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Empirical modeling of uncertainty in vision systems for industrial robotic applications

by

Francesco Finazzi, Alessandro Fassò, Davide Brugali

° Viale Marconi. 5, I – 24044 Dalmine (BG), ITALY, Tel. +39-035-2052339; Fax. +39-035-562779

Empirical modeling of uncertainty in vision systems for industrial robotic applications

Francesco Finazzi*, Alessandro Fassò† and Davide Brugali‡

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Abstract

Many industrial robotic applications concern a manipulator that pick up objects from a working area, being their position in space estimated through a vision system. Although many methods have been developed for carry out that kind of estimates, not as much effort has been focused on the uncertainty related with them. Example of factors that can influence the uncertainty are calibration errors, image quality, particular setting of the algorithm used, particular physical features of the objects and environmental conditions. With such factors it is clear that try to find out a precise and deterministic model for the uncertainty can be a difficult task to perform. In this paper, after analyzing the factors involved, we present an experimental methodology that allows to build an empirical model of the uncertainty and to identify the factors that actually influence it. The methodology is based on two statistical tools: Design of Experiment and Process Modeling. Due to the many constraints on the experimental space, the high number of factors involved and their different nature, we direct our attention to the so called "optimal" designs. By comparing different models and criteria, we obtained a model which allowed us to reduce the error on the estimate of objects position.

KEY WORDS: artificial vision, robotic applications, design of experiments, optimal designs, process modeling, sensitivity analysis.

*Corresponding author: F. Finazzi is Ph.D. student in Applied Statistic at the Department of Quantitative Methods of the University of Milan-Bicocca, Milan, 20126 - Italy `francesco.finazzi@unimib.it`

†A. Fasso' is Professor of Statistics and head of the Department of Information Technology and Mathematical Methods of the University of Bergamo, Dalmine, 24044 - Italy `alessandro.fasso@unibg.it`

‡D. Brugali is Assistant Professor at the Department of Information Technology and Mathematical Methods of the University of Bergamo, Dalmine, 24044 - Italy `davide.brugali@unibg.it`

1 Introduction

Without doubt, using a vision system to estimate the unknown position of an object inside a working area is the more efficient and simplest way to accomplish the task. On the other hand, the precision of this kind of estimates are affected by a lot of factors, the relevance of which are not known at prior. A significant fact is that these factors are not only related with the measure instrument (the vision system) but also with the environment and the object itself. For this reason, experimental one is the only way to obtain informations about the uncertainty (the error) connected with these estimates.

The problem here analyzed can be viewed under two aspects: identify the main factors that influence the error and quantify the error in order to compensate it and obtain better estimates. The former can be set into the framework of Sensitivity Analysis *(SA)*, while the latter concern the identification of a statistical model for the error. Although, from a mathematical point of view, the two aspects are strictly connected, they have different implication with respect to the Robotic Vision Application *(RVA)* at issue. While compensating the error only regards the elaboration of the data coming from the vision system, identifying the factors that actually influence the error allows to dimension the vision system itself and the environment where it works.

Consider, for example, the use of an off-the-shelf vision system in a *RVA*. Generally, the vision system allows to specify a set of algorithm parameters in order to obtain better or worse estimates, but it is not always clear if these parameters are really influential. The experimenter can set all the parameters at their high level but this could lead to an higher elaboration time without improve the precision of the estimates. There arises the necessity of a procedure able to identify and solve this situations.

The experimental methodology illustrated in this paper is just such a kind of procedure and can be applied to every specific *RVA* that has to be implemented. The paper is so structured: starting from a section of related works, we describe in section 3 the problem from a physical and mathematical point of view, justifying the use of the statistical tools reviewed in section 4. A conclusive section 6 follows section 5 that reports a case study where the methodology has been applied.

