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Modelling and testing for jumps in the prices of financial assets

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Dedication

To my loving parents.

Contents

1 Introduction

One of the most important areas of research in both finance and financial econometrics is to quantify and model risk related to investments in financial assets. During the past decade, a new strand of literature has emerged in the field, concerning the exploitation of high frequency data (transaction prices and quotes) in creating ex-post risk proxies. The new volatility measures statistically outperform all other previously used measures and in addition, are model free and very easy to compute.

One of the main features of the new tools to quantify risk is the development of techniques that are able to capture separately the so-called 'jumps', i.e. the sudden and significant changes in prices as a result of the arrival of information. It is now possible to distinguish between the persistent part of the risk, which can be modelled and forecast, and the unpredictable part, captured by jumps.

These two types of risk are different in nature and require a different treatment. They should be differently priced, hedged and managed. Consequently, being able to estimate them separately has great implications for the financial services industry, the wider economy and the academic world. The core of the thesis is to examine the financial market behaviour in the presence of jumps and to analyze the various tools available in the literature that allow to disentangle jumps from the continuous component in the prices of financial assets.

The research falls in the area of high frequency econometrics dealing with the construction and application of quantitative techniques based on high-frequency quotation and trading data to detect jumps in prices. The thesis is organized in three independent essays and two of its chapters are in the process to be submitted for a possible publication in leading academic journals in the field.

The first essay, entitled "The use of high frequency data in estimating volatility and detecting jumps in the prices of financial assets", includes a complete literature review of the latest nonparametric volatility estimators based on high frequency data, covering both robust and non-robust to jumps estimators, as well as the various jump detection procedures recently proposed in the literature. We consider both the univariate and the mutivariate frameworks.

The second essay, entitled "Identifying jumps in financial assets: a comparison between nonparametric jump tests", comprises a thorough comparison among five jump identification procedures proposed in the literature of high frequency econometrics over the last decade: the Andersen et al. (2007)-Lee and Mykland (2008), the Aït-Sahalia and Jacod (2008), the Barndorff-Nielsen and Shephard (2006a), the Jiang and Oomen (2008), and the Podolskij and Ziggel (2008) tests. The comparison is mainly based on an intensive Monte Carlo exercise, meant to assess the power and size properties of all these tests. We also extend the previous analysis to real high frequency data on US Treasury bonds and compare the behavior of the tests in this context. The objective of this chapter is to provide potential users of the tests with guidelines as to how and when to apply them. Several realistic scenarios concerning the price process are used in order to understand whether the performance of the procedures can be associated to different features of the data.

The third essay, entitled "Jumps and price discovery in the US Treasury market", explores different aspects related to the price discovery process for the US Treasury bonds when jumps occur. We use the Barndorff-Nielsen and Shephard (2006a) and Andersen et al. (2007) -Lee and Mykland (2008) nonparametric procedures to detect jumps and common jumps in the US Treasury market, for a period lagging from January 2003 to March 2004. We investigate the possible causes of jumps in the term structure, by considering a list of US macroeconomic announcements. We also examine different market activities, such as spread, depth, trading volume and order flow around the time of the jump. Finally, we investigate the informativeness of the order flow in the proximity of a jump. We are interested in finding out how the trading information has impact on prices in the nearness of a jump. Therefore, we measure the degree of informational asymmetry on different time windows before a jump occurrence, at the same time and after.

In what follows, we briefly describe the assumed data generating processes in the high-frequency econometrics literature and consequently, throughout this dissertation. First, we cannot observe the true, efficient price, which is time continuous, but we can observe a discrete version of it. Thus, the price is decomposed in the real one plus noise, which is customarily denominated as microstructure noise. Second, the efficient price can be decomposed in the diffusion part, which can have only continuous paths, and jumps. While the continuous part is usually a very general stochastic volatility model, the presence of the jump part is meant to take into account the sudden, unpredictable changes in prices. Due to this specification of the data generating process, variation in the observed price coming from the continuous part will be characterized by persistence and thus, predictability, while variation coming from the jump part of the process is unpredictable.

The seminal work that generated this new strand of literature is the paper by Andersen and Bollerslev (1998), who introduced the realized volatility (RV) as a new estimator for the ex-post volatility. The RV is defined as the sum of squared intraday returns and is proved to outperform any other previously used measures. When jumps are considered, the RV captures both the variations coming from the continuous component of the price and the jump part. Several other contributions to the literature show that risk can be broken down into the (partly) predictable and unpredictable pieces. This is in fact, the main focus of the thesis. However, in applying the various methods that disentangle jumps from the diffusion, we also have to take into account the fact that observed prices are always contaminated with microstructure noise.

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The use of high frequency data in estimating volatility and detecting jumps in the prices of financial assets

The prices of financial assets are complex processes, determined by various known and unknown factors, which display rather unpredictable evolutions and leave place to a lot of surprises. Thus, the efficient market hypothesis, at least in its weak form, is worldwide accepted and prices have been classically described by a Brownian motion. Moreover, the Brownian motion was used for a long period as an important assumption in pricing derivatives.

However, empirical evidence from the periods of financial turmoils shows that in periods of distress, prices tend to display larger variations. Therefore, a new model that could reflect this aspect started to be used: the stochastic volatility (SV) model. Its main characteristic is that it allows the volatility to vary with time. In this way, periods of financial distress are just periods of high volatility. Nevertheless, there were periods of market crashes during which the SV model proved itself unable to keep up with the variation in prices.

At this point, models used to characterize prices started to allow for discontinuities or jumps in prices. Merton (1976) was the first one to build up a jump-diffusion model and ever since then, jumps played an important role in the financial literature.

During the last decade, the literature in financial econometrics has developed several new tools to estimate volatility ex-post based on high-frequency data that were proved to be better than the ones previously used. This is a new class of nonparametric estimators, which consider the price process a generic semimartingale. Initially, this semimartingale was assumed to have only continuous sample paths. However, adding a jump process seemed extremely realistic. In such a set-up, the variation in the process will be induced by both the continuous and discontinuous components. As we show later on, the use of high frequency data enables us to decompose variation in prices in two different parts: the one coming from the diffusion component and the one caused by jumps. This is important because the first one is characterized by persistence, whereas the second is unpredictable.

This chapter is a comprehensive literature review on volatility estimators based on high frequency data. We mostly focus on estimators that either include jumps or are robust to them. We also describe in detail different jump detection procedures proposed in the literature. The analysis takes place in both a univariate and multivariate frameworks. Given that at high frequencies prices are contaminated with microstructure noise, we dedicate a paragraph in both the univariate and multivariate cases, to volatility measures that are robust to microstructure noise. However, we must mention that usually the literature treats noise and jumps separately.

2.1 The importance and the impact of jumps in finance

Taking jumps into consideration has a significant impact on both the financial literature and practice. Here are some of the multiple areas in finance that are influenced by the inclusion of jumps into models, as well as by the possibility of testing for jumps and jump estimation. Some of these areas are highlighted in Aït-Sahalia (2004). First of all, jumps are important in derivatives pricing and hedging, that must take into account the presence of a discontinuous part in prices. Second, in risk management, the possibility to identify and quantify jumps rises a lot of interesting questions concerning investors' remuneration for bearing risks. Is there a premium for the jump risk? Moreover and even more interesting, what kind of jumps are/ should bear a risk premium? Is there a risk premium for systemic jumps? Moreover, jumps imply large changes in asset prices, that increase the tails of the returns distribution. Thus, as outlined by Aït-Sahalia (2004), identified jumps can impact the modeling of different tail statistics like the value-at-risk. Third, jumps are important in portfolio allocation. The risk of financial assets can be decomposed into two different components: the one related to the continuous part of the price process and the one concerning the discontinuous part. Both these risks have to be considered in portfolio management. Moreover, measures for these two different risk components must be defined at a multivariate level. The jump component is totally unpredictable and its presence is triggered by the arrival of information on the market. Sometimes, the news that arrive on the market can concern and affect more assets or even the whole market. Thus, we can talk about assets that display common jumps. Last, as mentioned before, the presence of jumps can be translated into fatter tails of the returns distribution. Thus, in general, whenever researchers or practitioners would need nicely behaved return series for projects with different purposes, the best thing to do is to try to identify and estimate the jumps, and then, disentangle them from the continuous part of the process.

Regarding the impact of jumps on data, as already mentioned above, their presence generates excess unconditional and conditional kurtosis in the returns distribution. Based on this empirical observation, Johannes (2004) uses the conditional and unconditional fourth moments to test for the presence of jumps. In option pricing, as Eraker et al. (2003) show, the presence of jumps in returns steepens the slope of implied volatility curves. Moreover, the authors observe that adding jumps in volatility steepen furthermore the implied volatility curves, which is congruent with the empirical evidence. They show that jumps have a transient, impact on the returns, without leaving any 'marks' on the future distribution of the returns. Adding jumps to a SV model leads to gradual and temperated increases in volatility. Eraker et al. (2003) show that jumps in volatility must also be considered in order to generate fast dynamics in the volatility factor.

Once jumps started to be included in models describing the price generation processes for financial assets, the following questions had to be answered: how to identify and estimate jumps? and how to estimate models that include jumps? Customarily, a model including a drift, a volatility factor and finite activity jumps (compound Poisson processes) was considered, where both the volatility and the jump component were latent. However, the estimation of such jump diffusion models proved very difficult, as there are no closed forms of the likelihood function and in addition, the number of parameters to estimate is very high. The methods of estimation that can apply, such as the Efficient Method of Moments (EMM) proposed by Gallant and Tauchen (2002) and the Markov Chain Monte Carlo (MCMC) algorithms (Kim et al., 1998; Eraker, 2001; Chib et al., 2002, see) are computationally intensive and require the specification of a certain parametric form. Etimations of stochastic volatility models with jumps can be found in the following papers: Bakshi et al. (1997), Andersen et al. (2002), Chernov et al. (2003), Eraker et al. (2003) and Aït-Sahalia and Kimmel (2004).

Some of the latest advances in the literature of financial econometrics show that using high-frequency data enables us to "disentangle" the jump component from the diffusion component of the price process. Moreover, this can be done in a model-free framework and based on methods that are very easy to implement.

The idea of exploiting the richness of high-frequency data in order to understand how the price process is generated was first suggested by Andersen and Bollerslev (1998). They prove that the realized variance, computed as the sum of intraday squared returns, is a much better volatility estimator than the squared daily return that was formerly used to quantify ex-post volatility when assessing the forecasting ability of GARCH-type models. Following their seminal work, the interest in high frequency data has continuously increased, with researchers trying other ways to exploit the available huge datasets.

When jumps are added to a stochastic volatility process, the realized variance estimates the quadratic variation of a process, that includes both a variation due to the continuous part and the variation due to the jump part of the price process. This fact motivates researchers like Barndorff-Nielsen and Shephard (2004) to search for quantities that are able to estimate just the variation due to the continuous part. The reason for such an approach is that the continuous part is the one that is persistent, while the one due to jumps is unpredictable. The next step toward jump identification is evident: by using a non robust to jumps estimator and comparing it to a robust to jumps one, one can capture the variation caused by jumps as a difference. That is why, in this chapter, before getting to actual jump identification and estimation, we briefly describe some volatility estimators based on high frequency data. Some of them, such as Barndorff-Nielsen and Shephard (2004)' s realized bipower variation can be used for both jump identification and estimation, while others allow just jump estimation[1].

Aït-Sahalia (2004) offers an insight on how some of the usual estimation methods, like the maximum likelihood (MLE) and the general method of moments (GMM) work in identifying the various components of the price process. He shows that when the data generating process for the price is given by a simple constant diffusion model, RV is the maximum likelihood estimator of the integrated variance. Aït-Sahalia (2004) also considers a model with a constant volatility factor and both finite and infinite (Cauchy) activity jumps. He shows that in such a framework, both the maximum likelihood and the general method of moments (GMM) work well in "disen-

[1]For instance, the difference between the RV and Mancini (2004)'s threshold estimator does not directly allow to build a test for jumps

tangling diffusion from jumps"[2], that is disentangling variation due to the continuous part from variation due to the jump part, with GMM asymptotically approaching the efficiency of ML.

An important issue that must be signaled and which is mentioned by other authors, like Bollerslev et al. (2007), as well, is that both a jump and a SV model specification basically reflect variations in prices, implying a type of 'trade-off' between the two specifications. Thus, on periods characterized by law volatility, small moves in prices can be picked up as jumps, while during more volatile periods, jumps can be mistaken for variations caused by a high volatility component. In fact, as Bollerslev et al. (2007) suggests, the Barndorff-Nielsen and Shephard (2006a) jump statistic depends on the "overall level of volatility for the day".

2.2 Volatility estimators based on high frequency data

Starting with the second half of the 1990s, the low forecasting performance of the GARCH models led to an increasing interest toward alternative methods through which ex-post volatility could be estimated.

2.2.1 Realized volatility

Andersen and Bollerslev (1998) show that the poor forecasting ability of ARCH-type models is not due to the poor quality of the models, but rather to the noise inherent to the return generating process. This noise generates a bias when one tries to ex-post estimate the latent volatility process, $\sigma^2(t)$, by using the squared daily returns, r_t^2. Starting from a result in the theory of quadratic variation, they suggest using as an ex-post volatility measurement the squared sum of intraday returns. Andersen and Bollerslev (1998) show that by doing this, the forecasting ability of ARCH-type models noticeably

[2] Aït-Sahalia (2004); title of the article

improves. Let $Y(t)$ denote the natural logarithm of the price at time t, a mixture of a Brownian semimartingale plus jumps:

$$\mathrm{d}Y(t) = \mu(t)\mathrm{d}t + \sigma(t)\mathrm{d}W(t) + \mathrm{d}J(t), \qquad t \in [0, T], \tag{2.1}$$

where $\mu(t)$ and $\sigma(t)$ are respectively, the drift and the spot volatility. $J(t)$ represents the jump process at time t, assumed to have finite activity:

$$J(t) = \sum_{j=1}^{N_t} c_j, \tag{2.2}$$

where N_t has intensity of occurrence λ_t, while $c_j, j = 1, 2, \ldots, N_t$ measures the size of the jump at jump times $(\tau_j)_{j=1,2,\ldots,N_t}$ with c_j independent form N_t.

Given the above set-up, the quadratic variation of the process $Y(t)$ will be:

$$[Y](t) = \int_0^t \sigma^2(s)\mathrm{d}s + \sum_{j=1}^{N_t} c_j^2, \tag{2.3}$$

Now suppose that $h > 0$ is a fixed period, such as a trading day. The daily return for the ith day will be computed as:

$$y_i = Y(ih) - Y((i-1)h), \qquad i = 1, 2, \ldots, \tag{2.4}$$

Moreover, during each trading day prices are recorded at M equally spaced times, giving the following intraday (high-frequency) returns, with $j = 1, \ldots, M$ the time on day i when a return is computed:

$$y_{j,i} = Y((i-1)h + hjM^{-1}) - Y((i-1) + h(j-1)M^{-1}) \qquad j = 1, 2, \ldots, M, \tag{2.5}$$

The process of quadratic variation will be defined in the following manner:

$$[Y](t) = \operatorname*{plim}_{M\to\infty} \sum_{j=1}^{M} \{Y(t_j) - Y(t_{j-1})\}^2, \tag{2.6}$$

Starting from this definition of the quadratic variance (QV), it is straightforward that it could be estimated as:

$$\{Y_M\}_t = [Y_M]_i = \sum_{j=1}^{M} y_{j,i}^2, \tag{2.7}$$

The above estimator is called realized volatility (RV) and within the above framework of semimartingale plus jump process it consistently estimates the QV. However, in the no-jump case, the RV is proved to consistently estimate the integrated variance (IV). For this case, Barndorff-Nielsen and Shephard (2002) provide a central limit theorem (CLT) for the estimation error, that is the difference between RV and IV:

$$\sqrt{M}([Y_M]_i - \sigma_i^2)| \int_{(j-1)h}^{jh} \sigma^4(s)\mathrm{d}s \xrightarrow{\mathscr{L}} \mathscr{N}(0, 2h \int_{(j-1)h}^{jh} \sigma^4(s)\mathrm{d}s), \tag{2.8}$$

where

$$\sigma_i^2 = \sigma^2(hi) - \sigma^2(h(i-1)), \tag{2.9}$$

2.2.2 Other volatility estimators

Realized power variations

Having as a starting point the definition of the QV, Barndorff-Nielsen and Shephard (2006b) provide a generalization of this process and name it power variation. As it must be defined based on an equidistant time discretization, one has to assume to observe the price process at equidistant points in time. Thus, the equidistant returns can be defined in the following way:

$$y_j(t) = Y(j\delta) - Y((j-1)\delta), \qquad j = 1, \ldots, M, \tag{2.10}$$

where $\delta > 0$ is the time interval between any two consecutive observations.

The power variation of order r will be:

$$\{Y\}^{[r]}(t) = \operatorname*{plim}_{\delta \downarrow 0} \delta^{1-\frac{r}{2}} \sum_{j=1}^{[t/\delta]} |y_j(t)|^r, \tag{2.11}$$

When $r = 2$, the normalization in front of the power variation disappears and the quantity in (2.11) becomes the usual QV process. When $r > 2$, $1 - \frac{r}{2}$ tends to infinity as $\delta \downarrow 0$, while in the case $r < 2$, it goes to 0 as $\delta \downarrow 0$.

Barndorff-Nielsen and Shephard (2003) provide evidence that for a continuous semi-martingale stochastic volatility process, if the mean and the diffusion of the process, (μ, σ^2), are independent of the Brownian motion W, the above defined power variation will be equal to:

$$\{Y\}^{[r]}(t) = \mu_r \int_0^t \sigma^r(s) \mathrm{d}s, \tag{2.12}$$

where

$$\mu_r = E|u|^r = 2^{r/2} \frac{\Gamma(\frac{1}{2}(r+1))}{\Gamma(\frac{1}{2})}, \qquad r > 0, u \sim \mathcal{N}(0,1) \tag{2.13}$$

Given the above definition of power variation processes, one can build up a "realized" counterpart, based on data sampled as often as possible, as in the following equation:

$$\{Y_M\}^{[r]}(t) = \left(\frac{h}{M}\right)^{1-\frac{r}{2}} \sum_{j=1}^{M} |y_j(t)|^r \tag{2.14}$$

The probability limit of the realized power variation follows from equation (2.12). Limit distribution results on realized power variations are provided in Barndorff-Nielsen and Shephard (2003), Barndorff-Nielsen and Shephard (2006b), Barndorff-Nielsen et al. (2003) and Jacod (2008).

Range estimators

The emergence of this category of volatility estimators is mostly due to an earlier contribution of Parkinson (1980), who shows that the range or squared range, defined as high minus low during a trading day, is more efficient in estimating ex-post volatility than the absolute or squared daily returns.

Thus, Christensen and Podolskij (2005) propose a new estimator for the QV: the realized range based variance *RRV*. First of all, they define the intra-period range at sampling times t_{j-1} and t_j, $0 \leq t_{j-1} \leq t_j \leq t$, where $[0,t]$ is usually thought to be one trading day :

$$s_{Y_{t_j}} = \sup\{Y(t) - Y(s)\}_{t_{j-1} \leq s,t \leq t_j} \tag{2.15}$$

The realized range-based estimator will be defined in the following manner:

$$RRV^{\Xi} = \frac{1}{\lambda_2} \sum_{j=1}^{M} s^2_{Y_{t_j,\delta_j}} \tag{2.16}$$

or,

$$RRV^{\Delta} = \frac{1}{\lambda_2} \sum_{j=1}^{M} s^2_{Y_{t_j,\delta}}, \tag{2.17}$$

where RRV^{Ξ} is the estimator based on sampling times t_j, while RRV^{Δ} is based on equidistant sampling times.

Christensen and Podolskij (2005) show that for processes with continuous paths, RRV converges to IV and derive a central limit theorem:

$$M^{1/2}(RRV^{\Delta} - IV) \xrightarrow{\mathscr{L}} \mathscr{M}\mathscr{N}\big(0, \Lambda \int_0^1 \sigma_s^4 \mathrm{d}s\big) \tag{2.18}$$

2.2.3 Dealing with noise

The asymptotic properties of all nonparametric volatility measures reviewed here assume that the interval between consecutive observations tends to 0. However, at high frequencies, prices of financial assets do no seem to be sampled anymore from semimartingale processes. We observe that prices tend to either remain constant from one transaction to another or change drastically from one observation to another. Moreover, applying an equidistant sampling scheme only amplifies this phenomenon. We say that prices are contaminated with microstructure noise and write:

$$\widetilde{y}_{j,i} = y_{j,i} + \epsilon_{j,i}, \tag{2.19}$$

where $y_{j,i}$ stands for the equilibrium return on day i at time j, $\epsilon_{j,i}$ is the microstructure noise contamination, while $\widetilde{y}_{j,i}$ are the observed intraday returns.

The vast majority of the literature on volatility estimation in the presence of microstructure noise is centered on a single measure, the realized variance. At very high frequencies the RV becomes inconsistent for the QV, capturing only noise. Solutions to this problem concern either sampling less frequently or modelling and correcting for noise.

Bandi and Russell (2008) show that, when the price is contaminated with i.i.d. microstructure noise, over a time interval equal to δ, $y_{j,i}$ is of order $O_p(\sqrt{\delta})$, while $\epsilon_{j,i}$ is $O_p(1)$. This means noise has a smaller impact at lower frequencies. On one side, asymptotics for the RV requires to sample more frequently, while on the other side, the contamination with microstructure noise imposes less often frequencies. Obviously, a balance between these two effects needs to be found, so that an optimal sampling frequency can be computed. Bandi and Russell (2008) compute the mean squared error (MSE) for the RV in the presence of i.i.d. noise and show that an optimal sampling frequency is the one minimizing the MSE.

Zhang et al. (2005) observe that when all data is considered, in the presence of i.i.d. microstructure noise, RV consistently estimates $E[\epsilon^2_{j,i}]$, but is inconsistent for the integrated variance. Thus, they propose using two different time scales: a sparse one, for instance every 10 minutes, and another one based on all available transactions. Authors recommend sampling sparsely several times starting with different observations, compute more RVs and then average over them. In this way, all observations in the dataset are used. The bias-adjusted estimator of the quadratic variation of the process, $\widehat{[Y(t)]}$, is named the two-scale estimator and can be constructed as:

$$\widehat{[Y(t)]} = [Y^*(t)]^{(avg)} - \frac{\bar{n}}{n}[Y^*(t)]^{(all)}, \tag{2.20}$$

where $[Y^*(t)]^{(avg)}$ is the average RV computed as described above for a certain sparse sampling frequency, $[Y^*(t)]^{(all)}$ is the RV computed on all observations, n is the number of transactions, whereas $\bar{n}$ stands for the average number of observations in subsamples.

The two-scale estimator has a rate of convergence of $n^{-1/6}$. Zhang (2006) proposes to improve this speed of convergence by combining more than 2 scales. The new measure is called multi-scale estimator and is defined in the following way:

$$\widehat{[Y(t)]} = \sum_{i=1}^{K} \alpha_i [Y^*(t)]^{(n,K_i)}, \tag{2.21}$$

where $\widehat{[Y(t)]}$ is the multi-scale estimator, K is the number of time scales, α_i is the weight given to a certain time scale, i, and $[Y^*(t)]^{(n,K_i)}$ is the average RV computed over a number of K_i RV estimators built for the time scale i. The weights are estimated so that the variance of the estimator is minimum. This new estimator has a rate of convergence of $n^{-1/4}$. Ait-Sahalia et al. (2005) show that both the two- and multi- scale estimators work in the presence of serially dependent, but stationary microstructure noise.

As seen above, an alternative to sub-sampling is correcting RV for the bias induced by the presence of noise. The size of the bias depends on the behaviour of the noise process: i.i.d., heteroskedasticity, serial dependence. Several authors recommend filtering the data before computing volatility estimators by using either moving average or autoregressive filters (Andersen et al., 2001; Hansen et al., 2008; Bollen and Inder, 2002, see). Large (2005) proposes an alternation estimator which applies when prices move by a sequence of single ticks.

An important contribution to this part of the literature is owed to Barndorff-Nielsen et al. (2008a). They build up realized kernels as unbiased estimators for the integrated variance or quadratic variation of the price process. The realized kernel ($K(Y_\delta^*)$) is defined as follows:

$$K(Y_\delta^*) = \gamma_0(Y_\delta^*) + \sum_{h=1}^{H} k\left(\frac{h-1}{H}\right)\{\gamma_h(Y_\delta^*) + \gamma_{-h}(Y_\delta^*)\}, \qquad \boxed{2.22}$$

where δ is the time interval over which returns are computed, $k(\cdot)$ is the weight (kernel) function and γ_h, γ_{-h} are auto-covariances of lag h and -h, $h = 1 \dots H$, defined as:

$$\gamma_h(Y_\delta^*) = \sum_{j=1}^{M}(Y_{j\delta}^* - Y_{(j-1)\delta}^*)(Y_{(j-h)\delta}^* - Y_{(j-h-1)\delta}^*),$$

with $M = [t/\delta]$.

Barndorff-Nielsen et al. (2008a) derive limit distributions for the realized kernels and rates of convergence. The choice of the weight function is so that the variance of the estimator is minimum. Moreover, depending on the choice of k and H, these estimators can converge at the fastest possible speed and achieve the maximum likelihood lower bound. Moreover, the authors prove that realized kernels can be designed to be robust to dynamics in the noise process, endogenous noise and endogenous irregular spacing of the data.

All the above contributions concerning the treatment of data contaminated with microstructure noise assume the price processes are continuous semi-martingales. As an exception, Barndorff-Nielsen et al. (2008a) show that in the presence of rare jumps, realized kernels are consistent for the quadratic variation. However, there are no contributions in the literature concerning jump identification and estimation in the presence of noise.

2.2.4 Robust to jumps volatility estimators

The introduction of the realized variance into the volatility literature proved that high frequency data can be exploited in order to obtain more information on the volatility processes. Consequently, a series of many other volatility estimators were proposed by different authors.

Barndorff-Nielsen's and Shephard's power and bipower variations

Following the power variation, in Barndorff-Nielsen and Shephard (2004), another quantity that will prove itself very helpful in jump estimation is defined, namely the realized bipower variation (BPV):

$$\{Y\}^{[r,s]}(t) = \operatorname*{plim}_{\delta \downarrow 0} \delta^{1-\frac{r+s}{2}} \sum_{j=1}^{[t/\delta]-1} |y_j(t)|^r |y_{j+1}(t)|^s, \tag{2.23}$$

where y_j are equidistant intraday returns at time j. Further on, for the case of continuous semi-martingale stochastic volatility processes with (μ, σ^2) independent of the Brownian motion W, BPV defined as above is found equal to:

$$\{Y\}^{[r,s]}(t) = \mu_r \mu_s \int_0^t \sigma^{r+s}(u)\mathrm{d}u, \tag{2.24}$$

where μ_r and μ_s are defined as in the power variation case.

When jumps are added to the price process, for certain values of r and s, the following result is derived for the BPV:

$$\mu_r{}^{-1}\mu_s{}^{-1}\{Y\}^{[r,s]}(t) = \begin{cases} \int_0^t \sigma^{r+s}(u)\mathrm{d}u, & \text{if} \quad \max(r,s) < 2, \\ \text{some stochastic process}, & \text{if} \quad \max(r,s) = 2, \\ \infty, & \text{if} \quad \max(r,s) > 2. \end{cases}$$

Thus, if we choose $r \in (0,2)$ and $s = 2 - r$, the realized BPV based on M high-frequency returns, $\mu_r{}^{-1}\mu_{2-r}{}^{-1}\{Y_M\}_i^{[r,2-r]}$, will converge in probability to the integrated variance, $\int_{h(i-1)}^{hi} \sigma^2(u)\,\mathrm{d}u$. This implies that the quadratic variance of the jump component can be obtained by differencing the following two quantities:

$$\{Y_M\}_i^{[2]} - \mu_r{}^{-1}\mu_{2-r}{}^{-1}\{Y_M\}_i^{[r,2-r]} \xrightarrow{p} \sum_{j=N(h(i-1))+1}^{N(hi)} c_j^2 \tag{2.25}$$

For the case of continuous processes, the CLT-type result obtained for the realized variance (equation 2.8) is extended for the realized bipower variation. Moreover, the correlation between the two processes, RV and BPV, is computed, generating the following result (Barndorff-Nielsen and Shephard, 2006a):

$$\frac{\delta^{-1/2}}{\sqrt{\int_0^t \sigma^4(u)\,\mathrm{d}u}} \begin{pmatrix} \sum_{j=1}^{[t/\delta]-1} y_j^2 - \int_0^t \sigma^s(u)\mathrm{d}u \\ \mu_1^{-2} \sum_{j=1}^{[t/\delta]-1} |y_j||y_{j+1}| - \int_0^t \sigma^s(u)\mathrm{d}u \end{pmatrix} \xrightarrow{\mathscr{L}} \mathscr{N}\left\{ 0, \begin{pmatrix} 2 & 2 \\ 2 & 2.60907 \end{pmatrix} \right\} \tag{2.26}$$

Equation (2.26) is a very important result as it individualizes the interdependences between the two volatility measures. This result allowed the two authors to derive the asymptotic distribution for the difference between the two estimators, which will be the very basis for jump testing (see section 2.3).

Range estimators

Following the example of Barndorff-Nielsen and Shephard (2004), Christensen and Podolskij (2006) try to extend the results on the realized range to processes containing discontinuities. However, they discover that RRV does no longer converge to IV. Consequently, they try to define a new quantity that consistently estimates IV: the range-based bipower variation, RBV.

$$RBV^{\Delta}_{(r,s)} = \delta^{\frac{r+s}{2}-1} \frac{1}{\lambda_{r,2}} \frac{1}{\lambda_{s,2}} \sum_{j=1}^{M-1} s^r_{Y_{t_i},\delta} s^s_{Y_{t_{i+1}},\delta}, \tag{2.27}$$

Further on, the authors derive a CLT for the latter estimator and then, show that, just like in the case of the statistics built up by Barndorff-Nielsen and Shephard (2004), the difference between RRV and RBV consistently estimates the jump for a certain time period, like a trading day. Thus, these two range-based estimators could be used in testing for jumps.

Another range-based volatility estimator was developed by Dobrev (2006) and is a generalization of Parkinson (1980)'s range. This new estimator is defined as "the sum of the magnitudes of the largest k non-overlapping price moves". Let $[0,t]$ be a given time interval and $t_1, t_2, \ldots, t_{2k}$ partitions of this interval. We can compute the generalized range (GR) as:

$$GR_k(Y_{[0,t]}) = \max_{0 \leq t_1 \leq \ldots \leq t_{2k} \leq t} \sum_{i=1}^{k} |Y(t_{2i}) - Y(t_{2i-1})| \tag{2.28}$$

Dobrev (2006) proves the convergence of his estimator to the integrated variance. Moreover, he shows that GR is robust to a finite number of jumps and thus, can be used in jump estimation through comparisons to a non robust to jumps estimator, such as the realized variance.

Threshold estimators

Mancini (2004, 2009) develops another estimator for the integrated variance based on a property of the Brownian motion established by Lévi (Renó, 2007, see) which states that the following function represents a modulus of continuity [3] for the Brownian motion:

$$g(\delta) = \sqrt{2\delta \log \frac{1}{\delta}} \tag{2.29}$$

Thus, for the paths of a Brownian motion, we have:

$$a.s. \lim_{\delta \to 0} \sup_{|t-s| \leq \delta} \frac{|W(t) - W(s)|}{g(\delta)} \leq 1 \tag{2.30}$$

As the stochastic integral $\int_0^t \sigma(u)\, \mathrm{d}W(u)$ is a time-changed Brownian motion (Mancini, 2004), a threshold for the integrated variance can be estab-

[3] "A function $g(x)$ is called a modulus of continuity for the function $f(x)$ if, for all sufficiently small $\delta > 0$, $|t - s| \leq \delta$ implies $|f(t) - f(s)| < g(\delta)$. (Renó, 2007, see)

lished, giving rise to a new modality of estimating both components of the QV, i.e. the integrated variance and the quadratic variation of the jump component.

