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Optimizing modelling accuracy using variational mode decomposition and elastic net regression: Evidence in stock market prediction

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ABSTRACT

Accurate modelling of complex, nonlinear and nonstationary datasets remains a critical challenge in predictive analytics. This study introduces a novel variational mode decomposition-elastic net regression (VMD-Enet) framework that combines VMD with ENet to enhance prediction accuracy and interpretability. VMD first decomposes signals into intrinsic mode functions (IMFs), effectively denoising data and improving feature representation. ENet is then applied to select the most significant predictors while managing multicollinearity. The proposed approaches are evaluated using numerical simulations and real stock market data. The proposed VMD-ENet model demonstrated superior performance over the other methods. In the case of the stock market experimental analysis, VMD-ENet achieved the lowest errors, with RSS = 88.90, RMSE = 0.837, MAE = 0.668, $R^2 = 0.73$ and WQE = 0.0006. Compared to other regularization approaches, VMD-ENet significantly identifies key predictors without arbitrarily discarding correlated variables, ensuring model stability. These findings highlight the framework's robustness, interpretability and predictive superiority, making it a promising tool for financial market analysis and broader applications in complex data modelling.

1. Introduction

Estimating the prediction model is crucial across various fields of science, including medicine, economics, engineering, finance and the environment. Numerous statistical methods have been developed to detect the relationships between predictors and response variables. It is important that the model is estimated correctly to make more accurate decisions, identify patterns and extract meaningful insights from original or complex datasets.

In addition, signal processing is a crucial technique in nonstationary and nonlinear time series analysis, enabling researchers to filter noise, detect trends and extract meaningful patterns from data while keeping the characteristics of these data. There are several common signal processing techniques, such as Fourier decomposition [1], wavelet decomposition [2] and empirical mode decomposition [3]. These methods improve the accuracy of predictive models and enhance decision-making. On the other hand, to accurately predict a time series dataset, traditional regression models assume that predictors should be free from multicollinearity. Multicollinearity occurs when two or more predictors are highly correlated [4,5]; in addition, the time series variables are typically stationary [6]. Therefore, it is crucial to assess and address issues related to multicollinearity and nonstationarity when modelling the relationship.

To enhance the model's performance, we propose a new approach that integrates variational mode decomposition (VMD) [7] and elastic net regression (ENet) [8]. This proposed approach can address the challenges of nonstationarity and multicollinearity. VMD is employed for signal decomposition to effectively denoise the original data. It decomposes nonstationary and nonlinear signals into intrinsic mode functions (IMFs), each corresponding to a distinct frequency component, thereby ensuring stationarity. ENet mitigates multicollinearity and facilitates variable selection, which significantly influences the response variable.

On the other hand, the rapid globalization of financial markets has led to increasingly stronger linkages among stock markets and a higher degree of cross-country co-movement. Understanding the relationships among major stock market indices is important for investors and other financial sectors [9]. Financial stock markets are gradually becoming more intercorrelated on a global scale. The correlation of the stock market indices refers to a state that has a synchronized co-movement

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that leads to similar trends [10,11]. Stock markets are interconnected among some countries, based on financial integration, international investor participation, technological development and the elimination of cross-border capital movement barriers in most nations [12].

In the last few decades, emerging stock markets have grown remarkably in some countries and have established markets for their implications for global market integration [13]. One of the most integrated stock markets is the United States, while China, Japan and India are considered emerging markets. In 2023, the stock markets of Japan, China, the United States and India exhibited a complex web of interrelationships, influenced by geopolitical events, policy decisions and investor sentiments [14]. China, Japan and India hold significant economic positions in the Asian region, providing consistent economic growth in recent years. The United States is one of the largest economies in the world, Japan is the largest economic power player in Asia with its technologically advanced market and China has emerged as a dominant force in global finance and trade [15]. However, India is rapidly growing as a global economic power. Emerging economies, such as India, are highly linked and contagious to other global markets, with the United States playing a dominant role [16]. In the case of the Asian financial markets, it is often thought that, while the United States exerts its influence through funding cost, portfolio rebalancing and risk appetite channels, China affects these economies mainly through trade linkages [17]. Trade relations among these economies are intercorrelated with financial markets. During the period from late 2020 to early 2024, in emerging market indices, the weight of Chinese stocks decreased from 43 % to 35 %, but the weight of Indian stocks increased from 8 % to 28 %. These changes in emerging market indices highlight India's growing prominence in emerging markets and diversification away from the heavy weighting of Chinese stocks [16].

The contribution of this research is twofold: it proposes a new statistical machine learning approach by combining the advantage of VMD with ENet to enhance prediction accuracy and interpretability. However, in the application aspect, the proposed method is used for important economic–financial analyses of the stock market. It provides new evidence on assessing and estimating the co-movement and correlations among the stock markets of India with the three global markets – Japan, China and the United States. India's stock market is rising as an emerging market, and its market is becoming more integrated into global financial systems. It has been influenced by both domestic reforms and global financial trends, such as in leading Asian countries (China and Japan) and developed regions such as the United States. The importance of predicting stock market behaviour is for investors at the local or global level. The changes in stock markets are very important, as well as their relationships, especially in terms of trade, economic development and global financial markets. Investors are seeking better returns in these economies, making it essential to understand stock return forecasting and the transmission of volatility among global stock markets.

This paper is organized as follows. After the introduction, Section 2 provides the relevant literature. Thereafter, Section 3 explains the proposed VMD-ENet approach. Section 4 illustrates the data and application, as well as the results and discussion, and Section 5 presents the conclusion.

2. Literature review

Numerous studies have examined the co-movement and interdependence of global stock prices across foreign financial markets [10,18,19]. Some studies [20,21] have explained the causal linkage between the Indian stock market and Asian countries, while other studies have included the global effect of the market such as Baele et al. [22] discussed the increasing integration of financial markets, including stock markets, into the global economy. They analysed how countries in the Eurozone are interconnected and suggested methodologies for measuring financial integration, which can be applied to the relationships among major economies, such as the United States, China, Japan and India [22]. Bekaert et al. [23] examined the relationships between emerging markets (such as India and China) and developed markets (such as the United States and Japan). The authors discuss how financial markets become integrated over time, with contagion effects (i.e. how a financial shock in one market can spread to others) as a key feature of stock market interdependencies [23].

A few research studies have linked both Asian and global markets led by the US stock market. Lobo et al. [12] examined the co-integration between the stock market indices of India, China and the United States by applying a fractionally integrated vector error correction model. The results showed that the US stock market was highly correlated with the Indian stock market [12]. Sher et al. [24] studied the integration dynamics between the Chinese stock market and its major developed counterparts: Japan, the United Kingdom and the United States. Using a comprehensive analytical approach from 2012 to 2022, Granger causality tests revealed bidirectional causality, emphasizing mutual influence. Notably, no causal link exists between the United States and China, possibly due to regulatory disparities and trade wars. This study enhances the understanding of Chinese stock market dynamics, supporting global economic intertwining and urging the further openness of China's domestic shares for economic growth.

Kumar and Srividya [25] studied the cointegration and causality among the stock markets of India, the United States and Japan since the subprime crisis period by applying Johansen's co-integration test and Granger causality. The findings support the existence of cointegration among the Indian, US and Japanese stock markets [25]. In addition, there is unidirectional causality between the Japanese and US stock markets. Caporale et al. [14] examined stock market integration among the United States, China and other ASEAN countries from 2002 to 2020 using the cointegration method. The findings show that there is no cointegration between the United States and China. Ahmad et al. [26] examined the intercorrelation and causal relationship among the stock market index in the United States, Japan and India during the daily period from 1999 to 2004 by using the Johansen co-integration test and the Granger causality test. The results revealed that there is no long-term relationship between the Indian equity market and that of the US and Japanese equity markets. Furthermore, the US and Japanese stock markets had a very weak relationship in 2002–2004.

Kumar and Sharma [27] explored the long-term and short-term interdependence and cointegration of stock markets in NIKKEI Japan, SENSEX India and the NASDAQ USA over the period 2012–2022. The results support the fact that the trend of the stock market in both the United States and India moves in a similar flow, as they are highly correlated. Moreover, according to the Granger causality test, the NASDAQ index can predict the SENSEX index but not the NIKKEI index. Furthermore, applying the Granger causality test showed that the NASDAQ index predicted the SENSEX index with high precision, while the situation was not the same for the NIKKEI index. Mishra et al. [13] examined the stock market return between India and four leading Asian countries – China, Japan, Singapore and Hong Kong – and two global economics – the United States and the United Kingdom – by applying the multivariate GARCH-BEKK model, and they take into account the pre- and post-2008 global financial crisis periods. The findings support the fact that the tendency of the Indian stock market flows with the stock market index of both the United States and Hong Kong. Moreover, the results showed a clear distinction in terms of volatility between the Asian stock market and the global economy of the United States and the United Kingdom index.

From these studies, we can summarize that the stock markets of Japan, China, the United States and India are becoming increasingly interconnected due to globalization, economic integration and financial market liberalization. However, there are also notable differences in the degree and nature of these interconnections between Japan and the United States, which have more direct influences on each other than on other markets. China's market is more influenced by domestic policies, and India's market is still developing its relationship with global markets.

3. Methodology

Recent investigations have focused on integrating signal processing techniques with widely recognized statistical regression and forecasting methods. This combination has proven to be highly effective across multiple scientific fields [28].

Additionally, the support vector regression (SVR) method based on VMD has been explored in several studies [29–31]. Furthermore, an SVR model combined with the adaptive chaotic grey wolf optimization algorithm based on VMD has been investigated [32].