2 Related works

In literature we can find different works that explore the uncertainty problem in various directions. Völpel and Theimer (1995) developed a theory for treating uncertainty in localizing 3D objects using local, area-based stereo algorithms (see also [1], [9] and [11]). Zhang, Gu and Milios (2005) propose an approach based on the error propagation theory to obtain the uncertainty of robot self-localization through stereo camera. The work of Brandner and Thurner (2005) deals with the propagation of uncertainty within a quality control application using image based sensors. Tsin, Ramesh and Kanade (2001) treat the camera non-linearities that make the vision algorithms to be systematically biased. Studying the independent sources of error in the CCD sensor of the vision camera they provide a model that helps in obtaining accurate estimates of the camera response function, the scene radiance, and its uncertainty. Brandner (2006) discuss the evaluation of parameter uncertainties of an optical tracking application. The main contribution of his paper is the development of a "stochastic camera model" capable to analytically propagate parameter uncertainties and to take calibration uncertainties of the sensor into account. Liao and Pawlak (1996) investigate the influence of discretization and noise on moment accuracy as object descriptors. Kanatani (2004) examines the origin of uncertainty in geometric inference based on image feature points, introducing the covariance matrix of a feature point as a way to measure the spread of its potential positions.

While the above-mentioned works are restricted to only one algorithm type or only one source of uncertainty, our aim is to develop an algorithm-indipendent and vision system-indipendent methodology able to take into account a set of uncertainty sources.

3 Problem Formulation

3.1 Physical model

In every *RVA* some fixed *actors* can be identified, each of them influencing the error on position estimates. For a common application these *actors* are: the Vision System *(VS)*, the Object *(B)* and the Environment *(E)*. Often, the error cannot be associated to a single *actor*, but it arises by the interaction between two of them. Figure 1 shows the *actors* above-mentioned and all possible interactions.

The definition of *actor* is only conceptual. What we need in order to study the error is something that can be measured or evaluated. At each *actor* we then have to associate a set of *factors,* for instance all that properties of the *actor* that can physically characterize it (obviously only the properties we think can be useful in our analysis). Table 1 reports the commons *factors* that should be taken into consideration.

A first classification of the *factors* is between *fixed* and *variable*. We call *fixed* a *factor* which, although identified as having a possible influence on the error, is going to maintain a constant level during the working time of the *RVA*; *variable* otherwise. Consider, for example, the type of vision camera used in the *RVA*. Different cameras could surely lead to different estimates, but the experimenter could not have different cameras at his disposal (or the camera cannot be changed for other reasons).

Variable *factors* can be then distinguished between *controllable* and *uncontrollable*. A *factor* is said to be *controllable* if its level can be setted by the experimenter; *uncontrollable* if the experimenter can just measure it. The property of controllability is important in order to build experiments able to weight the influence of the *factor*.

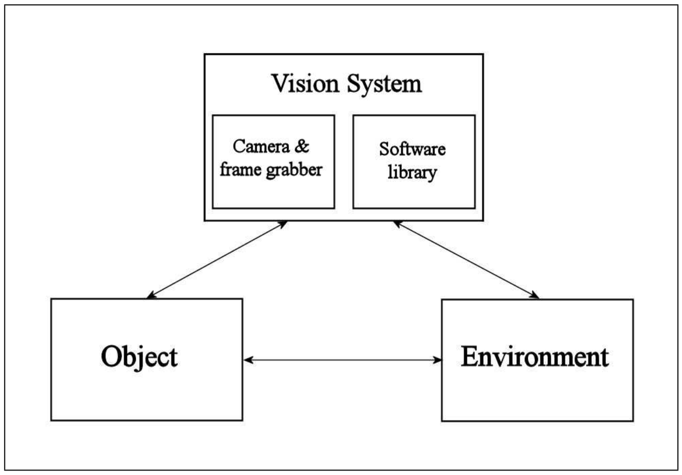

Figure 1 - Actors involved in a Robotic Visual Application

3.2 Mathematical Model

So far we have talked about position and error but we haven't yet specified how they are defined. In a *RVA* can be located up to four coordinate systems, respectively for the environment O_E, the robot manipulator O_R, the vision camera O_C and the object O_B. The true position in space of the object is defined as the position and orientation of O_B with respect to O_R. Assuming Cartesian coordinate systems we

Vision System
vision camera type
algorithm type
settings of the algorithm
Object
shape
material
position inside the working area
Environment
type of the sources of light
background type