Let us suppose that the sampling is done every δ periods, just like in Barndorff-Nielsen and Shephard (2004). Moreover, the rest of the notations are kept unchanged: i denotes the "lower frequency" sample points (trading days), while j the high-frequency ones. Let n be the number of possible jumps between two consecutive observations, t_{j-1} and t_j. Mancini (2004) defines the threshold as a function $r(\delta)$ that satisfies the following two conditions:

$$\lim_{\delta \to 0} r(\delta) = 0 \qquad \text{and} \qquad \lim_{\delta \to 0} \frac{\delta \log \frac{1}{\delta}}{r(\delta)} = 0 \tag{2.31}$$

Then, $\forall j = 1, 2, \ldots n$, $\mathrm{I}_{\{N_j - N_{j-1}\}} = \mathrm{I}_{\{y_{j,i}^2 \leq r(\delta)\}}$ a.s. and the threshold realized variance can be defined as:

$$TRV_\delta(Y)_t = \sum_{j=1}^{[t/\delta]} y_{j,i}^2 \mathrm{I}_{\{y_{j,i}^2 \leq r(\delta)\}} \tag{2.32}$$

Just like in the case of realized variance, power and bipower variations, Mancini (2004) derives a CLT result:

$$\frac{TRV_\delta(Y)_t - \int_0^t \sigma_u^2 \, \mathrm{d}u}{\sqrt{2\delta \int_0^t \sigma_u^4 \, \mathrm{d}u}} \xrightarrow{\mathscr{L}} \mathscr{N}(0, 1) \tag{2.33}$$

where $\int_0^t \sigma_u^4 \, \mathrm{d}u$ can be estimated by the following quantity:

$$\frac{1}{3\delta} \sum_{j=1}^{[t/\delta]} y_{j,i}^4 \mathrm{I}_{\{y_{j,i}^2 \leq r(\delta)\}} \tag{2.34}$$

Thus, every jump instant can be defined as:

$$\hat{\Delta}_j N = \mathrm{I}_{\{y_{j,i}^2 > r(\delta)\}}, \tag{2.35}$$

with $\Delta_j N = N_{t_j} - N_{t_{j-1}}$, while the jump size is given by:

$$\hat{\gamma}_j = y_{j,i}^2 \mathrm{I}_{\{y_{j,i}^2 > r(\delta)\}} \tag{2.36}$$

Mancini (2009) proves that for a finite activity jump process, the estimated jump size will converge to its true value with a speed of $\sqrt{n}$:

$$\sqrt{n} \sum_j (\hat{\gamma}_j - \gamma_j \mathrm{I}_{\{\Delta_j N \geq 1\}}) \xrightarrow{\mathscr{L}} \mathscr{M}\mathscr{N}\left(0, T \int_0^T \sigma_s^2 \, \mathrm{d}N_s\right) \tag{2.37}$$

Mancini (2009) extends the above results to the case of processes with infinite jump activity.

In empirical work, as showed by Mancini and Renó (2006), one can consider a time-varying threshold when estimating the integrated variance by means of the threshold estimator.

2.3 Testing for jumps in a univariate framework

As seen in section 2.2, a multitude of nonparametric volatility estimators have been developed. This whole literature flow was initially motivated by the need to consistently estimate the integrated volatility. However, as some of these estimators were proved to be nonrobust to jumps, more effort was put into defining jump robust quantities. This effort led to a co-product, that is the possibility to test whether during a certain time period (usually considered equal to a trading day), jumps occurred and, if so, to consistently estimate them.

2.3.1 Barndorff-Nielsen and Shephard (2006a) test

The first step toward this new direction that emerged in the field of financial econometrics was taken by Barndorff-Nielsen and Shephard (2006a), who observed that the difference or the ratio between realized volatility and realized quadratic variation can be thought as an estimator of the quadratic variation of the jump process. As seen in subsection 2.2.4, equation (2.25), an appropriate choice for the parameters r and s, that is $r < 2$ and $s = 2 - r$ makes the realized BPV consistently estimate IV and the difference between RV and realized BPV qualify for jump testing and estimation. In Barndorff-Nielsen and Shephard (2006a), the authors opt for $r = s = 1$ and prove the following CLT-type result for the hypothesis of a continuous process when $\delta \downarrow 0$:

$$\frac{\delta^{-1/2}(\mu_1^{-2}\{Y_M\}_t^{[1,1]}) - [Y_M]_t}{\sqrt{\int_0^t \vartheta \sigma_u^4 \, \mathrm{d}u}} \xrightarrow{\mathscr{L}} \mathscr{N}(0,1) \tag{2.38}$$

or

$$\frac{\delta^{-1/2}\left(\frac{\mu_1^{-2}\{Y_M\}_t^{[1,1]}}{[Y_M]_t} - 1\right)}{\sqrt{\vartheta \frac{\int_0^t \sigma_u^4 \, \mathrm{d}u}{\int_0^t \sigma_u^2 \, \mathrm{d}u^2}}} \xrightarrow{\mathscr{L}} \mathscr{N}(0,1) \tag{2.39}$$

The above results allows us to test for the following:

H0: no jumps, that is:

$$\delta^{-1/2}(\mu_1^{-2}\{Y_M\}_t^{[1,1]}) - [Y_M]_t \xrightarrow{p} 0$$

against the alternative:

H1: jumps present during $[0, t]$:

$$\delta^{-1/2}(\mu_1^{-2}\{Y_M\}_t^{[1,1]}) - [Y_M]_t \xrightarrow{p} -\sum_{j=1}^{N_t} c_j^2 \leq 0$$

The integral in the denominator of equation (2.38), named integrated quarticity, can be estimated by using the realized quadpower variation, which is proved to be robust to finite activity jumps:

$$\{Y_M\}_t^{[1,1,1,1]} = \delta^{-1} \sum_{j=1}^{[t/\delta]} |y_{j-3}||y_{j-2}||y_{j-1}||y_j| \xrightarrow{p} \mu_1^4 \int_0^t \sigma_s^4 \, \mathrm{d}u \tag{2.40}$$

Further on, Barndorff-Nielsen and Shephard (2006a) show that in (2.39), the ratio $\frac{\int_0^t \sigma_u^4 \,\mathrm{d}u}{\left(\int_0^t \sigma_u^2 \,\mathrm{d}u\right)^2} \geq 1/t$ with equality reached in the homoskedastic case. Thus, a correction is proposed to the initial test in equation (2.39):

$$\frac{\delta^{-1/2}}{\sqrt{\vartheta \max\left(t^{-1}, \frac{\{Y_M\}_t^{[1,1,1,1]}}{\left\{\{Y_M\}_t^{[1,1]}\right\}^2}\right)}} \left(\frac{\mu_1^{-2}\{Y_M\}_t^{[1,1]}}{[Y_M]_t} - 1\right) \xrightarrow{\mathscr{L}} \mathscr{N}(0,1) \tag{2.41}$$

Andersen et al. (2005) suggest using another estimator for the integrated quarticity: the realized tripower quarticity (TP_t), based on the observation that it is robust to jumps:

$$TP_t = M\mu_{4/3}^{-3}\left(\frac{M}{M-2}\right)\sum_{j=3}^{[t/\delta]} |y_{j-2}|^{4/3}|y_{j-1}|^{4/3}|y_j|^{4/3} \tag{2.42}$$

Huang and Tauchen (2005) compare through means of Monte Carlo simulation alternative statistics that can be used for performing the Barndorff-Nielsen and Shephard (2006a) test described above. Thus, the following different statistics are taken into consideration: the standard test statistic in equation (2.38) with the denominator estimated by using the quadpower quarticity, the standard statistic based on the tripower quarticity, the log versions of both these statistics and finally, the log plus adjusted versions, where the adjustment is the one proposed by Barndorff-Nielsen and Shephard (2006a) and discussed above (equation (2.41)). They show that all the above statistics perform quite well in picking up jumps, without major differences between alternative expressions. The authors also avert that one must pay attention when trying to perform the Barndorff-Nielsen and Shephard (2006a) test for a larger number of days. Basically, tiny daily biases are usually present on a daily basis and might be inflated a lot by the considered number of days when aggregation is completed in order to carry out the test for a longer than one day period. In addition, Huang and Tauchen (2005) are also interested by the effect of the microstructure noise on the test statistics. They reveal that, unless corrections to the estimators of the nominator and denominator of the test statistics are made, this kind of noise makes the latter downward biased. They indicate the use of staggered returns to adjust the realized variance, the realized bipower variation, as well as the estimator of the integrated quarticity.

2.3.2 Andersen et al. (2007) test

Another test that makes use of the Volatility estimators, but in a slightly different manner is the one in Andersen et al. (2007).They build up a very simple and intuitive rule that can be used to detect jumps. They start by considering a randomly selected intraday return, given a certain sampling frequency, δ:

$$y_{\xi\cdot\delta,\delta} = \sum_{j=1}^{[t/\delta]} y_j(t)\mathbb{I}_{\{\xi=j\}} \tag{2.43}$$

The authors prove a CLT theorem for the randomly selected returns:

$$\delta^{-1/2} y_{t+\xi\cdot\delta,\delta} \sim \mathcal{N}(0, IV_{t+1}) \tag{2.44}$$

where IV_{t+1} is the integrated variance at time $t+1$ and can be estimated by using Barndorff-Nielsen and Shephard (2004)'s realized bipower variation. Further on, one can detect multiple intraday jumps based on the rule:

$$c_j(\delta) = y_j \mathbb{I}_{\{|y_j|>\Phi_{1-\beta/2}\sqrt{\delta\cdot BV_{t+1}(\delta)}\}}, \qquad j = 1, 2, \ldots, \frac{1}{\delta} \tag{2.45}$$

where $BV_{t+1}(\delta)$ is the realized bipower variation estimated for a sampling frequency equal to δ and $\Phi_{1-\beta/2}$ is the corresponding critical value from the standard normal, with $\beta = 1-(1-\alpha)^{\delta}$ the test size for period δ and α the daily test size. The advantage of this testing procedure is that it allows performing the test over any time period, such that one can precisely identify the exact timing of each jump. Moreover, Andersen et al. (2007) provide simulation evidence that shows this test retains more power than the classical Barndorff-Nielsen and Shephard (2006a) procedure.

2.3.3 Aït-Sahalia and Jacod (2008)

In parallel with Barndorff-Nielsen and Shephard (2006b), Jacod (2008) worked on deriving the asymptotic properties of power variations. Part of this work contributed to another jump test he developed together with Aït-Sahalia (Aït-Sahalia and Jacod, 2008). The starting point for this new test consists in the following quantities:

$$A(r)_t = \int_0^t |\sigma_s|^r, \qquad B(r)_t = \sum_{j \leq t} |y_j|^r \tag{2.46}$$

where r is a positive number. The above expressions can be estimated by using the following estimator:

$$\widehat{B(r,\delta)}_t = \sum_{j=1}^{[t/\delta]} |y_j|^r \tag{2.47}$$

For different values of r, we can have the following convergences in probability:

$$\begin{cases} r > 2 & \Rightarrow \widehat{B(r,\delta)}_t & \xrightarrow{p} B(r)_t \\ r = 2 & \Rightarrow \widehat{B(r,\delta)}_t & \xrightarrow{p} [Y](t) \\ r < 2 & \Rightarrow \frac{\delta^{1-r/2}}{m_r}\widehat{B(r,\delta)}_t & \xrightarrow{p} A(r)_t \\ Y \text{ is continuous} & \Rightarrow \frac{\delta^{1-r/2}}{m_r}\widehat{B(r,\delta)}_t & \xrightarrow{p} A(r)_t \end{cases} \tag{2.48}$$

Aït-Sahalia and Jacod (2008) depart from the observation that in the above cases of convergence, the first and the normalized fourth do not depend on the sampling scale (δ) or, in other words, are invariant to scale modifications. They exploit this fact by building up two different tests: one with the null of jumps and the other with the null of a continuous specification.

The authors develop a family of test statistics with the following form:

$$\widehat{S(r,k,\delta)}_t = \frac{\widehat{B(r,k\delta)}_t}{\widehat{B(r,\delta)}_t} \tag{2.49}$$

where $k \in \mathbb{N}$ multiplies the scale. The above statistic converges in probability to 1, when jumps are present, as well as to $k^{p/2-1}$ in the continuity case. For both these cases, central limit theorems are proved and thus, the two tests can be put together.

2.3.4 Jiang and Oomen (2008) test

Another approach to jump identification was proposed by Jiang and Oomen (2008), with the null of no realization of jumps on the path between 0 and t. The idea behind their test stands in some theoretical issues concerning the pricing of the variance swap contracts. Thus, the pricing of these contracts is based on the so-called 'log-contract'. Neuberger (1994) shows that a short position in the log contract plus a long one in the underlying, with a delta of $1/Y_t$, will genrate the following payoff:

$$2(\,\mathrm{d}Y_t/Y_t - \,\mathrm{d}\ln Y_t) = \sigma^2\,\mathrm{d}t \tag{2.50}$$

where Y_t is the price at time t. In a discretized version, the payoff of the variance swap contract can be written as:

$$SwV_t(\delta) = 2\sum_{j=1}^{M}(R_j - r_j) \tag{2.51}$$

where $M = [t/\delta]$ is the number of intraday observations, R_j denotes the j-th intraday arithmetic return, while r_j the j-th log return. Given the above equations, the absence of jumps will make the difference between SwV and the realized variance equal to 0:

$$\operatorname*{plim}_{\delta\to 0}\left(SwV_t(\delta) - RV_t(\delta)\right) = \begin{cases} 0 & \text{no jumps in}[0,t] \\ 2\int_0^t \underline{J}_u\,\mathrm{d}q_u - \int_0^t J_u^2\,\mathrm{d}q_u & \text{jumps in}[0,t] \end{cases} \tag{2.52}$$

where $RV_t(\delta)$ is the estimated realized variance, $\underline{J}_u = exp(J_u) - J_u - 1$, with J the jump process and $\int_0^t J_u^2\,\mathrm{d}q_u$ is the jump variation between 0 and t.

A CLT can be established for SwV, resulting in the construction of a jump test:

$$\frac{M\int_0^t \sigma^2\,\mathrm{d}u}{\sqrt{\Omega_{SwV}}}\left(1-\frac{RV_t}{SwV_t}\right) \tag{2.53}$$

In the above equation, the integrated variance will be estimated by using Barndorff-Nielsen and Shephard (2006b) bipower variation, while Ω_{SwV} will be estimated using a multi- power variation (Barndorff-Nielsen et al., 2003; Barndorff-Nielsen et al., 2006):

$$\hat{\Omega}_{SwV} = \frac{\mu_6}{9}\frac{M^3\mu_{6/p}^{-p}}{M-p+1}\sum_{i=0}^{M-p}\prod_{k=1}^{p}|r_{i+k}|^{6/p} \tag{2.54}$$

where a suitable choice for p is 4 or 6. Just like the Aït-Sahalia and Jacod (2008) test, the above test is developed in a model free framework, taking into consideration an Ito semimartingale, without any assumptions on the drift, volatility or component. Furthermore, the authors modify the test in order to suit noisy data and prove that it retains power. While having a higher convergence rate than the standard Barndorff-Nielsen and Shephard (2006a) test, the conducted simulations reveal better size properties at high frequencies and more power if just 1 jump is considered in comparison with the above cited test.

2.3.5 Lee and Mykland (2008)

Lee and Mykland (2008) build up another jump test starting from the following question: given the trade-off between volatility and jumps, if there are high variations in prices, how can one be able to distingush when these were caused by jumps or by the volatility component? Thus, they imply that one can start from the simple log returns, standardize them properly by using a robust to jump volatility estimator and compare the resulting quantities with a proper threshold in order to detect jumps.

The following statistic is considered:

$$\mathscr{L}(j) = \frac{y_j}{\hat{\sigma}_j} \tag{2.55}$$

where $\hat{\sigma_j}$ is the realized bipower variation estimated on a K previous observations window.

The above statistic is used for jump identification at a given time t_j, unlike the case of the other tests where the hypotheses are formulated relative to a given period (time interval). Under the null of no jumps, the statistic will be asymptotically normal. However, in order to establish a more stringent rejection region, the authors try to find out how large the statistic can get when a jump is present. Thus, they consider the maximum of their statistic. They show that under the null, the maximum of the test statistic has an exponential cumulative distribution function:

$$\frac{\max\left(\mathscr{L}(j)\right) - C_M}{S_M} \to \xi, \qquad \mathbf{P}(\xi) = \exp(-e^{-x}) \tag{2.56}$$

where

$$C_M = \frac{(2\log M)^{1/2}}{\mu_1} - \frac{\log\pi + \log\left(\log M\right)}{2\mu_1(2\log M)^{1/2}} \tag{2.57}$$

and

$$S_M = \frac{1}{\mu_1(2\log M)^{1/2}} \tag{2.58}$$

Thus, the test can be conducted by simply replacing the maximum statistic above by the estimated value of $\mathscr{L}(j)$ and compare the resulting value with the threshold showed above. However, the findings in equation (2.56) are valid only under some mild assumptions on the drift and volatility coefficients of the models (Lee and Mykland, 2008). Simulations employed by the authors show the superiority of this test in terms of size and power in comparison with the standard bipower test of Barndorff-Nielsen and Shephard (2006a).

2.3.6 Podolskij and Ziggel (2008)

As seen in section 2.2.4, Mancini (2004, 2009) develops threshold volatility estimators that are robust to jumps in asset prices and provides CLT-type results for these quantities. Having both a robust and a non-robust (realized variance) volatility estimators, the obvious step forward is jump estimation and testing. However, a joint distribution law for both estimators could not be found. In order to overcome this difficulty, Podolskij and Ziggel (2008), choose to multiply the threshold estimator by some i.i.d. random variables, instead of considering the simple difference between the realized variance and the threshold estimator:

$$T(Y,p)_t^M = M^{\frac{p-1}{2}} \sum_{j=1}^{M} \left|y_j^M\right|^p (1 - \eta_j \mathbb{I}_{\{|y_j^M| \le cM^{-\bar{w}}\}}), \qquad p \ge 2 \tag{2.59}$$

where $c > 0$ a constant and $\bar{w} \in (0, 1/2)$. η_j are i.i.d. variables with $E(\eta_j) = 1$ and $E(\eta_j^2) < \infty$. The above estimator is proved to stably converge under the null of no jumps to the following quantity:

$$T(Y,p)_t^M \overset{F-stable}{\longrightarrow} \sqrt{Var(\eta_j)\mu_{2p}} \int_0^t |\sigma_s|^p \, \mathrm{d}W_s' \tag{2.60}$$

Under the alternative, this quantity diverges, allowing for the definition of the following test statistics:

$$S(p)_t^M = \frac{T(Y,p)_t^M}{\sqrt{Var(\eta_j)\mu_{2p}} \int_0^t |\sigma_s|^p \, \mathrm{d}W_s'} \tag{2.61}$$

where the integrated variation in the denominator can be estimated in a robust to jumps manner by using the same threshold estimators and μ_{2p} is given in equation (2.13). For the choice of the distribution of the random variables, the authors recommend the following one:

$$P^\eta = \frac{1}{2}(\delta_{1-\tau} + \delta_{1+\tau}) \tag{2.62}$$

with δ being the Dirac measure and the constant τ relatively small, e.g. $\tau = .1$ or $.05$. Simulations prove a better performance in terms of power in comparison with the Barndorff-Nielsen and Shephard (2006a) and the Aït-Sahalia and Jacod (2008) tests.

2.4 Extensions to a multivariate framework

In the previous section we showed it is possible to disentangle the variation in prices due to jumps from the one due to the diffusion component. Given the various contributions to the literature that deal with such issues in the univariate case, it is natural to try to extend the existing theory to a multivariate framework. This is extremely relevant from a practical point a view, as investors usually own portfolios of assets. From here, one can derive important implications in portfolio allocation, risk management, hedging and pricing, as different assets can display similar or divergent jumping patterns. Moreover, this can influence the way risk premia are attributed. If different jumps are proved to be systemic, involving the whole market, the corresponding risk should not be remunerated by the market. Only individual risks are expected to be awarded.

In the multivariate framework, we work with co-variations of different price processes taken two by two. However, unlike in the univariate case, an extra problem appears, namely the non-synchrony of transactions or quotations of any two assets. This problem was first characterized by Epps (1979), who shows that when observations for returns for any two assets are synchronized by applying the previous-tick scheme, the covariance tends to diminish at higher sampling frequencies. [4] The majority of the literature on estimating volatility in a multivariate framework based on high frequency data proposes estimators that require synchronizing the observations. However, in this case a special attention must be paid to the sampling scheme. The only estimator based on non-synchronous data is the one introduced by Hayashi and Yoshida (2005).

2.4.1 Non-robust to jumps estimators

Barndorff-Nielsen and Shephard (2007) generalize the realized volatility to a multiple-asset framework. Let y_j be a $(p \times 1)$ vector of returns at time j

[4] This phenomenon is known in the financial literature as the *Epps' effect*.

for p different assets, with j denoting, just as before, high-frequency sampling times during a trading day. Thus, the realized volatility is defined as:

$$[Y_M] = \sum_{j=1}^{M} y_j y_j' \xrightarrow[M\to\infty]{p} [Y] =$$

$$= \begin{pmatrix} [Y_{(1)}] & [Y_{(1)}, Y_{(2)}] & \dots & [Y_{(1)}, Y_{(p)}] \\ [Y_{(2)}, Y_{(1)}] & [Y_{(2)}] & \dots & [Y_{(2)}, Y_{(p)}] \\ \vdots & \vdots & \ddots & \vdots \\ [Y_{(p)}, Y_{(1)}] & [Y_{(p)}, Y_{(2)}] & \dots & [Y_{(p)}] \end{pmatrix} \tag{2.63}$$

where $Y_{(l)},\ l = 1\dots p$ is the price process for the l-th asset, The "covariation" terms in the above equation are estimated based on some equalities established for the quadratic covariation processes and given below:

$$[Y_{(l)}, Y_{(k)}] = \frac{1}{2}([Y_{(l)} + Y_{(k)}] - [Y_{(l)}] - [Y_{(k)}]) \tag{2.64}$$

or

$$[Y_{(l)}, Y_{(k)}] = \frac{1}{4}([Y_{(l)} + Y_{(k)}] - [Y_{(l)} - Y_{(k)}]), \tag{2.65}$$

where $[Y_{(l)} + Y_{(k)}]$ is the realized volatility for the sum of the prices of the l-th and k-th assets, $[Y_{(l)} - Y_{(k)}]$ is the realized volatility of the difference between the prices of the same assets, while $[Y_{(l)}]$ and $[Y_{(k)}]$ are the individual realized volatilities of the two assets.

Hayashi and Yoshida (2005) show that due to the *Epps' effect*, $[Y_{(l)}, Y_{(k)}]$ is underestimated when applying a synchronization scheme based on interpolation. In order to overcome this difficulty, they propose a new estimator, named the Cumulative Covariance Estimator, defined as:

$$U_n := \sum_{k,l} y^1(K^k) y^2(L^l) 1_{\{K^k \cap L^l \neq \Phi\}}, \tag{2.66}$$

where K^k and L^l are random intraday intervals, and y^1 and y^2 denote the return processes for asset 1 and asset 2, respectively. The above sum will include all changes in the prices of the two assets if and only if the intervals over which these changes are observed overlap.

Hayashi and Yoshida (2005) prove that $U_n \to \int_0^t \sigma_u^1 \sigma_u^2 \rho_u du$ in probability as $n \to \infty$, where n is the number of intersections, σ_u^1 and σ_u^2 are the diffusion parameters for assets 1 and 2, whereas ρ_u is the correlation between the returns of the two assets. Observations from the continuous time price processes are assumed to be made at random times. The conditions that these random times must fulfill restrict the possible choices to a narrow range of variables, from which the Poisson sampling scheme is the most plausible.

Both the above multivariate estimators do not take microstructure noise into account. Barndorff-Nielsen et al. (2008b) propose a generalization of the realized kernels to the multivariate framework, however based on synchronized observations. Voev and Lunde (2007) show that depending on the structure of the noise, a simple realized covariance might can be sometimes preferred to the cumulative covariance and propose a correction to the latter estimator. Sen and Xu (2007) propose subsampling when computing the Hayashi and Yoshida (2005) estimator in order to overcome the noise problem.

2.4.2 Robust to jumps estimators

Barndorff-Nielsen and Shephard (2007) also generalize the realized bipower variation to a multivariate framework. Just like the multivariate QV process and its estimator, the realized volatility, the BPV process can be estimated at a multivariate level based on its realized counterpart, defined in the following way:

$$\{Y_{\delta;q}\} = \begin{pmatrix} \{Y_{(1)\delta};q\} & \{Y_{(1)\delta},Y_{(2)\delta};q\} & \dots & \{Y_{(1)\delta},Y_{(p)\delta};q\} \\ \{Y_{(2)\delta},Y_{(1)\delta};q\} & \{Y_{(2)\delta};q\} & \dots & \{Y_{(2)\delta},Y_{(p)\delta};q\} \\ \vdots & \vdots & \ddots & \vdots \\ \{Y_{(p)\delta},Y_{(1)\delta};q\} & \{Y_{(p)\delta},Y_{(2)\delta};q\} & \dots & \{Y_{(p)\delta};q\} \end{pmatrix} \tag{2.67}$$

where the realized bipower variation for the l-th asset equals

$$\{Y_{(l)\delta}; q\} = \gamma_{q,\delta} \sum_{j=q+1}^{M} |y_{(l)j-q}||y_{(l)j}|, \tag{2.68}$$

with $\gamma_{q,\delta} = \frac{M}{M-q}$, $\gamma_{q,\delta} \downarrow 1$ as $\delta \downarrow 0$ and the realized bipower covariance for assets l and k is defined as:

$$\{Y_{(l)\delta}, Y_{(k)\delta}; q\} = \frac{\gamma_{q,\delta}}{4}(\{Y_{(l)\delta} + Y_{(k)\delta}; q\} - \{Y_{(l)\delta} - Y_{(k)\delta}; q\}) \tag{2.69}$$

For both quadratic and bipower covariation processes, a Central Limit Theorem is provided. However, the authors encounter problems in finding an estimator for the variance of the bipower variation that is robust to jumps.

Despite of the problems discussed above, the work of Barndorff-Nielsen and Shephard (2007) is valuable from two points of view. First, they pose the problem of jumps in a multivariate framework, revealing its importance in finance and giving rise to other theoretical and empirical work which tries to describe jumps in more than one asset. Second, they introduce a very interesting concept that, provided the statistical issues concerning multivariate jumps are solved, could prove itself to have important practical implications, especially in the field of asset allocation. Thus, they define the concept of co-jumping, which, similar to other co-features in the econometrics literature, such as cointegration, co-trending, co-breaking, implies that, given two or more assets that display a jump at a certain time, one can find a linear combination of their returns that does no longer jump.

Let us have a p- dimensional Brownian semimartingale plus jump process that we group in a p dimension vector, Y. The quadratic variation of Y will have the following form:

$$[Y]_t = \int_0^t \Sigma_u \, \mathrm{d}u + \sum_{j=1}^{N_t} C_j C_j'^{5} \tag{2.70}$$

[5] Notations are mostly taken from Barndorff-Nielsen and Shephard (2007)

where $\int_0^t \Sigma_u \, \mathrm{d}u$ is the multivariate integrated variance-covariance and C_j is a p dimension vector with the j-th jumps. Let $\tau_1, \tau_2, \ldots, \tau_{N_t}$ be the arrival times of the jump counting process. Pre-multiplying the vector of prices, Y, with a $k \times p$ matrix, D, containing elements that are cádlág and adapted to the filtration generated by Y, results in another process with the following quadratic variance-covariance:

$$\int_0^t D_u \sigma_u \sigma_u' D_u' \, \mathrm{d}u + \sum_{j=1}^{N_t} D_{\tau_j -} C_j C_j' D_{\tau_j -}' \tag{2.71}$$

We talk about co-jumping during 0 and t when some of the diagonal elements of $\sum_{j=1}^{N_t} D_{\tau_j -} C_j C_j' D_{\tau_j -}'$ are 0. Consequently, in order to establish whether co-jumping occurred or not, one has to check whether $\sum_{j=1}^{N_t} C_j C_j'$ is a reduced rank matrix, provided that matrix D is time invariant.

Similar to the generalizing approach of Barndorff-Nielsen and Shephard (2007), Gobbi and Mancini (2008b) extended Mancini (2009)'s threshold volatility estimators to a multivariate framework. Thus, given the price processes of two financial assets, they define a threshold estimator for the integrated quadratic covariation, $\int_0^t \rho_s \sigma_{(1)s} \sigma_{(2)s} \, \mathrm{d}s$, with ρ_s being the correlation coefficient between prices at time s and $\sigma_{(l)s}$, $l = 1, 2$ the diffusion parameter for the price process of asset l. Basically, they propose the following threshold estimator:

$$TRCOV_\delta(Y_{(1)}, Y_{(1)})_t = \sum_{j=1}^{[t/\delta]} y_{(1)j,i} \mathrm{I}_{\{y^2_{(1)j,i} \leq r(\delta)\}}, y_{(2)j,i} \mathrm{I}_{\{y^2_{(2)j,i} \leq r(\delta)\}}, \tag{2.72}$$

where $r(\delta)$ is the threshold and $I_{\{.\}}$ an indicator function.

Moreover, a central limit theorem is derived. The difference between the realized covariation and the threshold estimator consistently estimates the common variation in the jump processes of the two assets:

$$\sum_{j=1}^{[t/\delta]} y_{(1)j,i} y_{(2)j,i} - TRCOV_\delta(Y_{(1)}, Y_{(1)})_t \xrightarrow{p} \sum_{s \leq t} c_{(1)s} c_{(2)s}, \tag{2.73}$$

where $c_{(l)s}$, $l = 1, 2$ represents the jump at time s for asset l. Further on, Gobbi and Mancini (2008a) extend the above results to the case of infinite activity jumps.

2.4.3 Testing for common jumps in the prices of financial assets

Jacod and Todorov (2007) propose a test for the common arrival of jumps. This work is meant to extend to a bivariate framework univariate jump detection procedure in Aït-Sahalia and Jacod (2008). The assumed price processes are two stochastic volatility processes with jumps in both price and volatility. However, results are shown to be valid only when the price and the volatility processes do not jump together.

Supposing that one has already established based on an adequate testing procedure that the prices of two different financial assets are not continuous, the present test is meant to distinguish between the following two hypotheses:

- joint jumps, meaning that the prices of both assets jump together
- disjoint jumps, implying that jumps do not arrive together.

For this purpose, two different test statistics are used. In the case of a null of joint jumping, the idea that power variation should be invariant to scale modifications is exploited again, just like in Aït-Sahalia and Jacod (2008):

$$S(k,\delta)_t^{(joint)} = \frac{\sum_{j=1}^{[t/\delta]} \left(y_{(1)kj} y_{(2)kj}\right)^2}{\sum_{j=1}^{[t/\delta]} \left(y_{(1)j} y_{(2)j}\right)^2} \tag{2.74}$$

where $y_{(l)kj}$ represents the return of the l-th asset sampled every $k\delta$ times.

