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ORIGINAL ARTICLE

Mean–trend risk portfolio selection with non-dominated sorting asset preselection

David Neděla[1] · **Sergio Ortobelli**[2] · **Tomáš Tichý**[1]

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Abstract

In the vast landscape of financial markets, identifying potential investment assets such as stocks can be overwhelming and time-consuming. For portfolio managers, focusing on a specific selection of stocks through an effective filtering process can streamline this task. This paper introduces an efficient stock preselection method using multidimensional non-dominated sorting of selected return statistics. Unlike previous research, our approach leverages statistics derived from approximated return series through nonparametric regression and principal component analysis (PCA). We further explore the impact of this preselection on mean-variance and the newly proposed mean-trend risk large-scale portfolio selection strategies. By examining the efficient frontier of portfolios from various return and risk perspectives, our empirical analysis on US stock market data provides both ex-post and ex-ante results for 40 portfolio strategies. The findings suggest that for most risk-averse investors, mean-trend risk strategies with preselection significantly outperform both the same strategies without preselection and traditional mean-variance strategies.

Keywords Asset preselection · Large-scale optimization · Non-dominated sorting · Portfolio selection process · Mean–trend risk

Introduction

Traditionally, both individual and institutional investors have regarded prudent stock selection as a fundamental task in portfolio management. In general, given the vast number and different types of assets in the worldwide market, the portfolio selection task has evolved into a large-scale problem. As a consequence, investors place emphasis on a comprehensive analysis of assets, leading to filtering out the improper ones at a given time. Simultaneously, they choose the appropriate model to compile an optimal portfolio according to personal preferences.

This paper significantly enhances the portfolio selection literature by presenting a new and complex portfolio selection strategy. The first aim of this paper is to propose a strategy combining the return approximation and asset preselection processes, followed by the application of a mean-variability optimization approach (Markowitz 1952). For preselection, we use a non-dominated sorting method on the vector of asset parameters, based on chosen key indicators, statistics, or measures of the assets. Non-dominated sorting is a method used to rank units (e.g. stocks, funds, commodities) in multi-objective decision-making. A unit is "non-dominated" only if no other unit is better in all considered parameters. In contrast, a "dominated" unit is a unit that is worse or equal in all parameters compared to another unit and strictly worse in at least one parameter[1]. Inspired by the work of Juszczuk et al. (2022), the parameters considered of the asset return series are: the mean return, variance,

✉ David Neděla
david.nedela@vsb.cz

Sergio Ortobelli
sergio.ortobelli@unibg.it

Tomáš Tichý
tomas.tichy@vsb.cz

1 Department of Finance, VSB – Technical University of Ostrava, 17. listopadu 2172/15, 70800 Ostrava, Czech Republic

2 Department of Management, University of Bergamo, Via dei Caniana 2, 24127 Bergamo, Italy

[1] If a given unit is equal to another unit in some parameters, smaller in one or more parameters, and larger in one parameter, it belongs to neither the non-dominated or the dominated set.

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and aggregate pairwise covariance, enhanced by the Rachev ratio (RR).[2]

The innovation of this paper lies in combining the preselection and approximation processes, followed by the optimization part. In particular, we first approximate stock returns, compute their statistics and perform a preselection based on approximated statistics, in contrast to existing research (see, for instance, Juszczuk et al. 2022; Hosseinzadeh et al. 2023). The main merit of this framework is its dynamic perspective of particular parts that capture the temporal evolution of statistics over time. This fact can be appreciated from a practical perspective, not only by investors but also by researchers.

Addressing the challenge of large-scale optimization problems, when the number of assets becomes too large, it can be difficult to quickly find the optimum due to the fully dense covariance matrix. As noted by Hosseinzadeh et al. (2023), it's crucial to balance the number of observations with an acceptable statistical approximation by reducing the dimensionality of the portfolio selection problem. Individuals can apply two methods to reduce portfolio dimensionality. The first method is approximating the data (returns) using a factor model (Chen and Yuan 2016), and the second is an asset preselection process. It's crucial to emphasize that both methods should be applied simultaneously.

For the return approximation, we employ nonparametric polynomial locally weighted least squares regression (Ruppert and Wand 1994) based on a dynamic set of factors derived from principal component analysis (PCA). The benefit of including this approach in the complex portfolio optimization process lies in capturing nonlinear dependency in asset returns, improving the accuracy of estimation without relying on restrictive parametric assumptions, and higher portfolio performance (Ortobelli et al. 2019). Following the recent concept of Neděla et al. (2024), we employ the trend-dependent correlation to PCA to obtain the main factors of portfolio variability. This correlation is a dynamic measure that uses cumulative returns and compares deviations from the linear trend instead of its mean; see Ruttiens (2013). In this approach, we estimate (change) the trend at each time based on a historical rolling window of returns, which represents an adaptive process. Empirically, this type of correlation matrix has more appropriate properties for the application of PCA due to the higher explanatory power of the main principal components. This allows us to work with a lower number of factors while considering a different type of risk.

Historically, the most famous portfolio selection approach is Markowitz's mean-variability concept, which is regarded as the bedrock of the decision-making foundations of modern finance; see Markowitz (1952). Originally, the mean return and the variance of returns are considered. According to this approach, investors distribute their capital to the portfolio, which minimizes the variance for a given mean return. Thus, we can create an efficient frontier of varying risk portfolios. Following Markowitz's work, the standard mean–variance model has undergone many modifications to eliminate its shortcomings, i.e., the single period model, normally distributed returns, using variance as a risk measure, etc.; see, among others, Konno and Yamazaki (1991), Rockafellar and Uryasev (2002), Li et al. (2010), and Kalayci et al. (2019). Very recently, Neděla et al. (2024) introduced a mean–trend risk alternative to the mean–variance model that includes the trend-dependent covariance matrix. Generally, simple covariance uses the deviations of returns from their respective mean values. In contrast, trend-dependent covariance uses the deviations between the cumulative returns and the zero-volatile (linear) line, leading to the same value of final cumulative returns; see Fig. 1.

The background of trend-dependent correlation and covariance matrices is identical[3]. This new portfolio selection model leads to improvements in ex-post portfolio statistics. The rationale for this evidence is that trend-dependent risk represents a separate group of risk measures relying on the shape of the trend over time.[4]

For this reason, the second aim is to study the effect of the proposed preselection procedure based on the whole efficient frontier of mean–variability portfolio strategies. To justify the choice of this approach, we also compare the results with those of the classical mean–variance approach. It is worth recalling that recently, several studies have paid attention to the preselection of assets prior to the portfolio construction process; see, e.g., Chang et al. (2009), Juszczuk et al. (2022), Hosseinzadeh et al. (2023), and the

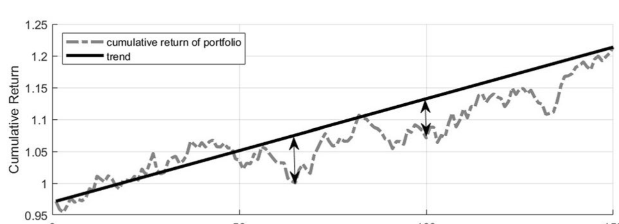

Fig. 1 Principle of trend-dependent risk and dependency measures

[2] The Rachev ratio measures how much potential profit a portfolio has compared to its worst possible losses expressed by Conditional Value-at-Risk Biglova et al. (2004). The mathematical formulation is provided further in the text in Equation (5).

[3] Note that if we change the order of the input data in the covariance calculation, the resulting value does not change. For trend measures, it is the order (representing time) that is important as it affects the cumulative returns and subsequent deviations from trend.

[4] Note that trend-dependent risk simultaneously reflects time-dependent risk, capturing an additional source of portfolio risk. This concept was originally proposed by Ruttiens (2013).

references therein. For investors, the proper selection of assets should be an important issue within the investment process, but it is very challenging to resolve. This process is highly influenced by the characteristics of market data, which are volatile in the long term and very difficult to predict for the future (Huang 2012). For example, Chang et al. (2009) conducted an analysis of portfolio optimization that accounts for different risk measures using different amounts of assets and found that portfolios with fewer assets outperform portfolios with more assets.

