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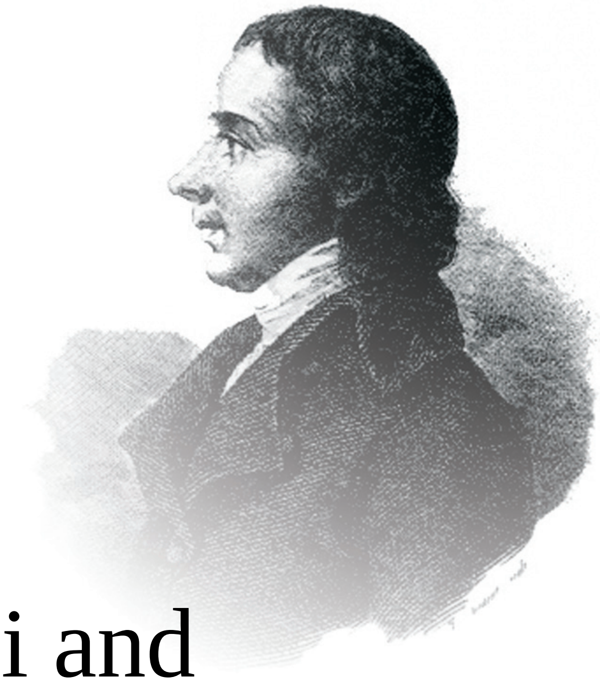

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Regularization of Non-Monotone Multi-valued Variational Inequalities with Applications to Partitionable Problems

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Abstract

We consider a general coercivity condition for multi-valued variational inequalities and show that this condition ensures both convergence of the Tikhonov-Browder regularization method and existence of solutions to the initial and perturbed problems. Applications of this approach to partitionable multi-valued variational inequalities utilizing extensions of order monotonicity properties are also described.

Key words: Multi-valued variational inequalities, regularization method, coercivity conditions, extended order monotonicity.

1 Introduction

Let X be a nonempty convex set in the real n-dimensional space R^n, and let $G : X \to \Pi(R^n)$ be a multi-valued mapping (Here and below $\Pi(A)$ denotes the family of all nonempty subsets of a set A). Then one can define the multi-valued *variational inequality problem* (MVI for short): Find a point $x^* \in X$ such that

$$\exists g^* \in G(x^*),\ \langle g^*, x - x^* \rangle \geq 0 \quad \forall x \in X. \tag{1}$$

In what follows we denote the solution set of this problem by X^*. Variational inequalities are used for formulation and solution of many equilibrium type problems arising in Mathematical Physics, Economics, Operations Research, Transportation and other fields. They are closely related with optimization, fixed point, saddle point and complementarity problems and their theory and methods are investigated extensively; see e.g. [1]–[4] and references therein. The general formulation (1) involves various subclasses. In particular, if G is of the form

$$G(x) = F(x) + \partial\varphi(x),$$

where $\varphi : X \to R$ is a convex continuous function, MVI (1) becomes equivalent to the mixed VI:

$$\langle F(x^*), x - x^* \rangle + \varphi(x) - \varphi(x^*) \geq 0 \quad \forall x \in X, \tag{2}$$

see [5, 4, 2] and references therein for more details.

Many problems arising in applications do not ensure monotonicity properties of the mapping G and this creates serious difficulties for well-definiteness of the corresponding VIs of form (1) or (2) and for suggestion of efficient solution methods. Usually, these methods require strengthened monotonicity properties of G. In the case when G is only monotone, we can apply the Tikhonov-Browder approach and replace the initial problem with a sequence of perturbed strongly monotone VIs whose solutions converge to the normal solution of the initial VI, even in infinite-dimensional settings; see e.g. [6].

However, one can not guarantee these nice convergence properties in the non-monotone case and then there exist several different approaches which enable one to keep some part of these properties; see [7]–[11]. Most of them are based on coercivity conditions which ensure the boundedness of the solution set. It was shown in [9] that the iteration sequence of the regularization method applied to single-valued order monotone VIs may be unbounded if X^* is so. Recently, a class of parametric coercivity conditions for such VIs was proposed and investigated in [12, 13]. These conditions admit the reduction of the feasible set so that each solution of the reduced VI solves the initial problem. Moreover, a very general coercivity condition, which does not yield the boundedness of the solution set, was proposed in [14] for single-valued VIs. In this paper, we intend to show that both the initial problem and all the perturbed VIs are well defined under this condition, together with the basic convergence property, thus extending essentially the field of applications of the Tikhonov-Browder regularization method. For instance, we apply this method to partitionable multi-valued problems utilizing extended order monotonicity concepts and obtain new convergence results. An example of applications to imperfectly competitive economic equilibrium problems is also described.

