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Abstract

Limits on information have deep economic impact and affect the conduct of economic policy. In the present paper we explore the effect of substantive uncertainty. A macro model is then derived in order to make this condition work at micro economic level too: the investment function implies an interaction between real and financial aspects; the labor market is ruled by imperfect competition; agents are boundedly rational and make their forecasts according to a Markov regime switching rule; and finally monetary authorities learns about the NAIRU. As a result we obtain a model which is mostly keynesian in nature, whose implications can nevertheless be compared with the new neoclassical synthesis models. Simulations are carried out and show the possible appearence of endogenous fluctuations, persistence of oscillations, and the emergence of a trade-off between the control of inflation and the cyclicality of the economy.

JEL Classification: E32, E37, E52

Keywords endogenous cycles, monetary policy, uncertainty, bounded rationality, learning

1 Introduction

The fact uncertainty deeply affects the economic systems either at micro and macro levels is indisputable. The meaning of the concept, its representation, its (possible) implementation in models is far more controversial. In the present paper we focus on "substantive" uncertainty[1] and on the impact it has on monetary policy rules within a small macroeconomic model, which, in turn, leads to the analysis of the dynamic behaviour of the economy.

As the key to the understanding of our work relates to the interaction between substantive uncertainty and monetary policy rules, few introductory remarks are worth to be drawn.

On the one hand the choice of a particular notion of uncertainty brings the attention to the occurrence of a theoretical debate where the challenge is focused on: i) the nature and existence of economic laws; ii) the nature of data generating processes (DGP) (which ultimately are the formal statistical expressions of the previous step); iii) the amount/quality of information available either at macro and micro level; iv) the treatment of information by heterogeneous agents.

On the other hand, the theme of monetary policy rules, at least requires to relate some stylized facts observed in the economy to the conduct of such policy by public authority.[2]

Both issues are grounded on their own flourishing reference literatures, which are (one might say unfortunately) quite separate from each other. While the rationale for separation is fairly obvious (the first relate to method, as the second refers to practical potential implementations), the attempt to create a connection sounds more ambitious and implies an evident trade-off: the more one tries to add to logical consistency some "universal" or "realistic" assumption on methodological side, the less he reaches "unique" or "stable" answers useful for policy suggestion. One can experience the effect of this trade-off on close inspection of the literature. On one side there is methodological debate open to "principle" discussions, where a variety of visions emerge, overlap, conflict and evolve, but always at risk of being self-referential.[3] On the other side there are "practitioners" who usually treat methodological issues as a black box, using mostly the same one, namely the one attracting highest consensus, despite the existence of alternatives which might be more appropriate in order to grasp the prevailing features of systems qualified by peculiar combinations of time and space[4].

Given this premise it is possible to understand better the nature of this paper. This is

[1] For more details see Egidi et al. (1991) and Variato (2004).

[2] See references listed in the next introductory paragraphs

[3] The interested reader can read, among others, Backhouse and Salanti (1999), Chick and Dow (2005), Goldsmith and Remmele (2005), Hindrick (2005), Lawson (2005a) (2005b)

[4] See, among others, Annand (2003), Hoover (2001), Hutchinson (2000), Moneta (2005) and Zouache (2004).

not supposed to be a place where methodological issues are treated in detail. Nevertheless, it is an occasion where methodological assumptions may be made more explicit. Here we take the Keynesian side[5]: a) having a question to answer in mind (i.e. the impact of monetary policy on system's dynamics); b) possessing a vision of the world (i.e. a structure of answers to the list of methodological questions seen in the points (i)-(iv) above mentioned); c) considering such vision not neutral (or fully objective), but simply as a convenient framework to "keep in the back of the mind"; d) we build a model meant to answer the question and (e) test it against empirical evidence.

Crucial to our analysis is step (b). A fully Keynesian model would require the assumption of "fundamental" uncertainty. In other words one should cast serious doubt on the very existence of economic laws enabled to be discovered; as a result one should avoid the use of ergodic DGP (e.g. Davidson (1988), (1994), (2001), and Lawson (1988),(1997)). At the moment such a choice would hinder the working of a useful formal model. Even though we think this is the proper assumption to deal with, the cost it imposes is too high: either one should to be able to change the instruments and methods used for macroeconomic policy design; or, by means conventional representative methods and instruments, one may find a way to match the complexities induced by the simultaneous presence of non-linear dynamic economic relations and non-ergodic stochastic processes. Both approaches, if proved successful, would radically affect the way Economics is handled, potentially leading in due time to a conceptual revolution. But such a time as not come yet.

As a result we take a rather compromising perspective knowing it is "not pure" from a methodological standpoint, but justifying it because it leads to the development of a model that while comparable with the "conventional view", contrast such position leading to more articulated policy suggestions.

Hence the use of "substantive" uncertainty. The attribute underlines the fact that agents have access to limited information: any time they have to undertake a decision they lack some piece of relevant information related to the decision itself; because of this they have to form expectations and therefore they are boundedly rational. The very fact information is limited at systemic level, matches with the presence of heterogeneous individuals and gives rise to asymmetric information. The use of substantive uncertainty simultaneously connects and distinguishes our theoretical set-up from the one of "fundamental" uncertainty, and from the one of "procedural" uncertainty. In fact, substantive and fundamental uncertainty are similar in the sense they both refer to a radical lack of information in the decisional process; on the converse they conflict because fundamental uncertainty strictly requires the a-priori use of non ergodic stochastic processes, while substantive uncertainty is compatible with any stochastic representation. Furthermore, both substantive and procedural uncertainty imply bounded rationality, and asymmet-

[5]See Keynes (1936), Arestis (1998) and Carabelli (1988).

ric information; on the other hands, while procedural uncertainty is due to individual heterogeneity, and in particular to a different capability to process the same available information (i.e. it is a micro economic issue with systemic implications), substantive uncertainty is due to an aggregate absence of information, which may be faced differently by heterogeneous agents (i.e. it is a macro economic issue with individual implications, which in turn feed-back to systemic effects).

Even though a realistic set-up would require the presence of both substantive and procedural uncertainty, in the present paper we deal only with substantive uncertainty; in other words we assume that agents are able to use information, when available. Anyway, individual heterogeneity together with substantive uncertainty prevent information to be symmetric. This justify why agents and monetary authority behave differently in the model.

Having discussed the methodological premises, we may now turn to the issue of monetary policy.

The role of monetary policy rules within small macroeconomic models has become a frequently studied topic in the literature. The macromodel is usually of the "new neoclassical synthesis" variety (see Goodfriend and King, 1997), which is a new, dynamic version of the old synthesis. In this framework, the rules are obtained either by means of an optimizing approach derived from optimal control techniques (see Ljungquist and Sargent, 2004) or by considering a policy reaction function, generally known as the Taylor rule (see Taylor, 1999), which is the option followed in this paper. As is well known, the Taylor rule (a linear feedback policy rule) is a policy of "leaning against the wind" that calls for nominal interest rates to be adjusted positively in response to inflation rates above target levels. The associated Taylor principle requires that this adjustment be more than one-to-one to assure stability. The logic of this policy is straightforward: by raising the nominal rate of interest by more than one-for-one in response to an increase in inflation, the central bank raises the real rate of interest, which, in turn, contributes to slowing down aggregate demand and thereby checking the inflationary process.