Subsequently, this paper presents an empirical analysis applied to US stock market data, offering ex-post and ex-ante evaluations of 40 distinct portfolio strategies situated along the mean–variance and mean–trend risk efficient frontiers. The first (last) strategy is the global minimum risk (maximum return) strategy, and between them are strategies that minimize risk for a given expected return set with an equidistant interval. The main findings of our analysis indicate that, for most risk-averse investors, the mean–trend risk model outperforms the classical mean–variance model. Remarkably, when the preselection process precedes the optimization step, the main benefit lies in a significant boost in the performance of individual portfolio strategies.

The remainder of this paper is structured as follows. In Section 2, we describe the methodology of the return approximation using PCA and nonparametric regression. Section 3 introduces the asset preselection method we use based on non-dominated sorting. Section 4 describes a portfolio optimization framework. In Section 5, we characterize the dataset and show the results of this study, along with an explanation of the results. The conclusions are summarized in Section 6.

Return approximation technique

This methodological section presents the description and formulation of essential aspects of the return approximation technique using nonparametric regressions and PCA.

First, assume that a portfolio consists of z risky assets characterized by a vector of log-returns $r = [r_1, r_2, \dots, r_z]$, thus, $Z = \{1, \dots, z\}$ is the set of risky assets. If a vector of asset weights is denoted as $x = [x_1, x_2, \dots, x_z]$, we denote the portfolio as $x'r$. Assuming a short-sale prohibition, then $x_i \geq 0$. To compute the log-return of the i-th asset for a particular moment t, we use the following equation:

$$r_{i,t} = \ln\left(\frac{P_{i,t}}{P_{i,t-1}}\right), \tag{1}$$

where $P_{t,i}$ is the adjusted closing price of the i-th asset at time t, for $t = 1, \dots, T$. We use log-returns due to the higher preference for them among researchers.

Since we use statistical parameter estimation in portfolio optimization, such as dependency measures, a larger sample of data can lead to greater estimation error. In this context, nonparametric statistical techniques have become quite popular among researchers and investment managers, as stated by (Ortobelli et al. 2019).

For the return approximation process using PCA, we define the set of z correlated return series $\{r_i\}_{i=1}^{z}$ by regressing them on several uncorrelated principal components $\{f_i\}_{i=1}^{z}$. Thus, we can say that each r_i is a function of f_i[5]. Since the aim is to reduce the dimensionality of the problem, we select only the first κ components (factors) from the PCA, which, on average, capture a sufficient portion of the portfolio variability.

As stated in Ortobelli and Tichý (2015), a frequently used approach for approximating returns from factors is classical linear regression. However, based on its inappropriate application to non-normally distributed data series, the literature recommends rather using a nonparametric regression approach. For example, the appropriate nonparametric regression approach is proposed by Ruppert and Wand (1994), using the locally weighted least squares method. Recently, this approach was applied to the return approximation process by Ortobelli et al. (2019). In this approach, conditional expectations and a multivariate kernel estimator are employed. Generally, if we obtain the matrix of factors F by performing PCA on a dependency matrix, the general formulation of the nonparametric regression is as follows:

$$\hat{r} = E(r \mid F = f) + \varepsilon = m(f) + \varepsilon, \tag{2}$$

where F represents the matrix with $f = (f_1, \dots, f_\kappa)$ vectors of κ uncorrelated factors (usually $\kappa < z$) and ε is the residual part of the estimation. To determine the function *m*(*f*), we can use several methods proposed in the literature, such as the locally weighted least squares regression introduced by Ruppert and Wand (1994), which is referred to as the RW estimator[6].

[5] Generally, when PCA is conducted on a broad and diversified asset universe, the extracted components primarily capture pervasive market-wide sources of common variation that are more generalizable across asset groups. In contrast, applying PCA within a narrower or sector-specific universe often produces factors that better reflect localized co-movements and risk structures, thereby improving explanatory power within that subset but limiting transferability beyond it.

[6] Alternatively, the estimator of Nadaraya (1964) and Watson (1964), or the Gasser–Müller kernel estimator (Gasser and Müller 1984), can be used.

The main issue in estimating the regression function *m*(*f*), which is basically unknown, lies in estimating the parameter *a* of the following minimization problem:

$$\min_{a,b} \sum_{i=1}^{T} [\hat{r}_i - a - b^T (f_i - f)]^2 K_H (f_i - f), \tag{3}$$

where *f* is the vector of factors with the *i*-th observation f_i and $K_H(\cdot)$ is a multivariate kernel estimator with a $\kappa \times \kappa$ symmetric positive definite matrix *H*. With the help of the weighted least squares matrix method, Ruppert and Wand (1994) presented the identification of the leading bias and variance terms for universal multivariate kernel weights. A more detailed derivation of the estimation process and the mathematical expression for finding the parameters *a* and *b* can be found in Ortobelli et al. (2019).

According to Hall and Kang (2005), the right choice of bandwidth is essential for the performance of the smoothed regression function. One type of bandwidth is presented by Scott (2015), who designed the rule-incorporating variance–covariance bandwidth $H = diag(h_1, \ldots, h_\kappa)$, which is formulated as follows:

$$\text{Scottx2019;s rule in } \mathbb{R}^\kappa : \hat{h}_i = \hat{\sigma}_i T^{-1/(\kappa+4)}, i = 1, \ldots, \kappa, \tag{4}$$

where $\hat{\sigma}_i$ represents the estimated standard deviation of the *i*-th factor f_i, and *T* is the length of the matrix of observations. Scott's bandwidth rule is used further in this paper. The task of this approach is to minimize the mean integrated squared error (MISE) obtained from the estimation. The analysis of bandwidth selection is also examined, for example, by Borrajo et al. (2017).

Non-dominated sorting asset preselection process

In this section, we characterize the approach for asset preselection inspired by the work of Juszczuk et al. (2022). Non-dominated sorting is a ranking method used to find Pareto-dominance relations as well as a Pareto set with respect to selected statistics. It can be used for multi-objective decision-making or optimization with a higher number of objectives than 1.

In this paper, the preselection process builds on the approximated characteristics of the stock returns and their dependence. In particular, we assume the covariance matrix of approximated returns $\hat{\Sigma}$ as well as the vector of mean approximated returns μ, potentially affect the contribution of individual assets to portfolio statistics, i.e., the mean return and variance, respectively. According to the logical concept of the mean–variance model, the higher mean return μ_i of the *i*-th asset represents its possibly higher impact on the mean return of a portfolio. Similarly, the lower value of the approximated variance of the *i*-th asset $\hat{\sigma}_i^2$, among other things, reflects its positive effect on the total variance of a portfolio.

Regarding the dependency between assets, the covariance between the *i*-th and *j*-th approximated return series $\hat{\sigma}_{i,j}$ for $i \neq j$ also ultimately affects the overall risk of the portfolio. It expresses the pairwise contributions of assets to the portfolio variance. Since each pair of assets has its covariance, Juszczuk et al. (2022) assumed the aggregated pairwise contribution indicator of the individual asset, which we denote by $\tau_i = \sum_{j \in Z \setminus \{i\}} \hat{\sigma}_{i,j}$. In other words, it represents the sum of covariances for an individual asset. Changes in this proxy measure τ_i of the asset *i* have the same effect on portfolio variance, similar to $\hat{\sigma}_i^2$. This means that the lower τ_i is preferred.

