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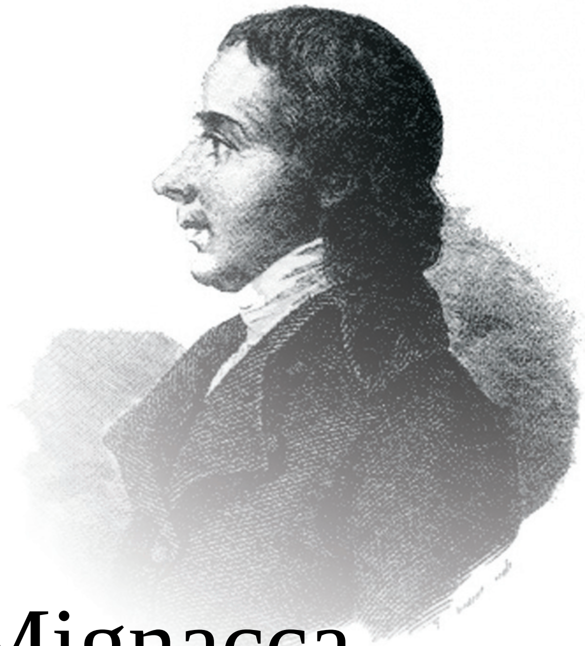

R. Giacometti, D. Mignacca, Using Black & Litterman framework for stress testing analysis in asset management

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Using Black & Litterman framework for stress testing analysis in asset management[1]

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In the classical Black and Litterman approach, by using reverse engineering, it is possible to obtain the expected assets equilibrium returns implied in the weights of the market portfolio, i.e. the benchmark. However, analysts may have different views on some of the expected returns implied in the benchmark's weights and it is possible to obtain the posterior distribution by combining analysts' views and prior market information.

In this paper we propose a methodology for a stress test analysis of the current managed portfolio, where two different shock types are combined. More precisely:

- we shock a set of factors which affect asset returns, imposing the analysts' views on their variation from the expected level;
- we assume that a mixture of normal distributions can describe the presence of hectic periods and quiet period. The asset correlation breakdown effect is well known i.e., ".. joint distributions estimated over periods without panics will misestimate the degree of correlation between asset returns during panics. " (Alan Greenspan[2]).

For this purpose, we introduce a number of macroeconomic factors which affect asset returns such as volatilities, interest rates, oil price etc. , At this stage, we do not perform a multi factor analysis, but we include the information in the covariance matrix. We assume that a mixture of normal distributions can describe the presence of high volatility periods and low volatility periods, taking into account extreme movements in the market. We derive the conditional moments of the posterior distribution by combining views on factors and market information.

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[2] Greenspan A,Remarks by Chairman Alan Greenspan, New challenges for monetary policy before a symposium sponsored by the Federal Reserve Bank of Kansas City in Jackson Hole, Wyoming August 27, 1999

Introduction

Stress testing approaches differ among institutions, due to the nature of the tested problem and the way in which stress scenarios have been selected. Scenario tests can be constructed on the basis of historical events (a crisis observed in the past) -historical stress scenarios-, or on scenarios that may be judged as possible in the future due to the changes of macroeconomic, socioeconomic or political factors -prospective or hypothetical stress scenarios-.

Events, commonly used to build historical scenarios are the large U.S. stock market declines in October 1987, the Asian financial crisis of 1997, the financial market fluctuations surrounding the Russian default of 1998, and the financial market developments following the September 11, 2001, terrorist attacks in the United States.

Prospective scenarios can be constructed according to an event-driven approach which identifies the risk sources, or factors, which cause changes in asset returns. By assuming large factors' movements, it is possible to investigate how much risk parameters change if such an event occurs. Therefore, stress scenarios are based on plausible but unlikely events and are suitable for a sensitivity analysis of the portfolio. Risk managers identify a portfolio's key financial drivers and then formulate scenarios in which these drivers are stressed.

One of the main problems in stress testing analysis is the simulation of consistent scenarios which are able to integrate historical and private information and to preserve the correlation structure in the data, as well as to capture direct effects of movements in the drivers and indirect effects due to correlations among portfolio assets. An appropriate framework is provided by the Black and Litterman[5,6,7] approach, which can be adapted to stress testing. Cherubini and Della Lunga[9] derive the expected value of the conditional distribution by imposing the views directly on asset classes and evaluate non linear and leverage positions. Meucci[24]assumes the presence of normally distributed underlying factors and applies the model on options trading and portfolio management problems. The underlying factors are normally distributed and the P&L function depends linearly or not linearly on these factors.

