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# The Coexistence of Mass and Elite Media in the Market for News 

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# The Coexistence of Mass and Elite Media in the Market for News 

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#### Abstract

In many countries, markets for news are characterized by the coexistence of sources of information which differ as to their contents (in particular, the entertainment/information mix). In this paper we model a market for news where two advertising-supported sources compete for the readers. Readers are divided into classes which differ regarding the informationprocessing skills and regarding the preferences for contents different from news. We show that there exists an equilibrium where one of the reports is read by all the population (mass source), while the second one is read only by one class of readers (élite source). Moreover, there exists a group reading both the mass report and the élite report. We also show that the ratio between news and other contents is lower in the mass source relative than the other one.


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## 1 Introduction

In many countries, markets for news are characterized by the coexistence of sources of information which differ as to their contents and as to the characteristics of their users. Even within the same type of medium, remarkable differences in content can be observed among different sources. As an example, refer to newspapers. Each newspaper offers a particular combination of contents, for example the news component vis à vis other contents or "services" ; moreover, we can observe variations in the quality of the contents provided: again, the "quality" of the news (i.e. accuracy, reliability, fineness, completeness ) can vary significantly across newspaper ( just for the sake of the example, The Sun versus The Times or Washington Post versus Washington Times). On the other hand, audiences are not homogeneous masses: besides strictly individual characteristics, readers differ by age, gender, place of living,
social and professional status and level of education. These differences imply different preferences, as regards the extent of the access to media, the most preferred contents mix and the quality of information.

In this paper, we focus mainly on the education level as a factor influencing the propensity to access the media (see also Battaggion-Vaglio (2009)). More precisely, we assume that the higher the education level, the higher the incentive to spend time and effort in reading, understanding and evaluating the information vehicled by media. Table 1 and 2 show some simple evidence which, at least prima facie, supports this view.
(TABLE 1 and TABLE 2 here)
The rapidly expanding literature on the economics of media has so far placed relatively little emphasis on these demand-related issues. As a matter of fact, on the one hand this literature has focused on the "upstream" relationship between media firms and pressure groups ("media capture" as an explanation of media bias). Influential papers, both theoretical and empirical, in this field are Brunetti-Weder (2003), Djankov, et al.. (2003), Gentzkow-Shapiro (2004), Besley-Prat (2005), Della VignaKaplan (2005), Baron (2006), Corneo (2006), Li-Mylovanov (2007). On the other hand, the literature concentrated on the competition among firms in media markets, stressing the two-sided nature of such markets (e.g.: Cancian, Bills, Bergstrom (1995); Vaglio (1995); Hackner, Nyberg (2001); Gal-Or-Dukes (2003); Strömberg (2004); Anderson-Coate (2005); Gabszewicz- Laussel-Sonnac (2006); Gabszewicz-Garella-Sonnac (2007)). An exception is represented by Mullainathan-Shleifer (2005), which is among the few papers directly addressing the issue of multiple-source readers and the heterogeneity of readers.

In our model, two sources compete for the readers. Revenue comes entirely from advertising. The report that each source provides is a combination of information and other services, which we summarize under the conventional heading of "entertainment". Readers are divided into two classes which differ as regards both the reading skills and the most preferred combination of information and entertainment. From the point of view of the modelling strategy, our framework considers both vertical differentiation (by "quality") and horizontal differentiation (information-entertainment), although not in the Hotelling location tradition, which however has had such large applications in the media market literature. In our framework, for each class of readers there exists a critical value of the entertainment/information ratio. Departure from this critical value, however, does not necessarily imply a utility loss for the reader, like it would happen in a Hotelling -type model of differentiation: such loss emerges only when the actual entertainment/information
ratio exceeds (for some types of readers) or falls below (for some other types of readers) the critical value. Then the rate of substitution among characteristics changes when their ratio exceeds some critical threshold. Not only the threshold varies across readers, both also the rate of substitution between news and entertainment changes in different directions in the different groups of readers.

We show that there may exist equilibria where one of the reports is read by all the population (mass source), while the second one is read only by one class of readers (élite source), which however read also the "mass" report. We shall also show that the ratio between information and entertainment is lower in the mass source relative than the other one. Crucial to the existence of the equilibrium are the sizes of the two classes of readers. This result confirms the relevance of the role of demand in the analysis of media markets and it exemplifies the insights that such approach might provide.

The paper is organized as follows: in Section 2 we introduce uncertainty and information, in Section 3 we describe the set-up of the model. Section 4 discusses the existence of the equilibrium. Some Conclusions will close the paper.

## 2 Value of information

Suppose that there exist two possible, relevant states of the world, called $A$ and $B$, with $\pi$ as the prior probability that the state of the world is $A$. Individuals must choose among two possible actions, one which is appropriate when the state of the world is $A$ and the other one which is appropriate in state $B$. However, the actual state of the world is not known to the individual, at the moment of choice. If the state is $A(B)$ and the appropriate action has been chosen, the ex post payoff is $\omega^{A}\left(\omega^{B}\right)$.If instead the wrong action has been chosen, the ex post outcome, in the two cases is respectively $l^{A}\left(l^{B}\right)$. We assume that

$$
\begin{gather*}
V^{I I}\left(q_{1}, q_{2}\right)=F\left(q_{2}\right) q_{1}+x^{A} q_{2}+e_{1}+e_{2}-t^{U}\left(q_{1}+q_{2}\right) \\
-\max \left[\varphi^{U}\left(e_{1}-q_{1} h^{U}\right), 0\right]-\max \left[\varphi^{U}\left(e_{2}-q_{2} h^{U}\right), 0\right]  \tag{1}\\
\pi \omega^{A}+(1-\pi) l^{B}>\pi l^{A}+(1-\pi) \omega^{B} \tag{2}
\end{gather*}
$$

i.e. the action which would be appropriate in state $A$ is optimal exante, on the basis of the prior. If we define: $x^{A}=\pi\left(\omega^{A}-l^{A}\right)$ and $x^{B}=(1-\pi)\left(\omega^{B}-l^{B}\right)$, the previous assumption translates into

$$
\begin{equation*}
x^{A}>x^{B} \tag{3}
\end{equation*}
$$

The expected value of utility from choosing $A$ is therefore (given the prior $\pi$ ):

$$
\begin{equation*}
V_{0}=\pi \omega^{A}+(1-\pi) l^{B} \tag{4}
\end{equation*}
$$

Before choosing, the individual can access, if it deems this worthwhile, two "reports". Each report contains a statement concerning the state of the world. Such statements are correct with probabilities, respectively, $q_{1}$ and $q_{2}$, which we call "accuracies" or "quality levels" of the two reports.