Table 1: Actors and factors for a common robotic vision application

have $O_E \equiv O_E(0,0,0)$, $O_R \equiv O_R(X_R, Y_R, Z_R)$, $O_C \equiv O_C(X_C, Y_C, Z_C)$ and $O_B \equiv O_B(X_B, Y_B, Z_B)$. In this case, the object position P is defined as

$$P \equiv P(x_R, y_R, z_R, \phi, \psi, \omega) \tag{1}$$

where ϕ, ψ and ω are the Yaw, Pitch and Roll rotation angles of O_B with respect to O_R. As said before, the true object position is unknown and it is estimated through the vision system. We then have on observed position P_o

$$P_o \equiv P_o(x_{Ro}, y_{Ro}, z_{Ro}, \phi_o, \psi_o, \omega_o) \tag{2}$$

generally different from P. The simplest way to handle the error is to consider each coordinate and rotation angle separately, and to define it as the difference between the observed and the true value

$$\begin{aligned}
\epsilon_X &= x_{Ro} - x_R \\
\epsilon_Y &= y_{Ro} - y_R \\
\epsilon_Z &= z_{Ro} - z_R \\
\epsilon_\phi &= \phi_o - \phi \\
\epsilon_\psi &= \psi_o - \psi \\
\epsilon_\omega &= \omega_o - \omega
\end{aligned} \tag{3}$$

Then we can think the error as separated into different components. In order to study the error, for each error component we want to identify an empirical model in the form

$$\epsilon_. = f_. \left(\mathbf{k}_E, \mathbf{k}_{VS}, \mathbf{k}_B; \boldsymbol{\beta}\right) + \varepsilon. \tag{4}$$

where $\mathbf{k}_E = (k_{1E}\ k_{2E}\ ...\ k_{n_1E})$, $\mathbf{k}_{VS} = (k_{1VS}\ k_{2VS}\ ...\ k_{n_2VS})$ and $\mathbf{k}_B = (k_{1B}\ k_{2B}\ ...\ k_{n_3B})$ are the vectors of the *variable factors* relative to the *actors* E, VS and B respectively, $\boldsymbol{\beta}$ is the vector of the unknown parameters of the model and ε is an independent and normally distributed error with zero mean and variance σ^2, including measurement errors and model inadequacy.

Although *factors* and parameters can be combined in many ways, we here consider only the subclass of linear polynomial models in the form

$$\begin{aligned}
\epsilon_. &= \beta_0 + \sum_{i=1}^{j} \beta_i k_i + \sum_{i=j+1}^{2j} \beta_i k_{i-j}^2 + \\
&\quad + \sum_{i=2j+1}^{2j+1+\binom{j}{2}} \beta_i k_m k_n + ... + \varepsilon. \\
(m &\neq n; 1 \leq m, n \leq j)
\end{aligned} \tag{5}$$

where $j = n_1 + n_2 + n_3$ and $k_1, k_2, ..., k_j$ are elements of the vector $\mathbf{k} = (\mathbf{k}_E \mid \mathbf{k}_{VS} \mid \mathbf{k}_B)$.

At this point it is important to note that the vector $\mathbf{k}$ can include the object position as a *factor*, namely we can write

$$\epsilon_{\cdot} = f_{\cdot}(x_R, y_R, z_R, \phi, \psi, \omega, k_7, k_8, ..., k_j; \beta) + \varepsilon. \quad (6)$$

In case we want to carry out a *SA* there are no problems, since it is quite indifferent whether to consider as *factor* the true position of the object or its observed position. In a different way, if we are to use the model to compensate the error, we must consider as *factor* the observed position. This is because, except for the experimental phase, we don't know the true object position. In this case we must consider a model in the form

$$\epsilon_{\cdot} = f_{\cdot}(x_{Ro}, y_{Ro}, z_{Ro}, \phi_o, \psi_o, \omega_o, k_7, k_8, ..., k_j; \beta) + \varepsilon. \quad (7)$$

Once the error models have been identified, they can be used to correct the observed position in this way

$$\begin{aligned} x_R &= x_{Ro} - (f_1(\mathbf{k}_E, \mathbf{k}_{VS}, \mathbf{k}_B; \boldsymbol{\beta}_1) + \varepsilon_1) \\ y_R &= y_{Ro} - (f_2(\mathbf{k}_E, \mathbf{k}_{VS}, \mathbf{k}_B; \boldsymbol{\beta}_2) + \varepsilon_2) \\ z_R &= z_{Ro} - (f_3(\mathbf{k}_E, \mathbf{k}_{VS}, \mathbf{k}_B; \boldsymbol{\beta}_3) + \varepsilon_3) \\ \phi &= \phi_o - (f_4(\mathbf{k}_E, \mathbf{k}_{VS}, \mathbf{k}_B; \boldsymbol{\beta}_4) + \varepsilon_4) \\ \psi &= \psi_o - (f_5(\mathbf{k}_E, \mathbf{k}_{VS}, \mathbf{k}_B; \boldsymbol{\beta}_5) + \varepsilon_5) \\ \omega &= \omega_o - (f_6(\mathbf{k}_E, \mathbf{k}_{VS}, \mathbf{k}_B; \boldsymbol{\beta}_6) + \varepsilon_6) \end{aligned} \quad (8)$$

