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'Soft' Growth and the Role of Monetary Policy in Selecting the Long-Run Equilibrium Path

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‘SOFT’ GROWTH AND THE ROLE OF MONETARY POLICY IN SELECTING THE LONG-RUN EQUILIBRIUM PATH

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ABSTRACT: Combining employment and growth theory with a cash-in-advance constraint, the model determines the balanced growth path (BGP). Under low intertemporal elasticity of substitution in consumption, the Friedman rule is consistent with the existence of an unique BGP: monetary rules leading to a steady-state rate of money expansion higher than that dictated by the Friedman rule lower balanced growth. Under high elasticity, the Friedman rule is consistent with multiple BGP: the authority can “select” the BGP exhibiting the highest growth rate by expanding money at the appropriate fixed rate, since balanced growth rises with the fixed rate of money expansion.

KEY WORDS: On-the-job training, skilled labor, strategic complementarity, indeterminacy, inflation targeting.

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1 INTRODUCTION

In the light of the disappointing growth performances of the main countries of the Euro area in the 1990s (especially vis-à-vis the robust growth of the U.S.), the so-called "dissenting" view (see IMF, 1999) blames the "too" restrictive macroeconomic policy that dominated all over Continental Europe in that decade. As far as monetary policy is concerned, this view set the "obsession" with inflation typical of the "orthodox" Bundesbank and of its successor, the ECB, against the "growth-oriented" approach of the "pragmatist" FED. The orthodox viewpoint replies by claiming that (within limits) central banks have control on market short-term interest rates, while the long-term real rate of interest -- the only one that can influence investment and growth – depends on economic "fundamentals" that cannot be affected by monetary policy.

This contrast is typically exemplified by the essay of Fitoussi (2001) and the comment on this essay by Streissler (2001). According to Fitoussi, "monetary policy was the prime mover in the process that led Europe into a situation of slow growth" (Fitoussi, 2001, p. 227). Indeed, over-restrictive monetary policies should be considered the ultimate culprit for the too high real interest rates that have been at the origin of the European unemployment problem and the "soft" growth in the 1990s. In particular, Fitoussi argues that monetary mismanagement prevented the decline in the real rate of interest which should have been brought about by the slackening in demand for money implied by the slowdown in economic growth. Streissler replies by noting that, if "restrictive" monetary policy means keeping the rate of inflation permanently at a negligible level, there is "no long-run effect of such a policy in raising real interest rates, but, if anything, rather a slight real interest *decreasing* effect" (Streissler, 2001, p.246, italics in the original).

The scope of this paper is to present a formal framework that can be used to discuss rigorously these opposite views. This model combines unemployment and growth theory with a cash-in-advance constraint, so that money can influence the balanced growth path (BGP) of the economy. In this context, the orthodox standpoint is vindicated when the intertemporal elasticity of substitution in consumption is low: under these circumstances, it is shown that any monetary rule leading to a steady state rate of money expansion higher than that dictated by the Friedman rule ("full liquidity") lowers the steady-state rate of output growth. Moreover, in this case, a higher long-term real interest rate may be consistent with a lower steady-state rate of output growth, and the steady-state rate of inflation may respond more than proportionally to an increase in the steady-state rate of money expansion, thus causing a less-than-proportional adjustment of the nominal interest rate to anticipated inflation.

However, the model provides support for the dissenting view when the intertemporal elasticity of substitution in consumption is high. In this case, indeed, the adoption of the Friedman rule is consistent with the existence of multiple BGP, while the BGP is unique under constant money growth. Hence, the monetary authority can "select" the BGP that under the Friedman rule exhibits the highest growth rate by expanding money at the appropriate fixed rate. If the Central Bank had chosen to expand money at a lower rate, it would have selected a BGP characterized by a lower growth rate, without gaining anything in terms of lower steady-state inflation. In other words, for the case in which the intertemporal elasticity of substitution is large, a “too” restrictive money policy may depress long-term growth without bringing about a lower inflation rate. This implies that a Central Bank that is optimistic about the growth potential of the economy may be willing to bet on the sustainability of a high-growth path, thus expanding money so as to accommodate the increase in the demand for liquidity due to the rapid expansion of economic activity. In contrast, a Central Bank that is pessimistic about the economy’s growth potential fears that a more accommodative monetary stance would generate inflationary pressures without any benefit in terms of long-term growth, thus validating its pessimism by increasing its money supply at a rate that is consistent only with a slow growth in economic activity.

One may conclude that the model presented here gives a theoretical foundation to the claim that over-restrictive monetary policy may damage long-term growth, although in “normal” circumstances (namely when the intertemporal elasticity of substitution is low) the orthodox propositions about the relationships linking growth and monetary policy still hold. Indeed, one should emphasize that the evidence seems to confirm that the intertemporal elasticity of substitution is relatively low.[1] Nevertheless, the role that monetary authorities may play in selecting one growth path among the many equilibrium trajectories along which the economy may possibly move can be relevant, especially when externalities and endogenous growth mechanisms cannot be neglected.

[1] For instance, Hall (1988) reports empirical evidence indicating that this elasticity is less than one.

As an example of these mechanisms, the paper introduces both training-on-the job and complementarity in physical and human capital investment. The idea is to model an environment in which jobs and career opportunities created by a fast-growing economy induce more individuals to acquire the basic knowledge that is necessary to be trained on the job,[2] thereby helping to ease the upward pressure on wages exerted by the increased labor demand for qualified workers and stimulating more investment and job creation. In this context, persistency is generated and more than one BGP may be consistent with market fundamentals. Moreover, the formal setup intends to mimic some significant features of the functioning of the economy in the Euro area. Indeed, it is assumed that: i) a unique Central Bank decides on the monetary policy in an economy consisting of many locations subject to idiosyncratic shocks, ii) in each location a single union establishes the wages to be paid by the local firms, iii) labor mobility across locations is imperfect, iv) the product and the capital markets are perfectly integrated across locations. Finally, the general equilibrium dynamic model presented in this paper allows to give a unified treatment of two crucial issues, namely the growth patterns and employment performances of the advanced economies, which are normally analyzed separately. In this way, it can capture the evidence showing that the different ability of countries to employ their labor forces has been correlated in the 1990s with the disparities in growth rates across the OECD countries (Bassanini et al., 2000).

[2] Even if formal education cannot substitute for learning by doing, it is a pre-condition for it: possession of basic formal education is necessary to be able to learn on the job. This dynamic complementarity between education and training is supported by many empirical studies relative to various countries. Among them, see OECD (1991); Lynch and Black (1995), Olson (1996) (both using U.S. data); Jacobs et al. (1996), Arulampalam and Booth (1997) (both using U.K. data), Brunello (2001) (using data of the

The paper is organized as follows: section 2 discusses the relevant literature; section 3 presents the model; section 4 characterizes the equilibrium paths of the non-monetary economy; section 5 studies the impact that alternative monetary rules have on the long-run equilibrium path of the economy, and section 6 concludes.

2 BACKGROUND

The analytical setup presented here models growth as a self-reinforcing process by which firms' capital investment and job creation enable an increasing number of individuals to be trained on the job, thus acquiring skills whose availability attracts more investment and boosts the growth potential of the economy.[3] Moreover, an important feature of this paper is that a mechanism causing persistence (training on the job) interacts with the existence of strategic complementary between investment in physical capital by firms and investment in active labor-market participation by workers. The presence of strategic complementarities between investment in physical capital, in R&D, or in job creation, on the one hand, and investment to acquire the required human capital and to conduct a job search on the other may generate multiple equilibria and lead to coordination failures ("traps"): in the absence of some institutional device coordinating the individual expectations and actions, decentralized

European Community Household Panel). These studies emphasize that on average less formal schooling seems to lead to more limited training opportunities and possibilities to augment human capital.

[3] Empirical data seem to confirm the contribution made to total factor productivity by the learning process which takes place on the job when machinery and technologies are used (see, for example, De Long and Summers, 1992). There is also empirical support for the hypothesis that a shortage of qualified workers has negative effects on productivity growth (for microeconometric evidence concerning the United Kingdom, see Haskel and Martin, 1996).

decision makers can give rise to Pareto-suboptimal outcomes (see Burdett and Smith, 1995; Acemoglu, 1996; Redding, 1996; Snower, 1996).

As the economy can gravitate around multiple equilibrium paths and agents' "animal spirits" are crucial in selecting the trajectory of the economy, the proposition that efficient capital markets are always able to coordinate intertemporal activities appropriately is challenged (see Leijonhufvud, 1998). Consequently, policies and institutions are advocated because of their role in averting a coordination failure and in achieving a Pareto-superior outcome (Hahn and Solow, 1995, chapt.7; Colander and van Ees, 1996). In accordance with this approach, the role of economic policy is to provide a consistent and reliable framework able to enforce the convention favorable to a high growth scenario, convincing the financial markets of its sustainability (see Ciocca and Nardozzi, 1995). However, the instruments and the mechanisms through which economic policy can select the efficient equilibrium trajectory are never well specified in this literature. The present paper aims at filling this void by modeling how the choice of a monetary rule may contribute to select the BGP along which the economy will move. As a matter of fact, this issue has been neglected even by the literature that has investigated the long-run effects of monetary expansion on the endogenously determined rate of economic growth (e.g. Gylfason, 1991; Wang and Yip, 1992; Gomme, 1993; Jones and Manuelli, 1993; van der Ploeg and Alogoskoufis, 1994; Palivos and Yip, 1995; Mino and Shibata, 1995; Chang and Lai, 2000; Jha et al., 2002), although multiple BGP and global (real) indeterminacy may arise in many endogenous growth setups.[4] To my knowledge, indeed, no other paper focuses on the role of monetary policy in selecting the long-run growth rate of the economy.[5]

3 THE MODEL

In the infinite-horizon economy under consideration, there are firms (that produce by renting physical capital and hiring workers), investors (that are the owners of the productive assets) and workers (that consume their entire income).

Individuals' mortality

Time is discrete, and individuals are finitely lived: they have a strictly positive and constant probability σ ($0<\sigma<1$) of dying in each period t. Thus, the probability of dying in a certain period is assumed to be independent of the age of the individual; and it is also assumed that the mortality rate of each large group of individuals does not fluctuate stochastically even though each individual's lifespan is uncertain. This implies that at the end of t a constant fraction σ of individuals belonging to each group and living in location i, $i\in[1,n]$, $n>0$, dies, while new individuals enter the economy at the beginning of t+1 (see the Appendix).

The firms

In the economy, there is a continuum--of measure $n>0$--of locations. In each location i there is a large number (normalized to be one) of identical firms. Locations differ with respect to the specific shock affecting them in each period. Indeed, in each period t the representative firm located in i produces some amount of Y_t, which is the

[4] For a discussion on endogenous growth models that are able to generate multiple balanced growth paths

unique good produced in this economy, according to the constant-returns-to-scale technology

$$Y_{it} = x_{it} K_{it}^{1-\alpha} (S_{it} + \Omega A_{it})^{\alpha}, \; 0 < \alpha < 1, 0 < \Omega < 1, \tag{1}$$

where x_{it} is a random variable taking a value in t which is specific to the i location, K_{it} is the physical capital that the i firm borrowed at the beginning of t to carry out production, S_{it} are the experienced workers (the "skilled workers") employed by the i firm in t, A_{it} are the newly hired workers (the "apprentices") of the i firm in t. Note that the apprentices are less productive than the experienced workers (Ω<1), and that aggregate output is given by $Y_t = \int_0^n Y_{it} di$.