Finally, according to the literature focused on asset preselection, eliminating assets with a lower Rachev ratio (RR) ultimately contributes to higher portfolio performance (Ortobelli and Tichý 2015). Due to its features, we should not ignore this performance indicator for evaluation since it examines the proportion of the tails of the asset return distribution. Generally, the RR is defined as the ratio between the expected tail return and the expected tail loss for a given threshold. Mathematically, RR for the *i*-th asset is formulated as follows:

$$RR_i = \frac{CVaR_\beta(r_b - r_i)}{CVaR_\alpha(r_i - r_b)}, \tag{5}$$

where r_b is a vector of risk-free return, and α and $\beta \in [0, 1]$ are significance levels in the CVaR calculation; see Biglova et al. (2004) and Rachev et al. (2008). The general formulation of the CVaR is given by

$$CVaR_\alpha(X) = \frac{1}{\alpha} \int_0^\alpha VaR_q(X) dq, \tag{6}$$

where

$$VaR_q(X) = -F_X^{-1}(q) = -\inf\{x \mid Pr(X \geq x) > q\}, \tag{7}$$

where $F_X^{-1}(q)$ is the inverse distribution function, see Artzner et al. (1999).

Using this methodology, it is straightforward to identify several fundamental but important parameters for each asset *i* that can be individually evaluated to express the suitability of the asset for the portfolio composition at a given time.

To preserve the explanatory power of the individual parameters, we use them separately for non-dominated sorting.

The Pareto dominance principle typically serves to sort alternatives in the population for non-dominated sorting. It is a very useful approach for the decision-making process based on multi-objective evolution. Likewise, using the concept of stochastic dominance (one-dimensional ordering), we can describe the non-dominated sorting as multidimensional. Generally, it states that an individual *A* dominates the individual *B* if and only if each parameter of *A* is not worse than the same parameter of *B* and at least one parameter of *A* is better than that parameter of *B*; see Bao et al. (2017). Comparing the set of individuals *Z*, all non-dominated individuals from the first phase are associated with the first non-dominated subset P_1. Then, all non-dominated individuals are removed from the initial set *Z*, and other non-dominated individuals from $Z \setminus P_1$ create the subset P_2. This process is repeated until no individual remains and we find the last subset P_l. In particular, each subset represents a single layer of the process. In general, considering only the first layer P_1, we perform the most rigorous preselection of assets, while when incorporating all layers P_l, we work with the initial set of assets. The decision-maker can decide how many subsets (layers) is accepted for his asset selection.

For the purpose of the analysis, we use a three- or four-dimensional (hereinafter referred to as 3D or 4D) vector for each asset $i = 1, \dots z$ composed of positive or negative values of parameters, that is, $(\mu_i, -\hat{\sigma}_i^2, -\tau_i)$ or $(\mu_i, -\hat{\sigma}_i^2, -\tau_i, RR_i)$, respectively. Hereafter in the text, we refer to each preselection strategy as, e.g., 3D for preselection involving three parameters $(\mu_i, -\hat{\sigma}_i^2, -\tau_i)$ with a single-layer procedure, and we use similar terms for the 4D variant.

Mean–trend risk portfolio selection

The most well-known model for portfolio selection is the Markowitz mean–variance model (Markowitz 1952). In the computation process, we work with the vector of expected mean returns of asset $\bar{r} = (\bar{r}_1, \dots, \bar{r}_z)$, where $\bar{r}_i = E(r_i)$ for $i = 1, \dots, z$, and the covariance matrix of the series of returns $\Sigma = (\sigma_{i,j})_{z \times z}$, where $\sigma_{i,j} = E[(r_i - \bar{r}_i)(r_j - \bar{r}_j)]$. According to this concept, we can distinguish two optimization frameworks, namely, minimizing the portfolio variance for a given mean expected return or maximizing the mean expected return. The general variance minimization framework is formulated as follows:

$$\begin{aligned} &\min_x \; x'\Sigma x \\ &\quad x'\bar{r} = S_M \\ &\quad x'\varphi = 1 \\ &\quad x_i \geq 0; i = 1, \dots, z, \end{aligned} \tag{8}$$

where φ is a *z*-dimensional vector with ones, and S_M is a predefined (required) value of the expected return of the portfolio.

As already mentioned, the Markowitz approach employs a classical asset covariance matrix and uses variance as a proxy of risk. This has been frequently criticized by many researchers and practitioners, and variance has been replaced by other indicators with more appropriate properties for asset risk measurement (Artzner et al. 1999; Rockafellar and Uryasev 2002; Guo et al. 2019).

For this reason, the literature is rich in various modifications of the variance, such as that proposed by Ruttiens (2013). There, the author presented the trend-dependent version of the variance (especially the standard deviations) to eliminate the main shortcomings, which are the static principle and the use of the simple mean for setting the deviations. In particular, the modification aimed to incorporate the effect of time in these risk measures, as well as other statistics by considering cumulative returns and required non-volatile trajectory. Generally, we assume that investor prefers to achieve their final cumulative wealth with stable returns, minimizing large fluctuations and ensuring consistent performance over time. Recently, similar to the classical covariance matrix, Neděla et al. (2024) proposed the application of the trend-dependent alternative in portfolio risk measurement. This covariance matrix is based on the trend-dependent (time-dependent) concept of risk measurement initially presented by Ruttiens (2013). The core of this concept lies in using cumulative return series and measuring deviations from the linear trends, described in Fig. 1. Such a trend connects the initial value of cumulative returns and its final value; see the mathematical representation below. Apart from that, we can derive dependency measures from this kind of risk indicator. Similarly to the classical covariance matrix Σ, we denote the trend-dependent matrix as $\Sigma^{TD} = (\nu_{i,j}^{TD})_{z \times z}$, where $\nu_{i,j}^{TD}$ is the trend-dependent covariance between the *i*-th and *j*-th return spread series. Mathematically, $\nu_{i,j}^{TD}$ is calculated using the concept of cumulative returns and their equally accrued returns, and it is given by

$$\nu_{i,j}^{TD} = E[(c_i - e_i)(c_j - e_j)], \tag{9}$$

where c_i is the cumulative return of asset *i* with the *t*-th observation, and it is calculated as $c_{i,t} = c_{i,t-1}(1 + r_{i,t})$, for $t = 1, \dots, T$, e_i represents the equally accrued return with the

t-th observation and it is given by $e_{i,t} = c_{i,0} + \frac{t}{T}(c_{i,T} - c_{i,0})$ for $c_{i,0} = 1$, which is the initial value of the cumulative return. In graphical form, e_i is the linear line from $c_{i,0}$ to $c_{i,T}$. A more detailed explanation with a graphical background can be found in Ruttiens (2013) and Neděla et al. (2024).

Following the concept of trend-dependent covariance, we can modify the classical mean–variance model (8) to create its mean–trend risk version by changing the covariance matrix to the trend-dependent one (Σ^{TD}), which is defined as follows:

$$\begin{aligned}\min_x\ & x'\Sigma^{TD}x \\ & x'\bar{r} = S_M \\ & x'\varphi = 1 \\ & x_i \geq 0; i = 1, \ldots, z,\end{aligned} \tag{10}$$

where the conditions of the model remain identical to those of the model (8). Note that this modification preserves the fact that it is still a convex quadratic programming problem like the original one.

Empirical analysis on US market data

An analysis is performed on US stock market data. In particular, the initial dataset consists of the daily historical adjusted close prices of selected stocks that were components of the S&P 500 index from January 3, 2005, to June 30, 2021 (approximately 4300 observations). Some assets of the index were eliminated due to an insufficiently long time series of data, meaning that they were not publicly traded on the stock exchange for the entire period analyzed. For this reason, the database contains only 374 assets, which still form a sufficient sample[7]. The components of the index were identified in early November 2021 from the Bloomberg database, and in this analysis, we do not cover changes in the index composition. Table 1 shows the general statistics of all asset series.[8]

The statistics of assets show that in some return series, we can find extreme values (increase of approximately 70%, decrease of approximately 94%). All of these have a significant impact on the final portfolio. In addition, we perform statistical tests, i.e. the Jarque-Bera and Kolmogorov-Smirnov tests, in order to examine if asset return series follow a normal distribution based on a 95% confidence level. From the observed results, the normality of the time series is rejected in all cases, with one exception.