In order to test for the null of disjoint jumps, the test statistic is:

$$S(k,\delta)_t^{(disjoint)} = \frac{\sum_{j=1}^{[t/\delta]} \left(y_{(1)j} y_{(2)j}\right)^2}{\sqrt{\sum_{j=1}^{[t/\delta]} \left(y_{(1)j}\right)^2 \sum_{j=1}^{[t/\delta]} \left(y_{(2)j}\right)^2}} \tag{2.75}$$

Critical regions for both cases (joint, disjoint) are determined and the asymptotic behavior of the above statistics is explored, leading to a CLT type result [6] that enables testing.

Unlike the above approach, which attempts to test for common jumps in a fixed number of asset prices, another interesting and simple procedure was introduced by Bollerslev et al. (2007). They observe that by applying the Barndorff-Nielsen and Shephard (2006a) jump detection procedure to a stock index, much fewer jumps are detected than when the procedure is applied for each individual stock. They split jumps into two categories: the common or systemic jumps and the idiosyncratic ones. The standard jump testing procedure is very sensitive to idiosyncratic jumps, and is not able to pick up common jumps, which are usually smaller.

Let p be the number of assets in portfolio. Bollerslev et al. (2007) consider the following statistic:

$$mcp_t = \sum_{j=1}^{M} \frac{1}{2p(p-1)} \sum_{k=1}^{p-1} \sum_{l=k+1}^{p} y_{(k)j} y_{(l)j} = \\ = \frac{1}{4(p-1)} \left[pRV_{EQW,t} - \frac{1}{p} \sum_{k=1}^{p} RV_{k,t} \right] \tag{2.76}$$

where $RV_{k,t}$ is the realized volatility for each individual asset in the portfolio and $RV_{EQW,t}$ is the realized volatility for an equally weighted portfolio and is defined in the following equation:

$$RV_{EQW,t} = \sum_{j=1}^{M} \left(\frac{1}{p} \sum_{k=1}^{p} y_{(k)j} \right)^2 \tag{2.77}$$

Just like in portfolio theory, the authors assume a very large number of assets, $p \to \infty$, which will lead to the disappearance of the last part of the mcp statistic containing the idiosyncratic variations. What is left will only reflect the common variation of the assets in the portfolio and, compared to

[6] A stable convergence in law is proved for each of these statistics.

a suitable threshold, can lead to jump identification. Bollerslev et al. (2007) standardize this statistic and make use of the bootstrap procedure to derive its distribution under the null of no jumps.

2.5 Conclusion

The objective of this chapter is to summarize some of the recent advances in the field of financial econometrics concerning volatility estimation based on high frequency data, as well as jump detection. The focus of the review is on the ability and importance of disentangling jumps from the diffusion component of the price processes. This is important because both components generate risks that have different requirements in terms of hedging, pricing and management. Moreover, due to the valuable implications in portfolio allocation and risk management, we also extensively describe the contributions to the literature in the multivariate case.

For both univariate and multivariate frameworks, the structure of our exposition follows four different directions. First, we describe the existing estimators for the quadratic variation of the processes. Second, given that prices are contaminated with microstructure noise, we present several possibilities to overcome the problems deriving from this contamination. Third, various robust to jumps estimators are exposed. Finally, the existing jump detection procedures are introduced.

As already mentioned, the focus in this chapter stays on disentangling jumps. It was not our intention here to exhaust all literature on volatility estimation. Therefore, the parts concerning contamination with microstructure noise are not described in such a detail as the other parts. However, their inclusion in our review is important, as real prices do not behave as observations from diffusions or jump diffusions and consequently, noise will always be an issue researchers will have to deal with.

Most of the contributions in the literature on how to deal with microstructure noise only concern estimators of the quadratic variance of the processes.

There are just a few contributions regarding robust to jumps estimators and testing for jumps. In general, we know sub-sampling works in this case, as well. One of the results obtained in Chapter 3 sheds some light on how to apply the jump detection procedures when prices are contaminated with microstructure noise. We propose combining different tests and sampling frequencies in order to optimize the performance of the tests.

3

Identifying jumps in financial assets: a comparison between nonparametric jump tests

There is a large consensus in the financial literature, theoretical and applied, that modeling return dynamics requires the specification of a stochastic volatility component, which accommodates the persistence in volatility, and of a jump component, which takes care of the unpredictable, large movements in the price process. The identification of the time and the size of jumps has profound implications in risk management, portfolio allocation, derivatives pricing (Aït-Sahalia, 2004). For this task, the use of jump diffusion models proved very difficult, as there are no closed forms of the likelihood function and in addition, the number of parameters to estimate is very high. One solution is to focus on the popular class of affine models (Duffie et al., 2000) which allow for tractable estimation, but impose a quite restrictive set of assumptions. An alternative approach is represented by nonlinear volatility models. However, the estimation procedure, based on simulation methods, such as the Gallant and Tauchen (2002)'s efficient method of moments, is computationally demanding and too much dependent on the choice of an auxiliary model (Chernov et al., 2003; Andersen et al., 2002, see, for instance).

One of the main advances in high frequency econometrics over the last decade was the development of several nonparametric procedures that allow to test for the presence of jumps in the path of a price process during a certain time interval or at certain point in time. Such methods are very simple to apply, they just require high frequency transaction prices or mid-quotes. Moreover, they are developed in a model free framework, incorporating different classes of stochastic volatility models. In addition to the seminal contribution of Barndorff-Nielsen and Shephard (2006a), in this chapter we consider four other tests proposed by Andersen et al. (2007)-Lee and Mykland (2008), Aït-Sahalia and Jacod (2008), Jiang and Oomen (2008)and Podolskij and Ziggel (2008). All tests are based on CLT-type results that require an intraday sampling frequency that tends to infinity. The test statistics are based on robust to jumps measures of variation in the price processes which are estimated by using either realized multi-power variations (Barndorff-Nielsen and Shephard, 2004, 2003, see) or threshold estimators (Mancini, 2009, see). The Andersen et al. (2007)-Lee and Mykland (2008) tests have the null hypothesis of continuity of the sample path at a certain moment, allowing for the exact identification of the time of a jump. The other procedures have a null of continuity within a certain time period, such as a trading day.

Given such a variety of nonparametric methodologies to identify jumps, one might wonder which procedure should be preferred, or whether there are data characteristics for which it is recommended to use one test instead of the others. The main objective of this chapter is to perform a thorough comparison among the five testing procedures, based on a comprehensive set of Monte Carlo simulations, which embodies important features of financial data. To quantify the size for all tests, our simulations are based on both constant and stochastic volatility models with varying persistence. To evaluate the power property, we consider stochastic volatility models with jumps of different sizes arriving with varying intensity. Based on the findings of the simulation exercise, we aim to provide a set of guidelines to users of nonparametric tests for jumps. It is important to establish whether the performance of the tests is related to some featuress of the data, such as different sampling frequencies, different levels of volatility, varying persistence in volatility, varying contamination with microstructure noise, varying jump size and jump intensity. Such characteristics vary between classes of assets, as well as between different time periods. For instance, equity prices are 'jumpier' than bond prices and markets in general have been more volatile and at the same time 'jumpier' during the last 2 years than before. Finally, we apply the tests to real tick data, using high frequency data on the US Treasury 2-, 5- 10- and 30- year bonds over a period lying between January 2003 and March 2004. We apply all tests on the data sampled at different frequencies.

The chapter is organized as follows. In Section 2, we review the 5 non-parametric tests for jumps available in the literature. Section 3 describes the Monte Carlo setup and reports the main findings of the simulations. Section 4 reports an empirical exercise using US Treasury data. Finally, Section 5 concludes and offers some guidelines to potential users.

3.1 Jump tests

In this section, we describe the available jump detection procedures. It is important to note that none of these procedures can test for the absence or presence of jumps in the model or data generating process. They merely supply us with information on whether within a certain time interval or at a certain moment, the realization of the process is continuous or not. Andersen et al. (2007) and Lee and Mykland (2008) assume the null of continuity of the sample path at time t_j. For all the other procedures, the null is of continuity of the sample path during a certain period, such as a trading day. The alternative hypothesis implies discontinuity of the sample path, that is the occurrence of at least one jump. For all procedures, under the null, the test statistics are asymptotically standard normal, though in some cases (Andersen et al., 2007; Lee and Mykland, 2008) standard normal thresholds, like 99% or 95% quantiles, appear too liberal and more restrictive thresholds need to be used.

Apart from the procedures proposed by Aït-Sahalia and Jacod (2008) and Podolskij and Ziggel (2008), all other procedures work only when a finite number of jumps occur within a certain time interval. This is due to the fact that in all cases, the construction of the test statistics is based on realized bipower variations estimators, which are robust only to a finite number of jumps. For this reason, in the simulation set-up, we only consider processes with a finite number jumps (compound Poisson) and compare tests under this scenario.

We turn now to the presentation of the various procedures.

3.1.1 Barndorff-Nielsen and Shephard (2006a) test (BNS henceforth)

Barndorff-Nielsen and Shephard (2006a) base their procedure on the possibility to build a consistent estimator for the integrated variance of a process. The test draws from previous reserach (Barndorff-Nielsen and Shephard, 2004), where authors show that the realized bipower variation (BV_t) consistently estimates the integrated variance in the presence of rare jumps:

$$BV_t = \operatorname*{plim}_{\delta \downarrow 0} \sum_{j=1}^{[t/\delta]} |y_j(t)||y_{j+1}(t)| \tag{3.1}$$

where δ is the intraday sampling frequency, with $[t/\delta]$ the number of intraday returns and y_j the j-th intraday return at time j, $j = 1...[t/\delta]$.

Thus, the difference between realized volatility and realized bipower variation qualifies for jump testing and estimation. Under the null hypothesis of continuous sample path from 0 to t, the test statistic is asymptotically standard normal:

$$\frac{\delta^{-1/2}(\mu_1^{-2} BV_t) - RV_t}{\sqrt{\int_0^t \vartheta \sigma_u^4 \, \mathrm{d}u}} \xrightarrow{\mathscr{L}} \mathscr{N}(0,1) \tag{3.2}$$

where RV_t is the realized volatility, and μ_1 and ϑ two constants, with $\mu_1 = \sqrt{2/\pi}$ and $\vartheta = (\pi^2/4) + \pi - 5$. The integral in the denominator of equation (4.2) is the integrated quarticity and can be estimated using the realized tripower quarticity (TP_t) (Andersen et al., 2005):

$$TP_t = M\mu_{4/3}^{-3}\left(\frac{M}{M-2}\right)\sum_{j=3}^{M}|y_{j-2}|^{4/3}|y_{j-1}|^{4/3}|y_j|^{4/3} \tag{3.3}$$

where M is the number of returns within a trading day and $\mu_{4/3} = E(|U|)^{4/3}$, with U being a standard normal variable.

The test statistic used in our simulation is the following:

$$z = \frac{1 - \frac{BPV_t}{RV_t}}{\sqrt{(\mu_1^{-4} + 2\mu_1^{-2} - 5)\delta max\left(1, \frac{TP_t}{BPV_t^2}\right)}} \to \mathscr{N}(0,1) \tag{3.4}$$

Note that equation (3.4) is the version of the BNS test that showed better finite sample properties according to the simulation exercise reported in Huang and Tauchen (2005).

3.1.2 Andersen et al. (2007) and Lee and Mykland (2008) tests (ABD and LM henceforth)

The papers by Lee and Mykland (2008) and Andersen et al. (2007) concurrently developed tests for jumps based on the standardization of intraday returns by robust to jumps volatility estimators. Both tests are constructed under the null that there is no jump in the realization of the process at a certain time, t_j. This enables users to identify the exact time of a jump, as well as the number of jumps within a trading day. We call these two procedures “intraday” tests, as they can detect jumps that occur any time during a trading day, whereas the other tests can only check for the discontinuity of the sample path at a daily level.

Andersen et al. (2007) build up a very simple and intuitive rule that can be used to detect jumps. They consider a randomly selected intraday return, $y_{\xi\cdot\delta,\delta}$, given a certain sampling frequency, δ:

$$y_{\xi\cdot\delta,\delta} = \sum_{j=1}^{[t/\delta]} y_j(t)\mathbb{I}_{\{\xi=j\}}, \tag{3.5}$$

where $\mathbb{I}_{\{\xi=j\}}$ is an indicator operator selecting the j-th observation from the data. The authors report a CLT theorem for the randomly selected returns which shows that:

$$\delta^{-1/2} y_{t+\xi\cdot\delta,\delta} \sim \mathcal{N}(0, IV_{t+1}) \tag{3.6}$$

where IV_{t+1} is the integrated variance at time $t+1$ and can be estimated by using Barndorff-Nielsen and Shephard (2004)'s realized bipower variation. One can detect multiple intraday jumps based on the rule:

$$c_j(\delta) = y_j \mathbb{I}_{\{|y_j| > \Phi_{1-\beta/2}\sqrt{\delta\cdot BV_{t+1}(\delta)}\}}, \qquad j = 1, 2, \ldots, \frac{1}{\delta} \tag{3.7}$$

where c_j stands for the critical region of the test, $BV_{t+1}(\delta)$ is the realized bipower variation estimated for a sampling frequency equal to δ at time $t+1$ and $\Phi_{1-\beta/2}$ is the corresponding critical value from the standard normal, with $\beta = 1-(1-\alpha)^{\delta}$ the test size for period δ and α the daily test size. Andersen et al. (2007) provide simulation evidence showing that this test has higher power than the classical Barndorff-Nielsen and Shephard (2006a) one. Moreover, they take into account the periodicity in the intraday volatility when applying the test on real data.

Lee and Mykland (2008) use the same realized bipower variation to standardize the returns, but they estimate it on a local window that precedes the time for which the test is performed.

Thus, the following statistic is considered:

$$\mathcal{L}(j) = \frac{y_j}{\hat{\sigma}_j} \tag{3.8}$$

where $\hat{\sigma_j}$ is the realized bipower variation estimated on a K previous observations window.

Under the null of continuity, the statistic is asymptotically normal. However, the usual 95% and 99% quantiles from the normal distribution prove themselves too permissive by leading to the identification of a very high number of jumps. As a result, the authors take into consideration the maximum of the $L(j)$ statistic over a given period, usually a day. The new standardized statistic will converge, for $\delta \to 0$, to a Gumbel variable:

$$\frac{\max\left(\mathscr{L}(j)\right) - C_M}{S_M} \to \xi, \qquad \mathbf{P}(\xi) = \exp(-e^{-x}) \tag{3.9}$$

where $C_M = \frac{(2\log M)^{1/2}}{\mu_2} - \frac{\log\pi + \log(\log M)}{2\mu_2(2\log M)^{1/2}}$ and $S_M = \frac{1}{\mu_2(2\log M)^{1/2}}$.

The test can be conducted by simply replacing in the test statistic in equation (3.9) $\mathscr{L}(j)$ by its estimated value and compare the resulting value with a critical value from the Gumbel distribution. Simulations employed by the authors show superiority of this test in terms of size and power in comparison with the standard bipower test of Barndorff-Nielsen and Shephard (2006a).

3.1.3 Aït-Sahalia and Jacod (2008) test (AJ henceforth)

Another procedure that enables the identification of discontinuities in prices is the one developed by Aït-Sahalia and Jacod (2008). The test considers the following processes which measure variation in the price:

$$A(r)_t = \int_0^t |\sigma_s|^r, \qquad B(r)_t = \sum_{j \le t} |y_j|^r \tag{3.10}$$

where r is a positive number, $A(r)$ is an r-order integrated variation of a continuous process, with σ_s the diffusion coefficient, while $B(r)$ measures the variation in a discontinuous process.

When a process contains both a diffusion component and jumps, the quantities in (3.10) can be estimated using the following estimator:

$$\widehat{B(r,\delta)}_t = \sum_{j=1}^{[t/\delta]} |y_j|^r \tag{3.11}$$

For different values of r, the following convergences in probability apply:

$$\begin{cases} r > 2 & \Rightarrow \widehat{B(r,\delta)}_t & \xrightarrow{p} B(r)_t \\ r = 2 & \Rightarrow \widehat{B(r,\delta)}_t & \xrightarrow{p} [Y](t) \\ r < 2 & \Rightarrow \frac{\delta^{1-r/2}}{\mu_r}\widehat{B(r,\delta)}_t & \xrightarrow{p} A(r)_t \\ Y \text{ is continuous} & \Rightarrow \frac{\delta^{1-r/2}}{\mu_r}\widehat{B(r,\delta)}_t & \xrightarrow{p} A(r)_t \end{cases} \tag{3.12}$$

where $\mu_r = E(|U|^r) = \pi^{-1/2}2^{r/2}\Gamma(\frac{r+1}{2})$, $U \sim \mathscr{N}(0,1)$.

Aït-Sahalia and Jacod (2008) notice that the first and the normalized fourth statistics do not depend on the sampling scale (δ) or, in other words, are invariant to scale modifications. Authors develop a family of test statistics of the following form:

$$\widehat{S(r,k,\delta)}_t = \frac{\widehat{B(r,k\delta)}_t}{\widehat{B(r,\delta)}_t} \tag{3.13}$$

where $k \in \mathbb{N}$ multiplies the scale.

The above statistic converges in probability to 1 when jumps are present, and to $k^{r/2-1}$ in the continuity case. This finding enables authors to propose two different tests. One has the null of continuity of the realization of the process within a certain time interval, with the alternative of discontinuity, while the other has the null of discontinuity, with at least one jump occurring and the alternative of a continuous sample path. In our exercise, we only implement the former test, which fits the framework of all other implemented tests.

3.1.4 Jiang and Oomen (2008) test (JO henceforth)

Another approach to jump identification is proposed by Jiang and Oomen (2008), with the null of no jumps in the sample path between 0 and t. The test draws from the pricing of the variance swap contracts, which is based on the so-called ‘log-contract’. Neuberger (1994) shows that a short position in the log contract plus a long one in the underlying, with a delta of $1/Y_t$, generates the following payoff:

$$2(\,\mathrm{d}Y_t/Y_t - \,\mathrm{d}lnY_t) = \sigma^2\,\mathrm{d}t \tag{3.14}$$

where Y_t is the price at time t. In a discretized version, the payoff of the variance swap contract ($SwV_t(\delta)$) can be written as:

$$SwV_t(\delta) = 2\sum_{j=1}^{[t/\delta]}(R_j - r_j) \tag{3.15}$$

where R_j denotes the arithmetic return j-th intraday return, while r_j the log return. The absence of jumps makes the difference between SwV and the realized variance equal to 0:

$$\operatorname*{plim}_{\delta\to 0}(SwV_t(\delta) - RV_t(\delta)) = \begin{cases} 0 & \text{no jumps in}[0,t] \\ 2\int_0^t \underline{J}_u\,\mathrm{d}q_u - \int_0^t J_u^2\,\mathrm{d}q_u & \text{jumps in}[0,t] \end{cases} \tag{3.16}$$

where $RV_t(\delta)$ is the estimated realized variance and $\underline{J}_u = exp(J_u) - J_u - 1$, with J the jump process.

CLT results are developed for SwV under the null, allowing for the construction of a jump test, defined as:

$$\frac{M\int_0^t \sigma^2\,\mathrm{d}u}{\sqrt{\Omega_{SwV}}}\left(1 - \frac{RV_t}{SwV_t}\right), \tag{3.17}$$

where $M = [t/\delta]$ is the number of intraday observations. In equation (3.17), the integrated variance is estimated using Barndorff-Nielsen and Shephard (2006b) bipower variation, while Ω_{SwV} is estimated using a multi- power variation (Barndorff-Nielsen et al., 2003; Barndorff-Nielsen et al., 2006):

$$\hat{\Omega}_{SwV} = \frac{\mu_6}{9} \frac{M^3 \mu_{6/p}^{-p}}{M-p+1} \sum_{i=0}^{M-p} \prod_{k=1}^{p} |r_{i+k}|^{6/p} \tag{3.18}$$

where a suitable choice for p is 4 or 6, as suggested by the authors. Furthermore, authors modify the test in order to fit noisy data and show that it still retains power. While having a higher convergence rate than the standard Barndorff-Nielsen and Shephard (2006a) test, simulations conducted in Jiang and Oomen (2008) reveal better size properties at high frequencies and more power if just 1 jump is considered in comparison with the above cited test.

3.1.5 Podolskij and Ziggel (2008) test (PZ henceforth)

This procedure is based on the same idea as the standard Barndorff-Nielsen and Shephard (2006a) test, the identification of jumps as a difference between a realized power variation and a robust to jumps estimator of the corresponding integrated quantity. Podolskij and Ziggel (2008)'s choice for the latter quantity is Mancini (2009)'s threshold estimator. However, since the derivation of a limiting theory for the simple differentiation between the two has proved particularly difficult, authors define the test statistics as a difference between a realized power variation estimator and a threshold estimator perturbed by some external positive i.i.d. random variables, $(\eta_j)_{1 \leq j \leq [t/\delta]}$, with $E[\eta_j] = 1$ and finite variation:

$$T(r,\delta)_t = M^{\frac{r-1}{2}} \sum_{j=1}^{[t/\delta]} |y_j|^r (1 - \eta_j \mathbb{I}_{\{|y_j| \leq c\delta^w\}}), \qquad r \geq 2, \tag{3.19}$$

where $M = [t/\delta]$, $1_{\{|y_j| \leq c*\delta^w\}}$ is an indicator function for absolute returns lower than a threshold fixed to $c * \delta^w$, with $c = 2.3\sqrt{BV_t}$ and $w = .4$.

The variance of the test statistic can be estimated using a threshold estimator:

$$\widehat{Var}(T(r,\delta)_t) = Var[\eta_i] M^{\frac{2r}{2}-1} \sum_{j=1}^{[t/\delta]} |y_j|^{2r} \mathbb{I}_{\{|y_j| \leq c\delta^w\}} \tag{3.20}$$

After standardization, T becomes asymptotically standard normal. For the perturbing variables, Podolskij and Ziggel (2008) recommend to sample them from the following distribution:

$$P^{\eta} = \frac{1}{2}(\varsigma_{1-\tau} + \varsigma_{1+\tau}), \tag{3.21}$$

where ς is the Dirac measure, and τ is constant chosen relatively small, e.g. $\tau = 0.1$ or 0.05.

3.2 Monte Carlo analysis

In this section we report and discuss results of a thorough comparison among five testing procedures, based on a comprehensive set of Monte Carlo simulations, which embodies specific features of financial data. To quantify the size for all tests, our simulations are based on both constant and stochastic volatility models with varying persistence. To evaluate the power property, we consider stochastic volatility models with jumps of different sizes arriving with varying intensity.

3.2.1 Simulation design

In this section we describe our Monte Carlo design. First, we simulate a simple stochastic process with constant volatility:

$$\mathrm{d}p(t) = \mu \mathrm{d}t + \sigma \mathrm{d}w_p(t), \tag{3.22}$$

where $\mu = .03bs$, and σ was fixed to 1.05bs. To evaluate the power properties of the tests, we also simulate a fixed number of jumps, which were randomly spaced under two different scenarios of fixed jump size, namely $.2\sigma$ and $.5\sigma$.

Second, following Huang and Tauchen (2005), we simulated several stochastic volatility processes with leverage effect, with or without jumps and different levels of persistence in volatility, as well as varying jump intensities and jump variances.

The benchmark model for our simulations is a stochastic volatility model with one volatility factor, to which we add jumps under the alternative hypothesis of discontinuous sample paths. The volatility factor enters the price equation in an exponential form, as suggested in Chernov et al. (2003):

$$\begin{aligned} \mathrm{d}p(t) &= \mu \mathrm{d}t + exp[\beta_0 + \beta_1 \upsilon(t)]\mathrm{d}w_p(t), \\ \mathrm{d}\upsilon(t) &= \alpha_\upsilon \upsilon(t)\mathrm{d}t + \mathrm{d}w_\upsilon(t), \qquad corr(\mathrm{d}w_p, \mathrm{d}w_\upsilon) = \rho \end{aligned} \tag{3.23}$$

where $p(t)$ is the log-price process, the w's are standard Brownian motions, $\upsilon(t)$ the volatility factor, μ the drift of the price process, α_υ the drift of the volatility process and ρ the leverage effect. This is the process that we simulate under the null hypothesis of no jumps.

Under the alternative, as in Huang and Tauchen (2005), we add a compound Poisson process with jump intensity λ and jump size distributed as $N(0, \sigma^2_{jump})$.

Chernov et al. (2003) show that it is possible to generate similar dynamics with the ones produced by a jump diffusion model by using a two factor stochastic volatility model. A first volatility factor controls for the persistence in the volatility process, while the second factor generates higher tails in a similar manner to a jump process. Moreover, by considering the volatility feedback component for the second factor, the model can sometimes accommodate market conditions even better than jump diffusions, as the volatility of volatility can capture the dynamics of extreme events.

Thus, we simulate a second stochastic volatility model with two volatility factors, one containing volatility feedback:

$$\begin{aligned} \mathrm{d}p(t) &= \mu \mathrm{d}t + s - exp[\beta_0 + \beta_1 \upsilon_1(t) + \beta_2 \upsilon_2(t)] \mathrm{d}w_p(t) \\ \mathrm{d}\upsilon_1(t) &= \alpha_{\upsilon_1} \upsilon_1(t) \mathrm{d}t + \mathrm{d}w_{\upsilon_1}(t) \\ \mathrm{d}\upsilon_2(t) &= \alpha_{\upsilon_2} \upsilon_2(t) \mathrm{d}t + [1 + \beta_{\upsilon_2} \upsilon_2(t)] \mathrm{d}w_{\upsilon_2}(t) \end{aligned} \tag{3.24}$$

with $corr(\mathrm{d}w_p, \mathrm{d}w_{\upsilon_1}) = \rho_{p,\upsilon_1}$ and $corr(\mathrm{d}w_p, \mathrm{d}w_{\upsilon_2}) = \rho_{p,\upsilon_2}$.

Chernov et al. (2003) show that the above model has solutions if the exponential function is spliced at very high levels of volatility based on appropriate growth conditions. Just as in Huang and Tauchen (2005), we consider a knot point of 100% annualized volatility.

While simulating the two stochastic volatility models described above can help assess the size of the tests, in order to analyze their performance, we augment the model in equation (3.23) with rare compound Poisson jumps. These jumps arrive at times sampled from a Poisson process with intensity λ and have sizes normally distributed with mean 0 and standard deviation σ_{jump}.

The values of the parameters of the two stochastic volatility models are the ones in Huang and Tauchen (2005) and are reported, for convenience, in Table 3.1. For the one factor stochastic volatility model (SV1F), Table 3.1 also reports the values of the jump parameters, λ and σ_{jump}:

In empirical applications it is customary to apply these tests at a daily level, in order to be able to conclude whether jumps occurred during the trading day. Therefore, we evaluate the statistical properties of all jump tests based on data simulated for 10000 trading days, for all models and under both hypotheses (continuity and discontinuity). For the simulation of each path, we use an Euler discretization scheme based on increments of 1 second. We then perform a sampling every minute and every 1, 5, 15 and 30 minutes and all our computations are carried out for all sampling frequencies plus the one at every second. For comparison purposes, all models with the same number of factors are based on the same Brownian motion(s). For instance, for all the models derived from the SV1F model, we use the same

	SV1F		SV2F
μ	0.030	μ	0.030
β_0	0	β_0	-1.200
β_1	0.125	β_1	0.040
α_v	{-0.137e^{-2}, -0.100, -1.386}	β_2	1.500
ρ	-0.620	α_{v_1}	-0.137 e^{-2}
λ	0 - 2	α_{v_1}	-1.386
σ_{jump}	0 - 2.50 by 0.50	β_{v_2}	0.250
		ρ_{p,v_1}	-0.300
		ρ_{p,v_2}	-0.300

Table 3.1: Parameter values for the 1 factor stochastic volatility models (SV1F) and for the 2 factor model(SV2F)

simulated Brownian motions to describe the dynamics of both the price and volatility factor.

Figures 3.1 and 3.2 report the simulated daily prices, volatility factors and returns from the two stochastic volatility models for 10,000 days. Data was sampled from 5 minute data. For the SV1F model, we assume a medium mean reversion of the volatility factor, $\alpha_v = -.1$.

Remarks:

(a) For the intraday jump detection procedures, given that all the other tests are applied on time intervals equal to one trading day, in order to allow proper comparisons, we compute the test statistics for every moment t_i within a trading day and then pick up the maximum statistic as the final test for that day. In order to contrast our results to the ones reported in Lee and Mykland (2008), we also adopted their strategy to calculate for this test both overall probabilities, as well as means and standard deviations of the intraday probabilities of spurious and nonspurious detection of jumps.