The random variable x_{it} is assumed to be uniformly distributed on the interval [0,n]. Moreover, it is identically distributed across locations and periods, and independently distributed across periods. In each t, x_{it} takes a different value in each location, with x_{it} varying continuously across locations. This implies that the average value of x_{it} across locations is not a random variable and does not fluctuate in time, even though individual firms are uncertain about their local x_{it} (no aggregate uncertainty).

The period net profits π_{it}^{n} (net of the cost of capital) of the i firm are given by:

$$\pi_{it}^{n} = \pi_{it}^{g} - (r_t + \delta) K_{it}, \; 0 < \delta < 1, \tag{2}$$

in conjunction with indeterminacy, see Benhabib and Farmer (1999).

[5] In Bosi (2001) the choice of an appropriate monetary rule can eliminate *local* (real) indeterminacy.

where $\pi^{g}_{it} = Y_{it} - v_{it}S_{it} - e_{it}A_{it}$ are the firm's gross profits, v_{it} is the real wage paid by the i firm to the skilled workers employed in t, e_{it} is the entry wage paid by the i firm to the apprentices hired in t, δ is a capital depreciation parameter, and r_t is the real interest rate, i.e., the market rate at which firms borrowed capital at the beginning of t. Interest payment and reimbursement of principal are due at the end of t. The interest rate is unique because capital is perfectly mobile across locations at the beginning of each period, while mobility is infinitely costly within the period: once borrowed and installed at the beginning of t, a firm's capital stock must remain fixed until the end of the period.

The investors

There is a large number (normalized to be one) of identical investors who are the firms' owners: for simplicity and without loss of generality, we assume that all investors are entitled to receive an equal share of the firms' net profits. Being the owners of the firms' productive assets, investors must decide in each t what fraction of their gross returns on wealth to spend on consumption rather than on buying productive assets to be lent at the beginning of t+1 to firms. Moreover, investors must finance their purchases of current consumption and productive assets out of money balances carried out from the previous period.[6] Hence, the problem of the representative investor amounts to deciding a contingency plan for consumption C^{in}_{t}, purchase of productive assets K_{t+1} and holding of nominal balances H^{d}_{t+1} in order to maximize the lifetime expected sequence of discounted utilities:

[6] In each period, indeed, investors must buy consumption goods and productive assets before receiving their gross return on wealth.

$$\sum_{t=0}^{\infty}\theta^{t} g(C_t^{in}), \theta\equiv\gamma(1\text{-}\sigma),\ \ 0<\gamma\leq 1, \tag{3}$$

subject to $K_{t+1}+C_t^{in}+\frac{H_{t+1}^{d}}{P_t}\leq(1+r_t)K_t+\pi_t^{n}+\frac{H_t+\xi_t}{P_t}$ and to $K_{t+1}+C_t^{in}\leq\frac{H_t^{d}}{P_t}$, where $g(C_t^{in})=\begin{cases}\frac{(C_t^{in})^{1\text{-}\zeta}}{1\text{-}\zeta} \text{ if } \zeta>0, \zeta\neq 1\\ \ln(C_t^{in}) \text{ if } \zeta=1,\end{cases}$ $K_t=\int_0^n K_{it}di$, $\pi_t^n=\int_0^n \pi_{it}^{n}di$, K_{i0} $\forall i$ and H_0 given. In (3), ζ is the inverse of the intertemporal elasticity of substitution, γ is a time-preference parameter, K_t and π_t^n are, respectively, aggregate capital and aggregate (net) profits, P_t is the money price of the homogeneous good and ξ_t denotes a lump-sum monetary transfer received at the end of t (thus, $H_{t+1}=H_t+\xi_t$ is the post-transfer nominal money held at the end of t). The motion of ξ_t is deterministic and follows a rule known to the agents. Expectations are rational, in the sense that they are consistent with the model and are generated by optimally processing the available information. Since there is uncertainty only at the local level, investors have perfect foresight on the behavior of aggregate variables. It is also worth to note that it is immaterial where the investors are located, since there is a single market for capital and a single market for the only good produced in this economy (no transportation cost). Finally--for simplicity and without loss of generality–it is ruled out the existence of actuarially fair annuities paid to the living investors by a financial institution collecting their wealth as they die: the wealth of someone who dies is inherited by some newly born individual (accidental bequests).

The skilled workers

Skilled workers are those who have been trained on the job while working in a firm for at least one period. In contrast, apprentices are workers with no work experience in the formal economy, but who have been hired by a firm after having invested to acquire the required basic knowledge. In their working lives, workers never lose the general skills that they have acquired. Being general, the skills acquired on the job are perfectly transferable. Thus, the skilled labor force evolves according to

$$M_{t+1}=(1-\sigma)(M_t+A_t),\ M_t=\int_0^n M_{it}\,di,\ M_0 \text{ given},\ A_t=\int_0^n A_{it}\,di, \tag{4}$$

where M_{it} are the skilled workers located in i during period t.

As in Blanchflower and Oswald (1994), workers choose location ex ante (at the beginning of each t), while firms decide on labor input once uncertainty is resolved. As for capital, labor is perfectly mobile across locations at the beginning of each period t, while mobility across locations is infinitely costly within one period.[7]

Once located in i, a skilled worker has the following period expected utility:

$$u_{it}^{sk}=E_t[p_{it}u(v_{it})+(1-p_{it})u(w)],\ u'>0, u''\leq 0, \tag{5}$$

where E_t is an expectation operator conditional on the information available in t as the realization of x_{it} is not yet known, w is the monetized value of home (informal) activities, and p_{it} is the fraction of the skilled workforce located in i that is employed in t:

[7] This short-term immobility implies that in period t a worker located in i does not work at all in the formal economy if s/he is not employed in that period by a firm of i.

$$p_{it} = \begin{cases} \dfrac{S_{it}}{M_{it}} & \text{if } S_{it} \leq M_{it} \\ 1 & \text{otherwise.} \end{cases} \tag{6}$$

At the beginning of each period, a skilled worker decides in what location to stay. Obviously, s/he locates where s/he can expect to enjoy the highest lifetime utility. Therefore, the discounted sequence of utilities that an optimizing skilled worker can expect (before the realization of x_{it}) to gain in the rest of his/her lifetime is given by

$$U^{sk}_{jt} = u^{sk}_{ji^*t} + \phi U^{sk}_{jt+1}, \ \phi \equiv \beta(1\text{-}\sigma), 0<\beta<1. \tag{7}$$

In (7), β is a time-preference parameter, and i* is the location where a skilled worker can have the best prospects (a "best location"):[8] $u^{sk}_{i^*t} \geq u^{sk}_{it} \ \ \forall i$.

The trainable workers

An investment in human capital at the beginning of period t in order to become “trainable” (or “employable” in the formal segment of the labor market) yields a strictly positive probability of being employed by a firm only in that period, since the basic knowledge acquired by a person is dissipated if it is not used on the job. Moreover, possession of the basic knowledge required by the firms has no value in the informal economy. Hence, the investment made in order to participate in the formal labor market will be lost, if within one period, the worker does not find an entry job paid at least as his/her reservation wage: after having invested in human capital, a trainable worker will accept any job offer paying an entry wage larger than his/her reservation wage e^{min}_{it}. Finally, also a trainable worker decides to stay in that location where s/he can expect to

[8] More than one location can share this status of best location.

enjoy the highest lifetime utility. Thus, the discounted sequence of utilities that an optimizing trainable worker can expect (before the realization of x_{it}) to gain in the rest of his/her lifetime is given by

$$U_t^{tr} = E_t \{ q_{i*t}[u(e_{i*t}) + \phi U_{t+1}^{sk}] + (1 - q_{i*t})[u(w) + \phi U_{t+1}^{un}] \}. \qquad (8)$$

In (8), U_{t+1}^{un} is the discounted sequence of utilities that an optimizing unskilled worker still alive at the beginning of t+1 can expect to get in the rest of his/her lifetime, $u(e_{it}) \geq u(e_t^{min}) \equiv u(w) - \phi(U_{t+1}^{sk} - U_{t+1}^{un})$, i* is a best location for a trainable worker,[9] and q_{it} is the fraction of the trainable workforce located in i that is hired in t:

$$q_{it} = \begin{cases} \dfrac{A_{it}}{L_{it}} & \text{if } A_{it} \leq L_{it} \\ 1 & \text{otherwise,} \end{cases} \qquad (9)$$

where L_{it} is the trainable workforce located in i.

The unskilled workers

At the beginning of each period, an unskilled worker must decide whether to incur the utility loss associated with participation in the formal labor market (one may interpret this disutility as due to the acquisition of the basic knowledge required by the firms operating in the formal economy and/or to the search of an entry job in the formal segment of the labor market) or to remain out of the formal labor market: an unskilled worker can be hired by a firm only if s/he becomes employable. An unskilled worker who decides not to invest in labor market participation has the same lifetime prospects

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[9] $E_t \{ q_{i*t}[u(e_{i*t}) + \phi U_{t+1}^{sk}] + (1 - q_{i*t})[u(w) + \phi U_{t+1}^{un}] \} \geq$
$\geq E_t \{ q_{it}[u(e_{it}) + \phi U_{t+1}^{sk}] + (1 - q_{it})[u(w) + \phi U_{t+1}^{un}] \} \ \forall i$.

as an employable worker who does not find an entry job after having incurred the utility loss entailed by this investment. Therefore, an optimizing unskilled worker can expect at the beginning of t to get the lifetime discounted sequence of utilities associated with the best available alternative:

$$U_t^{un} = \max\left\{- h(c) + U_t^{tr}, u(w) + \phi U_{t+1}^{un}\right\},\ h'>0, \tag{10}$$

where $-h(c)$ captures the disutility of investing in labor market participation (c is the monetized value of this disutility).

Wage determination

An insider-outsider scenario is considered. In each location, the wages are determined by negotiations held at the beginning of every period between a local union unconcerned about the interests of workers with no work experience and the local employers' association. In this context, it is immaterial whether the unions are only concerned about the workers employed in the previous period, or about both the latter and those experienced workers who were laid off in previous periods. In fact, even if the wage setters do not care about the interests of the skilled workers on layoff, the latter put pressure on them, insofar as they are perfect substitutes and thereby reduce the job security of the employed.

The union operating in i negotiates the real wage that all the firms of i must pay to the experienced workers in employment, while each individual firm takes its decisions on the demand for labor and capital in full autonomy. This negotiation also concerns the entry wage, which is established as the fixed fraction μ of the skilled workers' wage that firms must pay to the apprentices ($e_{it}=\mu v_{it}$). It is realistic to assume that the union does not allow the wage differential between skilled workers and apprentices fully to offset their productivity differential ($\Omega<\mu\leq 1$), so that any incentive for the employers to replace experienced workers with apprentices is suppressed.[10]

The bargaining process can be represented as if each union unilaterally sets the real wage in the awareness of its impact on the local firms' decisions. On the other hand, each union is aware that the effects of its wage policy on the economy as a whole is negligible. Similarly, each single firm perceives that its decisions on labor and capital input cannot influence the wage setting process because their impact is insignificant relatively to the size of the local labor market. Since the real wage, once negotiated, remains fixed for a certain lapse of time (a "period"), it is reasonable to assume that the wage is set by the union before the realization of the random variable that is relevant for that period.

Within this decentralized wage setting, in each t the local union operating in i must solve the following problem:

$$\max_{v_{it}} \; u_{it}^{sk} + \phi U_{t+1}^{sk} . \tag{11}$$

In each t the union has full control only over the current wage, if we maintain that current union membership cannot commit the workers who will manage the union in the future to the pursuit of policies not optimal from their own temporal perspective (a wage policy is feasible only if it is time consistent). Hence, the union's problem can be decomposed into a sequence of similar problems that can be solved recursively.

