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Asymmetric Monitoring of Multivariate Data with Nonlinear Dynamics

by

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Asymmetric Monitoring of Multivariate Data with Nonlinear Dynamics

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Abstract

We consider asymmetric change detection by generalizing the one-sided MEWMA control chart. In particular, we revise and extend the one-sided MEWMA algorithm to cope with mixed alternatives where some coordinates are allowed to increase and others may change in any direction. Moreover, in order to cope with nonlinear serially correlated data, we introduce one-sided MEWMA control charts based on semi-parametric stationary block bootstrap.

The motivating application is related to massive brake-disk production for the automotive industry. The example considers seven geometrical and dimensional parameters and shows how the proposed method discriminates between geometrical deformation and dimensional shifts.

Keywords: multivariate control charts; brake-disc quality control; block bootstrap; nonlinear multivariate process control

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1 Introduction

Multivariate statistical monitoring is becoming more and more important as long as multi-sensor monitoring is taking place in industry, especially for safety, fault detection and diagnosis, quality control and process control. Recently, these techniques have been applied to novel fields such as environmental monitoring [6] and computer intrusion detection [18]. Moreover, the multivariate approach, by adjusting for correlation among sensor measures, is the natural statistical technique for discriminating between sensor faults and systematic unwanted trends.

Statistical techniques for monitoring multidimensional data are not new and most multivariate control charts are essentially based on the contours of the multivariate normal distributions, for example the Multivariate Exponentially Weighted Moving Average (MEWMA) of Lowry *et al.* [11] and the Multivariate CUSUM (see *e.g.* [14]). This gives statistics and change detectors which are invariant along Gaussian contours. Variations of this approach, useful in certain cases, are based on regression residuals as discussed *e.g.* in [10] and references therein.

To cope with direction-sensitive changes, which is the focus of this paper, detectors based on CUSUM applied to linear combination of the data have been proposed in the past (see *e.g.* [8]) and their properties as general asymmetric detectors have been recently discussed in [7]. Another approach is based on the restricted maximum likelihood (RML) statistics and the corresponding detectors. Whenever computationally a little more demanding, the latter has been proven by Fassò, [6] and [7], to be more sensible in our case. Because of this, in the following sections, we extend the RML approach to more general asymmetric change detection problems by considering mixed change problems and nonlinear serially correlated data.

Control charts for autocorrelated data have received more and more attention in the last years, for example in [12] and [13], Lu and Reynolds consider both the MEWMA and the CUSUM univariate chart and, for the univariate setup, Jiang *et al.*. proposed the ARMA chart in [9] and Shu *et al.* proposed the Triggered Cuscore chart in [17]. Moreover, the debate between monitoring the original data and monitoring the residuals of some dynamical model is still open. See e.g. the recent papers of Runger [15] and Atienza *et al.* [1].

In this paper, we will use a detector based on the original data rather than residuals, because interpretation and directional change properties may be confounded when computing residuals from nonlinear multivariate models. Nevertheless, the dynamics over time of the multivariate process will be taken into account in order to get approximate unbiased estimates of thresholds and delay times.

To do this, in section 2, we introduce the asymmetric change model and the related detectors. In particular, in section 2.2, we review the change problem introduced by Fassò [6] and, in section 2.3, we introduce the new mixed change model which allows us to define the corresponding multivariate detector. This is useful when the increase of some quantities and the change in any direction of other correlated quantities are of interest. To cope with multivariate data under arbitrary nonlinear time dynamics and correlation, in section 2.4, semiparametric bootstrap control charts are introduced. In section 3, as a motivating example, we discuss an industrial problem related to massive brake-disc production, where geometrical deformation parameters are monitored against unidirectional shifts, and dimensional parameters against two sided shifts. Moreover, these data show some relevant nonliner dynamics.

2 Model Setup

In order to introduce symbols and concepts, consider industrial process quality control on multivariate data. In this case, data $y_t = (y_{t,1}, ..., y_{t,k})'$, say, are collected over time $t = 1, 2, ...$. The focus is on the vector parameter $\theta(t)$ characterizing the behavior of y_t. For the sake of simplicity, in this paper we consider the local mean level, $\theta(t) = E(y_t)$, but other cases, such as variability, correlation and dynamics may be similarly handled by joining the ideas of this paper and the approach used in [4] and [5].

