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# LARGE PLANETARY BODIES IMPACTS: A FIRST ORDER ANALYSIS

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## Abstract

We review situations where large scale impacts between bodies of planetary sizes may arise, or have been invoked to explain structures that seem to have arisen after such impacts. We are interested in the orbital variations for the special case the impacting object is in some sense absorbed by the impacted body, whose mass is larger. Under rather strong simplifications we calculate such variations using the condition of conservation of either the total energy or the total momentum and Kepler's third equation. Results are discussed in terms also of such events having possibly taken place in the solar system.

## 0.1 Introduction

Impacts between bodies in the galaxy and in solar system are events that certainly take place, albeit they are generally rare. Several orders more frequent are close passages of such bodies. However till recent times most astronomers have disregarded such events,

believing them so rare to be of practically no interest. This is one of the reasons why Immanuel Velikovsky book *Worlds in collision*, published in 1951, was so severely attacked by the astronomers community, who even threatened his publisher Doubleday that they would stop publishing with it if they continued producing Velikovsky book. Doubleday rescinded the contract, despite the book had an immense success and was for some months a best seller. Notice that Velikovsky did not deal with collisions, the title is misleading and was so chosen for editorial reasons, but only with close passages, within a context where planet Venus was claimed to be a recent offspring of Jupiter and present orbits of Venus and Mars were also claimed to have been attained rather recently, in the first half of the first millennium BC.

Evidence that collisions are necessary to explain features in the solar system has later accumulated and Velikovsky now would not be considered heretical, albeit his seminal role is usually neglected by the astronomers. He is still subject to ridiculous accusations by scholars who obviously have never read him, including Italian mathematician Piergiorgio Odifreddi, involved in a campaign against traditions and religion in name of a positivistic atheism; he wrote in one of his books that Velikovsky had presented a game of planets hitting themselves like birilli (skittles).

Need of collisions has appeared, inter alia, in the following astronomical situations:

- the standard theory for planetary formation has a vast cloud of gas and dust generating by aggregation, via impacts of initially gas and dust particles, larger and larger bodies; eventually impacts take place between bodies of diameter up to say hundred km. This theory, mainly developed by George Whetherill of Smithsonian Institute in the seventies and eighties, leads to a final planetary structure where up to half billion km from Sun only planets of Earth type are found, while the gas giant planets would be found farther away. The theory was considered for several years to be perfectly sound and satisfying.

However it underwent a deep crisis when the first extrasolar planets were been found, presently over two hundred of them being known. Such planets are mostly giant bodies of Jovian or even larger size (up to 8 times Jupiter mass). Many of them are very close to the central star, one at least revolving at about one million km from it. This discovery has shown that Whetherill scenario is partial and incomplete. Reasons why large Jovian type planets could revolve so close and not fall into the star have been provided by Del Popolo (1,2). Clearly the universe is more complex and surprising than astronomers believed, so that all theoretical scenarios, based upon some assumptions and simplifications, have only a limited range

- the discovery that Mercury has the most intense magnetic field among solar system bodies and the largest mass density as well is a strong argument to believe that it is not a proper planet, but the core of another planet. It is presently believed by most astronomers that an ancient previous Mercury was impacted, say one billion or more years ago, by a rather large body, which destroyed and expelled its mantle and crust, leaving the nucleus. Another theory by Ackerman (3,4), a physicist who has given important contribution to clarifying the background of Velikovsky scenario, is that Mercury was originally the core of Mars, lost via the Valles Marineris opening (about 5000 km long, Mercury size is about 4300 km), that opened after some 100 close passages of Mars near Earth. Here it is not the case to discuss this event longer, which is done in Ackerman on the basis also of the information provided by Rig Veda, and that will be discussed in a forthcoming monograph of Spedicato. We just state that the loss of Mars core took place circa 3160 BC, being the essential cause of the second of the great Platonic catastrophes, i.e. the Noachian flood. Intriguing information about this event is possibly given by Plutarch in his book *De facie quae in lunae orbe apparet*, where Mercury is said to have stolen one seventieth of Moon light and have given it to Earth whose year length passed from 360 to 365 days...

- the discovery, when lunar rocks were first analyzed, that Moon rock composition is incompatible with Earth's, due to different radionuclides ratios. So Moon must come from outside. Presently the most accepted theory is that long ago, say one billion or more years ago, a body of Martian size impacted tangentially Earth. Then Moon formed from condensation of the material expelled after the impact, consisting partly of Earth and mainly of the impacting body substance. However there are alternative theories, less destructive of Earth, where material from the impacted body should still be present on her crust, but, as far as known to us, has not been found. One is the capture of the Moon, supposed to be a solitary body passing near Earth. This phenomenon, studied in particular by Japanese in the seventies, see (5,6), requires however a large dense atmosphere to brake Moon, since pure capture is known to be impossible in a three body gravitational system. Such an atmosphere might have existed at the beginning of Earth history. More interesting is however the capture in a four body system, where e.g. the satellite of an object passing near Earth is captured by our planet. This event has no dynamical difficulty, only requires suitable conditions to hold at the moment of passage. Such process is being presently investigated by Petruzzi and Spedicato (7). Several traditions suggest that the event took place within memory of man, probably just at the end of the last Ice Age, i.e. during the first Platonic catastrophe, when the Atlantis civilization was destroyed. It is also possible that the event also removed a previous Earth satellite, located at circa 2 million km from a possible interpretation of a passage in Censorinus De die natali, which later became Mars

- impacts between bodies of which one is very small compared to the other are also of interest, this being the case when a planet like Earth is subject to impacts by comets, asteroids, Apollo objects..., whose size varies from some meters to order ten km. A single impact changes Earth radius by order one meter only, the cumulative effect of million

impacts since Earth formed could change the radius by a few km only, as analysis by Brunini (8) has shown.

In this paper we will consider the orbital changes due to impacts between bodies of different relative sizes, but in the range of the planetary sizes in solar system. Our analysis is admittedly rough, since a full analysis is beyond complete modelling and would take several men-years to be performed. The results are however quite interesting, especially since in many impact scenarios one never considers orbital changes. Of course also rotational changes should be evaluated, which further complicate the model. But see Spedicato (9) for a coupled revolution-rotation change after an event, the Earth axis inversion, that can be produced either by impacts, see Barbiero (10) or by the torque effects of a close passage of a sufficiently large body, see Baltensperger and Woelfli (11).

## **0.2 Orbital changes after large impacts in a planetary context**

We will consider in this paper the following three cases of impacts:

1. planet Jupiter impacted by another body of mass  $m$ , for different values of  $m$  relative to the mass of Jupiter
2. a planet P in the asteroid belt position with a certain default mass, impacted by another body of mass  $m$ , for different values of  $m$  relative to the mass of the planet P
3. planet Earth impacted by another body of mass  $m$ , for different values of  $m$  relative to the mass of Earth.

