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Iterative algorithm for finding equilibrium prices in a spatial electricity market

E. Allevi[1], A. Gnudi[2], I.V. Konnov[3], E.R. Kurbanova[3], and M.T. Vespucci[4]

Abstract. We consider a restructured electricity market divided in zones, where sell bids and purchase bids are presented by generating companies and customers respectively. The Market Operator has to fix the zonal prices as well as to decide the energy quantity to be accepted for each bid, in order to maximize the social surplus, while taking into account transmission constraints among zones. The market equilibrium is the result of the interactions among power producers, consumers and the Market Operator: this process generates a list of accepted purchase and sell quantities, clearing zonal prices and power flows among zones.

In the auction scheme introduced in [1] for the Italian market (see also [2]), if the price declared in a sell bid in zone k is higher than the price fixed by the Market Operator in that zone, then the offered energy is not accepted, while the whole offered volume is accepted if the bid price is lower than the zonal price. Purchase bids are treated analogously. If the bid price is exactly equal to the zonal price, then any energy volume between zero and the maximum offered quantity can be accepted by the Market Operator.

In this paper we consider a modified auction scheme where all bids participate to the auction, with the exception of sell bids with price greater than the zonal price and purchase bids with price less than the zonal price: for all other bids, any energy volume between zero and the maximum offered quantity can be accepted by the Market Operator.

With this modified auction, an algorithm is proposed, based on the Nelder and Mead simplex method, in order to find the global maximum of the social surplus function.

1 Introduction

In the new scenario introduced by the deregulation process, generating companies sell their own production by presenting bids on the day-ahead market for each hour of the following day. The Market Operator has to satisfy the hourly energy demand by choosing the cheapest suppliers, taking into account, at the same time, constraints on power flow exchanges among the master areas in which the network is divided (security network constraints).

Therefore in the recently restructured electricity markets, power generation is ensured on a competitive basis rather than being a privilege of a monopolistic utility. Independent power producers offer their production by presenting bids on the day-ahead market, aiming at maximizing their own

[1]Department of Quantitative Methods, Brescia University, Contrada S. Chiara, 50, Brescia 25122, Italy

[2]Department of Mathematics, Statistics, Computer Science and Applications, Bergamo University, Via dei Caniana 2, Bergamo 24127, Italy

[3]Department of Applied Mathematics, Kazan University, ul. Kremlevskaya,18, Kazan 420008, Russia

[4]Department of Engineering and Management, Bergamo University, Via Marconi 5, Dalmine 24044, Italy

profits and taking into account the rules set by the Market Operator. The Market Operator aims at maximizing the social benefit, while taking into account transmission constraints. The market equilibrium is the result of the interactions among power producers and the Market Operator. In paper [1], the authors proposed a model of the spatial electricity market which is reduced to a mixed integer optimization problem and the corresponding solution method. In this paper we consider a modification of this model formulated as a global continuous optimization problem and propose the simplex based search method to find its solutions.

An electricity market involves K zones; i.e. they are denoted by the index k, with $k = 1,\ldots, K$. The different markets are interconnected by H links, where link h represents a single transmission line between two zones. We denote the customers and the generating units by indices i and j, respectively, where, for zone k, i takes values in set I_k and j takes values in set J_k. Moreover, we introduce the following parameters:

- $\overline{QA}_i^k$ is the maximal quantity of energy, declared by customer i in zone k;
- $\overline{QV}_j^k$ is the maximal quantity of energy declared by supplier j in zone k;
- PA_i^k and PV_j^k are the purchasing and selling prices associated with customer i and supplier j in zone k;
- $\underline{TR}_h$ and $\overline{TR}_h$ are the lower and upper bounds on power flow on link h;
- s_h^k is the contribution of zone k to the power flow on link h. The values of this parameter depend on the network topology. They are limited to be either 0 or 1.

Purchase and sell bids are presented to the Market Operator, who establishes clearing energy prices in each zone. P_k is the clearing price for the electric power in zone k and $P = (P_1, P_2, \ldots, P_K)$ is the vector of clearing prices.