Since we present the effect of preselection on the whole efficient frontier of portfolios, we compute statistics for 40 portfolios that differ in the level of the mean expected return. The first portfolio strategy is known as a global minimum variance portfolio. Moreover, we assume that each portfolio is re-calibrated monthly (every 21 days), considering a one-year (252-day) rolling window of approximated returns. Short sales of stocks are prohibited for all of these strategies. To give more realistic results, we also include the impact of transaction costs, which are set as 20 basis points.

To provide a clear illustration, our proposed strategy, including the portfolio selection process with or without preselection, can be summed up by the following consecutive steps:

Step 1 Apply the PCA to the trend-dependent covariance matrix Σ^{TD} to obtain the κ factors (principal components) among all available assets' historical returns. By setting the parameter κ individually for each re-calibration, we aim to explain at least 85% of the total trend-dependent portfolio variability. In other words, κ is a dynamic parameter that varies from re-calibration time to re-calibration time. Then, approximate each return series $\hat{r_i}$ by applying the nonparametric RW estimator (3) presented in Section 2 to the set of κ principal components.

Step 2 For a complex strategy with the preselection process, compute the selected statistics for each asset using approximated returns from Step 1, that is, $(\mu_i, -\hat{\sigma}_i^2, -\tau_i)$ for the 3D vector or $(\mu_i, -\hat{\sigma}_i^2, -\tau_i, RR_i)$ for the 4D vector. Based on the non-dominated sorting approach described in Section 3, find all subsets with non-dominated assets $P_1, \ldots, P_l$.
If the preselection process is not included in the portfolio selection strategy, proceed directly to Step 3.

Table 1 Parameters computed using all daily log-returns for the considered dataset

	Mean(%)	SD(%)	Skew	Kurt	min	max
min	–0.0686	1.0866	–6.4453	8.0280	–0.9363	0.0926
average	0.0373	2.0886	–0.3547	19.6411	–0.2250	0.1958
max	0.1409	4.0955	2.3666	217.2310	–0.0841	0.7049

Note that (%) after the statistic indicates that the result is multiplied by 100

[7] The selection of assets on a particular date will not affect the observed conclusions of this analysis as no strategies/portfolios are matched that contain different assets at a given time. Therefore, survivorship bias is not relevant in our analysis.

[8] Note that we do not modify the original data series in any way; we only remove the incomplete data series.

Step 3 For the strategy with the preselection part, use a subset of preselected assets from Step 2. Then, determine the composition of the optimal portfolio x that forms a particular efficient frontier by solving the optimization framework using approximated return series $\hat{r_i}$. We use the classical mean–variance model (8) or its trend-risk modification (10) for individual strategies (1 to 40) with different levels of the expected return M.[9]
The strategy without the preselection process considers all assets from the original dataset with the same optimization process used for the previous strategy, using approximated returns from Step 1.

Step 4 Finally, compute the portfolio statistics and the ex-post final wealth (final W) while taking into account the proportional transaction costs of 20 basis points as follows:

$$W_{t_{k+1}} = (W_{t_k} - tc_{t_k})(x' r^{ex-post}_{t_{k+1}}), \tag{11}$$

where W_t is the portfolio wealth at time t, $r^{ex-post}_{t_{k+1}}$ is the vector of gross returns between the time periods t_k and t_{k+1} and $tc_{t_k} = W_{t_k} \cdot 0.002 \cdot \sum_{i=1}^{z} |x_i^{(k)} - \frac{x_i^{(k-1)} r_{i,t_k}}{\sum_{i=1}^{z} x_i^{(k-1)} r_{i,t_k}}|$ is the transaction costs at time t for the k-th re-optimization. The time $t_{k+1} = t_k + \zeta$, where $\zeta = 21$ days. The algorithm from Step 1 to Step 4 is then repeated until no more observations are available.

The main results obtained for different portfolio selection processes are shown in Tables 2, 3, and 4. We emphasize that the return approximation process and its use for portfolio optimization is a common foundational step across all strategies. As the analyzed statistics for each of the 40 portfolios, we choose the mean (%), standard deviation (%), $\mathrm{VaR}_{5\%}$(%) (7), $\mathrm{CVaR}_{5\%}$(%) (6), SR(%)[10], RR (5), and final W.[11]

Moreover, we show the ex-ante results of two portfolio diversification indicators, the turnover ϱ and the Herfindahl–Hirschman index (HHI). Essentially, the turnover ϱ quantifies the impact of re-optimization on the portfolio composition. Mathematically, the equation for the computation of ϱ_k, where k is the re-calibration moment, is formulated as follows:

$$\varrho_k = \sum_{i=1}^{z} |x_i^k - x_i^{k-1}|, \tag{12}$$

where x_i^k is the weight assigned to the asset i at the re-calibration time k, and $\varrho_k \in [0, 2]$. The boundary values of this interval represent the unchanged portfolio composition after the k-th re-calibration for $\varrho_k = 0$ and the total portfolio renewal for $\varrho_k = 2$.

The second indicator (HHI) was originally proposed by Hirschman (1964) and used to quantify market concentration. Due to its simplicity and ease of explanation, it can be used to assess portfolio diversification. In general, the HHI is calculated as a sum of the squares of the individual asset weights:

$$HHI = \sum_{i=1}^{z} x_i^2, \tag{13}$$

where $HHI \in [\frac{1}{z}, 1]$. If the HHI concentration approaches the value $\frac{1}{z}$, then the investment is allocated to a large set of assets. In contrast, $HHI = 1$ represents an investment in a single asset.

Studying the results obtained for the classical mean–variance portfolio, we can see that the most profitable strategies according to the mean and final wealth are labeled with the numbers 36–38. However, these strategies are inherently among the riskiest, which affects the results of the SR and RR performance measures. Most probably, from an investor's perspective, it is, therefore, more prudent to select strategies that do not exhibit excessive risk located at the beginning of the efficient set. In addition, these strategies are characterized by a higher diversification, which can be observed when analyzing the turnover ϱ and HHI, which have significantly lower values than for the risk strategies.

Furthermore, let us compare the statistics of the mean–variance efficient frontier (Table 2) with those of its mean–trend risk version in Table 3. Until now, we have compared strategies without the preselection process to study the inclusion of trend-dependent risk modification instead of variance.

When we study the statistics on the ex-post daily returns for mean–trend risk strategies, we observe that strategies targeting the lower level of risk substantially outperform the same strategies in the mean–variance frontier. The mean return, the final wealth, and the risk indicators (SD, $\mathrm{VaR}_{5\%}$, $\mathrm{CVaR}_{5\%}$) are higher, but the performance (SR, RR) of these portfolios is ultimately also higher. The difference compared to mean–variance portfolios is the more pronounced change in the portfolio composition during re-optimization, which is common for all strategies. Shifting our focus to the opposite end of the efficient set with risky strategies,

9 The first strategy (1st) is a global minimum variance portfolio, the last strategy (40th) is a maximum expected return portfolio, and the strategies between them are equally spaced.

10 In general, the Sharpe ratio expresses the expected return of a portfolio for a unit of risk, and it is formulated as $SR(x'r) = \frac{E(x'r) - r_b}{\sigma_{x'r}}$, where $\sigma_{x'r}$ represents the standard deviation of the portfolio and r_b represents the risk-free rate.

