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Dipartimento di Scienze Economiche
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Antitrust Policy and Collusion through Credible Covenants

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Abstract

This paper presents a political economy model of antitrust policy against horizontal price-fixing. The policy is implemented through discretion. In the event of collusion the public agency can enforce competition through fines and behavioral constraints. The paper shows that while fines do not constitute an incentive to investigate in the event of collusion when the policy is implemented through discretion, behavioral constraints are an effective tool in limiting collusion. However firms can strategically induce that no policy is implemented along the equilibrium path by making a credible “covenant” that little degree of collusion will be implemented today *and* in the future. Moreover, if firms have limited information about agency’s costs, social welfare rises up, while if the agency has limited information about production costs, the efficient cartel type increase its rents.

JEL classification: L13, L41

Keywords: credible covenants, collusion costs, antitrust policy, horizontal price-fixing, behavioral constraints.

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1 Introduction

Among the several activities classified as “antitrust policy”, one of the most important is that implemented to reduce the firms’ incentive to form a cartel. In the vast majority of the industrialized economies there exist, nowadays, laws which forbid, with exceptions¹, business practices that involve price-fixing, and public agencies in charge of the laws’ enforcement.² These agencies have fundamentally two weapons to fight horizontal price-fixing: *fin*es and *behavioral constraints*. The magnitude of the former varies according to the importance of the detected illegal behavior and is usually related to profits (especially in the US) or to sales (Euroland). Souam [1998,2000] has studied these two fine regimes and has shown that fines related to sales are more efficient, in welfare terms, than fines linked with profits when rents achievable through collusion are not high.³

Behavioral constraints are instead prohibitions to perform some illegal acts imposed to firms, and they are usually monitored by the public agencies for some time after the final decision.⁴ These constraints have a direct effect on firms’ market decisions, since they modify, through agency’s monitoring, market conditions. As shown firstly by Becker [1968], fines and behavioral constraints act especially as deterrent to behave illegally, i.e. in this case to form a cartel. However they have different effects on the agency’s objective function, i.e. on social welfare. Fines are a pure monetary transfer from colluding firms to consumers, and so they do not represent for the public agency an incentive to fight price-fixing once it has been observed.⁵ Behavioral constraints instead, by making collusion

¹Cooperation between firms might be allowed under antitrust laws when these firms demonstrate, during an investigation, that cooperation between competing firms on a particular strategy leads to an increase in social welfare. For instance, a trade association can issue price lists when it demonstrates that they reduce disruptive competition and protect quality; some companies might demonstrate that using a unique channel for purchasing inputs may reduce factors’ prices and so the price of the good they sell.

²The implementation of antitrust laws might be different from country to country; generally speaking, in all countries there exist public agencies in charge of competition policy. In Euroland these agencies have both detective and decisional power. In the US they have only detective power, since Courts are the subjects in charge of the decision.

³Achievable cartel profits are those obtained along the equilibrium path in a model where the public agency in charge of the policy has limited information about the cartel’s productive efficiency. When the efficiency gap between the most efficient cartel’s type and the less efficient one is small, the profit levels achievable through collusion are not high.

⁴For instance, the final decision might include the prohibition to exchange information about costs, or to issue price lists, or to impose vertical restraints to retailers in order to reduce incentive to deviate between colluding firms.

⁵A fine does not modify the social welfare. Hence even if collusion is detected, the social welfare is the same if antitrust policy is implemented or not.

more difficult since members' activities are monitored, yield an incentive for the agency to fight price-fixing even if the latter has occurred. These constraints have indeed a positive dynamic effect, since they increase the costs of collusion⁶ and so lead to an increase in social welfare.

The idea that behavioral constraints yield positive welfare effects finds confirmation in a rich empirical literature on the consequences of antitrust policy against price-fixing. Feinberg [1980]⁷ and [1987]⁸ found that capital-adjusted price-cost margins in 1970 were significantly lower, *ceteris paribus*, for firms indicted for price-fixing between 1955 and 1970. The estimated effect on after-indictment prices is lowering the Lerner Index by two percentage points. The same results have been found by Block, Nold and Sidak [1981];⁹ they underscored that increases in the Antitrust Division's inflation-adjusted budget have a significant negative effects on markups of white bread.¹⁰ Choi and Philippatos [1983] have obtained the same result in a study on a sample of U.S. large firms indicted for violations of Section I of the Sherman Act between 1958-72, matched with a group of unindicted firms.¹¹

To the best of our knowledge, the theory on antitrust economics has not taken into account yet this second weapon available to fight collusion, while it has instead concentrated the analysis on the role of fines. As shown by Besanko and Spulber [1989], Souam [1998,2000] and Martin [1998]¹² this assumption leads to analyze the competition policy

⁶See Bradburd and Over [1982] for a model where the costs of cartel formation and maintenance are assumed to increase with the number of firms in the industry. Alexander [1994] presents a model where these costs are also function of the antitrust activity.

⁷He analyzed a sample of U.S. 288 large manufacturing firms.

⁸He considers in this study the timing of antitrust effects on pricing, and identifies two types of effects: the deterrent effects of antitrust past indictments (firms think that the public agency will screen past offenders), and the firms' strategic reaction (minimizing the probability of conviction and the expected penalties) to an ongoing investigation and indictment.

⁹They studied the effects of antitrust indictments on prices in the U.S. white bread industry. They look at prices for white pan bread across 208 observations: 12 major cities for 12 years (1965-76) and other major cities for 8 years (1969-76).

¹⁰In addition, Department of Justice's price-fixing prosecutions in the bread industry have negative effects on markups in the *region* (and the year) in which the case is filed, and a larger (what they call remedial) negative effect in the city in which the action occurs, *the year following the start of the case*. Their interpretation about these results is that, once discovered and prosecuted, colluding firms remedy by reducing their markups in the following period.

¹¹They showed that indicted firms do suffer for a reduction in profits after the indictment, and that this result applies only if the firms are indicted for the first time. Once firms get used to the antitrust process, they do not care too much about it, and its enforcement power is much lower. It seems that the only effect is in this case the monetary fine.

¹²Besanko and Spulber supplied the first model of antitrust policy implemented by a public agency

implemented by a public agency as an optimal contract between the principal (the agency) and the agent (the cartel).¹³ The strategic interaction between them¹⁴ is then modeled as a sequential game, where the agency commits itself to a schedule of probability of investigation that is function of the price (or quantity) observed. This commitment is announced *ex-ante* to firms; the latter, knowing the above probability distribution, decide whether to form a cartel or not. Last, the agency sees the market price and implements *ex-post* the behavior announced *ex-ante*.

It is essential to underscore that under this approach the agency's threat to investigate, in order to be credible, has to be linked with the possibility to commit to an *ex-ante* announced behavior. Indeed in the above papers, where only the role of fines is considered, we will never observed an investigation in absence of commitment, since the social welfare is the same either in case of investigation and conviction or in case of *laissez faire*. Only the assumption of commitment makes credible the implementation *ex-post* of what has been announced at the beginning of the sequential game.

However this approach, on the one hand, does not take into account that the public agency has also a second weapon (behavioral constraints) available; on the other hand, its crucial assumption (commitment) looks rather unrealistic if we observe how the antitrust policy is implemented in reality. The latter is indeed carried out with discretion, after a signal received by the public agency. This signal normally coincides with a report made by a single consumer, or by an association, or by a competing firm.¹⁵ For instance, among the 31 illegal agreements convicted by the Italian antitrust authority during the period 1997–1999, 17 (54.8%) were started by private agents.¹⁶ This evidence is even more robust in the US, where the ratio between the so-called private litigation (antitrust cases initiated by private subjects) and public litigation is 10:1.¹⁷ Moreover, the unique announcement made

with limited information about the cartel's costs, Souam enlarged this model by introducing a continuum set of possible cartel's types and investigated the effects on social welfare of alternative regimes of fines, Martin analyzed the agency's optimal resource allocation in a framework where the public agency sets up a intervention threshold price.

¹³Harrington [2002a, 2002b] has recently provided some contributions about the effects of antitrust enforcement (i.e. damages and fines) on the cartel's optimal pricing in a dynamic setting. However in his work there is no strategic interaction among the firms and the public agency in charge of the competition policy.

¹⁴The agency and the cartel have conflicting goals, since the former wants to maximize the social welfare (i.e. it aims at marginal costs pricing) while the latter's objective is to reach the monopoly profits.

¹⁵Under some circumstances the agency itself starts an investigation after having observed a pattern of high prices, but not because it has announced it before.