In [1] the model MOD-ZON-STD has been introduced, in order to determine the market equilibrium. Such model is as follows:

$$\max_{QA_i^k, QV_j^k} \sum_{k=1}^{K} \left(\sum_{i\in I_k} PA_i^k QA_i^k - \sum_{j\in J_k} PV_j^k QV_j^k \right) \tag{1}$$

$$0 \le QA_i^k \le \overline{Q}A_i^k, \quad for\ all\ i,\ k \tag{2}$$

$$0 \le QV_j^k \le \overline{Q}V_j^k, \quad for\ all\ j,\ k \tag{3}$$

$$E_k = \sum_{j\in J_k} QV_j^k - \sum_{i\in I_k} QA_i^k, \quad for\ all\ k \tag{4}$$

$$\sum_{k=1}^{K} E_k = 0 \tag{5}$$

$$TR_h = \sum_{k=1}^{K} s_h^k E_k, \quad for\ all\ h \tag{6}$$

$$\underline{TR}_h \leq TR_h \leq \overline{TR}_h, \quad \textit{for all } h \tag{7}$$

This model determines the power quantities that have to be accepted by the Market Operator in order to obtain market equilibrium. It can be shown that the equilibrium prices, potentially different in market zones, are functions of the Lagrange multipliers λa_i^k of constraints (2) and λv_j^k of constraints (3). Indeed let us consider the case where all purchase bids in zone k are either totally accepted or totally rejected, while there exists a sell bid which is partially accepted: the price declared in this bid is the clearing price P_k of zone k and the generator which presented this bid is called marginal generator. Consider now a generator whose bid has been completely accepted (that is whose bid price is $PV_j^k < P_k$). If this generator offers one more energy unit at price PV_j^k, this unit will be accepted and will substitute one unit offered by the marginal generator. The effect of this substitution on the objective function is $\lambda v_j^k = -(PV_j^k - P_k)$. The same analysis can be made on the purchase side, obtaining $\lambda a_i^k = PA_i^k - P_k$.

In the following section we consider an extension of the model in [1].

2 Description of the model

Now we consider two schemes of this auction, which establish sale volumes QV_j^k, $k = 1, \ldots, K$, and purchase volumes QA_i^k, $k = 1, \ldots, K$. The first scheme determines the following rules for the first stage of the auction:

$$\begin{cases} QA_i^k = \overline{QA}_i^k \text{ if } P_k < PA_i^k \\ 0 \leq QA_i^k \leq \overline{QA}_i^k \text{ if } P_k = PA_i^k \\ QA_i^k = 0 \text{ if } P_k > PA_i^k \end{cases} \tag{8}$$

$$\begin{cases} QV_j^k = 0 \text{ if } P_k < PV_j^k \\ 0 \leq QV_j^k \leq \overline{QV}_i^k \text{ if } P_k = PV_j^k \\ QV_j^k = \overline{QV}_i^k \text{ if } P_k > PV_j^k \end{cases} \tag{9}$$

If the supplier's declared price is higher than the established one, he wouldn't take place in auction while if the declared price is lower than the established one, he would sell the whole volume of the electric power as an encouragement. The same situation is with the customer. If the customer's declared electric power price is lower than the established one of this region, then this customer wouldn't take place in auction, but if the declared price is higher than the established one, then customer's purchase request would be completely satisfied.

It means that sale and purchase volumes are fixed for the participants whose prices are not equal to the zonal prices. Volumes for other participants of the auction are established at the second stage, which consists in solving the following optimization problem:

$$B(P) = \max_{QA_i^k, QV_j^k} \sum_{k=1}^{K} \left(\sum_{i \in I_k} PA_i^k QA_i^k - \sum_{j \in J_k} PV_j^k QV_j^k \right) \tag{10}$$

$$E_k = \sum_{j \in J_k} QV_j^k - \sum_{i \in I_k} QA_i^k \tag{11}$$

$$\sum_{k=1}^{K} E_k = 0 \tag{12}$$

$$\sum_{k=1}^{K} P_k E_k = 0 \tag{13}$$

$$TR_h = \sum_{k=1}^{K} s_h^k E_k \tag{14}$$

$$\underline{TR}_h \leq TR_h \leq \overline{TR}_h \tag{15}$$

Constraint (11) defines E_k, the excess supply in zone k; constraint (12) reflects the equality of volumes of the energy sold and bought. Constraint (13) results from conservation of monetary balance in the system. Constraint (15) reflects the actual limitations for the volumes of transmitted energy in every power link. The purpose of this paper is to develop an iterative solution algorithm for finding equilibrium prices in the spatial electricity market, which involves problem (10) subject to (8), (9) and (11)–(15).