The data vector y_t is usually concomitantly cross-correlated, for example, y_t is supposed Gaussian, for each t, with covariance matrix given by Σ, namely

$$y_t \,\tilde{}\, N_k(\theta(t), \Sigma).$$

Moreover, y_t may be correlated over time, with some matrix correlation $\Gamma(h) = Cov(y_t, y_{t+h})$. Since we are mainly interested in the level $\theta(t)$, we assume that Σ and Γ are time invariant.

2.1 Real Time Change Detection

We want to detect in real time, the shifts of $\theta(t)$ from a reference set Θ_0 which defines the desired process performances. For example, in multivariate quality control Θ_0 defines standard operation or good quality.

From the statistical point of view, at each time t, using data $y_1, ..., y_t$, a decision has to be taken about the following no-change hypothesis:

$$H_0 : \theta(r) \in \Theta_0 \qquad r \leq t, \text{ "}recently\text{"}$$

against a change alternative

$$H_1 : \theta(r) \notin \Theta_0 \qquad \text{"}recently\text{"}.$$

In standard symmetric changes, we have that Θ_0 is a point and we have the following change hypoteses

$$H_0 : \theta = \theta^0 \qquad \text{vs} \qquad H_1 : \theta \neq \theta^0. \tag{1}$$

In this case, control charts are based on the following well known χ^2 test statistic

$$T^2 = Q_s(y) = \left(y - \theta^0\right)' \Sigma^{-1} \left(y - \theta^0\right)$$

which is constant along Gaussian ellipsoids $t^2 = (y - \theta^0)' \Sigma^{-1} (y - \theta^0)$ and reacts essentially in the same way to both positive and negative large values $|y - \theta^0| > 0$.

Since we are going to use exponentially weighted moving average detectors, we recall here the MEWMA detector, see e.g. [11], for the symmetric problem (1) above. This is based on the multivariate exponentially weighted moving average transform

$$z_t = \lambda y_t + (1 - \lambda) z_{t-1} \tag{2}$$

and claims alarm if

$$\frac{2 - \lambda}{\lambda} Q_s(z_t) > h_s. \tag{3}$$

In these equations, $0 < \lambda \leq 1$ is an appropriate smoothing factor and the threshold h_s can be chosen to control false alarm occurrence.

Depending on the shape of the reference set Θ_0 we have various cases. In particular, two relevant cases are discussed in the following subsections.

2.2 Increase Change Problems

Consider the non-increase reference set and the corresponding change hypotheses given by

$$H_0 : \theta \in \Theta_0 = \left\{\theta_j \leq \theta^0 \text{ for all } j\right\} \qquad \text{vs} \qquad H_1 : \theta_j > \theta^0 \text{ for some } j. \tag{4}$$

This is of interest, for example, when quality deteriorates as one or more coordinates of y_t increase. In this case, since H_0 is not a simple hypothesis, we use the generalized likelihood ratio (GLR) test statistic

$$Q(y) = \left(y - \hat{\theta}_0\right)' \Sigma^{-1} \left(y - \hat{\theta}_0\right) = \min_{\theta \in \Theta_0} (y - \theta)' \Sigma^{-1} (y - \theta)$$

where $\hat{\theta}_0$ is the so-called restricted maximum likelihood (RML) estimator and may be computed using standard quadratic programming (QP) techniques. Using the results in [6] and [7], exact computation of Q can be performed for $k = 2$ and for larger but moderate dimensions, $k \leq 12$, with formula (10) as explained in the appendix of this paper.

Various computation experiments have been performed on Pentium based personal computers and the Matlab software package. We concluded that, for large $k > 12$, QP techniques are appropriate. For intermediate complexity $k \in [6, 12]$, exact computations are feasible but slower than QP whilst, for small $k \leq 5$, the routine discussed in the Appendix is faster than QP routine *quadprog* () given by the Optimization Toolbox of the Matlab software package.