(b) In the case of the Andersen et al. (2007) procedure, the authors use a threshold from a normal but for a very low significance level. In our simulation exercise, if we consider nominal sizes of 1% or 5%, as for the other

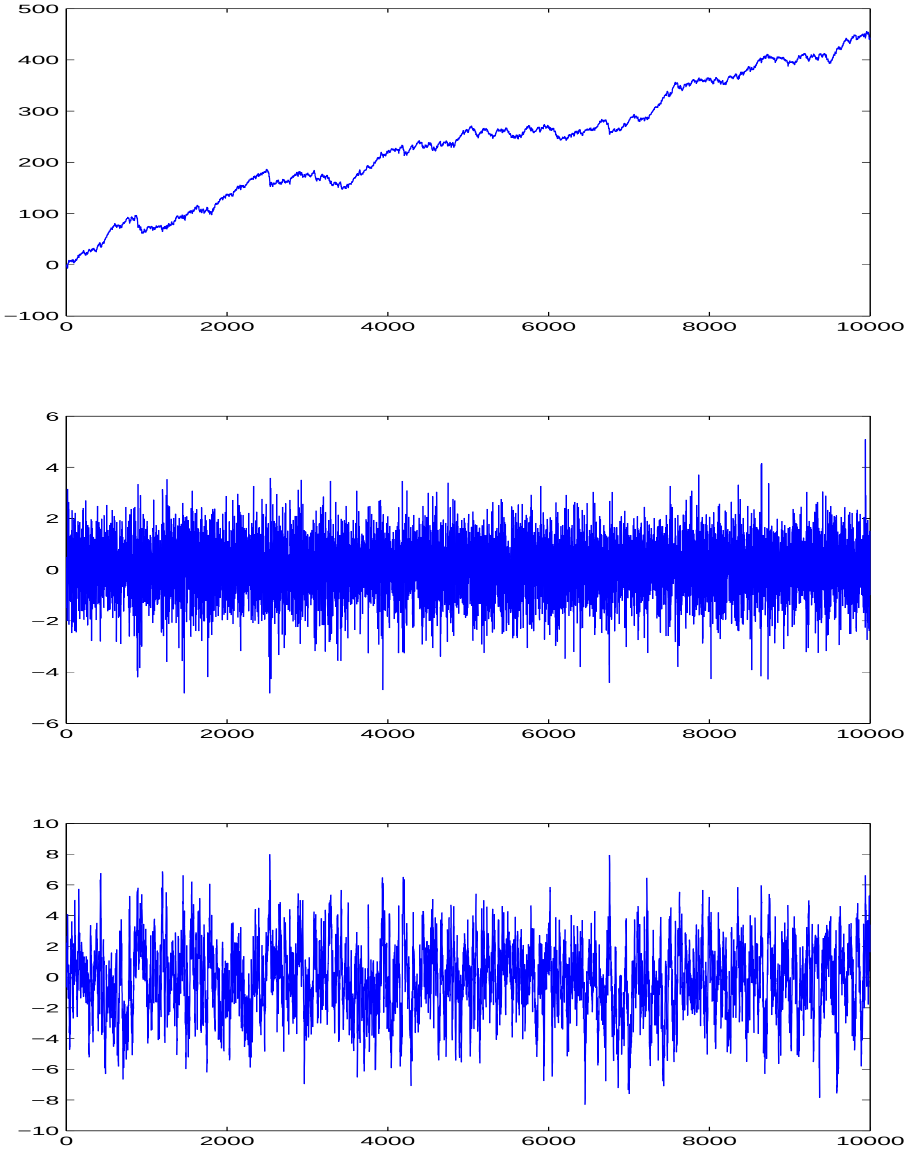


Figure 3.1: Simulated daily prices, returns and volatility factor respectively from the SV1F model with medium mean reversion

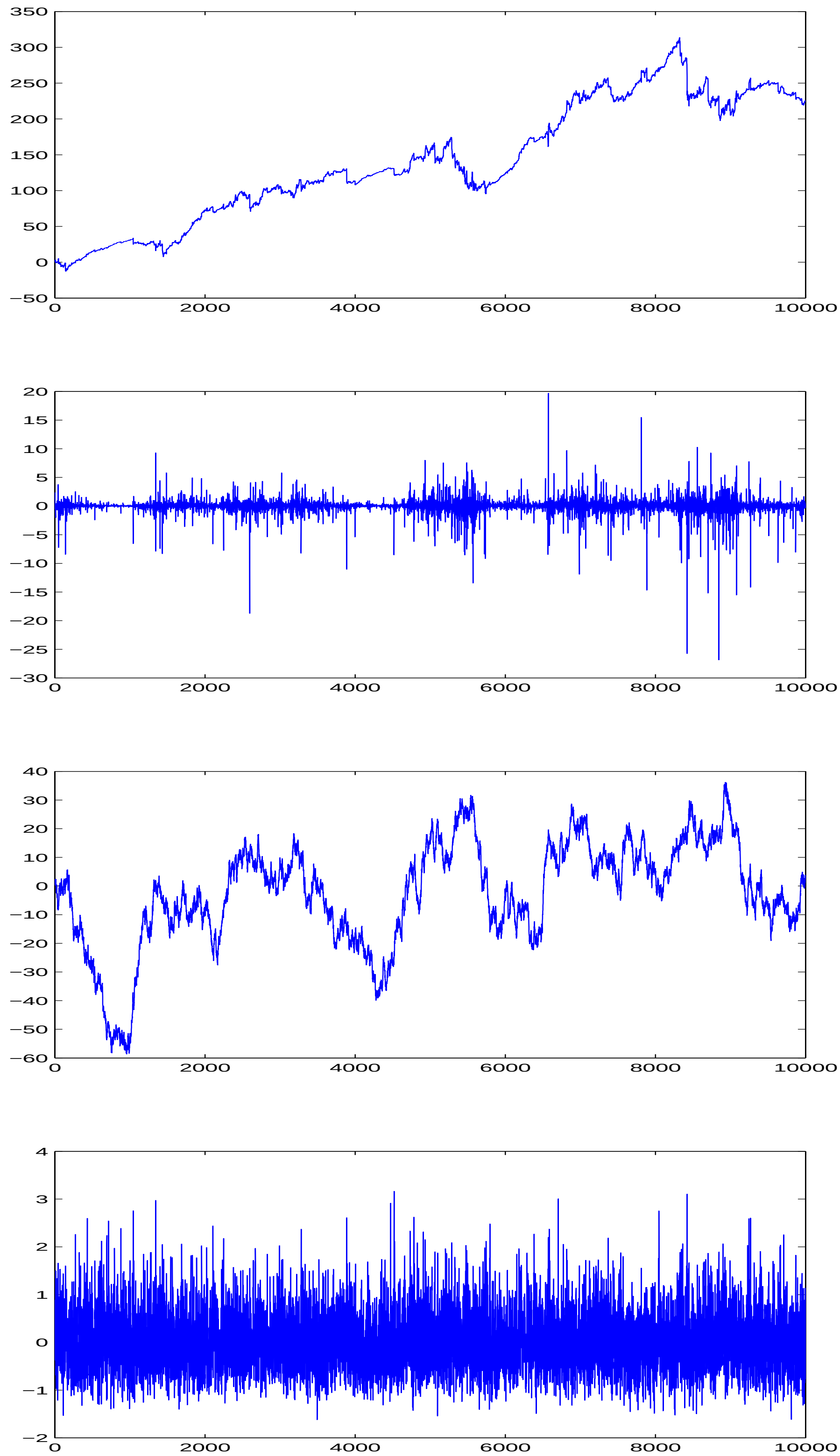


Figure 3.2: Simulated daily prices, returns and volatility factors respectively from the SV2F model

procedures, the test is highly oversized and is unable distinguish discontinuous from continuous sample paths. In order to enable comparisons with the other tests, we choose to handle this test just as Lee and Mykland (2008): we take the maximum of the statistic, standardize it and compare it to critical values taken from a Gumbel distribution. Consequently, as we show later in the chapter, results do not differ between the two procedures. Here, we computed the Andersen et al. (2007) test statistic based on the daily realized bipower variation, whereas for the Lee and Mykland (2008) test we standardized returns by a local estimator of volatility. However, this estimator is computed on a wide window and thus moves very slowly. It results, as shown in Boudt et al. (2009), that it is the periodicity of the volatility factor not accounted by us here that requires considering a varying volatility when detecting jumps.

(c) Aït-Sahalia and Jacod (2008) suggest two possible ways to estimate the variance of their test statistic in a robust to jumps manner. The first one is based on Mancini (2009)'s threshold estimators, while the other one on realized multipower variations (Barndorff-Nielsen and Shephard, 2004, 2006b, 2003, see). In our simulations, we employ both versions.

3.2.2 Monte Carlo findings

Constant volatility model

The nonparametric tests for jumps allow users to disentangle the unpredictable part of the returns from the persistent one. In order to compare the ability of the various tests to identify jumps, we first simulate a very simple stochastic process with a diffusion parameter that remains constant through time, as described in Section 3.2.1. When we evaluate the power of the tests, we add to the diffusion term jumps of different sizes. Under this setup, extreme dynamics are caused only by jumps. Consequently, this analysis enables us to understand how well tests can disentangle jumps at different sampling frequencies.

Size In Table 3.2, we report the empirical size of the tests from the simulations:

Procedure	Nominal size: 5%				
	1 sec	1 min	5 min	15 min	30 min
'Andersen et al'	0.044	0.040	0.040	0.040	0.051
'Ait-Sahalia & Jacod'	0.046	0.039	0.032	0.027	0.014
'Ait-Sahalia Power Var'	0.047	0.043	0.050	0.087	0.151
'BNS'	0.046	0.053	0.053	0.055	0.061
'Jiang & Oomen'	0.080	0.087	0.094	0.111	0.146
'Lee & Mykland'	0.044	0.052	0.052	0.045	0.047
'Podolskij & Ziggel'	0.046	0.064	0.086	0.102	0.117

Table 3.2: Size of daily jump tests. We assume a constant volatility model with the following annualized values for its parameters: $\sigma = .3$ and a constant drift, $\mu = .10$

As already noticed by Huang and Tauchen (2005), the BNS test is slightly oversized. At 1 second sampling frequency, its size is very close to the nominal one, whereas when we move toward lower frequencies, we observe a gradual increase in size. However, if we compare this dynamics with the ones for the other tests, the BNS procedure seems the most stable. The JO technique displays a size twice as big as the nominal one at very high frequencies, which then gradually increases even more as we sample less often.

The AJ test based on threshold estimators ('Ait-Sahalia & Jacod' in Table 3.2) is undersized for all sampling frequencies and becomes more and more undersized as we decrease the sampling frequency, while the version of the same test based on multi-power variation type estimators ('Ait-Sahalia Power Var'in Table 3.2) starts by being slightly undersized at 1 second, with a size of 4.7% and then becomes gradually oversized as we decrease the sampling frequency. Thus, the two versions of the same test behave differently as we vary the sampling frequency. This is an interesting finding also confirmed when the stochastic volatility models are considered.

The PZ test has a size close to the nominal one when data is sampled every second, but which increases quite abruptly afterward.

The intraday tests, ABD and LM, which we transformed in daily tests

	'Mean'	'Stdev'
'1 sec'	9.53E-08	3.09E-06
'1min'	6.86E-06	2.65E-05
'5min'	3.48E-05	6.02E-05
'15min'	9.44E-05	9.42E-05
'30min'	2.01E-04	1.38E-04

Table 3.3: Mean and standard deviation of the Lee and Mykland test at time t_i for a 5% significance level

by considering the supremum of all the statistics computed within a trading day, seem to be slightly undersized for some frequencies and without a clear dynamic across the sampling frequency. The differences between the two tests are due to the different approaches to estimate the local volatility estimator which standardizes the intraday returns. This local volatility proxy is computed based on all observations within a trading day for the ABD test or based on the number of observations recommended by the authors for the LM methodology, which varies with the sampling frequency.

As explained in Remark (a), for conformity with results in Lee and Mykland (2008) , we also calculate for this test overall probabilities, as well as means and standard deviations of the intraday probabilities of spurious detection of jumps. For the means and standard deviations of the intraday probabilities of spurious jumps we proceed in the following manner. For each time step, t_i, we compute the probability of spurious detection and then, we calculate its mean and standard deviation. For instance, at a 1 minute frequency, we have 389 returns per day and thus, we can compute 389 probabilities of spurious detection. The results are summarized in Table 3.3, for a 5% significance level. From Table 3.3, as expected, we observe a clear increase in the means of these probabilities as we lower the sampling frequency. Moreover, if we compute the overall probabilities of spurious jump detection, the same trend can be noticed. We do not report here these results, which are available upon request.

To summarize, in terms of the size criterion, the best test seems to be the classic BNS test, which displays no substantial changes over the sampling frequency. The intraday tests, ABD and LM, also seem to do well.

Power Under the discontinuity hypothesis, we add to the diffusion a fixed number of jumps randomly determined from a Poisson process with $\lambda = .5$. We consider different sizes for these jumps: $.2\sigma$, $.5\sigma$ and σ, where σ stands for the daily level of volatility.

In Table 3.4, we report the power for all tests for a 5% significance level. We include results only for the first two jump sizes, given that for a bigger jump size, the hierarchy of the tests based on their power does not change. For a jump size equal to 0.2σ, when sampling is performed every second, all

	Procedure	Nominal size: 5%				
		1 sec	1 min	5 min	15 min	30 min
	'Andersen et al'	1.0000	0.5545	0.0679	0.0423	0.0545
	Ait-Sahalia & Jacod'	1.0000	0.0990	0.0234	0.0295	0.0145
	'Ait-Sahalia Power Var'	1.0000	0.1357	0.0545	0.0973	0.1602
$size = 0.2\sigma$	'BNS'	1.0000	0.1374	0.0651	0.0545	0.0706
	Jiang & Oomen'	1.0000	0.3598	0.1646	0.1618	0.1830
	Lee & Mykland'	1.0000	0.5451	0.0862	0.0562	0.0606
	Podolskij & Ziggel'	1.0000	0.4549	0.1034	0.1034	0.1229
	'Andersen et al'	1.0000	0.9978	0.7887	0.2041	0.0940
	Ait-Sahalia & Jacod'	1.0000	0.7558	0.1051	0.0133	0.0122
	'Ait-Sahalia Power Var'	1.0000	0.7564	0.1874	0.1257	0.1557
$size = 0.5\sigma$	'BNS'	1.0000	0.9488	0.3565	0.1335	0.0945
	Jiang & Oomen'	1.0000	0.9972	0.7483	0.3805	0.3078
	Lee & Mykland'	1.0000	0.9522	0.7870	0.2536	0.1324
	Podolskij & Ziggel'	1.0000	0.9972	0.7792	0.2670	0.1663

Table 3.4: Power of daily jump tests. We assume a diffusion with parameters $\mu = .03$ and $\sigma = 1.05$ plus jumps with varying jump size

tests identify the jumps included in the price process. Then, power abruptly decreases as we diminish the sampling frequency. The ABD and LM tests, as well as the PZ procedure seem to display higher powers when sampling is performed every minute, followed by the JO technique. The procedures proposed by BNS and AJ perform very poorly in identifying small jumps. At even lower sampling frequencies, all tests lose their power in identifying jumps. The JO procedure maintains though a power of about 16% when we sample every 5 and 15 minutes, which does not decrease anymore with the sampling frequency, while the PZ still manages to detect about 10-12% of the jumps.

When the size of the jump increases to 0.5σ, all jumps are identified at a sampling frequency of 1 second, and a very high power is maintained at 1 minute for all tests. Power then gradually decreases for the intraday, the JO and the PZ tests and falls to values below 40% for BNS and below 20% for both versions of AJ test.

Note that the JO test can be written in Taylor series decomposition as the sum of returns raised to powers greater or equal to three. Thus, when jumps have the same sign, this procedure performs very well in detecting them. However, as showed later, when jumps have different signs and processes are more volatile, its performance worsens a lot, as returns with different signs tend to cancel themselves out.

In the next section, we evaluate the performance of the tests based on a stochastic volatility processes. Under the alternative, we consider different jump intensities and a nondeterministic jump size.

Stochastic volatility models

Size We simulate the stochastic processes as described in Section 3.2.1 for every second. Then, we sample the process every 1, 5, 15 and 30 minute(s) and compare the size of the tests.

In our simulation set-up in Table 3.1, for the process with one volatility factor (SV1F), we consider three different values for the mean reversion parameter of the volatility factor. We computed the empirical sizes for all three resulting models and for all tests. In all cases except the JO procedure, the empirical size tends to slightly decrease with the increase in the mean reversion parameter. The JO procedure is severely oversized when the volatility factor mean- reverts very slowly ($\alpha_v = -0.137e^{-2}$). In this case, the volatility process is clearly nonstationary and we suspect the test statistic explodes. In this case, the size at a sampling frequency equal to 1 second is about

25%, for a 5% nominal level, and decreases at lower sampling frequencies, as opposite to the usual behavior of increasing size when frequency diminishes. For the other simulated processes with medium and high mean reversion of the volatility, size at one second is around 9-10% and then increases at lower sampling frequencies.

Results are not affected by the values taken by the mean reversion parameter. Thus, for brevity, we report only results for a mean reversion parameter equal to -.1. The full set of results is available upon request.

An intuitive, useful way to see the dimension of the size distortions is to use QQ plots.

Consequently, Figures 3.3 and 3.4 contain QQ plots of the test statistics and the relevant theoretical distributions: the standard normal for the AJ, BNS, JO and PZ tests and the extreme values distribution in the case of the ABD and LM tests. As already documented in the literature, the BNS displays a size distortion as we decrease the sampling frequency. The JO technique, as expected given the results obtained within the previous section, becomes highly oversized as the sampling frequency diminishes. The AJ is undersized for all sampling frequencies and becomes more and more undersized as we increase the sampling frequency. For the intraday procedures, ABD and LM, size was computed here based on a maximum of all test statistics computed over a trading day. There is not a clear pattern of increase in size with the decrease in sampling frequency, as we would expect. In the case of the PZ procedure, size is very close to the nominal one when we sample every second and then, increases a lot as we lower the sampling frequency.

Table 3.5 reports the empirical size of the tests. If we look at all the sampling frequencies, the biggest size distortion is encountered in the case of the JO test, where, for a 1 second sampling frequency, we have a size equal to 0.095, which grows fast when we diminish the sampling frequency. A similar pattern can be seen for the PZ procedure, which displays a size close to the nominal one when sampling is performed every second, but then gets

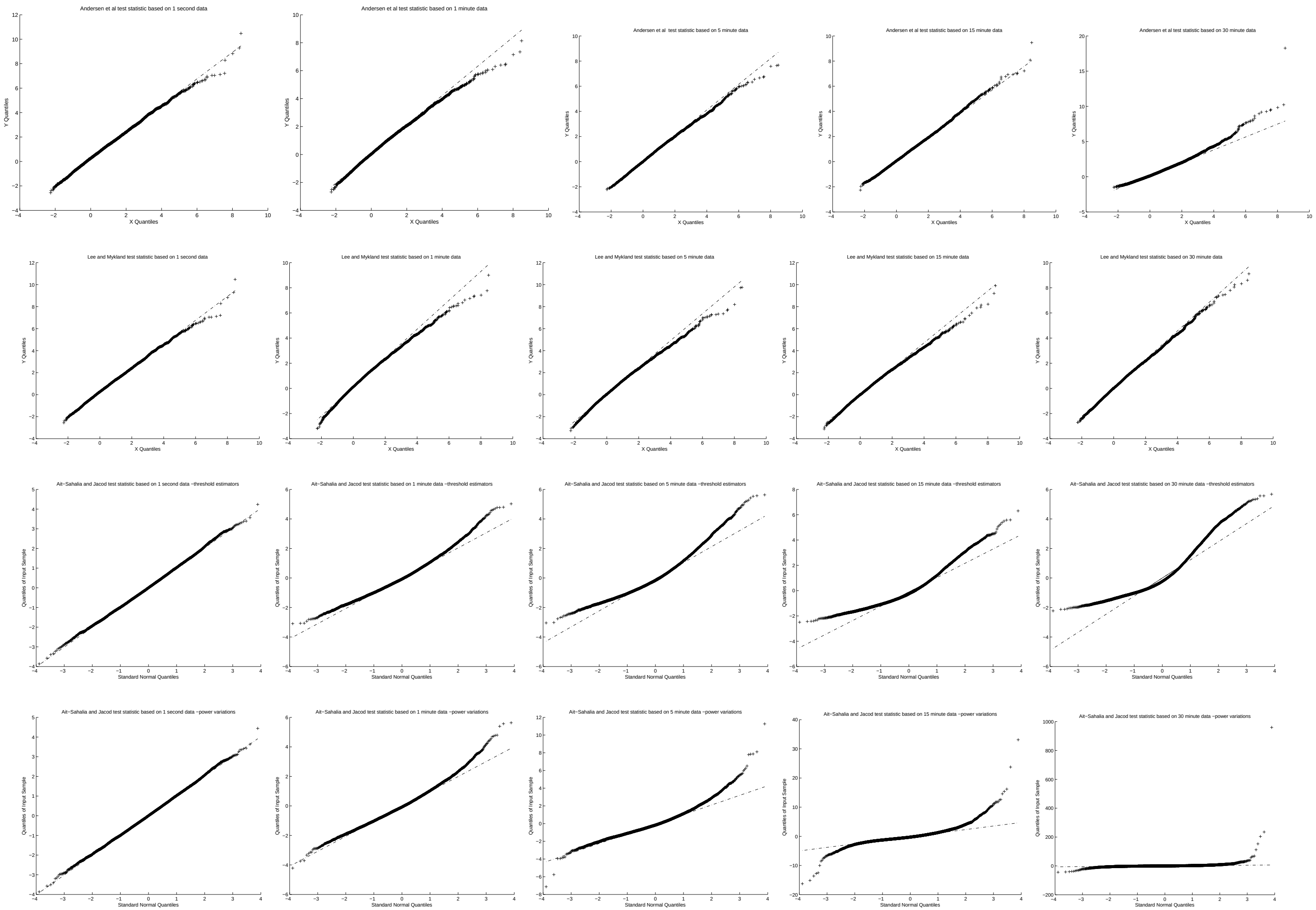


Figure 3.3: QQ plots of test statistics and a Gumbel (extreme values) distribution for the ABD and LM tests or a standard normal for the AJ, based both on threshold and power variations estimators

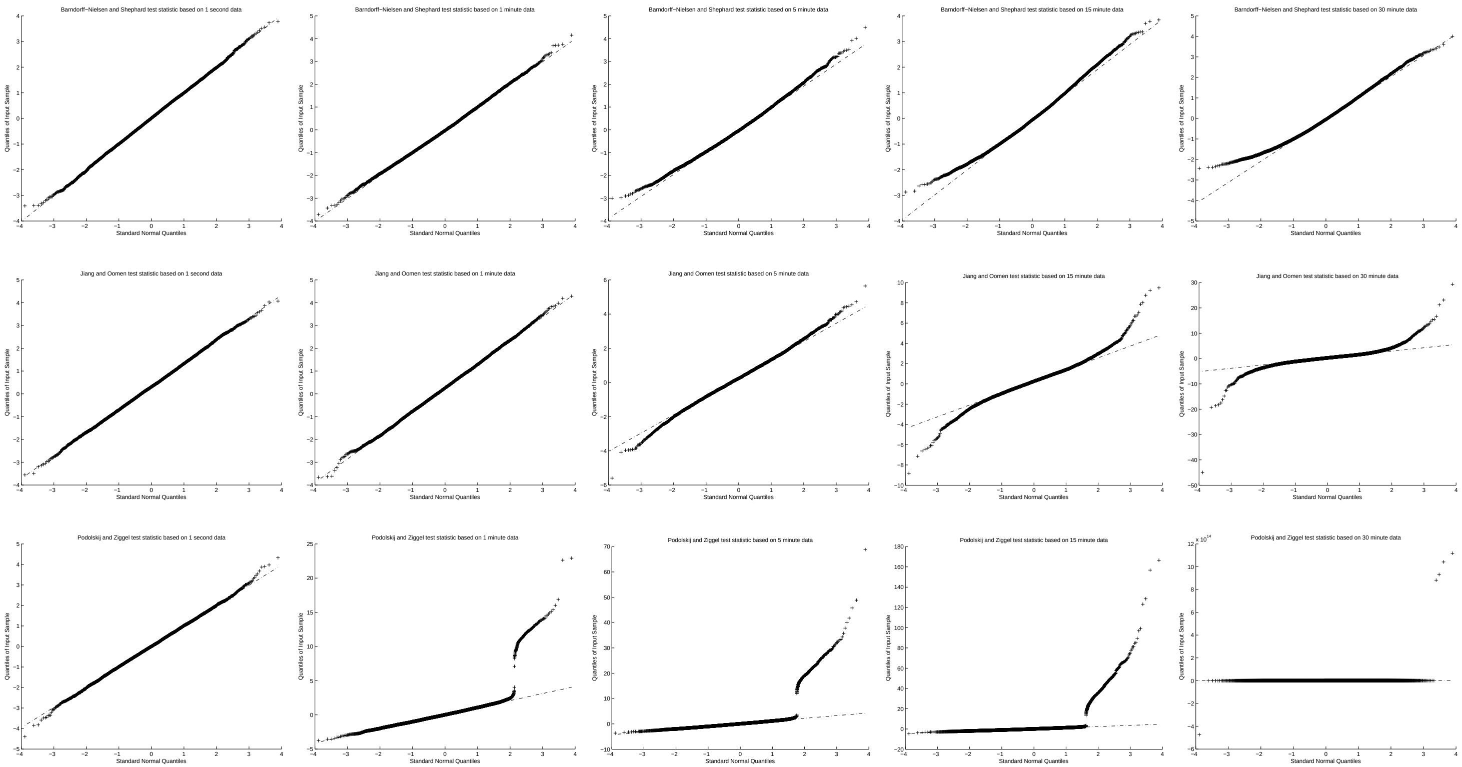


Figure 3.4: QQ plots of test statistics and a standard normal for the BNS, JO, and PZ tests

Procedure	Nominal size: 5%				
	1 sec	1 min	5 min	15 min	30 min
'Andersen et al'	0.074	0.048	0.044	0.041	0.054
Ait-Sahalia & Jacod'	0.047	0.038	0.031	0.027	0.014
'Ait-Sahalia Power Var'	0.048	0.046	0.051	0.088	0.150
'BNS'	0.048	0.054	0.053	0.057	0.063
Jiang & Oomen'	0.095	0.091	0.099	0.119	0.151
Lee & Mykland'	0.074	0.066	0.074	0.063	0.059
Podolskij & Ziggel'	0.049	0.065	0.083	0.100	0.121

Table 3.5: Size of the tests for jumps for the SV1F model with medium mean reversion

rapidly and highly oversized. Consequently, in practice, such tests should be applied on data sampled as frequent as possible, as the size distortion grows much faster than for the other tests. The best behavior in terms of size is found for the BNS classic test and for the intraday ABD and LM procedures. In all cases, size does not change very much over the sampling frequency and the size distortion is not very high. The AJ test statistic was standardized with standard deviations based on both power variations and threshold estimators. In both cases, at a sampling frequency of 1 second, the test seems slightly undersized. However, when diminishing the sampling frequency, the behavior of the test statistics differs. The test becomes rapidly oversized when its variance is based on realized power variations and severely undersized when threshold estimators are used to estimate its variance. This test is clearly another one which works well at very high frequencies, but can cause problems at lower frequencies.

If we turn back to the intraday procedures, there are no important differences between the ABD and LM test. The larger size at a 1 second sampling frequency is due to the fact that at this frequency, for both tests, we scaled the realized bipower variation which is used to standardize the simple log-return with a large number of observations (23399). This increases the test statistic, and given that we computed the maximum over a trading day, the test tends to be "artificially" oversized at this high frequency.

In order to get a better picture on the behavior of the intraday tests, we also compute, just as in the original LM paper, the mean and standard errors of the probability of spurious detection of jumps at time t_i for this test, which we report in Table 3.6, while Table 3.7 reports the overall probability of spurious jump detection.

	'Mean'	'Stdev'
1 sec'	1.65E-07	4.05E-06
'1min'	9.02E-06	3.00E-05
'5min'	5.10E-05	6.99E-05
'15min'	1.37E-04	1.15E-04
'30min'	2.61E-04	1.53E-04

Table 3.6: LM test: means and standard errors of the probability of spurious detection of jumps at time t_i, for different sampling frequencies

Frequency	Overall Size
'1sec'	0.00000
'1min'	0.00018
'5min'	0.00102
'15min'	0.00277
'30min'	0.00528

Table 3.7: LM test: Overall probability of spurious jump detection for different sampling frequencies for a 5% nominal size

We observe a clear trend of increasing size with the decrease in the sampling frequency for the LM test. However, when comparisons are made with other test statistics, it is desirable to use the maximum of the test statistic over a certain period.

Further on, for a more in-depth analysis of the size of the tests under the null, we simulated 10,000 days from the second stochastic volatility model (SV2F). This model not only generates extreme values, but also describes their dynamics. The empirical sizes for all the tests are reported in Table

3.8.

Procedure	Nominal size: 5%				
	1 sec	1 min	5 min	15 min	30 min
'Andersen et al'	0.993	0.755	0.438	0.242	0.165
Ait-Sahalia & Jacod'	0.127	0.094	0.039	0.020	0.008
'Ait-Sahalia Power Var'	0.052	0.077	0.121	0.205	0.255
'BNS'	0.054	0.073	0.097	0.113	0.119
Jiang & Oomen'	0.112	0.111	0.150	0.199	0.248
Lee & Mykland'	0.993	0.699	0.482	0.339	0.254
Podolskij & Ziggel'	0.701	0.648	0.448	0.305	0.239

Table 3.8: Size of the tests for jumps for the SV2F model, for a 5% significance level

If we look at all sampling frequencies, the best behavior in terms of size is displayed by the BNS test, with an effective size for a 1 second sampling frequency equal to 5.4%, which increases at lower sampling frequencies in a less drastic manner than for all the other tests. The AJ procedure based on realized power variations has a size close to the nominal one when sampling is done every second, but then becomes rapidly oversized. When applying the same methodology, but with threshold estimators, we observe that the test gets severely undersized at lower sampling frequencies. The PZ and the intraday procedures display by far the poorest performance , being severely oversized when we sample every second (97% for the intraday tests and 65.7% for PZ). For the LM test, if we look at the overall probability of spurious detection, as well as at the means and standard errors of the probability of spurious detection of jumps at a certain time, t_i, the size distortion is low (see Table 3.9).

Even though at a first glance percentages in Table 3.9 do not seem very high, they refer to all the observations in the sample and, combined with results in Table 3.8, indicate a very high rate of over-rejection of the null. For instance, if we consider a sampling frequency of 1 second, the LM spuriously detects jumps in .1% of cases. However, results in Table 3.8 indicate an empirical size of 99.3%, which means that the test spuriously identifies jumps almost every day.

Frequency	Overall Size
'1sec'	0.0016
'1min'	0.0099
'5min'	0.0202
'15min'	0.0280
'30min'	0.0334

Table 3.9: LM test: Overall probability of spurious jump detection for different sampling frequencies for a 5% significance

The size criterion confers us just half of the overall perspective that we need to attain over all these different jump detection procedures. The perspective will become complete as we consider another important criterion, the power properties of all these tests.

Power We now evaluate the power of the tests by adding to the continuous stochastic volatility process SV1F jump processes with different intensities and jump sizes.

Varying jump intensity In order to examine how jump detection changes as the number of jumps grows, we consider Poisson jump arrival times depending on the following varying jump intensities (λ): .014, .058, .089, .118, .5, 1, 1.5, 2, and 2.5. For all these scenarios, we consider a jump size that is normally distributed with mean 0 and standard deviation equal to 1.5%. We did not impose any restrictions on the maximum number of jumps per day. Thus, more than one jump can occur within a trading day.

In Table 3.10, we report the power of the tests by considering some scenarios for the jump intensity. The frequency of correctly identified jumps increases as the jump intensity raises.

The best tests in terms of power are the intraday procedures LM (with its simpler form based on constant volatility, ABD) and the PZ test. Let us consider the intraday procedures first. The power for these tests is around 98%

	Test	1 sec	1 min	5 min	15 min	30 min
	'Andersen et al'	0.989	0.892	0.766	0.614	0.440
	Ait-Sahalia & Jacod'	0.971	0.777	0.238	0.035	0.005
	'Ait-Sahalia Power Var'	0.969	0.775	0.280	0.185	0.236
$\lambda = 0.058$	'BNS'	0.954	0.819	0.685	0.496	0.366
	Jiang & Oomen'	0.478	0.445	0.386	0.353	0.321
	Lee & Mykland'	0.989	0.859	0.742	0.612	0.505
	Podolskij & Ziggel'	0.985	0.894	0.773	0.654	0.493
	'Andersen et al'	0.981	0.895	0.780	0.637	0.448
	Ait-Sahalia & Jacod'	0.971	0.779	0.208	0.031	0.007
	'Ait-Sahalia Power Var'	0.970	0.796	0.301	0.187	0.246
$\lambda = 0.118$	'BNS'	0.954	0.831	0.704	0.544	0.368
	Jiang & Oomen'	0.503	0.450	0.402	0.368	0.343
	Lee & Mykland'	0.981	0.863	0.762	0.651	0.521
	Podolskij & Ziggel'	0.975	0.895	0.783	0.667	0.513
	'Andersen et al'	0.987	0.915	0.805	0.667	0.494
	Ait-Sahalia & Jacod'	0.972	0.803	0.217	0.043	0.006
	'Ait-Sahalia Power Var'	0.972	0.811	0.323	0.220	0.246
$\lambda = 0.5$	'BNS'	0.959	0.854	0.729	0.561	0.402
	Jiang & Oomen'	0.492	0.461	0.421	0.379	0.347
	Lee & Mykland'	0.987	0.888	0.784	0.652	0.523
	Podolskij & Ziggel'	0.982	0.911	0.815	0.695	0.556
	'Andersen et al'	0.989	0.934	0.857	0.712	0.511
	Ait-Sahalia & Jacod'	0.982	0.833	0.206	0.040	0.005
	'Ait-Sahalia Power Var'	0.982	0.852	0.351	0.228	0.255
$\lambda = 1$	'BNS'	0.970	0.890	0.783	0.609	0.430
	Jiang & Oomen'	0.502	0.474	0.441	0.391	0.356
	Lee & Mykland'	0.989	0.912	0.828	0.688	0.535
	Podolskij & Ziggel'	0.988	0.932	0.861	0.738	0.570
	'Andersen et al'	0.996	0.962	0.908	0.789	0.566
	Ait-Sahalia & Jacod'	0.991	0.853	0.176	0.028	0.004
	'Ait-Sahalia Power Var'	0.992	0.899	0.410	0.259	0.279
$\lambda = 2$	'BNS'	0.984	0.933	0.854	0.687	0.488
	Jiang & Oomen'	0.535	0.516	0.484	0.443	0.397
	Lee & Mykland'	0.996	0.952	0.881	0.734	0.555
	Podolskij & Ziggel'	0.994	0.960	0.911	0.810	0.623

Table 3.10: Power of daily jump tests for a 5% significance level. We consider the SV1F model with medium mean reversion for the volatility factor and with a varying number of jumps, as a result of varying jump intensities

and 99% for a sampling frequency of 1 second and then gradually diminishes as the sampling frequency decreases. As the jump intensity diminishes, the power for these procedures ranges between 90% and 96%, for a sampling frequency of 1 minute, between 76% and 91% for 5 minutes data, between 61% and 81% for 15 minutes and finally between 44% and 62% for 30 minutes.