We consider the above three cases for the following reasons:

1a - according to the theory of Ackerman, which we consider the best one existing that is able to explain several problems in the solar system and is worth of greater investigation, Jupiter was impacted by a planetary size body of unknown mass at a time falling within human memory and which has left a memory especially in the Rig Veda (notice that Rig Veda is probably the oldest document of mankind, having formed in the 4th or 5th millennium BC, as is now claimed by many scholars, see Subhash Kak et al (12); the claim that it was much more recent was made in the 19th century by British scholars who wanted to degrade antiquity and quality of India's civilization, for reasons related to British domination of India). The impact took place where is now the so called red spot, which Ackerman claims to be an immense crater wherefrom gases still pour out. The material emitted by the crater partly went in orbit around Jupiter and later collapsed into the four Galilean satellites (that large bodies can form in a century or so has been recently found by a more sophisticated study of their formation, see (13); moreover this collapse exactly corresponds to the Moon formation after a large tangential impact on Earth as is claimed in the presently most accepted theory); other material went into deep space and partly hit Earth, this being probably the sevenfold impact catastrophes analyzed by Tollman et al (14), to be set at circa 7500 BC. A large chunk of material collapsed into a large body, initially extremely hot and of white colour (the Agni god of Rig Veda), then becoming less hot and of reddish colour (the Varuna god of Rig Veda) and now being planet Venus, still quite hot, but whose continuous decrease of temperature has been established by temperature measurement made years apart. It is therefore of importance to evaluate the orbital effects of such possible impact, especially because of the strange fact that Jupiter, which now is just a brilliant star whose diameter angle is too small for normal human vision, was so important in ancient religion, and especially it was noted

for its terrible lightning.

2a - the asteroid belt consists of a large number, over a billion depending on the size threshold, of bodies, the largest being Ceres, about 600 km diameter. They were discovered apparently only less than two hundred years ago and they move in a complex way, since Jupiter gravity affects them in a significant way. They have been considered for a while as the main source of Apollo objects, which may impact Earth at a rather worrying rate, but now they are seen as only a small source of such objects, that should derive mainly from the core of dead comets. Due to the so called Titus-Bode law, whose validity in terms of most stable orbits has been recently established, see Bass and Del Popolo (15) or Damgov and Spedicato (16), it is expected that one planet should revolve in the region of the asteroid belt. There are several explanations for a planet not being there, one that the planet exploded, either after an impact or due to the exotic causes considered by Van Flandern (16). The problem being open, it is certainly of interest to analyze an impact on a planet in this region

3a - apart from the above presented theory of the Moon having been generated by the debris of an impact on Earth of a Mars size body, we do not have reasons to believe that large impacts have affected Earth in at least the time since life has been present, so in the last one billion year about. Indeed such an impact would have left enormous geologic evidence and would moreover have destroyed almost certainly all forms of life. Earth has had plenty of catastrophes, but due generally to impacts with small size asteroids-comets-Apollos or close passages, even of large bodies (close passages are clearly much more probable, a factor perhaps 10 to 4). However it is instructive to look at the effects of an impact because Earth is located in the habitable region and an impact could send it out of this region, either too close to Sun or too far away, so that water would either evaporate or freeze, making life essentially impossible.



Impacts of the type considered have the following features:

- one or both bodies may fragment or hugely deform, or at least partly vaporize; how, this is virtually impossible to model, requiring detailed knowledge of the impacting bodies structure - part of the energy and momentum of both bodies would be transformed into heat and into deformation of the bodies - the bodies may coalesce, but part of their mass may be expelled, partly with escape velocity, partly remaining in orbit around the main body

It is clear that modelling the above effects is extremely complex, if not virtually impossible. Therefore in our approach we take a radical simplification assuming that the impacting body mass just adds itself to the main body, disregarding all heating, cratering etc effects.

Under the above simplification we should have conservation of total energy (kinetic and potential) and angular momentum (rotational and revolutional). As further simplification we ignore rotational energy and momentum, as is done virtually in all celestial mechanics papers.

We model the impact by a body approaching from behind or frontally, hence on a direction tangent to the orbit of the other body. We give initial speed and solar distance of the impacted body, obeying Kepler's third law, and assuming for simplicity that the orbit is circular. After impact the new speed and distance can be obtained by using Kepler's law, still under the simplifying hypothesis of circular orbit, and either conservation of energy or conservation of momentum. We have indeed two variables and two equations are enough to determine them, in general. This means that we are unable, due clearly to the model simplifications, to satisfy after impact both energy and momentum conservation. However it is interesting that the conservation law that we leave aside turns out to be

almost satisfied, the relative error being quite small, comparable to the error, always made in simulations, of neglecting rotation versus revolution.

### 0.3 Kepler's equation and the conservation equations

Our analysis of the problem about the impact between Jupiter and a body of planetary sizes, is based upon the following simplification:

1. we disregard the effects of the interaction over the body that is responsible for the event;
2. we disregard the effects on the other bodies of the solar system;
3. we consider Jupiter as a rigid spherical neutral body, neglecting for instance thermal effects, differential dynamical effects on the different layers of Jupiter and so on;
4. we assume that Jupiter and the impacting objects are on the same plane.

We assume that:

1. the total energy of the system does not change;
2. the total angular momentum of the system does not change.

Let  $m_G$ ,  $m_A$  and  $m_S$  be respectively the mass of Jupiter, the mass of the impacting body and the mass of the Sun. Let  $v_G$ ,  $\omega$  respectively the scalar value representing the revolutionary velocity, rotational velocity of Jupiter before the impact. Let  $T$  and  $d$  respectively the period of revolution of Jupiter and the distance of Jupiter from the Sun before the impact. We consider three equations: the Kepler's equation, the equation for

conservation of total momentum and the equation for the conservation of the total energy.

$$\left\{ \begin{array}{l} \text{Kepler's equation;} \\ \text{equation for conservation of momentum;} \\ \text{equation for the conservation of the energy;} \end{array} \right.$$

$$\left\{ \begin{array}{l} E_G(m_G, d, v_G) + E_A(m_A, d, v_A) = E'_G(m_G + m_A, d', v_G'); \\ M_G(m_G, d, v_G) + M_A(m_A, d, v_A) = M'_G(m_G + m_A, d', v_G'); \\ T'^2/d'^3 = 4\pi^2/G m_S = \text{constant}; \end{array} \right.$$

where we denote with  $v_A$  the scalar value representing the velocity of the impacting body A, with  $v'_G$  and  $\omega'$  respectively the scalar value representing the revolutionary velocity, rotational velocity of Jupiter after the impact, and with  $T'$  e  $d'$  respectively the period of revolution of Jupiter and the distance of Jupiter from the Sun after the impact. We have considered two cases, in the first the velocity's vectors of Jupiter and the impacting body have the same direction, while in the second they have opposite direction.

The momentum due to the orbital revolution before the impact is

$$m_G dv_G \pm m_A dv_A \tag{1}$$

where if the velocity's vectors of Jupiter and the impacting body A have the same direction we consider the sign +, while if the velocity's vectors have opposite direction we consider the sign -.