To conclude this section let us spend a few words about the units of measure associated with the models. Having decomposed the error in different components, we have to handle models where the error is specified with different units of measure, namely a length unit of measure and an angle unit of measure. To be able to compare different models we here introduce these transformations, that can be used if ϵ_ϕ, ϵ_ψ and ϵ_ω are closed to zero

$$\begin{aligned} \tilde{\epsilon}_\phi &= \tan(\epsilon_\phi) * \sqrt{y_{1B,far}^2 + z_{1B,far}^2} \\ \tilde{\epsilon}_\psi &= \tan(\epsilon_\psi) * \sqrt{x_{2B,far}^2 + z_{2B,far}^2} \\ \tilde{\epsilon}_\omega &= \tan(\epsilon_\omega) * \sqrt{x_{3B,far}^2 + y_{3B,far}^2} \end{aligned} \quad (9)$$

where $y_{1B,far}$ and $z_{1B,far}$ are the coordinates of the point $P_{1,far} \equiv P(y_{1B,far}, z_{1B,far})$ obtained as follows: each point belonging to the object surface is projected on the $Y_B Z_B$ plane; the farthest point of the object projection from O_B corresponds to $P_{1,far}$. In a similar manner, $P_{2,far} \equiv P(x_{2B,far}, z_{2B,far})$ and $P_{3,far} \equiv P(x_{3B,far}, y_{3B,far})$ are defined with respect to $X_B Z_B$ and $X_B Y_B$ planes respectively. Hence $\tilde{\epsilon}_\phi$, $\tilde{\epsilon}_\psi$ and $\tilde{\epsilon}_\omega$ are now expressed with a length unit of measure. The transformations have been defined in that way for this reason: suppose we want to pick-up an object by positioning the end-effector of the robot manipulator at a point of the object different from its origin (or we want to perform an action at that point); $\tilde{\epsilon}.$ tell us how distant is the farthest point (with respect to the specific plane) from its true position having only the error component $\epsilon.$. This then represents a worst case. Figure 2 shows the relation between $\tilde{\epsilon}.$ and $\epsilon.$ for a 2D case.

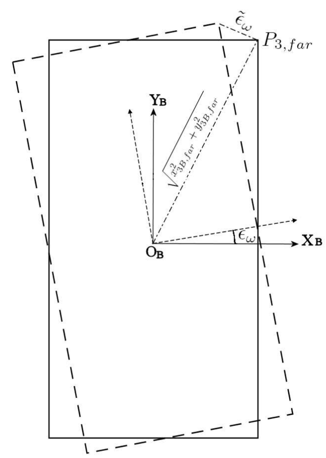

Figure 2 - Relation between $\tilde{\epsilon}.$ and $\epsilon.$

4 Statistical Tools

4.1 DOE - Design of Experiments

Design of Experiments method is here applied as a way to collect data in an efficient manner with respect to the number of *factors* involved and the model which is expected to fit the data. Although there exist many types of standard and well studied designs we pay our attention to computer-aided designs. The reasons of such a choice are:

- high number of involved factors
- high number of levels for each factor
- different number of levels between factors
- presence of uncontrollable factors
- possible constrained experimental region

In this case, the application of an algorithm based on an optimality criterion allows to generate a design with the desirable property of orthogonality and with a smaller number of experimental runs. Common optimality criterion are ([13] and [17]):

- D-Optimal: reduces the volume of the confidence ellipsoid to obtain accurate coefficients. Its value is calculated using the formula $D_{eff} = \frac{\ln(\det(X'X))}{j}$, where X is the regression matrix and j is the number of terms in the regression matrix.
- V-Optimal: minimizes the average prediction error variance to obtain accurate predictions. Its value is obtained using the formula $V_{eff} = \frac{1}{n_C} \sum_i x_i' \left(X_C' X_C{}^{-1}\right) x_i$, where x_i are rows of the regression matrix, X_C is the regression matrix of all candidate experimental points and n_C is the number of these points.
- A-Optimal: minimizes the average variance of the parameters and reduces the asphericity of the confidence ellipsoid. Its value is calculated through the formula $A_{eff} = trace\left((X'X)^{-1}\right)$.