We also compute the intraday probabilities of correct jump detection, following a similar method as in the case of the probabilities of spurious jump detection. These results do not confer any additional information to the ones in Table 3.10 and thus, we do not report them here, but they are available upon request.

The slightly different results between the LM and ABD procedures can be explained by the different number of observations which is used to compute the local volatility estimate entering the test statistic and which scales in square root form the latter.[1].

For the PZ procedure we observe a very high power (around .98 and .99 at 1 sec) which gradually decreases with the sampling frequency, but remains higher than for the other procedures (except the intraday tests).

Both versions of the AJ test display a high power at 1 second, which plummets at lower frequencies. For instance, if we look at the results for $\lambda = .5$, the power decreases at around 80% when sampling is done every minute, for both versions of the test, followed by a fall at a level of 21% for the version based on threshold estimators and 32% for the test based on power variations, for a sampling frequency of 5 minutes. If we look at lower frequencies, the test based on power variation-type estimators displays a gradual decrease in power, which gets to a value of 24% for a 30 minutes sampling frequency, while the version based on threshold estimators displays a very low power of 0.6% at 30 minutes.

The classical BNS test exhibits very good power properties, with a power ranging between .95 and .98 when sampling at every second, which then

[1]For the LM procedure, we use the optimal window length as proposed by the authors, while for the other test, the simple number of observations per day is taken into consideration

decreases with the sampling frequency, with values below the ones observed for the intraday and PZ tests. We believe it remains a very good choice for a user who wishes to avoid both intraday tests which might be time consuming and the PZ procedure, which is based on threshold estimators and might be thus sensitive to the choice of the threshold.

The last ranked test is JO's. The low power exhibited here contrast the relative good performance we noticed in Section 3.2.2, where we simulated a model with constant volatility and only positive jumps. The reason for this poor performance resides in the fact that the test statistic can be approximated in Taylor series by a sum of returns raised to powers higher or equal to 3, so that when both negative and positive jumps occur, they tend to cancel themselves out. Thus, for the SV1F model with jumps with random sizes and signs occurring at random times, the efficacy of the procedure to detect jumps is very low.

Varying jump size A further insight on the ability of all these procedures to identify jumps will be attained by varying the jump size. In this section, we fix the number of jumps for the entire sample and vary the jump size. However, we maintain its nondeterministic character, by drawing it from a normal distribution with mean 0 and a standard deviation that ranges between 0 and 2 bs with a growth rate of .5. Table 3.11 reports the power of the considered jump detection procedures.

Overall, the performance of all tests increases with the size of the jumps. The ranking of the tests is in line with what was found for the case of varying jump intensity.

There is a confirmation about the very good ability of the the LM - ABD and PZ tests to detect jumps, with powers around 98% and 99% at 1 second, which gradually decreases with the sampling frequency. With respect to the BNS procedure, we notice a power around 97% and 98% at 1 second, which decays when sampling less frequently, but to lower numbers than for the intraday and PZ test. The AJ does again very well for the highest frequency,

		1 sec	1 min	5 min	15 min	30 min
$\sigma = 0.5$	'Andersen et al'	0.967	0.739	0.477	0.224	0.101
	Ait-Sahalia & Jacod'	0.921	0.490	0.101	0.024	0.012
	'Ait-Sahalia Power Var'	0.921	0.509	0.159	0.123	0.171
	'BNS'	0.872	0.565	0.341	0.176	0.120
	Jiang & Oomen'	0.469	0.365	0.281	0.205	0.202
	Lee & Mykland'	0.967	0.719	0.491	0.288	0.167
	Podolskij & Ziggel'	0.950	0.725	0.509	0.303	0.200
$\sigma = 1$	'Andersen et al'	0.987	0.872	0.720	0.496	0.278
	Ait-Sahalia & Jacod'	0.972	0.713	0.189	0.029	0.006
	'Ait-Sahalia Power Var'	0.972	0.727	0.265	0.176	0.211
	'BNS'	0.943	0.779	0.612	0.416	0.267
	Jiang & Oomen'	0.487	0.419	0.366	0.318	0.278
	Lee & Mykland'	0.987	0.850	0.713	0.528	0.384
	Podolskij & Ziggel'	0.982	0.867	0.733	0.557	0.394
$\sigma = 1.5$	'Andersen et al'	0.986	0.919	0.812	0.641	0.427
	Ait-Sahalia & Jacod'	0.976	0.797	0.214	0.039	0.007
	'Ait-Sahalia Power Var'	0.976	0.815	0.332	0.216	0.245
	'BNS'	0.962	0.861	0.731	0.565	0.403
	Jiang & Oomen'	0.507	0.475	0.430	0.385	0.344
	Lee & Mykland'	0.986	0.890	0.794	0.648	0.517
	Podolskij & Ziggel'	0.984	0.914	0.819	0.691	0.534
$\sigma = 2$	'Andersen et al'	0.991	0.936	0.855	0.725	0.535
	Ait-Sahalia & Jacod'	0.983	0.847	0.223	0.038	0.004
	'Ait-Sahalia Power Var'	0.983	0.857	0.376	0.249	0.274
	'BNS'	0.970	0.890	0.799	0.659	0.503
	Jiang & Oomen'	0.519	0.489	0.459	0.418	0.385
	Lee & Mykland'	0.991	0.907	0.831	0.715	0.597
	Podolskij & Ziggel'	0.988	0.932	0.862	0.752	0.625
$\sigma = 2.5$	'Andersen et al'	0.990	0.947	0.887	0.778	0.613
	Ait-Sahalia & Jacod'	0.985	0.878	0.233	0.039	0.002
	'Ait-Sahalia Power Var'	0.984	0.890	0.418	0.267	0.288
	'BNS'	0.976	0.915	0.840	0.721	0.578
	Jiang & Oomen'	0.506	0.484	0.462	0.430	0.401
	Lee & Mykland'	0.990	0.922	0.861	0.759	0.639
	Podolskij & Ziggel'	0.987	0.945	0.893	0.811	0.690

Table 3.11: Power of daily jump tests for a 5% significance level. We consider the SV1F model with medium mean reversion for the volatility factor and with a varying jump variance

with a dramatic decrease in power at 5 and 15 minutes frequencies. Again, the JO procedure displays the worst performance.

We also performed a backup check by adding to the stochastic volatility process SV1F jumps from a normal distribution with lower standard deviations: .1 and .3. However, the ranking between tests does not change.

Finally, for the LM test, we also compute the means and standard deviations of the intraday probabilities of correctly detecting a jump, as well as the overall probabilities of correct jump identification. Again, we do not report here these results, but they are available upon request.

The behavior of the different tests for jumps in the presence of microstructure noise

The simulation comparison reported so far is based on the assumption that the simulated prices come from continuous time jump diffusion process. However, when we deal with real prices of financial assets, this is no longer the case. The observed price process is a discrete one. It is either constant, generating zero returns, or changes a lot from one transaction to another. As a result, transactions impact prices, and market participants can sometimes build strategies to exploit the short-term inefficiencies of the market (deviations from a random walk process). In the theoretical and empirical financial literature, there is a vast literature that tries to understand and exploit these inefficiencies, which are generically denominated microstructure effects. In this chapter, we treat these effects as simple noise that obstructs our viewing of the real price process.

Even if the impact of noise on realized variance has been very well documented in the literature, there is not much theoretical work concerning the impact of noise on jump detection. JO find a bias correction for the realized bipower variation in the presence of i.i.d. microstructure noise. Moreover, they show that their test statistic does not diverge in the presence of i.i.d. noise if the number of observations per day is large but remains finite. AJ derive the limit of their test statistic in the presence of i.i.d. noise, as well.

They also note that if the distance between observations is small, but not 0, the test statistic does not diverge. PZ prove the validity of the test even in the presence of some particular types of noise, such as i.i.d. and i.i.d. plus rounding processes.

In what follows, we simulate i.i.d. microstructure noise normally distributed with mean 0 and a varying variance. We then add this noise to the simulated process to understand how the presence of noise affects the statistical properties of our tests for jumps.

Size in the presence of noise The following values for the standard deviation of the noise were considered: 0, .027, .040, .052, 0.065 and 0.080. Table 3.12 reports the frequencies of spuriously detected jumps for all tests, under alternative sampling frequencies and noise variances.

Apart from the AJ and JO tests, all tests become severely undersized in the presence of microstructure noise with an increasing size distortion as the variance of the noise grows. In the case of the AJ procedure, in the presence of noise, the test statistic, just as the other tests, gets smaller and smaller, leading to an over-rejection of the null. Here, the version of the test based on threshold estimators does better than the one based on power variations at lower sampling frequencies. Thus, for the former version of the test, if sampling is made every 15 minutes, the size gets close to the nominal one. For instance in Table 3.12, when $\sigma_{noise} = 0.065$, size is 4.7% for the version based on threshold estimators, whereas for the other version of the test, it reaches a very high level of 12.06%. The JO procedure displays a very high size in the presence of noise, which increases with the variance of the noise. However, when sampling is done at lower frequencies (from 1 minute onward), size decreases abruptly in the beginning and then, moderately increases again.

The least affected by noise is the PZ procedure, which, at the highest sampling frequency, displays a size close to the nominal one even for the highest values of σ_{noise}. This is a consequence of its higher and rapidly increasing size, which turns out to be an advantage in this case, as it compensates the

		Nominal size:5%				
		1 sec	1 min	5 min	15 min	30 min
	'Andersen et al'	0.0742	0.0475	0.0439	0.0412	0.0535
	Ait-Sahalia & Jacod'	0.047	0.0376	0.0311	0.0267	0.0138
	'Ait-Sahalia Power Var'	0.048	0.0459	0.0512	0.088	0.1504
$\sigma_{noise} = 0$	'BNS'	0.0484	0.0537	0.0532	0.057	0.0625
	Jiang & Oomen'	0.0954	0.091	0.0985	0.1188	0.1509
	Lee & Mykland'	0.0742	0.0663	0.0735	0.0634	0.0586
	Podolskij & Ziggel'	0.0494	0.0675	0.0918	0.0993	0.1213
	'Andersen et al'	0.0133	0.0374	0.0438	0.0425	0.0514
	Ait-Sahalia & Jacod'	1	0.6016	0.0615	0.0305	0.0132
	'Ait-Sahalia Power Var'	1	0.5885	0.085	0.0946	0.1583
$\sigma_{noise} = 0.027$	'BNS'	0	0.0184	0.0514	0.0533	0.0623
	Jiang & Oomen'	0.0739	0.0596	0.095	0.1198	0.1502
	Lee & Mykland'	0.0133	0.0493	0.0688	0.0645	0.0599
	Podolskij & Ziggel'	0.0494	0.056	0.0862	0.1007	0.1187
	'Andersen et al'	0.0108	0.0318	0.0434	0.0431	0.0514
	Ait-Sahalia & Jacod'	1	0.8639	0.1031	0.0382	0.0163
	'Ait-Sahalia Power Var'	1	0.8487	0.1277	0.103	0.1621
$\sigma_{noise} = 0.040$	'BNS'	0	0.0059	0.0494	0.0596	0.0639
	Jiang & Oomen'	0.2961	0.0462	0.0881	0.1152	0.1497
	Lee & Mykland'	0.0108	0.0442	0.0644	0.0594	0.0561
	Podolskij & Ziggel'	0.047	0.0596	0.0859	0.1024	0.1186
	'Andersen et al'	0.0087	0.0295	0.0412	0.0417	0.0488
	Ait-Sahalia & Jacod'	1	0.9561	0.1604	0.0374	0.0143
	'Ait-Sahalia Power Var'	1	0.9476	0.1868	0.1087	0.1654
$\sigma_{noise} = 0.052$	'BNS'	0	0.002	0.0434	0.054	0.0612
	Jiang & Oomen'	0.6359	0.0382	0.0828	0.1103	0.1454
	Lee & Mykland'	0.0087	0.04	0.0609	0.0585	0.0588
	Podolskij & Ziggel'	0.0514	0.0592	0.0873	0.0985	0.1183
	'Andersen et al'	0.0107	0.0192	0.0338	0.0417	0.0526
	Ait-Sahalia & Jacod'	1	0.9851	0.2198	0.0471	0.0158
	'Ait-Sahalia Power Var'	1	0.9812	0.2397	0.1206	0.18
$\sigma_{noise} = 0.065$	'BNS'	0	0.0007	0.0297	0.053	0.0642
	Jiang & Oomen'	0.9068	0.0296	0.0755	0.1105	0.1494
	Lee & Mykland'	0.0107	0.0329	0.0524	0.0608	0.0534
	Podolskij & Ziggel'	0.0506	0.0534	0.0778	0.101	0.1192
	'Andersen et al'	0.0108	0.0187	0.0293	0.0363	0.0501
	Ait-Sahalia & Jacod'	1	0.9964	0.3044	0.0576	0.0182
	'Ait-Sahalia Power Var'	1	0.9942	0.3258	0.1482	0.1856
$\sigma_{noise} = 0.080$	'BNS'	0	0.0004	0.0251	0.0501	0.0646
	Jiang & Oomen'	0.9943	0.0275	0.063	0.1072	0.1486
	Lee & Mykland'	0.0108	0.0312	0.046	0.0547	0.0568
	Podolskij & Ziggel'	0.0495	0.057	0.0736	0.0956	0.1191

Table 3.12: Size of the tests in the presence of microstructure noise. We assume a SV1Fmodel with medium mean reversion for the volatility factor and noise drawn from normal distribution with varying variance

downward bias caused by the presence of noise. The intraday tests, ABD and LM, also behave very well in the presence of i.i.d. noise. This is because the Taylor series decomposition of the test statistic includes only simple returns which are less sensitive to large changes, unlike other tests that can be approximated by third or fourth order moments of returns. The BNS test is severely undersized at very high frequencies, but gets close to the nominal size if sampling is performed every 15 minutes.

Except the PZ test which has a size close to the nominal one at 1 second and 1 minute sampling frequency, as if the noise was not present, all other tests tend to get close to the nominal size as the sampling frequency diminishes: JO somewhere between the 5 and 15 minutes sampling frequencies, AJ and BNS at 15 minutes, and ABD - LM somewhere between 15 and 30 minutes.

Power in the presence of noise In this section we examine how the ability of the tests to detect jumps changes in the presence of microstructure noise. We simulated the SV1F stochastic volatility model with medium mean reversion, to which we added a jump process with intensity $\lambda = .5$ and jump sizes randomly drawn from a $\mathscr{N}(0, 1.5\%)$. Then, we contaminated the price with i.i.d. normally distributed noise, just as in the previous subsection. The probabilities of correct jump identification for all the tests and for different scenarios of noise contamination are reported in Table 3.13.

We observe that if we exclude the AJ test, which in the presence of noise has both its size and power equal to 1, the hierarchy of the tests in terms of power remains the same as if noise were not present, with a a clear decrease in power as the size of the noise increases. Thus, the PZ and the intraday procedures display again the best power, without a compensation in size. The BNS seems the worst performer at a sampling frequency of 1 second, but as we decrease the frequency it regains power. JO displays an increasing power with the increase in σ_{noise}. However, given its high size distortion in the presence of noise, it is not reliable, at least at very high frequencies.

		Nominal size:5%				
		1 sec	1 min	5 min	15 min	30 min
$\sigma_{noise} = 0$	'Andersen et al'	0.98607	0.919652	0.815672	0.661194	0.478856
	Ait-Sahalia & Jacod'	0.975622	0.796766	0.21393	0.038557	0.007463
	'Ait-Sahalia Power Var'	0.975622	0.814925	0.331841	0.21592	0.245274
	'BNS'	0.962438	0.860945	0.731095	0.565174	0.402985
	Jiang & Oomen'	0.507463	0.474627	0.4301	0.385075	0.344279
	Lee & Mykland'	0.98607	0.89005	0.794279	0.648259	0.517164
	Podolskij & Ziggel'	0.984577	0.914677	0.824876	0.689801	0.534577
$\sigma_{noise} = 0.027$	'Andersen et al'	0.923632	0.894776	0.806965	0.652736	0.474378
	Ait-Sahalia & Jacod'	1	0.84801	0.220896	0.040796	0.006965
	'Ait-Sahalia Power Var'	1	0.869652	0.329851	0.209453	0.243035
	'BNS'	0.207463	0.803483	0.712687	0.55995	0.398507
	Jiang & Oomen'	0.465423	0.458706	0.425373	0.381841	0.338308
	Lee & Mykland'	0.923632	0.8699	0.778607	0.640796	0.51194
	Podolskij & Ziggel'	0.90199	0.891542	0.814677	0.682836	0.528109
$\sigma_{noise} = 0.040$	'Andersen et al'	0.885323	0.873383	0.788557	0.644776	0.468905
	Ait-Sahalia & Jacod'	1	0.879104	0.229851	0.040796	0.006965
	'Ait-Sahalia Power Var'	1	0.89204	0.338557	0.214925	0.242289
	'BNS'	0.053483	0.734826	0.69204	0.540299	0.393284
	Jiang & Oomen'	0.480597	0.444527	0.414428	0.376119	0.338806
	Lee & Mykland'	0.885323	0.848259	0.764428	0.639303	0.507214
	Podolskij & Ziggel'	0.846269	0.865672	0.79403	0.677114	0.529851
$\sigma_{noise} = 0.052$	'Andersen et al'	0.845771	0.851244	0.778607	0.641542	0.462687
	Ait-Sahalia & Jacod'	1	0.892786	0.2301	0.039552	0.006716
	'Ait-Sahalia Power Var'	1	0.909453	0.354726	0.20796	0.247264
	'BNS'	0.010945	0.664925	0.671642	0.536567	0.387811
	Jiang & Oomen'	0.552239	0.432836	0.408706	0.375622	0.335572
	Lee & Mykland'	0.845771	0.826368	0.756965	0.630846	0.508458
	Podolskij & Ziggel'	0.809204	0.84602	0.789303	0.675373	0.519652
$\sigma_{noise} = 0.065$	'Andersen et al'	0.806716	0.823134	0.761194	0.630597	0.456716
	Ait-Sahalia & Jacod'	1	0.896766	0.259204	0.041791	0.00796
	'Ait-Sahalia Power Var'	1	0.914677	0.362687	0.213433	0.252736
	'BNS'	0.001493	0.591294	0.638806	0.528109	0.379602
	Jiang & Oomen'	0.651493	0.41393	0.402736	0.367413	0.333582
	Lee & Mykland'	0.806716	0.797264	0.740299	0.622388	0.497761
	Podolskij & Ziggel'	0.745274	0.815672	0.770647	0.666169	0.519154
$\sigma_{noise} = 0.080$	'Andersen et al'	0.757711	0.783831	0.738308	0.607214	0.452239
	Ait-Sahalia & Jacod'	1	0.906965	0.268159	0.044279	0.008458
	'Ait-Sahalia Power Var'	1	0.921393	0.379353	0.215423	0.255473
	'BNS'	0	0.502488	0.604975	0.494527	0.376617
	Jiang & Oomen'	0.747264	0.396269	0.388308	0.359453	0.330597
	Lee & Mykland'	0.757711	0.75995	0.715672	0.608209	0.492289
	Podolskij & Ziggel'	0.688806	0.777861	0.746269	0.638557	0.510945

Table 3.13: Power of the tests in the presence of microstructure noise:We assume a SV1F plus jump model with medium mean reversion for the volatility factor and noise drawn from normal distribution with varying variance

Cross-performances of the tests

Given the variety of jump tests with different characteristics in terms of size, power and behavior in the presence of microstructure noise, we wondered whether the simultaneous use of different tests could be helpful for users. We perform this analysis on data simulated based on the SV1F model, augmented by jumps and microstructure noise. Jumps arrive at times sampled from a Poisson distribution with intensity $\lambda = 0.5$ and have a size distributed as a $\mathscr{N}(0, 1.5)$, while the microstructure noise is sampled from a $\mathscr{N}(0, .04)$.

The main purpose of this exercise is to understand the effect of combining procedures on the performance of the tests, and not necessarily finding the optimal combination of tests, which might not even exist. Thus, we do not intend to exhaust all the possible combinations of testing procedures. Since the BNS test is the most utilized in applied work, we analyze here all combinations of this test with the other four tests. We consider both the intersection between results of two tests and the reunion and identify, for each of these situations, the percentages of correctly identified days with jumps, correctly identified days without jumps, as well as spuriously detected discontinuities. Thus, for instance, if we look at the first column in the upper panel of Table 3.14, the value .4940 means that 49.4% of jumps were identified by both the BNS and Lee and Mykland (2008) procedures, while the value .9082 indicates that in 90.82% of the days without jumps, both procedures did not identify jumps. Finaly, the last value shows that there are .69% spurious jumps detected when both procedures are simultaneuosly considered. If we look at the first column in the lower panel of the table, which considers the reunion of two jump detection criteria, we notice that in 68.56% of the days with jumps at least one of the two above procedures identifies jumps, in 99.31% of the days without jumps at least one of the two tests did not identify jumps, while in 45.5% of the days without jumps at least one of the two tests spuriously identifies jumps.

The results in Table 3.14 should be interpreted by contrasting them with the power and size properties of the tests reported in Tables 3.12 and 3.13. We

Intersection	'BNS-LM'	'BNS-Podolskij'	'BNS-JO'	'BNS-AitSahalia(power)'
'Jump'	0.4940	0.5323	0.2692	0.1418
'No Jump'	0.9082	0.0258	0.8507	0.8502
'Spurious'	0.0069	0.0237	0.0132	0.0064
Reunion	'BNS-LM'	'BNS-Podolskij'	'BNS-JO'	'BNS-AitSahalia(power)'
'Jump'	0.6856	0.6851	0.6473	0.6134
'No Jump'	0.9931	0.9289	0.9868	0.9936
'Spurious'	0.4550	0.4838	0.5125	0.5130

Table 3.14: Cross-performances of the BNS test coupled with the following tests: LM,PZ, JO and AJ, at a sampling frequency of 15 minutes

	'BNS5-LM15-BNS15'
'Jump'	0.6070
'No Jump'	0.9498
'Spurious'	0.0090

Table 3.15: Performance of the LM based on 15 minutes data coupled with BNS procedure based on 5 and 15 minutes data

observe that when we consider the reunion of two tests, there is an increase in the number of correctly classified cases (jumps/ no jumps), than when tests are taken separately. However, there is a huge proportion of spurious jumps (between 45% and 51%). When intersection between tests is considered, the proportion of spurious jumps is almost negligible, but with a considerable reduction in power.

Given the above results, our final useful exercise is to combine intersections and reunions across procedures and across frequencies. To illustrate this, we applied the BNS test on both 5 and 15 minutes simulated data, as well as the LM procedure based on 15 minutes data. We adopted the following decision rule: on a certain trading day, the path of the price process is considered discontinuous if one or more jumps is/ are detected by the LM test and at least by one of the two BNS tests. Results are summarized in Table 3.15.

We observe that the number of spuriously detected jumps becomes very low and is combined with high proportion of correctly identified jumps (approximately 60.70%) and a very high proportion (94.98%) of days rightly classified as without jumps. Thus, this kind of procedure can be very useful in practice, as it allows users to attain a very low proportion of spurious detection of jumps, without a considerable decrease in power (power of the individual tests in Table 3.13 is 54% for BNS and 64% for LM).

3.3 Empirical application

In this final section, we apply all non-parametric tests for jumps to real data. We report a short analysis based on high frequency data for four US Treasury bonds: the 2-, 5-, 10 and 30- year bonds. The data was provided by BrokerTec, an interdealer electronic trading platform and is made up of trade records and quotations.

In order to carry out the jump tests, we relied only on mid-quotes, for a period between January 2003 and March 2004. The prices are reported in 256th of a point and were maintained under this form throughout the analysis. Sampling was performed every 1, 5, 15 and 30 minutes.Table 3.16 reports the proportions of identified jumps.

The 30-Year bond is the least liquid one, which makes us suspect that results in this case are not totally reliable, as illiquidity can lead to spurious detection of jumps. As expected, the proportion of identified jumps decreases as maturity increases, for all tests and sampling frequencies. We observe a clear increasing tendency in the proportions of identified jumps from 1 to 5 minutes. This result could be due to the fact that, at higher frequencies, there is a higher level of contamination with microstructure noise, which tends to downward bias the test statistics. For the ABD, BNS, LM and PZ tests, the proportion of identified jumps is maximum when we sample every 5 minutes, followed by a gradual decrease at lower frequencies. This is expected as power of these statistics tends to decrease at lower frequencies. In the case of the JO procedure, we observe an increase in the proportion of identified jumps from 1 to 5 minutes, which then decreases and increases again. In order to

		Nominal size:5%			
		1 min	5 min	15 min	30 min
2-Year	'Andersen et al'	0.172	0.626	0.331	0.215
	Ait-Sahalia & Jacod'	0.076	0.079	0.023	0.013
	'Ait-Sahalia Power Var'	0.096	0.344	0.285	0.377
	'BNS'	0.169	0.437	0.209	0.179
	Jiang & Oomen'	0.043	0.185	0.162	0.205
	Lee & Mykland'	0.470	0.639	0.507	0.331
	Podolskij & Ziggel'	0.705	0.589	0.361	0.278
5-Year	'Andersen et al'	0.517	0.563	0.318	0.202
	Ait-Sahalia & Jacod'	0.060	0.050	0.030	0.020
	'Ait-Sahalia Power Var'	0.079	0.172	0.185	0.238
	'BNS'	0.182	0.219	0.162	0.113
	Jiang & Oomen'	0.132	0.172	0.149	0.169
	Lee & Mykland'	0.583	0.699	0.460	0.305
	Podolskij & Ziggel'	0.603	0.576	0.368	0.288
10-Year	'Andersen et al'	0.235	0.507	0.321	0.189
	Ait-Sahalia & Jacod'	0.053	0.066	0.020	0.020
	'Ait-Sahalia Power Var'	0.056	0.205	0.248	0.242
	'BNS'	0.126	0.245	0.159	0.139
	Jiang & Oomen'	0.070	0.179	0.162	0.172
	Lee & Mykland'	0.437	0.639	0.424	0.268
	Podolskij & Ziggel'	0.536	0.490	0.374	0.262
30-Year	'Andersen et al'	0.091	0.568	0.196	0.084
	Ait-Sahalia & Jacod'	0.034	0.014	0.000	0.000
	'Ait-Sahalia Power Var'	0.051	0.162	0.108	0.155
	'BNS'	0.078	0.419	0.135	0.057
	Jiang & Oomen'	0.014	0.166	0.118	0.098
	Lee & Mykland'	0.611	0.547	0.250	0.155
	Podolskij & Ziggel'	0.821	0.709	0.328	0.213

Table 3.16: Proportion of days with jumps for all maturities, at different sampling frequencies, as identified by the following procedures: ABD, AJ (both versions),BNS, JO, LM and PZ

get a better picture on the behavior of these tests when varying sampling frequency, we plotted in Figure 3.5 the proportion of jumps, as identified by the different procedures considered here, as a function of sampling frequency, for the 2-Year bond.

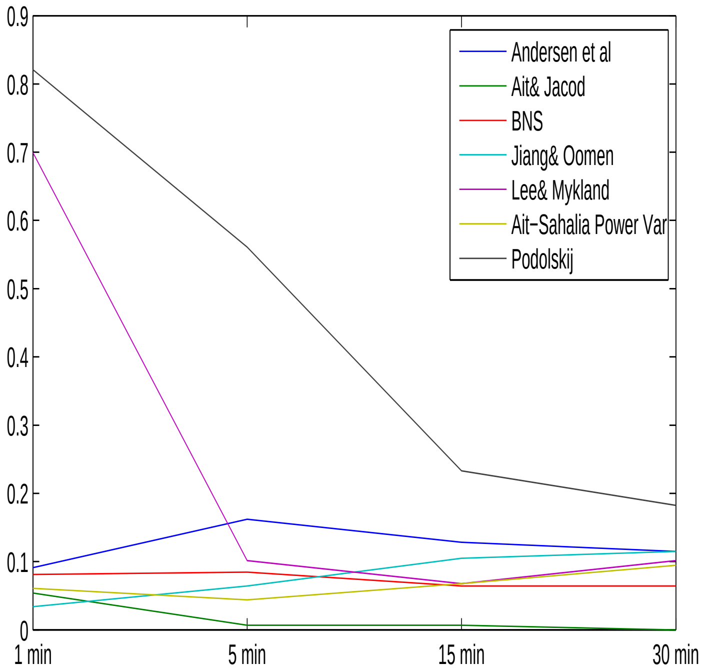


Figure 3.5: Proportion of days with jumps and sampling frequency for the 2-Year bond (1, 5, 15 and 30 minutes)

The AJ procedure behaves in a different manner in comparison with all the other tests. When sampling is performed every minute, it identifies between 3.4% and 7.6% days with jumps when the test statistic is based on threshold estimators and between 5.1% and 9.6% in the case the test statistic is built with multipower variation estimators, which is quite low in comparison to all the other tests. Finally, as we sample more sparcely, this proportion decreases for the first version of the test and increases for the second one. This is in line with the size behavior of the two versions of the test, as reported in Table 3.5.

3.4 Conclusion

The contribution of this research to the existing literature is twofold. First, we offer a robust and comprehensive comparison between alternative jump detection procedures based on high frequency data available in the literature, namely Andersen et al. (2007), Aït-Sahalia and Jacod (2008), Barndorff-Nielsen and Shephard (2006a), Jiang and Oomen (2008), Lee and Mykland (2008) and Podolskij and Ziggel (2008) tests. Second, we offer some useful guidelines to potential users on which test/ combinations of tests to use to detect jumps in the prices of financial assets.

To this end, we conducted a numerical analysis using alternative levels of volatility, different levels of persistence in the volatility factor, different jump intensities and jump sizes, different levels of microstructure noise contamination. We also report an empirical application using US Treasury high frequency data. We summarize the full set of results in Table 3.17.

Based on the overall results of our simulation, the intraday LM-ABD tests for jumps show the best performance. The procedures display a very high power, which is combined with a quite good size behavior. Thus, for the SV1F model, size remains relatively stable over different sampling frequencies. However, in the case of extremely volatile processes, like SV2F, there is the risk that the tests become highly oversized and consequently, their use might not be recommendable for very volatile data. The tests also perform very well in the presence of microstructure noise.

The Podolskij and Ziggel (2008) test displays high power and a very good behavior in the presence of noise, but is also quite oversized. Its size increases very rapidly when the sampling frequency diminishes. However, given its robustness to microstructure effects, it can be successfully applied at high frequencies, without worrying about the noise.

The classical BNS test shows very good size properties, with its size remaining quite stable over the varying sampling frequency. It also displays a quite good power. In the presence of microstructure noise, the BNS test statistic gets very downward biased and sampling at lower frequencies is obligatory.