In the presence of the small number of regions and participants, such problem may be solved by enumerating all the variants, but it is impossible if the number of zones is more than ten, and the number customers and suppliers in each regions is, for example, more than one hundred. Let us now consider the function of social surplus; its value $B(P)$ can be calculated algorithmically. At first, we can define the sets I'_k and J'_k, $k = 1, \ldots, K$, which denote, respectively, purchase bids and sell bids, whose declared prices are not equal to the zonal prices fixed by the Market Operator, i.e. $PA_i^k \neq P_k$, $PV_j^k \neq P_k$. For each purchase bid $i \in I'_k$ and each sell bid $j \in J'_k$ we can define sale and purchase volumes using conditions (8) and (9). Afterwards, we solve the following linear programming problem: we need to maximize the social benefit function subject to the above constraints for the variables $QA_i^k \leq \overline{QA}_i^k$, $i \in I_k \setminus I'_k$ and $QV_j^k \leq \overline{QV}_j^k$, $j \in J_k \setminus J'_k$, $k = 1, \ldots, K$, and conditions (11)-(15). If the linear programming problem is inconsistent, the value of the social surplus function is supposed to be equal to $-\infty$.

The domain of the social benefit function $B(P)$ is contained in the non-negative orthant, i.e. $P = (P_1, P_2, ..., P_K) \in \Re_+^K$. The so defined function is discontinuous and usually does not possess monotonicity properties. Therefore the gradient of this function cannot be calculated.

That is why gradient methods are not applicable for solving the maximization problem. In this case it is better to use methods of discrete optimization. For this reason, we decide to modify the condition of the auction. We replace conditions (8)–(9) with the following:

$$\begin{cases} 0 \leq QA_i^k \leq \overline{QA}_i^k \text{ if } P_k \leq PA_i^k \\ QA_i^k = 0 \text{ if } P_k > PA_i^k \end{cases} \tag{16}$$

$$\begin{cases} QV_j^k = 0 \text{ if } P_k < PV_j^k \\ 0 \leq QV_j^k \leq \overline{QV}_j^k \text{ if } P_k \geq PV_j^k \end{cases} \tag{17}$$

This small change appears to expand significantly the domain of continuity of the social cost surplus function, and allows us to apply iterative methods of continuous optimization for solving the modified maximization problem.

3 A mixed integer program for the modified auction

In this section we consider the mixed integer program which models the problem described in the previous section.

If $P_k < \min_j(PV_j^k)$, the total production in zone k is 0, also if $P_k > \max_i(PA_i^k)$ the total demand in zone k is 0. Therefore we only have to consider values of P_k in the following interval

$$\min_j(PV_j^k) \leq P_k \leq \max_i(PA_i^k).$$

The price PA_i^k, declared in purchase bid i presented in zone k, represents an upper bound on the zonal price P_k at which the customer is willing to buy energy: if δa_i^k is a binary variable such that

$$\delta a_i^k = \begin{cases} 0 & \text{do not buy energy} \\ 1 & \text{buy energy} \end{cases}$$

then constraints

$$\left(PA_i^k + \varepsilon\right)\left(1 - \delta a_i^k\right) \leq P_k \leq PA_i^k + M\left(1 - \delta a_i^k\right)$$

imply that $\delta a_i^k = 1$ if and only if $P_k \leq PA_i^k$ (therefore $\delta a_i^k = 0$ if and only if $P_k > PA_i^k$), with $M = \max_{i,j,k}(PA_i^k,\ PV_j^k)$ and ε a small positive number. As a consequence, by adding the constraint

$$0 \leq QA_i^k \leq \delta a_i^k\ \overline{QA}_i^k,\quad i \in I_k,\ k = 1, \ldots, K$$

we include in the model conditions (16).