The total momentum due to the orbital revolution after the impact is

$$(m_G + m_A)d'v_G' . \quad (2)$$

Consequently, the equation for the conservation of momentum is therefore

$$m_G dv_G \pm m_A dv_A = (m_G + m_A)d'v_G' . \quad (3)$$

The total energy due to orbital revolution before the impact is:

$$\frac{1}{2}m_G v_G^2 + \frac{1}{2}m_A v_A^2 - \frac{Gm_G m_S}{d} - \frac{Gm_A m_S}{d} , \quad (4)$$

while the total energy after the impact is:

$$\frac{1}{2}(m_G + m_A)v_G'^2 - \frac{G(m_G + m_A)m_S}{d'} . \quad (5)$$

Consequently, the equation for the conservation of energy is therefore

$$\frac{1}{2}m_G v_G^2 + \frac{1}{2}m_A v_A^2 - \frac{Gm_G m_S}{d} - \frac{Gm_A m_S}{d} = \frac{1}{2}(m_G + m_A)v_G'^2 - \frac{G(m_G + m_A)m_S}{d'} . \quad (6)$$

By the third law of Kepler we have,

$$d' = \left( \frac{Gm_S}{\omega'^2} \right)^{1/3} , \quad (7)$$

with  $G$  the gravitational constant. From the above we obtain

$$\omega' = \left( \frac{Gm_S}{d'^3} \right)^{1/2} . \quad (8)$$

Substituting in the equation (6) the equation (8), and by  $v_G' = \omega' d'$ , we obtain

$$\frac{1}{2}m_G v_G^2 + \frac{1}{2}m_A v_A^2 - \frac{Hm_G}{d} - \frac{Hm_A}{d} = -\frac{1}{2} \frac{(m_G + m_A)H}{d'},$$

where  $H = Gm_S$ . From the above we obtain the distance of Jupiter from the Sun after the impact:

$$d_G' = \frac{-\frac{1}{2}(m_G + m_A)H}{\frac{1}{2}m_G v_G^2 + \frac{1}{2}m_A v_A^2 - \frac{Hm_G}{d} - \frac{Hm_A}{d}}, \quad (9)$$

and we can calculate the error in the conservation of total momentum:

$$\begin{aligned} ErrAss^M &= |m_G dv_G \pm m_A dv_A - (m_G + m_A) d_E' v_G'| \\ &= |m_G dv_G + m_A dv_A - (m_G + m_A) d_E' \omega'|. \end{aligned}$$

Substituting in the equation (3) the equation (8) we obtain:

$$d_G' = \left( \frac{m_G dv_G \pm m_A dv_A}{(m_G + m_A) H^{1/2}} \right)^2, \quad (10)$$

and we can calculate the error in the conservation of total energy:

$$\begin{aligned} ErrAss^E &= \left| \frac{1}{2}m_G v_G^2 + \frac{1}{2}m_A v_A^2 - \frac{Hm_G}{d} - \frac{Hm_A}{d} - \frac{1}{2}(m_G + m_A) v_G'^2 + \frac{(m_G + m_A)H}{d_M'} \right| = \\ &= \left| \frac{1}{2}m_G v_G^2 + \frac{1}{2}m_A v_A^2 - \frac{Hm_G}{d} - \frac{Hm_A}{d} - \frac{1}{2}(m_G + m_A) d_M' \omega'^2 + \frac{(m_G + m_A)H}{d_M'} \right|. \end{aligned}$$

## 0.4 Numerical results about the impact between Jupiter and a body of planetary sizes

Our numerical results have been obtained using the package MATLAB and an appropriate algorithm for solving. Beginning we have calculated the distance  $d_G$  ' after the impact, using the equation for the conservation of energy and the Kepler's equation and considering the error in the equation for the conservation of momentum. In a second moment we have calculated the distance using the equation for the conservation of momentum and the Kepler's equation and considering the error in the equation for the conservation of energy. Finally we have considered the system consists of the three equation: the equation for the conservation of total energy, the equation for the conservation of the total momentum and the Kepler's equation, and we have solved this system using the method of least squares.

We have considered two cases, in the first the velocity's vectors of Jupiter and the impacting body have the same direction, while in the second they have opposite direction.

The numerical values for the parameters in the equation are the following:

$G = 6.6742e - 020 \text{ km}^3/\text{kg s}$  the gravitational constant,

$m_S = 1.9903e + 030 \text{ kg}$  the mass of the Sun,

$m_G = 1.90e + 027 \text{ kg}$  the mass of Jupiter.

We have considered three different values about the distant of Jupiter by the Sun before the impact:

$d_G = 778340000 \text{ km}$  , the current value,

$d_G = 5.1889e + 008 \text{ km}$  , 2/3 of current value,

$d_G = 1.0378e + 009 \text{ km}$  , 4/3 of current value .

The numerical values for the mass of impacting body is equal to appropriate fractions of the mass of Jupiter.

The numerical results lead to conclusion that the distance between Jupiter and the Sun after the impact change with increasing mass and velocity of the impacting body.

**Numerical results about the impact between Jupiter and a body of planetary sizes,**  
**where  $\vec{v}_A$  and  $\vec{v}_G$  have the same direction.**

$d_G=7.7834e+008$ km $v_G=13.06$ km/s $m_G=1.90e+027$ kg					
$v_A$	$v_G + 15$	km/s	$v_G + 30$	km/s	$v_G + 70$ km/s
$m_A$	$d'_G$	<i>Relative</i> <i>Error</i>	$d'_G$	<i>Relative</i> <i>Error</i>	$d'_G$ <i>Relative</i> <i>Error</i>
$1.90e + 030$					
$1.90e + 029$					
$1.90e + 028$					
$m_G = 1.90e + 027$					
$m_G/2 = 9.50e + 026$					
$1.90e + 026$	1.1591e+009	1.0501e-001	7.5375e+009	1.5745e+000	
$1.90e + 025$	8.0722e+008	6.9389e-003	8.6258e+008	2.9327e-002	1.2767e+009 2.1620e-001
$m_{TERRA} = 5.98e + 024$	7.8727e+008	2.1081e-003	8.0320e+008	8.5773e-003	8.8820e+008 5.0585e-002
$1.90e + 024$	7.8116e+008	6.6230e-004	7.8609e+008	2.6640e-003	8.1025e+008 1.4861e-002
$1.90e + 023$	7.7862e+008	6.5917e-005	7.7911e+008	2.6388e-004	7.8142e+008 1.4403e-003
$1.90e + 022$	7.7837e+008	6.5885e-006	7.7842e+008	2.6363e-005	7.7865e+008 1.4359e-004
$1.90e + 021$	7.7834e+008	6.5882e-007	7.7835e+008	2.6360e-006	7.7837e+008 1.4354e-005

Tabella 1: Numerical results obtained using the **equation for the conservation of total energy** and the **Kepler's equation**.



$d_G = 5.1889e+008$ km $v_G = 16.01$ km/s $m_G = 1.90e+027$ kg					
$v_A$	$v_G + 15$ km/s		$v_G + 30$ km/s		$v_G + 70$ km/s
$m_A$	$d'_G$ km <i>Relative Error</i>		$d'_G$ km <i>Relative Error</i>		$d'_G$ km <i>Relative Error</i>
$1.90e + 030$					
$1.90e + 029$					
$1.90e + 028$					
$m_G = 1.90e + 027$					
$m_G/2 = 9.50e + 026$	1.6840e+009 4.3976e-001				
$1.90e + 026$	6.3958e+008 3.9019e-002		1.1998e+009 3.1797e-001		
$1.90e + 025$	5.2978e+008 2.9516e-003		5.5308e+008 1.5412e-002	6.9830e+008 1.1384e-001	
$m_{TERRA} = 5.98e + 024$	5.2229e+008 9.0509e-004		5.2926e+008 4.6088e-003	5.6488e+008 2.9832e-002	
$1.90e + 024$	5.1997e+008 2.8518e-004		5.2215e+008 1.4409e-003	5.3270e+008 8.9943e-003	
$1.90e + 023$	5.1900e+008 2.8417e-005		5.1922e+008 1.4312e-004	5.2024e+008 8.8039e-004	
$1.90e + 022$	5.1890e+008 2.8407e-006		5.1893e+008 1.4302e-005	5.1903e+008 8.7852e-005	
$1.90e + 021$	5.1889e+008 2.8406e-007		5.1890e+008 1.4301e-006	5.1891e+008 8.7834e-006	

Tabella 2: Numerical results obtained using the **equation for the conservation of total energy** and the **Kepler's equation**.