[10] Burdett and Smith (1995) emphasize that the key assumption for the existence of a low skill trap is that an employer's profit flow is greater when employing a skilled worker than when employing an unskilled

A summary of the timing of events

Summarizing, in each t we have a sequence of events in the following order: i) a new cohort enters the economy; ii) unskilled workers decide whether to invest in order to participate in the formal labor market, the workers decide where to locate, firms borrow physical capital for carrying out production; iii) unions set the wage; iv) idiosynchratic shocks occur; v) firms atomistically determine their demand for skilled workers and apprentices, production takes place, apprentices are trained on the job, workers are paid and output is sold to both workers and investors, investors determine how much to invest in capital and the amount of money to hold for next-period purchases; vi) firms reimburse the principal and pay the interest on the capital borrowed at the beginning of the period, firms also pay the dividends to the shareholders, a fraction σ of each group of population dies.

4 CHARACTERIZATION OF AN EQUILIBRIUM PATH

Equilibrium conditions in the markets for product and physical capital

One can easily derive the conditions for equilibrium both in the product market and in the market for productive assets:

$$Y_t + (1-\delta)K_t = K_{t+1} + C_t^{in} + C_t^{w}, \tag{12a}$$

$$K_{t+1}^{s} = K_{t+1}^{d}, \tag{12b}$$

where $C_t^w = \int_0^n (v_{it}S_{it} + e_{it}A_{it})di$ is the amount of Y_t consumed by the workers.

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worker. Indeed, the fact that firms lay off unskilled workers before skilled workers is difficult to reconcile

Firms' optimality condition for capital accumulation

Firms of i determine their demand for capital at the beginning of t by satisfying the optimality condition

$$E_t\left[\frac{\partial \pi^g(x_{it}, M_{it}, k_{it}, s_{it}, v_{it})}{\partial K_{it}}\right] = r_t + \delta,\ \ k_{it}\equiv K_{it}/M_{it},\ s_{it}\equiv L_{it}/M_{it}, \quad (13)$$

where the firms' (gross) profit function $\pi^g(.)$ is given in (A3).

This optimality condition defines k_{it}, that is the physical capital/skilled labor ratio in the firms of i, as an implicit function of the trainable labor/skilled labor ratio of i, the wage and the real interest rate:

$$f(k_{it}, s_{it}, v_{it}) = r_t + \delta,\ \ f_1 < 0,\ f_2 > 0 \text{ and } f_3 < 0, \quad (14)$$

where $f(.) = \dfrac{(1-\alpha)v_{it}^2\left\{(\mu/\Omega)^2[1-(1+\Omega s_{it})^{2-\alpha}]-1\right\}}{2n\alpha(2-\alpha)(1-\tau)k_{it}^{2-\alpha}} + \dfrac{(1-\alpha)n}{2(1+\Omega s_{it})^{-\alpha}k_{it}^{\alpha}}$ and v_{it} is determined by the union operating in i according to the time-invariant wage rule (see the Appendix 2))

$$v_{it} = v(k_{it}),\ \ v_k > 0. \quad (15)$$

The lifetime well-being of a skilled worker along an equilibrium path

Using (15) and (A5), one can obtain the equation governing the equilibrium path of the lifetime well-being of a skilled worker:

$$U_t^{sk} = u^{sk}(v(k_t), k_t) + \phi U_{t+1}^{sk}, \quad (16)$$

where the subscripts denoting the location are dropped. Indeed, an equilibrium pair $\left(\{s_t\}_0^\infty, \{k_t\}_0^\infty\right)$ satisfying (14)-(16) and (A7) depends on structural parameters assumed

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with the contention that unskilled workers are more profitable.

to be equal across locations and on exogenously given trajectory of r_t. Thus, different locations display equal physical capital/skilled labor and trainable labor/skilled labor ratios. Hence, local unions are induced to set the same wage in all locations, and workers can be indifferent (ex ante) among locations, expecting the same well-being everywhere.[11]

Using (A7), one can rewrite (16) as

$$\Psi(s_{t+1},k_{t+1},s_t,k_t)=\frac{h(c)}{\phi q(v(k_t),k_t,s_t)}-\frac{h(c)}{q(v(k_{t+1}),k_{t+1},s_{t+1})}+\frac{u(w)}{\phi}-\frac{u(\mu v(k_t))}{\phi}-$$

$$-u^{sk}(v(k_{t+1}),k_{t+1})+u(\mu v(k_{t+1}))=0\,. \tag{17}$$

Determination of the equilibrium real interest rate

One can determine the time profile of the interest rate by solving the problem of the investors. The investors' optimal plan must satisfy:

$$1+r_{t+1}=\frac{(1+\Pi_{t+1})(C^{in}_{t+2})^{\zeta}}{\theta^2(C^{in}_t)^{\zeta}},\ \Pi_t\equiv\frac{P_{t+1}-P_t}{P_t},^{12} \tag{18}$$

$$\lim_{t\to\infty}\theta^t K_t(C^{in}_t)^{-\zeta}=0\,. \tag{19}$$

One can see in (18) that the intertemporal trade-off faced by the investors involves three period (instead of two as in the non-monetary economy), since the decision of hoarding money at time t affects the possibility of accumulating capital at t+1, on which depends consumption at t+2. This implies that the real rate of return on capital increases with the cost of holding money, that is with the inflation rate: the

[11] In other words, the equilibrium solution is symmetric across locations.
[12]The cash-in-advance constraint is always binding if the following condition is satisfied:

return on capital must compensate the investors for the cost that they incur by holding the liquidity required to accumulate capital. Hence, money is not superneutral since cash is required for investment (see Stockman, 1981).

Along an equilibrium path, one has:

$$C_t^{in} = M_t \mathcal{C}(k_t, s_t, k_{t+1}), \tag{20}$$

where $\mathcal{C}(.) = \frac{[v(k_t)]^2 k_t^{\alpha-1}}{2\alpha(2-\alpha)n}\left\{1+\left(\frac{\mu}{\Omega}\right)^2\left[(1+\Omega s_t)^{2-\alpha}-1\right]\right\} + \frac{k_t^{1-\alpha} n(1+\Omega s_t)^{\alpha}}{2}$ -

$-v(k_t)(1+\mu s_t)+(1-\delta)k_t-(1-\sigma)[1+s_t q(v(k_t),k_t,s_t)]k_{t+1}$.

Moreover, along an equilibrium path, the skilled workforce evolves according to

$$M_{t+1} = M_t(1+\rho_{M_T}),\ \rho_{M_t} \equiv \frac{M_{t+1} - M_t}{M_t},\ M_0 \text{ given}, \tag{21}$$

where $\rho_{M_T} = \rho(k_t,s_t) = (1-\sigma)[1+s_t q(v(k_t),k_t,s_t)]-1$.

Finally, along an equilibrium path, the inflation rate is given by

$$1+\Pi_t = \frac{(1+\Theta_t)\{\mathcal{C}(k_t,s_t,k_{t+1})+[1+\rho(k_t,s_t)]k_{t+1}\}}{[1+\rho(k_t,s_t)]\{\mathcal{C}(k_{t+1},s_{t+1},k_{t+2})+[1+\rho(k_{t+1},s_{t+1})]k_{t+2}\}},\ \Theta_t \equiv \frac{H_{t+1}-H_t}{H_t},\ H_0 \text{ given}, \tag{22}$$

where Θ_t is governed by a deterministic rule known by the public. In other words, the rate of money growth is the instrument controlled by the Central Bank, and a monetary policy is a pre-announced, possibly state-contingent rule, for setting Θ_t. Since the policy is announced and followed, there are no "surprise" and the possible adjustments of Θ_t can be considered as perfectly anticipated.

$\frac{\theta(C_{t+1}^{in})^{\zeta}}{(C_{t+2}^{in})^{\zeta}} \leq (1+\Pi_{t+1}), \forall t$.

Given (14), (15), (18), (20), (21) and (22), the condition for equilibrium in the capital market can be written as

$$\Phi(k_{t+3},s_{t+2},k_{t+2},s_{t+1},k_{t+1},s_t,k_t,\Theta_{t+1})=0, \tag{23}$$

where $\Phi(.)=f(k_{t+1}, s_{t+1}, v(k_{t+1}))+1-\delta-$

$$-\frac{(1+\Theta_{t+1})\{C(k_{t+1},s_{t+1},k_{t+2})+[1+\rho(k_{t+1},s_{t+1})]k_{t+2}\}\{[1+\rho(k_t,s_t)]C(k_{t+2},s_{t+2},k_{t+3})\}^{\zeta}}{[1+\rho(k_{t+1},s_{t+1})]^{1-\zeta}\theta^2\{C(k_{t+2},s_{t+2},k_{t+3})+[1+\rho(k_{t+2},s_{t+2})]k_{t+3}\}[C(k_t,s_t,k_{t+1})]^{\zeta}}.$$

Therefore, a general equilibrium path of k_t and s_t must satisfy (17), (19) and (23), with money growing according to a deterministic rule. Note that the equilibrium trajectories of k_t and s_t are independent of the scale of the formal segment of the economy, i.e., they do not depend on M_t and on K_t.

Balanced growth path (BGP)

If k_t and s_t reach their steady-state values $\bar{k}$ and $\bar{s}$, output (Y_t), employment in the formal economy (S_t+A_t) and skilled labor (M_t) follow their BGP. Along this path, Y_t, S_t+A_t and M_t grow at their steady-state rate, which is determined only by the parameters of the model:

$$\bar{\rho}_{Y_t}=\bar{\rho}_{M\,t}=\bar{\rho}_{S_t+A_t}=\rho(\bar{k},\bar{s}),\ \frac{\partial\rho(\bar{k},\bar{s})}{\partial\bar{k}}>0, \frac{\partial\rho(\bar{k},\bar{s})}{\partial\bar{s}}>0, \tag{24}$$

where:

$\rho_{Y_t}\equiv\dfrac{Y_{t+1}-Y_t}{Y_t}$, $\rho_{S_t+A_t}\equiv\dfrac{S_{t+1}+A_{t+1}-S_t-A_t}{S_t+A_t}$, $S_t+A_t=M_t[p(v(k_t),k_t)+s_t q(v(k_t),k_t,s_t)]$ and

$$Y_t=M_t\left\{\frac{[\mu v(k_t)]^2[1-(1+\Omega s_t)^{2-\alpha}]-[\Omega v(k_t)]^2}{2n\alpha(2-\alpha)\Omega^2 k_t^{1-\alpha}}+\frac{nk_t^{1-\alpha}(1+\Omega s_t)^{\alpha}}{2}\right\}.$$

One can rule out the possibility that the steady-state growth rate of the skilled population is higher than the steady-state growth rate of the workers' population by endogeneizing the birth rate of the workers' population, i.e., by assuming that in the long run it responds to economic conditions and/or (if the "economy" does not coincide with the world economy) by allowing immigration flows of unskilled workers (see the Appendix 3))

5 THE NON-MONETARY ECONOMY (FRIEDMAN RULE)

The path of the economy is the same as it is in the absence of the cash-in-advance constraint if the inflation rate is such that

$$1+\Pi_{t+1}=\frac{\theta(C^{in}_{t+1})^{\zeta}}{(C^{in}_{t+2})^{\zeta}}. \tag{25}$$

A monetary policy consistent with (25) follows the Friedman rule, which amounts to set

$$(1+\Theta_{t+1})=\frac{\theta[C(k_{t+1},s_{t+1},k_{t+2})]^{\zeta}\{C(k_{t+2},s_{t+2},k_{t+3})+[1+\rho(k_{t+2},s_{t+2})]k_{t+3}\}}{[1+\rho(k_{t+1},s_{t+1})]^{\zeta-1}[C(k_{t+2},s_{t+2},k_{t+3})]^{\zeta}\{C(k_{t+1},s_{t+1},k_{t+2})+[1+\rho(k_{t+1},s_{t+1})]k_{t+2}\}}. \tag{26}$$

Applying rule (26), (23) becomes

$$\Phi(k_{t+2},s_{t+1},k_{t+1},s_t,k_t)=f(k_{t+1},s_{t+1},v(k_{t+1}))+1-\delta-\frac{\{[1+\rho(k_t,s_t)]C(k_{t+1},s_{t+1},k_{t+2})\}^{\zeta}}{\theta[C(k_t,s_t,k_{t+1})]^{\zeta}}=0. \tag{27}$$

Balanced growth path

The system satisfying (17), (19) and (27) characterizes the equilibrium path of the non-monetary economy, i.e. the equilibrium path of the economy in the absence of the cash-in-advance constraint. By setting $k_t=k_{t+1}=k_{t+2}=k$ and $s_t=s_{t+1}=s$ in (17) and (27), one can obtain the steady-state values of the physical capital/skilled labor ratio and of the trainable labor/skilled labor ratio.[13] Indeed, a steady-state pair $(\bar{k}, \bar{s})$ must satisfy the system

$$\Psi(k,s) = 0, \tag{28a}$$

$$\Phi(k,s) = 0, \tag{28b}$$

where (28a) must hold in order to ensure long-term equilibrium in the (trainable) labor market, and (28b) must hold in order to ensure long-term equilibrium in the (physical) capital market.

