Extracting joint probability of default from CDS data

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July 1, 2011

Abstract

Systemic default risk -i.e. the risk of simultaneous default of multiple institutions- has caused great concern in recent past. However, the measure of systemic risk is not a trivial subject. The aim of this paper is to estimate the joint probability of default for couples of defaultable entities, belonging to different rating classes. Both bond and credit derivative markets convey information on the default probabilities. In particular, the price of Credit Default Swap (CDS) contracts involves counterparty risk i.e. the risk that the protection seller will fail to fulfill its obligations - usually either by failing to pay or by failing to deliver securities. The counterparty risk is reflected in the CDS price through the joint default probability of the reference entity and the protection seller.

In this paper, applying a no-arbitrage argument, we extract forward looking joint default probabilities of institutions operating in the CDS market. The analysis of the dynamics of the joint default probability can provide clear signals of an increase in systemic risk and danger of contagion.

JEL Classification Numbers: G01 , G32.
Keywords: Credit default swaps; counterparty risk; early warning signals; joint default probability.

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1 Introduction

As a first approximation, CDS prices reflect the expected loss of the reference entity given by its default probability and the recovery rate. Additional risk premia are required to compensate for an unexpected default. See Amadei et al [2] for a detailed discussion. These factors are actually the same that influence bond spreads: theoretically bond spreads should be equal to CDS premia for the same reference entity. Consider two financial agents α and β and let:

- $r(t, T)$ be the risk-free rate (Libor) in t for the maturity T.
- $s_\alpha(t, T)$ be the spread over the Libor of the issuance cost of α, prevailing in t for the maturity T.
- $w_{\alpha,\beta}(t, T)$ be the annual CDS premium to insure against the default of α within the period $[t, T]$ with β as the protection seller.
- $R_\alpha(t, T)$ be the zero-coupon bond (ZCB) yield in t for the maturity T.

In the equilibrium point, a portfolio composed by a zero coupon bond with maturity T and a CDS on that same bond with the same maturity, should replicate a synthetic risk-free asset. Hence the ZCB yield $R_\alpha(t, T)$ minus the CDS premium $w_{\alpha,\beta}(t, T)$ should be exactly equal to the risk-free rate $r(t, T)$, being then able to write the spread $s_\alpha(t, T)$ as the difference $R_\alpha(t, T) - r(t, T)$. However, in reality $w_{\alpha,\beta}(t, T) \neq s_\alpha(t, T)$ (see Amadei et al [2]). We define as "basis" the difference $w_{\alpha,\beta}(t, T) - s_\alpha(t, T)$. The invoked equilibrium is ensured by the two following arbitrage strategies:

1. *Case* $w_{\alpha,\beta}(t, T) < s_\alpha(t, T)$: the arbitrage strategy in this case consists in buying the bond, financing at the risk-free rate $r(t, T)$ and then buying the CDS by paying the premium $w_{\alpha,\beta}(t, T)$. The portfolio return is $(s_\alpha(t, T) + r(t, T)) - r(t, T) - w_{\alpha,\beta}(t, T) = s_\alpha(t, T) - w_{\alpha,\beta}(t, T)$ which is positive since $w_{\alpha,\beta}(t, T) < s_\alpha(t, T)$.
2. *Case* $w_{\alpha,\beta}(t, T) > s_\alpha(t, T)$: the arbitrage strategy in this case consists in short selling the bond, investing the proceeds at the risk-free rate of return $r(t, T)$ and selling protection in the CDS market with premium $w_{\alpha,\beta}(t, T)$. The portfolio return is $w_{\alpha,\beta}(t, T) + r(t, T) - (s_\alpha(t, T) + r(t, T)) = w_{\alpha,\beta}(t, T) - s_\alpha(t, T)$ with a positive return since $w_{\alpha,\beta}(t, T) > s_\alpha(t, T)$.

The portfolio payoffs are guaranteed for each strategy if and only if the positions are kept until bond maturity or until the credit event occurs. Otherwise the strategy faces a roll over risk in the financing/investing positions linked to the volatility of $r(t, T)$. In practice the basis is rarely zero because of market imperfections, differences in the liquidity of CDS and corporate bond markets and counterparty risk. In the following we concentrate our attention on this latter aspect: CDS market is affected by counterparty risk which is not present in bond market; when it increases the CDS premium decreases.