There is now a large literature that casts doubt on the general validity of these results, which are highly dependent on: i) the assumed specification of the model, ii) the amount of information available (see Woodford, 2003) and iii) the technicalities according to which the Taylor rule is formulated. The objective of the present paper is to investigate on these three fronts. First, the model assumes a stronger link between real and monetary aspects of the economy as stressed by Keynes (1936) in his vision of the "monetary theory of production". The interdependence depends not only on nominal rigidities in wage and price formation but also on the presence of debt and cash flows in the investment function as stressed by Minsky (1982). It follows that the IS equation is enriched in order to take these phenomena into consideration. Second, as the large amount of information implied became embarrassing, various degrees of imperfections and uncertainty have been built into the models. One of changes is the introduction of a measurement problem as far potential output is concerned. For instance, Orphanides and Williams (2002) express the Taylor rule as a function of inflation and unemployment. In this specification of the rule, they face the measurement problem related to the NAIRU. Bullard and Mitra (2002) go a step further by postulating that people know the model, but not the parameters, and so they learn. In this case, the dynamics are enriched by the concept of E-stability that helps determine whether rational expectations equilibria are stable under real time recursive learning dynamics. In a similar vein, Honkapohja and Mitra (2004) have shown how learning can converge to a sunspot equilibrium. In the present model, the assumption of rational expectation is dropped. Since agents do not know the model, they are assumed to be bounded rational forecasters that try to learn the values of the parameters by running regressions (for a discussion on model uncertainty, see Brock, Durlauf and West, 2003). Agents do not possess all the information required by the assumption of rational expectations even though they are more sophisticated than the backward-oriented agents of the past (e.g. Conlisk, 1996). People are supposed to behave like econometricians (e.g. Sargent, 1993), and their expectations should be consistent with outcomes (e.g. Hommes and Sorger, 1998). Finally, monetary authorities do not know the exact values of the parameters of their policy function and have to learn them.

This approach innovates the current literature in three ways. First, we employ a nonlinear approach as in Flaschel et al. (2001) rather than a log-linearized model. One advantage of this approach is that the model can have a greater variety of attractors (periodic or strange) as explained by Benhabib et al. (2003). The cost of nonlinearity is that the model does not yield closed-form solutions, and simulations must be carried out to study its properties. Parameters are calibrated to reflect values estimated in the relevant literature. Second, we suppose that people know neither the parameters nor the exact specification of the model, and they formulate expectations by econometric techniques. Agents, who do not know the true model, are assumed to form expectations according to a Markov-switching time series process. To form expectations they use probabilities of experiencing periods of good and bad times for growth and intervals of low and high inflation. Agents' beliefs are required to be consistent in the sense that, on average, their expectations match the outcomes of the economy. In addition, monetary authorities do not know the NAIRU (see Isard et al. 2001). Finally, even though current models are designed to examine inflation, its role is very limited. In some models a form of persistence is added to reinforce the role of inflation (see Wang and Wong, 2005). Others have started to consider it more intensively (see Wright, 2004) through the presence of debt in consumer spending. In our approach, inflation is intrinsic to the model through its impacts on cash flows; it is not brought into the analysis only through the objective function of the policy maker.

The following are the main results of the paper:

1. We have already shown that this type of model can create oscillations in the rate of growth of output and inflation that are persistent for a long period of time (see Fazzari et al, 2005). In this framework, the two main sources of endogeneity in the dynamics are analyzed: i) cash flows and debts are endogenously determined and have a powerful influence on investment, a primary factor in the creation of business cycles; ii) the process of learning induce further dynamics.

2. In the present paper, we show that these results are robust to different specifications of the Taylor rule and different expectational hypotheses. Because the model utilizes a Taylor rule, our results may shed light on the policy debate centered on that idea (e.g. Woodford, 2003, Clarida et al., 1999, Taylor, 1999, and Svensson, 2003). The Taylor principle in the "new neo-classical synthesis models" is based on an understanding of the monetary transmission mechanism that relies on price stickiness and substitution effects caused by changes in the real interest rate. The structure of our model is different because policy-induced changes in interest rates can also alter the values of such variables as cash flows and debts. In particular, the manipulation of the coefficient that relates to inflation in the Taylor rule can create stability only for a limited "corridor of stability". In contrast, manipulation of the coefficient relating to unemployment seems to be more stabilizing, a rather neglected result by the advocates of a more active policy. In fact, they usually support a stronger reaction to inflation than to unemployment.

3. A strategic element in shaping the dynamics of the system is how expectations are formulated. If one abandons the world of perfect foresight and introduces uncertainty and learning, the possibility of checking the cycle becomes even more difficult. Fluctuations persist despite changes in the parameters of the Taylor equations. These results are robust to changes in the specification of the Taylor rule and to the information structure of the model.

The structure of the paper is the following. In Section 2 some observations are presented on the "new neo-classical synthesis" models. In Section 3 a different model is introduced with the important addition of an investment function affected by interdependence between monetary and real aspects. Section 4 considers a specification of the monetary policy in a perfect foresight environment by means of simulations. In Section 5, the role of the parameters is considered in the linearized version of the model. Section 6 introduces expectations based upon a Markov regime-switching process. Section 7 considers learning both for agents and for monetary authorities, while Section 8 discusses the resulting dynamics. Section 9 contains conclusions.

2 The new neo-classical synthesis models: some criticisms

The canonical "new neo-classical synthesis" models consist of three equations: a IS equation, supplemented by a AS equation and a Taylor rule (see Woodford, 2003, Ch. 4). In these models the neo-classical analysis defines a dynamic path that represents, according to Woodford (2003, p. 9), "a sort of virtual equilibrium for the economy at each point of time, the equilibrium that one would have if wages and prices were not in fact sticky". At the same time the stickiness of prices and wages implies a short run determination of output akin to Keyenesian theory and therefore different from the tenets of the real business cycle literature.

These models aim at being microfounded and are usually based upon rational expectation, both aspects being essential to overcome the Lucas critique that may always lay an ambush in the case of economic policy exercises. It follows that the IS curve is mainly an Euler equation derived for the representative consumer in an intertemporal setting, while the AS curve is usually defined the new Keynesian Phillips curve. This curve is usually derived from the Calvo model (1983) that should be able to justify the presence of nominal price inertia in the equation.

This formulation, however, undergoes significant changes when it is inserted into econometric models. The necessity of fitting the data imposes some changes into the specification of the model that invariably introduce lagged variables in all the equations, so that the mixed forward-backward models, also named hybrid models, are the ones actually estimated (see Estrella and Fuhrer, 1998 and Ehrmann and Smets, 2003). In fact, the presence of lagged output and inflation in respectively the output and inflation equations seems to be necessary to fit the persistence in the data.

The justification of the hybrid formulation of the IS curve has not reached a vast consensus in the literature. What emerges is that the presence of lagged output in the IS equation has been motivated either by the presence of "rule of thumb" consumers who spend (with lags) all of their disposable income or by the presence of habit persistence in the agents' utility function (see Fuhrer, 2000). The type of interpretation chosen affects the values of the parameter on the lagged value of output and this can have important implications for the study of the dynamics.