Equation (28a) implicitly defines s as an increasing function of k:

$$s = a(k),\ a' > 0. \tag{29a}$$

Given the number of skilled workers existing in the economy, more physical capital is necessary to induce an increasing number of unskilled workers to participate in the labor market. Other things being equal, a larger number of unskilled workers searching for a job as apprentices depresses any single unskilled worker's expected returns on labor market participation. Thus, this larger s needs to be accomodated by a higher capital stock, which entails both a higher probability of being hired and better lifetime prospects for any single worker if hired.

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[13] Along a BGP, the Friedman rule dictates $1+\Theta = \theta(1+\rho_{Y_t})^{1-\zeta}$ and it is never the case that $1+\Theta < \theta(1+\rho_{Y_t})^{1-\zeta}$. An intuitive explanation for the non-existence of a BGP with $1+\Theta < \theta(1+\rho_{Y_t})^{1-\zeta}$ is obtained by adapting the argument in Abel (1985), p.58. Suppose this BGP exist, and consider consuming one unit less at time t and holding P_t more units of money. This money can be used to buy $(1+\Pi_t)^{-1}$ units of consumption at time t+1. Hence, this will change the net present value of utility by $-(C_t^{in})^{-\zeta} + \theta(1+\Pi_t)^{-1}(C_{t+1}^{in})^{-\zeta}$. Since along a BGP $1+\Pi_t=(1+\Theta)(1+\rho_{Y_t})^{-1}$ and $C_{t+1}^{in} = (1+\rho_{Y_t})C_t^{in}$, the change in the net present value of utility is $(C_t^{in})^{-1}[\theta(1+\Theta)^{-1}(1+\rho_{Y_t})^{1-\zeta} - 1]$. If $1+\Theta < \theta(1+\rho_{Y_t})^{1-\zeta}$ then this change is strictly positive and the growth path could not have been optimal.

In contrast, the equation that any combinations of k and s must satisfy in order to ensure steady-state equilibrium in the capital market implicitly defines s as a function of k which may be increasing or decreasing in its argument:

$$s=b(k),\ b' \lesseqgtr 0. \tag{29b}$$

This ambiguity depends on the forces acting on the two sides of the capital market. On the demand side, a rise in s has a positive effect on the firms' expected profits: at any level of the capital stock, the increment in expected profits due to a marginally higher k increases with s. Thus, firms demand more physical capital at any given interest rate as s becomes larger. On the supply side, a larger s has a positive wealth effect on investors, since it boosts future growth. Thus, at any given rate of return on capital, investors are willing to consume more and devote fewer resources to capital accumulation when they expect a larger s. Combining demand and supply forces, it follows straightforwardly that a larger s pushes up the equilibrium rate of return on capital, while the overall effect on k is ambiguous. If the investors' preference for smoothing consumption over time is relatively strong, i.e., if the intertemporal elasticity of substitution is low (ζ is relatively large), a large increase in r is required in order to convince the investors to follow a more upward sloping consumption path. In this case, an improvement in growth prospects generates a strong upward pressure on r, and the rise in s will depress k ($b'<0$). The opposite case holds if the investors' period utility is close to increase linearly in consumption (relatively small ζ).

Using (29a), one can state the condition that must be satisfied for long-run equilibrium in the capital market only in terms of k by considering exclusively those values of s that are consistent for given values of k with long-run equilibrium of the trainable-labor market. Hence,

$$f(\mathrm{k},a(\mathrm{k}),v(\mathrm{k}))-\delta=\frac{[1+\rho(\mathrm{k},a(\mathrm{k}))]^{\zeta}}{\theta}-1, \tag{30}$$

where for optimality on the demand side of the capital market one has $f(\mathrm{k},a(\mathrm{k}),v(\mathrm{k}))-\delta=\mathrm{r}$, and for optimality on the supply side of the capital market one has $\frac{[1+\rho(\mathrm{k},a(\mathrm{k}))]^{\zeta}}{\theta}-1=\mathrm{r}$.

Uniqueness of the BGP (large ζ)

It is evident from fig.1 that $b'<0$ is sufficient for having a unique steady-state pair $(\bar{\mathrm{k}}^*,\bar{\mathrm{s}}^*)$. We have seen that this condition is likely to hold if ζ is large, namely if the elasticity of capital supply with respect to r is low (see fig.2). By linearizing the system consisting of (17) and (27) about its steady state for the case in which ζ is large, numerical examples show that the linearized system can exhibit saddle-path stability: for any initial condition k_0 in a neighborhood of $(\bar{\mathrm{k}}^*,\bar{\mathrm{s}}^*)$, the linearized system characterizes a unique path converging to it (see the Appendix 4)).

The uniqueness of the BGP and the fact that along an equilibrium path the rate of growth does not depend on the scale of the formal economy have the important implication that economies with identical parameters' values (structurally similar), but with different initial endowments of skilled labor and physical capital, tend to differ permanently in their levels of M_t, K_t, A_t, S_t, and Y_t.[14]

[FIGURE 1]

[FIGURE 2]

Multiple BGP (small ζ)

The strategic complementarity between physical capital and trainable labor creates the possibility of multiple BGP. However, the potential for multiple BGP cannot actualize itself if the rise in rate of return on capital required by the investors to accelerate the accumulation process is too large relative to the increment in discounted (gross) profits generated by boosting growth. Therefore, the potential for multiple equilibrium paths is able to actualize itself only if the investors' preference for smoothing consumption over time is weak (ζ close to 0), so that they are willing to finance higher growth for a modest increase in the firms' cost of capital.[15]

It is evident from the previous discussion why the existence of multiple BGP requires that b'>0 for at least some range of values of k: along the curve giving the combinations of k and s consistent with the long-run equilibrium of the capital market, there must be an interval of values of k within which an increment of s increases the expected marginal profitability of capital more than the rate of return required by the investors. It is possible to verify that this is actually the case when multiple steady-state

[14] Even along the transition path, two economies with identical parameters but endowed with different initial stocks of skilled labor and physical capital can grow at an identical rate if their initial physical capital/skilled labor ratios are the same.

pairs of k and s exist (see fig. 3). This can be seen also in the (k,r) plane (see fig. 4): the condition (30) can be satisfied at more than one steady-state level of the interest rate. In fig. 4 one has both a steady state associated with lower k and r and a steady state associated with higher k and r: the dynamic externalities created by an increase in capital investment raise expected profitability so that firms are able to bear a higher cost of capital.

Indeed, for the case in which ζ is small, one may have two steady-state pairs $(\bar{k}^h, \bar{s}^h)$ and $(\bar{k}^l, \bar{s}^l)$ such that $\bar{k}^h > \bar{k}^l$ and $\bar{s}^h > \bar{s}^l$. Given (24), the rate of growth of employment and output is permanently higher at $(\bar{k}^h, \bar{s}^h)$ than it is along the BGP associated with $(\bar{k}^l, \bar{s}^l)$: $\bar{\rho}_Y^h = \bar{\rho}_{S+A}^h > \bar{\rho}_Y^l = \bar{\rho}_{S+A}^l$. The associated steady-state interest rates are such that $\bar{r}^h \geq \bar{r}^l$.[16] By linearizing the system consisting of (17) and (27) around these steady states, the system thus obtained characterizes a unique path of k_t and s_t converging to $(\bar{k}^h, \bar{s}^h)$ for any given k_0 in a neighborhood of $\bar{k}^h$ and a continuum of paths converging to $(\bar{k}^l, \bar{s}^l)$ for any given k_0 in a neighborhood of $\bar{k}^l$ (see the Appendix 5) and 6))·

Given the cumulative nature of the growth process, the levels of output and (formal) employment of an economy moving along the high-growth BGP diverge over time from the employment and output levels of a structurally similar economy

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[15] In the limiting case in which ζ=0 (constant marginal utility of consumption), different permanent rates of growth are consistent with the same equilibrium rate of return on capital.

[16] Again, one has $\bar{r}^h = \bar{r}^l = \theta^{-1} - 1$ if and only if $\zeta = 0$.

following the BGP characterized by $(\bar{k}^1, \bar{s}^1)$. Finally, it should be emphasized that a high-growth equilibrium path is always Pareto superior than a low-growth path. Indeed, in a high- growth regime i) investors enjoy a positive wealth effect, ii) the skilled workers are able to exploit a more favorable trade-off between real wage and the probability of being employed, thus increasing their expected lifetime sequence of discounted utilities, iii) the apprentices' wages are higher, iv) a larger number of workers have the opportunity to be trained on the job and increase their human wealth, and v) the lifetime prospects of an unskilled worker remain unchanged. It is worth noting that, when more than one long-run equilibrium path is possible, it is not necessarily the case that a long-term trade-off between employment and real wage emerges: along the high-growth BGP, both the employment level and the average real wage tend to be higher than when the economy follows the low-growth BGP. Since also the investors are better along a high-growth BGP, it is highly desirable to lead the economy toward a long-run equilibrium path along which the economy grows at a higher rate.

[FIGURE 3]

[FIGURE 4]

Global indeterminacy (ζ=0)

For the case in which $\zeta=0$, the system governing k_t and s_t reduces to a single first-order difference equation, since (17) can be rewritten as a first-order difference equation in k_t only by using the fact that (27) implicitly defines s_t as a function of k_t. Given that the motion of k_t and s_t is completely governed by forward-looking expectations, the initial condition on k_0 does not play any role in determining the dynamics of k_t and s_t for $t \geq 1$, and in $t=1$ the system can jump to $(\bar{k}^h, \bar{s}^h)$ or to one among the continuum of equilibrium paths converging to $(\bar{k}^l, \bar{s}^l)$ in a neighborhood of it (see the Appendix 7) and 8), together with fig. 5). This implies that--depending on the "animal spirits" of capital-market participants--structurally similar economies starting with equal initial endowments K_0 and M_0 may grow at different steady-state rates of growth (global indeterminacy).