From 2007 basis for corporate debt has been mainly negative for reference entities rated BBB and below and moderately positive for high quality reference entities. The persistence of negative basis can be motivated by the failure in implementing the arbitrage strategy 1 due to difficulties in:

1. buying the bond and financing the position at the risk-free rate, due to liquidity problems and high tensions in the interbank market.
2. buying the CDS, for a lack of protection sellers or for the perception of a high counterparty risk linked to these contracts.

An additional explanation of a non-zero basis can be a different reactivity of CDS and bond spread markets to new information on a issuers. A negative or positive basis can reflect a different degree of adjustment between the two markets that arbitrage strategies correct only in the long run.

2 A formula for the joint probability of default

In the previous section we have stated that the arbitrage free value of the basis is zero. This holds when the counterparty risk is not explicitly taken into consideration. In the following, we relax this assumption: a negative basis, representing the counterparty cost, can still be consistent with a arbitrage free valuation. Negative basis is typical of financial crisis periods and we will extract from it information on the joint default probability.

A formula for measuring the joint probability of default of two financial institutions is hereby derived. This is first pursued in a one-period framework.

2.1 The one-period case

Consider two risky financial institutions and for illustrative purpose denote them as α and β. Consider a third party called γ and, for the sake of argument, assume that it can not go bankrupt. Imagine that at time $t = 0$, the riskless entity γ builds a portfolio, according to the following uniperiodal strategy:

Strategy 1 (*one-period case*):

- Buy a 1-year zero coupon bond (ZCB) issued by α,
- buy a 1-year CDS from β, the protection seller, on the reference entity α,
- finance the positions on the market with a 1-year loan.

All the contracts have a face value of \$1. Since we assume that γ is risk-free, it can finance its positions at the 1-year Libor rate. On the contrary the interest rate offered by α on the bond issue is increased by a spread related to its rating class. So now let:

- RR_α, RR_β be the recovery rates of α and β, respectively.
- Π_t be the value at time t of the portfolio built by γ according to Strategy 1.

The present value of the bond issued by α is $e^{-r(0,1)-s_\alpha(0,1)} \approx 1 - r(0,1) - s_\alpha(0,1)$ and the amount of money been borrowed by γ is $e^{-r(0,1)} \approx 1 - r(0,1)$.

In Table 1 we report the value of the portfolio in two different points in time, $t = 0$ and $t = 1$, when all the cash flows are exchanged. The second column summaries, at time $t = 0$, the present value of the financial instruments constituting the portfolio held by γ. The portfolio value at time $t = 0$ is therefore:

$$\Pi_0 = w_{\alpha,\beta}(0,1) - s_\alpha(0,1) \tag{1}$$

The other columns of Table 1 report the cash flows in the different states of the world at time $t = 1$. The dash on the name of a financial institution stands for the institution being in default.

	$t=0$	$t=1$			
		α,β	$\overline{\alpha},\beta$	$\alpha,\overline{\beta}$	$\overline{\alpha},\overline{\beta}$
Loan	$-(1-r(0,1))$	-1	-1	-1	-1
ZCB	$1-r(0,1)-s_\alpha(0,1)$	1	RR_α	1	RR_α
CDS	$w_{\alpha,\beta}(0,1)$	0	$1-RR_\alpha$	0	$(1-RR_\alpha)RR_\beta$
Π_t	$w_{\alpha,\beta}(0,1)-s_\alpha(0,1)$	0	0	0	$-(1-RR_\alpha)(1-RR_\beta)$

Table 1: Cash flows of the uniperiodal Strategy 1 at time $t = 0$ and $t = 1$.

If α survives, independently from β, γ repays its loan using the money stemming from the zero coupon bond, while the CDS expires. If α defaults and β survives, the position of γ is hedged by the CDS and the portfolio value is null.