Acccording to the work of Madelbrot[20] and Fama[13], many empirical studies[3] show that in many cases, logaritmic returns are quite far from being normally distributed, especially for high frequency data. Recent papers[4,26,27], show that stable Paretian distributions are suitable for the autoregressive portfolio return process in the framework of asset allocation problem over a fixed horizon. The classical Black and Litterman approach is based on Gaussian distribution assumptions. Giacometti et al[16] investigate the hypothesis of non Gaussian distributions for the prior and different market perceptions of risk in the derivation of equilibrium returns. In the hypothesis of general distributions for the market and the views, it is not possible to derive an analytic solution for the posterior expected returns and it is necessary to proceed by the Montecarlo simulation (see Meucci [23]).

Therefore Black and Litterman model maintains its appeal among practitioners, since it permits to obtain an analytic expression of the posterior expected return. In order to preserve the structure and the simplicity of the original Black and Litterman model and, at the same time, to take in account the empirical evidence of fat tails, we introduce a mixture of normal distributions. This distribution is suitable to describe the presence of varying volatility periods - quiet periods and hectic periods - in financial markets.

Many authors assume the presence of two regimes in financial data. Chow et al.[10] recognize the presence of two regimes and identify as outliers the returns with a distance from the mean greater than a tolerance distance. They estimate two covariance matrices of the two samples (normal returns and outliers) and compute the blended covariance matrix as their linear combination. Aragones and Blanco[1] assume three regimes and compute correlation matrices in each of them,

[3] See Embrechts et al. [12], Rachev and Mittnik [31] and the references therein, Mittnick and Paolella [25], Panorska et al. [28], Tokat et al. [32], Tokat et al Schwartz [33].

classifying returns according to prefixed thresholds. Kim and Finger [18] suggest the use of a mixture of bivariate normal distributions in order to model core and peripherical assets and to compute the conditional covariance matrix repeating the procedure for each peripherical asset. Bee [2] extends the model to a multivariate mixture distribution. The introduction of a mixture of normal distributions presents many advantages:

- it permits to avoid the ex-ante classification of returns belonging to crisis or quiet periods,
- it permits to construct an integrated stress risk modelling process (see Berkowitz[3] for a discussion),
- it permits to overcome the limiting hypothesis of Guassian distribuions and to take into account the presence of heavy tails.

In this paper we propose a methodology for a stress test analysis of the current managed portfolio. The general idea of our approach is that asset returns depend on a number of financial or macroeconomic "core" factors that act as drivers. Therefore, it is possible to stress asset returns by imposing shocks in the drivers[4]. We model the presence of extreme movements in the market, by introducing a mixture of normal distributions and we compute the conditional moments of the posterior distribution by combining shocks on factors and prior market information.
We derive the conditional sensitivity of the expected asset returns to shocks in high/low volatile regimes and the extra return w.r.t the benchmark (called delta performance) of each asset in the managed portfolio.
The paper is organised as follows. In the first paragraph we briefly describe the classical Black and Litterman model (BL model hereafter); in the second paragraph we describe how Eurizon Capital SGR (Eurizon hereafter) has adopted the BL model for the *what if analysis* in order to monitor the portfolio's reaction to shocks. In the third paragraph we introduce a mixture of normal distributions to properly take into account extreme market movements. Finally, we present empirical results of the comparison between the original Eurizon model and the improved one.

1. Review of the classical Black and Litterman model

The BL model was mainly introduced to respond to two problems in asset allocation. The first is the need to overcome the critical step of expected return estimation, mainly critical for the presence of estimation errors. The second is the need to integrate subjective information, the experts' views, to the market information. The main idea of this approach is to extract the equilibrium returns, given the Sharpe ratio, as the returns implicit in the benchmark. BL argues that the only sensible definition of neutral returns is the set of expected returns that would clear the market if all investors had identical views. If the Capital Asset Pricing Model holds and if the market is in equilibrium, the weights based on market capitalizations are also the weights of the optimal portfolio. Afterwards, , via reverse optimization, one can recover the equilibrium returns (prior returns).The theoretical reason is that if the benchmark is a good proxy for the market portfolio, its composition is the solution of an optimization problem for a vector of unknown equilibrium returns.
The equilibrium returns $\mathbf{\Pi}$ of the stocks composing the benchmark, are obtained by solving the unconstrained maximization problem faced by an investor with quadratic utility function or by assuming normally distributed returns[6].

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[4]. This approach has been implemented by Domenico Mignacca and Paolo Protti and presented at the "Corso di alta formazione in finanza matematica" of the University of Bologna in 2004. This approach is currently used by Eurizon Capital SGR.

[6] More precisely $\Pi = 0.5\lambda\Sigma x$ where λ is the Sharpe ratio.