Moreover, Carvalho and Filho [33] examined gradient boosting regression models based on VMD. Zhang et al. [34] proposed a self-recurrent SVR with the VMD method, and Bian et al. [35] implemented a weighted multiscale SVR method based on VMD. Furthermore, the kernel ridge regression approach combined with the weighted mean of vectors based on multivariate VMD was proposed by Tao et al. [36]. Similarly, a multi-kernel robust ridge regression based on VMD was developed by Naik et al. [37]. The latest methods in this field, the multi-layer perceptron, SVR, LSTM and gated recurrent unit conjunction with multivariate VMD, have been implemented in the study of Wang et al. [38].

3.1. Variational mode decomposition

Based on the Wiener filter and Hibert transform methods, Dragomiretskiy and Zosso [7] introduced a new signal decomposition method known as the VMD method. This method was developed to address some of the limitations of traditional signal decomposition techniques. The main concept of VMD is to create and solve a variational problem [7]. VMD is nonrecursive such that it does not require iterative processes that can be computationally prone to errors. Therefore, it can decompose the nonlinear and nonstationary dataset or signal into multiple stable band-limited IMFs from the original signal or time series data [39]. It is designed to perform quasi-orthogonal decomposition, ensuring that the components it extracts are nearly independent and can be analysed separately. The method works by analysing signals in the frequency domain; one of the key features of VMD is its ability to decompose signals across multiple scales or frequency bands. VMD is robust to noise and excels at decomposing local features with closely related frequencies [40].

The VMD algorithm simultaneously decomposes a nonstationary and nonlinear signal $x(t)$ into K of the IMF components; each IMF is defined as $\mathtt{C}_k$ [30]. The original signal is completely reconstructed by summing the decomposition modes as follows:

$$x(t) = \sum_{k=1}^{K} \mathtt{C}_k(t) \tag{1}$$

Where $C_k(t)$ is the kth mode and has the amplitude-modulated (A_k) and frequency-modulation (ω_k) signal in equation (1)

$$C_k(t) = A_k(t)\cos(\varphi_k(t)) \tag{2}$$

Where φ_k is the phase of the kth mode. The $\omega_k(t)$ can be calculated as:

$$\omega_k(t) = \frac{\partial \varphi_k(t)}{\partial t} \tag{3}$$

The VMD method decomposes the time series through a constrained variational problem [7,30,41], with the following objective function:

$$\min_{\mathtt{C}_k,\omega_k} = \left\{ \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * \mathtt{C}_k(t) \right] e^{-j\omega_k t} \right\|_2 \right\} \quad ; \quad \sum_k^K \mathtt{C}_k(t) = x(t) \tag{4}$$

Where $\|.\|_2$ is the square and L_2 norm $\delta(t)$ is the Dirac function, * asterisk sign is the convolution operator, ∂_t stands for partial derivative respect to time t, and $j^2 = -1$.

A quadratic penalty term and a Lagrangian multiplier are introduced to convert the constrained optimization problem into an unconstrained one, allowing for the solution of this variational problem equation (4). The augmented Lagrangian is defined as follows:

$$\mathscr{L}(\{\mathtt{C}_k\},\{\omega_k\},\tau) := \rho \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * \mathtt{C}_k(t) \right] e^{-j\omega_k t} \right\|_2 + \left\| x(t) - \sum_k \mathtt{C}_k(t) \right\|_2 + \langle \tau(t), x(t) - \sum_k \mathtt{C}_k(t) \rangle. \tag{5}$$

where $\mathscr{L}$ augmented Lagrangian, ρ is the regularization parameter that represents the variance of the white noise, $\tau(t)$ is of the Lagrangian multiplier, $\langle .,. \rangle$ represents an inner product or expectation value.

The VMD analysis method depends on the selected value of two parameters, K and ρ. If K is too high or too low, the results are inaccurate. A large ρ loses bandwidth information, while a small α leads to repeated information. The Dung beetle optimization algorithm (DBO) method is used to adjust K and α simultaneously [42].

Equation (5) is solved using the Alternating Direction Method of Multipliers. The solution for each $\mathtt{C}_k$, ω_k, and $\tau(\omega)$ modes in the frequency domain can be written as:

$$\widehat{C}_k^{n+1}(\omega) = \frac{\widehat{x}(\omega) - \sum_{i<k} \widehat{C}_i^{n+1}(\omega) + \sum_{i>k} \widehat{C}_i^{n}(\omega) + \frac{\widehat{\tau}^n(\omega)}{2}}{1 - 2\rho(\omega - \omega_k)^2} \tag{6}$$

$$\widehat{\omega}_k^{n+1} = \frac{\int_0^\infty \omega \left| \widehat{C}_k^{n+1}(\omega) \right|^2 d\omega}{\int_0^\infty \left| \widehat{C}_k^{n+1}(\omega) \right|^2 d\omega} \tag{7}$$

$$\widehat{\tau}^{n+1}(\omega) = \widehat{\tau}^n(\omega) + \propto \left(\widehat{x}(\omega) - \sum_k \widehat{C}_k^{n+1}(\omega) \right) \tag{8}$$

Where n is the iteration counter, the superscripts $n+1$ and n indicate the results of the current and previous steps in the iteration process, respectively, and $\propto$ is the noise tolerance $\widehat{\mathtt{C}}_k(\omega), \widehat{x}(\omega),$ and $\widehat{\tau}(\omega)$ are the spectrum Fourier transforms of $\mathtt{C}_k(\omega), x(\omega),$ and $\tau(\omega)$, respectively.

3.2. Penalized regressions

One of the main techniques in statistical modelling that studies the relationships between predictors and regressors is regression. However, the standard ordinary least squares regression (OLS) faces several challenges in some cases, such as overfitting, multicollinearity among predictors and misestimation in high-dimensional data. These challenges can lead to inaccurate estimations of the coefficient and poor predictions. Penalized regression methods overcome these challenges by using an OLS loss function with a penalty term that constrains the size of the coefficients.

The general objective functions as follows form:

$$\widehat{\boldsymbol{\beta}} = argmin_{\beta} RSS + PR(\boldsymbol{\beta}) \tag{9}$$

Where

$$RSS = \sum_{i=1}^{n} (y_i - \widehat{y}_i)^2 \tag{10}$$

$$\widehat{y}_i = \beta_0 + \sum_{j=1}^{p} \beta_j x_{ij} \tag{11}$$

RSS is the residual sum of squares is the sum of the squared differences between the ith response variable y_i and estimated value $\widehat{y}_i$. x_{ij} is ith observation of the jth predictor variable, β_j is the regression coefficient of the jth predictor variable, and $PR(\boldsymbol{\beta})$ is the penalty applied to the coefficients $\boldsymbol{\beta}$. When $PR(\boldsymbol{\beta}) = 0$, the estimation corresponds to the OLS regression for the j-th element $\widehat{\boldsymbol{\beta}}$ is obtained [43].

Elastic Net (ENet) Regression was introduced by Zou and Hastie in 2005, ENet is a hybrid approach combining the Ridge (L_2 penalty) [44] and LASSO (L_1 penalty) methods [45]. The combining of two penalties leads to balancing coefficient shrinkage and variable selection. It is beneficial when dealing with highly correlated predictors. The ENet method is a penalized extension of the OLS approach used to estimate $\widehat{\boldsymbol{\beta}}$ incorporating both L_1 penalty and L_2 penalty regularization penalties to enhance stability and variable selection (Zou and Hastie in 2005) as follows form:

$$\widehat{\boldsymbol{\beta}}^{ENet} = argmin_{\beta} \sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{j=1}^{p} \beta_j x_{ij} \right)^2 + EP_{\lambda} \quad (12)$$

$EP_{\lambda}(\boldsymbol{\beta})$ is the ENet penalty function as follows form

$$EP_{\lambda}(\boldsymbol{\beta}) = \lambda_1 \|\boldsymbol{\beta}\|_1 + \lambda_2 \|\boldsymbol{\beta}\|_2^2 \quad (13)$$

$\|\boldsymbol{\beta}\|_1$ is the $L_1 -$ norm

$$\|\boldsymbol{\beta}\|_1 = \sum_{j=1}^{p} |\beta_j| \quad (14)$$

$\|\boldsymbol{\beta}\|_2^2$ is the $L_2 -$ norm

$$\|\boldsymbol{\beta}\|_2^2 = \sum_{j=1}^{p} (\beta_j)^2 \quad (15)$$

where $\lambda_1$and λ_2 are tuning parameters and $\lambda_1, \lambda_2 > 0$. The values of $\lambda_1$and λ_2 are dependent on the dataset, then they are selected by using K-CV algorithm. By denoting the tuning parameters λ_1, and λ_2 (Zou and Hastie in 2005) as follows:

$$\lambda_1 = 2n\lambda\alpha \quad (16)$$

$$\lambda_2 = n\lambda(1-\alpha) \quad (17)$$

Equation (12) becomes as following form:

$$\widehat{\boldsymbol{\beta}}^{ENet} = argmin_{\beta} \sum_{i=1}^{n} \left(y_i - \beta_0 - \sum_{j=1}^{p} \beta_j x_{ij} \right)^2 + \lambda \left(\alpha \sum_{j=1}^{p} |\beta_j| + \frac{(1-\alpha)}{2} \sum_{j=1}^{p} (\beta_j)^2 \right) \quad (18)$$

where λ and α $(0 < \alpha < 1)$ are the tuning and a regularization parameter, respectively. When $\alpha = 0$, the estimation corresponds to the Ridge regression estimator, while at $\alpha = 1$, it corresponds to the LASSO estimator.

3.3. The proposed VMD-ENet method

The proposed VMD-ENet method is designed to analyse the relationship between decomposed components and their respective significance in influencing the response variable involves decomposing signals into interpretable components, selecting relevant features, and modelling their effects. The methodology follows these key steps.