[FIGURE 5]

5 COMPARING DIFFERENT MONETARY POLICIES

We take into account four alternative monetary policies:

i) inflation targeting (Θ_t is adjusted so as to keep $\Pi_t = \overline{\Pi}\ \forall t$);[17]

ii) nominal interest pegging (Θ_t is adjusted so as to keep $i_t = \bar{i}\ \forall t$, where $(1+i_t)=(1+r_t)(1+\Pi_t)$);[18]

iii) constant money growth ($\Theta_t = \overline{\Theta}\ \forall t$);

iv) real interest pegging (Θ_t is adjusted so as to keep $r_t = \bar{r}\ \forall t$).[19]

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[17] This implies

$$(1+\Theta_{t+1}) = \frac{(1+\overline{\Pi})[1+\rho(k_{t+1}, s_{t+1})]\left\{C(k_{t+2}, s_{t+2}, k_{t+3}) + [1+\rho(k_{t+2}, s_{t+2})]k_{t+3}\right\}}{\left\{C(k_{t+1}, s_{t+1}, k_{t+2}) + [1+\rho(k_{t+1}, s_{t+1})]k_{t+2}\right\}}.$$

[18] This implies $$(1+\Theta_{t+1}) = \frac{\theta[1+\rho(k_{t+1}, s_{t+1})]\left\{C(k_{t+2}, s_{t+2}, k_{t+3}) + [1+\rho(k_{t+2}, s_{t+2})]k_{t+3}\right\}}{\left\{C(k_{t+1}, s_{t+1}, k_{t+2}) + [1+\rho(k_{t+1}, s_{t+1})]k_{t+2}\right\}} \bullet$$

$$\bullet \left\{\frac{C(k_t, s_t, k_{t+1})}{[1+\rho(k_{t+1}, s_{t+1})][1+\rho(k_t, s_t)]C(k_{t+2}, s_{t+2}, k_{t+3})}\right\}^{\zeta/2} \sqrt{(1+\bar{i})}.$$

Along an equilibrium path, the optimality condition that is satisfied on the supply side of the capital market changes with the policy rule adopted by the monetary authority. In particular, along a BGP, one has:

$$
\mathrm{r} = \begin{cases} (1+\overline{\Pi})\left\{\dfrac{[1+\rho(\mathrm{k}, a(\mathrm{k}))]^{\zeta}}{\theta}\right\}^2 - 1 & \text{under inflation targeting} \\ \dfrac{\sqrt{(1+\overline{\mathrm{i}})}[1+\rho(\mathrm{k}, a(\mathrm{k}))]^{\zeta}}{\theta} - 1 & \text{under nominal interest pegging} \\ \dfrac{(1+\overline{\Theta})}{[1+\rho(\mathrm{k}, a(\mathrm{k}))]}\left\{\dfrac{[1+\rho(\mathrm{k}, a(\mathrm{k}))]^{\zeta}}{\theta}\right\}^2 - 1 & \text{under constant money growth} \\ \overline{\mathrm{r}} & \text{under real interest pegging.} \end{cases} \tag{31}
$$

Monetary rules when the non-monetary economy has an unique BGP (large ζ)

We have seen that the BGP is likely to be unique in the non-monetary economy (i.e., under the Friedman rule) when ζ is relatively large. This uniqueness is certainly preserved by any monetary policy that makes the real return on capital required by the investors along a BGP at least as responsive to changes in the balanced growth rate of the economy as it is in the non-monetary economy.

This is the case both under inflation targeting and under nominal interest rate targeting. In particular, a monetary policy that adjusts Θ_t so as to keep $\Pi_t = \overline{\Pi}\ \forall t$ makes capital supply less elastic with respect to the real rate of interest than it is in the non-monetary economy. This is due to the fact that an increase in capital accumulation boosts growth, and the cost of holding money is not affected by the rate of growth of the

---

[19] This implies

$$
1+\Theta_{t+1} = \frac{(1+\overline{\mathrm{r}})[1+\rho(\mathrm{k}_{t+1}, \mathrm{s}_{t+1})]^{1-\zeta}\theta^2\{C(\mathrm{k}_{t+2}, \mathrm{s}_{t+2}, \mathrm{k}_{t+3}) + [1+\rho(\mathrm{k}_{t+2}, \mathrm{s}_{t+2})]\mathrm{k}_{t+3}\}[C(\mathrm{k}_t, \mathrm{s}_t, \mathrm{k}_{t+1})]^{\zeta}}{\{C(\mathrm{k}_{t+1}, \mathrm{s}_{t+1}, \mathrm{k}_{t+2}) + [1+\rho(\mathrm{k}_{t+1}, \mathrm{s}_{t+1})]\mathrm{k}_{t+2}\}\{[1+\rho(\mathrm{k}_t, \mathrm{s}_t)]C(\mathrm{k}_{t+2}, \mathrm{s}_{t+2}, \mathrm{k}_{t+3})\}^{\zeta}}.
$$

economy when monetary policy accommodates any change in the economy's growth rate so as to keep constant the inflation rate: under inflation targeting, any anticipated increase (or decrease) in the economy's growth rate has a stronger wealth effect than it has in the non-monetary economy because the intertemporal trade-off involves three instead of two periods.

Under constant money growth, $\zeta \geq 1$ implies that the capital supply is less elastic with respect to the interest rate than it is in the non-monetary economy and uniqueness is certainly preserved. As $\zeta<1$ but ζ is close to 1, it is likely that the BGP is unique even if the capital supply is more elastic with respect to the interest rate than it is in the non-monetary economy.

In contrast with the monetary rules discussed above, a monetary policy aimed at keeping constant the real interest rate may not preserve the uniqueness of the BGP. Indeed, this policy keeps invariant the real cost of capital in the face of changes in ρ_{Y_t}, thus making possible the existence of multiple BGP even if the BGP is unique in the absence of money (see fig. 6 and the numerical example in the Appendix 9)).

The best that the monetary authority can do for boosting growth along a BGP is to calibrate the monetary rules so as to let the economy grow at the same steady-state rate $\bar{\rho}^*_{Y_t} = \rho(\bar{k}^*, a(\bar{k}^*))$ that it can reach under the Friedman rule. This entails:

$\bar{\Pi}^* = \dfrac{\theta}{[1+\rho(\bar{k}^*, a(\bar{k}^*))]^{\zeta}} - 1$ under inflation targeting,

$\bar{i}^* = 0$ under nominal interest pegging,

$$\overline{\Theta}^* = \frac{\theta}{[1+\rho(\overline{k}^*, a(\overline{k}^*))]^{\zeta-1}} - 1 \text{ under constant money growth,}$$

$$\overline{r}^* = \frac{[1+\rho(\overline{k}^*, a(\overline{k}^*))]^{\zeta}}{\theta} - 1 \text{ under real interest pegging.}$$

Indeed, one can easily check that the adoption of an inflation target $\overline{\Pi} < \overline{\Pi}^*$ is inconsistent with the existence of a BGP (see footnote 14), while for any $\overline{\Pi} > \overline{\Pi}^*$ one has $\overline{\rho}_{Y_t} < \overline{\rho}^*_{Y_t}$ (see fig. 7 and the numerical example in the Appendix10)). Similarly, one can check that $\overline{i}^*$, $\overline{\Theta}^*$ and $\overline{r}^*$ are the targets, respectively, for nominal interest, constant money growth and real interest rate that maximize steady-state growth: again, $\overline{i} < \overline{i}^*$, $\overline{\Theta} < \overline{\Theta}^*$ and $\overline{r} < \overline{r}^*$ are inconsistent with the existence of a BGP, while $\overline{i} > \overline{i}^*$, $\overline{\Theta} > \overline{\Theta}^*$ and $\overline{r} > \overline{r}^*$ are associated with a steady-state rate of economic growth strictly lower than $\overline{\rho}^*_{Y_t}$.

Furthermore, one can note that a higher steady-state rate of inflation is associated with a higher $\overline{r}$ whenever the higher cost of holding money exerts an upward pressure on the equilibrium cost of capital which prevails over the depressing effect on it due to the lower steady-state rate of economic growth brought about by the increased cost of liquidity. Alternatively, a higher steady-state rate of inflation is associated with a lower $\overline{r}$ whenever the depressing effect on the equilibrium cost of capital due to the lower steady-state rate of economic growth prevails over the upward pressure on the equilibrium cost of capital caused by the higher cost of holding money. Both these possibilities can come true depending on the initial value of $\overline{\Pi}$ (see fig. 7 and the numerical example in the Appendix 10)). The first of these possibilities is consistent with a situation in which a permanent slowdown in economic growth does not bring about a decrease in the long-term real interest rate, while the second scenario is consistent with Fisher's conjecture and the evidence presented by Summers (1983), namely with a less-than-proportional adjustment of the nominal interest rate to anticipated inflation.

Monetary rules when the non-monetary economy has multiple BGP (small ζ)

In the non-monetary economy, one may have multiple BGP when the intertemporal elasticity of substitution is high (small ζ). The existence of multiple BGP tends to be preserved also when monetary policy aims at keeping $\Pi_t = \overline{\Pi}$, $i = \bar{i}$ or $r = \bar{r}\ \forall t$. This is because--under these rules--ζ close to 0 implies that changes in the rate of economic growth have a relatively small impact (or no impact at all if $r = \bar{r}\ \forall t$) on the real cost of capital.

In contrast, the adoption of the rule $\Theta_t = \overline{\Theta}\ \forall t$ is likely to be consistent with the existence of a unique BGP even if the non-monetary economy exhibits multiple BGP. This is because--under fixed money growth--ζ close to 0 implies that changes in the rate of economic growth determine relatively large movements in the real rate of return required by the investors. (the real cost of capital responds negatively to any increase in the economy's growth rate). Hence, in general only one steady-state rate of growth $\bar{\rho}_{Y_t} = \bar{\rho}_{M_t} = \bar{\rho}_{S_t+A_t} = \rho(\bar{k}, a(\bar{k}))$ is associated with the structural parameters of the economy and the value of $\overline{\Theta}$ set by the Central Bank. Moreover, $\bar{\rho}_{Y_t}$ tends to increase with $\overline{\Theta}$ for $\overline{\Theta} \leq \overline{\Theta}^h$, where $\overline{\Theta}^h = \dfrac{\theta}{[1+\rho(\bar{k}^h, a(\bar{k}^h))]^{\zeta-1}} - 1$ (ζ<1/2) is the (fixed) rate of money growth that maximizes the balanced growth rate, i.e., that is consistent with $\overline{\rho}^{h}_{Y_t}$ (see the Appendix 11) and figure 8).

Monetary rules when the non-monetary economy exhibits global indeterminacy (ζ=0)

In the light of the previous discussion, it is apparent that when the non-monetary economy may converge to two different BGP no matters what its initial state is, this global indeterminacy is not resolved by monetary policy under inflation targeting and under nominal or real interest rate pegging. In contrast, the monetary authority can select one of these long-term equilibrium paths and lead the economy to converge toward it by choosing the appropriate fixed rate of money growth. In this case, the choice of a higher rate of money growth may lead the economy to grow at a higher steady-state rate, without bringing about a higher steady-state rate of inflation. In particular, the Central Bank's choice of setting $\overline{\Theta} = \overline{\Theta}^{h}$ can lead the economy to grow along the BGP characterized by $\overline{\rho}_{Y_t} = \overline{\rho}^{h}_{Y_t}$, while the choice of the more restrictive monetary policy $\overline{\Theta} = \overline{\Theta}^{l}$ ($\overline{\Theta}^{l} < \overline{\Theta}^{h}$) would have led the economy toward the BGP characterized by $\overline{\rho}_{Y_t} = \overline{\rho}^{l}_{Y_t}$ ($\overline{\rho}^{h}_{Y_t} > \overline{\rho}^{l}_{Y_t}$), without any long-term benefit in terms of lower inflation ($\overline{\Pi}^{h} = \overline{\Pi}^{l}$) (see the Appendix 12)).