Analogously, the price PV_j^k, declared in sell bid j presented in zone k, represents a lower bound of the zonal price P_k at which the producer is willing to sell energy: if δv_j^k is a binary variable such that

$$\delta v_j^k = \begin{cases} 0 & \text{do not sell energy} \\ 1 & \text{sell energy} \end{cases}$$

then constraints

$$PV_j^k \cdot \delta v_j^k \leq P_k \leq PV_j^k - \varepsilon + (M + \varepsilon) \cdot \delta v_j^k$$

imply that $\delta v_j^k = 1$ if and only if $P_k \geq PV_j^k$ (therefore $\delta v_j^k = 0$ if and only if $P_k < PV_j^k$). By adding the constraint

$$0 \leq QV_j^k \leq \delta v_j^k\ \overline{QV}_j^k,\quad j \in J_k,\ k = 1, \ldots, K$$

we include conditions (17) in the model.

Therefore the model is the following:

$$B(P) = \max_{QA_i^k, QV_j^k} \sum_{k=1}^{K} \left(\sum_{i \in I_k} PA_i^k QA_i^k - \sum_{j \in J_k} PV_j^k QV_j^k \right) \tag{18}$$

$$\left(PA_i^k+\varepsilon\right)\left(1-\delta a_i^k\right)\leq P_k\leq PA_i^k+M\left(1-\delta a_i^k\right),\quad i\in I_k,\ k=1,\ldots,K \tag{19}$$

$$PV_j^k\cdot\delta v_j^k\leq P_k\leq PV_j^k-\varepsilon+(M+\varepsilon)\cdot\delta v_j^k,\quad j\in J_k,\ k=1,\ldots,K \tag{20}$$

$$0\leq QA_i^k\leq\delta a_i^k\,\overline{QA}_i^k,\quad i\in I_k,\ k=1,\ldots,K \tag{21}$$

$$0\leq QV_j^k\leq\delta v_j^k\,\overline{QV}_j^k,\quad j\in J_k,\ k=1,\ldots,K \tag{22}$$

$$E_k=\sum_{j\in J_k}QV_j^k-\sum_{i\in I_k}QA_i^k \tag{23}$$

$$\sum_{k=1}^{K}E_k=0 \tag{24}$$

$$\sum_{k=1}^{K}P_kE_k=0 \tag{25}$$

$$TR_h=\sum_{k=1}^{K}s_h^kE_k \tag{26}$$

$$\underline{TR}_h\leq TR_h\leq\overline{TR}_h \tag{27}$$

$$\delta a_i^k\in\{0,1\},\quad i\in I_k,\ k=1,\ldots,K \tag{28}$$

$$\delta v_j^k\in\{0,1\},\quad j\in J_k,\ k=1,\ldots,K \tag{29}$$

4 A new algorithm and numerical results

We have chosen the simplex search method by Nelder and Mead as a basis (see [3]). To implement the method, we need to choose the initial simplex thoughtfully. For example, when all the vertices of the initial simplex lie in the domain of uncertainty of the cost function (where it equals to $-\infty$) and if the first step gives us the vertex P for which $F(P)=-\infty$, the algorithm fails. That is why we must choose the initial approximation $P^{(m)}=(P^{1^{(m)}},\ldots,P^{K^{(m)}})$ carefully by using the condition

$$PA_i^k\leq P^{k^{(m)}}\leq PV_j^k,\quad k=1,\ldots,K,\ \ m=1,\ldots,K+1.$$

In addition, the distance between two neighboring vertices of an initial simplex should be fitted in such a way that the next vertex would not lie in the domain of uncertainty of the social benefit function.

For the analysis of the developed algorithm, two kinds of tests have been carried out. The first testing aimed at revealing features of the solution of the problem using the constructed algorithm.

For the pictorial presentation of results and for the simplicity of calculation, the following problem of dimensionality $K = 2$ was solved with one transmission line:

$|I_1| = 3$; $|I_2| = 3$; $|J_1| = 3$; $|J_2| = 3$; the matrices

$$PA_i^k = \begin{bmatrix} 10 & 8 & 6 \\ 12 & 9 & 6 \end{bmatrix}, \quad PV_j^k = \begin{bmatrix} 10 & 9 & 8 \\ 11 & 9 & 7 \end{bmatrix},$$

$$\overline{QA}_i^k = \begin{bmatrix} 10 & 5 & 10 \\ 5 & 15 & 5 \end{bmatrix}, \quad \overline{QV}_j^k = \begin{bmatrix} 2 & 10 & 5 \\ 12 & 6 & 14 \end{bmatrix},$$

represent declared prices and volumes;

$S_1^1 = 1, \quad S_1^2 = 1$;

$\underline{TR}_1 = -100$ and $\overline{TR}_1 = 100$.