¹⁶The remaining cases were initiated by the agency itself after having observed, as it is declared in the final report, some behaviors usually correlated to collusion (e.g. price parallelism).

¹⁷The analysis of private antitrust litigation has been done by Salant [1987], Baker [1988] and Besanko

ex-ante consists in what it is declared in the antitrust laws, where usually agreements made in order to fix prices are considered illegal (Grillo [2001]). Threshold intervention prices or commitments to a probability of investigation are not announced.¹⁸

A more comprehensive analysis of antitrust policy must then design it as a signaling game, where the sender (the cartel) delivers a signal and the receiver (the agency), after having observed it, decides whether to investigate or not. On the one hand this approach allows to implement the policy on a case-by-case ground, as it is generally accepted nowadays by policy makers; on the other hand, investigation *ex-post* can be a credible threat because the agency can also impose some behavioral constraints, which have a positive direct effect on welfare. This paper presents a model of antitrust economics against horizontal price-fixing based upon a two-stage signaling game, where in case of collusion the cartel has to pay a fine and it is subject to some behavioral constraints, that make collusion more expensive. Since collusion becomes, after a conviction in the first stage of the game, more expensive, social welfare in the second stage increases.

The analysis focuses on three scenarios: firstly, we study the optimal policy in case of perfect information. Then asymmetric information is added to the model: in one scenario the cartel has limited information about the agency's policy costs (while the agency has perfect information about the cartel efficiency state). This scenario supplies some insights on a situation where there exists some uncertainty about the public agency's ultimate objective when fighting price-fixing. Low (high) policy costs mean that the agency has strong (weak) preferences towards marginal costs pricing, and so allocating resources to fight collusion yields a low (high) opportunity cost. In the other scenario the agency has limited information about the cartel's efficiency state, so that firms can exploit this strategic advantage to increase their rents.

We will show that a little degree of collusion is *always* tolerated. However, in contrast with Besanko and Spulber [1989] and Souam [1998,2000], both in case of perfect and imperfect information the antitrust policy implemented through discretion is effec-

and Spulber [1990]. The first two articles highlight that antitrust policy against price-fixing is neutral in terms of social welfare when it is delegated to consumers in presence of perfect information. Indeed the perspective of a "treble damage reward" (as provided under the US laws) pushes consumers to get more damaged today in order to get a higher reward tomorrow. This "perverse" incentive makes private litigation ineffective. Besanko and Spulber [1990] showed instead that, in presence of limited information about production costs, private antitrust litigation are effective in terms of social welfare. The uncertainty about the final reward reduces the perverse incentive.

¹⁸Merger policy is an exception: in this case there might exist an *ex-ante* declaration. For instance in the US the Department of Justice has compiled the so-called Merger Guidelines, that establish ranges of the Herfindal Index (before and after the merger) which trigger an antitrust investigation. In this case the commitment assumption is realistic.

tive in limiting collusion. While previous contributions show that a case-by-case policy will never be implemented by a public agency without a commitment, the presence of behavioral constraints make fighting collusion a credible threat even without an *ex-ante* announcement. Furthermore, in presence of perfect information, the cartel can make positive profits only if it can credibly charge (produce) the same price (quantity) in both periods. This price is lower the smaller are the agency's policy costs. If the implicit "covenant" between the cartel and the agency to maintain the same price in the future is not credible the cartel will be investigated and convicted. This equilibrium can become less favorable, in social welfare terms, if the agency has limited information about production costs. Under this scenario there only exists a pooling equilibrium, where the most efficient type exploits the informational advantage, and enjoys an higher degree of collusion than that obtained with perfect information. This results is upset with limited information about the agency's costs; under this environment the social welfare increases if compared with the perfect information case. The more likely is the possibility that the agency has low costs the more the cartel will co-ordinate on a low price level.

The paper is organized with the following structure: Section 2 presents the model, Section 3 shows the optimal policy in case of perfect information, Section 4 presents the optimal policy when the cartel has limited information about the agency's policy costs, Section 5 displays the optimal policy in case of limited information about the cartel's efficiency state. Section 6 summarizes the paper, while in the Appendix are reported all the proofs.

2 The model

We consider a two-stage model where an industry composed by N risk neutral firms produces an homogeneous good, with market demand $p = a - bq$ ($q = \sum_{i=1}^N q_i$); the latter is the same in both periods and is common knowledge. Firms have a common cost function $C_i(q_i) = \theta q_i$ ($i = 1, \dots, N$). As in Besanko and Spulber [1989] and in Souam [1998], firms compete *à la* Bertrand and decide whether to collude or not both at $t = 1$ and at $t = 2$, the last period of the game. If they do not collude, as in a standard finitely repeated game, they replicate in each stage the Bertrand equilibrium of the static game, i.e. $p^c = \theta$, $q^c = \frac{a-\theta}{b}$, $q_i^c = \frac{a-\theta}{bN}$, and make normal profits.

In case of collusion they sustain, as in Alexander [1994], the costs of forming a cartel, equal to $c(q^c - q) + \Omega$. These costs are split into variable and fixed costs: variable costs, $c(q^c - q)$, are inversely related with production, i.e. they depend positively upon the degree of collusion. The higher is the latter the greater is the incentive to deviate and so the

higher are the costs of organization, control and maintenance of collusion. Fixed costs, Ω , are the costs of changing decision.¹⁹ Hence at $t = 1$, if a cartel is formed its costs are $\theta q_1 + c(q^c - q_1)$; at $t = 2$ they are instead

$$\begin{cases} \theta q_1 + c(q^c - q_1) & \text{if } q_2 = q_1 \\ \theta q_2 + c(q^c - q_2) + \Omega & \text{if } q_2 \neq q_1 \end{cases}$$

If cartels are legal firms maximize industry profits $\pi = (a - bq)q - \theta q - c(q^c - q)$; hence

$$\frac{d\pi}{dq} = a - 2bq - \theta + c = 0$$

Solving the above for q we get $\tilde{q} = \frac{a-\theta+c}{2b}$, with $q_1 = q_2 = \tilde{q}$. In each period cartel's profits are $\pi(\tilde{q}, \theta) = \frac{(a-\theta-c)^2}{4b}$, and total profits are $(1 + \delta)\pi(\tilde{q}, \theta)$, where δ is the discount factor, which is assumed to be the same for the cartel and the agency. If instead cartels are forbidden and a public agency is in charge of antitrust policy, the sequence of events in the two-stage game is the following: at $t = 1$ the industry decides whether to collude or not; if a cartel is formed the agency observes q_1 and, knowing market demand, chooses whether to investigate (action $\{i\}$) or not (action $\{ni\}$). If the agency investigates the cartel is convicted to pay a fine $A(q_1, \theta) = m[p(q) - \theta]q$ ($m > 1$), i.e. a multiple of its profits, to compensate the damage suffered by consumers.²⁰ Moreover, some behavioral constraints are imposed to the members, so that cartel's costs increase. Hence after a conviction at $t = 1$ cartel profits at $t = 2$ are equal to $\pi = (a - bq_2)q_2 - \theta q_2 - d(q^c - q_2) - \Omega$, with $d > c$. If the agency investigates it has to pay a fixed cost K . At $t = 2$, both the firms and the agency observe the outcome of the first stage and decide which actions to take in the last stage. First firms decide q_2 , then the agency sees it and chooses between $\{i\}$ and $\{ni\}$.

The agency's objective function is social welfare $W(q, \theta) = \int_0^q p(t)dt - \theta q$. If Bertrand competition takes on at both periods, social welfare is at its maximum, i.e. $W(q^c, \theta) = \frac{(a-\theta)^2}{2b} (1 + \delta)$; if cartels are legal, social welfare is $W(\tilde{q}, \theta) = \frac{a-\theta+c}{8b} [3(a - \theta) - c] (1 + \delta)$. A strategy for the cartel is then given by a pair of output levels $\{q_1, q_2\}$, a strategy for the agency is, in each period, the choice of a single action within the set $\{i, ni\}$. Before computing the optimal policy under perfect information, we state two Lemmas which simplify the analysis.

Lemma 1 *At $t = 2$ the agency best reply to every cartel's decision is $\{ni\}$.*

¹⁹When a cartel needs to change the price, its members have to meet, spend time to reach a new agreement and so on. Moreover, there also exist adjustment production costs.

²⁰The analysis can be extended to a regime of fine proportional to the cartel's sales; in this case $A'(q, \theta) = \alpha p(q)q$ ($\alpha > 0$).