11 For more information about SR and RR performance measures, see Sharpe (1994) and Rachev et al. (2008).

Table 2 Ex-post statistics computed for daily returns obtained for particular mean–variance strategies without preselection process

Str	Mean(%)	SD(%)	$VaR_{5\%}$(%)	$CVaR_{5\%}$(%)	SR(%)	RR	Final W	ϱ	HHI(%)
1	0.0253	0.7796	0.9595	1.6927	3.1556	0.9516	2.7475	0.4033	0.0637
2	0.0258	0.7803	0.9686	1.7013	3.2115	0.9476	2.7980	0.4174	0.0602
3	0.0262	0.7832	0.9626	1.7131	3.2514	0.9430	2.8437	0.4391	0.0568
4	0.0265	0.7891	0.9794	1.7268	3.2590	0.9398	2.8725	0.4634	0.0536
5	0.0266	0.7986	1.0102	1.7506	3.2320	0.9365	2.8834	0.4911	0.0508
6	0.0269	0.8103	1.0370	1.7814	3.2297	0.9351	2.9249	0.5208	0.0482
7	0.0274	0.8233	1.0702	1.8207	3.2383	0.9313	2.9826	0.5503	0.0459
8	0.0276	0.8386	1.1055	1.8650	3.2010	0.9273	3.0044	0.5802	0.0438
9	0.0274	0.8558	1.1745	1.9162	3.1131	0.9225	2.9804	0.6090	0.0419
10	0.0271	0.8761	1.2117	1.9774	3.0061	0.9169	2.9444	0.6341	0.0402
11	0.0269	0.8986	1.2853	2.0509	2.9115	0.9104	2.9241	0.6572	0.0389
12	0.0263	0.9227	1.3414	2.1281	2.7657	0.9061	2.8501	0.6801	0.0378
13	0.0258	0.9495	1.4133	2.2114	2.6356	0.9022	2.7941	0.7055	0.0370
14	0.0251	0.9785	1.4707	2.2989	2.4922	0.8993	2.7239	0.7305	0.0366
15	0.0245	1.0094	1.5459	2.3898	2.3535	0.8970	2.6564	0.7546	0.0365
16	0.0234	1.0429	1.6056	2.4906	2.1717	0.8928	2.5416	0.7761	0.0367
17	0.0222	1.0789	1.6749	2.6018	1.9925	0.8873	2.4275	0.7964	0.0371
18	0.0214	1.1171	1.7542	2.7156	1.8485	0.8833	2.3469	0.8144	0.0377
19	0.0211	1.1564	1.8365	2.8288	1.7640	0.8811	2.3235	0.8280	0.0386
20	0.0211	1.1968	1.9246	2.9432	1.7023	0.8796	2.3212	0.8384	0.0396
21	0.0213	1.2407	2.0163	3.0709	1.6539	0.8764	2.3348	0.8481	0.0407
22	0.0213	1.2857	2.0856	3.1967	1.5987	0.8746	2.3380	0.8569	0.0418
23	0.0217	1.3304	2.1802	3.3136	1.5757	0.8760	2.3764	0.8637	0.0431
24	0.0232	1.3781	2.2451	3.4347	1.6269	0.8771	2.5185	0.8744	0.0450
25	0.0242	1.4266	2.3235	3.5559	1.6463	0.8785	2.6280	0.8779	0.0465
26	0.0255	1.4796	2.4236	3.6815	1.6729	0.8810	2.7641	0.8825	0.0487
27	0.0260	1.5366	2.5150	3.8166	1.6431	0.8816	2.8193	0.8826	0.0512
28	0.0266	1.5965	2.6169	3.9562	1.6207	0.8817	2.8905	0.8815	0.0542
29	0.0273	1.6589	2.7183	4.1067	1.6029	0.8790	2.9743	0.8809	0.0577
30	0.0286	1.7236	2.8538	4.2673	1.6158	0.8727	3.1275	0.8780	0.0617
31	0.0300	1.7936	2.9810	4.4412	1.6299	0.8671	3.3049	0.8750	0.0662
32	0.0321	1.8700	3.1035	4.6300	1.6776	0.8631	3.5990	0.8650	0.0715
33	0.0346	1.9535	3.1874	4.8368	1.7344	0.8597	3.9782	0.8520	0.0777
34	0.0364	2.0554	3.2694	5.0768	1.7370	0.8566	4.2776	0.8392	0.0851
35	0.0395	2.1585	3.3774	5.3142	1.7947	0.8496	4.8281	0.8295	0.0933
36	0.0426	2.2717	3.5698	5.5797	1.8411	0.8449	5.4607	0.8073	0.1027
37	0.0435	2.4029	3.7495	5.8917	1.7792	0.8388	5.6663	0.7967	0.1136
38	0.0458	2.5513	3.9415	6.2576	1.7675	0.8317	6.2210	0.7907	0.1279
39	0.0379	2.7288	4.1871	6.7073	1.3620	0.8220	4.5346	0.8032	0.1469
40	0.0403	2.9673	4.5000	7.2689	1.3336	0.8210	4.9913	0.8018	0.1737

it is evident that profitability is slightly lower with similar risk compared to the results presented in Table 2. Generally, it can be concluded that the mean-trend risk optimization model is beneficial for risk-averse investors.

Furthermore, we include the preselection part in all strategies, as its involvement is the main objective of this paper. Basically, we examine preselection based on 3 and 4 parameters as discussed in Section 3. In Table 4, we present results for the three-dimensional non-dominated sorting method (3D), which is a more strict strategy than the second one. Thus, to provide a more comprehensive analysis, we also show the results for the four-dimensional vector (4D). The results of such a strategy are shown in Table 5.

The results show how various risk-averse investors can benefit from the preselection procedure before optimization. Looking at more details, it is evident that almost all the strategies employing non-dominated sorting preselection generate greater ex-post final wealth and performance ratios (SR, RR) compared to both previous complex portfolio selection frameworks. An interesting finding is that the best results are generated for the middle strategies, again supporting the perspective of less-risk-preferring investors. We can also notice slight differences in the risk indicators,

Table 3 Ex-post statistics computed for daily returns obtained for particular mean–trend risk strategies without preselection process

Str	Mean(%)	SD(%)	$VaR_{5\%}$(%)	$CVaR_{5\%}$(%)	SR(%)	RR	Final W	ϱ	HHI(%)
1	0.0384	1.0435	1.5807	2.4549	3.6085	0.9719	4.6238	0.7871	0.0522
2	0.0387	1.0464	1.5748	2.4590	3.6319	0.9724	4.6887	0.7973	0.0487
3	0.0389	1.0527	1.5685	2.4716	3.6254	0.9711	4.7193	0.8097	0.0454
4	0.0386	1.0631	1.5706	2.4879	3.5626	0.9717	4.6648	0.8267	0.0424
5	0.0382	1.0756	1.5823	2.5085	3.4805	0.9722	4.5840	0.8544	0.0396
6	0.0377	1.0898	1.5952	2.5364	3.3896	0.9698	4.4936	0.8835	0.0370
7	0.0372	1.1054	1.6256	2.5690	3.3008	0.9651	4.4138	0.9151	0.0347
8	0.0367	1.1234	1.6841	2.6109	3.2036	0.9615	4.3269	0.9473	0.0327
9	0.0366	1.1436	1.7194	2.6593	3.1319	0.9581	4.2971	0.9798	0.0308
10	0.0374	1.1667	1.7841	2.7161	3.1459	0.9550	4.4519	1.0132	0.0293
11	0.0384	1.1920	1.8468	2.7835	3.1600	0.9496	4.6259	1.0451	0.0280
12	0.0387	1.2192	1.8891	2.8638	3.1173	0.9407	4.6894	1.0753	0.0270
13	0.0382	1.2482	1.9583	2.9502	3.0018	0.9314	4.5896	1.1051	0.0263
14	0.0376	1.2779	2.0425	3.0341	2.8825	0.9247	4.4755	1.1305	0.0258
15	0.0372	1.3094	2.0968	3.1219	2.7854	0.9185	4.4108	1.1542	0.0256
16	0.0369	1.3430	2.1572	3.2127	2.6927	0.9135	4.3567	1.1808	0.0257
17	0.0363	1.3786	2.2140	3.3099	2.5815	0.9084	4.2583	1.2026	0.0261
18	0.0357	1.4163	2.2826	3.4133	2.4659	0.9029	4.1470	1.2254	0.0267
19	0.0347	1.4556	2.3403	3.5199	2.3325	0.8977	3.9891	1.2496	0.0277
20	0.0338	1.4963	2.4075	3.6282	2.2107	0.8936	3.8529	1.2727	0.0289
21	0.0331	1.5378	2.5013	3.7341	2.1050	0.8898	3.7454	1.2926	0.0304
22	0.0325	1.5802	2.5735	3.8436	2.0068	0.8854	3.6482	1.3107	0.0321
23	0.0318	1.6201	2.6494	3.9435	1.9199	0.8815	3.5608	1.3270	0.0340
24	0.0313	1.6602	2.7380	4.0398	1.8380	0.8790	3.4781	1.3386	0.0362
25	0.0301	1.7051	2.8126	4.1505	1.7217	0.8766	3.3211	1.3481	0.0386
26	0.0292	1.7505	2.8915	4.2635	1.6238	0.8736	3.2000	1.3578	0.0414
27	0.0285	1.7989	2.9622	4.3894	1.5404	0.8697	3.1102	1.3642	0.0444
28	0.0291	1.8486	3.0702	4.5198	1.5321	0.8677	3.1870	1.3690	0.0478
29	0.0300	1.9006	3.1684	4.6547	1.5372	0.8661	3.3026	1.3729	0.0517
30	0.0304	1.9549	3.2533	4.7981	1.5167	0.8630	3.3602	1.3759	0.0561
31	0.0310	2.0102	3.3426	4.9440	1.5036	0.8589	3.4383	1.3806	0.0610
32	0.0332	2.0687	3.4623	5.0968	1.5668	0.8552	3.7516	1.3865	0.0665
33	0.0358	2.1329	3.5558	5.2623	1.6418	0.8536	4.1626	1.3916	0.0729
34	0.0382	2.2050	3.6444	5.4454	1.6977	0.8515	4.5835	1.3945	0.0805
35	0.0387	2.2959	3.8026	5.6646	1.6542	0.8460	4.6842	1.3949	0.0891
36	0.0402	2.3984	3.9306	5.9056	1.6440	0.8411	4.9629	1.3949	0.0992
37	0.0402	2.5140	4.0908	6.1635	1.5707	0.8379	4.9747	1.3950	0.1112
38	0.0386	2.6499	4.2231	6.4745	1.4287	0.8320	4.6615	1.3938	0.1260
39	0.0356	2.8154	4.4172	6.8755	1.2397	0.8251	4.1435	1.3901	0.1459
40	0.0359	3.0265	4.5776	7.3698	1.1628	0.8230	4.1914	1.3853	0.1741