We consider a market of N assets whose returns are normally distributed. The expected returns E(R) are assumed to be normally distributed E(R) ~ N(Π, $\alpha\Sigma$) with the covariance matrix proportional to the historical one, rescaled by a shrinkage factor; since uncertainty of the mean/main is lower than the uncertainty of returns themselves, the value of α should be close to zero[7].

The equilibrium returns provide a neutral reference point for asset allocation. In case there are no views on market, there is no reason to deviate from the benchmark (the benchmark is a proxy of the equilibrium portfolio). However an active asset manager can deviate from the benchmark tracking strategy, according to his/her economic reasoning in the tactical asset allocation. BL model combines equilibrium returns with uncertain views about expected returns.
Assume that we have k views, expressed with a set of linear constraints (1).

(1) G=SE(R) + u, with u ~ N(0 , Ω) and Cov(u,E(R))=0;

where
S is a matrix k×N where each row corresponds to one view,
u is a random vector k×1 of errors of the views.,
Ω is the matrix k×k containing the covariance, or uncertainty, of the views.

The views are expressed on the expected returns and are normally distributed, so that the jointly distribution is:

$$(2)\quad \begin{bmatrix} E[R] \\ G \end{bmatrix} \sim N\left(\begin{bmatrix} \Pi \\ S\Pi \end{bmatrix}, \begin{bmatrix} \alpha\Sigma & \alpha\Sigma S' \\ \alpha S\Sigma & \alpha S\Sigma S' + \Omega \end{bmatrix} \right),$$

Using the Bayes' Theorem, it is possible to generate a vector of "posterior" returns for all asset returns. In particular the conditional distribution of the returns is:

$$[E(R) \mid G = g] \sim N\left(\tilde{\Pi}, \tilde{\Sigma}\right)$$

where

$$(3)\quad \tilde{\Pi} = \Pi + \alpha\Sigma S' (S\ \alpha\Sigma S' + \Omega)^{-1}(g - S\Pi)$$

$$\tilde{\Sigma} = \alpha\Sigma - \alpha\Sigma S' (S\ \alpha\Sigma S' + \Omega)^{-1} S\ \alpha\Sigma$$

The investor's views have the effect of modifying the equilibrium returns Π according to the degree of uncertainty. The greater the uncertainty, the less the deviation from neutral views.
The main results are that the views must not be expressed for each asset and the conditional expected returns do not suffer from typical problems of corner solution.

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[7] We assume that Π is the expected value of the distribution of the expected returns, following the original formulation of Black and Litterman. In other approaches [22,23,24] the expected returns are assumed to be constant. In both cases the analysis is similar and it is possible to derive the conditional distribution of the returns given the views and to compute the moments of the conditional distributions. In this paper we follow the original Black and Litterman approach.

An assumption of the BL model is the normal distribution of the uncertainty on the views. If all views are independent, the covariance matrix is diagonal. However, in practice, it can be difficult to specify the degree of confidence for each view. A more convenient approach is to consider that Ω is proportional to the assets volatility. The more volatile is an asset, the more uncertain is the view on its expected return. This can be easily implemented assuming that:

$$\Omega = \frac{\tau}{(1-\tau)} S\ \alpha\Sigma S' \qquad 0 \leq \tau \leq 1, \tag{4}$$

The conditional expected return, i.e. the equilibrium return adjusted by the views, can be easily expressed as:

$$\widetilde{\Pi} = \Pi + (1-\tau)\Sigma S' (S\Sigma S')^{-1}(g - S\Pi) \tag{5}$$

We can observe that for $\tau=0$, the views are certainty views; for $\tau=1$ the views are unreliable and $\widetilde{\Pi} = \Pi$. If S is invertible, i.e. we have a number of linearly independent views equal to the number of assets, (5) becomes

$$\widetilde{\Pi} = \Pi + (1-\tau)S^{-1}(g - S\Pi) \tag{6}$$

2. Black and Litterman model adapted/adopted for stress testing analysis

Following the event-driven approach, the risk managers identify a portfolio's key financial drivers and then formulate scenarios in which these drivers are stressed. In this section we explain how it is possible to include factors in the original model without affecting asset returns in absence of shocks and, at the same time, how we can shock factors and observe the effects on the asset returns. This framework solves one of the main problems in stress testing analysis i.e. the simulation of consistent scenarios able to integrate historical and private information and to preserve the correlation structure in the data, being able to capture the direct effects of movements in the driver and the indirect effects due to correlations among the portfolio assets.