If the reader reads just one report of quality $q$ is, his ex ante utility from trusting that report is

$$
\begin{equation*}
V^{I}(q)=\left[(1-\pi) l^{B}+\pi l^{A}\right]+q\left[x^{A}+x^{B}\right] \tag{5}
\end{equation*}
$$

When the reader reads both reports, the value of information is

$$
\begin{gather*}
V^{I I}\left(q_{1}, q_{2}\right)=\pi\left\{\left(\omega^{A}-l^{A}\right)\left[q_{1}+q_{2}\left(1-q_{1}\right)\right]+l^{A}\right\}+ \\
+(1-\pi)\left\{\left(\omega^{B}-l^{B}\right) q_{1} q_{2}+l^{B}\right\} \tag{6}
\end{gather*}
$$

and, more shortlly

$$
\begin{equation*}
V^{I I}\left(q_{1}, q_{2}\right)=x^{A}\left[q_{1}+q_{2}\left(1-q_{1}\right)\right]+x^{B} q_{1} q_{2}+\pi l^{A}+(1-\pi) l^{B} \tag{7}
\end{equation*}
$$

Defining $F(q)=x^{A}(1-q)+x^{B} q$, it is easy to see that: $F\left(q_{1}\right) q_{2}+$ $x^{A} q_{1}=F\left(q_{2}\right) q_{1}+x^{A} q_{2}$, while (7) can be rewritten as:

$$
\begin{gather*}
V^{I I}\left(q_{1}, q_{2}\right)=F\left(q_{1}\right) q_{2}+x^{A} q_{1}+\pi l^{A}+(1-\pi) l^{B}= \\
F\left(q_{2}\right) q_{1}+x^{A} q_{2}+\pi l^{A}+(1-\pi) l^{B} \tag{8}
\end{gather*}
$$

The expression (7) is derived from the following behavioral pattern: the reader trusts the sources when they agree about the state of the world, while he chooses the action appropriate to state A (the one consistent with the prior) when they disagree. Given $q_{i}(i: 1,2)$, the condition $q_{j} \geq 0.5(j \neq i)$ is necessary for the pattern just mentioned to be optimal; otherwise it would be dominated by an alternative pattern where the reader "interprets" the statement "The state of the world is A (B)" from report $j$ as if it were "The state of the world is B (A)" and then acts according to the first pattern. To avoid the unnecessary complications attached to this case, we shall assume from now on that $q_{i}$ $(i: 1,2) \geq 0.5$. Moreover, in what follows we shall assume for simplicity that $l^{A}=l^{B}=0$.

## 3 A model with sources as providers of information and entertainment

We consider a situation where there exist two media firms (or sources) which we call Source One and Source Two respectively. Each source receives an independent signal about the actual state of the world. The probability of the signal being correct is chosen by each firm and it is denoted as $q_{1}$ for Source One and $q_{2}$ for Source Two. The sources issue reports, which are bundles containing a truthful statement about the signal received by the source and additional characteristic which we call entertainment ( $e_{1}, e_{2}$ for the two firms respectively). A source choosing a value of $q$ for accuracy and $e$ for entertainment incurs a cost $c q+r e$, with $c, r>0$. Revenue comes entirely from advertising, and it equals the exogenous per-reader fee $w$ times the number of readers. We assume that there exists two populations of readers, which we call "educated" and "uneducated" readers, whose sizes are respectively $E$ and $U$. The two classes have different reading technologies and as well as different preferences about the information-entertainment mix. As regards the educated readers, we assume that do not consume time when reading entertainment, unless the entertainment-information ratio exceeds a given value $h^{E}$. Then, for an educated reader the cost of reading entertainment is:

$$
\begin{equation*}
\max \left[\varphi^{E}\left(e-q h^{E}\right), 0\right] \tag{9}
\end{equation*}
$$

where $\varphi^{E}>0$. At the same time, reading a piece of information whose level of accuracy is $q$ requires $t^{E} q$ units of time, with $t^{E}>0$. Then the net utility of the educated reader is:

$$
\begin{align*}
& N U_{d}^{E}=F\left(q_{2}\right) q_{1}+x^{A} q_{2}+k\left(e_{1}+e_{2}\right)-t^{E}\left(q_{1}+q_{2}\right)+ \\
& -\max \left[\varphi^{E}\left(e_{1}-q_{1} h^{E}\right), 0\right]-\max \left[\varphi^{E}\left(e_{2}-q_{2} h^{E}\right), 0\right] \tag{10}
\end{align*}
$$

when he reads two reports containing $q_{1}, e_{1}$ and $q_{2}, e_{2}$ respectively, and

$$
\begin{equation*}
N U_{s}^{E}=\left(x^{A}+x^{B}\right) q+k e-t^{E} q-\max \left[\varphi^{E}\left(e-q h^{E}\right), 0\right] \tag{11}
\end{equation*}
$$

when he reads one report containing $q, e$. The marginal utility of accuracy $q_{1}$ is $\left[F\left(q_{2}\right)-t^{E}+\varphi^{E} h^{E}\right]$ (double reading) and $\left(x^{A}+x^{B}-t^{E}+\varphi^{E} h^{E}\right)$, (single reading), when $\frac{e_{1}}{q_{1}}>h^{E}$; similarly, the marginal utility of entertainment $e_{1}$ is $\left(1-\varphi^{E}\right)$ (which we assume is positive) . If instead $\frac{e_{1}}{q_{1}}<h^{E}$, the marginal utilities are $\left[F\left(q_{2}\right)-t^{E}\right]$ (double reading), ( $x^{A}+x^{B}-t^{E}$ ) (single reading) while the marginal utility of entertainment is 1 .

Turning now to non-educated readers, they never consume time when they read entertainment. However, when the entertainmentinformation ratio is below the value $h^{U}$, then one uneducated reader incurs a reading time cost $\varphi^{U}\left(q h^{U}-e\right)$.

Then the net utility of uneducated reader is: $t^{U} q+\max \left[\varphi^{U}\left(q h^{U}-e\right), 0\right]$

$$
\begin{equation*}
N U^{U}=\left(x^{A}+x^{B}\right) q+k e-t^{U} q-\max \left[\varphi^{U}\left(q h^{U}-e\right), 0\right] \tag{12}
\end{equation*}
$$

The marginal utility of $q$ is $\left(x^{A}+x^{B}-t^{U}-\varphi^{U} h^{U}\right)$ when $\frac{e}{q}<h^{U}$ and $\left(x^{A}+x^{B}-t^{U}\right)$ otherwise. The corresponding marginal utilities of $e$ are $\left(1+\varphi^{U}\right)$ and 1 .

We assume that: $h^{U}>h^{E}$ and, as usual, $t^{U}>t^{E}$. Since we shall restrict to cases where uneducated readers read just one report, we do not report here their marginal utility value for double reading.

We assume that at time 0 the sources choose simultaneously their levels of $q$ and $e$. At time 1 the sources learn the signals, while the readers, knowing $q_{1}, q_{2}$ and $e_{1}, e_{2}$ (but not the signals) decide which reports to read.