which are marginally lower (higher) for low-risk (higher-risk) strategies. Regarding indicators of changes in portfolio composition, turnover is also lower for strategies with preselection compared to those that do not implement this tool.

For a better illustration, the mean returns and their variability expressed by standard deviation for the various strategies with and without preselection are presented in Fig. 2. The outcomes and benefits of the preselection mentioned above can be derived from this figure.

For a clearer illustration of wealth paths for individual portfolio strategies, we present Figs. 3, 4 and 5, where the final wealth is transformed into a log version for a clearer visualization of jumps. From the graphical representation of log-wealth paths, we can more easily analyze the behavior of different strategies in different states of the economic cycle.

The analysis above is performed for the three-dimensional vector with a single-layer process. However, we also compute the same statistics for preselection based on a four-dimensional vector (4D) extended by the RR. As an illustration, in Fig. 6, we show the log-wealth paths for mean–trend risk portfolios, including the 4D preselection process.

It is worth mentioning that this approach shows slightly inferior results for risk-seeking investors compared to the

Table 4 Ex-post statistics computed for daily returns obtained for particular mean–trend risk strategies with 3D preselection process

Str	Mean(%)	SD(%)	$VaR_{5\%}$(%)	$CVaR_{5\%}$(%)	SR(%)	RR	Final W	ϱ	HHI(%)
1	0.0354	1.0142	1.5122	2.3533	3.4159	0.9534	4.1008	0.7757	0.0651
2	0.0366	1.0289	1.5294	2.3915	3.4831	0.9534	4.3010	0.7679	0.0624
3	0.0377	1.0451	1.5642	2.4337	3.5348	0.9532	4.4943	0.7788	0.0599
4	0.0386	1.0630	1.5936	2.4799	3.5590	0.9515	4.6565	0.7970	0.0576
5	0.0392	1.0825	1.6539	2.5288	3.5561	0.9494	4.7814	0.8165	0.0555
6	0.0401	1.1035	1.6983	2.5809	3.5630	0.9479	4.9412	0.8416	0.0536
7	0.0411	1.1256	1.7401	2.6333	3.5811	0.9466	5.1401	0.8685	0.0518
8	0.0422	1.1491	1.7764	2.6900	3.6049	0.9449	5.3736	0.8967	0.0503
9	0.0434	1.1736	1.8030	2.7471	3.6369	0.9438	5.6503	0.9218	0.0489
10	0.0446	1.1998	1.8349	2.8091	3.6592	0.9430	5.9326	0.9442	0.0478
11	0.0460	1.2274	1.8782	2.8766	3.6860	0.9418	6.2583	0.9666	0.0469
12	0.0471	1.2570	1.9220	2.9509	3.6909	0.9397	6.5518	0.9887	0.0462
13	0.0483	1.2882	1.9678	3.0283	3.6898	0.9379	6.8564	1.0086	0.0458
14	0.0494	1.3211	2.0424	3.1119	3.6837	0.9359	7.1736	1.0249	0.0457
15	0.0504	1.3553	2.0969	3.2014	3.6665	0.9331	7.4728	1.0413	0.0459
16	0.0513	1.3904	2.1556	3.2926	3.6340	0.9307	7.7264	1.0568	0.0462
17	0.0520	1.4265	2.2537	3.3869	3.5955	0.9280	7.9648	1.0705	0.0468
18	0.0527	1.4643	2.3199	3.4870	3.5513	0.9254	8.1934	1.0811	0.0478
19	0.0531	1.5036	2.4181	3.5902	3.4816	0.9227	8.3080	1.0894	0.0490
20	0.0534	1.5440	2.4774	3.6949	3.4137	0.9204	8.4273	1.0970	0.0504
21	0.0538	1.5854	2.5624	3.8022	3.3496	0.9180	8.5630	1.1017	0.0522
22	0.0543	1.6278	2.6383	3.9121	3.2928	0.9155	8.7334	1.1050	0.0543
23	0.0545	1.6718	2.7160	4.0273	3.2134	0.9121	8.7762	1.1075	0.0566
24	0.0544	1.7179	2.7744	4.1483	3.1250	0.9082	8.7628	1.1106	0.0593
25	0.0540	1.7675	2.8664	4.2789	3.0106	0.9034	8.5992	1.1143	0.0623
26	0.0533	1.8189	2.9447	4.4141	2.8898	0.8983	8.3793	1.1185	0.0657
27	0.0528	1.8725	3.0191	4.5545	2.7797	0.8935	8.2099	1.1220	0.0693
28	0.0526	1.9285	3.0977	4.7013	2.6869	0.8888	8.1341	1.1241	0.0734
29	0.0528	1.9870	3.1633	4.8537	2.6206	0.8844	8.2170	1.1268	0.0778
30	0.0534	2.0473	3.2492	5.0137	2.5741	0.8791	8.4254	1.1308	0.0827
31	0.0537	2.1096	3.3329	5.1782	2.5114	0.8737	8.5201	1.1354	0.0881
32	0.0529	2.1807	3.4782	5.3641	2.3938	0.8656	8.2601	1.1400	0.0941
33	0.0521	2.2563	3.5688	5.5551	2.2749	0.8583	7.9774	1.1443	0.1006
34	0.0512	2.3367	3.6842	5.7501	2.1604	0.8530	7.7120	1.1473	0.1079
35	0.0503	2.4228	3.7583	5.9571	2.0467	0.8479	7.4418	1.1487	0.1163
36	0.0494	2.5189	3.9096	6.1825	1.9336	0.8435	7.1844	1.1490	0.1260
37	0.0495	2.6266	4.0328	6.4274	1.8570	0.8416	7.2052	1.1465	0.1374
38	0.0507	2.7522	4.1773	6.7103	1.8160	0.8405	7.5591	1.1441	0.1511
39	0.0496	2.8935	4.3827	7.0454	1.6890	0.8363	7.2322	1.1412	0.1685
40	0.0557	3.0835	4.6415	7.4758	1.7833	0.8371	9.2303	1.1367	0.1932

approach with the 3D vector. However, for risk-averse investors, the situation is completely different. The inclusion of this performance ratio allows us to achieve relatively high profitability and a low level of portfolio risk compared to previous strategies. Therefore, it can generally be stated that the use of the RR as one of the dimensions for non-dominated sorting is particularly suitable for investors who are not risk-seeking.