Starting from the classical BL model presented in the previous paragraph, we introduce K factors which can influence the portfolio performance and we assume that the percentage variations in the factor are jointly normally distributed. Therefore the factors prior distribution is $E(F) \sim N(\overline{F}, \alpha\Sigma_F)$. In order not to directly influence asset returns, we model centred factors, $E(F) - \overline{F} \sim N(0, \alpha\Sigma_F)$ so that the expected variation of the factors is null.
We can express the multivariate distribution of the N assets and the K factor as in (7), where the covariance matrix is defined in terms of blocks.This way we explicitly isolate correlations among the assets from the correlation among the exogenous factors and the cross correlations.

$$\begin{bmatrix} E(R) \\ E(F) - \overline{F} \end{bmatrix} \sim N\left(\begin{bmatrix} \Pi \\ 0 \end{bmatrix}, \begin{bmatrix} \alpha\Sigma_R & \alpha\Sigma_{RF} \\ \alpha\Sigma_{FR} & \alpha\Sigma_F \end{bmatrix} \right) \tag{7}$$

Now, we assume that experts express views only on the factors, in terms of deviation from the equilibrium level. In equilibrium views all are null. In other words, the views are the shocks on the factors.
The shocks are expressed as a set of linear constraints:

(8) $$G_F = S_F(E(F)-\overline{F})+u \qquad u \sim \text{N}(0\ ,\ S_F\alpha\Sigma S_F^{'} + \Omega\ )$$

S is a matrix k×F where each row corresponds to one shock,
u is a random vector k×1 of errors of the shock,
Ω is the matrix k×k containing the covariance, or uncertainty, of the shocks.

If we don't shock the factors we get the observed distributions for asset return. When we expect variations in the driver, we investigate the effects of these variations in asset returns. The expected value of the posterior distribution is:

(9) $$E\left(\begin{bmatrix} E(R) \\ E(F)-\overline{F}\end{bmatrix} | G_F = g_F\right) = \begin{bmatrix}\widetilde{\Pi} \\ \widetilde{F}\end{bmatrix} = \begin{bmatrix}\Pi + \alpha\Sigma_{RF}S_F^{'}\left(S_F\alpha\Sigma_F S_F^{'}+\Omega\right)^{-1} g_F \\ \alpha\Sigma_F S_F^{'}\left(S_F\alpha\Sigma_F S_F^{'}+\Omega\right)^{-1} g_F\end{bmatrix}$$

Since we are mainly interested in the reaction of the asset returns to the factor shocks, we focus our attention on the expected return (10)

(10) $$\widetilde{\Pi} = \Pi + \alpha\Sigma_{RF}S_F^{'}\left(S_F\alpha\Sigma_F S_F^{'}+\Omega\right)^{-1} g_F$$

We observe that assuming absence of shocks, a view given by a vector of zeros, we get the unconditional equilibrium returns.
The extra return due to a shock can be easily obtained as:

(11) $$\widetilde{\Pi} - \Pi = Hg_F \quad \textit{where} \quad H = \alpha\Sigma_{RF}S_F^{'}\left(S_F\alpha\Sigma_F S_F^{'}+\Omega\right)^{-1}$$

H contains all the information on the delta performance of each single asset. Moreover, if we assume $\Omega = \tau/(1-\tau)\ S\alpha\Sigma S'$, with $0 \le \tau \le 1$, (11) becomes

(12) $$H = (1-\tau)\Sigma_{RF}S_F^{'}\left(S_F\Sigma_F S_F^{'}\right)^{-1}$$

We point out that:

1) given any differential composition of an active managed portfolio w.r.t. the benchmark, we can compute the differential (portfolio/benchmark) impact, security by security, caused by the factors' shocks .

2) The covariance matrix of the asset returns, is not directly involved in the computation of the delta performance. This result is extremely important because it permits to disentangle the

assets correlation stress test from the factors correlation stress test. We can observe the effects of large movements in the factors by assuming that correlations among the factors are unchanged; at a second stage we can stress the correlations of the factors and finally we can stress the correlations of the returns as well.

3) When judging the usefulness of stress scenarios, one has to take into account both the size of the loss they cause and their plausibility[15,22]. Since the factors are elliptically distributed, we can compute the plausibility of a scenario by using the Mahalanobis distance of a scenario g_F from the present market situation. The square distance (13) is distributed as a χ^2 with k degrees of freedom, where k is the number of views.