Proof: see Appendix.

Lemma 1 shows that in the last stage of the game the agency has a dominant strategy: it will never investigate. Indeed the unique remedy in the event of collusion at the last stage of the game is the fine. Since the latter is a pure monetary transfer from producers to consumers the social welfare is not modified by the investigation; hence the agency has no *ex-post* incentive to investigate even if collusion is detected.

Lemma 2 *If at $t = 1$ $q_1 = q^c$ the agency chooses $\{ni\}$.*

Proof: see Appendix.

The agency knows market demand and so it can spot a competitive output; in this case any investigation is useless, since no fines and no behavioral constraints can be imposed. Hence producing the competitive output at $t = 1$ and then $q_2 = \tilde{q}$ is a permanent option available to the cartel, whose profits amount to, in this case, $\delta\pi(\tilde{q}, \theta)$.

3 Perfect information

The aim of this Section is to show the optimal policy when there is perfect information. First we compute the cartel's output level after a conviction for illegal price-fixing at $t = 1$. The total industry profit is maximized when

$$\frac{d\pi}{dq} = a - 2bq - \theta + d = 0$$

Solving for q we get $\hat{q} = \frac{a-\theta+d}{2b} > \tilde{q}$.²¹ The behavioral constraints imposed by the agency increase the collusion costs and so the cartel is forced to raise up its output; since the *post*-conviction output is greater than the *before*-conviction one, the society's deadweight loss is smaller after the investigation, i.e. $W(\hat{q}, \theta) \rightarrow W(q^c, \theta)$ as $d \rightarrow a - \theta$. However if $d < a - \theta$ then $\hat{q} < q^c$. The extensive form of the game is shown in Figure 1.

Solving the game by backwards induction, we have to identify the cartel decision at node I_3 .²² Here the decision is influenced by the presence of the fixed costs of changing decision, i.e. Ω . The cartel profits are indeed the following

$$\pi = \begin{cases} \frac{(a-\theta-c)^2}{4b} - \Omega & \text{if } q_2 \neq q_1 \\ (a - bq_1)q_1 - \theta q_1 - c(q^c - q_1) & \text{if } q_2 = q_1 \end{cases} \quad (1)$$

²¹We assume that $\pi(\hat{q}, \theta) > 0$, i.e. $\frac{(a-\theta-d)^2}{4b} > \Omega$.

²²At node I_2 the Nash equilibrium, since the agency will not investigate, is to play $q_2 = \tilde{q}$, while at node I_4 profit are maximized if $q_2 = \hat{q}$.

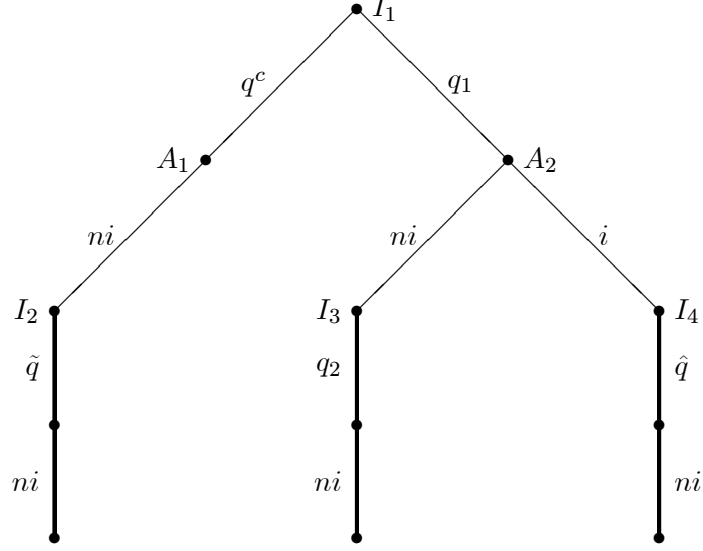


Figure 1: The extensive form of the antitrust game with perfect information

Note that if $q_2 \neq q_1$, by Lemma 1 the Nash equilibrium at I_3 yields $q_2 = \tilde{q}$. The latter will then be the cartel's optimal decision if the following condition is satisfied

$$\frac{(a - \theta - c)^2}{4b} - \Omega \geq (a - bq_1)q_1 - \theta q_1 - c(q^c - q_1) \quad (2)$$

Figure 2 plots condition (2); by inspection the cartel's best reply at I_3 is the following

$$q_2^* = \begin{cases} \tilde{q} & \text{if } 0 \leq q_1 < q_1^2 \text{ or } q_1^1 < q_1 \leq q^c \\ q_1 & \text{otherwise} \end{cases} \quad (3)$$

where (the suffix 1 indicates the first root of equation (2))

$$q_1^2 = \frac{a - \theta + c}{2b} - \left(\frac{\Omega}{b}\right)^{1/2} \quad (4)$$

and (the suffix 2 indicates the second root of equation (2))

$$q_1^1 = \frac{a - \theta + c}{2b} + \left(\frac{\Omega}{b}\right)^{1/2} \quad (5)$$

Note that if $\Omega \uparrow$ then $q_1^1 \rightarrow q^c$; the higher the fixed cost of changing decision the larger is the interval where the cartel finds optimal to keep the production level fixed at both periods.

Taking into account the cartel's best reply at I_3 shown in (3), we can now investigate the agency's decision at A_2 , the unique node where the antitrust policy can be

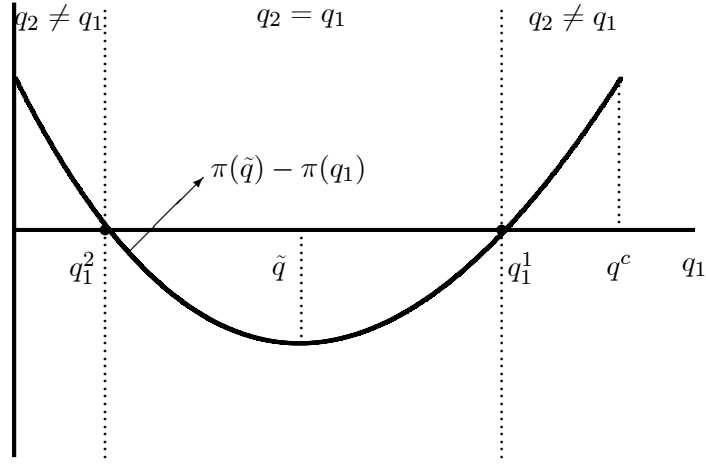


Figure 2: Profits differential at $t = 2$ if $q_2 \neq q_1$

implemented. If the agency chooses action $\{i\}$ social welfare is $W(q_1, \theta) - K + \delta W(\hat{q}, \theta)$, with

$$W(\hat{q}, \theta) = \frac{a - \theta + d}{8b} [3(a - \theta) - d]$$

while if it selects option $\{ni\}$ social welfare is function of q_1 , as described in (3), i.e.

$$\begin{cases} W(q_1, \theta) + \delta W(\tilde{q}, \theta) & \text{if } 0 \leq q_1 < q_1^2 \text{ or } q_1^1 < q_1 \leq q^c \\ (1 + \delta)W(q_1, \theta) & \text{otherwise} \end{cases} \quad (6)$$

Expression (6) shows that we have two possibilities at A_2 ; if $0 \leq q_1 < q_1^2$ or $q_1^1 < q_1 \leq q^c$ investigation is a best reply if the following condition holds

$$\delta[W(\hat{q}, \theta) - W(\tilde{q}, \theta)] > K \quad (7)$$

if instead $q_1^2 \leq q_1 \leq q_1^1$ to investigate is a best reply if

$$\delta[W(\hat{q}, \theta) - W(q_1, \theta)] > K \quad (8)$$

Note that the last condition depends upon the cartel's strategic behavior at $t = 1$; hence no investigation might be a best reply if the cartel can produce a level of output at the first period such that inequality (8) does not hold, and if $q_2 = q_1$. We can instead assume that inequality (7) is always fulfilled; indeed if the opposite is true the agency has not enough incentive to investigate today, and so getting a social benefit tomorrow where $q_2 = \hat{q}$, when it compares the difference between social welfare with behavioral constraints and social welfare with *laissez faire*. If inequality (7) is not fulfilled antitrust policy is a trivial

matter. Hence if $0 \leq q_1 < q_1^2$ or $q_1^1 < q_1 \leq q^c$ investigation is always a best reply.²³ We have instead to identify the q_1 -range where expression (8) is fulfilled. Rewriting it as

$$\delta[W(\hat{q}, \theta) - (a - \theta)q_1 + \frac{b}{2}q_1^2] > K$$

and solving for q_1 we get

$$q = \frac{(a - \theta)}{b} \pm \frac{\sqrt{\delta^2(a - \theta)^2 - 2\delta^2bW(\hat{q}, \theta) + 2\delta bK}}{\delta b} \quad (9)$$

and since no output higher than q^c will be produced, we define

$$\bar{q} = q^c - \frac{\sqrt{\delta^2(a - \theta)^2 - 2\delta^2bW(\hat{q}, \theta) + 2\delta bK}}{\delta b} \quad (10)$$

and inequality (8) is fulfilled if $0 \leq q_1 < \bar{q}$. Hence the agency's best reply at A_2 is the following

$$\begin{cases} \text{if} & 0 \leq q_1 < q_1^2 \quad \text{or} \quad q_1^1 < q_1 \leq q^c \quad \text{then} \quad \{i\} \\ \text{otherwise} & \begin{cases} \text{if} & 0 \leq q_1 < \bar{q} \quad \text{then} \quad \{i\} \\ \text{otherwise} & \{ni\} \end{cases} \end{cases} \quad (11)$$