To sum up and discuss the performance results obtained, in general, we observed the advantage of integrating preselection based on a multidimensional non-dominance sorting framework applied to either the mean–variance model or its mean–trend risk version. Choosing the number of dimensions (parameters) that are considered for decision-making relies heavily on the investor profile, as different combinations favor different groups of investors.

Finally, since an important preselection parameter is the number of assets that pass through the predefined filter of preselection, we record the values for both preselection settings considered above in Table 6. In particular, we distinguish whether a 3D or 4D vector is used. For a better illustration, we also show the shares of preselected assets as a percentage of the total assets in the initial dataset.

Table 5 Ex-post statistics computed for daily returns obtained for particular mean–trend risk strategies with 4D preselection process

Str	Mean(%)	SD(%)	$VaR_{5\%}$(%)	$CVaR_{5\%}$(%)	SR(%)	RR	Final W	ϱ	HHI(%)
1	0.0400	0.9828	1.4315	2.2522	3.9926	0.9672	4.9254	1.2041	0.0603
2	0.0404	0.9958	1.4740	2.2924	3.9831	0.9646	5.0094	1.2085	0.0573
3	0.0413	1.0103	1.5044	2.3351	4.0157	0.9601	5.1940	1.2072	0.0544
4	0.0419	1.0266	1.5555	2.3823	4.0123	0.9547	5.3239	1.2052	0.0518
5	0.0421	1.0455	1.5989	2.4342	3.9563	0.9500	5.3608	1.2035	0.0495
6	0.0420	1.0662	1.6243	2.4872	3.8653	0.9468	5.3285	1.2018	0.0474
7	0.0416	1.0884	1.6615	2.5406	3.7576	0.9437	5.2622	1.2008	0.0455
8	0.0415	1.1122	1.7142	2.5980	3.6657	0.9404	5.2353	1.1995	0.0438
9	0.0414	1.1377	1.7314	2.6595	3.5761	0.9377	5.2180	1.1972	0.0423
10	0.0413	1.1653	1.7765	2.7248	3.4832	0.9366	5.1983	1.1944	0.0412
11	0.0413	1.1948	1.8428	2.7940	3.3952	0.9354	5.1930	1.1914	0.0402
12	0.0418	1.2254	1.9131	2.8675	3.3469	0.9332	5.2868	1.1884	0.0395
13	0.0423	1.2569	1.9790	2.9447	3.3043	0.9305	5.3969	1.1861	0.0391
14	0.0428	1.2899	2.0346	3.0300	3.2585	0.9269	5.5060	1.1822	0.0389
15	0.0433	1.3248	2.1012	3.1210	3.2097	0.9236	5.6149	1.1790	0.0390
16	0.0436	1.3607	2.1783	3.2153	3.1490	0.9202	5.6881	1.1747	0.0394
17	0.0436	1.3974	2.2362	3.3122	3.0685	0.9170	5.6952	1.1699	0.0400
18	0.0435	1.4349	2.3074	3.4116	2.9817	0.9144	5.6735	1.1651	0.0409
19	0.0433	1.4737	2.3737	3.5131	2.8894	0.9117	5.6279	1.1604	0.0420
20	0.0430	1.5142	2.4437	3.6174	2.7923	0.9096	5.5607	1.1555	0.0435
21	0.0426	1.5559	2.4932	3.7237	2.6914	0.9082	5.4713	1.1505	0.0453
22	0.0422	1.5992	2.5613	3.8356	2.5900	0.9059	5.3731	1.1436	0.0474
23	0.0415	1.6445	2.6518	3.9548	2.4766	0.9023	5.2268	1.1333	0.0498
24	0.0407	1.6913	2.7551	4.0780	2.3606	0.8985	5.0622	1.1203	0.0525
25	0.0395	1.7395	2.8332	4.2034	2.2271	0.8953	4.8285	1.1049	0.0556
26	0.0385	1.7898	2.9376	4.3385	2.1089	0.8907	4.6405	1.0869	0.0589
27	0.0380	1.8417	3.0355	4.4793	2.0220	0.8861	4.5482	1.0675	0.0626
28	0.0377	1.8963	3.1137	4.6290	1.9505	0.8808	4.5025	1.0486	0.0667
29	0.0386	1.9529	3.1881	4.7799	1.9384	0.8768	4.6612	1.0280	0.0713
30	0.0395	2.0117	3.2162	4.9364	1.9286	0.8724	4.8394	1.0047	0.0763
31	0.0400	2.0733	3.3246	5.1039	1.8944	0.8671	4.9331	0.9796	0.0818
32	0.0393	2.1419	3.4402	5.2927	1.7996	0.8594	4.7910	0.9562	0.0879
33	0.0387	2.2161	3.5797	5.4887	1.7117	0.8529	4.6755	0.9317	0.0946
34	0.0382	2.2977	3.6754	5.6933	1.6304	0.8472	4.5886	0.9050	0.1021
35	0.0386	2.3857	3.8094	5.9001	1.5850	0.8435	4.6532	0.8760	0.1107
36	0.0391	2.4811	3.9166	6.1193	1.5443	0.8414	4.7474	0.8489	0.1208
37	0.0407	2.5867	4.1029	6.3567	1.5450	0.8413	5.0699	0.8260	0.1326
38	0.0428	2.7146	4.2233	6.6446	1.5476	0.8408	5.5013	0.8011	0.1467
39	0.0433	2.8717	4.4755	7.0049	1.4821	0.8367	5.6233	0.7856	0.1647
40	0.0496	3.0702	4.6106	7.4480	1.5919	0.8364	7.2332	0.7871	0.1900

Table 6 Amount of assets for different combinations of non-dominated sorting preselection

Preselection	min	average	max	min/374 (%)	average/374 (%)	max/374 (%)
3D	4	21.8789	45	1.0695	5.8500	12.0231
4D	6	35.2000	80	1.6043	9.4118	21.3904

In general, for each combination, the range between the minimum and maximum in which the given quantity of assets is located is relatively wide. Occasionally, the asset base intended for optimization is relatively large, and sometimes, we can only construct a portfolio from a few assets. Moreover, the amount of assets for different preselection combinations shows that as the dimension of the vector increases, the amount of non-dominated assets is higher, which confirms the theoretical assumptions.

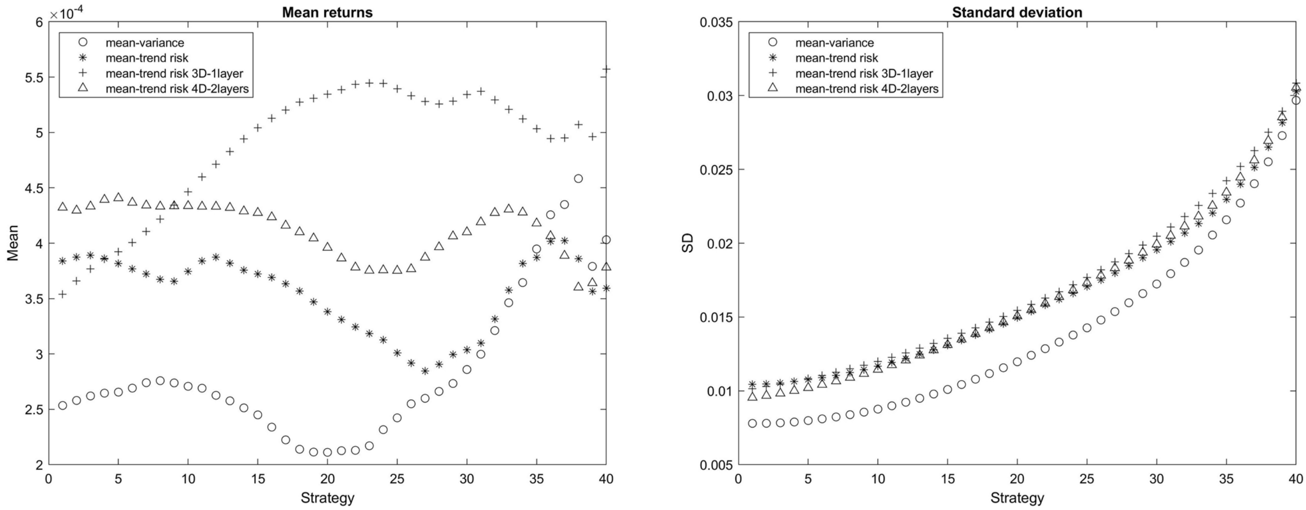

Fig. 2 Comparison of mean return and standard deviation for different portfolio selection strategies with and without preselection

