermatrix.

Table 8.5: XOR pairwise comparison for determining the importance of the three adopted dimensions.

Agriculture	Social	Economic	Operational
Social	1	1	3
Economic		1	3 XOR 4
Operational			1

Then, we move to the comparison of criteria with respect to the criteria. The main aim to this evaluation is to capture the relative importance of the criteria when another dependent criterion has already been evaluated. To achieve this aim, the following question is asked: "If your goal is to select the most impacted sector and you know that you are evaluating them against social, which other criterion, operational or economic, would be most important and by how much?" The related pairwise comparisons are illustrated in Tables 8.5-8.7. The estimated priority vectors are used as inputs in the criteria cluster blocks of the supermatrix.

Table 8.6: XOR pairwise comparison with respect to the social level.

Social	Social	Economic	Operational
Social	1	1/2	1/6
Economic		1	1/3
Operational			1

Table 8.7: XOR pairwise comparison with respect to the economic level.

Economic	Social	Economic	Operational
Social	1	1	1/9
Economic		1	1/9
Operational			1

Table 8.8: XOR pairwise comparison with respect to the operational level.

Operational	Social	Economic	Operational
Social	1	1	3
Economic		1	3
Operational			1

Next, we evaluate the pairwise comparisons of alternatives with respect to the criteria. The mechanism for filling these matrices is performed through the following question: “If your goal is to select the most impacted sector, how much is a certain sector impacted over another sector with respect to a particular criterion?” For example, to fill the comparison matrix (see Table 8.8), the experts are asked to give their estimate about how much more the tourism sector has been impacted than the industrial sector with respect to the social dimension. Due to the high cognitive effort needed to generate the right judgements, a lack of information, and the rapid evolution of the situation, an expert may not be entirely sure of the precise values of his or her judgments. Experts often have multiple preferences and are hesitant and irresolute in choosing among them; they express them based on XOR logic. For example, according to available data and reports, the tourism sector may be impacted by 6 XOR 7 times more than the industrial sector, according to the social level. Then, we fill the cell (4,1) with 6 XOR 7. The same reasoning holds for determining the impact of sectors according to each criterion, and related XOR pairwise comparison matrices are presented in Tables 8.8-8.10. The obtained priority vectors are used as inputs in related blocks of the supermatrix.

One of the main challenges in the evaluation process is the high uncertainty existing in the problem. This uncertainty produces a complex state of contradiction between optimism and pessimism. The rapidly changing situation and the amount of information being received creates optimistic and pessimistic views. Over the past few months, much information has supported our hopes and occasionally verified our fears. For example, are we optimistic or pessimistic about recovering quickly from the impact of COVID-19? Optimism and pessimism refer to positive and negative future expectations, respectively tations affect the evaluation process of experts in evaluating the impact of COVID-19 on a certain sector with respect to another sector. For example, say an expert is asked to provide his estimation of the impact of COVID-19 on the transport sector over the agriculture sector with respect the social dimension and responds with 4 XOR 5 XOR 6; however, he also notes that, according to the available data, indicators, and reports, the pessimism state is high and the expected impact is deeper in the transportation sector and may reach level 6. The related pairwise comparisons are reported in Table 8.10.

Table 8.9: XOR pairwise comparison for determining the degree of COVID-19 impact for each sector with respect to the economic level.

	Agriculture	Industrial	Transport	Tourism	Financial	Healthcare	Education
Agriculture	1	1/4	1/5	1/6 XOR 1/7	1/3	1	1/2
Industrial		1	1/3	1/5	3	5	4
Transport			1	1/3	4	6	5
Tourism				1	4 XOR 5	6 XOR 7	7
Financial					1	4	4
Healthcare						1	1
Education							1

Table 8.10: XOR pairwise comparison for determining the degree of COVID-19 impact for each sector with respect to the operational level.

	Agriculture	Industrial	Transport	Tourism	Financial	Healthcare	Education
Agriculture	1	1/5	1/6	1/7	1/4	1/7 XOR 1/8	1/4
Industrial		1	1/5	1/6	1/3	1/6	1/5
Transport			1	1/2	6	1/3	4
Tourism				1	4	1/2 XOR 1/3	5
Financial					1	1/6	1/4
Healthcare						1	6
Education							1

Table 8.11: XOR pairwise comparison for determining the degree of COVID-19 impact for each sector with respect to the economic level.

	Agriculture	Industrial	Transport	Tourism	Financial	Healthcare	Education
Agriculture	1	1/3 XOR 1/4	1/4 XOR 1/5 XOR 1/6	1/5	1/3	1/5	1/3
Industrial		1	1/5	1/6	3	1/4	2
Transport			1	1/2	5	3	4
Tourism				1	7	3 XOR 4	3
Financial					1	1/3	4
Healthcare						1	3
Education							1

Finally, to finish filling the supermatrix, the pairwise comparisons of criteria with respect to the alternatives are required. These comparisons capture the relative importance of the alternatives when another dependent alternative is already known. The mechanism for filling these matrices is performed through the following question: "If your goal is to select the most impacted sector and you know that the agriculture sector is also in the evaluation cluster, which other sectors are the most impacted?" The related pairwise comparisons are reported in Tables 8.11-8.17. The obtained priority vectors will be used as inputs in related blocks of the supermatrix.

Table 8.12: XOR pairwise comparison with respect to the Agriculture sector.

Agriculture	Social	Economic	Operational
Social	1	6	6
Economic		1	1
Operational			1

Table 8.13: XOR pairwise comparison with respect to the Industrial sector.

Industrial	Social	Economic	Operational
Social	1	1	1/
Economic		1	1/5
Operational			1

Table 8.14: XOR pairwise comparison with respect to the Transport sector.

Transport	Social	Economic	Operational
Social	1	2	1/4
Economic		1	1/4
Operational			1

Table 8.15: XOR pairwise comparison with respect to the Tourism sector.

Tourism	Social	Economic	Operational
Social	1	1/6	1/2
Economic		1	3
Operational			1

Table 8.16: XOR pairwise comparison with respect to the Financial sector.

Financial	Social	Economic	Operational
Social	1	1/8	1/2
Economic		1	4
Operational			1

Table 8.17: XOR pairwise comparison with respect to the Healthcare sector.

Healthcare	Social	Economic	Operational
Social	1	6	6
Economic		1	1
Operational			1

Table 8.18: XOR pairwise comparison with respect to the Education sector.

Education	Social	Economic	Operational
Social	1	9	3
Economic	1/9	1	1/6
Operational	1/3	6	1

After acquiring data, the XOR-W technique is used to derive the necessary priority vectors for all pairwise comparisons. Then, a supermatrix is constructed using the obtained priority vectors for the previous pairwise comparisons, as shown in the Table 8.18.

Table 8.19: Supermatrix

	Goal	Soc.	Econ.	Oper.	Agri.	Ind.	Trans.	Tour.	Fin.	Heal.	Edu.
Goal	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Soc.	0,4286	0,1111	0,1111	0,4286	0,7500	0,1429	0,2069	0,1111	0,0909	0,3273	0,6743
Econ.	0,4286	0,2222	0,1111	0,4286	0,1250	0,1429	0,1494	0,6667	0,7273	0,0909	0,0717
Oper.	0,1429	0,6667	0,7778	0,1429	0,1250	0,7143	0,6437	0,2222	0,1818	0,5818	0,2541
Agri.	0,0000	0,1027	0,0512	0,0985	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Ind.	0,0000	0,1068	0,0557	0,1325	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Trans.	0,0000	0,1232	0,1536	0,1808	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Tour.	0,0000	0,3285	0,2272	0,3255	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Fin.	0,0000	0,1068	0,0596	0,0787	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Heal.	0,0000	0,1232	0,3743	0,0920	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Edu.	0,0000	0,1088	0,0784	0,0920	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000

Next, the weighted supermatrix is obtained by ensuring all columns sum up to unity. Thus, this matrix must be normalized. In order to normalize the above supermatrix, the elements of a column should be summed, and then all of the elements should be divided by the summation. These calculations must be done for every column to obtain the normalized supermatrix. For example, the sum of the second column of the super matrix is equal to 2, according to the mentioned method; every element of this column must be divided by 2. As a result, the summation of the first column of the normalized super matrix is equal to 1, as shown in Table 8.19.

Table 8.20: Weighted Supermatrix

	Goal	Soc.	Econ.	Oper.	Agri.	Ind.	Trans.	Tour.	Fin.	Heal.	Edu.
Goal	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Soc.	0,4286	0,0556	0,0556	0,2143	0,7500	0,1429	0,2069	0,1111	0,0909	0,3273	0,6743
Econ.	0,4286	0,1111	0,0556	0,2143	0,1250	0,1429	0,1494	0,6667	0,7273	0,0909	0,0717
Oper.	0,1429	0,3333	0,3889	0,0714	0,1250	0,7143	0,6437	0,2222	0,1818	0,5818	0,2541
Agri.	0,0000	0,0514	0,0256	0,0493	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Ind.	0,0000	0,0534	0,0279	0,0663	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Trans.	0,0000	0,0616	0,0768	0,0904	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Tour.	0,0000	0,1643	0,1136	0,1628	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Fin.	0,0000	0,0534	0,0298	0,0394	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Heal.	0,0000	0,0616	0,1872	0,0460	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Edu.	0,0000	0,0544	0,0392	0,0460	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000

Finally, the limited super matrix can be calculated in order to select and rank the most impacted sector. The limited supermatrix is obtained by raising it to the k-th power until the columns stabilize and become identical in each block. The limiting power of the supermatrix is reached at the 15th stage, as shown in Table 8.20.