Note that \bar{q} is positively related to d , since the greater are the behavioral constraints the higher is \hat{q} and so the greater is \bar{q} . Moreover, \bar{q} is inversely related to K , as we will see deeply in Section 4. The last decision to analyze is the cartel's choice at I_1 . There, producing q^c yields $\delta[\pi(\tilde{q}, \theta) - \Omega]$, i.e. the cartel sacrifices some profits at $t = 1$ to enjoy discounted (almost) monopoly profits later. Now, let us assume that $m > \frac{\pi(\tilde{q}, \theta) + \delta\pi(\hat{q}, \theta)}{\pi(\bar{q}, \theta)}$; then if $0 \leq q_1^2$ or $q_1^1 < q_1 < q^c$ cartel profits, equal to $(1 - m)\pi(q_1, \theta) + \delta[\pi(\hat{q}, \theta) - \Omega]$, are negative. Last, if instead $q_1^2 \leq q_1 \leq q_1^1$ profits are $(1 + \delta)\pi(q_1, \theta)$ if $\bar{q} \leq q_1 \leq q_1^1$ (the agency does not investigate) and $(1 - m)\pi(q_1, \theta) + \delta[\pi(\hat{q}, \theta) - \Omega]$ (again negative) if $q_1^2 \leq q_1 < \bar{q}$. Hence the cartel first decision is the following (note that $q_1 = \bar{q}$ maximizes $(1 + \delta)\pi(q_1, \theta)$)

$$\begin{cases} q^c & \text{if} \quad \delta > \frac{\pi(\bar{q}, \theta)}{\pi(\bar{q}, \theta) - \pi(\tilde{q}, \theta) - \Omega} \\ \bar{q} & \text{otherwise} \end{cases} \quad (12)$$

The above analysis underscores that trying to induce the agency not to investigate is better than competing and then colluding if the short-run gain from collusion and induced no investigation are better than the long-run difference between maximum profits and collusive profits at $t = 1$. If we assume that the discount factor does not satisfy the inequality shown in (12), we can now establish the equilibrium under perfect information.

²³This assumption rules out the possibility that $q_1 = \tilde{q}$; in this case $\{i\}$ dominates $\{ni\}$ if $\delta[W(\hat{q}, \theta) - W(\tilde{q}, \theta)] > K$, which the same condition presented in expression (7).

Proposition 1 *In case of perfect information there exist two equilibria:*

- (1) *when $\bar{q} > q_1^1$ then $q_1^* = q^c$ since the agency will always investigate at $t = 1$ if $q_1 \neq q^c$ and $q_2^* = \tilde{q}$;*
- (2) *if $\tilde{q} < \bar{q} \leq q_1^1$ then $q_1^* = q_2^* = \bar{q}$ iff $\delta \leq \frac{\pi(\bar{q}, \theta)}{\pi(\tilde{q}, \theta) - \pi(\bar{q}, \theta) - \Omega}$, while the agency will not investigate at $t = 1$; otherwise $q_1^* = q^c$ and $q_2^* = \tilde{q}$, and again the agency selects $\{ni\}$ at $t = 1$.*

Proof: see Appendix.

Proposition 1 shows that in case of perfect information a little degree of collusion is always tolerated even if the agency can impose behavioral constraints that increase the collusion costs. In case of high behavioral constraints $\bar{q} > q_1^1$ and the agency can prevent collusion at the first period, while the collusive agreement cannot be limited when this weapon is no longer available (at $t = 2$); in case of low behavioral constraints collusion is always limited. The latter is the interesting case: the cartel can induce the agency not to fight horizontal price-fixing by making a *credible covenant*: even in the future, when the agency never investigates, the cartel will be engaged in a “little” degree of collusion. Notice that antitrust policy is effective against collusion even if the policy is implemented in a discretionary way.

4 Limited information on agency’s policy costs

If the public agency has a strategic advantage due to asymmetric information about its costs, social welfare increases if compared with that obtained with the perfect information case. To show this we assume that the agency’s costs can take two levels, i.e. $K \in \{K_A, K_B\}$, with $K_A < K_B$. More precisely, our goal is to explore if the possibility that the agency might have smaller costs can provide some welfare enhancements (i.e. there exists a positive probability that agency’s costs are lower than K_B). Since K has an effect on \bar{q} we have that $\bar{q}_A > \bar{q}_B$ (\bar{q}_A (respectively \bar{q}_B) indicates the output produced at $t = 1$ to make indifferent the type A (type B) agency between $\{i\}$ and $\{ni\}$), and two possible cases.

Case 1. $\tilde{q} < \bar{q}_B \leq q_1^1 < \bar{q}_A$ (the high agency costs differential case);

Case 2. $\tilde{q} < \bar{q}_B < \bar{q}_A \leq q_1^1$ (the low agency costs differential case);

Moreover, the sequence of actions in the signaling game has a modification: Nature moves first and tells only to the agency its costs; then the industry, with a unique efficiency state and prior information about the agency’s costs (i.e. $\eta = Prob(K = K_A)$), chooses q_1 , and so on as in the perfect information case. Hence the information set when the cartel

moves at $t = 1$ is not a singleton. We will now identify the optimal strategies under the two above cases, by computing a *Perfect Bayesian Equilibrium*.

4.1 Equilibrium with high agency costs differential

If $\tilde{q} < \bar{q}_B \leq q_1^1 < \bar{q}_A$ the type- A agency's best reply at $t = 1$ is $\{ni\}$ if $q_1^1 < q_1 \leq q^c$ and $\{i\}$ if $0 \leq q_1 \leq q_1^1$, while type B best reply is $\{ni\}$ if $\bar{q}_B \leq q_1 \leq q^c$ and $\{i\}$ if $0 \leq q_1 < \bar{q}_B$. Hence the cartel decision at $t = 1$ depends on the prior probability that the agency has low costs (type A). The following Proposition identifies the equilibrium.

Proposition 2 *In case of high costs differential between the two agency's types, the cartel will produce $q_1^* = \bar{q}_B = q_2^*$ iff the following condition holds*

$$\eta \leq \frac{(1 + \delta)\pi(\bar{q}_B, \theta) - \delta\pi(\tilde{q}, \theta)}{(m + \delta)\pi(\bar{q}_B, \theta) - \delta\pi(\hat{q}, \theta)} \quad (13)$$

If condition (13) is not fulfilled the cartel will set $q_1^ = q^c$, $q_2^* = \tilde{q}$. In both cases the agency will not investigate.*

Proof: see Appendix.

The above Proposition shows that the optimal cartel's strategy at $t = 1$ has a “bang-bang” property, as shown in Figure 3: if the probability that the agency has low costs is small the cartel will induce an equilibrium where price-fixing is maintained at the same level in both periods. If instead the opposite is true a first-best outcome is reached at the first period. This result underscores that the agency can benefit from the presence of uncertainty about its opportunity costs in fighting collusion: a sufficiently high chance that the agency has strong preferences towards marginal costs pricing will force the cartel to lose some profits, since in case of perfect information $q_1^* = \bar{q}_B$.

4.2 Equilibrium with low agency costs differential

If there is uncertainty between the agency's costs but the latter are supposed to be rather low for both agency types, we have that $\tilde{q} < \bar{q}_B < \bar{q}_A < q_1^1$. Hence also the low costs agency type has $\{ni\}$ as best reply for any output within the interval $[\bar{q}_A, q_1^1]$. The equilibrium is stated in the following Proposition.