One important drawback that has to be considered is that a design so generated can be optimal with reference to the design matrix but not always satisfying from an application point of view. The experimenter must then keep an eye on the design to be sure that experimental points cover the experimental region quite uniformly.

4.2 Process Modeling

In [15], Process Modeling is defined as the description of the total variation of a quantity by partitioning it into a deterministic component (usually under the form of a mathematical function) and a random component with a particular probability distribution. The main parts that characterize process modeling are then the response variable y, the mathematical function $f(x;\beta)$ and the random error ε. The general form of the model is

$$y = f(\boldsymbol{x};\boldsymbol{\beta}) + \varepsilon \tag{10}$$

The random error that is included in the model makes the relationship between the response variable and the predictor variables a statistical relationship. This means that the functional relationship holds only on average and not for each data point. The mathematical function consists of two parts, namely the predictor variables $\boldsymbol{x}$ and the parameters $\boldsymbol{\beta}$. The predictor variables are observed along with the response variable, while the parameters are quantities estimated during the modeling process. Parameters and predictor variables are combined in different ways to obtain the function that describes the deterministic variation in the response variable. Although there exist infinite of these combinations, polynomial models like (5) are the most frequently used empirical models for fitting functions. This is because polynomial models are simple, have moderate flexibility of shapes and are not dependent on the underlying metric. Moreover, the estimation of the unknown $\boldsymbol{\beta}$ parameters can be obtained through any method base on linear least squares.

4.3 Sensitivity Analysis

SA is here applied to rank *factors* with respect to their influence to the error in position estimate. Since we have models in the form of (5) and since we deal with orthogonal design where *factors* are uncorrelated, we can base our analysis on the sensitivity index S_i as defined in [5]:

$$S_i = \frac{\sigma^2_{h_i}\beta^2_i}{\sigma^2_{\epsilon.}} \tag{11}$$

where h_i is the generic polynomial term associated with β_i. The above-said index has the property that

$$\sum_i S_i = 1 - \frac{\sigma^2_{\varepsilon.}}{\sigma^2_{\epsilon.}} = R^2 \tag{12}$$

See [4] and [6] for an general introduction about *SA* on data modelling.

5 Case Study

5.1 Case description

In this section it is illustrated a case study where the methodology has been applied. The case study concerns with a robot manipulator that has to pick-up objects from a driven band conveyor. The objects have the same rectangular parallelepiped shape (3x16x63 mm) but different colors, and they fall down on the conveyor at random position and with random orientation. The vision camera is placed above the conveyor and its position can be fixed at different heights from the conveyor's plane. Since objects lean on the conveyor's plane and since it is supposed they always present the same face at the vision camera, the object position can be defined as $P \equiv P(x_R, y_R, z_C, \phi_C, \psi_C, \omega)$, where z_C, ϕ_C and ψ_C are constant parameters. The vision camera is connected to a Personal Computer where the images are elaborated to obtain the objects positions. The algorithm is base on the method of moments ([7], [14], [16] and [18]) and it can work at sub-pixel precision with different accuracy. Moreover, the analysis can be based on single or multiple images. The environment is lighted by two separated neon lamps and by the natural light coming through a window. While natural light is approximately fixed, neon lamps can be both switched on, both switched off or only one switched on.

The *factors* identified for this *RVA* are listed in Table 2. Camera shutter time and camera gain are fixed with respect to the conveyor speed in order to avoid blurred image.

Since the object position on the band conveyor has been individuated as a possible *factor*, during the experimental phase we needed a way to move the object at different and well known positions. We then decided to fix the object at the end-effector of the robot manipulator so to simulate different positions and orientations of it.

Actors and Factors	**F**	**C**	**S**
Vision System			
camera resolution	x	x	V_R
camera type (color/mono)	x	x	V_T
camera lens type	x	x	V_L
camera shutter time	x	x	V_S
camera gain	x	x	V_G
camera height above conveyor		x	V_H
algorithm accuracy		x	A_A
number of acquired images		x	A_N
Object			
shape	x		B_S
material	x		B_M
color			B_C
observed position along X axis		x	Bx_o
observed position along Y axis		x	By_o
observed orientation		x	$B\omega_o$
Environment			
type of the sources of light	x		L_T
light intensity			L_I
conveyor speed	x	x	C_S
conveyor band color	x		C_C

Table 2: Actors and factors identified in the case study. Legend: F: fixed; C: controllable; S: symbol

Table 3 reports the variability range of each *factor*, the number of levels and their distribution.