### 3.1 Decision problems

Let us consider time 0 . The decision problem of a source consists in choosing $q$ and $e$ given the choice of the opponent, and taking into account the choice which the readers will make at time 1 regarding how many reports to read and, in case they read only one, which of the two. As explained in the introduction, we shall focus on a situation where the report of one of the sources (which we assume is source One) is read by all individuals, while the report of the other source (by assumption, source Two) is read only by the educated. Moreover, we consider a situation where
$\frac{e_{1}}{q_{1}}>h^{U}>h^{E} \quad$ and $\frac{e_{2}}{q_{2}}<h^{E}$, i.e. source One chooses an entertainment/accuracy mix which is closer to the tastes of the uneducated readership, whereas the entertainment/accuracy mix of source Two is more favorable to the educated population. Our purpose is to show that such a situation can be an equilibrium for appropriate parameter values. The decision problem for source One is

$$
\begin{equation*}
\max w(E+U)-\left(\frac{c}{2} q_{1}^{2}+r e_{1}\right) \tag{13}
\end{equation*}
$$

s.t.

$$
\begin{gather*}
F\left(q_{2}\right) q_{1}+x^{A} q_{2}+k\left(e_{1}+e_{2}\right)-t^{E}\left(q_{1}+q_{2}\right)+ \\
-\max \left[\varphi^{E}\left(e_{1}-q_{1} h^{E}\right), 0\right]-\max \left[\varphi^{E}\left(e_{2}-q_{2} h^{E}\right), 0\right]  \tag{14}\\
\geq\left(x^{A}+x^{B}\right) q_{2}+k e_{2}-t^{E} q_{2}-\max \left[\varphi^{E}\left(e_{2}-q_{2} h^{E}\right), 0\right]
\end{gather*}
$$

$$
\begin{gather*}
\left(x^{A}+x^{B}\right) q_{1}+k e_{1}-t^{U} q_{1}-\max \left[\varphi^{U}\left(q_{1} h^{U}-e_{1}\right), 0\right] \geq \\
\left(x^{A}+x^{B}\right) q_{2}+k e_{2}-t^{U} q_{2}-\max \left[\varphi^{U}\left(q_{2} h^{U}-e_{2}\right), 0\right] \tag{15}
\end{gather*}
$$

Condition (14) requires that it is better, for an educated reader, to read both reports instead of reading just Report Two. Condition (15) states that it is better for the uneducated reader to read report One instead of report Two. Let us remind that the choice is also subject to the bounds on the value of $q_{1}$ and to a non-negativity constraint on $e_{1}$.

$$
\begin{gather*}
q_{1} \geq \frac{x^{A}}{x^{A}+x^{B}-t^{U}}  \tag{16}\\
1-q_{1} \geq 0  \tag{17}\\
e_{1} \geq 0 \tag{18}
\end{gather*}
$$

The constraint (16) requires that $q_{1}$ is large enough to make the information value from reading report One larger than the value from not reading at all, for an uneducated reader $\left(\frac{x^{A}}{x^{A}+x^{B}-t^{U}}\right.$, the minimum value of $q_{1}$ which meets this last requirement, is also larger than 0.5 ).

We shall be particularly interested in solutions where the constraint (15) is satisfied as a strict inequality. It is easy to see why. Suppose we have determined the values $q_{1}, e_{1}$ and $q_{2}, e_{2}$ which simultaneously solve the decision problems above, and suppose that the constraint (15) holds as an equality. Then it would be sufficient for source Two to increase even by a negligible amount $q_{2}, e_{2}$ (or both), to steal the uneducated market from source One, while keeping the educated market. Then the situation described would not be an equilibrium.

The decision problem of Source Two instead is:

$$
\max w E-\left(\frac{c}{2} q_{2}^{2}+r e_{2}\right)
$$

s.t.

$$
\begin{gather*}
F\left(q_{1}\right) q_{2}+x^{A} q_{1}+k\left(e_{1}+e_{2}\right)-t^{E}\left(q_{1}+q_{2}\right)+ \\
-\max \left[\varphi^{E}\left(e_{1}-q_{1} h^{E}\right), 0\right]-\max \left[\varphi^{E}\left(e_{2}-q_{2} h^{E}\right), 0\right]  \tag{19}\\
\geq\left(x^{A}+x^{B}\right) q_{1}+k e_{1}-t^{E} q_{1}-\max \left[\varphi^{E}\left(e_{1}-q_{1} h^{E}\right), 0\right]
\end{gather*}
$$

The constraint requires that the educated readers prefers reading both reports, rather than just report One. Moreover:

$$
\begin{equation*}
q_{2} \geq \frac{x^{A}}{x^{A}+x^{B}-t^{E}} \tag{20}
\end{equation*}
$$

$$
\begin{gather*}
1-q_{2} \geq 0  \tag{21}\\
e_{2} \geq 0 \tag{22}
\end{gather*}
$$

If we restrict to the case where $\frac{e_{1}}{q_{1}}>h^{U}>h^{E}>\frac{e_{2}}{q_{2}}$, the first-order conditions for Source One are:

$$
\begin{gather*}
-c q_{1}+\lambda_{1}\left(F\left(q_{2}\right)-t^{E}+\varphi^{E} h^{E}\right)+\mu\left(x^{A}+x^{B}-t^{U}\right)+\theta_{1}-\theta_{2}=0 \\
-r+\lambda_{1}\left(k-\varphi^{E}\right)+\mu+\theta_{3}=0  \tag{23}\\
\mu\left[\left(x^{A}+x^{B}-t^{U}\right) q_{1}+k e_{1}-\left(x^{A}+x^{B}-t^{U}-\varphi^{U} h^{U}\right) q_{2}-e_{2}\left(k-\varphi^{U}\right)\right]=0 \\
\lambda_{1}\left\{\left[F\left(q_{2}\right)-t^{E}+\varphi^{E} h^{E}\right] q_{1}+e_{1}\left(k-\varphi^{E}\right)-x^{B} q_{2}\right\}=0 \tag{24}
\end{gather*}
$$

The first-order conditions for Source Two are instead:

$$
\begin{gather*}
-c q_{2}+\lambda_{2}\left(F\left(q_{1}\right)-t^{E}\right)+\theta_{4}-\theta_{5}=0  \tag{25}\\
-r+\lambda_{2} k+\theta_{6}=0  \tag{26}\\
\lambda_{2}\left(F\left(q_{1}\right) q_{2}+k e_{2}-t^{E} q_{2}-x^{B} q_{1}\right)
\end{gather*}
$$

where $\lambda_{1}$ and $\lambda_{2}$ are the multipliers associated to the constraints (14) and (19); $\mu$ is the multiplier associated to (15); $\theta_{1}, \theta_{2}$, and $\theta_{4}, \theta_{5}$ are associated to the constraints $q_{1}, q_{2} \epsilon\left[q_{\text {min }}, 1\right] . \theta_{3}, \theta_{6}$ are associated to the non-negativity constraints on $e_{1}$ and $e_{2}$. Suppose now that the constraints (15), (16),(17), (18), (20), (21), (22) are not binding (i.e. let us set $\left.\mu=\theta_{1}=\theta_{2}=\theta_{3}=\theta_{4}=\theta_{5}=\theta_{6}=0\right)$ the conditions (24) and (25) reduce to

$$
\begin{align*}
\left(F\left(q_{2}\right)-t^{E}+\varphi^{E} h^{E}\right) & =\left(k-\varphi^{E}\right) \frac{c}{r} q_{1}  \tag{27}\\
\left(F\left(q_{1}\right)-t^{E}\right) & =\frac{c}{r} k q_{2} \tag{28}
\end{align*}
$$

So that we can get the following reaction functions, respectively for source One and for source Two:

$$
\begin{gather*}
q_{1}=\frac{r}{c\left(k-\varphi^{E}\right)}\left(x^{A}-t^{E}+\varphi^{E} h^{E}\right)-\frac{r\left(x^{A}-x^{B}\right)}{c\left(k-\varphi^{E}\right)} q_{2}  \tag{29}\\
q_{2}=\frac{r}{k c}\left[\left(x^{A}-t^{E}\right)-\left(x^{A}-x^{B}\right) q_{1}\right] \tag{30}
\end{gather*}
$$