Proposition 3 *In case of low costs differential between the two agency's types, the cartel will produce $q_1^* = \bar{q}_B = q_2^*$ iff the following condition holds*

$$\eta \leq \frac{(1 + \delta)[\pi(\bar{q}_B, \theta) - \pi(\bar{q}_A, \theta)]}{(m + \delta)\pi(\bar{q}_B, \theta) - \delta\pi(\hat{q}, \theta)} \quad (14)$$

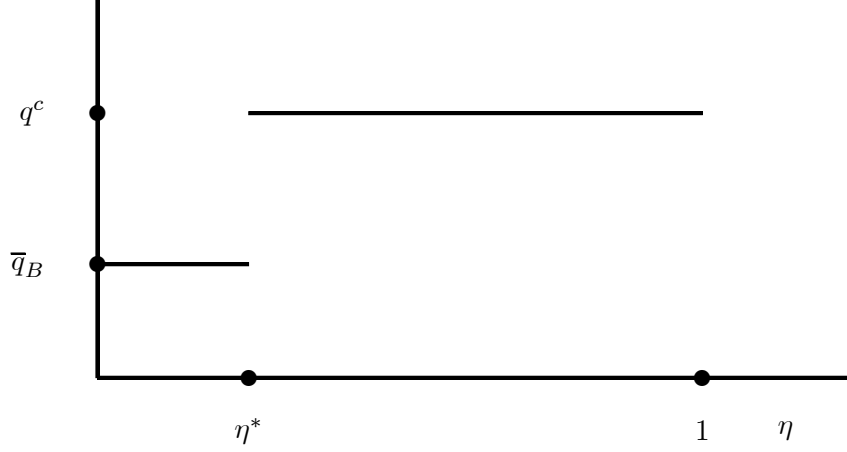


Figure 3: Equilibrium with high costs differential

If condition (14) is not fulfilled the cartel will set $q_1^* = \bar{q}_A = q_2^*$. In both cases the agency will not investigate.

Proof: see Appendix.

Under this case the agency cannot replicate a situation where in the first stage of the game marginal costs pricing is induced along the equilibrium path; in all periods price-fixing is tolerated. If the chance that the agency costs are low is sufficiently high then a low degree of collusion is tolerated, if instead the latter costs are likely to be high, society has to incur an higher deadweight loss. Again the cartel plays a “bang-bang” strategy, according to the probability distribution about the agency’s costs (see Figure 4). Hence we can highlight that the agency benefits from limited information about its costs if it can increase the prior probability that its costs are low, so that the low costs agency type finds optimal to investigate even if the cartel can credibly propose a “covenant” where the same output produced today will be maintained in the future.

5 Limited information on cartel’s efficiency

The equilibrium changes if there is, as in Besanko and Spulber [1989] and in Souam [1998, 2000], limited information about the cartel’s efficiency state. We assume that the latter is identified by two values of the constant marginal costs of production, i.e. $\theta \in \{\theta_1, \theta_2\}$, with $\theta_1 < \theta_2$. Hence the θ_1 -state (θ_2) is the efficient (inefficient) one. In this scenario, we have that (under linear demand and costs) $q_1^c > q_2^c$, $\tilde{q}_1 > \tilde{q}_2$, $\bar{q}_1 > \bar{q}_2$, $\hat{q}_1 > \hat{q}_2$, $q_1^1 > q_2^1$, and $q_1^2 > q_2^2$. To simplify the analysis we assume that $\bar{q}_i < q_i^1$ ($i = 1, 2$) and that $\delta \leq \frac{\pi(\bar{q}_i, \theta_i)}{\pi(\bar{q}_i, \theta_i) - \pi(\bar{q}_i, \theta_i) - \Omega}$. This leads to two possible cases:

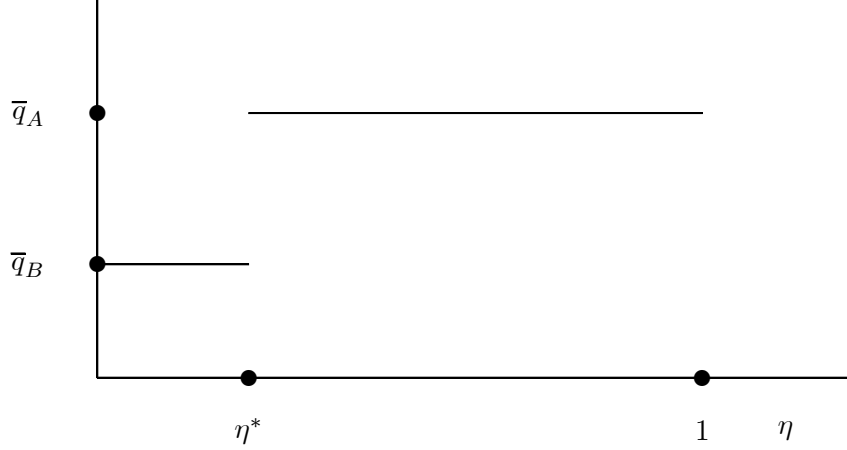


Figure 4: Equilibrium with low costs differential

Case 1. $\bar{q}_1 < q_2^1$ (the small efficiency gap case);

Case 2. $\bar{q}_1 \geq q_2^1$ (the high efficiency gap case).

We will now identify the optimal strategies under the two above cases, by computing again a *Perfect Bayesian Equilibrium*. We assume that the agency has a prior knowledge about the realization of the efficiency state, such that $\gamma = \text{Prob}(\theta = \theta_1)$, and we define $\mu(q|\theta_i)$ as the posterior probability that the efficiency state is θ_i ($i = 1, 2$) when output q is observed at $t = 1$, with $\mu(q|\theta_1) + \mu(q|\theta_2) = 1$.

The sequence of events in the signaling game presents another change: Nature moves first and selects the efficiency state θ_i ($i = 1, 2$), which is communicated to the cartel and not to the agency. Then the remainder of the game is as before; however, unless output levels greater than q_2^c are produced, the agency information set when it has to select whether to investigate or not is not a singleton.²⁴ Notice that, by Lemma 1, if $q_1 \neq q_2$ then $q_{2i}^* = \tilde{q}_i$, i.e. the two cartel's types produce different outputs at $t = 2$, where q_{2i} is the output produced by type i at $t = 2$.

Two candidate equilibria must be checked: *separating* (the two types produce different outputs at $t = 1$) and *pooling* (they produce the same output at $t = 1$). However a separating equilibrium does not exist. Indeed, the unique candidate for a separating equilibrium is that the θ_1 -type produces \bar{q}_1 and the θ_2 -type \bar{q}_2 , while the agency does not investigate. However this will be an equilibrium only if no player has an incentive to deviate from it; this is not the case, since the θ_1 -type has an incentive to produce \bar{q}_2 if the agency does not investigate there. But if this happens the agency will investigate and so a separating equilibrium does not exist. Hence only pooling equilibria exist, so that

²⁴The θ_2 -type will never produce an output greater than q_2^c , since it yields a loss; hence the agency knows that such output levels, if observed, are produced by the efficient type.

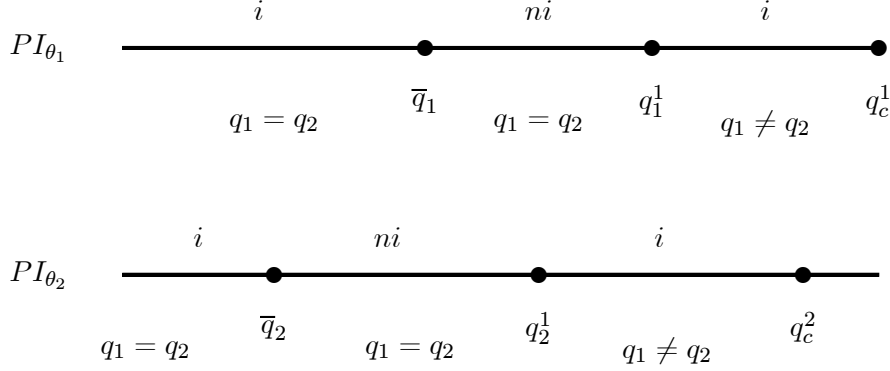


Figure 5: Players' moves in the small efficiency gap case

$$\mu(q|\theta_1) = \gamma.$$

5.1 Equilibrium with small efficiency gap

Figure 5 shows the agency's best reply and the cartel's output at $t = 2$ for each feasible level of output produced at $t = 1$ in case of perfect information about state i (PI_{θ_i}). Note that, by overlapping the two lines, in the small efficiency gap case both cartel types will set $q_1 = q_2$ only if $q_1 \in [\bar{q}_2, q_2^1]$.

