A seismic swarm as a dynamic ergodic stochastic process: a case study of the L'Aquila's earthquake in 2009

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Abstract: The lethal earthquake of 6 April 2009 in L'Aquila, Central Italy, re-opened the discussion about the earthquake prediction due to the several precursory phenomena described in association to the event. One of the most important precursors that preceded L'Aquila main-shock was the foreshock activity. Papadopoulos et al. (NHESS, 2010) reported that a foreshock activity was there in the last months before the main-shock but the foreshock signal became very strong in the last 10 days with drastic changes in space-time-size domains of local seismicity. The importance of short-term foreshocks for the prediction of the main-shock was noted since the 1960's. However, foreshocks appear to precede only some main shocks and not others, while there are also foreshocks too small to detect by routine seismic analysis. In this context, the aim of the paper is to analyse the phenomenon of swarm as a dynamic ergodic stochastic process with particular reference to mean time of transition of a certain class of earthquake swarms (belonging to a certain state) to other classes of varying intensity. This kind of analysis can be referred to some indicators such as the mean first passage time and the mean time to return with their respective probabilities, that constitute an important interpretive tool in forecasting.

Keywords: seismic swarm, markovian processes, ergodicity.

1. Introduction

The available data consists of a data set of more than 15.000 shocks that occurred in the province of L'Aquila during the entire calendar year 2009 from 1st January to 31th December and were drawn by the Italian Seismic Bulletin (ISIDE).
A preliminary descriptive space time analysis of available data shows that the random phenomenon can be considered as a dynamic continuous parameter stochastic process and as such dealt with probability theory for the analysis of events random increments.
As known, the problem of ergodicity of a dynamic stochastic process has been addressed for the first time by physicists in the study of the kinetic theory of gases. For example, when a mass of gas is subject to random changes as a result of subsequent changes in status, the reiteration of these changes tends to create some regularity of behavior in the long run.
In our study of the swarm we will try to find out if there are similarities in its behavior to that of the kinetic theory of gases, using similar methods of analysis.

2. Methods and results

The dynamic characteristics of a destructive earthquake swarm, are known as being characterized by a foreshock (frequent shocks that occur before the main shock) and the main-shock, The shock of magnitude 6.3 that occurred on 6 th April can be placed within a sequence of four time intervals characterized as follow:

- Interval 1 - from January 1 to December 31, 2009 for a total of 15890 shocks;
- Interval 2 - from March 2 to May 2 (one month before and one month after the main-shock) for a total of 8611 shocks;
- Interval 3 - from March 22 to April 21, 2009 (fifteen days before and fifteen days after the main-shock) for a total of 6781 shocks
- Interval 4 - from 1 to 13 April 2009 (one week before and one week after the main-shock) for a total of 4369 shocks

For each of the four time intervals, we have defined five transition states corresponding to the following classes of earthquake magnitude:

State 1- (S_1)- shock with a magnitude of less than 1;
State 2 -(S_2) -shock with a magnitude between 1 and 1.4;
State 3 -(S_3) -shock with a magnitude between 1.4 and 1.8;
State 4 -(S_4) -shock with a magnitude between 1.8 and 2.4;
State 5-(S_5) -shock with a magnitude greater than 2.4.

From the above time intervals, we have estimated four 5x5 ergodic transition matrices and we have calculated the limit vectors and the corresponding matrix of the mean of first passage .
Dynamic processes are related to the time evolution and apply when the time factor (t) is a fundamental entity influencing the process.
In our study, states constitute a finite sequence of events not referred to the time at which they occurred.
An evolutionary system of random events is able to move between h incompatible transition states S_1, S_2, S_3, ..., S_i, ..., S_j, ..., S_h . At a given time the system may be in one and a only of these states. Once a certain state is reached at time (t_h), the system stays there until (t_k), with k steps of random transition , passes to the new state S_j .
In this case study, we are in the presence of a random evolutionary process and would like to know what is the probability that the system is in a generic state (S_i) with probability $p(S_i)$, regardless of the instant at which this happens, taking into account the type state previously occurred.
This can also be defined as the probability of transition from state S_i to S_j or P_{ij}. These probabilities are obtained studying the statistical behavior of the phenomenon: the frequencies with which state changes define an array whose elements correspond to the estimated transition probabilities, if normalized by row.
The “inheritance property” of few steps of transition, even if partial implying the system “memory”, may be limited.
For some classes of earthquake intensity the observed regularity allows to predict the future of the phenomenon and to conclude that some memory mechanism exists.