For this function, values at the points of a grid of size 20×20 have been calculated with Step 1. The results obtained are given in Table A.1 and in Figure B.1. Using the selected points, we present the graph of the social benefit function $B(P)$. (See Figures B.2 and B.3). These plots confirm the assumption that the domain of the social benefit function is a discrete set of local areas where the function is continuous. The solution of the test problem, found by using the developed algorithm, is now presented in Table A.2. The problem was solved several times for different initial simplexes. Finally, two local maximum different points of the social benefit function were obtained as solutions of a test problem.

Also, several series of tests were carried out in order to analyze the dependence of the algorithm convergence rates from various parameters of the problem (e.g. number of regions K or of transmission lines H) and from the choice of the initial point $P^{(0)}$.

A sequence of test problems was solved with increasing dimensionality $K = 2, \ldots, 99$ keeping the number of sell and purchase bids in each zone constant. The declared prices were in the interval [1, 30], the declared volumes were in the interval [1, 20]. The results of the tests are presented in Table A.3, from which we conclude that the algorithm convergence rate depends at a greater extent on input data than on dimensionality.

Next, the same sequence of test problems with increasing dimensionality $K = 2, \ldots, 12$ and the same numbers of bids (3 suppliers and 3 customers) in each region was also solved. The declared prices were in the interval [1, 30], the declared volumes were in the interval [1, 20]. Here in two cases the parameter H was constant and equal to 1, but the point of the initial approximation varied. Also, in the third case the parameter H varied: when the number of regions increased, the number of electricity transmission lines $H = K - 1$ increased, too. Results of testing are presented in Table A.4. Comparing the results of the first and third test, we found, that the growth of the number of transmission lines has not lead to essential growth if the number of iterations of Nelder - Mead's method. But in spite of such preservation of the number of steps, time expenses for the solution of the problem with big values of the parameter H were rather considerable. It is explained by the growth of the dimensionality of the linear programming problem (3) - (6).

We carried out additional series of experiments in order to investigate the influence of constraint (13). The first example involved three zones (K=3), the other data are given in Table A.5. First we solved the general problem, i.e. the value B(P) was calculated as a solution of problem (10) subject to (8), (9), (11)–(15). The results of computations with Nelder-Mead's Method and the Coordinate Descent Method are presented in Table A.6.

Next, we solved the relaxed problem, where the value B(P) was calculated as a solution of problem (10) subject to (8), (9), (11),(12),(14),(15), i.e. we removed constraint (13). The results of computations with Nelder-Mead's Method and the Coordinate Descent Method are presented in Table A.7. They showed essential influence of constraint (13) on convergence of both the methods.

We investigated properties of both the methods on an example with real data. It involved 19 regional markets and 18 transmission links; see Figure B.4. The auction involved 72 participants. The problem was solved with both Nelder-Mead's Method and the Coordinate Descent Method. Usually, their separate work yielded points of local maximum of function B (P). However, the combined using of these methods gave the precise solution.

5 Conclusion

The results of the testing show that 1) problem 3 is a problem of global optimization; 2) the given algorithm in most cases finds a local maximum which is close enough to global; 3) the obtained value has significant improvement in comparison with initial points. Hence, the given approach is useful for solving the task under consideration. However, to obtain more exact results it is necessary to consider an opportunity of using other methods of solving the problem. This is planned to be carried out in the future work.

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Part A

Table A1. Values of the social surplus function.