We can now set the agency's best reply for each output observed at $t = 1$. First note that $\forall q \in [q_1^1, q_c^1]$, the agency's best reply is $\{i\}$, while $\forall q \in [q_2^1, q_1^1]$ her best reply is

$$\begin{cases} \{i\} & \text{iff } \gamma \leq \gamma_1 \\ \{ni\} & \text{iff } \gamma > \gamma_1 \end{cases}$$

where

$$\gamma_1 = \frac{\overbrace{W(\tilde{q}_2, \theta_2) - W(\bar{q}_2, \theta_2)}^{<0}}{\underbrace{W(\bar{q}_1, \theta_1) - W(q, \theta_1)}_{<0} + \underbrace{W(\tilde{q}_2, \theta_2) - W(\bar{q}_2, \theta_2)}_{<0}}$$

Instead $\forall q \in [\bar{q}_1, q_2^1]$ the agency's best reply is $\{ni\}$, while $\forall q \in [\bar{q}_2, \bar{q}_1]$ her best reply is

$$\begin{cases} \{i\} & \text{iff } \gamma > \gamma_2 \\ \{ni\} & \text{iff } \gamma \leq \gamma_2 \end{cases}$$

where

$$\gamma_2 = \frac{W(q, \theta_2) - W(\bar{q}_2, \theta_2)}{W(\bar{q}_1, \theta_1) - W(q, \theta_1) - [W(q, \theta_2) - W(\bar{q}_2, \theta_2)]} \quad (15)$$

Last $\forall q \leq \bar{q}_2$ we have $\{i\}$. The following Proposition shows the necessary condition for a pooling equilibrium to exist.

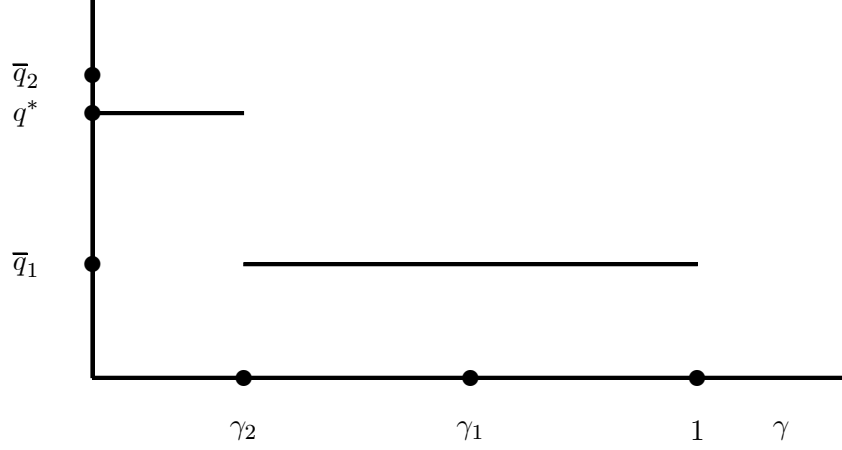


Figure 6: Pooling equilibrium with small efficiency gap

Proposition 4 *In case of small efficiency gap two pooling equilibria exist:*

- (1) $q_{11}^* = q_{12}^* = q^*$ and no investigation iff $\gamma \leq \gamma_2$, where q_1^* is the smallest output satisfying (15) belonging to the interval $[\bar{q}_2, \bar{q}_1]$;
- (2) $q_{11}^* = q_{12}^* = \bar{q}_1$ and no investigation iff $\gamma > \gamma_2$.

Proof: see Appendix.

The above Proposition presents the typical result of an economic situation where one agent enjoys a strategic advantage; the most efficient cartel type can make higher profits than in case of perfect information (it produces $q_1^* \leq \bar{q}_1$), while the inefficient type makes lower profits. Social welfare in equilibrium is equal to $(1+\delta)[\gamma W(q^*, \theta_1) + (1-\gamma)W(q^*, \theta_2)]$. The final outcome about the relevant pooling equilibrium depends upon the exogenous parameter γ . Figure 6 shows the output produced in the pooling equilibrium as a function of γ .

5.2 Equilibrium with high efficiency gap

If the efficiency gap is high we have, as shown in Figure 7, that $q_2^1 < \bar{q}_1$. Then if $q_2^1 < q_1 < \bar{q}_1$ the θ_1 -type will set at $t = 2$ $q_2 = q_1$, while the θ_2 -type $q_2 = \tilde{q}_2$. Hence now the agency's best reply is the following

$$\left\{ \begin{array}{ll} \forall q \in [q_1^1, q_1^c] & \rightarrow \{i\} \\ \forall q \in [\bar{q}_1, q_1^1] & \rightarrow \begin{cases} \{i\} & \text{iff } \gamma \leq \gamma_1 \\ \{ni\} & \text{iff } \gamma > \gamma_1 \end{cases} \\ \forall q \in [q_2^1, \bar{q}_1] & \rightarrow \{i\} \\ \forall q \in [\bar{q}_2, q_2^1] & \rightarrow \begin{cases} \{i\} & \text{iff } \gamma \geq \gamma_2 \\ \{ni\} & \text{iff } \gamma > \gamma_2 \end{cases} \\ \forall q \leq \bar{q}_2 & \rightarrow \{i\} \end{array} \right. \quad (16)$$

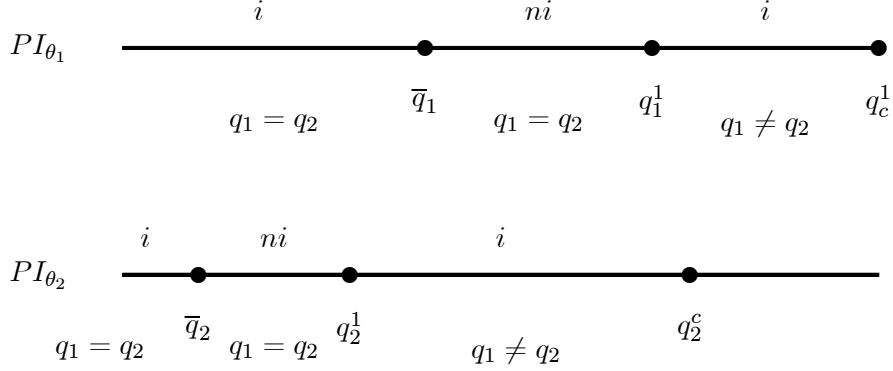


Figure 7: Players' moves in the high efficiency gap case

Notice that the agency in case of high efficiency gap enlarges the output interval where it always investigates. The pooling equilibrium under this scenario is shown in Proposition 5.

Proposition 5 *In case of small efficiency gap two pooling equilibria exist:*

- (1) $q_{11}^* = q_{12}^* = q^*$ and no investigation, iff $\gamma \leq \gamma_2$, where q_1^* is the smallest output satisfying (15) belonging to the interval $[\bar{q}_2, q_2^1]$;
- (2) $q_{11}^* = q_{12}^* = \bar{q}_1$ and no investigation iff $\gamma > \gamma_2$.

Proof: See the proof of Proposition 4 in the Appendix.

These two cases show that under limited information about the cartel's efficiency the antitrust policy is less effective in fighting horizontal price-fixing than in case of perfect information; nevertheless the policy, always implemented through discretion, is welfare improving, and it can even guarantee an higher welfare than that obtained with perfect information and an industry with low efficiency (the equilibrium output with the pooling equilibrium is always higher than \bar{q}_2).

6 Summary and conclusions

Since collusion shrinks social welfare, it is important to identify the tools that make competition policy more effective in fighting it. At the same time it is convenient to build an antitrust model based upon discretion, since the latter is the approach usually adopted by competition authorities in real world price-fixing cases. This article provides a political economy model where the strategic interaction between a public agency in charge of antitrust policy and a cartel is designed as a signaling game. In contrast with previous

contributions, we consider that the agency can impose, in case of collusion, in addition to fines, some behavioral constraints, that will be monitored for some time after the final decision, and that increase the costs of collusion. Fines, however they are applied (the literature has shown the circumstances where a certain regime of fine is more effective than another), do not constitute an incentive to investigate in the event of collusion. Since they are a pure monetary transfer from producers to consumers, they leave social welfare unchanged at the collusive level, and so the agency will never find optimal to fight collusion once this has been observed, as long as there exists a social costs of antitrust activity. Under this scenario, as shown by the literature, the unique way to fight horizontal price-fixing is by assuming that the agency can announced its policy at the beginning of the strategic interaction and then committing to it, so that an investigation will be done in the event of collusion even if it is not a best reply.