Different camera height corresponds to different field of view of the camera. The less the height the less the area viewed by the camera. This leads to constraints that have to be taken into consideration during the experiment definition phase. The object, in fact, must always lie inside the camera field of view. Table 4 reports the two constraints of this case study.

Factor	UoM	Range	Levels
V_H	m	[1.6, 2.5]	3 - Equidistant
A_A	sub-pixel	[1, 11]	3 - $\{1, 5, 11\}$
A_N	#	[1, 20]	20 - Equidistant
B_C	gray-value	[140, 240]	3 - $\{140, 170, 240\}$
Bx_o	mm	[0, 120]	24 - Equidistant
By_o	mm	[0,75]	15 - Equidistant
$B\omega_o$	°	[0, 90]	18 - Equidistant
L_I	%	[55, 100]	3 - $\{55, 83, 100\}$

Table 3: Factors description

$\boldsymbol{V}_H$[m]	$\boldsymbol{Bx}_o$ ***range***[mm]	$\boldsymbol{By}_o$ ***range***[mm]
1.60	$[0, 65]$	$[0, 40]$
2.05	$[0, 95]$	$[0, 60]$
2.50	$[0, 120]$	$[0, 75]$

Table 4: Constraints

The aim of the case study was then to identify three different models, namely

$$\begin{aligned}
\epsilon_X &= f_1\left(V_H, A_A, A_N, B_C, Bx_o, By_o, B\omega_o, L_I; \boldsymbol{\beta}_1\right) + \varepsilon_1 \\
\epsilon_Y &= f_2\left(V_H, A_A, A_N, B_C, Bx_o, By_o, B\omega_o, L_I; \boldsymbol{\beta}_2\right) + \varepsilon_2 \\
\epsilon_\omega &= f_3\left(V_H, A_A, A_N, B_C, Bx_o, By_o, B\omega_o, L_I; \boldsymbol{\beta}_3\right) + \varepsilon_3
\end{aligned} \tag{13}$$

5.2 Data acquisition

As far as concerns the definition of the experiment we relied on the Matlab® Model-Based Calibration Toolbox, which supplies all the tools that are needed to design an experiment and to process the collected data.

To describe the error we considered a linear quadratic polynomial model with a total number of 45 terms: namely a constant term, 8 linear terms and 36 second order terms. We then defined two different experimental designs, Γ_1 and Γ_2, so to have two distinct data sets, one to identify the model and one to validate it. Each design is composed by 120 experimental runs and has been generated through an algorithm based on the *V-Optimal* optimality criterion, taking into account the factors levels specified in Table 3 and relative constraints. Refer to [8] for further details about the experimental designs of this case study.

The experiment has been executed by considering Γ_1 and Γ_2 in sequence. Tables 5 and 6 reassume the collected data about the error in position estimate after eliminating outliers.

	$\boldsymbol{\epsilon}_X$ [mm]	$\boldsymbol{\epsilon}_Y$ [mm]	$\boldsymbol{\epsilon}_\omega$ [°]	$\tilde{\boldsymbol{\epsilon}}_\omega$ [mm]
Min	-0.382	-2.053	-0.335	-0.190
Max	$+0.851$	$+0.281$	$+0.086$	$+0.049$
Mean	$+0.286$	-0.642	-0.104	$+0.059$
Std	$+0.276$	$+0.535$	$+0.086$	$+0.049$

Table 5: Experimental collected data based on first data-set

	$\boldsymbol{\epsilon}_X$ [mm]	$\boldsymbol{\epsilon}_Y$ [mm]	$\boldsymbol{\epsilon}_\omega$ [°]	$\tilde{\boldsymbol{\epsilon}}_\omega$ [mm]
Min	-0.411	-2.039	-0.304	-0.172
Max	$+0.905$	$+0.310$	$+0.049$	$+0.028$
Mean	$+0.286$	-0.633	-0.120	-0.068
Std	$+0.261$	$+0.531$	$+0.076$	$+0.043$

Table 6: Experimental collected data based on second data-set

5.3 Models identification

The two data sets have then been used in this way: at first step, for each position component has been identified two distinct models, one based on Γ_1 and the other based on Γ_2. The models so obtained included all the 45 terms and a stepwise method has been applied to maintain significative terms only. In particular we applied a stepwise method based on the minimization of the PRESS[1] statistic. Finally, we derived a third model on Γ_1 by comparing the two pruned models and by considering only the common *factors*. Tables 7,8 and 9 report the final models identified for the three position components. The β coefficients are referred to the *factors* normalized in the range [-1, 1] and each *factor* is accompanied by its sensitivity index as defined by equation (11).