Table 8.21: Limited Supermatrix

	Goal	Soc.	Econ.	Oper.	Agri.	Ind.	Trans.	Tour.	Fin.	Heal.	Edu.
Goal	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Soc.	0,1745	0,1745	0,1745	0,1745	0,1745	0,1745	0,1745	0,1745	0,1745	0,1745	0,1745
Econ.	0,2022	0,2022	0,2022	0,2022	0,2022	0,2022	0,2022	0,2022	0,2022	0,2022	0,2022
Oper.	0,2899	0,2899	0,2899	0,2899	0,2899	0,2899	0,2899	0,2899	0,2899	0,2899	0,2899
Agri.	0,0284	0,0284	0,0284	0,0284	0,0284	0,0284	0,0284	0,0284	0,0284	0,0284	0,0284
Ind.	0,0342	0,0342	0,0342	0,0342	0,0342	0,0342	0,0342	0,0342	0,0342	0,0342	0,0342
Trans.	0,0525	0,0525	0,0525	0,0525	0,0525	0,0525	0,0525	0,0525	0,0525	0,0525	0,0525
Tour.	0,0988	0,0988	0,0988	0,0988	0,0988	0,0988	0,0988	0,0988	0,0988	0,0988	0,0988
Fin.	0,0268	0,0268	0,0268	0,0268	0,0268	0,0268	0,0268	0,0268	0,0268	0,0268	0,0268
Heal.	0,0619	0,0619	0,0619	0,0619	0,0619	0,0619	0,0619	0,0619	0,0619	0,0619	0,0619
Edu.	0,0308	0,0308	0,0308	0,0308	0,0308	0,0308	0,0308	0,0308	0,0308	0,0308	0,0308

One of the key objectives of this study is to assess the impact of the COVID-19 pandemic on Moroccan performance at the social, economic, and operational levels, sector by sector. This information could be extremely useful to policymakers in terms of prioritizing impacted sectors for intervention and funding, thus contributing to the well-being of the Moroccan region. According to the obtained results, especially Figures 4–5, the tourism, healthcare, and transport sectors were the most impacted by COVID-19, with 30.99%, 21.81%, and 17.88% respectively.

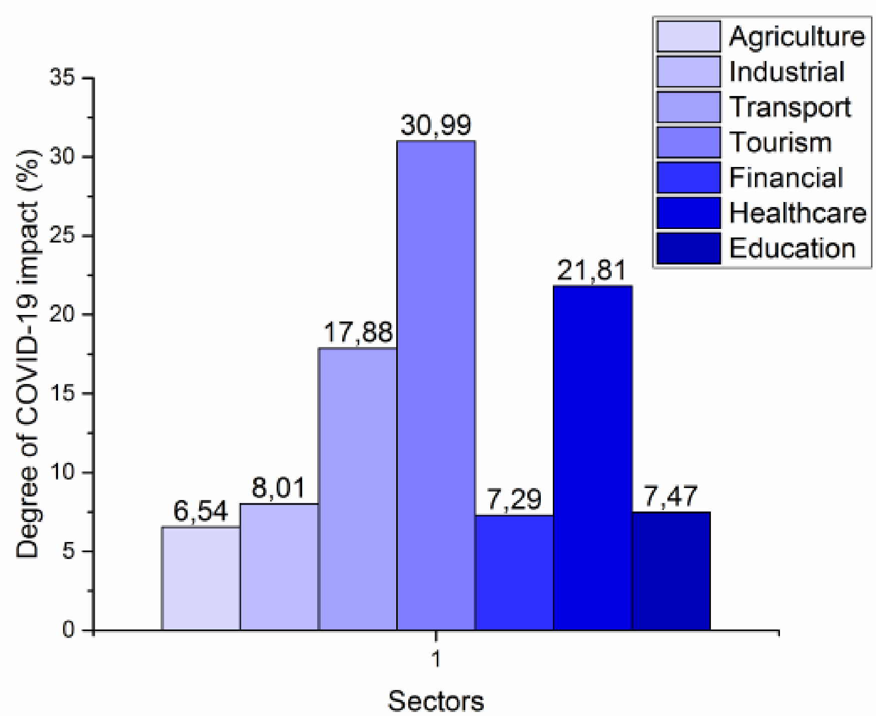


Figure 8.6: Global sectorial impact of COVID-19.

These results indicate that the third and the eighth sustainable development goals (SDG) of the

United Nations (UN) agenda for 2030, "Healthy Lives" and "Decent Work," have strongly impacted Morocco due to the pandemic crisis. These three sectors have a deep effect on the social, operational, and economic levels, where it is clear that the operational level has received the highest impact of 44.41%. This reflects the vital role that these sectors play in the Moroccan economy. The high impact observed on the operational level is essentially due to the direct role of the healthcare sector in fighting against the pandemic. For instance, Moroccan private health sector has not had a great response while public health system has borne most of the expenses. We can also observe that the financial, education, industrial, and agriculture sectors have received relatively less impact from COVID-19, with 7.29%, 7.47%, 8.01%, and 6.54% respectively. This could be explained by the fact that these sectors are in the second line in fighting against the pandemic.

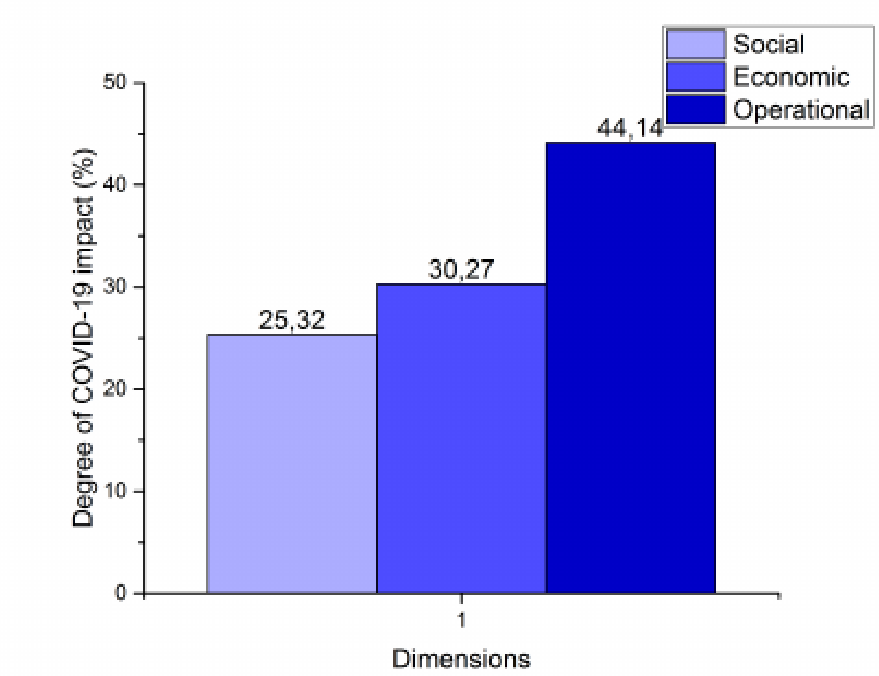


Figure 8.7: The local dimensions impact of COVID-19.

Figure 8.8 depicts how the COVID-19 pandemic quickly took on a regional dimension, which was expressed not only in the contagion's geography but also in the government's response to contain the side effect of the pandemic. The three most impacted regions not only conglomerate Moroccan population but also concentrate most Moroccan economic activities. Furthermore, Figure 8.8 can be used as a visual tool to link the spatial Moroccan impact from the global to regional or local perspectives. For instance, it shows how Moroccan cities like Casablanca and Tanger have suffered economically as a result of their high population density. Despite the fact that Casablanca region was the epicenter of the health crisis, Marrakech region has paid the highest economic and social price for the lockdown steps due to its high dependency to tourism activities. Current unemployment figure estimates for 2020 show the Casablanca region set to lose around 4.3% of its jobs, while Marrakech region is projected to lose approximately 6%. This would not only be an economic disaster, also risks destabilizing fragile social conditions in many Moroccan regions.

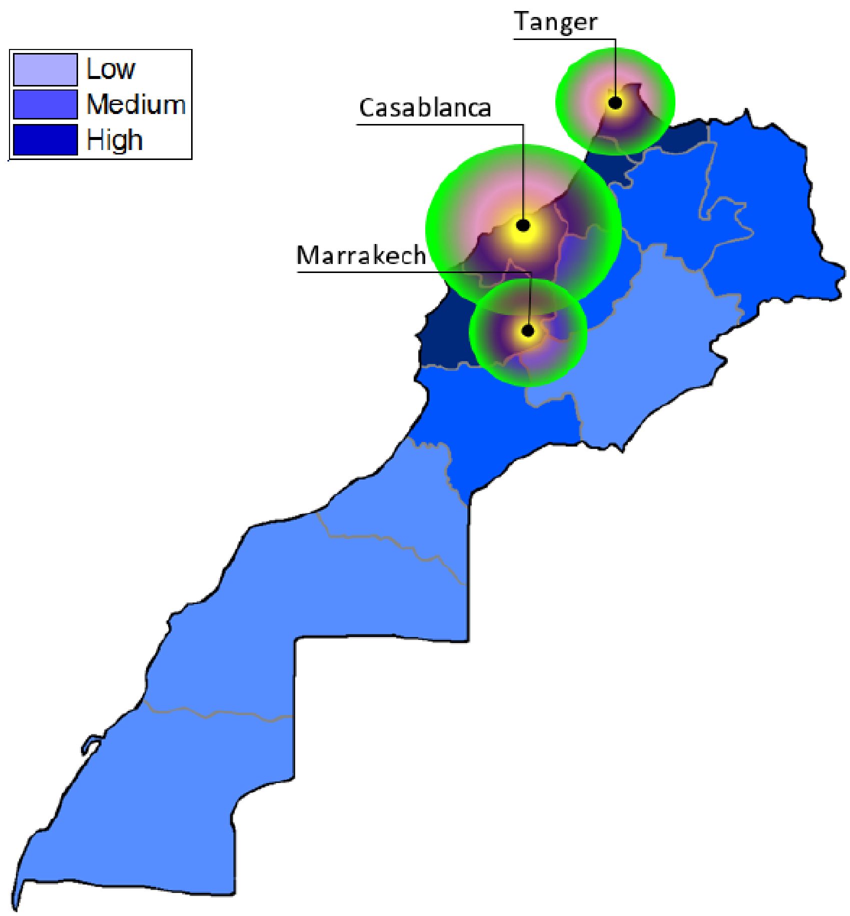


Figure 8.8: Spatial Moroccan impact of COVID-19.