6 CONCLUSIONS

The general equilibrium dynamic model presented in this paper combines employment and growth theory with a cash-in-advance constraint in order to help addressing three related questions: i) Is it possible to give a solid theoretical foundation to the proposition according to which an over-restrictive monetary policy may lower the long-run growth potential of the economy? ii) Can monetary policy lead the economy along a higher balanced growth trajectory? iii) Can monetary policy intervene to correct a possible capital market failure in its role of coordinating intertemporal decisions?

In the paper, one can give positive answers to these questions only when the intertemporal elasticity of substitution in consumption is implausibly high. In contrast, for empirically more significant values of this elasticity, the negative answers that the orthodox monetary theory is used to give to these questions still hold.

APPENDIX

1) Derivation of the firms' (gross) profit function π^g

Given the perfectly transferable nature of the general skills acquired by an apprentice, each employer is aware that there is no guarantee that a newly hired worker will remain with his/her firm in the future. This is why an employer does not consider the future returns accruing from the on-the-job training of an apprentice: since the forthcoming benefit of adding a skilled worker to the stock of human capital available to the economy as a whole cannot be appropriated privately, the employer can ignore it as an insignificant externality. Therefore, the selection of the optimal labor policies by a firm amounts in each t to solving the static decision problem of maximizing (2) with respect to S_{it} and A_{it}. Given its optimal labor policies, a firm is able to determine at the beginning of t the amount of K_{it} to borrow and install. As the local shock is favorable, the aggregate demand for either trained labor or apprentices by firms in location i may be rationed. In the aggregate, it is always the case that:

$$S_{it} \le M_{it}, \tag{A1a}$$

$$A_{it} \le L_{it}. \tag{A1b}$$

When labor demand happens to be rationed, it is reasonable to assume that the scarce supply of labor is evenly distributed among firms of the same location. Note that the union wages are not determined at the firm level and that employers cannot compete for labor in short supply by raising the relevant wages in order to keep and poach workers, even if skills are perfectly transferable among firms (see Soskice, 1990). Therefore, with one as the normalized number of firms of location i, we can take (A1) to be the constraints faced by each individual firm as the union wages induce all the available skilled and trainable workers to accept a job offer. Hence, the firm's choice of the labor inputs amounts to solving the static decision problem of maximizing (2) subject to (1) and (A1), from which one can derive the optimal labor policies:

$$S_{it}=S(x_{it},M_{it},k_{it},v_{it})=\begin{cases} M_{it}k_{it}\left[\dfrac{\alpha x_{it}}{v_{it}}\right]^{1/(1-\alpha)} & \text{if } x_{it} \le \dfrac{v_{it}k_{it}^{\alpha-1}}{\alpha},\ k_{it} \equiv \dfrac{K_{it}}{M_{it}} \\ M_{it} & \text{otherwise,} \end{cases} \tag{A2a}$$

$$A_{it}=A(x_{it},M_{it},k_{it},s_{it},v_{it})=\begin{cases} 0 & \text{if } x_{it} \le \dfrac{\mu v_{it}k_{it}^{\alpha-1}}{\alpha\Omega} \\ M_{it}s_{it} & \text{if } x_{it} > \dfrac{\mu v_{it}(1+\Omega s_{it})k_{it}^{\alpha-1}}{\alpha\Omega},\ s_{it} \equiv \dfrac{L_{it}}{M_{it}} \\ M_{it}k_{it}\left[\dfrac{\alpha x_{it}}{\Omega^{-\alpha}\mu v_{it}}\right]^{1/(1-\alpha)} - \dfrac{M_{it}}{\Omega} & \text{otherwise.} \end{cases} \tag{A2b}$$

Using (2), (3) and (A2), one can obtain the (gross) profit function

$$\pi^g=\pi^g(x_{it},M_{it},k_{it},s_{it},v_{it}), \tag{A3}$$

where

$$
\pi^{g}(.) = \begin{cases} (1-\alpha)M_{it}k_{it}\left[\left(\dfrac{\alpha}{v_{it}}\right)^{\alpha} x_{it}\right]^{1/(1-\alpha)} & \text{if } x_{it} \le \dfrac{v_{it}k_{it}^{\alpha-1}}{\alpha} \\ x_{it}M_{it}k_{it}^{1-\alpha} - M_{it}v_{it} & \text{if } \dfrac{v_{it}k_{it}^{\alpha-1}}{\alpha} < x_{it} \le \dfrac{\mu v_{it}k_{it}^{\alpha-1}}{\alpha\Omega} \\ x_{it}M_{it}k_{it}^{1-\alpha}(1+\Omega s_{it})^{\alpha} - M_{it}v_{it} - s_{it}M_{it}\mu v_{it} & \text{if } x_{it} > \dfrac{\mu v_{it}(1+\Omega s_{it})^{1-\alpha}}{\alpha\Omega k_{it}^{1-\alpha}} \\ (1-\alpha)M_{it}k_{it}\left[\left(\dfrac{\alpha\Omega}{\mu v_{it}}\right)^{\alpha} x_{it}\right]^{1/(1-\alpha)} + \dfrac{(\mu-\Omega)M_{it}v_{it}}{\Omega} & \text{otherwise.} \end{cases}
$$

2) Derivation of the equilibrium condition for the trainable labor market and of the wage rule

Having the optimal demand for skilled labor in (A2a), one can compute the probability of a skilled worker located in i (before the realization of x_{it}) to be employed in period t:

$$
p(v_{it}, k_{it}) = 1 - \frac{v_{it}k_{it}^{\alpha-1}}{n\alpha(2-\alpha)}, \quad p_{v} < 0, p_{k} > 0. \tag{A4}
$$

By using (5) and (A4), one can write the period utility expected (before the realization of x_{it}) by a skilled worker located in i:

$$
u^{sk}(v_{it}, k_{it}) = \left[1 - \frac{v_{it}k_{it}^{\alpha-1}}{n\alpha(2-\alpha)}\right]u(v_{it}) + \frac{v_{it}k_{it}^{\alpha-1}}{n\alpha(2-\alpha)}u(w). \tag{A5}
$$

Similarly, one can use (A2b) to compute the probability that a trainable worker located in i (before the realization of x_{it}) will be hired in period t:

$$
q(v_{it}, k_{it}, s_{it}) = 1 - \frac{\mu v_{it}k_{it}^{\alpha-1}[(1+\Omega s_{it})^{2-\alpha} - 1]}{n\alpha(2-\alpha)\Omega^{2}s_{it}}, \quad q_{v} < 0, q_{k} > 0, q_{s} < 0. \tag{A6}
$$

Note that $q(.)$ diminishes as there is a larger number of trainable workers, remaining constant both the size of the skilled workforce and the stock of capital located in i. In equilibrium, the number of unskilled workers who become trainable in location i must be such that an unskilled worker is indifferent between investing in (formal) labor-market participation or staying in the informal economy:

$$
h(c) = q(v_{it}, k_{it}, s_{it})[u(\mu v_{it}) - u(w) + \phi(U_{t+1}^{sk} - U_{t+1}^{un})], \tag{A7a}
$$

where along an equilibrium path

$$U_t^{un} = u(\mathrm{w}) + \phi U_{t+1}^{un}. \tag{A7b}$$

The period utility function of a skilled worker depends on the real wage and on the physical capital/skilled labor ratio, which is a predetermined variable when an union sets the wage. Given the forward looking behavior of firms and unskilled workers, the current wage policy of an union could affect the union's future policy and the utility of its members only if it had a significant impact on the investors' behavior. However, this is not the case because of the continuum of unions operating in the economy. Thus, the problem of a single union amounts to solve the following sequence of static problems:

$$\max_{v_{it}} u^{sk}(v_{it}, k_{it}), \tag{A8}$$

from which one obtains the following sequence of first-order conditions:

$$\frac{\partial u^{sk}(v_{it}, k_{it})}{\partial v_{it}} = 0, \tag{A9}$$

defining implicitly the time-invariant wage rule (15).

3) Population's dynamics

Assume that the workers' population evolves according to:

$$N_{t+1} = (1-\sigma+\xi)N_t + G_t,\ \xi > 0, \tag{A10}$$

where N_t is the total number of workers in period t, ξ is the birth rate and G_t is the net inflow of (unskilled) immigrants at the end of period t who join the unskilled workforce at the beginning of t+1. Given (A10), the rate of growth of the workers' population is the following:

$$\chi_t = \xi - \sigma + a_t,\ \chi_t \equiv \frac{N_{t+1} - N_t}{N_t},\ a_t \equiv \frac{G_t}{N_t}. \tag{A11}$$

Finally, assume that a_t evolves according to:

$$a_{t+1} = a_t + j(\rho_{S_t + A_t} - \chi_t), \tag{A12}$$

where the function $j(.)$ increases monotonically with $\rho_{S_t + A_t} - \chi_t$ and is such that

$$j(.) = \begin{cases} a_{t+1} - a_t > 0 \text{ if } \rho_{S_t+A_t} > \chi_t \\ 0 \text{ if } \rho_{S_t+A_t} = \chi_t \\ a_{t+1} - a_t < 0 \text{ if } \rho_{S_t+A_t} < \chi_t. \end{cases}$$

Given (A11) and (A12), a balanced growth path must be characterized by $\bar{a} = \bar{\rho}_{S+A} - \xi + \sigma$ (entailing $\bar{\chi} = \bar{\rho}_{S+A}$).

4) Numerical example showing that the system obtained by linearizing (17) and (27) around $(\bar{k}^*, \bar{s}^*)$ *can be saddle-path stable*

Let h(c)=.027577, $u(v_t)=v_t$ and u(w)=w=.5, n=1, $\Omega = \mu = .5$, δ=.01238, ζ=.71, σ=.01, α=2/3, β=.80808, γ=.97954. Given these parameter values, we get: $\bar{k}^*$=3.138, $\bar{s}^*$=.2 and $\bar{v}^*$=.9006815. Using the fact that $k_t = k(v_t) = \left[\frac{2v_t - w}{n\alpha(2-\alpha)}\right]^{1/(1-\alpha)}$, one can rewrite (17) and (27) as a system of difference equations in s_t and v_t. Linearizing the system thus obtained around $(\bar{v}^*, \bar{s}^*)$, one can derive the following characteristic equation of the linearized system: $\lambda^3 - 3.1862072\lambda^2 + 3.3441234\lambda - 1.1571009 = 0$, where $\lambda_1 = .8834503, \lambda_2 = 1.2775535$ and $\lambda_3 = 1.0252034$ are the solving characteristic roots, implying saddle-path stability.