Solving the system composed by (29) and (30) we get the two equilibrium values for $q_{1}$ and $q_{2}$

$$
\begin{gather*}
q_{1}=\frac{r}{c} \frac{\left(1-\frac{r}{c} \frac{\left(x^{A}-x^{B}\right)}{k}\right)\left(x^{A}-t^{E}\right)+\varphi^{E} h^{E}}{\left(k-\varphi^{E}\right)-\left(\frac{r}{c}\right)^{2} \frac{\left(x^{A}-x^{B}\right)^{2}}{k}}  \tag{31}\\
q_{2}=\frac{r}{k c} \frac{\left[\left(x^{A}-t^{E}\right)\left[\left(k-\varphi^{E}\right)-\left(x^{A}-x^{B}\right) \frac{r}{c}\right]-\frac{r}{c}\left(x^{A}-x^{B}\right) \varphi^{E} h^{E}\right]}{\left(k-\varphi^{E}\right)-\left(\frac{r}{c}\right)^{2} \frac{\left(x^{A}-x^{B}\right)^{2}}{k}} \tag{32}
\end{gather*}
$$

The equilibrium values of $e_{1}$ and $e_{2}$ can then be obtained from constraints (14) an d (19). Instead of their explicit expressions, consider the formulation as functions of $q_{1}$ and $q_{2}$.

$$
\begin{gather*}
e_{1}=\frac{x^{B} q_{2}-\left(F\left(q_{2}\right)-t^{E}+\varphi^{E} h^{E}\right) q_{1}}{k-\varphi^{E}} \\
e_{2}=\frac{x^{B} q_{1}-\left(F\left(q_{1}\right) q-t^{E}\right) q_{2}}{k} \tag{33}
\end{gather*}
$$

which, thanks to (27) and (28), can be rewritten as:

$$
\begin{gather*}
e_{1}=\frac{x^{B}}{k-\varphi^{E}} q_{2}-\frac{c}{r} q_{1}^{2}  \tag{34}\\
e_{2}=\frac{x^{B}}{k} q_{1}-\frac{c}{r} q_{2}^{2} \tag{35}
\end{gather*}
$$

from (34) and (35) it is easy to see that $q_{2}>q_{1}$ implies $e_{2}<e_{1}$ and therefore also $\frac{e_{1}}{q_{1}}>\frac{e_{2}}{q_{2}}$

## 4 Equilibrium

### 4.1 Existence conditions

The solutions (31), (32), (34), (35) must satisfy the requirements

1. $q_{2}>q_{1}$
2. $\frac{x^{A}}{x^{A}+x^{B}-t^{U}}<q_{1}<1$
3. $\frac{x^{A}}{x^{A}+x^{B}-t^{E}}<q_{2}<1$
4. $e_{1}, e_{2}>0$
5. $\frac{e_{1}}{q_{1}}>h^{U}>h^{E}>\frac{e_{2}}{q_{2}}$
6. $\left(x^{A}+x^{B}-t^{U}\right) q_{1}+k e_{1}-\left(x^{A}+x^{B}-t^{U}-\varphi^{U} h^{U}\right) q_{2}-\left(k+\varphi^{U}\right) e_{2}>$
0

In this paragraph we provide a discussion of the parameter restrictions which satisfy these requirement, together with some numerical results showing that, for an appropriate selection of parameter values, the solutions described in the previous paragraph make sense from both the economic and mathematical viewpoint.

Lemma 1 Requirement 1. is satisfied if and only if

$$
\left(k-\varphi^{E}\right)-\left(\frac{r}{c}\right)^{2} \frac{\left(x^{A}-x^{B}\right)^{2}}{k}<0
$$

Proof. Suppose that $\left(k-\varphi^{E}\right)-\left(\frac{r}{c}\right)^{2} \frac{\left(x^{A}-x^{B}\right)^{2}}{k}>0$.In this case $q_{2}>$ $q_{1}$ implies $-\frac{r}{c}>\frac{k \varphi^{E} h^{E}+\left(x^{A}-t^{E}\right) \varphi^{E}}{\left(x^{A}-x^{B}\right) \varphi^{E} h^{E}}$, which is impossible. If instead $\left(k-\varphi^{E}\right)-\left(\frac{r}{c}\right)^{2} \frac{\left(x^{A}-x^{B}\right)^{2}}{k}<0, q_{2}>q_{1}$ implies $-\frac{r}{c}<\frac{k \varphi^{E} h^{E}+\left(x^{A}-t^{E}\right) \varphi^{E}}{\left(x^{A}-x^{B}\right) \varphi^{E} h^{E}}$, which is always true.

Assuming $q_{2}>q_{1}$ is satisfied, simple calculations show that requirement 2 implies that $h^{E}>0$ lies in the interval $\left(L_{1}, U_{1}\right)$, where:

$$
\begin{aligned}
L_{1} & \equiv\left[\left(k-\varphi^{E}\right)\left(\frac{c}{r}\right)+\left(\frac{r}{c}\right) \frac{\left(x^{A}-x^{B}\right)}{k}\left(x^{B}-t^{E}\right)-\left(x^{A}-t^{E}\right)\right] \frac{1}{\varphi^{E}} \\
U_{1} & \equiv \frac{1}{\varphi^{E}}\left[q_{U}\left[\left(k-\varphi^{E}\right)\left(\frac{c}{r}\right)-\frac{r}{c} \frac{\left(x^{A}-x^{B}\right)^{2}}{k}\right]+\left(\frac{r}{c} \frac{\left(x^{A}-x^{B}\right)}{k}-1\right)\left[\left(x^{A}-t^{E}\right)\right]\right]
\end{aligned}
$$

Similarly, Requirement 3. implies that $h^{E}>0$ belongs to the interval $\left(L_{2}, U_{2}\right)$, where:

$$
\begin{aligned}
L_{2} & \equiv\left[\left(x^{A}-t^{E}\right)\left(\frac{\left(k-\varphi^{E}\right)}{\left(x^{A}-x^{B}\right)} \frac{c}{r}-1\right)-q_{E}\left(\frac{k\left(k-\varphi^{E}\right)}{\left(x^{A}-x^{B}\right)}\left(\frac{c}{r}\right)^{2}-\left(x^{A}-x^{B}\right)\right)\right] \frac{1}{\varphi^{E}} \\
U_{2} & \equiv\left\{\left[\left(x^{A}-t^{E}\right)\left[\frac{\left(k-\varphi^{E}\right)}{\left(x^{A}-x^{B}\right)} \frac{c}{r}-1\right]\right]-k \frac{\left(k-\varphi^{E}\right)}{\left(x^{A}-x^{B}\right)}\left(\frac{c}{r}\right)^{2}+\left(x^{A}-x^{B}\right)\right\} \frac{1}{\varphi^{E}}
\end{aligned}
$$

The point is now whether or not the intersection among the intervals $\left(L_{1}, U_{1}\right),\left(L_{2}, U_{2}\right)$ and $(0, \infty)$ is empty. To this end, it is sufficient that:
a) The minimum between $U_{1}$ and $U_{2}$ is positive; b) one of the two following cases occurs: b1) $\left(L_{1}<L_{2}<U_{1}\right)$, b2) $\left(L_{1}<U_{2}<U_{1}\right)$.