The usefulness of this study is to determine the sectorial impact levels of the pandemic and the implications for social, economic, and operational performance. This could be of great help to policymakers looking to prioritize the most impacted sectors in terms of intervention and financing. The design of new policies for recovery, economic growth, and support of the SDGs is key to emergence from this critical state. First, we need to develop innovative methods to collect and filter data. Data management in times of crisis has tremendous importance for informing and supporting policymakers. However, the lack of data and information available during the pandemic in Morocco makes it more difficult to assess the impact. After obtaining the necessary data and information, we need to analyze and understand the impact of the crisis on the various sectors and design economic and social policies in response. For example, due to the high correlation between the transport and tourism industry, governments and industry have to focus their efforts on lifting travel restrictions, applying new health protocols for safe travel, restoring traveler confidence, and stimulating demand with new safety and cleanliness labels for the sector. In this regard, tourist information applications and domestic tourism promotion initiatives can be beneficial. Once borders start reopening, the tourism industry will start to recover and the government must support it by preparing short and long-term tourism recovery plans, rebuilding destinations, and rethinking the tourism sector. This may both assist in the sector's recovery and strengthen it in the long term, particularly in regions and cities where the sector supports many jobs and businesses. Investing in transportation infrastructure is crucial for economic recovery and job creation. The government must strike a balance between building a more connected urban network between the north and south regions. This is especially beneficial for the healthcare sector to avoid major bottlenecks and for a successful vaccination campaign against COVID-19. For the industry sector, the government has to provide quick financial support to small and medium enterprises in terms of direct capital injections or subsidiaries such as loans, taxes relief, and promoting new investments. Since the agriculture sector has been slightly impacted by this pandemic and its contribution to Moroccan GDP, it has an outsized impact on the overall economic growth. In this scenario, the adoption of technology-enabled systems seems to be quite ambitious for making the agriculture sector more resilient and responsive to COVID-19 consequences. While for the education sector, the government is urged to intensify e-learning methods, develop new platforms, increase school's infrastructure, and implement learning recovery programs. Finally, investing in private and other public sectors and services during and after the crisis is also a good strategy for long-term and sustainable recovery.

8.5 CONCLUSION

COVID-19 has been the matter of the moment from the day it was declared a pandemic, it has led to the termination of economic activities globally. Scientists and policymakers across continents are joining forces for innovative tie-ups in response: pharmaceutical giants and medical start-ups are working together to develop efficient vaccines and policymakers in all countries are setting policies to relaunch the economy. Morocco has taken a good path in fighting against the COVID-19 pandemic. But despite the ongoing efforts, many questions remain unanswered. What is the right policy to relaunch the economy? The process of economic recovery is complex, with many stages. The most interesting question that arises: what sectors have been most impacted by COVID-19 and require priority in intervention from the central government? This is the main motivation behind this study. However, due to the lack of information and a high level of uncertainty, decision support becomes very challenging and requires more advanced techniques. In this study, first, we develop a new technique for dealing with uncertainty and complexity called XOR-ANP. The main benefit to using this technique is its ability to deal with uncertainty stemming from hesitation or reluctance and the flexibility to incorporate the DM's preferences by using three types of XOR matrices: optimistic, pessimistic, and neutral. Moreover, to contribute to the COVID-19 literature from an operational research perspective, the developed XOR-ANP technique is adopted to determine the economic sectors most impacted by COVID-19 to assist policymakers in mitigating the impact of the pandemic and prioritizing their interventions to relaunch the Moroccan economy. The results show that tourism and transport are the most impacted sectors, followed by healthcare. Based on these results, the government needs to give high priority to these sectors by investing more in them. Finally, since the COVID-19 pandemic has affected the entire globe, we can simulate the proposed study on other countries, such as Italy and Algeria, to see which sectors have been the most impacted there and helps to learn the main reasons why certain countries may respond better than others. The proposed XOR-ANP model can also be easily applied to a variety of different research fields, such as renewable energy, banking and portfolio selection, and healthcare planning.

9 XOR-Best Worst Method

The more original a discovery, the more obvious it seems afterward.
-Arthur Koestler-

This chapter develops a new technique for dealing with decision-making problems under uncertainty using exclusive-or (XOR) logic, called the XOR-Best Worst Method (XOR-BWM). Then, the proposed technique is adopted to assess the impact of COVID-19 in Italy on seven relevant sectors (tourism, transport, industrial, financial, agriculture, education, healthcare) by considering social, operational, and economic dimensions.

9.1 INTRODUCTION

Recently, Hocine and Kouaissah (2020) have proposed a new technique for dealing with uncertain decision-making problems using XOR logic called XOR analytic hierarchy process (XOR-AHP). Conceptually, XOR is an uncertain logic that describes a situation in which there is only one choice between two or more competitive actions, and neither is strong enough to overcome the others. Unlike classical logic, which requires a deep understanding of a system, precise numeric values, and exact formulations, XOR logic offers a more free way of thinking and utilizes a higher level of abstraction—originating from our knowledge and experience—to model complex systems. In contrast to existing uncertainty theories, the XOR analysis framework provides alternative insights and an intuitive and natural way to model the imprecision and uncertainty existing in decision-making problems. XOR analysis, at least in some cases, is the most natural representation of imprecision and uncertainty where no assumptions are required (such as distribution functions, fuzzy membership functions, or lower and upper bound values). For instance, it is well-recognized that COVID-19 poses economic and social risks. In this regard, when experts are asked to assess the economic impact of COVID-19 on, for example, the tourism and agriculture sectors using a numerical scale ranging from 1 (equal impact) to 9 (significantly greater impact), it is very difficult for them to provide an exact estimate. Experts are forced to express their estimations using XOR logic, saying, for example: "On an economic level, the tourism sector is more impacted than the agriculture sector by 7 or 8 times." Mathematically, this statement can be expressed as follows: 7 XOR 8. Thus, problems featuring these characteristics can be modeled using an XOR framework as suggested in Hocine and Kouaissah (2020). In practical applications of the XOR-AHP model, for n criteria, using reciprocity, at least $n(n-1)/2$ pairwise comparisons must be conducted by an expert to obtain the final solution; this represents one of the main limitations of the pairwise comparisons of AHP methodology. Rezaei (2015) has shown that these multiple calculations are not necessary, and that this limitation stems from the unstructured way of conducting pairwise comparisons. Following the same line of reasoning, however, in many real-life decision problems, conducting all the required pairwise comparisons would demand impractically high cognition efforts from the DMs and experts (especially in uncertain situations such as the COVID-19 crisis where the evolution of pandemic is very rapid and related information is insufficient or absent). Motivated by these concerns, Rezaei (2015) developed a new technique called the Best Worst Method (BWM) to address the existing limitations of pairwise comparisons. In this technique, the best (the most desirable or the most important) and the worst (the least desirable or the least important) criteria are first identified by the DM. Pairwise comparisons are then executed between each of these two criteria (best and worst) and the other criteria. In doing so, the number of pairwise comparisons can be reduced to 2n-3, which is a very desirable improvement that provides more flexibility and applicability in dealing with real-life decision-making problems. Although this technique offers high applicability (see, e.g., Rezaei et al., 2016; Gupta et al., 2017; Kheybari et al., 2019; Liang et al., 2020; Mohammadi & Rezaei, 2020), recent state-of-the-art work by Mi et al. (2019) shows that the issue of uncertainty still has not been tackled broadly. To fill this gap, this study aims to extend and enhance the classical BWM (Rezaei, 2015) to deal with decision-making problems in the presence of uncertainty that stems from XOR data as suggested by Hocine and Kouaissah (2020). The technique thus developed is called XOR-Best Worst Method (XOR-BWM). The proposed model contributes to the literature for the following reasons: (i) This model provides the ability to deal with uncertainty, such as that surrounding COVID-19, especially when DMs are hesitant or reluctant in expressing their judgments. This contribution enhances the algorithm of the BWM to cope with real scenarios, especially extreme events such as the COVID-19 pandemic. (ii) Since the standard version of the BWM does not take into consideration the DM's preferences, the proposed XOR-BWM allows incorporation of the DM's preferences by using three types of preferences: optimistic, pessimistic, and neutral. These contributions play a key role in supporting DMs and are suitable for real-world scenarios. (iii) It contributes to the COVID-19 literature from the social, economic, and operational research perspectives. It considers a real-world scenario of determining the economic sectors most impacted by COVID-19 to assist policymakers in mitigating the impact of the pandemic and prioritizing their intervention to relaunch the Italian economy.

9.2 RELATED WORKS

The Best Worst Method (BWM) is a relatively new MCDM approach. It was designed to estimate the relative importance vectors using pairwise comparisons' philosophy. This method was proposed by Rezaei (2015; 2016) and further developed in distinguished papers by Kocak et al. (2018), Rezaei (2020), Liang et al. (2020), Mohammadi, and Rezaei (2020), and others. Since the development of the

BWM, it has attracted the attention of many scholars and several research papers have confirmed its high potential applicability to real-life problems in different fields such as supply chain management (Rezaei et al., 2016), risk assessment (Torabi et al., 2016), the airline industry (Rezaei et al., 2017), clean energy (Lo et al., 2018) the semantic web (Mohammadi & Rezaei, 2020), and other fields (for a recent state-of-the-art survey on integrations and applications of the BWM in decision-making problems refer to Mi et al., 2019). According to some statistics, the paper published by Rezaei in 2015 is the third most cited article published in Omega journal, and recent developments suggest that the research on the BWM will continue to increase in the future. For more recent advances in the BWM and its applications, readers are referred to a recent survey of Mi et al. (2019). Methodologically, the BWM is very similar to the AHP. Pairwise comparisons are still the key to running the necessary computations to generate the weights based on two evaluation vectors: the best criterion against the other criteria, and the other criteria against the worst criterion. Through this process, the DM's preferences are converted from linguistic expressions to numerical values using a 1–9 scale. Compared to one of the most popular pairwise comparison methods, AHP, the main benefit of using the BWM is that it requires fewer comparison data while being able to generate more consistent comparisons. Thus, the main idea behind developing this technique is to lift the limitation of making $n(n-1)/2$ pairwise comparisons to $2n-3$ pairwise comparisons. BWM not only leads to more consistent comparisons but also reduces the number of comparisons that must be made. Thus, the pairwise comparison results are two vectors based on which a linear/non-linear mathematical programming problem should be designed to derive the priority weights. For more details about the theoretical background, mathematical concepts, and best practices of the BWM, see, e.g., Mi et al. (2019) and Rezaei et al. (2020). Since the main philosophy behind the BWM is the pairwise comparisons of experts' judgments, it inherently involves subjectivity, uncertainty, and imprecision, which can impact the accuracy of the input data. For this reason, the weights produced by the traditional BWM may not necessarily be accurate, and they are associated with a certain degree of uncertainty. Consequently, uncertainty spreads over the criteria and alternatives, resulting in uncertain local and global weights, reducing the confidence of the results. Several papers have been published addressing this issue using various uncertainty theories, including fuzzy set theory, interval analysis, and probability (or stochastic) theory. Three main streams of research have been proposed, namely, interval BWM, fuzzy BWM, and probability BWM (see e.g., Rezaei, 2016; Guo et al., 2017; Mi et al., 2019; Mohammadi & Rezaei, 2020). For example, the interval BWM can be used to deal with rounding and measurement errors. Consider the following interval number $x = [l, u]$, where l and u are real numbers representing the lower and upper limit of the interval. Determining these two boundary values is very crucial in interval analysis, which contains an infinite number of possibilities with no information whatsoever on the behavior of the data between those two boundary values. Although it is typically accepted that random variables are captured through stochastic processes and probability theory, consensus fails on whether historical data is sufficient to formulate suitable probabilistic distributions (see Kirkwood, 1992). Unfortunately, under some circumstances such as COVID-19, historical data is lacking or non-existent, and uncertainty may arise from linguistic sources. In these cases, the fuzzy BWM can be adopted to deal with imprecise numerical quantities. However, the adoption of fuzzy numbers presents difficulties related to the issue of overlapping and the defuzzification procedure may lose a certain amount of information existing in the original data. In general, it can be argued that no approach is superior to the others in all cases; all approaches have their advantages and limitations, and the choice of the right approach should be guided by the analysis. Thus, sometimes, releasing a fair comparison of uncertain BWM methods is difficult or almost impossible because each uncertain method is based on different assumptions, uses different algorithms and formulas, and requires different information for its application.