2 Preliminary Results

We first recall several definitions (see e.g. [2, Section 2.1]). The mapping $G : X \to \Pi(R^n)$ is said to be a *K-mapping* if it is upper semicontinuous and has nonempty, convex and compact values. Also, the mapping $G : X \to \Pi(R^n)$ is said to be

(a) *strongly monotone* with constant $\kappa > 0$ if for each pair of points $x, x'' \in X$ and for all $g' \in G(x'), g'' \in G(x'')$ it holds that

$$\langle g' - g'', x' - x'' \rangle \geq \kappa \|x' - x''\|^2;$$

(b) *monotone*, it for each pair of points $x', x'' \in X$ and for all $g' \in G(x'), g'' \in G(x'')$ it holds that

$$\langle g' - g'', x' - x'' \rangle \geq 0.$$

We consider MVI (1) under the following blanket assumptions.

(A1) *The set X is nonempty, convex and closed.*

(A2) *$G : X \to \Pi(R^n)$ is a K-mapping.*

(A3) *There exists a nonempty compact set D such that, for any point $x \in X \setminus D$, there is a point $y \in X \bigcap D$ such that*

$$\langle g, y - x \rangle \geq 0 \quad \forall g \in G(y).$$

Condition (A3) is one of the weakest coercivity ones which provide solvability of the initial problem (see e.g. [15, 1]) and extends condition A in [14] from the single-valued case. Clearly, if does not imply the boundedness of the set X^* (see for instance Example 1 in [14]).

For the sake of convenience, we give the proof of the existence result for MVI (1) under the above conditions since the known proofs are presented in somewhat different settings. We begin from recalling the classical existence result; see [16, 17].

Proposition 1. *If (A1) and (A2) are fulfilled and X is bounded then MVI (1) has a solution.*

The assertion follows from the known Kakutani fixed point theorem applied to the mapping $x \mapsto \pi_X[x - G(x)]$, where $\pi_X[\cdot]$ denotes the projection mapping onto X.

Let $B_r = \{x \in R^n \mid \|x\| \leq r\}$ denote the closed ball of radius r centered at zero and let $X(r) = X \bigcap B_r$. We need the following auxiliary property (see [18, 17]).

Lemma 1. *If*

$$\langle q, x - x' \rangle \geq 0 \quad \forall x \in X(r)$$

for some elements $q \in R^n$ and $x' \in \mathrm{int} B_r \bigcap X$, then

$$\langle q, x - x' \rangle \geq 0 \quad \forall x \in X. \tag{3}$$

Proof. PROOF. Let $x \in X$ be an arbitrary point. If $x \in B_r$, then (3) is satisfied. Let $x \notin B_r$. Then, for sufficiently small $\alpha > 0$, we have $x(\alpha) = \alpha x + (1 - \alpha)x' \in B_r$. Indeed,

$$\|x(\alpha)\|^2 = \alpha\|x\|^2 + (1 - \alpha)\|x'\|^2 - \alpha(1 - \alpha)\|x - x'\|^2 \leq r^2$$

for sufficiently small $\alpha > 0$. Moreover, $x(\alpha) \in X$ for $\alpha \in [0, 1]$ since X is convex. Then we have $\langle q, x(\alpha) - x' \rangle \geq 0$ or

$$\langle q, x - x' \rangle \geq 0,$$

as required. □

Proposition 2. *If assumptions (A1)–(A3) are fulfilled, then MVI (1) has a solution.*

Proof. PROOF. Let $r > 0$ be chosen large enough so that $D \subset \mathrm{int} B_r$. Consider the problem of finding a point $\tilde{x} \in X(r)$ such that

$$\exists \tilde{g} \in G(\tilde{x}), \langle \tilde{g}, x - \tilde{x} \rangle \geq 0 \quad \forall x \in X(r). \tag{4}$$

Since $X(r)$ is compact, the problem is solvable due to Proposition 1. We consider two possible cases.

Case 1: $\tilde{x} \in \mathrm{int} B_r$.
Then, Lemma 1 with $q = \tilde{g}$ and $x' = \tilde{x}$ implies that $\tilde{x} \in X^*$.