2.3 Mixed Change Problems

Often, we are faced with monitoring problems where we need warning or alarms for the increase of some coordinates, say θ_j, for $j = 1, ..., h$, and changes in any directions of the other coordinates, for $j = h + 1, ..., k$. For example, in Section 3, we consider an industrial multivariate problem where quality deteriorates as at least one of the four geometric deformation parameters increase and as at least one of the remaining dimensional parameter errors increase both in the positive and negative directions. We then have the following mixed change hypotheses

$$\begin{aligned} &H_0 : \theta \in \Theta^* = \left\{\theta_j = \theta^0, \quad j = 1, .., h, \quad \theta_j \leq \theta^0, \quad j = h + 1, ..., k\right\} \\ &H_1 : \theta \notin \Theta^* \end{aligned} \tag{5}$$

and the corresponding GLR statistics, namely

$$Q\left(y\right)^* = \min_{\theta \in \Theta^*} \left(y - \theta\right)' \Sigma^{-1} \left(y - \theta\right),$$

is computed by standard QP routines *e.g.* by the Matlab's *quadprog*() function as in the previous section.

In the sequel, we consider the detectors and charts based on $Q_t = Q\left(z_t\right)$ and $Q_t^* = Q^*\left(z_t\right)$, which are obtained applying statistics Q and Q^*, respectively, to the EWMA transform z_t given by (2).

2.4 Threshold and Mean Time Computations

In order to have detectors which are properly defined for the change problems above, we need appropriate thresholds that control the false alarm occurrence and the delay of correct detection. When data y_t are independent over time, standard results for symmetric MEWMA detector (3) have been worked out using the Markov chain approach (see *e.g.* [16]). These allow us to compute mean time between false alarms and mean delay time after change and, hence, to properly design the detector.

Since the applications we have in mind refer to correlated data, we have two possibilities. The first one is based on filtering the data by some time series model and applying the Q chart to the residuals. The second is based on applying directly the Q chart to the original (serially correlated) data.

The former approach is preferred when standard results are available for independent data. In this case the distribution of Q is known to be of the so-called chi-bar ($\bar{\chi}$) type. This is a mixture of χ^2 distributions with a probability mass on 0 and weights given by multivariate normal integrals. Except special cases, such as $k = 2$ or 3 and the uncorrelated case $\Sigma = diagonal$, these integrals have to be computed by Monte Carlo simulation. Moreover, filtering a multivariate time series may result in a difficult interpretation of the original one-sided structure of the null hypotheses (4) and (5).

Hence, we choose the latter approach and compute the percentiles of Q and the mean time between exceedances by semi-parametric bootstrap simulation based on an appropriate stochastic model for the time series at hand. We then use a long series of simulated data y_t^B and a correspondingly long series of Q_t^B which are the bases for computing thresholds.

In particular, in order to get simulated replications of y_t, we use a model given by

$$y_t = g\left(y_{t-1}, ..., y_{t-p}\right) + \eta_t \tag{6}$$

where $g\left(\right)$ defines a linear autoregression. Moreover, η_t describes the additional nonlinear dynamics which is supposed stationary. Whenever this dynamics is not directly modelled, it is taken into account by the stationary block bootstrap of the residuals of model (6).

To do this first, we use an appropriate piece of data, where the reference dynamics H_0 is supposed to be working, for estimating $g\left(\right)$ and computing estimated residuals $\hat{\eta}_t$. Then, following Davison and Hinkley [2], § 8.2.3, we apply block stationary bootstrap to the residuals $\hat{\eta}_t$ to reproduce the nonlinear dynamics not captured by $\hat{g}\left(\right)$. To this end, we construct a sequence of simulated errors η_t^B by concatenating blocks $\hat{\eta}_{s+1}, ..., \hat{\eta}_{s+L}$ from original residuals, each block with random starting point s and random length L. The block lengths have a geometric distribution with mean larger than three times the main high frequency periodicity. Finally, we get the simulated y_t^B by recursively adding $g\left(\right)$ and η_t^B in equation (6).

3 Monitoring Dimension and Deformation

This case study is concerned with quality control of brake disc mass production. In particular, we are interested in monitoring dimensional parameters, which are important for car assembling efficiency. For example, the correct diameter of the center hole is required to avoid problems in car assembling. Moreover, we are interested in geometric deformation parameters, which are important for braking efficiency; for example, high braking band oscillation or high disc thickness variation may result in whistles, vibrations and slip-stick phenomena.

We will consider three dimensional parameters, namely: the diameter of the center hole (Ø), braking surface (BS) thickness and braking band off set (B off set); moreover, we will consider four geometric parameters, i.e. disc thickness variation (DTV), braking band A oscillation (BB-A run out), braking band B oscillation (BB-B run out) and braking band radial parallelism (BB radial $||$).