As far as the AS curve is concerned, Ball, Mankiw and Reis (2005) point out that "because the new Keynesian Phillips curve lacks any source of inflation inertia, it makes absurdly counterfactual predictions about the effects of monetary policy"(page 3). One way of overcoming this difficulty is to suppose that those firms that are unable to re-optimize their price, adjust their price charged in the previous period, by a lagged inflation rate (see Gali and Gertler, 1999). In this case, the new Keynesian Phillips curve assumes a hybrid nature.

According to Ball, Mankiw and Reis (2005), however, this research strategy is far from being satisfactory because "by taking a weighted average of the two flawed models, the hybrid model of the Phillips curve ends up with the flaws of each"(page 4). They rather propose a behavioural approach that justifies why people are slow to process widely available macroeconomic information, which, according to the authors, is at the root of the monetary nonneutrality.

In what follows we shall follow a different approach. First of all, we assume that the problem is not only "sticky information" due to the necessity of filtering complex data, but the presence of uncertainty about the overall model of the economy. In this context, we assume that the agents try to be bounded rational forecasters, the exact meaning of the term being defined later on.

In this context of uncertainty, it becomes more natural to justify the presence of leads and lags in the equations. For instance, the consumption function can be justified on the basis of the permanent income hypothesis, where, however, the mixed of past and future is a changing average in time due to the process of learning of the consumers.[6] At the same time, in this context is indispensable to separate consumption from investment. In the present model, the IS curve is enriched by the presence of an investment function which is specified for a short- term analysis, where any link with the existing value of capital is neglected (for a study of this link, see Bullard and Eusepi, 2005 and Woodford, 2005). The investment function is based upon the real and the financial accelerator principles (see Bernanke et al., 1999, Fazzari et al., 1988, and Zarnovitz, 1999), where, due to uncertainty and the presence of asymmetric information, the presence of cash flow becomes relevant. And since the cash flow depends on debt, a new equation referring to this state variable must be introduced.

2.1 The Model

In order to compare our approach and the "new neo-classical synthesis" models in a more complete way, some further technical differences should be stressed. First, our model is set in terms of the rate of growth and not in log terms. This implies, among other things, a different formulation of the Taylor rule along with the introduction of a labor demand equation. Second, our model is nonlinear, whereas log-linearization is more commonly assumed in the literature. As a result, most solutions are not available in a closed form and simulations must be carried out. In such models it is important to introduce a structure of information that indicates when agents make their decisions.

[6] The relevance of credit constraints on consumption in creating nominal rigidities in the IS curve is stressed by Wright (2004), while Carroll (2001) stresses the relationship between uncertainty and precautionary saving, in raising the sensitivity of consumption to current income, as captured by the rule of thumb specification.

Third, the model is generic in terms of expectations and therefore need not necessarily use the limiting case of rational expectations. Finally, some equations are presented in intensive form.

The model consists of 7 equations, where the "hat" symbol denotes expectations. The investment i_t equation, which is cast in intensive form (investment is expressed as a fraction of nominal output), emphasizes the role of the accelerator through its dependence on the rate of growth g_t and of cash flow, the latter being a function of income distribution ω, the percentage of retained profits θ, and the debt service $(\frac{\eta_2 R_t d_t}{1+\hat{\pi}_t})$, where R_t is the nominal interest rate, d_t is debt divided by income, π_t is the inflation rate, and all variables are adjusted for the different dates of information availability. The debt equation, also expressed in intensive form, supposes that debt rolls over in each period. Consumption is assumed to depend on past and expected income and on the real rate of interest, as in models of the synthesis; it is contained in the g_t equation. Employment l_t is assumed proportional to the rate of growth for a given productivity τ, while the rate of unemployment is $u_t = 1 - l_t$, since labor supply is fixed at 1. The AS equation is based on the NAIRU formulation and is of a mixed backward-forward nature as discussed by Fuhrer and Moore (1995). The Taylor rule is formulated in one of the many possible specifications found in the literature (see in particular Walsh, 2003 and Bullard and Eusepi, 2005); the form we adopt is particularly convenient because it allows us to state the productivity gap in terms of the rate of growth. The basic model follows:

$$\pi_t = \alpha\hat{\pi}_t - \sigma_1(u_t - u^*) + (1-\alpha)\pi_{t-1} \tag{1}$$

$$R_t = R_t^* + \psi_1(\pi_t - \pi_0) + \psi_2(g_t - g_0) + \psi_3 R_{t-1} \tag{2}$$

$$d_t = \left[\frac{1+R_{t-1}}{(1+g_{t-1})(1+\pi_{t-1})}\right] d_{t-1} + \frac{i_{t-1}}{(1+g_{t-1})} - \theta(1-\omega_0) \tag{3}$$

$$i_t = \eta_0 + \eta_1\hat{g}_t + \eta_2\theta(1-\omega_0)(1+\hat{g}_t) - \frac{\eta_2 R_t d_t}{1+\hat{\pi}_t} \tag{4}$$

$$g_t = i_t + c_0(1+\hat{g}_t) + c_1 - c_2(\frac{1+R_t}{1+\hat{\pi}_t} - 1) - 1 \tag{5}$$

$$l_t = \frac{l_{t-1}(1+g_t)}{1+\tau} \tag{6}$$

$$u_t = 1 - l_t \tag{7}$$

The model has 8 unknowns: $\pi(t), d(t), i(t), g(t), l(t), u(t), R(t)$ and $R^*(t)$ in 7 equations, given the expectations about $\hat{\pi}_t$ and $\hat{g}_t$. It can be closed in several different ways. One way to close the model is to assume that the optimal nominal rate of interest is given, $R_t^* = R^*$. Furthermore, by the Fisher equation, one can also assume that the real rate of interest (r^*) is given, leading to

$$1 + R_0 = (1+r^*)(1+\pi_0),$$

where, through the Taylor rule, $R_0 = R^*/(1-\psi_3)$, and steady state values are indicated by the subscript 0. It follows that inflation rate π_0 is determined by this equation in a Wicksellian way as a bridge between the real and nominal rates of interest.

The determination of the steady state values of the other variables can be found in the following way.[7] Equation (1) determines u_0, the NAIRU. Equation (7) determines l_0, while equation (6) implies $g_0 = \tau$. The investment steady state i_0 is found in the growth equation,

$$i_0 = g_0(1-c_0) + c_2 * r_0 + (1 - c_0 - c_1).$$

The debt equation and the investment equation determine ω_0 and d_0.

By defining

$$A = i_0 - \eta_0 - \eta_1 g_0 + \eta_2 \frac{R_0}{(1+\pi_0)(g_0 - r_0)} i_0$$
$$B = \eta_2 \theta(1+g_0) + \eta_2 \theta \frac{R_0}{1+\pi_0} \frac{1+g_0}{g_0 - r},$$

we have $\omega_0 = 1 - \frac{A}{B}$. In this case ω_0 is endogenous, which makes sense in our model because investment and aggregate demand determine supply, investment determines saving and profits and therefore the share. In the present model, however, it remains constant because the dynamics of prices and wages are not considered separately as in the Goodwin tradition (see Velupillai, 2004). The steady-state debt ratio is

$$d_0 = \frac{i_0 - (1+g_0)(1-\omega_0)}{g_0 - r}.$$

Three observations on the steady state debt are worth mentioning. First, in accordance with the no Ponzi game assumption, d_0 must be bounded to avoid an infinite amount of debt. Second, the steady state value must be greater than zero because we want to analyze an economy with debt. The final restriction is that $R \geq 0$: there is a lower bound on the nominal rate of interest (see Benhabib et al., 2002).