9.3 MODEL FORMULATION

To proceed with BWM, a numerical scale from 1 to 9 is used. This numerical scale reflects the preferences of the DM in the comparison process. Let Let $X = \{x_1, x_2, \ldots, x_n\}$ be a set of criteria. We denote $XORa_{ij}$ as the value of the comparison between criterion x_i and x_j, where $XORa_{ij} = a_1 XORa_2 XOR \ldots XORa_k$. When criteria x_i and x_j have equal relative importance, $XORa_{ij}$ will be 1. In the case of $XORa_{ij} > 1$, the relative importance of criterion x_i is more than or equal to x_i. An extreme preference of x_i to x_j is represented by 9. The XOR-BWM comparison matrix, $XOR - A = (a_{ij})_{n \times n}$, can be formulated as follows:

$$XOR - A = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{pmatrix} \underset{k=1,\ldots,m}{XOR}(a_{11}^k) & \underset{k=1,\ldots,m}{XOR}(a_{1j}^k) & \cdots & \underset{k=1,\ldots,m}{XOR}(a_{1n}^k) \\ \underset{k=1,\ldots,m}{XOR}(a_{i1}^k) & \underset{k=1,\ldots,m}{XOR}(a_{ij}^k) & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ \underset{k=1,\ldots,m}{XOR}(a_{n1}^k) & \cdots & \cdots & \underset{k=1,\ldots,m}{XOR}(a_{nn}^k) \end{pmatrix} \end{array}. \tag{9.1}$$

Following Rezaei (2015, 2016) and Hocine and Kouaissah (2020), the XOR-BWM can be modeled through the following steps:

Step 1: Determine a set of decision criteria.

In this step, we consider the most important criteria that affect decision analysis. For instance, in the case of assessing the sectorial impact of COVID-19, the decision criteria could be {social (x_1),,economic (x_2),and operational (x_3)}.

Step 2: Determine the best criterion and the worst criterion.

The best criterion is the most desirable or most important and the worst criterion is the least desirable or least important among the set of decision criteria. In this step, the DM identifies the best and the worst criteria. In general, no comparison is made at this stage. For example, for a specific purpose, economic and operational may be the best and the worst criteria, respectively.

Step 3: Determine the pairwise compression of the best criterion.

In this step, pairwise comparisons are performed for each criterion involved in the decision problem. This process can be done by asking experts to express their judgments numerically using a scale from 1 (equal impact) to 9 (extreme impact). A generic question that can be used is how much the best criterion is better than other criteria. For example, in the context of COVID-19, one can ask how much important the economic dimension is greater than the operational dimension in detecting the impact of COVID-19. The resulting Best-to-Others vector can be represented as $XOR - A_B = (XORa_{B1}, \ldots, XORa_{Bn})$, where $XORa_{Bj}$ represents the XOR preference of the best criterion B over criterion j. It is clear that $XORa_{BB} = 1$.

Step 4:Determine the pairwise compression of all the criteria over the worst criterion.

In this step, the pairwise comparisons are performed in a similar way as in the previous step with a different generic question: how much the other criteria are better than the worst criterion. The resulting Others-to-Worst vector can be represented as: $XOR - A_W = (XORa_{1W}, \ldots, XORa_{nW})^T$, where $XORa_{jW}$ represents the XOR preference of the criterion j over the worst criterion.

Step 5: Incorporate the decision-maker's preferences.

Preference modeling plays a fundamental role in management science and operational research when the DM's desires are considered. Let us consider this example: suppose a certain government is willing to buy a COVID-19 vaccine. Since there is still a high level of uncertainty and competition around the available COVID-19 vaccine doses, identifying the most viable vaccine is a critical issue, especially in developing countries where resources are limited. In this case, experts may try to provide initial estimations, based on the available data and information, to evaluate the existing vaccines. An expert with multiple competing reliable judgments may prefer to express them in a realistic way using XOR logic. For example, an expert asked to provide his or her judgment about the effectiveness of a certain vaccine over another may say: based on trial data and the data available about the approaches, techniques, and technologies used to develop each vaccine, Vaccine A seems more effective that Vaccine B by 2 XOR 3 XOR 4 times. Based on this statement, a DM taking an optimistic view would favor Vaccine A. However, DMs are often unwilling to take risks or make decisions based on an optimistic view; they desire the ideal vaccine to be selected through an interactive process that involves different interests, dimensions, and policies. Thus, to serve the DM's preferences, according to Hocine and Kouaissah (2020), the pairwise comparison corresponding to real-world scenarios assumes: i) optimistic (positive) if $XOR - A = (a_{ij})_{n\times n} = \max\limits_k \left[XOR - A = (a_{ij}^k)_{n\times n} \right]$, ii) pessimistic (negative) if $XOR - A = (a_{ij})_{n\times n} = \min\limits_k \left[XOR - A = (a_{ij}^k)_{n\times n} \right]$, and iii) neutral does not follow any mandatory direction (max or min) for all $i, j \in \{1, 2, ..., n\}$.

Step 6:Generate the optimal weights.

After obtaining all the necessary pairwise comparisons and incorporating the DM's preferences, the weight vector is estimated to prioritize the criteria and rank the alternatives. The estimation of these weights is performed by solving a mathematical programming model called the XOR weighting technique (XOR-W) which is formulated as follows:

$$\min \quad \pi + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i\neq j}}^{n} (\sigma_{ij}^{+} + \sigma_{ij}^{-}) + \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i\neq j}}^{n} (\beta_{ij}^{+} + \beta_{ij}^{-}) \tag{9.2}$$

$$\text{s.t.} \quad \frac{w_B}{w_j} - \underset{k=1,\ldots,m}{XOR} (a_{Bj}^k) + v_{Bj}^{-} - v_{Bj}^{+} = 0 \;\; (i, j \in N,\; i \neq j,\; k \in M) \tag{9.3}$$

$$\frac{w_j}{w_W} - \underset{k=1,\ldots,m}{XOR} (a_{jW}^k) + v_{jW}^{-} - v_{jW}^{+} = 0 \;\; (i, j \in N,\; i \neq j,\; k \in M) \tag{9.4}$$

$$\psi_{ij} \underset{k=1,\ldots,m}{XOR} (a_{Bj}^k) - \psi_{ij} \min_k \left[\underset{k=1,\ldots,m}{XOR} (a_{Bj}^k) \right] + \sigma_{Bj}^{-} - \sigma_{Bj}^{+} = \delta \; (i, j \in N,\; i \neq j,\; k \in M) \tag{9.5}$$

$$\psi_i \underset{k=1,\ldots,m}{XOR}(a^k_{jW}) - \psi_i \min_k \left[\underset{k=1,\ldots,m}{XOR}(a^k_{jW})\right] + \sigma^-_{jW} - \sigma^+_{jW} = \delta \; (i,j \in N,\ i \neq j,\ k \in M) \tag{9.6}$$

$$XOR(a^k_{il}) \times XOR(a^k_{lj}) + \beta^-_{ij} - \beta^+_{ij} = XOR(a^k_{ij}) \; (i,j \in N,\ i \neq j,\ k \in M) \tag{9.7}$$

$$\underset{k=1,\ldots,m}{XOR}(a^k_{ij}, \lambda_k) = a^1_{ij}\lambda_1 + a^2_{ij}\lambda_2 + \ldots + a^m_{ij}\lambda_m = 1 \; (i,j \in N,\ i \neq j,\ k \in M) \tag{9.8}$$

$$v^-_{Bj} + v^+_{Bj} \leq \pi \; (i,j \in N,\ i \neq j) \tag{9.9}$$

$$v^-_{jW} + v^+_{jW} \leq \pi \; (i,j \in N,\ i \neq j) \tag{9.10}$$

$$\sum_{i=1}^{n} w_i = 1,\ w > 0 \; (i \in I) \tag{9.11}$$

$$\psi_i = \frac{1}{\max_k \left[\underset{k=1,\ldots,m}{XOR}(a^k_{ij})\right] - \min_k \left[\underset{k=1,\ldots,m}{XOR}(a^k_{ij})\right]}$$

$$\lambda_k = \{0,1\} \quad (k \in M)$$

$$\sigma^+_{ij}, \sigma^-_{ij}, \beta^+_{ij} \text{ and } \beta^-_{ij} \geq 0 \; (i,j,B,W \in N,\ i \neq j)$$

where $N = \{1,2,\ldots,n\}$ and $M = \{1,2,\ldots,m\}$, π is the maximum deviation, $XORa_{ij}$ is the element of a $(n \times n)$ pairwise comparison matrix $XOR - A$ that summarizes the relative importance of the *best* criterion against the *other* criteria and the other criteria against the worst criterion, W_i is the derived weight given to the i-th criterion, $\underset{k=1,\ldots,m}{XOR}(a^k_{ij}, \lambda_k)$ is the function of XOR judgment logic, and λ_k is a binary variable. The objective function (Equation (9.2)) ensures that the maximum absolute deviations of the inconsistencies (β^+ and β^-) of the XOR pairwise comparison matrix and the unwanted deviations (σ^+ and σ^-) from the aspiration level δ are minimized. Equations (3–4) define a hyperplane in the n-dimensional priority space. Equations (9.5-9.6) represent the DM's preferences, where δ is a psychological parameter that gauges optimism/pessimism assessment and ranges from 0 to 1 as shown in Table 9.2. Equation (9.7) allows inconsistency to be controlled. Equation (9.8) guarantees that only one evaluation will be selected for the i-th and j-th pairwise comparisons, which is the most appropriate evaluation among the potential evaluations that will achieve the highest level of DM consistency. Equations (9.9-9.10) formulate the Chebyshev norm. Equation (9.11) satisfies the normalization condition.