Case 2: $\tilde{x} \notin \mathrm{int} B_r$, i.e. $\|\tilde{x}\| = r$.
Then, by (A3), there is a point $y \in D \subset \mathrm{int} B_r$ such that

$$\langle \tilde{g}, \tilde{x} - y \rangle \geq 0.$$

However, $y \in X(r)$, hence (4) gives

$$\langle \tilde{g}, \tilde{x} - y \rangle = 0 \tag{5}$$

and

$$\langle \tilde{g}, x - y \rangle \geq 0 \quad \forall x \in X(r).$$

Applying now Lemma 1 with $q = \tilde{g}$ and $x' = y$, we obtain

$$\langle \tilde{g}, x - y \rangle \geq 0 \quad \forall x \in X.$$

Using (5), we conclude that

$$\langle \tilde{g}, x - \tilde{x} \rangle \geq 0 \quad \forall x \in X,$$

i.e. $\tilde{x} \in X^*$, as required. □

3 Convergence Properties

So, MVI (1) is solvable under conditions (A1)–(A3). We now consider the classical Tikhonov-Browder regularization method (see e.g. [6]), which consists in replacing problem (1) by the sequence of the perturbed problems of the form: Find $x^\varepsilon \in X$ such that

$$\exists g^\varepsilon \in G(x^\varepsilon), \langle g^\varepsilon + \varepsilon x^\varepsilon, x - x^\varepsilon \rangle \geq 0 \quad \forall x \in X, \tag{6}$$

where $\varepsilon > 0$ is the regularization parameter. We denote by X^*_ε the solution set of problem (6). The next theorem establishes the solvability of this problem.

Theorem 1. *If assumptions (A1)–(A3) are fulfilled, then MVI (6) has a solution for each $\varepsilon > 0$.*

Proof. PROOF. Set

$$\tilde{D} = \left\{ x \in X \mid \max_{y \in D} \min_{g \in G(x)} \langle g, x - y \rangle < 0 \right\}$$

and

$$\tilde{D}_\varepsilon = \left\{ x \in X \mid \max_{y \in D} \min_{g \in G(x)} \langle g + \varepsilon x, x - y \rangle < 0 \right\}.$$

Condition (A3) implies that $\tilde{D}$ is bounded, moreover, $\tilde{D} \subseteq D \bigcap X$. We fix $\varepsilon > 0$ and show that $\tilde{D}_\varepsilon$ is bounded as well. On the contrary, suppose that there exists a sequence $\{x^k\} \subset \tilde{D}_\varepsilon$ such that $\|x^k\| \to \infty$ as $k \to \infty$. For each point x^k we can determine the point y^k as a solution of the problem

$$\min_{g \in G(x^k)} \langle g, x^k - y^k \rangle = \max_{y \in D} \min_{g \in G(x^k)} \langle g, x^k - y^k \rangle.$$

Then we have

$$\begin{aligned} c\langle g^k + \varepsilon x^k, x^k - y^k \rangle &= \min_{g \in G(x^k)} \langle g + \varepsilon x^k, x^k - y^k \rangle \\ &\leq \max_{y \in D} \min_{g \in G(x^k)} \langle g + \varepsilon x^k, x^k - y^k \rangle < 0 \end{aligned}$$

for some $g^k \in G(x^k)$. It follows that

$$\langle g^k, x^k - y^k \rangle < \varepsilon \langle x^k, y^k - x^k \rangle \leq \varepsilon \|x^k\| \left(\|y^k\| - \|x^k\| \right) < 0$$

for k large enough, i.e. $x^k \in \tilde{D}$, which contradicts the boundedness of $\tilde{D}$. Therefore, $\tilde{D}_\varepsilon$ is also bounded and condition (A3) is satisfied for MVI (6) where D is defined as $\overline{\text{conv}} \left\{ \tilde{D}_\varepsilon \bigcup D \right\}$. □

Now we state the main result on the convergence of the regularization method.

Theorem 2. *If assumptions (A1)–(A3) are satisfied, then the following assertions are true:*

(i) MVI (1) has a solution;

(ii) for any $\varepsilon > 0$, MVI (6) has a solution;

(iii) any sequence $\{x^{\varepsilon_k}\}$ with $x^{\varepsilon_k} \in X^*_{\varepsilon_k}$ has limit points as $\{\varepsilon_k\} \to 0$ and all these limit points belong to X^*.