3 The perfect foresight model

When studying the dynamics of the above system, one has to consider a couple of preliminary aspects. First of all, it is better to exclude that the dynamics are driven exclusively

[7] There are, of course, different ways to close the model. In a previous paper (see Fazzari et al., 2005), we assumed that r^* and ω_0 are given. In this case, the steady-state inflation rate π_0 cannot be determined by the supply curve, which is based upon the NAIRU hypothesis, but is determined simultaneously with the debt ratio by the investment and debt equations. This way of determining inflation resembles, *mutatis mutandis*, what has been called the fiscal theory of price determination (see Woodford, 2001). In the present case, it should be named the financial theory of inflation because it fixes the nominal amount of debt service in keeping with the requirements of the investment ratio. Another hypothesis is to assume that R^* and ω_0 are exogenous, while r_0 is endogenous.

by the learning process. In this case, it is better to start with a perfect foresight model. In a model with t-1 dating of expectations (see Evans and Honkapohja,2001), one obtains that

$$\begin{aligned} \hat{\pi}_{t|t-1} &= \pi_t \\ \hat{g}_{t|t-1} &= g_t. \end{aligned}$$

In the second place, the presence of nonlinearity implies the necessity of running simulations which, in turn, are made easier by the presence of some form of recursivity. In the present Section this will be obtained by introducing a lag in the rate of unemployment in the AS curve so that it becomes:

$$\pi_t = -\frac{\sigma_1}{1-\alpha}(u_{t-1} - u^*) + \pi_{t-1}, \tag{8}$$

Later on, this hypothesis will be dropped and recursivity will be obtained in a different way.

In this context, R_t, g_t and i_t are solved simultaneously in matrix-vector form as

$$\begin{pmatrix} 1 & -\psi_2 & 0 \\ \frac{c_2}{(1+\pi_t)(1-c_0)} & 1 & -\frac{1}{1-c_0} \\ \frac{\eta_2 d_t}{1+\pi_t} & -(\eta_1 + \eta_2\theta(1-\omega)) & 1 \end{pmatrix} \begin{pmatrix} R_t \\ g_t \\ i_t \end{pmatrix} = \begin{pmatrix} R^* + \psi_1(\pi_t - \pi_0) - \psi_2 g_0 + \psi_3 R_{t-1} \\ \frac{c_0+c_1+c_2-1}{1-c_0} - \frac{c_2}{(1-c_0)(1+\pi_t)} \\ \eta_0 + +\eta_2(1-\omega) \end{pmatrix},$$

or

$$L_t x_t = M_t,$$

from which we have $x_t = L_t^{-1} M_t$. The equations for d_t, l_t, and u_t are equations (3), (6), and (7), respectively.

The model has been simulated. The results are illustrated in Figure 1, and parameter values are given in Appendix A. We make three observations about these results. First, the parameter values are in line with econometric studies (see, for instance, Ehrman and Smets, 2003 and Woodford, 2003). Since many models do not explicitly include investment, the values of the coefficients are taken from Fazzari et al., 2005). Second, the response of our model to an impulse is similar to what happens in other models (see Christiano et al., 1997). For instance, if the rate of interest undergoes a temporary shock, unemployment rises, g falls along with investment, and inflation falls with a lag. Third, because these relationships are nonlinear, the system oscillates. Given these parameters, the system tends to fluctuate for long periods even though it is shocked for only one period.

The system is still fluctuating at $N = 10000$, which indicates that the endogenous forces are very strong.

Persistent fluctuations in the economy with myopic perfect foresight

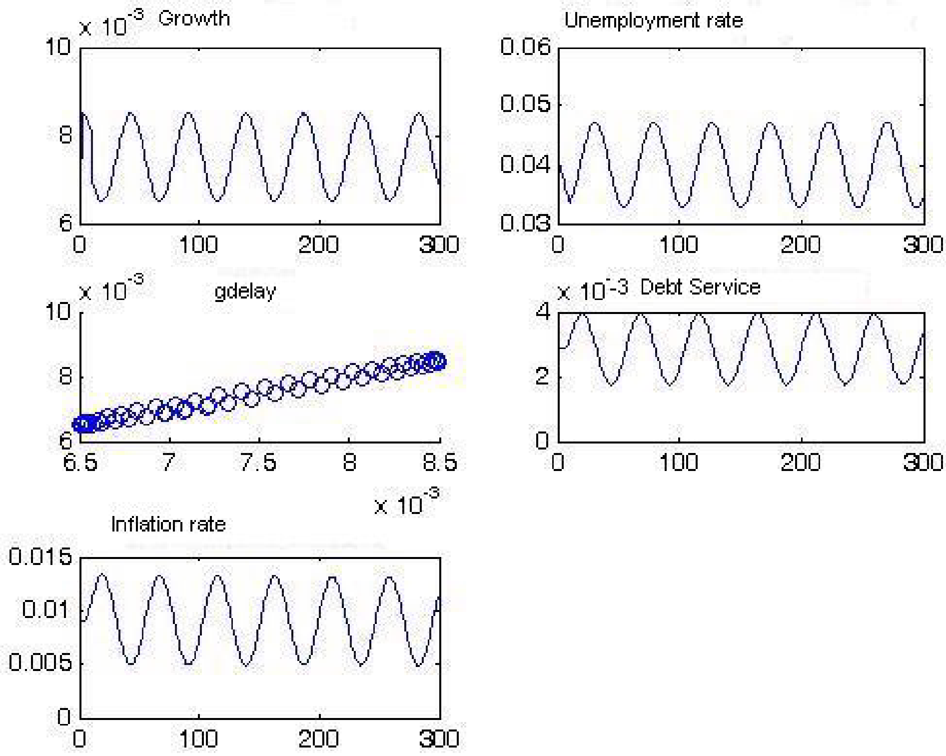

Figure 1: Persistent fluctuations in the economy with myopic perfect foresight

These results generalize those obtained in Fazzari et al. (2005), not only because a Taylor rule has been included, but also because they are compatible with the hypothesis of perfect foresight.

4 The Linearization of the model

In order to study the dynamics more carefully along with the role of the various parameters it is useful to consider a linearization of the model around its steady state. The following specification results, where the values of the multipliers are specified in Appendix B:

$$\begin{aligned} d_t &= d_0 + D_1(d_{t-1} - d_0) + D_2(i_{t-1} - i_0) - D_3(g_{t-1} - g_0) \\ &\quad + D_4(R_{t-1} - R_0) - D_5(\pi_{t-1} - \pi_0) \end{aligned} \tag{9}$$

$$i_t = i_0 + I_1(g_t - g_{0)} - I_2(R_t - R_0) - I_3(d_t - d_0) + I_4(\pi_t - \pi_0) \tag{10}$$

$$g_t = G_1 i_t - G_2 R_t + G_3 \pi_t + \frac{c_1 - 1}{1 - c_0} \tag{11}$$

$$\pi_t = P_1(u_{t-1} - u^*) + \pi_{t-1} \tag{12}$$

$$l_t = l_{t-1} + L_1(g_t - g_0) \tag{13}$$

$$R_t = R_t^* + \psi_1(\pi_t - \pi_0) + \psi_2(g_t - g_0) + \psi_3 R_{t-1} \tag{14}$$