The former dimensional parameters have to be checked against changes in both directions but the latter, being deformations, have to be checked only against one-sided changes. We then have the following change hypotheses:

$$\begin{aligned} &H_0 : \theta_j = \theta_j^0,\ j = 1, 2, 3 \quad \text{and} \quad \theta_j \leq \theta_j^0, \quad j = 4, ..., 7 \\ &H_1 : \theta_j \neq \theta_j^0,\ \text{for some } j = 1, 2, 3 \quad \text{or} \quad \theta_j > \theta_j^0,\ \text{for some } j = 4, ..., 7. \end{aligned} \tag{7}$$

The observed data of Figure 1 are collected about every $20'$ and, for the sake of this paper, are divided into two subsets. The first 525 observations are from a certain production setup i.e., from our point of view, are considered as before change data and used for estimating the *in control* process level θ^0 and dynamics. The remaining 246 observations come from another setup i.e. are considered as stationary random observations after a step change and will be used for illustrating the after change performances.

In order to properly introduce $Q-$ chart computations, first we address the correlation structure and dynamics of the disc production process before change. To do this, from the correlation matrix of Table 1, we see that, whenever correlations between the first and the second group are definitely low, geometric parameters are highly cross-correlated, as expected, and, among the dimensional ones, we find a moderate but significant correlation between surface thickness and offset. Overall, the correlation matrix determinant is quite small being given by 0.096. It is then strongly recommended to use multivariate control charts instead of multiple univariate control charts.

Moreover, the serial correlations reported in Figure 2 suggest that a moderate inertia is present in the production plant and, together with the corre-

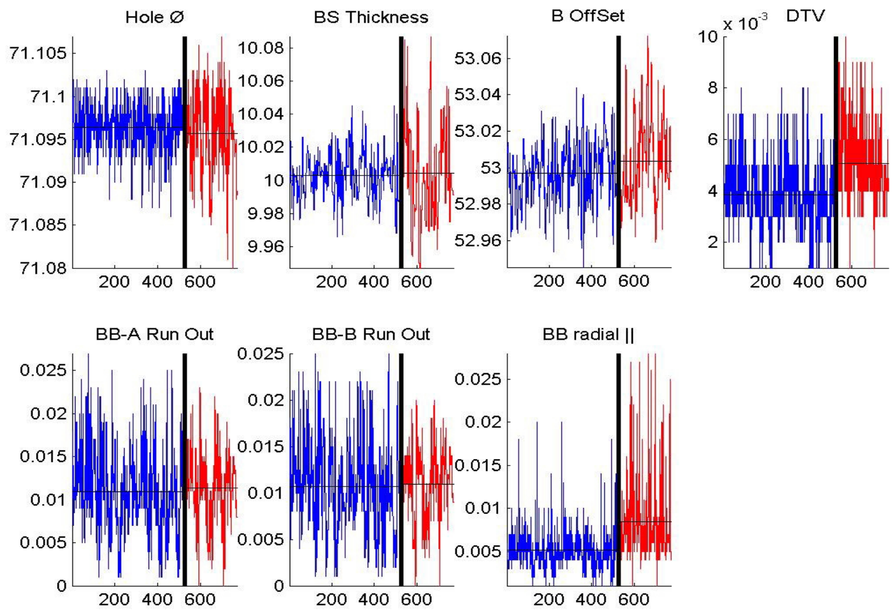

Figure 1: Disk data: thick lines denote plant change time $t = 526$.

lation matrix of Table 1, suggest that the process dynamics may be approximated by a vector autoregression of order p denoted by $k-\mathrm{VAR}(p)$.

Then, in order to get appropriate bootstrap estimates of thresholds, we fit a $7-\mathrm{VAR}(p)$ model and choose the model order $p = 5$ by minimizing the AIC criterion. Moreover, we prune all coefficients with p-value greater than 1% and find the following reduced VAR(5) model relating the zero mean deviates:

$$\begin{aligned}
\hat{y}_{1,t} &= 0.23 y_{1,t-1} \\
\hat{y}_{2,t} &= 0.78 y_{2,t-1} - 0.44 y_{2,t-3} + 0.21 y_{3,t-3} + 0.35 y_{2,t-4} - 0.17 y_{3,t-4} \\
\hat{y}_{3,t} &= 0.73 y_{3,t-1} - 0.19 y_{3,t-3} + 0.13 y_{3,t-4} \\
\hat{y}_{4,t} &= 0.19 y_{4,t-2} + 0.20 y_{4,t-3} \\
\hat{y}_{5,t} &= 0.20 y_{1,t-1} + 0.47 y_{5,t-1} \\
\hat{y}_{6,t} &= 0.20 y_{1,t-1} + 0.03 y_{2,t-1} + 0.44 y_{5,t-1} + 0.14 y_{5,t-5} \\
\hat{y}_{7,t} &= 0.07 y_{6,t-3}
\end{aligned} \tag{8}$$