Proof. PROOF. Assertions (i) and (ii) have been established in Proposition 2 and Theorem 1, respectively. To prove (iii), take sequence $\{\varepsilon_k\} \to 0$ and any corresponding sequence $\{x^{\varepsilon_k}\}$. If $x^{\varepsilon_k} \notin D$, then, by (A3), we have

$$\forall g \in G(x^{\varepsilon_k}), \langle g, x^{\varepsilon_k} - y^k \rangle \geq 0$$

for some point $y^k \in D$. Moreover, by definition,

$$\exists g^k \in G(x^{\varepsilon_k}), \langle g^k + \varepsilon_k x^{\varepsilon_k}, y^k - x^{\varepsilon_k} \rangle \geq 0.$$

It follows that

$$\varepsilon_k \langle x^{\varepsilon_k}, y^k - x^{\varepsilon_k} \rangle \geq \langle g^k, x^{\varepsilon_k} - y^k \rangle \geq 0$$

and

$$\|y^k\| \geq \|x^{\varepsilon_k}\|,$$

hence $\{x^{\varepsilon_k}\}$ is bounded and has limit points. Taking the limit $k \to \infty$ in (6) now gives that any limit point is a solution of the initial problem. □

4 Application to Partitionable Variational Inequalities

Many equilibrium problems formulated as VIs possess a decomposable structure; see e.g. [1]–[3]. Usually, the feasible set of such VIs represents a Cartesian product of sets and we can obtain existence and uniqueness results under order monotonicity properties which are weaker than the usual monotonicity ones. It should be noted that most works on partitionable VIs are devoted to the case where all the sets constituting the Cartesian product lie in one-dimensional spaces. Recently, extensions of several order monotonicity concepts for the general case were proposed in [19, 13]. Being based on the corresponding results from [13], we now adjust the regularization method for partitionable multi-valued VIs under extended order monotonicity conditions.

Let M be the index set $\{1, \dots, m\}$. We suppose that the feasible set is of the form

$$X = \prod_{s \in M} X_s, \tag{7}$$

where $x_s \in X_s \subseteq R^{n_s}$ for every $s \in M$, i.e. it is the Cartesian product of sets corresponding to the partition of the real Euclidean space R^n associated to M, namely,

$$R^n = \prod_{s \in M} R^{n_s} = R^{n_1} \times \cdots \times R^{n_m}, \quad \sum_{s \in M} n_s = n \tag{8}$$

and, for each point $x \in R^n$ we can define its partition

$$x = x_1 \times \cdots \times x_m, \tag{9}$$

where $x_s \in R^{n_s}$ for $s \in M$ or $x = (x_s \mid s \in M)$ for brevity. In this case we say that the set X admits the partition associated to M and can rewrite (1) equivalently as follows: Find $x^* = (x^*_s \mid s \in M) \in X$ such that

$$\exists g^* = (g^*_s \mid s \in M) \in G(x^*) : \sum_{s \in M} \langle g^*_s, x_s - x^*_s \rangle \geq 0 \tag{10}$$

$$\forall x_s \in X_s,\ \forall s \in M.$$

We now recall the extended order monotonicity concepts associated to this partition; see [13, Definition 1].

Definition 1. Let M be an index set such that (8), (9) hold, and let $G : X \to \Pi(R^n)$ be a mapping with the partition associated to M. Then the mapping G is said to be

(a) a $P_0(M)$*-mapping*, if for each pair of points $x', x'' \in X$, $x' \neq x''$, and for all $g' = (g'_s \mid s \in M) \in G(x')$, $g'' = (g''_s \mid s \in M) \in G(x'')$, there exists an index i such that $x'_i \neq x''_i$ and

$$\langle x'_i - x''_i, g'_i - g''_i \rangle \geq 0;$$

(b) a $P(M)$*-mapping*, if for each pair of points $x', x'' \in X$, $x' \neq x''$, and for all $g' = (g'_s \mid s \in M) \in G(x')$, $g'' = (g''_s \mid s \in M) \in G(x'')$, there exists an index i such that

$$\langle x'_i - x''_i, g'_i - g''_i \rangle > 0;$$

(c) a *strict* $P(M)$*-mapping*, if there exists $\gamma > 0$ such that $G - \gamma I_n$ is a $P(M)$-mapping, where I_n is the identity map in R^n.