Scale		Definition
Cost Criteria	Benefit Criteria	
0	1	Optimistic
0.2	0.8	Moderately Optimistic
0.5	0.5	Neutral
0.8	0.2	Moderately Pessimistic
1	0	Pessimistic

Table 9.1: Scales for optimism/pessimism.

Step 7: Check the consistency.

Once judgments have been entered, it is necessary to check that they are consistent. The consistency ratio (CR) is an important indicator to check the degree of consistency of pairwise comparisons. The consistency level of pairwise judgments is measured by calculating a consistency ratio (CR). The following formula is used in this calculation:

$$CR = \frac{\pi^*}{CI} \tag{9.12}$$

where π^* is is the optimal solution of the XOR-W program and CI is the consistency index, determined based on Table 9.3 (Razei, 2015).

Table 9.2: Consistency Index (CI) table.

	1	2	3	4	5	6	7	8	9
Consistency Index (CI)	0.00	0.44	1.00	1.63	2.30	3.00	3.73	4.47	5.23

It is clear that CR takes values between interval $[0, 1]$. Therefore, the more consistent is the most near to 0.

9.4 REAL-LIFE APPLICATION: ASSESSMENT OF COVID-19'S SECTORIAL IMPACT IN ITALY USING XOR-BWM

Viruses have a tremendous potential to become life-threatening and to cause human beings unrecoverable losses. The human race hardly learns to deal with one strain of virus when another appears and presents a risk to humanity's future (Inhorn & Brown, 1990; Smith et al., 2019). The new strain of severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2), never previously identified in human history, presents just such a threatening situation (Packham, 2020). It is a contagious viral infection that can be spread through close person-to-person contact, airborne transmission, and contact with contaminated surfaces. Infection with COVID-19 is characterized by widely differing pathological manifestations, which range from asymptomatic or mild influenza-like symptoms to severe pneumonia and acute respiratory distress syndrome. Biologically, coronavirus is an enveloped, positive-sense, single-stranded RNA virus with a genome of nearly 30,000 nucleotides. It encodes four structural proteins, Nucleocapsid (N) protein, Membrane (M) protein, Spike (S) protein, Envelop (E) protein, and several non-structural proteins (see Figure 9.1.1a). CoVs are named due to their crown-like surface projections, thought to resemble the Sun's corona (see Figure 9.1.1a) (see V'kovski et al., 2020). A 3D representation of SARS-CoV-2 is shown in Figure 9.1.1b.

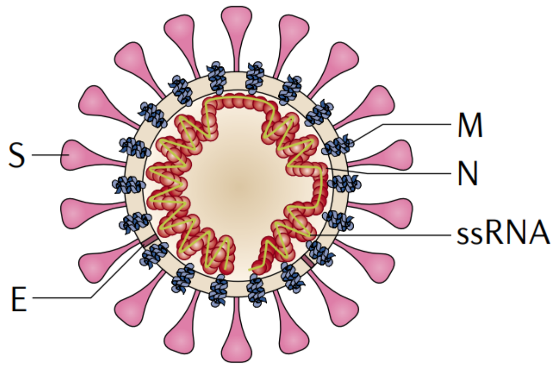


Fig. 1a. SARS-CoV-2 main components.

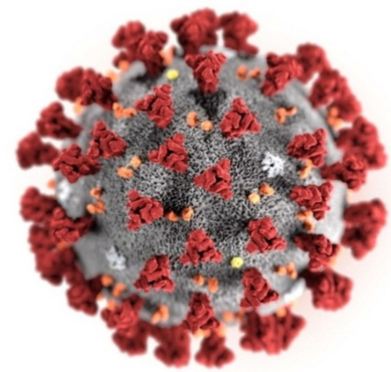


Fig. 1b. SARS-CoV-2 3D representation.

Figure 9.1: SARS-Cov-2

COVID-19 has been declared a pandemic by the WHO and has and continues to have severe effects on both human well-being and economic conditions around the world. COVID-19 is rapidly spreading around the globe; by end of December 2020, at the time of writing this paper, more than 80 million people had been infected. More than 57 million of those had recovered, but the virus had caused approximately 1.8 million deaths. Most infected patients exhibit mild to moderate symptoms, but approximately 15% progress to severe pneumonia, and about 5% eventually develop acute respiratory distress syndrome, septic shock, and/or multiple organ failure (Cao, 2020). Figure 8.2 shows the worldwide spread of the COVID-19 infection as of the end of December 2020. Italy was one of the first Western countries to experience a severe COVID-19 emergency, with a spiral of infections and deaths. The country quickly faced one of the highest case counts in the world; in May 2020, the Italian Ministry of Health reported more than 220,000 confirmed cases and approximately 30,000 deaths. By the end of December 2020, Italy had registered over 2 million infected people and more than 70 thousand deaths; for updated information on the state of the situation visit the WHO website.

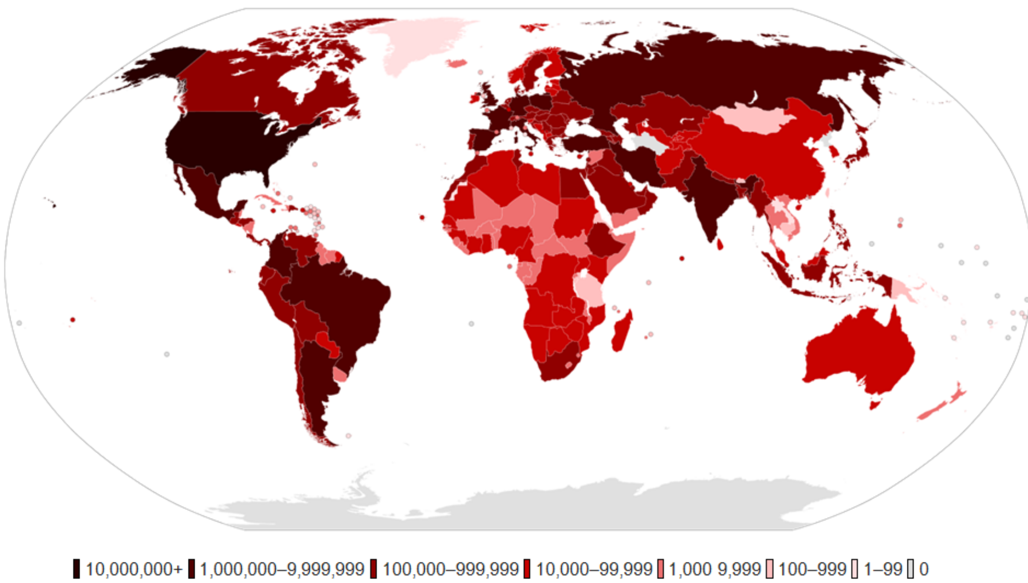


Figure 9.2: World map of confirmed COVID-19 cases, December 2020. Source: Data derived from Johns Hopkins University CSSE, The Centers for Disease Control and Prevention, New York Times, CNBC.

Beginning early in the epidemic, the Italian government implemented a wide range of measures to balance the complex trade-offs between public health, social well-being, and economic conditions. Italy's national exit strategy plan began on 4th May 2020 with the gradual relaxation of containment measures carried out in different phases. However, uncertainty and complexity are ubiquitous and inherent features of decision-making processes related to this pandemic. Generally, this uncertainty arises from a lack of information, the rate at which the pandemic is spreading and changing, and the way decision-making components interact. This feature is well clear from the impact of COVID-19. In this regard, policymakers require reliable decision-making supports that improve and validate their decisions and policies. For example, to effectively measure and mitigate the impact of the pandemic and relaunch their economies, policymakers will need to determine which sectors have been most impacted by the pandemic. The following analyses aim to investigate and assess the impact of COVID-19 on seven relevant sectors (tourism, transport, industrial, financial, agriculture, education, healthcare) by considering social, operational, and economic dimensions.

After the confirmation of the first infection in the country on 26th February 2020, Italy launched a series of increasingly strict restrictions to respond to the COVID-19 outbreak. Restrictions included the cancellation of large events, border closures, and finally confinement of movement and closure of non-essential shops on 23rd March. The prevalence of the pandemic was particularly severe in the northern regions, moderate in the central regions, and mild in the southern regions of Italy. As COVID-19 spread over the country, its impacts increased and began to appear in all aspects of life. For example, at end of May 2020, the first quarter of 2020, Italian GDP had fallen by 5.4%. In the second quarter, it fell by 12.4% and the national industrial production index decreased by

17.5%, with production falling to an all-time low (OECD, 2020; ISTAT, 2020). The Italian government implemented a lockdown strategy to limit the spread of COVID-19. Major automobile companies temporarily shut their factories. Ferrari S.p.A. halted production for a couple of weeks. In the tourism sector, it was estimated 48 million fewer tourists arrived in Italy between January and August 2020, compared to the same period of 2019. With roughly 9.3 million fewer arrivals year-over-year, Veneto experienced the largest drop. The second-highest decrease was reported by Lombardy, which saw around 6.6 million fewer tourist arrivals than in 2019 (UNWTO, 2020). The transportation sector also saw a negative impact; according to the international air transport association, the pandemic could trigger a loss in revenues of 25 to 55% compared to the previous year (IATA, 2020). With the healthcare sector at the front line or the epicenter of the pandemic, most public and private hospitals witnessed a remarkable increase in demand, even reaching their capacity. Many interventions, at both the governmental and regional level, aim to balance the complex trade-offs between public health, social well-being, and economic conditions, while attempting to relaunch the economy under these conditions. The impacts of COVID-19 have left governments and policymakers counting costs and wondering what recovery could look like. In this context, the design of an efficient policy for recovery and economic relaunch is a high priority and requires all related sectors and dimensions to be taken into consideration. The flow chart in Figure 9.4 illustrates the main steps of the XOR-BWM, as follows: define the goal, determine criteria and alternatives, incorporate the DM's preferences (optimistic, pessimistic, and consistency control), use the W-XOR method to generate weights, and finally, obtain the overall impact ranking. A typical XOR-BWM decision structure is shown in Figure 8.4, where the goal of the decision problem is to detect the most impacted sector among seven sectors (i.e., agriculture, industrial, transport, tourism, financial, healthcare, and education) based on three dimensions or criteria (social, economic, and operational).