$d_G = 1.0378e+009$ km $v_G = 11.31$ km/s $m_G = 1.90e+027$ kg					
$v_A$	$v_G + 15$ km/s		$v_G + 30$ km/s		$v_G + 70$ km/s
$m_A$	$d'_G$ km <i>Relative Error</i>		$d'_G$ km <i>Relative Error</i>		$d'_G$ km <i>Relative Error</i>
$1.90e + 030$					
$1.90e + 029$					
$1.90e + 028$					
$m_G = 1.90e + 027$					
$m_G/2 = 9.50e + 026$					
$1.90e + 026$					
$1.90e + 025$	1.0936e+009 1.1693e-002		1.1977e+009 4.5249e-002		2.1790e+009 3.6342e-001
$m_{TERRA} = 5.98e + 024$	1.0548e+009 3.5199e-003		1.0836e+009 1.2934e-002		1.2443e+009 7.3623e-002
$1.90e + 024$	1.0432e+009 1.1027e-003		1.0520e+009 3.9905e-003		1.0957e+009 2.1049e-002
$1.90e + 023$	1.0383e+009 1.0962e-004		1.0392e+009 3.9421e-004		1.0433e+009 2.0198e-003
$1.90e + 022$	1.0378e+009 1.0955e-005		1.0379e+009 3.9373e-005		1.0383e+009 2.0116e-004
$1.90e + 021$	1.0378e+009 1.0954e-006		1.0378e+009 3.9368e-006		1.0378e+009 2.0108e-005

Tabella 3: Numerical results obtained using the **equation for the conservation of total energy** and the **Kepler's equation**.

$d_G=7.7834e+008$ km						
$v_G=13.06$ km/s						
$m_G=1.90e+027$ kg						
$v_A$	$v_G + 15$	km/s	$v_G + 30$	km/s	$v_G + 70$	km/s
$m_A$	$d'_G$	<i>Relative</i>	$d'_G$	<i>Relative</i>	$d'_G$	<i>Relative</i>
	km	<i>Error</i>	km	<i>Error</i>	km	<i>Error</i>
$1.90e + 030$	3.5870e+009	1.0831e+000	8.4442e+009	1.0104e+000	3.1410e+010	1.0006e+000
$1.90e + 029$	3.5529e+009	1.0850e+000	8.3397e+009	1.0106e+000	3.0940e+010	1.0007e+000
$1.90e + 028$	3.2504e+009	1.1048e+000	7.4189e+009	1.0132e+000	2.6827e+010	1.0008e+000
$m_G = 1.90e + 027$	1.9282e+009	1.5004e+000	3.5913e+009	1.0551e+000	1.0535e+010	1.0039e+000
$m_G/2 = 9.50e + 026$	1.4879e+009	3.5584e+000	2.4257e+009	1.1402e+000	6.0412e+009	1.0106e+000
$1.90e + 026$	9.4926e+008	2.2105e-001	1.1372e+009	5.6282e+000	1.7212e+009	1.1750e+000
$1.90e + 025$	7.9613e+008	1.3926e-002	8.1413e+008	5.9515e-002	8.6311e+008	4.7913e-001
$m_{TERRA} = 5.98e + 024$	7.8396e+008	4.2207e-003	7.8959e+008	1.7228e-002	8.0473e+008	1.0373e-001
$1.90e + 024$	7.8013e+008	1.3250e-003	7.8191e+008	5.3350e-003	7.8669e+008	2.9943e-002
$1.90e + 023$	7.7852e+008	1.3184e-004	7.7870e+008	5.2782e-004	7.7917e+008	2.8827e-003
$1.90e + 022$	7.7836e+008	1.3177e-005	7.7838e+008	5.2726e-005	7.7842e+008	2.8719e-004
$1.90e + 021$	7.7834e+008	1.3176e-006	7.7834e+008	5.2720e-006	7.7835e+008	2.8709e-005

Tabella 4: Numerical results obtained using the **equation for the conservation of total momentum** and the **Kepler's equation**.

$d_G = 5.1889e+008$ km $v_G = 16.01$ km/s $m_G = 1.90e+027$ kg						
$v_A$	$v_G + 15$	km/s	$v_G + 30$	km/s	$v_G + 70$	km/s
$m_A$	$d'_G$	<i>Relative</i>	$d'_G$	<i>Relative</i>	$d'_G$	<i>Relative</i>
	km	<i>Error</i>	km	<i>Error</i>	km	<i>Error</i>
$1.90e + 030$	1.5945e+009	1.3031e+000	3.7535e+009	1.0264e+000	1.3961e+010	1.0015e+000
$1.90e + 029$	1.5824e+009	1.3108e+000	3.7116e+009	1.0270e+000	1.3761e+010	1.0015e+000
$1.90e + 028$	1.4736e+009	1.3970e+000	3.3411e+009	1.0332e+000	1.2006e+010	1.0019e+000
$m_G = 1.90e + 027$	9.8370e+008	1.4954e+001	1.7675e+009	1.1384e+000	4.9725e+009	1.0087e+000
$m_G/2 = 9.50e + 026$	8.1239e+008	1.0729e+000	1.2688e+009	1.3783e+000	2.9815e+009	1.0228e+000
$1.90e + 026$	5.9244e+008	7.9561e-002	6.9072e+008	7.3706e-001	9.8964e+008	1.3858e+000
$1.90e + 025$	5.2667e+008	5.9119e-003	5.3642e+008	3.1062e-002	5.6285e+008	2.4064e-001
$m_{TERRA} = 5.98e + 024$	5.2135e+008	1.8110e-003	5.2441e+008	9.2389e-003	5.3263e+008	6.0554e-002
$1.90e + 024$	5.1968e+008	5.7044e-004	5.2065e+008	2.8839e-003	5.2325e+008	1.8070e-002
$1.90e + 023$	5.1897e+008	5.6836e-005	5.1907e+008	2.8626e-004	5.1933e+008	1.7616e-003
$1.90e + 022$	5.1890e+008	5.6815e-006	5.1891e+008	2.8605e-005	5.1894e+008	1.7571e-004
$1.90e + 021$	5.1889e+008	5.6813e-007	5.1890e+008	2.8603e-006	5.1890e+008	1.7567e-005

Tabella 5: Numerical results obtained using the **equation for the conservation of total momentum** and the **Kepler's equation**.