There is only a moderately good fit, as can be assessed by the correlations between observed y_t and fitted $\hat{y}_t$ which are in the range $0.156 \div 0.727$, but

Hole Ø	1.00						
BS Thickness	-0.04	1.00					
B Off set	-0.03	0.37	1.00				
DTV	0.03	-0.03	0.00	1.00			
$BB-A$ Run Out	0.07	0.05	-0.09	0.36	1.00		
$BB-B$ Run Out	0.09	0.08	-0.06	0.36	0,90	1.00	
BB radial ||	0.06	-0.01	0.01	0.53	0.26	0.24	1.00

Table 1: Correlation Matrix of Disk Parameters.

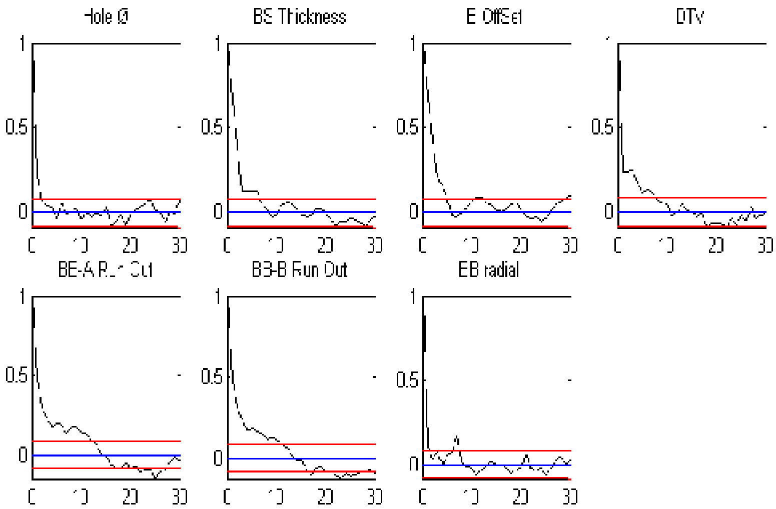

Figure 2: Autocorrelations of Disk Data before Change.

this is not the main concern because here, we are not interested in a forecasting model. Moreover, residual autocorrelations of Figure 3 show that the linear component has been filtered out and model (8) reproduces the gross periodicity of the process. Nevertheless, the autocorrelations among squared residuals reported in Figure 4 suggest that some significant nonlinear dynamics is present in the data, motivating the semi-parametric block bootstrap introduced in section 2.4.

Although modelling this nonlinear component in closed form may be of interest in some cases, here, we avoid this and get a long simulation ($m = 100'000$) of the disc production process y_t by the approach of equation (6) with 7-dimensional residual jointly block-bootstrapped, using a block length longer than 5 hours on average. The simulated trajectory preserves the linear

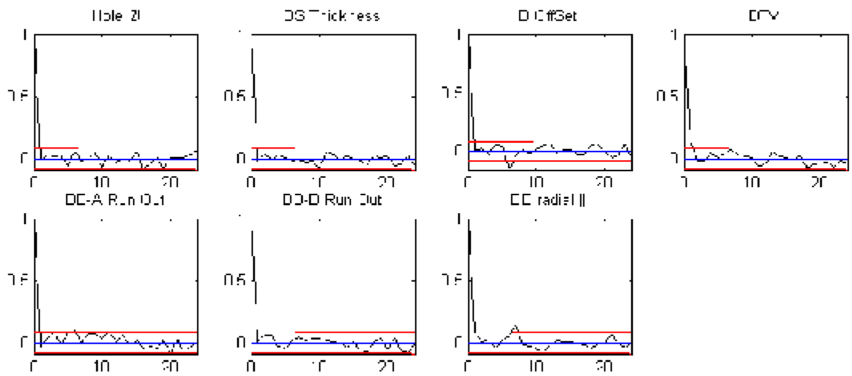

Figure 3: Autocorrelations of residuals from model (8).