We have dropped the unemployment equation and formulated the AS curve in terms of employment because the labor supply is constant. Given the above structure, it is possible to study the bifurcation properties of the various parameters. Table 1 shows the bifurcation pattern for the parameters of the Taylor rule. In particular, it is useful to study whether a changing value of a parameter succeeds in making the absolute value of a complex eigenvalue become 1. In that case, a closed orbit is created.[8]

Table 1: The bifurcation pattern for the parameters of the Taylor curve

Modulus of max eigenvalue	ψ_1	ψ_2	ψ_3
1	1.1	0.95	0.1
1.009	0.5	0.95	0.1
0.998	0.9	0.95	0.1
1.0013	3.0	0.95	0.1
0.99	1.1	0.95	0
0.99	1.1	4.0	0.1

The results of varying ψ_1, which reflects the reaction of the Taylor rule to the rate of inflation, are of particular interest. A very low value of ψ_1 is destabilizing in the sense that the system tends to explode, which is in agreement with the current literature. Increasing its value beyond a threshold value, however, also generates a cyclical destabilizing pattern. In between, there is an interval of stability in which fluctuations remain persistent because they converge slowly to the steady state. The reason for this non monotonic pattern is that the role of the rate of interest in our model is more complex than in other models.

[8] In order to study the dynamic properties of the closed orbit, a discrete version of the so called Hopf bifurcation theorem must be considered. This, in turn, requires consideration of the coefficient of the Taylor expansion up to the third order. See Azariadis (1993) and Benhabib et al. (2003).

It checks aggregate demand, but it also stimulates the growth of debt, and this creates fluctuations.[9]

In contrast, changes in ψ_2 and ψ_3 have monotonic effects, at least in the range of values considered. Increases in the former are stabilizing, while increases in the latter are destabilizing. The role of ψ_2 is noteworthy: it helps stabilize the system, but it is not the parameter usually considered by supporters of the zero inflation target. To understand the limit to the manipulation of this parameter, the assumption of perfect foresight must be dropped. Before doing this, we further explore dynamic results. Table 2 contains the results of some simulation exercises that explore sensitivity to parameter values.

Table 2: Some sensitivity exercises

Parameter	**Benckmark**	**New**	$\boldsymbol{\psi_1}$	$\boldsymbol{\psi_2}$
α	0.4	0.45	1	2
σ_1	0.05	0.1	0.9	2
θ	1	0.9	0.75	1.5
c_2	0.015	0.06	0.9	1
η_2	0.338	0.34	0.9	2

(Benchmark values: $\psi_1 = 1.1$, $\psi_2 = 0.95$)

The changes in ψ_1 and in ψ_2 are those necessary to obtain the threshold level at which the maximum absolute value of the eigenvalue equals 1. For instance, an increase in η_2, the coefficient in the investment function measuring the impact of cash flow, increases the absolute value of the eigenvector. To counter this tendency, the value of ψ_1 must become smaller than the benchmark value, or ψ_2 must become larger.

We draw two conclusions from these results. The first is that also the value of ψ_2 may play a role in checking fluctuations. The second is that the dynamics results are model specific, a well known fact in the literature (see Woodford, 2003). They depend on the specification of the Taylor rule and the dynamic nature of the overall model which governs the numbers of lags. These caveats hold true in the present model, even though we shall show how the results become more robust in the presence of learning.

[9]The results of our previous model (see Fazzari et al, 2005) can be obtained by setting $c_2 = 0$, $\theta = 1$, $\psi_1 = 1$, $\psi_2 = \psi_3 = 0$.

5 Alternative hypotheses about expectations

The hypothesis of perfect foresight requires too much information.[10] In an uncertain world, people do not know the true model. And if they do, they do not know the values of the parameters. We suppose that, over a medium-run perspective, people expect a dynamic pattern characterized by differences in performance between "good times" and "bad times." This state of knowledge is specified as a two-state Markovian model with high growth and low growth states (see Hamilton, 1989) and periods of "high" and "low" inflation. In this perspective we suppose that agents form their expectations according to a particular form of bounded rationality.[11] Hommes and Sorger (1998) argue that expectations must be consistent with the data in the sense that agents do not make systematic errors; e.g., the forecasts and the data should have the same mean and autocorrelations (see also Grandmont, 1998).

To this purpose, let us consider a model with contemporaneous expectations, i.e.

$$\hat{\pi}_{t+1} = E_t^* \pi_{t+1} \quad \text{and} \quad \hat{g}_{t+1} = E_t^* g_{t+1},$$

where the operator E_t^* stands for expectation at time t.[12] In this case, at the end of period t, agents believe that the growth rate in period $t+1$ will be (see also Clements and Hendry, 1999)

$$g_{t+1}^e = \alpha_1 + \beta_1 s_{t+1} + (\rho_1 + \mu_1 s_{t+1}) g_t + \epsilon_{1,t+1}, \tag{15}$$

where ϵ is a random variable with the properties assumed by Hamilton (1988) and s_{t+1} is a random variable that assumes the value 0 in the low state and 1 in the high state. It evolves according to the following transition probabilities:

$$\begin{aligned}
\Pr(s_{t+1} = 0 \mid s_t = 0) &= a_1 \\
\Pr(s_{t+1} = 1 \mid s_t = 0) &= 1 - a_1 \\
\Pr(s_{t+1} = 0 \mid s_t = 1) &= 1 - b_1 \\
\Pr(s_{t+1} = 1 \mid s_t = 1) &= b_1.
\end{aligned}$$

Given (15) as the forecasters' perceived law of motion, we follow Evans and Honkapohja

[10] Benhabib et al. (2003) study the impact of the Taylor rule in a perfect foresight model in continuous time. The authors note the presence of a cyclical solution along with the traditional saddle path solution stressed by the literature.

[11] While "rationality" implies that people maximize, "bounded" implies that they have limited information and cannot fully maximize (e.g. Sargent, 1993, Conlisk, 1996, Grandmont, 1998, and Evans and Honkapohja, 2001). Differences between the various approaches to modeling bounded rationality lie in the amount of information assumed.

[12] In case E_t stands for the mathematical expectations, one could obtain a rational expectational model. See Soderlind (1999).

(2003) and Honkapohja and Mitra (2004) in assuming that s_t and g_{t-1} are known and g_t is unknown at the time expectations are formed for g_{t+1}. We also assume

$$\hat{E}(g_t|s_t, g_{t-1}) = \alpha_1 + \beta_1 s_t + (\rho_1 + \mu_1 s_t) g_{t-1}.$$

Accordingly, for $s_t = 1$,

$$\begin{aligned}\hat{E}(g_{t+1}|s_t = 1, g_{t-1}) &= \alpha_1 + \beta_1 E(s_{t+1}|s_t = 1) \\ &\quad + [\rho_1 + \mu_1 E(s_{t+1}|s_t = 1)]\hat{E}(g_t|s_t = 1, g_{t-1}) \\ &= \alpha_1 + \beta_1 b_1 + [\rho_1 + \mu_1 b_1][\alpha_1 + \beta_1 + (\rho_1 + \mu_1) g_{t-1}] \\ &= [\alpha_1 + \beta_1 b_1 + (\rho_1 + \mu_1 b_1)(\alpha_1 + \beta_1)] \\ &\quad + [(\rho_1 + \mu_1 b_1)(\rho_1 + \mu_1)] g_{t-1},\end{aligned}$$

where the operator E is written as $\hat{E}$ to indicate its subjective character, which is not necessarily equal to the rational expectations objective conditional expectation.