$d_G = 1.0378e+009$ km $v_G = 11.31$ km/s $m_G = 1.90e+027$ kg						
$v_A$	$v_G + 15$	km/s	$v_G + 30$	km/s	$v_G + 70$	km/s
$m_A$	$d'_G$	<i>Relative</i>	$d'_G$	<i>Relative</i>	$d'_G$	<i>Relative</i>
	km	<i>Error</i>	km	<i>Error</i>	km	<i>Error</i>
$1.90e + 030$	6.3760e+009	1.0393e+000	1.5011e+010	1.0055e+000	5.5838e+010	1.0004e+000
$1.90e + 029$	6.3084e+009	1.0401e+000	1.4814e+010	1.0057e+000	5.4982e+010	1.0004e+000
$1.90e + 028$	5.7098e+009	1.0494e+000	1.3085e+010	1.0070e+000	4.7494e+010	1.0005e+000
$m_G = 1.90e + 027$	3.1423e+009	1.2096e+000	5.9926e+009	1.0302e+000	1.8052e+010	1.0023e+000
$m_G/2 = 9.50e + 026$	2.3145e+009	1.6253e+000	3.8870e+009	1.0764e+000	1.0062e+010	1.0062e+000
$1.90e + 026$	1.3359e+009	4.6107e-001	1.6348e+009	3.8095e+000	2.5793e+009	1.1056e+000
$1.90e + 025$	1.0684e+009	2.3524e-002	1.0963e+009	9.2545e-002	1.1722e+009	8.5892e-001
$m_{TERRA} = 5.98e + 024$	1.0474e+009	7.0522e-003	1.0561e+009	2.6035e-002	1.0795e+009	1.5267e-001
$1.90e + 024$	1.0409e+009	2.2066e-003	1.0436e+009	7.9969e-003	1.0510e+009	4.2542e-002
$1.90e + 023$	1.0381e+009	2.1925e-004	1.0384e+009	7.8857e-004	1.0391e+009	4.0436e-003
$1.90e + 022$	1.0378e+009	2.1910e-005	1.0378e+009	7.8747e-005	1.0379e+009	4.0237e-004
$1.90e + 021$	1.0378e+009	2.1909e-006	1.0378e+009	7.8737e-006	1.0378e+009	4.0217e-005

Tabella 6: Numerical results obtained using the **equation for the conservation of total momentum** and the **Kepler's equation**.

$d_G=7.7834e+008$ km $v_G=13.06$ km/s $m_G=1.90e+027$ kg			
$v_A$	$(v_G + 15)$ km/s	$(v_G + 30)$ km/s	$(v_G + 70)$ km/s
$m_A$	$d'_G$ km	$d'_G$ km	$d'_G$ km
$1.90e + 030$	1.6444e+009	4.1782e+009	1.5695e+010
$1.90e + 029$	1.6255e+009	4.1255e+009	1.5460e+010
$1.90e + 028$	1.4549e+009	3.6606e+009	1.3402e+010
$m_G = 1.90e + 027$	4.8167e+008	1.6967e+009	5.2465e+009
$m_G/2 = 9.50e + 026$		1.0428e+009	2.9885e+009
$1.90e + 026$	1.0542e+009	4.3374e+009	7.1001e+008
$1.90e + 025$	8.0168e+008	8.3836e+008	1.0699e+009
$m_{TERRA} = 5.98e + 024$	7.8561e+008	7.9640e+008	8.4647e+008
$1.90e + 024$	7.8064e+008	7.8400e+008	7.9847e+008
$1.90e + 023$	7.7857e+008	7.7890e+008	7.8030e+008
$1.90e + 022$	7.7836e+008	7.7840e+008	7.7854e+008
$1.90e + 021$	7.7834e+008	7.7835e+008	7.7836e+008

Tabella 7: Numerical results obtained using the **method of least squares**.

$d_G = 5.1889e+008$ km $v_G = 16.01$ km/s $m_G = 1.90e+027$ kg			
$v_A$	$(v_G + 15)$ km/s	$(v_G + 30)$ km/s	$(v_G + 70)$ km/s
$m_A$	$d'_G$ km	$d'_G$ km	$d'_G$ km
$1.90e + 030$	5.5559e+008	1.8272e+009	6.9701e+009
$1.90e + 029$	5.4527e+008	1.8057e+009	6.8699e+009
$1.90e + 028$	4.4429e+008	1.6150e+009	5.9916e+009
$m_G = 1.90e + 027$		7.6145e+008	2.4646e+009
$m_G/2 = 9.50e + 026$	1.2482e+009	3.9440e+008	1.4569e+009
$1.90e + 026$	6.1601e+008	9.4527e+008	3.0391e+008
$1.90e + 025$	5.2822e+008	5.4475e+008	6.3058e+008
$m_{TERRA} = 5.98e + 024$	5.2182e+008	5.2684e+008	5.4876e+008
$1.90e + 024$	5.1982e+008	5.2140e+008	5.2798e+008
$1.90e + 023$	5.1899e+008	5.1914e+008	5.1979e+008
$1.90e + 022$	5.1890e+008	5.1892e+008	5.1898e+008
$1.90e + 021$	5.1889e+008	5.1890e+008	5.1890e+008

Tabella 8: Numerical results obtained using the **method of least squares**.

$d_G = 1.0378e+009$ km $v_G = 11.31$ km/s $m_G = 1.90e+027$ kg			
$v_A$	$(v_G + 15)$ km/s	$(v_G + 30)$ km/s	$(v_G + 70)$ km/s
$m_A$	$d'_G$ km	$d'_G$ km	$d'_G$ km
$1.90e + 030$	$3.0629e+009$	$7.4638e+009$	$2.7909e+010$
$1.90e + 029$	$3.0277e+009$	$7.3651e+009$	$2.7481e+010$
$1.90e + 028$	$2.7140e+009$	$6.4966e+009$	$2.3736e+010$
$m_G = 1.90e + 027$	$1.2418e+009$	$2.9059e+009$	$2.3736e+010$
$m_G/2 = 9.50e + 026$	$4.3361e+008$	$1.7950e+009$	$5.0000e+009$
$1.90e + 026$	$1.6438e+009$		$1.1534e+009$
$1.90e + 025$	$1.0810e+009$	$1.1470e+009$	$1.6756e+009$
$m_{TERRA} = 5.98e + 024$	$1.0511e+009$	$1.0699e+009$	$1.1619e+009$
$1.90e + 024$	$1.0420e+009$	$1.0478e+009$	$1.0733e+009$
$1.90e + 023$	$1.0382e+009$	$1.0388e+009$	$1.0412e+009$
$1.90e + 022$	$1.0378e+009$	$1.0379e+009$	$1.0381e+009$
$1.90e + 021$	$1.0378e+009$	$1.0378e+009$	$1.0378e+009$

Tabella 9: Numerical results obtained using the **method of least squares**.