It should be noted that the above concepts in (a) and (b) admit equivalent definitions via relative monotonicity ones; see [20]–[22]. In fact, G is a $P_0(M)$-mapping if and only if, for each pair of points $x', x'' \in X$ and for all $g' \in G(x')$, $g'' \in G(x'')$ there exist numbers $\alpha_1 \geq 0, \dots, \alpha_m \geq 0$ such that $\alpha_i \neq 0$ for at least one i with $x'_i \neq x''_i$ and

$$\sum_{s \in M} \alpha_s \langle g'_s - g''_s, x'_s - x''_s \rangle \geq 0.$$

Moreover, G is a $P(M)$-mapping, if and only if, for each pair of points $x', x'' \in X$ and for all $g' \in G(x'), g'' \in G(x'')$ there exist nonnegative (or equivalently, positive) numbers $\alpha_1, \dots, \alpha_m$ such that

$$\sum_{s \in M} \alpha_s \langle g'_s - g''_s, x'_s - x''_s \rangle > 0.$$

We now replace the basic assumptions (A1) and (A2) with the following.

(A1′) *X admits the partition associated to M, i.e. (7) holds where X_s is a nonempty, convex and closed subset of R^{n_s} for each $s \in M$.*

(A2′) *$G : X \to \Pi(R^n)$ is a K- and $P_0(M)$-mapping.*

We now give three properties established in [13, Corollary 1, Proposition 5 and Theorem 1].

Proposition 3. *Let (A1′) be satisfied.*

(i) If G is a $P_0(M)$-mapping, then $G + \varepsilon I_n$ is a strict $P(M)$- mapping for each $\varepsilon > 0$.

(ii) If G is a $P(M)$-mapping, then MVI (1), (7) has at most one solution.

(iii) If G is a strict $P(M)$- and K-mapping, then MVI (1), (7) has a unique solution.

The regularization method can be now defined as follows. Given a number $\varepsilon > 0$, we define the perturbed MVI: Find $x^{\varepsilon} = (x_s^{\varepsilon} \mid s \in M) \in X$ such that

$$\exists g^{\varepsilon} = (g_s^{\varepsilon} \mid s \in M) \in G(x^{\varepsilon}) : \sum_{s \in M} \langle g_s^{\varepsilon} + \varepsilon x_s^{\varepsilon}, x_s - x_s^{\varepsilon} \rangle \geq 0 \qquad (11)$$

$$\forall x_s \in X_s, \ \forall s \in M.$$

We establish convergence of this method.

Theorem 3. *Suppose that assumptions (A1'), (A2'), and (A3) are satisfied. Then:*

(i) MVI (1), (7) has a solution;

(ii) for each $\varepsilon > 0$, the perturbed problem (11) has the unique solution x^{ε};

(iii) The sequence $\{x^{\varepsilon_k}\}$ has limit points as $\{\varepsilon_k\} \to 0$ and all these points are solutions of the initial problem.

Proof. PROOF. Due to Proposition 3 (i) and (iii), MVI (11) must have a unique solution and (ii) holds. Parts (i) and (iii) follow now from Theorem 2. □

Observe that the cost mapping G may not be monotone under the above assumptions.

5 Application to Oligopolistic Problems

We consider an example of an oligopolistic market structure in which several firms supply m products so that each firm supplies only one product, but the price p_k of the k-th product depend on the quantities of the products $i = 1, \dots, k$. Let I_k denote the indices of the sellers supplying the k-th product. For each supply vector $x = (x_j)_{j \in I_k, k=1,\dots,m}$ we set $\sigma^{(k)}(x) = \sum_{j=1}^{k} \sum_{i \in I_j} x_i$ and $\tau^{(k)}(x) = \sum_{i \in I_k} x_i$, so that $p_k = p_k\left(\sigma^{(k)}(x)\right)$. The i-firm's profit is defined by

$$\varphi_i(x) = x_i p_k \left(\sigma^{(k)}(x)\right) - f_i(x_i),$$

where $f_i(x_i)$ is the i-th firm's total cost of supplying x_i units, for $i \in I_k$.

We suppose that each supply volume level is bounded, i.e., there exist scalars α_i, β_i, $i \in I_k, k = 1, \dots, m$ such that

$$0 \leq \alpha_i \leq x_i \leq \beta_i \leq +\infty$$

and that the total volume of supply for each product can be in principle bounded, i.e., there exist positive scalars γ_k, $k = 1, \dots, m$ such that

$$\tau^{(k)}(x) \leq \gamma_k \leq +\infty$$

We denote by X_k the set of all feasible quantities for the k-th product, i.e. all the vectors $x_{(k)} = (x_j)_{j \in I_k}$ satisfying the above constraints.