There is not a clear-cut behaviour with respect to the other two tests considered in our analysis, JO and AJ. The former displays a high size which increases rapidly as we sample less often, accompanied by a low power. When jumps with random signs and sizes are considered, positive and negative changes tend to compensate themselves, resulting in a reduction of power. The latter test works well in terms of both size and power only at high frequencies (1 second in our simulation exercise). However, for lower frequencies, there is evidence of a substantial decrease in power, combined with an increase/ decrease in size, depending on how the statistic is computed: based on multi-power variations or threshold estimators. Moreover, this test becomes extremely over sized at high frequencies in the presence of noise and thus, a very frequent sampling scheme, which could preserve good size and power properties, is not possible.

Finally, we applied all six jump detection procedures on real US Treasury high frequency data. We were interested in the behavior of the tests across sampling frequencies. We observed in most of the cases lower proportions of detected jumps at the highest frequency, which can be interpreted as a consequence of the high level of contamination with microstructure noise at this frequency. The proportion of jumps increases when sampling every 5 minutes and then gradually decreases again with the decrease in the sampling frequency.

Based on our results, we recommend to potential users the LM -ABD intraday procedure, as well as the PZ test, which have good power properties combined with a manageable size. Moreover, these tests are the most robust to microstructure noise. However, when the price processes are very volatile, as it might happen for some assets such as some derivatives, stocks, they become highly oversized. In this case, we recommend the use of the BNS test, as its size distortion is smaller and more stable across frequencies.

We show that potential users of these procedures can gain advantages by combining them through both reunion and intersection across procedures and across sampling frequencies. In this way, one manages to minimize the proportion of spurious jump detection without a significant loss of power.

One limitation of the present analysis is that in the simulation design, we take into consideration only processes that can generate a finite number of jumps within a certain time interval. This is due to the fact that most of the tests considered here (the only exceptions being AJ and PZ) are based on multipower variation-type estimators, which are robust only to a finite number of jumps. We leave this extension to future research.

Procedure	***Size***	***Power***	***Noise***
'Andersen et al'	slightly undersized; size varies across the frequency	high power decreasing gradually	undersized in the presence of noise; maintains quite good power properties
Ait-Sahalia & Jacod'	slightly undersized; size decreases at lower frequencies	high power at high frequencies which diminishes abruptly at lower frequencies	extremely oversized at very high frequencies, followed by drastic decreases in size from 1 min onward; very high power which decreases abruptly
'Ait-Sahalia Power Var'	oversized; size rapidly increases across the frequency	high power at high frequencies which diminishes abruptly at lower frequencies	extremely oversized at very high frequencies, followed by drastic decreases in size from 1 min onward; very high power which decreases abruptly
'BNS'	oversized; size increases slightly across the frequency; stable in comparison with the others	high power decreasing gradually; lower numbers than the intraday and 'Podolskij & Ziggel' tests	severely undersized at high frequencies; low power in the presence of noise
Jiang & Oomen'	oversized; size increases rapidly across the frequency	low power, as jumps with different sizes tend to cancel out	extremely oversized at very high frequencies; behaves like in the absence of noise from 1 min onward; low power
Lee & Mykland'	oversized; size varies across the frequency	high power decreasing gradually	undersized in the presence of noise; maintains quite good power properties
Podolskij & Ziggel'	oversized; size increases rapidly across the frequency	high power decreasing gradually	becomes quickly oversized even in the presence of noise; maintains quite good power properties

Table 3.17: Summary of our results: size and power properties and behavior in the presence of microstructure noise for all the tests

4

Jumps and price discovery in the US Treasury market

The advances made during the past decade in the field of high frequency econometrics conferred both researchers and practitioners the possibility to detect and estimate jumps in asset prices through simple nonparametric techniques, as an alternative to simulation based estimation methods of stochastic volatility models with jumps. As a result, a new strand in the empirical finance literature has intensively documented on the determinants of jumps, as well as on how their identification impacts on different research areas in finance, especially volatility and price discovery.

Our research lies in the above framework, as well. We identify and estimate jumps in the US Treasury 2-,5-,10- and 30-year bonds both on a daily basis, by using the standard Barndorff-Nielsen and Shephard (2004) test for jumps, as well as at an intraday level, by applying the Lee and Mykland (2008) procedure with a correction for periodicity to our local volatility estimates proposed by Boudt et al. (2009). We find that US Treasury bonds exhibit jumps in their prices in 14.5% of the days for the 2-year maturity, in 10.6% for the 5-year bond, 9.6% for the 10-year and finally in 17.91% of the days for the 30-year bond. Then, we examine how different characteristics of the market, such as liquidity measures, trading volume, order flow behave when jumps occur. We find that in the 5 minutes before a jump, liquidity withdraws, and then it raises again. The trading volume falls in the 5-10 minutes before a jump takes place and increases again afterward. In addition, we endeavor to see to what extent jumps in the term structure are generated by macroeconomic announcements and find that around 90% of the jumps occur at the same time or slightly after an announcement. Moreover, the standardized announcement surprise is found to be an important determinant of the probability of jump occurrence. Finally, we examine the impact of the trading activity on prices in the proximity of jumps.

To our knowledge, there is no other paper that examines the informational role of trading around jump times. Using a methodological framework similar to that of Green (2004), based on a price formation model proposed by Madhavan et al. (1997), we estimate the degree of informational asymmetry at the jump time, before and after. We estimate, for the 2- and 5-year bonds, a very high degree of information asymmetry in the immediate proximity of a jump (-/+ 2 minutes). Moreover, this high degree of order flow informativeness does not dissipate immediately, but remains quite high up to 20 minutes after a jump takes place. There is a low level of information asymmetry before the jump, which, given that most of the jumps take place as a result of macroeconomic news announcements, is consistent with a low degree of information leakage before announcements, as indicated in Green (2004). Results for the 30-year bond follow the same lines as for the first two maturities, but with a lot of the coefficients being insignificant, due to liquidity problems, while results for the 10-year maturity are quite different and will be described in the appropriate section.

In the empirical literature, there are three more papers which deal with similar issues. Jiang et al. (2008) identify jumps in the US-Treasury bonds, by using data covering 2 years, 2005 and 2006. They show that most of the jumps take place on scheduled macroeconomic announcement days. Moreover, before jumps, liquidity on the market withdraws and volatility increases. The authors run Probit regressions in order to identify what variables contribute to the increase in the probability of a jump taking place. First, they find surprises in macroeconomic announcements as explaining the occurrence of jumps. Further on, they add to the regression liquidity shocks before jumps and find the latter highly significant. Moreover, they show that the previous variable, surprises in macroeconomic announcements, loses its power in explaining jumps when liquidity shocks are considered. In terms of price discovery, they show that after a jump takes place, the order flow seems less informative than in the case when no jumps take place.

Lahaye et al. (2008) identify jumps and cojumps using the intraday Lee and Mykland (2008) procedure, modified by Boudt et al. (2009), in the prices of 8 different financial assets: four dollar exchange rates, three stock index futures (Nasdaq, Dow Jones and S&P500) and 30-year U.S. Treasury bond futures. They run Tobit and Probit regressions to quantify the impact of news surprises on jumps and co-jumps. They find that bond markets are the most sensitive to news surprises. When co-jumps are considered, the news surprises seem to have a much larger impact on the probability of common jumping them in the case of individual jumps.

Dungey et al. (2007) use the classical Barndorff-Nielsen and Shephard (2004) methodology to identify jumps and co-jumps in the 2, 3, 5, 10 and 30 year US Treasury bonds, for a period lagging from 2002 to 2006. They find that co-jumps are caused in 2/3 of the cases by macroeconomic news announcements.

Up to a certain point, our research work situates itself close to the Jiang et al. (2008) paper, as we follow some of the steps in their empirical analysis concerning the study of market activities around jumps and association of jumps with macroeconomic announcements. However, our research work is different in many ways from theirs. First, we use the Lee and Mykland (2008)-Andersen et al. (2007) jump detection procedures, which allow us to identify the exact time of a jump within a trading day, as well as the sizes of the detected jumps. Second, while Jiang et al. (2008) consider illiquidity as the most important determinant of the probability of jump occurrence, we argue here that the liquidity withdrawal and the occurrence of jumps are endogenous and essentially express the same thing. This is because the vast majority of jumps are caused by news releases, before which markets are usually characterized by a liquidity withdrawal. Third, when examining the impact of macroeconomic announcements on the jump probability, we consider the entire sample and not just the days with announcements. Finally and more importantly, we dedicate a vast part of this chapter to examine the impact of trading on prices and consequently to estimate the degree of informativeness of the order flow when jumps occur, in agreement with the literature on price discovery.

The rest of the chapter is structured in the following way. Section 4.1 describes the interdealer market on which US Treasury bonds are exchanged and the data in our sample. Section 4.2 summarizes the jump detection techniques we considered, the classical Barndorff-Nielsen and Shephard (2006a) procedure, as well as Andersen et al. (2007) -Lee and Mykland (2008) procedure, which enables us to intradaily detect jumps. Finally, Section 4.3 is the widest and contains the majority of the findings of this chapter. Here, after having identified jumps in prices in the previous section, we analyze different market activities around the time of the jump, we quantify the impact of macroeconomic announcements on the jump likelihood and we examine the informativeness of the trading activity in the nearness of jumps.

4.1 Data and market description

Our analysis is based on high frequency data for four US Treasury bonds: the 2-, 5-, 10 and 30- year bonds. The data was provided by BrokerTec, an interdealer electronic trading platform and is made up of trade records, quotations and order cancellations, as well as a work-up part.

4.1.1 The interdealer brokerage market

The secondary market for US Treasury bonds is an interdealer over-the - counter market, where, as shown in Fleming and Mizrach (2008) and Fleming

(1997), there are 22-23 hours of trading activity per day, with most of it being placed during new-York hours, that is between 7:30 a.m. and 5:00 p.m. . There are some trading peaks between 10:00 and 10:30 a.m. and between 14:30 and 15:00 (Fleming and Mizrach, 2008).

The largest part of the transactions in the interdealer market for the US Treasury bonds takes place through two large interdealer brokerage firms: ICAP PLC with about 60% of market share and Cantor Fitzgerald with 28% (Mizrach and Neely, 2006). Before 2000, both the actors on the marketplace provided voice-assisted brokerage services. All the market data from ICAP was collected in the GovPX database and was customarily used for the studies concerning the US Treasury bonds. However, as noted by Mizrach and Neely (2006), Boni and Leach (2004), Fleming (2003) and Barclay et al. (2006), with the foundation of electronic trading platforms, most of the trading with US Treasury bonds migrated from the voice-assisted to the electronic platforms. Thus, as shown in Barclay et al. (2006), e-Speed, the electronic platform of Cantor Fitzgerald was inaugurated in March 1999, while its main competitor, BrokerTec, was set up in June 2000 and was purchased by ICAP in 2003. These electronic trading platforms are characterized by lower transaction costs and by a higher level of liquidity, also due to the fact that electronic systems match opposite orders automatically, making the whole trading process more fluid. The voice-assisted platforms remain though important for their use for more "customized", complex transactions that require human intermediaries to perform negotiations between parties. Mizrach and Neely (2006) show that after the introduction of the electronic trading platforms, the average trading volume almost tripled from $200 billion in 1999 to $ 575 billion in 2005 .

4.1.2 Market characteristics

The interdealer market is an expandable limit order one, where transactions typically pass through three different phases (Boni and Leach, 2004, see). Traders post limit orders, that can be automatically matched by the electronic system. They can also respond to an already existing order (becoming "aggressive"). However, as noted by Boni and Leach (2004), there is an incentive on the market to provide liquidity through limit orders, as commissions are paid only for responding orders.

The next phase of a transaction is the so-called "work-up" process. This type of market provides the traders with the right of refusal to trade additional quantities, provided that the other party desires this. Thus, traders usually enter limit orders in order to find counter-parties and then increase quantities during the work-up process. Moreover, there is the possibility to post "iceberg" orders, that have hidden quantities.

Once the parties agree on quantities, the trades are perfected and they appear in the Trade section. Boni and Leach (2004) show that the right and not the obligation to further increase traded quantities reduces the costs associated with information leakage and stale limit orders, unlike the usual limit order markets, where large orders might cause free-riding on the signal. Fleming and Mizrach (2008), using BrokerTec reveal that liquidity is greater than the one reported by studies using data from voice-assisted brokerage platforms. Moreover, the iceberg orders are sparse and are mostly used during volatile periods.

4.1.3 Dataset

The data contains intraday observations covering the orderbook, with both order submissions and cancellations, the trade section and the work-up process for the 2-, 5-, 10- and 30- year bonds and covering a period between January 2003 and March 2004. While the first three bonds are very liquid, for the 30-year one the competitor brokerage platform, E-Speed, owns a bigger market share.

Prices are reported in 256th of a point and are maintained under this form throughout the analysis, as they do not influence the results, given that we work with log-returns.

For each trading day, we keep in our sample just the data comprised between 7:30 a.m. EST and 5:00 p.m. EST, when trading is more active.

Sampling is done every 5 and 15 minutes. Based on the information in the order book , we compute mid-quotes, spread and depth of the market at the best bid and ask quotes. We rely on information in the trade section to compute orderflow and trade volumes.

4.2 Testing for the presence of jumps

One of the important advances in the field of high frequency econometrics during the last decade was the development of several nonparametric procedures that allow testing for the presence of jumps in the path of a price process during a certain time interval or at certain point in time. The pioneers in this area were Barndorff-Nielsen and Shephard (2006a). Following their seminal contribution, several other researchers pursued this topic: Andersen et al. (2007)-Lee and Mykland (2008), Aït-Sahalia and Jacod (2008), Jiang and Oomen (2008)and Podolskij and Ziggel (2008). Except Andersen et al. (2007)-Lee and Mykland (2008), all the other procedures have nulls that assume the continuity of the price path within a certain time period, such as a trading day. The Andersen et al. (2007)-Lee and Mykland (2008) procedure tests for the absence of jumps at a certain moment, allowing thus for the exact identification of the time of a jump.

The choice of the jump identification procedures that we use is based on simulations carried out by the authors in Dumitru and Urga (2009). We choose to apply here the Andersen et al. (2007)-Lee and Mykland (2008) test because it is one of the procedures that display the highest power, combined with a manageable size and it allows for the exact identification of the jump time. One of the conclusions of Dumitru and Urga (2009) is that combining tests and sampling frequencies through both reunion and intersection can grant users a better performance in terms of both size and power. Consequently, here we combine the results of the Andersen et al. (2007)-Lee and Mykland (2008) test with the ones of the Barndorff-Nielsen and Shephard

(2006a) procedure, which is also reported in Dumitru and Urga (2009) as having good power and size properties.

We compute the Barndorff-Nielsen and Shephard (2006a) test statistic based on both 5 and 15 minutes data. However, for the version based on the higher frequency, we make use of staggered returns when calculating the realized bipower variation, as in Andersen et al. (2007). In applying the intraday procedure we make use of data sampled every 15 minutes. We consider as final jumps the ones identified with the Andersen et al. (2007) -Lee and Mykland (2008) test if they were also detected by the Barndorff-Nielsen and Shephard (2006a) procedure on either 5 or 15 minutes data. If more than one jump was detected within one day, all of them were taken into consideration. We make use of 99% critical values for both tests applied here.

4.2.1 Jump tests

Barndorff-Nielsen and Shephard (2006a) test

Barndorff-Nielsen and Shephard (2006a) base their procedure on the possibility to build a consistent estimator for the integrated variance of a process. Thus, in Barndorff-Nielsen and Shephard (2004), they prove that the realized bipower variation consistently estimates the integrated variance:

$$BV_t = \operatorname*{plim}_{\delta \downarrow 0} \delta^{1-\frac{r+s}{2}} \sum_{j=1}^{[t/\delta]} |y_j(t)||y_{j+1}(t)| \tag{4.1}$$

where δ is the intraday sampling frequency, with $[t/\delta] = n$ the number of intraday returns, and y_j the j-th intraday returm at time j, $j = 1...[t/\delta]$.

In consequence, the difference between realized volatility and realized bipower variation will qualify for jump testing and estimation. Barndorff-Nielsen and Shephard (2006a) formulate a null of no jumps during a certain time period, such as one trading day, against the alternative of jumps being present, and base their testing on a CLT-type result developed under the null:

$$\frac{\delta^{-1/2}(\mu_1^{-2}BV_t) - RV_t}{\sqrt{\int_0^t \vartheta\sigma_u^4 \,\mathrm{d}u}} \xrightarrow{\mathscr{L}} \mathscr{N}(0,1) \tag{4.2}$$

where $\mu_r = E|u|^r = 2^{r/2}\frac{\Gamma(\frac{1}{2}(r+1))}{\Gamma(\frac{1}{2})}, \quad r > 0, \ u \sim \mathscr{N}(0,1)$ and the integral present in the denominator of equation (4.2), named integrated quarticity, can be estimated by using the the realized tripower quarticity (Andersen et al., 2005):

$$TP_t = n\mu_{4/3}^{-3}\left(\frac{n}{n-2}\right)\sum_{j=3}^{[t/\delta]} |y_{j-2}|^{4/3}|y_{j-1}|^{4/3}|y_j|^{4/3} \tag{4.3}$$

Huang and Tauchen (2005) consider different versions for the test statistic of this test and compare them by means of Monte Carlo simulations. Based on their work, as well as on simulations carried on by the authors themselves (Barndorff-Nielsen and Shephard, 2006a), the most appropriate form for the test statistic is the following:

$$z = \frac{1 - \frac{BPV_t}{RV_t}}{\sqrt{(\mu_1^{-4} + 2\mu_1^{-2} - 5)\delta max\left(1, \frac{TP_t}{BPV_t^2}\right)}} \rightarrow \mathscr{N}(0,1) \tag{4.4}$$

Lee and Mykland (2008)-Andersen et al. (2007) tests

Both research papers by Lee and Mykland (2008) and Andersen et al. (2007) concurrently developed tests for jumps based on the standardization of the intraday returns by robust to jumps volatility estimations. In Andersen et al. (2007), returns are standardized by the square root of realized bipower variation, which is estimated on the observations of that trading day. However, when they apply the test to real data, they notice the need to take into consideration the intraday periodicity of the volatility. Lee and Mykland (2008) use the same realized bipower variation to standardize the returns, but estimate it on a local window that precedes the time the test is performed. Both tests have the null hypothesis of continuity of the sample path at a certain time, t_j and thus, users are enabled to identify the exact time of a jump, as well as the number of jumps within a trading day. For brevity, beneath we describe the Lee and Mykland (2008) test. The following statistic is considered:

$$\mathscr{L}(j) = \frac{y_j}{\hat{\sigma_j}}, \qquad j = 1 \dots n, \tag{4.5}$$

where $\hat{\sigma_j}$ is the realized bipower variation estimated on a K previous observations window.

Under the null of no jumps, the statistic will be asymptotically normal. However, the usual normal thresholds (like the 99% quantile) prove themselves to be too permissive. The two papers considered here provide different solutions to this problem, which lead, however, to very similar critical values. Andersen et al. (2007) choose the size at a daily level , α, say 1% , which is then distributed such that for each intraday time interval of length δ, the size is given by $\beta = 1-(1-\alpha)^{\delta}$. Lee and Mykland (2008) choose to take into consideration the maximum of the $L(j)$ statistic over a given period, usually a day. The new properly standardized statistic will display an extreme value distribution (Gumbel distribution):

$$\frac{\max\left(\mathscr{L}(j)\right) - C_n}{S_n} \to \xi, \qquad \mathbf{P}(\xi) = \exp(-e^{-x}), \qquad \forall j = 1, 2, \dots, n \tag{4.6}$$

where

$$C_n = \frac{(2\log n)^{1/2}}{\mu_1} - \frac{\log \pi + \log\left(\log n\right)}{2\mu_1 (2\log n)^{1/2}} \tag{4.7}$$

and

$$S_n = \frac{1}{\mu_1 (2\log n)^{1/2}} \tag{4.8}$$

Thus, the test can be conducted by simply replacing the maximum statistic above by the estimated value of $\mathscr{L}(j)$ and compare the resulting value with the threshold showed above.

The problem with both these intraday tests is that volatility estimated as described above changes very slowly, while for real data, volatility tends to cluster and peak during the same time intervals within a trading day, such as after macroeconomic news announcements in the morning or some time before the closing of the market. Boudt et al. (2009) show that volatility moves within the trading day mostly as a result of intra-week and intra-day periodicity and consequently, propose parametric and nonparametric estimators of the periodicity factor that are robust to the presence of jumps. Here, we only describe the latter approach, which we will further use in correcting the Lee and Mykland (2008) test statistic.

Boudt et al. (2009) write the returns as being described by the following discrete model:

$$y_j = f_j s_j u_j + a_j, \qquad j = 1 \dots n, \tag{4.9}$$

where s_j is the average bipower variation, estimated on a local window around j, $f_j = \sigma_j / s_j$, the periodicity factor, with σ_j the spot volatility, $u_j \sim i.i.d. \mathcal{N}(0,1)$. Then, the periodicity factor is estimated based on the following several steps: First, we standardize all intraday returns by the squared root of the properly scaled realized bipower variations estimated on a local window around j, as shown before. We denote by $r_{1,j}, r_{2,j}, \dots r_{n_j,j}$ all standardized returns that refer to the same day of the week and the same time of the day, j.

Second, Rousseeuw and Leroy (1988)' "shortest half scale estimator" is computed in the following manner. We construct the corresponding order statistics for the above sequence of standardized returns, resulting in $r_{(1),j} \leq r_{(2),j} \leq \dots \leq r_{(n_j),j}$. The shortest half scale estimator will be the smallest length of all ordered subsequences consisting of $h_j = [n_j/2] + 1$ observations:

$$ShortH_j = 0.741 min\{r_{(h_j),j} - r_{(1),j}, \dots, r_{(n_j),j} - r_{(n_j - h_j + 1),j}\}, \tag{4.10}$$

where 0.741 is a correction for consistency under normality. The above estimator can be standardized in following manner:

$$f_j^{ShortH} = \frac{ShortH_j}{\frac{1}{n}\sqrt{ShortH_j^2}}, \tag{4.11}$$

with n the number of intraday intervals.

Last, we compute the final periodicity estimator, which will be the standardized Weighted Standard Deviation (WSD), as resulting from the following equations:

$$f_j^{WSD} = \frac{WSD_j}{\frac{1}{n}\sqrt{WSD_j^2}}, \tag{4.12}$$

with

$$WSD_j = \sqrt{1.081 \frac{\sum_{l=1}^{n_j} w_{l,j} r_{l,j}^2}{\sum_{l=1}^{n_j} w_{l,j}}}, \tag{4.13}$$

where the weights $w_{l,j}$ are given by:

$$w_{l,j} = \begin{cases} 1, & \text{if} \quad \left(\frac{r_{l,j}}{\hat{f}_j^{ShortH}}\right)^2 \leq 6.635 \\ 0, & \text{otherwise} \end{cases} \tag{4.14}$$

In equation 4.14, the threshold of 6.635 is the 99% quantile of the χ^2 distribution.

4.2.2 Results: detected jumps and co-jumps

We find that the 2-year bonds jump in 14.5% of the days, the 5-year in 10.6%, the 10-year in 9.6% and finally the 30-year in 17.91% of the days. As expected, if we donot consider the result for the 30-Year bond, we observe a decrease in the proportion of identified jumps with the increase in the maturity. The 30-year bond is highly illiquid during the period we considered in our sample and thus we expect the high proportion of detected jumps is spuriously generated.

Table 4.1 summarizes some descriptive statistics on the estimated jump sizes for all the maturities, while Table 4.2 reports the same indicators but for jump sizes that were previously standardized by a local volatility estimator, just as in the Andersen et al. (2007) -Lee and Mykland (2008) procedure. The biggest size is encountered in the case of the 30-Year bond, but we suspect this finding is due to liquidity issues, just as the high proportion of jumps identified for this maturity. If we ignore this bond, when we look at all central tendency parameters in Table 4.1, we observe, similar to Jiang et al. (2008), that the 10-year bond displays the highest jump size, followed by the 5-Year and the 2 Year. It seems that the latter jumps more frequently, but less abruptly than the other bonds. However, if we standardize these jumps by robust to jumps local volatility estimators, we observe in Table 4.2 that the previous hierarchy disappears, clearly indicating that the shorter maturity bonds are less volatile than the others.

	Size			
	Mean	Median	Mode	Standard deviation
Y2	0.081%	0.063%	0.023%	0.052%
Y5	1.787%	0.181%	0.000%	10.394%
Y10	1.795%	0.292%	0.000%	9.814%
Y30	2.127%	0.474%	0.211%	9.639%

Table 4.1: Estimated jumps for the 2, 5, 10 and 30-year US Treasury bonds

	Standardized Size			
	Mean	Median	Mode	Standard deviation
Y2	31.15	23.57	16	19.39
Y5	28.38	22.25	16.07	15.67
Y10	28.36	24.19	15.91	13.24
Y30	30.19	21.8	16.04	30.15

Table 4.2: Estimated standardized jumps for the 2, 5, 10 and 30-year US Treasury bonds

Table 4.3 reports the number of common jumps between maturities, when taken two by two. We observe a clear prevalence of common jumps at the shorter end of the term structure. Thus, we have the largest number of co-jumps for combinations of the 2-year maturity with the other bonds.

	Y2	Y5	Y10	Y30
Y2	44	28	24	23
Y5		32	21	17
Y10			29	17
Y30				53

Table 4.3: Number of jumps and co-jumps for the 2, 5, 10 and 30-year bonds, taken two by two

The analysis in this subsection shows us that the shorter maturity bonds jump more frequently than the others, making us suspect that they react to a larger proportion of macroeconomic announcements than the others, that is they are more sensitive to events occurring in the economic environment. The other bonds are less sensitive, displaying a lower probability of jump occurrence, but, given the long maturity, they transpose the incertitude in the environment in larger moves in the prices.

4.3 Jumps and price discovery

This section accommodates all our results which cover on one side, issues concerning the informational content of jumps in the US Treasury term structure and, on the other side, relate the detected jumps with different market characteristics, especially liquidity measures and to the price discovery process. Moreover, as we will see later on, a great amount of these results, especially those which relate jumps with market characteristics are purely descriptive, while other are the result of a regression analysis that attempts to find factors that explain the probability of a jump, as well as the size of the jump.

4.3.1 Market activities in the proximity of jumps

Once we identified in the previous section the individual and common jumps for all the bonds, we are interested here to see what exactly 'happens' in the market when jumps occur. Thus we measure several market activities on a window of -/+ 25 minutes around the time of the jump and examine their behavior within this interval. We consider the depth of the market at the best bid and ask quotes, the spread, the trading volume and the order flow.

Figure 4.1 illustrates the different market activities around the time of the jump. As the depth indicators refer to the best bid and ask quotes, they are both expressed in number of contracts, while the order flow, computed as a difference between total buy volume minus total sell volume within 5 minutes, is expressed in units of a $, just as the quotes.

Both the depth at the best ask quote and the one at the best bid quote fall in the 5-10 minutes that precede the jump, indicating a withdrawal of liquidity before the jump occurrence. Given that the majority of the jumps take place on public announcement days, this is an expected phenomenon, as also indicated in Jiang et al. (2008). Moreover, spread peaks within the 5 minutes interval before the occurrence of the jump, indicating the same liquidity withdrawal. The trade volume has a general increasing tendency throughout day. However, its dynamic before a jump is the same for all maturities, as seen in Figure 4.1 for the 2 -Year bond: it falls in the 5-10 minutes before the jump and then experiences an abrupt increase.

The order flow tends to decrease before the jump, following up to a certain extent the dynamics of the depth indicators. However, this is more visible for the 2-Year bond, indicating an asymmetry between the bid and ask traded quantities. This behavior cannot be confirmed for the other maturities, where the asymmetry is not that clear.