5) Conditions that the system obtained by linearizing (17) and (27) around a steady-state pair $(\bar{k}, \bar{s})$ *must satisfy for having a unique equilibrium path converging to* $(\bar{k}^h, \bar{s}^h)$ *and a continuum of equilibrium paths converging to* $(\bar{k}^l, \bar{s}^l)$

If there exist two steady-steady pairs $(\bar{k}^h, \bar{s}^h)$ and $(\bar{k}^l, \bar{s}^l)$, it is the case that

$$\frac{-(\Phi_{K_{t+2}} + \Phi_{K_{t+1}} + \Phi_{K_t})}{\Phi_{S_{t+1}} + \Phi_{S_t}} > \frac{-(\Psi_{K_{t+1}} + \Psi_{K_t})}{\Psi_{S_{t+1}} + \Psi_{S_t}}, \tag{A13}$$

as all derivatives are evaluated at $(\bar{k}^h, \bar{s}^h)$, and that

$$\frac{-(\Phi_{K_{t+2}} + \Phi_{K_{t+1}} + \Phi_{K_t})}{\Phi_{S_{t+1}} + \Phi_{S_t}} < \frac{-(\Psi_{K_{t+1}} + \Psi_{K_t})}{\Psi_{S_{t+1}} + \Psi_{S_t}}, \tag{A14}$$

as all derivatives are evaluated at $(\bar{k}^l, \bar{s}^l)$. One can check that (A13) entails

$$a_2 > -a_1 - a_3 - 1, \tag{A15}$$

while (A14) entails

$$a_2 < a_1 - a_3 - 1, \tag{A16}$$

where

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0 \tag{A17}$$

is the characteristic equation of the system obtained by linearizing (17) and (27) around steady-state values of k_t and s_t. Together with $a_1<-2$, $0<a_3<1$ and $a_2>0$, (A15) entails $\lambda_1>\lambda_2>1>0>\lambda_3>-1$, while (A16) entails $\lambda_1>1>\lambda_2>0>\lambda_3>-1$, where λ_1, λ_2 and λ_3 are the characteristic roots.

6) Numerical example showing that the system obtained by linearizing (17) and (27) around a steady-state pair $(\bar{k}, \bar{s})$ *can have a unique equilibrium path converging to* $(\bar{k}^h, \bar{s}^h)$ *and a continuum of equilibrium paths converging to* $(\bar{k}^l, \bar{s}^l)$

Let h(c)=9.2576516^{-03}, $u(v_t)=v_t$, $u(w)=w=.1$, $\Omega = .25$, $\mu = .2681247$, σ=.03, α=.75, β=.4333714, γ=.8617753, ζ=.025, n=1, δ=.28255. Given these parameter values, we get: $\bar{k}^h = .1$, $\bar{s}^h = .2$, $\bar{\rho}^h_{Y_t}$ =.008338, $\bar{k}^l = .0988142$, $\bar{s}^l = .1$ and $\bar{\rho}^l_{Y_t}$ =-.010630. By solving (A17) in a neighborhood of $(\bar{k}^h, \bar{s}^h)$, one obtains $\lambda_1 = -.0247185$, $\lambda_2 = 1.0147784$ and λ_3=7.6340984, where a_1=-8.6241583, a_2=7.5331318 and a_3=.1914909; while by solving (A17) in a neighborhood of $(\bar{k}^l, \bar{s}^l)$, one obtains $\lambda_1 = -.0359049$, $\lambda_2 = .9858373$ and λ_3=8.1317157, where a_1=-9.0816481, a_2=7.6891839 and a_3=.2878336.

7) Proof that the system obtained by linearizing (17) around a steady-state value $\bar{k}$ can jump to $\bar{k}^h$ or to one among the continuum of paths converging to $\bar{k}^l$

Linearizing the first-order difference equation (17) around $\bar{k}$ yields the characteristic equation $\lambda + \left(\dfrac{\Psi_{K_t} - \Psi_{s_t} \dfrac{\Phi_k}{\Phi_s}}{\Psi_{K_{t+1}} - \Psi_{s_{t+1}} \dfrac{\Phi_k}{\Phi_s}} \right) = 0$, where all derivatives are evaluated at $(\bar{k}, \bar{s})$. Since at $(\bar{k}^h, \bar{s}^h)$ one has $-\left(\dfrac{\Psi_{k_{t+1}} + \Psi_{k_t}}{\Psi_{s_{t+1}} + \Psi_{s_t}} \right) < -\dfrac{\Phi_k}{\Phi_s}$, while at $(\bar{k}^l, \bar{s}^l)$ one has $-\left(\dfrac{\Psi_{k_{t+1}} + \Psi_{k_t}}{\Psi_{s_{t+1}} + \Psi_{s_t}} \right) > -\dfrac{\Phi_k}{\Phi_s}$, it is straightforward that $\lambda>1$ if (17) is linearized around $(\bar{k}^h, \bar{s}^h)$ and $\lambda<1$ if it is linearized around $(\bar{k}^l, \bar{s}^l)$.

8) Numerical example showing that the system obtained by linearizing (17) around a steady-state value $\bar{k}$ *can jump to* $\bar{k}^h$ *or to one among the continuum of paths converging to* $\bar{k}^l$

Let h(c)=.0123832, $u(v_t)=v_t$, $u(w)=w=.1$, $\Omega = .25$, n=1, μ=.2681247, σ=.03, α=.75, β=.4906097, γ=.8247422, ζ=0, δ=.2290823. Given these parameter values, we get: $\bar{k}^h$=.1, $\bar{s}^h = .2$, $\bar{\rho}^h_Y$=.0083379, $\bar{k}^l = .0986489$, $\bar{s}^l = .1$ and $\bar{\rho}^l_Y$=-.0106358. The characteristic root obtained by linearizing (17) around $\bar{k}^h = .1$ is λ=1.0210701, while the characteristic root obtained by linearizing (17) around $\bar{k}^l$=.0986489 is λ=.9812596.

9) Numerical example in which the economy has an unique BGP in the absence of money and multiple BGP under real interest pegging

Let $h(c)=.0123832$, $u(v_t)=v_t$ and $u(w)=w=.1$, $\Omega=.25$, $\zeta=n=1$, $\mu=.2681247$, $\sigma=.03$, $\alpha=.75$, $\beta=.4906097$, $\gamma=.8247422$, $\delta=.21866$. Given these parameter values, in the non-monetary economy there is a unique BGP along which $\bar{k}^*=.1$, $\bar{s}^*=.2$, $\bar{\rho}^*_{Y_t}=.0083379$ and $\bar{r}^*=.2604223$. If the monetary authority sets $r_t=\bar{r}^*\ \forall t$, one has two BGP, that are characterized, respectively, by $\bar{k}^h=\bar{k}^*$, $\bar{s}^h=\bar{s}^*$, $\bar{\rho}^h_{Y_t}=\bar{\rho}^*_{Y_t}$ and by $\bar{k}^l=.0986489$, $\bar{s}^l=.1$, $\bar{\rho}^l_{Y_t}=-.0106358$.

10) Numerical example in which there is an unique BGP in the absence of money and $\bar{\rho}_{Y_t}$ decreases with $\bar{\Pi}$ (while $\bar{r}$ may increase or decrease with $\bar{\Pi}$) under inflation targeting

Take the same parameter values of 9), that are consistent with the existence of an unique BGP in the non-monetary economy. If the monetary authority sets $1+\Pi_t=1+\bar{\Pi}^*=.7933848\ \forall t$, there is an unique BGP along which $\bar{k}^*=.1$, $\bar{s}^*=.2$, $\bar{\rho}^*_{Y_t}=.0083379$ and $\bar{r}^*=.2604223$. If the monetary authority sets $1+\Pi_t=1+\bar{\Pi}'=.8024395\ \forall t$, there is an unique BGP along which $\bar{k}'=.0995596$, $\bar{s}'=.17$, $\bar{\rho}'_{Y_t}=.0026845$ and $\bar{r}'=.2605525$. Finally, if the monetary authority sets $1+\Pi_t=1+\bar{\Pi}''=.824107\ \forall t$, there is an unique BGP along which $\bar{k}''=.0986489$, $\bar{s}''=.1$, $\bar{\rho}''_{Y_t}=-.0106358$ and $\bar{r}''=.2604223$.

11) Numerical example in which the economy has multiple BGP in the absence of money and $\bar{\rho}_{Y_t}$ increases with $\bar{\Theta}$ (for $\bar{\Theta}\le\bar{\Theta}^h$) under constant money growth

Take the same parameter values of 6), that are consistent with the existence of two BGP in the non-monetary economy. If the monetary authority sets $1+\Theta_t=1+\bar{\Theta}^h=.8427168\ \forall t$, there is an unique BGP along which $\bar{k}^h=.1$, $\bar{s}^h=.2$, $\bar{\rho}^h_{Y_t}=.008338$. If the monetary authority sets $1+\Theta_t=1+\bar{\Theta}^l=.8272565\ \forall t$, there is an unique BGP along which $\bar{k}^l=.0988142$, $\bar{s}^l=.1$ and $\bar{\rho}^l_{Y_t}=-.010630$.

12) Numerical example in which there is global indeterminacy in the absence of money and the authority's choice of the (fixed) rate of money growth "selects" the BGP followed by the economy

Take the same parameter values of 8), that are consistent with the existence of two BGP in the non-monetary economy. The existence of both these BGP is preserved under inflation targeting ($1+\Pi_t = 1+\overline{\Pi} = .8 \;\forall t$). In contrast, under $1+\Theta_{t+1} = 1+\overline{\Theta}^h = \theta(1+\overline{\rho}_Y^h) = .8066703 \;\forall t$, there exists only one BGP (the high-growth BGP of the non-monetary economy). Linearizing the system consisting of (17) and (27) about its unique steady state ($\overline{k}^h = .1,\quad \overline{s}^h = .2$), one can obtain the following characteristic roots: λ_1=3.6264 and λ_2=.1838. Thus, the linearized system characterizes a continuum of paths converging to ($\overline{k}^h = .1,\quad \overline{s}^h = .2$). Similarly, under $1+\Theta_{t+1} = \overline{\Theta}^l = \theta(1+\overline{\rho}_Y^l) = .7914913 \;\forall t$, there exists only one BGP (the low-growth BGP of the non-monetary economy). Again, linearizing the system about its unique steady state ($\overline{k}^l = .0986489,\quad \overline{s}^l = .1$), one can obtain the following characteristic roots: λ_1=3.6904 and λ_2=.1594. Thus, the linearized system characterizes a continuum of paths converging to ($\overline{k}^l = .0986489,\quad \overline{s}^l = .1$). Note that along both BGP the inflation rate is the same ($1+\overline{\Pi}^h = 1+\overline{\Pi}^l = 0.8$).