(13) $$d^2(g_F) = g_F'\left(\frac{S_F \alpha \Sigma_F S_F'}{1-\tau}\right)^{-1} g_F$$

If the distance is zero, the plausibility is one, and as the distance increases the plausibility decreases. We define the index of plausibility P(.) as:

(14) $$P(g_F) = 1 - F_{\chi^2(k)}(d^2(g_F))$$

Ideally we can consider plausible stress test scenarios at a determined confidence level p=5%, which satisfies

(15) $$P(g_F) \geq p$$

Finally, for each view we can compute the sensitivity of the plausibility index to each marginal shock:

(16) $$\frac{\partial P(g_F)}{\partial g_F} = \frac{\partial(1 - F_{\chi^2(k)}\left((1-\tau) g_F (S_F \Sigma_F S_F')^{-1} g_F'\right)}{\partial g_F} = -2 f_{\chi^2(k)}(d^2(g_F))(1-\tau)(S_F \Sigma_F S_F')^{-1} g_F'$$

This approach has been implemented and is currently used by Eurizon Capital SGR.

3. Stress testing analysis and extreme movements

Moving from this approach, we want to improve the methodology presented so far along the line of distributional assumption The model presented in the previous section is based on the assumption of normally distributed returns.

In order to preserve the structure of the original model and include an heavy tail distribution we assume that returns are generated from a finite mixture of multivariate normal distributions with S components (see McLachlan and Basford [21] for an accurate discussion on mixture models).

(17) $$f_X(x) = \sum_{i=1}^{S} \pi_i f_{X|i}(x), \;\; \pi_i \in (0,1), \quad \sum_{i=1}^{S} \pi_i = 1$$

In particular, we assume that the density distribution of assets and factors is a mixture of two[8] multivariate normal distributions (S=2). The probability distribution function is:

[8] The choice of two distributions in the mixture is due to the analysis explained in computational results.

(18) $$f_{\begin{bmatrix} E(R) \\ E(F)-\overline{F} \end{bmatrix}}(.) = \pi f_{\begin{bmatrix} E(R)^{(1)} \\ E(F)^{(1)}-\overline{F}^{(1)} \end{bmatrix}}(.) + (1-\pi)\, f_{\begin{bmatrix} E(R)^{(2)} \\ E(F)^{(2)}-\overline{F}^{(2)} \end{bmatrix}}(.)$$

where

(18) $$\begin{bmatrix} E(R) \\ E(F)-\overline{F} \end{bmatrix} = \begin{cases} \begin{bmatrix} E(R)^{(1)} \\ E(F)-\overline{F}^{(1)} \end{bmatrix} \sim N\left(\begin{bmatrix} E(R^{(1)}) \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_r^{(1)} & \Sigma_{rF}^{(1)} \\ \Sigma_{Fr}^{(1)} & \Sigma_F^{(1)} \end{bmatrix} \right) & \textit{with probability } \pi \\ \begin{bmatrix} E(R)^{(2)} \\ E(F)-\overline{F}^{(2)} \end{bmatrix} \sim N\left(\begin{bmatrix} E(R^{(2)}) \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_r^{(2)} & \Sigma_{rF}^{(2)} \\ \Sigma_{Fr}^{(2)} & \Sigma_F^{(2)} \end{bmatrix} \right) & \textit{with probability } 1-\pi \end{cases}$$

The benefit of such a specification is that it allows the possibility that occasionally the returns are generated from a distribution with a higher variance, while simultaneously maintaining the structure of normal densities. The mixture distributions of the factors lead to the mixture distributions of the views:

. (20) $$G = \begin{cases} G^{(1)} \sim N\left(0, S_F \Sigma_F^{(1)} S_F^{'}\right) & \textit{with probability } \pi \\ G^{(2)} \sim N\left(0, S_F \Sigma_F^{(2)} S_F^{'}\right) & \textit{with probability} 1-\pi \end{cases}$$

We finally derive the conditional sensitivity of the expected asset returns to the views, reflecting the reaction of the asset returns in low and high volatility periods.

(21) $H^{(i)} = (1-\tau)\Sigma_{rF}^{(i)} S_F^{'} \left(S_F \Sigma_F^{(i)} S_F^{'} \right)^{-1}$

The delta performance is the linear combination of the two delta performances.

(22) $H = \pi_1 H^{(1)} + (1-\pi_1) H^{(2)}$

We can move further in our stress test analysis, if we believe that the historical returns do not contain sufficient information to adequately stress the portfolio. In principle, we can apply two different approaches: we can stress either the volatilities or the volatilities and the correlation matrix. The first approach assumes that the correlation matrix estimated in hectic periods captures the correlation breakdown effect adequately. The stress is applied uniquely on the volatility vector. Let $\Sigma_F^{(2)}$ be the current estimated covariance matrix in hectic periods. By decomposing $\Sigma_F^{(2)}$ in $\Sigma_F^{(2)s} = V^{(2)}{}' C_F^{(2)} V^{(2)}$, we can stress volatilities as in (22) (see Kupiec [19]).