As regards point a), further simple calculations show that $U_{1}$ is positive if $\frac{r}{c}>\bar{\rho}$ or $\frac{r}{c}<\underline{\rho}$ while $U_{2}$ is positive if $\bar{\varepsilon}>\frac{r}{c}>\underline{\varepsilon}$, where:

$$
\begin{gathered}
\underline{\rho} \equiv \frac{\left(x^{A}-t^{E}\right)-\sqrt{\left(x^{A}-t^{E}\right)^{2}-4\left[\left(x^{A}-t^{E}\right)-q_{U}\left(x^{A}-x^{B}\right)\right] \frac{x^{A}-x^{B}}{k} q_{U}\left(k-\varphi^{E}\right)}}{2\left[\left(x^{A}-t^{E}\right)-q_{U}\left(x^{A}-x^{B}\right)\right] \frac{x^{A}-x^{B}}{k}} \\
\underline{\rho} \equiv \frac{\left(x^{A}-t^{E}\right)+\sqrt{\left(x^{A}-t^{E}\right)^{2}-4\left[\left(x^{A}-t^{E}\right)-q_{U}\left(x^{A}-x^{B}\right)\right] \frac{x^{A}-x^{B}}{k} q_{U}\left(k-\varphi^{E}\right)}}{2\left[\left(x^{A}-t^{E}\right)-q_{U}\left(x^{A}-x^{B}\right)\right] \frac{x^{A}-x^{B}}{k}} \\
\bar{\varepsilon} \equiv \frac{2 \frac{k\left(k-\varphi^{E}\right)}{\left(x^{A}-x^{B}\right)}}{\left(x^{A}-t^{E}\right) \frac{\left(k-\varphi^{E}\right)}{\left(x^{A}-x^{B}\right)}-\sqrt{\left(\left(x^{A}-t^{E}\right) \frac{\left(k-\varphi^{E}\right)}{\left(x^{A}-x^{B}\right)}\right)^{2}-4 \frac{k\left(k-\varphi^{E}\right)}{\left(x^{A}-x^{B}\right)}\left(x^{B}-t^{E}\right)}} \\
\underline{\varepsilon} \equiv \frac{2 \frac{k\left(k-\varphi^{E}\right)}{\left(x^{A}-x^{B}\right)}}{\left(x^{A}-t^{E}\right) \frac{\left(k-\varphi^{E}\right)}{\left(x^{A}-x^{B}\right)}+\sqrt{\left(\left(x^{A}-t^{E}\right) \frac{\left(k-\varphi^{E}\right)}{\left(x^{A}-x^{B}\right)}\right)^{2}-4 \frac{k\left(k-\varphi^{E}\right)}{\left(x^{A}-x^{B}\right)}\left(x^{B}-t^{E}\right)}}
\end{gathered}
$$

As regards cases b1) and b2), the former occurs if

$$
\underset{c}{r} \epsilon\left(\frac{q_{E} k}{x^{B}-t^{E}}, \frac{q_{E} k}{\left(x^{A}-t^{E}\right)-q_{U}\left(x^{A}-x^{B}\right)}\right)
$$

while the latter occurs if

$$
\stackrel{r}{c} \epsilon\left(\frac{k}{x^{B}-t^{E}}, \frac{k}{\left(x^{A}-t^{E}\right)-q_{U}\left(x^{A}-x^{B}\right)}\right)
$$

The previous discussion outlines a procedure for finding numerical values of the parameters that ensure the existence of the solution described. The procedure starts by finding a set of values for $x^{A}, x^{B}, t^{E}, t^{U}, k$ such that the intersection of intervals $\left(\frac{q_{E} k}{x^{B}-t^{E}}, \frac{q_{E} k}{\left(x^{A}-t^{E}\right)-q_{U}\left(x^{A}-x^{B}\right)}\right),\left(\frac{k}{x^{B}-t^{E}}, \frac{k}{\left(x^{A}-t^{E}\right)-q_{U}\left(x^{A}-x^{B}\right)}\right)$, $(\bar{\varepsilon}, \underline{\varepsilon})$ and one of $(-\infty, \underline{\rho}),(\bar{\rho}, \infty)$ is non-empty. Then, by choosing a value of $\frac{r}{c}$ in this intersection,one determines a set of admissible values for $h^{E}$.Given a value for $h^{E}$, one can determine the set of values $\varphi^{E}$ which imply $\left(k-\varphi^{E}\right)-\left(\frac{r}{c}\right)^{2} \frac{\left(x^{A}-x^{B}\right)^{2}}{k}<0$ (the condition ensuring $\left.q_{2}>q_{1}\right)$. Given these inputs, it is possible to compute the equilibrium values of $q_{1}, q_{2}$ and $e_{1}, e_{2}$. While the values of $q_{1}, q_{2}$ so computed certainly satisfy requirements $1 ., 2$. and 3 ., requirements $4 ., 5$. and 6 .
must be further checked and iterated adjustments in the previously chosen parameters are in general necessary. Notice that once $h^{U}$ has been chosen to satisfy requirement 5 ., requirement 6 . is satisfied by a sufficiently large value of $\varphi^{U}$, which does not enter in any other existence condition. Fortunately the process converges very rapidly. In Table 3, we provide a numerical example of the solutions obtained by the previously described procedure, for a given choice of $x^{A}, x^{B}, t^{E}, t^{U}, \varphi^{E}, \varphi^{U}, h^{E}, h^{U}, \frac{r}{c}$ and for different values of $k$.
(TABLE 3 here)

### 4.2 Deviations

In the last proposition, we have shown that the configuration (31), (32), (34) and (34) makes mathematical and economic sense, for appropriate parameter values . To show that it is an equilibrium, we must show that, under appropriate assumptions, it is also deviation-proof. Starting with source One, there are two possible deviations:

1. Source One gives up the educated market and keeps the uneducated market only
2. Source One gives up the uneducated market and keeps the educated market only

To investigate these deviations, define:

$$
\begin{gather*}
\bar{y}_{1}=\frac{x^{B} q_{2}-\left(F\left(q_{2}\right)-t^{E}+\varphi^{E} h^{E}\right)\left(q_{1}+\Delta q_{1}\right)}{k-\varphi^{E}}-e_{1} \text { if } \Delta e_{1}+e_{1}>\left(q_{1}+\Delta q_{1}\right) h^{E} \\
\underline{y}_{1}=\frac{x^{B} q_{2}-\left(F\left(q_{2}\right)-t^{E}\right)\left(q_{1}+\Delta q_{1}\right)}{k}-e_{1} \quad \text { if } \Delta e_{1}+e_{1}<\left(q_{1}+\Delta q_{1}\right) h^{E} \tag{36}
\end{gather*}
$$

$\bar{y}_{1}$ and $\underline{y}_{1}$ are the two branches of the constraint (14). $\bar{y}_{1}$ and $\underline{y}_{1}$ coincide at the $\Delta q_{1}$ value $\delta_{1}=\frac{x^{B} q_{2}}{\left(F\left(q_{2}\right)-t^{E}+k h^{E}\right)}-q_{1}$, while $\bar{y}_{1}>\underline{y}_{1}$ if $\Delta q_{1}<\delta_{1}$.