[1] With n runs in the data set, the model equation is fitted to n-1 runs and a prediction taken from this model for the remaining one. PRESS is the sum of squares of the prediction residuals. See [13] for further details.

Term	$\boldsymbol{\beta}$	$\boldsymbol{std\beta}$	$\boldsymbol{S}$[%]
(1)	+0.3182	0.0325	
By_o	+0.3037	0.0147	66.42
Bx_o	+0.0834	0.0201	5.08
L_I^2	+0.1209	0.0266	2.99
Bx_oV_H	−0.0664	0.0220	2.71
V_H^2	−0.1077	0.0269	2.52
V_H	−0.0345	0.0160	1.12
$B\omega_oA_A$	+0.0398	0.0142	1.12
$B\omega_o^2$	+0.0546	0.0255	0.68
$B\omega_o$	+0.0137	0.0128	0.16

Table 7: Model for error on X component

Term	$\boldsymbol{\beta}$	$\boldsymbol{std\beta}$	$\boldsymbol{S}$[%]
(1)	−0.2444	0.0321	
Bx_o	−0.5280	0.0143	47.08
A_A	+0.3554	0.0119	33.76
A_A^2	−0.4115	0.0247	9.24
V_H	−0.0786	0.0147	1.50
L_I	−0.0718	0.0109	1.41
A_AV_H	+0.0764	0.0131	1.28
By_oV_H	−0.0894	0.0214	1.21
V_H^2	−0.1049	0.0254	0.58
By_o	+0.0370	0.0199	0.24
$B\omega_o^2$	−0.0574	0.0249	0.17
B_C	−0.0162	0.0109	0.07

Table 8: Model for error on Y component

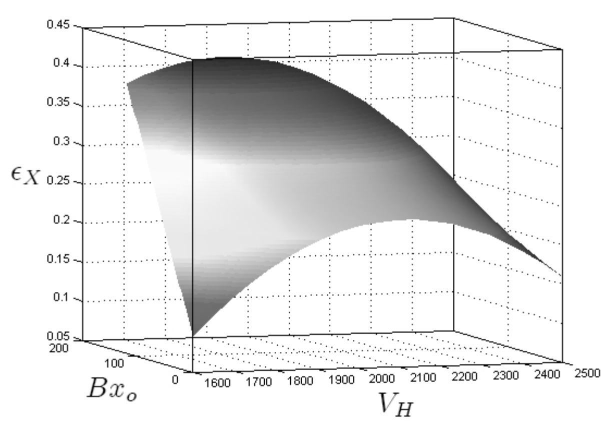

Figure 3 - Response surface of ϵ_X with reference to factors Bx_o and V_H

Term	β	$std\beta$	$S[\%]$
(1)	-0.1445	0.0134	
$B\omega_o$	-0.0340	0.0076	11.65
Bx_o	-0.0341	0.0080	10.21
$B\omega_o A_A$	-0.0277	0.0082	6.26
$By_o B\omega_o$	-0.0294	0.0101	4.98
By_o	-0.0195	0.0080	3.38
V_H^2	$+0.0252$	0.0146	1.65

Table 9: Model for error on angle component

5.4 Models validation

Validate the model means to establish if it is representative of the underlying phenomenon and if it meets the requested properties of accuracy and predictability. The methods we used to validate the models are internal validation, external validation and residual analysis. While internal validation is based on statistics computed on the same data-set used to build the model, external validation is based on an independent data-set. Table 10 reports for each model the relative statistics. In particular, Valid. RMSE is the RMSE[2] computed using the independent data-set.