If $s_t = 0$, the conditional forecasting rule is

$$\hat{E}(g_{t+1}|s_t = 0, g_{t-1}) = [\alpha_1 + \beta_1(1 - a_1) + (\rho_1 + \mu_1(1 - a_1))\alpha_1] + [\rho_1 + \mu_1(1 - a_1)]\rho_1 g_{t-1}.$$

A similar forecasting rule is applied to inflation, where the random state variable is denoted by z_t. Its perceived law of motion is

$$\pi^e_{t+1} = \alpha_2 + \beta_2 s_{t+1} + (\rho_2 + \mu_2 s_{t+1})\pi_t + \epsilon_{2,t+1},$$

and its transition probabilities are: [13]

$$\begin{aligned}\Pr(z_{t+1} = 0 \mid z_t = 0) &= a_2 \\ \Pr(z_{t+1} = 1 \mid z_t = 0) &= 1 - a_2 \\ \Pr(z_{t+1} = 0 \mid z_t = 1) &= 1 - b_2 \\ \Pr(z_{t+1} = 1 \mid z_t = 1) &= b_2.\end{aligned}$$

We also assume

$$\hat{E}(\pi_t|z_t, g_{t-1}) = \alpha_2 + \beta_2 z_t + (\rho_2 + \mu_2 z_t)\pi_{t-1}.$$

[13] The eigenvalues of the transition matrix are 1 and $-1 + \alpha_1 + b_1$. See Hamilton (1994). It follows that the values assigned to the probabilities in the Markov process must avoid explosive patterns. The results also depend on the values of the coefficient (α,β,ρ, and μ) that have been used in the expectation equations.

The forecast for this variable is, for $z_t = 1$,

$$\begin{aligned}\hat{E}(\pi_{t+1}|z_t = 1, \pi_{t-1}) &= \alpha_2 + \beta_2 E(z_{t+1}|z_t = 1) \\ &\quad + [\rho_2 + \mu_2 E(z_{t+1}|z_t = 1)]\hat{E}(\pi_t|z_t = 1, \pi_{t-1}) \\ &= \alpha_2 + \beta_2 b_2 + [\rho_2 + \mu_2 b_2][\alpha_2 + \beta_2 + (\rho_2 + \mu_2)\pi_{t-1}] \\ &= [\alpha_2 + \beta_2 b_2 + (\rho_2 + \mu_2 b_1)(\alpha_2 + \beta_2)] \\ &\quad + [(\rho_2 + \mu_2 b_2)(\rho_2 + \mu_2)]\pi_{t-1}\end{aligned}$$

If $z_t = 0$, the conditional forecasting rule is

$$\hat{E}(\pi_{t+1}|z_t = 0, \pi_{t-1}) = [\alpha_2 + \beta_2(1 - a_2) + (\rho_2 + \mu_2(1 - a_2))\alpha_2] + [\rho_2 + \mu_2(1 - a_2)]\rho_2\pi_{t-1}.$$

Two features of this approach are worth stressing. First, different stochastic variables for growth and inflation are introduced. The case of $s_t = z_t$ is a special case. Second, s_t and z_t are unobserved (latent) random variables that introduce regime switching (In our case, however, s_{t-1} and z_{t-1} are known. For a different hypothesis, see Schorfheide, 2005).This does not imply that they have no economic meaning.[14] The use of regime-switching can be interpreted as a convenient device to apply time series analysis to the problem of forecasting, and, in view of its popularity among forecasters, it may reflect their practices.

6 Learning unknown parameters

The value of the parameters of the expectations function are learned by the agents as assumed by Sargent (1999) and Akerlof, Dickens, and Perry (2000). Learning takes place by means of rolling regressions.[15] The Hamilton-type forecasts are embodied in the simulation model in the following way. To get the model started, naive expectations for the first 50 periods are assumed. After the first 50 periods, to make a forecast for period $t+1$, s_t is first considered. If, for example, it equals 1, a first order autoregressive regression with a constant is fitted to the previous observations on g_{t+1} for which $s_t = 1$, but no more than 50 observations are included in the regression. The parameters estimated by the regression and the current value g_{t-1} are then used to compute $\hat{g}_{t+1}$. An analogous computation is used to forecast $\hat{g}_{t+1}$ when $s_t = 0$ and to forecast π_{t+1}.

Before examining the impact of introducing Markovian regime-switching on the nature of fluctuations, we consider the possibility that the monetary authorities may be uncertain about the parameters of the policy reaction function. In particular, we suppose that the

[14] An association with 'animal spirits' is made by Howitt and McAfee (1992). See also Farmer (1999).
[15] For an analysis of the stability of learning process with this method, see Bullard and Mitra (2002).

monetary authorities want to implement the following rule,

$$R_t = R_t^* + \psi_1(\pi_t - \pi_0) - \psi_2(u_t - u_0) + \psi_3 R_{t-1}, \tag{16}$$

but may be uncertain over the value of the NAIRU, as a stream of literature has discussed (see Orphanides and Williams, 2002, and Isard et al., 2001).

In this case we again assume that the authorities learn this value from rolling regressions. We suppose that they observe the AS equation for 50 periods and then estimate its coefficient to infer the value of the NAIRU, which is $u_0 = -\sigma_3/\sigma_1$, so that the Taylor rule becomes:

$$R_t = R_t^* + \psi_1(\pi_t - \pi_0) - \psi_2(u_t - \hat{u}_0) + \psi_3 R_{t-1}. \tag{17}$$

By substituting this Taylor rule, along with the expectations functions and the learning process of the previous section into the initial model, we obtain a recursive structure if the change the specification of the IS, which in view of the presence of uncertainty and of asymmetric information may be made more dependent on the past. In particular, the cash flow in the investment equation is lagged one period as the empirical analysis by Chirinco (1993). The system then acquires the following structure of information. First, aggregate demand is determined on the basis of expectations of growth and inflation. Second, firms fix prices and monetary authorities set their policy. Finally, debt is adjusted and the labor market variables are determined. This model has a recursive structure that eliminates simultaneity.

7 The overall dynamics

The nonlinearity of the model again requires the use of simulations; these are presented in Figure 2 (see Appendix C for the parameter values). The results suggest a number of observations. First, even though expectations are not rational they tend to be consistent, as appears in Figure 2. In the second place, the presence of persistent fluctuations is confirmed. This result is not unexpected, because learning process introduce further dynamics into the system. (See Evans and Honkapohja, 2003, and Honkapohja and Mitra, 2004).