**Numerical results about the impact between Jupiter and a body of planetary sizes,**  
**where  $\vec{v}_A$  and  $\vec{v}_G$  have opposite direction.**

$d_G = 7.7834e+008$ km $v_G = 11.31$ km/s $m_G = 1.90e+027$ kg						
$v_A$	$-(v_G + 15)$	km/s	$-(v_G + 30)$	km/s	$-(v_G + 70)$	km/s
$m_A$	$d'_G$	<i>Relative Error</i>	$d'_G$	<i>Relative Error</i>	$d'_G$	<i>Relative Error</i>
	km		km		km	
$1.90e + 030$	$3.5803e+009$	$1.0833e+000$	$8.4340e+009$	$1.0104e+000$	$3.1390e+010$	$1.0006e+000$
$1.90e + 029$	$3.4873e+009$	$1.0866e+000$	$8.2391e+009$	$1.0108e+000$	$3.0746e+010$	$1.0007e+000$
$1.90e + 028$	$2.6977e+009$	$1.1263e+000$	$6.5708e+009$	$1.0149e+000$	$2.5191e+010$	$1.0009e+000$
$m_G = 1.90e + 027$	$2.5639e+008$	$4.7631e+000$	$1.0258e+009$	$1.1930e+000$	$5.5860e+009$	$1.0074e+000$
$m_G/2 = 5.90e + 026$	$1.8913e+006$	$2.0137e+003$	$1.4527e+008$	$3.3416e+000$	$1.6424e+009$	$1.0390e+000$
$1.90e + 026$	$3.9660e+008$	$1.9225e+000$	$2.8910e+008$	$2.5073e+001$	$8.5326e+007$	$4.5303e+000$
$1.90e + 025$	$7.3058e+008$	$1.0490e-001$	$7.1353e+008$	$2.0889e-001$	$6.6907e+008$	$9.0812e-001$
$m_{TERRA} = 5.98e + 024$	$7.6304e+008$	$3.1746e-002$	$7.5750e+008$	$6.0329e-002$	$7.4282e+008$	$1.9572e-001$
$1.90e + 024$	$7.7345e+008$	$9.9650e-003$	$7.7167e+008$	$1.8677e-002$	$7.6694e+008$	$5.6473e-002$
$1.90e + 023$	$7.7785e+008$	$9.9147e-004$	$7.7767e+008$	$1.8478e-003$	$7.7720e+008$	$5.4364e-003$
$1.90e + 022$	$7.7829e+008$	$9.9097e-005$	$7.7827e+008$	$1.8458e-004$	$7.7823e+008$	$5.4161e-004$
$1.90e + 021$	$7.7834e+008$	$9.9092e-006$	$7.7833e+008$	$1.8456e-005$	$7.7833e+008$	$5.4141e-005$

Tabella 10: Numerical results obtained using the **equation for the conservation of total momentum** and the **Kepler's equation**.

$d_G = 5.1889e+008$ km $v_G = 11.31$ km/s $m_G = 1.90e+027$ kg						
$v_A$	$-(v_G + 15)$	km/s	$-(v_G + 30)$	km/s	$-(v_G + 70)$	km/s
$m_A$	$d'_G$	<i>Relative</i>	$d'_G$	<i>Relative</i>	$d'_G$	<i>Relative</i>
	km	<i>Error</i>	km	<i>Error</i>	km	<i>Error</i>
$1.90e + 030$	1.5909e+009	1.3038e+000	3.7479e+009	1.0264e+000	1.3950e+010	1.0015e+000
$1.90e + 029$	1.5467e+009	1.3180e+000	3.6568e+009	1.0274e+000	1.3655e+010	1.0015e+000
$1.90e + 028$	1.1728e+009	1.4988e+000	2.8794e+009	1.0385e+000	1.1116e+010	1.0021e+000
$m_G = 1.90e + 027$	7.3699e+007	1.8725e+002	3.7104e+008	1.6592e+000	2.2788e+009	1.0190e+000
$m_G/2 = 9.50e + 026$	3.4964e+006	4.8064e+002	2.7548e+007	1.8426e+001	5.8715e+008	1.1155e+000
$1.90e + 026$	2.9161e+008	1.1932e+000	2.2908e+008	4.2376e+000	9.9166e+007	4.8504e+000
$1.90e + 025$	4.9098e+008	7.9018e-002	4.8166e+008	1.4828e-001	4.5723e+008	5.2725e-001
$m_{TERRA} = 5.98e + 024$	5.0997e+008	2.4176e-002	5.0694e+008	4.4020e-002	4.9893e+008	1.3219e-001
$1.90e + 024$	5.1604e+008	7.6141e-003	5.1507e+008	1.3738e-002	5.1249e+008	3.9431e-002
$1.90e + 023$	5.1861e+008	7.5862e-004	5.1851e+008	1.3636e-003	5.1825e+008	3.8439e-003
$1.90e + 022$	5.1886e+008	7.5834e-005	5.1886e+008	1.3626e-004	5.1883e+008	3.8342e-004
$1.90e + 021$	5.1889e+008	7.5831e-006	5.1889e+008	1.3625e-005	5.1889e+008	3.8332e-005

Tabella 11: Numerical results obtained using the **equation for the conservation of total momentum** and the **Kepler's equation**.

$d_G = 1.0378e+009$ km $v_G = 11.31$ km/s $m_G = 1.90e+027$ kg						
$v_A$	$-(v_G + 15)$	km/s	$-(v_G + 30)$	km/s	$-(v_G + 70)$	km/s
$m_A$	$d'_G$	<i>Relative</i>	$d'_G$	<i>Relative</i>	$d'_G$	<i>Relative</i>
	km	<i>Error</i>	km	<i>Error</i>	km	<i>Error</i>
$1.90e + 030$	6.3658e+009	1.0393e+000	1.4995e+010	1.0055e+000	5.5807e+010	1.0004e+000
$1.90e + 029$	6.2075e+009	1.0408e+000	1.4659e+010	1.0057e+000	5.4683e+010	1.0004e+000
$1.90e + 028$	4.8589e+009	1.0580e+000	1.1780e+010	1.0078e+000	4.4976e+010	1.0005e+000
$m_G = 1.90e + 027$	5.6842e+008	2.1588e+000	2.0428e+009	1.0885e+000	1.0434e+010	1.0039e+000
$m_G/2 = 9.50e + 026$	2.6585e+007	5.5438e+001	3.7609e+008	1.7895e+000	3.2900e+009	1.0190e+000
$1.90e + 026$	4.8500e+008	3.0243e+000	3.2905e+008	1.4958e+001	6.0617e+007	5.4948e+000
$1.90e + 025$	9.6750e+008	1.3029e-001	9.4137e+008	2.7230e-001	8.7345e+008	1.4947e+000
$m_{TERRA} = 5.98e + 024$	1.0152e+009	3.8993e-002	1.0067e+009	7.6397e-002	9.8418e+008	2.6430e-001
$1.90e + 024$	1.0306e+009	1.2199e-002	1.0278e+009	2.3460e-002	1.0206e+009	7.3612e-002
$1.90e + 023$	1.0371e+009	1.2120e-003	1.0368e+009	2.3133e-003	1.0361e+009	6.9965e-003
$1.90e + 022$	1.0377e+009	1.2112e-004	1.0377e+009	2.3101e-004	1.0376e+009	6.9619e-004
$1.90e + 021$	1.0378e+009	1.2112e-005	1.0378e+009	2.3098e-005	1.0378e+009	6.9584e-005

Tabella 12: Numerical results obtained using the **equation for the conservation of total momentum** and the **Kepler's equation**.