In order to define a solution in this market structure, we consider the oligopolistic market as a case of the Nash equilibrium concept for noncooperative games; see e.g. [23]. We intend to transform this game problem into a suitable VI of form (10). To this end, we consider the following set of assumptions.

The price function p_1 is continuous, concave and nonincreasing, the profit function φ_1 is concave, $\gamma_1 = +\infty$; the price functions p_k, $k = 2, \dots, m$ are affine and nonincreasing, for each $i \in I_k$, $k = 1, \dots, m$ the cost function f_i is convex and continuous.

Under these assumptions, the game equilibrium problem becomes equivalent to the following VI: Find $x^* \in X = \prod_{i=1}^{m} X_i$ such that

$$\exists g^* \in G(x^*), \;\; \langle g^*, x - x^* \rangle \geq 0 \quad \forall x \in X,$$

where

$$G(x) = \prod_{k=1}^{m} G_k(x),$$

$$G_k(x) = \left(\partial f_i(x_i) - p_k \left(\sigma^{(k)}(x) \right) - x_i p_k' \left(\sigma^{(k)}(x) \right) \right)_{i \in I_k}, \tag{12}$$

see also [2, 24]. Observe that we do not suppose the differentiability of the functions f_i and p_1.

Denote by n_k the number of indices in I_k. We consider the following partition of the initial space into $n_1 + m - 1$ subspaces corresponding to the vector $x = (x_1, \dots, x_{n_1}, x_{(2)}, \dots, x_{(m)})$, hence $M = \{1, \dots, n_1 + m - 1\}$. Now we present conditions which provide order monotonicity properties for the cost mapping. We denote by ∂f the usual subdifferential of the convex function f and define also the subdifferential $\partial p = -\partial(-p)$ of the concave function p.

Proposition 4. *The mapping G, defined in (12), is a $P_0(M)$-mapping.*

Proof. **Proof.** Take arbitrary points x', x'', $x' \neq x''$. Due to [24, Proposition 11], $G_1(x) = G_1(x_{(1)})$ is a P_0-mapping. Hence, if $x'_{(1)} \neq x''_{(1)}$, then there exists an index $i \in I_1$ and elements $g'_i \in \partial f_i(x'_i) - p_1 \left(\tau^{(1)}(x') \right) - x'_i \partial p_1 \left(\tau^{(1)}(x') \right)$ and $g''_i \in \partial f_i(x''_i) - p_1 \left(\tau^{(1)}(x'') \right) - x''_i \partial p_1 \left(\tau^{(1)}(x'') \right)$ such that

$$(g'_i - g''_i)(x'_i - x''_i) \geq 0.$$

Let $x'_{(1)} = x''_{(1)}$. Without loss of generality we suppose that $\sum_{j=1}^{m} \sum_{i \in I_j} x'_i \geq \sum_{j=1}^{m} \sum_{i \in I_j} x''_i$, then there exists an index $k > 1$ such that $x'_{(k)} \neq x''_{(k)}$, $\sigma^{(k)}(x') \geq \sigma^{(k)}(x'')$ and $\tau^{(k)}(x') \geq \tau^{(k)}(x'')$. Then for $g'_k \in G_k(x')$, $g''_k \in$

$G_k(x'')$ we have

$$\begin{aligned}\langle g'_k - g''_k, x'_{(k)} - x''_{(k)}\rangle &= \sum_{i\in I_k}(q'_i - q''_i)(x'_i - x''_i) + \\ &+(p_k(\sigma^{(k)}(x'')) - p_k(\sigma^{(k)}(x')))(\tau^{(k)}(x') - \tau^{(k)}(x'')) + \\ &+\sum_{i\in I_k}(x''_i b - x'_i b)(x'_i - x''_i) \\ &\geq -b\sum_{i\in I_k}(x'_i - x''_i)^2 \geq 0,\end{aligned}$$

where $q'_i \in \partial f_i(x'_i)$, $q''_i \in \partial f_i(x''_i)$, $b = p'_k(\sigma^{(k)}(x')) = p'_k(\sigma^{(k)}(x''))$. The first inequality follows from monotonicity of ∂f_i and $-p_k$, the second inequality follows from non-positivity of b since p is nonincreasing. Thus, G is a $P_0(M)$-mapping. □

Using Theorem 2 we see that the for each $\varepsilon > 0$ the perturbed problem will have a unique solution under suitable assumptions, therefore the above regularization method can be applied to find a solution.

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