(23) $\Sigma_F^{(2)s} = (D^{(2)} + \Delta)' C_F^{(2)} (D^{(2)} + \Delta)$

where

$V^{(2)}$ is the $K \times 1$ vector of the factors volatility,
Δ is the $K \times 1$ vector of the volatilities' shocks,

$C_F^{(2)}$ is the K×K correlations matrix.

The second approach is relevant for the *what if analysis* and allows the introduction of specific exogenous shocks in the factors correlation matrix $C_F^{(2)}$ of hectic periods. The shocked matrix must be well specified i.e positive semi definite. Even under this condition the complete stressed correlation matrix may not be well specified (Finger [14]). There are different approaches to adjust the correlation matrix such as the shrinkage technique, Kupiec [19], the hypershere decomposition of Rebonato and Jackel[30] and the solution of an unconstrained convex optimisation problem to determine the nearest well specified correlation matrix, Qi ad Sun [29] .

3 Computational results

In this section we want to compare the two different models presented in this paper. In particular, we consider:

- the Eurizon model presented in paragraph 3, (model 1.A) and a variation of it where we substitute the estimated covariance matrix with an EWMA covariance matrix (model 1.B),
- the mixture model presented in paragraph 4 (model 2.A) and a variation of it with an additional stress of the covariance matrix (model 2.B): we increase the factors volatility in hectic periods by 3 times their original value, according to (23).

The period of the analysis ranges from the beginning of 1996 to the end of 2008; many extreme movements are present in the historical data, which include many financial crisis (the 1997 Bhat crisis, and the 1998 Russian ruble crisis in August 1998 followed by the collapse of a major hedge fund (Long-term Capital Management) in September 1998, the terrorist attack of September 11, 2001, and the subprime mortgage crisis that began in the summer of 2007 and the subsequent financial collapse in 2008.
We apply our model to a portfolio composed of the ten most capitalised shares of the S&P. We rescale their weights in order to sum up to 1. We use the new weights to construct a synthetic benchmark. We consider as drivers 4 factors: a stock index, the S&P, a bond index, RIX 10 year future bund, and index of volatility, the VIX, and a commodity, the Brent.
The analysis is performed on weekly returns in accordance to the Eurizon practice.

According to the Bera-Jarque and Kolmogoroff Smirnoff test, we strongly reject the null hypothesis of normal distribution at the 5% significance level. However, by using a rolling window of 104 weeks for 80% of assets, we cannot reject the null hypothesis except for the last 20 weeks of the sample period, corresponding to the most recent and deep financial crisis. For this reason, we decide to compute the historical covariance matrix of model 1.A on a rolling window at 104 weeks (basically 2 years). The EWMA is computed on the complete data set with λ=0.94. (model 1.B)
The maximum likelihood estimates of mixture distribution parameters, are obtained by using the EM algorithm (see Dempster et al.[11]). In table 1, we report the Akaike Information Criterion (AIC) and the Bayes Information Criterion (BIC) for different numbers of components of mixture distributions.

Number of components (S)	AIC	BIC
1	-3.9540	-3.9001
2	-4.0797	-3.9716
3	-4.0989	-3.9365

4	-4.1099	-3.8932
5	-4.1123	-3.8412

Table 1 Information for different numbers of components of the mixture (rescaled by 1.0e+4)

We conclude that S=2 (two components) is the number that provides the best performance in terms of BIS. The greatest marginal decrease of the AIC criterion is obtained with two components. In order to determine a starting point for the EM algorithm, we estimate a regime switching model on the S&P data on the entire sample period.

In table 2 we report the expected return and variance of the two regimes (t-statistic in brackets). Table 3 reports the transition matrix. The probability of remaining in state 1 (low volatility) and 2 (high volatility) is respectively 98% and 87%.

	μ	σ
State 1	0.0019 (2.3017)	0.0189 (21.5757)
State 2	-0.0058 (-1.0861)	0.0491 (9.2646)

Table 2 Parameters of the two regimes

Figure 1 shows the time series of the S&P, the posterior distributions of each weekly return and the EWMA weekly volatility on the sample period. The presence of two regimes is clearly evident and can be easily interpreted as high and low volatility periods. The volatilities are significantly different from zero.

Transition Probability	State1	State2
State 1	0.98	0.02
State 2	0.13	0.87

Table 3 Transition matrix

Hereinafter, we investigate if the inclusion of a mixture distribution improves the ability of the model to anticipate the asset reaction to the factors' shocks. The idea is to perform a back testing analysis. We divided the data into two samples: the first 12 years of weekly data for the parameter estimation and the remaining 52 weeks for an out-of-sample analysis. The out-of sample analysis is repeated for 52 consecutive weeks .