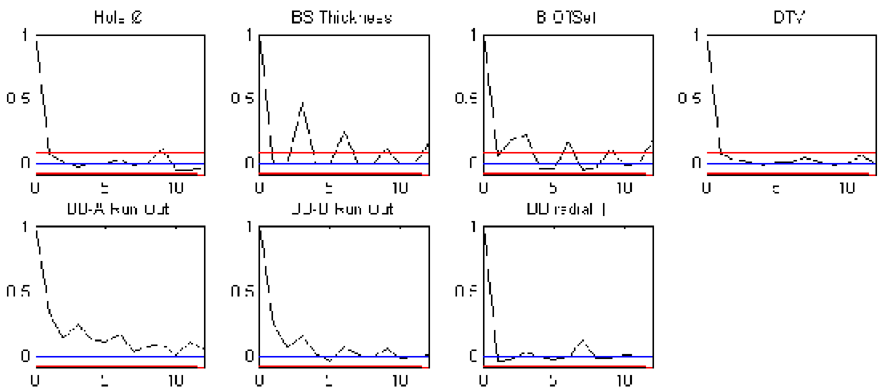

Figure 4: Autocorrelations of squared residual from model (8).

structure of VAR model (8), the nonlinear dynamics and the innovation cross-correlation.

In Figure 5, we see the Q chart for the dimension-deformation hypotheses (7) and the bootstrap thresholds corresponding to mean time between false alarms of 20, 100 and 365. Comparing the Q chart with the equivalent multiple univariate EWMA control chart of Figure 6 shows that, for the same smoothing factor $\lambda = 0.1$ and the same false alarm probability $\alpha = 5\%$, the Q chart gives a much clearer monitoring system. More details on Monte Carlo optimality assessment are given, for the general case, in [6] and [7].

Moreover, in order to identify which parameters are the most likely to be responsible for a detected out of control signal, we may go into details using various steps of after detection diagnostics. The number of steps depends on the complexity of the system and, in our case, two are enough.

At the first stage, we are interested in contrasting geometric deformation (GD) and dimension shifts (DS). In particular, when subgroups are independent, *e.g.* as in [3], we have additivity of Q. Then, using the approximate block independence of Table 1, we have

$$Q \cong Q_{DS} + Q_{GD}$$

with obvious symbol meaning. The corresponding charts, not reported here for brevity, clearly show that the geometric deformation chart gives an earlier out-of-control signal. Since it anticipates the shift of dimension chart, on purely statistical grounds, it could be interpreted as the cause. Of course this conclusion is quite preliminary and in-depth assignable causes analysis should be performed.

At the second stage, we are interested in which individual coordinate is responsible for a certain alarm in the Q chart. We then use the seven univariate EWMA charts of Figure 6 and, consistently with above Q decomposition, we conclude that DTV and braking band radial parallelism are the main causes.

4 Conclusions

In complex systems monitoring, it is important to have efficient detectors having short delay time after a change takes place. Moreover, careful analysis of change hypoteses often show that the standard approach based on Gaussian contours is not feasible.

In this paper, we have shown that the asymmetric Q chart is a flexible tool that can be used for multivariable change detection and diagnostics. Although analytical results for thresholds and delay times are not known in

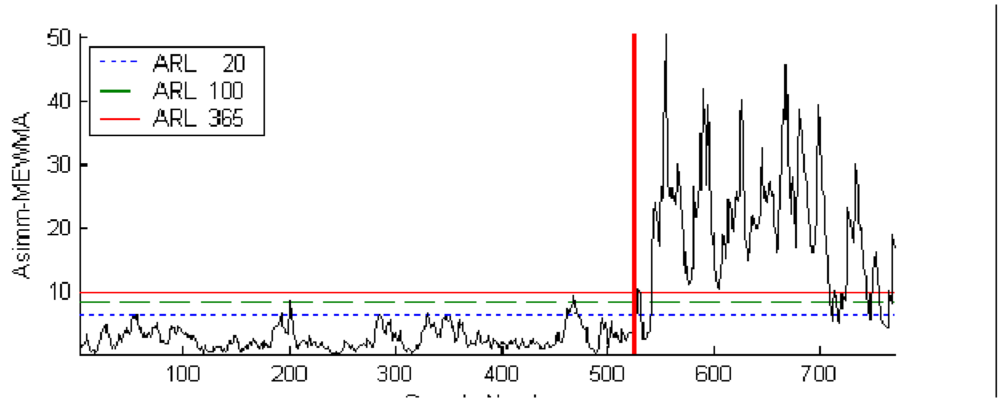

Figure 5: Q Chart with Bootstrap Thresholds. Thick line denotes change time $t = 526$.