	3	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
3	0	0	0	0	25	25	15	15	25	25	0	0	0	0	0	0	0	0
4	0	0	0	0	25	25	15	15	25	25	0	0	0	0	0	0	0	0
5	0	0	0	0	25	25	15	15	25	25	0	0	0	0	0	0	0	0
6	0	0	0	0	25	25	15	15	25	25	0	0	0	0	0	0	0	0
7	0	0	0	0	**52**	25	15	15	25	25	0	0	0	0	0	0	0	0
8	10	10	10	10	25	**58**	15	15	25	25	10	10	10	10	10	10	10	10
9	10	10	10	10	25	25	**58**	15	25	25	10	10	10	10	10	10	10	10
10	10	10	10	10	25	25	25	**54**	25	25	10	10	10	10	10	10	10	10
11	0	0	0	0	25	25	25	25	25	25	0	0	0	0	0	0	0	0
12	0	0	0	0	25	25	25	25	25	25	0	0	0	0	0	0	0	0
13	0	0	0	0	25	25	25	25	25	25	0	0	0	0	0	0	0	0
14	0	0	0	0	25	25	25	25	25	25	0	0	0	0	0	0	0	0
15	0	0	0	0	25	25	25	25	25	25	0	0	0	0	0	0	0	0
16	0	0	0	0	25	25	25	25	25	25	0	0	0	0	0	0	0	0
17	0	0	0	0	25	25	25	25	25	25	0	0	0	0	0	0	0	0
18	0	0	0	0	25	25	25	25	25	25	0	0	0	0	0	0	0	0
19	0	0	0	0	25	25	25	25	25	25	0	0	0	0	0	0	0	0
20	0	0	0	0	25	25	25	25	25	25	0	0	0	0	0	0	0	0

Table A2. (*W_0* is the initial point)

Step number	Top number (w)	W_0=(10,10)			W_0=(1,1)		
		P_1	P_2	F(w)	P_1	P_2	F(w)
0	0	10	10	54	1	1	0
	1	19.659	12.588	0	10.659	3.588	0
	2	12.588	19.659	0	3.588	10.659	15
1	0	10	10	54	2.294	5.83	0
	1	14.83	11.294	25	7.124	7.124	52
	2	11.294	14.83	0	3.588	10.659	15
2	0	10	10	54	8.418	11.953	25
	1	14.83	11.294	25	7.124	7.124	52
	2	12.975	8.556	25	3.588	10.659	15
3	0	10	10	54	8.418	11.953	25
	1	14.83	11.294	25	7.124	7.124	52
	2	12.135	11.693	25	5.679	10.099	15
4	0	10	10	54	7.771	9.539	15
	1	12.415	10.647	25	7.124	7.124	52
	2	11.067	10.846	25	6.402	8.611	25
5	0	10	10	54	7.447	8.331	25
	1	11.207	10.324	25	7.124	7.124	52
	2	10.534	10.423	25	6.763	7.868	25
6	0	10	10	54	7.285	7.727	25
	1	10.604	10.162	25	7.124	7.124	52
	2	10.267	10.212	25	6.943	7.496	25
7	0	10	10	54	7.205	7.426	25
	1	10.302	10.081	25	7.124	7.124	52
	2	10.133	10.106	25	7.033	7.31	25
8	0	10	10	54	7.164	7.275	25
	1	10.151	10.04	25	7.124	7.124	52
	2	10.067	10.053	25	7.079	7.217	25
9	0	10	10	54	7.144	7.199	25
	1	10.075	10.02	25	7.124	7.124	52
	2	10.033	10.026	25	7.101	7.17	25
10	0	10	10	54	7.134	7.161	25
	1	10.038	10.01	25	7.124	7.124	52
	2	10.017	10.013	25	7.112	7.147	25
11	0	10	10	54	7.129	7.143	25
	1	10.019	10.005	25	7.124	7.124	52
	2	10.008	10.007	25	7.118	7.135	25
12	0	**10**	**10**	**54**	**7.126**	**7.133**	**52**
	1	10.009	10.003	25	7.124	7.124	52
	2	10.004	10.003	25	7.121	7.13	52

Table A3. (*d* is the distance between the points of the initial simplex, *h* – is the number of transmission lines)

	the number of regions, N	the number of steps	result, F(w*)
w_0=(10,10) d=10 h=1	2	14	54
	7	17	952
	12	14	1790
	17	32	2822
	22	21	3906
	27	47	4963
	32	44	5937
	37	16	7641
	42	24	8413
	47	16	9125
	50	13	9303
	99	24	18720

Table A4.

the number of regions, N	the number of steps			result, F(w*)		
	w_0=(10,10) d=10 h=1	w_0=(0,0) d=10 h=1	w_0=(10,10) d=10 h=N-1	w_0=(10,10) d=10 h=1	w_0=(0,0) d=10 h=1	w_0=(10,10) d=10 h=N-1
2	14	13	14	54	54	54
4	10	23	10	430	373	430
6	12	39	12	623	578	623
8	16	19	16	1172	1172	1172
10	15	23	17	1423	1423	1423
12	14	20	14	1790	1557	1790

Table A5.