Also sensitivity analysis applied to this model brings about interesting results. First of all, changes in the parameter η_2, which represents the strength of financial aspects in the investment function, alter the structure of the power spectrum. In particular, the bigger its the value, the more low frequencies become relevant. In the second place, the results seem to be more robust to changes in the structure of information or in the specification in the Taylor rule (for instance, one can use expected inflation in the equation).This constitutes an important difference with respect both to the linearized model previously

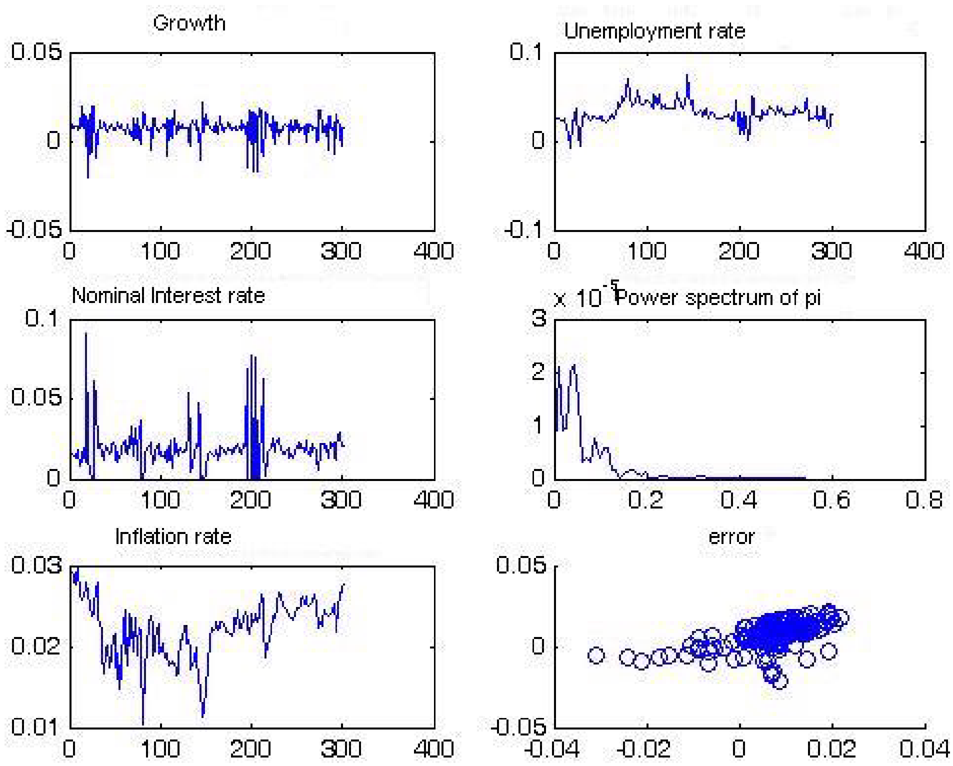

Figure 2: Simulation in the presence of uncertainty

discussed and the literature (see Bullard and Mitra, 2002). Thirdly, the dynamics can be affected by the parameters of the Taylor rule. In this environment of uncertainty, however, fluctuations remain persistent in spite of changes in the parameters of the Taylor equation. In the present case, they can only modify the structure of the power spectrum. Finally, learning can modify the power spectrum. Cogley and Sargent (2005) have interpreted the spectrum at zero as a measure of inflation persistence. Learning can increase this tendency and this can shed new light on the debate of the hybrid formulations discussed earlier as the only possible means of getting inertia.

8 Concluding remarks

In this paper, we have modified the "new neoclassical models" that are utilized to study the impact of monetary policy in two ways. Because of the presence of "substantive" uncertainty, the goods market takes an investment function where liquidity problems create a constraint to frims' behavior. On the other, we have dropped the hypothesis of rational expectations introducing bounded rationality and learning by the agents.

The particular form of uncertainty adopted has important implications on the results obtained. On one side, the interaction between nominal and real rigidities not only within an equation (or in a market, as is usually done in the literature) but between markets (investment that depends on cash flows and nominal rigidities in the working of the labor market the system) tends to display endogenous persistent fluctuations for reasonable values of the parameters, which remain within the range of estimates obtained in econometric research. On the other, the presence of learning generates further dynamics which can justify the presence of persistence in the system. This process of learning is compatible with the principle of rationality because agents are minimizing an objective function, while at the same time there must be consistency between expectations and results measured in some statistical sense.

We have investigated how the presence of the Taylor rule and different hypotheses about expectations can qualify the working of the monetary policy. Three main results have been obtained:

1. Because the literature on the Taylor principle has emphasized the possibility that the rate of interest may overreact to inflation, the emphasis has been on ψ_1, the coefficient linked to inflation. The more conservative is the monetary authority, the faster inflation is tamed. In the present model this is no longer true, because inflation and the rate of interest have more complex roles—they also have an impact on debt. A result is that an increase in ψ_1 can lead to an increase in the amplitude of the business cycle. In other words, there can be a trade-off between the control of inflation and the variability of the cycle.

2. The role of ψ_2, the coefficient linked to the real variable, can be stabilizing if it is applied either to growth or to employment/unemployment. This is a paradoxical result because much of the literature has concluded that a very conservative monetary policy is needed to tame a cycle.

3. These results heavily depend on the structure of information and the specification of the Taylor rule. When naive expectations are replaced by a more sophisticated scheme and a process of learning is introduced, an additional source of dynamics is imposed on the model that generates greater complexity, and the results become more robust in a double sense. On the one hand, fluctuations tend to persist with a different structure of information. On the other, they do not disappear for a broader range of values of ψ_1 and ψ_2, which can only modify the structure of the power spectrum. This implies that, in the presence of debt, monetary policy alone cannot pursue the two targets of checking inflation and taming the business cycle.

The analysis of the paper can be deepened in several directions. First, it is possible to enrich technical specifications of the model. For instance, one may add more equations, in order to capture: i) the impact of various degree of openess in the economy; ii) the interaction between monetary and fiscal policy; iii) the relationship between debt and other monetary and financial assets; and iv) a generalization of the Hamilton model (e.g. Aoki, 1996) where the probabilities of the Markov scheme might be endogenized (e.g. Filardo, 1994), or the learning mechanism introduces VAR models (e.g. Evans and Honkapohja, 2003), and/or the hypothesis that the (past) latent variables are known is suppressed (see Schorfheide, 2005).

Second, there remain two broad themes that probably require a longer gestation. As noted in the introduction, in a world of uncertainty, one should consider not only the way agents form expectations, but also the foundations of the equations themselves. As we mentioned, in this context the presence of persistence could be further justified. Whether the principles of behavioral macroeconomics (see Akerlof, 2002) can be used to justify both the workings of markets in a macro model and the presence of different agents (consumers, entrepreneurs and monetary authorities) with different amount of information remains an open question. The evaluation of the effects on equilibrium due the adoption of a "fundamentally" or at least "substantively" uncertain world is the second important methodological aspect that remains to be discussed. Among the possible line of reflection, one could jointly consider the endogenous formation of expectation and an explicit coordination mechanism. This aspect has been recently deepened by the approach relating on the so called agent-based models (see Leijonhufvud, 2006) and seems to open new interesting perspectives to macroeconomics.

A APPENDIX

A) The following are the parameters for the simulations summarized in Figure 1.

$N = 250$	$\tau = 0.075$	$\sigma_1 = 0.03$	$\sigma_2 = \sigma_1 * u_0$	$\theta = 1$	$c_0 = 0.4$
$c_1 = 0.4$	$c_2 = 0.025$	$\eta_0 = 0.1344$	$\eta_1 = 0.15$	$\eta_2 = 0.345$	$\alpha = 0.60$
$r_0 = 0.0015$	$u_0 = 0.04$	$\psi_3 = 0.05$	$\psi_1 = 1.1$	$\psi_2 = 0.65$	$R^* = 0.0010$

The system has been shocked in g by an amount equal to .001 for 10 periods.