Behavioral constraints instead directly affect the cartel's optimal decision after a conviction, and so they are, as shown in this article, an effective tool in limiting collusion when antitrust policy is implemented via discretion. Behavioral constraints do not completely eliminate horizontal price-fixing; the social costs of the policy force the agency to compare the marginal benefits obtained through an incremental effort in fighting collusion with marginal costs. Hence a "little" degree of collusion may be tolerated since it produces a lower social costs than that necessary to eliminate it, and so it can be forgiven. However behavioral constraints are effective in limiting collusion since they improve social welfare if compared with the *laissez faire* level. This result is obtained also in the more unfavorable situation where the agency can operate, i.e. in case of limited information about production costs. Under some circumstances behavioral constraints may lead to an equilibrium where collusion takes place only in the future while marginal costs pricing is enforced in the first stage of the game; in this way the deadweight social loss is reduced, since it is discounted, while it is completely eliminated at the first period, when it could be very large.

This article also underscores that the cartel may strategically induce the agency not to investigate by making a credible "covenant" that a little degree of collusion will take place today *and* in the future. If so the antitrust policy acts as a deterrent, since it is not observed along the equilibrium path but prevents an higher degree of collusion. The presence of limited information about the agency's costs leads to an increase in social welfare, while in case of limited information about production costs the efficient type cartel can increase its rents. This is an interesting insight for the policy maker: a little degree of uncertainty about the social costs of fighting collusion allows for an incremental social benefit, since the cartel takes into account that a low costs agency may find optimal

to investigate when an high costs agency will not fight collusion. Hence a reputation effect about the agency's preference towards social optimum (and so low opportunity costs of fighting collusion) may act as an important deterrent.

7 Appendix

Proof of Lemma 1:

Suppose that $q_2 < q^c$, i.e. the agency observes a collusive output. If the agency chooses $\{i\}$ social welfare is $W(q_2, \theta) - K$; under the alternative action social welfare is $W(q_2, \theta)$; then $\{ni\}$ dominates $\{i\}$.

□

Proof of Lemma 2:

If the agency selects $\{i\}$ at $t = 1$ when $q_1 = q^c$ social welfare is $W(q^c, \theta) - K + \delta W(\tilde{q}, \theta)$. Indeed at $t = 2$ by Lemma 1 the agency will not investigate and so $q_2 = \tilde{q}$. If the agency chooses $\{ni\}$ social welfare is $W(q^c, \theta) + \delta W(\tilde{q}, \theta)$, $\rightarrow \{ni\}$ dominates $\{i\}$ when $q_1 = q^c$.

□

Proof of Proposition 1:

We can have two cases: either $\bar{q} > q_1^1$ (case (1)), or $\bar{q} \leq q_1^1$ (case (2)). Under case (1) the agency's best reply at A_2 from (11) is always $\{i\}$, since \bar{q} lies in the output range $q_1^1 < q_1 \leq q^c$; hence the cartel knows that if at I_1 $q_1 \neq q^c$ is chosen its profits will be

$$\underbrace{(1 - m)\pi(q_1, \theta)}_{<0} + \delta[\pi(\hat{q}, \theta) - \Omega] < 0$$

By choosing instead q^c its profits are $\delta[\pi(\tilde{q}, \theta) - \Omega] > 0$. Hence $q_1^* = q^c$. Under case (2), from (11) the agency does not investigate if $\bar{q} \leq q_1 \leq q_1^1$. Since $\frac{d\pi}{dq_1} < 0$ if $q_1 > \tilde{q}$, $q_1^* = \bar{q}$ if $(1 + \delta)\pi(\bar{q}, \theta) \geq \delta[\pi(\tilde{q}, \theta) - \Omega]$, i.e.

$$\delta \geq \frac{\pi(\bar{q}, \theta)}{\pi(\tilde{q}, \theta) - \pi(\bar{q}, \theta) - \Omega}$$

□

Proof of Proposition 2:

The cartel can select at $t = 1$ four different output levels: (1) $q_1 = \bar{q}_B$, (2) $q_1 \in]\bar{q}_B, q_1^1]$, (3) $q_1 \in]q_1^1, q^c[$, (4) $q_1 = q^c$. Its profits are

Case (1) $q_1 = \bar{q}_B$

$$\eta[(1 - m)\pi(\bar{q}_B, \theta) + \delta\pi(\hat{q}, \theta)] + (1 - \eta)(1 + \delta)\pi(\bar{q}_B, \theta) \quad (17)$$

Case (2) $q_1 \in]\bar{q}_B, q_1^1]$

$$\eta[(1 - m)\pi(q_1, \theta) + \delta\pi(\hat{q}, \theta)] + (1 - \eta)(1 + \delta)\pi(q_1, \theta) \quad (18)$$

Case (3) $q'_1 \in]q_1^1, q^c[$

$$(1 - m)\pi(q_1, \theta) + \delta\pi(\hat{q}, \theta) \quad (19)$$

Case (4) $q_1 = q^c$

$$\delta\pi(\tilde{q}, \theta) \quad (20)$$

The cartel's best reply is that one which yields the highest return between the four above cases. Now (17) \geq (18) if the following condition holds

$$\eta \leq \frac{1 + \delta}{m + \delta} \quad (21)$$

(17) \geq (19) if

$$\eta \leq \frac{(1 + \delta)\pi(\bar{q}_B, \theta) - (1 - m)\pi(q_1, \theta) - \delta\pi(\hat{q}, \theta)}{(m + \delta)\pi(\bar{q}, \theta) - \delta\pi(\hat{q}, \theta)} \quad (22)$$

while (17) \geq (20) if

$$\eta \leq \frac{(1 + \delta)\pi(\bar{q}_B, \theta) - \delta\pi(\tilde{q}, \theta)}{(m + \delta)\pi(\bar{q}, \theta) - \delta\pi(\hat{q}, \theta)} \quad (23)$$

Moreover, (18) \geq (19) if

$$\eta \leq \frac{(1 + \delta)\pi(q_1, \theta) - (1 - m)\pi(q'_1, \theta) - \delta\pi(\hat{q}, \theta)}{(m + \delta)\pi(q_1, \theta) - \delta\pi(\hat{q}, \theta)} \quad (24)$$

where $q'_1 \in]q_1^1, q^c[$. (18) \geq (20) if

$$\eta \leq \frac{(1 + \delta)\pi(q_1, \theta) - \delta\pi(\tilde{q}, \theta)}{(m + \delta)\pi(q_1, \theta) - \delta\pi(\hat{q}, \theta)} \quad (25)$$

while, last, (19) is always less than (20) since

$$\underbrace{(1 - m)\pi(q'_1, \theta)}_{<0} < \delta \underbrace{[\pi(\tilde{q}, \theta) - \pi(\hat{q}, \theta)]}_{>0} \quad (26)$$

which is always fulfilled. Hence case (3) can be dropped since it is always dominated by at least case (4). Note that all the above fractions are greater than 0 and smaller than 1, since η is a probability. Furthermore, it is possible to show that

$$\frac{1 + \delta}{m + \delta} > \frac{(1 + \delta)\pi(\bar{q}_B, \theta) - \delta\pi(\tilde{q}, \theta)}{(m + \delta)\pi(\bar{q}, \theta) - \delta\pi(\hat{q}, \theta)}$$

and that

$$\frac{(1+\delta)\pi(\bar{q}_B, \theta) - \delta\pi(\tilde{q}, \theta)}{(m+\delta)\pi(\bar{q}, \theta) - \delta\pi(\hat{q}, \theta)} > \frac{(1+\delta)\pi(q_1, \theta) - \delta\pi(\tilde{q}, \theta)}{(m+\delta)\pi(q_1, \theta) - \delta\pi(\hat{q}, \theta)}$$

Hence if $0 \leq \eta \leq \frac{(1+\delta)\pi(q_1, \theta) - \delta\pi(\tilde{q}, \theta)}{(m+\delta)\pi(q_1, \theta) - \delta\pi(\hat{q}, \theta)}$ case (1) dominates (2) and (4) (case (2) dominates (4)) and it is the cartel's best reply at $t = 1$. If $\frac{(1+\delta)\pi(q_1, \theta) - \delta\pi(\tilde{q}, \theta)}{(m+\delta)\pi(q_1, \theta) - \delta\pi(\hat{q}, \theta)} < \eta \leq \frac{(1+\delta)\pi(\bar{q}_B, \theta) - \delta\pi(\tilde{q}, \theta)}{(m+\delta)\pi(\bar{q}, \theta) - \delta\pi(\hat{q}, \theta)}$ case (1) still dominates (2) and (4) (case (4) dominates (2)) and so $q_1^* = \bar{q}_B$. If $\frac{(1+\delta)\pi(\bar{q}_B, \theta) - \delta\pi(\tilde{q}, \theta)}{(m+\delta)\pi(\bar{q}, \theta) - \delta\pi(\hat{q}, \theta)} < \eta \leq \frac{1+\delta}{m+\delta}$ case (1) dominates (2) but it is dominated by case (4), and so $q_1^* = q^c$. Last if $\frac{1+\delta}{m+\delta} < \eta \leq 1$ case (4) still dominates (1) and (2).