Step 1: Using the VMD method decompose the original predictors. Extract IMFs from each predictor to capture frequency components. Where Each predictor $x_l(t)$; $l = 1, 2, .., L$ is decomposed into K intrinsic mode functions IMFs $\{IMF_k\}_{k=1}^{K}$. This objective is accomplished by solving the following constrained optimization problem:

$$\min_{IMF_{l,k}, \omega_k} = \left\{ \sum_k \left\| \partial_t \left[\left(\delta(t) + \frac{j}{\pi t} \right) * IMF_{l,k}(t) \right] e^{-j\omega_k t} \right\|_2 \right\} \quad (19)$$

Subject to

$$x_l(t) = \sum_{k=1}^{K} IMF_{l,k}(t)$$

In our study, the K and ρ parameters were selected based on combination of prior knowledge of the data characteristics the approach of the Dung beetle optimization algorithm (DBO) [42]. Sensitivity analysis showed that variations of ± 1 in K had a negligible impact on model accuracy, confirming the robustness of the chosen value.

Thus, we obtain a new feature set consisting of IMFs from original predictors as follows:

$$\begin{aligned} VMD(x_1(t)) &= \{IMF_{1.1}(t), IMF_{1.2}(t), \ldots, IMF_{1.K}(t)\} \\ VMD(x_2(t)) &= \{IMF_{2.1}(t), IMF_{2.2}(t), \ldots, IMF_{2.K}(t)\} \\ &\vdots \\ VMD(x_L(t)) &= \{IMF_{L.1}(t), IMF_{L.2}(t), \ldots, IMF_{L.10}(t)\} \end{aligned} \quad (20)$$

Step 2: Preprocessing of IMFs and the response variable, feature Selection, and Preparing for Elastic Net regression.

i. **Standardization**: Each $IMF_{l.k}(t); l = 1, 2, .., L$ and $k = 1, 2, \ldots, K$ extracted via VMD in step 1 and the response variable $y(t)$ are standardized to have a mean of 0 $(\mu = 0)$ and unit standard deviation $(\sigma = 1)$ as follows form:

$$\widetilde{IMF}_{i.k}(t) = \frac{\left(IMF_{i.k}(t) - \mu_{IMF_{i.k}(t)} \right)}{\sigma_{IMF_{i.k}(t)}} \quad (21)$$

$$\tilde{y} = \frac{(y_i - \mu_y)}{\sigma_y} \quad (22)$$

ii. Use all $\widetilde{IMF}_{i.k}(t)$ components as new predictor variables to explain $\tilde{y}(t)$.

$$\tilde{y}(t) = \sum_{l=1}^{L} \sum_{k=1}^{K} \widetilde{IMF}_{i.k}(t) \beta_{ik} + \varepsilon(t) \quad (23)$$

iii. For each $IMF_{i.k}(t)$ check the **Multicollinearity** via the Variance Inflation Factor (VIF):

$$VIF_{\widetilde{IMF}_{i.k}(t)} = \frac{1}{1 - R^2_{\widetilde{IMF}_{i.k}(t)}} \quad (22)$$

where $R^2_{\widetilde{IMF}_{i.k}(t)}$ is the coefficient of determination.

iv. **Split Data:** Each dataset is divided into two parts: 70 % for training and the remaining 30 % for testing.

$\widetilde{IMF} \in \mathbb{R}^{n*p}$: Matrix of IMF features (n samples, p features)
$\tilde{y} \in \mathbb{R}^n$: Response variable
$n_{train} = \lfloor 0.7 \times n \rfloor$ and $n_{test} = n - ntrain$
where $\lfloor \cdot \rfloor$ denotes the floor function.
Partition the data:

$$\widetilde{IMF}_{train} = \widetilde{IMF}_{1:n_{train}}, \widetilde{IMF}_{test} = \widetilde{IMF}_{n_{train}+1:n}$$

$$\tilde{y}_{train} = \tilde{y}_{1:n_{train}}, \tilde{y}_{test} = \tilde{y}_{n_{train}+1:n}$$

where $\widetilde{IMF}_{train}$, and $\tilde{y}_{train}$ are training data. $\widetilde{IMF}_{test}$ and $\tilde{y}_{test}$ are test data.

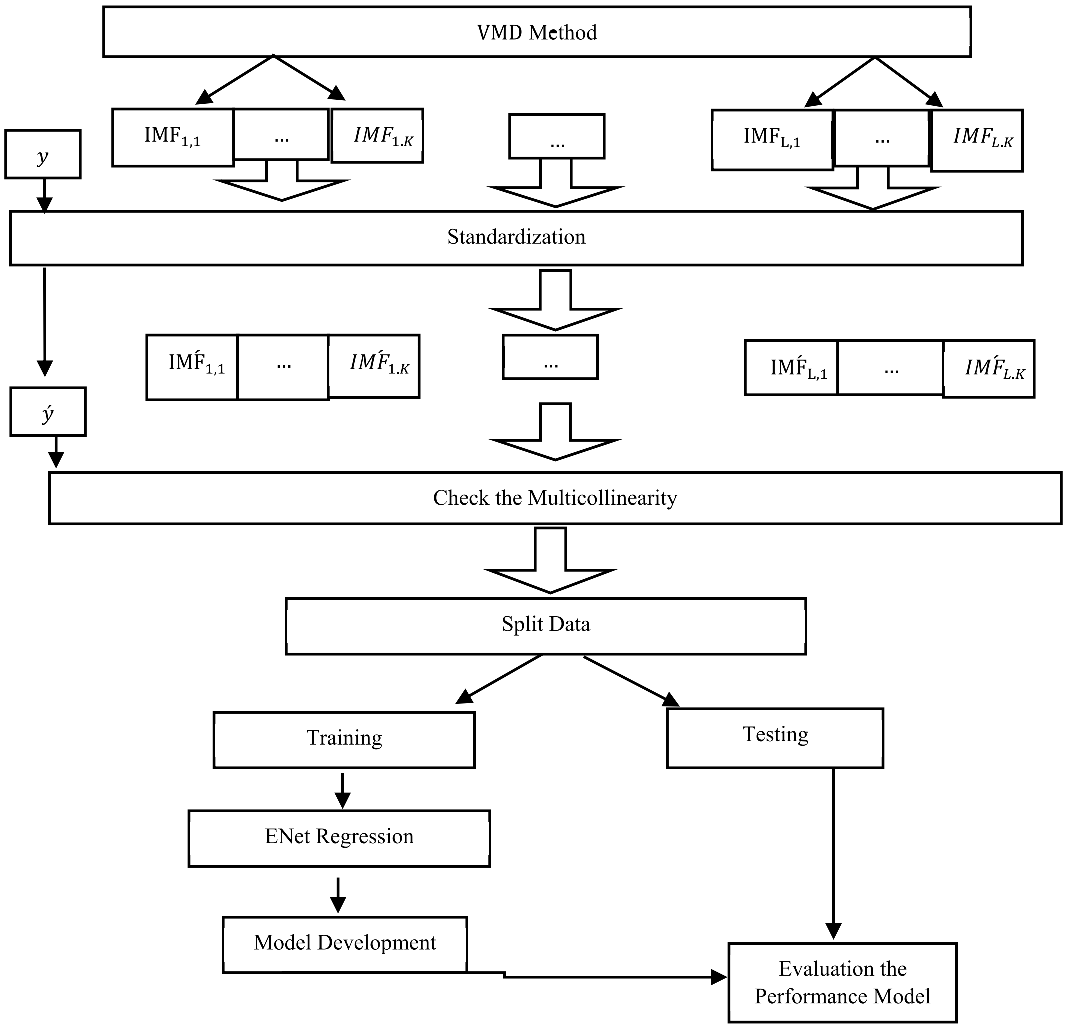

Fig. 1. Flowchart of the proposed VMD-ENet modelling framework.

Step 3: Implement Elastic Net Regression (ENet): Model the relationship between decomposed $IM\acute{F}_{i.k}(t)$ via VMD in the last steps and the response variable $\acute{y}$ via the ENet method as follows steps and forms:

i. The VMD-ENet objective function is defined as follows:

$$\widehat{\beta}^{VMD-ENet} = argmin_{\beta} \sum_{i=1}^{n_{train}} \left(\widetilde{y}_i - \beta_0 - \sum_{l=1}^{L} \sum_{k=1}^{K} \widetilde{IMF}_{i.k}(t)\beta_{l.k} \right)^2 + \lambda(EP_\alpha); \tag{24}$$

$$EP_\alpha = \alpha \left(\sum_{l=1}^{L} \sum_{k=1}^{K} |\beta_{L.k}| \right) + \frac{(1-\alpha)}{2} \left(\sum_{l=1}^{L} \sum_{k=1}^{K} (\beta_{l.k})^2 \right)$$

ii. Using the K - fold cross-validation ($K-CV$) at $K = 10$ method select the optimized hyperparameters (α, λ) to determine the best balance between Ridge and LASSO penalties [46].

iii. Using the training dataset. Fit the VMD-ENet model.

Step 4: Identify the most influential IMFs using the nonzero regression coefficients $(\beta_{l.k} \neq 0)$ from VMD-ENet method.

Step 5: Reconstructed the predictor signals based on the non-zero coefficient regression to interpret relationships as follows:

$$\xi = \{l.k | \beta_{l.k} \neq 0\} \tag{25}$$

where ξ is the set of important IMFs that significantly contribute to y. Let ξ_l be the subset of **selected IMFs** for predictor x_l, such that:

$$\overline{\overline{x}}_{l.Reconst.}(t) = \sum_{k \in \xi_l} \widetilde{IMF}_{i.k}(t) \tag{26}$$

Where $\overline{\overline{x}}_{l.Reconst.}$ is he **reconstructed version** of predictor x_l. $IM\acute{F}_{l.k}(t)$ is the IMFs that have non-zero regression coefficients $(\beta_{l.k} \neq 0)$.