$$
\begin{align*}
& \bar{z}_{1}=\frac{\left(x^{A}+x^{B}-t^{U}-\varphi^{U} h^{U}\right) q_{2}+\left(k+\varphi^{U}\right) e_{2}-\left(x^{A}+x^{B}-t^{U}\right)\left(q_{1}+\Delta q_{1}\right)}{k}-e_{1} \\
& \text { if } \Delta e_{1}+e_{1}>\left(q_{1}+\Delta q_{1}\right) h^{U}  \tag{37}\\
& \underline{z}_{1}=\frac{\left(x^{A}+x^{B}-t^{U}-\varphi^{U} h^{U}\right) q_{2}-\left(x^{A}+x^{B}-t^{U}-\varphi^{U} h^{U}\right)\left(q_{1}+\Delta q_{1}\right)}{\left(k+\varphi^{U}\right)}+e_{2}-e_{1} \\
& \text { if } \Delta e_{1}+e_{1}<\left(q_{1}+\Delta q_{1}\right) h^{U}
\end{align*}
$$

For the sake of intuition, see Figure 1.
(FIGURE 1 here)
The solid kinked lined represents the two branches of (36), while the dashed kinked line represents the two branches of (37). The increasing
straight lines correspond to the expressions $\Delta e_{1}=\Delta q_{1} h^{E}+\left(q_{1} h^{E}-e_{1}\right)$ and $\Delta e_{1}=\Delta q_{1} h^{U}+\left(q_{1} h^{U}-e_{1}\right)$ The curve represents the differential isocost of the family:

$$
\begin{equation*}
\frac{c}{2}\left(q_{1}+\Delta q_{1}\right)^{2}+r\left(e_{1}+\Delta e_{1}\right)-\frac{c}{2} q_{1}^{2}-r e_{1}=\Delta C \text { ost } \tag{38}
\end{equation*}
$$

Notice that the isocost is tangent to $\bar{y}_{1}$ at $\Delta q_{1}=\Delta e_{1}=0$.
Deviation 1. requires that Source One choose $\Delta q_{1}$ and $\Delta e_{1}$ such that

$$
\begin{equation*}
\min \frac{c}{2}\left(q_{1}+\Delta q_{1}\right)^{2}+r\left(e_{1}+\Delta e_{1}\right) \tag{39}
\end{equation*}
$$

s.t.

$$
\begin{gather*}
\Delta q_{1} \in\left[q_{U}-q_{1}, 1-q_{1}\right] \\
\Delta e_{1} \geq-e_{1} \\
\Delta e_{1}<\bar{y}_{1} \text { if } \Delta e_{1}+e_{1}>\left(q_{1}+\Delta q_{1}\right) h^{E} \\
\Delta e_{1}<\underline{y}_{1} \text { if } \Delta e_{1}+e_{1}<\left(q_{1}+\Delta q_{1}\right) h^{E}  \tag{40}\\
\Delta e_{1} \geq \bar{z}_{1} \text { if } \Delta e_{1}+e_{1}>\left(q_{1}+\Delta q_{1}\right) h^{U} \\
\Delta e_{1} \geq \underline{z}_{1} \text { if } \Delta e_{1}+e_{1}<\left(q_{1}+\Delta q_{1}\right) h^{U} \tag{41}
\end{gather*}
$$

Under deviation 2., the constraints (40) and (41) are replaced by

$$
\begin{gather*}
\Delta e_{1} \geq \bar{y}_{1} \text { if } \Delta e_{1}+e_{1}>\left(q_{1}+\Delta q_{1}\right) h^{E}(a) \\
\Delta e_{1} \geq \underline{y}_{1} \text { if } \Delta e_{1}+e_{1}<\left(q_{1}+\Delta q_{1}\right) h^{E}(b)  \tag{42}\\
\Delta e_{1}<\bar{z}_{1} \text { if } \Delta e_{1}+e_{1}>\left(q_{1}+\Delta q_{1}\right) h^{U} \\
\Delta e_{1}<\underline{z}_{1} \text { if } \Delta e_{1}+e_{1}<\left(q_{1}+\Delta q_{1}\right) h^{U} \tag{43}
\end{gather*}
$$

We can now state and prove the following proposition.
Proposition 2 i) Deviation 1. is not profitable if $E$ is sufficiently large. ii) Deviation 2. is never profitable.

Proof. i) Suppose that both deviations are feasible. In the case of Deviation 1, this means that there exists some pair $\Delta \widetilde{q}_{1}, \Delta \widetilde{e}_{1}$ such that the constraints (40) and (41) are satisfied. Profit from deviation is then $w U-\frac{c}{2}\left(q_{1}+\Delta \widetilde{q}_{1}\right)^{2}-r\left(e_{1}+\Delta \widetilde{e}_{1}\right)$. This is smaller than the equilibrium profit $w(E+U)-\frac{c}{2} q_{1}^{2}-r e_{1}$ if

$$
E>\frac{\frac{c}{2} q_{1}^{2}+r e_{1}-\frac{c}{2}\left(q_{1}+\Delta \widetilde{q}_{1}\right)^{2}-r\left(e_{1}+\Delta \widetilde{e}_{1}\right)}{w}
$$

ii) If Deviation 2. is feasible, there exists at least one pair $\Delta \widehat{q}_{1}, \Delta \widehat{e}_{1}$ such that constraint (42) is satisfied (while (43) is not binding) . By definition, $\Delta q_{1}=\Delta e_{1}=0 \quad$ minimize cost under the constraint (42-a); At $\Delta q_{1}=0, \quad$ the slope of the isocost $\frac{c}{2}\left(q_{1}+\Delta q_{1}\right)^{2}+r\left(e_{1}+\Delta e_{1}\right)-$ $\frac{c}{2} q_{1}^{2}-r e_{1}=0 \quad$ is $\quad \frac{d \Delta e_{1}}{d \Delta q_{1}}=-\frac{c}{r} q_{1}=-\frac{F\left(q_{2}\right)-t^{E}+\varphi^{E} h^{E}}{k-\varphi^{E}}$, i.e. the isocost is tangent to (42-a). Since $\frac{d \Delta e_{1}}{d \Delta q_{1}}$ is decreasing in absolute value, the isocost lies entirely below $\bar{y}_{1}$, for any $\Delta q_{1}$. For $\Delta q_{1}>\delta_{1}$ the relevant branch of the (42) constraint is (b). But for $\Delta q_{1}>\delta_{1}, \underline{y}_{1}$ lies entirely above $\quad \bar{y}_{1}$. Then, independently of whether $\left(\Delta \widehat{q}_{1}+q_{1}\right) h^{E}$ is smaller or larger then $\quad e_{1}+\Delta \widehat{e}_{1}$, the cost of the deviation, i.e. $\frac{c}{2}\left(q_{1}+\Delta \widehat{q}_{1}\right)^{2}+r\left(e_{1}+\Delta \widehat{e}_{1}\right)$ is not smaller than $\frac{c}{2} q_{1}^{2}+r e_{1}$. Then the profit from deviation is $w E-\frac{c}{2}\left(q_{1}+\Delta \widehat{q}_{1}\right)^{2}+r\left(e_{1}+\Delta \widehat{e}_{1}\right)$ which is smaller than the equilibrium profit: $w(E+U)-\frac{c}{2} q_{1}^{2}-r e_{1}$. This result holds for any value of $E$.