$d_G = 7.7834e+008$ km $v_G = 11.31$ km/s $m_G = 1.90e+027$ kg			
$v_A$	$-(v_G + 15)$ km/s	$-(v_G + 30)$ km/s	$-(v_G + 70)$ km/s
$m_A$	$d'_G$ km	$d'_G$ km	$d'_G$ km
$1.90e + 030$	1.6444e+009	4.1782e+009	1.5695e+010
$1.90e + 029$	1.6255e+009	4.1255e+009	1.5460e+010
$1.90e + 028$	1.4549e+009	3.6606e+009	1.3402e+010
$m_G = 1.90e + 027$	4.8167e+008	1.6967e+009	5.2465e+009
$m_G/2 = 9.50e + 026$		1.0428e+009	2.9885e+009
$1.90e + 026$	1.0542e+009	4.3374e+009	7.1001e+008
$1.90e + 025$	8.0168e+008	8.3836e+008	1.0699e+009
$m_{TERRA} = 5.98e + 024$	7.7515e+008	7.8035e+008	8.1551e+008
$1.90e + 024$	7.8064e+008	7.8400e+008	7.9847e+008
$1.90e + 023$	7.7857e+008	7.7890e+008	7.8030e+008
$1.90e + 022$	7.7836e+008	7.7840e+008	7.7854e+008
$1.90e + 021$	7.7834e+008	7.7835e+008	7.7836e+008

Tabella 13: Numerical results obtained using the **method of least squares**.

$d_G = 5.1889e+008$ km $v_G = 11.31$ km/s $m_G = 1.90e+027$ kg			
$v_A$	$-(v_G + 15)$ km/s	$-(v_G + 30)$ km/s	$-(v_G + 70)$ km/s
$m_A$	$d'_G$ km	$d'_G$ km	$d'_G$ km
$1.90e + 030$	5.5559e+008	1.8272e+009	6.9701e+009
$1.90e + 029$	5.4527e+008	1.8057e+009	6.8699e+009
$1.90e + 028$	4.4429e+008	1.6150e+009	5.9916e+009
$m_G = 1.90e + 027$		7.6145e+008	2.4646e+009
$m_G/2 = 9.50e + 026$	1.2482e+009	3.9440e+008	1.4569e+009
$1.90e + 026$	6.1601e+008	9.4527e+008	3.0391e+008
$1.90e + 025$	5.2822e+008	5.4475e+008	6.3058e+008
$m_{TERRA} = 5.98e + 024$	5.1613e+008	5.1810e+008	5.3191e+008
$1.90e + 024$	5.1982e+008	5.2140e+008	5.2798e+008
$1.90e + 023$	5.1899e+008	5.1914e+008	5.1979e+008
$1.90e + 022$	5.1890e+008	5.1892e+008	5.1898e+008
$1.90e + 021$	5.1889e+008	5.1890e+008	5.1890e+008

Tabella 14: Numerical results obtained using the **method of least squares**.

$d_G = 1.0378e+009$ km $v_G = 11.31$ km/s $m_G = 1.90e+027$ kg			
$v_A$	$-(v_G + 15)$ km/s	$-(v_G + 30)$ km/s	$-(v_G + 70)$ km/s
$m_A$	$d'_G$ km	$d'_G$ km	$d'_G$ km
$1.90e + 030$	3.0629e+009	7.4638e+009	2.7909e+010
$1.90e + 029$	3.0277e+009	7.3651e+009	2.7481e+010
$1.90e + 028$	2.7140e+009	6.4966e+009	2.3736e+010
$m_G = 1.90e + 027$	1.2418e+009	2.9059e+009	9.0058e+009
$m_G/2 = 1.90e + 026$	4.3361e+008		5.0000e+009
$1.90e + 026$	1.6438e+009	1.7950e+009	1.1534e+009
$1.90e + 025$	1.0810e+009	1.1470e+009	1.6756e+009
$m_{TERRA} = 5.98e + 024$	1.0350e+009	1.0452e+009	1.1142e+009
$1.90e + 024$	1.0420e+009	1.0478e+009	1.0733e+009
$1.90e + 023$	1.0382e+009	1.0388e+009	1.0412e+009
$1.90e + 022$	1.0378e+009	1.0379e+009	1.0381e+009
$1.90e + 021$	1.0378e+009	1.0378e+009	1.0378e+009

Tabella 15: Numerical results obtained using the **method of least squares**.

## 0.5 Numerical results about the impact between the Earth and a body of planetary sizes

Our analysis of the problem about the impact between the Earth and a body of planetary sizes, is based upon the same simplification made in the analysis of the impact between Jupiter and a body.

We have considered two cases, in the first the velocity's vectors of the Earth and the impacting body have the same direction, while in the second they have opposite direction.

The numerical values for the parameters in the equation are the following:

$G = 6.6742e - 020 \text{ km}^3/\text{kg s}$  the gravitational constant,

$m_S = 1.9903e + 030 \text{ kg}$  the mass of the Sun,

$m_T = 5.98e + 024 \text{ kg}$  the mass of the Earth.

$d_T = 1.496e + 008 \text{ km}$  the distance between the Earth and the Sun before the impact,

The numerical values for the mass of impacting body is equal to appropriate fractions of the mass of the Earth.

The numerical results lead to conclusion that the distance between the Earth and the Sun after the impact change with increasing mass and velocity of the impacting body.

Numerical results about the impact between the Earth and a body of planetary sizes, where  $\vec{v}_T$  and  $\vec{v}_A$  have the same direction.

$d_T = 1.496e+008$ km $v_T = 29.8$ km/s $m_T = 5.98e+024$ kg					
$v_A$	$v_T + 15$ km/s		$v_T + 30$ km/s		$v_T + 70$ km/s
$m_A$	$d'_T$ km <i>Relative Error</i>		$d'_T$ km <i>Relative Error</i>		$d'_T$ km <i>Relative Error</i>
$5.98e + 027$					
$5.98e + 026$					
$5.98e + 025$					
$m_T = 5.98e + 024$	4.0441e+008	3.1355e-001			
$m_T/2 = 2.99e + 024$	2.5795e+008	1.2445e-001			
$m_{MARTe} = 6.42e + 023$	1.7042e+008	1.7657e-002	2.1174e+008	8.3902e-002	1.5723e+010 7.3503e+000
$5.98e + 023$	1.6896e+008	1.6219e-002	2.0640e+008	7.6105e-002	2.1003e+009 2.0876e+000
$5.98e + 022$	1.5149e+008	1.3069e-003	1.5422e+008	5.3102e-003	1.6644e+008 3.0794e-002
$5.98e + 021$	1.4979e+008	1.2710e-004	1.5005e+008	5.0920e-004	1.5114e+008 2.7892e-003
$5.98e + 020$	1.4962e+008	1.2674e-005	1.4965e+008	5.0702e-005	1.4975e+008 2.7621e-004
$5.98e + 019$	1.4960e+008	1.2670e-006	1.4960e+008	5.0681e-006	1.4962e+008 2.7594e-005
$5.98e + 018$	1.4960e+008	1.2670e-007	1.4960e+008	5.0678e-007	1.4960e+008 2.7592e-006

Tabella 16: Numerical results obtained using the **equation for the conservation of total energy** and the **Kepler's equation**