We consider the conditional expected return computed according to the 4 models as forecasts of future returns. We use as weekly shocks the variations of the factors returns from their mean/main values of the last 104 weeks and as Sharpe ratio the excess return of the S&P over its variance in the same period.

Each week we compute the mean squared errors for the ten shares. Errors are the difference between the expected return computed according to the models and the realization of the day. Note that we reestimate the parameters of the distributions each week.

In figure 3 and 4 we report the weekly mean squared errors and their standard deviation.

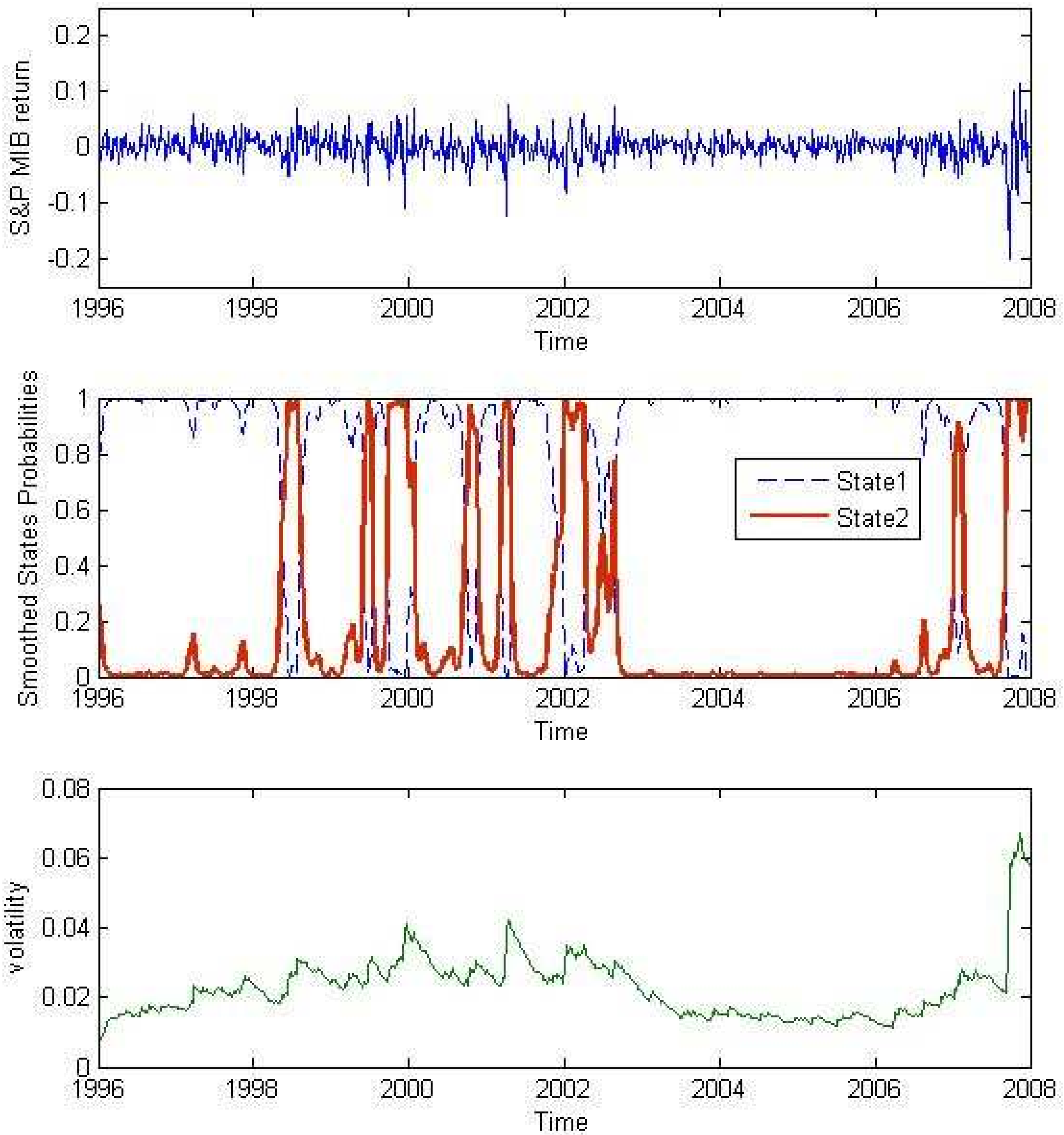

Figure 1: S&P Returns, posterior probabilities and EWMA for the period 1996-2008;

The dynamic of the mean squared errors is very similar for the first half of the year.
In general, model 1.B seems to overreact to the factors shocks, introducing an unacceptable volatility. The difference among the models becomes evident only after July 2008. We observe a dramatic increase in the mean squared errors between the 35th week and the 40th week (October 2008). The best performance in the second half of 2008 is provided by model 2.A and model 2.B, with a smaller mean and smaller standard deviation of squared errors. During the chaotic period, model 2.B seems to more accurately describe the reactions of the expected return to the shocks. However, on average, model 2.A gives the smaller mean errors and standard deviations.