A non-zero cash flow is generated only when both α and β default, that is with a probability equal to the joint default probability of α and β. Under the hypothesis that $0 < RR_\alpha < 1$ and $0 < RR_\beta < 1$, this would result in a negative cash flow. It follows that γ will implement such a strategy whenever the basis is negative, hence $\Pi_0 < 0$. Thus within our framework we admittedly exclude the presence of a positive basis, which typically conveys information linked with market microstructure noises, here not explicitly modelled.

Hence, the expected value of the portfolio at $t = 1$ is:

$$\mathbf{E}[\Pi_1] = -P_{\overline{\alpha},\overline{\beta}}(0,1)(1 - RR_\alpha)(1 - RR_\beta) \tag{2}$$

where we denote $P_{\overline{\alpha},\overline{\beta}}(0,1)$ as the risk-neutral 1-year joint default probability of α and β.

Recall that in an arbitrage-free world, for every security with value f_t at time $t \geq 0$, it must hold that:

$$f_t = \mathbf{E}_Q[f_T]e^{-r(t,T)(T-t)} \qquad \forall t < T \tag{3}$$

where Q is the risk-neutral probability measure.

That is, excluding arbitrage possibilities, in our case we must have:

$$\Pi_0 = \mathbf{E}[\Pi_1]e^{-r(0,1)}, \quad \Pi_0 < 0 \tag{4}$$

Combining Eq. 1, 2 and 4, we obtain:

$$P_{\overline{\alpha},\overline{\beta}}(0,1) = \frac{\Big(s_\alpha(0,1) - w_{\alpha,\beta}(0,1)\Big)^+}{(1 - RR_\alpha)(1 - RR_\beta)} e^{r(0,1)} \tag{5}$$

where $(\cdot)^+ \equiv \max(\cdot, 0)$.

According to Eq. 5 the joint probability of default is explained by:

- *The negative basis.* In absence of counterparty risk and arbitrage opportunities, this difference is supposed to be zero[1]. The presence of counterparty risk reduces the premium of the CDS and motivates a negative basis . The wider is this difference, the higher is the counterparty risk and eventually the higher is the joint probability of default of the bond issuer α and the protection seller β. If the basis is positive we set the joint default probability equal to zero.

- *A discounting factor*, due to the fact that the cash flows considered pertain different instants in time.

- *The recovery rates of the financial institutions considered.* The higher the recoveries, the higher is the probability of joint default, other things being equal. That is, if increasing expectations on recoveries don't lead to a decrease of the spreads, a joint default is more likely to happen.

For further development, it can be useful to give a marginal default probability formula, which is coherent with our framework. First define $P_{\overline{\alpha}}$ and $P_{\overline{\beta}}$ as the risk-neutral probabilities of default of α and β accordingly.

Applying the risk-neutral principle stated in Eq. 3 to the case of the CDS considered we have[2]:

$$w_{\alpha,\beta}(0,1) = \Big(P_{\overline{\alpha},\beta}(0,1)(1 - RR_\alpha) + P_{\overline{\alpha},\overline{\beta}}(0,1)(1 - RR_\alpha)RR_\beta\Big)e^{-r(0,1)} \tag{6}$$

from which we get:

$$P_{\overline{\alpha},\beta}(0,1) = \frac{w_{\alpha,\beta}(0,1)e^{r(0,1)}}{1 - RR_\alpha} - P_{\overline{\alpha},\overline{\beta}}(0,1)RR_\beta \tag{7}$$

Thus, using the trivial equality $P_{\overline{\alpha}}(\cdot,\cdot) = P_{\overline{\alpha},\overline{\beta}}(\cdot,\cdot) + P_{\overline{\alpha},\beta}(\cdot,\cdot)$, together with Eq. 5 and 7, after some rearranging, we get:

$$P_{\overline{\alpha}}(0,1) = \frac{s_\alpha(0,1)e^{r(0,1)}}{1 - RR_\alpha} \tag{8}$$

[1] See for instance Hull et al. [16].

[2] See the fifth row in Table 1.

With a similar argument we can state:

$$P_{\overline{\beta}}(0,1) = \frac{s_\beta(0,1)e^{r(0,1)}}{1 - RR_\beta} \tag{9}$$

Note how the previous two formulæ are consistent with the literature[3].