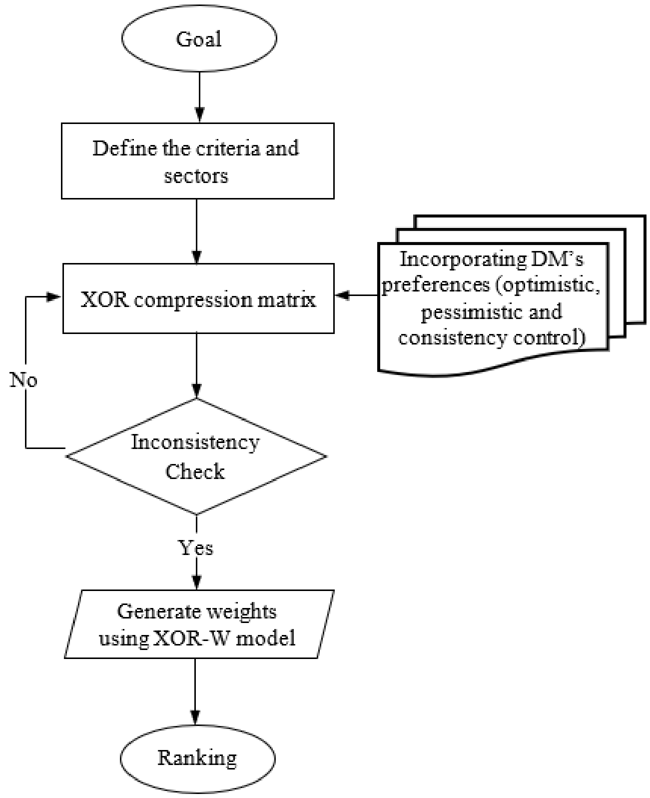


Figure 9.3: The XOR-BWM model.

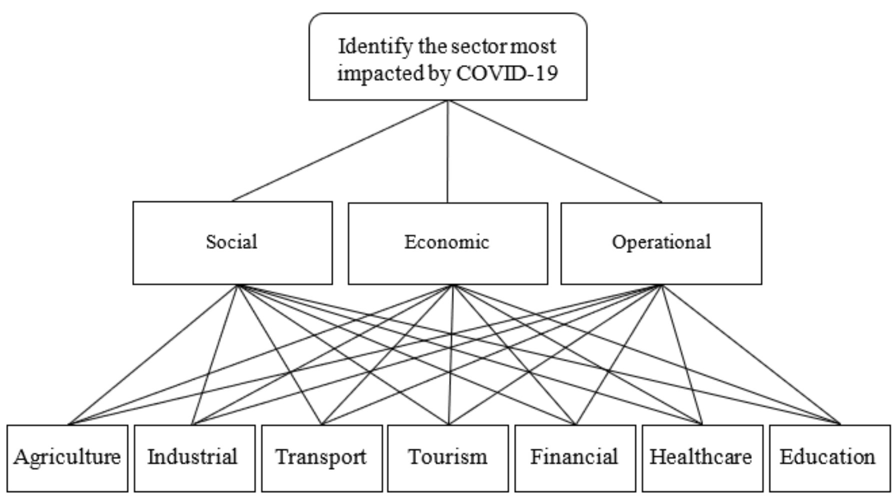


Figure 9.4: COVID-19 XOR-BWM diagram.

To proceed with the XOR-BWM, as an initial step before going through the pairwise comparison, it is required to specify the best and the worst criterion. There is no comparison at this step. In this case, the economic and operational are determined to be, respectively, the best and worst criteria.

Then, an expert was asked to determine the preference ratio pairwise comparison using a measurement scale of 1 to 9. To elicit expert judgments, the following question is asked: "*With respect to the goal of selecting the most impacted sector, by how much is the best criterion better than the others, and by how much are the other criteria better than the worst criterion?*" Tables 9.9-9.4 display the pairwise comparison.

Table 9.3: XOR pairwise comparison for determining the importance of the best criterion over the other criteria.

Best	Social	Economic	Operational
Economic	2 XOR 3	1	3 XOR 4

Table 9.4: XOR pairwise comparison for determining the importance of all criteria over the worst criterion.

Operational	Worst criterion (Operational)
Social	6 XOR 7 XOR 8
Economic	2 XOR 4
Operational	1

Using the software Lingo (Sharge, 2019), the weights of the criteria and the objective function are obtained as follows: $w_{soc.} = 0.4194$, $w_{econ.} = 0.4750$, $w_{oper.} = 0.1056$, and $\pi^* = 1.5$. The binary variables set is $\lambda_{1,\ldots,7} = \{1,0,1,0,1,0,0\}$. Based on the binary variable set, the XOR function output for each judgment is as follows: $XOR(a_{eco./soc.}) = 2$, $XOR(a_{eco./oper.}) = 3$, and $XOR(a_{soc./oper.}) = 6$. For the consistency ratio, since the maximum output is 6, the corresponding consistency index for this problem is 3 (see Table 9.3), and the consistency ratio is $1.5/3 = 0.5$, which implies a high degree of consistency. This is due to the fact that consistency control preferences that have been integrated into the constraint system of the program.

Next, the pairwise comparisons of sectors with respect to the criteria should be determined. As an initial step before conducting these pairwise comparisons, the best and worst alternatives (i.e., sectors) must be specified. Due to the nature of the economic system of Italy and its response to the COVID-19 pandemic, the best and worst sectors are Industrial and Agriculture respectively. The mechanism for filling these matrices is performed through the following question: "*If your goal is to detect the most impacted sector, by how much more is the best sector impacted, with respect to a particular criterion, relative to other sectors, and by how much are the other sectors impacted relative to the worst sector?*"" For example, to fill the pairwise comparison (see Table 9.6), the experts are asked to estimate how significantly the Industrial sector is impacted relative to the other sectors with respect to the economic dimension. Due to a lack of information and the rapid evolution of the situation, an expert may not be entirely sure of the precise values of some of his judgments. The expert may have multiple preferences and be hesitant and irresolute in choosing among them; he or she may express them using XOR logic. For example, according to the available data and reports, the Industrial sector was impacted by 6 XOR 7 times as much as the Education sector with respect to the economic dimension. So, we fill the cell (1,7) with 6 XOR 7. The same reasoning holds for determining the impact on each other sector relative to each criterion; related XOR pairwise comparison matrices are presented in Tables 9.5-9.10.

Table 9.5: XOR pairwise comparison for determining the degree of COVID-19 impact for each sector with respect to the economic dimension based on the best alternative.

Best	Agriculture	Industrial	Transport	Tourism	Financial	Healthcare	Education
Industrial	7 XOR 8	1	4	3 XOR 4	2	5	6 XOR 7

Table 9.6: XOR pairwise comparison for determining the degree of COVID-19 impact for each sector with respect to the economic dimension based on the worst alternative.

Worst	Agriculture
Agriculture	1
Industrial	7 XOR 8
Transport	4
Tourism	5
Financial	6 XOR 7
Healthcare	3
Education	2

After acquiring data from different sources (i.e., index reports, literature, and experts), the XOR-W technique is used to derive the necessary weights for all pairwise comparisons. Using the software Lingo (Sharge, 2019), the degree of impact on each sector is obtained, as follows: $w_{agri.} = 0.0350$, $w_{ind.} = 0.3152$, $w_{trans.} = 0.1051$, $w_{tour.} = 0.1576$, $w_{fin.} = 0.2452$, $w_{health.} = 0.0788$, $w_{edu.} = 0.0631$ and the objective function is $\pi^* = 1$. The binary variables set is $\lambda_{1,\dots,8} = \{0,1,1,0,1,0,1,0\}$. Based on the binary variable set, the XOR function output for each judgment is as follows:
$XOR(a_{ind./agri.}) = 8$, $XOR(a_{ind./Tour.}) = 3$, $XOR(a_{ind./edu.}) = 6$ and $XOR(a_{fin./agri.}) = 6$. For the consistency ratio, since the maximum output is 8, the corresponding consistency index for this problem is 4.47 (see Table 9.3), and the consistency ratio is $1/4.47 = 0.2237$, which implies the values are very consistent.

Table 9.7: XOR pairwise comparison for determining the degree of COVID-19 impact on each sector with respect to the operational dimension based on the best alternative.

Best	Agriculture	Industrial	Transport	Tourism	Financial	Healthcare	Education
Tourism	9	5	2 XOR 3	1	7	3	8 XOR 9

Table 9.8: XOR pairwise comparison for determining the degree of COVID-19 impact on each sector with respect to the operational dimension based on the worst alternative.

Worst	Agriculture
Agriculture	1
Industrial	6
Transport	8
Tourism	9
Financial	3
Healthcare	6 XOR 7
Education	2

After acquiring data from different resources (i.e., index reports, literature, and experts), the XOR-W technique is used to derive the necessary weights for all pairwise comparisons. Using the software Lingo (Sharge, 2019), the degree of impact on each sector is obtained: $w_{agri.} = 0.0247$, $w_{ind.} = 0.1482$, $w_{trans.} = 0.2717$, $w_{tour.} = 0.2964$, $w_{fin.} = 0.1482$, $w_{health.} = 0.0741$, $w_{edu.} = 0.0367$, and the objective function is $\pi^* = 3$. The binary variables set is $\lambda_{1,\ldots,6} = \{1,0,1,0,1,0\}$. Based on the binary variables set, the XOR function output for each judgment is as follows: $XOR(a_{tour./trans.}) = 2$, $XOR(a_{tour./edu.}) = 8$, $XOR(a_{health./agri.}) = 6$. For the consistency ratio, since the maximum output is 8, the corresponding consistency index for this problem is 4.47 (see Table 9.3) and the consistency ratio is $3/4.47 = 0.6711$.