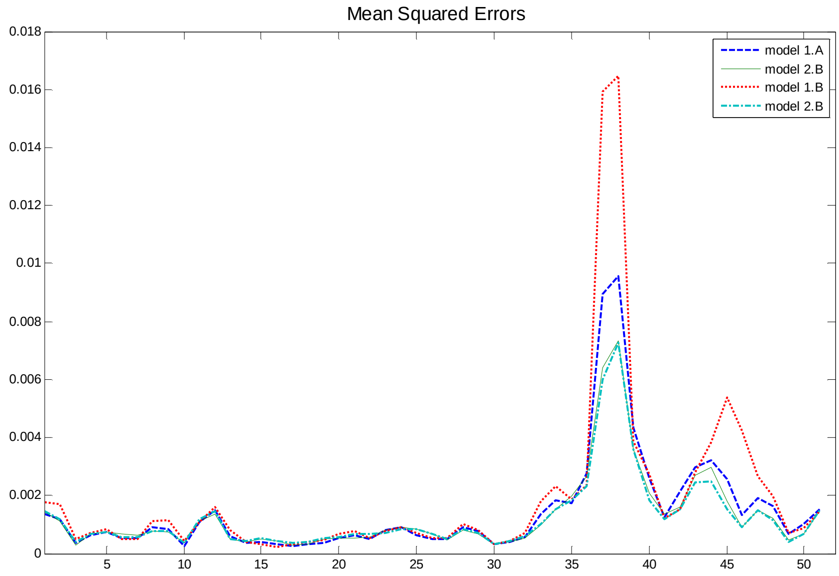

Figure 2: Weekly mean squared errors

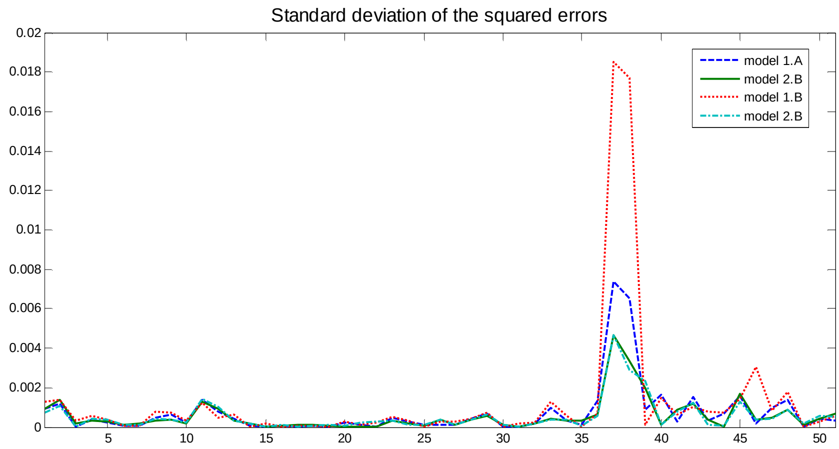

Figure 3 Weekly standard deviation of the squared errors

Conclusion

The purpose of our work is twofold. Firstly to present the stress model used in Eurizon Capital SGR, secondly to enhance the model by using a mixture distribution.

Some results of this model need to be outlined. Given any differential composition of an active managed portfolio w.r.t. the benchmark, we can compute the differential (portfolio/benchmark) impact, security by security, caused by the factors' shocks. Moreover, the covariance matrix of asset returns is not directly involved in the computation of the delta performance. This result is extremely important because it permits to disentangle the assets correlation stress test from the factors correlation stress test. We can observe the effects of large movements in the factors assuming that correlations among factors are unchanged. When judging the usefulness of stress scenarios, one has to take into account both the size of the loss they cause and the plausibility. Since the factors are elliptically distributed, we can compute the plausibility of a scenario by using the Mahalanobis distance of a scenario g_F from the present market situation.

The introduction of a mixture distribution improves the ability of the model to capture the reaction of expected returns to extreme shocks. Finally, in our opinion the model can also be used in the optimisation process where returns for optimisation are derived from the scenario and the result is a portfolio which maximises its sensitivity to the scenario itself.

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