2.2 Multi-period case

To extend the formula in Eq. 5 over the generic time sequence $0 = t_0 < t_1 < \ldots < t_n$, we consider the case in which γ pursues the following multi-period strategy:

Strategy 2 (*Multi-period case*): At time t_i with $i = 0, \ldots, n-1$, if both α and β are alive, then:

- Buy a $(t_{i+1} - t_i)$-year zero coupon bond issued by α,
- buy a $(t_{i+1} - t_i)$-year CDS [4] from β on α,
- finance the positions on the market with a $(t_{i+1} - t_i)$-year loan,

otherwise close the position.

On the basis of the usual no-arbitrage argument, we state that the actual expected cost of the strategy must equate the present value of its outcomes, that is:

$$\begin{aligned} &\Big(s_\alpha(0,t_1) - w_{\alpha,\beta}(0,t_1)\Big)^+ t_1 + \\ &+ \sum_{i=1}^{n-1} P_{\alpha,\beta}(t_{i-1},t_i)\Big(s_\alpha(t_i,t_{i+1}) - w_{\alpha,\beta}(t_i,t_{i+1})\Big)^+ (t_{i+1}-t_i)e^{-r(0,t_i)t_i} = \\ &= (1-RR_\alpha)(1-RR_\beta)\sum_{i=1}^{n} P_{\overline{\alpha},\overline{\beta}}(t_{i-1},t_i)e^{-r(0,t_i)t_i} \end{aligned} \tag{10}$$

The left-hand side of the Eq. 10 refers to the expected profit of rolling Strategy 2, while the right-hand side is the expected present value of the flows generated in the case both α and β go bankrupt. According to Strategy 2, the amount $\$\Big(s_\alpha(t_i,t_{i+1}) - w_{\alpha,\beta}(t_i,t_{i+1})\Big)$ has to be understood as the profit for speculating against the joint default of α and β in the time interval $[t_i, t_{i+1}]$. As before, we argue that, as long as the recovered amounts RR_α and RR_β are positive, γ

[3] Ignoring the discount factor we are exactly in line with Hull [15, Chap. 20].

[4] The quotes $w_{\alpha,\beta}(t_{i+1},t_i)$ of the CDS involved in Strategy 2 are expressed in annual terms. Imagine that they are single-premium contracts and let this unique premium to be paid in t_i, at the beginning of the life of the contract. Thus the premium due in t_i is given by $w_{\alpha,\beta}(t_{i+1},t_i)(t_{i+1}-t_i)$.

won't implement such a strategy whenever $s_\alpha(t_i,t_{i+1}) \le w_{\alpha,\beta}(t_i,t_{i+1})$ [5]. This is the reason why in Eq. 10 we consider negative bases only.

From Eq. 10 we get:

$$\begin{aligned} P_{\overline{\alpha},\overline{\beta}}(t_{n-1},t_n) &= \tilde{P}_{\overline{\alpha},\overline{\beta}}(0,t_1)e^{r(t_1,t_n)(t_n-t_1)} + \\ &\sum_{i=1}^{n-1}\Big[P_{\alpha,\beta}(t_{i-1},t_i)\tilde{P}_{\overline{\alpha},\overline{\beta}}(t_i,t_{i+1})e^{r(t_{i+1},t_n)(t_n-t_{i+1})} + \\ &- P_{\overline{\alpha},\overline{\beta}}(t_{i-1},t_i)e^{r(t_i,t_n)(t_n-t_i)}\Big] \end{aligned} \tag{11}$$

in which we define:

$$\tilde{P}_{\overline{\alpha},\overline{\beta}}(t_i,t_{i+1}) \equiv \frac{\Psi(t_i,t_{i+1})}{(1-RR_\alpha)(1-RR_\beta)} \quad i=0,\ldots,n-1 \tag{12}$$

where:

$$\Psi(t_i,t_{i+1}) \equiv \Big(s_\alpha(t_i,t_{i+1}) - w_{\alpha,\beta}(t_i,t_{i+1})\Big)^+ (t_{i+1}-t_i)e^{r(t_i,t_{i+1})(t_{i+1}-t_i)} \tag{13}$$