□

Proof of Proposition 3:

The cartel can select at $t = 1$ four different output levels: (1) $q_1 = \bar{q}_B$, (2) $q_1 \in]\bar{q}_B, \bar{q}_A[$, (3) $q_1 = \bar{q}_A$, (4) $q_1 = q^c$. Its profits are

Case (1) $q_1 = \bar{q}_B$

$$\eta[(1-m)\pi(\bar{q}_B, \theta) + \delta\pi(\hat{q}, \theta)] + (1-\eta)(1+\delta)\pi(\bar{q}_B, \theta) \quad (27)$$

Case (2) $q_1 \in]\bar{q}_B, \bar{q}_A[$

$$\eta[(1-m)\pi(q_1, \theta) + \delta\pi(\hat{q}, \theta)] + (1-\eta)(1+\delta)\pi(q_1, \theta) \quad (28)$$

Case (3) $q_1 = \bar{q}_A$

$$(1+\delta)\pi(\bar{q}_A, \theta) \quad (29)$$

Case (4) $q_1 = q^c$

$$\delta\pi(\tilde{q}, \theta) \quad (30)$$

The cartel's best reply is that one which yields the highest return between the four above cases. Now $(27) \geq (28)$ if the following condition holds

$$\eta \leq \frac{1+\delta}{m+\delta} \quad (31)$$

$(27) \geq (29)$ if

$$\eta \leq \frac{(1+\delta)[\pi(\bar{q}_B, \theta) - \pi(\bar{q}_A, \theta)]}{(m+\delta)\pi(\bar{q}, \theta) - \delta\pi(\hat{q}, \theta)} \quad (32)$$

while (27) \geq (30) if

$$\eta \leq \frac{(1+\delta)\pi(\bar{q}_B, \theta) - \delta\pi(\tilde{q}, \theta)}{(m+\delta)\pi(\bar{q}, \theta) - \delta\pi(\hat{q}, \theta)} \quad (33)$$

Moreover, (28) \geq (29) if

$$\eta \leq \frac{(1+\delta)[\pi(q_1, \theta) - \pi(\bar{q}_A, \theta)]}{(m+\delta)\pi(q_1, \theta) - \delta\pi(\hat{q}, \theta)} \quad (34)$$

(28) \geq (30) if

$$\eta \leq \frac{(1+\delta)\pi(q_1, \theta) - \delta\pi(\tilde{q}, \theta)}{(m+\delta)\pi(q_1, \theta) - \delta\pi(\hat{q}, \theta)} \quad (35)$$

while, last, (29) is always less than (30) since

$$(1-m)\pi(\bar{q}_A, \theta) \geq \delta\pi(\tilde{q}, \theta) \quad (36)$$

by Proposition 1. Hence case (4) can be dropped since it is always dominated by at least case (3). Note that all the above fractions are greater than 0 and smaller than 1, as required since η is a probability. Furthermore, it is possible to show that

$$\frac{1+\delta}{m+\delta} > \frac{(1+\delta)[\pi(\bar{q}_B, \theta) - \pi(\bar{q}_A, \theta)]}{(m+\delta)\pi(\bar{q}_B, \theta) - \delta\pi(\hat{q}, \theta)}$$

and that

$$\frac{(1+\delta)[\pi(\bar{q}_B, \theta) - \pi(\bar{q}_A, \theta)]}{(m+\delta)\pi(\bar{q}_B, \theta) - \delta\pi(\hat{q}, \theta)} > \frac{(1+\delta)[\pi(q_1, \theta) - \pi(\bar{q}_A, \theta)]}{(m+\delta)\pi(q_1, \theta) - \delta\pi(\hat{q}, \theta)}$$

Hence if $0 \leq \eta \leq \frac{(1+\delta)[\pi(q_1, \theta) - \pi(\bar{q}_A, \theta)]}{(m+\delta)\pi(q_1, \theta) - \delta\pi(\hat{q}, \theta)}$ case (1) dominates (2) and (3) (case (2) dominates (3)) and it is the cartel's best reply at $t = 1$. If $\frac{(1+\delta)[\pi(q_1, \theta) - \pi(\bar{q}_A, \theta)]}{(m+\delta)\pi(q_1, \theta) - \delta\pi(\hat{q}, \theta)} < \eta \leq \frac{(1+\delta)[\pi(\bar{q}_B, \theta) - \pi(\bar{q}_A, \theta)]}{(m+\delta)\pi(\bar{q}_B, \theta) - \delta\pi(\hat{q}, \theta)}$ case (1) still dominates (2) and (3) (case (3) dominates (2)) and so $q_1^* = \bar{q}_B$. If $\frac{(1+\delta)[\pi(\bar{q}_B, \theta) - \pi(\bar{q}_A, \theta)]}{(m+\delta)\pi(\bar{q}_B, \theta) - \delta\pi(\hat{q}, \theta)} < \eta \leq \frac{1+\delta}{m+\delta}$ case (1) dominates (2) but it is dominated by case (3), and so $q_1^* = \bar{q}_A$. Last if $\frac{1+\delta}{m+\delta} < \eta \leq 1$ case (3) still dominates (1) and (2).

□

Proof of Proposition 4:

First we show that if $q_1 = \bar{q}_2$ the agency, in case of perfect information will always investigate. Indeed the move $\{i\}$ yields

$$\gamma[W(\bar{q}_2, \theta_1) - K + \delta W(\hat{q}_1, \theta_1)] + (1-\gamma)[W(\bar{q}_2, \theta_2) - K + \delta W(\hat{q}_2, \theta_2)]$$

while $\{ni\}$ gives

$$\gamma(1 + \delta)W(\bar{q}_2, \theta_1) + (1 - \gamma)(1 + \delta)W(\bar{q}_2, \theta_2)$$

and $\{i\}$ dominates $\{ni\}$ if, by substituting for $K = \delta[W(\hat{q}_i, \theta_i) - W(\bar{q}_i, \theta_i)]$ ($i = 1, 2$)

$$\gamma \delta \underbrace{[W(\bar{q}_1, \theta_1) - W(\bar{q}_2, \theta_1)]}_{>0} > 0$$

that is always true. Then for any $q_1 \in]\bar{q}_2, \bar{q}_1]$, we have that $\{i\}$ gives

$$\gamma[W(q_1, \theta_1) - K + \delta W(\hat{q}_1, \theta_1)] + (1 - \gamma)[W(q_1, \theta_2) - K + \delta W(\hat{q}_2, \theta_2)]$$

while $\{ni\}$ gives

$$\gamma(1 + \delta)W(q_1, \theta_1) + (1 - \gamma)(1 + \delta)W(q_1, \theta_2)$$

and $\{ni\}$ dominates $\{i\}$ if, by substituting for K

$$\gamma \underbrace{[W(\bar{q}_1, \theta_1) - W(q_1, \theta_1)]}_{>0} + (1 - \gamma) \underbrace{[W(\bar{q}_2, \theta_2) - W(q_1, \theta_2)]}_{<0} \leq 0$$

solving for γ we get (15). Last we know that since in $q_1 \in]\bar{q}_2, q_2^1]$ $\frac{d\pi_i}{dq} < 0$, q_1^* solves the following equality

$$\gamma = \frac{W(q_1^*, \theta_2) - W(\bar{q}_2, \theta_2)}{W(\bar{q}_1, \theta_1) - W(q_1^*, \theta_1) + W(q_1^*, \theta_2) - W(\bar{q}_2, \theta_2)}$$

The same procedure applies to all $q \in]q_2^1, q_1^1]$.

□

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