Step 6: Evaluate the Model Performance, by goodness of fit criteria; residual sum of squares (RSS), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), Mean absolute scaled error (MASE), Mean absolute percentage error (MAPE), Theil's **U statistic (TheilU),** coefficient of determination (R^2). Researchers can use the Weighted Quality Evaluation (WQE) index proposed by Ref. [47], which combines RMSE, MAE, MAPE, and R^2 into a single measure, where smaller WQE values indicate better model performance.

Fig. 1 illustrates the main steps of the proposed method. The raw time series $x(t)$ is first decomposed into finite number IMF components using the VMD method. These IMF components and the response variable are standardized, checked for multicollinearity among the components and then split, including the response variable and IMF components, into training and testing sets. The training data are used to build the fitting model with elastic net regression, and the testing data are used to evaluate how well the model performs.

Table 1
The results of the performance criteria in the numerical experiment.

#	Method	*RSS*	*RMSE*	*MAE*	*TheilU*	*MASE*	*MAPE*	R^2	*WQE*
1	OLS	196.14	1.6158	1.207	2.1758	2.6522	29.479	0.16	0.622
2	SVR	260.17	1.8606	1.407	2.327	3.0918	32.511	0.54	0.5823
3	SCAD	196.14	1.6158	1.207	2.1758	2.6522	29.479	0.16	0.6221
4	MCP	196.14	1.6158	1.207	2.1758	2.6522	29.479	0.16	0.6221
5	Ridge	207.36	1.6614	1.2452	2.1889	2.7363	30.000	0.23	0.6165
6	LASSO	197.02	1.6194	1.2099	2.1759	2.6587	29.506	0.17	0.6217
7	adLASSO	197.45	1.6211	1.2113	2.1762	2.6617	29.525	0.17	0.6213
8	ENet	197.06	1.6195	1.21	2.176	2.6589	29.511	0.17	0.6217
9	VMD-OLS	42.434	0.6756	0.4862	3.2976	2.0273	268.89	0.04	0.5952
10	VMD-SVR	50.695	0.8115	0.6275	**1.7824**	2.6165	**162.77**	0.66	0.328
11	VMD-SCAD	29.384	0.5515	0.4301	2.7696	1.7933	230.77	0.38	0.4050
12	VMD-MCP	29.036	0.5465	0.4264	2.7597	1.7781	230.23	0.38	0.4025
13	VMD-Ridge	11.798	0.3836	0.3134	2.4319	1.3067	199.14	0.75	0.1952
14	VMD-LASSO	13.993	0.412	0.3323	2.2535	1.3855	193.53	0.68	0.2218
15	VMD-adLASSO	15.198	0.427	0.3427	2.3147	1.429	185.54	0.71	0.2071
16	VMD-ENet	**10.462**	**0.3624**	**0.2951**	2.2413	**1.2306**	**178.44**	**0.78**	**0.1555**

4. Numerical experiment (simulation)

To demonstrate the effectiveness of the proposed VMD-ENet method, we create a numerical experiment by using sine and cosine functions to generate nonstationary and nonlinear signals in the simulated dataset, including three predictor variables denoted by x_1, x_2 and x_3 , while the white noise errors are generated as $x_l(t)+\varepsilon$; $\varepsilon \sim$ *iid* $N(0,1), l = 1,2,3$. This white noise component ensures variability similar to that in real datasets. However, the response variable (y) is generated based on a combination of one or two predictor components [48]. To have a more reliable experiment, we simulate data for a sample size of $n = 250$, spanning a time sequence from zero to nine. Moreover, 2,000 replications of the dataset were generated to ensure consistency. The functional test formula is as follows:

$$y(t) = 0.5t + sin(\pi t) + sin(2\pi t) + cos(6\pi t) + cos(\pi t)$$

$$x_1(t) = -0.2t + sin(2\pi t) + cos(\pi t) + sin(5\pi t) + sin(9\pi t) + \varepsilon$$

$$x_2(t) = -0.2t + cos(6\pi t) + sin(\pi t) + cos(9\pi t) + \varepsilon$$

$$x_3(t) = -0.7t + sin(\pi t) + cos(7\pi t) + sin(8\pi t) + cos(13\pi t) + \varepsilon$$

Table 1 presents a comparative evaluation of various methods, namely VMD-OLS, VMD-SVR, VMD-SCAD, VMD-MCP, VMD-Ridge, VMD-LASSO, VMD-adLASSO and VMD-ENet. Among the studied methods, and based on the goodness-of-fit criteria, VMD-ENet outperforms the others by achieving the lowest error values across all goodness-of-fit criteria, as highlighted in Table 1. This finding supports

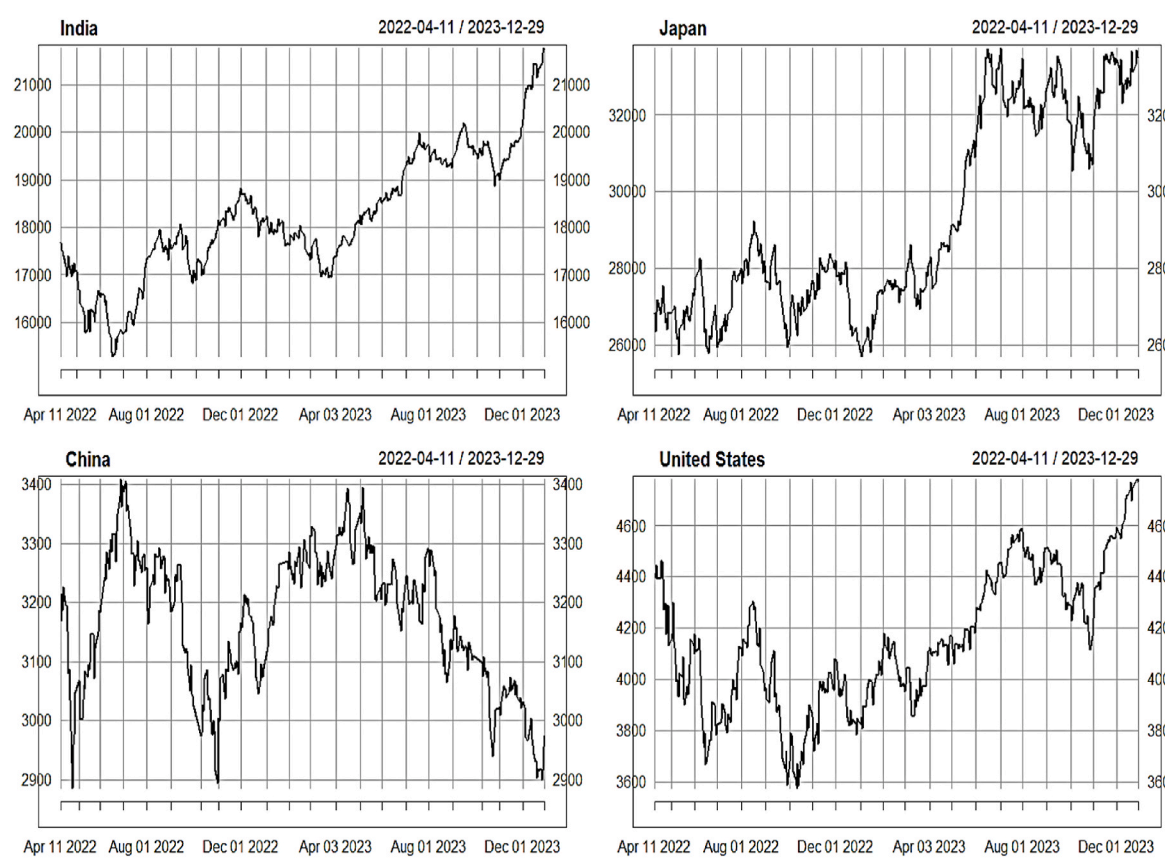

Fig. 2. Plots the observation for each stock market index.