Let us consider the possible deviations for source Two.
3. Source Two keeps the educated reader market and steals the uneducated reader market from Source One
4. Source Two gives up the educated reader market and steals the uneducated reader market from Source One

Let us define

$$
\begin{gather*}
\underline{y}_{2}=\frac{x^{B} q_{1}-\left(F\left(q_{1}\right)-t^{E}\right)\left(\Delta q_{2}+q_{2}\right)}{k}-e_{2} \quad \text { if }\left(e_{2}+\Delta e_{2}\right)<\left(\Delta q_{2}+q_{2}\right) h^{E} \\
\bar{y}_{2}=\frac{x^{B} q_{1}-\left(F\left(q_{1}\right)-t^{E}+\varphi^{E} h^{E}\right)\left(\Delta q_{2}+q_{2}\right)}{k-\varphi^{E}}-e_{2} \quad \text { if }\left(e_{2}+\Delta e_{2}\right)>\left(\Delta q_{2}+q_{2}\right) h^{E} \tag{44}
\end{gather*}
$$

$\bar{y}_{2}$ and $\underline{y}_{2}$ are the two branches of the constraint (19). $\bar{y}_{2}$ and $\underline{y}_{2}$ coincide at the $\Delta q_{2}$ value $\delta_{2}=\frac{x^{B} q_{1} h^{E}-\left(F\left(q_{1}\right)-t^{E}+k h^{E}\right) e_{2}}{F\left(q_{1}\right)-t^{E}+k h^{E}}$, while $\bar{y}_{2}>\underline{y}_{2}$ if $\Delta q_{2}<\delta_{2}$.

Define also:

$$
\begin{align*}
& \underline{z}_{2}=\frac{\left(x^{A}+x^{B}-t^{U}\right) q_{1}+k e_{1}-\left(x^{A}+x^{B}-t^{U}-\varphi^{U} h^{U}\right)\left(\Delta q_{2}+q_{2}\right)}{k+\varphi^{U}}-e_{2} \\
& \text { if } \varphi^{U}\left(\left(\Delta q_{2}+q_{2}\right) h^{U}-\left(e_{2}+\Delta e_{2}\right)\right)>0 \\
& \quad \bar{z}_{2}=\frac{\left(x^{A}+x^{B}-t^{U}\right) q_{1}-\left(x^{A}+x^{B}-t^{U}\right)\left(\Delta q_{2}+q_{2}\right)}{k}+e_{1}-e_{2}  \tag{45}\\
& \quad \text { if } \varphi^{U}\left(\left(\Delta q_{2}+q_{2}\right) h^{U}-\left(e_{2}+\Delta e_{2}\right)\right)<0
\end{align*}
$$

See Figure 2.
(FIGURE 2 here)
The solid kinked lined represents the two branches of (44), while the dashed kinked line represents the two branches of (45). The increasing straight lines correspond to the expressions $\Delta e_{1}=\Delta q_{1} h^{E}+\left(q_{1} h^{E}-e_{1}\right)$
and $\Delta e_{1}=\Delta q_{1} h^{U}+\left(q_{1} h^{U}-e_{1}\right)$ The curve represents the differential isocost of the family:

$$
\begin{equation*}
\frac{c}{2}\left(q_{2}+\Delta q_{2}\right)^{2}+r\left(e_{2}+\Delta e_{2}\right)-\frac{c}{2} q_{2}^{2}-r e_{2}=\Delta \text { Cost }_{2} \tag{46}
\end{equation*}
$$

Notice that the isocost $\Delta$ Cost $_{2}=0$ is tangent to $y_{2}$ at $\Delta q_{2}=\Delta e_{2}=$ 0.

Deviation 3. requires that source Two chooses $\Delta q_{2}, \Delta e_{2}$ such that

$$
\min \left(\frac{c}{2}\left(\Delta q_{2}+q_{2}\right)^{2}+r\left(e_{2}+\Delta e_{2}\right)\right)
$$

s.t.

$$
\begin{align*}
& \Delta e_{2} \geq \underline{z}_{2} \text { if } \varphi^{U}\left(\left(\Delta q_{2}+q_{2}\right) h^{U}-\left(e_{2}+\Delta e_{2}\right)\right)>0 \\
& \Delta e_{2} \geq \bar{z}_{2} \text { if } \varphi^{U}\left(\left(\Delta q_{2}+q_{2}\right) h^{U}-\left(e_{2}+\Delta e_{2}\right)\right)<0 \tag{47}
\end{align*}
$$

and,

$$
\begin{align*}
& \Delta e_{2} \geq \underline{y}_{2} \text { if }\left(e_{2}+\Delta e_{2}\right)<\left(\Delta q_{2}+q_{2}\right) h^{E}(a) \\
& \Delta e_{2} \geq \bar{y}_{2} \text { if }\left(e_{2}+\Delta e_{2}\right)>\left(\Delta q_{2}+q_{2}\right) h^{E}(b) \tag{48}
\end{align*}
$$

Under deviation 4. (48) is no longer binding.
Proposition 3 i) Deviation 3 is not profitable for $U$ sufficiently small. ii) Deviation 4. is not profitable if $E$ is sufficiently large.

Proof. i) If Deviation 3. is feasible, then there exists at least one pair $\Delta \widetilde{q}_{2}, \Delta \widetilde{e}_{2}$ satisfying (47) and (48). Such a deviation is not profitable if the differential revenue from deviation $(w U)$ is smaller than the differential cost associated to the deviation. Then it must be $U<$ $\frac{\left(\frac{c}{2}\left(\Delta \widetilde{q}_{2}+q_{2}\right)^{2}+r\left(e_{2}+\Delta \widetilde{e}_{2}\right)\right)-\left(\frac{c}{2} q_{2}^{2}+r e_{2}\right)}{w}$. Notice that $\Delta q_{2}=\Delta e_{2}=$ 0 minimizes cost under (48-a)), and that the isocost $\left(\frac{c}{2}\left(\Delta q_{2}+q_{2}\right)^{2}+r\left(e_{2}+\Delta e_{2}\right)\right)$ lies entirely below $\underline{y}_{2}$ for any $\Delta q_{2}$ value. Since $\underline{y}_{2}$ lies below $\bar{y}_{2}$ for $\Delta q_{2}<\delta_{2}$, then $\frac{c}{2}\left(\Delta \widetilde{q}_{2}+q_{2}\right)^{2}+r\left(e_{2}+\Delta \widetilde{e}_{2}\right) \geq\left(\frac{c}{2} q_{2}^{2}+r e_{2}\right)$.
ii) If Deviation 4. is feasible, then there exists at least one pair $\Delta \bar{q}_{2}, \Delta \bar{e}_{2}$ satisfying (47). Such a deviation is not profitable if the differential revenue from deviation $(w(U-E))$ is smaller than the differential cost associated to the deviation. Then it must be $E>$ $U-\frac{\left(\frac{c}{2}\left(\Delta \bar{q}_{2}+q_{2}\right)^{2}+r\left(e_{2}+\Delta \bar{e}_{2}\right)\right)-\left(\frac{c}{2} q_{2}^{2}+r e_{2}\right)}{w}$

Intuitively, the core implication of the equilibrium we described is that the existence of a sufficiently large class of educated readers is necessary to ensure the coexistence of two sources with different information/entertaiment mixes. Conversely, somehow surprisingly, the
existence conditions impose an upper bound to the size of the uneducated readers population only. More precisely the incentive to deviate of source 1 is not affected by the size of uneducated population: notice that source 1 is already on both markets at the minimum required cost, so that giving up the uneducated market entails a loss of revenue without any saving in costs. There is another interesting point. At the equilibrium, the uneducated readers are strictly better off when reading report One than when reading Report Two. Suppose that $E$ is not sufficiently large, so that deviation 1. becomes profitable. Then Source One would choose levels of $q_{1}$ and $e_{1}$ which would make the uneducated readers utility exactly equal to the one they would attain by reading report Two: then they would be worse off relative to the equilibrium. The existence of a large class of educated readers is therefore beneficial also for the uneducated.