The expert's preferences are highly valuable information in decision analysis related to COVID-19. This is because various factors interact, such as the changeable nature of the virus, socio-economic parameters, etc. Generally, these factors are uncertain and generate a volatile, dynamic state of psychological preferences for DMs and experts. By psychological preferences we mean whether the DM is optimistic, pessimistic, or somewhere in between. The constant flow of new knowledge related to the virus is one of the most significant factors influencing DM optimism or pessimism. The high volume of new evidence released each day since the pandemic began has both supported our aspirations and occasionally confirmed our fears. For instance, are we optimistic or pessimistic about recovering quickly from the effects of COVID-19? Optimism and pessimism refer to positive and negative future expectations respectively (see, e.g., Scheier et al., 1994). For example, take an expert asked to provide an estimate of the relative impacts of COVID-19 on the tourism sector and agriculture sector with respect to the social dimension. The expert responds with 4 XOR 5 XOR 6; however, she notes that according to the available data, indicators, and reports, the pessimism state is high and the expected impact on the tourism sector may reach level 6. Therefore, to make the analysis process more realistic, it is highly beneficial to incorporate and embed these psychological preferences in the decision analysis. The corresponding pairwise comparisons are shown in Table 9.9-9.10.

Table 9.9: XOR pairwise comparison for determining the degree of COVID-19 impact on each sector with respect to the social dimension based on the best alternative.

Best	Agriculture	Industrial	Transport	Tourism	Financial	Healthcare	Education
Tourism	4 XOR 5 XOR 6	7	2	1	8 XOR 9	2	3

Table 9.10: XOR pairwise comparison for determining the degree of COVID-19 impact on each sector with respect to the social dimension based on the worst alternative.

Worst	Financial
Agriculture	6
Industrial	3
Transport	6
Tourism	8 XOR 9
Financial	1
Healthcare	8
Education	7

After acquiring data from different sources (i.e., index reports, literature, and experts), the XOR-W technique is used to derive the necessary importance weights. Using the software Lingo (Sharge, 2019), the degree of impact on each sector is determined, as follows: $w_{agri.} = 0.0236$, $w_{ind.} = 0.0700$, $w_{trans.} = 0.1586$, $w_{tour.} = 0.2294$, $w_{fin.} = 0.0537$, $w_{health.} = 0.2386$, $w_{edu.} = 0.2261$, and the objective function is $\pi^* = 3.7250$. The binary variables set is $\lambda_{1,...,5} = \{0, 0, 1, 1, 0\}$. Based on the binary variables set, the XOR function output for each judgment is as follows: $XOR(a_{tour./agri.}) = 6$, $XOR(a_{tour./trans.}) = 8$. The XOR function of the Tourism/Agriculture comparison produced its maximum value, meaning that the DM's optimistic expectation was achieved. For the consistency ratio, since the maximum output is 8, the corresponding consistency index for this problem is 4.47 (see Table 9.3) and the consistency ratio is 3.7250/4.47 = 0.8333.

Finally, to determine the global impact, the additive weighted value function is used. The local impacts for each sector are multiplied by the importance degree of criteria as follows:

$$V_i = \sum_{j=1}^{J} w_j v_{ij} \tag{9.13}$$

where V_i is the global impact of the sector i, v_{ij} is the local impact about the criterion j, and w_j is the importance weight of criterion j. The final results are reported in Table 9.11 and Figures 9.4-9.64.

Table 9.11: Local and global impact of COVID-19.

Sectors	Local Impact			Global Impact
	Social	Economic	Operational	
Agriculture	0,0236	0,0350	0,0247	0,0291
Industrial	0,0700	0,3152	0,1482	0,1948
Transport	0,1586	0,1051	0,2717	0,1451
Tourism	0,2294	0,1576	0,2964	0,2024
Financial	0,0537	0,2452	0,1482	0,1546
Healthcare	0,2386	0,0788	0,0741	0,1453
Education	0,2261	0,0631	0,0367	0,1287

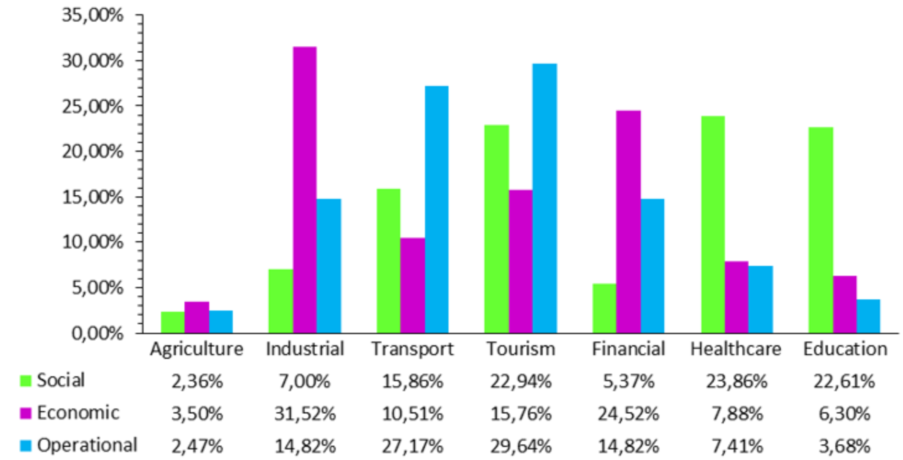


Figure 9.5: Local sectorial impact of COVID-19.

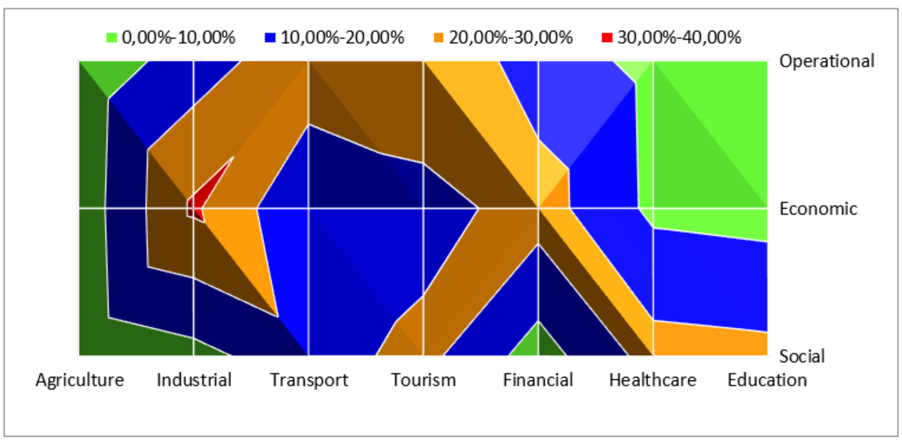


Figure 9.6: Degree heatmap of Covid-19 multi-dimensional impact.

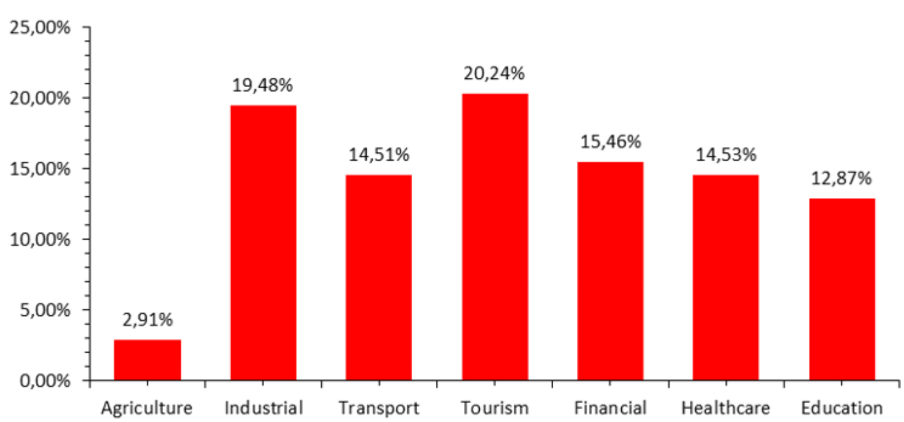


Figure 9.7: Global impact of COVID-19.

On 26th March 2020, United Nations (UN) Secretary-General António Guterres remarked at the G-20 virtual summit on the COVID-19 pandemic that we are at war with a virus, highlighting three critical areas for concerted G-20 action: first, suppress the transmission of COVID-19 as quickly as possible; second, collaborate to minimize its social and economic impact; and third, work together to set the stage for a recovery that builds a more sustainable, inclusive, and equitable economy, guided by our shared promise: the 2030 Agenda for Sustainable Development. While the first action has not yet been truly achieved, as the virus continues to spread rapidly around the globe, most countries are looking forward to planning for and beginning to implement the second and third priorities. To this end, and to effectively measure and mitigate the impact of the pandemic and relaunch its economy, the Italian government and policymakers need to determine which of the country's economic sectors have been most impacted. As made clear by the XOR-BWM results, especially those depicted in Figures 9.5-9.7, the tourism, industrial, and financial sectors have been most impacted by COVID-19, with 20.24%, 19.48%, and 15.46% respectively. These sectors play a vital role in Italy at many levels such as social, economic, and operational. The three sectors have direct, indirect, and induced impacts on the Italian economy and society and the social and economic costs of contractions in these sectors can be quantified using measures such as the unemployment rate, FTSE MIB index value, and GDP reduction. For instance, lockdowns and halting the production process in many key sectors have led to drops in input and output that, in turn, have generated a lock of about 52% of the total circulating value added (Giammetti et al., 2020). Furthermore, considering the industry-level shocks in aggregate, OECD (2020) estimates that the shutdown measures could have a potential immediate impact on GDP of around 25%; with the decline in tourist and leisure activities being important relatively. We can also observe that the healthcare, transport, education, and agriculture sectors are impacted by COVID-19, with 14.53%, 14.51%, 12.87%, and 2.91% respectively. All these sectors have deep effects on the social, operational, and economic levels, where the impact changes from one sector to another (see Figures 9.5-9.6).

One of the main aims of this study is to determine the sector-by-sector impact of the pandemic and its implications on the social, economic, and operational levels of Italian performance. This could be of great help to policymakers to prioritize the impacted sectors in terms of intervention and financing, which will contribute to the regional well-being of Italy. Figure9.8 shows how the COVID-19 pandemic immediately assumed a regional dimension, which was reflected not only in the geography of the contagion but also in how the increasing demands from the northern and southern regions were treated by the government. For example, in Figure 9.8 we can observe how northern cities such as Milan and Bergamo suffered socially due to their high population density. However, even though the north was the epicenter of the health crisis, the south is set to pay the highest price for the lockdown measures in economic and social terms. This is related to unemployment levels, which have always been higher in the south than in the north. Coronavirus arrived in this context, striking most violently the portion of the service sector that had managed to recover after the financial crisis. As a result, current unemployment figure estimates for 2020 show the north set to lose 3.5% of its jobs, while the south is projected to lose 6%—approximately 380,000 jobs, equivalent to losing in one year the same number of jobs that were lost between 2009 and 2013 (ISTAT, 2020). This would not only be an economic disaster but also risks destabilizing fragile social conditions in the South.