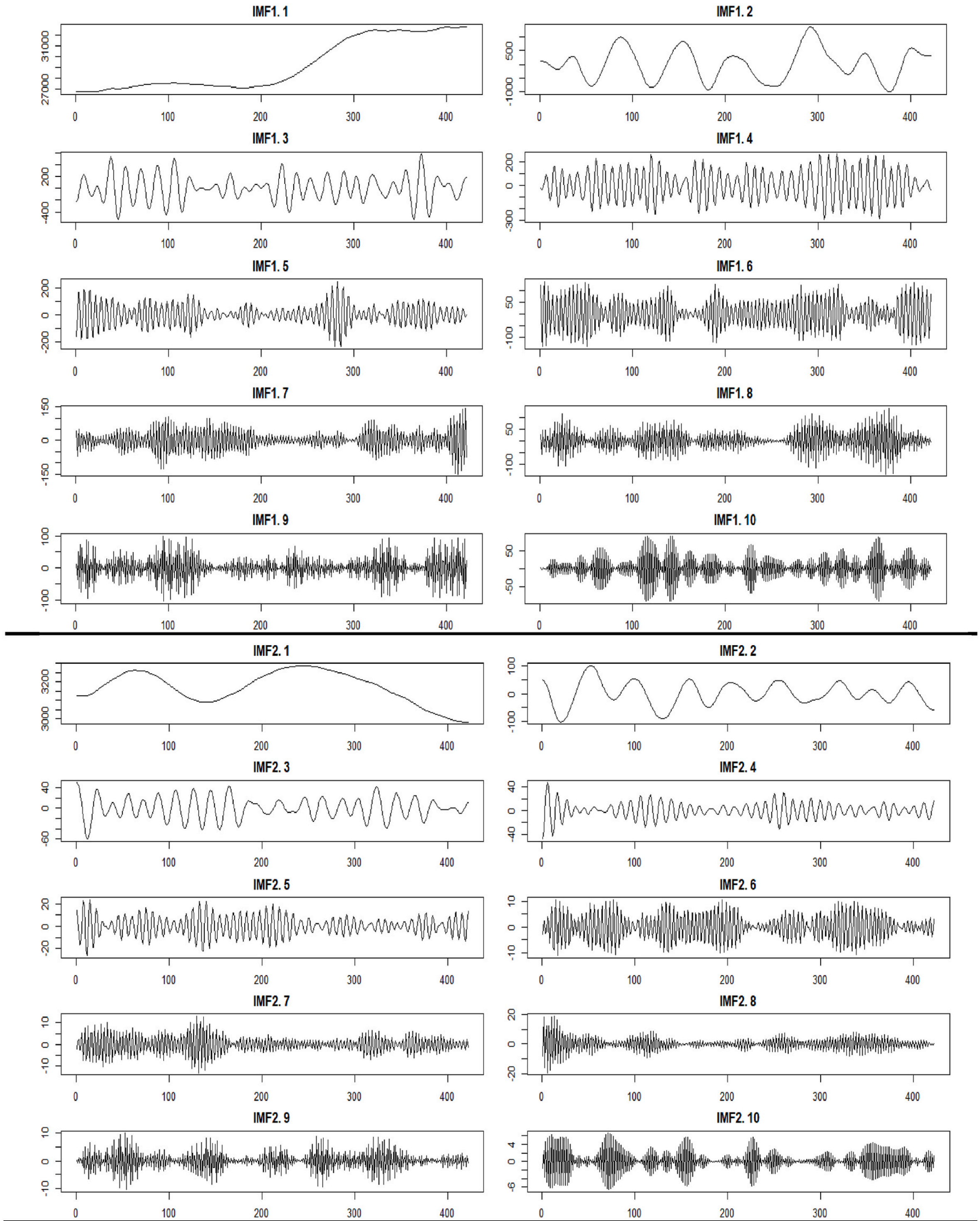

Fig. 3. Decomposition of the stock market data for Japan, China and the United States signals via VMD.

the high prediction accuracy of the proposed VMD-ENet method, followed by VMD-Ridge. Penalty-based methods, such as SCAD, offer moderate performance, while OLS and SVR show the poorest performance. In addition, the proposed VMD-ENet method has the advantage of optimal model selection, particularly in cases requiring high predictive accuracy and low error rates.

5. Stock market application

To evaluate the performance of the proposed VMD-ENet method by using a real dataset, we choose the application related to the daily stock market for four countries. The predictors are the stock markets of Japan, China and the United States, while the stock market of India is used as a response variable over the period from April 11, 2022, to December 29, 2023. All datasets were obtained from the *Yahoo Finance database* [49]. The dataset is divided into two sets: 70 % of the data are used for

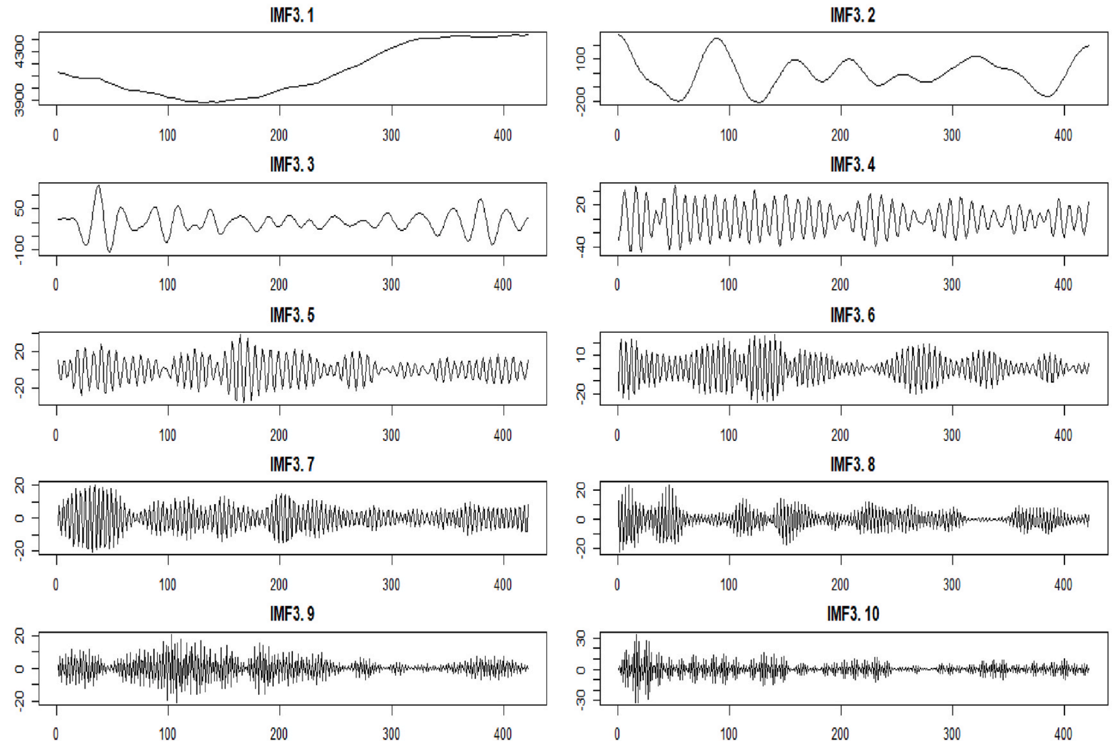

Fig. 3. (*continued*).

Table 2
VIF test.

$\widetilde{IMF}_{1.1}$	$\widetilde{IMF}_{1.2}$	$\widetilde{IMF}_{1.3}$	$\widetilde{IMF}_{1.4}$	$\widetilde{IMF}_{1.5}$	$\widetilde{IMF}_{1.6}$	$\widetilde{IMF}_{1.7}$	$\widetilde{IMF}_{1.8}$	$\widetilde{IMF}_{1.9}$	$\widetilde{IMF}_{1.10}$
10.40	1.58	1.18	1.03	1.05	1.03	1.02	1.07	1.03	1.15
$\widetilde{IMF}_{2.1}$	$\widetilde{IMF}_{2.2}$	$\widetilde{IMF}_{2.3}$	$\widetilde{IMF}_{2.4}$	$\widetilde{IMF}_{2.5}$	$\widetilde{IMF}_{2.6}$	$\widetilde{IMF}_{2.7}$	$\widetilde{IMF}_{2.8}$	$\widetilde{IMF}_{2.9}$	$\widetilde{IMF}_{2.10}$
1.31	1.15	1.08	1.05	1.06	1.06	1.04	1.02	1.02	1.13
$\widetilde{IMF}_{3.1}$	$\widetilde{IMF}_{3.2}$	$\widetilde{IMF}_{3.3}$	$\widetilde{IMF}_{3.4}$	$\widetilde{IMF}_{3.5}$	$\widetilde{IMF}_{3.6}$	$\widetilde{IMF}_{3.7}$	$\widetilde{IMF}_{3.8}$	$\widetilde{IMF}_{3.9}$	$\widetilde{IMF}_{3.10}$
10.18	1.57	1.13	1.03	1.05	1.06	1.04	1.03	1.07	1.02

training the model, while the remaining 30 % of the data are used to evaluate or test the model. Fig. 2 provides plots of the daily closing stock market indices for India, China, Japan and the United States. The plots provide clear market trends, volatility and interconnections among the four countries' indices. The existence of nonstationary and nonlinear patterns of the four stock market indices is notable. Therefore, it is important to understand trends and the co-integration of stock indices. Moreover, the result of the Granger causality test determined that movements in three stock markets predicted movements in the Indian stock market. The findings will provide insights into market integration and the potential for predictive modelling in financial analysis for investors.

6. Results and discussion

Fig. 3 provides the decomposition by using the VMD algorithm of the nonstationary and nonlinear data of the three stock market indices: Japan, China and the United States. This algorithm decomposes each original signal of the predictor into 10 IMFs, each characterized by distinct properties, such as frequency and wavelength. The *high-frequency* components capture short-term volatility driven by market fluctuations and speculative trading, while the *low-frequency* components reflect broader market trends and structural changes influenced by macroeconomic factors.

Table 2 presents the variance inflation factor (VIF) test results for the IMFs. This analysis evaluates the degree of multicollinearity among the new IMF predictors. The results indicate that $\acute{IMF}_{1.1}$ ($VIF = 10.40$) and $\acute{IMF}_{3.1}$ ($VIF = 10.18$) have high multicollinearity, where ($VIF > 10$). In contrast, the remaining predictors show low to moderate VIF values. To enhance model stability, the ENet method is integrated into the proposed method to address multicollinearity.

Fig. 4 illustrates how the model's error (MSE) varies with different values of α in the ENet. The MSE is lowest when $\alpha = 0.04$, which means the model is doing the best when it behaves mostly similar to ridge regression, whereas as α increases towards 1, that is, more like LASSO, the MSE gets slightly higher. At $\alpha = 0.04$, the ENet model gives the most accurate results for these data.