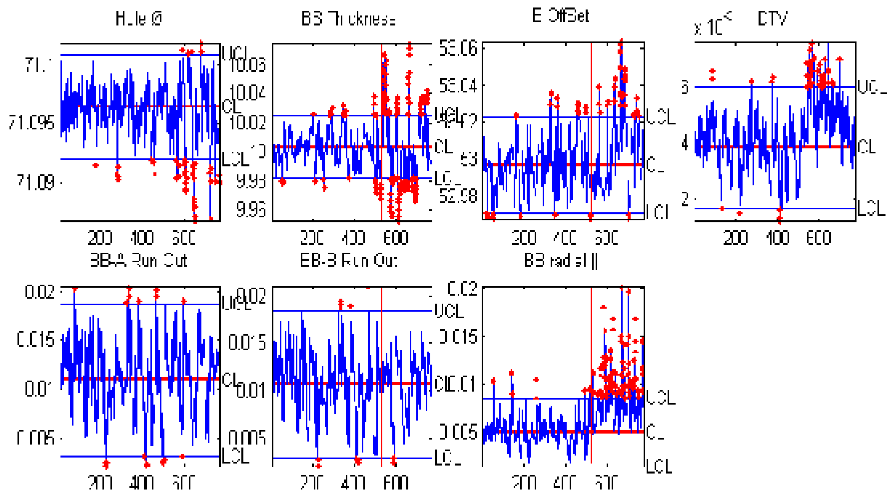

Figure 6: EWMA Charts with 95% bands. Thick line denotes change time $t = 526$.

closed form at least to these authors, the control chart can be easily adapted to the case under study by using estimated values which are obtained by Bootstrap or Monte Carlo simulation of the data generating process.

5 Appendix: Exact Computation of Q

Following [6] and [7], the restricted maximum likelihood (RML) estimator of $\theta \in \Theta_0 = \{\theta_j \leq 0\}$ is the projection of y into Θ_0 minimizing

$$\left(y - \hat{\theta}_0\right)' \Sigma^{-1} \left(y - \hat{\theta}_0\right) \tag{9}$$

for $\hat{\theta}_0 \in \Theta_0$.

To introduce the closed form solution, first we consider the simple bivariate case of next section.

5.1 The Bivariate Case

Let us consider $k = 2$ and, without loss of generality, suppose that y has unit variances and correlation coefficient denoted by ρ. Minimization of (9) gives

$$\hat{\theta}(y) = yI_{(0)} + \hat{\theta}_{(1)} I_{(1)} + \hat{\theta}_{(2)} I_{(2)}$$

where

$$I_{(0)} = I(y \in \Theta_0),$$

$$I_{(1)} = I\left(y \in \Re^2 : y_1 > 0,\ y_2 < \rho y_1\right),$$

$$I_{(2)} = I\left(y \in \Re^2 : y_2 > 0,\ y_1 < \rho y_2\right)$$

and

$$\hat{\theta}_{(1)} = \begin{pmatrix} 0 \\ y_2 - \rho y_1 \end{pmatrix} \qquad \text{and} \qquad \hat{\theta}_{(2)} = \begin{pmatrix} y_1 - \rho y_2 \\ 0 \end{pmatrix}.$$

Note that when $y \in \Theta_3 = \Re^2 - \Theta_0 - \Theta_1 - \Theta_2$, the RML estimate $\hat{\theta}(y)$ vanishes.