and $s_\alpha(t_i,t_{i+1})$ and $w_{\alpha,\beta}(t_i,t_{i+1})$ might be understood as on forward basis. Thus, within our framework, $\tilde{P}_{\overline{\alpha},\overline{\beta}}(t_i,t_{i+1})$ corresponds to the joint default probability seen in $t=0$ for the time interval $[t_i,t_{i+1}]$, conditionally to the survive of both α and β up to time t_i (compare Eq. 12 with Eq. 5). Henceforth we will refer to $\tilde{P}_{\overline{\alpha},\overline{\beta}}(\cdot,\cdot)$ as the conditional risk-neutral probability of joint default.

To grasp a better insight of the formula provided, notice that Eq. 11 leads to the following recursive relation:

$$P_{\overline{\alpha},\overline{\beta}}(t_i,t_{i+1}) = \begin{cases} \tilde{P}_{\overline{\alpha},\overline{\beta}}(0,t_1) & \text{for } i=0 \\ P_{\alpha,\beta}(t_{i-1},t_i)\tilde{P}_{\overline{\alpha},\overline{\beta}}(t_i,t_{i+1}) & \text{for } i=1,\ldots,n-1 \end{cases} \tag{14}$$

which corresponds to the standard definition of unconditional probability. Thus, using Eq. 14 we can build a time structure of the risk-neutral probability of joint default between α and β. Notice that, in order to make Eq. 14 feasible in practice we need to use the trivial relation:

$$P_{\alpha,\beta}(\cdot,\cdot) = 1 - P_{\overline{\alpha}}(\cdot,\cdot) - P_{\overline{\beta}}(\cdot,\cdot) + P_{\overline{\alpha},\overline{\beta}}(\cdot,\cdot) \tag{15}$$

where:

$$P_{\overline{\alpha}}(t_i,t_{i+1}) = \begin{cases} \tilde{P}_{\overline{\alpha}}(0,t_1) & \text{for } i=0 \\ P_\alpha(t_{i-1},t_i)\tilde{P}_{\overline{\alpha}}(t_i,t_{i+1}) & \text{for } i=1,\ldots,n-1 \end{cases} \tag{16}$$

with $P_\alpha(t_{i-1},t_i) \equiv 1 - P_{\overline{\alpha}}(t_{i-1},t_i)$ and in analogy with Eq. 8 we define:

$$\tilde{P}_{\overline{\alpha}}(t_i,t_{i+1}) \equiv \frac{s_\alpha(t_i,t_{i+1})}{1-RR_\alpha}(t_{i+1}-t_i)e^{r(t_i,t_{i+1})(t_{i+1}-t_i)} \tag{17}$$

as the marginal default probability within t_i and t_{i+1}, seen in $t=0$. The same holds for the marginal probabilities referred to β.

[5] This corresponds to the case of positive basis.

2.3 The impact of the recovery risk

All the variables entering our definition of joint default probability stated in Eq. 14 are directly observable on the markets, with the only exception of the recovery rates RR_α and RR_β. It would be desirable for our estimates not to be affected by arbitrary assumptions on them. The only hypothesis we are going to embrace is a widely accepted one, that is $0 \le RR_\alpha < 1$ and $0 \le RR_\beta < 1$.

Notice that in order for Eq. 12 to assume a probability meaning, we need to ensure that:

$$\Psi(t_i, t_{i+1}) \le (1 - RR_\alpha)(1 - RR_\beta) \tag{18}$$

being the right-hand side unknown. Thus we seek a logistic transform $\mathcal{L}(\cdot)$ of $\Psi(t_i, t_{i+1})$ of the type:

$$\mathcal{L}\Big(\Psi(t_i, t_{i+1})\Big) \equiv \frac{a}{1 + \exp\{-\Psi(t_i, t_{i+1})\}} + b \tag{19}$$

where a and b are chosen such that:

$$\begin{cases} \mathcal{L}(0) = 0 \\ \lim\limits_{\Psi(t_i,t_{i+1}) \to +\infty} \mathcal{L}\Big(\Psi(t_i, t_{i+1})\Big) = (1 - RR_\alpha)(1 - RR_\beta) \end{cases}$$