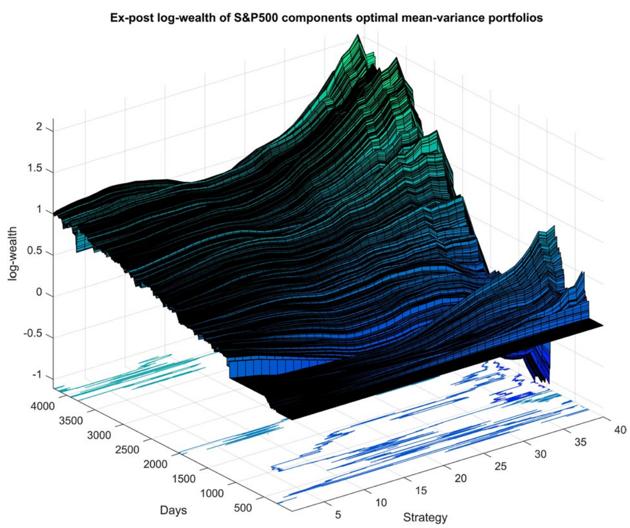

Fig. 3 Ex-post log-wealth of mean–variance strategies without preselection process

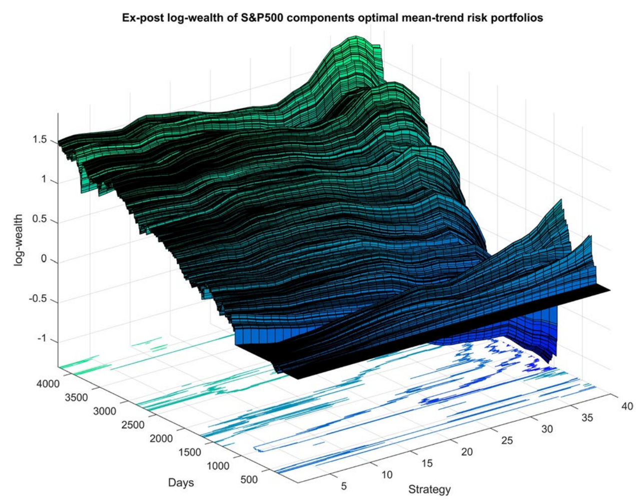

Fig. 4 Ex-post log-wealth of mean–trend risk strategies without preselection process

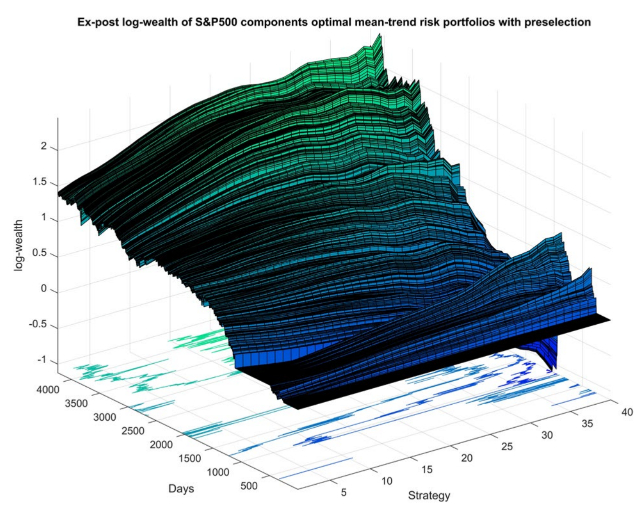

Fig. 5 Ex-post log-wealth of mean–trend risk strategies with 3D preselection process

Conclusion

In this study, we explore the preselection process using a non-dominated sorting approach within complex mean–variance and mean–trend risk portfolio selection strategies. Our preselection method involves three-dimensional or four-dimensional sorting of features related to selected assets. Unlike previous research, we apply this approach to approximated return series using trend-dependent principal component analysis (PCA) and nonparametric regression. These approximated returns are then used for further optimization, enhancing portfolio performance. We emphasize the importance of integrating trend (time) in all segments of this strategy. Our analysis uses components of the S&P 500 index from early 2005 to mid-2021.

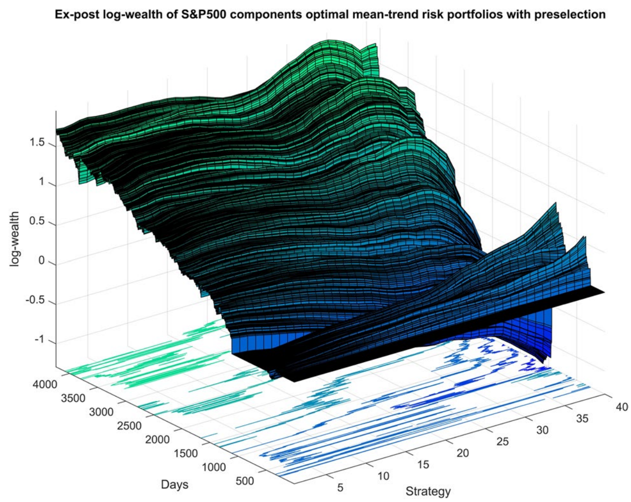

Fig. 6 Ex-post log-wealth of mean–trend risk strategies with 4D preselection process

Our empirical analysis shows that preselection based on three-dimensional non-dominated sorting significantly improves the wealth and performance of various risky portfolio strategies compared to those without preselection. Additionally, incorporating the Rachev ratio into the asset preselection framework results in higher ex-post profitability and lower risk, which is particularly beneficial for risk-averse investors. We also find that using fewer filtered assets in optimization leads to better performance than strategies without appropriate preselection. Furthermore, the mean–trend risk optimization framework proves more effective than the classical mean-variance framework for investors with different risk profiles.

For portfolio managers, this study offers a practical tool to enhance portfolio optimization by simplifying the stock selection process and improving decision-making efficiency. By focusing on a preselected subset of stocks, managers can implement their preferred strategies more effectively, leading to better performance and alignment with investor risk preferences.

In conclusion, our method simplifies the stock selection process and helps portfolio managers achieve better results with less effort. This makes it easier to create investment portfolios that meet the needs of risk-averse investors. Future research could explore incorporating systemic risk into asset reduction and developing a dynamic preselection model that adapts to changing market conditions in real time.

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Declarations

Competing interests The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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David Neděla is affiliated with the Faculty of Economics, VSB– Technical University of Ostrava. He specialises in portfolio selection, risk management, and quantitative investment strategies. His research interests include risk-based asset allocation, multi-objective optimization, and systemic risk assessment. His work focuses on developing data-driven methods for improving portfolio robustness and performance.

Sergio Ortobelli Lozza is a full professor in Applied Mathematics at the Department of Management of the University of Bergamo. His research focuses on the application of probability theory and operational research to Economics and Finance. He has published extensively in international peer-reviewed journals on portfolio theory and financial market analysis.

Tomáš Tichý is a full professor at the Faculty of Economics, VSB – Technical University of Ostrava. His research focuses on different areas from finance, for example, quantitative finance, portfolio optimization, prediction models, and risk management, multi-criteria decision-making methods. His work explores the application of advanced analytical and optimization techniques to improve portfolio performance and robustness under uncertainty.