Transmission links:			
Link number	Zone	Min flow	Max flow
1	1-2	-100	100
2	1-3	-100	100
Sell bids:			
Sell bid number	Zone	Price (€/MWh)	Quantity (MWh)
1	1	10	7
2	1	17	8
3	2	15	9
4	3	12	3
5	3	16	3
Purchase bids:			
Purchase bid number	Zone	Price (€/MWh)	Quantity (MWh)
1	1	16	10
2	1	17	8
3	2	17	9
4	2	15	7
5	3	10	6
6	3	12	1
7	3	15	1

Table A6.

Coordinate Descent Method		Nelder-Meed's Method	
Link number	Flows	Link number	Flows
1	-3	1	-3
2	0	2	0
Sell bids:			
P	Q	P	Q
17	7	16	7
17	0	16	0
15	0	15.22	0
15	3	15.22	3
15	0	15.22	0
Purchase bids:			
P	Q	P	Q
17	0	16	0
17	7	16	7
15	3	15.22	3
15	0	15.22	0
15	0	15.22	0
15	0	15.22	0
15	0	15.22	0
B(P)=64		B(P)=64	
P=(17;15;15)		P=(16;15.22;15.22)	

Table A.7.

Coordinate Descent Method		Nelder-Meed's Method	
Link number	Flows	Link number	Flows
1	-3	1	-6
2	6	2	3
Sell bids:			
P	Q	P	Q
10	7	15.39	7
10	0	15.39	0
15	0	1.25	0
15	3	26.99	3
15	0	26.99	3
Purchase bids:			
P	Q	P	Q
10	0	15.39	0
10	1	15.39	4
15	9	1.25	9
15	0	1.25	0
15	0	26.99	0
15	0	26.99	0
15	0	26.99	0
B(P)=64		B(P)=67	
P=(10;15;15)		P=(15.39;1.25;;26.99)	

Part B

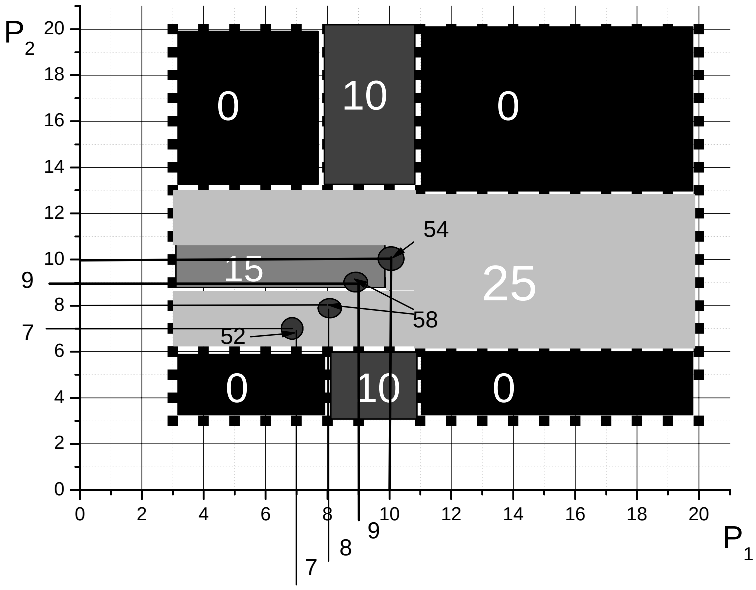

Figure B1.