Fig. 5 shows the D-fold cross-validated (D-CV) estimate at $D = 10$ of the MSE as the log (λ) function for the Enet regression at $\alpha = 0.04$. The first vertical dotted line from the right represents the location of the minimum MSE, while the second vertical dotted line represents the location of the point chosen at the minimum *MSE* with the one-standard-error rule. It has a significant and strong effect on the predictors; thus, the final mode will be empty, unlike the first line, which has 17 predictors in the final model. In this study, the selection of the λ value is based on the optimal minimum *MSE* value, which is the first dotted line.

Table 3 compares the variable selection and estimated coefficients across the studied models; each has specific tuning parameters that have been selected based on the $10 - CV$ algorithm. The OLS-SVR has most of the variables in the model. The VMD-Ridge shrinkage regression has all the predictors, whereas the VMD-MCP, VMD-SCAD, VMD-LASSO and

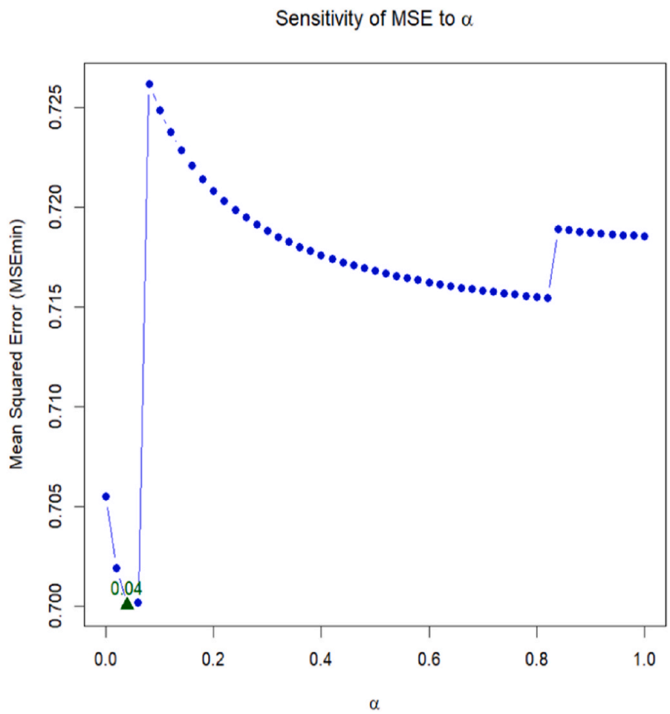

Fig. 4. Variation of MSE with elastic net parameter α

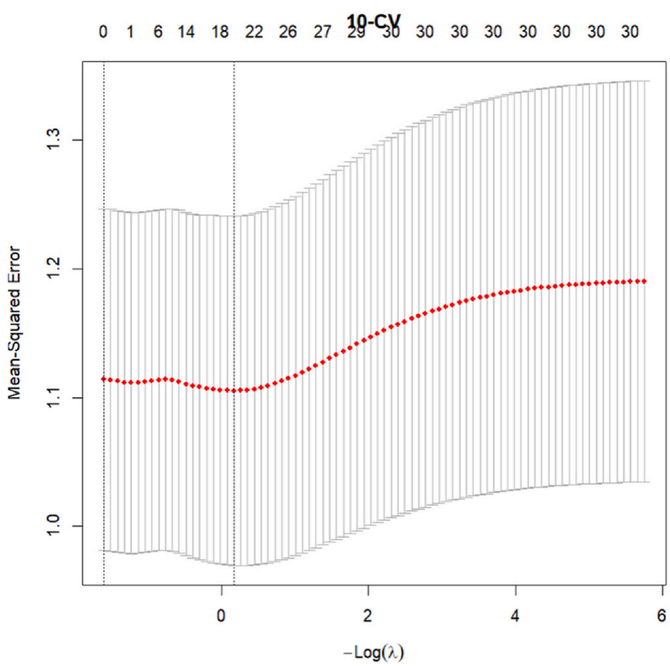

Fig. 5. 10-CV estimation of the mean squared error as the $log(\lambda)$ at $\alpha = 0.04$.

VMD-adLASSO eliminate many coefficients, retaining only one or two variables in the estimated model. However, VMD-ENet has the most significant feature in the model, and it is particularly effective in handling multicollinearity and ensuring that correlated variables are not arbitrarily removed, such as $\acute{IMF}_{1.1}$, which make it an ideal model among the studied approaches that tackle multicollinearity and offer both feature selection and model stability.

From Table 3, many of the IMF components from x_1 (Japan) and x_3 (the United States) have relatively great positive coefficients ($\widetilde{IMF}_{1.4}$, $\widetilde{IMF}_{1.6}, \widetilde{IMF}_{3.4}, \widetilde{IMF}_{3.5}$ *and* $\widetilde{IMF}_{3.8}$), which means that the frequency from medium to low modes of these stock markets holds predictive strength for India's stock market returns. These components typically reflect long-term structural trends and macroeconomic factors; that is, India's stock market co-shifts with these economies. In contrast, the coefficients of IMF components for x_2 (Chine) are generally smaller and mixed in sign, reflecting lower and minus stable moving from the Chinese to the Indian market.

Table 4 and Fig. 6 provide a performance comparison of the studied models. The comparison is based on RSS, RMSE, MAE, MASE and Theil's U-statistic, with each model tuned using specific regularization parameters. Among the methods, VMD-ENet ($\lambda = 0.845833, \propto = 0.04$) achieves the lowest error criteria, as highlighted in Table 4. Again, we can see that the proposed VMD-ENet method is the most effective model in terms of predictive accuracy and its ability to handle multicollinearity. VMD-SVR shows strong predictive performance but falls short of regularization-based models, such as VMD-ENet. The remaining regularization methods perform well but lag behind ENet. VMD-OLS, lacking regularization, has the highest errors, underscoring the need for feature selection and shrinkage techniques.

Fig. 7 illustrates the reconstruction of predictor signals $\overline{\overline{x}}_{l.Reconst.}; l = 1, 2, 3$ based on the most significant features selected through the ENet regression method. The process involves decomposing the original predictor signals into IMFs using the VMD method and retaining only those with nonzero regression coefficients, which are then summed to form the final reconstructed predictors. This approach ensures that only meaningful components are preserved, reducing noise and improving interpretability. As shown in Fig. 7, the reconstructed predictors provide a clearer representation of the essential features influencing the dependent variable, making this method particularly useful in scenarios with multicollinearity. By visualizing the reconstructed predictors, Fig. 7 highlights key patterns and relationships within the data, enhancing model performance and interpretation.

As shown in Fig. 7, the Indian market grew strong in 2023 after a period of variation, while the reconstructed trends in Japan and the US market were upward. This signals that long-term modifications in these developed markets have a strong effect on India's market performance, which matches the results of ENet in Table 3, as the largest coefficients for Japan and the US components provide the most useful information for predicting India's market.

In contrast, the reconstructed components for China are more stable, with smaller and mixed coefficients, which indicates that the impact on the Indian market is weak. Overall, in our study, the results indicate that India's stock market is more affected by long-term global trends in developed economies, such as Japan and the US markets, than by short-term fluctuations.

7. Conclusion

This research proposed an integrated method of VMD and ENet regression for signal processing to enhance predictive modelling. VMD effectively decomposes complex, nonstationary and nonlinear signals into IMFs, enhancing feature extraction and noise reduction. Moreover, ENet regression improves model robustness by addressing multicollinearity and feature selection challenges. To demonstrate the performance of the proposed VMD-ENet method, we applied it in numerical experiments and real data applications. The application about predicting the Indian stock market by three predictors: stock markets of Japan, China and the United States. The VMD method was applied to the three predictor variables, extracting their IMFs to capture essential market patterns and reduce noise. These refined IMFs were then used as new predictors in the ENet regression model to enhance predictive accuracy by tackling multicollinearity, identifying the most significant decomposition components for the response variable and improving interpretability. The findings of both numerical experiments and real data applications demonstrated that the proposed VMD-ENet method

Table 3
Variable selection and estimated coefficients.

	VMD-OLS	VMD-SVR	VMD-Ridge	VMD-MCP	VMD-SCAD	VMD-LASSO	VMD-adLASSO	VMD-ENet
β_0	−0.16	0.06	−0.04	−0.04	−0.03	−0.04	−0.04	−0.02
x_1								
$\widetilde{IMF}_{1.1}$	0.27	5.11	0.05	0	0	0	0	0.05
$\widetilde{IMF}_{1.2}$	0.06	4.57	0.01	0	0	0	0	0
$\widetilde{IMF}_{1.3}$	0.07	5.79	0.02	0	0	0	0	0.01
$\widetilde{IMF}_{1.4}$	0.24	15.80	0.08	−0.04	0.06	0.05	0.15	0.10
$\widetilde{IMF}_{1.5}$	−0.10	−9.43	−0.03	0	0	0	0	−0.03
$\widetilde{IMF}_{1.6}$	0.14	9.26	0.05	0	0	0	0.04	0.06
$\widetilde{IMF}_{1.7}$	0.04	11.31	0.01	0	0	0	0	0
$\widetilde{IMF}_{1.8}$	0.01	5.76	0.02	0	0	0	0	0
$\widetilde{IMF}_{1.9}$	0.04	8.88	0.02	0	0	0	0	0.01
$\widetilde{IMF}_{1.10}$	0.04	1.15	0.01	0	0	0	0	0
x_2								
$\widetilde{IMF}_{2.1}$	0.16	6.01	0.03	0	0	0	0	0.01
$\widetilde{IMF}_{2.2}$	−0.03	5.25	0.01	0	0	0	0	0.00
$\widetilde{IMF}_{2.3}$	−0.02	0.44	0.01	0	0	0	0	0.00
$\widetilde{IMF}_{2.4}$	−0.12	−3.98	−0.03	0	0	0	0	−0.03
$\widetilde{IMF}_{2.5}$	0.04	1.74	0.01	0	0	0	0	0.00
$\widetilde{IMF}_{2.6}$	0.06	5.87	0.03	0	0	0	0	0.02
$\widetilde{IMF}_{2.7}$	0.09	5.14	0.03	0	0	0	0	0.03
$\widetilde{IMF}_{2.8}$	0.01	−1.73	0.01	0	0	0	0	0.00
$\widetilde{IMF}_{2.9}$	−0.07	−7.97	−0.02	0	0	0	0	−0.02
$\widetilde{IMF}_{2.10}$	0.10	10.48	0.03	0	0	0	0	0.03
x_3								
$\widetilde{IMF}_{3.1}$	−0.37	−8.66	−0.03	0	0	0	0	0
$\widetilde{IMF}_{3.2}$	−0.03	−4.63	−0.01	0	0	0	0	0
$\widetilde{IMF}_{3.3}$	−0.04	−0.03	−0.01	0	0	0	0	0
$\widetilde{IMF}_{3.4}$	0.08	7.84	0.03	0	0	0	0	0.03
$\widetilde{IMF}_{3.5}$	0.07	1.33	0.03	0	0	0	0	0.03
$\widetilde{IMF}_{3.6}$	0.03	−0.80	0.01	0	0	0	0	0
$\widetilde{IMF}_{3.7}$	−0.01	−3.21	0.01	0	0	0	0	0
$\widetilde{IMF}_{3.8}$	0.10	10.97	0.03	0	0	0	0	0.04
$\widetilde{IMF}_{3.9}$	0.07	6.28	0.02	0	0	0	0	0.02
$\widetilde{IMF}_{3.10}$	−0.09	−12.75	−0.03	0	0	0	0	−0.04