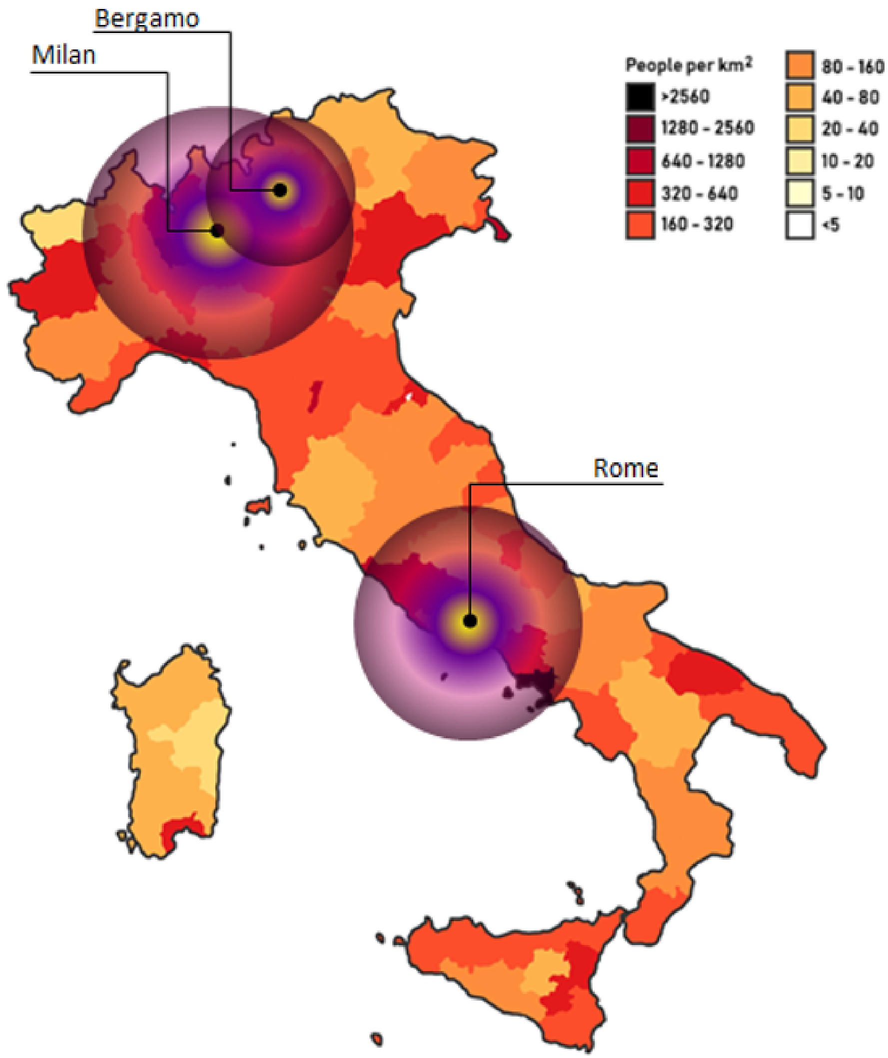


Figure 9.8: Spatial Italian impact of COVID-19.

The design of policies for recovery and economic growth that support the UN sustainable development goals is a key issue as Italy shifts out of this critical state. Future government interventions should be modified based on the data obtained thus far to mitigate or reduce social, economic, and operational impacts, both in the short and long term. For instance, it is well-known that the Italian economic structure relies mainly on services and manufacturing. Industry accounts for about 25% of Italy's total production and employs more than 30% of the total workforce. In this respect, the Italian government may provide more subsidiaries, financing support, tax relief to businesses, and individual social aid to the most vulnerable. Another important sector that has seen a devastating impact from COVID-19 is tourism, in which Italy is one of the leading countries worldwide. Indeed, among the world's top tourism destinations, Italy occupies the fifth position by international tourism receipts (UNWTO, 2020). The Italian government has to work closely with the private sector to mitigate the pandemic's impact on jobs and economies while also preparing for tourism's restart and emphasizing continued support for gradually restarting all tourism-related activities. Based on these analyses, helping the industrial, tourism, financial, health, and other public sectors and services during and after the crisis is a good strategy for long-term and sustainable recovery. Essential public services are the only social safety net for some population groups, especially the most vulnerable.

9.5 CONCLUSION

Since COVID-19 was declared a pandemic, it has led to the universal termination of socio-economic activities, causing deep damages to public safety and the health system. Across the globe, scientists and politicians are joining hands with both pharmaceutical companies and medical start-ups to develop effective vaccines and design efficient policies to relaunch world economies. In fighting the pandemic, Italy has, like many countries around the world, made considerable progress. Despite the ongoing efforts, several questions remain to be answered. What is the most suitable policy for promoting economic recovery? The mechanism of economic recovery is complex, with several different phases. The most interesting question that arises: what sectors have been most impacted by COVID-19 and need central government intervention to be prioritized? This is the key impetus for this research to be carried out. Decision support, however, is very difficult and requires use of sophisticated methods to address situations involving a lack of knowledge and a high degree of uncertainty. First, in this study, we develop a new technique called XOR-BWM to cope with uncertainty. The primary advantage of using this approach is its ability to deal with uncertainty resulting from hesitation or reluctance and its ability to integrate the expectations of DMs by using three forms of preferences: optimistic, pessimistic, and neutral. Moreover, the proposed XOR-BWM is adopted to contribute to the COVID-19 literature from the operational research perspective and determine the most impacted economic sectors. This process is applied to assist Italian policymakers in mitigating the impact of the pandemic and prioritizing their economic interventions aimed at relaunching the Italian economy. The findings show that Italy's most impacted sectors are the tourism, industrial, and financial sectors. The COVID-19 pandemic has affected the entire world; the proposed study can be simulated in the same way to see which sectors have been most affected in other countries as well, such as the US and Algeria. Furthermore, the proposed XOR-BWM can be easily applied to a variety of different research fields, such as renewable energy, supply chain management, and healthcare planning.

10 General Summary and Conclusion

This thesis is concerned with the design and develop an effective solutions and techniques to deal with hybrid uncertainty in MCDM problems. The study contributes to the knowledge of MCDM by developing new MODM and MADM methods that incorporates the DM preferences. In doing so, Chapter 3 proposed a novel method called: weighted-additive fuzzy multi-choice goal programming (WA-FMCGP) to fuzzifier the MCAL of LHS of each fuzzy goal. The main contribution of this model is using an objective function that minimizes the weighted-additive summation of the normalized deviations; thus, the model can adopt any minimization process from any goal programming (GP) variant. To verify the applicability of this technique, a real application to select the best wind farm location has been offered. In Chapter 4, a novel method called: fuzzy interval goal programming (FIGP) has been proposed to deal with interval coefficients in the LHS of each fuzzy goal. This model generalizes the classic FGP such that each decision variable supports the setting of interval coefficients, and considers the most common types of fuzzy membership functions (i.e., left, right, triangular, and trapezoidal) that we formulate as fuzzy interval membership functions. The application of the model is verified by using a real-world scenario of selecting the optimal RE portfolio for electricity generation in Italy. In case to deal with the interval AL in RHS, the Chapter 5 propose a novel model called fuzzy goal programming with interval target (FGP-IT). To provide a practical perspective, the proposed approach is used to determine the optimal sustainable wind energy portfolio decision in Algeria. To overcome the limitations of 'single-tolerance' during FGP modeling, Chapter 6 proposes a novel model called multi tolerances fuzzy goal programming (MT-FGP) to being unable to fit complicated 'multi-tolerance' decision contexts and running the risk of ignoring solutions that achieve better optimality. An empirical application is considered for an RE portfolio decision that is critical for sustainable RE development in Algeria. Chapter 7 develops a new model called XOR data envelopment analysis (XOR-DEA) to solve MADM problems using XOR logic. The necessary background information for proceeding with XOR numbers and how ordering and ranking XOR numbers is proposed. The concept of "the output mechanism of XOR function" to support the analyst to control uncertainty based on controlled and uncontrolled channels is proposed. To validate the XOR-DEA modelling, a real-life application of ranking renewable energy technologies has been conducted. Chapter 8 develops a new model called XOR analytic network process (XOR-ANP) to solve MADM problems using XOR logic. This contribution enhances the understanding of the role of DMs' preferences in decision-making processes. In particular, it allows us to explore the implications generated by the optimism/pessimism attitude toward a particular decision-making problem. To validate the effectiveness and feasibility of this technique, a real-life application to measure the impact of the COVID-19 pandemic in Morocco has been proposed. Chapter 8 presents a new model called the XOR-best-worst method (XOR-BWM) to solve MADM problems using XOR logic. To validate this uncertain MADM technique, a real-life application to measure the impact of the COVID-19 pandemic in Italy has been adopted. The details summary of each developed technique is given in the subsection of each chapter. In summary, the overall aims of this thesis were to develop MCDM methodologies to deal with:

- ✓ To develop a model that quantifies the DM's preferences, the linguistic uncertain term of "approximately … or …".
- ✓ To develop a model that quantifies the DM preferences, the linguistic uncertain term of "around between … and …".
- ✓ To develop a model that quantifies the DM's preferences, the Exclusive-or (XOR) term of "… or …".
- ✓ To validate the effectiveness and the feasibility of the proposed techniques, several real-life applications have been adopted.

Through this thesis, all these aims been achieved. There are some other issues which were not fully addressed in the current study. Since the effect of randomness was unexplored in the empirical case study, this would be a topic worthy of further exploration (e.g., using and further integrating the chance constraint technique to explore whether a solution can be further improved upon or not). The probabilistic chance constraint programming (CCP) concept initiated in the 1970s (Charnes and Cooper, 1961; Prekopa, 1970) has been applied to GP later and enriched after year 2000 (De et al., 1982; Ballestero, 2001; Aouni et al., 2012). As can be imagined, integrating the chance constraints into the proposed model and replacing the FGP part of the model with stochastic GP are both viable options. This would lead to a 'Stochastic-MCGP' model, which would (still) be a GP model worthy of inquiry.

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