Hence we get $a = 2(1 - RR_\alpha)(1 - RR_\beta)$ and $b = -(1 - RR_\alpha)(1 - RR_\beta)$. If we substitute $\Psi(t_i, t_{i+1})$ with its logistic transform, from Eq. 12 we get:

$$\tilde{P}_{\overline{\alpha},\overline{\beta}}(t_i, t_{i+1}) = \frac{2}{1 + \exp\{-\Psi(t_i, t_{i+1})\}} - 1 \tag{20}$$

as our ultimate formula for the unconditional risk-neutral probability of joint default within $[t_i, t_{i+1}]$.

Recalling Eq. 14, we can state that the these logistic transformations are sufficient to guarantee the consistency of the unconditional probability of joint default since $0 \le P_{\alpha,\beta}(t_i, t_{i+1}) \le 1$.

For the same reasoning as above, we apply the logistic transform even to the marginal probabilities $\tilde{P}_{\overline{\alpha}}(t_i, t_{i+1})$ and $\tilde{P}_{\overline{\beta}}(t_i, t_{i+1})$ to get:

$$\tilde{P}_{\overline{\alpha}}(t_i, t_{i+1}) = \frac{2}{1 + \exp\{-\Psi_\alpha(t_i, t_{i+1})\}} - 1 \tag{21}$$

$$\tilde{P}_{\overline{\beta}}(t_i, t_{i+1}) = \frac{2}{1 + \exp\{-\Psi_\beta(t_i, t_{i+1})\}} - 1 \tag{22}$$

where

$$\Psi_\alpha(t_i, t_{i+1}) \equiv s_\alpha(t_i, t_{i+1})(t_{i+1} - t_i)e^{r(t_i,t_{i+1})(t_{i+1}-t_i)} \tag{23}$$

$$\Psi_\beta(t_i, t_{i+1}) \equiv s_\beta(t_i, t_{i+1})(t_{i+1} - t_i)e^{r(t_i,t_{i+1})(t_{i+1}-t_i)} \tag{24}$$

3 Conclusions

A measure of systemic risk is provided by the joint default probability amongst different financial institutions. A methodology for estimating it has been proposed. It is based on a strong argument under the hypothesis of no-arbitrage. Furthermore, any dependence from recovery rate assumptions is avoided.

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4 Appendix

A Derivation of forward spreads

In the definition provided in Eq. 12 we introduced $s_\alpha(t_i, t_{i+1})$ and $w_{\alpha,\beta}(t_i, t_{i+1})$. The former is given by the standard relation:

$$s_\alpha(t_i, t_{i+1}) = \frac{t_{i+1}}{t_{i+1} - t_i} s_\alpha(0, t+1) - \frac{t_i}{t_{i+1} - t_i} s_\alpha(0, t) \tag{25}$$

For the latter, consider for simplicity the case of annual time instants $t = 0, 1, \ldots, T$ and CDS contracts in which a premium is paid annually. Thus in a free arbitrage world the following relation can be stated:

$$w_{\alpha,\beta}(0, T) \left(1 + \sum_{t=1}^{T-1} P_{\alpha,\beta}(t-1, t) e^{-r(0,t)t}\right) = \sum_{t=1}^{T} w_{\alpha,\beta}(t-1, t) e^{-r(0,t-1)(t-1)} \tag{26}$$

that is the expected actual value of the payments in a T-year contract must equate the expected actual value of T annual forward CDS. From Eq. 26 we get:

$$\begin{aligned} w_{\alpha,\beta}(t, t+1) = {} & w_{\alpha,\beta}(0, t+1) e^{r(0,t)t} + \\ & \sum_{\tau=1}^{t} \Big[w_{\alpha,\beta}(0, t+1) P_{\alpha,\beta}(\tau-1, \tau) e^{-r(\tau-1,\tau)} + \\ & - w_{\alpha,\beta}(\tau-1, \tau) e^{-r(0,\tau-1)(\tau-1)} \Big] \end{aligned} \tag{27}$$

with $t = 0, \ldots, T-1$.