Table 4
Comparison of different methods.

#	Method	Tuning Parameters	*RSS*	*RMSE*	*MAE*	*TheilU*	*MASE*	*MAPE*	R^2	*WQE*
1	OLS	–	173518.0	116.888	90.661	2.157	0.783	5.333	0.03	0.7807
2	SVR	$\epsilon = 0.1, c = 1, \Gamma = 0.033$	194872.0	123.872	97.712	1.283	0.843	16.240	0.09	0.9720
3	SCAD	$\lambda = 9.436629, \Gamma = 3.7$	175240.0	117.466	91.107	1.838	0.787	5.333	0.02	0.7864
4	MCP	$\lambda = 10.84983, \Gamma = 3$	175303.0	117.488	90.889	2.014	0.785	5.333	0.02	0.7860
5	Ridge	$\lambda = 5399.986, \propto = 0$	181152.0	119.431	94.141	1.115	0.813	6.092	0.02	0.8110
6	LASSO	$\lambda = 8.598304, \propto = 1$	175223.0	117.461	90.950	1.943	0.785	5.433	0.02	0.7876
7	adLASSO	$\lambda = 6.319821, \propto = 1$	176065.0	117.743	90.860	2.105	0.785	5.437	0.01	0.7896
8	ENet	$\lambda = 14.82466, \propto = 0.58$	175210.0	117.456	90.998	1.903	0.786	5.419	0.02	0.7875
9	VMD-OLS	–	130.063	1.012	0.844	1.240	1.016	1.341	0.72	0.0075
10	VMD-SVR	$\epsilon = 0.1, c = 1, \Gamma = 0.033$	98.217	0.879	0.686	1.129	0.826	8.459	0.59	0.1702
11	VMD-SCAD	$\lambda = 0.1397997, \Gamma = 3.7$	90.911	0.846	0.674	0.991	0.811	1.258	0.71	0.0089
12	VMD-MCP	$\lambda = 0.1499026, \Gamma = 3$	90.506	0.844	0.674	0.982	0.811	1.257	0.71	0.0089
13	VMD-Ridge	$\lambda = 1.891557, \propto = 0$	89.589	0.840	0.671	0.925	0.807	1.414	0.63	0.0386
14	VMD-LASSO	$\lambda = 0.1499026, \propto = 1$	91.255	0.847	0.674	0.999	0.812	1.265	0.71	0.0086
15	VMD-adLASSO	$\lambda = 0.5878023, \propto = 1$	90.412	0.844	0.682	0.959	0.822	1.546	0.57	0.0630
16	VMD-ENet	$\lambda = 0.845833, \propto = 0.04$	**88.897**	**0.834**	**0.669**	**0.827**	**0.805**	1.296	**0.73**	**0.0006**

outperformed the other studied models based on goodness-of-fit criteria.

Furthermore, the findings of the application indicate that this integrated approach enhances stock market modelling by capturing significant features from global markets that influence the Indian stock market. It contributes to the understanding of the movement of stock markets in India according to Japan, China and the United States and highlights their interdependence and cointegration, emphasizing the need for a global perspective when analysing and managing financial assets. Understanding market linkages helps investors and policymakers plan properly and have the actual effects of economic policy changes in India based on other economics. This information is essential in time series analysis.

CRediT authorship contribution statement

Abdullah S. Al-Jawarneh: Writing – original draft, Visualization, Software, Methodology, Investigation, Formal analysis, Conceptualization. **Ahmed R.M. Alsayed:** Writing – original draft, Software,

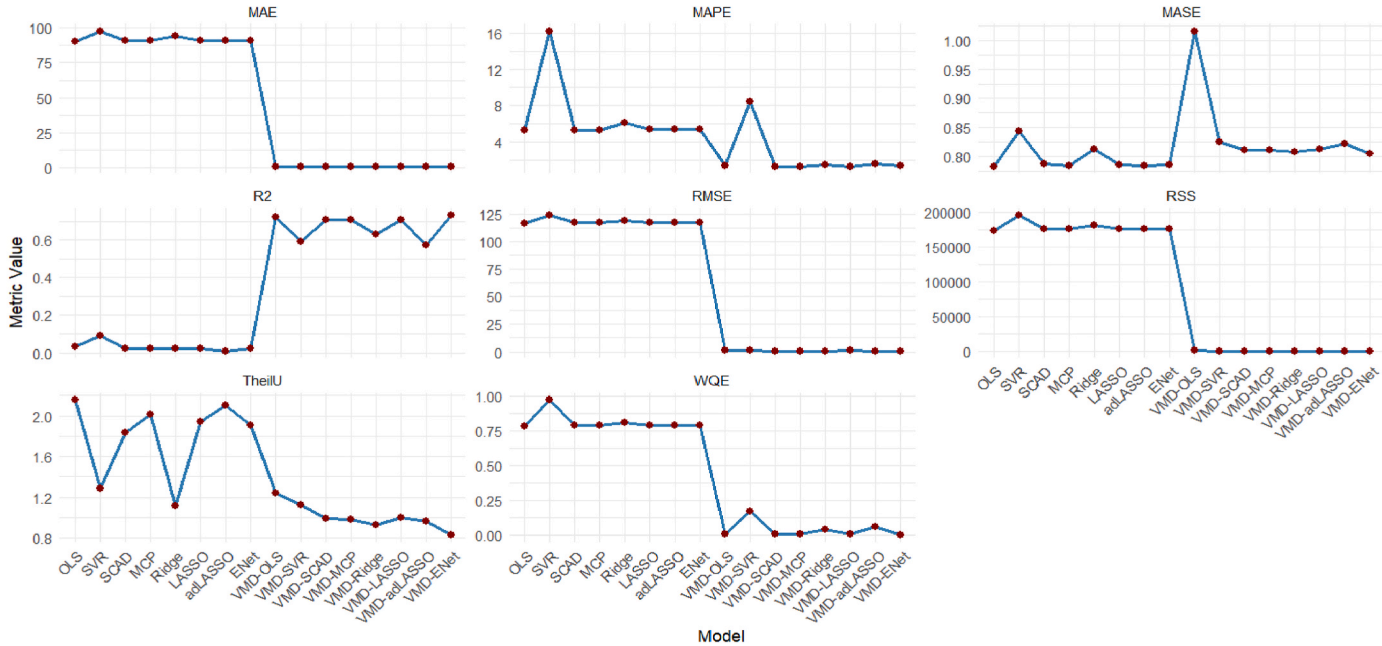

Fig. 6. Performance comparison of estimated models.

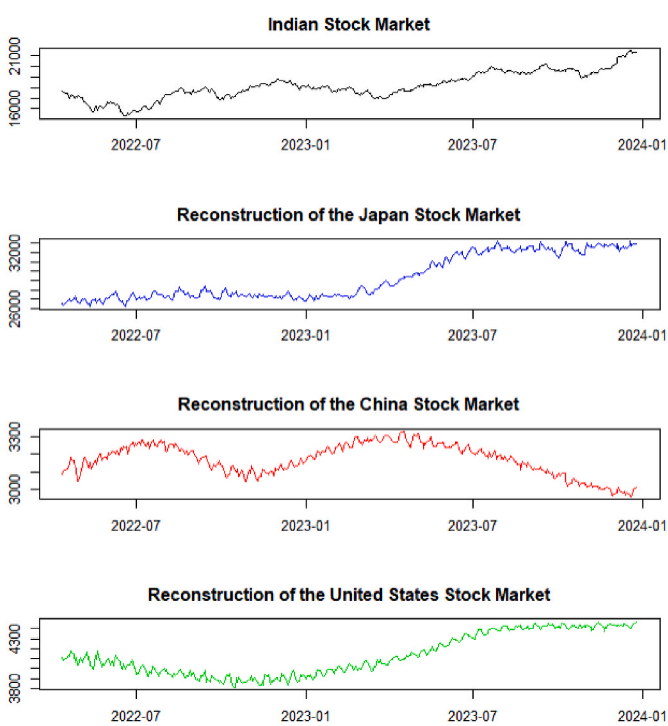

Fig. 7. Indian Stock Market vs. Reconstruction Predictors of the Stock Market.