Now the estimated score is simply given by

$$Q(y) = y_1^2 I_{(1)} + y_2^2 I_{(2)} + y' \Sigma^{-1} y I_{(3)}$$

where

$$I_{(3)} = 1 - I_{(0)} - I_{(1)} - I_{(2)}.$$

5.2 The Multivariate Q

In this section we define the detector Q in closed form for $y \in \Re^{k>2}$. To do this, we need some more notations. Let the permutation vector

$$\pi = (\pi_1, ..., \pi_k)$$

be a permutation of integers $(1, ..., k)$ and, correspondingly, for a column vector $y = (y_1, .., y_k)'$ let

$$x = \pi (y) = (y_{\pi_1}, ..., y_{\pi_k})'$$

be the permuted vector, defined through the permutation operator $\pi\,()$. Similarly for a row vector, we define $\pi (y') = (y_{\pi_1}, ..., y_{\pi_k})$ and for a square matrix $A = (a_{ij})$, we define

$$\pi (A) = \left(a_{\pi_i \pi_j}\right).$$

Moreover, let π^{-1} be the reverse permutation so that $y = \pi^{-1} (\pi (y))$.

In order to define the projection $\hat{\theta} \in \Theta_0$ note that, since Θ_0 is convex, $\hat{\theta}$ is unique with probability one. Moreover, following [4], consider the frontier

$$\bar{\Theta}_0 = \{\theta \in \Theta_0 : \theta_j = 0 \text{ for some } j = 1, ..., k\}$$

and its partition into the following $p = 2^k - 1$ hyperplanes defined using permutations $\pi_s = (\pi_{s,1}, ..., \pi_{s,k})$:

$$R_s = \left\{\theta \in \Theta_0 : \theta_{\pi_{s,i}} = 0, i = 1, ..., q_s \text{ and } \theta_{\pi_{s,i}} < 0, i = q_s + 1, ..., k\right\}.$$

For each hyperplane R_s and $\theta \in \Re^k$, let

$$\eta_{(s)} = \pi_s (\theta) = \left(\theta_{\pi_{s,1}}, ..., \theta_{\pi_{s,k}}\right)'.$$

In the sequel the vector $\eta_{(s)}$ will be partitioned according to the first q_s coordinates which are zero in R_s, i.e.

$$\eta_{(s)} = \left(\theta_{\pi_{s,1}}, ..., \theta_{\pi_{s,q_s}}, \theta_{\pi_{s,q_s+1}}, ..., \theta_{\pi_{s,k}}\right)' = \left(\alpha'_{(s)}, \beta'_{(s)}\right)'$$

say, where $\alpha_{(s)} \in \Re^{q_s}$. Now, let the vector

$$x_{(s)} = \pi_s (y)$$

and its covariance

$$\Delta_{(s)} = \pi (\Sigma) = E\left(x_{(s)} x'_{(s)}\right)$$

be conformably partitioned, i.e.

$$x_{(s)} = \pi_s (y) = \left(a'_{(s)}, b'_{(s)}\right)',$$

where $a_{(s)} \in \Re^{q_s}$, and

$$\Delta_{(s)} = \begin{pmatrix} \Delta_{(s)aa} & \Delta_{(s)ab} \\ \Delta_{(s)ba} & \Delta_{(s)bb} \end{pmatrix}.$$

Finally, let 0_q be the origin in $\Re^q$ and let $I()$ be the set indicator function. Then, for permutations π_s, $s = 1, ..., p$, consider partial projections

$$\hat{\eta}_{(s)} = \begin{pmatrix} 0_{q_s} \\ b_{(s)} - \Delta_{(s)ba}\Delta^{-1}_{(s)aa}a_{(s)} \end{pmatrix}$$

and projecting subsets

$$I_{(0)} = I\left(y \in \Theta_0\right),$$

$$\begin{aligned} I_{(s)} = \ & I[a_j > 0,\ j = 1, ..., q_s,\ \text{and} \\ & (b_{(s)} - \Delta_{(s)ba}\Delta^{-1}_{(s)aa}a_{(s)})_i < 0,\ i = 1, ..., k - q_s + 1]. \end{aligned}$$

It follows that the score can be written as

$$Q = \begin{cases} \min_s \left(Q_{(s)} > 0\right) & \text{if } y \notin \Theta_0 \\ 0 & \text{else,} \end{cases} \tag{10}$$

where

$$Q_{(s)} = a'_{(s)}\Delta^{-1}_{(s)aa}a_{(s)}.$$

Then $Q = Q_{(r)}$ for some r and the RML estimate for $\hat{\theta} \in \Theta_0$ is given by

$$\hat{\theta} = yI_{(0)} + \hat{\theta}_r\left(1 - I_{(0)}\right).$$

Note that these results for $\hat{\theta}$ and Q revise equations (11) and (12) of [6] and equation (11) of [7].

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