## 5 Conclusions

In this paper we present a model which analyzes, as an equilibrium outcome, a market configuration where the audience for a medium, entirely financed by advertising, is split between two sources, one being a "mass" source and the other one an "élite" source. We do not claim it is the only equilibrium for this model, while we focus on its interesting properties and implications. The élite report differs from the mass one in that it provides higher-quality information, while the mass report is richer in services other than information. The main result is that the equilibrium is supported if the size of the educated reader population is sufficiently large. One further result states that the size of the uneducated population must not be too large: the intuitive reason is that a large uneducated population would induce all the sources to prefer the uneducated market, while neglecting the educated one. The existence of a large class of educated readers has at least two beneficial effects on the performance of the media market. First, as we stressed in the last section, a large population of educated readers positively affects even the utility of the uneducated population. Second, educated readers benefit from information coming from two independent sources: this corresponds to what, in a less formal language, we call the virtues of pluralism. This concept often appears in the media economics literature, but it is usually referred to the supply side structure (Battaggion and Vaglio (2007)). Here we show that the extent to which pluralism is actually exploited depends on the structure of the demand side.

Our results rest on some simplifying restrictions, described in the previous sections. Here we focus on the assumption of quadratic cost function for information quality, vis à vis the linearity assumption re-
garding the cost function for entertainment. The cost structure we assumed is at the same time very simple and it allows for interior solutions, in particular as regards the $q$ values. Interior solutions would be instead ruled out by the assumption a linear cost function to quality. A more crucial set of assumptions refers to reader preferences. We assume that the propensity to read depends on the reading skills of the individual and that individuals differ both as regards the reading skills and the most preferred entertainment/information mix. These assumptions provide a behavioral foundation for a relatively standard indifference curve representation, characterized by a decreasing marginal rate of substitution between entertainment and information quality. We assumed also that sources are entirely financed by advertising, while readers bear just the opportunity costs related to reading. Many media markets actually work this way (think for example of TV and free press). However our analysis could be refined by introducing a positive report price on top of the opportunity cost.

Finally, our model offers a number of suggestions for empirical inquiry. While the structure of the model is inspired by a set of simple stylized facts (heterogeneity of sources and readers and reading-education relationship) much remains to be investigated at the empirical level. For instance, the size and the composition of the population accessing multiple sources of information should be measured; the preferences over the information/entertainment mix need a detailed description; the notion of entertainment itself should defined in a more operational way. A last remark is in order as regards the range of media to which the model is reasonably suited. In the paper compared different newspapers for ease of exposition, but in principle the model might apply to the comparison between, say, two broadcasters or again, to a newspaper versus a TV channel.

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| TABLE 1 |  |  |
| :--- | :--- | :--- |
|  | \% who Read <br> Daily | \% who Read <br> Sunday |
| Post graduate | 68 | 74 |
| College Graduate | 60 | 69 |
| Some College | 56 | 66 |
| High School graduate | 51 | 60 |
| Less than High School <br> graduate | 34 | 40 |
| Source: Newspaper Association of America 2004 |  |  |


| TABLE 2 |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Sections | Master's <br> Degree | Bachelor's <br> Degree | Some <br> College | High school <br> graduate |
| Main News/Front <br> Page | $94 \%$ | $91 \%$ | $88 \%$ | $86 \%$ |
| International/National <br> News | $73 \%$ | $66 \%$ | $52 \%$ | $48 \%$ |
| Editorial Page | $57 \%$ | $47 \%$ | $39 \%$ | $38 \%$ |
| Comics | $34 \%$ | $34 \%$ | $41 \%$ | $42 \%$ |
| TV listings | $29 \%$ | $29 \%$ | $29 \%$ | $37 \%$ |
| Advertising | $53 \%$ | $54 \%$ | $65 \%$ | $69 \%$ |
| Source: Newspaper Association of America 2008 |  |  |  |  |


| TABLE 3 |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $x^{A}=100, x^{B}=10, r / c=3,333, t^{E}=2,5, t^{U}=5, h^{E}=0,88, h^{U}=0,883, \phi^{E}=0,5, \phi^{U}>0,271538$ |  |  |  |  |  |  |  |  |  |
| K | $\mathrm{q}_{1}$ | $\mathrm{q}_{2}$ | $\mathrm{e}_{1}$ | $\mathrm{e}_{2}$ | $\mathrm{e}_{1} / \mathrm{q}_{1}$ | $\mathrm{e}_{2} / \mathrm{q}_{2}$ | VII | VI1 (E) | VI2(E) |
| 25,4* |  |  |  |  |  |  |  |  |  |
| 25,5 | 0,997487 | 0,999859 | 0,09843 | 0,08827 | 0,098682 | 0,088239 | 185,0049 | 107,4848 | 107,2298 |
| 25,6 | 0,997177 | 0,999549 | 0,096905 | 0,086765 | 0,097179 | 0,086804 | 184,9507 | 107,4515 | 107,1965 |
| 25,7 | 0,996867 | 0,99924 | 0,095389 | 0,085316 | 0,095689 | 0,085381 | 184,8966 | 107,4183 | 107,1632 |
| 25,8 | 0,996558 | 0,99893 | 0,093887 | 0,08388 | 0,094211 | 0,08397 | 184,8424 | 107,3850 | 107,1299 |
| 25,9 | 0,996248 | 0,998621 | 0,092397 | 0,082457 | 0,092745 | 0,08257 | 184,7883 | 107,3518 | 107,0967 |
| 26 | 0,995939 | 0,998312 | 0,090921 | 0,081045 | 0,091292 | 0,081 182 | 184,7343 | 107,3186 | 107,0634 |
| 26,1 | 0,99563 | 0,998004 | 0,089458 | 0,079646 | 0,08985 | 0,079805 | 184,6802 | 107,2854 | 107,0302 |
| 26,2 | 0,995321 | 0,997695 | 0,088007 | 0,078259 | 0,088421 | 0,078439 | 184,6262 | 107,2522 | 106,9970 |
| 26,3* |  |  |  |  |  |  |  |  |  |
| VII: value of the information for the educated reader reading two reports (One and Two) VI1(E): value of information for the educated reader reading report One VI2(E): value of information for the educated reader reading report Two <br> * Existence conditions are not satisfied. |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |



FIGURE 1


FIGURE 2