An effective way to verify the assumptions just mentioned, is to try to assess the situation after n successive steps of the transition process the ergodic behavior at the limit of its evolution.
Let P_{ii} denote the probability of transition from a single step, estimated with the observed data, the corresponding probability of transition P_{ii} (n) from i to j, in n steps. The transition may occur, in different ways, namely by following multiple mutually incompatible route A , B, or C, ...
The probability P_{ij} (n) is calculated as the sum of the probabilities of each route P_{ij} (n) = (p_{ij} (A)) + (p_{ij} (B)) + (p_{ij} (C)), where P_{ij} (n) gives rise a recurrence relation that consent us to distinguish some important features of the process during its evolution, such us the average transition time from one state to another or the average time to return to the starting state or even the time of permanence in a state, as well as the process configuration limit.
When the process is able to achieve any state of the system starting from any other during its evolution, it satisfies the conditions for ergodicity.
From an analysis of the indicators of the mean time of first passage for the four interval, we can see a substantial confirmation of the characteristics of the phenomenon in terms of probability of switching from one state to another.

Limit vector (15809 shocks)

S_1	S_2	S_3	S_4	S_5
0.01	0.67	0.13	0.10	0.07

Limit vector (8611 shocks)

S_1	S_2	S_3	S_4	S_5
0.01	0.14	0.34	0.30	0.21

Limit vector (6781 shocks)

S_1	S_2	S_3	S_4	S_5
0.01	0.14	0.34	0.30	0.21

Limit vector (4369 shocks)

S_1	S_2	S_3	S_4	S_5
0.046	0.082	0.295	0.344	0.274

Mean first passage time matrix (15809 shocks)

	S_1	S_2	S_3	S_4	S_5
S_1	13.16	4.48	3.94	6.072	9.22
S_2	13.12	4.18	3.78	6	9.21
S_3	13.19	4.31	3.67	5.83	9.19
S_4	13.2	4.5	3.8	5.93	9.06
S_5	13.2	4.52	3.92	6	9.03

Mean first passage time matrix (8611 shocks)

	S_1	S_2	S_3	S_4	S_5
S_1	156.24	1.24	8.54	13.87	24.48
S_2	169.27	1.49	8.01	12.88	23.46
S_3	175.42	1.76	7.63	11.59	22.86
S_4	176.25	2	9.01	9.9	20.16
S_5	176.84	2.17	10.32	11.09	14.28

Mean first passage time matrix (6781 shocks)

	S_1	S_2	S_3	S_4	S_5
S_1	12.19	3.32	3.2	6.63	24.48
S_2	12.64	3.7	3.05	6.44	23.46
S_3	12.75	4.22	3.03	5.91	22.86
S_4	12.82	4.74	3.65	5.05	20.16
S_5	12.94	5.08	4.29	5.83	14.28

Mean first passage time matrix (4369 shocks)

	S_1	S_2	S_3	S_4	S_5
S_1	217.38	10.95	3.67	4.71	1.15
S_2	227.02	12.19	3.27	4.72	1.04
S_3	227.84	13.7	3.33	4.4	0.9
S_4	228.25	14.88	4.13	3.98	0.18
S_5	228.72	15.78	4.9	4.67	0.08

For example, if we consider the 15.900 shocks, occurred in 2009, it would take 13 transitional stages to reach the transition state S_1 of lowest hazard from any previous

state, 5 steps to reach the state S_2, and so on, until 9 steps to be in the most dangerous state S_5 with a magnitude greater then 2.4.
A very different behaviour is observed for the seismic swarm of 4369 shocks occurred a week before and one after 6 April 2009.
During this time interval, shocks belonging to the state S_1 occurring very rarely and reaching the lowest value of the magnitude contemplated in the S_1 state took more than 200 stages of transition (shock). On the other hand, only one stage of transition is necessary in order to have two successively shocks of the highest magnitude S_5.
The number of stages of transition which determine the mean time to return from certain level to same level could be defined as an indicator dangerous due to recursion of this type of shock.

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