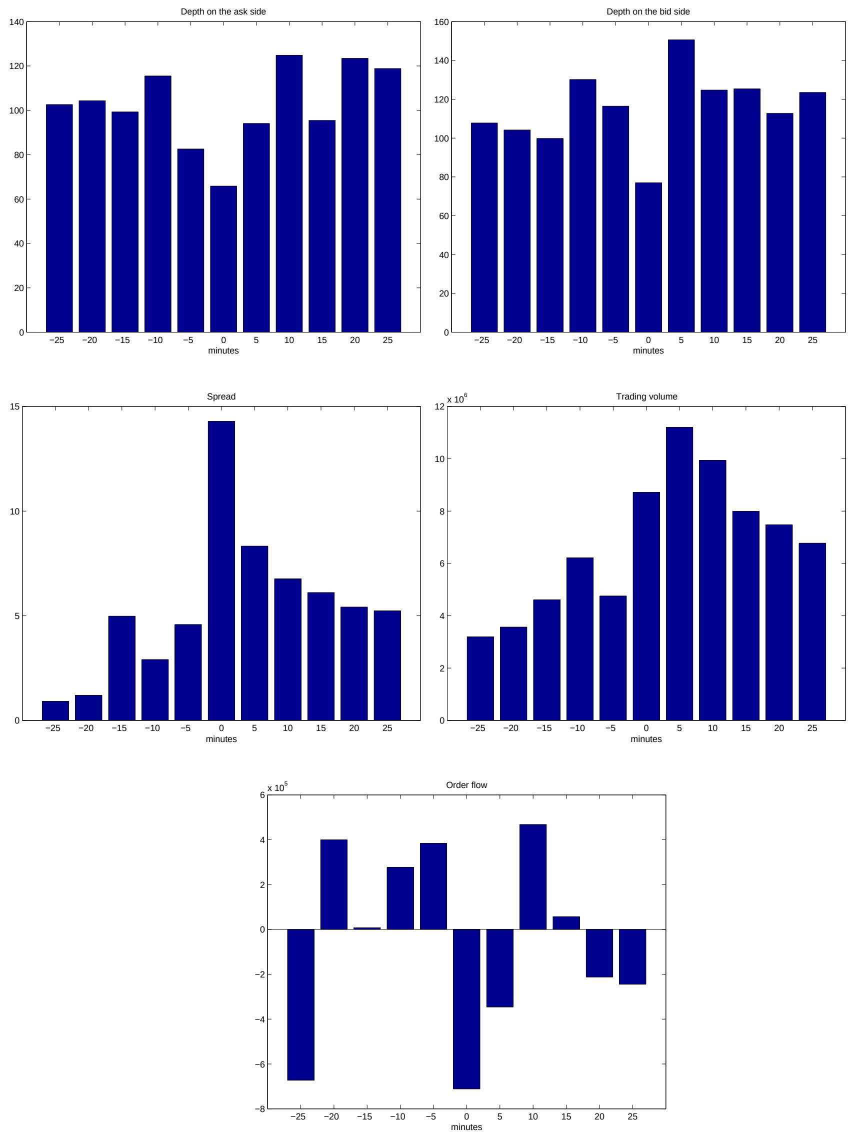
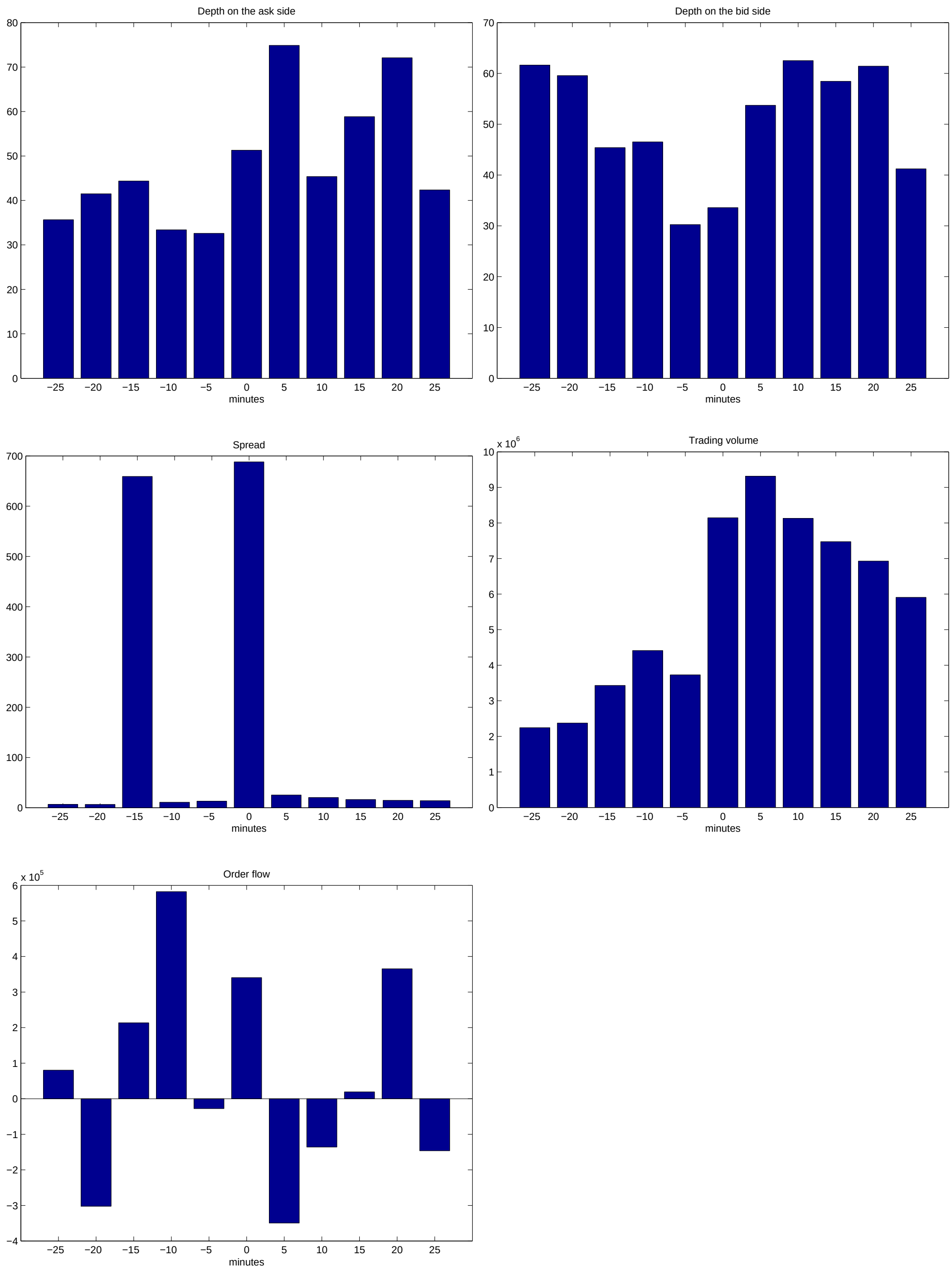
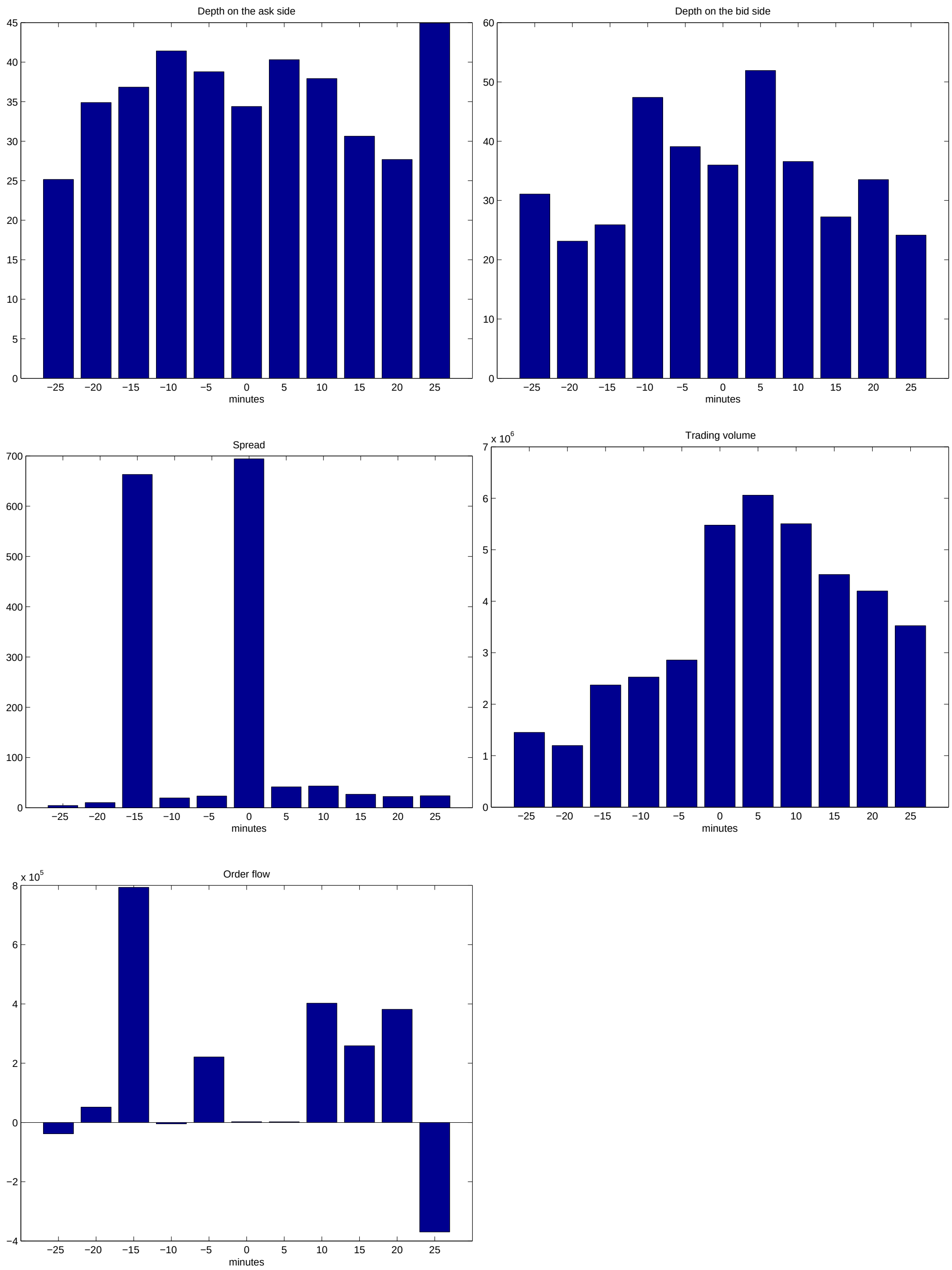
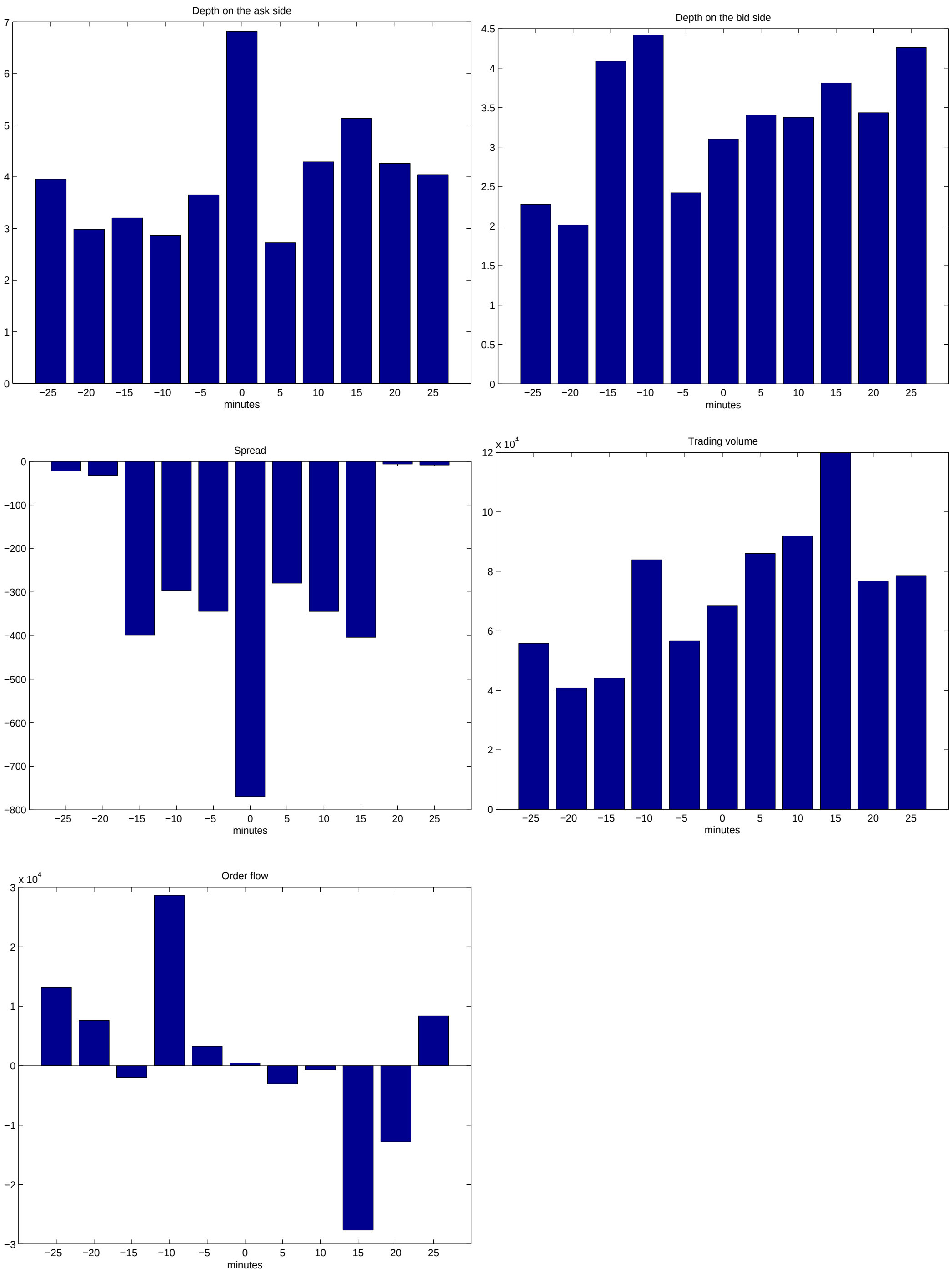


Figure 4.1: Market activities around the time of jump for the 2-Year bond


Depth on the ask side
minutes
Depth on the bid side
minutes
Spread
minutes
Trading volume
x 10^6
minutes
Order flow
x 10^5
minutes


Depth on the ask side
minutes
Depth on the bid side
minutes
Spread
minutes
x 10^6
Trading volume
minutes
x 10^5
Order flow
minutes



Depth on the ask side
minutes
Depth on the bid side
minutes
Spread
minutes
Trading volume
x 10^4
minutes
Order flow
x 10^4
minutes

4.3.2 Jumps and macroeconomic announcements

The fact that bond prices tend to experience large moves when macroeconomic news are released has been well documented in the empirical literature for a long time. Fleming and Remolona (1997),Fleming and Remolona (1999b), Fleming and Remolona (1999a), Balduzzi et al. (2001), Green (2004), Pasquariello and Vega (2007) and others studied the impact of macroeconomic news announcements on prices and other market characteristics, such as liquidity, trading volume, order flow. However, the development of diverse nonparametric jump detection techniques based on high frequency data allows us to define jumps based on clear statistical criteria. Being able to identify jumps and their exact timing helps us improve our knowledge on the possible causes of jumps, too.

For each maturity and for each jump, we checked whether on the day and around the time of the jump there were any macroeconomic announcements. Thus, similar to Jiang et al. (2008), we identify the macroeconomic announcements that cause jumps in the prices of US Treasury bonds. Data on announcements is taken from Yahoo! Finance, that reports some of the data provided by Briefing.com. A complete list of all the announcements we found relevant is included in Table 4.4.

For each maturity, we computed the number and percentage of jumps to which macroeconomic announcements can be associated, as well as jumps which cannot be matched with any news releases. Results are summarized in Table 4.5. For the 2, 5, 10 -year bonds, more than 90% of the jumps were generated by the release of public information on the market. For the 30-year bond, this percentage equals only 83.73%, which we believe is due to the spurious detection of jumps that cannot be matched with announcements.

Once we identified all news releases that can impact on bond prices, we computed the absolute value of the announcement surprise, provided data on this was available. We standardized each surprise by the standard deviation of all surprises available for the same announcement during the period considered in our sample. We selected all standardized surprises larger than the

'Auto Sales'	'Factory Orders'	'Nonfarm Payrolls'
'Average Workweek'	'Fed''s Beige Book'	'NY Empire State Index'
'Building Permits'	'FOMC Meeting'	'Personal Income'
'Business Inventories'	'FOMC Minutes'	'Personal Spending'
'Capacity Utilization'	GDP-Adv & Final'	'Philadelphia Fed'
Chain Deflator-Adv & final'	'Help-Wanted Index'	'PPI'
'Construction Spending'	'Hourly Earnings'	'Productivity-Prel'
'Consumer Confidence'	'Housing Starts'	'Retail Sales'
'Consumer Credit'	'Industrial Production'	'Retail Sales ex-auto'
CPI' & Core CPI	'Initial Claims'	'Trade Balance'
'Current Account'	'ISM Index'	'Treasury Budget'
'Durable Orders'	'ISM Services'	'Truck Sales'
'Employment Cost Index'	'Leading Indicators'	'Unemployment Rate'
'Existing Home Sales'	'Mich Sentiment-Prel.'	'Wholesale Inventories'
'Export Prices ex-ag.'	'New Home Sales'	

Table 4.4: Macroeconomic announcements that generate jumps in the term structure

	Y2		Y5		Y10		Y30	
Match	94	*92.16%*	79	*100.00%*	60	*93.75%*	72	*83.72%*
No match	8	*7.84%*			4	*6.25%*	14	*16.28%*
Total	102	*100.00%*	79	*100.00%*	64	*100.00%*	86	*100.00%*

Table 4.5: Number and percentages of jumps matched with macroeconomic announcements

	Y2		Y5		Y10		Y30	
Big surprise	64		64		64		64	
Big surprise & no jump	57	89.06%	57	8/9	56	87.50%	64	100.00%

Table 4.6: Number of big announcement surprises and number and percentages of those not associated with any jumps

95% quantile of the normal distribution and examined whether jumps took place on the corresponding days and times. Surprisingly, for all maturities, we identified a very low percentage of 'big' surprises associated with jumps. Our findings are summarized in Table 4.6.

While more than 90% of the jumps are generated by announcements, it seems that approximately 90% of the announcements do not cause jumps. These results can be explained by several factors. First, given our sample of just 15 months, for each type of news release, we have just a few surprises. Thus, the standard deviation of each type of surprise is computed on a very small sample, generating biases. Second, there are certain announcements that are more important than others and jumps can occur even if surprises are not very big. Third, as suggested by Hess (2004), the "timeliness" of the news releases might be important, as well. If several announcements reveal similar information, the earlier ones should have a greater impact on the prices.

4.3.3 Regression analysis: determinants of jumps

Within this subsection, we try to identify factors that increase the probability of a jump taking place. Jiang et al. (2008) consider in their paper several factors that can determine the probability of a jump in an announcement day: the announcement surprise, volatility, the absolute order flow, the order unbalance and finally, depth and spread, as measures of liquidity withdrawal before a jump takes place. The results of their regressions indicate liquidity measures as the most important determinants of jumps.

As seen in Figure 4.1, liquidity measures tend to plunge (depth) or peak (spread) just before a jump takes place. Thus, their dynamic is almost the same with that of the returns around a jump. For this reason, we believe liquidity measures and jumps are somehow endogenously determined or measure the same thing. This fact however, is valid only for bonds, which jump in 90% of the cases as a result of news releases.

In order to quantify the impact of announcement surprises on jumps, we estimate an extreme value (Gumbel) binary choice model in which we consider as determinants of the probability of jump occurrence the announcement surprise and the square root of the bipower variation estimate for the corresponding trading day, based on 5 minutes staggered returns. The inclusion of the volatility estimator has two major rationales. First, it is sensible to believe that if jumps occurred within one day, volatility might have increased as well. However, we consider here just the volatility coming from the continuous part of the price process and not the one including the jumps. Second, we believe that a volatility proxy might capture other unknown factors that could contribute to the price dynamics but which might be hard to identify and observe. In our analysis, we take into consideration all the days in our sample, independent of whether news were released or not on that day. The binary dependent variable is set to 1 if at least one jump occurred on a certain day and to 0 otherwise.

Table 4.7 includes part of the estimation output for these binary choice regressions. The choice of the extreme value distribution is based on the reported Akaike, Schwartz and Hannan-Quinn information criteria.

We observe that the surprise is significant at a 1% significance level for the 2, 5 and 10 -year bonds and at a 5% significance level for the less liquid 30 -year bond. Our proxy for voltatiliy is also found highly significant (1%) for the 2, 5 and 30- year bonds, while for the 10-year one we have significance only at a 5% significance level. In the same table we report results for the Hosmer-Lemeshow goodness-of-fit test for binary choice models, which compares values predicted by the model with the real values of the dependent

		Coefficient	p-value	Goodness of fit	
Y2	*C*	-2.53	0.0000	*H-L Statistic*	5.74
	Surprise	0.29	0.0021	*Prob. Chi-Sq(8)*	0.68
	Volatility (BV)	1881.19	0.0000		
Y5	*C*	-2.37	0.0000	*H-L Statistic*	4.36
	Surprise	0.40	0.0002	*Prob. Chi-Sq(8)*	0.82
	Volatility (BV)	432.54	0.0002		
Y10	*C*	-1.62	0.0000	*H-L Statistic*	8.28
	Surprise	0.30	0.0014	*Prob. Chi-Sq(8)*	0.41
	Volatility (BV)	107.26	0.0262		
Y30	*C*	-1.33	0.0000	*H-L Statistic*	11.91
	Surprise	0.18	0.0284	*Prob. Chi-Sq(8)*	0.16
	Volatility (BV)	88.96	0.0066		

Table 4.7: Results from regressing the probability of a jump on the announcement surprise and volatility

variable. Results in Table 4.7 suggest that differences between actual and predicted values are not significant, indicating a good fit.

4.3.4 Jumps and the impact of trading on prices

While the previous section shows that US Treasury bond prices react as a result of the release of public information, there is also a rich literature on price formation suggesting that the trading activity itself is a source of information for the market participants. When there are investors who detain private information, their trading activity might reveal some information to the market. This idea has been formalized within many theoretical and empirical models which describe the price formation process of the financial assets and which imply that transaction prices can be predicted from current and previous order flow information (Kyle, 1985; Glosten and Milgrom, 1985; Madhavan et al., 1997; Hasbrouck, 1991; Dufour and Engle, 2000, see).

In this section, we examine the informational role of the order flow and estimate the information asymmetry when jumps occur or around the jump times, in comparison with days when no jumps are detected. Just as in Green (2004), who examines the impact of trading on bond prices around news releases, we start from Madhavan et al. (1997)' model of price formation (denoted as MRR):

$$p_{t_i} - p_{t_{i-1}} = (\phi + \theta)x_{t_i} - (\phi + \rho\theta)x_{t_{i-1}} + e_{t_i}, \tag{4.15}$$

where t_i are the times when trades take place, $i = 1 \dots N$, with N the total number of trades, x_{t_i} is the order flow at time t_i, with $x_{t_i} = 1$ if the transaction is buyer initiated and $x_{t_i} = -1$ if the initiator was the seller, ϕ captures the compensation for providing liquidity, including all order processing costs, but also the effects of dealer inventories, ρ is the autocorrelation in the order flow, while θ measures the information asymmetry. The latter is the most important parameter in our analysis and assesses the impact of the surprise in the order flow $(x_{t_i} - \rho x_{t_{i-1}})$ on price changes.

In order to analyze how the parameters of the above model change in the presence of jumps, we transform equation 4.3.4, by adding several dummies, resulting in the following five models:

Model 1

$$\begin{aligned} p_{t_i} - p_{t_{i-1}} =& (\phi_J + \theta_J) I_{J,t_i} x_{t_i} - (\phi_J + \rho\theta_J) I_{J,t_i} x_{t_{i-1}} + (\phi_{NJ} + \theta_{NJ}) I_{NJ,t_i} x_{t_i} - \\ & (\phi_{NJ} + \rho\theta_{NJ}) I_{NJ,t_i} x_{t_{i-1}} + e_{t_i}, \end{aligned} \tag{4.16}$$

where the parameters are estimated separately for the days with jumps ($I_{J,t_i} = 1$) and for those without jumps ($I_{NJ,t_i} = 1$).

Model 2

$$\begin{aligned} p_{t_i} - p_{t_{i-1}} =& (\phi_{J0} + \theta_{J0}) I_{J,t_i} I_{J0,t_i} x_{t_i} - (\phi_{J0} + \rho\theta_{J0}) I_{J,t_i} I_{J0,t_i} x_{t_{i-1}} + (\phi_B + \theta_B) I_{J,t_i} I_{B,t_i} x_{t_i} - \\ & (\phi_B + \rho\theta_B) I_{J,t_i} I_{B,t_i} x_{t_{i-1}} + (\phi_A + \theta_A) I_{J,t_i} I_{A,t_i} x_{t_i} - (\phi_A + \rho\theta_A) I_{J,t_i} I_{A,t_i} x_{t_{i-1}} + \\ & (\phi_{NJ} + \theta_{NJ}) I_{NJ,t_i} x_{t_i} - (\phi_{NJ} + \rho\theta_{NJ}) I_{NJ,t_i} x_{t_i - 1} + e_{t_i}, \end{aligned} \tag{4.17}$$

where, for the days with jumps, we differentiate between the moment of the jump, J0 and the periods before (B) and after (A) the jump.

Model 3

$$\begin{aligned} p_{t_i} - p_{t_{i-1}} =& (\phi_{J0} + \theta_{J0}) I_{J,t_i} I_{J0,t_i} x_{t_i} - (\phi_{J0} + \rho\theta_{J0}) I_{J,t_i} I_{J0,t_i} x_{t_{i-1}} + \\ & (\phi_{B5} + \theta_{B5}) I_{J,t_i} I_{B5,t_i} x_{t_i} - (\phi_{B5} + \rho\theta_{B5}) I_{J,t_i} I_{B5,t_i} x_{t_{i-1}} + \\ & (\phi_{A5} + \theta_{A5}) I_{J,t_i} I_{A5,t_i} x_{t_i} - (\phi_{A5} + \rho\theta_{A5}) I_{J,t_i} I_{A5,t_i} x_{t_{i-1}} + \\ & (\phi_{other} + \theta_{other}) I_{other,t_i} x_{t_i} - (\phi_{other} + \rho\theta_{other}) I_{other,t_i} x_{t_{i-1}} + e_{t_i} \end{aligned} \tag{4.18}$$

where, for the days with jumps we consider a window of +/- 5 minutes around the jump and estimate parameters at the jump time (J0), for the 5 minutes that precede the jump (B5), for the 5 minutes after the jump (A5) and for the rest of the data (other).

Model 4

$$\begin{aligned} p_{t_i} - p_{t_{i-1}} = & (\phi_{J0} + \theta_{J0}) I_{J,t_i} I_{J0,t_i} x_{t_i} - (\phi_{J0} + \rho\theta_{J0}) I_{J,t_i} I_{J0,t_i} x_{t_{i-1}} + \\ & (\phi_{B10} + \theta_{B10}) I_{J,t_i} I_{B10,t_i} x_{t_i} - (\phi_{B10} + \rho\theta_{B10}) I_{J,t_i} I_{B10,t_i} x_{t_{i-1}} + \\ & (\phi_{A10} + \theta_{A10}) I_{J,t_i} I_{A10,t_i} x_{t_i} - (\phi_{A10} + \rho\theta_{A10}) I_{J,t_i} I_{A10,t_i} x_{t_{i-1}} + \\ & (\phi_{other} + \theta_{other}) I_{other,t_i} x_{t_i} - (\phi_{other} + \rho\theta_{other}) I_{other,t_i} x_{t_{i-1}} + e_{t_i}, \end{aligned} \tag{4.19}$$

just as model 3, but the window is of +/- 10 minutes around the jump time.

Model 5

$$\begin{aligned} p_{t_i} - p_{t_{i-1}} = & (\phi_{J0} + \theta_{J0}) I_{J,t_i} I_{J0,t_i} x_{t_i} - (\phi_{J0} + \rho\theta_{J0}) I_{J,t_i} I_{J0,t_i} x_{t_{i-1}} + \\ & (\phi_{B20} + \theta_{B20}) I_{J,t_i} I_{B20,t_i} x_{t_i} - (\phi_{B20} + \rho\theta_{B20}) I_{J,t_i} I_{B20,t_i} x_{t_{i-1}} + \\ & (\phi_{A20} + \theta_{A20}) I_{J,t_i} I_{A20,t_i} x_{t_i} - (\phi_{A20} + \rho\theta_{A20}) I_{J,t_i} I_{A20,t_i} x_{t_{i-1}} + \\ & (\phi_{other} + \theta_{other}) I_{other,t_i} x_{t_i} - (\phi_{other} + \rho\theta_{other}) I_{other,t_i} x_{t_{i-1}} + e_{t_i}, \end{aligned} \tag{4.20}$$

just as model 3, but the window is of +/- 20 minutes around the jump time.

To estimate the above models, we use all the transaction data available, without any previous sampling. Given that jumps are identified based on 5/ 15 minutes data, we cannot perfectly match the times of the jumps with the times of the trades. Thus the indicator function I_{J0,t_i} selects a window of +/- 2 minutes around the jump time. All the other indicator functions that select observations around the times of the jumps are adapted accordingly. For instance, I_{B10,t_i} selects all observations preceding with 12 to 2 minutes a jump time.

Just as Madhavan et al. (1997) and Green (2004), we use the Generalized Method of Moments to estimate the above equations. We exemplify here only the estimation of model 1, as the estimation for the others is very similar. Let $\beta = (\alpha, \rho, \phi_J, \theta_J, \phi_{NJ}, \theta_{NJ})$ be the vector of parameters to estimate for model 1, with α the intercept added to the model. In order to find the estimates for the components of this vector, the following moment conditions are used:

$$E\begin{bmatrix} x_{t_i}x_{t_{i-1}} - x_{t_i}^2\rho \\ e_{t_i} - \alpha \\ (e_{t_i} - \alpha)I_{J,t_i}x_{t_i} \\ (e_{t_i} - \alpha)I_{J,t_i}x_{t_{i-1}} \\ (e_{t_i} - \alpha)I_{NJ,t_i}x_{t_i} \\ (e_{t_i} - \alpha)I_{NJ,t_i}x_{t_{i-1}} \end{bmatrix} = 0 \tag{4.21}$$

Just as in Green (2004), our estimates are robust to ARCH-type heteroskedasticity. Results for the 2-, 5- and 10-year bonds are summarized in tables 4.8, 4.9 and 4.10. Results for the 30 year maturity are affected by the low liquidity that characterizes the data for this maturity. Consequently, we do not find them reliable and report them only in Appendix A, Table A.1. The estimated coefficients for this maturity behave, in terms of size, for all models, very much alike the estimates for the 2- and 5- year maturities. However, for the days with jumps, coefficients are usually not significant, probably due to the low number of observations used to estimate them.

As mentioned before, the most important parameters in the above equations are the θ-s, which represent the adverse selection parameters and consequently account also for the informational role of trading. This is why in our comments we will mostly focus on these parameters.

In general, if we look at results for Model 1 for all maturities, we observe that the estimates that account for information asymmetry tend to increase in size with the increase in maturity. Thus, for the 2-year bond, $\widehat{\theta}$ takes value .38 for days with jumps and .33 for days without jumps, for the 5-year bond the same estimated parameters are about .85 and .84, while for the 10-year bond the values are 1.32 and 1.36. This increase in the coefficients with the maturity is due to the fact that price changes tend to be higher for longer maturities, which is also consistent with the fact that jump sizes are bigger for higher maturities. Green (2004) also shows that announcements with greater price impacts also translate into higher information asymmetry.