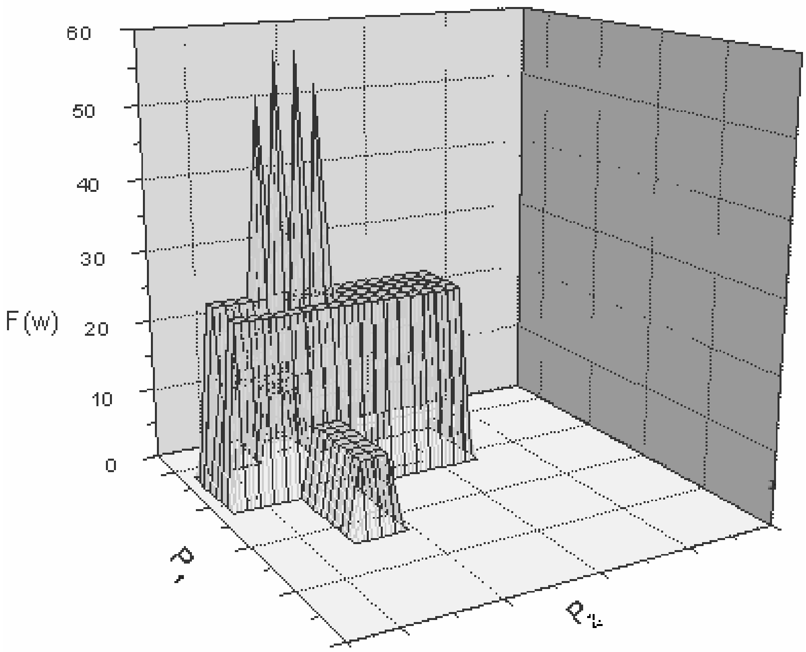

Figure B2.

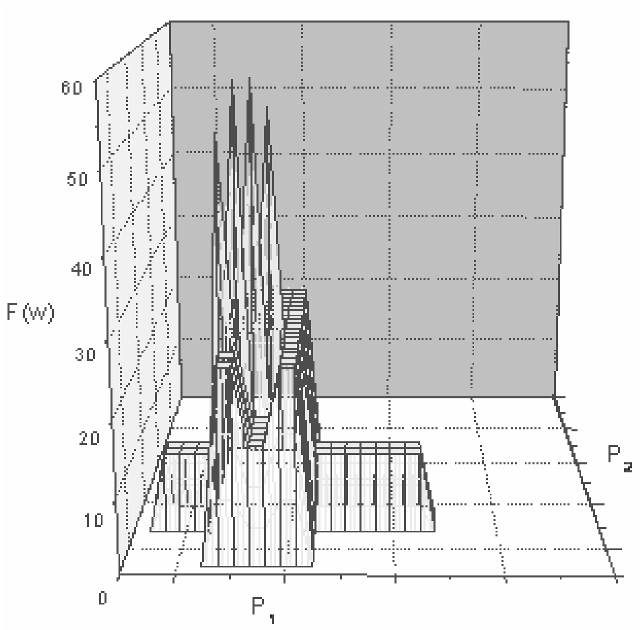

Figure B3.

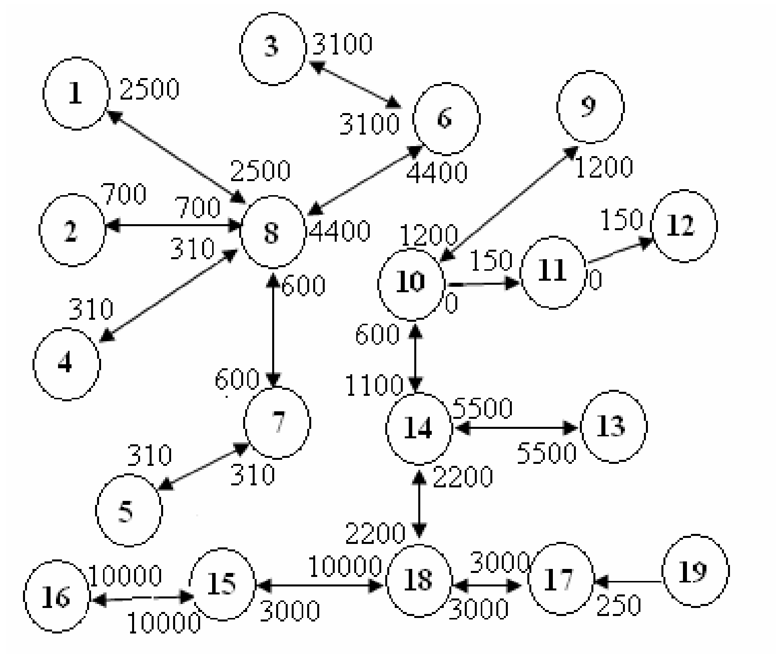

Figure B4.

Redazione

Dipartimento di Matematica, Statistica, Informatica ed Applicazioni
Università degli Studi di Bergamo
Via dei Caniana, 2
24127 Bergamo
Tel. 0039-035-2052536
Fax 0039-035-2052549

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