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FIGURE 1
Uniqueness of the BGP when *b*'<0

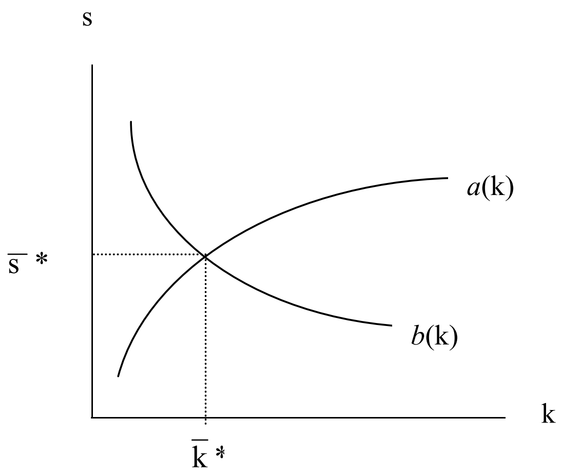

FIGURE 2
Uniqueness of the BGP in the non-monetary economy

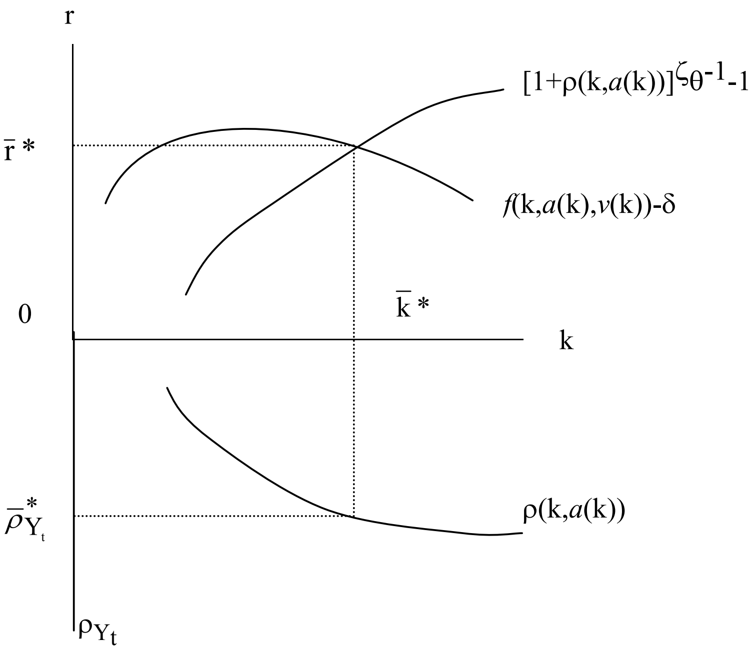

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FIGURE 3
Multiple BGP when b'>0

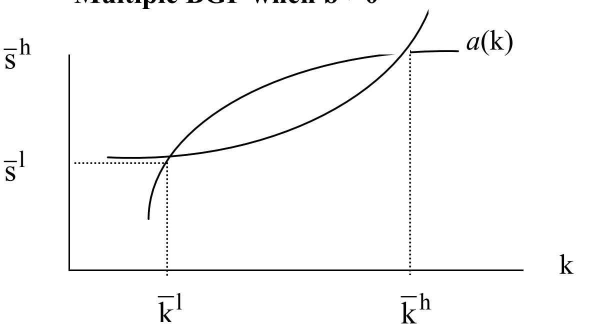


FIGURE 4
Multiple BGP in the non-monetary economy

0

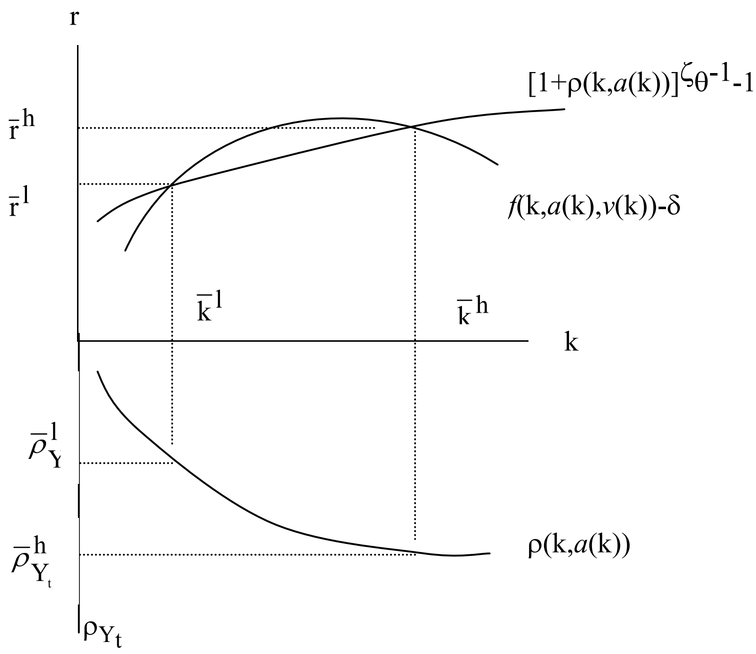


FIGURE 5

Phase line of eq. (17) (ζ=0)

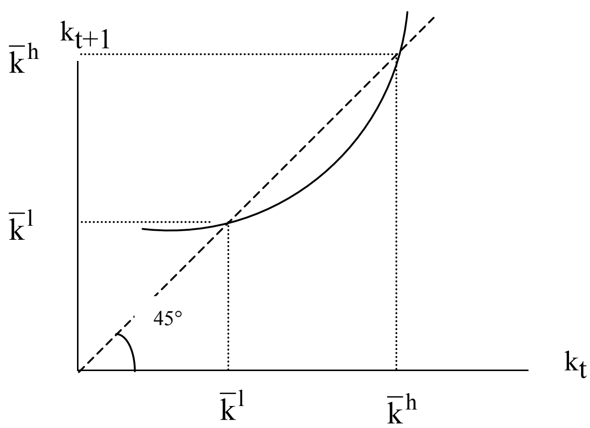
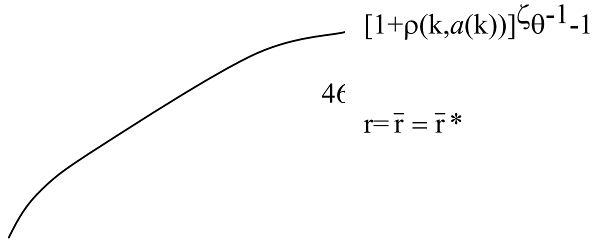


FIGURE 6

Real interest targeting when the BGP is unique in the non-monetary economy



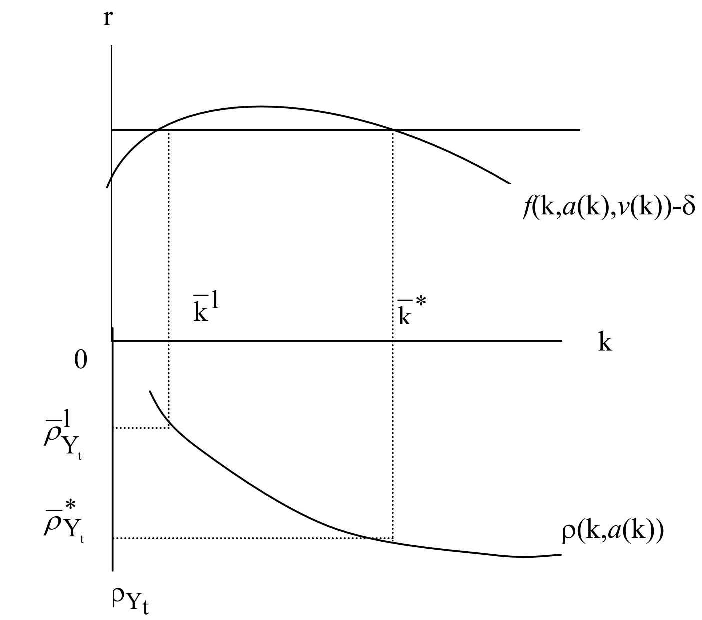


FIGURE 7

BGP under inflation targeting ($\overline{\Pi}'' > \overline{\Pi}' > \overline{\Pi}^*$)

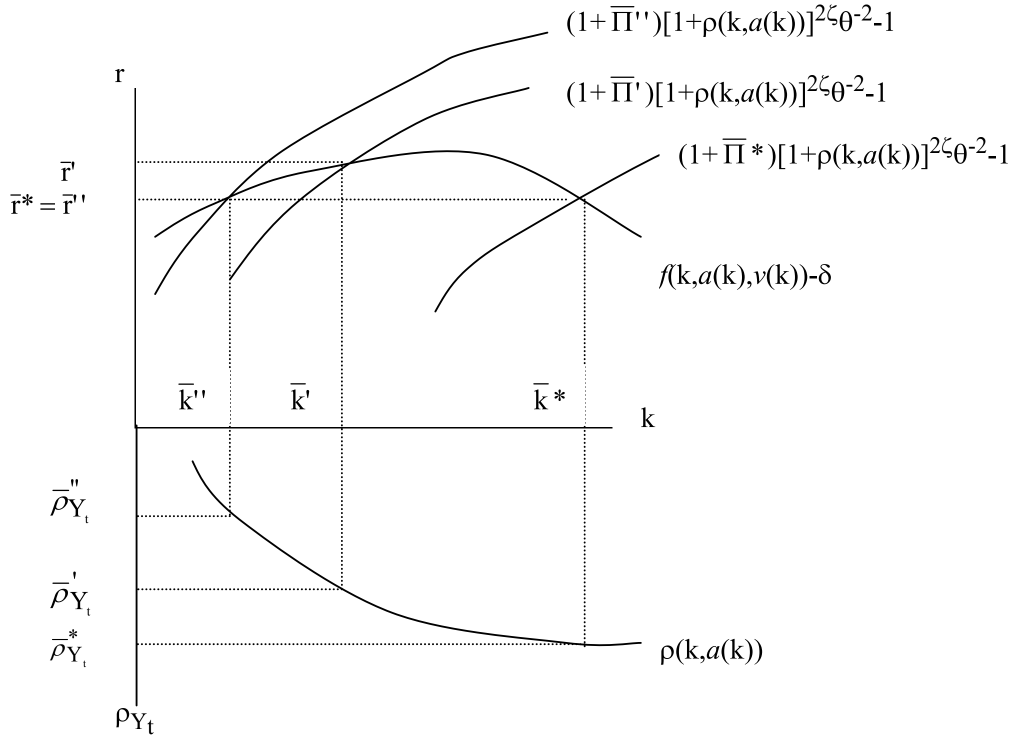
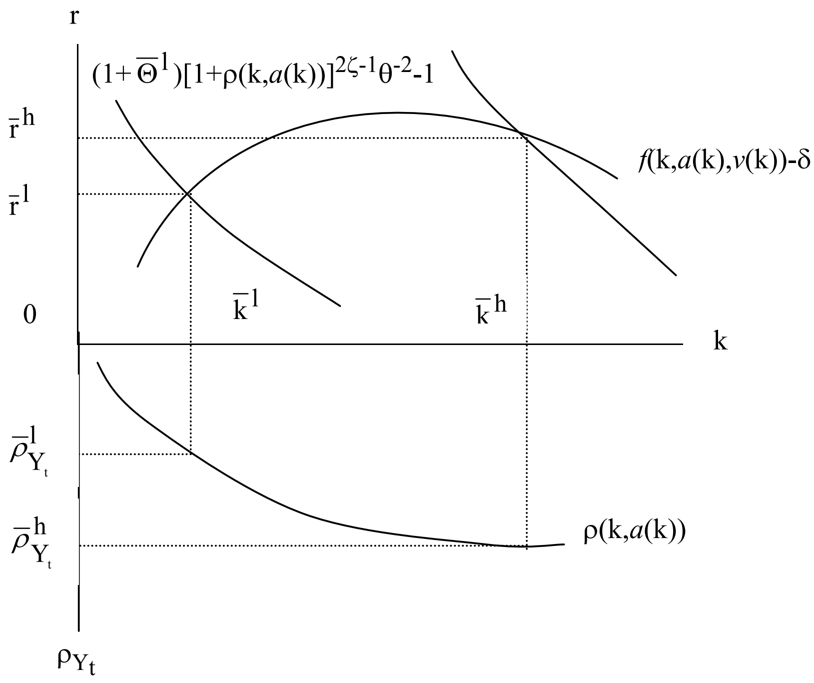


FIGURE 8

BGP under fixed money growth (ζ<1/2 and $\overline{\Theta}^h > \overline{\Theta}^1$)

$(1+\overline{\Theta}^h)[1+\rho(k,a(k))]^{2\zeta-1}\theta^{-2}-1$



r
$(1+\overline{\Theta}^{1})[1+\rho(k,a(k))]^{2\zeta-1}\theta^{-2}-1$
$\bar{r}^{h}$
$\bar{r}^{1}$
$f(k,a(k),v(k))-\delta$
0
$\bar{k}^{1}$
$\bar{k}^{h}$
k
$\bar{\rho}^{1}_{Y_t}$
$\bar{\rho}^{h}_{Y_t}$
$\rho(k,a(k))$
ρ_{Y_t}