B) The linearized version of the model can be expressed in the following way. The symbol "wave hat" denotes a deviation of the a variable from its steady state values. Constant values have been omitted. The system has been made compact through substitution of the variables into 6 equations.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\psi_1 & 0 & 1 & 0 & -\psi_2 & 0 \\ -I_4 & I_3 & I_2 & 1 & -I_1 & 0 \\ -G_3 & 0 & G_2 & G_1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -L_1 & 1 \end{pmatrix} \begin{pmatrix} \widetilde{\pi}_t \\ \widetilde{d}_t \\ \widetilde{R}_t \\ \widetilde{i}_t \\ \widetilde{g}_t \\ \widetilde{l}_t \end{pmatrix} =$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & P_1 \\ -D_4 & D_1 & D_5 & D_2 & -D_3 & 0 \\ 0 & 0 & \psi_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \widetilde{\pi}_{t-1} \\ \widetilde{d}_{t-1} \\ \widetilde{R}_{t-1} \\ \widetilde{i}_{t-1} \\ \widetilde{g}_{t-1} \\ \widetilde{l}_{t-1} \end{pmatrix},$$

where the coefficient of the matrices represent the following multipliers (all variables are defined as deviations from the steady state values):

$$
\begin{aligned}
P_1 &= \frac{\partial \pi_t}{\partial l_{t-1}} = \frac{\sigma_1}{(1-\alpha)}; \\
D_1 &= \frac{\partial d_t}{\partial d_{t-1}} = \frac{1+R_0}{(1+g_0)(1+\pi_0)}; \\
D_2 &= \frac{\partial d_t}{\partial i_{t-1}} = \frac{1}{1+g_0}; \\
D_3 &= \frac{\partial d_t}{\partial g_{t-1}} = \frac{i_0+(1+r_0)d_0}{(1+g_0)^2}; \\
D_4 &= \frac{\partial d_t}{\partial \pi_{t-1}} = \frac{(1+R_0)d_0}{[(1+g_0)(1+\pi_0)]^2}; \\
D_5 &= \frac{\partial d_t}{\partial R_{t-1}} = \frac{d_0}{(1+g_0)(1+\pi_{0)}}; \\
I_1 &= \frac{\partial i_t}{\partial g_{t-1}} = \eta_1 + \theta\eta_2(1-\omega); \\
I_2 &= \frac{\partial i_t}{\partial R_t} = \frac{\eta_2}{(1+\pi_0)}; \\
I_3 &= \frac{\partial i_t}{\partial d_t} = \frac{\eta_2 R_0}{(1+\pi_0)}; \\
I_4 &= \frac{\partial i_t}{\partial \pi_{t-1}} = \frac{\eta_2 R_0 d_0}{(1+\pi_0)^2}; \\
G_1 &= \frac{\partial g_t}{\partial i_t} = \frac{1}{1-c_0}; \\
G_2 &= \frac{\partial g_t}{\partial R_t} = \frac{c_2}{(1-c_0)}; \\
G_3 &= \frac{\partial g_t}{\partial \pi_t} = G_2; \\
L_1 &= \frac{\partial l_t}{\partial i_t} = \frac{l_0}{1+\tau_0}
\end{aligned}
$$

The following parameters are used for Tables 1 and 2:

$N = 250$	$\tau = 0.075$	$\sigma_1 = 0.05$	$\sigma_2 = \sigma_1 * u_0$	$\theta = 1$	$c_0 = 0.4$
$c_1 = 0.4$	$c_2 = 0.015$	$\eta_0 = 0.1344$	$\eta_1 = 0.15$	$\eta_2 = 0.338$	$\alpha = 0.40$
$r_0 = 0.0025$	$u_0 = 0.04$	$\psi_3 = 0.1$	$\psi_1 = 1.1$	$\psi_2 = 0.95;$	$R^* = 0.0028$

The modulus of maximum (complex) eigenvalue of the system $z_t = X^{-1}Y z_{t-1}$ with the above parameters is 1.

For Table 3 the upper matrix has been modified in the following way

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\psi_1 & 0 & 1 & 0 & 0 & -\psi_2 \\ -I_4 & I_3 & I_2 & 1 & -I_1 & 0 \\ -G_3 & 0 & G_2 & G_1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -L_1 & 1 \end{pmatrix}$$

The parameters used are the following:

$$\begin{array}{cccccc} N = 250 & \tau = 0.075 & \sigma_1 = 0.05 & \sigma_2 = \sigma_1 * u_0 & \theta = 1 & c_0 = 0.4 \\ c_1 = 0.4 & c_2 = 0.025 & \eta_0 = 0.1344 & \eta_1 = 0.15 & \eta_2 = 0.34 & \alpha = 0.40 \\ r_0 = 0.0025 & u_0 = 0.04 & \psi_3 = 0.717 & \psi_1 = 1.2 & \psi_2 = 0.237; & R^* = 0.0028 \end{array}$$

C) The following are the parameters behind Figure 2.

$$\begin{array}{cccccc} N = 200 & \tau = 0.075 & \sigma_1 = 0.05 & \sigma_2 = \sigma_1 * u_0 & \theta = 1 & c_0 = 0.4 \\ c_1 = 0.4 & c_2 = 0.025 & \eta_0 = 0.1344 & \eta_1 = 0.15 & \eta_2 = 0.40 & \alpha = 0.20 \\ r_0 = 0.0025 & u_0 = 0.04 & \psi_3 = 0.1 & \psi_1 = 1.15 & \psi_2 = 0.65; & R^* = 0.0075 \end{array}$$

The parameters of the stochastic components are the following:

$$\begin{aligned} \alpha_1 &= \frac{g_0 \left[1 - (\rho_1 + \mu_1 b_1)(\rho_1 + \mu_1)\right] - \beta_1 b_1 - \beta_1 (\rho_1 + \mu_1 b_1)}{1 + \rho_1 + \mu_1 b_1} \\ \alpha_2 &= \frac{\pi_0 \left[1 - (\rho_2 + \mu_2 b_2)(\rho_2 + \mu_2)\right] - \beta_2 b_2 - \beta_2 (\rho_2 + \mu_2 b_2)}{1 + \rho_2 + \mu_2 b_2} \end{aligned}$$

these are obtained by setting $s = z = 1$ (resp., $s = z = 0$) and solving from the steady state expectation formula.

The other parameters are:

$$\begin{array}{ccccc} a_1 = 0.4 & a_2 = 0.45 & b_1 = 0.6 & b_2 = 0.8 & \beta_1 = 0.001 \\ \beta_2 = 0.0002 & \rho_1 = 0.55 & \rho_2 = 0.5 & \mu_1 = 0.43 & \mu_2 = 0.49 \end{array}$$

A.1 List of Definitions

$d_t = \frac{D_t}{p_{t-1} y_{t-1}}$ = debt per unit of nominal income at the beginning of period t;

$g_t = \frac{y_t}{y_{t-1}} - 1$ = output rate of growth;

$i_t = \frac{I_t}{p_{t-1} y_{t-1}}$ = gross investment per unit of income.

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