$d_T = 1.496e+008$ km $v_T = 29.8$ km/s $m_T = 5.98e+024$ kg						
$v_A$	$v_T + 15$	km/s	$v_T + 30$	km/s	$v_T + 70$	km/s
$m_A$	$d'_T$	<i>Relative</i> <i>Error</i>	$d'_T$	<i>Relative</i> <i>Error</i>	$d'_T$	<i>Relative</i> <i>Error</i>
	km		km		km	
$5.98e + 027$	3.3789e+008	2.7102e+000	6.0185e+008	1.1228e+000	1.6756e+009	1.0097e+000
$5.98e + 026$	3.3588e+008	2.7983e+000	5.9648e+008	1.1256e+000	1.6548e+009	1.0099e+000
$5.98e + 025$	3.1785e+008	4.2328e+000	5.4875e+008	1.1556e+000	1.4708e+009	1.0123e+000
$m_T = 5.98e + 024$	2.3438e+008	7.2541e-001	3.3812e+008	1.8616e+000	7.0741e+008	1.0515e+000
$m_T/2 = 2.99e + 024$	2.0402e+008	2.6438e-001	2.6686e+008	6.3099e+001	4.7561e+008	1.1308e+000
$m_{MARTe} = 6.42e + 023$	1.6456e+008	3.5625e-002	1.8023e+008	1.7484e-001	2.2550e+008	6.8727e+001
$5.98e + 023$	1.6361e+008	3.2702e-002	1.7824e+008	1.5800e-001	2.2032e+008	8.5332e+000
$5.98e + 022$	1.5109e+008	2.6155e-003	1.5260e+008	1.0649e-002	1.5664e+008	6.2536e-002
$5.98e + 021$	1.4975e+008	2.5422e-004	1.4990e+008	1.0187e-003	1.5030e+008	5.5863e-003
$5.98e + 020$	1.4962e+008	2.5347e-005	1.4963e+008	1.0141e-004	1.4967e+008	5.5250e-004
$5.98e + 019$	1.4960e+008	2.5340e-006	1.4960e+008	1.0136e-005	1.4961e+008	5.5190e-005
$5.98e + 018$	1.4960e+008	2.5339e-007	1.4960e+008	1.0136e-006	1.4960e+008	5.5184e-006

Tabella 17: Numerical results obtained using the **equation for the conservation of total momentum** and the **Kepler's equation**.

$d_T = 1.496e+008$ km $v_T = 29.8$ km/s $m_T = 5.98e+024$ kg			
$v_A$	$v_T + 15$ km/s	$v_T + 30$ km/s	$v_T + 70$ km/s
$m_A$	$d'_T$ km	$d'_T$ km	$d'_T$ km
$5.98e + 027$		$2.6397e+008$	$8.2969e+008$
$5.98e + 026$		$2.6079e+008$	$8.1918e+008$
$5.98e + 025$		$2.3168e+008$	$7.2638e+008$
$m_T = 5.98e + 024$	$3.1939e+008$	$2.3404e+007$	$7.2638e+008$
$m_T/2 = 2.99e + 024$	$2.3098e+008$		$2.0671e+008$
$m_{MARTe} = 6.42e + 023$	$1.6749e+008$	$1.9598e+008$	$7.9745e+009$
$5.98e + 023$	$1.6628e+008$	$1.9232e+008$	$1.1603e+009$
$5.98e + 022$	$1.5129e+008$	$1.5341e+008$	$1.6154e+008$
$5.98e + 021$	$1.4977e+008$	$1.4998e+008$	$1.5072e+008$
$5.98e + 020$	$1.4962e+008$	$1.4964e+008$	$1.4971e+008$
$5.98e + 019$	$1.4960e+008$	$1.4960e+008$	$1.4961e+008$
$5.98e + 018$	$1.4960e+008$	$1.4960e+008$	$1.4960e+008$

Tabella 18: Numerical results obtained using the **method of least squares**.

**Numerical results about the impact between the Earth and a body of planetary sizes, where  $\vec{v}_T$  and  $\vec{v}_A$  have opposite direction.**

$d_T = 1.496e+008$ km $v_T = 29.8$ km/s $m_T = 5.98e+024$ kg						
$v_A$	$-(v_T + 15)$	km/s	$-(v_T + 30)$	km/s	$-(v_T + 70)$	km/s
$m_A$	$d'_T$	<i>Relative</i>	$d'_T$	<i>Relative</i>	$d'_T$	<i>Relative</i>
	km	<i>Error</i>	km	<i>Error</i>	km	<i>Error</i>
$5.98e + 027$	3.3699e+008	2.7147e+000	6.0065e+008	1.1231e+000	1.6736e+009	1.0097e+000
$5.98e + 026$	3.2706e+008	2.8468e+000	5.8471e+008	1.1281e+000	1.6351e+009	1.0100e+000
$5.98e + 025$	2.4350e+008	5.2199e+000	4.4951e+008	1.1900e+000	1.3052e+009	1.0138e+000
$m_T = 5.98e + 024$	9.4768e+006	4.1673e+001	3.7907e+007	8.6848e+000	2.0638e+008	1.1764e+000
$m_T/2 = 2.99e + 024$	4.0996e+006	6.1922e+001	7.5942e+002	2.1821e+007	3.0254e+007	3.0556e+000
$m_{MARTe} = 6.42e + 023$	8.5796e+007	9.8635e-001	7.5094e+007	1.8197e+000	5.0041e+007	3.1321e+002
$5.98e + 023$	8.9256e+007	8.9292e-001	7.8994e+007	1.6129e+000	5.4690e+007	3.7404e+001
$5.98e + 022$	1.4228e+008	6.4763e-002	1.4083e+008	9.5130e-002	1.3699e+008	2.1491e-001
$5.98e + 021$	1.4885e+008	6.2874e-003	1.4870e+008	9.0862e-003	1.4830e+008	1.9148e-002
$5.98e + 020$	1.4953e+008	6.2690e-004	1.4951e+008	9.0451e-004	1.4947e+008	1.8938e-003
$5.98e + 019$	1.4959e+008	6.2671e-005	1.4959e+008	9.0411e-005	1.4959e+008	1.8917e-004
$5.98e + 018$	1.4960e+008	6.2669e-006	1.4960e+008	9.0406e-006	1.4960e+008	1.8915e-005

Tabella 19: Numerical results obtained using the **equation for the conservation of total momentum** and the **Kepler's equation**.

$d_T = 1.496e+008$ km $v_T = 29.8$ km/s $m_T = 5.98e+024$ kg			
$v_A$	$-(v_T + 15)$ km/s	$-(v_T + 30)$ km/s	$-(v_T + 70)$ km/s
$m_A$	$d'_T$ km	$d'_T$ km	$d'_T$ km
$5.98e + 027$		$2.6337e+008$	$8.2869e+008$
$5.98e + 026$		$2.5490e+008$	$8.0935e+008$
$5.98e + 025$		$1.8206e+008$	$6.4357e+008$
$m_T = 5.98e + 024$	$2.0694e+008$		$8.4985e+007$
$m_T/2 = 2.99e + 024$	$1.3103e+008$		
$m_{MARTe} = 6.42e + 023$	$1.2811e+008$	$1.4342e+008$	$7.8867e+009$
$5.98e + 023$	$1.2911e+008$	$1.4270e+008$	$1.0775e+009$
$5.98e + 022$	$1.4688e+008$	$1.4752e+008$	$1.5171e+008$
$5.98e + 021$	$1.4932e+008$	$1.4938e+008$	$1.4972e+008$
$5.98e + 020$	$1.4957e+008$	$1.4958e+008$	$1.4961e+008$
$5.98e + 019$	$1.4960e+008$	$1.4960e+008$	$1.4960e+008$
$5.98e + 018$	$1.4960e+008$	$1.4960e+008$	$1.4960e+008$

Tabella 20: Numerical results obtained using the **method of least squares**.

## References

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