Methodology, Investigation, Formal analysis, Conceptualization. **Heba N. Ayyoub:** Writing – original draft, Software, Methodology, Investigation, Formal analysis.

Consent for publication

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Data availability

we have attached the link of the data source in the manuscript

References

[1] Titchmarsh EC. Introduction to the theory of Fourier integrals, vol. 950. Clarendon Press Oxford; 1948.
[2] Chui C. Wavelet basics. Kulwer Academic Publishers; 1995.
[3] Huang NE. Introduction to the Hilbert–Huang transform and its related mathematical problems. In: In Hilbert Huang transform and its applications. World Scientific; 2014. p. 1–26.
[4] Jim F. Multicollinearity in regression analysis: problems, detection, and solutions. 2017.
[5] Hussein E, Al-Qudah Y, Hamadameen AO, Al-Obaidi RH, Al-Jawarneh AS, Al-Sharqi F, Owledat A. Matrices and correlation coefficient for possibility interval-valued neutrosophic hypersoft sets and their applications in real-life. Int J Neurotrosophic Sci 2025;26(1):254–65.
[6] Paolella MS. Linear models and time-series analysis: regression, ANOVA, ARMA and GARCH. John Wiley & Sons; 2018.
[7] Dragomiretskiy K, Zosso D. Variational mode decomposition. IEEE Trans Signal Process 2013;62(3):531–44.
[8] Zou H, Hastie T. Regularization and variable selection via the elastic net. J Roy Stat Soc B 2005;67(2):301–20.
[9] Shakil R, Islam S, Shohan YA, Mia A, Rajbongshi A, Rahman MH, Akter B. Addressing agricultural challenges: an identification of best feature selection technique for dragon fruit disease recognition. Array 2023;20:100326.
[10] Alsayed AR. Turkish stock market from pandemic to Russian invasion, evidence from developed machine learning algorithm. Comput Econ 2023;62(3):1107–23.
[11] Liu Y, Huang X, Xiong L, Chang R, Wang W, Chen L. Stock price prediction with attentive temporal convolution-based generative adversarial network. Array 2025: 100374.
[12] Lobo BJ, Wong W-K, Chen H. Links between the Indian, U.S. and Chinese stock markets. Departmental Working Papers wp0602, Links between the Indian, US And Chinese Stock Markets, Departmental Working Papers wp0602, National University of Singapore, Department of Economics, Singapore 2016:1–27.

[13] Mishra AK, Agrawal S, Patwa JA. Return and volatility spillover between India and leading Asian and global equity markets: an empirical analysis. J Econ Finance Admin Sci 2022;27(54):294–312.
[14] Caporale GM, Gil-Alana LA, You K. Stock market linkages between the ASEAN countries, China and the US: a fractional integration/cointegration approach. Emerg Mark Finance Trade 2022;58(5):1502–14.
[15] Nakayama J, Yokouchi D. Relationship between Japanese stock market behavior and category-based news. Risks 2025;13(3):50.
[16] Samarakoon LP. Stock market interdependence, contagion, and the U.S. financial crisis: the case of emerging and Frontier markets. J Int Financ Mark Inst Money 2011;21(5):724–42.
[17] Salisu AA, Gupta R. Commodity risk and forecastability of international stock returns: the role of oil returns skewness. Risks 2025;13(3):49.
[18] Manzi G, Alsayed A, Martins H, Saibene G. Regional economic convergence in federation contexts: a comparative analysis of Brazil and the European Union. Regional Studies 2023;57(11):2269–84.
[19] Bhuiyan MSM, Rafi MA, Rodrigues GN, Mir MNH, Ishraq A, Mridha MF, Shin J. Deep learning for algorithmic trading: a systematic review of predictive models and optimization strategies. Array 2025:100390.
[20] He F, Liu-Chen B, Meng X, et al. Price discovery and spillover dynamics in the Chinese stock index futures market: a natural experiment on trading volume restriction. Quant Finance 2020;20(12):2067–83.
[21] Stawiarski B. Granger causality and cointegration during stock bubbles and market crashes. Workshop Nonstationary Syst Appl 2021:93–107.
[22] Baele L, Ferrando A, Hördahl P, Krylova E, Monnet C. Measuring financial integration in the euro area (No. 14). ECB occasional paper; 2004.
[23] Bekaert G, Harvey CR. Market integration and contagion. 2003.
[24] Sher A, Haizhong A, Khan MK, Sági J. Chinese stock market integration with developed world: a portfolio diversification analysis. Heliyon 2024;10(9):e29413.
[25] Kumar AS, Srividya V. Cointegration among the stock markets of India, USA and Japan. Int J Econ Res 2014;11(3).
[26] Ahmad KM, Ashraf S, Ahmed S. Is the Indian stock market integrated with the US and Japanese markets? An empirical analysis. South Asia Econ J 2005;6(2): 193–206.
[27] Kumar JP, Sharma NM. The interdependence and cointegration of stock markets: evidence from Japan, India and USA. Stat J IAOS 2024;40(2):435–47.
[28] Alsayed A, Manzi G. A validation forecast using robust estimators into environmental application. Int J Energy Environ Econ 2018;26(3):183–95.
[29] Chu J, Dong Y, Han X, Xie J, Xu X, Xie G. Short-term prediction of urban PM 2.5 based on a hybrid modified variational mode decomposition and support vector regression model. Environ Sci Pollut Control Ser 2021;28:56–72.
[30] Zuo G, Luo J, Wang N, Lian Y, He X. Two-stage variational mode decomposition and support vector regression for streamflow forecasting. Hydrol Earth Syst Sci 2020;24(11):5491–518.
[31] Wang X, Yu Q, Yang Y. Short-term wind speed forecasting using variational mode decomposition and support vector regression. J Intell Fuzzy Syst 2018;34(6): 3811–20.
[32] Zhang Z, Hong WC. Application of variational mode decomposition and chaotic grey wolf optimizer with support vector regression for forecasting electric loads. Knowl Base Syst 2021;228:107297.
[33] Carvalho TMN, de Assis de Souza Filho F. Variational mode decomposition hybridized with gradient boost regression for seasonal forecast of residential water demand. Water Resour Manag 2021;35(10):3431–45.
[34] Zhang Z, Hong WC, Li J. Electric load forecasting by hybrid self-recurrent support vector regression model with variational mode decomposition and improved cuckoo search algorithm. IEEE Access 2020;8:14642–58.
[35] Bian X, Wu D, Zhang K, Liu P, Shi H, Tan X, Wang Z. Variational mode decomposition weighted multiscale support vector regression for spectral determination of rapeseed oil and rhizoma alpiniae offcinarum adulterants. Biosensors 2022;12(8):586.
[36] Tao H, Ahmadianfar I, Goliatt L, Kazmi SSUH, Yassin MA, Oudah AY, Yaseen ZM. PM2. 5 concentration forecasting: development of integrated multivariate variational mode decomposition with kernel Ridge regression and weighted mean of vectors optimization. Atmos Pollut Res 2024;15(6):102125.
[37] Naik J, Dash PK, Dhar S. A multi-objective wind speed and wind power prediction interval forecasting using variational modes decomposition based Multi-kernel robust ridge regression. Renew Energy 2019;136:701–31.
[38] Wang Z, Chen L, Chen H, ur Rehman N. Monthly ship price forecasting based on multivariate variational mode decomposition. Eng Appl Artif Intell 2023;125: 106698.
[39] Liu H, Chen C. Data processing strategies in wind energy forecasting models and applications: a comprehensive review. Appl Energy 2019;249:392–408.
[40] Chen Y, Hu X, Zhang L. A review of ultra-short-term forecasting of wind power based on data decomposition-forecasting technology combination model. Energy Rep 2022;8:14200–19.
[41] Danandeh Mehr A, Reihanifar M, Alee MM, Vazifehkhah Ghaffari MA, Safari MJS, Mohammadi B. VMD-GP: a new evolutionary explicit model for meteorological drought prediction at ungauged catchments. Water 2023;15(15):2686. https://doi.org/10.3390/W15152686.
[42] Xu DM, Li Z, Wang WC. An ensemble model for monthly runoff prediction using least squares support vector machine based on variational modal decomposition with dung beetle optimization algorithm and error correction strategy. J Hydrol 2024;629:130558.
[43] Montgomery DC, Peck EA, Vining GG. Introduction to linear regression analysis, vol.821. John Wiley & Sons; 2012.
[44] Hoerl AE, Kennard RW. Ridge regression: biased estimation for nonorthogonal problems. Technometrics 1970;12(1):55–67.
[45] Tibshirani R. Regression shrinkage and selection via the lasso. J Roy Stat Soc B 1996;58(1):267–88.
[46] Al-Jawarneh AS, Alsayed AR, Ayyoub HN, Ismail MT, Sek SK, Ariç KH, Manzi G. Enhancing model selection by obtaining optimal tuning parameters in elastic-net quantile regression, application to crude oil prices. J Risk Financ Manag 2024;17 (8):323.
[47] Zhou Y, He X, Montillet JP, Wang S, Hu S, Sun X, Ma X. An improved ICEEMDAN-MPA-GRU model for GNSS height time series prediction with weighted quality evaluation index. GPS Solut 2025;29(3):113.
[48] Al-Jawarneh AS, Ismail MT, Awajan AM, Alsayed AR. Improving accuracy models using elastic net regression approach based on empirical mode decomposition. Commun Stat Simulat Comput 2022;51(7):4006–25.
[49] Yahoo. https://finance.yahoo.com/; 2024 (access, 2025).