Statistic	ϵ_X	ϵ_Y	ϵ_ω	$\tilde{\epsilon}_\omega$
Observations	117	116	115	115
Parameters	10	12	7	7
PRESS RMSE	0.114	0.111	0.063	0.036
Fit RMSE	0.109	0.104	0.061	0.035
Valid. RMSE (Γ_2)	0.110	0.141	0.076	0.043
R^2	0.828	0.966	0.381	0.381
$R^2 adj$	0.794	0.957	0.347	0.347

Table 10: Statistics associated with models

As far as concerned the residual analysis, we obtained for each model well-behaved residuals, normally distributed, with zero mean and constant variance. Table 11 summarized the residual error so defined

$$
\begin{aligned}
{}_2\epsilon_X &= x_R - (x_{Ro} - \epsilon_X) \\
{}_2\epsilon_Y &= y_R - (y_{Ro} - \epsilon_Y) \\
{}_2\epsilon_\omega &= \omega - (\omega_o - \epsilon_\omega)
\end{aligned}
\tag{14}
$$

[2] root mean square error

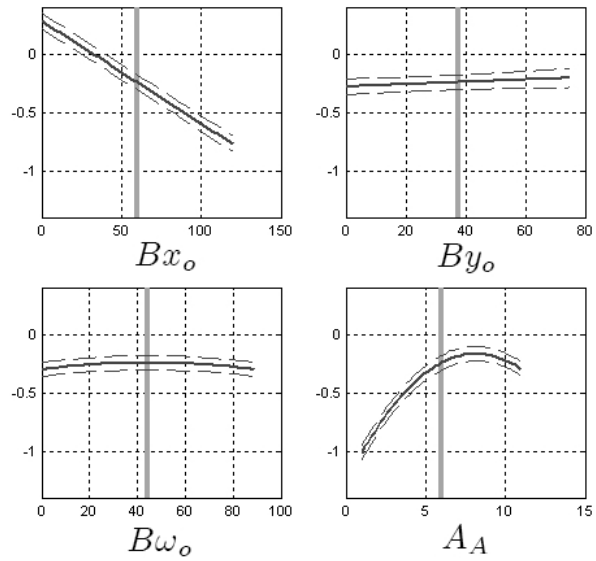

Figure 4 - Toolbox output: model evaluation for ϵ_Y

5.5 Results interpretation

A first look at the R^2 statistic tell us that only models related with ϵ_X and ϵ_Y are able to explain the error in position estimates. The model related with ϵ_ω has a R^2 littler then 0.4, and ω_o itself has low sensitivity indexes associated with it in the other two models. However, error ϵ_ω was limited from the start with respect to ϵ_X and ϵ_Y (compare the std in Table 5 and 6). We can conclude that the vision system supplies, in this case, accurate estimates of the orientation angle and that the particular angle value doesn't influence the other error components.

As far as concerned the other *factors*, we can say that the most influential ones are the observed object position (Bx_o, By_o) and the algorithm accuracy A_A, followed by the camera height above the conveyor V_H and the light intensity L_I. *Factors* that are not significative on the error are the object color B_C and the number of acquired images A_N. With reference to Table 5, 6 and 11 we can see that, by compensating the error through the models, the error component ϵ_X has been reduced by 57% while ϵ_Y by about 80% (see ERF[3] in Table 11). Moreover, Fit RMSE, PRESS RMSE and Validation RMSE statistics of Table 11 are such to suggest that models are not overfitted.

[3] error reduction factor. ERF = $\sigma_{2\epsilon.}/\sigma_{\epsilon.}$

	${}_2\epsilon_X$ [mm]	${}_2\epsilon_Y$ [mm]	${}_2\tilde{\epsilon}_\omega$ [mm]
Min	−0.390	−0.227	−0.095
Max	+0.301	+0.377	+0.078
Mean	−0.004	+0.002	+0.001
Std	+0.111	+0.104	+0.033
ERF	0.42	0.19	0.76

Table 11: Residual error

6 Conclusion

The experimental methodology defined in this paper is our first contribution in the direction of providing a measure of uncertainty to be associate with the position in space of an object estimated through a vision system. The methodology tries to describe the uncertainty in terms of physical *factors* reassumed in a set of mathematical models and, even when the models are not able to explain the uncertainty with respect to the *factors*, the methodology is able to quantify it. The application of the methodology to a case study allowed us to quantify the uncertainty and to significatively reduce it. Although our results are encouraging, there are aspects that are not explained by our analysis. The most difficult task is to translate the mathematical models in physical justification; for instance, understand why the same *factor* acts in different way within different models.

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