	Model 1			
	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0035	0.0012	2.94	0.0032
ϕ_J	0.0310	0.0141	2.20	0.0275
θ_J	0.3759	0.0118	31.81	0.0000
ϕ_{NJ}	0.0456	0.0040	11.33	0.0000
θ_{NJ}	0.3258	0.0036	90.58	0.0000

	Model 2					Model 3			
	Coefficient	Std. Error	t-Statistic	Prob.		Coefficient	Std. Error	t-Statistic	Prob.
α	0.0034	0.0012	2.90	0.0037	α	0.0034	0.0012	2.88	0.0039
ϕ_{J0}	-0.8520	0.5397	-1.58	0.1144	ϕ_{J0}	-0.8421	0.5381	-1.57	0.1176
θ_{J0}	1.3779	0.4303	3.20	0.0014	θ_{J0}	1.5269	0.4259	3.59	0.0003
ϕ_B	0.0653	0.0246	2.66	0.0079	ϕ_{B5}	0.1910	0.1397	1.37	0.1716
θ_B	0.2858	0.0194	14.76	0.0000	θ_{B5}	0.3446	0.0605	5.70	0.0000
ϕ_A	0.0316	0.0125	2.53	0.0112	ϕ_{A5}	-0.1346	0.1262	-1.07	0.2860
θ_A	0.3595	0.0111	32.43	0.0000	θ_{A5}	0.6672	0.0944	7.07	0.0000
ϕ_{NJ}	0.0456	0.0040	11.33	0.0000	ϕ_{other}	0.0460	0.0038	12.19	0.0000
θ_{NJ}	0.3258	0.0036	90.58	0.0000	θ_{other}	0.3292	0.0034	97.70	0.0000

	Model 4					Model 5			
	Coefficient	Std. Error	t-Statistic	Prob.		Coefficient	Std. Error	t-Statistic	Prob.
α	0.0034	0.0012	2.87	0.0041	α	0.0034	0.0012	2.88	0.0039
ϕ_{J0}	-0.8421	0.5381	-1.56	0.1176	ϕ_{J0}	-0.8420	0.5403	-1.56	0.1192
θ_{J0}	1.5276	0.4259	3.59	0.0003	θ_{J0}	1.4193	0.4316	3.29	0.0010
ϕ_{B10}	0.1536	0.0797	1.93	0.0541	ϕ_{B20}	0.1235	0.0618	2.00	0.0455
θ_{B10}	0.2460	0.0476	5.16	0.0000	θ_{B20}	0.2660	0.0424	6.27	0.0000
ϕ_{A10}	-0.0834	0.0770	-1.08	0.2789	ϕ_{A20}	-0.0559	0.0472	-1.18	0.2364
θ_{A10}	0.5729	0.0614	9.33	0.0000	θ_{A20}	0.5064	0.0400	12.65	0.0000
ϕ_{other}	0.0465	0.0038	12.30	0.0000	ϕ_{other}	0.0473	0.0038	12.47	0.0000
θ_{other}	0.3279	0.0034	97.58	0.0000	θ_{other}	0.3262	0.0034	96.68	0.0000

Table 4.8: Estimated coefficients, their standard errors and t-tests for Models 1-5 for the 2-Year bond. Throughout all the models, we use a unique correlation coefficient for the order flow: $\widehat{\rho} = 0.6609$

Model 1

	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0044	0.0017	2.58	0.0098
ϕ_J	-0.1710	0.0216	-7.92	0.0000
θ_J	0.8575	0.0197	43.63	0.0000
ϕ_{NJ}	-0.1767	0.0056	-31.29	0.0000
θ_{NJ}	0.8454	0.0073	115.18	0.0000

Model 2

	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0044	0.0017	2.56	0.0106
ϕ_{J0}	-1.6179	0.9121	-1.77	0.0761
θ_{J0}	2.9582	0.8663	3.41	0.0006
ϕ_B	-0.1746	0.0421	-4.15	0.0000
θ_B	0.7612	0.0336	22.64	0.0000
ϕ_A	-0.1303	0.0208	-6.26	0.0000
θ_A	0.7883	0.0179	44.14	0.0000
ϕ_{NJ}	-0.1767	0.0056	-31.29	0.0000
θ_{NJ}	0.8454	0.0073	115.18	0.0000

Model 3

	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0044	0.0017	2.56	0.0105
ϕ_{J0}	-1.6712	0.9138	-1.83	0.0674
θ_{J0}	3.2768	0.8655	3.79	0.0002
ϕ_{B5}	-0.2157	0.1475	-1.46	0.1435
θ_{B5}	0.7726	0.1386	5.57	0.0000
ϕ_{A5}	-0.0168	0.2863	-0.06	0.9533
θ_{A5}	1.1441	0.1336	8.57	0.0000
ϕ_{other}	-0.1731	0.0053	-32.63	0.0000
θ_{other}	0.8410	0.0067	125.50	0.0000

Model 4

	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0043	0.0017	2.53	0.0113
ϕ_{J0}	-1.6698	0.9138	-1.83	0.0676
θ_{J0}	3.2773	0.8655	3.79	0.0002
ϕ_{B10}	-0.2633	0.1051	-2.51	0.0122
θ_{B10}	0.7290	0.1034	7.05	0.0000
ϕ_{A10}	-0.1813	0.1683	-1.08	0.2813
θ_{A10}	0.9441	0.0836	11.29	0.0000
ϕ_{other}	-0.1721	0.0053	-32.48	0.0000
θ_{other}	0.8404	0.0067	125.03	0.0000

Model 5

	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0043	0.0017	2.54	0.0112
ϕ_{J0}	-1.6072	0.9128	-1.76	0.0783
θ_{J0}	3.0325	0.8731	3.47	0.0005
ϕ_{B20}	-0.3145	0.1502	-2.09	0.0363
θ_{B20}	0.8049	0.1085	7.42	0.0000
ϕ_{A20}	-0.1205	0.1018	-1.18	0.2366
θ_{A20}	0.8408	0.0639	13.16	0.0000
ϕ_{other}	-0.1718	0.0053	-32.37	0.0000
θ_{other}	0.8397	0.0068	124.32	0.0000

Table 4.9: Estimated coefficients, their standard errors and t-tests for Models 1-5 for the 5-Year bond. Throughout all the models, we use a unique correlation coefficient for the order flow: $\widehat{\rho} = 0.6928$

Model 1

	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0034	0.0033	1.02	0.3101
ϕ_J	-0.1040	0.0430	-2.42	0.0155
θ_J	1.3216	0.0419	31.55	0.0000
ϕ_{NJ}	-0.2017	0.0119	-16.93	0.0000
θ_{NJ}	1.3647	0.0104	130.61	0.0000

Model 2

	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0035	0.0033	1.04	0.2988
ϕ_{J0}	-0.8359	1.7737	-0.47	0.6374
θ_{J0}	0.0004	1.5499	0.00	0.9998
ϕ_B	-0.0302	0.0803	-0.38	0.7073
θ_B	1.0373	0.0964	10.76	0.0000
ϕ_A	-0.0882	0.0418	-2.11	0.0347
θ_A	1.2630	0.0410	30.82	0.0000
ϕ_{NJ}	-0.2017	0.0119	-16.93	0.0000
θ_{NJ}	1.3647	0.0104	130.61	0.0000

Model 3

	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0034	0.0033	1.02	0.3061
ϕ_{J0}	-0.9096	1.7708	-0.51	0.6075
θ_{J0}	0.5169	1.5494	0.33	0.7387
ϕ_{B5}	0.5159	1.2391	0.42	0.6772
θ_{B5}	2.3117	0.9639	2.40	0.0165
ϕ_{A5}	0.1530	0.2963	0.52	0.6056
θ_{A5}	1.7152	0.2743	6.25	0.0000
ϕ_{other}	-0.1902	0.0112	-16.92	0.0000
θ_{other}	1.3566	0.0099	136.74	0.0000

Model 4

	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0034	0.0033	1.03	0.3049
ϕ_{J0}	-0.9097	1.7711	-0.51	0.6075
θ_{J0}	0.5198	1.5496	0.34	0.7373
ϕ_{B10}	0.2787	0.4956	0.56	0.5739
θ_{B10}	1.5666	0.6596	2.37	0.0176
ϕ_{A10}	-0.0195	0.2731	-0.07	0.9430
θ_{A10}	1.5090	0.2892	5.22	0.0000
ϕ_{other}	-0.1909	0.0112	-17.05	0.0000
θ_{other}	1.3562	0.0098	137.85	0.0000

Model 5

	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0036	0.0033	1.07	0.2864
ϕ_{J0}	-0.8725	1.7801	-0.49	0.6240
θ_{J0}	-0.0529	1.5444	-0.03	0.9727
ϕ_{B20}	-0.3082	0.4060	-0.76	0.4478
θ_{B20}	1.9321	0.5167	3.74	0.0002
ϕ_{A20}	0.1541	0.1648	0.94	0.3498
θ_{A20}	1.3630	0.1551	8.79	0.0000
ϕ_{other}	-0.1951	0.0115	-17.00	0.0000
θ_{other}	1.3591	0.0103	132.20	0.0000

Table 4.10: Estimated coefficients, their standard errors and t-tests for Models 1-5 for the 10-Year bond. Throughout all the models, we use a unique correlation coefficient for the order flow: $\widehat{\rho} = 0.6725$

This model estimates coefficients separately for days with jumps and days without jumps. The results in Table 4.8 on page 114, Table 4.9 on page 115 and Table 4.10 on the preceding page indicate that coefficient estimates do not vary much between days with jumps and days without jumps. For the 2- and 5-year maturities, estimates for θ_J are bigger than those for θ_{NJ}, while for the 10-year maturity, the results are reversed.

Results from Model 1 confer us limited information concerning what happens when jumps take place. In order to gain further insights on how information asymmetry changes around the jumps, we need to examine a more narrow window around the jump occurrence. In Model 2, we separately estimate coefficients for the 'jump window', which is -/+ 2 minutes around the jump time. Moreover, we split the days with jumps in intervals that precede jumps and periods that follow them. Evidence for the 2- and 5- year maturities indicate that information asymmetry raises up when jump occurs ($\widehat{\theta_{J0}} = 1.38$ for the 2-year bond and $\widehat{\theta_{J0}} = 2.96$ for the 5-year bond) and descends to lower levels for before and after the jumps and for days with no jumps. For both maturities, θ takes higher values after the jump than before. For the 2-year maturity, we have $\widehat{\theta_B} = 0.29$ and $\widehat{\theta_B} = 0.36$, while for the 5 -year maturity we have $\widehat{\theta_B} = 0.76$ and $\widehat{\theta_B} = 0.79$. This is consistent with findings in Green (2004), who shows that after macroeconomic news are released, information asymmetry in the US Treasury market does not decline immediately, but with a lag of 15 minutes following an announcement. In addition, as indicated by Green (2004) the lower level of information asymmetry before a jump occurrence might be indicative of no information leakage before an announcement.

For the 10-year maturity, evidence for Model 2 is in part contradictory with the findings for the first two maturities. The coefficient that captures information asymmetry for the 'jump window' is not significant at 1% or 5% significance levels, with p-value of .99 (see Table 4.10 on the previous page). What is consistent with the other findings is that the estimates of θ are higher after the jump than before. We believed this might be because the 'jump window' (-/+ 2 minutes around the jump time) we used was too narrow. Consequently, we extend this window to -/+ 5 minutes around the jump time and widen all the other windows accordingly. Estimates of all the models for the 10-year maturity are reported in Table A.2 in Appendix A. If we look at results for Model 2, we observe that the estimate of θ_{J0} increases to 1.14 and is significant at a 5% significance level, but remains lower than $\widehat{\theta_A}$ and $\widehat{\theta_{NJ}}$.

Further on, in Models 3- 5, we narrow our analysis to those intervals of time that are very close to the time of the jump. Thus, we maintain the 'jump window' of -/+ 2 minutes around identified time of the jump, and we further select wider windows before and after the jump. Thus, Model 3 captures the informativeness of the order flow 5 minutes before and after the jump, Model 4 takes windows of 10 minutes before and after, while Model 5 considers windows of 20 minutes. For the 2- and 5 -year maturities, we notice again very similar behaviors. Thus, for all the three models, we observe that $\widehat{\theta}$ are very high at the jump time, they are quite low before the jump and remain at higher levels after the jump. For the 2-year bond, for instance, $\widehat{\theta_{J0}} = 1.52$ for all the three models, $\widehat{\theta_B}$ is quite low, varying from 0.12 for Model 5 to .34 for Model 3, while $\widehat{\theta_A}$ is the highest for Model 3 and then it diminishes as the window around the jump is enlarged, but remains however, above the estimator for all the other observation, $\widehat{\theta_{other}}$. For the 2-year maturity, $\widehat{\theta_A}$ is .66 for Model 3, .57 for Model 4 and .51 for Model 5, while for the 5- year maturity the estimate decreases from 1.44 for Model 3, to .94 for Model 4 and then to .84 for Model 5. Thus, evidence on these two maturities shows that the occurrence of jump, as a result of new information arriving on the market generates a lot of information asymmetry, which persists but at lower levels as we get away from the jump time.

For the 10-year bond, results for Models 3-5 are reversed from the ones for the first two maturities, but also from the ones for Model 2 for the same maturity. θ_{J0} is again insignificant, while if we look at the estimated coefficients for 5, 10 or 20 minutes before and after the jump occurrence, we observe that those estimated for the observations that precede the jump are higher. This contradicts what we expect based on results for the other maturities and also based on findings in Green (2004). However, Green (2004) includes in his analysis only 5-year bonds and public announcements, not jumps.

The order processing cost parameter, ϕ , captures dealers' compensation for providing liquidity and theory suggests it should be positive. However, our results are mixed. For the 5-year bond, estimates are negative for all the models, just as in Green (2004). For the other maturities, coefficients are sometimes positive and sometimes negative. A $\phi < 0$ indicates that dealers consume liquidity in the interdealer market and thus exhibit a sub-optimal behavior, which Green (2004) suggests it might be due to the fact that they are sufficiently compensated in the retail market.

If we look at all maturities and all models, we observe that the $\widehat{\phi}$-s are not significant not even at a 5% significance levels, for the jump windows, as well as for the windows that precede or follow jumps. Unlike Green (2004), who finds that $\widehat{\phi}$ is higher before an announcement takes place than after, we find mixed evidence when comparing the ϕ estimates before and after the jump. $\widehat{\phi}$ before the jump is consistently higher than the estimate after the jump for the 2- and 10- year maturities, but the situation is reversed for the 5-year bond.

The above estimation procedure assumes and computes a constant correlation of the order flow throughout the sample, which is reported within the caption for each table. In Table 4.11 on the following page we report the order flow autocorrelation coefficients for groups of observations formed on the basis of the indicator functions from Models 1 - 5. When such data groups are considered, we notice some variations in the correlation coefficients between the different sets of observations. Results for Model 1 for the 2-, 5- and 10- year maturities indicate that order flow seems to be more autocorrelated in days with jumps than in days without jumps. If we split the days with jumps in before and after intervals, as in Models 2 - 5, we notice that in all cases autocorrelation within the 'jump window' is lower than before and af-

		2-Year	*5-Year*	*10-Year*	*30-Year*
Model 1	$I_{J,t}$	0.671	0.711	0.689	0.428
	$I_{NJ,t}$	0.659	0.690	0.671	0.446
Model2	$I_{J,t}I_{J0,t}$	0.638	0.731	0.725	0.680
	$I_{J,t}I_{B,t}$	0.670	0.702	0.690	0.461
	$I_{J,t}I_{A,t}$	0.673	0.717	0.692	0.416
	$I_{NJ,t}$	0.659	0.690	0.671	0.446
Model 3	$I_{J,t}I_{J0,t}$	0.638	0.731	0.725	0.680
	$I_{J,t}I_{B5,t}$	0.656	0.740	0.770	0.438
	$I_{J,t}I_{A5,t}$	0.716	0.800	0.795	0.312
	$I_{other,t}$	0.661	0.692	0.672	0.443
Model 4	$I_{J,t}I_{J0,t}$	0.638	0.731	0.725	0.680
	$I_{J,t}I_{B10,t}$	0.677	0.741	0.745	0.433
	$I_{J,t}I_{A10,t}$	0.725	0.796	0.794	0.374
	$I_{other,t}$	0.660	0.692	0.671	0.443
Model 5	$I_{J,t}I_{J0,t}$	0.638	0.731	0.725	0.680
	$I_{J,t}I_{B20,t}$	0.670	0.721	0.711	0.386
	$I_{J,t}I_{A20,t}$	0.713	0.790	0.778	0.421
	$I_{other,t}$	0.660	0.691	0.671	0.443

Table 4.11: Autocorrelation coefficients of the signed order flow. Different coefficients are computed for different groups of observation. The grouping criteria is given in column 2 by the indicator functions

ter the jump, and is highest after the jump. This is consistent with the fact that once relevant information arrives on the market, the trading activity explodes, with traders interpreting news based on the observed order flow (Green, 2004). This is why high levels of autocorrelation are also associated with high levels of information asymmetry.

Given the differences in the autocorrelation of the order flow between different time windows considered, we wondered whether this could affect the results of the estimations of Models 1 - 5. Consequently, for the 2- year bond, we re-estimated all the models, by considering varying autocorrelation coefficients, as reported in Table 4.11. The estimation output is included in Table A.3 on page 131 in Appendix A. The main consequence of considering different correlation coefficients is that for the 'jump window', as a result of a lower autocorrelation in the order flow, the information asymmetry parameter decreases as well. For instance, for Model 2, $\widehat{\theta_{J0}}$ was 1.37 when we used a unique autocorrelation coefficient in the estimation, which decreases to 1.29 when different correlation coefficients are used. Apart from this, within each model, the hierarchy of the coefficients in terms of size does not change. Thus, the θ coefficients are the highest for the 'jump window' and jump days and are higher after a jump takes place than before its occurrence. For brevity, we do not report results based on varying correlation coefficients for the other maturities here.

4.4 Conclusions

The present chapter attempts to clarify different aspects concerning the behavior of some market characteristics when jumps occur in the US Treasury market, as well as to identify causes and possible predictors of jumps.

We detect and estimate jumps in the US Treasury 2-,5-,10- and 30-year bonds both on a daily basis, by using the standard Barndorff-Nielsen and Shephard (2004) test for jumps, as well as at an intraday level, by applying the Andersen et al. (2007)-Lee and Mykland (2008) procedure with a correction for periodicity to our local volatility estimates proposed by Boudt et al. (2009). We find that the 2-year bonds jump in 14.5% of the days, the 5-year in 10.6%, the 10-year in 9.6% and finally the 30-year in 17.91% of the days. Moreover, when taken two by two, bonds tend to co-jump in approximately 20 days. This is because bonds usually experience jumps in their prices as a result of the same news announcements. However, when all 4 maturities are considered, we find only 5 overall co-jumps.

We also examine the behavior of several market activities in the proximity of a jump. We consider here liquidity measures, such as spread and depth of the market, as well as the trading volume and the orderflow . We find that in the 5 minutes before a jump, liquidity withdraws, and then it raises again. However, we do not consider this decrease in liquidity as predictive of jumps, but as another measure of the same thing. In other words, we consider them endogenous, and having both other determinants, such as announcement surprises. The trading volume falls before in the 5-10 minutes before a jump takes place and increases again afterward. For the orderflow, which we expect to have some predictive power for the next jumps, we cannot find a clear pattern in all cases.

When we consider the different news releases during January 2003 to March 2004, we find that the majority (around 90%) of the jumps occur at the same time or slightly after an announcement. Moreover, the standardized announcement surprise is found to be an important determinant of the probability of jump occurrence.

Finally, we dedicate a very large part of this chapter to examine the impact of trading on bond prices in the nearness of jumps. We find that for the 2- and 5- year maturities, the level of information asymmetry increases immediately after jumps occur, due to the arrival of new information on the market, and then remains at a high level up to 20 minutes after the jump. Before a jump takes place, there is a low degree of informational asymmetry, consistent with a low extent of information leakage. For the 10-year bond, results are a bit contradictory with the ones for shorter maturities, as we detect a higher level of information asymmetry before than after the jump. However, this parameter remains always higher for the windows around the jump times than when no jumps occur.

5

Conclusions and further developments

The present thesis enriches the existing literature in the field of high frequency econometrics through contributions in the area of jump identification and estimation from both a methodological and applied point of view. From a methodological point of view, the thesis presents various volatility estimators and jump detection procedures and offers viable solutions as to how users should identify jumps in the prices of financial assets. From an applied point of view, the present research reports very interesting novel results concerning the price formation process for the US Treasury bonds when jumps occur on the market. The thesis is made up of three independent essays.

The first essay, entitled “The use of high frequency data in estimating volatility and detecting jumps in the prices of financial assets”, confers rationales concerning the importance of taking jumps into consideration in the financial literature and reviews the latest nonparametric volatility estimators based on high frequency data. This chapter emphasizes methodologies that contribute to jump detection and estimation. Thus, the literature review covers both robust and non-robust to jumps estimators, as well as various jump detection procedures recently proposed in the literature. Both the univariate and the mutivariate frameworks are considered. Moreover, given that real prices are always contaminated with microstructure noise, we also describe the solutions in the existing literature concerning volatility estimation in the presence of noise.

The second essay, entitled “Identifying jumps in financial assets: a comparison between nonparametric jump tests”, comprises a thorough comparison among five jump identification procedures proposed in the literature of high frequency econometrics over the last decade: the Andersen et al. (2007)-Lee and Mykland (2008), the Aït-Sahalia and Jacod (2008), the Barndorff-Nielsen and Shephard (2006a), the Jiang and Oomen (2008), and the Podolskij and Ziggel (2008) tests. To this end, we simulate different price processes, closely following the simulation design in Huang and Tauchen (2005), and evaluate the size and power properties for all procedures. One of the main objectives of this research is to understand whether the performance of the tests can be related to some characteristics of the data. Thus, within the simulation design, we vary the sampling frequencies, the levels of volatility, the persistence in volatility, the degree of contamination with microstructure noise, the jump size and intensity. The analysis is also extended to real high frequency data on US Treasury bonds, in order to compare the behavior of the tests in this context. Results reveal the Lee and Mykland (2008) and Andersen et al. (2007) intraday tests, as well as the Podolskij and Ziggel (2008) one as the best procedures, as they combine good power properties with a manageable size and more robustness to microstructure noise. We provide a comprehensive table that summarizes the results, but mainly helps users in choosing one of these tests according to some particular characteristics of the data. Another contribution of this chapter deals with the problem of jump identification in the presence of finite samples and microstructure noise. In this respect, we show that potential users of these procedures can gain advantages by combining them through both reunions and intersections across procedures and across sampling frequencies. Such an approach delivers better results in terms of the combined size and power criteria.

The third essay, entitled “Jumps and price discovery in the US Treasury market”, explores different aspects related to the price discovery process for the US Treasury bonds when jumps occur. We identify and estimate jumps in the US Treasury 2-,5-,10- and 30-year bonds both on a daily basis, by using the standard Barndorff-Nielsen and Shephard (2004) test for jumps, as well as at an intraday level, by applying the Lee and Mykland (2008) procedure with a correction for periodicity to our local volatility estimates proposed by Boudt et al. (2009). Results show that US Treasury bonds exhibit jumps in their prices in 14.5% of the days for the 2-year maturity, in 10.6% for the 5-year bond, 9.6% for the 10-year and finally in 17.91% of the days for the 30-year bond. Then, the behaviour of different market characteristics, such as liquidity measures, trading volume and order flow, when jumps occur is examined. We find that in the 5 minutes before a jump, liquidity withdraws, and then it raises again. The trading volume falls in the 5-10 minutes before a jump takes place and increases again afterward. In addition, we examine the causes of jumps in the term structure and find that around 90% of the jumps occur at the same time or slightly after macroeconomic announcements. Moreover, the standardized announcement surprise is revealed to be an important determinant of the probability of jump occurrence. Finally, we dedicate a large part of this chapter for exploring the impact of the trading activity on prices in the proximity of jumps. The interesting results can be summarized as follows. The trading information is found to have a low impact on prices before a jump takes place, which increases to a very high level immediately after the jump and then dissipates gradually, maintaining itself at quite high levels up to 20 minutes after a jump occurs.

Further research We plan to broaden the research undertaken in this thesis in two different directions. The first direction closely follows the work in the third chapter of the thesis. On one hand, we plan to extend the analysis on US Treasury bonds to a larger database. We intend to use data on the 2-, 5-, 10, 30- year US Treasury bonds for a period between 2003 and 2008. This might reveal additional highlights on the microstructure of the market when jumps take place and might allow for the set-up of trading strategies. On the other hand, we would like to perform an analysis similar to the one in Chapter 4 on stock data. Such an endeavor is of great importance as different markets have different microstructure particularities and react very differently to various types of information.

The second direction concerns the construction of a robust to jumps nonparametric covariance estimator. Transferring the different nonparametric volatility estimators from a univariate to a multivariate context is not straightforward. This because, for any two financial assets, observations are not synchronous, that is they are not observed at the very same time, which can lead to an underestimated covariance between the two (Epps effect). We are interested in defining a covariance estimator that overcomes the problem of nonsynchronicity and that is also robust to jumps. We plan to develop the limit theory for this estimator, as well as to study its finite sample properties and its behaviour when applied to real data. Disentangling from the overall covariance the part generated by jumps leaves us with the persistent part that can be modelled and forecast. This is extremely valuable for portfolio allocation, risk management and hedging.

Appendices

A

More results on the price formation process when jumps occur

Model 1	Coefficient	Std. Error	t-Statistic	Prob.
α	-0.0257	0.1051	-0.24	0.8069
ϕ_J	-0.7431	0.5968	-1.25	0.2131
θ_J	3.8223	0.5850	6.53	0.0000
ϕ_{NJ}	-1.1816	0.2124	-5.56	0.0000
θ_{NJ}	3.4496	0.2053	16.80	0.0000

Model 2	Coefficient	Std. Error	t-Statistic	Prob.
α	-0.0237	0.1049	-0.23	0.8215
ϕ_{J0}	-24.9491	17.4855	-1.43	0.1536
θ_{J0}	16.9362	13.7566	1.23	0.2183
ϕ_B	0.6611	1.1087	0.60	0.5510
θ_B	2.9124	1.0484	2.78	0.0055
ϕ_A	-0.8181	0.7015	-1.17	0.2436
θ_A	3.7285	0.6788	5.49	0.0000
ϕ_{NJ}	-1.1817	0.2124	-5.56	0.0000
θ_{NJ}	3.4497	0.2053	16.80	0.0000

Model 3	Coefficient	Std. Error	t-Statistic	Prob.
α	-0.0273	0.1045	-0.26	0.7938
ϕ_{J0}	-24.9511	17.7134	-1.41	0.1590
θ_{J0}	19.1459	13.8811	1.38	0.1678
ϕ_{B5}	-0.2331	6.8353	-0.03	0.9728
θ_{B5}	6.5974	2.3795	2.77	0.0056
ϕ_{A5}	-4.7729	15.2462	-0.31	0.7542
θ_{A5}	23.9930	15.0539	1.59	0.1110
ϕ_{other}	-1.0746	0.1983	-5.42	0.0000
θ_{other}	3.4392	0.1918	17.93	0.0000

Model 4	Coefficient	Std. Error	t-Statistic	Prob.
α	-0.0239	0.1044	-0.23	0.8190
ϕ_{J0}	-24.9523	17.7136	-1.41	0.1589
θ_{J0}	19.1466	13.8809	1.38	0.1678
ϕ_{B10}	2.6739	5.4653	0.49	0.6247
θ_{B10}	6.1792	4.3859	1.41	0.1589
ϕ_{A10}	-6.7657	7.8044	-0.87	0.3860
θ_{A10}	14.4657	7.3015	1.98	0.0476
ϕ_{other}	-1.0610	0.1986	-5.34	0.0000
θ_{other}	3.4417	0.1917	17.96	0.0000

Model 5	Coefficient	Std. Error	t-Statistic	Prob.
α	-0.0282	0.1045	-0.27	0.7875
ϕ_{J0}	-24.0574	17.2641	-1.39	0.1635
θ_{J0}	14.9826	13.0245	1.15	0.2500
ϕ_{B20}	3.0304	3.7612	0.81	0.4204
θ_{B20}	5.3797	2.9308	1.84	0.0664
ϕ_{A20}	-5.7990	4.7319	-1.23	0.2204
θ_{A20}	8.6947	4.4831	1.94	0.0525
ϕ_{other}	-1.0608	0.1986	-5.34	0.0000
θ_{other}	3.4414	0.1917	17.96	0.0000

Table A.1: Estimated coefficients, their standard errors and t-tests for Models 1-5 for the 30-Year bond. Throughout all the models, we use a unique correlation coefficient for the order flow: $\widehat{\rho} = 0.4430$

Model 2

	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0035	0.0033	1.04	0.2988
ϕ_{J0}	-0.1817	0.7828	-0.23	0.8164
θ_{J0}	1.1431	0.5225	2.19	0.0287
ϕ_{B}	-0.0606	0.0870	-0.70	0.4865
θ_{B}	1.0102	0.0911	11.09	0.0000
ϕ_{A}	-0.0927	0.0413	-2.25	0.0247
θ_{A}	1.2527	0.0402	31.15	0.0000
ϕ_{NJ}	-0.2017	0.0119	-16.93	0.0000
θ_{NJ}	1.3647	0.0104	130.61	0.0000

Model 3

	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0034	0.0033	1.02	0.3076
ϕ_{J0}	-0.1950	0.7817	-0.25	0.8030
θ_{J0}	1.6079	0.5288	3.04	0.0024
ϕ_{B5}	-0.6672	0.9869	-0.68	0.4990
θ_{B5}	2.1253	1.1136	1.91	0.0563
ϕ_{A5}	0.0205	0.3248	0.06	0.9497
θ_{A5}	1.3736	0.3089	4.45	0.0000
ϕ_{other}	-0.1906	0.0112	-17.04	0.0000
θ_{other}	1.3562	0.0098	137.98	0.0000

Model 4

	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0035	0.0033	1.05	0.2914
ϕ_{J0}	-0.1597	0.7964	-0.20	0.8411
θ_{J0}	1.2609	0.5186	2.43	0.0151
ϕ_{B10}	-0.3350	0.7356	-0.46	0.6488
θ_{B10}	1.7748	0.6770	2.62	0.0088
ϕ_{A10}	0.0997	0.2567	0.39	0.6978
θ_{A10}	1.3958	0.2513	5.55	0.0000
ϕ_{other}	-0.1928	0.0112	-17.25	0.0000
θ_{other}	1.3567	0.0098	137.98	0.0000

Model 5

	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0036	0.0033	1.07	0.2852
ϕ_{J0}	-0.1475	0.8011	-0.18	0.8540
θ_{J0}	1.0923	0.5119	2.13	0.0328
ϕ_{B20}	-0.6106	0.4776	-1.28	0.2010
θ_{B20}	1.8977	0.5145	3.69	0.0002
ϕ_{A20}	0.1240	0.1614	0.77	0.4422
θ_{A20}	1.3062	0.1535	8.51	0.0000
ϕ_{other}	-0.1934	0.0112	-17.22	0.0000
θ_{other}	1.3570	0.0099	137.44	0.0000

Table A.2: Estimated coefficients, their standard errors and t-tests for Models 2-5 for the 10-Year bond, for a 'jump window' of -/+ 5 minutes. The other time windows around the jump time are adjusted accordingly to the 'jump window'. For instance, for Model 3, which considers a -/+ 5 minutes window around the jumps, the real window is of -/+ 10 minutes around the jump time, as identified in Section 4.2.2 on page 97. Throughout all the models, we use a unique correlation coefficient for the order flow: $\widehat{\rho} = 0.6609$.

Model 1	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0035	0.0012	2.94	0.0032
ϕ_J	0.0189	0.0143	1.32	0.1872
θ_J	0.3880	0.0122	31.81	0.0000
ϕ_{NJ}	0.0478	0.0040	11.91	0.0000
θ_{NJ}	0.3237	0.0036	90.58	0.0000

Model 2	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0034	0.0012	2.90	0.0037
ϕ_{J0}	-0.7647	0.5206	-1.47	0.1419
θ_{J0}	1.2906	0.4030	3.20	0.0014
ϕ_B	0.0576	0.0249	2.31	0.0209
θ_B	0.2935	0.0199	14.76	0.0000
ϕ_A	0.0178	0.0127	1.39	0.1635
θ_A	0.3734	0.0115	32.43	0.0000
ϕ_{NJ}	0.0478	0.0040	11.91	0.0000
θ_{NJ}	0.3236	0.0036	90.58	0.0000

Model 3	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0034	0.0012	2.88	0.0039
ϕ_{J0}	-0.7454	0.5193	-1.44	0.1512
θ_{J0}	1.4301	0.3989	3.59	0.0003
ϕ_{B5}	0.1959	0.1394	1.40	0.1600
θ_{B5}	0.3398	0.0596	5.70	0.0000
ϕ_{A5}	-0.2651	0.1397	-1.90	0.0577
θ_{A5}	0.7977	0.1128	7.07	0.0000
ϕ_{other}	0.0463	0.0038	12.26	0.0000
θ_{other}	0.3290	0.0034	97.70	0.0000

Model 4	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0034	0.0012	2.87	0.0041
ϕ_{J0}	-0.7453	0.5193	-1.44	0.1512
θ_{J0}	1.4308	0.3989	3.59	0.0003
ϕ_{B10}	0.1415	0.0809	1.75	0.0802
θ_{B10}	0.2581	0.0500	5.16	0.0000
ϕ_{A10}	-0.2156	0.0871	-2.48	0.0133
θ_{A10}	0.7051	0.0756	9.33	0.0000
ϕ_{other}	0.0472	0.0038	12.50	0.0000
θ_{other}	0.3273	0.0034	97.58	0.0000

Model 5	Coefficient	Std. Error	t-Statistic	Prob.
α	0.0034	0.0012	2.88	0.0039
ϕ_{J0}	-0.7520	0.5212	-1.44	0.1491
θ_{J0}	1.3293	0.4043	3.29	0.0010
ϕ_{B20}	0.1160	0.0624	1.86	0.0629
θ_{B20}	0.2735	0.0436	6.27	0.0000
ϕ_{A20}	-0.1471	0.0523	-2.81	0.0049
θ_{A20}	0.5976	0.0472	12.65	0.0000
ϕ_{other}	0.0483	0.0038	12.74	0.0000
θ_{other}	0.3252	0.0034	96.68	0.0000

Table A.3: Estimated coefficients, their standard errors and t-tests for Models 1-5 for the 2-Year bond. Throughout all the models, we use different values for the autocorrelation coefficient for the order flow, as resulting from Table 4.11 on page 120

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